

Performance Analysis of Fault-Tolerant Multistage Interconnection Networks

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IN
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by

SANDEEP SHARMA

Supervised by

Dr. P.K. Bansal

DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING
THAPAR INSTITUTE OF ENGINEERING & TECHNOLOGY
(Deemed University)
PATIALA – 147 001

2000

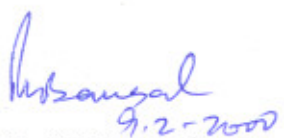
CERTIFICATE


I hereby certify that the work presented in the thesis entitled "**Performance Analysis of fault-tolerant Multistage Interconnection Networks**" in fulfillment of the requirements for the award of the Degree of Master of Engineering in Computer Science is being submitted in the Department of Computer Engineering, Patiala. It is further certified that the work carried out for the thesis is an authentic record done under the supervision of Dr. P.K.Bansal.

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other university.


(SANDEEP SHARMA)

This is to certify that the above statement made by the candidate is correct to the best of our knowledge.


(Dr. P.K.Bansal)
Professor
Department of
Computer Sc. & Engg.
TIET, Patiala


Head
Department of
Computer Sc. & Engg.
TIET, Patiala


Dean
Academic Affairs
TIET, Patiala

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ABSTRACT

The effectiveness of a parallel or distributed system is often determined by its communication network. In order to operate more efficiently a network is required to provide low latency and be able to handle large amount of traffic. Multistage Interconnection Network plays a vital role on the performance of these multiprocessor system. In this thesis Irregular MIN's (MFT, FT, RMDOT, MDOT) are analyzed in terms of reliability, bandwidth, probability of acceptance and cost effectiveness. Also the existing FT and MDOT networks have been modified and named as MFT & RMDOT for better performance

The permutation passable determines the data routing capability in the presence and absence of faults. This parameter is also analyzed in terms of Identity and Incremental permutation layout for regular MINs i.e. for ABN, ASEN-2 & irregular MIN's i.e for MFT and FT networks and the result shows that ABN network is better than ASEN and MFT is better than FT.

The reliability of a network is evaluated in terms of MTTF. MTTF has been evaluated for MDOT, FT & MFT and results of MTTF shows that MFT provides an improvement over the performance of the other networks. It is also found that probability of acceptance, bandwidth and cost effectiveness parameters are better for RMDOT and MFT as compared to MDOT & FT.

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CHAPTER –1

INTRODUCTION

1.1 Trends Towards Parallel Processing

Today is the Era of parallel processing and according to Sidney Fernbach: “Today’s large computers (mainframes) would have been considered ‘Supercomputers’ 10 to 20 years ago.” From the applications point of view, there are levels of sophistication.

- Data processing
- Information processing
- Knowledge processing
- Intelligence processing

Parallel processing is an efficient form of information processing which emphasize the exploitation of concurrent events in the computing process. To achieve parallel processing is required the development of more capable and cost-effective computer systems. Earlier computer manufactures started with the development of systems with a single central processor, called a uniprocessor system. Today ,we can extend the computer structure to include multiprocessors with shared memory space and peripherals, named multiprocessor systems.

1.2 Multiprocessor System

A computer system having two or more CPU’s is called a multiprocessor system. We can say it is the advent of the microprocessor that has encouraged the development of

multiple processor systems. As microprocessors are inexpensive and occupy little space, it is feasible to interconnect a large no. of processors into one composite system under a common control. Multiprocessor is a computer system that consists of several CPU's or processing elements (PE's) .

Multiprocessor system organization is determined primarily by the interconnection structure to be used between the memories and processors. There are three ways to provide interconnection:

- 1) Time shared common bus
- 2) Cross bar switch network
- 3) Multiport memories

1.2.1 Classification Of Multiprocessor

Multiprocessor can be classified by the orientation of their main memory systems. There are two ways to represent multiprocessor:

- 1) Shared memory system
- 2) Distributed memory system

In shared memory system there is a long common memory accessed by all the processing elements (PE's) is shown in Fig. 1.1 In distributed memory system, each processor has its local main memory .In another way we can say distributed are loosely coupled system as shown in Fig 1.2.

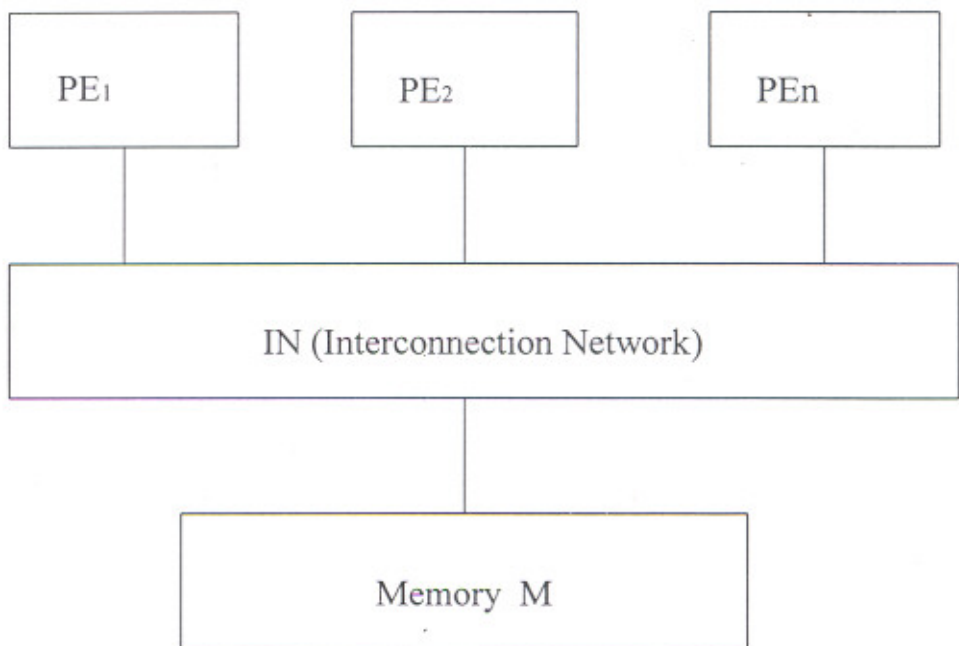


Fig 1.1 Shared Memory system

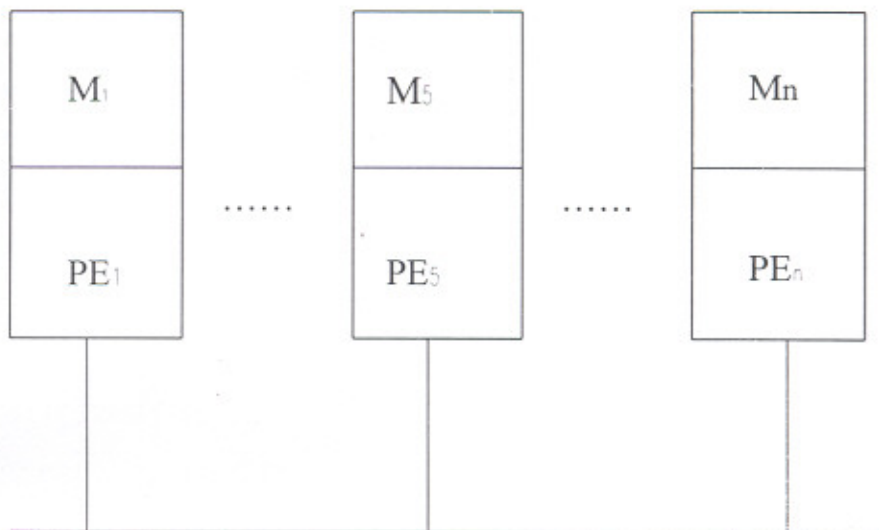


Fig 1.2 Distributed Memory system

1.3 Network

A network is an interconnected collection of autonomous computers. Two computers are said to be interconnected if they are able to exchange information. The connection can make up through links.

A typical IN consists of a no. of switching elements(SE's) and interconnection links. A SE is a $n \times n$ crossbar switch (i.e n number of inputs and n number of outputs) shown in Fig. 1.6 .Interconnection functions are realized by properly setting control of switching elements.

1.3.1 Interconnection Network

In a complex collection of switches (nodes) and links (edges) permitting CPU's in a multiprocessors system to communicate among them selves or with memory modules to provide fast, reliable, efficient and fault-tolerant communication at a reasonable cost are the main consideration for designing an IN. Three interconnection network structures using multiprocessors are shown in Fig 1.3,1.4,1.5.

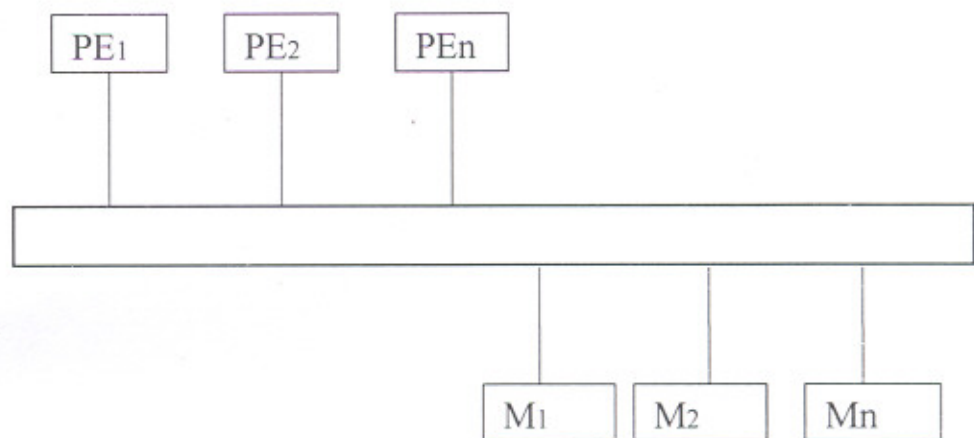


Fig 1.3 Single bus

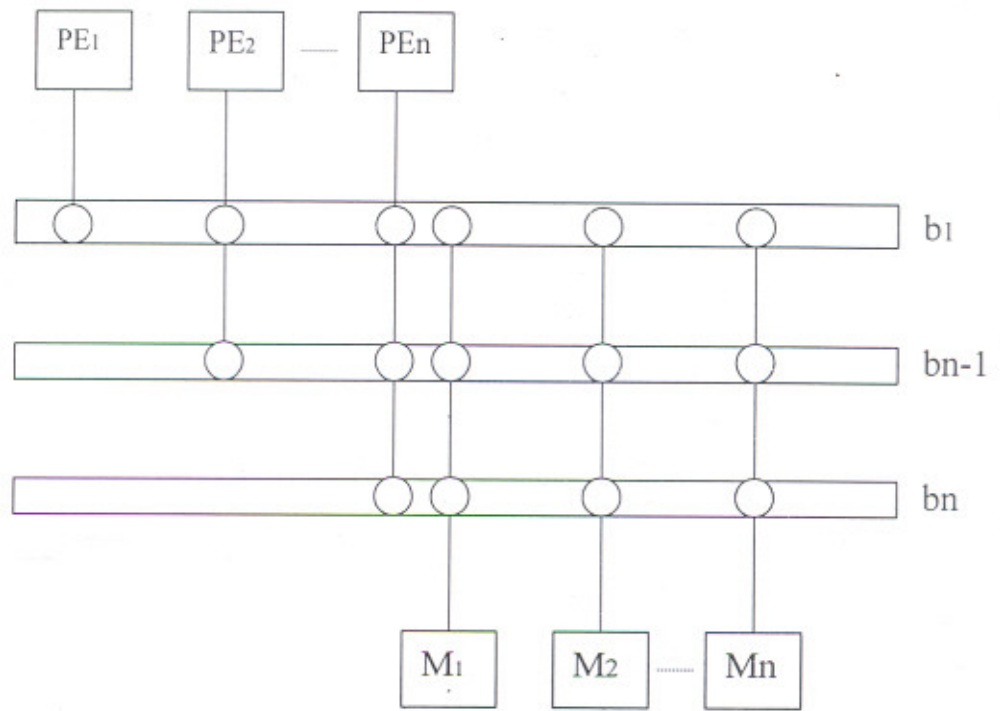


Fig 1.4 Multiple buses

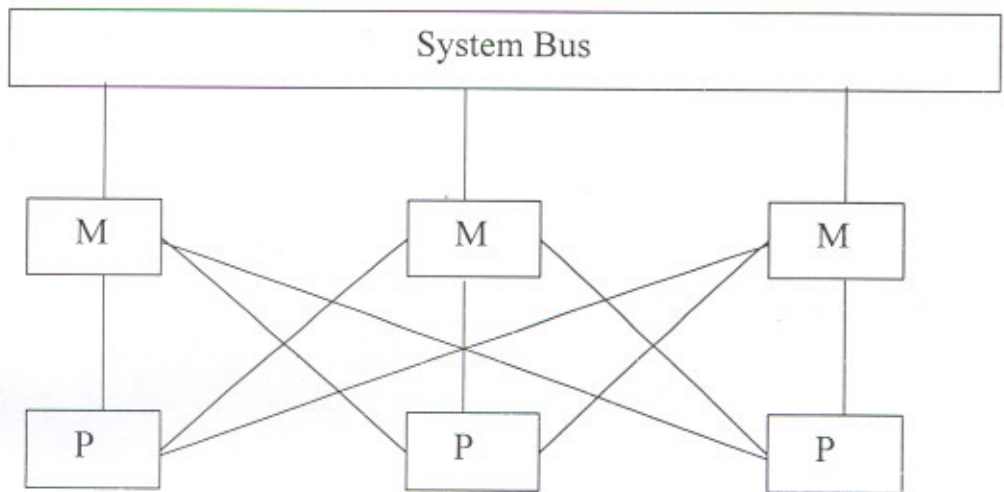


Fig 1.5 Crossbar Network

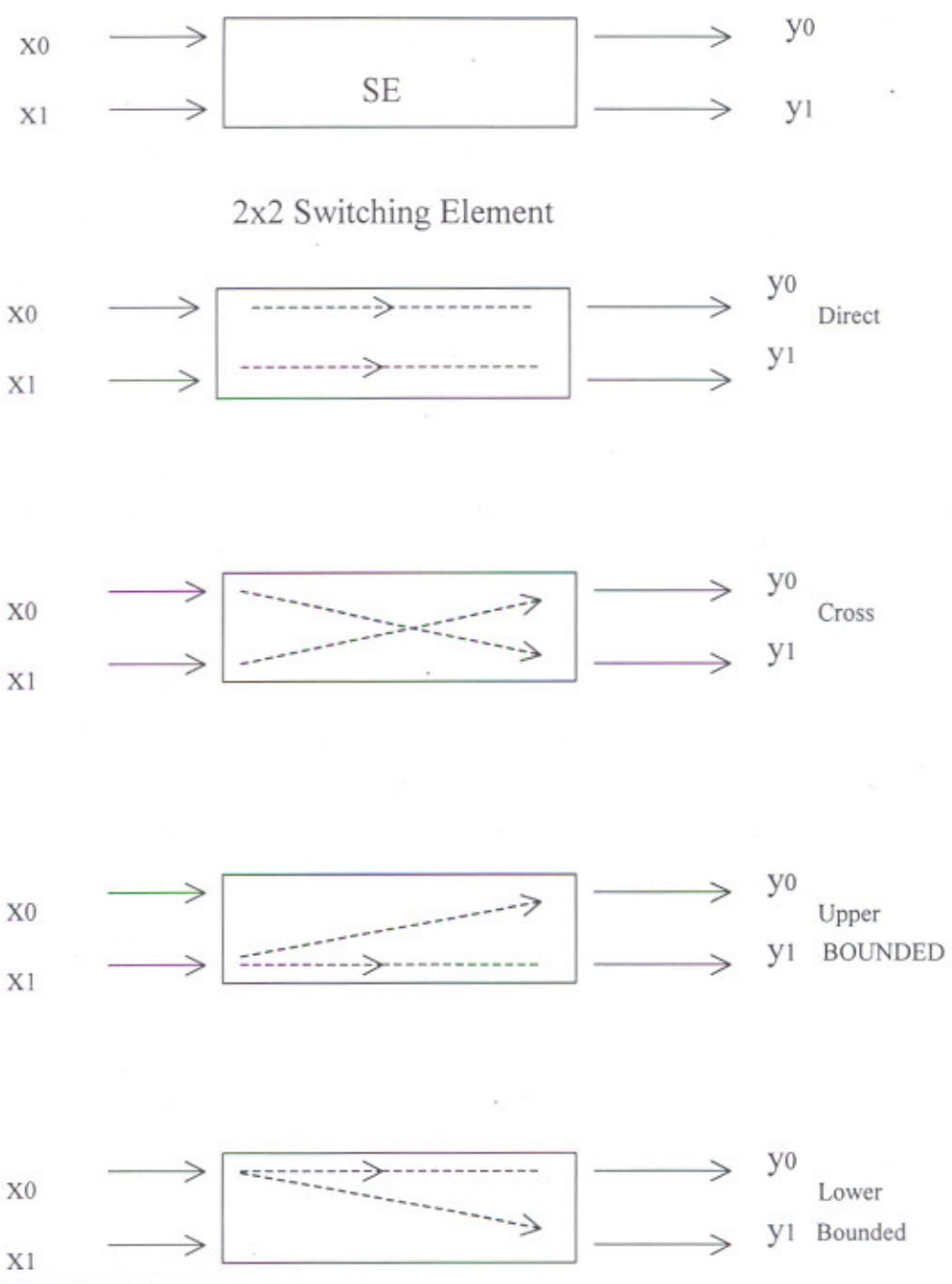


Fig 1.6 2x2 Switching Element with four Interconnection states

1.3.1.1 Goal Of IN

Design a network that combines full connection capability with graceful degradation in spite of the existence of faults.

1.3.1.2 Operation Characteristics Of IN

There are three basic fundamental decisions on which the architecture of IN depended.

1) Communication Mode

The timing control or communication of an IN can be either synchronus or asynchronus. In synchronus mode a single central global clock broadcast the clock signal to all devices on the IN , whereas in asynchronus mode, system works without a global clock perform an independent operation.

2) Control Strategy

Distributed control or centralized control can manage the switching elements. In distributed control all the requests are handled independently, but in centralized a global controller receives all requests and transmits the message in the IN's.

3) Data Transmission

There are two switching methodologies – circuit and packet used for the transmission of data in an IN. In circuit switching a physical path is established between source to destination but in case of packet switching the information is broken in small pieces and is send in the form of packets.

1.3.2 Multistage Interconnection Networks

As a compromise between two extremes (time-shared and crossbar networks) and various operation characteristics of an IN, multistage interconnecting network were introduced. A multistage network is capable of connecting an arbitrary i/p terminal to an arbitrary o/p terminal. Generally a MIN consists of more than one stages of small interconnection networks called switching elements (SEs).

A fundamental requirement of a MIN is that it should be able to connect every source to every destination.

1.3.2.1 Connection Issues

The following factors are the performance measures of MINs

- (a) Bandwidth
- (b) Probability of acceptance
- (c) Through put
- (d) Reliability
- (e) Permutation
- (f) Cost
- (g) Path length
- (h) Fault - tolerance

(a) **Bandwidth**

The network bandwidth is defined as the number of PE requests honoured per unit of time. A high network bandwidth is often desired at reasonably low network cost.

For a NxN MIN

$$BW = N * P_a * P_i$$

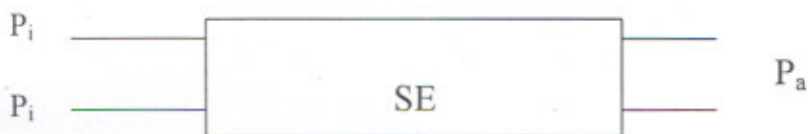
Where P_i —Probability of issuing a request at the input stage of the network by a source.

P_a -- Probability of acceptance.

N -- No. of I/p's = No. of O/p's

(b) **Probability Of Acceptance**

The Ratio of expected no. of successful requests to the expected number of requests submitted by the sources without getting blocked by other requests or connection in the network called probability of acceptance



$$P_a = 1 - (1 - P_i/2)^2$$

(c) **Through put**

The average no. of packets delivered from source to destination by network in unit time called Through put.

(d) **Reliability**

Reliability is one of the major design parameters for any network. Reliability means that in the presence of multiple CPU's if one goes down, the others may be able to take over its work, although at reduced performance. Let T be the time till the failure of a network occurs, then the possibility that it will not fail before time t defines the reliability

$$R(t) = P(T > t)$$

Where R denotes the reliability and P the probability.

(e) **Permutations**

It is a set of source to destination connections. Generally to minimize the communication overhead when routing, a permutation has to be scheduled. Scheduling a permutation consists of partitioning the set of nodes into n subset, E1, E2-----, En for some n, such that for every i = 1,2,3 -----n, the paths originating from the Ei do not conflict over links i.e. they can be established simultaneously and their correspondence message can be delivered in parallel.

1.3.2.2 **Reliability Model**

Reliability of a network can be measure in two ways:

- (1) Series Reliability
- (2) Parallel Reliability

(i) **Series Reliability Model**

A system is said to be in series if all the components of this are involved during the operation. The failure of any component means failure of whole system. The important characteristic of a series model is that its reliability is always worse than the poorest component in it. Reliability of a series type of system shown below .

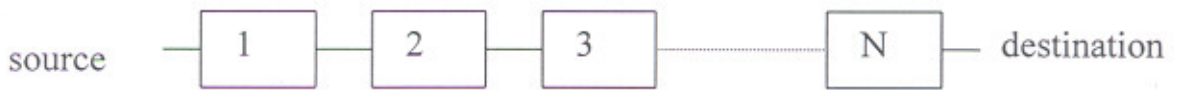


Fig 1.7 Series system

It is given by the product of reliabilities of individual components in it.

$$R_{\text{series}}(t) = P_1(t) \cdot P_2(t) \cdot P_3(t) \dots\dots\dots P_n(t) = \prod P_i(t)$$

Where P_i is the Reliability of i th components

(ii) **Parallel Reliability Model**

A system is said to be in parallel if the successful functioning of any one of the component leads to the success of the system. System fails only when all the component fails shown in Fig. 1.8

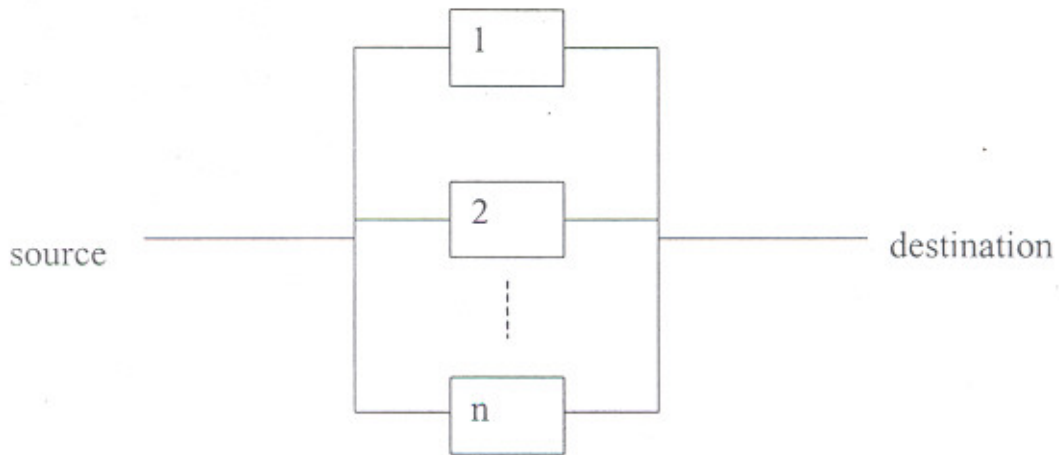


Fig 1.8 Parallel system

Reliability of a parallel type of system is given by:

$$R_{\text{parallel}}(t) = 1 - \sum_{i=1}^n (1 - P_i(t))$$

In general, if the time to failure of components is exponentially distributed then:

$$P_i(t) = e^{-\lambda_i t}$$

λ is the failure rate of i th component.

$$R_{\text{series}}(t) = \sum_{i=1}^n [e^{-\lambda_i t}]$$

and

$$R_{\text{parallel}}(t) = 1 - \sum_{i=1}^n [1 - e^{-\lambda_i t}]$$

1.4 Formulation of Problem

There are number of design parameters on which a network is based and are discussed earlier. Probability of acceptance, Reliability and Permutation passable ,fault tolerance and cost are the major design parameters of network. Much work has already been done in the design and analysis of regular MIN's but little works has been done on irregular MIN's . The major advantage of irregular MIN's are that they provide reasonably good performance at a considerably low cost in terms of the network goal as discussed above.

The following parameters are to be analyzed in this problem of analysis of irregular MIN's :

- 1). The system used for the interconnection of various component should be reliable in nature. Thus, the reliability analysis has been for the various irregular MINs in terms of mean time to failure (MTTF) is an important factor to be determined & analyzed. But, a survey of design is a prerequisite for such an analysis.

- 2). Besides, design & reliability considerations there is an other important parameter i.e permutation passability. (how many of the input requests are simultaneously able to pass through a given MIN , mature and reach successfully at the intended destination , as they occur in different permutations at different instant of time). Some of the requests will pass through the most favorable path ,others have to be routed through an alternate available path, in case the former is already busy in serving to some other request, remaining have to be simply dropped due to the unavailability of an alternate path between the given source & the destination. Thus it is an important parameter to be analysed. In this thesis, this parameter has been analysed for some of the standard irregular networks & some suggestions have been made to make some

changes in the design of these in order to boost the performance as far as this parameter is concerned. This is a parameter which is of great concern in networks with heavy load.

3). In this thesis some modifications in design of the existing irregular networks have been suggested. The pros & the cons of these modifications in terms of the performance of the networks have been analyzed.

1.5 Organization Of Thesis

The thesis is organized as :

Chapter 1 contains the introduction. Chapter 2 contains a brief survey of regular & irregular networks. It also contains some modified irregular networks such as MFT(Modified FT) and RMDOT (Remodified DOT) . Chapter 3 deals with the analysis of permutation passable of ABN & ASEN-2. This includes a detail comparison of two MIN's both in terms of identical as well as incremental permutations. It also includes the analysis of modified network. Chapter 4 analyses the reliability , probability of acceptance , bandwidth and cost of Irregular MIN's both existing and modified networks. Finally , Conclusion & further scope of work has been presented in Chapter 5.

OVERVIEW OF MULTISTAGE INTERCONNECTION NETWORKS

2.1 Introduction

A large no of variety of MIN's (Multistage interconnection networks) have been proposed & analyzed by various researches are used to provide cost effective, high bandwidth communication in multiprocessor systems. A brief description of the various MIN's is given in this chapter. Broadly MIN's can be classified as shown in Fig. 2.1

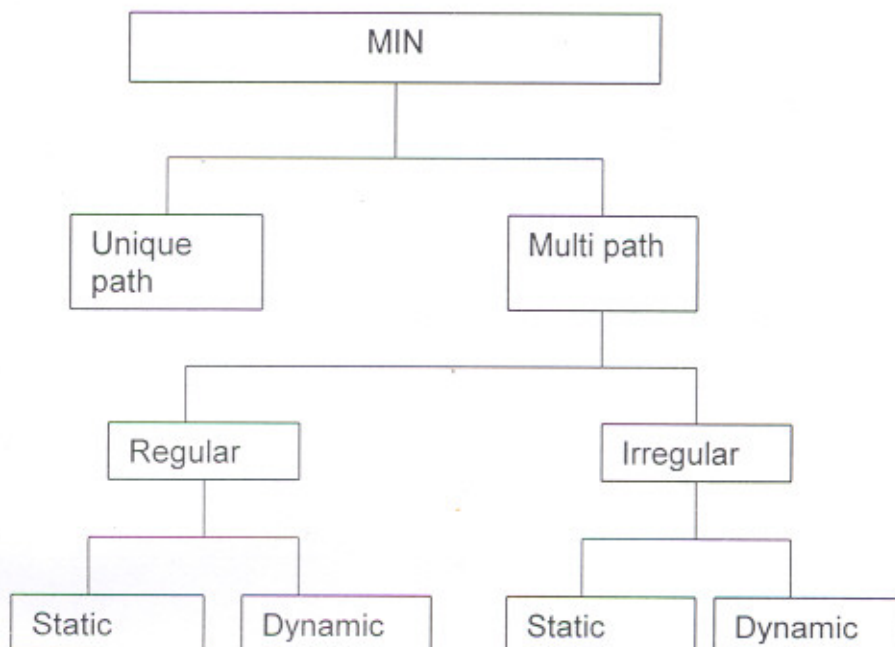


Fig 2.1 Classification of various MIN's

2.2 Unique Path MINs

Unique path MINs are characterized by the presence of the single unique path between any source-destination pair. MINs with $\log N$ stages have two important properties, a unique path exists from any source to destination and distinct source/destination paths have common links. Both of these lead to two serious disadvantages

1. The failure of even a single link or switch in the network disconnect several I/O paths, leading to lack of fault-tolerance and low reliability.
2. An I/O connection may be blocked by previously established connection (even if the output involved are distinct) thereby causing poor performance in a random access environment.

To overcome the problems of unique path MIN's several multi-path MIN's have been proposed to allow an alternate path to be chosen if conflicts arise with other connection or if faults develop in the network. The MIN's proposed in the literature are briefly discussed :

2.2.1 Generalized Cube

The generalized cube[2] network is a $N \times N$ MIN with $\log_2 N$ number of stages with N links connected to $N/2$ switches. Each switch is a 2×2 interchange box, individually controlled, and can be set to one of four states (straight, exchange, lower and upper broadcast). The generalized cube network of size 8×8 is shown in Fig. 2.2.

2.3 MULTI-PATH REGULAR STATIC Mins

In these types of networks there is no fork available at each of the SE, & hence the data is backtrack to the source for an alternate path if some fault occurs. The Extra Stage Cube (ESC) [2] and INDRA network [15] are the example of multi-path regular static MINs.

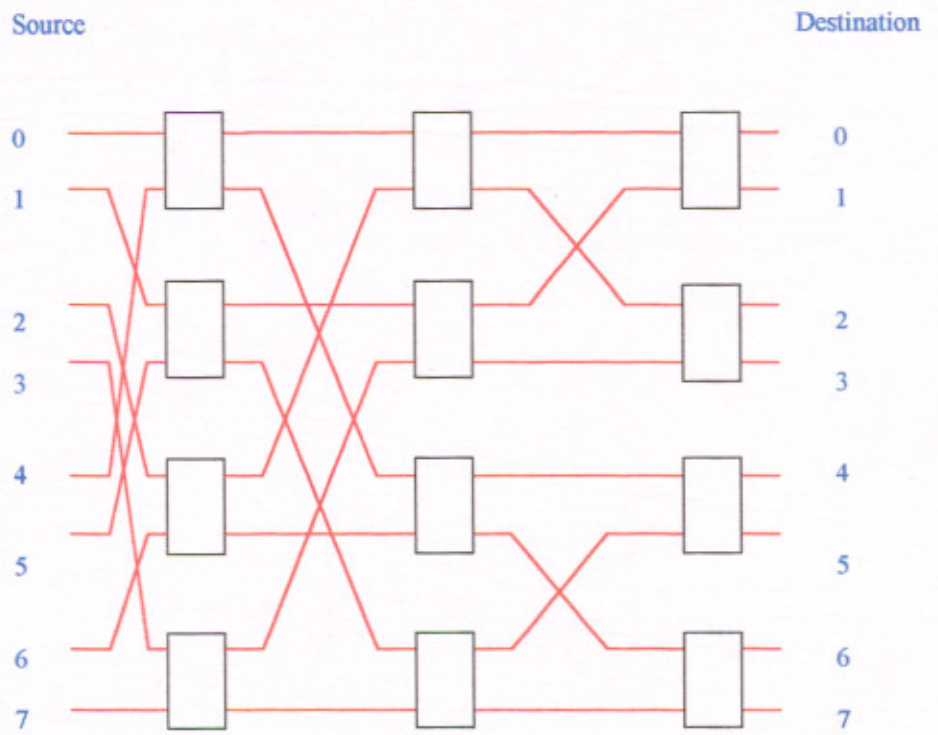


Fig. 2.2 Generalized Cube Network of size $N=8$

2.3.1 Extra Stage Cube Network (ESC)

ESC is formed from the generalized cube network by adding an extra stage to the input side of the network along with multiplexers and demultiplexers at the input and output stage respectively. In case of a fault in a given stage, the extra stage is enabled and the request gets routed through the fault free SEs of this extra stage but in the absence of faults, extra stage remains disabled. ESC is single switch fault-tolerant and robust in the presence of multiple faults. Due to the availability of the extra stage, both performance and reliability improves, but at the expense of increase in network cost. $N \times N$ ESC network has $\log_2 N + 1$ number of stages with N links connected to $N/2$ switches. ESC network of size 8×8 is shown in Fig. 2.3.

2.3.2 INDRA Network

INDRA networks incorporates multiple copies of a basic network typically Omega, but preface the copies with a distribution stage of switches. Stages are numbered 0 through n from input to output ($n = \log_R N$). Without using R redundant links to stage 0, there exists R paths between source – destination pair. By using redundant links R^2 paths, though not all disjoint, exists between any source to destination pair. In case of fault in any stage, data can be routed through one of the R^2 alternate path. The INDRA network is $(R - 1)$ fault-tolerant and is quite reliable and efficient in general. $N \times N$ INDRA network has $\log_R N + 1$ number of stages with RN links/stages. INDRA network of size 8×8 is shown in Fig. 2.4.

Extra Group Network (EGN) [6] and 3-Replicated network (3-rep) [6] MIN's also falls in this category.

2.4 Multi-Path Regular Dynamic MINs

In this type of MINs, a fork exists at every point in a stage, which can reroute the data to an alternate available path without resorting to backtracking. It also reduces the unnecessary time delay and is generally achieved by providing extra links in the network.

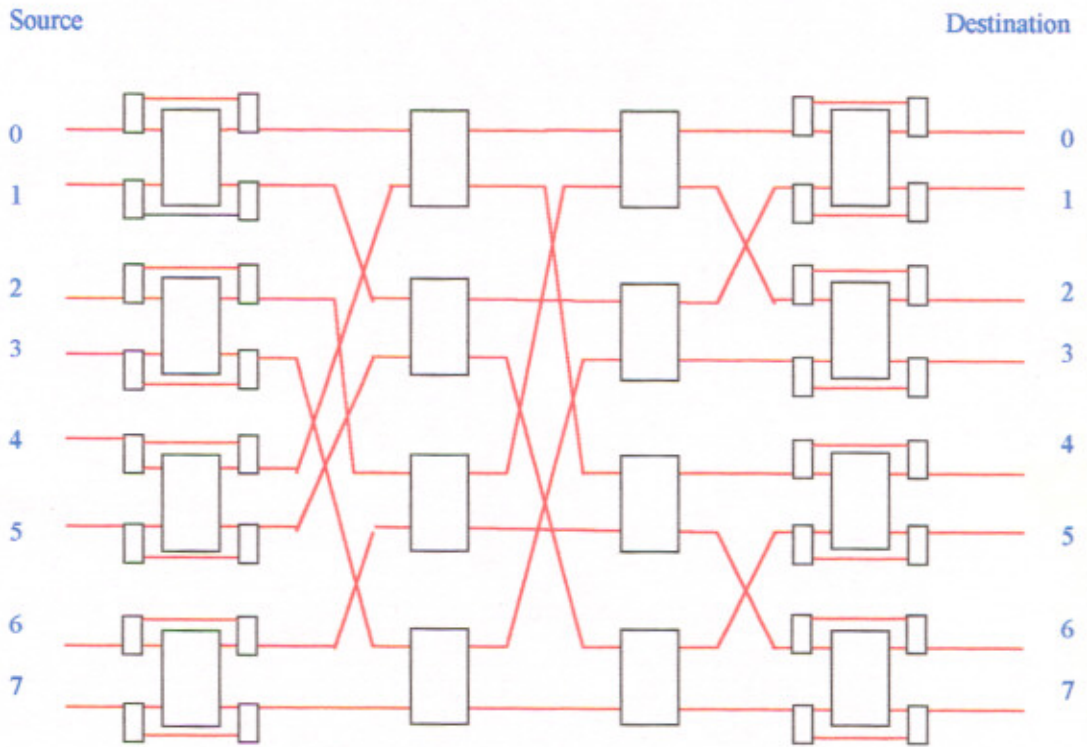


Fig. 2.3 : ESC MIN for N=8

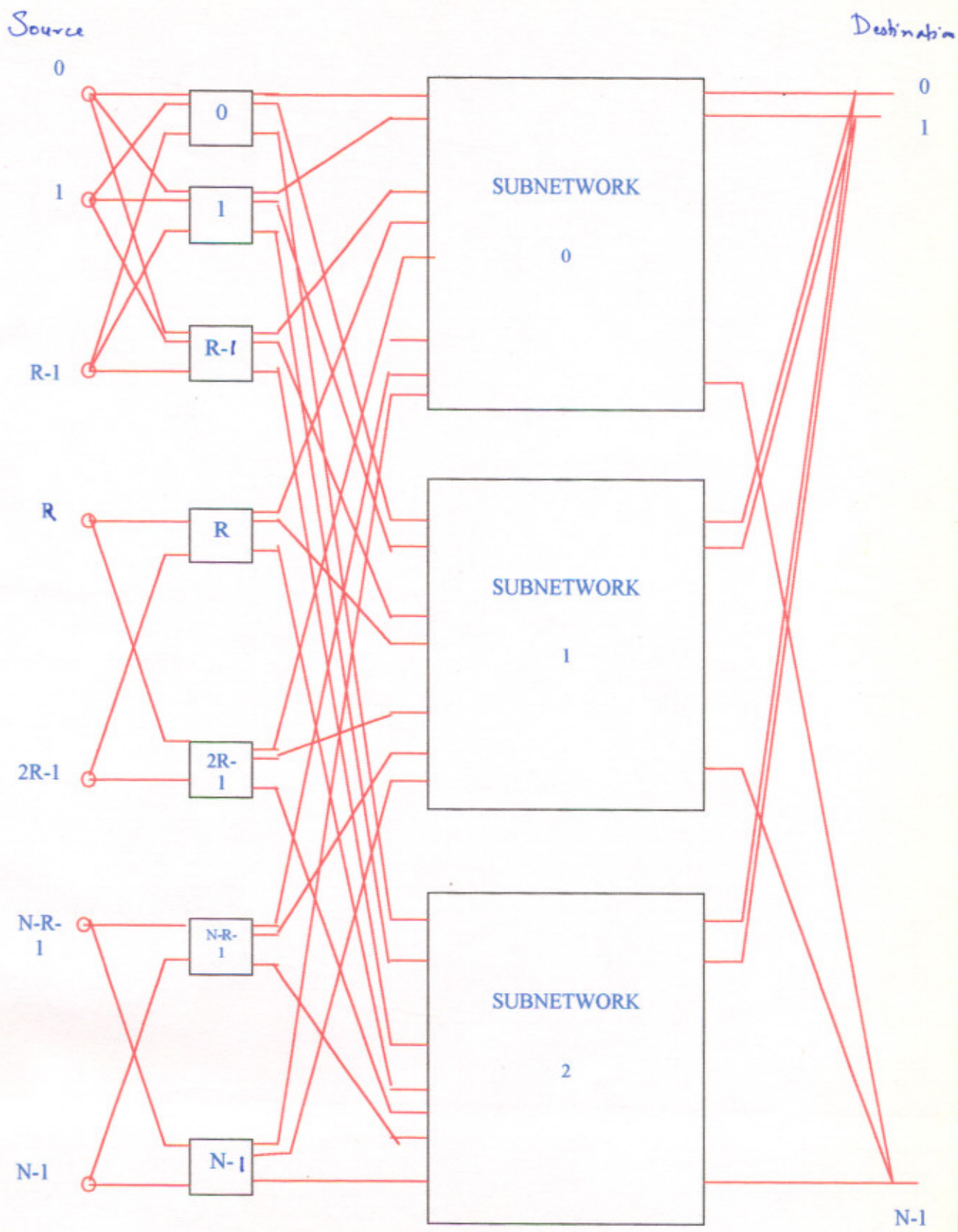


Fig. 2.4 INDRA Network as the union of R subnetworks.

The Inverse Augmented Data Manipulator (IADM) [6], F-network [13], Augmented Shuffle Exchange Network (ASEN) [11] and Augmented Baseline Network (ABN) [4] are the examples of multi-path regular dynamic MINs.

2.4.1 INVERSE AUGMENTED DATA MANIPULATOR (IADM)

Each switch of this network is 3 x 3 selector. To improve upon the reliability and performance aspects of unique path MINs extra intrastage links are provided. Unlike the generalized cube, delta, and omega networks, the IADM can tolerate some faults because of multiple paths between a source S and destination D, if $S \neq D$. $N \times N$ IADM network has $\log_2 N$ number of stages with $3N$ links per stage connected to N switches. IADM network of size 8x8 is shown in Fig. 2.5.

2.4.2 F-Network

F-network basically adds links to the generalized cube network. At each stage except the output stage, two different switches can be selected while maintaining the same destination. F-network assumes :

- 1) Failure of only internal switches
- 2) failed switches are unusable
- 3) faults occur independently.

F-network is single switch fault-tolerant and robust in the presence of multiple faults. $N \times N$ F-network has $\log_2 N + 1$ number of stages with $4N$ links per stage connected to N switches. F-network of size 8x8 is shown in Fig. 2.6.

2.4.3 AUGMENTED SHUFFLE EXCHANGE NETWORK (ASEN-2)

ASENs have $(\log_2 N - 1)$ stages (two stages of $N/2$ SEs per stage of size 3x3 and one stage of $N/2$ SEs of size 2x2), plus N 2x1 multiplexers and N 1x2 demultiplexers.

In this network, some loops are introduced to avoid faults and busy roots, the loops formed in all stages except the final stage are such that for every loop there exists another loop which is connected to the same set of switches in the next stage. Such pairs of loops are called conjugate loops. A general principle to be followed in forming the loops is to

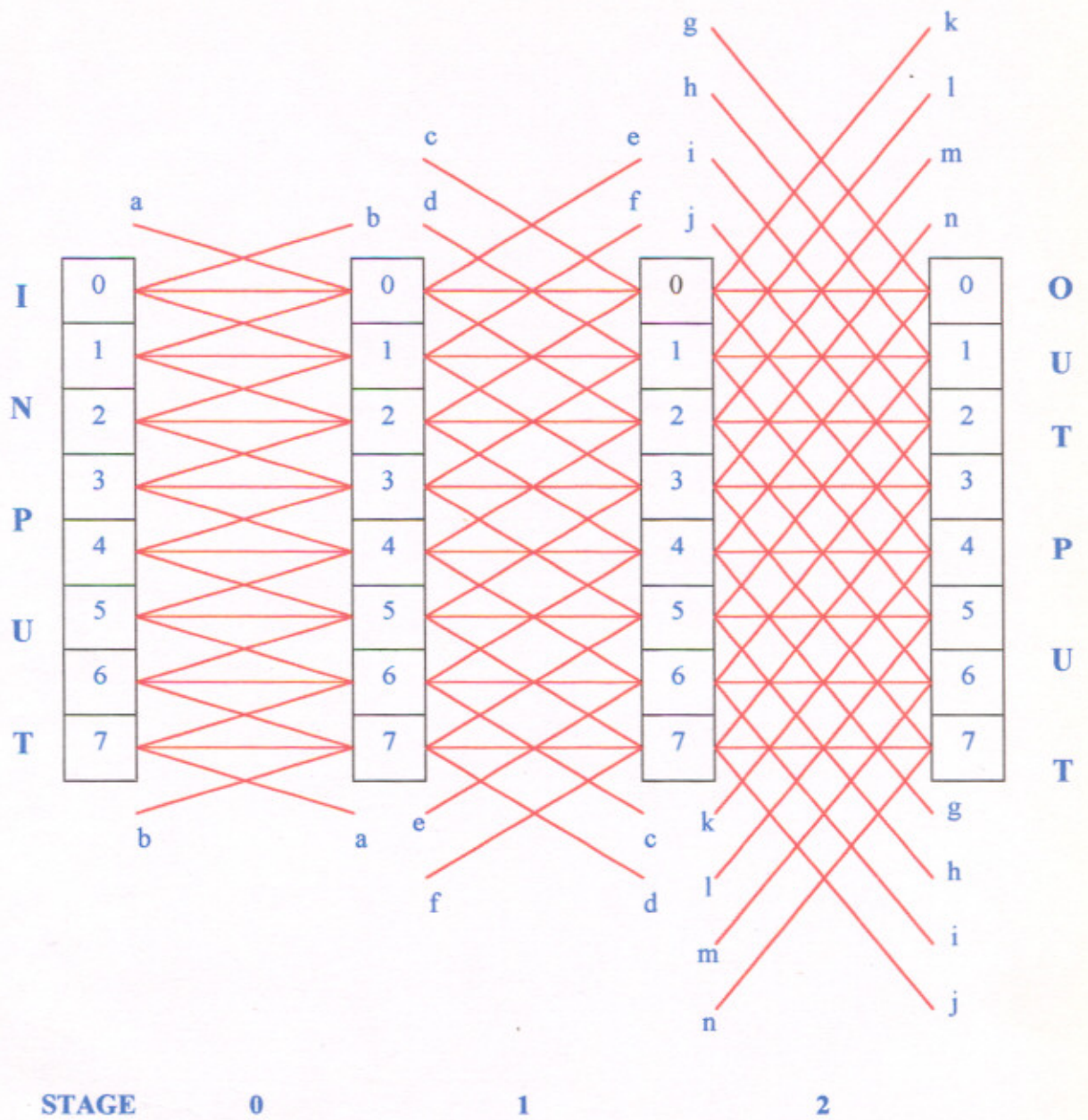
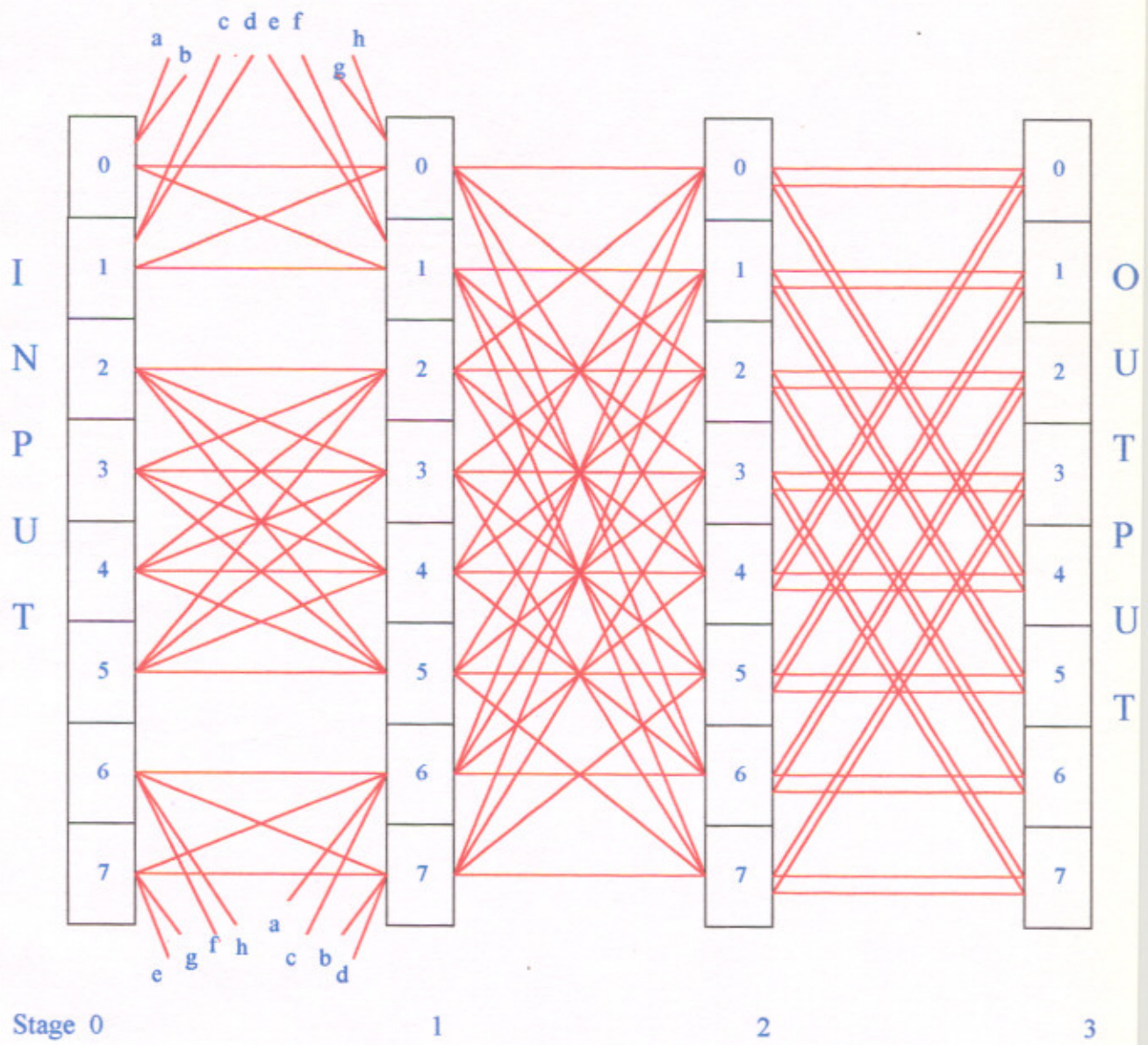


Fig. 2.5 IADM MIN of size $N=8$



**Fig. 2.6 F – network of size
N=8**

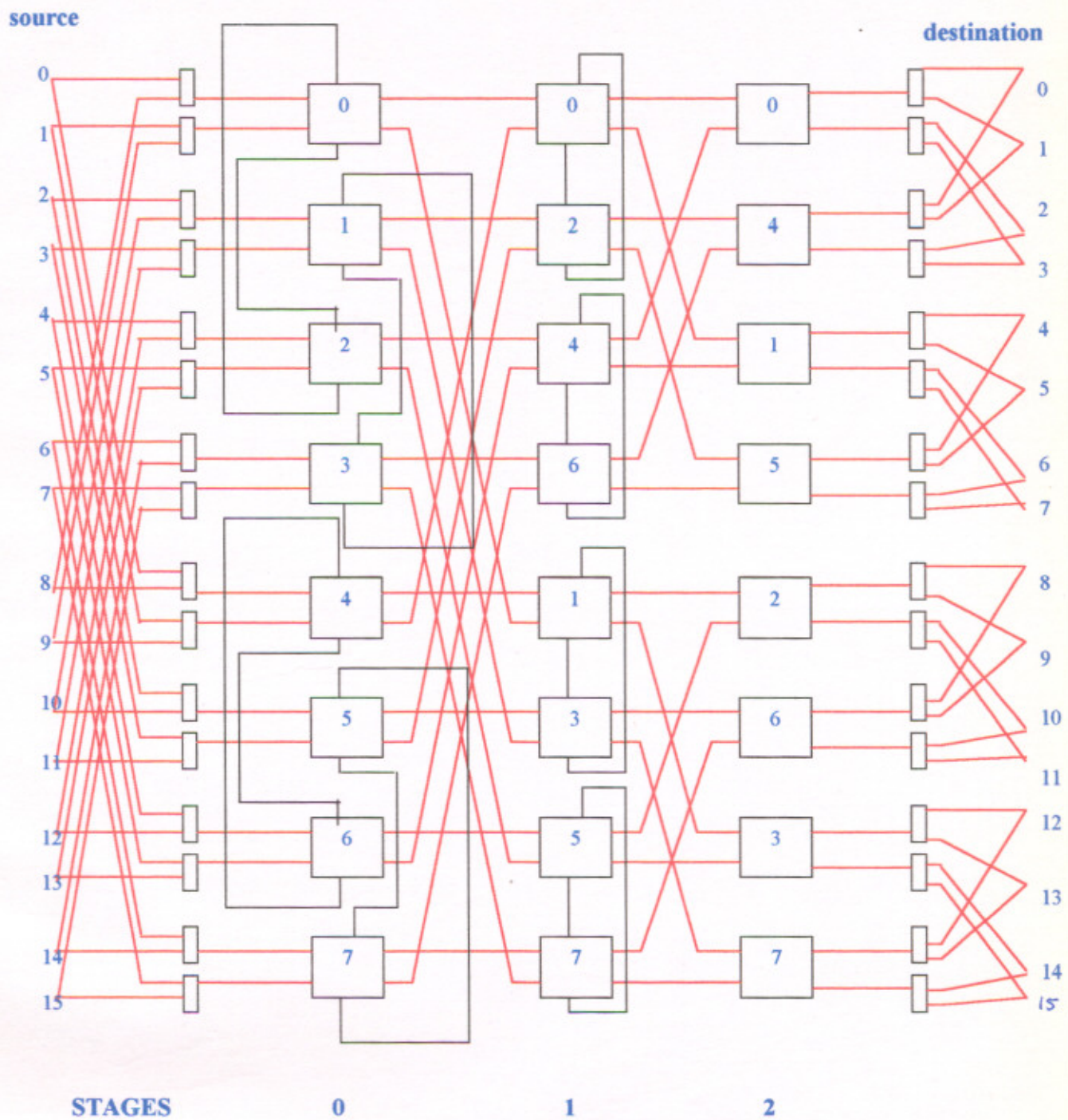


Fig. 2.7 : ASEN-2 MIN for N=16

ensure that if two switches are part of a loop, their conjugate switches will also be part of a loop. ASEN-2 network of size 16x16 is shown in Fig. 2.7.

2.4.4 AUGMENTED BASELINE NETWORK (ABN)

The NxN network has $\log_2 N - 2$ stages. The switches in the last stage are of size 2 x 2 and the remaining switches in stage 1 through $\log_2 N - 3$ are of size 3 x 3. There is one 4 x 1 multiplexers for each input link of a switch in stage 1 and one 1 x 2 Demultiplexers for each input link of a switch in stage $n - 2$. This network provides two disjoint paths for each source-destination pair, so ABNs are single switch fault-tolerant. When both the switches to which a source or a destination is connected becomes faulty, then that source or destination is disconnected from the rest of the network. Since one of the input stage has been replaced by 4:1 type of MUX, having low hardware complexity as compared to 3 x 3 SEs, this also reduces the cost of the network and thereby improving ratio of MTTF to cost. Reliability and performance of ABN is better than other regular MINs because it has one stage less than ASEN, and two stages less from SEN. ABN network of size 16x16 is shown in Fig. 2.8.

2.5 Irregular MINs

Irregular MINs are classified as either unique path and multi path.

2.5.1 Unique Path MINs

The unique-path irregular MIN is MDOT [6] network.

2.5.1.1 MODIFIED DOUBLE TREE NETWORK (MDOT)

This network has the advantage of flip control, (i.e. individual stage control) which effects the performance and reliability of the entire system, and distributed control (i.e. individual switch control). MDOT is also used in the construction of FT & FDOT -2 network. NxN MDOT network has $2n - 1$ (where $n = \log_2 N$) number of stages with $2^n +$

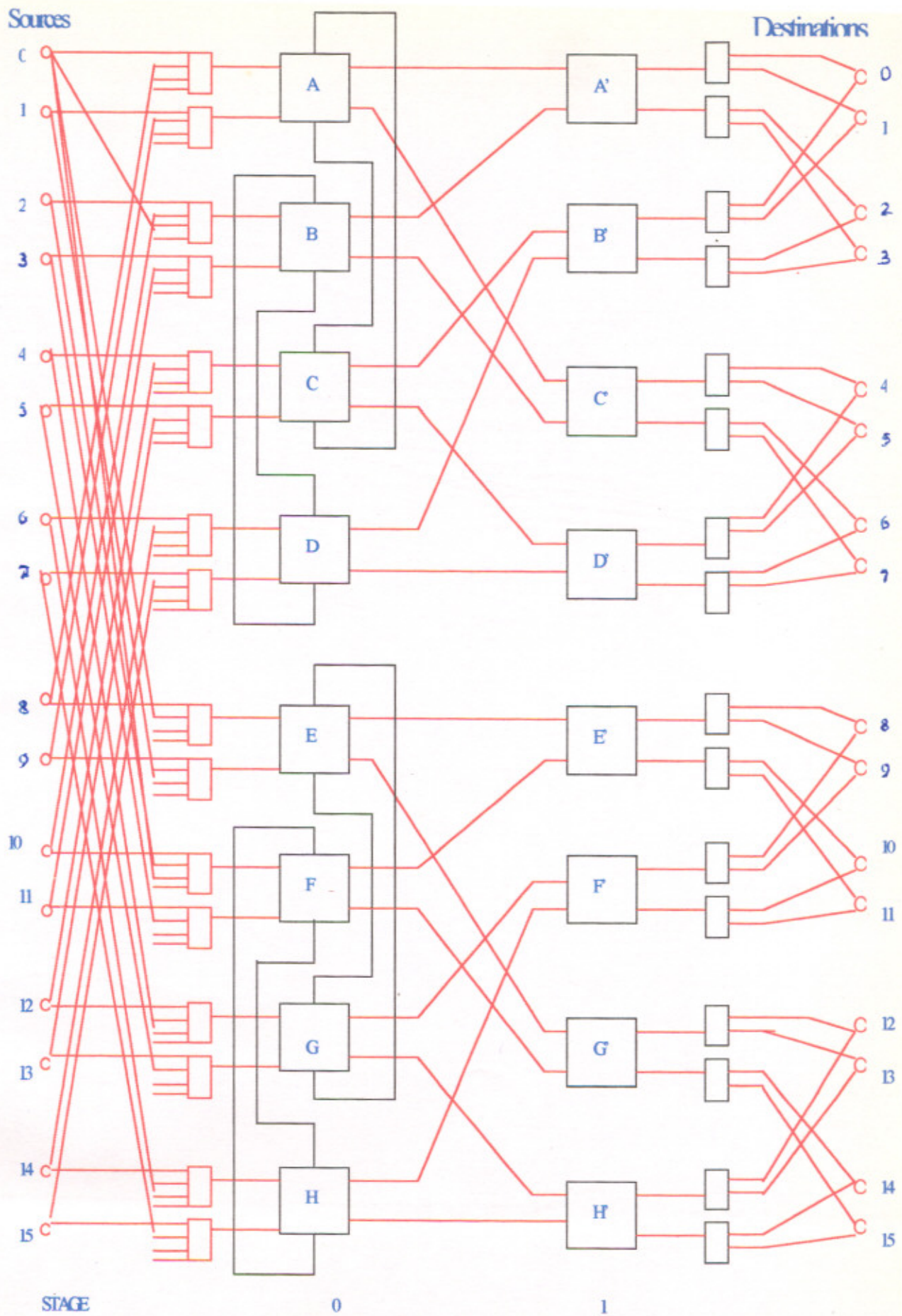


Fig. 28 16X16 Augmented Baseline Network (ABN)

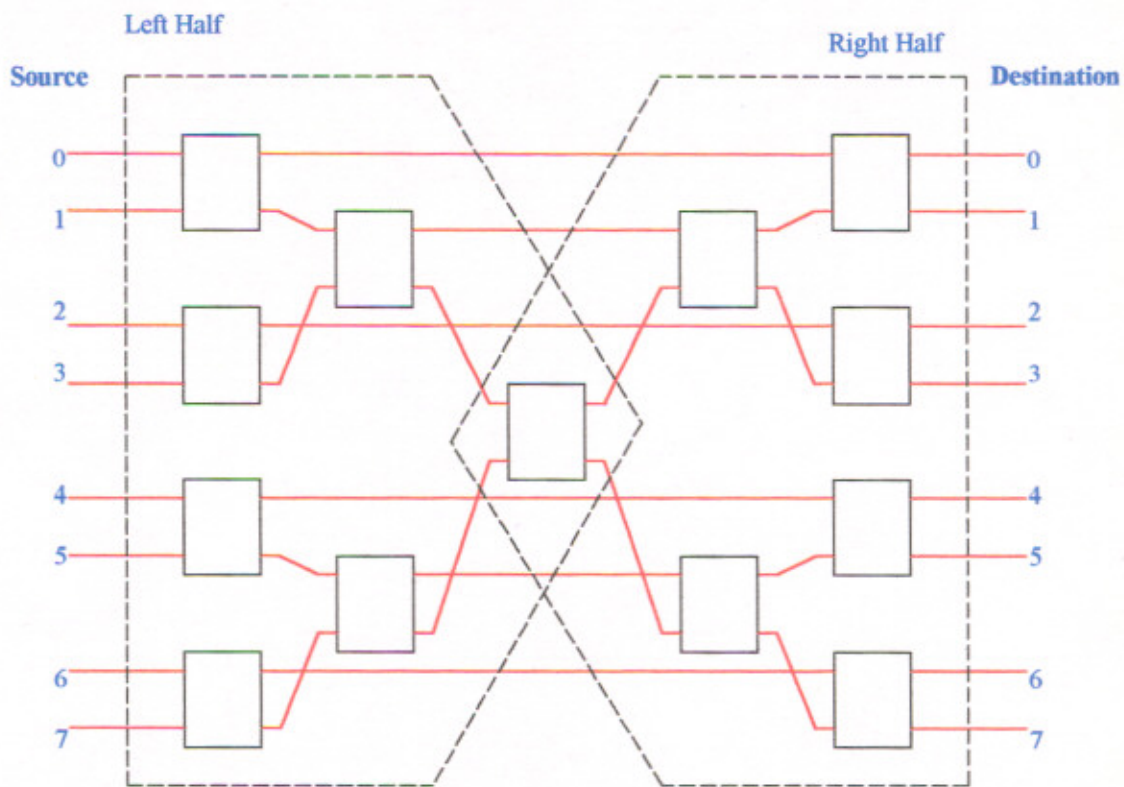


Fig 2.9 MDOT MIN of size N=8

(n+2) number of switches . First and last stage has 2^{n-1} switches of size 2x2. MDOT network of size 8x8 is shown in Fig. 2.9.

2.5.1.2 RMDOT (RE-MODIFIED DOUBLE TREE NETWORK)

RMDOT is a modification of the existing MDOT. In this network the stage(2n-3) i.e the central switch of MDOT has been removed which is the major bottleneck and gave severe routing problem. The other four switches are arranged in a such order to improve the permutation passability (in incremental case), reliability and probability of acceptance. The cost of this network is also less than MDOT. NxN RMDOT network has 2n-1 (where $n = \log_2 N$) number of stages with $2^n + (n+1)$ number of switches . First and last stage has 2^{n-1} switches of size 2x2. RMDOT network of size 8x8 is shown in Fig. 2.10.

2.5.2 Multi-Path Irregular MINs

Irregular Networks, though inherently multi-path, may or may not be fault-tolerant. The various multi-path MINs are FDOT (Fault Tolerant Double Tree Network)[3] and FT (Four Tree Network) [6]

2.5.2.1 Multi-Path Irregular Static MINs

Fault-tolerant Double Tree Network (FDOT) falls in this category.

2.5.2.1.1 FAULT-TOLERANT DOUBLE TREE NETWORK (FDOT)

FDOT have $3.N/2$ multiplexers or demultiplexers. FDOT-k network is an irregular type of network, with k independent sub-networks of size $(N/K \times N/k)$ and an extra one. The extra sub-network helps to enhance fault-tolerant capability and to keep a desired level of performance even in the presence of faults. FDOT-k network is k fault-tolerant. Depending on the source-destination pair, an irregular network supports multiple paths of different path lengths. Thus chances of blocking of a request are further reduced. As a result of which, performance of FDOT-k network tends to be better than other MINs.

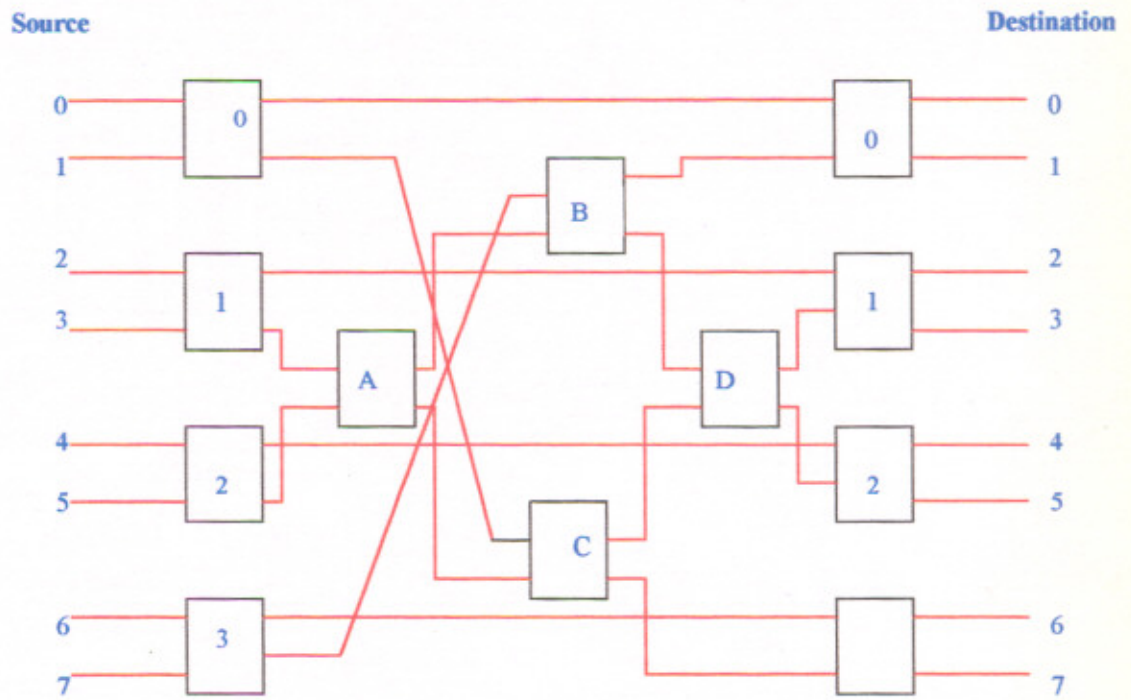


Fig . 2.10 RMDOT MIN of size N=8

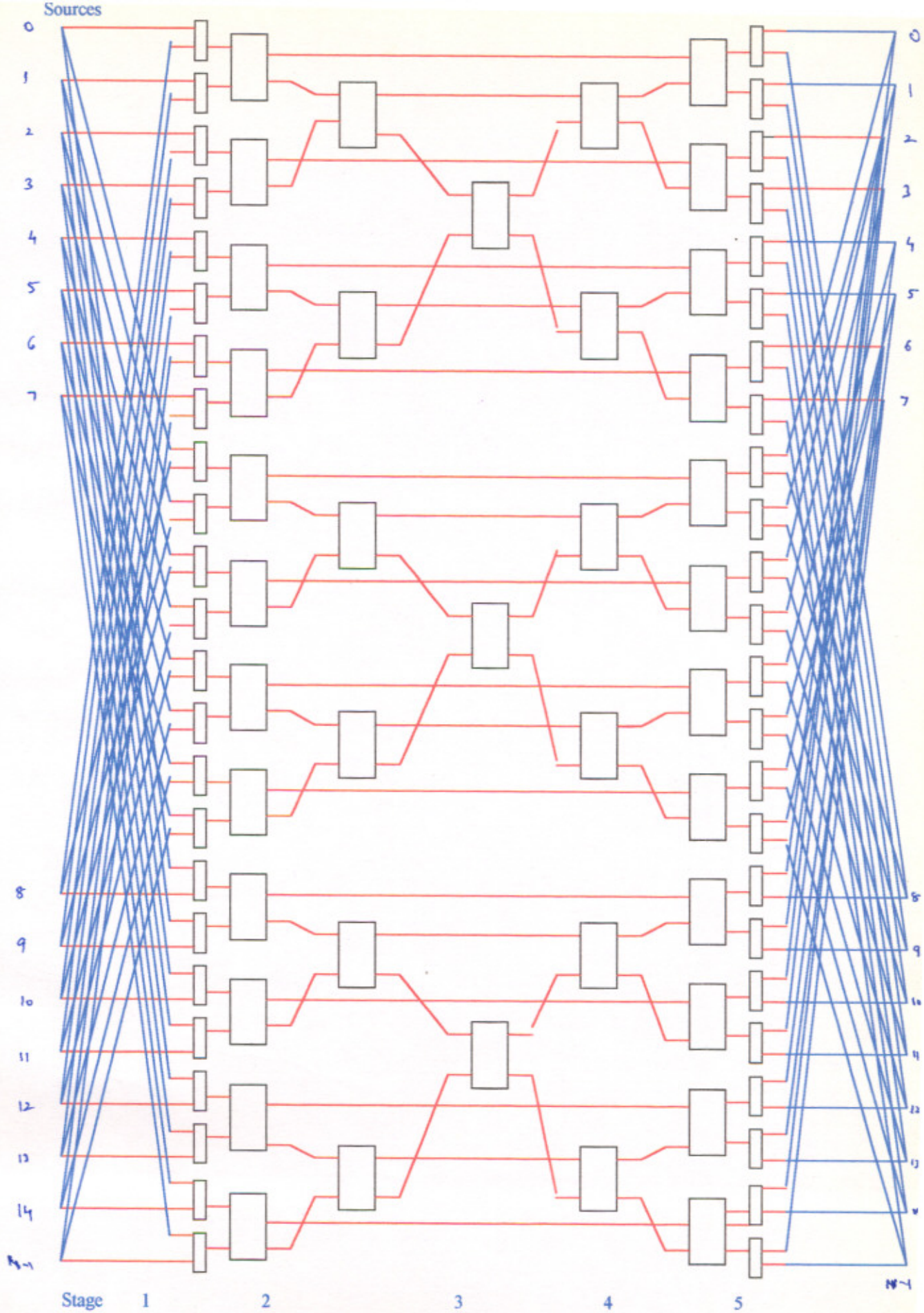


Fig. 2.11 FDOT-2 MIN for $N=16$

$N \times N$ FDOT network has $2n-1$ (where $n = \log_2 N/2$) number of stages with $3 \cdot 2^{n+1} - n$ number of switches. FDOT network of size 16×16 is shown in Fig. 2.11.

2.5.2.2 Multi-Path Irregular Dynamic MINs

The example of multi-path irregular dynamic MINs are Four Tree network (FT).

2.5.2.2.1 FOUR TREE NETWORK (FT)

A FT network of size $N \times N$ has $2 \log_2 N/2 - 1$ stages. Out of total $2^{n+1} + (3 \cdot n + 1)$ switches 2^{n-1} are of size 2×2 and the rest are of size 3×3 . There are 2^n 2×1 MUX and an equal number of 1×2 DEMUX. Each stage has exactly 2^{n-i} switches where $i=1,2,3,\dots,4$ and last stage has 2^{n-1} switches. FT network being an irregular network supports multiple paths of different path lengths.

This network is constructed with the help of two identical groups, each consisting of MDOT network of size $2^{n-1} \times 2^{n-1}$, which are arranged one above the other. The two groups are formed based on the most significant bit (MSB) of the source-destination terminals. Every 3×3 SE in a stage forms a loop with the corresponding numbered 3×3 SE of other sub-network in the same stage. Every source and destination is connected to both the subgroups by means of multiplexers and demultiplexers. In case the primary path is busy or faulty, requests will be routed through secondary path in the sub-network. Thus a fork exists at every point in a stage except the last which makes alternate routing feasible. FT network is single switch fault-tolerant. If both switches in a loop are simultaneously faulty then clearly some sources are disconnected from some destinations. FT network is more cost-effective than other multi-path irregular MIN with high reliability and performance. FT network of size 16×16 is shown in Fig. 2.12.

2.5.2.2.2 Proposed Modified Four Tree Network (MFT)

A MFT network of size $N \times N$ has $2 \log_2 N/2 - 1$ stages. Both stage i and stage $(2n-3)$ has exactly 2^{n-i} switches where $i=1,2,\dots,n+1$. A MFT network being an irregular network supports multiple paths of different path lengths. It also inherits the property of regular

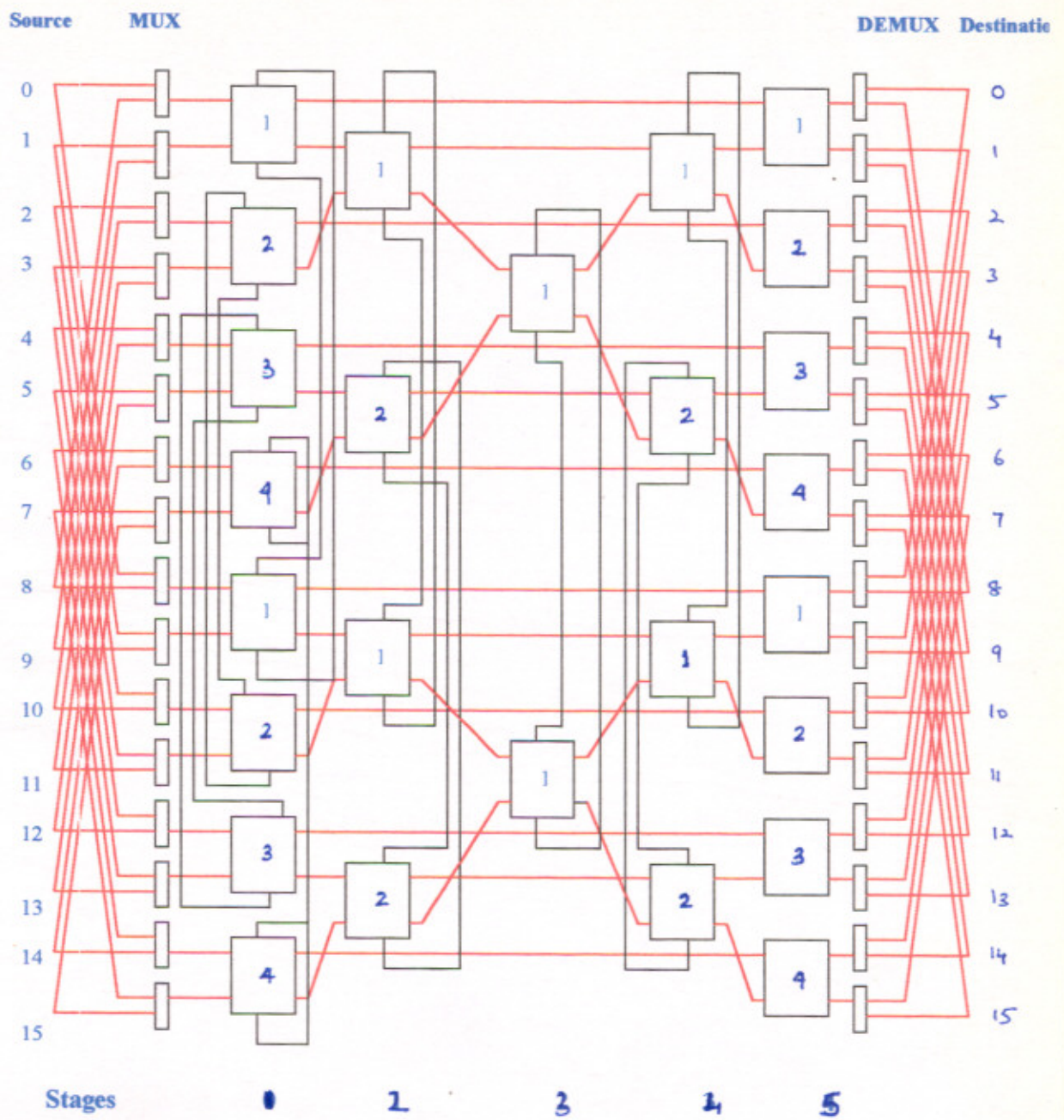


Fig. 2.12 FT MIN of size N=16

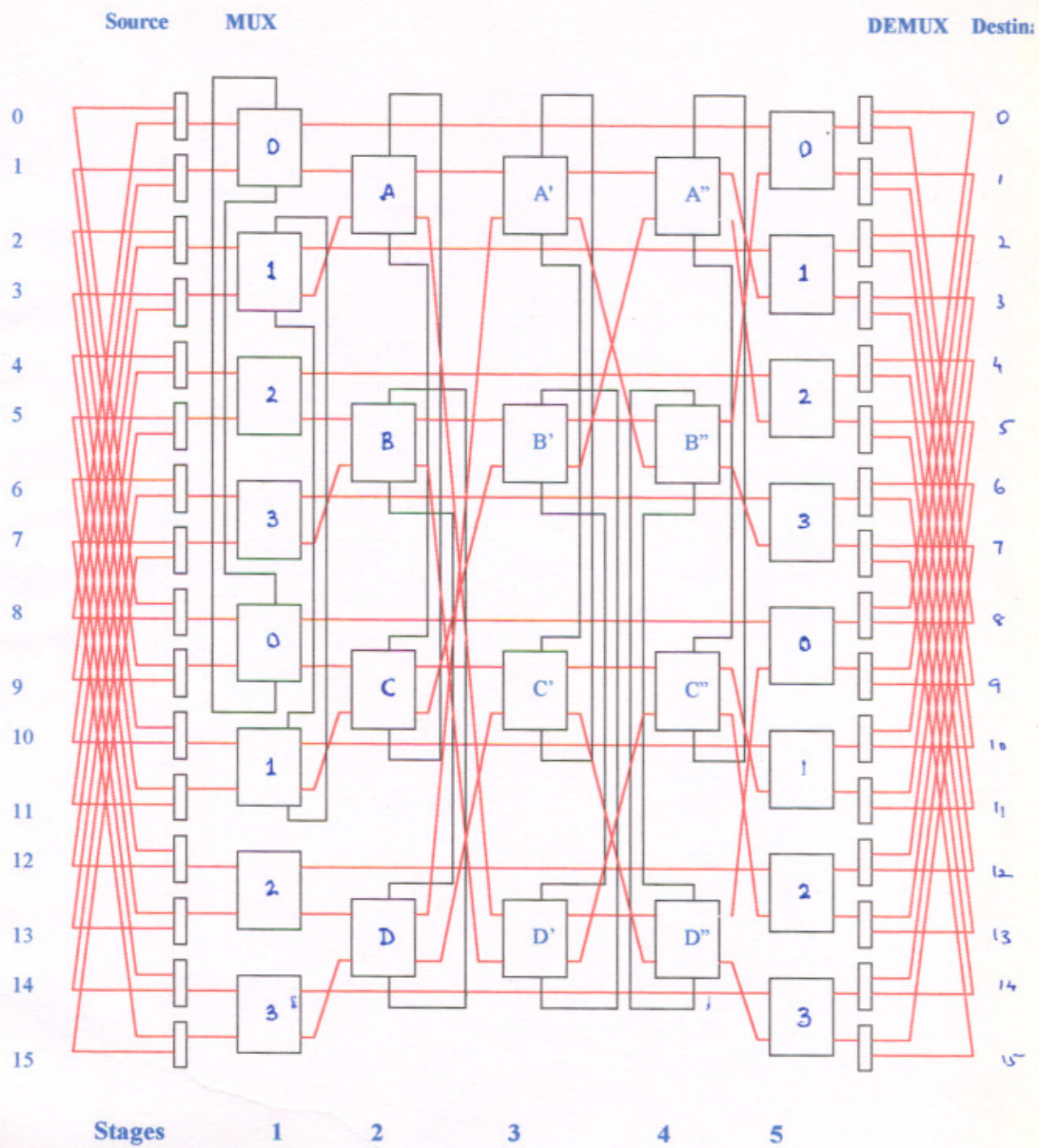


Fig. 2.13 MFT MIN of size N=16

networks since the number of switches are same in all the stages except the last and first stage of a network. Every 3x3 SE in a stage forms a loop with the corresponding numbered 3x3 SE of other sub-network in the same stage. Every source and destination is connected to both the subgroups by means of multiplexers and demultiplexers. The advantage of this network is - if both switches in a loop are simultaneously faulty then even some sources are connected to the destinations. FT network is more cost-effective than MFT but the performance (in terms of permutation passable) of MFT is better than FT. MFT network of size 16x16 is shown in Fig. 2.13.

2.6 Conclusion

This chapter deals with the overview of the existing regular and irregular MIN's and is very helpful for design purpose of any network. It also provides a full constructional features of all unique and multi-path, regular and irregular networks. It not only provides the present state of the art technology in interconnection networks but also gives an insight on the various steps/stages, to ultimate reach the present status. It also helps to know the various openings in the field, in which further work can be carried out.

CHAPTER -3

Permutation measures of Regular & Irregular Networks

A one to one correspondence between source to destination is called Permutation. Permutations has been analyzed by the various researchers in case of the MIN's but so far, nothing has been done to discuss the permutation passable in case of ASEN & ABN.

The request always pass from the most suitable path available if such path is busy then the request is pass through an alternate path. If no alternate path is available then the request has to be simply dropped or said to be having clash. One of the major aspects of permutation parameter is that it varies with pathlength.

To find out this parameter for a network, it is assumed that source and destination is represented by :

$$S_i \quad (\text{where } i=0,1,\dots,N-1)$$

$$D_i \quad (\text{where } i=0,1,\dots,N-1)$$

Permutation can be evaluated in two ways :

Identity Permutations

A one-to-one correspondence between same source and destination number is called Identity Permutation. In terms of source and destination this can be expressed by:

$$S_i = D_i$$

For $i= 0,1,\dots,N-1$

For example : A connectivity between source to destination for identity is shown in fig 3.1 and represented by :

Source (0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15)
Destination (0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15)

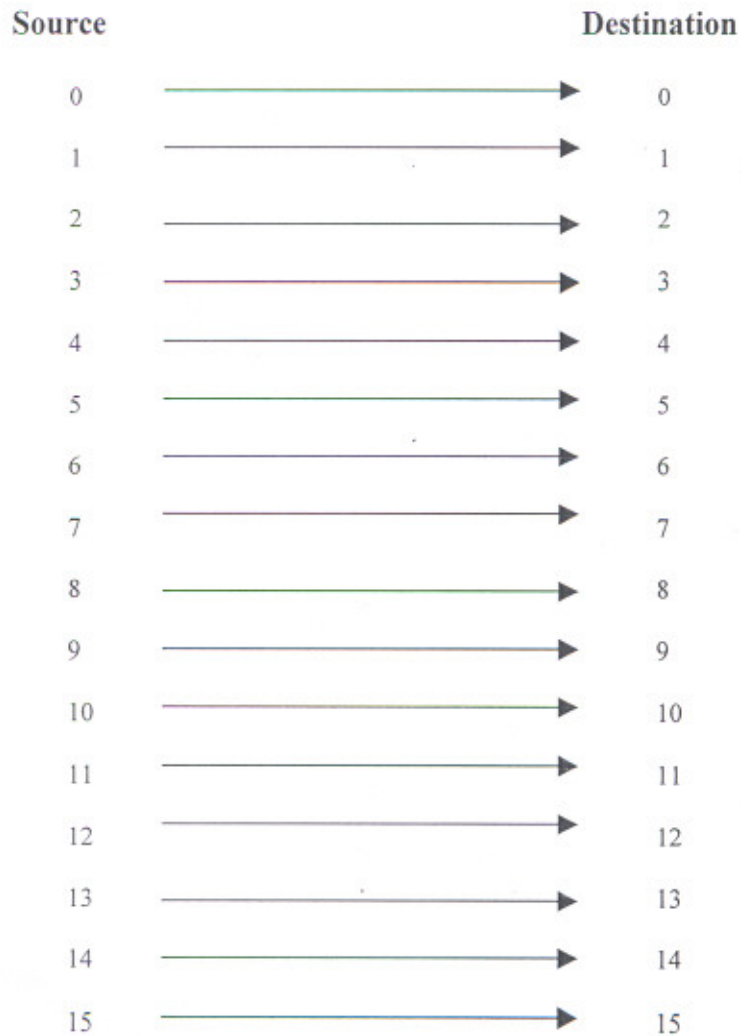


Figure 3.1 Layout of identity permutation for network of size 16

Incremental Permutations

Incremental means that each source is connected to destination in a circular chain. The layout is shown in fig. 3.2 and represented as :

Source	(0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15)
Destination	(4 5 6 7 8 9 10 11 12 13 14 15 0 1 2 3)

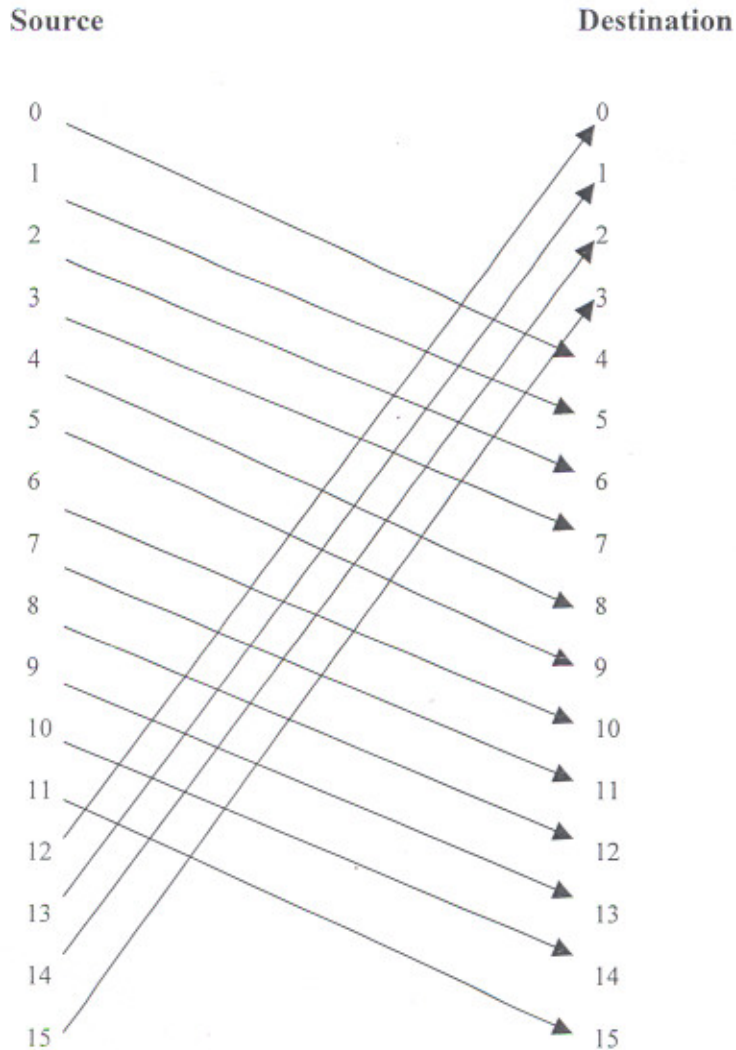


Figure 3.2 Incremental permutation layout for network of size N=16

There are two cases to find out the permutations

- If a fault is present in a single switch (Non-Critical Case)
- If the switches are faulty in a loop (Critical Case)

It is assumed that probability of issuing a request is 1.0.

3.1 Permutations For Regular MINs

3.1.1 Identity Permutation For ASEN-II

Case 1: Without fault

	Source	Destination	PL PATH LENGTH
0 → 0	:: 0	→ 0 → 0 → 0 → 0	3
1 → 1	:: 1	→ (4→6)→(4→6) → 4 → 1	5
2 → 2	:: 2	→ 2 → 1 → 2 → 4 → 2	3
3 → 3	:: 3	→ (5 → 7) → (6 → 4) → 0 → 3	5
4 → 4	:: 4	→ 2 → 4 → 1 → 4	3
5 → 5	:: 5	→ (6→4) → (0→2)→5→ 5	5
6 → 6	:: 6	→ 3 → 6 → 5 → 6	3
7 → 7	:: 7	→ (7 → 5) → (2 → 0) → 1 → 7	5
8 → 8	:: 8	→ 4 → 1 → 2 → 8	3
9 → 9	:: 9	→ (0→ 2)→(5 → 7)→ 6 → 9	5
10 → 10	:: 10	→ 5 → 3→6 → 10	3
11 → 11	:: 11	→ (1 → 3) → (7 → 5) → 2 → 11	5
12 → 12	:: 12	→ 6 → 5 → 3 → 12	3

13 → 13	::	13	→ (2→0)→(1 → 3)→7 →	13	5
14 → 14	::	14	→ 7 → 7 → 7 →	14	3
15 → 15	::	15	→ (3 → 1) → (3 → 1) → 3 →	15	5

Total Path length = 64

Total No. of request passes = 16

Average Path Length = $64/16 = 4.0$ Percentage Passable = 100

Case 2: A Single MUX Faulty

	Source		Destination	PL	
0 → 0	::	0	CLASH	0	
1 → 1	::	1	→ (4→6)→(4→6) → 4 →	1	5
2 → 2	::	2	→ 1 → 2 → 4 →	2	3
3 → 3	::	3	→ (5 → 7) → (6 → 4) → 0 →	3	5
4 → 4	::	4	→ 2 → 4 → 1 →	4	3
5 → 5	::	5	→ (6→4) → (0→2)→5→	5	5
6 → 6	::	6	→ 3 → 6 → 5 →	6	3
7 → 7	::	7	→ (7 → 5) → (2 → 0) → 1 →	7	5
8 → 8	::	8	→ 4 → 1 → 2 →	8	3
9 → 9	::	9	→ (0→2)→(5 → 7)→ 6 →	9	5
10 → 10	::	10	→ 5 → 3→6 →	10	3
11 → 11	::	11	→ (1 → 3) → (7 → 5) → 2 →	11	5
12 → 12	::	12	→ 6 → 5 → 3 →	12	3
13 → 13	::	13	→ (2→0)→(1 → 3)→7 →	13	5
14 → 14	::	14	→ 7 → 7 → 7 →	14	3

15 → 15 :: 15 → (3 → 1) → (3 → 1) → 3 → 15 5

Total Path length = 61 Total No. of request passes = 15

Average Path Length = 61/15 = 4.067 Percentage Passable = 93

Case 3: SE0 Faulty at stage 0 (Non-Critical Case)

	Source	Destination	PL
0 → 0	:: 0	CLASH	
1 → 1	:: 1	→ (4 → 6) → (4 → 6) → 4 → 1	5
2 → 2	:: 2	→ 2 → 1 → 2 → 4 → 2	3
3 → 3	:: 3	→ (5 → 7) → (6 → 4) → 0 → 3	5
4 → 4	:: 4	→ 2 → 4 → 1 → 4	3
5 → 5	:: 5	→ (6 → 4) → (0 → 2) → 5 → 5	5
6 → 6	:: 6	→ 3 → 6 → 5 → 6	3
7 → 7	:: 7	→ (7 → 5) → (2 → 0) → 1 → 7	5
8 → 8	:: 8	→ 4 → 1 → 2 → 8	3
9 → 9	:: 9	→ (0 → 2) → (5 → 7) → 6 → 9	5
10 → 10	:: 10	→ 5 → 3 → 6 → 10	3
11 → 11	:: 11	→ (1 → 3) → (7 → 5) → 2 → 11	5
12 → 12	:: 12	→ 6 → 5 → 3 → 12	3
13 → 13	:: 13	CLASH	
14 → 14	:: 14	→ 7 → 7 → 7 → 14	3
15 → 15	:: 15	→ (3 → 1) → (3 → 1) → 3 → 15	5

Total Path length = 56 Total No. of request passes = 14

Average Path Length = 56/14 = 4.0 Percentage Passable = 87

Case 4: SE0 Faulty in a loop (Critical Case)

	Source	Destination	PL
0 → 0	:: 0	CLASH	
1 → 1	:: 1	→ (4→6)→(4→6) → 4 → 1	5
2 → 2	:: 2	→ 1 → 2 → 4 → 2	3
3 → 3	:: 3	→ (5 → 7) → (6 → 4) → 0 → 3	5
4 → 4	:: 4	CLASH	
5 → 5	:: 5	→ (6→4) →(0→2)→5→ 5	5
6 → 6	:: 6	→ 3 → 6 → 5 → 6	3
7 → 7	:: 7	→ (7 → 5) → (2 → 0) → 1 → 7	5
8 → 8	:: 8	→ 4 → 1 → 2 → 8	3
9 → 9	:: 9	CLASH	
10 → 10	:: 10	→ 5 → 3 → 6 → 10	3
11 → 11	:: 11	→ (1 → 3) → (7 → 5) → 2 → 11	5
12 → 12	:: 12	→ 6 → 5 → 3 → 12	3
13 → 13	:: 13	CLASH	
14 → 14	:: 14	→ 7 → 7 → 7 → 14	3
15 → 15	:: 15	→ (3 → 1) → (3 → 1) → 3 → 15	5

Total Path length = 48

Total No. of request passes = 12

Average Path Length = $48/12 = 4.0$ Percentage Passable = 75

Similarly other cases can be analyzed and shown in table 1 where $s_0...s_2$ are various stages . 'A' denotes the switches in non critical case (if fault occurs in a single switch) and 'B' denotes the switches in a critical case (if fault occurs in a loop)

Faults	Total path length	Total no of passes	Average path length	% passable
WITHOUT	64	16	4.0	100
MUX	61	15	4.067	93
S0 A	56	14	4.0	87
S0 B	48	12	4.0	75
S1 A	56	14	4.0	87
S1 B	48	12	4.0	75
S2 A	56	14	4.0	87
DEMUX	61	15	4.067	93

Table 1 : Identity permutation measures of ASEN - II

3.1.2 Incremental Permutation For ASEN -II

Case 1: Without Fault

	Source	Destination	PL
0 → 3	:: 0	→ 0 → 0 → 0 →	3
1 → 4	:: 1	→ (0 → 2) → 4 → 1 →	4
2 → 5	:: 2	→ 1 → 2 → 5 →	3
3 → 6	:: 3	→ (1 → 3) → 6 → 5 →	4
4 → 7	:: 4	→ (6 → 4) → 0 → 1 →	4
5 → 8	:: 5	→ 2 → 5 → 2 →	3
6 → 9	:: 6	→ 3 → 7 → 6 →	3
7 → 10	:: 7	→ (3 → 1) → 3 → 6 →	4
8 → 11	:: 8	→ 4 → 1 → 2 →	3
9 → 12	:: 9	→ (4 → 6) → 5 → 3 →	4
10 → 13	:: 10	→ 5 → 3 → 7 →	3
11 → 14	:: 11	→ (5 → 7) → 7 → 7 →	4
12 → 15	:: 12	→ (2 → 0) → 1 → 3 →	4
13 → 0	:: 13	→ 6 → (4 → 6) → 4 →	4

14 → 1	::	14	→ 7 → (6→4) → 0 →	1	4
15 → 2	::	15	→ (7 → 5) → 2 → 4 →	2	4

Total Path length = 58 Total No. of request passes = 16

Average Path Length = $58/16 = 3.625$

Case 2: A Single MUX Faulty

		Source		Destination	PL
0 → 3	::	0	CLASH		
1 → 4	::	1	→ (0→2) → 4 → 1 →	4	4
2 → 5	::	2	→ 1 → 2 → 5 →	5	3
3 → 6	::	3	→ (1 → 3) → 6 → 5 →	6	4
4 → 7	::	4	→ (6 → 4) → 0 → 1 →	7	4
5 → 8	::	5	→ 2 → 5 → 2 →	8	3
6 → 9	::	6	→ 3 → 7 → 6 →	9	3
7 → 10	::	7	→ (3 → 1) → 3 → 6 →	10	4
8 → 11	::	8	→ 4 → 1 → 2 →	11	3
9 → 12	::	9	→ (4 → 6) → 5 → 3 →	12	4
10 → 13	::	10	→ 5 → 3 → 7 →	13	3
11 → 14	::	11	→ (5 → 7) → 7 → 7 →	14	4
12 → 15	::	12	→ (2 → 0) → 1 → 3 →	15	4
13 → 0	::	13	→ 6 → (4→6) → 4 →	0	4
14 → 1	::	14	→ 7 → (6→4) → 0 →	1	4
15 → 2	::	15	→ (7 → 5) → 2 → 4 →	2	4

$$\text{Total Path length} = 55 \quad \text{Total No. of request passes} = 15$$

$$\text{Average Path Length} = 55/15 = 3.67$$

Similarly other cases can be analyzed and shown in table 2 where $s_0 \dots s_2$ are various stages. 'A' denotes the switches in non critical case (if fault occurs in a single switch) and 'B' denotes the switches in a critical case (if fault occurs in a loop)

Faults	Total path length	Total no of passes	Average path length	% passable
WITHOUT	58	16	3.625	100
MUX	55	15	3.67	93
S0 A	51	14	3.64	87
S0 B	51	12	4.25	75
S1 A	51	14	3.64	87
S1 B	43	12	3.58	75
S2 A	51	14	3.64	87
DEMUX	54	15	3.6	93

Table 2 : Incremental permutation measures of ASEN - II

3.1.3 Identity Permutation For ABN

Case 1: Without fault

	Source	Destination	PL
$0 \rightarrow 0$:: 0	$\rightarrow A \rightarrow A' \rightarrow$	2
$1 \rightarrow 1$:: 1	$\rightarrow (A \rightarrow C) \rightarrow B' \rightarrow$	3
$2 \rightarrow 2$:: 2	$\rightarrow B \rightarrow A' \rightarrow$	2
$3 \rightarrow 3$:: 3	$\rightarrow (B \rightarrow D) \rightarrow B' \rightarrow$	3
$4 \rightarrow 4$:: 4	$\rightarrow C \rightarrow D' \rightarrow$	2
$5 \rightarrow 5$:: 5	$\rightarrow (C \rightarrow A) \rightarrow C' \rightarrow$	3
$6 \rightarrow 6$:: 6	$\rightarrow D \rightarrow D' \rightarrow$	2

7 → 7	::	7	→ (D → B) → C' →	7	3
8 → 8	::	8	→ E → E' →	8	2
9 → 9	::	9	→ (E → G) → F' →	9	3
10 → 10	::	10	→ F → E' →	10	2
11 → 11	::	11	→ (F → H) → F' →	11	3
12 → 12	::	12	→ G → H' →	12	2
13 → 13	::	13	→ (G → E) → G' →	13	3
14 → 14	::	14	→ H → H' →	14	2
15 → 15	::	15	→ (H → F) → G' →	15	3

Total Path length = 40 Total No. of request passes = 16

Average Path Length = $40/16 = 2.5$

Case 2: A Single MUX Faulty

		Source		Destination	PL
0 → 0	::	0	CLASH		
1 → 1	::	1	→ (A → C) → B' →	1	3
2 → 2	::	2	→ B → A' →	2	2
3 → 3	::	3	→ (B → D) → B' →	3	3
4 → 4	::	4	→ C → D' →	4	2
5 → 5	::	5	→ (C → A) → C' →	5	3
6 → 6	::	6	→ D → D' →	6	2
7 → 7	::	7	→ (D → B) → C' →	7	3
8 → 8	::	8	→ E → E' →	8	2
9 → 9	::	9	→ (E → G) → F' →	9	3

10 → 10	::	10	→ F → E' →	10	2
11 → 11	::	11	→ (F → H) → F' →	11	3
12 → 12	::	12	→ G → H' →	12	2
13 → 13	::	13	→ (G → E) → G' →	13	3
14 → 14	::	14	→ H → H' →	14	2
15 → 15	::	15	→ (H → F) → G' →	15	3

Total Path length = 38 Total No. of request passes = 15

Average Path Length = $38/15 = 2.53$

Similarly other cases can be analyzed and shown in table 3 where s0 .. s1 are various stages . 'A' denotes the switches in non critical case (if fault occurs in a single switch) and 'B' denotes the switches in a critical case (if fault occurs in a loop).

Faults	Total path length	Total no of passes	Average path length	% passable
WITHOUT	40	16	2.5	100
MUX	38	15	2.5	93
S0 A	35	14	2.5	87
S0 B	30	12	2.5	75
S1 A	36	14	2.5	87
DEMUX	38	15	2.5	93

Table 3 : Identity permutation measures of ABN

3.1.4 Incremental Permutation For ABN

Case 1: Without fault

	Source	Destination	PL
0 → 3	:: 0	→ B → A' → 3	2
1 → 4	:: 1	→ A → C' → 4	2

2 → 5	::	2	→ C → D' →	5	2
3 → 6	::	3	→ B → C' →	6	2
4 → 7	::	4	→ D → D' →	7	2
5 → 8	::	5	→ G → F' →	8	2
6 → 9	::	6	→ E → E' →	9	2
7 → 10	::	7	→ H → F' →	10	2
8 → 11	::	8	→ F → E' →	11	2
9 → 12	::	9	→ E → G' →	12	2
10 → 13	::	10	→ G → H' →	13	2
11 → 14	::	11	→ F → G' →	14	2
12 → 15	::	12	→ H → H' →	15	2
13 → 0	::	13	→ C → B' →	0	2
14 → 1	::	14	→ A → A' →	1	2
15 → 2	::	15	→ D → B' →	2	2

Total Path length = 32 Total No. of request passes = 16

Average Path Length = $32/16 = 2.0$

Case 2: A Single MUX Faulty

		Source		Destination	PL
0 → 3	::	0	→ B → A' →	3	2
1 → 4	::	1	→ A → C' →	4	2
2 → 5	::	2	→ C → D' →	5	2
3 → 6	::	3	→ B → C' →	6	2
4 → 7	::	4	→ D → D' →	7	2
5 → 8	::	5	→ G → F' →	8	2

6 → 9	::	6	→ E → E' →	9	2
7 → 10	::	7	→ H → F' →	10	2
8 → 11	::	8	→ F → E' →	11	2
9 → 12	::	9	→ E → G' →	12	2
10 → 13	::	10	→ G → H' →	13	2
11 → 14	::	11	→ F → G' →	14	2
12 → 15	::	12	→ H → H' →	15	2
13 → 0	::	13	→ C → B' →	0	2
14 → 1	::	14	CLASH		
15 → 2	::	15	→ D → B' →	2	2

Total Path length = 30 Total No. of request passes = 15

Average Path Length = $30/15 = 2.0$

Similarly other cases can be analyzed and shown in table 4 where s0 .. s1 are various stages . 'A' denotes the switches in non critical case (if fault occurs in a single switch) and 'B' denotes the switches in a critical case (if fault occurs in a loop).

Faults	Total path length	Total no of passes	Average path length	% passable
WITHOUT	32	16	2.0	100
MUX	30	15	2.0	93
S0 A	28	14	2.0	87
S0 B	24	12	2.0	75
S1 A	28	14	2.0	87
DEMUX	30	15	2.0	93

Table 4 : Incremental permutation measures of ABN

From table 1,2,3 & 4 it has been analyzed that ABN is better than ASEN in terms of average path length both in identity as well as incremental case.

3.2 Permutation for Irregular MINs

3.2.1 Comparison Of FT (Four Tree Network) & MFT (Modified Four Tree Network)

MFT is the modification of FT and the comparable permutation layouts are given below , the other cases are shown in table 5 & 6.

3.2.1.1 Incremental Permutation For FT

Case 1: Without fault

	Source	Destination	PL
0 → 4	:: 0 → 1 → 1 → 1 → 2 → 3 →	4	5
1 → 5	:: 1 → 1 → 1 → 1 → 2 → 3 →	5	5
2 → 6	:: 2	CLASH	
3 → 7	:: 3	CLASH	
4 → 8	:: 4 → 3 → 2 → 1 → 1 → 1 →	8	5
5 → 9	:: 5	CLASH	
6 → 10	:: 6	CLASH	
7 → 11	:: 7	CLASH	
8 → 12	:: 8 → 1 → 1 → 1 → 2 → 3 →	12	5
9 → 13	:: 9	CLASH	
10 → 14	:: 10	CLASH	
11 → 15	:: 11	CLASH	
12 → 0	:: 12	CLASH	
13 → 1	:: 13	CLASH	
14 → 2	:: 14	CLASH	

15 → 3 :: 15 CLASH

Total Path length = 20 Total No. of request passes = 4

Average Path Length = 20/4 = 5.0

Case 2: SE1 faulty at stage 3 (Non-Critical Case)

	Source	Destination	PL
0 → 4	:: 0 → 1 → 1 → 1 → 2 → 3 →	4	5
1 → 5	:: 1 → 1 → 1 → 1 → 2 → 3 →	5	5
2 → 6	:: 2	CLASH	
3 → 7	:: 3	CLASH	
4 → 8	:: 4	CLASH	
5 → 9	:: 5	CLASH	
6 → 10	:: 6	CLASH	
7 → 11	:: 7	CLASH	
8 → 12	:: 8 → 1 → 1 → 1 → 2 → 3 →	12	5
9 → 13	:: 9	CLASH	
10 → 14	:: 10	CLASH	
11 → 15	:: 11	CLASH	
12 → 0	:: 12	CLASH	
13 → 1	:: 13	CLASH	
14 → 2	:: 14	CLASH	
15 → 3	:: 15	CLASH	

Total Path length = 15 Total No. of request passes = 3

$$\text{Average Path Length} = 15/3 = 5.0$$

Case 3: SE1 faulty in loop at stage 3 (Critical Case)

	Source	Destination	PL
0 → 4	:: 0	CLASH	
1 → 5	:: 1	CLASH	
2 → 6	:: 2	CLASH	
3 → 7	:: 3	CLASH	
4 → 8	:: 4	CLASH	
5 → 9	:: 5	CLASH	
6 → 10	:: 6	CLASH	
7 → 11	:: 7	CLASH	
8 → 12	:: 8	CLASH	
9 → 13	:: 9	CLASH	
10 → 14	:: 10	CLASH	
11 → 15	:: 11	CLASH	
12 → 0	:: 12	CLASH	
13 → 1	:: 13	CLASH	
14 → 2	:: 14	CLASH	
15 → 3	:: 15	CLASH	

Total Path length = 0 Total No. of request passes = 0

$$\text{Average Path Length} = 0$$

Faults	Total path length	Total no of passes	Average path length	% passable
WITHOUT	20	4	5	25
MUX	20	4	5	25
S1 A	20	4	5	25
S1 B	20	4	5	25
S2 A	15	3	5	18
S2 B	10	2	5	12
S3 A	10	2	5	12
S3 B	0	0	0	0
S4 A	15	3	5	18
S4 B	10	2	5	12
S5	20	4	5	25
DEMUX	20	4	5	25

Table 5 : Incremental permutation measures of FT

3.2.1.2 Incremental Permutation For MFT

Case 1: Without fault

	Source	Destination	PL
0 → 4	:: 0 → 0 → A → A' → A'' → 2 →	4	5
1 → 5	:: 1 → 0 → C → C' → C'' → 2 →	5	5
2 → 6	:: 2 → 1 → C → B' → B'' → 3 →	6	5
3 → 7	:: 3 → 1 → A → D' → D'' → 3 →	7	5
4 → 8	:: 4 → 2 → D → A' → B'' → 0 →	8	5
5 → 9	:: 5 → 2 → (B → D) → C → D'' → 0 →	9	5
6 → 10	:: 6 → 3 → B → B' → A'' → 1 →	10	5
7 → 11	:: 7 → 3 → (D → B) → D → C'' → 1 →	11	5
8 → 12	:: 8	CLASH	
9 → 13	:: 9	CLASH	
10 → 14	:: 10	CLASH	

11 → 15	::	11	CLASH
12 → 0	::	12	CLASH
13 → 1	::	13	CLASH
14 → 2	::	14	CLASH
15 → 3	::	15	CLASH

Total Path length = 40 Total No. of request passes = 8

Average Path Length = $40/8 = 5.0$

Case 2: SEA faulty at stage 3 (Non-Critical Case)

		Source	Destination	PL
0 → 4	::	0	CLASH	
1 → 5	::	1 → 0 → C → C' → C'' → 2 →	5	5
2 → 6	::	2 → 1 → C → B' → B'' → 3 →	6	5
3 → 7	::	3 → 1 → A → D' → D'' → 3 →	7	5
4 → 8	::	4	CLASH	
5 → 9	::	5 → 2 → (B → D) → C → D'' → 0 →	9	5
6 → 10	::	6 → 3 → B → B' → A'' → 1 →	10	5
7 → 11	::	7 → 3 → (D → B) → D → C'' → 1 →	11	5
8 → 12	::	8	CLASH	
9 → 13	::	9	CLASH	
10 → 14	::	10	CLASH	
11 → 15	::	11	CLASH	
12 → 0	::	12	CLASH	
13 → 1	::	13	CLASH	
14 → 2	::	14	CLASH	

15 → 3 :: 15 CLASH

Total Path length = 30 Total No. of request passes = 6

Average Path Length = 30/6 = 5.0

Case 3: SEA faulty in loop at stage 3 (Critical Case)

	Source	Destination	PL
0 → 4 ::	0	CLASH	
1 → 5 ::	1	CLASH	
2 → 6 ::	2 → 1 → C → B' → B'' → 3 →	6	5
3 → 7 ::	3 → 1 → A → D' → D'' → 3 →	7	5
4 → 8 ::	4	CLASH	
5 → 9 ::	5	CLASH	
6 → 10 ::	6 → 3 → B → B' → A'' → 1 →	10	5
7 → 11 ::	7 → 3 → (D → B) → D → C'' → 1 →	11	5
8 → 12 ::	8	CLASH	
9 → 13 ::	9	CLASH	
10 → 14 ::	10	CLASH	
11 → 15 ::	11	CLASH	
12 → 0 ::	12	CLASH	
13 → 1 ::	13	CLASH	
14 → 2 ::	14	CLASH	
15 → 3 ::	15	CLASH	

Total Path length = 20 Total No. of request passes = 4

Average Path Length = 20/4 = 5.0

Faults	Total path length	Total no of passes	Average path length	% passable
WITHOUT	40	8	5	50
MUX	40	8	5	50
S1 A	35	7	5	43
S1 B	30	6	5	37
S2 A	30	6	5	37
S2 B	20	4	5	25
S3 A	30	6	5	37
S3 B	20	4	5	25
S4 A	30	6	5	37
S4 B	20	4	5	25
S5	35	7	5	43
DEMUX	40	8	5	50

Table 6 : Incremental permutation measures of MFT

It has been analyzed from above table 5 & 6 that permutation passable of MFT is better than existing FT, for example the permutations passable of FT at SE3 are zero if it gets faulty in loop but in MFT requests served are 0.25.

3.3 Conclusion

From the analyses it has been observed from table 1,2,3 & 4 that the permutations passable of ABN & ASEN are almost equal but the average path length of ABN is less than ASEN.

It has been observed from the analysis that the permutation passable of irregular modified FT is better than existing irregular FT.

CHAPTER 4

Reliability & Performance Analyses of Irregular Network

This chapter provides probability of acceptance, reliability analysis of various irregular MIN's. The full chapter provides the comparison of existing FT & MDOT with modified FT and RMDOT (remodified double tree) network. The analyses on these shown that the various parameters of proposed network are better than existing network.

Considerations Of Reliability

The following assumptions [6] are made during the analysis :

1. All the switch elements are not dependent on each other. So it means failure of any does not affect the reliability of other.
2. The failures of switch occur independently in a network with a failure rate of λ ($=10^6$ per hour) per unit time. Based on the gate count failure rate for a 2 x 2 SE, $\lambda_2 = \lambda$ and for a 3 x 3 SE $\lambda_3 = 2.25 \lambda$ for m:1 multiplexer $\lambda_m = \lambda_d = \lambda / 4 * m =$ failure rate of 1:m demultiplexer. (where λ_m - failure rate of multiplexer, λ_d - failure rate of demultiplexer, λ_2 - failure rate of 2x2 SE, λ_3 - failure rate of 3x3 SE)

4.1 Reliability Analyses Of Irregular Networks

Irregular networks are multipath inherently means there are number of paths available from desired source to destination. Higher the path length, more will be the number of switches comes in the way, so reliability or Mean Time To Failure (MTTF) needs to be calculated at all the existing path lengths separately.

4.1.1 Modified Double Tree Network (MDOT)

In MDOT all the SE's are essential so there are no redundant SE's comes at a given path length. The block diagram of reliability analysis is shown in Fig. 4.1. All the SE's comes in series so the reliability and MTTF expression of this is given by :

$$R_{MDOT}(t) = (e^{-\lambda t})^{N+(N/2)^{\spadesuit} + (N/4)^{\spadesuit\spadesuit} + \dots + 1^{\infty}}$$

$$MTTF_{MDOT}(t) = \int_0^{\infty} R_{MDOT}(t) \cdot dt$$

The results of equation are given in table 7.

-
- ♦ term to be added for path length > 2
 - ♦♦ term to be added for path length > 4
 - ∞ term to be added for maximum path length

N/w P. L.	8x8	16x16	64x64	128x128	512x512
2	124958	62500	15625	7812	1954
4	83333	41667	10473	5208	1303
5	71429	-	-	-	-
6	-	35714	8928	4464	1116
7	-	34480	-	-	-
8	-	-	8333	4100	1003

Table 7 : MTTF Vs. Size for MDOT
(failure rate $\lambda = 10^{-6}$ per hour)

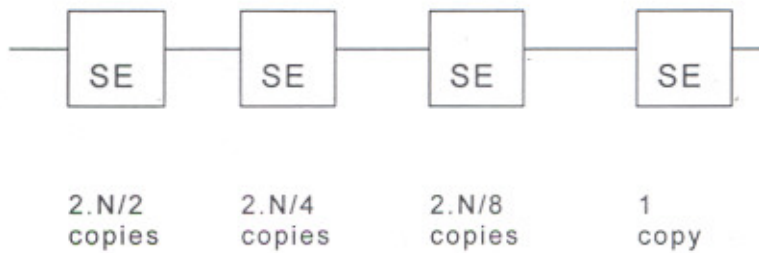


Fig 4.1 Reliability block diagram of MDOT

4.1.2 Four Tree Network (FT)

One of the special features of FT network is that some forks are introduced at each stage except last one to avoid various faults. So it means in the presence of fork each request has two ways to reach to destination.

In case of FT network redundancy has been added into the system via intrastage auxiliary links of 3 x 3 SEs. This redundancy has been introduced at the component level, which is generally more effective than the unit level redundancy. So the reliability of FT network is expected to be better.

4.1.2.1 Pessimistic (lower bound) analyses of FT

To obtain the pessimistic reliability of FT, it has been assumed that the multiplexers attached with each input side of the SE is taken as a series system. The failure of any one of the two multiplexers attached to a given SE will be considered as a failure of that particular SE. The pessimistic

reliability block diagram is as shown in Fig. 4.2 and the corresponding expression is given by:

$$R_{\text{pess, FT}}(t) = [1-(1-e^{-\lambda t})^2]^{N/4} \cdot \{ [1-(1-e^{-\lambda t})^2]^{N/4+N/8+\dots+1} \} \cdot [1-(1-e^{-\lambda t})^2]^{N/4}$$

&

$$\text{MTTF}_{\text{FT}}(t) = \int_0^{\infty} R_{\text{pess, FT}}(t) dt$$

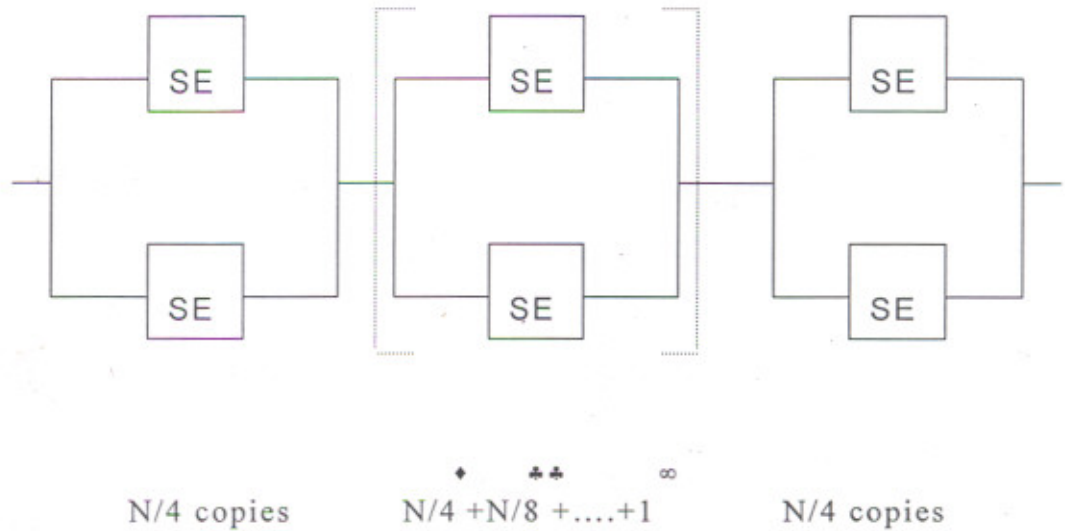


Fig 4.2 Lower bound reliability block diagram of FT

The results of above equation are given in table 8.

4.1.2.2 Optimistic (upper bound) analyses of FT

For the optimistic reliability analyses, it has been assumed that a SE remains operational as long as any one of the two multiplexers attached to it remains operational at the input side. The optimistic reliability model for FT is as shown in Fig 4.3 and the reliability expression is given by:

$$R_{\text{opti, FT}}(t) = [1-(1-e^{-\lambda t})^2]^{N/2} \cdot [1-(1-e^{-\lambda t})^2]^{N/4+N/4+N/8+\dots+1^\infty} \cdot [1-(1-e^{-\lambda t})^2]^{N/4}$$

&

$$\text{MTTF}_{\text{FT}}(t) = \int_0^\infty R_{\text{opti, FT}}(t) dt$$

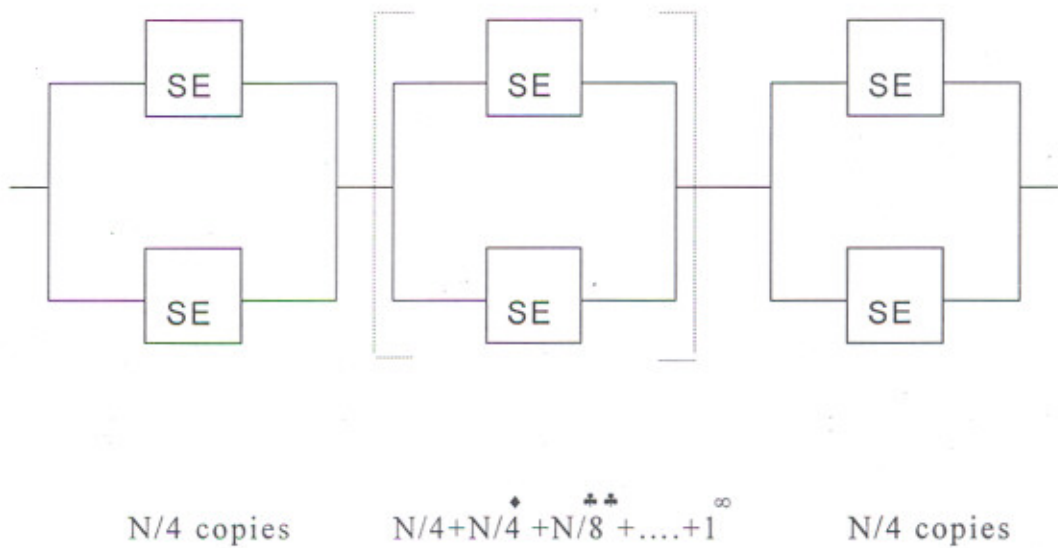


Fig 4.3 Upper bound reliability block diagram of FT

The results of above equation are given in table 9.

4.1.3 Proposed Modified Four Tree Network (MFT)

Similar to FT, MFT network also having a fault tolerant capability in the presence of fork at every stage except last one in the network. As a result there exists two ways of routing an input request to the output stage. The

N/w P. L.	<u>16x16</u>	<u>32x32</u>	<u>64x64</u>	<u>128x128</u>	<u>512x512</u>
2	174854	116418	78819	54021	25997
4	136096	91675	62605	43179	20944
5	124120	-	-	-	-
6	-	83924	57469	39717	19312
7	-	80700	-	-	-
8	-	-	55325	38267	18626
9	-	-	54338	-	-
10	-	-	-	37277	18157
11	-	-	-	37119	-
12	-	-	-	-	18082
13	-	-	-	-	-
14	-	-	-	-	18027
15	-	-	-	-	18018

Table 8 : Lower Bound MTTF Vs. Size for FT
(failure rate $\lambda = 10^{-6}$ per hour)

N/w P. L.	<u>16x16</u>	<u>32x32</u>	<u>64x64</u>	<u>128x128</u>	<u>512x512</u>
2	190618	127944	87119	59952	28993
4	144316	97600	66338	46190	22458
5	130560	-	-	-	-
6	-	88549	60768	42060	20489
7	-	84843	-	-	-
8	-	-	58278	40364	19679
9	-	-	57140	-	-
10	-	-	-	39216	19131
11	-	-	-	39034	-
12	-	-	-	-	19044
13	-	-	-	-	-
14	-	-	-	-	18979
15	-	-	-	-	18968

Table 9 : Upper Bound MTTF Vs. Size for FT
(failure rate $\lambda = 10^{-6}$ per hour)

reliability of MFT is expected to be lower than that of FT network as the number of switches remains same in the inner stages.

4.1.3.1 Pessimistic (lower bound) analyses of MFT

To obtain the pessimistic reliability of MFT, it has been assumed that the multiplexers attached with each input side of the SE is taken as a series system. The failure of any one of the two multiplexers attached to a given SE will be considered as a failure of that particular SE. The reliability model for MFT is shown in Fig. 4.4 and is given by the following expression:

$$R_{\text{pess, MFT}}(t) = [1 - (1 - e^{-\lambda t})^2]^{N/4} \cdot [1 - (1 - e^{-\lambda t})^2]^{N/4 + (n-1)} \cdot [1 - (1 - e^{-\lambda t})^2]^{N/4}$$

&

$$\text{MTTF}_{\text{MFT}}(t) = \int_0^{\infty} R_{\text{pess, MFT}}(t) dt$$

4.1.3.2 Optimistic (upper bound) analyses of MFT

For the optimistic reliability analyses, it has been assumed that a SE remains operational as long as any one of the two multiplexers attached to it remains operational at the input side. The reliability expression for MFT is shown in Fig. 4.5 and is given by the following expression:

$$R_{\text{opti, MFT}}(t) = [1 - (1 - e^{-\lambda t})^2]^{N/2} \cdot [1 - (1 - e^{-\lambda t})^2]^{N/4 + N/4 + (n-1)} \cdot [1 - (1 - e^{-\lambda t})^2]^{N/4}$$

&

$$\text{MTTF}_{\text{MFT}}(t) = \int_0^{\infty} R_{\text{opti, MFT}}(t) dt$$

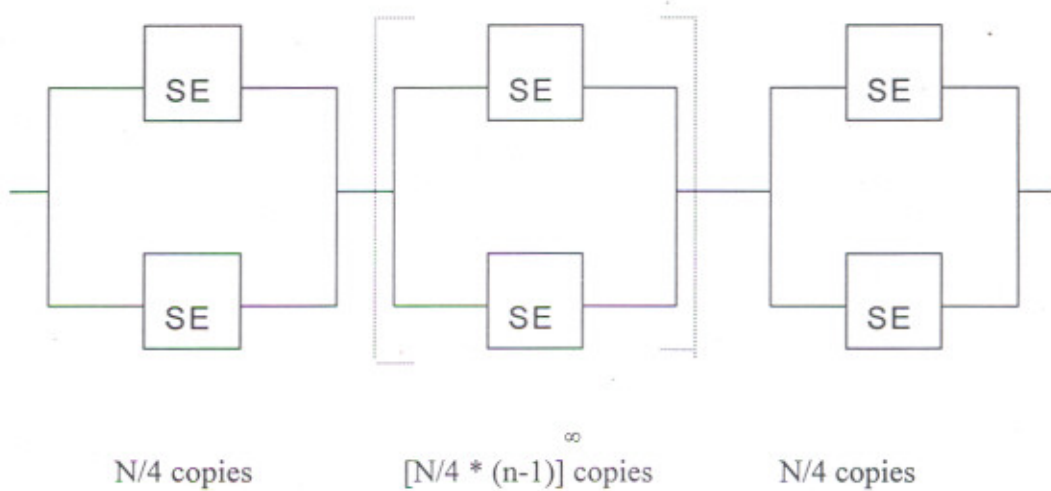


Fig 4.4 Lower bound reliability block diagram of MFT

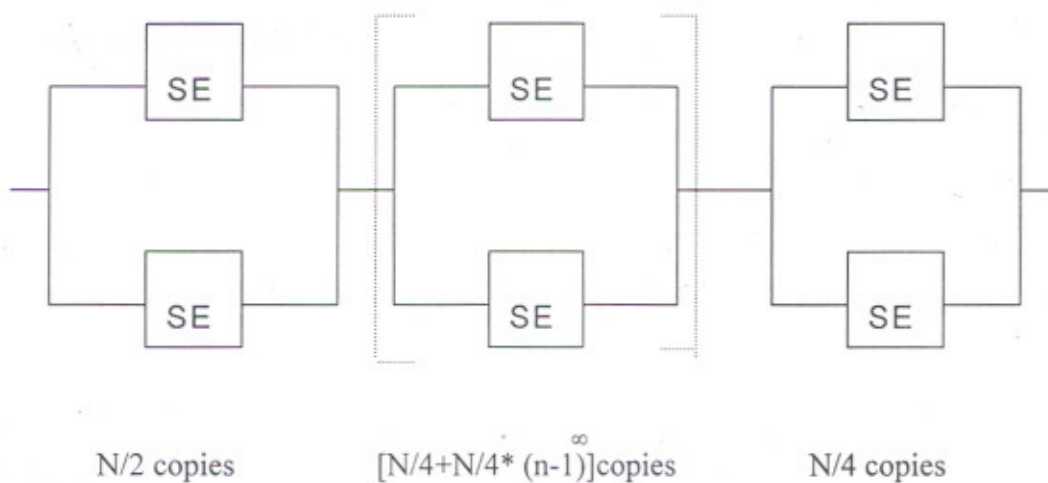


Fig 4. 5 Upper bound reliability block diagram of MFT

The results of both upper & lower reliability of MFT in comparison with FT are given in table 10.

Size	N=16	N=32	N=64	N=128	N=512
FT (UB)	155164	99734	66028	44469	20967
(LB)	145023	93179	61711	41596	19645
MFT (UB)	134501	75939	54078	38159	18873
(LB)	120081	68685	47293	32814	16034

UB – Upper Bound MTTF

LB – Lower Bound MTTF

Table 10 : MTTF Comparisons for FT and Modified FT

4.2 Performance Analysis of Irregular Networks

The other most important parameter after reliability is Performance of a multistage interconnection network. A network should be able to provide maximum possible interconnections between various sources & destinations, but within some cost constraints .Among the various multipath network discussed in chapter 2,networks using intrastage links are found to be more effective in improving the performance. This chapter contains the comparison of performance analysis of MDOT,FT with RMDOT & MFT networks. Keeping in view the inherent multipath nature of irregular networks, they seem to be the right candidate foresight performance, the possibility of which has been explored in the present chapter .

Assumptions

The following assumptions[6] are made for the performance analysis which are identical to those made for the regular networks by various researches .

- 1). Each source generates random & independent requests with equal probability i.e. requests are uniformly distributed over all the destination modules.
- 2). At the beginning of every cycle each source generates new requests with probability P_0 . Blocked or unaccepted requests of previous cycle are ignored.
- 3). Requests are generated for each destination with equal probability. Thus , incoming requests to a switch are routed to each interstage output of the switch with equal probability.
- 4).Intrastage requests are given lower priority over interstage requests.

In case of fault dependent performance analysis of FT & MFT networks ,the analysis has been confined to single switch faults per stage & simple loop faults per stage .Link faults can be considered as a part of switch failures. The probability of carrying a request at the input-output links of a faulty switch is taken as zero .In case of a loop fault all the SEs in a loop are considered faulty.

4.2.1 Calculation Of Probability Of Acceptance

Probability of acceptance at the output of a mxm crossbar switch :

Let

P_i = Probability of presence of a request at the i th input link of a SE

Q_j = Probability of presence of a request at the j th output link of a SE.

$P_{i,n-1}$ = Probability of issuing a request from the $(n-1)$ th stage at the i th input link of a SE in the n th stage.

$Q_{j,n}$ = Probability of presence of a request at the j th output link of a SE in the n th stage.

Since all the output links are accessed by a request with an equal probability, therefore;

$$Q_{j,n} = 1 - \Sigma(1-(P_{i,n-1})/m)$$

Where $i,j=1,2,3,\dots,m$

and m is the number of interstage links of the given SE .Fig. 4.6.

For output links, connected to faulty switch (Fig.4.7), no request will be routed through them, as a result ,the probability for such a link is:

$$Q_{j,n} = P_{j,n} = Q_{j,n+1} = 0 \text{ for all } i \ \& \ j$$

Probability of acceptance with demultiplexers at the final stage :

If destinations are connected to final stage SEs through 1:k demultiplexers (Fig.4.8) then probability of acceptance of request by a destination is given by:

$$P_a = Q_{j,n} / k$$

Because there is no blocking in case of demultiplexers , input request is equally likely to be routed to all output links of a demultiplexer. Performance of MDOT ,FT,RMDOT,MFT networks is calculated in terms of probability of acceptance(P_a),by making use of condition without faulty. Performance analysis results of 8x8(MDOT,RMDOT) and 16x 16 (FT,MFT) networks are given in table 11 and shown in Fig. 4.9.

	P_a	BW
MDOT	0.521734	8.347736
RMDOT	0.549153	8.786448
FT	0.687	10.9929
MFT	0.68765	10.9947

Table 11 : Probability of acceptance for request and bandwidth offered by Irregular MIN's request generation probability = 1.0

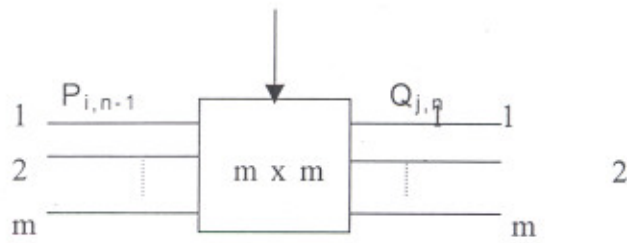


Fig. 4.6 P_a at the input & output of an n stage SE

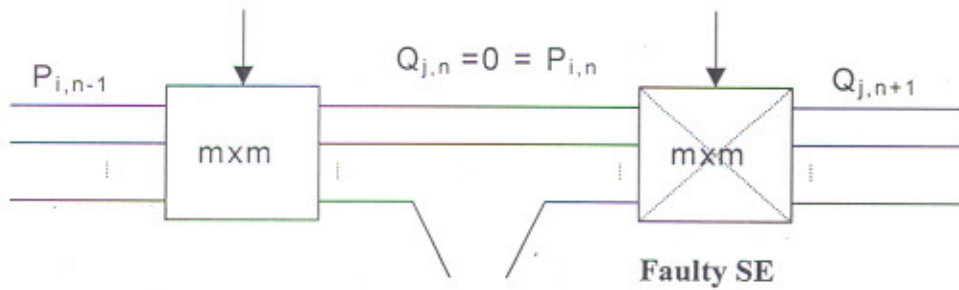


Fig. 4.7 An $m \times m$ SE connected to a faulty SE

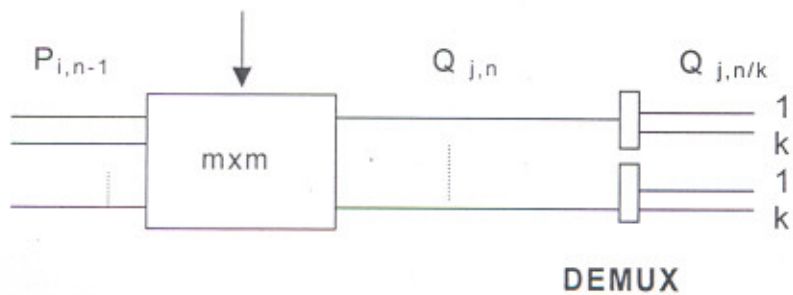


Fig. 4.8 An output stage SE connected to demultiplexers

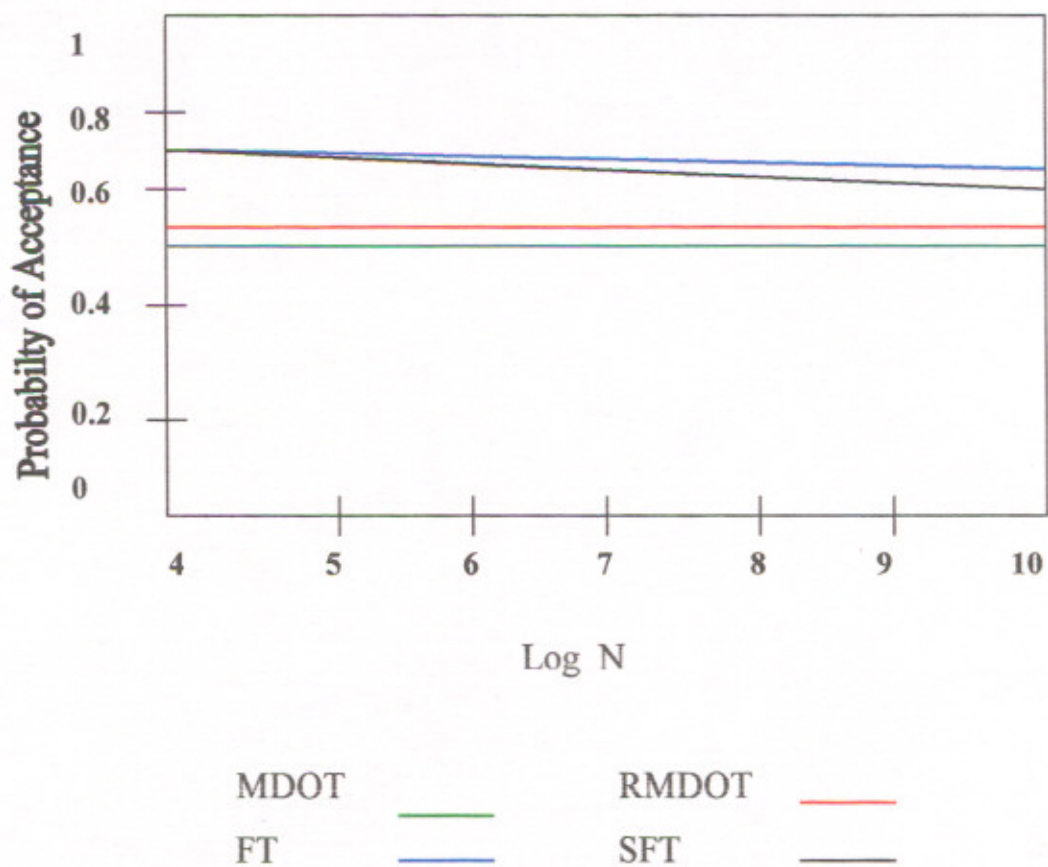


Fig.4.9 Probability of Acceptance (with $P_o = 1.0$) Vs size for various irregular networks (existing Vs modified)

4.3 Cost-Effectiveness

Assumption :

Component		Cost
2x2		4 unit
3x3		9 unit
Multiplexer	m : 1	m
Demultiplexer	1 : m	m

A measure of cost-effectiveness for reliability can be given by comparing MTTF and the cost of the network. Cost-effectiveness of FT & MFT are evaluated and compared with each other and results are shown in Figure 4.10. From the results, we can observe that FT network is almost comparable to that of proposed MFT network for a shorter network.

4.4 Conclusion

Results of the reliability of MDOT, FT & modified FT-irregular MINs are presented in terms of MTTF for all possible path lengths and for all network sizes.

It is observed that FT network is more reliable than MDOT and almost comparable to that of proposed modified FT network.

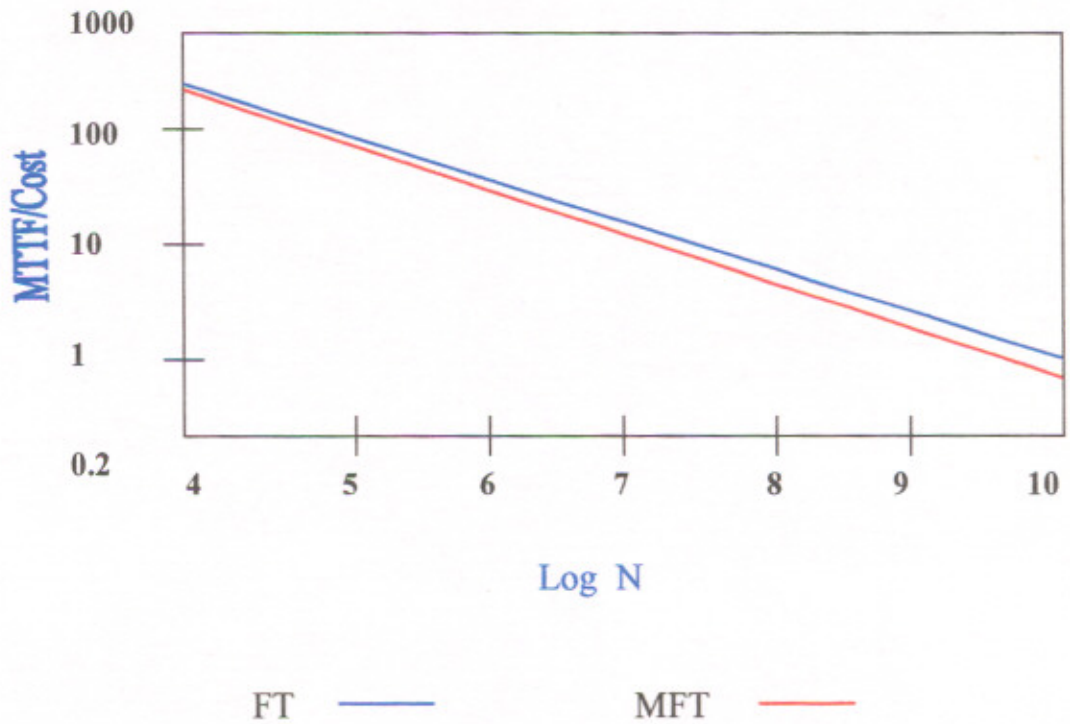


Fig. 4.10 MTTF/Cost Comparison of FT & MFT (LB)

There is wide variation in optimistic and pessimistic results in network. Optimistic results prove it to be the most reliable of all. While pessimistic results contradict this fact. The reason is that under pessimistic assumptions every SE has got 2 redundant counterparts which can replace it in case of a failure and the probability of failure of all the three SEs in a group is very low.

Probability of acceptance results of RMDOT is better than MDOT and it has also low cost because of a middle switch removal from MDOT. The performance of both the FT and MFT is almost equal because the path length of both the networks are equal and cost of the network is just comparable.

The various results which are present in the form of table shows that after a certain path length, MTTF(mean time to failure) degrades only slightly at higher path lengths. The reason being as the as the path length increases the number of switches comes in the way start decreasing.

CONCLUSIONS & FURTHER SCOPE

5.1 Conclusions

The thesis analyses Regular and Irregular Networks in terms of Reliability, Permutation Passability , Probability of Acceptance , bandwidth and Cost-effectiveness. The work presented here can be summarized as under:

- ◆ Survey of existing Regular and Irregular MINs has been presented.
- ◆ FT network has been modified called MFT . It involves lesser number of stages but with marginally increased number of switches as compared to FT network. The analyses of the proposed MFT establishes that it is better than FT network in terms of Permutation Passable and is as good as the FT network as far as the Reliability and Cost effectiveness is concerned.
- ◆ The Permutation Passable , in terms of Identity and Incremental Permutation layouts, of various Regular (ABN & ASEN-2) and Irregular (MDOT,RMDOT,FT & MFT) MINs has been analysed considering a number of sample cases. This analyses shows that, in general, proposed MFT provides better performance than the various Irregular MINs discussed and also ABN gives better performance than ASEN-2.
- ◆ The reliability of various Irregular.(MDOT, FT, MFT) has been worked out in terms of MTTF. It is observed that Probability of acceptance of RMDOT is better than MDOT , while analyses of FT network shows that it is better than the existing MDOT, especially at higher network sizes.

5.2 Further Scope

The field of irregular networks can be further explored in the light of the following suggestions.

- ◆ The designing of regular & irregular networks having better reliability and permutations can be explored.
- ◆ A new concept of buffering of request can also be introduced in the Multistage Interconnection Network.
- ◆ The networks can be applied for various applications like ATM , Internet routing.
- ◆ The detailed probability of acceptance and bandwidth analyses can be done for all the Irregular MINs.

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