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MULTIAXIAL BEHAVIOUR OF CONCRETE

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Certified that the Seminar entitled," Multiaxial Behaviour of Concrete" which is being submitted by Mr.Kanwal Jit Singh in partial fulbfilment for the award of the Degree of M.Sc.Engineering in Civil Engineering (Structural Engineering) is a record of student's own work carried out by him under my supervision and guidance.

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S Y N O P S I S

In this Seminar entitled," Multiaxial Behaviour of Concrete", the behaviour of concrete under uniaxial, Biaxial and Triaxial system of stress has been discussed. Concrete has to encounter biaxial as well as triaxial state of stress under different forms of loading, Several authors have carried out lot of experimental research on this subject with a view to evolve some definite line of action to tackle this unpredictable behaviour of concrete. It has been tried in the present study to bring forth different studies carried out by different authors on the subject with a special emphasis on the strength of concrete under Biaxial State of stress. Triaxial state of stress has also been discussed besides creep prediction for concrete under Multiaxial state of stress.

NOTATIONS

STRESSES:

σ	Principal Stress
σ_m	Mean normal stress at discontinuity expressed in Kgf/cm^2
K, α	Ratio of principal stresses
σ_p	Ultimate strength of concrete plate under biaxial compression
σ_0	Ultimate strength of concrete plate under uniaxial compression.

STRAINS:

ϵ_{ld}	Principal tensile strain at discontinuity expressed in units of microstrain
ϵ	Principal Strain
ν	Poisson's ratio
ϵ_p	Strain at ultimate stress
$C_{d\infty}$	Limited delayed elastic strain per unit stress
ϵ_d	Delayed elastic strain at start of increment
C_f	Irrecoverable strain per unit stress
$\delta\epsilon_c, \delta\epsilon_d, \delta\epsilon_f$	Increments of ϵ_c, ϵ_d, ϵ_f etc. during increments of time δt.
ϵ_c	Total creep strain
ϵ_d	Recoverable creep or delayed elastic strain
ϵ_e	Elastic strain
ϵ_f	Irrecoverable creep or flow
ν_s	Poisson's ratio found in static lest

OTHER:

Es	Secant modulus at ultimate load
E	Modulus of elasticity of concrete
Q	Parameter controlling rate of occurence
Ed	Dynamic modulus of elasticity
δt	Increment in time.

CHAPTER 1

INTRODUCTION:

The behaviour of concrete under multiaxial state of stress is of vital concern to the structural concrete designer. Concrete has to resist biaxial as well as triaxial state of stress under different forms of loading and the form in which the concrete is being used. End blocks of pre-stressed concrete beams, girders and piers of curved and skew bridges are some of many applications where the concrete is subjected to complex stress system. Thin slabs and thin walled shell structures are subjected to two-axial stress system. Thick walled slabs and thick walled spatial shells, as a rule, are subjected to three-dimensional stress. Concrete is a heterogeneous material and its behaviour under multiaxial state of stress has recently been explored to some extent, lot of research in this field is still required to be done. Despite considerable research endeavour, a clear picture of the strength of concrete under multiaxial loading has not yet emerged.

In all countries of the world the maximum permissible stresses are based on knowledge of the uniaxial compressive and tensile strength and on shear strength. In the case of multiaxial loading of a structural element, the maximum permissible stresses are increased to values arrived at on the basis of purely theoretical considerations.

Actually, however the strength of concrete is dependent upon the type of stress field and inter relationship of the stresses set up. It may be above or below the uniaxial strength. In the case of the former, the strength of concrete would not be fully utilised which would be economical, whereas in the case of later, the strength might be too low to provide the necessary safety against failure. Prestressed concrete vessels are invariably subject to multiaxial stresses hence all countries engaged in this new technology have been making every effort to develope analytial methods that take into account multiaxial stress in order to meet the exacting safety requirement imposed on such structures. However, knowledge of multiaxial failure and strain behaviour of concrete has not kept pace with this rapid development. Only after this gap has been filled, it will be possible to arrive at optimum solution in respect of safety and economy. This does not only apply to the construction of prestressed concrete pressure vessels but also to the construction of impounding dams and structural member of bridges.

In this report efforts have been made to give all the available details about the strength, elasticity and creep of concrete under multiaxial loading.

CHAPTER 2

PRESENT STATE OF ART

Due to the common occurence of complex stress systems in concrete structures, the understanding of the strength of concrete under multiaxial stresses is of vital interest to the structural designer. Notwithstanding considerable research over the past two decades the position still remains confused and generally the structural designer does not have clear and simple guidelines to decide whether a particular combination of multiaxial stresses is acceptable and so he is apt to be overly conservative.

The experimental investigations into the complex problem have tended to yield widely varying results. In addition to significant differences in the properties of concrete of the test specimens, the method of test, the sequence of loading, the size and shape of the specimen, and the confining effects of the loading platens are largely responsible for the apparent disparity.

The considerable research carried out in the past two decades has not yielded any conclusive evidence. The experimental data are particularly inadequate in triaxial systems, especially when atleast one of the three principal stresses is tensile. The lack of adequate experimental data combined with the large disparity in observed results has made it extremely difficult to specify strength criteria for all possible stress combinations. In the circumstances, it appears rational and useful to define reasonable lower bonds of concrete strength.

Although the research on the strength of concrete under multiaxial loading started as early as 1928, it is during the last twenty four years, that the problem has received notable attention. The results of these researchers have shown widely different trends. For instance, the observed strength in the case of two equal compressive stresses along the two principal directions reported in literature varies from 80 to 350% of the uniaxial compressive strength of an identical test specimen. A significant part of this apparent disagreement in observed strengths can be ascribed to the confining effect of the bearing platens of the testing machine. The friction between the specimen and the bearing platens not only causes confinement of the concrete but in the case of multiaxial load tests, part of the applied load may be transfered directly to the platens which enclose the test specimen. As a result, most of the earlier experimental investigations on the strength of concrete under multiaxial compression wereprone to over-estimate the strength. An ingenious device to eliminate the confining effect of the platens effectively was developed by Kupfer et.al(3) in the form of brush bearing platens. The disparity in the observed strength may be expected to be considerably narrowed down if the confining effect of loading platens were effectively eliminated.

There are many failure theories which have been put forward in the past. But there seems to be no single criterion for failure adequate to describe failure under all possible load combinations. The limiting energy of distortion or the octahedral shear has often been used as one of the main failure criteria for concrete. Unfortunately, octahedral stresses cannot define the strength of concrete unless different empericals constants are used in different zones of stress combinations. The emperical constants restrict the general validity of the strength relationship.

CHAPTER 3

3.0 STRESS SYSTEMS

The possible stress combinations can be classified in terms of the principal stresses σ_1, σ_2 and σ_3 into uniaxial, biaxial and triaxial stress systems.

3.1 UNIAXIAL STRESS: ($\sigma_2 = \sigma_3 = 0$)

The non-zero principal stress σ_1 may be compressive or tensile. Now, the standard compression test on cubes or cylinders is generally accepted as a measure of the uniaxial compressive strength of concrete (σ_c'). Although none of the several tension tests (direct, indirect, modulus of rupture, cube split, prism split, punch and ring tests) enjoys the same status as the standard compression test, the indirect tension (split) test on cylinders which is also known as the Brazilian test is probably taken as the best measure of the uniaxial tensile strength of concrete,(σ_t',)

3.2 BIAXIAL STRESS ($\sigma_3 = 0$)

Most of the experimental search so far has been devoted to the study of biaxial stress due to its common occurence. There are three possible stress combinations in biaxial stress depending upon whether both the principal stresses are compressive, one compressive and the other tensile, or both tensile.

3.2.1 BIAXIAL COMPRESSION ZONE:

Extensive research has been carried out to study the stress condition in which both the principal stresses are compressive. The earlier tests were generally inclined to overestimate the strength in biaxial compression. Lateral confinement by the solid loading platens appears to have contributed significantly to this overestimate on the basis of the existing tests in biaxial compression, the author has recommended the following inter-action relationship as a resonable lower bound.

$$\frac{\sigma_1}{\sigma_c'} = 1 + \frac{1}{6}\frac{\sigma_2}{\sigma_c'}, \quad \text{if } \frac{\sigma_2}{\sigma_c'} \leq 1.2 \qquad \ldots(1)$$

$$\frac{\sigma_2}{\sigma_c'} = 1 + \frac{1}{6}\frac{\sigma_1}{\sigma_c'}, \quad \text{if } \frac{\sigma_1}{\sigma_c'} \leq 1.2 \qquad \ldots(2)$$

It is interesting to note that equation (1) can be readily derived from the limiting strain criterion by equating the strain in the direction of σ_1 to the strain due to σ_c^1 acting by itself and assuming Poisson's ratio to 1/6(which is a reasonable average value). Also we can write

$$\frac{\sigma_1}{E} - \frac{1}{6}\frac{\sigma_2}{E} = \frac{\sigma_c'}{E} \qquad \ldots(3)$$

3.2.2 COMPRESSION - TENSION ZONE:

The stress condition in which one of the principal stresses is compressive and the other tensile is very common. It occurs in reinforced and pre-stressed concrete wherever significant flexural shear or Torsion exists. Several investigators have carried out experiments to study this stress condition in concrete. Although considerable scatter exists, the trend of the test results indicates that the strength in the compression-tension zone is less than the respective uniaxial strengths. Tests by Kupfer et. al(3) and McHenry and Karni (8) show an S-shaped inter-action curve with very little curvature except near the point representing simple compression. Considering all test results, a simple lower bound for concrete strength in the compression-tension zone may be represented by the straight line joining the points representing the strengths in simple compression and simple tension. In the non-dimensional form, the interaction relationship is expressed as follows:

$$\frac{\sigma_1}{\sigma_c'} + \frac{\sigma_2}{\sigma_t'} = 1 \qquad \ldots.(4)$$

In equation (4), σ_1 and σ_2 are the numerical values of the compressive and tensile principal stresses respectively. It is interesting to note that equation (4) can be readily derived from the limiting strain criterion by equating the strain in the direction of principal tensile stress σ_2 to the strain due to σ_t' acting alone and assuming $\frac{\mu\sigma_c'}{\sigma_t'} = 1$ where μ is the effective Poisson's ratio in the compression-tension zone.

3.2.3 BIAXIAL TENSION ZONE:

The test data are very inadequate in this zone due to the difficulty in carrying out the biaxial tension test properly and probably because this stress condition occurs infrequently. There seems to be two compensating factors affecting the strength in biaxial tension. If the limiting strain criterion governed failure, the smaller principal stress would tend to increase the large principal stress at failure due to the Poisson's ratio effect. On the other hand, the critical strain in biaxial tension appear to be lower than in simple tension tending to reduce the strength in biaxial tension. As the two factors have opposite influences on the strength, there is very little, if any, interaction in the range of biaxial tension. Hence interaction relationship may be expressed as

$$\sigma_1 = \sigma_t' \quad \text{if} \quad \sigma_2 \leq \sigma_t' \qquad \ldots(5)$$

$$\sigma_2 = \sigma_t' \quad \text{if} \quad \sigma_1 \leq \sigma_t' \qquad \ldots(6)$$

3.3 TRIAXIAL STRESS:

Four possible stress combinations arise in triaxial stress systems as all the three, two, one or none of the three principal stresses may be compressive. Very meagre experimental research seems to have been carried out on the triaxial strength of concrete, and most of it has been confined to triaxial compression. The analytical work has attempted to express triaxial strength of concrete in terms of the octahedral stresses and the stress invariants. The influence of the intermediate principal stress on triaxial strength has received some attention. Using a well known Hencky-Huber-Von Mises criterion of minimum energy of distortion, it can be readily shown that the energy of distortion is minimum when the intermediate principal stress is equal to half the sum of the major and minor principal stresses. Extending the limiting strain criterion, represented by equations (1) and (2) for the case of biaxial compression, to the case of triaxial compression:

$$\frac{\sigma_1}{\sigma_c'} = 1 + \frac{1}{6}\frac{(\sigma_2 + \sigma_3)}{\sigma_c'}, \sigma_1 \geq \sigma_2 \geq \sigma_3 \quad \ldots(7)$$

Where σ_1 is the major principal stress (the largest of the three compressive principal stresses). Equation (7) is of the same form as equation (1) except that the smaller principal stress σ_2 has been replaced by the sum of the intermediate and minor principal stresses ($\sigma_2 + \sigma_3$).

CHAPTER 4

4.0 STRENGTH OF CONCRETE UNDER BIAXIAL STRESSES

Studies of the behaviour of concrete under multiaxial stress states are essential to develope a universal failure criterion for concrete. Biaxial stresses act in the shear region of flexural members as well as in shells, plates and various containment structures. Most of the studies has been limited to tests in the range of biaxial compression and no data on the bahaviour of concrete under biaxial tension are available. One of the major problems in conducting tests on concrete subjected to biaxial stresses is the development of a well defined and uniform biaxial stress state in the specimen.

Among all reported investigations about the behaviour of concrete under biaxial stresses uptill now, the work done by Kupfer, Hilsdorf, and Rusch (3) is most reliable one. They covered the entire range of stress combinations from biaxial compression to biaxial tension.

Concrete cubes or plates were used for studies of the biaxial compressive strength of concrete by Foppl, Wastlund, Glonib, Weigler and Becker(3), Iyengar(1) Vile and Robinson(3) Foppl (3) showed that a prismatic specimen subjected to uniaxial or biaxial compressive loads may be confined along its loaded surfaces due to friction between the bearing platens of the testing machines and concrete.

It is well known that such restraint may result in an increase of the apparent strength of the test piece. Foppl (3) therefore tried to eliminate confinement by applying lubricant to the loaded surfaces of the specimen. He showed, however that such treatment may lead to the opposite effect, soft packings or lubricating agents between specimen and bearing platens cause lateral tensile stresses and a non-uniform stress distribution in the specimen resulting in a reduction of its apparent strength. Later investigators have continued to use test set up with conventional bearing platens and in some instances employed various surface treatments of the concrete or soft packings between the bearing platens and the specimen to eliminate restraint. Hilsdorf(3) has shown that friction between test specimen and bearing platens not only causes confinement of the concrete, but that part of the applied load may be sustained by the bearing platens which enclose the test specimen. If the load sustained by the bearing platens is not taken into account in determining the concrete stress, the strength of the test specimen will be overestimated.

A more detailed review of previous investigations has been presented in Hilsdorf (3) in his paper entitled experimental determination of the biaxial strength of concrete'. He has concluded that square concrete plates subjected to in-plane loading appear to be suitable specimens to determine the biaxial strength of concrete over the entire range of biaxial stress combinations. It is proposed to load such specimens without restraint by replacing the solid bearing platens of conventional testing machine with "Brush Bearing Platens". These platens consist of a series of closely spaced small steel bars which are flexible enogh to follow the concrete deformations without generating appreciable restraint on the test piece. Nevertheless their buckling stability is sufficient to transmit the required compressive forces into the concrete test piece. For tensile tests, the filament can be glued to the concrete. Various calibration tests showed the effectiveness of brush bearing platens in eliminating restraint. No adverse effects could be found such as local stress concentrations in the concrete near the tips of small steel bars.

4.1 <u>BRUSH BEARING PLATENS</u>:

The platens consist of individual steel filaments with a cross-section of 3x5 mm. The length of the filaments from 100 to 140 mm, depending on the maximum concrete stress for which the particular brush bearing platen can be used without buckling of the filaments. The higher the strength of the concrete to be tested, the shorter the individual filaments. The use of shorter brush bearing platen for higher strength concrete does not significantly increase the restraint of the test piece since the concrete strains at a given stress decrease as the strength of concrete increases. The individual filaments are spaced approximately 0.2mm apart and are soldered together over a length of 35 mm so that a solid block is formed. The lateral flexibility of the filaments is such that for biaxial compression or biaxial tension, the average principal stresses in the specimen do not deviate by more than 0.5 percent from the values calculated under the assumption of no restraint. For tests in the range of compression-tension, this error may be upto 3 percent. The flatness of the surface of the brush bearing platens was maintained within $2x10^{-3}$mm. For the tensile tests, the brush bearing platens were glued to the concrete specimens using epoxy resins. Penetration of the glue between the brush filaments was avoided by sealing these spaces with a rubber cement. This treatment had no measurable effect on the flexibility of the filaments.

To verify effectiveness and reliability of the brush bearing platens, concrete prisms with various height to side length ratio including cubes as well as concrete plates 20x20x5 cms were loaded in uniaxial compression with and without brush bearing platens. If brush bearing platens wereused, the strength of the specimens was independent of their shape and equal to strength of presmatic specimens with a height to side length ratio of 4.0. This appears to provide sufficient proof that end restraint of concrete specimens can be eliminated by brush bearing platens.

4.2 LOADING FRAME:

In this case, individual frames were designed for the two principal stress directions. One frame is stationary while the other frame can move freely. The later frame is suspended from the stationary frame by means of long, hinged steel rods and four vertical springs so that it can follow small movements of the specimen in any direction without generating appreciable secondary stresses in the specimen. This also facilitates alignment of the test set up prior to loading. Both frames consist of precast, prestressed concrete elements with a compressive strength of 600 Kg/cm^2(Fig.2)

The test specimen is placed in the centre of the two crossing loading frames. It rests on an adjustable plateform which is lowered after a small preload has been applied to the specimen. Double-acting hydraulic loading jacks fitted into the loading frame can generate maximum loads of 75,000 Kg in compression and 40,000 Kg in tension. Solid bearing platens with special seats to which the brush bearing platen can be mounted are attached to the loading frames.

4.2.1 CONTROL OF THE RATION σ_1/σ_2:-

The ratio of the applied stresses σ_1/σ_2 can be maintained constant throughout a test by a load distribution frame. Hydraulic jack No.1 which is connected to a pump applies a load to a beam which is supported by two additional hydraulic jack Nos .2 & 3 with the hydraulic jacks in the main testing machine. The position of the hydraulic jack No.1 is adjustable along the beam and controls the ratio of applied stresses σ_1/σ_2 (Fig.2)

4.3 EXPERIMENTAL STUDY:

Experimental study was carried out by M.Ebrahim Tasuji, Floyd O Slate and Arthur H.Nilson (7) for concrete plates with 12.7x12.7x1.3cm size and subjected to biaxial stress, compression-compression, compression-tension and tension-tension, were included. Tension was tekan as positive and $\sigma_1 > \sigma_2 > \sigma_3$ .For compression-Compression ,stress ratios (σ_2 / σ_3 = 0) (uniaxial compression), 0.2, 0.5 and 1.0, for compression -Tension, stress ratios were (σ_1 / σ_3) -0.05, -0.10 & -0.25, and for tension-tension, stress ratios were (σ_1 / σ_2) = 1.0, 2.0, and ∞ (uniaxial tension) The specimens were loaded so that the largest stress increased at a rate of (27.4 Kgf/cm^2/min) for the uniaxial and biaxial compression tests and (9.8 Kgf/cm^2/min) for the tests involving direct tension.

The specimens were tested at the ages of 13,14 or 15 days in a biaxial loading frame. Loads were applied using comb-like platens, rather than solid plates, in order to avoid confinement of the specimen due to friction. For the tensile loads, the concrete specimens were glued to the comb-like platens using epoxy adhesives. The ratio of the principal stresses was maintained constant as loads increased. Loads and deformations were recorded continuously Deformation in the plane of the specimens were measured with

10.2cm electrical stream gages, and the deformation normal to the plane of the specimen were measured with a Fotonic sensor.

4.3.1 OBSERVATIONS AND RESULTS:

The nondimensionalized ultimate strength data are reported in terms of a biaxial stress envelope. Stresses are expressed in terms of uniaxial compressive strength σ_{co} The average uniaxial compressive strength (σ_{co}) was -339.3 Kgf/cm^2 and the average uniaxial tensile strength (σ_{to}) was 29.4 Kgf/cm^2.(Fig.3) shows the biaxial compression-tension and biaxial tension portions of the envelope to a larger scale.

The test results show that the ultimate strength of concrete in biaxial compression is greater than in Uniaxial compression and is dependent on the principal stress ratio. A maximum strength increase of approximately 22 percent was achieved at a stress of $\sigma_2/\sigma_3 = 0.5$. Under biaxial compression-tension, the compressive strength decreased almost linearly as the applied tensile stress was increased. Under biaxial tension, the strength increased compared to uniaxial tensile strength. A maximum increase in the order of 10 to 20 percent was recorded at a stress ratio of $\sigma_1/\sigma_2 = 2.0$. This strength increase resulted in a slight convexity in the ultimate strength envelope in the region of biaxial tension (Fig.4).

4.3.2 DEFORMATIONS:

Fig.(5) show the typical relationship of "normalized" stress to actual strain for the various states of biaxial loading. The principal strains are such that $\epsilon_1 > \epsilon_2 > \epsilon_3$ with tensile strains being positive. In uniaxial and biaxial compression, the average value of the maximum principal compressive strain at the ultimate stress was about -2500 microstrain and the average value of the maximum tensile strain was about +1000 microstrain. In biaxial compression-tension, the magnitude at failure of both the principal compressive strain and the principal tensile strain at the ultimate stress was about +150 microstrain.

The test results indicate that the introduction of a second principal stress significantly affects the effective elastic modulus of the concrete specimen in the direction of the first principal stress.(Fig.5-7) shows the variations in strain ϵ at a given level of stress (σ). The change in the elastic modulic is not due solely to the Poisson's effect, it is also related to microcrack confinement.

The material constants obtained in the experimental study were somewhat different in uniaxial compression and uniaxial tension. In uniaxial compression, the average elastic modulus was 2.00×10^5 Kgf/cm^2 and average poisson's ratio was 0.22, while in uniaxial tension the corresponding values were 2.13×10^5 Kg/cm^2 and 0.16 respectively.

4.3.3 FAILURE MODES:

For all biaxial and uniaxial tests, failure occured by tensile splitting, with the fractured surface (s) orthognal to the direction of the maximum tensile strain. Specifically, under biaxial compression, failure took place by cracking along a plane parallel to the unloaded surface of the specimen; under uniaxial compression, fracture occured by the formation of cracks parallel to the applied load and perpendicular to the larger unloaded surface of the specimen. For tests under combined compression and tension, one continous crack normal to the principal tensile stress was formed except for smaller ratios of tension to compression ($\sigma_1 / \sigma_3 = -0.05$) for which several cracks were observed at fracture. Under biaxial and uniaxial tension failure was by the formation of a single crack perpendicular to the direction of the maximum tensile stress; for equal biaxial tension, there was no preferred direction for the fracture surface except that cracks were always normal to the unstressed surface of the specimen.

4.3.4 DISCONTINUITY:

The behaviour of the concrete specimen was considered at the discontinuity level as well as at the ultimate strength. The discontinuity level represents the onset of major microcracking of concrete as defined by Newman (9) and is apparently related to the begining of extensive mortar cracking.

For the uniaxial and biaxial compression tests, discontinuity was defined as the point at which the ratio of the principal tensile strain to the principal compressive strain began to increase. For uniaxial compression, this was equivalent to the point at which Pisson's ratio started to increase. For biaxial tests involving direct tension- discontinuity was defined as the point at which the curve of stress versus the maximum tensile strain began to deviate from linearity. Discontinuity occured at about 75 percent of the ultimate load in the uniaxial and biaxial compression tests; and at about 60 percent of the ultimate load in tests involving direct tension. Fig.(8) shows the discontinuity envelope together with the ultimate strength envelope. The discontinuity stress envelope, like the ultimate strength envelope, is convex outward in the region of biaxial compression and is nearly linear in the region of biaxial compression-tension. In the region of biaxial tension, the discontinuty stress envelope in contrast to the ultimate strength envelope is practically square. The average uniaxial compressive stress at discontinuity was -250.1 Kg/cm^2.

4.3.5 DIFFERENT OPINIONS:

There is general aggrement that the strength of concrete in biaxial compression may be considerably higher than in uniaxial compression and that in the biaxial compression-tension portion, the ultimate strength envelope is almost a straight line.

The difference between the results of Iiae and Tasuji(10) (done in the same laboratory on the same equipment and using similar materials), as shown in Fig.(9) seem to indicate that the increase in biaxial compressive strength for concrete is higher for concrete with a lower uniaxial compressive strength and vice versa.

Only a few studies have been made regarding the behaviour of concrete in biaxial tension. The results of the present study indicate that there is a definite increase in the biaxial tensile strength of concrete as compared with its uniaxial tensile strength. Such an increase is expected according to the maximum strain theory. Kupfer et al (3) and Nelissen observed no such strength increase in biaxial tension. They indicated that the biaxial tensile strength of concrete is equal to its uniaxial tensile strength. Some investigators have reported a decrease in the biaxial tensile strength as compared with the uniaxial tensile strength.

For uniaxial and biaxial compression tests, all curves of volumetric strain versus major principal compressive stress contained an inflection point which was at about 80 percent of the ultimate load. It appears that the inflection point is close to the stage at which major micro-cracking of concrete (discontinuity) is initiated.

The observed modes of failure indicate that tensile deformation are of crucial importance in the failure mechanism of concrete, reinforcing the work of previous investigators. Fig.(10) shows the relationship between the principal tensile strain and mean normal stress at discontinuity for the concrete plate. The best fit linear relationships are:

$$\epsilon_{1d} = -112 - 4.98\,\sigma_m\,(kgf/cm^2) \quad \ldots(7)$$

$$\epsilon_{1d} = 110 - 2.38\,\sigma_m\,(kgf/cm^2) \quad \ldots(8)$$

Where

ϵ_{1d} = principal tensile strain at discontinuity expressed in units of microstrain.

σ_m = $(\sigma_1 + \sigma_2 + \sigma_3)/3$ = mean normal stress at discontinuity expressed in Kgf/cm^2.

Eq.(1) is for the uniaxial and biaxial compression tests and Eq.(2) is for the tests involving direct tension. The former, having a slope considerably steeper than the latter indicates that a larger amount of indirect tensile strain (due to Poisson's effect) can be sustained by concrete than direct tensile strains (due to direct tensile stresses). Thus, it is evident that the magnitude of the failure strains (at discontinuity and consequently at ultimate fracture) is not constant but increases with the degree of compression.

4.2.8 DESIGN RECOMMENDATIONS:

On the basis of this experimental study, a biaxial stress-strain relationship for plain concrete can be expressed in the following form:

$$\sigma = \frac{\epsilon E}{(1-\nu K)\left[1 + \left(\frac{1}{1-\nu K}\frac{E}{E_s} - 2\right)\left(\frac{\epsilon}{\epsilon_0}\right) + \left(\frac{\epsilon}{\epsilon_0}\right)^2\right]} \quad \ldots(9)$$

Eq.(9) takes into account the nonlinear behaviour of concrete due to microcracking and is based on the observed shape of biaxial stress-strain curves. Eq.(9) was proposed by Lin et al for concrete in biaxial compression. Its application is extended herein for concrete in all states of biaxial loading.

Based on the experimental results, the uniaxial tensile strength of concrete may be estimated from its uniaxial compressive strength by

$$\sigma_{to} = 1.59\sqrt{\sigma_{co}} \quad (Kgf/cm^2) \qquad \ldots(10)$$

σ_{to} & σ_{co} are both expressed in units of Kgf/cm^2

4.3.8 CONCLUSIONS:

The following conclusions are based on the experimental work in this study.

1. The ultimate strength of concrete under biaxial compression is greater than that under uniaxial compression. A maximum strength increase of approximately 22 percent is achieved at a stress ratio of $\sigma_2/\sigma_3 = 0.5$. Under biaxial compression-tension, the compressive strength decreases almost linearly as the applied tensile stress is increased. Under biaxial tension, the strength of concrete increases as compared to its uniaxial tensile strength. A maximum strength increase in the order of 10 to 20 percent is achieved at a stress ratio of $\sigma_1/\sigma_2 = 2.0$

2. The stress-strain curves of concrete possess the same general shape in both compression and tension.

3. In uniaxial and biaxial compression, the compressive strain at maximum load is about -2500 microstrain. In uniaxial and biaxial tension, the tensile strain at maximum load is about +150 microstrain.

4. The elastic modulus in uniaxial tension is slightly greater than that in uniaxial compression while the Poisson's ratio in tension is somewhat smaller than that in compression.

5. The elastic modulii in biaxial compression(tension) are significantly greater than those in uniaxial compression (tension).

6. For all biaxial and uniaxial tests, failure occurs by tensile splitting, with the fractured surface orthognal to the direction of the maximum tensile strain.

7. The onset of major microscracking of concrete, referred to as discontinuity, occurs at about 75% of the ultimate load in uniaxial and biaxial compression while it takes place at about 60% of the ultimate load in stress states involving direct tension.

8. The maginitude of the tensile strain at failure is not constant, but increases with the degree of compression indicating that concrete can sustain significantly higher indirect tensile strains than direct tensile strains.

9. A biaxial ultimate strength criterion for concrete, in the form of a simple stress envelope, is recommended for design purposes.

CHAPTER 5

5.0 STRESS STRAIN RESPONSE AND FAILURE OF CONCRETE IN UNIAXIAL AND BIAXIAL COMPRESSION:

Although concrete strength under conditions of combined stress has been under investigation for years, the load deformation behaviour of multiaxially loaded concrete prior to failure is less clearly defined. Yet this information is fundamental and represents the basic input for refined analysis by the finite element method of reinforced concrete members and structures. Further progress in developing such methods for application to concrete is closely linked to the development of quantitative information on the load-deformation behaviour of concrete in the state of combined stress.

Some work has been carried out at Cornell University to provide more complete understanding of internal conditions such as w stress, displacement and microcracking of concrete under biaxial compression to develope practically useful relationship between stress and strain, and to establish failure criteria for concrete under combined stress. As part of these investigations, both idealised models of concrete, and real concrete specimens were studied experimentally, under uniaxial and biaxial short-term compressive loading through the full range from zero to failure load.

5.1 EXPERIMENTAL STUDY:

Thin square plates were subjected to uniaxial and biaxial compression using idealised models of concrete. Principal variables were ratio of principal stresses, water cement ratio, aggregate-cement ratio and maximum size of aggregate. Uniaxial compressive strength of real concrete was from 210 to 350 Kg/cm^2.

The model was composed of a mortar matrix in which were embedded randomly 30 circular aggregate discs of three different sizes. Both model and real concrete specimens were thin in order to obtain as nearly as possible a state of plane stress and to obtain X-ray information on cracking during loading without the necessity of disturbing the specimen. Special efforts was made to minimise the frictional confinement introduced between the bearing platen and the concrete. The brush bearing platens designed by Buyukoztink et al were used in the investigation. The principal stress ratios were kept constant throughout the test for each combination of biaxial compression.

5.2 TEST RESULTS:

The formation and propagation of microcracks have been recognised as major factors in the failure of concrete. The following X-ray information on the formation and propagation of microcracks was obtained from the uniaxially loaded concrete models.

1. Shrinkage cracks and cracks caused by bleeding around the aggregates (bond cracks) clearly do exist. In nearly every case these initial cracks spread to cause major cracks at failure and thereby exprementally support the theoretical deduction by Hsu and slate and Matheus that shrinkage cracks, are instrumental in the failure of concrete.

2. Bond cracks occur first around the larger aggregates. This is consistent with the observation of Hsu and of Alexander and Wardlaw. At loads lower than 65% of ultimate, no appreciable change occured at the interfaces and with increasing loads the bond cracks increased in width and length along the interface. Fig.(11).

3. At about 85% of the ultimate load, mortar cracks were initiated as shown in Fig.(11). The mortar cracks tend to bridge bond cracks on larger aggregates in preference to those on small aggregates.

4. At ultimate load, the cracks form parallel to the applied load. Such cracks eventually caused the splitting modes of failure observed in the tests. Fig.(11).

No interfacial or mortar cracks were observed on biaxially loaded specimens.

5. The test results from both model and real concrete specimens show that considerably higher strength is obtained in biaxial compression than in uniaxial loading.

Based on the experimental results, and with the requirement of simplicity, an empirical failure envelope for concrete under biaxial compression is proposed as shown in

Fig.(12). It may be written as

$$\left.\begin{array}{ll} \alpha < 0.2, & \frac{\sigma_p}{\sigma_0} = 1 + \frac{\alpha}{1.2-\alpha} \\ 1.0 \geq \alpha \geq 0.2, & \frac{\sigma_p}{\sigma_0} = 1.2 \end{array}\right\} \quad \ldots(10)$$

Eqn.(10) is recommended for design purposes.

5.3 STRESS DEFORMATION CHARCTERISTICS:

The typical stress-deformation relations of real concrete in biaxial compression are plotted on dimensionless axes. The shape of the stress-deformation curves obtained, in general were similar for all mixes . It can be seen that for a given value of σ_1/σ_0 , the strain in the corresponding direction ϵ_L is reduced by the presence of σ_2 stress.

For example, for the ratio $\sigma_2/\sigma_1 = 1$, the decrease in strain ϵ_1, compared with uniaxial case is about 35 percent at 30% of uniaxial ultimate load. If the concrete were considered as homogeneous and elastic at that stage of biaxial loading, the equation would be

$$\epsilon_1 = \frac{\sigma_1}{E} - \frac{\nu\sigma_2}{E} \quad \ldots(11)$$

If poisson's ratio for concrete is assumed to be 0.2, for the ratio $\sigma_2/\sigma_1 = 1$ the decrease in strain due to poisson's effect is only 20 percent according to equation(2) compared with the observed 35 percent. Microcracking observations indicate that the main cause of the actual larger strain reduction is the confinement of potential microcracking in the presence of biaxial compression.

Eqn.(11) may be written in general form

$$\sigma = \frac{\epsilon E}{1 - \nu\alpha} \qquad \ldots(12)$$

Where σ & ϵ are respectively, the stress and strain in either direction 1 or direction 2.

An expression similar to Eqn.(12) modified to account for nonlinear behaviour is proposed for concrete in biaxial compression.

$$\sigma = \frac{A + B\epsilon E}{(1 - \nu\alpha)(1 + C\epsilon + D\epsilon^2)} \qquad \ldots(13)$$

Where the parameters A,B,C and D are found from the following conditions on the stress-strain curve in compression.

(a) For $\epsilon = 0$, $\sigma = 0$

(b) For $\epsilon = 0$, $\dfrac{d\sigma}{d\epsilon} = \dfrac{E}{1 - \nu\alpha}$

(c) For $\epsilon = \epsilon_p$, $\sigma = \alpha_p$

(d) For $\epsilon = \epsilon_p$, $\dfrac{d\sigma}{d\epsilon} = 0$

substituting the resulting values in equation (13) and introducing the secent modulus at peak stress

$E_s = \dfrac{\sigma_p}{\epsilon_p}$, one obtains

$$\sigma = \epsilon E \Big/ \left\{ (1 - \nu_\alpha)\left[1 + \left(\frac{1}{1 - \nu_\alpha}\frac{E}{E_s} - 2\right) \times \left(\frac{\epsilon}{\epsilon_p}\right) + \left(\frac{\epsilon}{\epsilon_p}\right)^2\right]\right\} \ldots(14)$$

The derivation of Eq.(14) and the recommended governing parameters used i.e. $E, \sigma_p, \epsilon_p, \nu$ were presented by Liu. The suggested equation provides good agreement between experimental and analytical stress-strain curve.

The strain at maximum stress for concrete in the major principal stress direction ϵ_p was found to be about 0.0025 for the concrete investigated.

The strain in the minor principal direction ϵ_2 indicated elongation for stress ratios from 0. to 0.2, and indicated contraction for stress ratios from 0.2 to 1.0

The ratio of ϵ_2 to ϵ_1 in uniaxial compression remained practically constant upto about 60 percent of the ultimate load. At higher stress levels, this ratio increased owing to cracking within the specimen. In the biaxial compression case, ϵ_2/ϵ_1 remained constant at the average values of 0.02 , 0.2 and 1.0 for $\sigma_2/\sigma_1 = 0.2$, 0.5 and 1.0 respectively. This probably occurs because the compressive stresses are applied in both principal directions and thus there are no cracks in the principal directions where the strains are measured.

5.4 MODES OF FAILURE:

In biaxial compression, failure of the concrete cannot occur by separation along the line of the major load because transverse extension is prevented. The failure occurs by resulting deformation in the direction perpendicular to the plane of loading. Contrary to the shear mode observed by Rosenthal and Gluklich (5) the eventual failure is by tensile splitting, but in a plane parallel to the plane of specimen, where the third dimension is unrestrained. This suggests that tensile deformation is a vital factor governing the strength of concrete as suggested by Slate and Meyers.

5.5 CONCLUSIONS:

1. Test results show that the strength of concrete under biaxial compression is higher than under uniaxial loading. The strength increase under biaxial compression is dependent, on the principal stress ratio.

2. In Biaxial loading, strains are significantly less than predicted by theoretical elastic analysis presumably because microcracks are prevented from occuring.

3. Prediction of initiation and propagation of micro-cracks by theoretical analysis of Buyukozturk et al based on a simple model was found to apply also to both complex models and real concrete.

4. The principal strain ratio for uniaxial compression remains practically constant upto 60% of the ultimate load. At higher stress levels, the principal strain ratio increases owing to the cracking within the specimen. In the biaxial compression case, the principal strain ratios remain practically consistant throughout the range of loading.

5. To account for the increased strength of concrete when it is subjected to biaxial compression, a simple equation is proposed.

6. A stress strain relation for concrete under biaxial compression is suggested. The suggested equation provides good agreement between experimental and analytical stress-strain curves.

7. In uniaxial loading, for the flat specimen tested, ultimate failure occurs by splitting in the plane ll to the load and icular to the face of the specimen. In the biaxial compression case, ultimate failure occurs by splitting along planes ll to the load and ll to the face of the specimen. The mode of failure suggests that tensile deformation is vital in the failure mechanism of the concrete.

CHAPTER 6

6.0 STRAIN AND ULTIMATE STRENGTH OF CONCRETE UNDER TRIAXIAL STRESS:

Four possible stress combinations arise in triaxial stress systems as all three, two, one or none of the three principal stresses may be compressive. Very meagre experimental research seems to have been carried out on the triaxial strength of concrete and most of it has been confined to triaxial compression. The analytical work has attempted to express triaxial strength of concrete in terms of the Octahedral stresses and the stress invariants. The influence of the intermediate principal stress on triaxial strength has received some attention. Certainly the strength is higher with triaxial compressive loading than with uniaxial loading, smaller however with combined loading by compression and tension.

6.1 TRIAXIAL STRENGTH CRITERIUM FOR CONCRETE:

The paper reported by F.Bremes (11) on the above is also worth describing which is as follows:

6.1.1 TEST PROGRAMME:

The objective of the investigation was to determine the strength behaviour and to a lesser degree the strain behaviour, under the effect of random stress fields and random stress conditions. The cube is the most suitable shape to produce three dimensional stress condition- work by Rusch, weigler and Becker indicated however that the restriction of transverse strain must be avoided under any curcumstances. After numerous, initial tests a method of load application was developed which enables the residual friction between the load transmitting plate and the test specimen to be reduced to less than 1 percent while compressive stress is applied. Tensile forces are applied through a very rigid steel plate cemented to the test specimen. The thickness of the adhesive layer and its transverse strain were optimized to such a degree that the monoaxial tensile strength determined by this method for the cube corresponding to 95% of that determined for the long prismatic beam. So, all test results are rather conservative.

In order to establish a relationship between multiaxial and monoaxial strength, the associated monoaxial strength values were determined using the same test equipment and procedures.

6.1.2 TEST PROCEDURE:

The test equipment developed has three independently controlled trains capable of generating compressive as well as tensile forces. All power transmitting elements have universal end joints to avoid any constraint. An electric load measuring device is installed in each train between the hydraulic system and the load transmitting plate. This eliminates any irregularities in the hydraulic system. The compressive forces applied are recorded by a line recorder.

Particularly in the case of triaxial tests, it is very difficult to recognise a fracture while the experiment is in progress. From the desired lines it is evident that the actual line turns sharply into the vertical direction which indicates the occurence of a fracture. At the point where the desired line becomes nearly a horizontal the fracture is complete. The forces acting while the fracture occured were used for evaluating and the results obtained. Calcitic aggregate and a grade PZ 375 portland cement were used for making the concrete. As regards it monoaxial desired strength, the concrete tested corresponds to a B 450 concrete as defined in DIN 1045. When tested the actual dimensions of the test specimen were 100 mm in each direction. It is a known fact that over a few mm the strength of the marginal zones is less than that of the core. The smaller the test specimen, the greater is the influence of such zones of reduced strength. To eliminate this effect, the test specimen were prepared with 106mm long sides. Prior to the test, they were then ground to the exact desired dimension, using a plane parallel grinder with diamond wheels. The test specimen were taken from their forms after curing for 24 hours. Thereafter they were stored for 90 days under standard climatic conditions. A storage period of this length is necessary to eliminate the influence of various time dependent post-curing effects. Nine test specimens were prepared for each test series. For each series of tests, the strength was measured monoaxially, always using the same test method to provide a reference basis. In addition, the strength was measured without any friction reducing intermediate layer to enable comparisons to be made with the results of conventional test method. In all cases, it was found that the strength values obtained by the newly developed method amounted to only 80% of the compressive strength values obtained by the conventional methods. This proves that the actual strength of the continuum has been measured.

6.1.3. TEST RESULTS:

Owing to the unhindred latral strain, all failures occured in the direction that free of any load or in the case of three-dimensional stress, in the direction of the least compressive stress, or in the direction of a tensile stress percent at the same time. This permits the conclusion that concrete is subjected to strain failures rather than stress failures. In three dimensional test it was established that, contrary to previous belief, the effect of the mean principal stress is of decisive importance for the development of strength.

It was indeed found that the strength increases rapidly if the mean principal stress is relatively small and that the strength decreases rapidly if compressive and tensile forces are acting at the same time. It can be seen that under three-dimensional compressive stress conditions, strength values are rapidly reached that today cannot be utilised in practice. This is why test aiming in this direction were limited for the time being and all efforts concentrated on determining the influence of compressive/tensile stress fields.

6.1.4 CONCLUSIONS:

In view of the strong dependence of the strength upon the ratio between two or three effective stresses, it appears inadvisable in the case of multiaxial stress to work with fixed maximum stress values no longer. On the contrary, the economy and safety of a structure will be enhanced if factors are used that are dependent upon the particular type of loading. However, whether the high strength values obtained in the case of three-dimensional compressive stress should be utilised is another question. It appears advisable to assume a fictitious maximum strength for the time being where this limit should be, will depend upon type of structure. Application simultaneous compressive and tensile stresses.

Tests have shown that the use of slack reinforcement will significantly increase the fracture load level under combined compressive and tensile loads. Slack reinforcement with a cross sectional area of 0.5 percent of the area under tensile stress will increase the ultimate strength (principal compressive stresses) by not less than 15 percent. If the cross-sectional area of the reinforcement is increased to

1 percent of the area under tensile stress, the compressive strength will increase by 35 percent. To ensure optimum economy and safety, extensive experimental work will still have to be carried out concerning the effect of slack reinforcement on the compressive strength of concrete. The same applies to the efforts made to determine the influence of the geological origin of the aggregates on the development of strength. Initial tests conducted by Bremer indicated that with quartzitic aggregates, strength development is slower than with calcitic aggregates used. Under purely compressive stress, strength development is approximately 10-15 percent slower if quartzitic aggregates are used, while with combined compressive and tensile loads, the loss of strength is somewhat faster than with calcitic aggregates.

CHAPTER 7

7.0 CREEP PREDICTION FOR CONCRETE UNDER MULTIAXIAL STRESS:

Previous tests indicate that a reasonable although not exact assumption in the prediction of creep strain resulting from multiaxial stresses is that the Poisson's ratio for creep is constant and equal to the static elastic value. To predict creep under multiaxial loading, this assumption was combined with the rate of flow method, developed essentially for the prediction of axial creep strain under uniaxial loading. Creep predictions on the basis of independently obtained control data, are compared with the experimental strain obtained from multiaxially loaded concrete specimen. Only moderate stress levels were considered and all specimen were sealed. The results of the comparison indicate satisfactory agreement between the proposed method and the test results. Second order effects not taken into account in the analysis are discussed.

Considerable attention has been given in the past to the prediction of creep under uniaxial variable stress, and all the methods developed are based on curves of creep or creep recovery for the concrete under constant stress, where approximate predictions are sufficient, these curves may be manufactured from the factored data given in the design recommendations, for better accuracy, control test should be mounted in which the mix and environmental conditions simulate the concrete in the real structure. Here for brevity the curves on which the predictions are based are referred to simply as control data and they are assumed to be derived from control tests.

The simplest methods of prediction for uniaxial stress are " rate of creep" and "effective modulus" both of which approximate the dependence of creep on the age of loading and which can be considerably in error if the stress varies at all greatly.

Two other methods " Superposition" and "rate of flow" recognise the dependence on age of loading more fully and have been shown to give better predictions of creep at the expense of more data for control tests.

It is clearly most convenient for the creep control tests to be performed under uniaxial conditions and thus points the way to the simplest means of calculating creep under multiaxial stress, namely the use of a poisson's ratio for creep. It has been found to be approximately constant and equal to the static elastic value on loading, there is some evidence that it may decline, somewhat with time, however there is no convincing evidence that it is zero or that it increases with time. Poisson's effects are generally small and the reported data provides a basis for making acceptable approximation in the value of Poisson's ratio. It must be recognised, however, that there are circumstances in which the Poisson's effect assumes much greater importance and some more detailed findings that could then be of significance are mentioned later.

Here, a method of predicting creep under multiaxial stress is described and tested against the strains measured in tests in which one of the stresses were cycled. The stresses considered are below the discontinuity level which marks the onset of severe microcracking.

7.1 PREDICTION PROCEDURE:

In brief, the rate of flow method is extended to three dimensions by taking a Poisson's ratio for creep equal to the static value on first loading.

In summary creep is considered to consist of two components (a) irrecoverable creep flow ϵ_f , the rate of which is independent of any previous stress history and (b) Recoverable creep or delayed elastic strain ϵ_d, which tends to a limiting value that is independent of the age of concrete. The rate of occurence is related to the concurrent flow. Both components are directly propor-tional to stress.

When the stress varies with time, the period under load is divided into a number of increments during each of which the stress is considered constant, that is, the stress history is represented by a step function. Taking a typical time increment δt during which the stress is σ the flow during the increment $\delta\epsilon_f$ is given by

$$\delta\epsilon_f = \sigma\ \delta c_f \qquad ...(15)$$

Where δc_f is the flow per unit stress during δt. The delayed elastic strain during the increment, $\delta \epsilon_d$ is given by

$$\delta \epsilon_d = (\sigma C_{d\infty} - \epsilon_d)(1 - e^{-\delta c_f / Q}) \quad (16)$$

Where

$C_{d\infty}$ = limited delayed elastic strain per unit stress

ϵ_d = delayed elastic strain at start of increment

Q = Parameter controlling rate of occurence.
It is, like $C_{d\infty}$, a constant for a particular mix and environment.

The total creep during δt is therefore

$$\delta \epsilon_c = \delta \epsilon_f + \delta \epsilon_d \quad (17)$$

The creep under a triaxial system of stress if found in the direction of the three principal stresses σ_1, σ_2, σ_3 by applying the creep, Poisson's ratio (constant and equal to the static elastic value).

Then in direction 1

$$\delta \epsilon_{c_1} = \left[(\delta \epsilon_{f_1} + \delta \epsilon_{d_1}) - \nu_s (\delta \epsilon_{f_2} + \delta \epsilon_{d_2}) + (\delta \epsilon_{f_3} + \delta \epsilon_{d_3})\right] \quad (18)$$

and similar expression drop out for the strains $\delta \epsilon_{c_2}$ and $\delta \epsilon_{c_3}$ in direction 2 and 3 by cyclial rotation of suffices.

The full history of strain is found by repeating the procedure for all time increments and adding the elastic strains in the normal manner.

As discussed earlier, control tests are desirable and the necessary data can be accumulated as follows:

1. The elastic modulus, dynamic or static measured over the period of loading.

2. The static elastic poisson's ratio at any time during the period of loading, in this case the dynamic value is not an alternative to the static.

3. A curve of creep render constant uniaxial compressive stress, preferably starting at the same age as the history of the variable stress. This is used to deduce the flow/unit stress as described later with reference to Fig.15.

4. A curve of creep recovery measured after a sufficient period under constant uniaxial compressive stress. This is used to deduce the limiting delayed elastic strain per unit stress $C_{d\infty}$ and the rate parameters Q.

7.2 EXPERIMENTAL VERIFICATION OF PROPOSED METHOD:

Three tests were performed on identical concretes sealed and maintained at the same constant temperature. The mix proportion by weight were as follows:

Water cement ratio = 0.40, Aggregate-cement ratio = 3.20 10 to 5 mm coarse aggregate, sand = 1.67. The strength at 7 and 14 days found using 100 mm cubes were 32 and 42 N/mm^2 respectively.

The stress histories are summarized in Fig.19 and, as can be seen, they were as follows:

1. Uniaxial, with repeated stress increments and decrements.
2. Triaxial with three different initial stresses, again with further cycling of the stress σ_1,
3. Biaxial, with different initial applied stresses, one of which (σ_1) was further cyclid.

The control tests were not at all performed at the same time as the tests under variable stress, but the mix and environmental conditions were identical with one exception. The control creep test commenced at 14 days while the tests under variable stress started at 7 days. Prediction from 7 to 14 days are therefore based on the measured strains in the uniaxial test under variable stress. The control creep curve is shown in Fig.15 and of Q , by fitting Eq.(16) to the observed results.

The values used in the predictions are

$C_{d\infty}$ = 6.5×10^{-6} N/mm^2

Q = 0.45×10^{-6}

ν_s = 0.165

Age days	7	21	42	77	133	240
Dynamic Modulus	0.0370	0.039	0.0396	0.0398	0.0418	0.0420

The means of determining the flow is shown in Fig.15. The measured and predicted strains for the three tests are given in Fig.16, 17 and 18. The general shapes of the creep curves are reproduced well by the predictions. At most, the error in the predicted strain is of the order of 40 microstrain and the difference between measured and predicted strains is generally less than 20 microstrain. This is considered satisfactory since the experimental strain were determined with 95% confidence limits of about $\pm$ 10 percent. These limits refer to a variation in the experimental results which is for the most part a randomly occuring factor by which a particular strain could be in error throughout the test. The predictions of smaller strain corresponding to the difference of the two terms in Eq.(18) can have high percentage errors, but only because the strains themselves are small. For instance see ε_3 in Fig.18.

In addition to the random errors, some second-order effects have been detected experimentally. There have been ignored in the predictions but they are responsible for some lack of precision.

7.3 SECOND ORDER EFFECTS:

It should be added that recognition of these effects is not difficult but to quantify them is quite another matter and it is not possible here to do more than indicate the trends. The first area of second-order effects is connected with the assumptions of the method.

1. The proportionality of stress and strain may not hold as the discontinuity level of stress is approached. Furthermore, there is evidence that Poisson's ratic changes as the stress increases.

2. The limiting delayed elastic strain may change somewhat with age; and the occurence of the delayed elastic strain is not ideally represented by the single exponential terms of Eq.(16), the sum of several such terms giving a distinct improvement.

3. The precise value of poisson's ratio is not usually of great importance in the tests reported here a change of 0.01 in poisson's ratio makes a difference of not more than about 10 microstrain to the predictions - but two further experimental findings are worth mentioning. Firstly Poisson's ratio for unrecovered strain after removing a sustained load declines from the value observed during the period under load. Secondly, both the static elastic and creep values have been found to be lower for biaxial state of stress than for uniaxial.

It is likely that (a) the value of creep Poisson's ratio drops if the concrete is drying under load and (b) the value of the limited delayed elastic strain will change if the moisture content of the concrete goes to extremes.

The two creep components can be considered as functions of temperature, humidity, age of loading, and time under load. The expressions that result are rather complicated and while this development may be worthwhile for special purposes, it can hardly be envisaged for general application.

7.4 CONCLUSIONS:

1. The creep of concrete under variable multiaxial state of stress can be satisfactorily predicted on the assumption of a constant Poisson's ratio equal to the static elastic value, combined with a creep prediction method, such as rate of flow which is known to give reasonable results for uniaxial state of stress.

2. The method is simple in concept and requires only straightforward prediction data which are best found from control tests under constant uniaxial stress. Alternatively they may be manufactured from collated research and design data.

7.5 NUMERICAL EXAMPLE:

The method of prediction is illustrated below by considering the calculation of typical increments of creep and elastic strain. The specimen under triaxial stress is chosen and the time increment is that from 21 to 28 days ($1 N/mm^2$ = 14.5 psi)

At the end of the previous increment (14 to 21 days) the stresses were

$$\sigma_1 = 5.05 \quad \sigma_2 = 7.22 \quad , \quad \sigma_3 = 3.57 \; N/mm^2$$

The delayed elastic strains were

$$Ed_1 = 32.8, \quad Ed_2 = 47.0, \quad Ed_3 = 23.2 \text{ microstrain}$$

The total strain were

$$\varepsilon_1 = 153.4, \quad \varepsilon_2 = 272.1, \quad \varepsilon_3 = 72.5 \text{ microstrain}$$

Elastic strains:- The stress in direction 1 increases at 21 days to 10.30 N/mm^2 with $Ed = 0.039 \times 10^6$ N/mm^2 and $\nu_s = 0.165$. The changes in elastic strain are

In direction 1

$$\delta\epsilon_{e_1} = \frac{\delta\sigma_1}{Ed} = \left(\frac{10.30 - 5.05}{0.039} \right) = 134.2 \text{ microstrain}$$

In Direction 2

$$\delta\epsilon_{e_2} = 22.2 \text{ microstrain}$$

In Direction 3

$$\delta\epsilon_{e_3} = -22.2 \text{ microstrain}$$

The total strains after the change of stress are

$$\varepsilon_1 = 153.4 + 134.2 = 287.6$$

$$\varepsilon_2 = 272.1 - 22.2 = 249.9$$

$$\varepsilon_3 = 72.5 - 22.2 = 50.3 \text{ microstrain}$$

<u>CREEP</u>:

The increment of flow per unit stress during the time is

$$\delta C_f = 3.0 \text{ microstrain per } N/mm^2$$

Hence the flows given by (σ) (δC_f) are

In Direction 1

$$\delta\epsilon_{f_1} = 10.30 \times 3.0 = 30.9 \text{ microstrain}$$

In Direction 2 :

$\delta\epsilon_{j_2}$ = 7.22 x 3.0 = 21.7 microstrain

In Direction 3:

$\delta\epsilon_{j_3}$ = 3.57 x 3.0 = 10.70 microstrain

Similarly the delayed elastic strains given by

$$(\sigma C_{d\infty} - \epsilon_d)\ (1 - e^{-\delta\epsilon_j/Q})$$ with $C_{d\infty}$ = 6.5

microstrain per N/mm^2, Q = 0.45 x 10^{-6}

In Direction 1

$$\delta\epsilon_{d_1} = (10.30\times6.5 - 32.8)\ (1 - e^{-3.0/0.45})$$

$$= 34.2 \text{ microstrain}$$

In Direction 2

$$\delta\epsilon_{d_2} = (7.22 \times 6.5 - 47.0)\ (1 - e^{-3.0/0.45}) = 0$$

In Direction 3

$\delta\epsilon_{d_3} = 0$

In direction 2 & 3, the delayed elastic strain are already fully developed.

Hence, the total increments of creep, for each stress taken independently are

In Direction 1

$\delta\epsilon_{f_1} + \delta\epsilon_{d_1}$ = 30.9 + 34.2 = 65.1 microstrain

In Direction 2

$\delta\epsilon_{f_2} + \delta\epsilon_{d_2}$ = 21.7

In Direction 3

$$\delta\epsilon_{f_3} + \delta\epsilon_{d_3} = 10.7$$

The creep for the combined stresses are:

In Direction 1

$$\delta\epsilon_{c_1} = 65.1 - 0.165\ (10.7 + 21.7) = 59.4 \text{ microstrain}$$

In Direction 2

$$\delta\epsilon_{c_2} = 21.7 - 0.165\ (65.1 + 10.7) = 9.2 \text{ microstrain}$$

In Direction 3

$$\delta\epsilon_{c_3} = 10.7 - 0.165\ (65.1 + 21.7) = -3.6 \text{ microstrain}$$

The total strains at the end of the increment, at 28 days are:

$$\epsilon_1 = 287.6 + 59.4 = 347.0 \text{ microstrain}$$

$$\epsilon_2 = 249.9 + 9.2 = 259.1 \text{ microstrain}$$

$$\epsilon_3 = 50.3 - 3.6 = 46.7 \text{ microstrain.}$$

CONCLUDING REMARKS:

From the experimental research conducted by eminent research workers it is depicted that the ultimate strength of concrete under biaxial compression is greater than under uniaxial compression. Under Biaxial compression-tension, the compressive strength decreases almost linearly as the applied tensile stress is increased. Under biaxial tension, the strength of concrete increases as compared to its uniaxial tensile strength. For all biaxial or uniaxial tests, failure occurs by tensile splitting with the fractured surface orthognal to the direction of the maximum tensile strain.

In Biaxial compression, failure of concrete cannot occur by separation along the line of the major load because transverse extension is prevented. The failure occurs by resulting deformation in the direction perpendicular to the plane of loading. In uniaxial loading, for the flat specimen, failure occurs by splitting in a plane parallel to the load and perpendicular to the face of the specimen. The mode of failure suggests that tensile deformation is vital in the failure mechanism, of the concrete.

It is concluded that the slack reinforcement will significantly increase the fracture load level under combined compressive and tensile loads. Slack reinforcement with a X-sectional area of 0.5 percent of the area under tensile stress will increase the ultimate strength by at least 15 percent.

R_E_F_E_R_E_N_C_E_S

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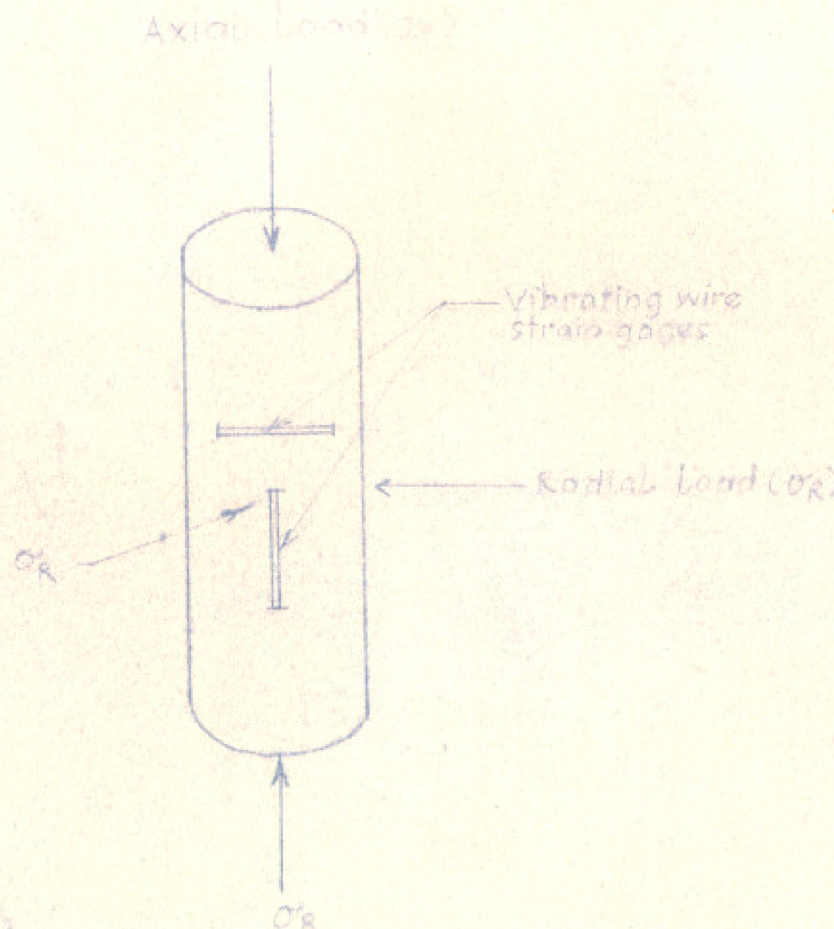
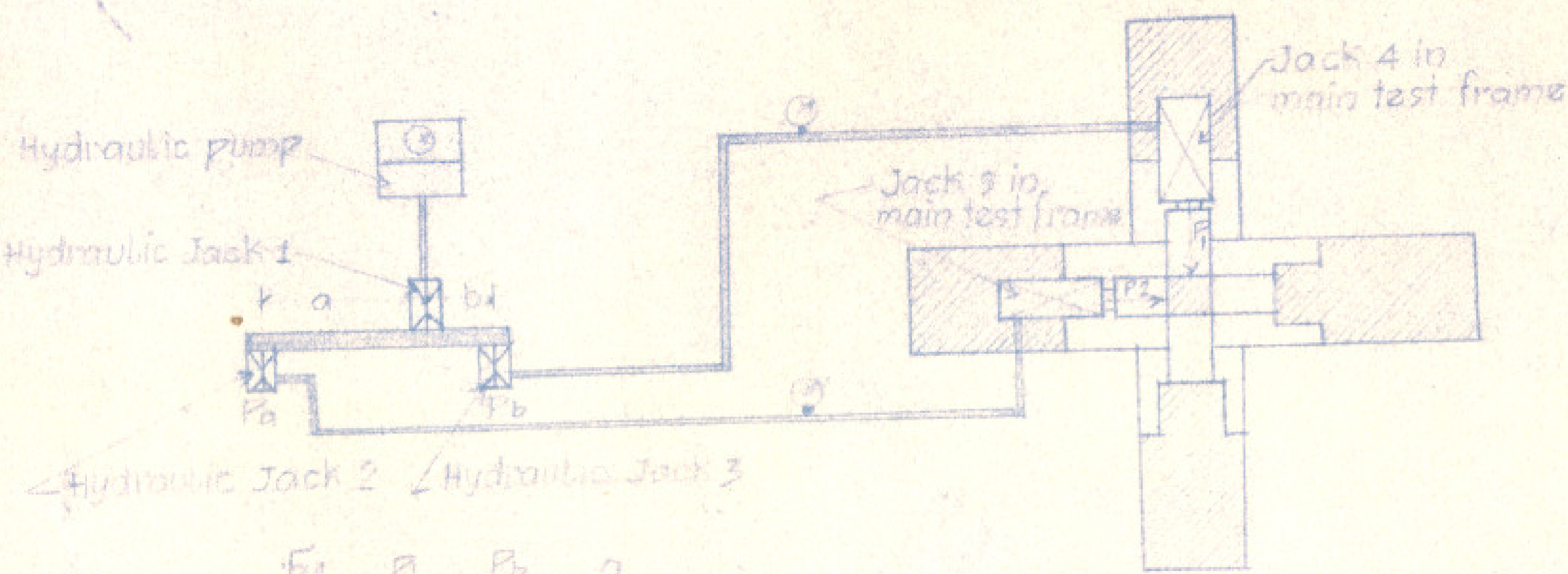


FIG. 1 LOADING CONDITION AND GAGE PLACEMENT



$$\frac{\sigma_1}{\sigma_2} = \frac{P_1}{P_2} = \frac{P_b}{P_a} = \frac{a}{b}$$

FIG. 2 - HYDRAULIC SYSTEM TO MAINTAIN CONSTANT RATIO σ_1/σ_2

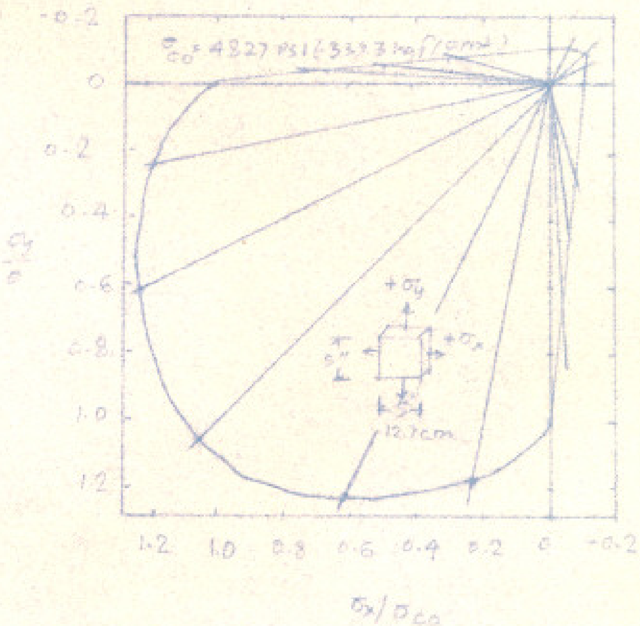


FIG. 3 - BIAXIAL ULTIMATE STRENGTH ENVELOPE OF CONCRETE

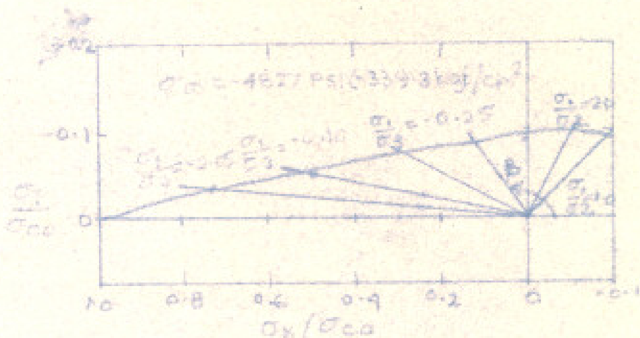


Fig - 4 Ultimate strength envelope of concrete under compression-tension and tension-tension biaxial stresses

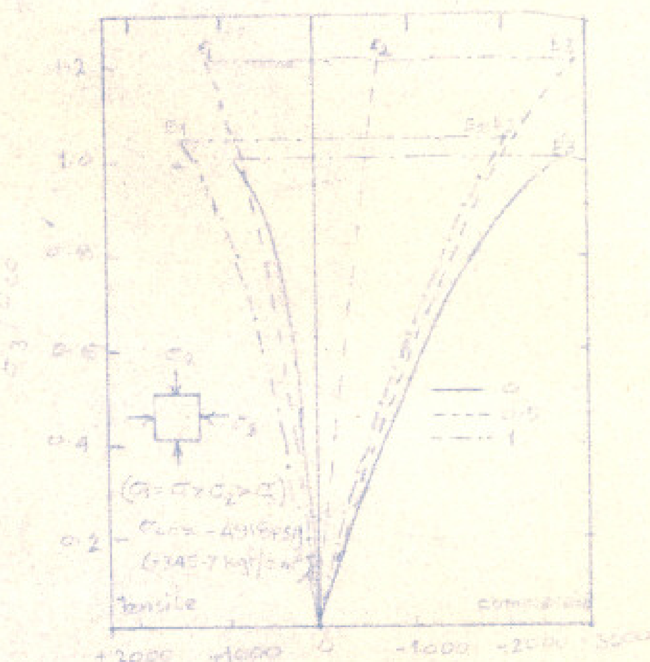


Fig-5 Stress-strain relationships of concrete under [illegible] compression

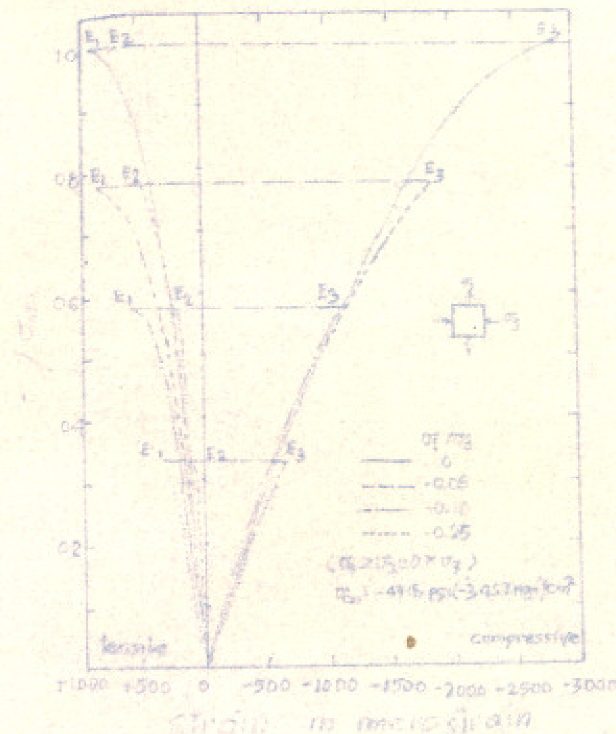


FIG-6 Stress strain relationship of concrete under biaxial compression - tension

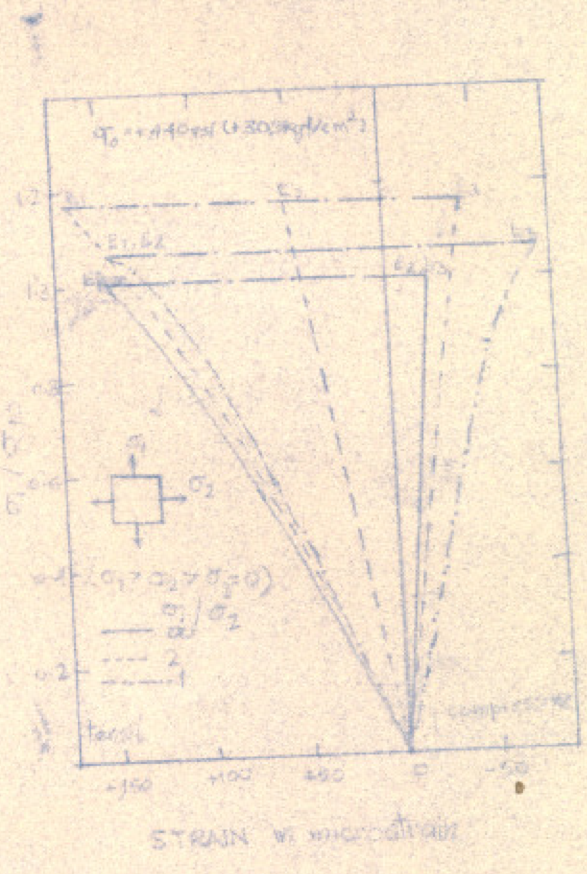


FIG-7 Stress-strain relationships of concrete

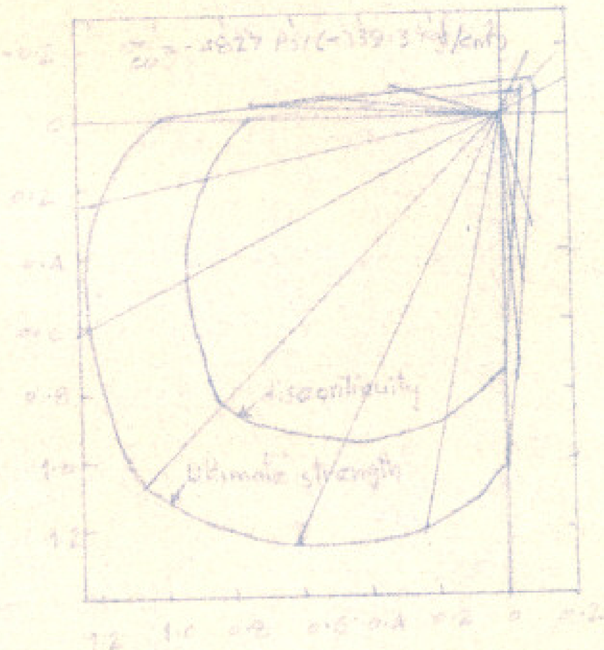


Fig-8 Biaxial discontinuity and ultimate strength envelopes of concrete.

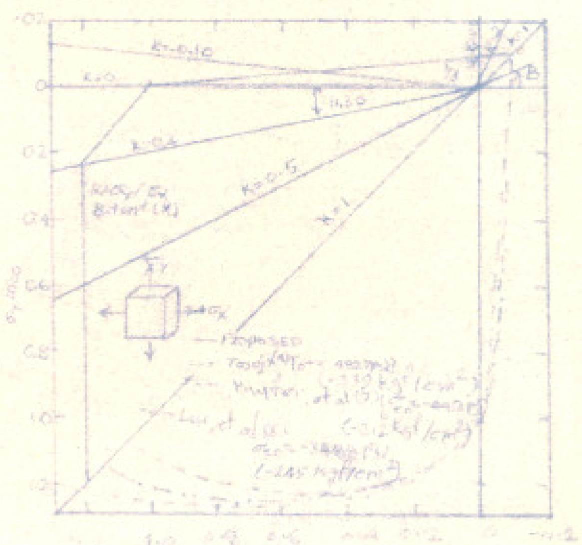


FIG-9 Proposed and experimental biaxial ultimate strength envelopes.

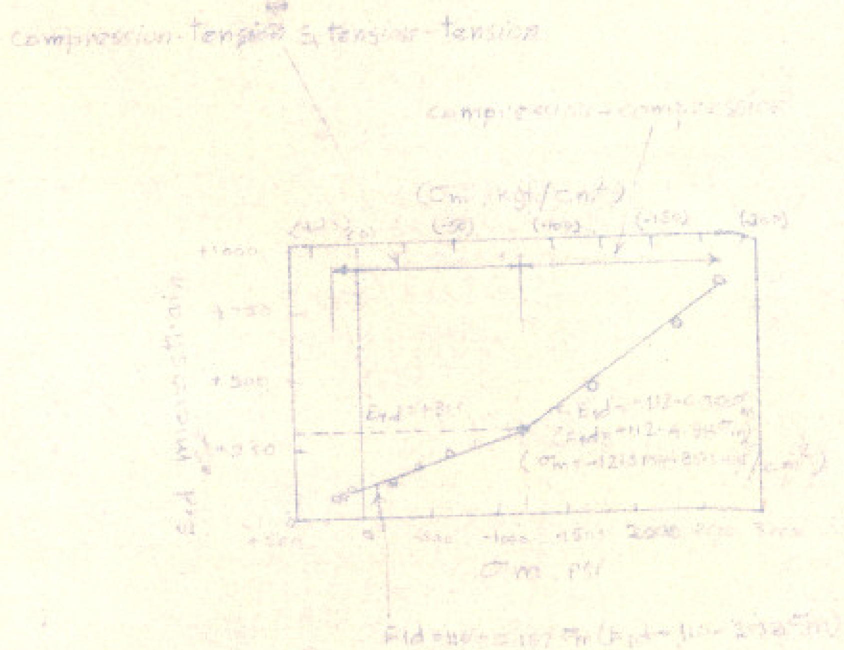


FIG-10 Linear relationships between principal tension strain ε_{td} and mean normal stress σ_m at discontinuity

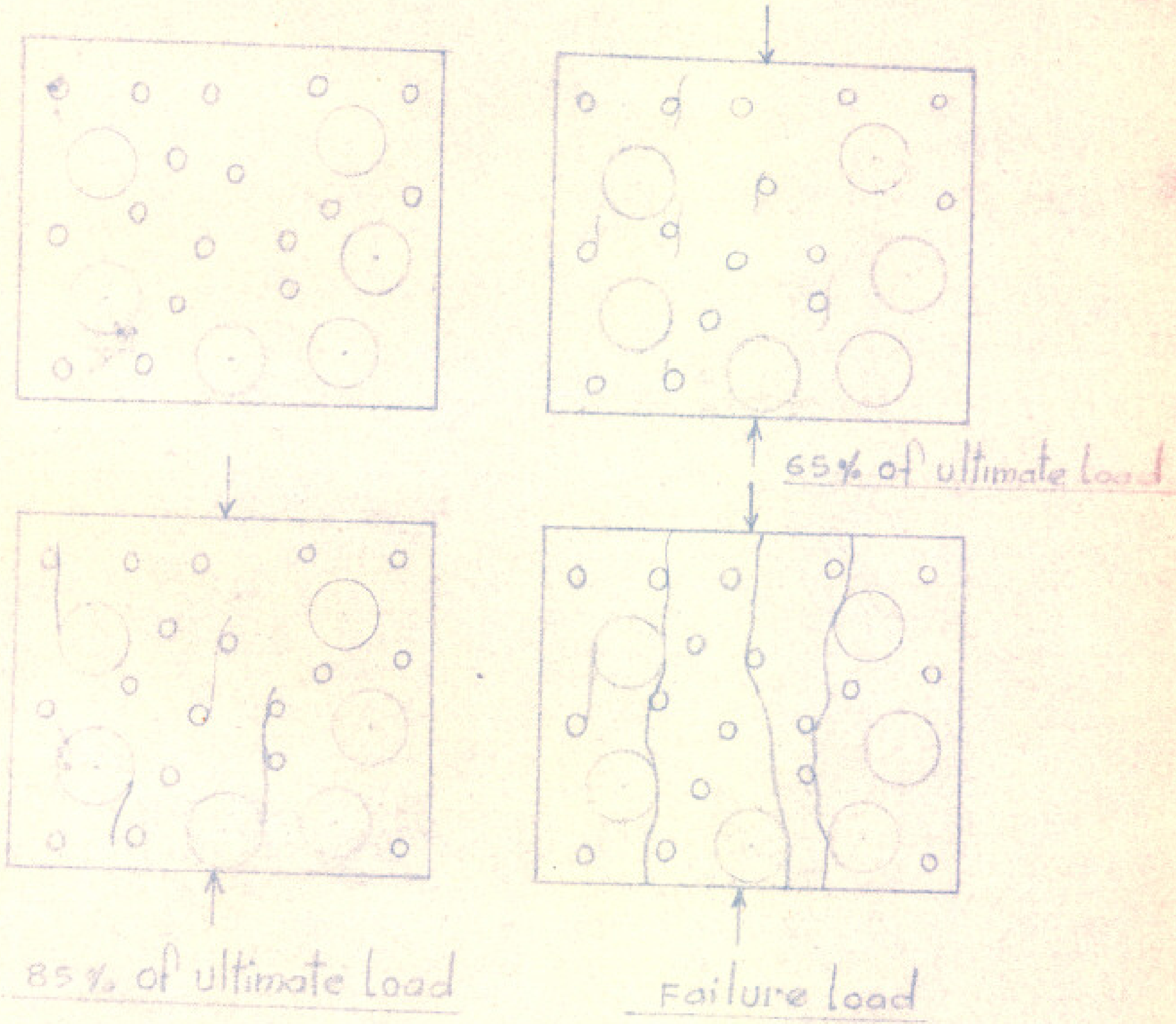


Fig — 11 FAILURE MECHANISM UNDER DIFFERENT LOADINGS

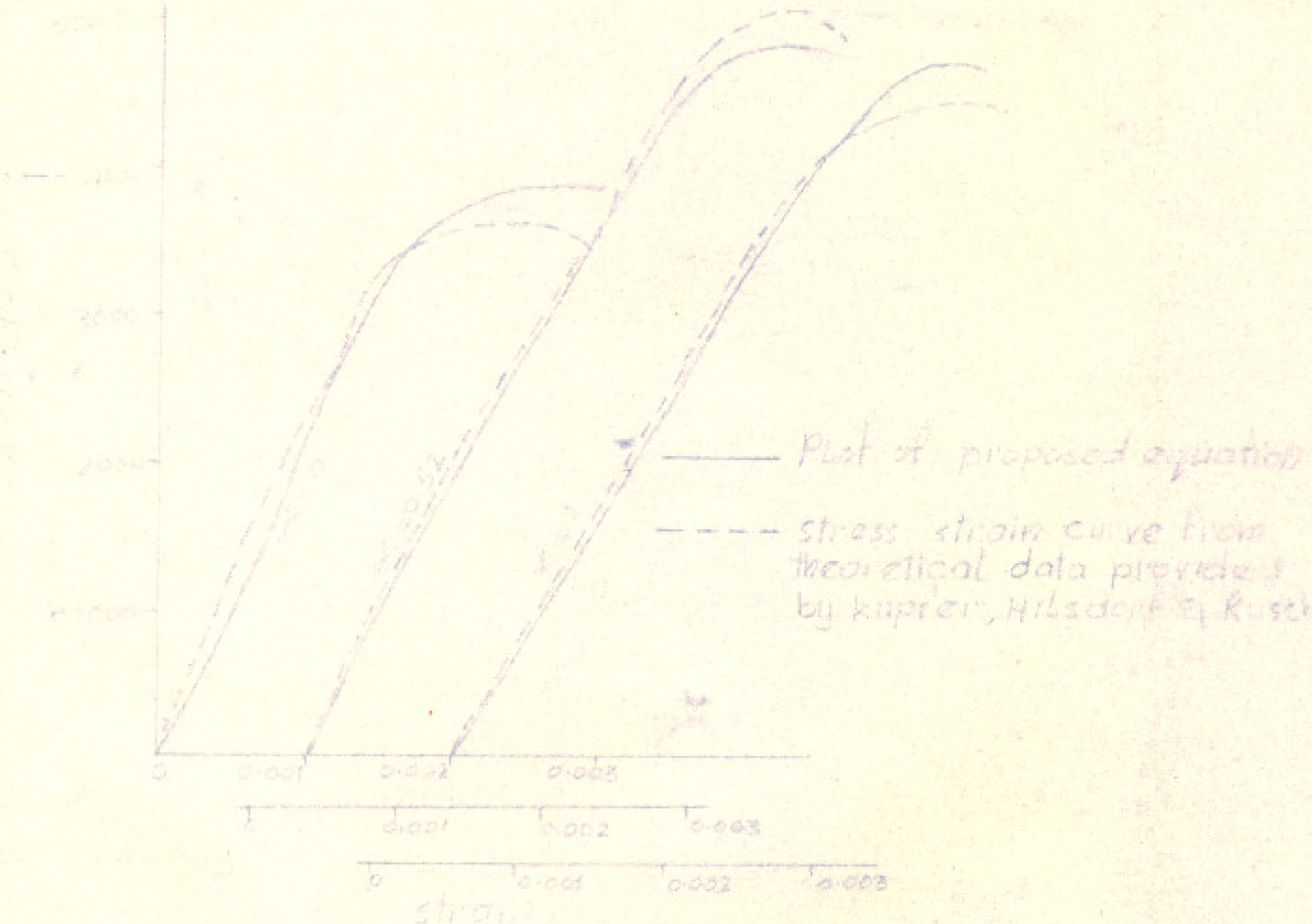


Fig -12 Compression of the plot of the proposed equation with those obtained from biaxial compression tests by kapfer, Hilsdrof & Rusch

— Proposed failure envelope

- - - - Experimental failure envelope

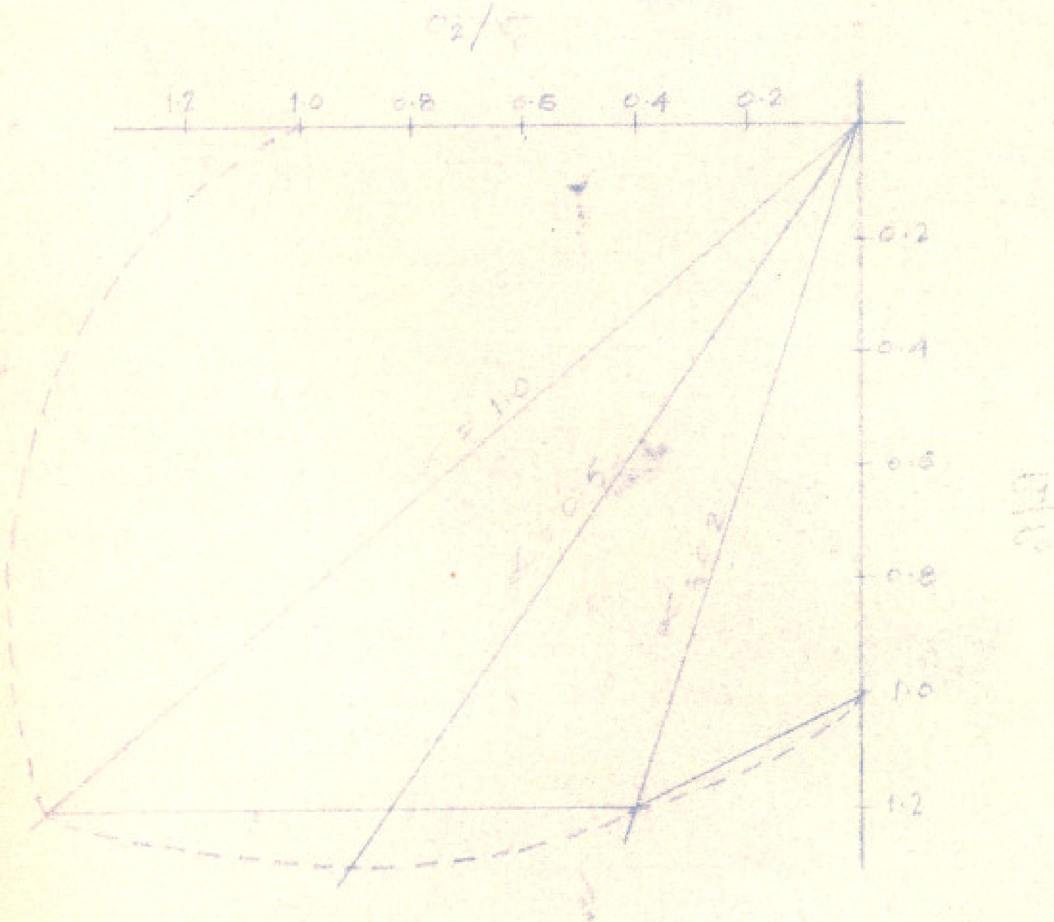


Fig - 13

Proposed failure envelope for concrete under biaxial compression

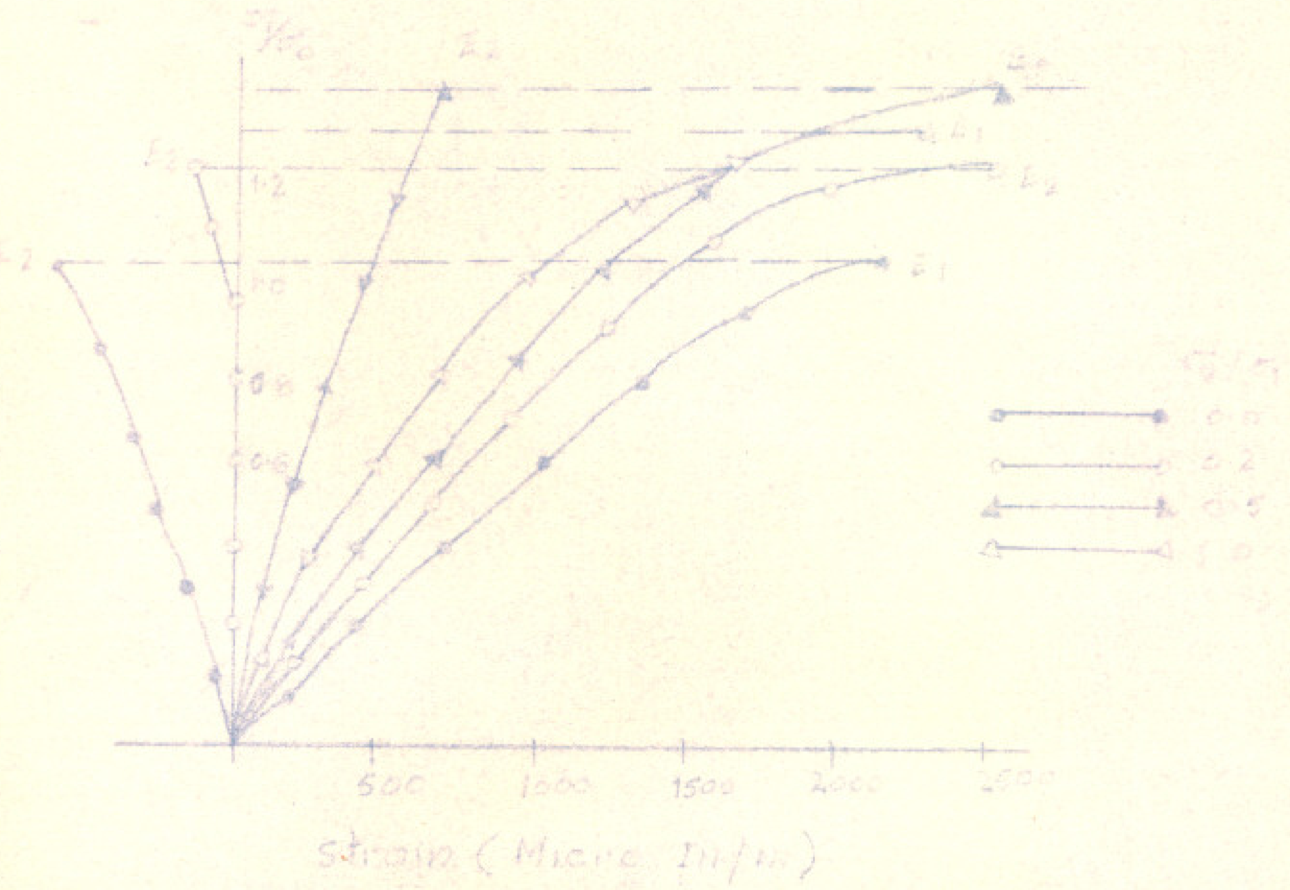


Fig-14

Typical stress-strain relation of concrete under biaxial compression.

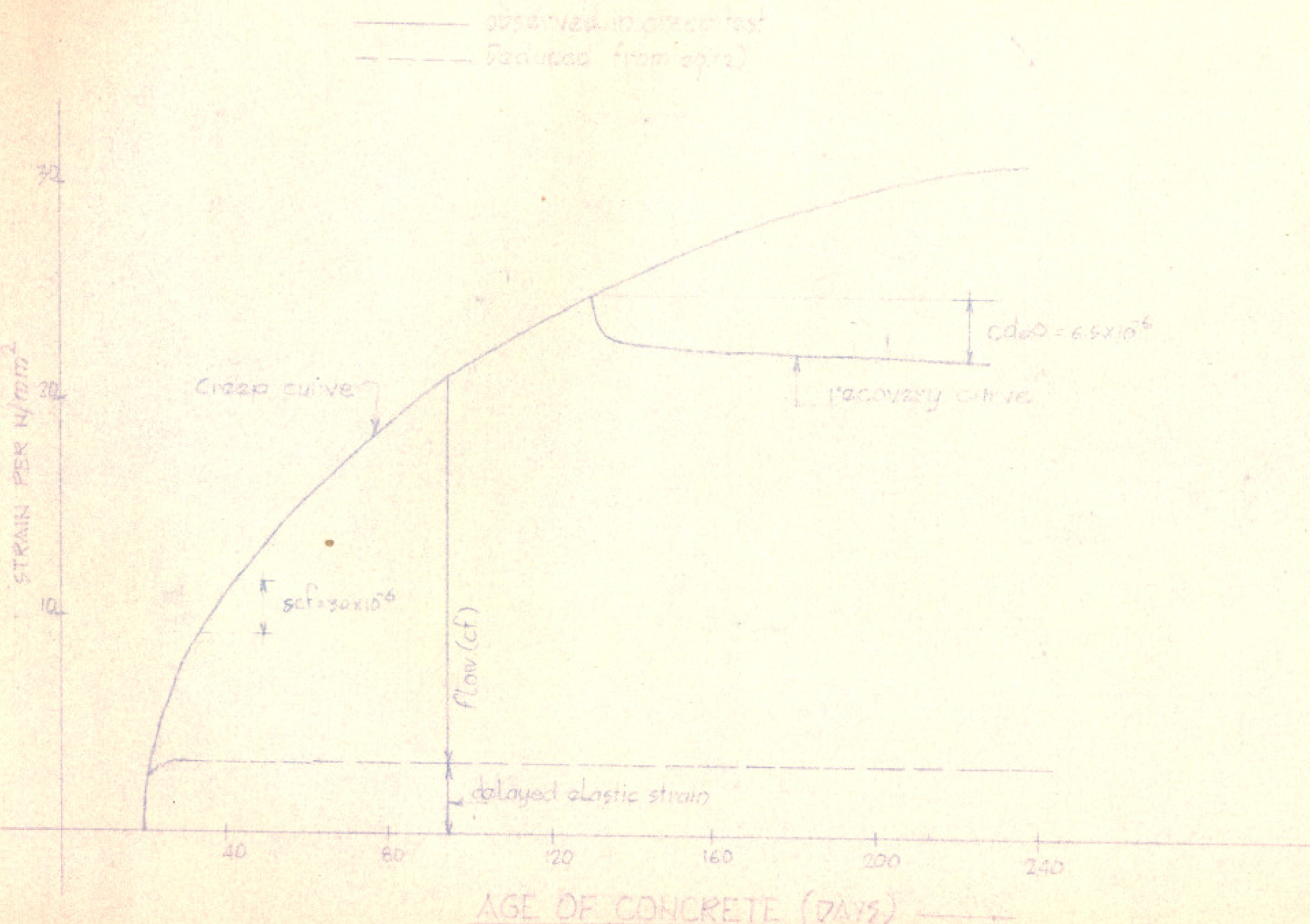


Fig.-15 - CONTROL TEST RESULTS AND THE DETERMINATION OF PREDICTION DATA

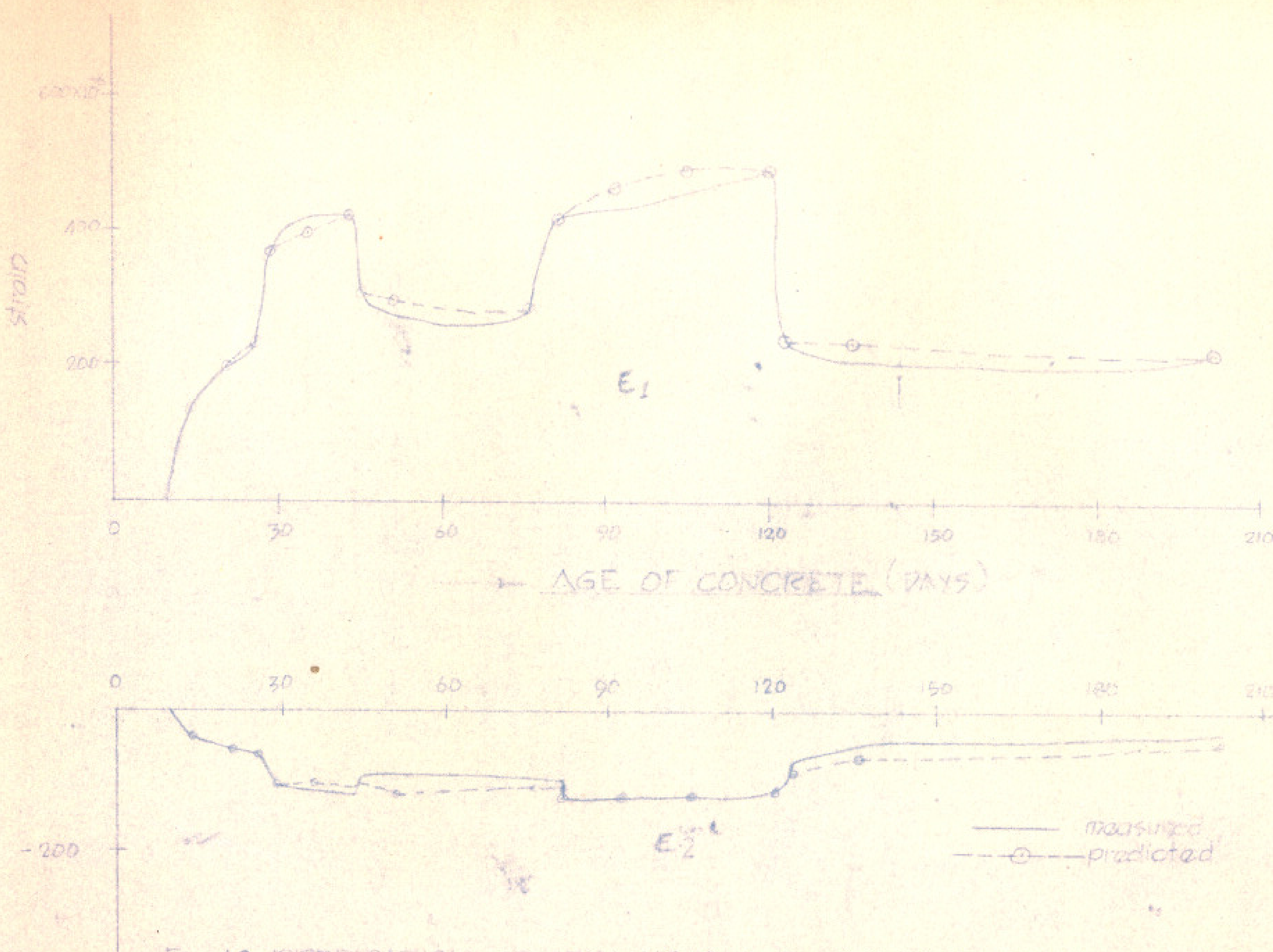
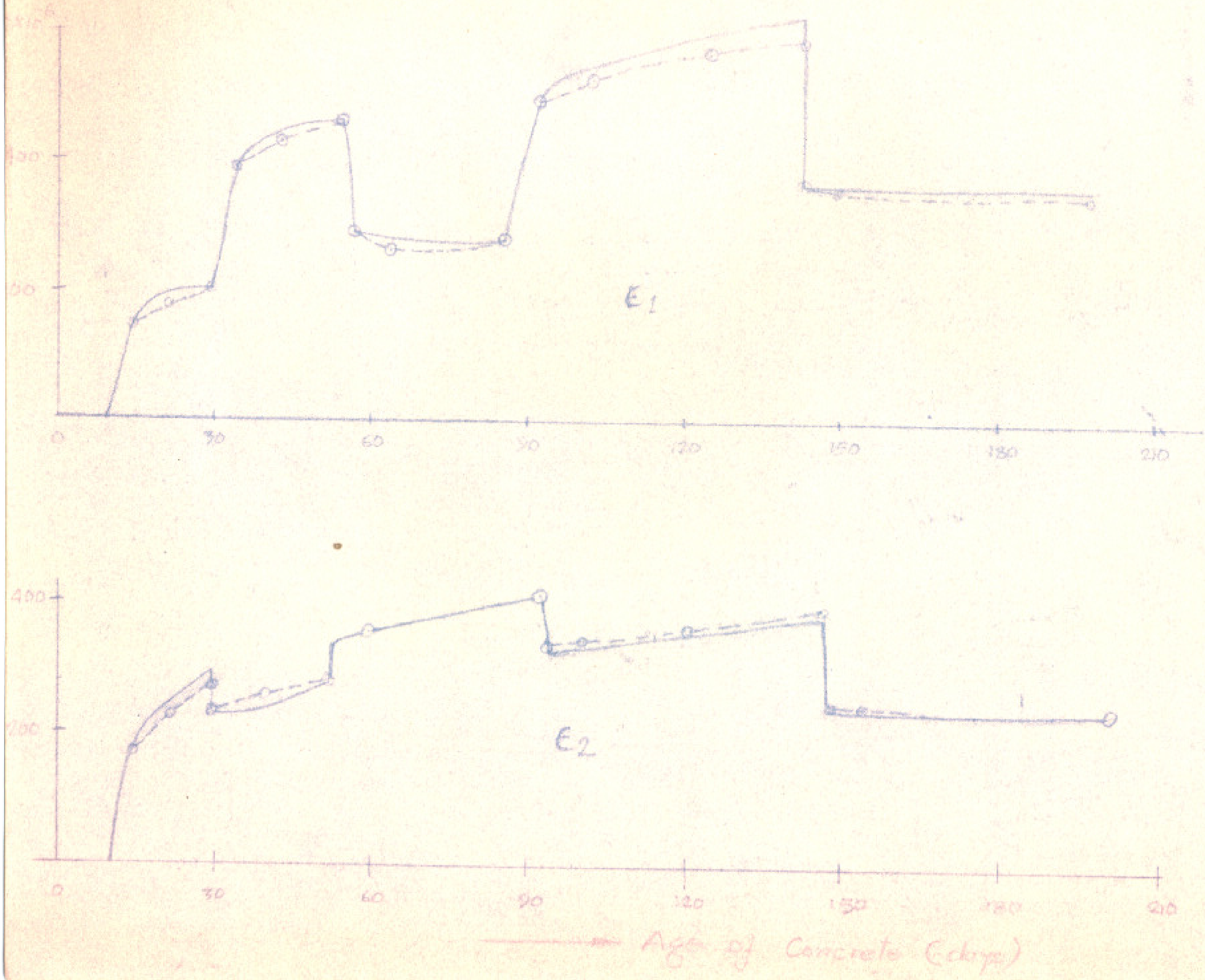


Fig. 16 PREDICATION OF STRAINS DUE TO VARIABLE UNIAXIAL STRESS SYSTEM



ϵ_1
ϵ_2
400
200
0
30
60
90
120
150
180
210
Age of Concrete (days)

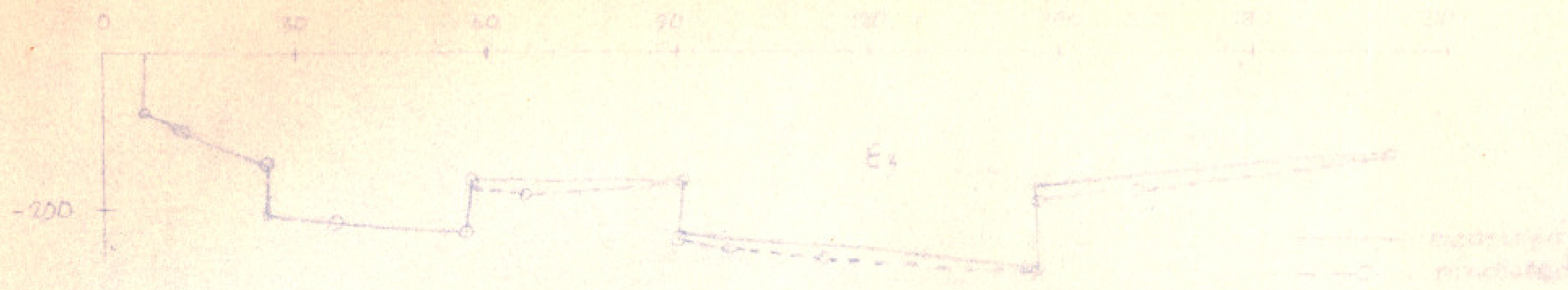
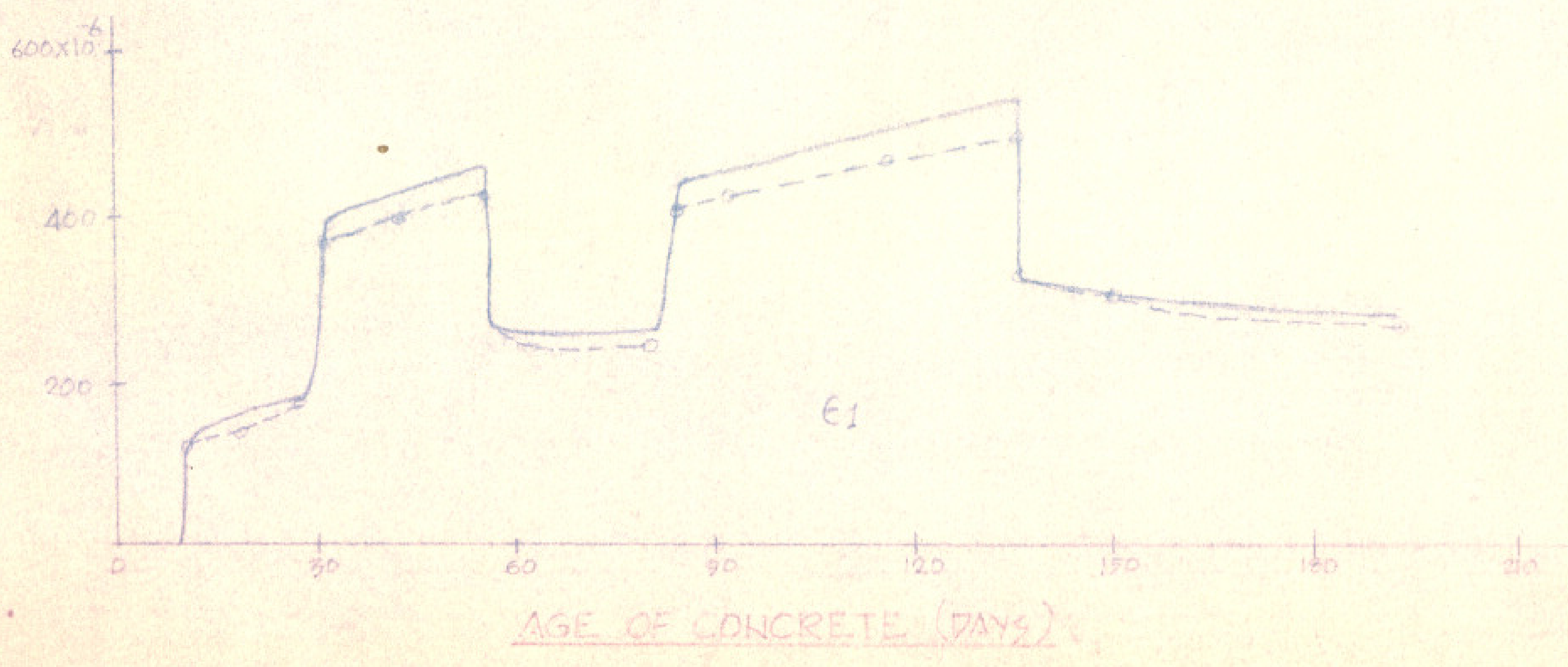


Fig. 17 PREDICTION OF STRAINS DUE TO VARIABLE BIAXIAL STRESS SYSTEM



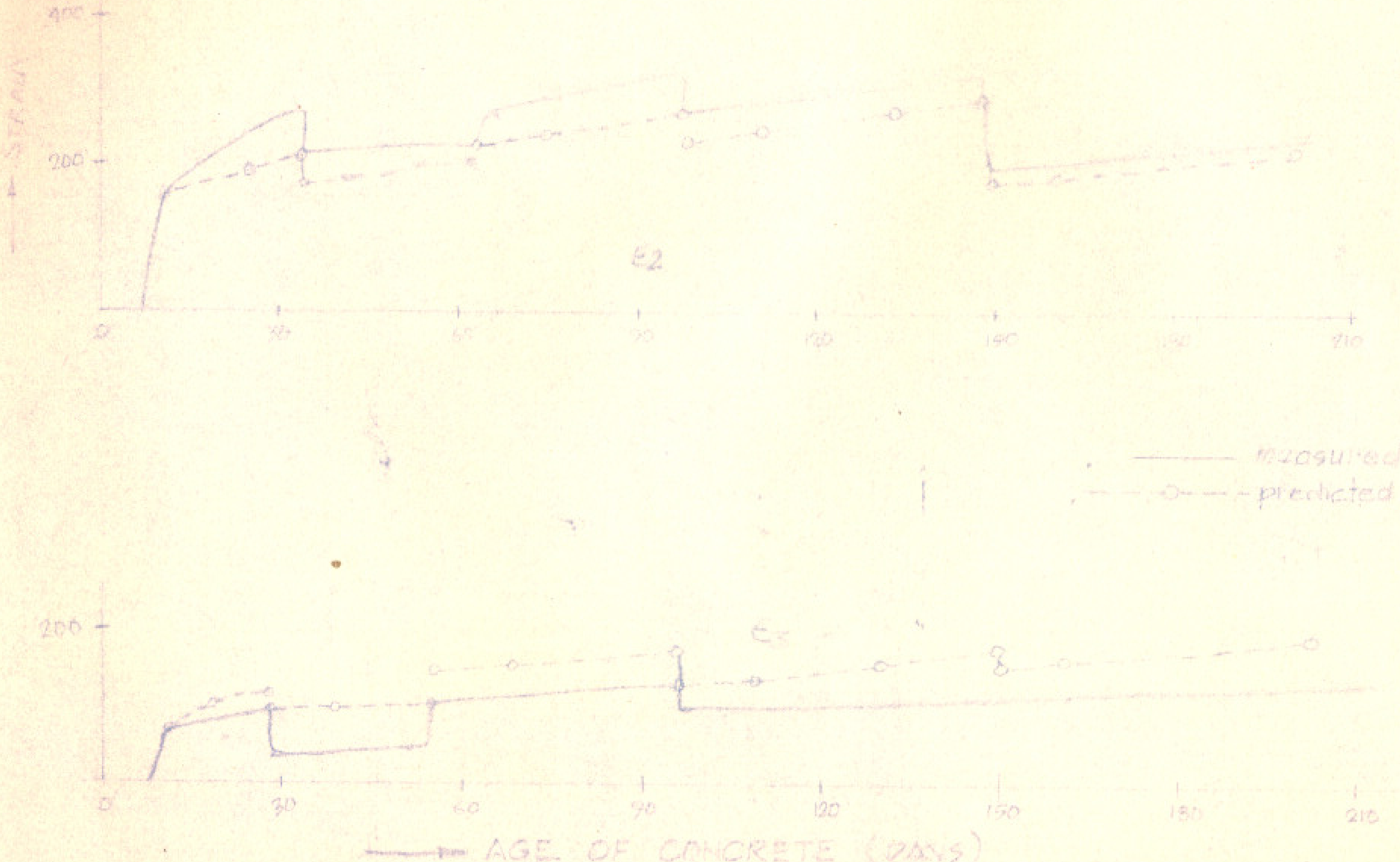
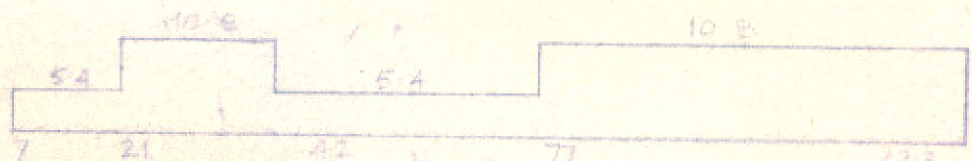


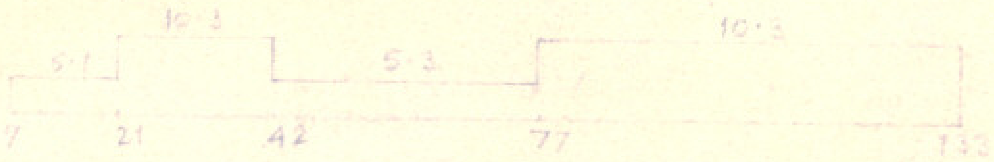
Fig 18 PREDICTION OF STRAINS DUE TO VARIABLE TRIAXIAL STRESS SYSTEM

σ_1 (N/mm²) 1 N/mm² = 145 psi

Uniaxial load ($\sigma_2 = \sigma_3 = 0$)



Biaxial load ($\sigma_2 = 6.7$ N/mm², $\sigma_3 = 0$)



Triaxial Load ($\sigma_2 = 7.2$, $\sigma_3 = 3.6$ N/mm^2)

Fig. 10 Three systems of variable multiaxial stress used in the experiments