A NEW HEURISTIC ALGORITHM FOR MULTIOBJECTIVE BULK TRANSPORTATION PROBLEM

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DEDICATED

TO

GOD, MY PARENTS

AND

MY SUPERVISOR

CERTIFICATE

I hereby certify that the work present in the thesis entitled, "A NEW SIMPLE HEURISTIC ALGORITHM OR MULTI-OBJECTIVE BULK TRANSPORTATION PROBLEM" which is being presented for the award of degree of Master of Science, School of Mathematics and Computer Applications, Thapar University, Patiala is an authentic record of my own work carried out under the supervision of Dr. Mahesh Kumar Sharma.

The matter presented in this thesis has not been submitted for the award of any other degree of this for any other university.

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ABSTRACT

The classical transportation problem (TP) is one of the many well-structured problems in operations research that has been extensively studied in literature. The TP is one of the subclass of the linear programming problems for which simple and practical computational procedures have been developed that take the advantage of the special structure of the problem. The TP is amongst the most important special linear programming problem – in terms of the frequency with it appears in the applications and also in the simplicity of the procedure developed for its solutions.

Transportation problem has been paid much attention and classified into several types of transmutations and one of the variants of the TP is bulk transportation problem (BTP) which differs from the TP that it stipulates that the total requirement of each destination has to be met from single source only, however a some source can supply to any number of destination subject to the availability of the commodity at the source.

The present thesis consists of three chapters. Chapter one is introducing in nature in which multi-objective optimization has been described and brief survey of the literature to the topic has been discussed. In the second chapter two algorithms for multi-objective bulk transportation problem (MBTP) given by Prakash et al. (2007, 2009) have been reviewed. In chapter three, a heuristic algorithm has been developed for MBTP which is a combination the algorithms review in chapter second.

CONTENTS

CHAPTER - 1

INTRODUCTION

The classical transportation problem (TP) is one of the many well-structured problems in operations research that has been extensively studied in literature. The TP is one of the subclass of the linear programming problems for which simple and practical computational procedures have been developed that take the advantage of the special structure of the problem. The TP is amongst the most important special linear programming problem – in terms of the frequency with it appears in the applications and also in the simplicity of the procedure developed for its solutions. The importance of TP can also be gauged from the fact that this laid the formulation for further theoretical and algorithmic developed of the minimal cost network flow optimization problems and the applicability in the real-life problem of distributing scheduling, etc.

The single-objective model was the first to be developed and thus it was received considerably more exposure, been put to more use, and is generally considered to be a relatively high level of refinement. Thus the implication is simple, well-tested tool is available and we may be inclined to fit the problem to this model despite the assumptions required. But in the real life there are many problems with more than one objective for which the multi-objective models are required.

Dantzig's initial concept was concerned about the development of the linear programing model but with a single objective. This so set the tone for development of traditional linear programming that many (if not most) linear programming texts completely ignore even the possibility of more than one objective. Unlike the traditional single objective optimization problem wherein it is settle on optimizing single objective function, there is no single universally accepted approach for solving multi-objective optimization problems due to usually conflicting nature of objective functions leading To the situation where the optimization of one of these may adversely affect the optimization of others. So in the case of multi-objective optimization problems, there is no need to access the decision maker's utility fuction that may vary from decision maker to decision maker.

1.1 MULTI-CRITERIA DECISION MAKING:

The process of decision making is the selection of an act or courses of action from among alternative acts or courses of actions such that it will produce optimal results under some criteria of optimization. Before the problem can be considered well-defined, the set of alternative and the set of criteria have to be known and established first; only then can the selection process commence. What makes multiple criteria decision making complex is the plurality of the criteria involved in the problem. In a single objective problem the selection process can be managed with relative ease even if there are a large number of alternatives. As a matter of fact, solution procedure for single-criterion problems with several alter-natives are widely available. The degree of difficulty of decision making is far more sensitive with the number of criteria.

In the decision analysis of complex systems, such terms as "multiple criteria", "multiple objectives", or "multiple attributes" are used to describe decision situations. Often, these terms are used interchangeably. Certainly, there are no universal definitions of these terms. Multiple criteria decision making has seemed to emerge as the accepted nomenclature for all models and techniques dealing with multiple objective decision making (MODM) or multiple attribute decision making (MADM). These are the two brand categories of MCDM problems. MODM methods are often used with reference to problems with large set of alternatives, while MADM methods are meant to select the best from a small explicit list of alternatives. MODM, therefore, is a problem of design, and mathematical techniques of optimization are needed in solving it. On the other hand, MADM is a problem of choice and classical mathematical programming tools need to be used.

1.2 CONFLICTING CRETERIA:

A necessary condition of MCDM is the presence of more than one criterion. The sufficient condition is that the criteria must be conflicting in nature. Therefore the following definitions can be stated: "A problem can be considered as that of MCDM if and only if there appears at least two conflicting criteria and there are at least two alternative solutions." Criteria are said to be in conflict if the full satisfaction of one will result in impairing or precluding the full satisfaction of the other(s). Criteria are considered to be "strictly" conflicting if the increase in satisfaction of one results in a decrease in satisfaction of the other.

The sufficient condition of MCDM, however, does not necessarily stipulate "strictly" conflicting criteria. Conflict may arise due to intrapersonal and interpersonal reasons. A consumer purchasing a car is often confronted with conflicting criteria caused intra-personally. For example he may wish to have cheap, comfortable, fuel-efficient, and luxurious car. Usually no single car model can satisfy all these criteria. A similar situation is in the case of an industrial executive trying to fill up a general managerial position in his company. On the other hand, a family looking for a house to reside is a typical example where conflicts of criteria may be due to interpersonal reasons. In view of the conflicting nature of the criteria involved in MCDM, choosing the "best" alternative is indeed a difficult task for the decision maker. Consequently there is a need for methods to systematically resolve the conflicts among criteria (or objective) in order to reach acceptable compromises and come up with satisfying (or often as "satisficing") solutions.

Most decision making scenarios in transportation problem involve multiple objectives that often conflict. Multi-objective optimization is not purely a maximizing or minimizing problem. It is a mixture of several conflicting maximum or minimum problems, which boils down to that of "satisficing" these conflicting objectives. Multi-objective optimization is a process of simultaneously optimizing two or more conflicting objective subject to certain constraints.

1.3 MULTI OBJECTIVE OPTIMIZATION

Multi-objective optimization can be defined as:

"a vector of decision variables which satisfies constraints and optimizes a vector function whose element represent the objective functions. Hence the term "optimizes" means finding means finding the solution which would give the values of all the objective functions acceptable to the decision maker."

This model can be formulated as:

$$Optimize\ f(X) = \left(f_1(X), f_2(X), \dots, f_k(X)\right)$$

$$subject\ to\ g_j(X) \leq=\geq b_j, j = 1, 2, \dots, m$$

$$X \geq 0$$

$$X = (x_1, x_2, \dots, x_n)^T$$

Where, $f(X)$ is the objective function to optimize. $\left(f_1(X), f_2(X), \dots, f_k(X)\right)$ are k number of distinct objective functions subject to m constraints. X is a vector consists of decision making variables $x_1, x_2, \dots, x_n$.

Multi-objective optimization problems can be found in various fields: product and process design, finance, aircraft design, the oil and gas industry, automobile design, or wherever optimal decisions need to be taken in the presence of trade-offs between two or more conflicting objectives. Maximizing profit and minimizing the cost of a product; maximizing performance and minimizing fuel consumption of a vehicle; and minimizing weight while maximizing the strength of a particular component are examples of multi-objective optimization problems.

If a multi-objective problem is well formed, there should not be a single solution that simultaneously minimizes each objective to its fullest. In each case an objective must have reached a point such that, when attempting to optimize the objective further, other objectives suffer as a result.

1.3.1 FUNDAMENTALS OF MULTICRITERIA (OBJECTIVE) OPTIMIZATION:

Multi-objective optimization is not pure minimizing or maximizing problem, it is peculiar mixture of several conflicting maximum or minimum problem which boils down to that of satisfying "satisficing" (meaning, attaining a satisfactory solution) these conflicting objective. This kind of problem is nowhere dealt with in classical mathematics. Rather, it exits as a brand of mathematical programming in the mixed objective and subjective modes.

A number of techniques of multi-objective optimization have been developed. Some are to simplified; other invoke lots of assumption; quit a number of possess mathematically rigor but tend to unfortunately lose individual or organizational preferences and value. It is more flexible in approaching complex problems, and it absorbs the exiting single objective methodology as a special case.

A general constraint optimization problem is defined as a problem of optimizing (maximizing or minimizing) a function $f(X)$ subject to the constraints $g_1(X) \le v_1, g_2(X) \le v_2, ..., g_m(X) \le v_m$, where X is a vector of non-negative real numbers. When, there are k objectives to be optimized simultaneously, one thus in contours a multi-objective optimization problem which can be formulated in the following form:

$$\begin{aligned} &\text{Maximize} && z_l = f_l(x) && l = 1,2,...,k \\ &\text{Subject to} && g_i(x) \le b_i && i = 1,2,...,m \\ & && x \ge 0 && \end{aligned}$$

It is always possible to express the objective function in their "maximize" form since a minimization problem can always be transformed to a maximization problem by proper sign manipulations. Likewise for the constraints, "greater than" an "equal" are always convertible to their equivalent "less than". The matrix version of the above model can be conveniently written as:

$$\begin{aligned} \text{Maximize} \quad & Z = CX \\ \text{Subject to} \quad & AX \le B \\ & X \ge 0 \end{aligned}$$

Where Z, X and B are $k \times 1, n \times 1$ and $m \times 1$ matrix respectively; c is a $k \times n$ matrix; and a is an $m \times n$ matrix.

In multi-criterion optimization, the concept of a unique optimum no longer holds for it is impossible to simultaneously maximize all the conflicting objectives. In other words, the increase in any one of the objective will decrease the others.

1.4 CONCEPT OF OPTIMAL AND EFFICIENT SOLUTIONS:

1.4.1 Optimal Solution

An optimal solution in the classical sense is one which attains the maximum value of all the objectives simultaneously. The solution x^* is optimal to the problem defined if and only if $x^* \in S$ and $f_l(x^*) \ge f_l(x)$ for all l and for all $x \in S$, where S is the feasible region.

In general, there is no optimal solution to a multi-objective problem. Therefore, optimality replaced by the concept of "satisficing" or the best compromise solution, which depends on the decision makers preferences with respect to the object. Optimality is not an illusion only when the objectives are non-conflicting. Therefore, one must be satisfied with obtaining efficient solutions in multi-objective problem.

1.4.2 Efficient or Non-Dominated solutions:

A set of solutions is said to be efficient if there exists no solution that is superior to it with respect to at least one objective function but is not inferior to it with respect to any of the objective functions.

If x_1 and x_2 are two solutions, then these can have any of two possibilities-one dominates the other or non-dominates the other. In a minimization problem, without the loss of generality, a solution x_1 dominates x_2 iff the following two conditions are satisfied:

$$\forall \; i \in \{1, 2, \dots, N_{obj}\} : \; f_i(x_1) \le f_i(x_2)$$

$$\exists \; j \in \{1, 2, \dots, N_{obj}\} : \; f_j(x_1) < f_j(x_2)$$

Where, $f(x_1)$ and $f(x_2)$ are the objective functions

If any of the above conditions is violated, the solution $x_1$does not dominate the solution x_2. If x_1 dominates the solution x_2, x_1 is called the non-dominated solution with in the set $\{x_1, x_2\}$. The solutions that are non-dominated with in the entire search space are denoted as Pareto-optimal and constitute the Pareto-optimal set or Pareto-optimal front. From the entire set off efficient (non-dominated) solutions the decision maker can select the solution one believed most attractive.

1.5 CLASSICAL TRANSPORTATION PROBLEM:

Classical transportation problem is particular class of linear programming, which is associated with day-to-day activities in our real life and mainly deals with logistics. In general, distribution of product from depot to customer is called “Transportation Problem” (TP) which first developed by F. L. Hitchcock since 1941. It usually aims to minimize the total transportation cost. Other objectives that can be set are a minimization of the total delivery time, a maximization of the profit, etc. Transportation problems are also linear programming problems and can be solved by simplex method but because of practical significance the transportation problems are of special interest and it is tedious to solve them through simplex method.

In the classical transportation problem the cost of transportation is directly proportional to the number of units of the commodity transported. The transportation problem networks shows in figure 1.1.

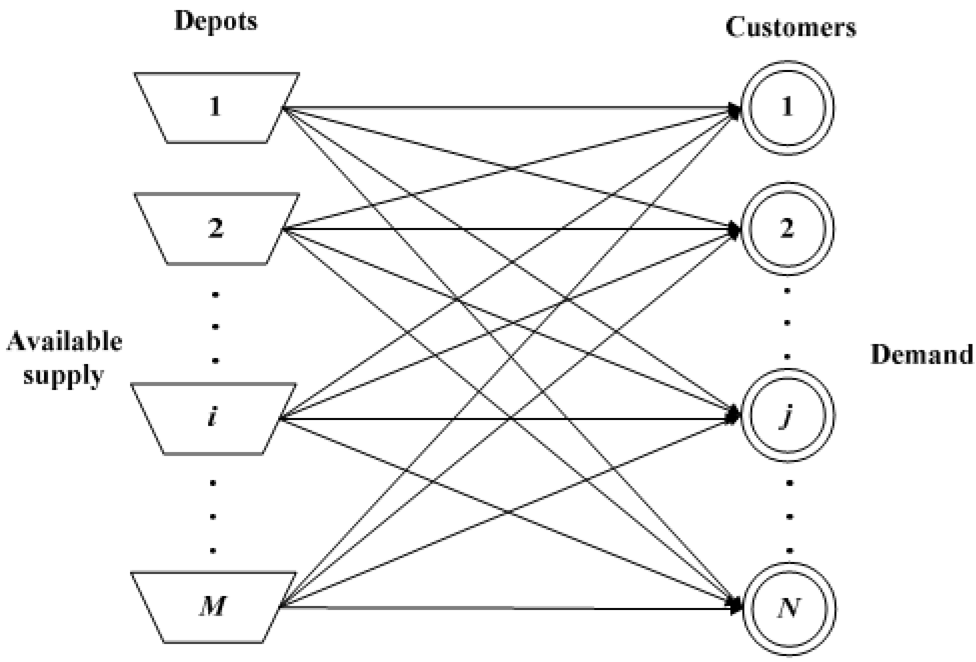

Figure – 1.1 **THE TRANSPORTATION PROBLEM NETWORK**

The classical transportation problem is a single objective transportation problem, which is extensively used. For the problem associated with more than one objective, the decision maker need to simultaneously take other objectives apart from the minimization objective of transportation cost. The other objectives for transportation problem may related to delivery time, quantity of goods delivered, unfulfilled demand, underused capacity, reliability of delivery, safety of delivery, etc.

The formulation of the transportation problem is

$$Minimize\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$subject\ to \quad \sum_{j=1}^{n} x_{ij} \leq a_i, \qquad i = 1,2,\dots,m$$

$$\sum_{j=1}^{n} x_{ij} \geq b_j, \qquad j = 1,2,\dots,n$$

$$x_{ij} \geq 0 \text{ for all } i \text{ and } j.$$

Where $i = 1,2,\dots,m$ are the origins.

$j = 1,2, \dots, n$ are the destinations.

$a_i =$ amount available at the i^{th} origin.

b_j = demand of the j^{th} destination.

x_{ij} = amount transported from the i^{th} origin to the j^{th} destination.

$$if \qquad \sum_{i=1}^{m} a_i = \sum_{j=}^{n} b_j$$

Then the problem is a balanced transportation problem, otherwise it is unbalanced.

The aim is to minimize the objective function satisfying the above constraints. In classical transportation problem in linear programming the traditional objective is to minimize to the total cost

1.6 MULTI-OBJECTIVE TRANSPORTATION PROBLEM:

The multi-objective transportation model is set to solve the transportation problem simultaneously associated with several objectives. Normally, existing multi-objective transportation models use a minimization of the total cost objective as one of their objectives. The other objectives may concern about quantity of goods delivered, underused capacity, energy consumption, total delivery time, etc. Kasana and Kumar (2003) formulate the multi-objective transportation problem as follows:

Consider m origins and n destinations and also the quantities available at each origin and the quantities to be transported to each destination. The total quantities required at the destinations may differ from the total quantities available at the origins. For such situations, the problem is balanced by introducing fictitious origin or destination; whichever is needed in order to get precisely the same quantities at the origins and the destinations. Specifically, a balanced transportation problem is considered as it amounts to no loss of generality.

Suppose x_{ij} = amount transported from the ith origin to the jth destination and for each fixed k: $k = 0,1, \dots, p-1$, α_{ij}^{k} , $i = 1,2, \dots, m$, $j = 1,2, \dots, n$ be the units of parameter required for transporting one unit of the quantity from origin i to destination. What is to be determined is the routing from origin i to destination j satisfying p objectives. The starting object is termed as primary and the other are classified as secondary.

The primary objective is to minimize

$$Z_0 = \sum_{i=1}^{m} \sum_{j=1}^{n} \alpha_{ij}^{k} x_{ij}$$

and for $k = 1,2,\ldots,(p-1)$, also to minimize

$x_k = max\{\alpha_{ij}^{k} : x_{ij} \geq 0, i = 1,2,\ldots,m, j = 1,2,\ldots,n\}$

in order of the priorities to be assigned under the constraints

$$\sum_{j=1}^{n} x_{ij} = a_i \quad (i = 1,2,\ldots,m)$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad (j = 1,2,\ldots n)$$

The problem formulated above has p objective functions given by equations. A transportation problem with m supply nodes and n demand nodes contains mn variables and $m+n$ constraint equations.

1.7 BULK TRANSPORTATION PROBLEM:

In the bulk transportation problem, the entire requirement of the each destination is to be met from one source only; but a source can supply to any number of destinations subject to the availability of the commodity at it. The transportation starts simultaneously. Let a_i $(i = 1,2,\ldots,m)$ be the units of the commodity available at source i, $b_j (j = 1,2,\ldots,n)$ the units of the commodity required at destination j, c_{ij} $(i = 1,2,\ldots,m\,; j = 1,2,\ldots,n)$ the units of cost bulk transportation of the commodity from source i to j, and x_{ij} $(i = 1,2,\ldots,m\,; j = 1,2,\ldots,n)$ the variables asumming the value 0 or 1 according as the entire requirement of destination j is not met or met from source i. All the parameters $a_i's$, $b_j's$ and $c_{ij}'s$ are free to take non-negative real values. And C denote the total cost of bulk transportation problem.

The mathematical formulation of the problem is

$$C = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij},$$

Without according priorities to them subject to constraints

$$\sum_{j=1}^{n} b_j x_{ij} \leq a_i \;\; (i = 1,2,\ldots,m)$$

$$\sum_{i=1}^{m} x_{ij} = 1 \quad (j = 1,2, \dots n),$$

$$x_{ij} = 0 \text{ or } 1 \ (i = 1,2, \dots, m \,; j = 1,2, \dots, n).$$

1.8 LITERATURE SURVEY:

The present work deals with multi-objective optimization problems. Multi-objective problems differ from single-objective problems in the sense that the former problems have more than one objective whereas the latter problems have only one objective. There are many approaches for solving multi-objective optimization problems whereas there is universally accepted single approach seeking to optimize the single objective function. The various approaches for solving multi-objective optimization problems are lexicographic / prioritized and Paerto optimal / efficient / non-dominated solution approach. A discussion about them can be found in the works of Zeenlay (1974), Prakash (1981), Igznio (1982), Sharma and Prakash (1986), steuer (1986),Prakash, Aggarwal and Shah (1988), Prakash and Pradeep (1991), Prakash, Balaji and Tuteja (1999), Taha (2008). The first two approaches reduce the multi-objective optimization to single-objective optimization problems while the last two approaches do not alter the nature of the problems. Ther are two main approaches for solving the multi-objective optimization problems –(a) analytic (b) heuristic. The analytic approach yields an exact solution of the problem whereas the heuristic approach yields a satisfying solution which can also be at times an exact solution. There are many problems which either cannot be solved or even if can be solved through analytical approach, the amount of time and labor spent on solving them through analytic approach renders it unsuitable. In such as cases, heuristic approach comes to our rescue. The heuristic approach does not require specialized knowledge of the subject. It is based on intuition, experience and judgment, thereby making it easy to apply and has wider applicability. A discussion about heuristic approach used for solving problems can be found in the works of Ignizio (1982), Glover (1989, 1990), Reeves (1993).

TP with a different single-objective to minimize the duration of transportation has been studied by many researcher as Sharma and Swarup (1977), Seshan and Tikekar (1980), Prakash (1982), Sonia and Puri (2004), Sonia , Khandelwal and Puri (2008), etc. TP with multiple objectives has also been discussed by Prakash (1981), Purushotam, Prakash and Dhyani (1984), Aggarwal and Shah (1988).

Transportation problem has been paid much attention and classified into several types of transmutations since it was first proposed by Hitchcock (1941). One of the variants of the transportation problem is bulk transportation problem (BTP) which differs from the transportation problem that it stipulates that the total requirement of each destination has to be met from single source only; however, a source can supply to any number of destinations subject to the availability of the commodity at the source. The bulk transportation was first formulated by Maio and Roveda (1971) with the single objective to minimize the total transportation cost. Srinivasan and Thompson (1973) presented an algorithm consisting of the two phases to solve this problem. Verma and Puri (1996) has applied branch and bound method to minimize the cost of the bulk transportation problem. Murthy (1976) proposed a method based on the principle of lexicographic minimum to solve the bulk transportation problem. Prakash and Ram (1995) have considered a bulk transportation as primary and secondary objectives respectively.

1.9 Present Work

The present thesis consists of three chapters. Chapter one is introducing in nature in which multi-objective optimization has been described and brief survey of the literature to the topic has been discussed. In the second chapter two algorithms for multi-objective bulk transportation problem (MBTP) given by Prakash et al. (2007, 2009) have been reviewed. In chapter three, A Heuristic algorithm has been developed for MBTP which is a combination the algorithms review in chapter second.

Chapter - 2

PARETO OPTIMAL SOLUTIONS OF A COST-TIME TRADE-OFF BULK TRANSPORTATION PROBLEM

2.1 INTRODUCTION

In this chapter, the algorithms given by Parkash et. Al. (2008, 2009) for bi-criterion transportation problem have been reviewed in section 2.3 and2.7 respectively. The normal transportation problem wherein the requirement of each destination can be met from one or more than one source with the single objective to minimize the total cost of transportation has long been studied and is well known.

There is another type of the normal transportation problem called the bulk transportation problem which differs from the normal transportation problem in that it stipulates that the requirement of each destination has to be met from one source only; however a source can supply to any number of destinations subject to availability of the commodity at it.

A cost time trade-off bulk transportation problem with the objectives to minimize the total cost and duration of bulk transportation without according priorities to them is considered. The entire requirements of each destination is to be met from one source only; however a source can supply to any number of destinations subject to the availability of the commodity at it.

Sometimes it happens that the decision maker is not able to assign priorities to his/her objectives. In such a situation, help can be provided to the decision maker by presenting him/her with a set of Pareto optimal solutions. He/she can pick up that solution out of this set which suits him/her most with regard to his/her objectives and liking.

2.2 FORMULATION OF MULTIOBJECTIVE BULK TRANSPORTATION PROBLEM

Suppose that there are m sources and n destinations. Given amounts of a commodity are available at the sources and specified amounts of the commodity are required at the destinations. The entire requirement of each destination is to be met from one source only; but a source can supply to any number of destinations subject to the availability of the commodity at it. The transportation starts simultaneously. Let $a_i(i = 1,2, \dots, m)$ be the units of the commodity available at source i, b_j $(j = 1,2, \dots, n)$ the units of the commodity required at destination j, c_{ij} $(i = 1,2, .., m\, ; j = 1,2, \dots, n)$ the units of cost of bulk transportation of the commodity from source i to destination j, t_{ij} $(i = 1,2, .., m\, ; j = 1,2, \dots, n)$ the units of time of bulk transportation of the commodity from source i to destination j, and x_{ij} $(i = 1,2, .., m\, ; j = 1,2, \dots, n)$ the variable assuming the value 0 or 1 according as the entire requirement of destination j is not met or met from source i. All the parameters $a_i's, b_j's, c_{ij}'s$ and $t_{ij}'s$ are free to take any non-negative real values.

Let C and T denote the total cost and duration of bulk transportation respectively. The mathematical formulation of the problem is as follows.

Determine $x_{ij}'s$ which minimize the two-objective functions:

$$C = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{2.1}$$

$$T = \max\{t_{ij} x_{ij} : i = 1, 2, \dots, m;\ j = 1, 2, \dots, n\} \tag{2.2}$$

without according priorities to them subject to the constraints

$$\sum_{j=1}^{n} b_j x_{ij} \le a_i\, (i = 1, 2, \dots, m) \tag{2.3}$$

$$\sum_{i=1}^{m} x_{ij} = 1\ (j = 1, 2, \dots, n) \tag{2.4}$$

$$x_{ij} = 0 \text{ or } 1\ (i = 1, 2, \dots, m;\ j = 1, 2, \dots, n) \tag{2.5}$$

It is required to find the set of Pareto optimal solutions of the above problem. For the purpose of listing the Pareto optimal solutions of the formulated cost–time trade-off bulk transportation problem, we call a Pareto optimal solution $\bar{x}^{(1)}$, the 1st Pareto optimal solution, if it is the optimal solution of the formulated problem with the minimization of C as the first priority objective and that of T as the second priority objective. We call a Pareto optimal solution $\bar{x}^{(2)}$, the 2nd Pareto optimal solution, if there exists no Pareto optimal solution $\bar{y}$ of the formulated problem satisfying the conditions:

(i) $C(\bar{x}^{(1)}) < C(\bar{y}) < C(\bar{x}^{(2)})$ and

(ii) $T(\bar{x}^{(1)}) > T(\bar{y}) > T(\bar{x}^{(2)})$

Proceeding as we did to define the 2nd Pareto optimal solution, we define the 3rd and subsequent Pareto optimal solutions.

2.3 Solution Procedure

Prakash et. al. (2008) proposed an algorithm for obtaining the set of Pareto optimal solution of the problem (2.1) – (2.5) which require a sequence of single objective bulk transportation problem to be solved. This algorithm employ a branch and bound method for solving the sequence of single objective bulk transportation problem. This algorithm makes use of the previously available of the information as done in all the branch and bound methods. The algorithm for obtaining the set of Pareto optimal solution cost-time trade-off bulk transportation problem together with the branch and bound method employed for solving the sequence of single objective bulk transportation problem.

2.4 ALGORITHM

2.4.1 Algorithm Using Preemptive Priority Factors

The algorithm using preemptive priority factors obtains the set of Pareto optimal solutions of the cost–time trade-off bulk transportation problem through solving a sequence of single-objective bulk transportation problems. The single-objective bulk transportation problems whose optimal solutions yield Pareto optimal solutions of the cost–time trade-off bulk transportation problem are derived making use of the preemptive priority factors as done by Prakash et al. (1999). Procedures to obtain the 1st and subsequent Pareto optimal solutions are outlined below.

The 1st Pareto optimal solution of the cost–time trade-off bulk transportation problem is the optimal solution of the problem with the minimization of *C* provided by Eq. (1) as the first priority objective and that of T provided by Eq. (2) as the second priority objective. So for obtaining the 1st Pareto optimal solution, the cost–time trade-off bulk transportation problem with prioritized objectives is solved. For this purpose, we reduce it to an equivalent single-objective bulk transportation problem which is designated as the 1st single-objective bulk transportation problem of the cost–time trade-off bulk transportation problem. Its optimal solution yields the 1st Pareto optimal solution $\bar{x}^{(1)}$ of the cost–time trade-off bulk transportation problem.

Step 1: The set $\{t_{ij} : i = 1, 2, \ldots, m\, ; j = 1, 2, \ldots, n\}$ is partitioned into subsets $L_k(k = 1, 2, \ldots, q)$ in the following way. Each of the subsets $L_k{}'s$ consists of the $t_{ij}{}'s$ having the same numerical value. L_1 consists of the t_{ij} having the largest numerical value, L_2 consists of the t_{ij} having next largest value, and so on. L_q consists of the t_{ij} having the smallest numerical value.

Step 2: Preemptive priority factors $M_0, M_1, \ldots, M_q$ are assigned to C, $\sum_{L_1} x_{ij}, \ldots, \sum_{L_q} x_{ij}$ respectively. Here $\sum_{L_k} x_{ij}$ is the sum of the $x_{ij}{}'s$ corresponding to the t_{ij} belonging to L_k. All the priority factors $M_k{}'s$ are fixed positive real numbers and are such that the expression $\sum_{k=0}^{q} \alpha_k M_k$, where $\alpha_k{}'s$ are real numbers which can be negative or zero or positive, has the same sign as the non-zero α_k with the smallest subscript in it irrespective of the values of other $\alpha_k' s$. This implies that $M_0, M_1, \ldots, M_q$ are such that $M_0 \gg M_1 \gg \cdots \gg M_q$. (The symbol $\gg$ indicates that the quantity on its left side arbitrarily large compared to right hand side).

Step 3: After this, the cost time bulk transportation problem with C and T as the first and second priority objectives respectively is reduced to an equivalent single-objective bulk transportation problem seeking to determine $x_{ij}{}'s$ which minimize

$$Z = M_0 \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} + \sum_{k=1}^{q} M_k \sum_{L_k} x_{ij}$$

Subject to the given constraints in the formulation.

2.4.2 Branch and Bound Method for solving 1st and any Single-Objective Bulk Transportation Problem

Step 4: Now derive a branch and bound method for solving the first single objective bulk transportation problem minimizing Z subject to the given constraints. This very method would solve any single-objective bulk transportation problem because all single-objective bulk transportation problems differ only in the numerical values of the parameters. The tableau representation of the 1st single objective bulk transportation problem is shown in Table – 2.1.

Table – 2.1

Sources	Destinations				Availability
	D_1	D_2	$\cdots$	D_m	a_i
S_1	$c_{11}M_0 + M_k$	$c_{12}M_0 + M_k$	$\cdots$	$c_{1n}M_0 + M_k$	a_1
S_2	$c_{21}M_0 + M_k$	$c_{22}M_0 + M_k$	$\cdots$	$c_{2n}M_0 + M_k$	a_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	$c_{m1}M_0 + M_k$	$c_{m2}M_0 + M_k$	$\cdots$	$c_{mn}M_0 + M_k$	a_m
Requirement b_j	b_1	b_2	$\cdots$	b_n	

In this table, cell $(i, j)'s$ 0correspond to the variables $x_{ij}'s(i = 1, 2, \dots, m\,; j = 1, 2, \dots, n)$. M_k included in the expression $c_{ij}M_0 + M_k$ inside the cell$(i,\ j)$ is M_1 or M_2 or…or M_q according as the t_{ij} corresponding to the cell $(i,\ j)$ belongs to L_1 or L_2 or…or L_q.

Step 5: As the entire requirement of each destination D_j is to be met from one source only, all the variables $x_{ij}'s$ corresponding to the cells in the column associated with D_j except one would be zero. So a set consisting of one x_{ij} at level 1 corresponding to a cell $(i,\ j)$ in each of the columns associated with a destination would provide a solution of the 1st single-objective bulk transportation problem provided it satisfies the given constraints.

Step 6: This set would contain the number n of the variables $x_{ij}'s$ at level 1, each with a different value of j, and the remaining ones at level 0. Now a branch and bound method is developed for solving the 1st single-objective bulk transportation problem. Different branch and bound methods are developed by choosing in different ways

1. The lower bound of the objective function at a node
2. The node at each stage for branching the set of solutions, and
3. The variable with respect to which branching at the chosen node is to be done.

Step 7: Lower bound LB at the topmost node S in the tree is the lower bound of the objective function of the set of all solutions of the 1st single-objective bulk transportation problem and is computed as follows. First, rows associated with sources $S_i's$ containing the $a_i's$ having values smaller than those of all the $b_i's$ are deleted from above table, because the sources corresponding to these rows cannot be used to meet the requirements of any of the destinations. Also every cell (i, j) corresponding to which $a_i < b_j$ is deleted from above table, because then the source corresponding to the row containing the cell cannot meet the requirement of the destination corresponding to the column containing the cell.

Step 8: After this, the smallest entry in each column associated with a destination is subtracted from all the entries of that column yielding a reduced cost table in which each column associated with a destination contains a zero cost in a cell. The lower bound of the objective function at the node S is computed by adding the subtracted smallest entries of each of the columns yielding the reduced cost table. Lower bound of the objective function at any other node is computed in the same way as done in the case of the traveling salesman problem but ignoring the condition to eliminate sub-tours.

Step 9: The reduced cost table and the lower bound at the node other than the node S are obtained in the same way as done at the node S after updating the table of the node. By updating the table, we mean that all the columns associated with the destinations whose requirements have been met are deleted from it. Then the amounts of the commodity which have been used to meet the entire requirements of the destinations from the sources are subtracted from the amounts previously available at them resulting in updating the commodity available at the sources. Having deleted the columns associated with the destinations whose requirements have been met and updating the commodity available at the sources, rows associated with the sources at which the commodity is less than the requirements of all the destinations are deleted. Also each of the cells $(i, j)'s$ corresponding to which the updated commodity available at the source S_i is less than that required at the destination D_j is deleted. The table thus obtained is updated table of the node. After this, the smallest entry in each column associated with a destination is subtracted from all the entries of that column yielding a reduced cost table of the updated table of the node. The lower bound at the node other than the node S is the lower bound of the immediately preceding node plus the sum of the subtracted smallest entries of each of the columns of the updated table of the node.

Step 10: The node chosen at each stage for branching the set of solutions is the node among the terminal unfathomed nodes having the smallest lower bound of the objective function. The variable with respect to which branching at the chosen node is to be done is determined as follows. In the reduced cost table of the chosen node, least cost of exclusion is entered in the cells with zero costs. The least cost of exclusion for each cell with a zero cost in a column is the next lower cost in a cell of the column. However if there are more than one cell in a column with a zero cost then the least cost of exclusion for each of these cells is zero. Further if there is only one cell with a zero cost in a column and all other cells in that column have been deleted, then the least cost of exclusion for this cell is ∞.

For if the requirement of the destination associated with the column containing this cell is not met from the source associated with the row containing this cell, then the requirement of the destination associated with the column containing this cell can never be met. The least cost of exclusion for a cell (i,j) indicates the least cost to be incurred if the requirement b_j of destination D_j is not met from source S_i. Branching is done at the chosen node with respect to the variable x_{ij} corresponding to the cell (i,j) having the greatest least cost of exclusion among all the cells with zero costs. The terminal unfathomed node containing the number n of the variables $x_{ij}'s$ at level 1, each with a different value of j, along the chain from the topmost node S to it and having the smallest lower bound of the objective function among all the terminal unfathomed nodes at a stage provides the optimal solution of the 1st single-objective bulk transportation problem.

2.4.3 Procedure for obtaining 2nd and subsequent Pareto optimal solutions

After having obtained the 1st Pareto optimal solution $\bar{x}^{(1)}$ of the cost–time trade-off bulk transportation problem, its 2nd Pareto optimal solution is obtained. For this purpose, we obtain the problem by deleting all the variables $x_{ij}'s$ corresponding to the $t_{ij}'s \geq T(\bar{x}^{(1)})$ in the 1st single-objective bulk transportation problem seeking to determine $x_{ij}'s$ which minimize the objective function Z subject to the given constraints. The problem thus obtained is designated as the 2nd single-objective bulk transportation problem. The optimal solution of this problem is obtained exactly in the same way as done for the 1st single-objective bulk transportation problem. This optimal solution yields the 2nd Pareto optimal solution $\bar{x}^{(2)}$.

Now we explain why the optimal solution of the 2nd single-objective bulk transportation problem would yield the 2nd Pareto optimal solution $\bar{x}^{(2)}$. The reason is this that all the $x_{ij}'s$ corresponding to the $t_{ij}'s \geq T(\bar{x}^{(1)})$ are zero in the 2nd single-objective bulk transportation problem. So the optimal solution of this problem will have no $x_{ij}'s$ at level 1 corresponding to the $t'_{ij}s \geq T(\bar{x}^{(1)})$. This will result into yielding the 2nd Pareto optimal solution $\bar{x}^{(2)}$ for which the duration $T(\bar{x}^{(2)})$ of bulk transportation will be less than $T(\bar{x}^{(1)})$. For obtaining the 3rd Pareto optimal solution, we obtain the 3rd single-objective bulk transportation problem from the 2nd single-objective bulk transportation problem by deleting all the variables $x_{ij}'s$ corresponding to the $t'_{ij}s \geq T(\bar{x}^{(2)})$. therein. The optimal solution of the 3rd single-objective bulk transportation problem is obtained exactly in the same way as done for the 1st single-objective bulk transportation problem. This optimal solution yields the 3rd Pareto optimal solution $\bar{x}^{(3)}$ for which the duration $T(\bar{x}^{(3)})$ of bulk transportation will be less than $T(\bar{x}^{(2)})$. Subsequent Pareto optimal solutions are obtained by proceeding exactly in the same way as done for obtaining the Pareto optimal solutions $\bar{x}^{(2)}$ and $\bar{x}^{(3)}$ This process of obtaining Pareto optimal solutions is terminated after encountering a single-objective bulk transportation problem whose tree contains all terminal nodes fathomed without having arrived at a node containing the number n of the variables $x_{ij}'s$ at level 1, each with a different value of j, along a chain from the topmost node S to it, indicating that it is no longer possible to find a new Pareto optimal solution with lesser duration of bulk transportation. Thus, the total number of the single-objective bulk transportation problems to be solved for obtaining the set of Pareto optimal solutions is only one more than that of the Pareto optimal solutions.

2.5 Numerical Example

We shall illustrate the algorithm through solving the numerical problem. We apply the algorithm for obtaining the set of Pareto optimal solutions of the numerical problem obtained by taking $m = 4, n = 5$ and assigning numerical values to all the other parameters in the problem. The tableau representation of the numerical problem is shown in Table -2.2. In this table, the cells $(i,j)'s$ correspond to the variables $x'_{ij}s$ $(i = 1,2,3,4\,; j = 1,2,3,4,5)$. The upper and lower entries of the cell (i,j) depict the units of cost and time of bulk transportation from source S_i to destination D_j respectively. The marginal row and column depict the units of the commodity required at the destinations and available at the source respectively.

The numerical problem seeks to determine $x_{ij}'s$ which minimize the two-objective functions:

Table – 2.2

Sources	D_1	D_2	D_3	D_4	D_5	Availability (a_i)
S_1	2 4	3 4	3 10	7 8	1 7	5
S_2	4 4	1 7	1 12	2 14	8 8	4
S_3	1 8	7 2	11 4	1 4	5 4	3
S_4	20 4	30 6	10 7	2 2	5 2	2
Requirement (b_j)	3	3	2	2	1	

$$C = 2x_{11} + 3x_{12} + 3x_{13} + 7x_{14} + x_{15} + 4x_{21} + x_{22} + x_{23} + 2x_{24} + 8x_{25} + x_{31} + 7x_{32} + 11x_{33} + x_{34} + 5x_{35}$$

$$+ 20x_{41} + 30x_{42} + 10x_{43} + 2x_{44} + 5x_{45}$$

$$T = \max\{t_{ij}x_{ij} : i = 1, 2, 3, 4;\ j = 1, 2, 3, 4, 5\}$$

Without according priorities to them subject to the given constraints after assigning numerical values to all the parameters therein.

Solution of numerical problem trough algorithm using preemptive priority factors

We apply the above algorithm using preemptive priority factors for obtaining the set of Pareto optimal solutions of the numerical problem. Using step 1 to step 4 to obtain the single objective bulk transportation problem is given by equation (6), we find $q = 8$. The subsets forming the partition of the set $\{t_{ij} : i = 1, 2, 3, 4\ ; j = 1, 2, 3, 4, 5\}$ are as follows:

$$L_1 = \{t_{24}\}, \qquad L_2 = \{t_{23}\}, \qquad L_3 = \{t_{13}\}$$

$$L_4 = \{t_{14}, t_{25}, t_{31}\}, \qquad L_5 = \{t_{15}, t_{22}, t_{43}\}, \qquad L_6 = \{t_{42}\}$$

$$L_7 = \{t_{11}, t_{12}, t_{21}, t_{33}, t_{34}, t_{35}, t_{41}\}, \qquad L_8 = \{t_{32}, t_{44}, t_{45}\}$$

The $t_{ij}'s$ belonging to L_1, L_2, L_3, L_4, L_5, L_6 ,L_7 and L_8 have the numerical values $14, 12, 10, 8, 7, 6, 4$ and 2 respectively.

The 1st single-objective bulk transportation problem of the numerical problem seeks to determine $x_{ij}'s$ which minimize

$$Z = M_0(2x_{11} + 3x_{12} + 3x_{13} + 7x_{14} + x_{15} + 4x_{21} + x_{22} + x_{23} + 2x_{24} + 8x_{25} + x_{31} + 7x_{32} + 11x_{33} + x_{34} + 5x_{35}$$
$$+20x_{41} + 30x_{42} + 10x_{43} + 2x_{44} + 5x_{45})$$
$$+ M_1(x_{24}) + M_2(x_{23}) + M_3(x_{13}) + M_4(x_{14} + x_{25} + x_{31}) + M_5(x_{15} + x_{22} + x_{43}) + M_6(x_{42}) \quad ...(2.6)$$
$$+ M_7(x_{11} + x_{12} + x_{21} + x_{33} + x_{34} + x_{35} + x_{41}) + M_8(x_{32} + x_{44} + x_{45})$$

subject to the given constraints after assigning numerical values to all the parameters therein.

The optimal solution of the 1st single-objective bulk transportation problem of the numerical problem yields the 1st Pareto optimal solution $\bar{x}^{(1)}$ of the numerical problem. The tableau representation of the 1st single-objective bulk transportation problem of the numerical is shown in table-3. In this table, the cells $(i, j)'s$ $(i = 1, 2, 3, 4\,; j = 1, 2, 3, 4, 5)$ depict the cost which resemble in nature the cost associated with an artificial variable in the Big-M method of linear programming. The $M_k's$ are handled almost in the same manner as M is handled in the Big-M method. While minimizing Z, it should be kept in mind that $M_0 \gg M_1 \gg \cdots \gg M_8$. Some or all of the $M_k's$ can be present here in the optimal expression of Z in contrast to the Big-M method where M cannot be present in the optimal expression of the objective function.

The matrix form of objective function given by equation (2.6) is presented in table – 2.3.

Table – 2.3

Sources	Destinations					Availability
	D_1	D_2	D_3	D_4	D_5	a_i
S_1	$2M_0+M_7$	$3M_0+M_7$	$3M_0+M_3$	$7M_0+M_4$	M_0+M_5	5
S_2	$4M_0+M_7$	M_0+M_5	M_0+M_2	$2M_0+M_1$	$8M_0+M_4$	4
S_3	M_0+M_4	$7M_0+M_8$	$11M_0+M_7$	M_0+M_7	$5M_0+M_7$	3
S_4	$20M_0+M_7$	$30M_0+M_6$	$10M_0+M_5$	$2M_0+M_8$	$5M_0+M_8$	2
Requirement b_j	3	3	2	2	1	

Applying the branch and bound method, a tree is drawn for solving the first single objective bulk transportation problem of the numerical problem and are shown in figures.

Table – 2.4

Sources	Destinations					Availability
	D_1	D_2	D_3	D_4	D_5	a_i
S_1	M_0+M_7 $-M_4$	$2M_0+M_7$ $-M_5$	$2M_0+M_3$ $-M_2$	$6M_0+M_4$ $-M_7$	0	5
S_2	$3M_0+M_7$ $-M_4$	0	0	M_0+M_1 $-M_7$	$7M_0+M_4$ $-M_5$	4
S_3	0	$6M_0+M_8$ $-M_5$	$10M_0+M_7$ $-M_2$	0	$4M_0+M_7$ $-M_5$	3
S_4	∞	∞	$9M_0+M_5$ $-M_2$	M_0+M_8 $-M_7$	$4M_0+M_8$ $-M_5$	2
Requirement b_j	3	3	2	2	1	

$$S = M_0 + M_4 + M_0 + M_5 + M_0 + M_2 + M_0 + M_7 + M_0 + M_5$$
$$= 5M_0 + M_2 + M_4 + 2M_5 + M_7$$

The cells which contains zero are $(1,5), (2,2), (2,3), (3,1), (3,4)$. Calculate the penalties to corresponding these cells. The cell (1,5) is selected because the penalty on this cell is the largest and make the allocation in cell (1,5). Applying branch and bound algorithm $x_{15} = 0$ or 1. In case $x_{15} = 0$, then the cell (1,5) will be blocked and make the reduce matrix given in table – 2.5.

Table – 2.5

Sources	Destinations					Availability
	D_1	D_2	D_3	D_4	D_5	a_i
S_1	$M_0 + M_7 - M_4$	$2M_0 + M_7 - M_5$	$2M_0 + M_3 - M_2$	$6M_0 + M_4 - M_7$	∞	5
S_2	$3M_0 + M_7 - M_4$	0	0	$M_0 + M_1 - M_7$	$7M_0 + M_4 - M_5$	4
S_3	0	$6M_0 + M_8 - M_5$	$10M_0 + M_7 - M_2$	0	$4M_0 + M_7 - M_5$	3
S_m	∞	∞	$9M_0 + M_5 - M_2$	$M_0 + M_8 - M_7$	$4M_0 + M_8 - M_5$	2
Requirement b_j	3	3	2	2	1	

$$S = 5M_0 + M_2 + M_4 + 2M_5 + M_7$$
$$LB = \overline{(1,5)} = 5M_0 + M_2 + M_4 + 2M_5 + M_7 + 4M_0 + M_8$$
$$= 9M_0 + M_2 + M_4 + M_5 + M_7 + M_8$$

At the level zero, we got *LB* as show above and show its branch in figure – 2.1.

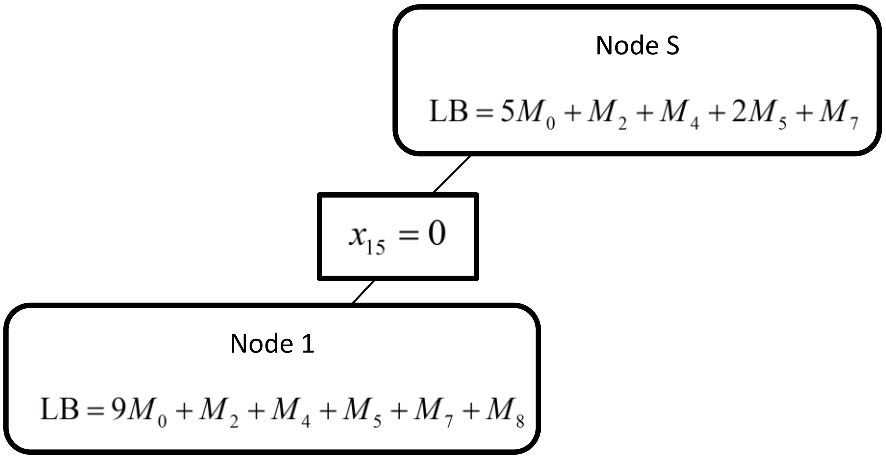

Figure - 2.1

In case of $x_{15} = 1$ then we apply branch and bound algorithm for cell $(1,5)$ and block the corresponding row and column and make the reduced matrix given in table – 2.6.

Table – 2.6

Sources	Destinations					Availability
	D_1	D_2	D_3	D_4	D_5	a_i
S_1	-----	-----	-----	-----	-----	5
S_2	$3M_0 + M_7 - M_4$	0	0	$M_0 + M_1 - M_7$	-----	4
S_3	0	$6M_0 + M_8 - M_5$	$10M_0 + M_7 - M_2$	0	-----	3
S_m	∞	∞	$9M_0 + M_5 - M_2$	$M_0 + M_8 - M_7$	-----	2
Requirement b_j	3	3	2	2	1	

Using step 5 to step 7, *LB* of node *S* is obtained.

$S = 5M_0 + M_2 + M_4 + 2M_5 + M_7 + 0$

$LB = (1,5) = 5M_0 + M_2 + M_4 + 2M_5 + M_7$

At the level one, we got LB as show above and show its branch in figure – 2.2.

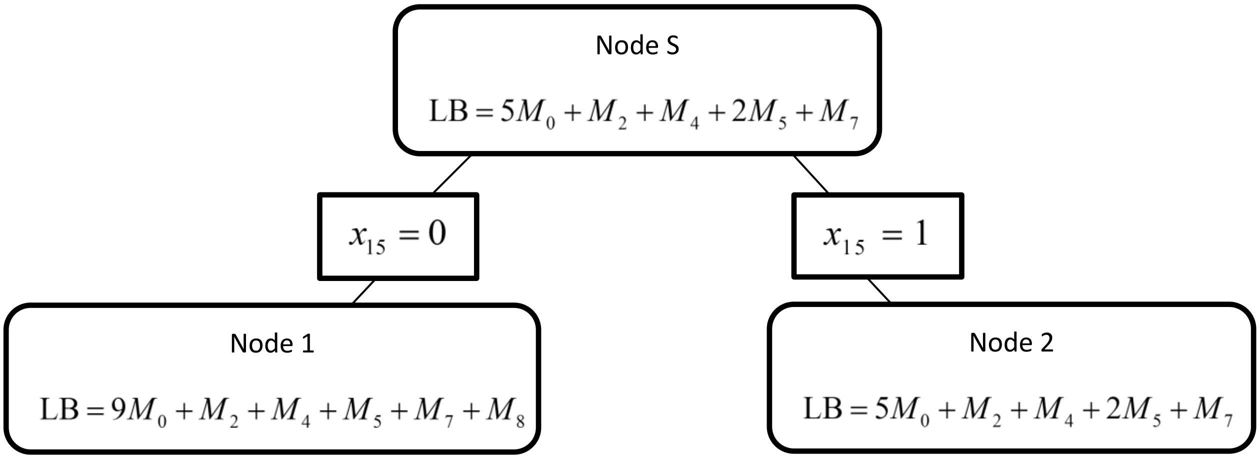

Figure – 2.2

Procedure is repeated until the requirements of all destinations have been completed. Computations are given from table – 7 to table – 13 and the complete solution for the Lower Bound's shown in figure – 2.6.

Table – 2.7

Sources	Destinations				Availability
	D_1	D_2	D_3	D_4	a_i
S_1	$M_0 + M_7 - M_4$	$2M_0 + M_7 - M_5$	$2M_0 + M_3 - M_2$	$6M_0 + M_4 - M_7$	4
S_2	$3M_0 + M_7 - M_4$	0	0	$M_0 + M_1 - M_7$	4
S_3	0	$6M_0 + M_8 - M_5$	$10M_0 + M_7 - M_2$	0	3
S_m	∞	∞	$9M_0 + M_5 - M_2$	$M_0 + M_8 - M_7$	2
Requirement b_j	3	3	2	2	

If we can choose cell (2, 3) then the solution is $C(\bar{x}^{(1)}) = 8$ and $T(\bar{x}^{(1)}) = 12$ that is not sufficient for the further solution. The further solution is $C(\bar{x}^{(1)}) = 8$ and $T(\bar{x}^{(1)}) = 10$.

Now next cell to choose (2,2).

Table – 2.8

Sources	Destinations				Availability
	D_1	D_2	D_3	D_4	a_i
S_1	$M_0+M_7-M_4$	$2M_0+M_7-M_5$	$2M_0+M_3-M_2$	$6M_0+M_4-M_7$	4
S_2	$3M_0+M_7-M_4$	∞	0	$M_0+M_1-M_7$	4
S_3	0	$6M_0+M_8-M_5$	$10M_0+M_7-M_2$	0	3
S_m	∞	∞	$9M_0+M_5-M_2$	$M_0+M_8-M_7$	2
Requirement b_j	3	3	2	2	

Table – 2.9

Sources	Destinations				Availability
	D_1	D_2	D_3	D_4	a_i
S_1	$M_0+M_7-M_4$	0	$2M_0+M_3-M_2$	$6M_0+M_4-M_7$	4
S_2	$3M_0+M_7-M_4$	∞	0	$M_0+M_1-M_7$	4
S_3	0	$4M_0+M_8-M_7$	$10M_0+M_7-M_2$	0	3
S_4	∞	∞	$9M_0+M_5-M_2$	$M_0+M_8-M_7$	2
Requirement b_j	3	3	2	2	

$$LB = 5M_0 + M_2 + M_4 + 2M_5 + M_7 + 2M_0 + M_7 - M_5$$
$$= 7M_0 + M_2 + M_4 + M_5 + 2M_7$$

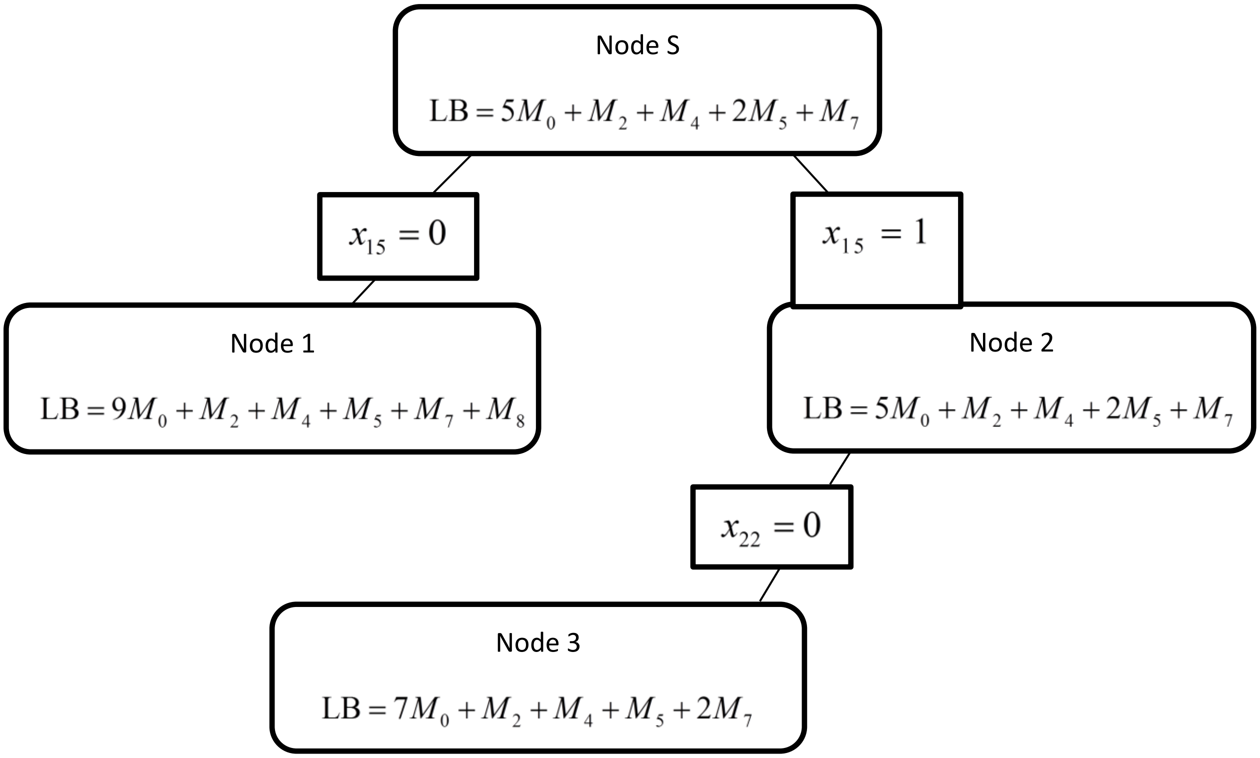

Figure – 2.3

Table – 2.10

Sources	Destinations				Availability
	D_1	D_2	D_3	D_4	a_i
S_1	$M_0 + M_7 - M_4$	-----	$2M_0 + M_3 - M_2$	$6M_0 + M_4 - M_7$	4
S_2	-----	-----	-----	-----	4
S_3	0	-----	$10M_0 + M_7 - M_2$	0	3
S_m	∞	-----	$9M_0 + M_5 - M_2$	$M_0 + M_8 - M_7$	2
Requirement b_j	3	3	2	2	

Table – 2.11

Sources	Destinations				Availability
	D_1	D_2	D_3	D_4	a_i
S_1	$M_0+M_7-M_4$	-----	0	$6M_0+M_4-M_7$	4
S_2	-----	-----	-----	-----	4
S_3	0	-----	$8M_0+M_7-M_3$	0	3
S_m	∞	-----	$7M_0+M_5-M_3$	$M_0+M_8-M_7$	2
Requirement b_j	3	3	2	2	

$$LB = 5M_0 + M_2 + M_4 + 2M_5 + M_7 + 2M_0 + M_3 - M_2$$
$$= 7M_0 + M_3 + M_4 + 2M_5 + M_7$$

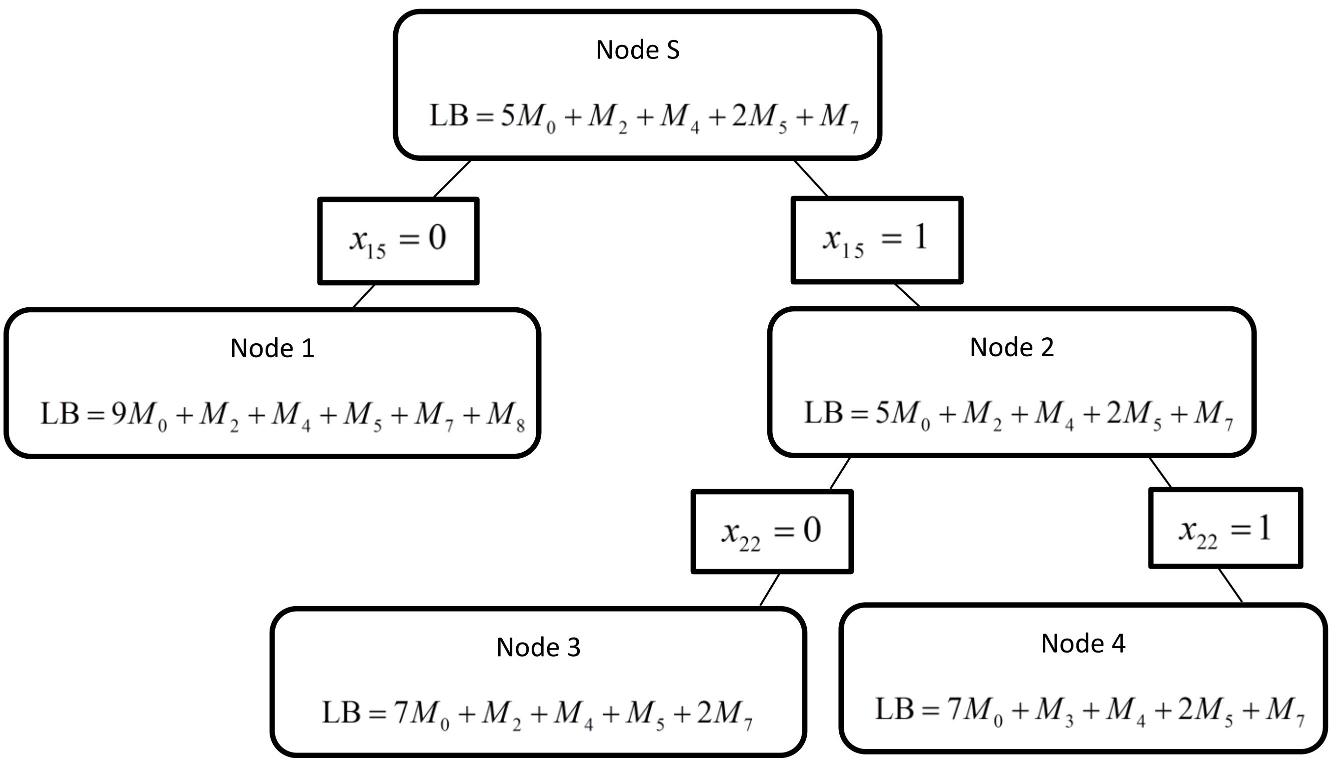

Figure – 2.4

Table – 2.12

Sources	Destinations				Availability
	D_1	D_2	D_3	D_4	a_i
S_1	$M_0+M_7-M_4$	∞	$2M_0+M_3-M_2$	$6M_0+M_4-M_7$	4
S_2	∞	∞	0	$M_0+M_1-M_7$	4
S_3	0	$4M_0+M_8-M_7$	$10M_0+M_7-M_2$	0	3
S_4	∞	∞	$9M_0+M_5-M_2$	$M_0+M_8-M_7$	2
Requirement b_j	3	3	2	2	

Table – 2.13

Sources	Destinations				Availability
	D_1	D_2	D_3	D_4	a_i
S_1	$M_0+M_7-M_4$	∞	$2M_0+M_3-M_2$	$6M_0+M_4-M_7$	4
S_2	∞	∞	0	$M_0+M_1-M_7$	4
S_3	0	0	$10M_0+M_7-M_2$	0	3
S_4	∞	∞	$9M_0+M_5-M_2$	$M_0+M_8-M_7$	2
Requirement b_j	3	3	2	2	

$LB=11M_0+M_2+M_4+M_5+M_7+M_8$

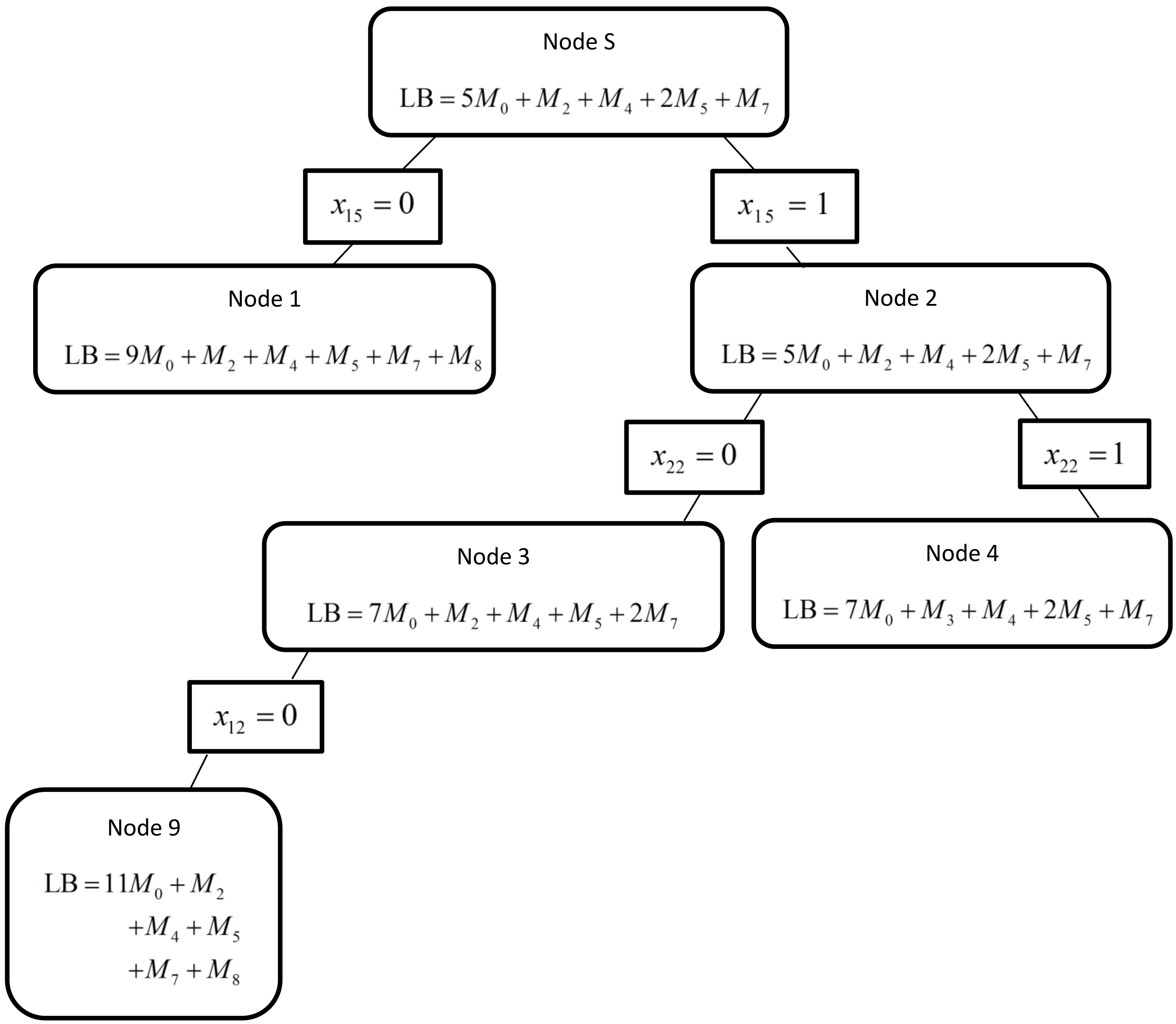

Figure – 2.5

Similarly, we can obtain the next allocations and all LBs as shown in figure – 2.6.
The last node is 16 which is at level one,

$$\text{Z} = 8M_0 + M_3 + M_4 + 2M_5 + M_8$$

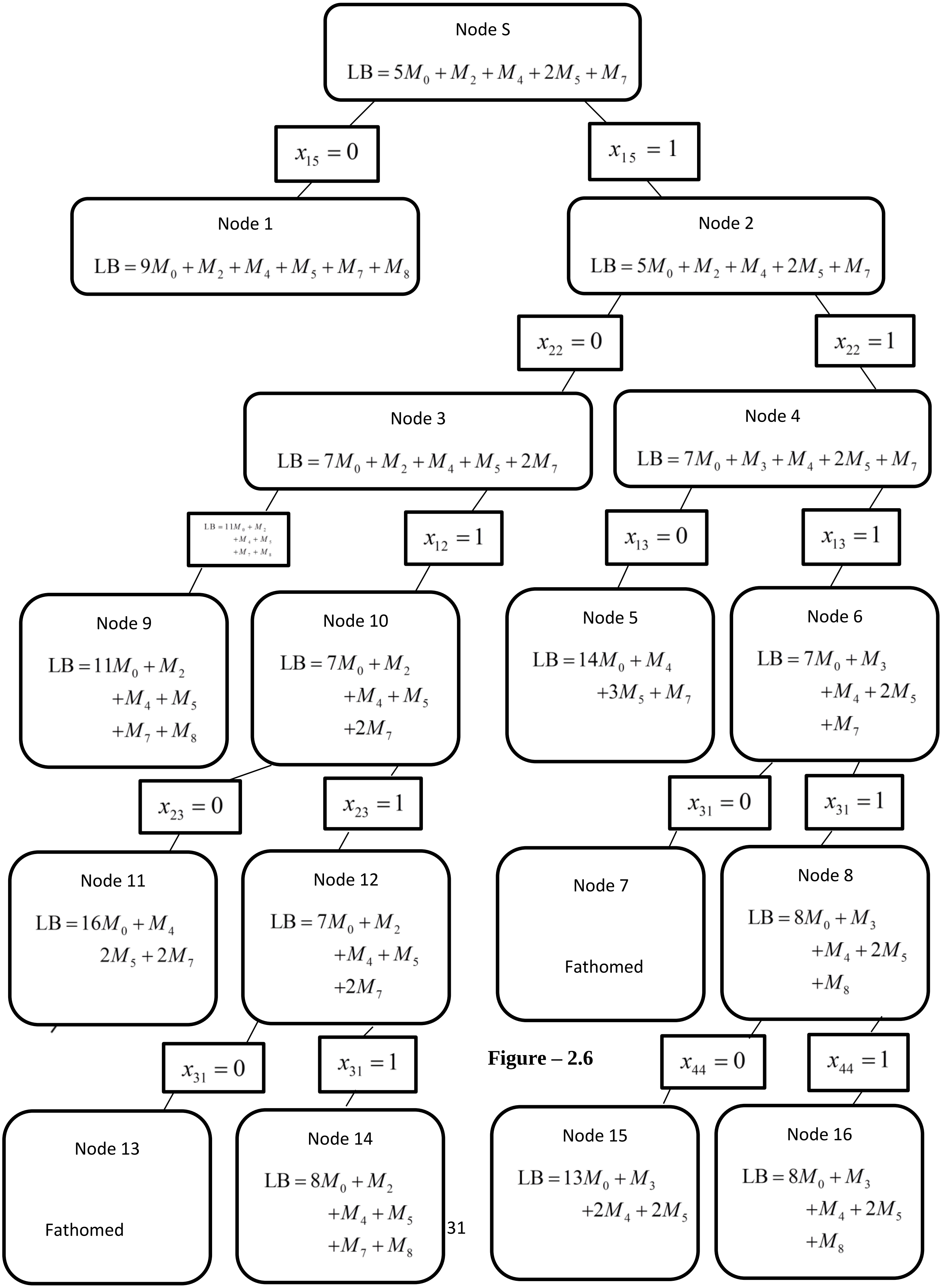
Node S
LB = 5M0 + M2 + M4 + 2M5 + M7
x15 = 0
x15 = 1
Node 1
LB = 9M0 + M2 + M4 + M5 + M7 + M8
Node 2
LB = 5M0 + M2 + M4 + 2M5 + M7
x22 = 0
x22 = 1
Node 3
LB = 7M0 + M2 + M4 + M5 + 2M7
Node 4
LB = 7M0 + M3 + M4 + 2M5 + M7
LB = 11M0 + M2 + M4 + M5 + M7 + M8
x12 = 1
x13 = 0
x13 = 1
Node 9
LB = 11M0 + M2 + M4 + M5 + M7 + M8
Node 10
LB = 7M0 + M2 + M4 + M5 + 2M7
Node 5
LB = 14M0 + M4 + 3M5 + M7
Node 6
LB = 7M0 + M3 + M4 + 2M5 + M7
x23 = 0
x23 = 1
x31 = 0
x31 = 1
Node 11
LB = 16M0 + M4 2M5 + 2M7
Node 12
LB = 7M0 + M2 + M4 + M5 + 2M7
Node 7
Fathomed
Node 8
LB = 8M0 + M3 + M4 + 2M5 + M8
x31 = 0
x31 = 1
x44 = 0
x44 = 1
Node 13
Fathomed
Node 14
LB = 8M0 + M2 + M4 + M5 + M7 + M8
Node 15
LB = 13M0 + M3 + 2M4 + 2M5
Node 16
LB = 8M0 + M3 + M4 + 2M5 + M8

Figure – 2.6

The LB at any other node in the tree is the lower bound of objective function of the subset of all solution of the single objective bulk transportation problem containing the variables $x_{ij}'s$ at level 0 or 1 along the chain from the node S to it. Since the unfathomed terminal node 16 in the tree contains five variables. $x_{15}, x_{22}, x_{13}, x_{31}, x_{44}$ at level 1 along the chain from the node S to it and the LB being:

$$Z = 8M_0 + M_3 + M_4 + 2M_5 + M_8$$

at it is the least among the LB's at all the unfathomed terminal nodes, the node 16 provides the optimal solution of the 1st single objective bulk transportation problem of the numerical problem. This optimal solution of the 1st single objective bulk transportation problem yields the 1st Pareto optimal solution $\bar{x}^{(1)}$ of the numerical problem. For the 1st Pareto optimal solution $\bar{x}^{(1)}$, the variable x_{ij} at level 1 are $x_{15}, x_{22}, x_{13}, x_{31}, x_{44}$; the total cost and duration of bulk transportation are $C(\bar{x}^{(1)}) = 1 + 1 + 3 + 1 + 2 = 8$ and $T(\bar{x}^{(1)}) = \max\{7, 7, 10, 8, 2\} = 10$ units respectively.

The remaining Pareto optimal solution of the numerical problem are obtained by following the above same procedure. For obtaining the 2nd Pareto optimal solution, we obtain the 2nd single-objective bulk transportation problem from the 1st single-objective bulk transportation problem of numerical problem by deleting all the variable x_{ij} corresponding to $t_{ij} \geq T(\bar{x}^{(1)}) = 10$ therein, given in table – 2.14.

Table – 2.14

Sources	Destinations					Availability
	D_1	D_2	D_3	D_4	D_5	a_i
S_1	$2M_0+M_7$	$3M_0+M_7$	∞	$7M_0+M_4$	M_0+M_5	5
S_2	$4M_0+M_7$	M_0+M_5	∞	∞	$8M_0+M_4$	4
S_3	M_0+M_4	$7M_0+M_8$	$11M_0+M_7$	M_0+M_7	$5M_0+M_7$	3
S_4	$20M_0+M_7$	$30M_0+M_6$	$10M_0+M_5$	$2M_0+M_8$	$5M_0+M_8$	2
Requirement b_j	3	3	2	2	1	

Similarly we can obtain 2nd Pareto optimal solution $\bar{x}^{(2)}$, the variable $x_{ij}'s$ at level 1 are $x_{15}, x_{22}, x_{34}, x_{11}, x_{43}$; the total cost and duration of the bulk transportation are $C(\bar{x}^{(2)}) = 1 + 1 + 1 + 2 + 10 = 15$ and $T(\bar{x}^{(2)}) = \max\{7, 7, 4, 4, 7\} = 7$ units respectively.

Similarly, we can obtain 3rd Pareto optimal solution $\bar{x}^{(3)}$, the variable $x_{ij}'s$ at level 1 are $x_{33}, x_{12}, x_{21}, x_{44}, x_{35}$; the total cost and duration of the bulk transportation are $C(\bar{x}^{(3)}) = 11 + 3 + 4 + 2 + 5 = 25$ and $T(\bar{x}^{(3)}) = \max\{4, 4, 4, 2, 4\} = 4$ units respectively.

For obtaining the further Pareto optimal solution, we can obtain the 4th single objective bulk transportation problem from the 3rd single objective bulk transportation problem of the numerical problem by deleting all the variables x_{ij} corresponding to $t_{ij}'s \geq T(\bar{x}^{(3)}) = 4$ therein. This has been shown in table – 2.15.

Table – 2.15

Sources	Destinations					Availability
	D_1	D_2	D_3	D_4	D_5	a_i
S_1	∞	∞	∞	∞	∞	5
S_2	∞	∞	∞	∞	∞	4
S_3	∞	$7M_0 + M_8$	∞	∞	∞	3
S_4	∞	∞	∞	$2M_0 + M_8$	$5M_0 + M_8$	2
Requirement b_j	3	3	2	2	1	

The numerical problem is found to have only three Pareto optimal solutions, the set of Pareto optimal solutions of the numerical problem together with the variables $x_{ij}'s$ at level 1, the total cost and duration of bulk transportation are shown in table – 2.16.

Table – 2.16

Pareto optimal solutions	Variables at level 1	Total Cost	Total Duration
$\bar{x}^{(1)}$	$x_{13}, x_{15}, x_{22}, x_{31}, x_{44}$	$C\left(\bar{x}^{(1)}\right) = 8$	$T\left(\bar{x}^{(1)}\right) = 10$
$\bar{x}^{(2)}$	$x_{11}, x_{22}, x_{43}, x_{34}, x_{15}$	$C\left(\bar{x}^{(2)}\right) = 15$	$T\left(\bar{x}^{(1)}\right) = 7$
$\bar{x}^{(3)}$	$x_{21}, x_{12}, x_{33}, x_{44}, x_{35}$	$C\left(\bar{x}^{(3)}\right) = 25$	$T\left(\bar{x}^{(1)}\right) = 4$

2.6 A HEURESTIC ALGORITHM FOR MULTI-OBJECTIVE BULK TRANSPORTATION PROBLEM

In this section a heuristic algorithm for the same multi-objective bulk transportation problem; given by Prakash et al.(2009) has been reviewed.

2.7 Solution Procedure

The cost-time trade-off bulk transportation problem formulated in section 2.2 is an integer nonlinear problem. This is so because of the second objective function provided by Eq. (2.2) is nonlinear and the decision variables $x_{ij}'s$ assume the values 0 or 1.Kasana and Kumar (2004) have applied Extremum Difference Method (EDM) to the transportation problem has obtained better result than obtained by applying Vogel's Approximation Method (VAM). The method is based on the principle that if an allocation is not made at the least cost cell of a row or column having the largest extremum difference, then the cost penalty will be higher for any other choices of rows and columns with other extremum differences. This cause the increase in the value of the objective function of the minimum type which is not desired. EDM method is applied to the said problem and obtain a set of efficient solutions that give more flexibility to the decision maker than that given by solutions obtained by applying the techniques of Prakash et al (2008) and that too wish lesser computational effort. Procedure to obtain the set of efficient solutions is discussed below.

2.8 ALGORITHM

2.8.1 Procedure to Obtained 1ST Efficient Solution

In the 1st efficient solution, the first priority is assigned to minimization of C provided by Eq. (2.1) and second priority assigned to the minimization of T provided by Eq. (2.2). Thus, the cost-time trade-off BTP is solved with priorities to obtain the 1st efficient solution of the BTP.the table representation of the problem m sources $S_1, S_2, \ldots, S_m$ with their respective capacities $a_1, a_2, \ldots, a_m$ and n destinations $D_1, D_2, \ldots, D_n$ with their respective demands $b_1, b_2, \ldots, b_n$ together with the unit cost c_{ij} and time of transportation t_{ij} from i^{th} source to j^{th} destination is given in table -2.1

Table – 2.17

Sources	Destinations				Availability
	D_1	D_2	$\cdots$	D_n	a_i
S_1	c_{11} t_{11}	c_{12} t_{12}	$\cdots$	c_{1n} t_{1n}	a_1
S_2	c_{21} t_{21}	c_{22} t_{22}	$\cdots$	c_{2n} t_{2n}	a_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	c_{m1} t_{m1}	c_{m2} t_{m2}	$\cdots$	c_{mn} t_{mn}	a_m
Requirements b_j	b_1	b_2	$\cdots$	b_n	

Since the entire requirement of the destination D_j is to be met from a single source only, all the variables $x_{ij}'s$ corresponding to the cells in the column associated to D_j except one which would be zero. Thus, a set consisting of one non-zero x_{ij} for each column would determine the solution of the 1st bulk transportation problem provided it satisfies constraints (2.3)—(2.5). The algorithm to obtain the solution is explained below:

Step 1. Obtained a row reduced matrix by deleting those rows wherein the availability a_i is less than the requirement b_j for each j. This is done since the requirement of a single destination is to be met from a single source only.

Step 2. In the reduced matrix delete those cells (i, j) from the row for which the availability a_i is less than b_j.

Step 3. Calculate the cost penalty P_j $(j = 1,2, \ldots, n)$ for each destination by Extremum Difference Method (EDM) i.e. the difference of maximum and minimum costs. If the maximum and minimum cost of transportation to a destination j is c_{Mj} and c_{mj} respectively, then the cost penalty for destination j is given by $P_j = c_{Mj} - c_{mj}$. If the cost of transportation to a destination j from all the sources is same, then the penalty for the destination j will be zero.

Step 4. Select the destination k having penalty P_k where $P_k = \max\{P_j : j = 1,2, \dots, m\}$. Let (i, k) be the least cost cell for selected destination k and allocate the amount $b_k$to the selected cell i.e. $x_{ik} = 1$. Drop the destination k from the table and update the availability of the source i from a_i to $a_i - b_k$. In case of tie of cost penalty, the first priority is given to the destination that has the least cost lower than that of the other destination in tie and the second priority is given to the destination having lower duration of transportation as well. In case of tie at both these aspects, it is an arbitrary choice of the decision maker. If the demand of the selected destination cannot be satisfied at the least cost then do not allocate the destination.

Step 5. Apply the previous step till all the destination have been considered. If the demands of all the destinations have been met by applying the above step, an optimal solution has been obtained and is designated the 1st efficient solution $\bar{x}^{(1)}$. Otherwise, apply the heuristic moves as explained in the next step.

Step 6. Retrace from the final table of the problem in which the solution has not been obtained, i.e., the demand of all the destination have not been met following the constraints(2.3)—(2.5). Shift the assignment of the last destination allocated to some other source at the next least cost. Check whether a solution has been obtained by applying step .4. If still not, retrace the penultimate table and shift the allocation of the destination made in that table to some other source with the next least cost. Check whether a solution has been obtained by step .4. If not, apply this step again in the same backward approach till the solution is obtained.

The retracing approach has been considered since the allocations were made considering the decreasing penalties calculated by EDM, i.e., keeping in view the largest penalties that would be incurred if the allocations are not made at the least cost. So th retracing approach allows us to obtain the solution at the minimum of the maximum penalties.

2.8.2 Procedure to Obtained 2nd Efficient Solution

Having, thus, obtained 1st efficient solution $\bar{x}^{(1)}$ of the 1st cost-time trade-off bulk transportation problem, 2nd efficient solution $\bar{x}^{(2)}$ is obtained by deleting all the cells (i, j) corresponding to the $t_{ij} \geq T(\bar{x}^{(1)})$. The resultant problem is designated the 2nd efficient solution $\bar{x}^{(2)}$. Further, the 3rd efficient solution is obtained by deleting those cells (i, j), in the 2nd BTP, that correspond to the $t_{ij} \geq T(\bar{x}^{(2)})$. Subsequent efficient solutions are obtained by proceeding exactly in the same way as for $\bar{x}^{(2)}$ and $\bar{x}^{(3)}$.

2.9 Numerical Example

Here, we consider the same numerical problem, given in a table 2.18 below with 4 sources and 5 destinations.

Table – 2.18

Sources	D_1	D_2	D_3	D_4	D_5	Availability (a_i)
S_1	2 4	3 4	3 10	7 8	1 7	5
S_2	4 4	1 7	1 12	2 14	8 8	4
S_3	1 8	7 2	11 4	1 4	5 4	3
S_4	20 4	30 6	10 7	2 2	5 2	2
Requirement (b_j)	3	3	2	2	1	

The upper and lower entries of a cell (i, j) depict the units of cost and time of bulk transportation from source S_i to destination D_j respectively. The marginal row and column depict the units of the commodity required at the destinations and variables at the sources respectively. The numerical problem seeks to determine x_{ij} which minimize the two objective functions:

$$C = \sum_{i=1}^{4}\sum_{j=1}^{5} c_{ij}x_{ij} \tag{2.7}$$

$$T = \max\{t_{ij}x_{ij} : i = 1,2,3,4; j = 1,2,3,4,5\} \tag{2.8}$$

Without according priorities to them subject to the constraints (2.3) – (2.5), after assigning numerical values to all the parameters therein. Since none of the sources have the availability less than the requirements of the each of the destinations thus no row is drop from the table; where the cells (4, 1) and (4, 2) are dropped since $a_4 < b_1$and $a_4 < b_2$ to obtain table 2.19.

Table – 2.19

Sources	D_1	D_2	D_3	D_4	D_5	Availability (a_i)
S_1	2 4	3 4	3 10	7 8	1 7	5
S_2	4 4	1 7	1 12	2 14	8 8	4
S_3	1 8	7 2	11 4	1 4	5 4	3
S_4	-----	-----	10 7	2 2	5 2	2
Requirement (b_j)	3	3	2	2	1	

It can be easily verified that the penalties using EDM for the destinations D_1, D_2, D_3, D_4 and D_5 are 3, 6, 10, 6 and 7 respectively. Since the largest penalty corresponds to destination D_3, so make an allocation in the cell (2, 3), i.e., $x_{23} = 1$, since it has the least cost for D_3. Drop the destination D_3 and update the availability of source S_2 to 2 units to obtain Table – 2.20

Table – 2.20

Sources	D_1	D_2	D_4	D_5	Availability (a_i)
S_1	2 4	3 4	7 8	1 7	5
S_2	-----	-----	2 14	8 8	2
S_3	1 8	7 2	1 4	5 4	3
S_4	-----	-----	2 2	5 2	2
Requirement (b_j)	3	3	2	1	

It should be noted here that the cells (2, 1) and (2, 2) have been dropped from table – 2.20. Since the supply of source S_2 is now less than the demands of destination D_1 and D_2.

Recalculates the penalties using EDM in table – 2.20 and proceed in the same manner. It can be easily verified that the penalty using EDM for the destination $D_1, D_2, D_4,$ and D_5 are 1, 4, 6, 7 respectively. Since the largest penalty corresponds to destination D_5, so make an allocation in the cell (1, 5), i.e., $x_{15} = 1$. Since it has the least cost for D_5, drop the destination and update the availability of the source. S_1 to 1 unit to obtain table – 2.21.

Table – 2.21

Sources	D_1	D_2	D_4	Availability (a_i)
S_1	2 4	3 4	7 8	4
S_2	----	----	2 14	2

S_3	1 8	7 2	1 4	3
S_4	-----	-----	2 2	2
Requirements (b_j)	3	3	2	

Again check $a_i \geq b_j$ and find the penalties of the table – 2.21.

The penalties using EDM for the destinations D_1, D_2 and D_4 are 1, 4 and 6 respectively. Choose the largest penalty. The largest penalty is 6 at destination D_4. Drop the destination D_4 and update the availability of source S_3 to 2 units to obtain table – 2.22.

Table – 2.22

Sources	D_1	D_2	Availability (a_i)
S_1	2 4	3 4	4
S_2	----	----	2
S_3	----	----	1
S_4	----	----	2
Requirements (b_j)	3	3	

Again check $a_i \geq b_j$ and find the penalty of the table – 2.22.

The penalties using EDM for the destination D_1 and D_2 is 2 for both destinations. Choose arbitrarily first destination D_1 so make an allocation in the cell (1, 1), i.e., x_{11} = 2, since it has least cost for D_1. Wherein it can be seen that the destination D_1 is allocated to source S_1 and the destination S_1 is not allocated to any of sources, i.e., a solution to the 1st bulk transportation problem has not been obtained yet. The status of allocation so far is given in table – 2.23.

Table – 2.23

Destination	Source
D_1	S_1
D_2	Not Assigned
D_3	S_2
D_4	S_3
D_5	S_1

Now we apply the heuristic moves as explained in step 4 of the algorithm. Starting from table – 2.20, rather than the allocating at the cell (1, 1), make an allocation at the cell (1, 2); it can still be seen that the solution is not obtained, since in this case destination D_1 remains unallocated to the any sources. Thus retracing back to table – 2.20, wherein the allocation was made in the cell (3, 4), if the allocation made in the next least cost cell, for destination D_4, i.e., (4, 4), the reduced matrix is given in table – 2.24.

Table – 2.24

Sources	D_1	D_2	Availability (a_i)
S_1	2 4	3 4	4
S_2	----	----	2
S_3	1 8	7 2	3
S_4	-----	-----	0
Requirements (b_j)	3	3	

After calculating the penalties of both the destinations D_1 and D_2 are 1 and 5 respectively in this table, the allocation is made in the cell (1, 2) and hence the final allocation is made in the cell (3, 1).

Table 2.25

Destination	Source
D_1	S_3
D_2	S_1
D_3	S_2
D_4	S_4
D_5	S_1

Thus, keeping in view all the constraints (3) - (5), all the destinations have been allocated to certain sources, thereby obtaining the 1st non-dominated solution $\bar{x}^{(1)}$ for which the variables x_{ij} at level 1 are $x_{23}, x_{15}, x_{44}, x_{12}, x_{31}$; the total cost and duration of the bulk transportation are
$C(\bar{x}^{(1)})$ = 1 + 1 + 2 + 3 + 1 = 8 units and
T(($\bar{x}^{(1)}$) = max{12,7,2,4,8}=12 units respectively.

The remaining efficient solutions are obtained following the procedure explained in section 2.8 for obtaining the 2nd non-dominated solution, we obtain the 2nd BTP by dropping all those cells (i, j) for which the $t_{ij} \geq T(\bar{x}^{(1)}) = 12$. The remaining table is

Table – 2.26

Sources	D_1	D_2	D_3	D_4	D_5	Availability (a_i)
S_1	2 4	3 4	3 10	7 8	1 7	5
S_2	4 4	1 7	-----	-----	8 8	4

S_3	1 8	7 2	11 4	1 4	5 4	3
S_4	-----	-----	10 7	2 2	5 2	2
Requirement (b_j)	3	3	2	2	1	

And applying the same procedure to find the 2nd efficient solution is obtained for which variables x_{ij} at level 1 are x_{13}, x_{15} , x_{44} , x_{22} , x_{31} the total cost and duration of the bulk transportation are

C($\bar{x}^{(2)}$) = 3 + 1 + 2 + 1+ 1 = 8 units and

Time is T($\bar{x}^{(2)}$) = max {10, 7, 2, 7, 8} = 10

Compare two solutions obtained it can be easily checked that the 2nd solution dominates the 1st one. Thus, redesign ate the 2nd efficient solution $\bar{x}^{(2)}$ as the 1st efficient solution $\bar{x}^{(1)}$.

For obtaining the 2nd efficient solution, we reduce the 1st BTP to the 2nd BTP by dropping the cells(i, j) for which the $t_{ij} \geq T\left(\bar{x}^{(1)}\right) = 10$ units. Applying the same procedure as above, we obtain the 2nd efficient solution $\bar{x}^{(2)}$ for which the variables at level 1 are given by $x_{11}, x_{22}, x_{43}, x_{34}, x_{15}$; the total cost and the duration of the bulk transportation are $C\left(\bar{x}^{(2)}\right) = 2 + 1 + 10 + 1 + 1 = 15$ and $T(\bar{x}^{(2)}) = \max\{4,7,7,4,7\} = 7$ units respectively.

Similarly, the 3rd efficient solution $\bar{x}^{(3)}$ is obtained by dropping the cells(i, j) for which the $t_{ij} \geq T\left(\bar{x}^{(2)}\right) = 7$. Applying the same procedure, we obtain the 3rd efficient solution $\bar{x}^{(3)}$ for which the variables at level 1 are given by$x_{21}, x_{12}, x_{33}, x_{44}, x_{35}$; the toatal cost and the duration of the bulk transportation problem are $C(\bar{x}^{(3)}) = 4 + 3 + 11 + 2 + 5 = 25$ and $t(\bar{x}^{(3)}) = \max\{4,4,4,2,4\} = 4$.

To obtain the 4th Pareto optimal solution, we drop all the cells(i, j) in the 3rd BTP for which $t_{ij} \geq T\left(\bar{x}^{(3)}\right) = 4$ units. It can be seen that as a result, the destination D_3 cannot be allocated to any of the source and hence there exists no efficient solution of the numerical problem for which the duration of the bulk transportation problem is less than 4 units and thus, the process of obtaining further efficient solutions is terminated. Thus, the numerical problem is found to have three efficient solutions. The set of efficient solutions of the numerical problem together with the variables $x_{ij}'s$ at level 1, total cost and the duration of the bulk transportation are as shown in Table – 2.27

Table – 2.27

Efficient solutions	Variables at level 1	Total Cost	Total Duration
$\bar{x}^{(1)}$	$x_{13}, x_{15}, x_{22}, x_{31}, x_{44}$	$C\left(\bar{x}^{(1)}\right) = 8$	$T\left(\bar{x}^{(1)}\right) = 10$
$\bar{x}^{(2)}$	$x_{11}, x_{22}, x_{43}, x_{34}, x_{15}$	$C\left(\bar{x}^{(2)}\right) = 15$	$T\left(\bar{x}^{(1)}\right) = 7$
$\bar{x}^{(3)}$	$x_{21}, x_{12}, x_{33}, x_{44}, x_{35}$	$C\left(\bar{x}^{(3)}\right) = 25$	$T\left(\bar{x}^{(1)}\right) = 4$

(Set of efficient solutions of the numerical problem)

Chapter – 3

A NEW SIMPLE HEURESTIC ALGORITHM FOR MULTIOBJECTIVE BULK TRANSPORTATION PROBLEM

3.1 A NEW SIMPLE HEURESTIC ALGORITHM FOR MULTIOBJECTIVE BULK TRANSPORTATION PROBLEM

In this chapter the multi-objective bulk transportation problem given by equation (2.1 to 2.5) has been considered and a new algorithm has developed to find the Pareto optimal solutions.

The Pareto optimal solution the two objectives of the problem are converted into single objective by using preemptive priority factors given by Prakash et al. (2008) and a new heuristic algorithm is developed is given below.

Step 1: Obtain a column reduce matrix (the matrix which contains at least one zero in each destination). Delete those rows wherein the availability a_i is less than the requirement b_j for all j. This is done since the requirement of a single destination is to be met from a single source only i.e., drop the source S_i if $a_i < b_j \ \forall j$.

Step 2: In the column reduce matrix, block those cells for which availability a_i is less than requirement b_j. b_j assigning ∞ to those cells.

Step 3: Calculate the cost penalties of each destination (corresponding to those cells which contain zero cost) by taking the difference of minimum costs of corresponding to row and column.

Step 4: Select the cell for which the penalty is largest and allocate b_j to this cell. In case of tie at both these aspects, it is an arbitrary choice of the decision maker.

Step 5: Drop that destination for further consideration. Assign (∞) to the cells for which availability a_i is less than requirement b_j ($a_i < b_j$) and calculate the new penalties for the remaining sub-matrix as done in step – 3. Repeat the same procedure till the requirement of all destination are completed.

3.2 Procedure to Obtain 2nd and Subsequent Efficient Solutions

Having, thus, obtained 1st efficient solution $\bar{x}^{(1)}$ of the 1st cost-time trade-off bulk transportation problem, 2nd efficient solution $\bar{x}^{(2)}$ is obtained by deleting all the cells (i,j) corresponding to the $t_{ij} \geq T(\bar{x}^{(1)})$. The resultant problem is designated the 2nd efficient solution $\bar{x}^{(2)}$. Further, the 3rd efficient solution is obtained by deleting those cells (i,j), in the 2nd BTP, that correspond to the $t_{ij} \geq T(\bar{x}^{(2)})$. Subsequent efficient solutions are obtained by proceeding exactly in the same way as for $\bar{x}^{(2)}$ and $\bar{x}^{(3)}$.

3.3 Numerical Example

Here, we consider a numerical problem of 4 sources and 5 destinations and apply the procedure explained above to obtain the set of efficient solutions. The tableau representation of the numerical problem is given in Table-3.1.

Table – 3.1

Sources	D_1	D_2	D_3	D_4	D_5	Availability (a_i)
S_1	2 4	3 4	3 10	7 8	1 7	5
S_2	4 4	1 7	1 12	2 14	8 8	4
S_3	1 8	7 2	11 4	1 4	5 4	3
S_4	20 4	30 6	10 7	2 2	5 2	2
Requirement (b_j)	3	3	2	2	1	

Now we apply the algorithm for obtaining the set of Pareto optimal solutions of the numerical problem obtained by taking $m = 4, n = 5$ and assigning numerical values to all the other parameters in the problem formulated above. The table representation of the numerical problem is shown in Table - 2.

In this table, cells (i, j)*'s* correspond to the variables $x'_{ij}s$ $(i = 1,2,3,4; j = 1,2,3,4,5)$. The upper and lower entries of a cell (i, j) depict the units of cost and time of bulk transportation from source S_i to destination D_j respectively. The marginal row and column depict the units of the commodity required at the destinations and available at the sources respectively.

$$C = 2x_{11} + 3x_{12} + 3x_{13} + 7x_{14} + x_{15} + 4x_{21} + x_{22} + x_{23} + 2x_{24} + 8x_{25} + x_{31} + 7x_{32} + 11x_{33} + x_{34} + 5x_{35}$$

$$+ 20x_{41} + 30x_{42} + 10x_{43} + 2x_{44} + 5x_{45}$$

$$T = \max\{t_{ij}x_{ij} : i = 1, 2, 3, 4; \ j = 1, 2, 3, 4, 5\}$$

Without according priorities to them subject to the given constraints after assigning numerical values to all the parameters therein.
Using the procedure given in chapter 2, Prakash et al (2008) converted the two objectives in to a single-objective given by the following equation.

$$Z = M_0(2x_{11} + 3x_{12} + 3x_{13} + 7x_{14} + x_{15} + 4x_{21} + x_{22} + x_{23} + 2x_{24} + 8x_{25} + x_{31} + 7x_{32} + 11x_{33} + x_{34} + 5x_{35}$$

$$+20x_{41} + 30x_{42} + 10x_{43} + 2x_{44} + 5x_{45})$$

$$+ M_1(x_{24}) + M_2(x_{23}) + M_3(x_{13}) + M_4(x_{14} + x_{25} + x_{31}) + M_5(x_{15} + x_{22} + x_{43}) + M_6(x_{42}) \qquad ...(3.1)$$

$$+ M_7(x_{11} + x_{12} + x_{21} + x_{33} + x_{34} + x_{35} + x_{41}) + M_8(x_{32} + x_{44} + x_{45})$$

subject to the given constraints after assigning numerical values to all the parameters therein.
The optimal solution of the 1st single-objective bulk transportation problem of the numerical problem yields the 1st Pareto optimal solution $\bar{x}^{(1)}$ of the numerical problem. The tableau

representation of the 1st single-objective bulk transportation problem of the numerical is shown in table-3. In this table, the cells $(i,j)'s\ (i = 1,2,3,4\ ;\ j = 1,2,3,4,5)$ depict the cost which resemble in nature the cost associated with an artificial variable in the Big-M method of linear programming. The $M_k's$ are handled almost in the same manner as M is handled in the Big-M method. While minimizing Z, it should be kept in mind that $M_0 \gg M_1 \gg \cdots \gg M_8$.

Table-3.2

Sources	Destinations					Availability
	D_1	D_2	D_3	D_4	D_5	a_i
S_1	$2M_0+M_7$	$3M_0+M_7$	$3M_0+M_3$	$7M_0+M_4$	M_0+M_5	5
S_2	$4M_0+M_7$	M_0+M_5	M_0+M_2	$2M_0+M_1$	$8M_0+M_4$	4
S_3	M_0+M_4	$7M_0+M_8$	$11M_0+M_7$	M_0+M_7	$5M_0+M_7$	3
S_4	$20M_0+M_7$	$30M_0+M_6$	$10M_0+M_5$	$2M_0+M_8$	$5M_0+M_8$	2
Requirement b_j	3	3	2	2	1	

Now, the algorithm (step 1 to step 5) is applied the matrix given in table -3.3. Since none of the sources have the availability less than the requirements of the each of the destinations thus no row is blocked (by putting ∞ in the cell) from the table; whereas the cells (4,1) and (4,2) are blocked since $a_4 < b_1$ and $a_4 < b_2$ shows in Table -3.3.

Table-3.3

Sources	Destinations					Availability
	D_1	D_2	D_3	D_4	D_5	a_i
S_1	$2M_0+M_7$	$3M_0+M_7$	$3M_0+M_3$	$7M_0+M_4$	M_0+M_5	5
S_2	$4M_0+M_7$	M_0+M_5	M_0+M_2	$2M_0+M_1$	$8M_0+M_4$	4
S_3	M_0+M_4	$7M_0+M_8$	$11M_0+M_7$	M_0+M_7	$5M_0+M_7$	3
S_4	∞	∞	$10M_0+M_5$	$2M_0+M_8$	$5M_0+M_8$	2
Requirement b_j	3	3	2	2	1	

Find the column reduced matrix (In the table every column contains at least one zero) and given in table -3.4.

Table – 3.4

Sources	Destinations					Availability
	D_1	D_2	D_3	D_4	D_5	a_i
S_1	$M_0+M_7-M_4$	$2M_0+M_7-M_5$	$2M_0+M_3-M_2$	$6M_0+M_4-M_7$	0	5
S_2	$3M_0+M_7-M_4$	0	0	$M_0+M_1-M_7$	$4M_0+M_7-M_5$	4
S_3	0	$6M_0+M_8-M_5$	$10M_0+M_7-M_2$	0	$4M_0+M_7-M_5$	3
S_4	∞	∞	$9M_0+M_5-M_2$	$M_0+M_8-M_7$	$4M_0+M_8-M_5$	2
Requirement	3	3	2	2	1	

b_j						

Calculate the penalties of each destination (corresponding to those cells which contain zero cost) by taking difference of minimum costs of corresponding to rows and columns, and in this case the penalties are: ($3M_0 + M_4 + M_8 - M_5 - M_7$, $2M_0 + M_7 - M_5$, $2M_0 + M_3 - M_2$, $M_0 + M_7 - M_4$, $M_0 + M_8 - M_7$) corresponding to cells (1,5), (2,2), (2,3), (3,1) and (3,4) respectively. Choose the largest penalty allocate the in the corresponding cell. Make allocation b_5 in the cell (1,5), this we get $x_{15} = 1$. Drop the destination D_5 and update the availability of source S_1 to 4 units to obtain table -3.5.

Table – 3.5

Sources	Destinations				Availability
	D_1	D_2	D_3	D_4	a_i
S_1	$M_0 + M_7 - M_4$	$2M_0 + M_7 - M_5$	$2M_0 + M_3 - M_2$	$6M_0 + M_4 - M_7$	4
S_2	$3M_0 + M_7 - M_4$	0	0	$M_0 + M_1 - M_7$	4
S_3	0	$6M_0 + M_8 - M_5$	$10M_0 + M_7 - M_2$	0	3
S_4	∞	∞	$9M_0 + M_5 - M_2$	$M_0 + M_8 - M_7$	2
Requirement b_j	3	3	2	2	

Repeat the process (step 1 to step 5) until all the requirements of each destination is completed. Since there is none of the cell can be blocked, find the penalties.

The penalties are given as ($2M_0 + M_7 - M_5$, $2M_0 + M_3 - M_2$, $M_0 + M_7 - M_4$, $M_0 + M_8 - M_7$) corresponding to the cells (2,2), (2,3), (3,1) and (3,4) respectively.

Since largest penalty is corresponding to the cell (2,3), so make the allocation b_2 to the cell (2,3). Drop the destination D_2 and update the availability of S_2 to units to obtain in table -3.6.

Table-3.6

Sources	Destinations			Availability
	D_1	D_2	D_4	a_i
S_1	$M_0+M_7-M_4$	$2M_0+M_7-M_5$	$6M_0+M_4-M_7$	4
S_2	$3M_0+M_7-M_4$	0	$M_0+M_1-M_7$	2
S_3	0	$6M_0+M_8-M_5$	0	3
S_4	∞	∞	$M_0+M_8-M_7$	2
Requirement b_j	3	3	2	

Now the cells (2,1) and (2,2) are blocked since $a_2 < b_1$ and $a_2 < b_2$.

Convert the matrix into reduced matrix (note in table -3.6 the cell (2,2) is blocked by ∞ and the destination D_2 does not contain any zero) shows in table -3.7.

Table – 3.7

Sources	Destinations			Availability
	D_1	D_2	D_4	a_i
S_1	$M_0+M_7-M_4$	0	$6M_0+M_4-M_7$	4
S_2	∞	∞	$M_0+M_1-M_7$	2
S_3	0	$4M_0+M_8-M_7$	0	3
S_4	∞	∞	$M_0+M_8-M_7$	2
Requirement b_j	3	3	2	

Penalties are $3M_0 + M_8 - 2M_7 + M_4$, $M_0 + M_7 - M_4$, $M_0 + M_8 - M_7$ are corresponding cells (1, 2), (3, 1) and (3, 4) respectively. Since largest penalty is corresponding to cell (1, 2), so make allocation b_1 to the cell (1, 2). Drop the destination D_2 and update the availability of S_1 to 1 unit to obtain table -3.8

Table – 3.8

Sources	Destinations		Availability
	D_1	D_4	a_i
S_1	$M_0 + M_7 - M_4$	$6M_0 + M_4 - M_7$	1
S_2	∞	$M_0 + M_1 - M_7$	2
S_3	0	0	3
S_4	∞	$M_0 + M_8 - M_7$	2
Requirement b_j	3	2	

Now the cells (1, 1) and (1, 4) are blocked since $a_1 < b_1$ and $a_1 < b_4$.
This shows in the table -3.9.

Table – 3.9

Sources	Destinations		Availability
	D_1	D_4	a_i
S_1	∞	∞	1
S_2	∞	$M_0 + M_1 - M_7$	2
S_3	0	0	3
S_4	∞	$M_0 + M_8 - M_7$	2
Requirement b_j	3	2	

Find the penaties.
Penalties are $(\infty, M_0 + M_8 - M_7)$ corresponding to cells (3, 1) and (3, 4) respectively.

Since largest penalty is corresponding to cell (3, 1), so make the allocation b_3 to the cell (3, 1). Drop the destination D_1 and update the availability of S_3 to 0 unit to obtain table -3.10.

Table – 3.10

Sources	Destinations	Availability
	D_4	a_i
S_1	∞	1
S_2	$M_0 + M_1 - M_7$	2
S_3	0	0
S_4	$M_0 + M_8 - M_7$	2
Requirement b_j	2	

Now the cells (3, 4) are blocked since $a_3 < b_4$. This shows in table -3.11.

Table – 3.11

Sources	Destinations	Availability
	D_4	a_i
S_1	∞	1
S_2	$M_0 + M_1 - M_7$	2
S_3	∞	0
S_4	$M_0 + M_8 - M_7$	2
Requirement b_j	2	

Convert the matrix into reduced matrix (not table -3.11 the cell (3, 4) is blocked by ∞ and this column does not containing any zero) and this shows in table -3.12

Table – 3.12

Sources	Destinations	Availability
	D_4	a_i
S_1	∞	1
S_2	$M_1 - M_8$	2
S_3	∞	0
S_4	0	2
Requirement b_j	2	

Hence keeping in view, all the constraints (3)—(5), all the destinations have been allocated to certain sources, thereby obtaining the 1st efficient solution xbar1 for which the variables $x_{ij}'s$ at level 1 are $x_{23}, x_{15}, x_{44}, x_{12}, x_{31}$; the total cost and the duration of the bulk transportation are $C\left(\bar{x}^{(1)}\right) = 1 + 1 + 2 + 3 + 1 = 8$ and $T\left(\bar{x}^{(1)}\right) = \max\{12,7,2,4,8\} = 12$ units respectively.

The remaining efficient solutions are obtained following the procedure explained in the **Procedure to Obtain 2nd and subsequent Efficient Solutions.** For obtaining the 2nd efficient solution, we obtain the 2nd BTP by dropping those cells (i, j) for which the $t_{ij} \geq T\left(\bar{x}^{(1)}\right) = 12$. Subsequently, the following allocations are made as in the case of the 1st BTP. Thus, the 2nd efficient solution for $\bar{x}^{(2)}$ is obtained for which the variables $x_{ij}'s$ at level 1 are $x_{13}, x_{15}, x_{44}, x_{22}, x_{31}$; the total cost and the duration of the bulk transportation problem are $C(\bar{x}^{(2)}) = 3 + 1 + 2 + 1 + 1 = 8$ and $T(\bar{x}^{(2)}) = \max\{10,7,2,7,8\} = 10$ units respectively. Comparing the two solutions obtained it can be easily checked that the 2nd solution dominates the 1st one. Thus, redesign ate the 2nd efficient solution $\bar{x}^{(2)}$ as the 1st efficient solution $\bar{x}^{(1)}$.

For obtaining the 2^{nd} efficient solution, we reduce the 1^{st} BTP to the 2^{nd} BTP by dropping the cells(i, j) for which the $t_{ij} \geq T\left(\bar{x}^{(1)}\right) = 10$ units. Applying the same procedure as above, we obtain the 2^{nd} efficient solution $\bar{x}^{(2)}$ for which the variables at level 1 are given by $x_{11}, x_{22}, x_{43}, x_{34}, x_{15}$; the total cost and the duration of the bulk transportation are $C\left(\bar{x}^{(2)}\right) = 2 + 1 + 10 + 1 + 1 = 15$ and $T(\bar{x}^{(2)}) = \max\{4,7,7,4,7\} = 7$ units respectively.

Similarly, the 3^{rd} efficient solution $\bar{x}^{(3)}$ is obtained by dropping the cells(i, j) for which the $t_{ij} \geq T\left(\bar{x}^{(2)}\right) = 7$. Applying the same procedure, we obtain the 3^{rd} efficient solution $\bar{x}^{(3)}$ for which the variables at level 1 are given by$x_{21}, x_{12}, x_{33}, x_{44}, x_{35}$; the toatal cost and the duration of the bulk transportation problem are $C(\bar{x}^{(3)}) = 4 + 3 + 11 + 2 + 5 = 25$ and $t(\bar{x}^{(3)}) = \max\{4,4,4,2,4\} = 4$.

To obtain the 4^{th} Pareto optimal solution, we drop all the cells(i, j) in the 3^{rd} BTP for which $t_{ij} \geq T\left(\bar{x}^{(3)}\right) = 4$ units. It can be seen that as a result, the destination D_3 cannot be allocated to any of the source and hence there exists no efficient solution of the numerical problem for which the duration of the bulk transportation problem is less than 4 units and thus, the process of obtaining further efficient solutions is terminated. Thus, the numerical problem is found to have three efficient solutions. The set of efficient solutions of the numerical problem together with the variables $x_{ij}'s$ at level 1, total cost and the duration of the bulk transportation are as shown in Table – 3.14.

Table – 3.14

Efficient solutions	Variables at level 1	Total Cost	Total Duration
$\bar{x}^{(1)}$	$x_{13}, x_{15}, x_{22}, x_{31}, x_{44}$	$C\left(\bar{x}^{(1)}\right) = 8$	$T\left(\bar{x}^{(1)}\right) = 10$
$\bar{x}^{(2)}$	$x_{11}, x_{22}, x_{43}, x_{34}, x_{15}$	$C\left(\bar{x}^{(2)}\right) = 15$	$T\left(\bar{x}^{(1)}\right) = 7$
$\bar{x}^{(3)}$	$x_{21}, x_{12}, x_{33}, x_{44}, x_{35}$	$C\left(\bar{x}^{(3)}\right) = 25$	$T\left(\bar{x}^{(1)}\right) = 4$

(Set of efficient solutions of the numerical problem)

Conclusion

The algorithm developed in this chapter is simple and straight forward. Application of the algorithm to the numerical problem yields the same three efficient solutions as shown in table 3.15 that were obtained by Prakash et al (2008, 2009). The proposed algorithm reduced the computational work and still obtained the identical solution.

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