

# **Impact of Transceiver Impairments on Capacity and Optimum Number of Users for Massive MIMO Systems**

*Dissertation submitted in the partial fulfillment of requirements for the award of  
degree of*

**Master of Engineering  
in  
Electronics and Communication Engineering**

Submitted by  
**Surender Redhu**  
**Roll No. 801361028**

Under guidance of  
**Dr. Sanjay Sharma**  
**Professor and Head**



**ELECTRONICS AND COMMUNICATION ENGINEERING  
DEPARTMENT**

**THAPAR UNIVERSITY**

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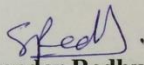
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## Declaration

I, hereby declare that the work, which is being presented in the dissertation, entitled “**Impact of Transceiver Impairments on Capacity and Optimum Number of Users for Massive MIMO Systems**” in partial fulfilment of the requirements for the award of degree of Master of Engineering in Electronics and Communication Engineering submitted at Electronics and Communication Engineering Department of Thapar University, Patiala, is an authentic record of my own work carried out under the guidance of **Dr. Sanjay Sharma (Professor and Head)**, Electronics and Communication Department and refers other research’s work which are duly listed in reference section. The matter presented in this dissertation has not been submitted in any other University/Institute for the award of degree.

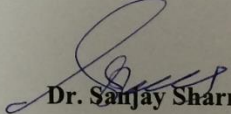
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Roll No. 801361028

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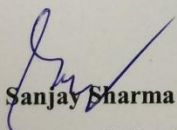
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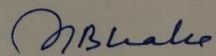
  
Dr. Sanjay Sharma

Professor and Head

ECED, Thapar University

Countersigned by:

  
Dr. Sanjay Sharma  
Professor and Head (ECED)  
Thapar University, Patiala

  
Dr. S. S. Bhatia  
Dean, Academic Affairs  
Thapar University, Patiala

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MIMO systems with more than tens of antennas in communication terminals, referred to as large-MIMO systems or Massive MIMO system. Growing the antennas at the base stations is a native methodology for increasing the capacity of wireless system. The usage of such large no. of antennas can improve the energy and/or spectral efficiency of the wireless communication systems because of the significant improvements in array gain and spatial resolution. But, the impact of hardware impairments at the transmitter and receiver on the massive MIMO systems has gained slight attention in these days, even though large number of antenna might just be attractive for the network organization if every antenna having the reasonable equipment. The employment of transmitter and receiver contains of numerous types of hardware modules and every one of them have diverse influence on the signal.

This thesis studies a new model for the massive MIMO systems that includes overall hardware impairments at transmitter contains large no. of antennas and receiver side. At the receiver side, there is user equipment such as single antenna users. In comparison to the general case, in which transceiver have ideal hardware, we proved that the non-ideality of hardware makes an upper limit on the accuracy of channel estimation and also on the capacity of each user.

In this thesis work, we also demonstrated that the massive MIMO system does not provide the full advantages to all the users i.e. we proved that how the capacity decreases as we increase the no. of users after a certain limit, because of more training time requirement as the number of user increases. We show that how many are the optimum users to which this system provides full advantages. We also proved that how the transceiver impairments affects the accuracy of channel estimation and impact of that on the capacity and optimum no. of users in massive MIMO system.

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## List of acronyms

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MIMO	:	Multiple Input Multiple Output
4G	:	Fourth Generation
SNR	:	Signal to Noise Ratio
SINR	:	Signal to Noise plus Interference Ratio
QoS	:	Quality of Service
SE	:	Spectral efficiency
EE	:	Energy Efficiency
UE	:	User equipment
i.i.d	:	Independent and Identical Distribution
RF	:	Radio Frequency
BS	:	Base Station
D.O.F	:	Degree of Freedom
E.S.D	:	Empirical Spectral Distribution
I/Q	:	In-phase and Quarter-phase
FDD	:	Frequency Division Duplexing
TDD	:	Time Division Duplexing
5G	:	Fifth Generation
MAC	:	Media Access Control
WCDMA	:	Wideband Code Division Multiple Access
MRC	:	Maximum Ratio Combining
ZF	:	Zero Forcing
MF	:	Match Filter
MMSE	:	Minimum Mean Square Error
RZF	:	Regularized Zero Forcing
GMI	:	Generalized Mutual Information

MISO	:	Multiple Input Single Output
SC	:	Selection Combining
GSEC	:	Group Switch and Examine Combining
BER	:	Bit Error Rate
PSAM	:	Pilot Symbol Aided Modulation
ISI	:	Inter Symbol Interference
MSE	:	Mean Square Error
CSI	:	Channel State Information
AOD	:	Angles of Departure
AOA	:	Angles of Arrival
NIC	:	Number of Independent Channels
CSIT	:	Channel State Information at Transmitter
CSIR	:	Channel State Information at Receiver
ZMSW	:	Zero-Mean Spatially White
SVD	:	Singular Value Decomposition
UL	:	Uplink
ULA	:	Uniform Linear Array
SISO	:	Single Input Single Output
3G	:	Third Generation

# Chapter 1

## Introduction

---

Multiple-input multiple-output technique has concerned much consideration in wireless communications for more than ten years. Because, it can offer major growths in data transmission capacity and link consistency without extra bandwidth and increasing the power to be needed for transmission.

The advantages of MIMO technology become an answer for the present and future wireless communication loads [1]. In order to scale up these benefits, very large MIMO, also named as massive MIMO, has been first put onward in 2010. MIMO systems with more than tens of antennas in communication terminals, stated to as large-MIMO systems or Massive MIMO system. The methodology of growing the number of antennas is a native methodology for enhancing the network capability. The inspiration to study such Massive MIMO systems is to employ the theoretically expected profits of MIMO systems, in terms of both increased diversity orders as well as high spectral efficiencies.

### 1.1 Motivation of Thesis

As the era of high data is coming in the future, the demands of users regarding the capacity and accuracy of the wireless communication system increasing day by day. So we need a system or technology that would fulfill all these requirements. In the current 3G and 4G wireless communication technology, MIMO technology plays a great role. So, we can think of scaling the MIMO systems with more than tens of antennas in communication terminals, referred to as large-MIMO systems or Massive MIMO system. Growing the antennas at the base stations is a native methodology for increasing the capacity of wireless system.

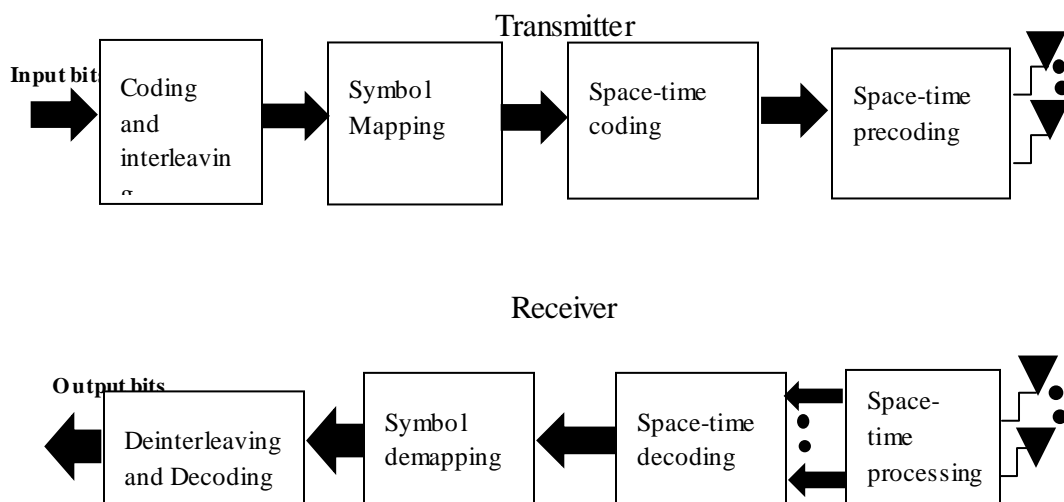
Massive MIMO wireless systems are the key technology for the 5G communication technology. Because they will improve the capacity of wireless channels up to a large extent. And also in many research papers, these system are said to be backbone for the future of wireless communication system.

So, massive MIMO systems are having the large scope for the researchers in wireless field. We can study the various issues in these systems, like pilot contamination, inter-user interference, hardware impairment impacts etc.

The study of impact of impairments due to non-ideal hardware at the transmitter and receiver is a wide area for the research, because these are studied not very well till now. So, in this work we have chosen the hardware impairment issue and its impact on the wireless communication systems.

## 1.2 Wireless Communication with MIMO

By means of the many antennas at the transmitter side and receiver side in wireless communication systems, traditionally known as MIMO technology, has attractiveness during the last ten years because of its powerful performance improving abilities, one of the key feature is enabling 4G technology [1]. Data transmission in wireless communication channels is compromised largely by multi-path fading. The term multi-path, is the receiving of the transmitted signal at a projected receiver over different angles or time delay paths or differing frequency (i.e., Doppler) shifts due to the scattering of electromagnetic waves in the atmosphere. Due to all these, the power of received varies in space because of angle spread and/or frequency because of delay spread and/or time because of Doppler spread by the imposing of random multi-path components superposition[1]. This variation which is random in signal level is called as fading and can harshly affect the accuracy and consistency of wireless communication. In addition, the task of achieving high data rates and high reliable wireless communication systems become extremely challenging due to the limitations imposed by power constraint and unusual frequency bandwidth [2].



**Figure 1.1: Basic block diagram of MIMO wireless communication system**

MIMO technology set up a revolution in wireless communication system design. The technology offers a large number of advantages that support to meet the challenges

posed by both the impairments in the wireless channel along with resource limitations. Additionally the time and frequency dimensions that are exploited in predictable single-antenna wireless communication systems, and the influences of MIMO are understood by developing the spatial dimension (delivered by the many number of antennas at the both side).

### **1.3 Advantages of MIMO technology**

The advantages of MIMO technology, which helps in achieving such noteworthy performance gains are named as array gain, spatial diversity gain, spatial multiplexing gain and also interference reduction. All these terms are defined in short-lived below [2].

- **Array gain**

The increase in the received SNR that comes from a coherent combining effect of the signals at a receiver side is known as array gain. The term coherent combining, may be understood by the help of spatial processing at the receive antenna array or may be using the spatial pre-processing by the array of antenna at the transmitter. The array gain increases resistance of noise, by this means refining the coverage and the range of a wireless network.

- **Spatial diversity gain**

As mentioned previously, the signal level at a receiver in a wireless system varies or fades. Spatial diversity gain diminishes fading and is recognized by providing the receiver with multiple (preferably independent) copies of the transmitted signal in space, frequency or time. With an increasing number of independent copies (the number of copies is often denoted as the diversity order), the probability that however one of the copies is not suffering a deep fade increases, thus improving the value and reliability of reception [2]. A MIMO channel with  $M_T$  transmit antennas and  $M_R$  receive antennas potentially offers  $M_T \times M_R$  independently fading links, and hence a spatial diversity order of  $M_R \times M_T$ .

- **Spatial multiplexing gain**

MIMO systems offer a linear increase in data rate through spatial multiplexing, which means transmitting multiple independent data streams within the bandwidth of operation. Under appropriate channel conditions, for example rich scattering in the

environment, the receiver can distinct the data streams. Moreover, each data stream experiences at least the same channel quality that would be practiced by a SISO system, so by a multiplicative factor equal to the number of streams, the capacity is efficiently improved [2]. Overall, the maximum no. of data streams those can be unfailingly sustained by channels of wireless MIMO system, equals to the minimum of the number of antennas at the transmitter and the no. of antennas at the receiver, that means,  $\min(M_R, M_T)$ . This gain will increases with the capacity of a wireless communication network.

- **Interference reduction and avoidance**

The term interference, in wireless networks when the many number of users share the channel at the same time and also use the frequency resources. The interference may be diminished in wireless MIMO communication systems to increase the separation between the users, by taking advantage of the spatial dimension. As the array gain increases the lenience to noise in addition to the interference power, therefore refining the signal-to-noise-plus-interference ratio (SINR) in this communication system [2]. In addition, spatial dimension also influenced for the dedications of interference prevention. Interference decline and evasion improve the coverage and range of a wireless network. Altogether, it may be impossible to exploit concurrently all the assistances defined above due to incompatible demands on the spatial degrees of freedom. But, using the various combinations of the advantages through a wireless network will result in enhanced capacity, coverage and reliability.

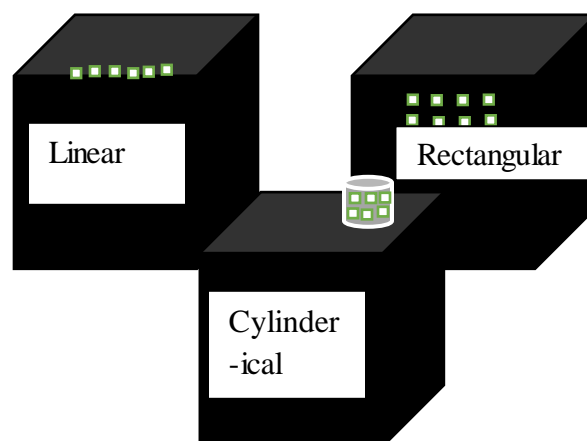
#### **1.4 Massive MIMO**

Massive MIMO which is also named as large-scale MIMO is a developing technology that is scaling up the MIMO technology by feasible amount of magnitude as compared to current wireless MIMO systems. Using massive MIMO, we consider systems those uses antenna array with many antennas may be no. of tens or no. of hundreds, instantaneously serving many users in the same time and frequency resources of the communication system [3]. The very basic or main idea behind the massive or large-scale MIMO is to obtain all the advantages on greater scale of the current wireless MIMO systems. Generally, massive MIMO wireless communication systems are the enabler for the expansion of future wireless communication networks for both i.e. fixed and mobile that will use the spectrum very efficiently, will be

energy-efficient and also robust. The necessities for high rate of wireless communication networks raise exponentially with the uses of smart terminals. Thus, the capacity of the networks has to be increased in order to assure the quality of service (QoS) of mobile applications. Improvement in the spectrum efficiency (SE) is one of great practical way to achieve the proper network capacity [4]. The main features of massiv MIMO systems are:

- Due to high array gain the propagation losses will be diminished with the help of coherent combining.
- Because of channel estimation errors, the Interference can be wiped out from the wireless systems in vector space of the large dimensions.
- Here, the complexity of signal processing algorithms are very low which are proved asymptotically optimum.
- The interference between the users is simply mitigated with the effect of high beamforming resolution.

In addition, with the extreme power depletion of wireless communication networks, both the carbon productions and operator expenses rise every year. Hence, green communication has increased more courtesy in the educational and industrial fields, in addition the energy efficiency (EE) has been regarded as alternative dynamic metric for assessing the new technique along with the spectrum efficiency [4]. Numerous diverse arrangements and placement settings used by a massive MIMO system can be anticipated from fig. 1.2. Every antenna terminal should be small and active, rather fed with an optical or electric digital bus.



**Figure 1.2: Various arrangements for Massive MIMO arrays**

### 1.5 Massive MIMO Features

In massive MIMO system, hundreds of antennas are used to serve tens of user equipment (UEs) simultaneously. Theoretical and measurement results indicate that massive MIMO systems can significantly improve the spectrum efficiency and simultaneously reduce the radiated power [5]. All the features of massive MIMO system are shortly summarized in Table 1.

**Table 1: Massive MIMO Features**

	<b>Features</b>	<b>Reasons</b>
<b>Advantages</b>	<ul style="list-style-type: none"> <li>• High spectrum efficiency</li> <li>• High energy efficiency</li> </ul>	<ul style="list-style-type: none"> <li>• Large multiplexing gain and array gain</li> <li>• Radiated energy can be concentrated on UE</li> </ul>
	<ul style="list-style-type: none"> <li>• High reliability</li> <li>• Efficient linear precoder/detector</li> </ul>	<ul style="list-style-type: none"> <li>• Large diversity gain</li> <li>• Favorable propagation condition for i.i.d. Rayleigh channel</li> </ul>
	<ul style="list-style-type: none"> <li>• Weak inter-user interference and enhanced physical security</li> </ul>	<ul style="list-style-type: none"> <li>• Orthogonal UE channels and extremely narrow beam</li> </ul>
	<ul style="list-style-type: none"> <li>• Simple scheduling scheme</li> <li>• Robust to individual element failure</li> </ul>	<ul style="list-style-type: none"> <li>• Channel harden phenomenon averages out the fast fading</li> <li>• Large number of antenna array elements</li> </ul>
<b>Disadvantages</b>	<ul style="list-style-type: none"> <li>• Pilot contamination</li> <li>• High signal processing complexity</li> </ul>	<ul style="list-style-type: none"> <li>• Limited orthogonal pilots as of bounded coherent interval and bandwidth</li> <li>• Large number of antennas and multiplexing UE</li> </ul>

	<ul style="list-style-type: none"> <li>• Sensitive to beam alignment</li> <li>• Poor broadcast channel</li> </ul>	<ul style="list-style-type: none"> <li>• Extremely narrow beam is sensitive to UE moving or antenna array swaying</li> <li>• Be blind to UE positions</li> </ul>
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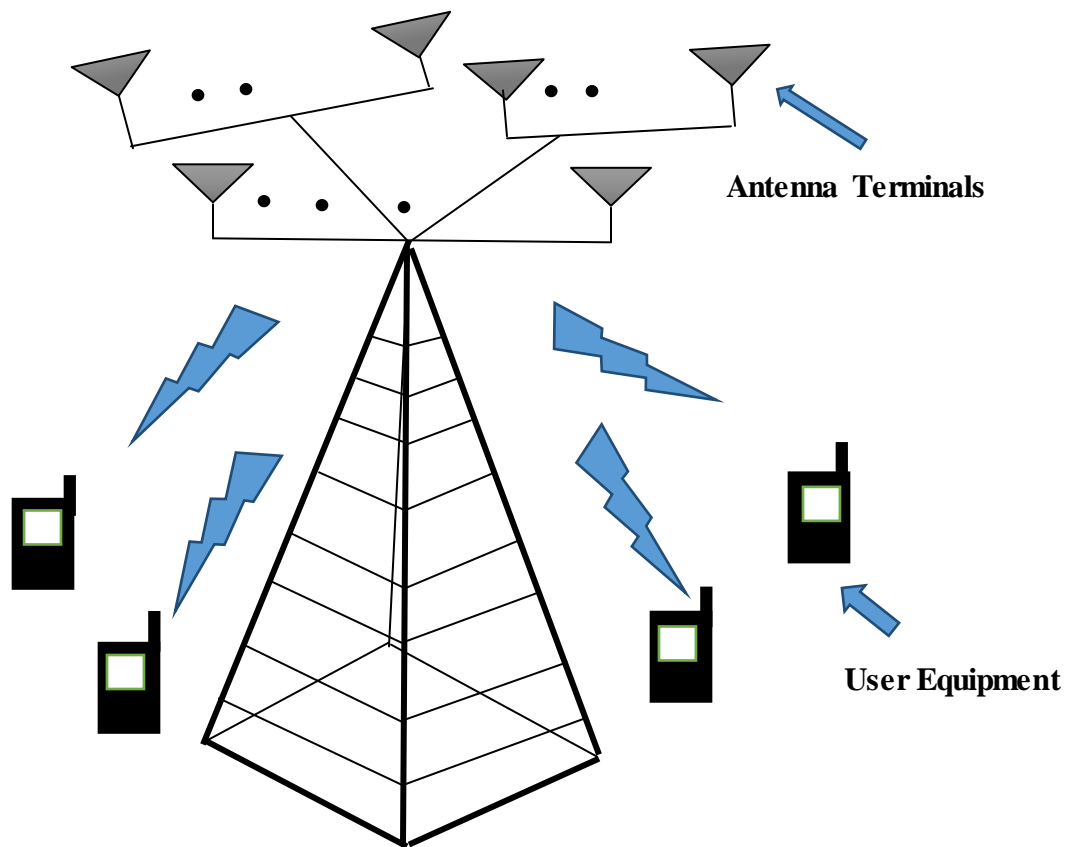
But, the benefits of MIMO technology come with a price. Usually, if the number of antennas are increased then that means we are increasing the no. of RF chains that will increase the processing load. So, there are many specific disadvantages for example: essential space between so many antennas, the battery consumption is also increased due to the processing load of large no. of antennas at the user end and etc. Consequently, it might be accomplished that the no. of antennas in a massive MIMO wireless communication system is restricted by above stated disadvantages, but it is not the complete story. Massive MIMO, which have a base station with a large no. of antennas helps all the users concurrently, with the identical frequency and time means. In the wireless massive MIMO system, the base station (BS) equipped with 10 or even 100 to 1000 antennas. These so much antennas at the base station, not only provides little bit additional spatial Degree of Freedom (DOF), but also consequences will be decrease in the transmit power constraints and the interference level.

Likewise, when the dimensions of a MIMO system become large, the consequences of random matrix theory can be applied. According to this theory, as the dimensions of a matrix (with some minor conventions on the information of the entries) grow large, the scattering of empirical spectral distribution (ESD) of the matrix becomes deterministic [8].

So, this expected behavior permits many generalizations in processing, due to that even the matched filter (MF) can be applied for recognition in an idyllic conditions. Though, the final performance under the positive conventions of the channel and arrangements of the system hardware for massive MIMO wireless system can be attained. Conversely, this is an interesting thing to do and typically comes with price of harsher hardware which are used in the system and directly above the necessities.

### **1.6 Transceiver Impairment Issue in Massive MIMO System**

The impact of impairments of the no-ideal hardware which are installed on the transmitter and receiver of these systems has received slight attention up to now, even



**Figure 1.3: Basic diagram for massive MIMO system**

though large number of antenna might just be attractive for the network organization if every antenna having the reasonable equipment. The employment of transmitter and receiver contains of numerous types of hardware modules and every one of them have diverse influence on the signal. For instance the hardware deficiency can't be evade so we are expending model [9] that reflect the collective influence of all the hardware limitations not the effect of separate components. According to the model collective residual transceiver impairments can be demonstrated by independent additive white distortion noises at the transmitter and receiver sides (BS and UE). This model is well explained in next section and used for transceiver impairment analysis of Massive MIMO system.

Inexpensive hardware modules are mainly liable with the impairments those are occurs in any MIMO system for example, amplifiers with non-linearity behavior, in-phase and quarter- phase imbalance, phase noise due to components, and the various other errors [9]. This impact which is caused by impairments of the hardware is

commonly diminished with the help of analog and digital signal processing algorithms. All these techniques can't eliminate the impairments entirely, some parts of the impairments of the receiver and transmitter side hardware, as hardware characteristics varies with time or age it can't parameterized and estimated completely, since all types of noise are random with different nature. These impairments at the transmitter and receiver are known for putting various limits on the capacity, spectral efficiency, energy efficiency, optimum no. of users, but, as there is limited no. of publications those examines the behavior of massive MIMO wireless communication.

As we know that the hardware imperfection can't be avoid so we are using a proposed model [9] that consider the aggregate effect of all the hardware imperfections not the effect of individual components.

In this thesis, we examines the impact of impairments that is caused by all the different types of hardware at the transmitter and receiver side of the massive MIMO system, in contrast to the perfect hardware. Here, we are assuming that the appropriate analog and digital signal processing algorithms have been applied and we will find out the impact of residual hardware impairments of the system. So, Inspired from the various analysis a new system model is considered that have the aggregate effect of all the hardware imperfections not the effect of individual components. This model is well explained in next section and used for transceiver impairment analysis of Massive MIMO system.

### **1.7 Optimum Number of Users**

The channel for the wireless communication system may be estimated by user equipment (UE) or BS, i.e. the channel learning can be done using frequency division duplex (FDD) scheme or time division duplex scheme. The channel estimation method TDD is based on property of downlink and uplink channels which is reciprocity property i.e. downlink channel is transpose of uplink channel. To study the channel using TDD, transmitter sends known training signals during some percentage of the transmission interval. In early study of the training time requirement in multiple-antenna wireless links [10], it is proved that, under some conditions, to maximize the capacity, one generally spends training interval equal to the no. of transmit antenna. Here we are addressing a different problem of a cellular network: given a massive MIMO wireless link with  $M$  no. of users,  $N$  antennas at the BS(Base

Station) , a signal-to-noise ratio (SNR)  $\rho$  and coherence time of length  $T$  (in symbols), then we found that massive MIMO system have several advantages but with some limits.

As massive MIMO system are known for high rate systems for the 5G technology, but capacity decreases as the no. of users in a cell exceeds beyond a certain limit. In multi-antenna system when the power of training and data signal is equal then the no. of optimum users are equal to the training period [10]. The same concept can be applied for the massive MIMO system to find out the optimum no. of users to which massive MIMO system provides the maximum rate. For this, firstly we calculate the lower bound on the capacity that is attainable with TDD schemes according to [10],[11], because, with the lower bound we can calculate the optimal amount of training as a function of,  $M$ , and  $N$ .

### **1.8 Problem Formulation**

According to the various research articles there are lot of gaps in the massive MIMO systems those can be studied. After the survey of all those research articles, we have found the following issues that van be studied.

- Increasing antennas at the base stations is an instinctive methodology for improving the network capacity. If we are using the large no. of antennas at the base stations, then they can improve the energy efficiency and spectral efficiency of the wireless communication systems due to the significantly enhanced spatial resolution and array gain of the massive MIMO systems.
- In comparison to conventional system with ideal hardware at the transmitter and receiver, we can prove that impairments of the hardware make finite upper limit on the accuracy of the channel estimators and on the capacity of downlink/uplink channel for every user.
- We can prove that how the capacity decreases after a certain limit as we increase the no. of users, because requirement of training time increases as the number of users increases.
- We can also prove that how the transceiver impairments affects the accuracy of channel estimation and impact of that on the capacity and optimum no. of users in massive MIMO system.

### **1.9 Objectives**

Till now, we have successfully understand the gaps or issues in the massive MIMO systems, which is a key technology for the 5G wireless communication systems. After all that we have concluded the following objectives for this thesis work:

- Firstly, we will study that how the capacity of wireless communication networks increases with the increase in no. of antennas at the transmitter or receiver side.
- Then we will examines the performance of channel estimator for the massive MIMO systems under the impact of hardware impairments.
- After that we will find that how many no. of users are optimum in a massive MIMO system to which these systems provides the full advantages,
- Lastly, we will study the impact of non-ideal hardware on the no. of optimum users in the massive MIMO systems.

### **1.10 Organization of Dissertation**

This dissertation comprises of six chapters including introduction which is as follows:

- *Chapter 2* deals with literature review where study is done upon existing massive MIMO wireless systems and their various issues are also discussed. All the work done in this field is summarized in this chapter.
- *Chapter 3* provides the detailed analysis of the various estimators of the wireless communication channels, the accuracy of channel estimators with the impact of hardware impairments, various types of correlations in the wireless systems and study of these correlation with the help of specific correlation model.
- *Chapter 4* is dedicated to the analysis of the channel models of the massive MIMO systems, capacity of these channels, bounds on the capacity due to the impact of hardware impairments. This section also contains the study of no. of optimum users in these systems under the hardware impairments.
- *Chapter 5* provides the numerical illustrations of the mathematical or theoretical work of the previous chapters. In this chapter, we have done the simulation of all the work done in the chapter 3 and chapter 4 with the help of MATLAB tool. The results proves the mathematical work of pervious chapters.
- *Chapter 6* highlights the conclusion of the dissertation and tells about future prospects.

This section contains the detailed summary of recent research done in the wireless communication. All the study related to future aspects and issues about wireless communication in recent years is collected in this section. Firstly we focus on the future of wireless communication, after that we focus on issues in wireless communication future.

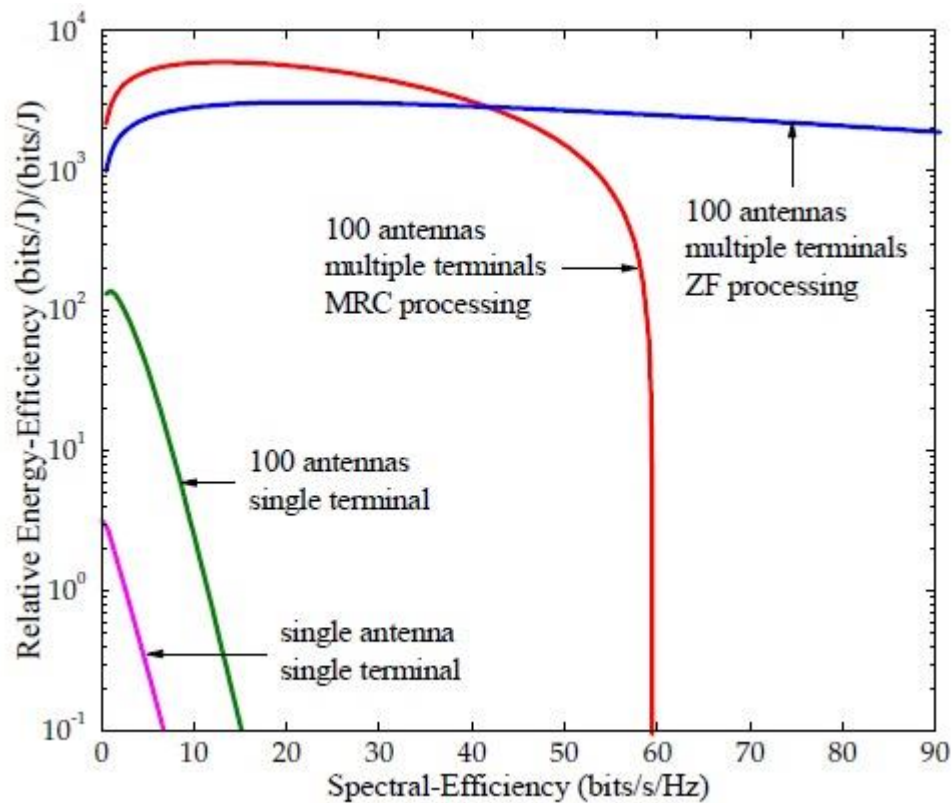
#### 2.1 Future of wireless communication

The next generation wireless networks need to accommodate 1000 times more data traffic than contemporary networks. Since the spectrum is scarce in the bands suitable for coverage, the main improvements need to come from spatial reuse of spectrum; many concurrent transmissions per area unit. This is made possible by the massive MIMO technology, where the access points are equipped with hundreds of antennas. These antennas are phase-synchronized and can thus radiate the data signals to multiple users such that each signal only adds up coherently at its intended user. Over the last couple of years, massive MIMO system has gone from being a theoretical concept to becoming one of the most promising ingredients of the emerging 5G technology. This is because it provides a way to improve the area spectral efficiency (bit/s/Hz/area) under realistic conditions, by upgrading existing base stations. In other words, massive MIMO system is a commercially attractive solution since 100 times higher efficiency is possible without installing 100 times more base stations.

**In [3] T. L. Marzetta** proposed a cellular base station serves a multiplicity of single-antenna terminals over the same time-frequency interval. Time division duplex operation combined with reverse link pilots enables the base station to estimate the reciprocal forward and reverse link channels. The conjugate transpose of the channel estimates are used as a linear precoder and combiner respectively on the forward and reverse links. Specifically, the impact of non-correlated noise disappears, and the throughput of the wireless communication system are independent of the size of the cells, also the no. of terminals. As the spectral efficiency does not depends on the bandwidth of the channel, and the essential transmitted energy per bit become extinct. Only remaining impairment is inter-cellular interference produced by re-use of the

pilot sequences in other cells (pilot contamination) which does not disappear with very large number of antennas.

In [4] E. G. Larsson *et al.* proposed that massive MIMO systems takes the advantage of large no. of antennas at the base stations and also the TDD operation. Additional antennas in the antenna array helps in directing energy into every small regions in the space so that it can take huge improvements in capacity and energy efficiency of the massive antenna systems. Additional advantages of massive MIMO systems consist of wide use of reasonable low-power components, reduced potential and popularization of the media access control (MAC) layer. And also the strength to intended jamming is improved by these systems. The expected throughput determined by the environment of propagation provided that orthogonal channels to the terminals in experiments not revealed any boundaries.



**Figure 2.1: Spectral Efficiency and Energy efficiency of Massive MIMO under Various conditions, like MRC and ZF processing**

In [13] E. Telatar proposed the use of multiple antennas will prominently increase the attainable rates on fading channels if the channel parameters can be predictable at the receiver and if the path gains between dissimilar antenna pairs behave

unconventionally. The second of these necessities can be met with comparative easiness and is slightly technical in nature. The many conditions to be justified in certain communication scenarios and not in others. Then the original writing of this paper in late 1994, there has been some work in which the statement of the obtainability of channel state information is swapped with the hypothesis of a gradually varying channel.

**In [14] L. Lu *et al.*** proposed that Massive multiple-input multiple-output (MIMO) wireless communications states to the knowledge providing cellular base stations (BSs) with a very large number of antennas, and has been made known to allow the orders of magnitude upgrading in spectral and energy efficiency using comparatively linear processing. They offered a widespread overview of high-tech research on the topic that has recently attracted significant attention. They initiate with an information theoretic study to demonstrate the estimated advantages of massive MIMO, and then addresses employment issues associated to channel estimation, detection and precoding schemes. They predominantly focus on the potential impact of pilot contamination produced by the usage of non-orthogonal pilot sequences by users in neighboring cells. They also study the energy efficiency accomplished by massive MIMO systems, and reveal how the degrees of freedom provided by massive MIMO systems permit efficient single-carrier transmission. Lastly, the challenges and opportunities linked with applying massive MIMO in 5G wireless communications systems are deliberated.

**In [16] M. Mueck *et al.*** introduced a novel flexible mobile device reconfiguration classes (MDRCs) framework which is likely to be applicable to future generation radio devices. This framework sets the scene for the idea of a plane advancement of reconfigurable features in mobile devices, flooring the way from present, essentially static employment choices in the direction of a fully flexible device platform environment. On behalf of this flexible mobile device framework, baseband interfaces are presented as they are presently discussed in reconfigurable radio systems regulation. Certainly, the future studied radio equipment and telecommunications terminal equipment directive is likely to permit for such innovative reconfiguration, empowering users to attain and install so-called radio apps software components which may disturb the passivity of a mobile device to the crucial requirements of the Directive.

**In [18] M. Frodigh *et al.*** discussed future generation wireless networks. Two complementing main trends are recognized: unified roaming between unlike air interfaces, leading greatest connected idea, and the continuous expansion of the present third-generation (3G) standards. The development of WCDMA in the direction of high-speed downlink packet access, targeting for highest rates in the range of 8-10 Mb/s, is defined as an example of air-interface development. Fourth generation technologies such as ad hoc networking and multi-hop networks, still in the research, are discussed and their influence on wireless communication systems is discussed.

**In [19] A. Nosratinia *et al.*** proposed that transmit diversity usually needs more than one antenna at the transmitter. Though, many wireless devices are restricted by size or hardware complexity to one antenna. In recent times, a new class of methods called cooperative communication has been suggested that permits single-antenna mobiles in a multi-user environment to share their antennas and make a virtual multiple-antenna transmitter that permits them to attain transmit diversity. This article discussed a summary of the progresses in this developing field.

## **2.2 Massive MIMO system and issues**

Significant benefits can be realized if a larger number of antennas is used; e.g., large MIMO systems with tens to hundreds of antennas can enable multi-gigabit rate transmissions at high spectral efficiencies of the order of tens to hundreds of bps/Hz. However, several technical challenges need to be addressed in realizing such large MIMO systems. Some of the key challenges include:

- The availability of independent spatial dimensions in real-world channels,
- The placement of a large number of antennas and RF chains,
- Large MIMO signal processing at practically implementable low complexities,
- Issues in multi-cell operation.

**In [9] E. Bjornson *et al.*** proposed that as the no. of antennas at the base stations will rises, the various parameters of the wireless communication systems are increases. As the antennas increases, the large gains for the signals are attainable in addition to a little bit interference between the users. In this paper it is studied that all the benefits of massive MIMO system are asymptotic, so it become essential to examine that the predictable system channel models are optimum up to what extent, in comparison to

the practical system. In this paper, a new system model is proposed that includes universal impairments due to hardware at the transmitter (having antenna array) and receiver (user equipments) side. Equally, in opposition to the conventional or general case of the system having hardware with full ideality. In this paper, it is indicated that impairments due to non-ideal hardware generates finite upper limit on the accuracy of the channel estimators of the wireless communication system and also on the capacity of the downlink/uplink channel of every user. It is proved that the capacity of the system is primarily restricted by the hardware, which is used at the UE, though the effect of hardware impairments in the large arrays system disappears asymptotically and the pilot contamination issue becomes insignificant.

**In [23] T. L. Marzetta *et al.*** discussed that demand for wireless communications is expected to raise by more over the succeeding five years. A probable technology for meeting this demand is Massive MIMO, also called Large-Scale Antenna Systems, Large-Scale MIMO, a system of multiuser multiple antenna wireless which promises orders-of-magnitude enhancements in spectral-efficiency over 4G technology, and associated developments in emitted energy-efficiency. The distinctive feature of massive MIMO system is that a large number of service-antennas maybe hundreds or even thousands work for a meaningfully smaller number of active independent terminals. Upsetting the traditional parity between service antennas and terminals in this manner is a game-changer: The modest multiplexing precoding and decoding algorithms can be almost optimal, costly ultra-linear forty-Watt power amplifiers are changed by many low-power units, and the auspicious action of the law of large numbers can significantly enable power-control and resource distribution.

**In [24] T. K. Truong *et al.*** proposed that Multiple-input multiple-output (MIMO) communication may provide high spectral efficiency through the deployment of a very large number of antenna elements at the base stations. The gains from massive MIMO communication come from the use of multiuser MIMO on the uplink and downlink, but with a large excess of antennas at the base station compared to the number of served users. Initial work on massive MIMO system did not fully address several practical issues associated with its deployment. They considers the impact of channel aging on the performance of massive MIMO systems. The effects of channel variation are characterized as a function of different system parameters assuming a simple model for the channel time variations at the transmitter. Channel prediction is

proposed to overcome channel aging effects. The analytical results on aging show how capacity is lost due to time variation in the channel. Numerical results in a multi-cell network show that massive MIMO works even with some channel variation and that channel prediction could partially overcome channel aging effects.

**In [25] F. Rusek *et al.*** proposed that as the number of antennas at the base station grows, then the wireless system is almost limited by the use of pilot or known symbols again in the neighboring cells, this issue is known as pilot contamination in the wireless communication systems. Pilot contamination is a challenging issue for all the wireless communication technology, also in the very large MIMO system design also known as massive MIMO systems, which permits future research in the wireless communication field. They also proved that the interaction between antenna elements or correlation between various antenna elements in the array can incur significant losses, to the channel orthogonality and capacity of the wireless link in these systems. So, to reduce or to overcome these effects in large MIMO systems, the spacing between the various antenna elements in the large array must be reduced. Moreover, the antenna or mutual coupling problem of the wireless system also depends on the arrangement of the antennas, e.g., linear array, planar array and cylindrical array.

**In [26] H. Ngo *et al.*** proposed that the radiated power of the antenna in a cellular or wireless system will be inversely proportional to the square-root of the no. of antennas at the base station. Further they said that if channel state information is perfectly presented, then power of the antenna could be finished inversely proportional to the no. of antennas at the base station in case of massive MIMO systems. Lower bounds on the capacity of massive MIMO systems for maximum ratio combining (MRC), zero-forcing (ZF) and minimum mean-square error (MMSE) detection are studied in this article. A receiver with MRC methodology usually performs worse than the ZF and MMSE receiver. Though, as we decreases the power levels then the cross-talk or interference is introduced by the substandard maximum-ratio receiver. After that they prove that that performance falls below and this humble receiver becomes a feasible option. Then, in this paper they found the compromise or trade-off between the energy efficiency and spectral efficiency.

**In [27] J. Hoydis *et al.*** proposed that estimates of attainable rates with several linear precoders and detectors which are confirmed to be asymptotically close-fitting, but

precise for realistic system dimensions, are shown in simulations. It is recognized from earlier work assuming non-correlated channels, that as no. of antennas are very large while RF chains are fixed, the system performance is restricted by pilot contamination, the modest precoders/detectors. In specific, they derive that how many number of antennas per UE are desirable to achieve efficiency of the ultimate performance bound with very large no. of antennas and how many more antennas are required with MF to attain the performance of minimum mean-square error (MMSE) detection and regularized zero-forcing (RZF), respectively.

**In [28] W. Zhang**, proposed a common theoretical framework for examining the Gaussian channels by transmitting the information over them, as they are having memory-less transceiver distortion. These system covers many nonlinear distortion models as well as transmit-side clipping and also the receive-side analog-to-digital conversion. This structure or arrangement is mainly based on the so-called generalized mutual information (GMI). After that, in this paper it is analyzed that the specific benefits from the setup or installation of Gaussian codebook group and nearest-neighbor decoding. For which, it is accepted that the GMI is a general form in analogous to the channel capacity of the non-distorted Gaussian channels. As, this distortion model is applied to exact to study the wireless system, then the various results of engineering significance are obtained for these systems. Then, it is proved that the channels which are having the distortion only at the transmitter, then a general approach, which gives the distorted signal as the sum of the original signal part can be applied.

**In [29] H. Huh *et al.*** proposed a structural design for the MIMO network that realizes the better spectral efficiencies as comparable to the massive MIMO systems. In this paper, the suggested architecture is centered on a family of MIMO network schemes, which is defined by small clusters or groups of cooperating base stations. After that they proved that the zero-forcing precoding of multiuser MIMO wireless systems with suitable inter-cluster interference limitations. They studied the uplink pilot signals reuse across cells, and frequency reuse. After that the key notion involves the splitting of the user population geographically then determines the bins. The bins are such that all the users in the same bin are statistically equal. For each bin they used the optimal MIMO-network communication system architecture in the family.

In [30] J. Kotecha *et al.* proposed the optimal estimation of correlated multiple-input multiple-output (MIMO) channels with the help of pilot signals, supposing knowledge of the second-order channel statistics at the transmitter. Assuming a block fading channel model and minimum mean square error (MMSE) estimation at the receiver, they enterprise the transmitted signal to improve two criteria, MMSE and the conditional mutual information between the MIMO channel and the received signal. This analysis is based on the newly planned virtual channel representation that resembles to beamforming in static virtual directions and exposes the structure and the accurate degrees of freedom in the correlated channel. Still, the design framework is valid to more general channel models, which contain known channel models, like transmitter and receiver correlated model. They proved that optimal signaling is in a block form, where the block length depends on the signal-to-noise ratio (SNR) along with the matrix of channel correlation.

### 2.3 Massive MIMO capacity with Transceiver impairments

As massive MIMO system are known for high rate systems for the 5G technology, but capacity decreases as the no. of users in a cell exceeds beyond a certain limit. In multi-antenna system when the power of training and data signal is equal then the no. of optimum users are equal to the training period [10]. The same concept can be applied for the massive MIMO system to find out the optimum no. of users to which massive MIMO system provides the maximum rate. For this, firstly we calculate the lower bound on the capacity that is attainable with TDD schemes according to [10],[12], because, with the lower bound we can calculate the optimal amount of training as a function of,  $M$ , and  $N$ . Then transceiver impairments affects the accuracy of channel estimation and impact of that on the capacity and optimum no. of users in massive MIMO system.

In [10] B. Hassibi *et al.* proposed that the training marks the capacity of a fading channel, if the training is very less, then channel is inappropriately learned, and if too much training, then there is no time available for data transmission before the channel changes. In this, a lower bound on the capacity of a channel is calculate, which is knowledgeable by training, and then that bound is maximized as a function of the received signal-to-noise ratio (SNR), coherence time, and number of transmitter antennas at the BS. It is proved that if the training and data powers are permitted to vary then optimal number of training symbols is equal to the number of transmit

antennas. When the training and data powers are instead necessary to be equal, the optimal number of symbols may be greater than the number of antennas. It is shown that training-based schemes can be optimal at high SNR, but not at low SNR.

**In [31] E. Bjornson *et al.*** proposed that the obtainable energy can be focused very precisely at the proposed destinations without producing much inter-user interference. Then these results be dependent on asymptotic, it is essential to investigate whether the conventional system models are quiet reasonable in the asymptotic systems. It was investigated that the fundamental limits of large-scale multiple-input single-output communication systems using a global system model that accounts for transceiver hardware impairments. In comparison to the case of ideal hardware, it is shown that practical impairments make finite upper limit on the estimation correctness and capacity of large-scale MISO systems. Amazingly, the performance is only bounded by the hardware at the single-antenna user terminal, though the impact of impairments at the large-scale array disappears asymptotically. Additionally, it is also shown that randomly high energy efficiency can be accomplished by dropping the power while growing the number of antennas.

**In [32] J. Jose *et al.*** proposed that Time-Division Duplexing (TDD) permits to estimate the downlink channels for a randomly large number of base station antennas from a limited number of orthogonal uplink pilot signals, by using channel reciprocity. According to this opinion, a newly proposed "Massive MIMO" system was shown to attain extraordinary spectral efficiency in realistic situations of distance-dependent path loss and channel coherence time and bandwidth. The core focus and involvement of this paper is a better-quality MIMO TDD structural design achieving spectral efficiencies analogous with "Massive MIMO", with order of magnitude less antennas per active user per cell. The offered structural design is based on a family of Network-MIMO schemes defined by small clusters of cooperating base stations, zero-forcing multiuser MIMO precoding with appropriate inter-cluster interference mitigation constraints, uplink pilot signals allocation and frequency reuse across cells. The main indication consists of dividing the users into equivalence classes, improving the Network-MIMO scheme for each equivalence class, and allowing a scheduler to assign the channel time-frequency dimensions to the different classes in order to maximize an appropriate network service function that captures a wanted concept of fairness. This results in a mixed-mode Network-MIMO structural

design, where different schemes are optimized for the assisted user equivalence class and also multiplexed in time-frequency. Next they said that, to carry out the performance examination and the optimization of the projected architecture in an organized and computationally effective way, they study the large-system where the number of users and the number of antennas go to infinity with stable ratios.

**In [33] E. Bjornson *et al.*** proposed that the capacity of ideal MIMO channels has a high SNR slope that equals the minimum of the number of transmit and receive antennas. In this literature it is analyzed that these result holds when there are alterations from physical transceiver impairments. It is analytically proved that such physical MIMO channels have a limited upper capacity limit, for any channel distribution and SNR. So, high-SNR slope collapses to zero. This appears depressing, but they proved that encouraging results for the relative capacity gain of employing MIMO is at least as large as with ideal transceivers.

**In [34] Q. Wang *et al.*** proposed that increasing the antenna quantity can degrade the system performance due to mutual coupling. Antenna selection systems have better performance and lower hardware cost than full-MIMO systems. However, the conventional selection combining (SC) scheme consumes a great amount of training overhead and has high operational complexity in the presence of mutual coupling. This paper proposes a group switch-and-examine combining (GSEC) scheme for massive MIMO systems with the spatial correlation and mutual coupling existing at both the transmitter and receiver. Simulation results demonstrate that the proposed GSEC scheme provides better effective capacity performance and lower operational complexity than the conventional selection combining (SC) and full-MIMO scheme.

**In [35] M. Medard** proposed a model for time-varying communication single-access and multiple-access channels without feedback. They consider the difference between mutual information when the receiver knows the channel perfectly and mutual information when the receiver only has an estimate of the channel. They relate the variance of the channel measurement error at the receiver to upper and lower bounds for this difference in mutual information. They illustrate the use of our bounds on a channel modeled by a Gauss–Markov process, measured by a pilot tone. They relate the rate of time variation of the channel to the loss in mutual information due to imperfect knowledge of the measured channel.

In [36] D. Aktas *et al.* provided scaling results for the sum capacity of the multiple access, uplink channel are provided for a flat-fading environment, with multiple-input-multiple-output links, when there is interference from other cells. The standard MIMO scaling regime is measured in which the number of antennas per user and per base station grow large together. Employing the known characterizations of the restrictive eigenvalue distributions of large random matrices, the asymptotic performance of the sum capacity of the system is considered for a structural design in which the base stations cooperate in the joint decoding process of all users. Then, this asymptotic sum capacity is matched with that of the conventional setup in which the base stations only decode the users in their cells. For the case of base station cooperation, an exciting "resource pooling" phenomenon is perceived: in some cases, the restrictive performance of a macro-diversity multiuser network has the same asymptotic performance as that of a single-user MIMO link with an corresponding amount of pooled received power. This resource pooling phenomenon allows us to derive an elegant closed-form expression for the sum capacity of a new version of Wyner's classical model of a cellular network, in which MIMO links are merged into the model.

In this section we will study the channel model or channel behavior of Massive MIMO system. Firstly, we assume that the channel follows the simple block-fading properties, Further we assume that both the data transmission and channel estimation is to be done within the interval  $T$ .

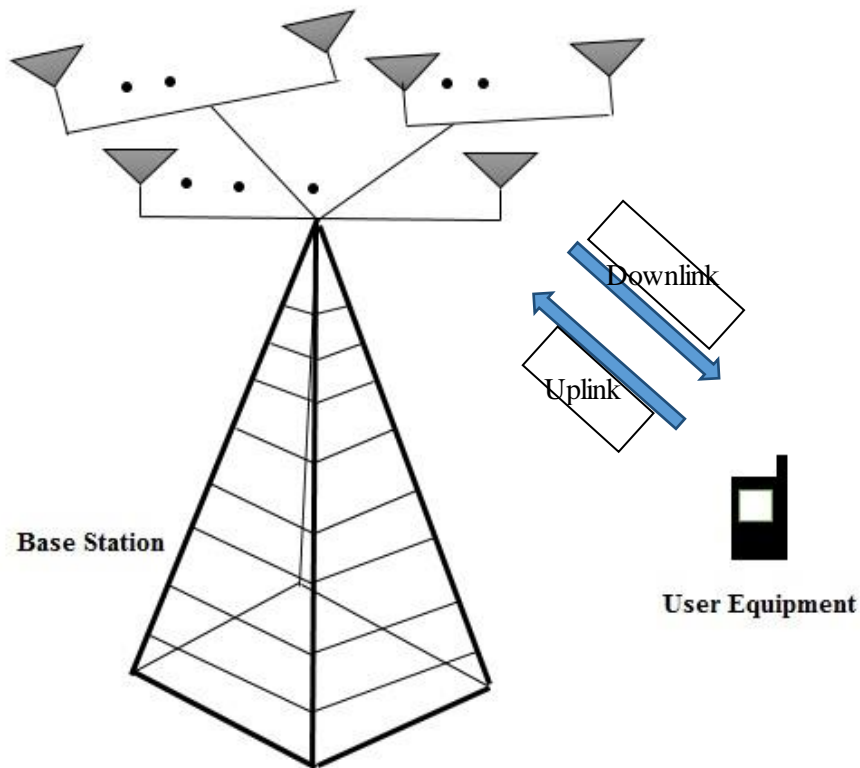


Figure 3.1: TDD channel between BS and UE

The realization or estimation of wireless channels are done randomly and are independent between the blocks. During a block of  $T$  symbols, the cellular model can be given as

$$\mathbf{Y} = \sqrt{\rho} \mathbf{X} \mathbf{H} + \mathbf{V} \quad (3.1)$$

Where  $\mathbf{Y}$  is a  $T \times N$  complex valued matrix received at the BS and  $N$  are the number of antennas at the BS. The matrix  $\mathbf{X}$  is of  $T \times M$  and sent from the transmitter where  $M$  represents the number of users in a particular cell. The  $M \times N$  matrix  $\mathbf{H}$  is the channel matrix connecting the  $M$  users to the  $N$  receive antennas at the BS, and  $\mathbf{V}$  is

a  $T \times N$  matrix of additive white Gaussian noise (AWGN). Thus,  $\rho$  is the expected received SNR at each BS antenna. Let the additive noise  $\mathbf{V}$  and the channel  $\mathbf{H}$  have zero-mean unit-variance independent complex Gaussian entries.

Firstly, we will do the estimation of channel for massive MIMO system, according to that we will calculate the capacity of the system and analyses the performance of this system. Here we are using the TDD channel estimation scheme which is explained below.

### 3.1 Channel Estimators

As stated earlier, during the propagation of signals or information, the wireless channels introduces the several impairments. These impairments or distortions contains the phase variations and also the time varying attenuations that alters the transmitted data signals. Generally, a receiver mitigates or overcome these distortions for a specific demodulation technique of the transmitted data signals. Then, at the receiver, the performance of the non-coherent demodulation, which is inferior as compared to the coherent demodulation. This is measured in terms of the bit error rate (BER). The BER is defined with the help of amount of error in the received data, which is accomplished for a specific amount of transmit power in a communication system [39]. Thus, most of the communication systems in now a days prefers the coherent reception scheme in the wireless systems. The receiver schemes which are coherent in nature, needs the channel state information or knowledge to recompense distortions which are generated by the channel. The procedure or technique of getting the channel state information (CSI) or knowledge is called as channel estimation. The channel estimation is an essential part of all the modern wireless communication receivers. The channel estimation is done with the help of known symbols called “pilot” symbols. Firstly these pilot symbols are transmitted with the information or data signals, then these symbols are received at the receiver. Then, with the help of these pilot signals the channel of the wireless communication systems are estimated. This estimation scheme is called pilot symbol aided modulation (PSAM).

Then the refined signal processing algorithms are applied by the estimator to get the channel knowledge and track it using pilot symbols. The problem of channel estimation for cellular channels has been extensively and successfully addressed in the literature. Estimation schemes accomplished of tracking the fading channel in

cellular systems are an important part of standardized systems. There are following type of channel estimation schemes.

### 3.1.1 Zero Forcing Estimator

Zero Forcing Equalization scheme is a linear equalization algorithm used in communication systems which inverts the frequency response of the channel. This algorithm was proposed by Robert Lucky. The Zero-Forcing Equalizer applies the inverse of the channel to the received signal, to restore the signal before the channel. The name Zero forcing corresponds to bringing down the Inter Symbol Interference (ISI) to zero in a noise free case. This will be useful when ISI is more predominant when comparing to the noise. By assuming  $M_T = M_R$  and  $\mathbf{H}$  is a full rank square matrix, we can calculate the covariance matrix of the effected noise as:

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{N}$$

Where,

$\mathbf{Y}$  = received Symbol Matrix,

$\mathbf{H}$  = Channel matrix,

$\mathbf{X}$  = Transmitted symbol Matrix,

$\mathbf{N}$  = Noise matrix,

To solve for  $\mathbf{X}$ , we need to find a matrix  $\mathbf{W}$  which satisfies  $\mathbf{W}\mathbf{H} = \mathbf{I}$ . The zero forcing detectors for meeting this constraint is given by

$$\mathbf{W} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \quad (3.2)$$

Where  $\mathbf{W}$  is Equalization Matrix,  $\mathbf{H}$  is Channel Matrix. This matrix is known as the Pseudo inverse for the general  $M \times N$  matrix. Zero forcing equalizer tries to null out the interfering terms when performing the equalization. While doing so, there can be amplification of noise. Hence Zero forcing equalizer is not the best possible equalizer. However, it is simple and reasonably easy to implement.

### 3.1.2 Minimum Mean Square Error

A Minimum Mean Square Error (MMSE) estimator describes the approach which minimizes the mean square error (MSE), which is a common measure of estimator quality [40]. The main feature of MMSE equalizer, is that it does not usually eliminate ISI completely but, minimizes the total power of the noise and ISI components in the output. For a system defined by

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{N}$$

The Minimum mean square error approach tries to find a coefficient  $\mathbf{W}$  which minimizes the criterion

$$E \left\{ \left[ \mathbf{W}_{y-x} \right] \left[ \mathbf{W}_{y-x} \right]^H \right\}$$

To solve for  $\mathbf{X}$ , we need to find a matrix  $\mathbf{W}$  which satisfies  $\mathbf{W}\mathbf{H} = \mathbf{I}$ . The Minimum Mean Square Error (MMSE) detector for meeting this constraint is given by,

$$\mathbf{W} = \left[ \mathbf{H}^H \mathbf{H} + \mathbf{M}_T \mathbf{I} \right]^{-1} \mathbf{H}^H \quad (3.3)$$

This matrix is known as the Pseudo inverse for the general  $M \times N$  matrix. Mobile channels have the faster fading as compared to cellular channels due to the increased mobility. The channel estimation schemes those can help in tracking these fast fading channels are required. Linear Minimum Mean Square Error (LMMSE) estimation scheme is much better than the Zero Forcing (ZF) scheme, so we use the LMMSE estimator in our work. The comparison of ZF and LMMSE is also shown in results section.

### 3.2 Training based channel estimated scheme

In the TDD scheme a known signal or called training signal is send from the transmitter. Here we are considering the case for imperfect channel state information (CSI) so, a part of the transmitted matrix  $\mathbf{X}$  to be known at the receiver from which we can estimate the channel  $\mathbf{H}$ [41]. Training based channel estimated schemes have following two phases.

1) **Training Phase:** The model for this can be written as

$$\begin{aligned} \mathbf{Y}_\tau &= \sqrt{\rho_\tau} \mathbf{X}_\tau \mathbf{H} + \mathbf{V}_\tau \\ \mathbf{X}_\tau &\in \mathbb{C}^{T_\tau \times M}, \text{tr} \mathbf{X}_\tau \mathbf{X}_\tau^* = MT_\tau \end{aligned} \quad (3.4)$$

Where  $\mathbf{Y}_\tau \in \mathbb{C}^{T_\tau \times N}$  is received matrix,  $\rho_\tau$  is SNR during training phase, and  $\mathbf{X}_\tau$  is the training matrix for the period of  $T_\tau$  and also known to the receiver.

2) **Data Transmission Phase:** Model for this phase can be written as

$$\begin{aligned} \mathbf{Y}_d &= \sqrt{\rho_d} \mathbf{X}_d \mathbf{H} + \mathbf{V}_d \\ \mathbf{X}_d &\in \mathbb{C}^{T_d \times M}, E \text{tr} \mathbf{X}_d \mathbf{X}_d^* = MT_d \end{aligned} \quad (3.5)$$

Where  $\mathbf{Y}_d \in \mathbb{C}^{T_d \times N}$  is received matrix,  $\rho_d$  is SNR during training phase, and  $\mathbf{X}_d$  is the training matrix for the period of  $T_d$ . The above two phases can be incorporated in the single model as

$$\mathbf{Y} = \begin{pmatrix} \mathbf{Y}_\tau \\ \mathbf{Y}_d \end{pmatrix}, \quad \mathbf{X} = \begin{pmatrix} \sqrt{\frac{\rho_\tau}{\rho}} \mathbf{X}_\tau \\ \sqrt{\frac{\rho_d}{\rho}} \mathbf{X}_d \end{pmatrix}, \quad \mathbf{V} = \begin{pmatrix} \mathbf{V}_\tau \\ \mathbf{V}_d \end{pmatrix} \quad (3.6)$$

Total time and SNR can be related as  $T = T_\tau + T_d$ ,  $\rho T = \rho_\tau T_\tau + \rho_d T_d$ . To find out the optimum parameters we put a lower bound on the capacity for calculating the worst performance of this cellular system. As we know that the accuracy of estimated channel depends on the transmitted and received training sequence. So we can write this as  $\hat{\mathbf{H}} = f(\mathbf{X}_\tau, \mathbf{Y}_\tau)$ . For linear minimum mean-square error estimator (LMMSE), the estimated channel can be written as

$$\hat{\mathbf{H}} = \sqrt{\frac{1}{\rho_\tau}} \left( \frac{1}{\rho_\tau} I_M + \mathbf{X}_\tau^* \mathbf{X}_\tau \right)^{-1} \mathbf{X}_\tau^* \mathbf{Y}_\tau \quad (3.7)$$

Now, we will study the channel estimation under the transceiver impairments. Then we analyse the performance of massive MIMO system with the impact of hardware impairments. The model for this is explained below.

### 3.3 Transceiver Hardware Impairments

Till now, all the research on massive MIMO system is done by considering the ideal hardware at the transmitter and receiver. But practically, both are suffered from non-ideal hardware impairments that causes following disturbances

- a) Produce a divergence between the transmitted signal and what is really generated.
- b) Distort the received signal in the detecting process.

There are different types of hardware components at the transmitter and receiver side in the wireless communication system and each of them distorts the information signal in its own behavior. In recent times, a system model for these types of wireless

systems has been recommended in which the overall impact of impairments of the hardware at transmitter and receiver side are demonstrated with the help of independent additive distortion noise at the BS and also with the UE. The system model for this can be written as

$$\mathbf{Y} = \sqrt{\rho}(\mathbf{X} + \boldsymbol{\eta}_t^{\text{UE}})\mathbf{H} + \boldsymbol{\eta}_r^{\text{BS}} + \mathbf{V} \quad (3.8)$$

In this model two additive noise terms are extra than the conventional model which are  $\boldsymbol{\eta}_t^{\text{UE}}$  and  $\boldsymbol{\eta}_r^{\text{BS}}$ . Where  $\boldsymbol{\eta}_t^{\text{UE}}$  and  $\boldsymbol{\eta}_r^{\text{BS}}$  describe the level of transceiver impairments at the user equipment (UE) and BS, respectively. And these are modeled as  $\boldsymbol{\eta}_t^{\text{UE}} \in \mathbb{C}$ ,  $\boldsymbol{\eta}_r^{\text{BS}} \in \mathbb{C}^{N \times M}$ . The distribution for  $\boldsymbol{\eta}_t^{\text{UE}} \sim \text{CN}(0, v_t^{\text{UE}})$  and  $\boldsymbol{\eta}_r^{\text{BS}} \sim \text{CN}(0, \gamma_r^{\text{BS}})$ . And the covariance matrices for these are modeled as

$$v_t^{\text{UE}} = \kappa_t^{\text{UE}} \rho^{\text{UE}}$$

$$\gamma_r^{\text{BS}} = \kappa_r^{\text{BS}} \rho^{\text{UE}} \text{diag}(|H_{11}|^2, \dots, |H_{NM}|^2)$$

The quality or ideality of hardware at transmitter and receiver are characterized by  $\kappa_t^{\text{UE}}$  and  $\kappa_r^{\text{BS}}$ . For this system the channel can be estimated by LMMSE estimator as

$$\hat{\mathbf{H}} = \underbrace{\mathbf{X}_d^* \mathbf{R} \bar{\mathbf{Y}}^{-1}}_{\mathbf{W}} \mathbf{Y} \quad (3.9)$$

Where  $\bar{\mathbf{Y}}$  is covariance matrix of  $\mathbf{Y}$  and denoted as

$$\bar{\mathbf{Y}} = \text{E}\{\mathbf{Y}\mathbf{Y}^*\} = \rho^{\text{UE}}(1 + \kappa_t^{\text{UE}})\mathbf{R} + \rho^{\text{UE}}\kappa_r^{\text{BS}}\mathbf{R}_{\text{diag}} + \mathbf{X} + \sigma_{\text{BS}}^2\mathbf{I}$$

And  $\mathbf{R}_{\text{diag}} = \text{diag}(r_{11}, \dots, r_{NN})$ . Where  $\mathbf{R}$  is the covariance matrix of the channel  $\mathbf{H}$ .

The total mean square error can be given as  $MSE = \text{E}\left\{\|\hat{\mathbf{H}} - \mathbf{H}\|_2^2\right\} = \text{tr}(\mathbf{Z})$ , where the error covariance matrix  $\mathbf{Z}$  is

$$\mathbf{Z} = \text{E}\left\{\left(\hat{\mathbf{H}} - \mathbf{H}\right)\left(\hat{\mathbf{H}} - \mathbf{H}\right)^H\right\} = \mathbf{R} - \rho^{\text{UE}}\mathbf{R}\bar{\mathbf{Y}}^{-1}\mathbf{R} \quad (3.10)$$

The LMMSE estimator has the form  $\hat{\mathbf{H}} = \mathbf{W}\mathbf{Y}$  where  $\mathbf{W}$  minimizes the MSE. The MSE is given as

$$MSE = tr(\mathbf{R} - \mathbf{X}_d \mathbf{W} \mathbf{R} - \mathbf{X}_d^* \mathbf{R} \mathbf{W}^H + \mathbf{W} \bar{\mathbf{Y}} \mathbf{W}^H) \quad (3.11)$$

This error result in decrease in accuracy of the channel estimation. Then, decrease in accuracy will affect the capacity and other parameter of the massive MIMO system which are studied in further sections and the affects can be viewed from the fig.5.6, fig.5.7, and fig. 5.10.

### 3.4 Channel correlation in Massive MIMO

Massive multiple-input–multiple-output (MIMO) systems have the potential to achieve very high capacities, depending on the propagation environment. In the wireless communication system, the available channels from transmitter side to receiver side are assumed to be independent and non-correlated, but in reality they are correlated to each other up to some extent. So, we need to measure the correlation between them to find out the capacity for the wireless system. Capacity increases as channel correlation decreases. The high spectral efficiency is diminished if the signals arriving at the receivers are correlated. The system capacity decreases as the distance from the transmitter increases. Indeed the transmitter correlation increases as the distance increases [43].

However, correlation analysis within the context of MIMO systems necessitates the investigation of additional characteristics of the wireless channel, such as its complex (amplitude and phase) and double directional (transmit and receive angular spectra) nature. Channel correlation is a measure of match or likeness among the channels.

#### 3.4.1 Correlation types

In reviewing wireless channels, a main aspect is the obtainability of channel state information (CSI) at the transmitter side and receiver side. It is realistic to accept that CSI is existing at the receiver via training. Though we having CSI at the transmitter permits for better performance, this may not be possible in practice, particularly in MIMO channels, due to quick dissimilarities and partial feedback bandwidth. However, it is realistic to assume that the channel statistics are known at the transmitter since these statistics change over much larger time scales than the channel gains. Because of unsatisfactory spacing between antenna elements and limited scattering in the environment, the elements of the channel matrix are not always

independent. The channel correlation of a MIMO system is primarily due to two components:

**(a) Spatial correlation**

In a real-world multipath wireless communication environment, the wireless channels are not independent from each other but due to scatterings in the propagation paths, the channels are related to each other with different degrees. This kind of correlation is called spatial correlation [44]. For a given channel matrix  $H$ , the spatial correlation between the channels are defined as

$$\rho_{ij,pq} = \frac{E\{h_{ij}h_{pq}^*\}}{\sqrt{E\{h_{ij}h_{ij}^*\}E\{h_{pq}h_{pq}^*\}}} \quad (3.12)$$

Where  $i,p = 1,2,\dots, M_T$  and  $j,q = 1,2,\dots, M_R$ .

$ij$  and  $pq$  denotes the two different channel between transmitter and receiver. The spatial correlation depends on the multipath signal environment. Multipath signals tend to leave the transmitter in a range of angular directions (called angles of departure, AOD) rather than a single angular direction. This is the same for the multipath signals arriving at the receiver (called angles of arrival, AOA). Usually, the spatial correlation increases when AOD and AOA are reduced and vice versa.

**(b) Antenna mutual coupling**

The electromagnetic interface between the antenna elements in an antenna array is called mutual coupling. Due to its nature, mutual coupling shows differently in transmitting and receiving antenna arrays and so has to be treated differently. The consequence of mutual coupling is severe if the element spacing is minor [45]. It will disturb the antenna array primarily in the following ways:

- variation of array radiation pattern
- modify the array manifold (the received element voltages)
- change the matching characteristic of the antenna elements (change the input impedances)

**3.4.2 Effect of correlation on capacity**

The capacity of a MIMO system not only depends on the number of channels ( $M_R \times M_T$ ), nonetheless also depends on the correlation between the channels. On the whole,

the greater the channel correlation, the lesser is the channel capacity. In reviewing wireless channels, a significant phase is the accessibility of channel state information (CSI) at the transmitter and receiver. It is equitable to assume that CSI is accessible at the receiver through training. Whereas having CSI at the transmitter allows for better performance, this may not be possible in practice, particularly in MIMO channels, due to fast variations and partial feedback bandwidth. However, it is sensible to assume that the channel statistics are known at the transmitter since these statistics change over much larger time scales than the channel gains [46].

Due to inadequate spacing between antenna elements and imperfect scattering in the environment, the elements of the channel matrix are not at all times independent. Though, low correlation between antenna elements does not promise high spectral efficiency. Correlation between antenna elements can be decreased by separating antennas spatially.

As an alternative of measuring overall correlation, we could approach the problem from the opposite view and inquire, how many independent channels are there? The idea of the number of independent channels (NIC) has clear importance, both as a practical measure and as a source of theoretical insight into channel behavior [47].

### 3.5 Kronecker model

The correlation between the fading of two different antenna pairs is the product of the analogous transmitter correlation and receiver correlation matrix. This correlation model is stated as the *Kronecker* model in the literature. Inappropriately, such a correlation structure is still fairly limiting, and can only be defensible in scenarios where the scattering is nearby rich at either the transmitter or the receiver. Conferring to kronecker model, the correlation between channels or signals transmitted given by the kronecker product of transmitter and receiver correlation matrices [47]. Mathematically the kronecker product is characterized as:

$$\mathbf{R}_H = \mathbf{R}_T \otimes \mathbf{R}_R \quad (3.13)$$

Where  $\mathbf{R}_H$  represents the channel correlation matrix and  $\mathbf{R}_T$  represents the correlation matrix for the transmitter. And  $\mathbf{R}_R$  represents the correlation matrix for the receiver. Symbol ‘ $\otimes$ ’ denotes the kronecker product of  $\mathbf{R}_T$  and  $\mathbf{R}_R$ .

## Capacity Analysis of Massive MIMO System

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### 4.1 Capacity of MIMO system

Let's take a MIMO system having  $M$  transmit antennas, and a receiver with  $N$  antennas. The channel can be represented by the  $N \times M$  matrix. The  $N \times 1$  received signal  $\mathbf{y}$  is given as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v} \quad (4.1)$$

The channel can be represented by the  $N \times M$  matrix  $\mathbf{H}$  of channel gains  $h_{ij}$  representing the gain from transmit antenna  $j$  to receive antenna  $i$ . where  $\mathbf{x}$  is the  $M \times 1$  transmitted vector and  $\mathbf{v}$  is the  $N \times 1$  additive white circularly symmetric complex Gaussian noise vector normalized so that its covariance matrix is the identity matrix. Here we take a wireless channel with bandwidth  $B$ , the zero mean complex Gaussian noise and covariance matrix  $\sigma_v^2 \mathbf{I}_N$ , where  $\sigma_v^2 = N_0 B$ .

The power constraint given below in the wireless system implies that the transmitted symbols satisfy the equation (4.1)

$$\sum_{i=1}^M E[x_i x_i^*] = \rho \quad (4.2)$$

or we can also define this as  $\text{Tr}(\mathbf{R}\mathbf{x}) = \rho$ , where  $\text{Tr}$  denotes the trace of input covariance matrix  $\mathbf{R}\mathbf{x} = E[\mathbf{x}\mathbf{x}^T]$ .

Different observations can be made about channel state information or knowledge of the channel gain matrix  $\mathbf{H}$  at the transmitter and receiver. Those are mentioned as the channel state information at the transmitter and channel state information at the receiver, respectively. Further, the fixed channel CSIR is usually presumed, subsequently the gain of the channels can be found simply by transmitting a pilot sequence with the data signals for estimating the channel of the wireless communication system. As, if a feedback path is available, then the CSIR from the receiver will be feedback to the transmitter to improve the CSIT. The CSIT is also accessible in time-division duplexing (TDD) wireless communication systems without feedback. It is done by exploiting the reciprocal properties of wireless channel propagation. If the channel is not known at either the transmitter or receiver side, then distribution of the channel gain matrix is presumed initially. Most common model of the wireless communication for this type of distribution is a zero-mean spatially white

(ZMSW) model, in which the entries or values of the channel matrix  $\mathbf{H}$  are assumed to be i.i.d. zero mean, unit variance, complex circularly symmetric Gaussian random variables.

In equation (4.1), using the random matrix theory, any channel matrix  $\mathbf{H}$  for a wireless communication can be written using singular value decomposition (SVD) as

$$\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{A}^H \quad (4.3)$$

Where  $\mathbf{U}$  and  $\mathbf{A}$  are unitary matrices and  $\mathbf{\Sigma}$  is an  $N \times M$  diagonal matrix of singular values  $\{\sigma_i\}$  of  $\mathbf{H}$ . These singular values have the property that  $\sigma_i = (\lambda_i)^{1/2}$  for  $\lambda_i$  the  $i$ th eigenvalue of  $\mathbf{H}\mathbf{H}^H$ , and  $r_H$  of these singular values are nonzero, where  $r_H$  is the rank of the matrix  $\mathbf{H}$ . Since  $r_H$  can't exceed the number of columns or rows of  $\mathbf{H}$ ,  $r_H \leq \min(N, M)$ . If  $\mathbf{H}$  is full rank, which is from time to time denoted as a rich scattering environment, then  $r_H = \min(M, N)$ . Other environments may lead to a low rank  $\mathbf{H}$ : a channel with high correlation amongst the gains in  $\mathbf{H}$  may have rank 1.

Capacity of the wireless channels depend on the knowledge about the channel gain matrix or its distribution at the transmitter and/or receiver side. Precisely, the capacity is given in terms of the mutual information between the channel input vector  $\mathbf{x}$  and output vector  $\mathbf{y}$  is given by the equation (4.4)

$$C = \max I(\mathbf{X}; \mathbf{Y}) = \max [H(\mathbf{Y}) - H(\mathbf{Y}|\mathbf{X})] \quad (4.4)$$

for  $H(\mathbf{Y})$  and  $H(\mathbf{Y}|\mathbf{X})$  the entropy in  $\mathbf{y}$  and  $\mathbf{y}|\mathbf{x}$ . The mutual information of  $\mathbf{y}$  depends on its covariance matrix, so for the narrowband MIMO model can be given by equation (4.5)

$$\mathbf{R}_y = E[\mathbf{y}\mathbf{y}^H] = \mathbf{H}\mathbf{R}_x\mathbf{H}^H + \mathbf{I}_N \quad (4.5)$$

where  $\mathbf{R}_x$  is the covariance of the MIMO channel input.  $H(\mathbf{Y}) = B \log_2 \det[\pi e \mathbf{R}_y]$  and  $H(\mathbf{V}) = B \log_2 \det[\pi e \mathbf{I}_N]$ , resulting in the mutual information

$$I(\mathbf{X}; \mathbf{Y}) = B \log_2 \det[ \mathbf{I}_N + \mathbf{H}\mathbf{R}_x\mathbf{H}^H ] \quad (4.6)$$

Now,

$$C = \max I(\mathbf{X}; \mathbf{Y}) = \max B \log_2 \det[ \mathbf{I}_M + \mathbf{H}\mathbf{R}_x\mathbf{H}^H ] \quad (4.7)$$

As  $\mathbf{R}_x = \frac{\rho}{M} \mathbf{I}_M$ ,

$$I = B \log_2 \det [ \mathbf{I}_N + \frac{\rho}{M} \mathbf{I}_M \mathbf{H}\mathbf{H}^H ] \quad (4.8)$$

The capacity gain of MIMO channels as in equation (4.7), derived under the idealistic assumption that the channel matrix entries are independent complex Gaussian variables, might be reduced on real channels.

#### 4.2 Performance of LMMSE Estimator

Now, we examine the performance of LMMSE channel estimator for the massive MIMO systems that has the form  $\hat{h} = Az$ . Where A minimizes the mean square error (MSE). The MSE definition from (3.11) gives

$$MSE = tr\left(\mathbf{R} - \mathbf{X}_d \mathbf{W} \mathbf{R} - \mathbf{X}_d^* \mathbf{R} \mathbf{W}^H + \mathbf{W} \bar{\mathbf{Y}} \mathbf{W}^H\right)$$

The channel of the massive MIMO system can be disintegrated as  $\mathbf{H} = \hat{\mathbf{H}} + \boldsymbol{\varepsilon}$ , where  $\hat{\mathbf{H}}$  is the LMMSE estimate of the channel at the receiver as in equation (3.9) and  $\boldsymbol{\varepsilon} \in \mathbb{C}^{N \times 1}$  amount of unknown estimation error at the receiver. The covariance matrices are  $\mathbf{E}\{\hat{\mathbf{H}}\hat{\mathbf{H}}^H\} = \mathbf{R} - \boldsymbol{\Xi}$  and  $\mathbf{E}\{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^H\} = \boldsymbol{\Xi}$ . So, there might be a non-linear estimator that gives lesser MSEs than the LMMSE estimator or which can improve the channel estimation accuracy. Though, the change in MSE performance should be minor, meanwhile the dependent distortion noises are comparatively weak.

Consider the case of  $\mathbf{R} = \lambda \mathbf{I}$  and  $\mathbf{S} = 0$ . Then error covariance matrix in (3.11) becomes

$$\boldsymbol{\Xi} = \lambda \left( 1 - \frac{p^{UE} \lambda}{p^{UE} \lambda (1 + \kappa_r^{BS} + \kappa_t^{UE}) + \sigma_{BS}^2} \right) \mathbf{I} \quad (4.9)$$

For the high SNR,

$$\lim_{p^{UE} \rightarrow \infty} \boldsymbol{\Xi} = \lambda \left( 1 - \frac{1}{(1 + \kappa_r^{BS} + \kappa_t^{UE})} \right) \mathbf{I} \quad (4.10)$$

This outcome carries important visions on the average estimation error per element in  $\mathbf{H}$ . The error adjustment is given by the factor in front of the identity matrix in (4.10). It is independent of the number of antennas N, therefore allowing N grow large neither increases nor decreases the estimation error per element. The estimation error is obviously a decreasing function of the pilot power  $p^{UE} = |d|^2$  but opposing to the ideal hardware case the error variance is not touching to zero as  $p^{UE} \rightarrow \infty$ .

### 4.3 Capacity Bounds due to Hardware Impairments and Optimum no. of Users

Now we will calculate the capacity of the system using the estimated channel and see the result for different cases. As  $\hat{\mathbf{H}}$  is the estimate of  $\mathbf{H}$  at the BS then the received signal from equation (3.5) can be written as

$$\mathbf{Y}_d = \sqrt{\rho_d} \mathbf{X}_d \hat{\mathbf{H}} + \underbrace{\sqrt{\rho_d} \mathbf{X}_d \tilde{\mathbf{H}} + \mathbf{V}_d}_{\mathbf{V}'_d} \quad (4.11)$$

Where  $\tilde{\mathbf{H}} = \mathbf{H} - \hat{\mathbf{H}}$  is the channel estimation error, and  $\mathbf{V}'_d$  is the combines the estimation and additive noise. The channel capacity of any training based system is permanently limited by the capacity of the known channel [49], with power constraints as

$$\sigma_{\mathbf{V}'_d}^2 = \frac{1}{NT_d} \text{tr} E \mathbf{V}'_d \mathbf{V}'_d{}^*$$

Then capacity bound can be written as in [10]

$$C_\tau \geq C_{\text{worst}} = \min_{\mathbf{R}_V, \text{tr} \mathbf{R}_V = N} \max_{\mathbf{R}_X, \text{tr} \mathbf{R}_X = M} E \frac{T - T_\tau}{T} \log \det \left( \mathbf{I}_N + \frac{\rho_d}{1 + \rho_d \sigma_{\tilde{\mathbf{H}}, \mathbf{R}_X}^2} \frac{\mathbf{R}_V^{-1} \hat{\mathbf{H}}^* \mathbf{R}_X \hat{\mathbf{H}}}{M} \right)$$

Where  $\mathbf{R}_V = E \mathbf{V} \mathbf{V}^*$ ,  $\mathbf{R}_X = E \mathbf{X} \mathbf{X}^*$  and  $\sigma_{\tilde{\mathbf{H}}, \mathbf{R}_X}^2 \cong \frac{1}{NM} E \text{tr} \tilde{\mathbf{H}}^* \mathbf{R}_X \tilde{\mathbf{H}}$ . So the co-efficient

$T - T_\tau$  shows the fact that data transmission phase have the duration of only

$T_d = T - T_\tau$ . We can define  $\sigma_{\hat{\mathbf{H}}}^2$  as  $\sigma_{\hat{\mathbf{H}}}^2 = \frac{1}{NM} E \text{tr} \hat{\mathbf{H}}^* \hat{\mathbf{H}}$ . And by the principle of

orthogonality for MMSE  $\sigma_{\tilde{\mathbf{H}}}^2 = 1 - \sigma_{\hat{\mathbf{H}}}^2$ . Where  $\sigma_{\tilde{\mathbf{H}}}^2 = \frac{1}{NM} E \text{tr} \tilde{\mathbf{H}}^* \tilde{\mathbf{H}}$ . Normalized channel

estimate can be defined as  $\bar{\mathbf{H}} \cong \frac{1}{\sigma_{\hat{\mathbf{H}}}} \hat{\mathbf{H}}$ . Then the capacity bound can be written as

$$C_\tau \geq \min_{\mathbf{R}_V, \text{tr} \mathbf{R}_V = N} \max_{\mathbf{R}_X, \text{tr} \mathbf{R}_X = M} E \frac{T - T_\tau}{T} \log \det \left( \mathbf{I}_N + \frac{\rho_d \sigma_{\hat{\mathbf{H}}}^2}{1 + \rho_d \sigma_{\tilde{\mathbf{H}}, \mathbf{R}_X}^2} \frac{\mathbf{R}_V^{-1} \bar{\mathbf{H}}^* \mathbf{R}_X \bar{\mathbf{H}}}{M} \right)$$

The factor  $\rho_{\text{eff}} = \frac{\rho_d \sigma_{\hat{\mathbf{H}}}^2}{1 + \rho_d \sigma_{\tilde{\mathbf{H}}, \mathbf{R}_X}^2}$  can be considered as the effective SNR. As the  $\mathbf{R}_X = \mathbf{I}_M$

and  $\mathbf{R}_V = \mathbf{I}_N$  then we have

$$C_\tau \geq \mathbb{E} \frac{T-T_\tau}{T} \log \det \left( \mathbf{I}_M + \frac{\rho_d \sigma_{\hat{\mathbf{H}}}^2}{1 + \rho_d \sigma_{\hat{\mathbf{H}}, \mathbf{R}_X}^2} \frac{\bar{\mathbf{H}} \bar{\mathbf{H}}^*}{M} \right) \quad (4.12)$$

Co-variance matrix for MMSE estimate can be given as  $\mathbf{R}_{\hat{\mathbf{H}}} = \mathbf{R}_{\mathbf{H}} - \mathbf{R}_{\mathbf{H}\mathbf{Y}_\tau} \mathbf{R}_{\mathbf{Y}_\tau}^{-1} \mathbf{R}_{\mathbf{Y}_\tau \mathbf{H}}$ .

Where  $\mathbf{R}_{\mathbf{H}\mathbf{Y}_\tau} = \mathbb{E}(\text{vec} \mathbf{H})(\text{vec} \mathbf{Y}_\tau)^*$ ,  $\text{vec} \hat{\mathbf{H}} = \mathbf{R}_{\mathbf{H}\mathbf{Y}_\tau} \mathbf{R}_{\mathbf{X}_\tau}^{-1} (\text{vec} \mathbf{X}_\tau)$  and

$\mathbf{R}_{\mathbf{Y}_\tau} = \mathbb{E}(\text{vec} \mathbf{Y}_\tau)(\text{vec} \mathbf{Y}_\tau)^*$ . According to choice of the training signal  $\mathbf{X}_\tau \mathbf{X}_\tau^* = T_\tau \mathbf{I}_M$ .

Then

$$\mathbf{R}_{\hat{\mathbf{H}}} = \frac{1}{1 + \rho_\tau T_\tau} \mathbf{I}_M \otimes \mathbf{I}_N, \quad \mathbf{R}_{\hat{\mathbf{H}}} = \frac{\rho_\tau T_\tau}{1 + \rho_\tau T_\tau} \mathbf{I}_M \otimes \mathbf{I}_N$$

So the equation (4.12) can be given as

$$C_\tau \geq \mathbb{E} \frac{T-T_\tau}{T} \log \det \left( \mathbf{I}_M + \rho_{\text{eff}} \frac{\bar{\mathbf{H}} \bar{\mathbf{H}}^*}{M} \right) \quad (4.13)$$

$$\text{And } \rho_{\text{eff}} = \frac{\rho_d \rho_\tau T_\tau}{1 + \rho_d + \rho_\tau T_\tau}.$$

Now, for optimizing over  $T_\tau$  let us consider the power for training and data signals are equal i.e.  $\rho_\tau = \rho_d = \rho$ . Then equation (4.13) become

$$C_\tau \geq \mathbb{E} \frac{T-T_\tau}{T} \log \det \left( \mathbf{I}_M + \frac{\rho^2 T_\tau}{1 + (1+T_\tau)\rho} \frac{\bar{\mathbf{H}} \bar{\mathbf{H}}^*}{M} \right) \quad (4.14)$$

Form the above derived equation we can see that if we are increasing the  $T_\tau$ , our channel estimation improves which improves capacity but after a certain limit it decreases the capacity as the time available for data transmission decreases. All this can also be noticed from the above equation, decrease in capacity is linear by increasing training interval but increase in capacity is logarithmic.

It can also be shown that if the SNR is high then training interval is equal to no. of users and if the SNR is low the training interval is half of the total interval.

Case 1) at high SNR:

$$C_\tau \geq E \frac{T-T_\tau}{T} \log \det \left( \mathbf{I}_M + \frac{\rho}{1+\frac{1}{T_\tau}} \frac{\bar{\mathbf{H}}\bar{\mathbf{H}}^*}{M} \right) \quad (4.15)$$

Case 2) at low SNR:

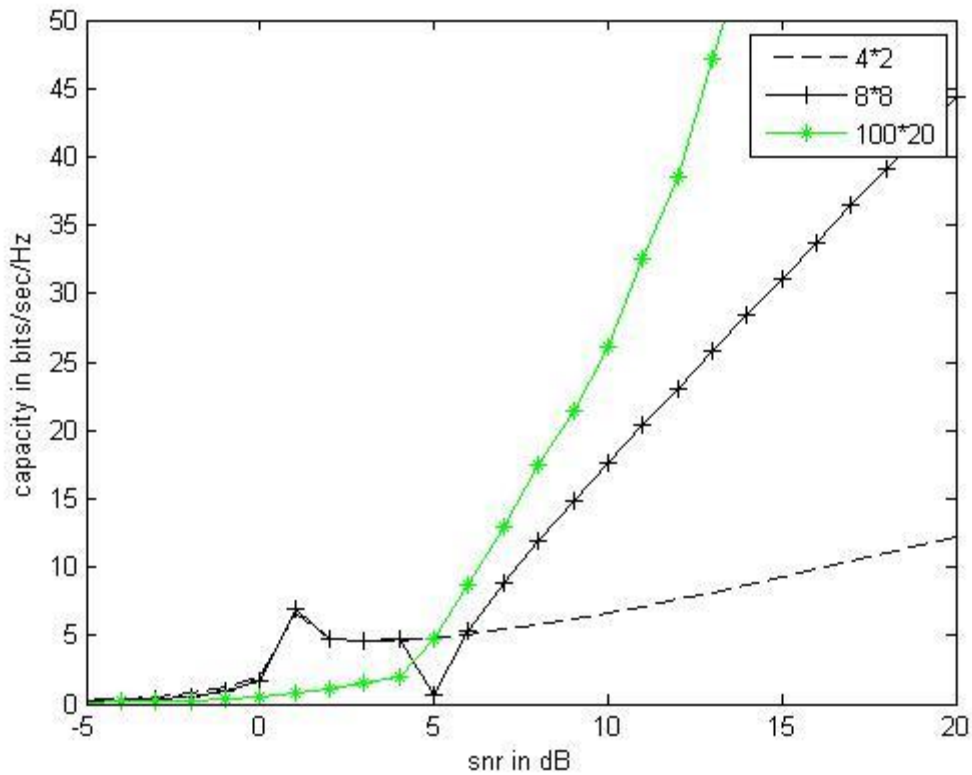
$$C_\tau \geq E \frac{T-T_\tau}{T} \text{tr} \log \left( \mathbf{I}_M + \rho^2 T_\tau \frac{\bar{\mathbf{H}}\bar{\mathbf{H}}^*}{M} \right) \approx \frac{T-T_\tau}{T} E \text{tr} \rho^2 T_\tau \log e \frac{\bar{\mathbf{H}}\bar{\mathbf{H}}^*}{M} \quad (4.16)$$

**5.1 Capacity of various MIMO Systems**

Firstly we assume that  $\mathbf{H}$  has CN (0,1) independent entries. The following graph shows that as we increases the no. of antennas at the BS and UE, there is a significant increase in the channel capacity as shown in the fig. 5.1.

**Table 2. Capacity of Various MIMO Systems**

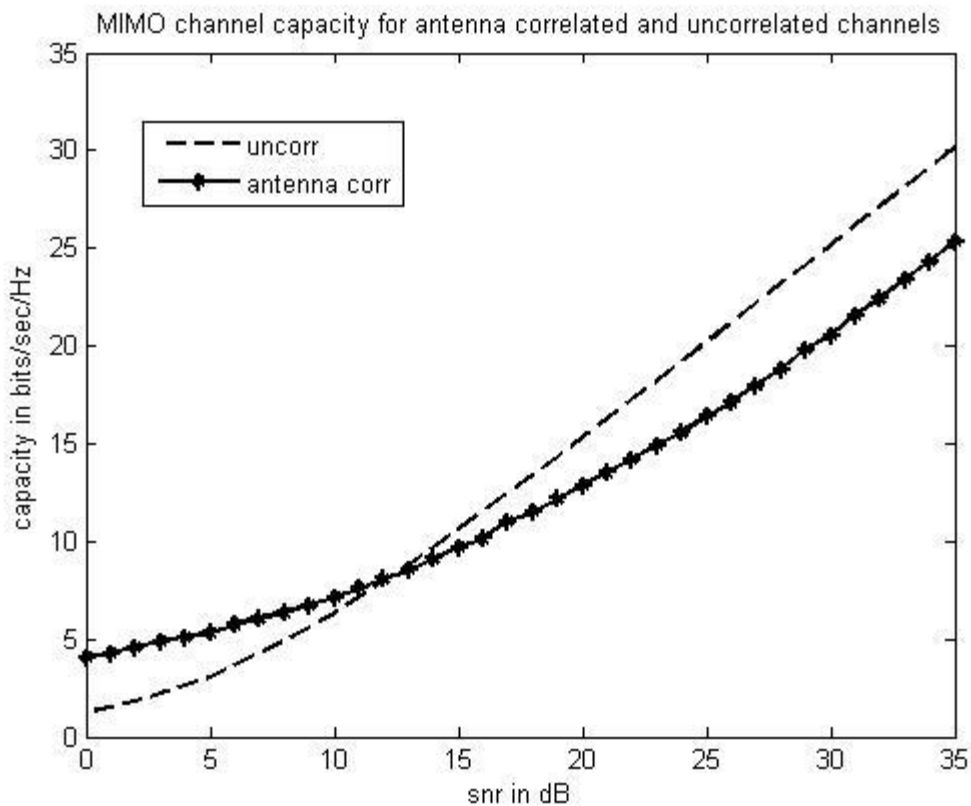
SNR in dB	Capacity (Bits/sec/Hz)		
	4*2 antenna system	8*8 antenna system	100*20 antenna system
5	3	3	6
7	4	10	20
10	6	17	30



**Figure 5.1: Increase in Channel Capacity with Varying Antennas**

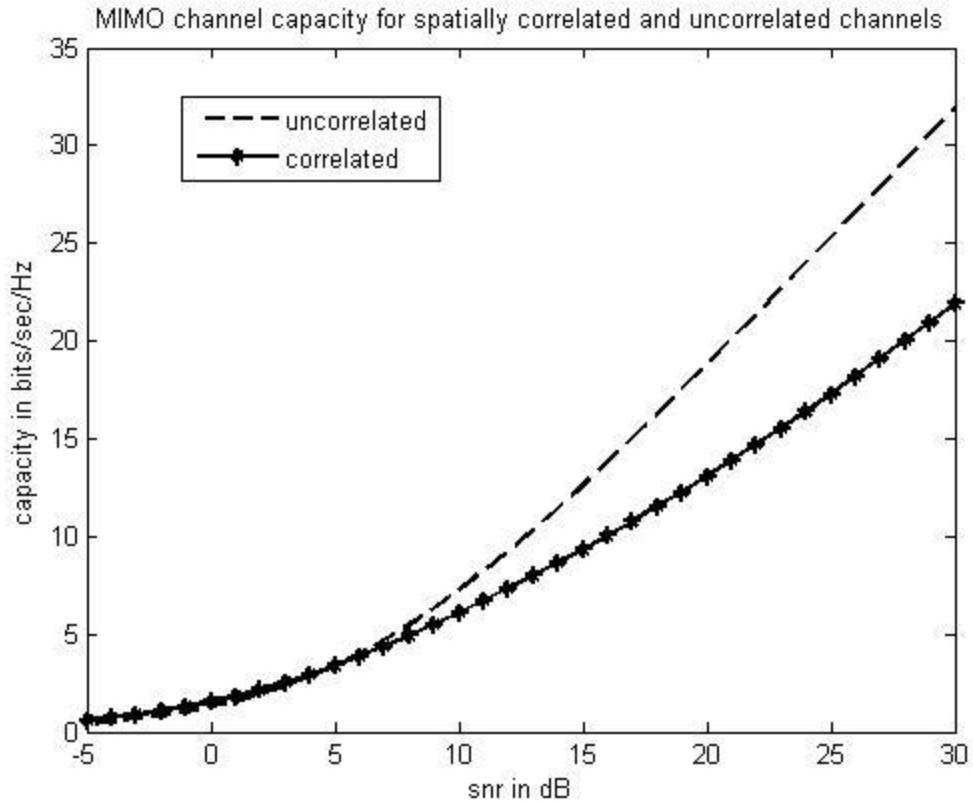
## 5.2 Effects of Various Channel Correlation on MIMO Systems

As we studied earlier that due to inadequate spacing between antenna elements and imperfect scattering in the environment, the elements of the channel matrix are not at all times independent. Though, low correlation between antenna elements does not promise high spectral efficiency. Correlation between antenna elements can be decreased by separating antennas spatially. Firstly, we studied the spatial correlation and compare this with the non-correlated environment. The difference can be seen from the fig. 5.2, which shows that capacity of MIMO system without correlation is higher than the correlated system.



**Figure 5.2: Capacity of Spatially Correlated and Uncorrelated for 4\*4 MIMO System**

Further, we simulated the antenna correlation, which is discussed in previous chapters. This simulation can be examined from the fig. 5.3, which explains how the antenna correlation affects the capacity of the MIMO system. Here we have taken the 3x3 MIMO system for simplification of the calculations.

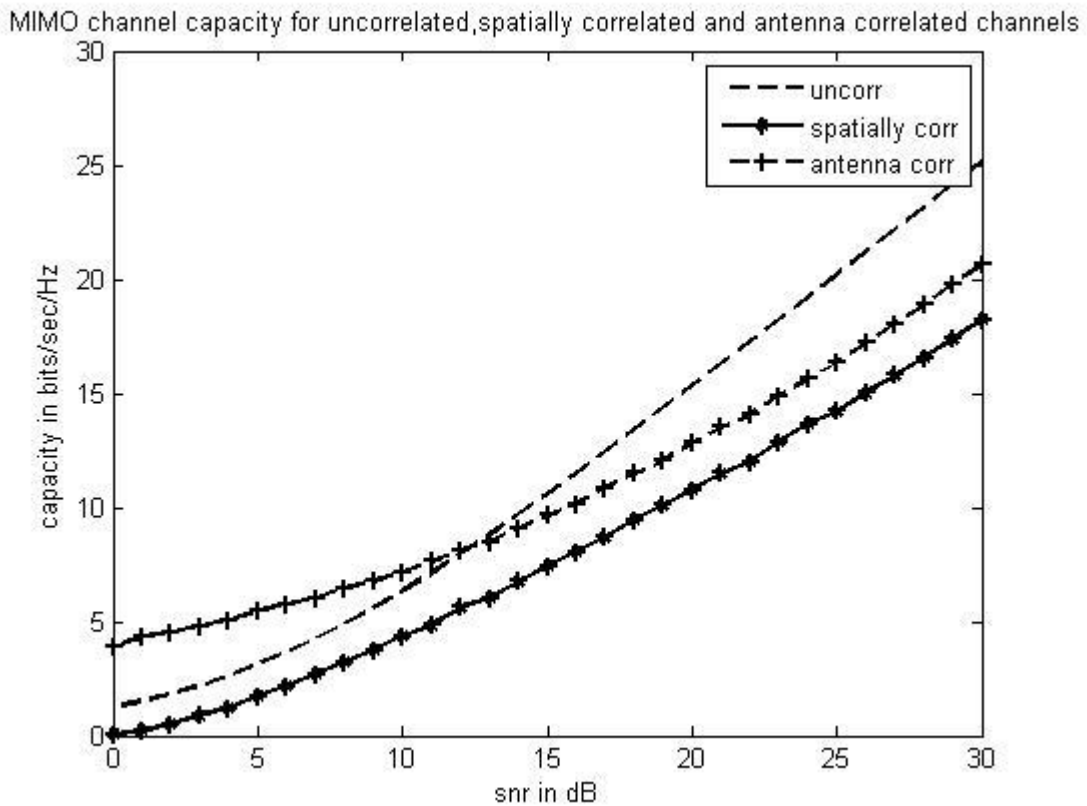


**Figure 5.3: Capacity of Antenna correlated and uncorrelated for 3\*3 MIMO System**

Now, the effects of all types of correlation i.e. antenna correlation and spatial correlation in comparison with non-correlated system is studied. Table 3 shows that how the capacity of 3×3 MIMO system is increases with the increase in SNR for all types of correlation. Then we found that spatial correlation affects the system capacity more than antenna correlation. And, antenna correlation do not affects the capacity at low SNR.

**Table 3. Capacity of Spatially Correlated, Antenna Correlated and Uncorrelated 3\*3 MIMO System**

SNR in dB	Capacity (Bits/sec/Hz)		
	Spatially correlated system	Antenna correlated system	Uncorrelated system
5	2	5	3
15	6	8	10
25	13	15	20



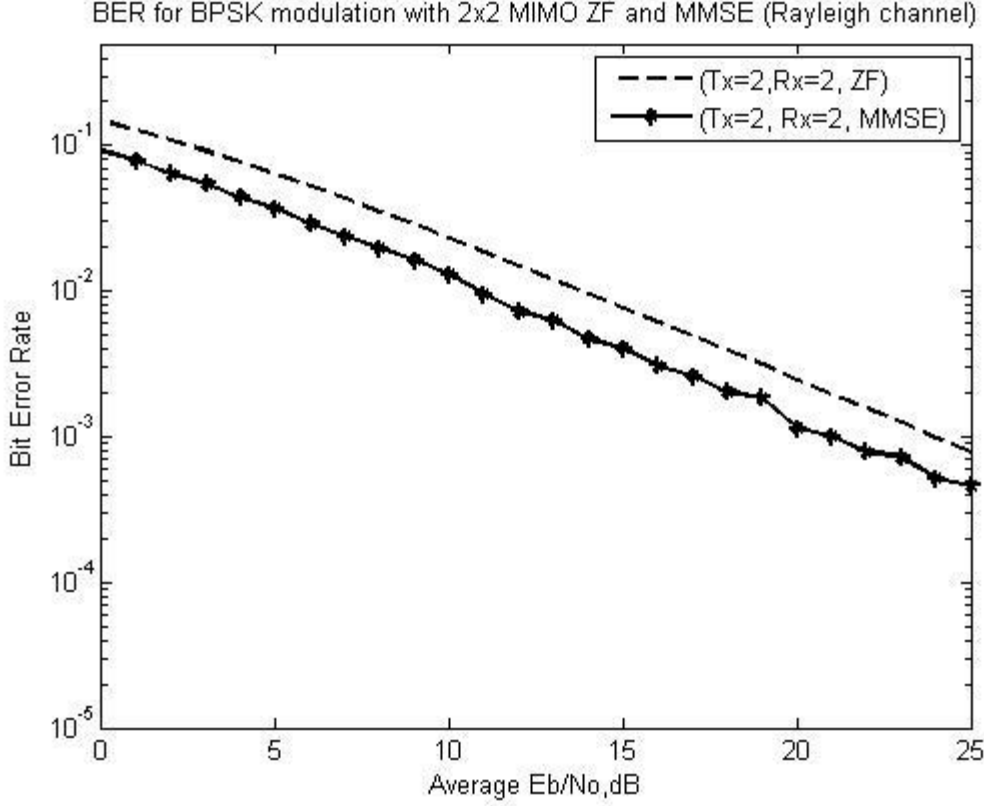
**Figure 5.4: Capacity of Spatially Correlated, Antenna Correlated and Uncorrelated 3\*3 MIMO System**

### 5.3 ZF and LMMSE Estimator performance

In this section, simulation is performed to verify the effectiveness of estimation technique over MIMO receivers. The effect of zero forcing equalizer is compared with the effect of MMSE for the respective system. The BER performance is evaluated for ZF and MMSE case with respect to signal to Noise Ratio (SNR). The performance is reflected from the fig. 5.5. The BER is always seems to be on a bit higher side for MMSE case as compared to that for ZF case.

### 5.4 Effect of Transceiver Impairments on LMMSE Estimator and Capacity Bounds

Now, we will examine the impact of impairments due to non-ideal hardware at the transmitter and receiver side on the accuracy of the estimator of wireless communication channel. In fig. 5.6, antennas at the BS are 50 and we assumed that there is no interference. The channel covariance matrix  $\mathbf{R}$  is produced by the exponential correlation model, which means that the (i, j) th element of  $\mathbf{R}$  is given as:



**Figure 5.5: Performance Comparison of Zero Forcing (ZF) and MMSE estimator**

$$[\mathbf{R}]_{i,j} = \begin{cases} \delta r^{j-i}, & i \leq j \\ \delta (r^{i-j})^*, & i > j \end{cases} \quad (5.1)$$

Where  $\delta$  is an arbitrary scaling factor. For the massive MIMO system, this model fundamentally refers to a uniform linear array (ULA). In this model, the correlation factor between the adjacent antennas at the base station is given by  $|r|$  which has the value between 0 and 1. Here we are taking the correlation coefficient of  $r = 0.7$ . Fig.

5.6 shows the comparative estimation error per channel element,  $MSE_{rel} = \frac{MSE}{tr(\mathbf{R})}$ , as a

function of the average SNR in the UL, defined as

$$SNR^{UL} = p^{UE} \frac{tr(\mathbf{R})}{N\sigma_{BS}^2} \quad (5.2)$$

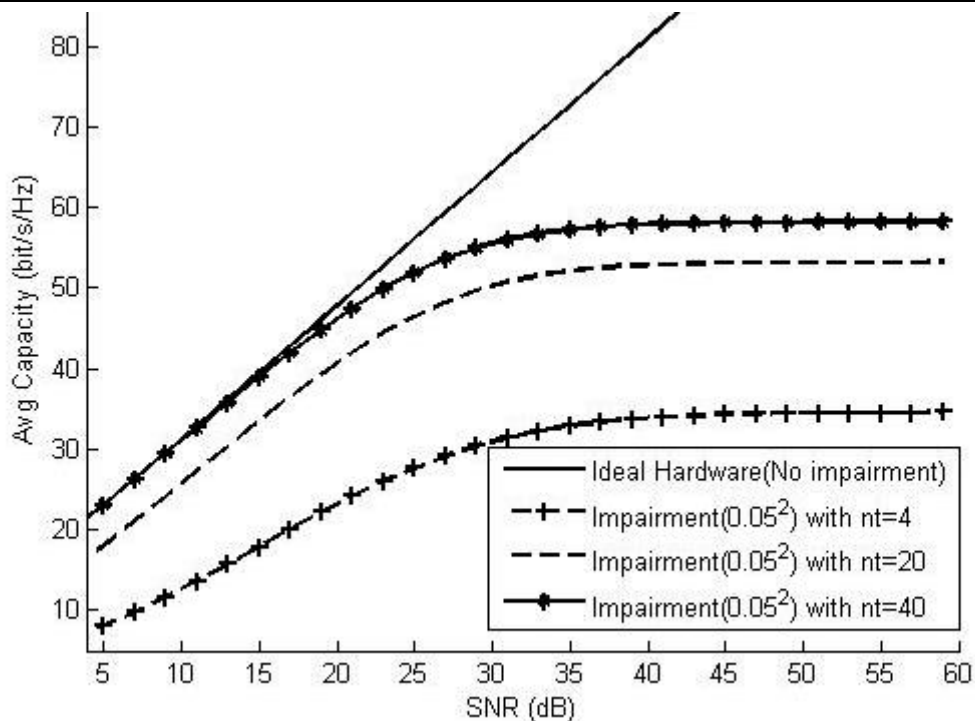
Where  $p^{UE}$  is the SNR for the user equipment. Here, we consider three different levels of impairments:  $\kappa_i^{UE} = \kappa_r^{BS} \in \{0, 0.05^2, 0.15^2\}$ .

Table 4 shows the simulation results of the three levels of impairments. It is clear from the table 4 that as the impairments increases the MSE also increases with the increase in SNR. So, the error in channel estimation increases with the hardware impairments. Fig. 5.7 shows the MATLAB simulation results for all the three level of impairments.

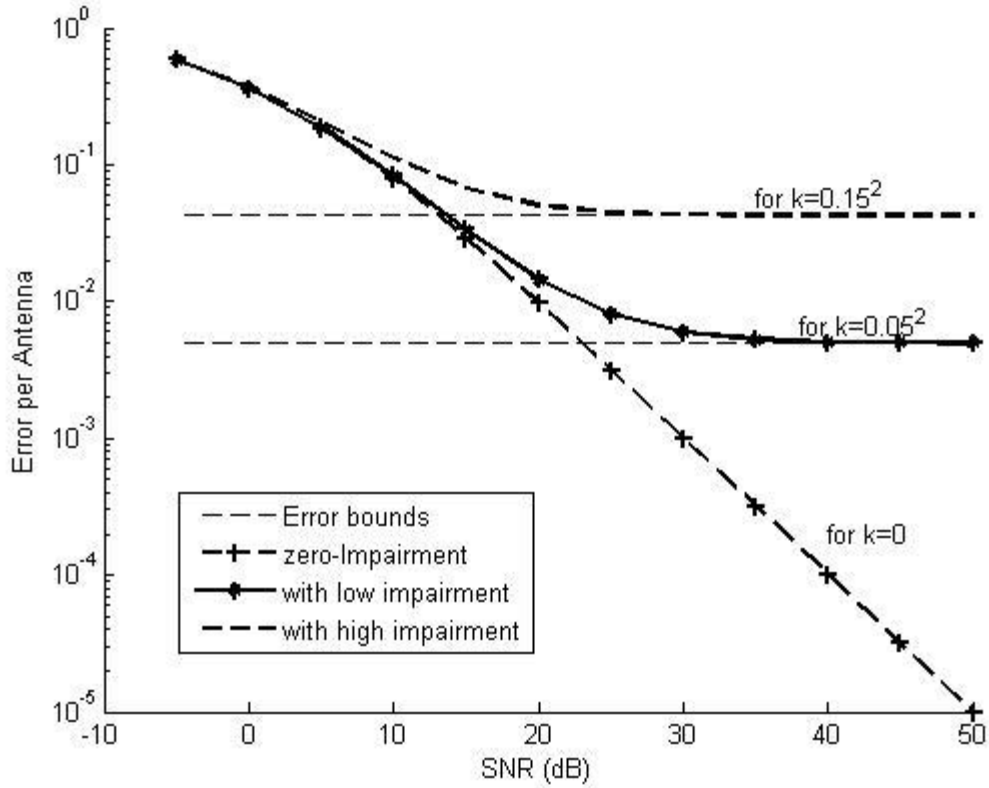
Now, study a channel with  $N_t$ ,  $N_r$  (no. of transmitter and receiver) and varying SNR. As we are varying the number of antennas at the BS with hardware impairments, then the average capacity increases. This can be shown from fig. 5.7. Here, we are considering the level of impairment at both transmitter and receiver  $\kappa_t$  and  $\kappa_r = 0.05$ . As we increase the SNR, then the capacity limits to a certain value, this can be studied from the table 5. Table 5 shows the results of various cases with different no. of antennas at the base station. Fig. 5.7 shows the MATLAB result, as the SNR increases beyond a certain value, then the capacity is bounded by some value for each case.

**Table 4. Comparison of Error provided by Different levels of Impairments**

SNR in dB	Error per antenna		
	$\kappa_t^{UE} = \kappa_r^{BS} = 0$	$\kappa_t^{UE} = \kappa_r^{BS} = 0.05^2$	$\kappa_t^{UE} = \kappa_r^{BS} = 0.15^2$
0	$10^{-0.6}$	$10^{-0.6}$	$10^{-0.7}$
10	$10^{-0.9}$	$10^{-1.0}$	$10^{-1.1}$
30	$10^{-1.5}$	$10^{-2.3}$	$10^{-3.1}$



**Figure 5.6: Estimation error per antenna due to various hardware impairments**



**Figure 5.7: Capacity Bound of Massive MIMO System with Hardware Impairments**

**Table 5. Capacity Bounds with different no. of Antennas at BS**

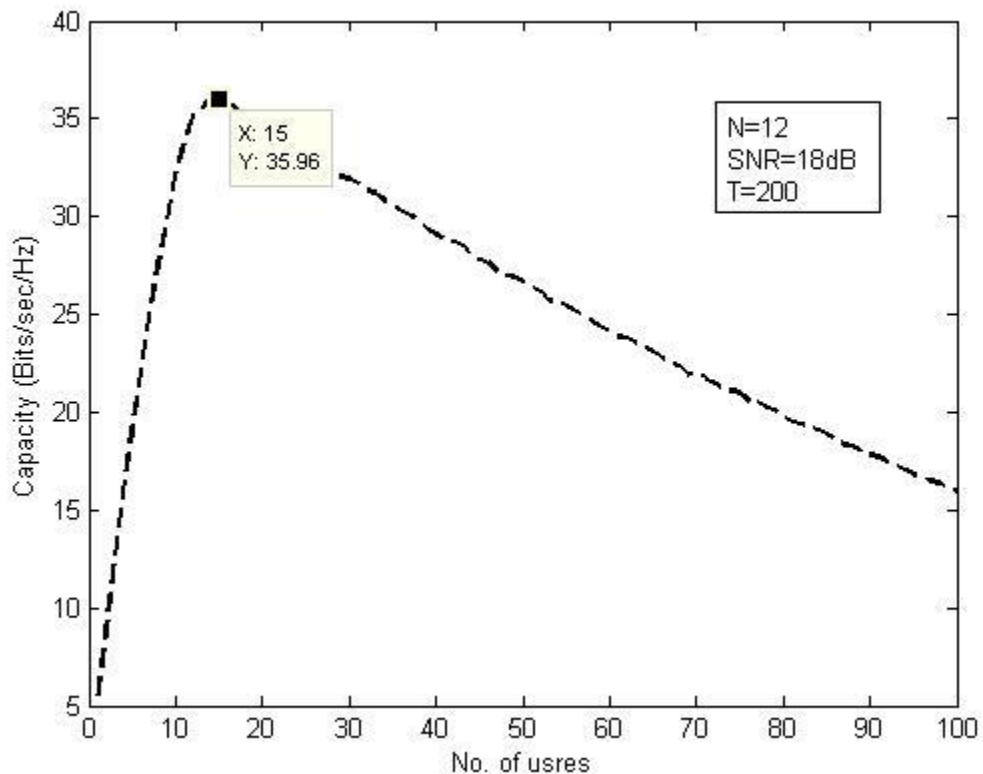
SNR in dB	Capacity Limits with Hardware Impairments (Bits/sec/Hz)		
	BS Antennas = 4	BS Antennas = 20	BS Antennas = 40
20	21	40	45
35	32	49	55
60	32	49	55

### 5.5 Optimum Number of Users for MIMO and Massive MIMO

Here if we have a given SNR  $\rho$ , coherence time interval  $T$ , and the total no of antennas at the BS are  $N$ , then we can find out the capacity of the massive MIMO system as a function of no. of users  $M$ . If we are taking equal training and data power  $\rho_\tau = \rho_d = \rho$ , then the optimal training time is equal to or greater than no. of users ( $M$ ) so, results are based on the fact that we are varying training time with no of users i.e.  $T_\tau = M$ . The capacity is low when no. of users are less as the antennas are less to

estimate the channel, and if  $M$  is nearly equal to the total time then also the capacity is very less because the whole time spend on training interval. So, we can seek the optimum no of users that maximizes the capacity of this system.

In the fig.13 we are taking less no of antennas in equation (4.13) at the BS to compare the capacity with the case of large no of antennas. In fig. 5.8, no. of users varies from 1 to 100, SNR=18dB, and no of antennas ( $N$ ) at the BS are 12 and the total time interval is 100. Then we found that capacity is maximum at  $M \approx 15$ , and the variation of capacity can be viewed from fig. 5.8. The maximum value of capacity at  $M \approx 15$  is 29 Bits/sec/Hz.



**Figure 5.8: Capacity of Massive MIMO systems with less antennas at the BS.**

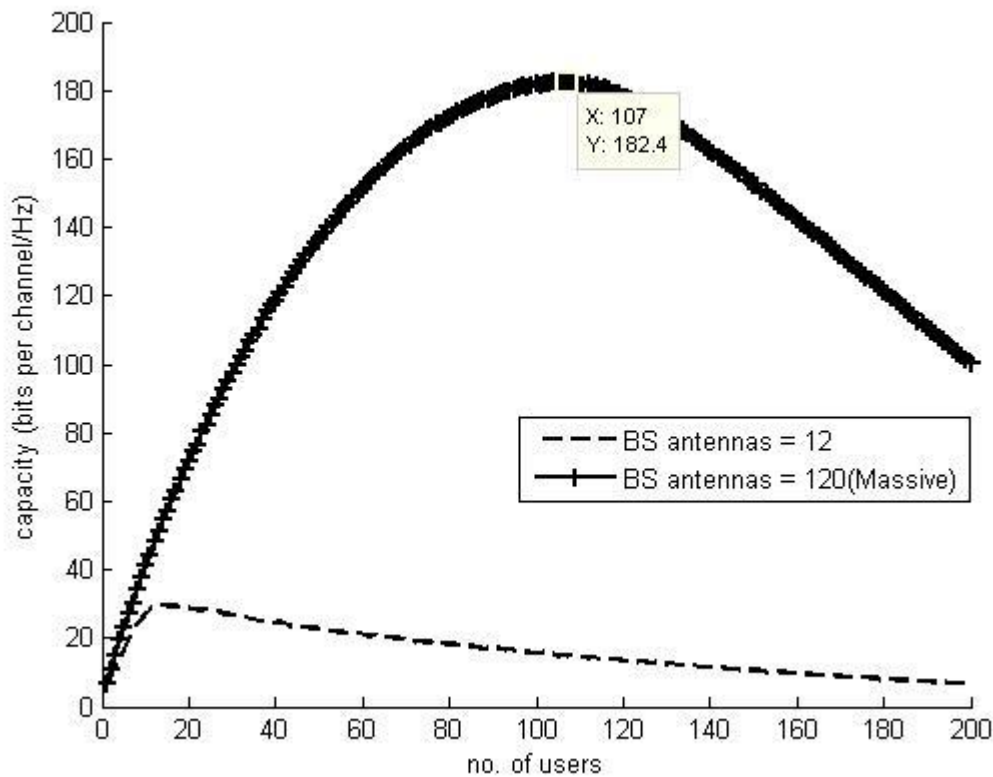
But in the massive MIMO, the no of antenna at the base station are very large. So we consider a case for  $N=120$ , and no of users varies from 1 to 200, for the same SNR as in the above case for  $T=300$ . Capacity is maximum at  $M \approx 107$  and the value is  $\approx 182$  bits/sec/Hz. So we can see that as the no of antennas at the Base station are increased the capacity increased nearly 6 times and also the optimum no. of users increased. So, fig.14 shows the comparison of MIMO and massive MIMO system.

## 5.6 Impact of Hardware Impairments on Optimum Number of Users

As we studied, mean square error in equation (11) shows the impact of hardware impairment on the accuracy of channel estimation and can be seen from the fig.11.

**Table 6. Optimum No. of Users with different no. of Antennas at BS**

BS Antennas	Optimum No. of Users
12	15
120	107

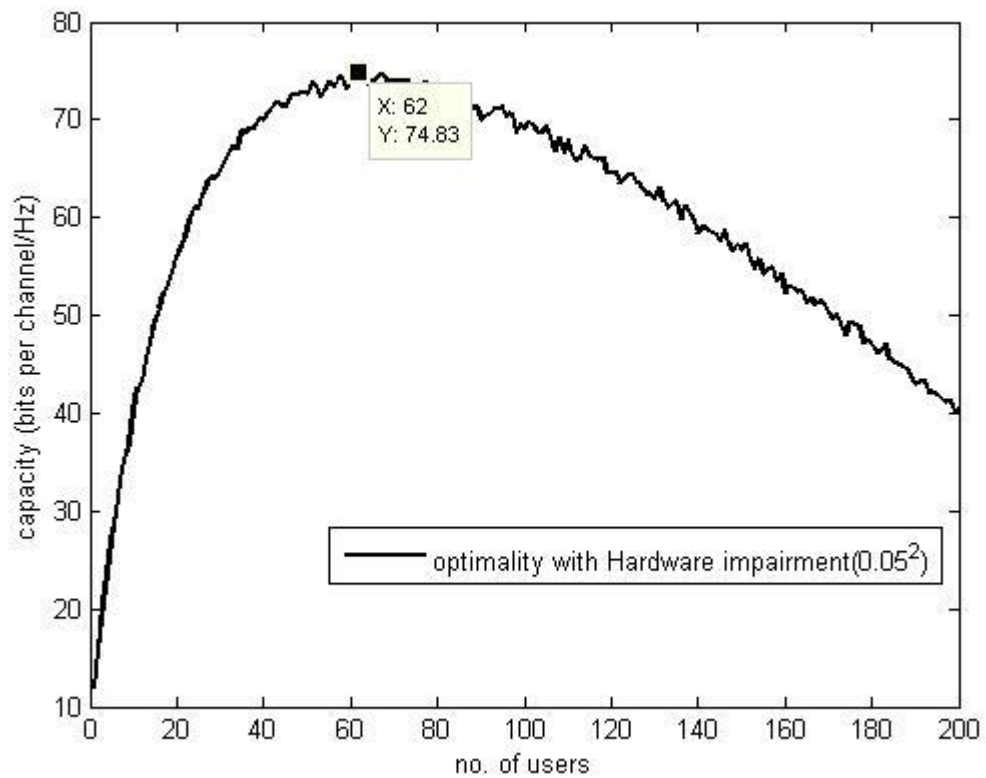


**Figure 5.9: Capacity of Massive MIMO systems with large no of antennas at the BS**

From the fig.11 we can see that as the hardware impairment comes into picture then the accuracy of LMMSE estimator decreases.

The next result shows the impact of hardware impairment on the capacity and optimum no of users in the massive MIMO system in fig.15. In this case we consider the estimated channel from equation (3.9) for calculating the capacity. Here the

capacity reduces to  $\approx 75$  bits/sec/Hz with 62 optimum no. of users. Thus the capacity reduces to nearly half of the ideal or conventional system.



**Figure 5.10: Impact of Transceiver impairments on Optimum no. of Users**

#### 6.1 Conclusion

The next generation wireless networks need to accommodate 1000 times more data traffic than contemporary networks. Since the spectrum is scarce in the bands suitable for coverage, the main improvements need to come from spatial reuse of spectrum; many concurrent transmissions per area unit. This is made possible by the massive MIMO technology, where the access points are equipped with hundreds of antennas. Over the last couple of years, massive MIMO system has gone from being a theoretical concept to becoming one of the most promising ingredients of the emerging 5G technology. This is because it provides a way to improve the area spectral efficiency (bit/s/Hz/area) under realistic conditions, by upgrading existing base stations.

The research on the massive MIMO wireless communication increasing rapidly, but the effect of impairments due to non-ideal hardware at transmitter and receiver has received slight attention up to now, but very less research has done on this problem. So, this is a new area for the research point of view.

In thesis, firstly we studied the case of ideal hardware, then we examines the impact of impairments that is caused by all the different types of hardware at the transmitter and receiver side of the massive MIMO system, in contrast to the perfect hardware. Here, we are assuming that the appropriate analog and digital signal processing algorithms have been applied and we will find out the impact of residual hardware impairments of the system. So, Inspired from the various analysis a new system model is considered that have the aggregate effect of all the hardware imperfections not the effect of individual components. We found that these impairments puts finite upper limit on the accuracy of the channel estimator and also on the capacity of downlink/uplink channel for each UE.

All these analysis are based on a system model that have the aggregate effect of all the hardware imperfections not the effect of individual components. This model is mathematically manageable, as this is proved experimentally in this thesis.

We proved that impairments due to non-ideal hardware at the both side i.e. base station antennas and user equipment, create non-zero estimation error floors on the

accuracy of the channel estimator. This stands in contrast to the very optimistic asymptotic results previously reported for ideal hardware. Under the transceiver impairments, we noticed that non-ideal or practical hardware at the transmitter and receiver side degrades the accuracy of the channel estimator as in fig.11.

We also show that how the capacity decreases after the certain no. of users because of more training time requirement as the no of user increases. Due to too little training, the channel is inappropriately learned and due to too much training, there is no time left for data transmission before the channel variations according to block fading. So, there is a trade-off between the channel estimation accuracy and capacity of the massive MIMO systems. When the training and data powers are equal then the optimum training time is equal to the no. of users at which capacity of the system is maximum i.e.  $T_{\tau} = M$ . Then we examine that how the channel estimation under transceiver impairments affects the capacity of a channel. Then we see that capacity and optimum no. of users are affected by transceiver impairments with considerable amount from fig.5.7 and fig. 5.10. So these results help us during implementation of massive MIMO system which is a key technology for the future 5G wireless networks.

## **6.2 Future Scope**

The basic idea behind massive MIMO is to obtain all the benefits of conservative MIMO, but on a much greater scale. Generally, massive MIMO is an enabler for the development of future broadband (fixed and mobile) networks which will be energy-efficient, secure, and robust, and will use the spectrum efficiently. The necessities for high rate of wireless communication networks raise exponentially with the uses of smart terminals.

Massive MIMO system is seen as key technology enablers for future 5G wireless mobile networks. This technology can be incorporated with millimeter wave (MMW) communication. The deficiency of un-fragmented available spectrum resources below 6 GHz and the significant progress of MMW radio technologies over the last few years have started a strong interest for the exploitation of MMW bands in future wireless cellular networks for both backhauling and access.

With massive MIMO systems a new model has come up, promising the solution of many of the challenges by over provisioning of antenna elements, such as inter-cell interference as one of the main limitations of cellular networks can be reduced to any

wanted level by applying sufficiently large no. of antennas. For more realistic number of antennas inter-cell interference will probably play a significant role also in the future, especially in case an increasing number of small cells has to be integrated into the overall network. Suitable combinations of massive MIMO with well-known interference mitigation techniques like coordinated scheduling, coordinated beamforming, joint transmission precoding, or network MIMO might provide a good trade-off between overall effort, complexity, and performance.

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## List of Publications

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- S. Redhu and S. Sharma, “Impact of Transceiver Impairments on Optimum Number of Users for Massive MIMO Systems,” Communicated for publication in *Springer Wireless Personal Communication*, May 2015.