PERFORMANCE OF SPACE-TIME BLOCK CODED COMMUNICATION UNDER TIME-VARYING FADING CHANNELS

Thesis submitted in the partial fulfillment of requirement for the award of the Degree of

MASTER OF ENGINEERING

IN

ELECTRONICS AND COMMUNICATION ENGINEERING

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JUNE-2009

DECLARATION

I hereby declare that the work, which is being presented in the thesis, entitled "Performance of space-time block coded communication under time-varying fading channels," in partial fulfillment of the requirements for the award of degree of Master of Engineering in Electronics and Communication Engineering at Electronics and Communication Engineering Department of Thapar University, Patiala, is an authentic record of my own work carried out under the guidance of Dr. Amit Kumar Kohli.

I have not submitted the matter presented in the thesis for the award of any other degree of this or any other university.

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This is to certify that the above statement made by the student is correct to the best of my knowledge and belief.

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ACKNOWLEDGEMENT

I would like to express my gratitude to Dr. Amit Kumar Kohli, Assistant Professor, Electronics and Communication Engineering Department, Thapar University, Patiala for his patient guidance and support throughout this thesis work. I am truly very fortunate to have the opportunity to work with him. He has provided me help in technical writing and presentation style, and I found this guidance to be extremely valuable.

I am very thankful to the Head of the Department, Dr. A. K. Chatterjee, for his encouragement, support and providing the facilities for the completion of this thesis.

I am also thankful to entire faculty and staff members of Electronics and Communication Engineering Department for their unyielding encouragement.

I am greatly indebted to all my friends, who have graciously applied themselves to the task of helping me with ample morale support and valuable suggestions. Finally, I would like to extend my gratitude to all those persons who directly or indirectly helped me in the process and contributed towards this work.

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ABSTRACT

Broadband wireless communication must cope with critical performance limiting challenges that include time-selective and frequency-selective channels, as well as power and bandwidth constraints. Space-time (ST) coding offers an effective transmit-antenna diversity technique to combat fading.

This thesis presents a simple two branch transmit diversity scheme and detection of these transmitted signals by using channel estimation based on "least squares with two types of variable forgetting factor i.e. LSn-VFF and LSn-NVFF (LSn with new VFF)" algorithm at the receiver. The data symbols are encoded using space-time block codes, and the encoded data is split into *n* streams, which are simultaneously transmitted using *n* transmitter antennas. The received signal at receiver antenna is a linear superposition of *n* transmitted signals perturbed by noise. Maximum likelihood decision rule at receiver for these received signals is used to choose the signal with minimum Euclidean distance between received and transmitted signal. A complex orthogonal design and the corresponding space-time block code, which provide the full diversity and full transmission rate is not possible for more than two antennas. Therefore, we have used space-time block codes with complex constellation only for two transmitter antenna system and for more than two antennas quasi-orthogonal design is used. Further, space-time block codes (STBC) with real constellation for two, four, eight (higher-order) transmitter antennas are used for advanced wireless communication systems.

The channel state information is essential requirement for space-time block code detector/receiver. It is necessary to formulate the new channel estimation algorithms to improve the performance of the space-time codes in the time-varying environment using variable forgetting factor. In this research work, different channel estimation techniques are proposed. The performance characteristics of different algorithms (Least mean squares, Recursive least square, Least square of order one and two, Recursive least squares with variable forgetting factor and least square algorithm of order one and two, with both type of variable forgetting factor i.e. VFF and NVFF) are analyzed and compared. These channel estimates are further used for space-time block

coding in different simulation examples. Simulation results depict that LSn-NVFF performs well in all time-varying channel models and orthogonal STBC's performance is better than quasi-orthogonal STBC. LSn-NVFF performance is best for orthogonal (2×2) STBC and quasi-orthogonal (4×4) STBC. For 8×8 STBC system, LSn outperforms the all algorithms because LSn-NVFF and LSn-VFF fail to track eight channels simultaneously due to increase in computation complexity of the algorithms and increase in number of parameter adjusted by single VFF algorithms for all channels.

CONTENTS

LIST OF ABBREVIATIONS

AF	adaptive filter
AR1	autoregressive model of first order
AWGN	additive white Gaussian noise
BER	bit error rate
CDMA	code division multiple access
DFE	decision feedback equalizer
EDGE	enhanced data rates for GSM evolution
EW-RLS	exponentially windowed RLS
FIR	finite impulse response
G_c	coding gain
G_d	diversity gain
GSM	global system for mobile communication
LMS	least mean square
LSn	least-square of order one
LSn-NVFF	LSn with new variable forgetting factor
LSn-VFF	LSn with variable forgetting factor
LSn2	least-square of order two
LSn2-NVFF	LSn2 with new variable forgetting factor
LSn2-VFF	LSn2 with variable forgetting factor
MIMO	multiple input multiple output
ML	maximum likelihood
MMRC	maximal ratio receive combining
MMSE	minimum mean square error
MSE	mean square error
NADC	Northern American digital cellular
OFDM	orthogonal frequency division multiplexing
PSK	phase shift key
QPSK	quadrature phase shift key
QSTBC	quasi-orthogonal STBC

RF	radio frequency
RLS	recursive least square
RLS-VFF	RLS with variable forgetting factor
SNR	signal to noise ratio
ST	space-time
STBC	space-time block codes
STC	space-time codes
STTC	space-time trellis codes
TDMA	time division multiple access
TU	typical urban
VFF	variable forgetting factor
W-CDMA	wideband CDMA
WSSUS	wide sense stationary uncorrelated scattering

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

Diversity

The fundamental phenomenon which makes reliable wireless transmission difficult is time-varying multipath fading. It is this phenomenon which makes wireless transmission a challenge when compared to fiber, coaxial cable, line-of-sight microwave or even satellite transmission. Severe attenuation in a multipath wireless environment makes it extremely difficult for the receiver to determine the transmitted signal [1]. In additive white Gaussian noise (AWGN), using typical modulation and coding schemes, reducing the effective bit error rate (BER) from 10^{-2} to 10^{-3} may require only 1 or 2 dB higher signal to noise ratio (SNR). Achieving the same in multipath fading environment, however, may require up to 10 dB improvement in SNR [2].

Signal is reached to receiver from transmitter from various paths due to reflection, diffraction etc. in wireless communication. Each path has a different attenuation, time delay and phase shift, the signal from different paths add constructively sometimes or destructive sometimes, resulting in fluctuation signal strength. This phenomenon is known as multipath fading. Channel fading cause's the performance degradation and renders reliable high data rate transmission a challenging problem in 4G wireless communication.

MIMO systems can be defined as "for any wireless communication system, transmitting and receiving end are equipped with multiple antennas". Key feature of MIMO systems is the ability to turn multipath propagation, traditionally a pitfall of wireless transmission, into a benefit for the user. MIMO effectively takes advantage of random fading [3], [4], [5] and when available, multipath delay spread [6], [7], for multiplying transfers rates. Core Idea in MIMO system is space-time signal processing in which time (the natural dimension of digital communication data) is complemented with the spatial dimension inherent in the use of multiple spatially distributed antennas. Band limited wireless channels are narrow pipes that do not accommodate rapid flow of data. Deploying multiple transmit and receive antennas broadens this data pipe. Information theory [3], [5] provides measures of capacity, and the standard approach to increasing data flow is linear processing at the receiver [8], [9]. In fact, the advantages of MIMO are far more fundamental. The underlying mathematical nature of MIMO, where data is transmitted over a matrix rather than a vector channel, creates new and enormous opportunities beyond just the added diversity or array gain benefits. This was shown in [4], where the author shows how one may under certain conditions transmit min(M,N) independent data streams simultaneously over the eigenmodes of a matrix channel created by N transmitter and M receiver antennas.

Diversity is the key topic in mobile communications. It is principally used to combat fading. The basic idea is that if different copies of the same signal are available then there is a high probability that at least one of them is of a good quality. Of course choosing the best copy and rejecting all others is not the optimal solution. This brings us to the problem of choosing the best way to combine all of them [10].

The popular forms of diversity are:

1. Temporal diversity: Replicas of the information bearing signal are transmitted in different time slots, where the separation between the time slots is greater than the coherence of the channel.
2. Frequency diversity: In this case, replicas of the information bearing signal are transmitted in different frequency bands, where the separation between the frequencies is greater than the coherence bandwidth of the channel.
3. Antenna (spatial) diversity: It has been observed that antennas with a spacing of more than half a wavelength leads to spatially uncorrelated channels. The transmission of the replicas of the information bearing signal over these uncorrelated spatial channels leads to spatial diversity.

Note that not all kinds of diversity are always feasible. For example a slowly fading channel (with a long coherence time) can not support temporal diversity with practical interleaving depths. Similarly, frequency diversity is not feasible when the coherence bandwidth of the channel is comparable to the bandwidth of the signal employed. However, irrespective of the channel characteristics, antenna diversity can always be exploited as long as there is sufficient spacing between the antennas.

Temporal and frequency types of diversity normally introduce redundancy in time and/or frequency domain, and therefore induce loss in bandwidth efficiency. Typical examples of spatial diversity are multiple transmit and/or receiver, multiple antenna communication rely on space diversity to mitigate fading without necessarily sacrificing precious bandwidth resources; thus, they become attractive solution for broadband wireless application. Compare to single antenna transmission, multiple antenna transmissions increase the channel capacity.

A simple space diversity scheme is to use multiple antennas at the receiver. This scheme does not involve loss of band width. The optimal way to combine the outputs of different antennas is the Maximal Ratio combining. At the same time remote unit are supposed to be small light weighted pocket communicators. Inevitably, the pocket communicators must remain simple. Also, antenna diversity at a mobile handset is more difficult to implement because of electromagnetic interaction of antenna elements on small platforms and the expense of multiple down conversion RF paths. Furthermore, the channels corresponding to different antennas are correlated, with the correlation factor determined by the distance as well as the coupling between the antennas. Typically, the second antenna is inside the mobiles handset, resulting in signal attenuation at the second antenna. This can cause some loss in diversity benefit. All these factors motivate the use of multiple antennas at the base station for transmission [11]. This method is called transmit diversity.

Two major obstacles to implement transmit antenna diversity are as follows:

i) Unlike the receiver, the transmitter does not have instantaneous information about the fading channels.
ii) The transmitted signals are mixed spatially before they arrive at the receiver.

A number of transmit antenna diversity scheme have been proposed, and can be divided into two categories: open loop (e.g., [12], [13], [14]), and closed loop (e.g., [15], [16]). The difference between open and closed loop schemes is that the former does not require channel knowledge at the transmitter. On the other hand, the latter relies on some channel information at the transmitter that is acquired through feedback channels. Although feedback channels are present in most wireless systems (for power control purposes), mobility may cause fast channel variation. As a result, the transmitter may not be capable of acquiring and tracking the channel variations. Thus, usage of open loop transmit antennas diversity schemes is well motivated for future broadband wireless system, which are characterized by high mobility.

Transmit diversity can be classified into the following categories based on the availability of the channel information at the transmitter and receiver:

a) Schemes with feedback and feed forward information (giving complete channel state information to both the transmitter as well as receiver).
b) Schemes with feed forward information but no feed back (receiver has the channel state information but the transmitter does not have any information.
c) Schemes without any channel state information at the transmitter or the receiver.

First category uses, implicit or explicit feedback of information from the receiver to the transmitter to configure the transmitter. In time division duplex systems [17], the same antenna weights are used for reception and transmission, so feedback is implicit in the appeal to channel symmetry. These weights are chosen during reception to maximize the SNR, and during transmission to weight the amplitudes of the transmitted signals. Explicit feedback includes switched diversity with feedback [16] based on the feedback of the channel response. However, in particular, vehicle movements or interference causes a mismatch between the state of channel perceived by transmitter and that perceived by receiver.

Second category is with feed forward without feedback technique. This technique uses linear processing at the transmitter to spread the information across the antennas. At the receiver, information is obtained by either linear processing or maximum likelihood decoding techniques. Feed forward information, is required to estimate the channel from the transmitter to receiver. These estimates are used to compensate for the channel response at the receiver. This scheme was first proposed by Wittneben [18].

The third category does not require feedback or feed forward information. Instead, it uses multiple transmit antenna combined with channel coding to provide diversity. Example of this approach is to combine phase sweeping transmitter diversity of [19] with channel coding. Another scheme is to encode information by a channel code and transmit the code symbols using different antennas in orthogonal manner. This can be done either by frequency multiplexing [20], time multiplexing [12], or by using orthogonal spreading sequences for different antenna [21]. The disadvantage of these schemes over previous two schemes is the loss in bandwidth efficiency due to the use of channel code. Using appropriate coding, it is possible to relax the orthogonality requirement need in these schemes and obtain the diversity as well as coding advantage offer without sacrificing bandwidth. This is possible when the whole system is viewed as a MIMO system and suitable codes are used.

In transmit diversity multiple antennas are used to achieve space diversity and to achieve time diversity the transmission is repeated n times. Scope of diversity is expanded by using multiple antennas at both the receiver and transmitter. A major conclusion of these works is that the capacity of multi-antenna systems far exceeds that of a single antenna system. In particular, the capacity grows at least linearly with the number of transmit antennas as long as number of received antenna is greater than or equal to number of transmit antennas [11].

Space-time Codes (STC) were first introduced by Tarokh et al. from AT&T research labs [4] in 1998 as a novel means of providing transmit diversity for the multiple antenna fading channel. There are two main types of STC:

a) Space-time Trellis Codes (STTC)
b) Space-time Block Codes (STBC)

The original STTC were introduced by Tarokh et al. in [11], there has been extensive research aiming the improving the performance of the original STTC design. These original STTC design were hand crafted and therefore, are not optimum design. In the recent years, a large number of research proposals have been published which propose new code construction or perform systematic searches for different convolution STTC or some variant of the original design criteria proposed by Tarokh et al. Examples of such work can be found in [22]-[27].

When the number of antennas is fixed, the decoding complexity of space-time trellis coding (measured by the number of trellis states at the decoder) increases exponentially as a function of diversity level and transmission rate [11]. In addressing the issue of decoding complexity, Alamouti [2] discovered a remarkable space-time block coding for transmission with two antennas. This scheme supports maximum likelihood (ML) detection based only on linear processing at the receiver. The very simple structure and linear processing of the Alamouti construction makes it a very attractive scheme that is currently part of both the W-CDMA and CDMA-2000 standards [28]. Key advantage of STTC over STBC is the provision of coding gain. Their disadvantage [29] is that they are extremely hard to design and generally require high complexity encoders and decoders.

Space-time block codes operate on a block of input symbols, producing a matrix output whose columns represent time and rows represents antennas. It contrast to single antenna block codes for the AWGN channel, space-time block codes do not generally provide coding gain, unless concatenated with an outer code. There main feature [1], [2] is the provision of full diversity with a very simple decoding scheme. On the other hand, space-time trellis codes operate on one input symbol at a time, producing a sequence of vector symbols whose length represents antennas.

Both STTC and STBC were first designed assuming a narrowband wireless system, i.e. a flat fading channel. However, when used over frequency selective channels a channel equalizer has to be used at the receiver along with the space-time decoder. Initial attempts to address the problem for STTC made use of whatever structure was available in the space-time coded signal [30]-[32], where the structure of the code was used to convert the problem into one that can be solved using known equalization schemes. For the STBC, the channel equalization problem was addressed by modifying the original Alamouti scheme in such a way that the use over frequency selective channel, and hence the equalization, is a much easier task. For example, in [33], STBC was used in conjunction with OFDM. OFDM is used to convert the frequency selective channel into a set of independent parallel frequency-flat subchannels. The Alamouti scheme is then applied to two consecutive subcarriers (or two consecutive OFDM block) [28].

In this thesis, STBC codes are used because of their simple decoding algorithm. The decoding of space-time block codes requires the knowledge of the channel at the receiver. The channel state information can be obtained at the receiver by sending the pilot symbols from each of the transmitter antennas to the receiver antenna. Thus, channel estimation is the backbone of STBC system and it is necessary to formulate the new channel estimation algorithms to improve the performance of the space-time block codes in the time-varying environment.

Overview of Channel Estimation Algorithms

With the widespread application of digital techniques in the area of mobile, indoor and personal radio communications, it becomes increasingly important to analyze the existing adaptive algorithms and develop new adaptive algorithms for identifying time-varying channels. For many wireless systems, independent of whether time division multiple access (TDMA) or code division multiple access (CDMA) is employed, estimation of a fading channel with high accuracy is a key factor in the receiver design. For some systems, channel fading is relatively slow compared with the frame rate, and, hence, channel estimates can be updated frame by frame. This approach is based on the assumption that the channel coefficients will not change significantly within a frame. However, for many systems, this assumption does not hold. Thus, a symbol by symbol channel estimate updating is necessary to maintain an acceptable performance [34].

It is obvious that a more generalized channel model is necessary if we wish to develop a more general means square error (MSE) analysis method for channel tracking problems. A widely used model is wide sense stationary uncorrelated scattering (WSSUS) model [35] [36]. With this model, the different paths of the channels are assumed to be uncorrelated. The coefficient of each path, which is represented as a random variable, is assumed to be wide sense stationary. Thus, the autocorrelation function of every coefficient of the channel is time invariant. The assumption of wide sense stationarity is somewhat controversial. For example, in a mobile radio channel, which can be well described by Jake's model [37] n many cases, the statistics of the channel are determined by the maximum Doppler frequency, which is proportional to vehicle speed. Thus, when vehicle speed is changing, he channel can not be considered as wide sense stationary. But there are several reasons that make us adopt the assumption in spite of the changing vehicle speed. First, in a practical system, such as the Northern American Digital Cellular (NADC) system [38], each data slot lasts 6.7 ms, so that the vehicle speed within such a short interval can be considered as a constant and the channel can be viewed as wide sense stationary. Secondly, only with the assumption of wide sense stationarity, the problem become mathematically tractable and the MSE analysis becomes meaningful. The third and the most important reason is that the results obtain with the assumption of wide sense stationary are applicable also to non-stationary situation via robust design [34]. The analytical and simulation results presented in [39] depict that the first order Markov channel provides a mathematically tractable model for time-varying Rayleigh fading channels. This model is given by following equation.

$$h(n) = \alpha h(n-1) + w(n)$$

where w(n) is the white noise. α is the parameter of the model. h(n) is the n^{th} channel coefficient.

This model is a stationary model as long as value of $\alpha < 1$, and it becomes non-stationary model for $\alpha \geq 1$.

For smaller value of α, channel is called slow fading channel and for value of α closer to one, channel is called fast fading channel. Slow fading channel is mainly estimated by Least Mean Square (LMS) or the exponentially windowed recursive least squares (EW-RLS) algorithms [40]. When channel fading is very slow, the channel variation within a data block is small and the LMS or EW-RLS algorithms can track the channel variations quite well. However, when the fading is fast, these algorithms fail to track the channel variation effectively [41].

An adaptive filter (AF) operating in a non-stationary environment needs to be able to track variations in input statistics. If implemented with infinite precision arithmetic, an AF has four sources of misadjustment error [42].

(1) First, the error comes from the noise, i.e. measurement noise.
(2) Second is the estimation error which arises from basing estimates on windowed data sequences. This causes the estimates of signal statistics to never converge in the mean square sense.
(3) Lag noise, the second source of misadjustment, is caused by the reaction time of the adaptation algorithm to changes in its environment. Lag error is minimized by rapidly discounting the past and basing estimates predominantly on recent data. These two errors place conflicting demands on the adaptation algorithm.
(4) Finally, model error could be contained in prediction error.

Different channel estimation algorithms had been proposed. In the field of adaptive signal processing, least mean square (LMS) is extensively explored algorithm and has found wide applications for the stationary environments due to its implementation simplicity [43]. However, the performance of LMS substantially degrades in the time-varying environment (non-stationary cases) [44]-[47], [43]. Machhi et al. have evaluated the performance of the LMS algorithm in the non-stationary environment [48], where the non-stationary is introduced using the complex chirp exponential signal buried in additive white Gaussian noise (AWGN) [49], [50]. For AR1 channel model, the LMS algorithm performs better than the RLS algorithm under some typical conditions [43], as shown later in this thesis work.

The optimum algorithm for the non-stationary environment is Kalman filtering algorithm. A distinctive feature of the Kalman filter is that its mathematical formulation is described in terms of the state-space concepts. Another novel feature of the Kalman filter is that its solution is computed recursively, applying without modification to stationary as well as non-stationary environments [43]. In multiuser scenario, Chen et al. have presented the Kalman filtering algorithm for channel estimation [51]. However, the computation complexity and requirement of the knowledge of system model may often preclude the above Kalman filter based approaches.

Thus we can see that LMS is suitable for stationary environment with less complexity, but its performance degrades in non-stationary environment. And Kalman filter is suitable for both stationary and non-stationary environment, but it has too much computation complexity.

The RLS algorithm is the algorithm which lies between LMS algorithm and Kalman filtering algorithm, i.e. it has less complexity than Kalman filtering algorithm and performs better than in LMS in non-stationary environment. For this reason, in this thesis work, we have chosen RLS algorithm with some modification, to track the channels in non-stationary environment. In [52], Sayed et al. have presented the RLS algorithm as a special case of Kalman algorithm. In non-stationary environment, the substantial degradation in the tracking performance of the RLS algorithms observed due to the design constraints. Subsequently the extended RLS algorithm has been proposed in [53], which provides the better tracking performance than LMS algorithm.

In RLS algorithm, the use of weighting factor is intended to ensure that the data in distant past are "forgotten" in order to afford the possibility of following the statistical variation of the observable data when the filter operates in a non stationary environment. A special form of weighting that is commonly used is the exponential weighting factor or forgetting factor, defined by $\beta=\lambda^{(n-i)}$ [43].

For stationary environment $\lambda=1$.

For non stationary environment $0\leq\lambda<1$.

An important requirement of recursive estimators for adaptive control and adaptive signal processing lies in their ability to track parameter changes. From this viewpoint, the famous standard RLS algorithm [43], which is known to have the optimal properties in stationary environments, is unsuitable for non stationary environments. Thus many attempts have been directed to the development of modified versions of the RLS algorithm to include tracking capability in time-varying environments. Among these modified RLS algorithms, the best known is an exponential data weighting RLS algorithm using a forgetting factor. However, in certain situations, this algorithm can lead to a problem often referred to as the blow-up problem. Also the lower the value of the forgetting factor, the higher the tracking velocity but the higher the influence of the noise, that is, the larger the parametric errors. Recursive least squares algorithms have been used extensively in adaptive filtering, self-tuning control systems and system identification. RLS is well known for its good convergence property and small mean square error when the system is time-invariant. However RLS is shown in [43] not effective for tracking time-varying parameters because it is difficult to find a suitable forgetting factor to provide good tracking in dealing with large model variations. To avoid these difficulties, the idea of a variable forgetting factor was introduced [54].

The general strategy for the control of variable forgetting factor (VFF) can be described as follows. *Large forgetting factor (effectively large memory of data) is used when the learning is in the steady state and also there is no obvious model variation, while small one (to fade away the very old data) is applied when the model error is large*. In time-varying environment, the control should be able to sense the change of the model and reduce the disturbance from the noise.

In the environment with impulsive noise, at the incident of large error signal, there could be two possibilities. The error is due to either large model variation or impulse noise. In case the former one occurred, the forgetting factor should be adjusted to make the filter response to the change; otherwise, the forgetting factor should remain large to neglect the effect of the impulse noise. In order to avoid the disturbance by the impulse noise, we can use the autocorrelations of nonzero lags to measure the model error and control the forgetting factor for those model errors not very large [55].

Problem Statement

This thesis presents the following work

1). Firstly, channel estimation algorithms for the estimation of different types of channels are introduced and these algorithms are modeled and compared for different channels.
2). Then, we will evaluate the performance of these channel estimation algorithm for the proposed STBC systems.

Organization of Thesis

This thesis is organized in four chapters

Chapter I summarize the basic problem statement of the research work and give the overview of the diversity and channel estimation algorithms.
In Chapter II, we review the related work done in the area of STBC. We also present the different types of STBC codes for complex and real data.
In Chapter III, we compare the different channel estimation techniques with transmitter diversity.
In Chapter IV, simulation results for different channel estimation algorithms and the corresponding Bit Error Rates (BER) for different STBC codes are given. All the simulations are done using MATLAB.
Finally we conclude our work for different channel estimation algorithms and corresponding BER with Space-time block codes.

CHAPTER 2

PERFORMANCE CRITERIA OF SPACE-TIME CODES

Space-time block codes combine all the copies of the signal in such a way that the entire signals can be extracted from any of the received signal. The pioneer work in this field is done by Alamouti [2] and Tarokh [56]. Most of the Space-time codes have been proposed for flat fading channel. But for high rates (in broadband wireless communication), flat fading model in not well justified. This thesis focuses on space-time block codes for Time-varying Channels.

Data is encoded using a space-time block code and the encoded data is split into n streams which are transmitted using n transmit antennas. The received signal at each antenna is a linear superposition of the n transmitted signals perturbed by noise. Maximum likelihood decoding is achieved in a simple way through decoupling of signals transmitted from different antennas rather than joint detection. This uses the orthogonal structure of the space-time block code and gives a maximum likelihood decoding which is based only on the linear processing at the receiver.

In this chapter, we first review the space-time block code given by Alamouti [2]. Complex orthogonal design and corresponding space-time block code which provides full diversity and full transmission is not possible for more than two antennas, so here space-time block codes are given which provide ½ of the maximum possible transmission rate for any number of transmit antennas, with full diversity. Then comparison of space-time block codes and convention codes is given.

2.1 General Principles of MIMO Systems and Need of Space-time Codes

Current transmission schemes over MIMO channels typically fall into two categories [28]:

(1) data rate maximization

(2) diversity maximization

The first kind focuses on improving the average capacity behavior. This can be explained by following example:

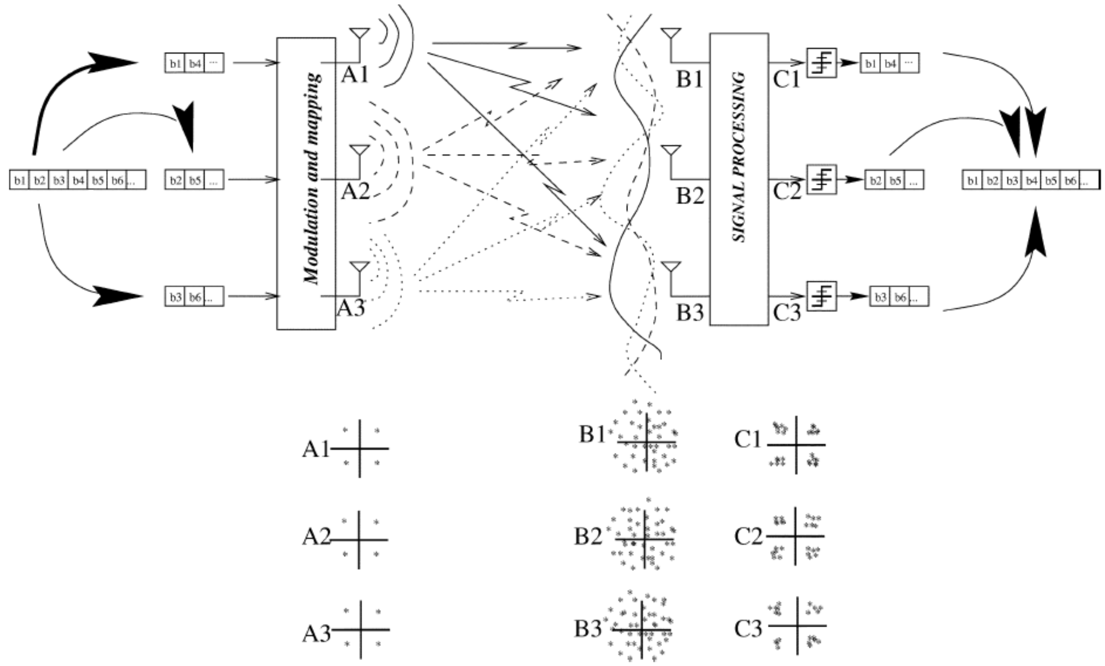


Fig 2.1.1 Basic Spatial Multiplexing Scheme with Three Transmitter and Three Receiver Antennas Yielding Three-fold Improvement [28]

In figure 2.1.1, a high rate bit stream is decomposed into three independent 1/3 rate bit sequences which are then transmitted simultaneously using multiple antenna, thus consuming one third of normal spectrum. At the receiver, after having identified the mixing channel matrix through training symbols, the individual bit stream is separated and estimated. This occurs in the same way as three unknowns are resolved from a linear system of three equations.

More generally, individual streams should be encoded jointly in order to protect transmission against errors caused by channel fading and noise plus interference. This leads to a second kind of approach in which one tries to minimize the outage probability, or equivalently maximize the outage capacity. If the level of redundancy is increased between the transmitter antennas through joint coding, the amount of independence between the signals decreases. Ultimately, it is possible to code the signals so that the effective data rate is back to that of a single antenna system.

The set of schemes aimed at realizing joint encoding of multiple transmitter antennas are called ***Space-time Codes (STC).*** In these schemes a number of coded symbols equal to transmitter antennas are generated and transmitted simultaneously, one symbol from each antenna. These symbols are generated by the space-time encoder such that by using the appropriate signal processing and decoding procedure at the receiver, the diversity gain and/or the coding gain is maximized. Figure 2.2.1 shows a simple block diagram for STC.

2.2 Space-time Codes

Traditionally, the intelligence of the multiantenna system is located in the weight selection algorithm rather than in the coding side although the development of space-time codes is transforming this view. Space-time codes were first given by Tarokh et al. [11] in 1998 as a novel means of providing transmit diversity for the multiple antenna fading channels. The space-time coding scheme proposed in [56] is essentially a joint design of channel coding, modulation; transmit diversity and the optional receiver diversity.

2.2.1 System Model and Transmission Rate

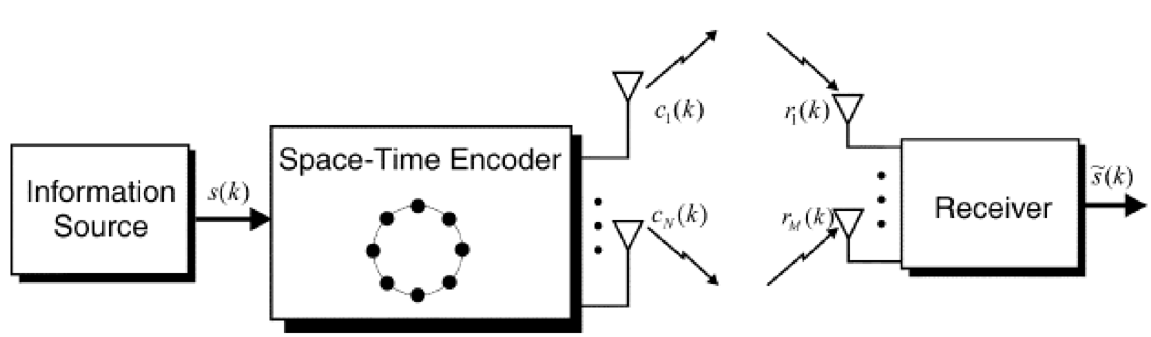
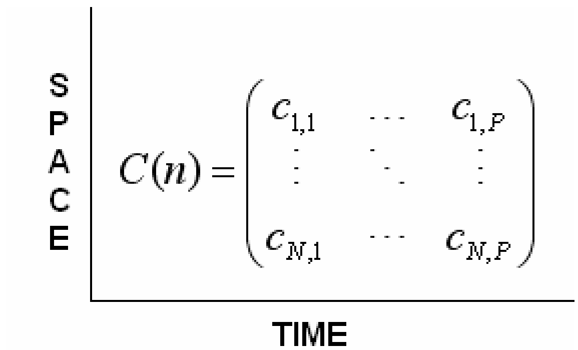


Figure 2.2.1 Space-time Coding [28]

Figure 2.2.1 shows the discrete time baseband model of a single user communication system with N transmit antenna and M receive antenna, and K is the size of signal **s**(n) to be transmitted.

$$\mathbf{s}(n) = [s_{n,0}\ s_{n,1}\ \ldots\ s_{n,k-1}]^T \tag{2.2.1}$$

The space-time encoder maps s(n) to the following N x P matrix:



(2.2.2)

P is the time slots used to transmit K symbols, so the overall transmission rate R is [57]:

$$R = K/P \tag{2.2.3}$$

Example: let G represents the code which utilize two transmit antenna and is defined by

(i) $$G = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix}$$

here K = 2 and P = 2 , thus transmission rate R = 1.

(ii) $$G = \begin{pmatrix} x_1 & -x_2^* & \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} \\ x_2 & x_1^* & \frac{x_3^*}{\sqrt{2}} & -\frac{x_3^*}{\sqrt{2}} \\ \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} & \frac{(-x_1 - x_1^* + x_2 - x_2^*)}{2} & \frac{(x_2 + x_2^* + x_1 - x_1^*)}{2} \end{pmatrix}$$

here K=3 and P=4, thus transmission rate R = ¾.

Above two examples shows the orthogonal space-time block codes, example (i) have R=1, i.e. full transmission rate, but example (ii) provides the transmission rate ¾ of the full transmission rate.

2.2.2 Performance in Independent Fading Coefficients

The channel is assumed to be a flat fading channel and the path gain from i[th] transmitter to j[th] receiver is define as $\alpha_{i,j}$. the path gain are modeled as samples of independent complex Gaussian random variables with variance 0.5 per dimension with non zero mean and known to receiver. This is equivalent to the assumption that signals transmitted from different antennas undergo independent fades. The wireless channel is assumed to be quasi-static so that the path gains are constant over a frame and vary form one frame to another. It is also assumed that receiver has the complete knowledge of the channel. Channel knowledge is either acquired through training or by employing blind estimation algorithms. Let us assumed that each element of the signal constellation is contracted by a scale factor $\sqrt{E_s}$, so that the average energy of the constellation elements is 1. Maximum Likelihood decoding is performed at the receiver. The signal received at the antenna j at time t is given by [11].

$$d_t^j = \sum_{i=1}^{n} \alpha_{i,j} c_t^j \sqrt{E_s} + \eta_t^j \qquad (2.2.2.1)$$

where $$1 \le i \le N$$ $$1 \le j \le M$$

η_t^j is the noise at time 't' modeled as independent samples of a zero mean complex Gaussian random variable with variance $N_0/2$ and c_t^j j=1, 2 … n are the signals transmitted simultaneously from n transmit antennas.

We assume that probability that a maximum-likelihood receiver decides erroneously in favor of a signal

$$\mathbf{e} = e_1^1 e_1^2 \cdots e_1^N e_2^1 e_2^2 \cdots e_2^N \cdots e_h^1 e_h^2 \cdots e_h^N \quad (2.2.2.2)$$

assume that

$$\mathbf{c} = c_1^1 c_1^2 \cdots c_1^N c_2^1 c_2^2 \cdots c_2^N \cdots c_h^1 c_h^2 \cdots c_h^N \quad (2.2.2.3)$$

The probability of transmitting **c** and deciding in favor of **e** at the decoder is well approximated by

$$P(\mathbf{c} \to \mathbf{e} \mid \alpha_{i,j}, i = 1,2\cdots N, j = 1,2\cdots M) \le \exp(-d^2(\mathbf{c},\mathbf{e})E_s / 4N_0) \quad (2.2.2.4)$$

where $d^2(\mathbf{c},\mathbf{e}) = \sum_{j=1}^{M}\sum_{t=1}^{h} \left| \sum_{i=1}^{N} \alpha_{i,j}(c_t^i - e_t^i) \right|^2$ (2.2.2.5)

or

$$d^2(\mathbf{c},\mathbf{e}) = \sum_{j=1}^{M}\sum_{i=1}^{N}\sum_{i'=1}^{N} \alpha_{i,j}\overline{\alpha_{i',j}} \sum_{t=1}^{h} (c_t^i - e_t^i)\overline{(c_t^{i'} - e_t^{i'})} \quad (2.2.2.6)$$

let $\Omega_j = (\alpha_{1,j}, \alpha_{2,j} \cdots \alpha_{N,j})$ (2.2.2.7)

Substituting this value in equation (2.2.2.6),

$$d^2(\mathbf{c},\mathbf{e}) = \sum_{t=1}^{h} \Omega_j \mathbf{A}(\mathbf{c},\mathbf{e}) \Omega_j^* \quad (2.2.2.8)$$

where $A_{pq} = \sum_{t=1}^{h} (c_t^p - e_t^p)\overline{(c_t^q - e_t^q)}$ (2.2.2.9)

Substituting above equation in equation (2.2.2.4),

$$P(\mathbf{c},\mathbf{e} \mid \alpha_{i,j}, i = 1,2\cdots N, j = 1,2\cdots M) \le \prod_{j=1}^{M} \exp(-\Omega_j \mathbf{A}(\mathbf{c},\mathbf{e}) \Omega_j^* E_s / 4N_0)$$

(2.2.2.10)

Before solving further, we will see some properties of matrix:

- Any matrix **A** with a square root **B** is nonnegative definite.
- For ay nonnegative Hermitian matrix **A**, there exists a lower triangular square matrix **B** such that **BB***=**A**.
- Given a Hermitian matrix **A**, the eigenvectors of **A** span $\mathbb{C}n$, the complex space of n dimensions and it is easy to construct an orthonormal basis of $\mathbb{C}n$ consisting of eigenvectors **A**. Furthermore, there exists a unitary matrix **V** and a real diagonal matrix **D** such that **VAV***=**D**. the row of **V** are an orthonormal basis of $\mathbb{C}n$ given by eigenvectors of **A**. the diagonal elements of **D** are the eigenvalues λ_i, i = 1,2,…,n of **A**.
- The eigenvalues of Hermitian matrix are real.
- The eigenvalues of nonnegative definite Hermitian matrix are non negative

Since **A**(**c**,**e**) is hermitian matrix, there exists a unitary matrix **V** and a real matrix **D** such that

$$\mathbf{VA(c,e)V^* = D.} \tag{2.2.2.11}$$

* denotes the hermitian conjugate.

From the above properties of matrix ,

- ✓ The rows ($\mathbf{v}_1$, $\mathbf{v}_2$, …,$\mathbf{v}_N$) of **V** are a complete orthonormal basis of $\mathbb{C}n$ given by eigenvectors of **A**.
- ✓ The diagonal elements of **D** are the eigenvalues λi, i = 1,2,…,N of **A**.
- ✓ It can be seen that square root of **A**(**c**,**e**) is

$$B(\mathbf{c},\mathbf{e}) = \begin{pmatrix} e_1^1 - c_1^1 & e_2^1 - c_2^1 & \cdots & e_h^1 - c_h^1 \\ e_1^2 - c_1^2 & e_2^2 - c_2^2 & \cdots & e_h^2 - c_h^2 \\ \vdots & \vdots & \vdots & \vdots \\ e_1^N - c_1^N & e_2^N - c_2^N & \cdots & e_h^N - c_h^N \end{pmatrix} \tag{2.2.2.12}$$

- ✓ Thus eigenvalues of **A**(**c**,**e**) are nonnegative real numbers.

Let $(\beta_{1,j}, \beta_{2,j} \cdots \beta_{N,j}) = (\Omega_j \mathbf{V}^*)$ (2.2.2.13)

Since **V** is the unitary matrix (from equation 2.2.2.11), hence

$$\mathbf{V^* = V^{-1}} \tag{2.2.2.14}$$

Substitute this value in equation 2.2.2.13,

$$(\beta_{1,j}, \beta_{2,j} \cdots \beta_{N,j})\mathbf{V} = \Omega_j$$

Substitute this value in equation 2.2.2.8,

$$d^2(\mathbf{c},\mathbf{e}) = [(\beta_{1,j}, \beta_{2,j} \cdots \beta_{N,j})]\mathbf{VA}(\mathbf{c},\mathbf{e})\mathbf{V}^* \begin{pmatrix} \overline{\beta_{1,j}} \\ \overline{\beta_{2,j}} \\ \vdots \\ \overline{\beta_{N,j}} \end{pmatrix} \tag{2.2.2.15}$$

From the properties of matrix shown above, **VA(c,e)V*** is a diagonal matrix having eigenvalues as a diagonal elements.

$$d^2(\mathbf{c},\mathbf{e}) = [(\beta_{1,j}, \beta_{2,j} \cdots \beta_{N,j})] \begin{pmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_n \end{pmatrix} \begin{pmatrix} \overline{\beta_{1,j}} \\ \overline{\beta_{2,j}} \\ \vdots \\ \overline{\beta_{N,j}} \end{pmatrix} \tag{2.2.2.16}$$

$$d^2(\mathbf{c},\mathbf{e}) = |\beta_{1,j}|^2 \lambda_1 + |\beta_{2,j}|^2 \lambda_2 \cdots + |\beta_{N,j}|^2 \lambda_N \tag{2.2.2.17}$$

$$d^2(\mathbf{c},\mathbf{e}) = \sum_{i=1}^{N} |\beta_{i,j}|^2 \lambda_i \tag{2.2.2.18}$$

thus equation 2.2.2.4 becomes

$$P(\mathbf{c} \to \mathbf{e} \mid \alpha_{i,j}, i = 1,2 \cdots N, j = 1,2 \cdots M) \le \exp(-\sum_{i=1}^{N} |\beta_{i,j}|^2 \lambda_i E_s / 4N_0) \tag{2.2.2.19}$$

$\beta_{i,j}$ is the complex Gaussian random variable with variance 0.5 per dimension and mean $\mathbf{K}^j$.

where $\mathbf{K}^j$=(E $\alpha_{1,j,}$ E$\alpha_{2,j}$ … E$\alpha_{N,j}$), i.e mean of $\alpha_{i,j}$.

Thus |E $\beta_{i,j}$ |^2 have Rician distribution with pdf

$$p(|\beta_{i,j}|) = 2|\beta_{i,j}|\exp(-|\beta_{i,j}| - K_{i,j}) I_0(2|\beta_{i,j}|\sqrt{K_{i,j}} \tag{2.2.2.20}$$

for $\beta_{i,j} \geq 0$, where I_0( ) is the zero order modified Bessel function of first kind.

For M receiving antenna, the probability of error will be:-

$$P(\mathbf{c} \to \mathbf{e} \mid \alpha_{i,j}, i = 1,2\cdots N, j = 1,2\cdots M) \leq \prod_{j=1}^{M} \exp(-\sum_{i=1}^{N} |\beta_{i,j}|^2 \lambda_i E_s / 4N_0) \tag{2.2.2.21}$$

2.2.3 Design Criteria for Optimum Performance

The goal is to design optimal ST encoder so that $P(\mathbf{c} \to \mathbf{e})$ is minimized.

Let us denote the space-time code matrix **C** and **C'** mapped from **c** and **e** respectively and define N X M error matrix Δ**C** as [58]:

$$\Delta\mathbf{C} = (\mathbf{C}-\mathbf{C}')(\mathbf{C}-\mathbf{C}')^{\mathbf{H}} \tag{2.2.3.1}$$

At high SNR (E_s/N_0 >> 1), it has been shown that $P(\mathbf{c} \to \mathbf{e})$ upper bounded by

$$P(\mathbf{c} \to \mathbf{e}) \leq (\prod_{i=1}^{rank(\Delta C)} \lambda_i)^{-M} (E_s / 4N_0)^{-rank(\Delta C)M} \tag{2.2.3.2}$$

where λ_i , i = 1, …, rank(Δ**C)** are the nonzero eigenvalues of Δ**C**.

Conventionally the performance of the ST coded systems is measured by two parameters:

i) diversity gain

ii) coding gain

Diversity gain is given by $G_d = rank(\Delta\mathbf{C}).M$ (2.2.3.3)

and Coding gain is given by $G_c = (\prod_{i=1}^{rank(\Delta C)} \lambda_i)^{1/rank(\Delta C)}$ (2.2.3.4)

Substitute above two value in equation 2.2.3.2,

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq (G_c E_s / 4N_0)^{-G_{\mathbf{d}}} \quad (2.2.3.5)$$

We have two types of design criteria as given below:

i) **Rank Criterion** : in order to achieve maximum coding gain, the minimum determinant of $\Delta\mathbf{C}$ or equivalently, the matrix **C-C'** has to be full rank over all possible distinct **C** and **C'** (**C≠C'**).

ii) **Determinant Criterion:** Suppose $\Delta\mathbf{C}$ is full rank. In order to achieve the maximum coding gain, the minimum determinant of $\Delta\mathbf{C}$ over all possible and **C'** (**C≠C'**) should be maximized.

Using these two design criteria, we may note the following:

1) At the high SNR, the diversity gain G_d plays more important role than the coding Gain G_c in reducing the upper bound. Thus, ST coding designs should first satisfy the rank criterion, and then the determinant criterion. If a tradeoff has to be made, the coding gain should be the candidate.
2) In order to achieve the maximum diversity gain, we infer from the dimensionality of and $\Delta\mathbf{C}$ that one should select P≥N. This implies that time domain processing is

indispensable for ST coding. Recall also that ST coding does not require channel knowledge at the transmitter.

3) Although multiple receive antennas (M>1) are helpful to increase the diversity gain, they are optional in ST coding designs. All design criteria are meaningful even for a single receive antenna provide that SNR is high.
4) These ST coding design criteria are based on maximum likelihood (ML) coding. However, the promised diversity and coding gain may not be necessarily achieved for a specific (non ML) ST decoding Scheme. The basic method to check the achieved diversity, and code gains is to derive the error probability expression in 2.2.3.5.

2.2.4 Example of Space-time Codes

(a) Delay Diversity Code

Delay diversity code was the first coding scheme, proposed by a multiple transmit antenna [12]. In this scheme, time delay versions of the symbols are transmitted across the antenna. This multiple input channel has an equivalent representation as a single input channel with memory. In a fading channel, this is a equivalent representation as a signal input channel with memory. In a fading channel, this is a equivalent to transforming a fading channel into a frequency selective channel. The delay diversity scheme can achieve a diversity gain of the order of the number of a transmit antennas.

Limitation: The coding gain is not optimized by this scheme.

(b) Space-time Trellis Codes

The delay diversity scheme was further extended by Tarokh in [11] to improve the coding gain to a certain extent. It was shown that the space-time trellis code constructed in [11] have maximum diversity gain (given by number of transmit antennas) and a coding gain greater than that of the delay diversity scheme. In particulars, trellis codes of different complexities (measured by the number of states in the trellis and the number of transitions from each state) and different constellations (QPSK, 8 PSK among others) were constructed. This was the first attempt n the direction of improving the coding gain of codes for multiple transmitter antennas apart from ensuring maximum diversity gain. However, it should be noted that no systematic approach has been laid out to maximize the coding gain or to ensure a minimum coding gain.

(c) Space-time Block Codes

I) Alamouti code [2] was the first code and probably well known STBC. He proposed a simple transmit diversity scheme (fig. 2.2.4.1), which improves the signal quality at the receiver on the one side of the link by simple processing across two transmit antennas at the opposite end. *STBC has advantage of no bandwidth expansion, not requiring the knowledge at the transmitter, and simple maximum likelihood decoding at the receiver using only linear processing technique.* Now, we derive the mathematical description of two transmit antenna and one receive antenna, called as two branch transmit diversity with one receiver scheme as shown in figure 2.2.4.1.

Transmission: In this scheme two different are simultaneously transmitted from each antenna at a given symbol period. During first time period the signal from transmit antenna zero is denoted by s_0 and from transmit antenna one is denoted by s_1. During second symbol period the signal from transmit antenna zero is denoted by ($-s_1^*$) and from transmit antenna one is denoted by s_0^*.

Table 2.2.4.1

Time	Antenna 0	Antenna 1
T	s_0	s_1
t + T	$-s_1^*$	$-s_0^*$

Encoding and transmission sequence for two branch transmit diversity scheme.
T is the symbol period.

The channel for transmit antenna zero is h_0 and for transmit antenna one is h_1.

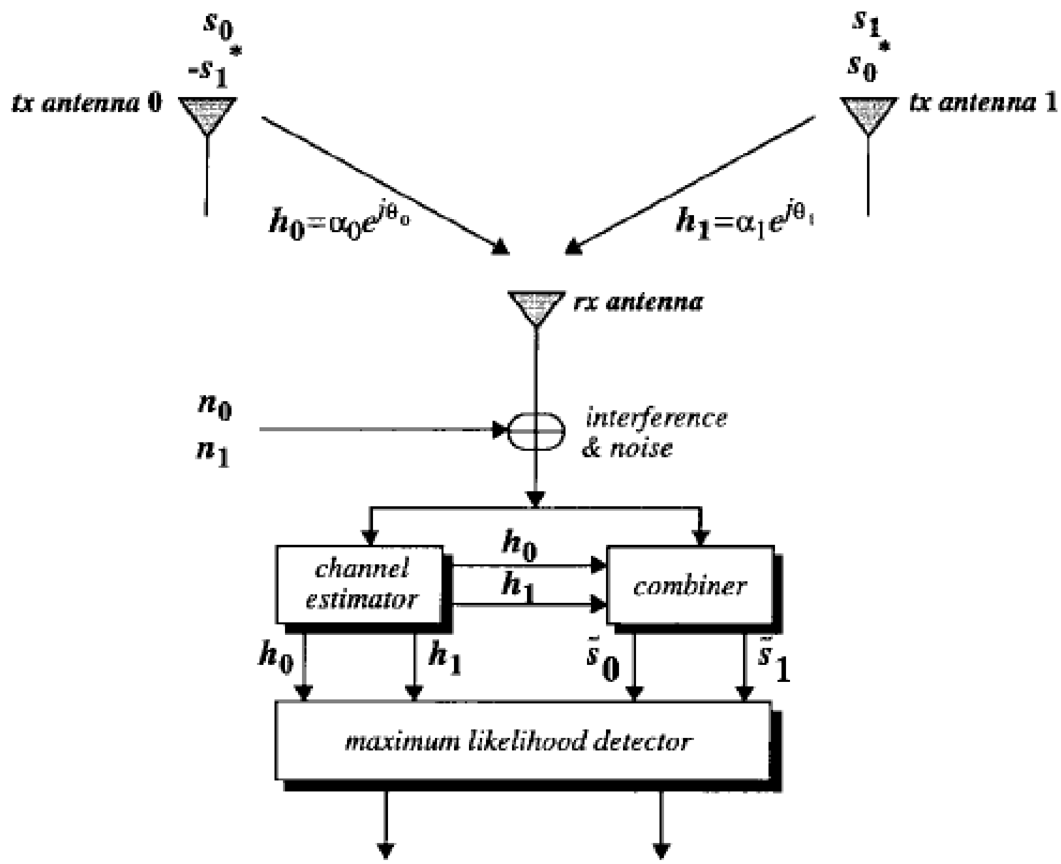


Fig 2.2.4.1 Two Branch Transmit Diversity with One Receiver [2]

Reception: received signal can be express as:

$$r_0 = h_0 s_0 + h_1 s_1 + n_0 \qquad (2.2.4.1)$$

$$r_1 = -h_0 s_1^* + h_1 s_0^* + n_1 \qquad (2.2.4.2)$$

r_0 and r_1 are the received at time t and t+T. The combiner combines the received signal as follows:

$$\tilde{s}_0 = h_0^* r_0 + h_1 r_1^* \qquad (2.2.4.3)$$

$$\tilde{s}_1 = h_1^* r_0 - h_0 r_1^* \qquad (2.2.4.4)$$

Substituting the value of r_0, r_1 from equation 2.2.4.1 and 2.2.4.2 respectively and h_0 and h_1 from equation A.1 (Appendix A) above two equation becomes,

$$\tilde{s}_0 = (|\alpha_0|^2 + |\alpha_1|^2)s_0 + h_0^* n_0 + h_1 n_1^* \tag{2.2.4.5}$$

$$\tilde{s}_1 = (|\alpha_0|^2 + |\alpha_1|^2)s_1 - h_0 n_1^* + h_1^* n_0 \tag{2.2.4.6}$$

Maximum likelihood Decision Rule

These combined signal are then sent to maximum likelihood detector which uses the decision rule expressed in A.7 (in Appendix A). Note that the resulting combined signals are same to those obtained from two branch MRRC (see Appendix A).

The only difference is the phase rotation on the noise components, which do not degrade the effective SNR. Thus orthogonality of the code imposes an 'artificial' orthogonality on the channel.

Characteristics of these schemes are as follows:

i) No feedback is required from receiver to transmitter.
ii) No bandwidth expansion (as redundancy is applied in space across multiple antennas, not in time or frequency).
iii) Low complexity decoders.

II) orthogonal space-time block code

In this section we will describe two type of orthogonal design which are as follows:

(A) real orthogonal design
(B) complex orthogonal design

(A) Real orthogonal design

A real orthogonal design of size n is a n X n orthogonal matrix with entries $\pm x_1, \pm x_2, \ldots, \pm x_n$. A linear processing orthogonal design in variable $x_1, x_2, \ldots, x_n$ is an n X n matrix ξ such that [1, definition 3.4.1]:

- Entries of ξ are real linear combination of variables $x_1, x_2, \ldots, x_n$.
- $\xi^T\xi = D$, where D is the diagonal matrix with $(i,i)^{th}$ diagonal element of the form $(l_1^i\ x_1^2 + l_2^i\ x_2^2 + \ldots\ldots\ldots\ldots + l_n^i\ x_n^2)$ with coefficients $l_1^i, l_2^i, \ldots\ldots\ldots, l_n^i$ all strictly positive numbers. An orthogonal design of size n exists if and only in n = 2, 4, 8 [1, corollary 3.5.1]. Advantage of linear processing orthogonal design is, there is no loss of bandwidth, in the sense that orthogonal designs provide the maximum possible transmission rate at full diversity. A linear processing orthogonal design ξ in variable $x_1,x_2,\ldots,x_n$ exists if and only if there exists a linear processing orthogonal design L such that

  $LL^T = L^TL = (x_1^2+x_2^2+ \ldots + x_n^2)\, I$, where I is the identity matrix [1, theorem 3.4.1].

Example of orthogonal design is as follows:

The 2×2 design

$$\begin{pmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{pmatrix} \qquad (2.2.4.7)$$

The 4×4 design

$$\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \end{pmatrix} \qquad (2.2.4.8)$$

The 8×8 design

$$\begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ -x_2 & x_1 & x_4 & -x_3 & x_6 & -x_5 & -x_8 & x_7 \\ -x_3 & -x_4 & x_1 & x_2 & x_7 & x_8 & -x_5 & -x_6 \\ -x_4 & x_3 & -x_2 & x_1 & x_8 & -x_7 & x_6 & -x_5 \\ -x_5 & -x_6 & -x_7 & -x_8 & x_1 & x_2 & x_3 & x_4 \\ -x_6 & x_5 & -x_8 & x_7 & -x_2 & x_1 & -x_4 & x_3 \\ -x_7 & x_8 & x_5 & -x_6 & -x_3 & x_4 & x_1 & -x_2 \\ -x_8 & -x_7 & x_6 & x_5 & -x_4 & -x_3 & x_2 & x_1 \end{pmatrix} \qquad (2.2.4.9)$$

(B) Complex orthogonal design

Channel matrix in section III is a complex orthogonal matrix of order 2×2 given by Alamouti. It has been proved that complex orthogonal design only exists in two dimensions. This means Alamouti Scheme is in unique in some sense. Space-time block code proposed by Alamouti is given below:

$$\begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix} \qquad (2.2.4.10)$$

It is not possible to achieve full diversity along with full transmission rate for more than two antennas [1, theorem 5.3.1]. There are block codes which provide full diversity but not the full transmission rate for more than two antennas [1]. In [1] block codes are given which achieve ½ of the full transmission rate along with full diversity for any number of antennas and also proposed block code which provide the ¾ of the full transmission rate for the specific cases of three and four transit antennas. STBC having half of the full transmission rate is shown here.

$$\begin{pmatrix} x_1 & x_2 & x_3 \\ -x_2 & x_1 & -x_4 \\ -x_3 & x_4 & x_1 \\ -x_4 & -x_3 & x_2 \\ x_1^* & x_2^* & x_3^* \\ -x_2^* & x_1^* & -x_4^* \\ -x_3^* & x_4^* & x_1^* \\ -x_4^* & -x_3^* & x_2^* \end{pmatrix}$$

(C) Complex Quasi-orthogonal design

For more than two transmit antennas some codes are known achieving full diversity but lower rates [1]. The basic technique to overcome the rate limitation of orthogonal STBCs for more than two transmit antennas is to allow a small amount of non-orthogonality in the STBC matrix. As a result several full rate quasi-orthogonal STBC (QSTBC) schemes have been introduced. QSTBC suffers from a loss in diversity order due to coupling between the symbols in codeword. QSTBC of dimension 4×4 and 8×8 are shown below. The rate of 4×4 QSTBC is one whereas rate of 8×8 QSTBC is ¾ [59].

The 4×4 design

$$\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & -x_3 & -x_2 & x_1 \end{pmatrix}$$

The 8×8 design

$$\begin{pmatrix} x_1 & x_2 & x_3 & 0 & x_4 & x_5 & x_6 & 0 \\ -x_2^* & x_1^* & 0 & -x_3 & x_5^* & -x_4^* & 0 & x_6 \\ x_3^* & 0 & -x_1^* & -x_2 & -x_6^* & 0 & x_4^* & x_5 \\ 0 & -x_3^* & x_2^* & -x_1 & 0 & x_6^* & -x_5^* & x_4 \\ -x_4 & -x_5 & -x_6 & 0 & x_1 & x_2 & x_3 & 0 \\ -x_5^* & x_4^* & 0 & x_6 & -x_2^* & x_1^* & 0 & x_3 \\ x_6^* & 0 & -x_4^* & x_5 & x_3^* & 0 & -x_1^* & x_2 \\ 0 & x_6^* & -x_5^* & -x_4 & 0 & x_3^* & -x_2^* & -x_1 \end{pmatrix}$$

2.2.5 Advantages of ST Codes with Conventional Codes

i) Channel coding induces bandwidth efficiency loss due to the redundancy introduced in time domain. On the contrary ST coding is performed without sacrificing bandwidth efficiency by taking advantage of the additional space domain.

ii) Channel coding is typically designed for AWGN channels, and it relies on the interleaving to be effective in fading channels, that causes undesirable decoding delay. ST coding is especially designed for fading, and does not induce decoding delay.

iii) The use of additional antennas is assumed to come for free as compared to the billion dollars spent on frequency band auction. Thus, ST coding is cheaper and more efficient than channel coding.

CHAPTER 3

TIME-VARYING CHANNEL ESTIMATION ALGORITHMS

As we have seen in previous chapter that for decoding of Space-time codes requires the channel state information. Channel estimation is usually difficult, usually in time-varying fading channels. Simple algorithm i.e. steepest gradient, RLS etc. [43], can only track the mean value of the channel; these algorithms can not track the variation in the channel. Polynomial model for representing the time-varying channel is shown in [41]. In this chapter we use the first order polynomial model for channel. This provide the advantage that along with mean value of channel, it also estimate the rate of change of channel with time (i.e. first order derivative of Taylor's series is calculated while higher order derivatives are neglected). Estimation of first order derivative will provide better estimation of channel. This is shown in simulation results in Chapter 5.

3.1 Basic Terminology for Channel Estimation Algorithms

let there are L multipath channels, then channel matrix is defined as

$$\mathbf{c}_n = [c_{n,0}, c_{n,1} \cdots c_{n,L-1}]^H \tag{3.1.1}$$

and known signal matrix is defined as

$$\mathbf{s}_n = [s_n, s_{n-1} \cdots s_{n-L+1}]^H \tag{3.1.2}$$

transmitted signal is recovered from the received signal, defined as

$$y_n = \sum_{l=0}^{L-1} c_{n,l} s_{n-l} + w_n$$

or

$$y_n = \mathbf{s}_n^H \mathbf{c}_n + w_n \tag{3.1.3}$$

where w_n is the additive White Gaussian Noise.

We will simulate the various algorithms for following channels:

1. Channel with linear variation:

$$\mathbf{c}_n = \frac{1}{2} + \frac{n}{2000} \qquad \text{where } n = 1, 2 \cdots 1000 \qquad (3.1.4)$$

2. Channel with sinusoidal variation:

$$\mathbf{c}_n = \frac{1}{2} + \frac{1}{2}\cos(\frac{2\pi n}{1000}) \qquad \text{where } n = 1, 2 \cdots 1000 \qquad (3.1.5)$$

3. Channel with complex variation:

$$\mathbf{c}_n = \frac{1}{2} + \frac{1}{2}\exp(\frac{j2\pi n}{1000}) \qquad \text{where } n = 1, 2 \cdots 1000 \qquad (3.1.6)$$

4. General model for time-varying channel (AR1 model):

$$\mathbf{c}_n = \mathbf{c}_{n-1} + v_n \qquad \text{where } n = 1, 2 \cdots 1000 \qquad (3.1.7)$$

where $\mathbf{v_n}$ is the AWGN.

We have to estimate $\mathbf{c}_n$, we will estimate the value of $\mathbf{c}_n$ with the help of new channel vector $\mathbf{C}_n$, whose value will be change according to the algorithm. We have used the $\mathbf{C}_n$ and $\mathbf{S}_n$, as new vectors, so that algorithm can be generalized, as shown below.

and estimation error is defined as

$$e_n = y_n - \mathbf{C}_{n-1}^H \mathbf{S}_n \qquad (3.1.8)$$

Above parameters will be same for all the channel estimation algorithms.

Polynomial channel model is proposed by Borah [41] for time varying channels. This model is based on Taylor's series which is expressed mathematically as follows.

$$X(n) = x_0(n) + n\, x_1(n)/1! + n^2\, x_2(n)/2! + \ldots\ldots\ldots$$

This expression represent the time varying channel model as shown in section 3.2 (c, d, f, g, h, i). This model can be used for tracking the time varying channel. Polynomial model become more complex as the order increases and become mathematically complex and can not be implement practically. So we have used this model upto second order.

3.2 Mathematical Description of Different Channel Estimation Algorithms

Different algorithms proposed are as follows:

a) **LMS** (Least mean square) algorithm
b) **RLS** (Recursive least square) algorithm
c) **LSn** (Least square of order 1) algorithm
d) **LSn2** (Least square of order 2) algorithm
e) **RLS-VFF** (RLS with Variable Forgetting Factor) algorithm
f) **LSn-VFF** (LSn with Variable Forgetting Factor) algorithm
g) **LSn2-VFF** (LSn2 with Variable Forgetting Factor) algorithm
h) **LSn-NVFF** (LSn with New Variable Forgetting Factor) algorithm
i) **LSn2-NVFF** (LSn2 with New Variable Forgetting Factor) algorithm

Before describing the above algorithms, we will see an old optimization technique known as Steepest Descent Algorithm. This method is basic to all other algorithms.

Let cost function is **J(C)**, and we want to find the optimal solution $\mathbf{C_0}$ that satisfy the condition

$$\mathbf{J}(\mathbf{C}_0) \le \mathbf{J}(\mathbf{C}) \text{ for all } \mathbf{C}. \tag{3.1.9}$$

starting with an initial guess denoted by **C**(0), generate a sequence of weight vector **C**(1), **C**(2) . . . **C**(n), such that the cost function **J**(**C**) is reduced at each iteration of the algorithm, that is,

$$\mathbf{J}(\mathbf{C}(n+1)) < \mathbf{J}(\mathbf{C}(n)) \tag{3.1.10}$$

for such solution, steepest descent algorithm is described by

$$\mathbf{C}(n+1) = \mathbf{C}(n) - \frac{1}{2}\mu\mathbf{g}(n) \tag{3.1.11}$$

where $\mathbf{g}(n)$ is the gradient of the $\mathbf{J}(\mathbf{C}(n))$ and μ is the step size parameter.

$$\mathbf{g}(n) = \nabla\mathbf{J}(\mathbf{C}(n)) = -2\mathbf{p} + 2\mathbf{R}\mathbf{C}(n) \tag{3.1.12}$$

In reality exact measurement of the gradient vector are not possible, because it require prior knowledge of correlation matrix **R** of the tap input vectors and cross correlation vector **p** between the tap inputs and the desired response. Consequently, the gradient vector should be estimated form the available data when we operate in an unknown environment. This method is known as LMS algorithm.

a) LMS algorithm

Simplest way to estimate the **R** and **p** is as follows:

$$\begin{aligned}\widehat{\mathbf{R}}(n) &= \mathbf{s}(n)\mathbf{s}^H(n)\\ \hat{\mathbf{p}}(n) &= \mathbf{s}(n)y^*(n)\end{aligned} \tag{3.1.13}$$

from equation 3.1.8, 3.1.11, 3.1.12 and 3.1.13, we have

$$\mathbf{C}(n+1) = \mathbf{C}(n) + \mu\mathbf{s}(n)e^*(n) \tag{3.1.14}$$

This is known as LMS algorithm. It had been seen that, in non stationary environment error can be minimized by rapidly discounting the past and estimate predominantly on recent data. This can be done by using a forgetting factor as given in the next algorithm known as recursive least algorithm (RLS).

b) RLS algorithm

Let initial value of $\mathbf{C}_n$ is $\mathbf{C}_0$, which is defined as:

$\mathbf{C}_0$ **=zeros(1,L) and**

$$\mathbf{S}_n = \mathbf{s}_n \tag{3.1.15}$$

Autocorrelation for tap input vector with forgetting factor is given as [43]

$$\Phi_n = \lambda\Phi_{n-1} + \frac{\mathbf{S}_n\mathbf{S}_n^H}{\sigma_n^2} \tag{3.1.16}$$

where σ_n^2 is the noise variance.

Correlation matrix is given by

$$\mathbf{Z}_n = \lambda \mathbf{Z}_{n-1} + \frac{\mathbf{S}_n y_n^*}{\sigma_n^2} \tag{3.1.17}$$

where * denotes the conjugate.

Matrix inverse lemma:

$$\begin{aligned} \mathbf{A} &= \mathbf{B}^{-1} + \mathbf{C}\mathbf{D}^{-1}\mathbf{C}^H \\ \mathbf{A}^{-1} &= \mathbf{B} - \mathbf{B}\mathbf{C}(\mathbf{D} + \mathbf{C}^H\mathbf{B}\mathbf{C})^{-1}\mathbf{C}^H\mathbf{B} \end{aligned} \tag{3.1.18}$$

comparing with equation (3.1.16)

$$\begin{aligned} \mathbf{A} &= \Phi_n \\ \mathbf{B}^{-1} &= \lambda\Phi_{n-1} \\ \mathbf{C} &= \mathbf{S}_n \\ \mathbf{D} &= \sigma_n^2 \end{aligned} \tag{3.1.19}$$

inverse of Φ_n can be written as

$$\Phi_n^{-1} = \lambda^{-1}\Phi_{n-1}^{-1} - \frac{\lambda^{-1}\Phi_{n-1}^{-1}\mathbf{S}_n\mathbf{S}_n^H\lambda^{-1}\Phi_{n-1}^{-1}}{\sigma_n^2\mathbf{I} + \mathbf{S}_n^H\lambda^{-1}\Phi_{n-1}^{-1}\mathbf{S}_n} \tag{3.1.20}$$

I is the identity matrix.

let

$$\frac{\lambda^{-1}\Phi_{n-1}^{-1}\mathbf{S}_n}{\sigma_n^2\mathbf{I} + \mathbf{S}_n^H\lambda^{-1}\Phi_{n-1}^{-1}\mathbf{S}_n} = \mathbf{k}_n \tag{3.1.21}$$

and $\Phi_n^{-1} = \mathbf{P}_n$ (3.1.22)

from above three equation,

$$\mathbf{P}_n = \lambda^{-1}[\mathbf{P}_{n-1} - \mathbf{k}_n\mathbf{S}_n^H\mathbf{P}_{n-1}] \tag{3.1.23}$$

from equation 3.1.21 and 3.1.22,

$$\mathbf{k}_n = \frac{\mathbf{P}_n \mathbf{S}_n}{\sigma_n^2} \tag{3.1.24}$$

according to Wiener-Hopf Equation for optimal solution,

$$\Phi_n \mathbf{C}_n = \mathbf{Z}_n \tag{3.1.25}$$

or $\mathbf{C}_n = \mathbf{P}_n \mathbf{Z}_n$ (3.1.26)

Substitute the value of $\mathbf{Z}_n$ from equation 3.1.17, above equation becomes

$$\mathbf{C}_n = \mathbf{P}_n \lambda \mathbf{Z}_{n-1} + \frac{\mathbf{P}_n \mathbf{S}_n y_n^*}{\sigma_n^2} \tag{3.1.27}$$

Substituting the value of $\mathbf{P}_n$ in above equation from equation 3.1.23,

$$\mathbf{C}_n = \mathbf{P}_{n-1}\mathbf{Z}_{n-1} - \mathbf{k}_n \mathbf{S}_n^H \mathbf{P}_{n-1}\mathbf{Z}_{n-1} + \frac{\mathbf{P}_n \mathbf{S}_n y_n^*}{\sigma_n^2} \tag{3.1.28}$$

from equation 3.1.8, 3.1.24 and 3.1.26, above equation becomes,

$$\mathbf{C}_n = \mathbf{C}_{n-1} + \mathbf{k}_n e_n^* \tag{3.1.29}$$

now, we summarize the algorithm:

$$\begin{aligned}
\mathbf{S}_n &= \mathbf{s}_n \\
e_n &= y_n - \mathbf{C}_{n-1}^H \mathbf{S}_n \\
\mathbf{k}_n &= \frac{\mathbf{P}_{n-1}\mathbf{S}_n}{\lambda \sigma_n^2 \mathbf{I} + \mathbf{S}_n^H \mathbf{P}_{n-1} \mathbf{S}_n} \\
\mathbf{C}_n &= \mathbf{C}_{n-1} + \mathbf{k}_n e_n^* \\
\mathbf{P}_n &= \lambda^{-1}[\mathbf{P}_{n-1} - \mathbf{k}_n \mathbf{S}_n^H \mathbf{P}_{n-1}]
\end{aligned} \tag{3.1.30}$$

here $\mathbf{C}_n$ is the estimated channel.

and initialization is done as:

$$\mathbf{C}_0 = zeros(L,1)$$
$$\mathbf{P}_0 = \delta^{-1}\mathbf{I} \tag{3.1.31}$$

where **I** is the identity matrix of dimension L X L.

δ is small positive constant for high SNR and large positive constant for low SNR.

c) LSn algorithm

An algorithm using the first order channel model is called the LSn estimation. Derivation of the Algorithm is same as the RLS algorithm with some modification in $\mathbf{C}_n$, $\mathbf{S}_n$ and estimated channel.

Here $\mathbf{S}_n$ and $\mathbf{C}_n$ are defined as:

$$\mathbf{S}_n = [\mathbf{s}_n^H \quad n\mathbf{s}_n^H] \tag{3.1.32}$$

and $\mathbf{C}_{n-1} = [\mathbf{h}_{0,n-1}^H \quad \mathbf{h}_{1,\text{n-1}}^{\text{H}}]^{\text{H}}$ (3.1.33)

The algorithm is as follows:

$$\mathbf{S}_n = [\mathbf{s}_n^H \quad n\mathbf{s}_n^H]$$
$$e_n = y_n - \mathbf{C}_{n-1}^H\mathbf{S}_n$$
$$\mathbf{k}_n = \frac{\mathbf{P}_{n-1}\mathbf{S}_n}{\lambda\sigma_n^2\mathbf{I} + \mathbf{S}_n^H\mathbf{P}_{n-1}\mathbf{S}_n}$$
$$\mathbf{C}_n = \mathbf{C}_{n-1} + \mathbf{k}_n e_n^* \tag{3.1.34}$$
$$\mathbf{h}_n = \mathbf{h}_{0,n} + n\mathbf{h}_{1,n} + n\mathbf{h}_{2,n}$$
$$\mathbf{P}_n = \lambda^{-1}[\mathbf{P}_{n-1} - \mathbf{k}_n\mathbf{S}_n^H\mathbf{P}_{n-1}]$$

and initialization is done as:

$$\mathbf{C}_0 = zeros(2L,1)$$
$$\mathbf{P}_0 = \delta^{-1}\mathbf{I} \tag{3.1.35}$$

where **I** is the identity matrix of dimension 2L X 2L.

δ is small positive constant for high SNR and large positive constant for low SNR.

Here we have used the channel model of first order polynomial; this increases the minimum mean square error (MMSE). The first order polynomial model can not give the accurate model of the channel. To overcome this problem, variable forgetting factor is used, which plays a compensatory role in the inaccurate description of the channel [40].

d) LSn2 algorithm

This algorithm is similar to the LSn algorithm with following changes:

$$\begin{aligned} \mathbf{S}_n &= [\mathbf{s}_n^H \quad n\mathbf{s}_n^H \quad n^2\mathbf{s}_n^H] \\ \mathbf{C}_{n-1} &= [\mathbf{h}_{0,n-1}^H \quad \mathbf{h}_{1,\text{n-1}}^{\text{H}} \quad \mathbf{h}_{2,\text{n-1}}^{\text{H}}]^{\text{H}} \\ \mathbf{h}_n &= \mathbf{h}_{0,n} + n\mathbf{h}_{1,n} + n\mathbf{h}_{2,n} \end{aligned} \tag{3.1.36}$$

and initialization is done as:

$$\begin{aligned} \mathbf{C}_0 &= zeros(3L,1) \\ \mathbf{P}_0 &= \delta^{-1}\mathbf{I} \end{aligned} \tag{3.1.37}$$

where **I** is the identity matrix of dimension 3L X 3L.

e) RLS with Variable Forgetting Factor

Forgetting factor is adapted as a similar manner to the LMS algorithm, given by

$$\lambda_n = \lambda_{n-1} - \alpha \hat{\nabla}_{\lambda_n} \tag{3.1.38}$$

α is a small, positive learning rate parameter. $\hat{\nabla}_{\lambda_n}$ is the scalar estimate of the cost function ∇_{λ_n} ,where

$$\nabla_{\lambda_n} = \frac{\partial \mathbf{J}_n}{\partial \lambda} \tag{3.1.39}$$

and

$$\mathbf{J}_n = \mathbf{E}[\frac{|e_n|^2}{\sigma_n^2}] = \frac{1}{\sigma_n^2}\mathbf{E}[e_n e_n^*] \tag{3.1.40}$$

$$\nabla_{\lambda_n} = \frac{1}{\sigma_n^2}\mathbf{E}[\frac{\partial e_n}{\partial \lambda}e_n^* + \frac{\partial e_n^*}{\partial \lambda}e_n] \tag{3.1.41}$$

from equation 3.1.8,

$$\frac{\partial e_n}{\partial \lambda} = -\frac{\partial \mathbf{C}_{n-1}\mathbf{S}}{\partial \lambda} \tag{3.1.42}$$

$$\text{let } \frac{\partial \mathbf{C}_n}{\partial \lambda} = \mathbf{D}_n \tag{3.1.43}$$

equation 3.1.42 becomes

$$\frac{\partial e_n}{\partial \lambda} = -\mathbf{D}_{n-1}^H\mathbf{S} \tag{3.1.44}$$

$$\text{and } \frac{\partial e_n^*}{\partial \lambda} = -\mathbf{S}_n^H\mathbf{D}_{n-1} \tag{3.1.45}$$

Substituting above two values in equation 3.1.41,

$$\nabla_{\lambda_n} = -\frac{1}{\sigma_n^2}\mathbf{E}[\mathbf{D}_{n-1}^H\mathbf{S}_n e_n^* + \mathbf{S}_n^H\mathbf{D}_{n-1}e_n] \tag{3.1.46}$$

$$\hat{\nabla}_{\lambda_n} = -\frac{1}{\sigma_n^2}\mathrm{Re}[\mathbf{D}_{n-1}^H\mathbf{S}_n e_n^*] \tag{3.1.47}$$

updating equation from equation 3.1.38, will become

$$\lambda_n = \lambda_{n-1} + \frac{\alpha}{\sigma_n^2}\mathrm{Re}[\mathbf{D}_{n-1}^H\mathbf{S}_n e_n^*]_{\lambda^-}^{\lambda^+} \tag{3.1.48}$$

where λ^+ and λ^- are the upper and lower bound of the algorithm. Value of λ should not go beyond these limits.

The algorithm is as follows:

$$
\begin{aligned}
&\mathbf{S}_n = \mathbf{s}_n \\
&e_n = y_n - \mathbf{C}_{n-1}^H \mathbf{S}_n \\
&\mathbf{k}_n = \frac{\mathbf{P}_{n-1}\mathbf{S}_n}{\lambda\sigma_n^2\mathbf{I} + \mathbf{S}_n^H \mathbf{P}_{n-1}\mathbf{S}_n} \\
&\mathbf{C}_n = \mathbf{C}_{n-1} + \mathbf{k}_n e_n^* \\
&\mathbf{P}_n = \lambda^{-1}[\mathbf{P}_{n-1} - \mathbf{k}_n \mathbf{S}_n^H \mathbf{P}_{n-1}] \\
&\mathbf{D}_n = [\mathbf{I} - \mathbf{k}_n \mathbf{S}_n^H]\mathbf{D}_{n-1} + \frac{1}{\sigma_n^2}\mathbf{M}_n \mathbf{S}_n e_n^* \\
&\mathbf{M}_n = \lambda^{-1}[\mathbf{I} - \mathbf{k}_n \mathbf{S}_n^H]\mathbf{M}_{n-1}[\mathbf{I} - \mathbf{S}_n \mathbf{k}_n^H] + \lambda^{-1}\mathbf{P}_n + \sigma_n^2 \lambda^{-1}\mathbf{k}_n \mathbf{k}_n^H \\
&\lambda_n = \lambda_{n-1} + \frac{\alpha}{\sigma_n^2}\mathrm{Re}[\mathbf{D}_{n-1}^H \mathbf{S}_n e_n^*]_{\lambda^-}^{\lambda^+}
\end{aligned}
\tag{3.1.49}
$$

where $\frac{\partial \mathbf{P}_n}{\partial \lambda} = \mathbf{M}_n$

Initialization is as follows:

$$
\begin{aligned}
&\mathbf{C}_0 = zeros(L,1) \\
&\mathbf{P}_0 = \delta^{-1}\mathbf{I} \\
&\mathbf{D}_n = zeros(L,1) \\
&\mathbf{M}_n = zeros(L,L)
\end{aligned}
\tag{3.1.50}
$$

Recursion for updating of $\mathbf{D}_n$ and $\mathbf{M}_n$ is shown in Appendix B.

f) LSn with Variable Forgetting Factor

The derivation of this algorithm is exactly same as the previous algorithm, except the size of $\mathbf{C}_n$ and $\mathbf{S}_n$ will be change as given in LSn algorithm (c). Here adaptation of fogtetting factor is done using LMS, because of its simple mathematical calculation. Other algorithms are not used because these algorithm are mathematically complex, and required more time than LMS algorithm, and it is also possible that these algorithm may not track the forgetting factor in time.

the algorithm is as follows:

$$\begin{aligned}
&\mathbf{S}_n = [\mathbf{s}_n^H \quad n\mathbf{s}_n^H] \\
&e_n = y_n - \mathbf{C}_{n-1}^H \mathbf{S}_n \\
&\mathbf{k}_n = \frac{\mathbf{P}_{n-1}\mathbf{S}_n}{\lambda\sigma_n^2 \mathbf{I} + \mathbf{S}_n^H \mathbf{P}_{n-1}\mathbf{S}_n} \\
&\mathbf{C}_n = \mathbf{C}_{n-1} + \mathbf{k}_n e_n^* \\
&\mathbf{h}_n = \mathbf{h}_{0,n} + n\mathbf{h}_{1,n} \\
&\mathbf{P}_n = \lambda^{-1}[\mathbf{P}_{n-1} - \mathbf{k}_n \mathbf{S}_n^H \mathbf{P}_{n-1}] \\
&\mathbf{D}_n = [\mathbf{I} - \mathbf{k}_n \mathbf{S}_n^H]\mathbf{D}_{n-1} + \frac{1}{\sigma_n^2}\mathbf{M}_n \mathbf{S}_n e_n^* \\
&\mathbf{M}_n = \lambda^{-1}[\mathbf{I} - \mathbf{k}_n \mathbf{S}_n^H]\mathbf{M}_{n-1}[\mathbf{I} - \mathbf{S}_n \mathbf{k}_n^H] + \lambda^{-1}\mathbf{P}_n + \sigma_n^2 \lambda^{-1}\mathbf{k}_n \mathbf{k}_n^H \\
&\lambda_n = \lambda_{n-1} + \frac{\alpha}{\sigma_n^2}\mathrm{Re}[\mathbf{D}_{n-1}^H \mathbf{S}_n e_n^*]_{\lambda^-}^{\lambda^+}
\end{aligned} \qquad (3.1.51)$$

Initialization is as follows:

$$\begin{aligned}
&\mathbf{C}_0 = zeros(2L,1) \\
&\mathbf{P}_0 = \delta^{-1}\mathbf{I} \\
&\mathbf{D}_n = zeros(2L,1) \\
&\mathbf{M}_n = zeros(2L,2L)
\end{aligned} \qquad (3.1.52)$$

g) LSn2 with Variable Forgetting Factor

This algorithm is same as the previous algorithm with following changes

$$\begin{aligned}
&\mathbf{C}_0 = zeros(3L,1) \\
&\mathbf{P}_0 = \delta^{-1}\mathbf{I} \\
&\mathbf{D}_n = zeros(3L,1) \\
&\mathbf{M}_n = zeros(3L,3L)
\end{aligned} \qquad (3.1.53)$$

Along with changes made in equations 3.1.34 and 3.1.35

h) LSn with New Variable Forgetting Factor

Let the extended prediction error is defined by

$$Q_n = \frac{1}{M}\sum_{i=0}^{M} e_{n-i}^2 \tag{3.1.54}$$

And asymptotic memory length is define as [43]

$$N = \frac{1}{(1-\lambda)} \tag{3.1.55}$$

Since the error coming from the noise is a random process, an appropriate averaging (M) of the error in (3.1.54) is helpful to minimize the effect of a spurious large additive noise error. However, M should be should be small as compare with the minimum asymptotic memory length, $N_{\min}$ [60].

A strategy for choosing forgetting factor may now be define by letting

$$\lambda_n = 1 - \frac{1}{N_n} \tag{3.1.56}$$

$$N_n = \frac{\sigma_e^2 N_{\max}}{Q_n} \tag{3.1.57}$$

where σ_e^2 is the expected noise variance based on real knowledge of the process. Since this forgetting factor adaptation scheme does not guarantee that the λ_n does not become negative, a reasonable lower limit $\lambda_{\min}$ is place on the forgetting factor [60].

λ is updated using equation (3.1.56) and (3.1.57) and remaining equation is same as the LSn algorithm (c).

i) LSn2 with New Variable Forgetting Factor

λ is updated by equation (3.1.56) and (3.1.57) and remaining algorithm is same as the LSn2 algorithm described in (d).

CHAPTER 4

SIMULATION RESULTS

Simulation results shown below are verified using Matlab. This chapter is organized as follows.

Section 4.1 shows the channel estimation algorithms' results.
Section 4.2 shows the BER performance of STBC using different channel estimation algorithms.

4.1 Channel Estimation Algorithms' Results

In the following simulation, BPSK data were generated as an input to the channel model at every symbol period. The number of multipath is set to three. The minimum mean square error (MMSE) is given by:

$$\varepsilon = E[|\mathbf{c}(n) - \mathbf{w}(n)|^2] \tag{4.1}$$

For all the channel simulation value of λ^+ and λ^- is set to 0.99 and 0.75, the step size is empirically chosen as α =0.005, λ=0.99 and the number of channels tracked by each algorithm is three. Averaging is done for M=3 in LSn-NVFF and LSn2-VFF for all simulation.

We have used four types of channels for verifying the results of various estimation algorithms. Different channels are as follows:

1. Channel with linear variation
2. Channel with sinusoidal variation
3. Channel with complex variation
4. General model for time-varying channel (AR1 model)

It is widely accepted that received signal to noise ratio (SNR), as side information is beneficial, especially when channel is time-varying. First order Morkov model (AR1) uses only the received SNR of the symbol immediately preceding the current one. It is proved that given the information corresponding to the previous symbol, the amount of uncertainty remaining in the current symbol is negligible, thus first order Morkov model is sufficient to mathematically represent the time-varying channel.

Simulation results of channel estimation algorithms are shown in three sections.

In section 4.1.1, channel tracking performances and corresponding MMSE plots are shown. SNR is 30 dB and fdT product is 0.001, for all simulations of this section. Fig. 4.1.1 shows the channel tracking performance. Fig. 4.1.2 shows the mean square error performance of the algorithm for linear channel. In similar order, graphs for other channels are shown. In this section different algorithms' result are shown for four types of channels. Firstly, results of different algorithms are shown for channel with linear variation. Then, results are shown for channel with sinusoidal variation, channel with complex variation and AR1 channel model, respectively.

In section 4.1.2, MMSE plots of different channel estimation algorithms are shown, in different fdT conditions. Channel used for this simulation is AR1 model of channel. In this section, SNR is 25 dB and different values of fdTs are 0.05, 0.005 and 0.0005.

In section 4.1.3, MMSE plots of different channel estimation algorithms are shown, in different SNR conditions. AR1 model is used for simulation. Here fdT product is constant i.e. 0.001.

4.1.1 Channel Tracking Performance and MMSE

4.1.1.1. Channel with Linear Variation

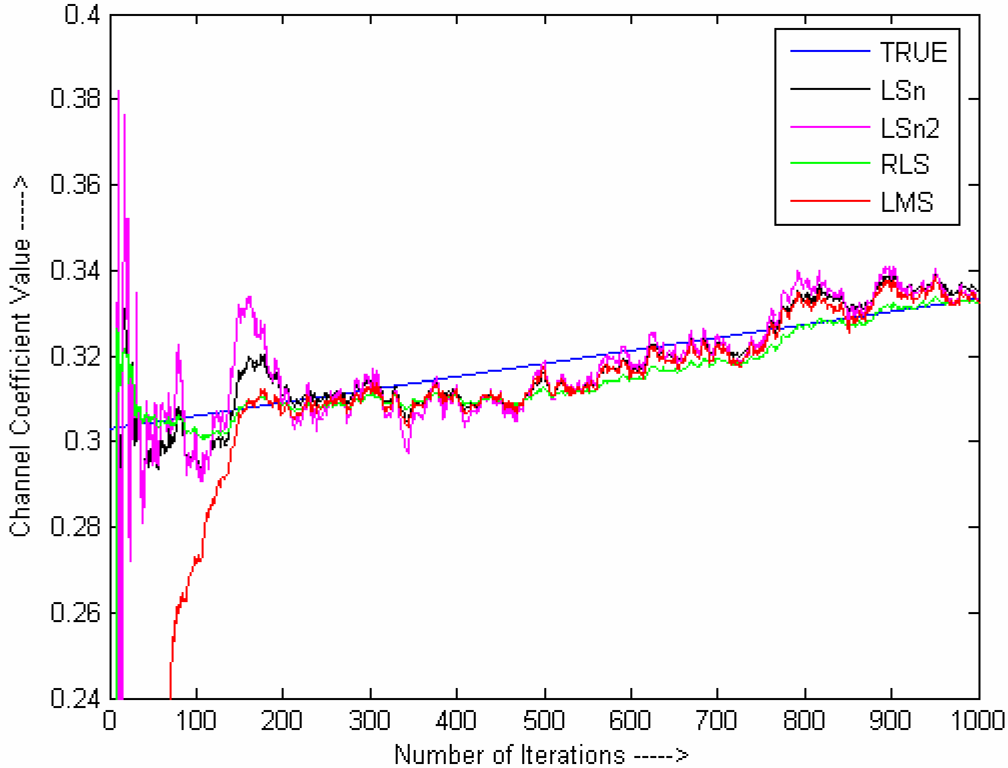


Fig. 4.1.1.1 Channel Tracking Performance

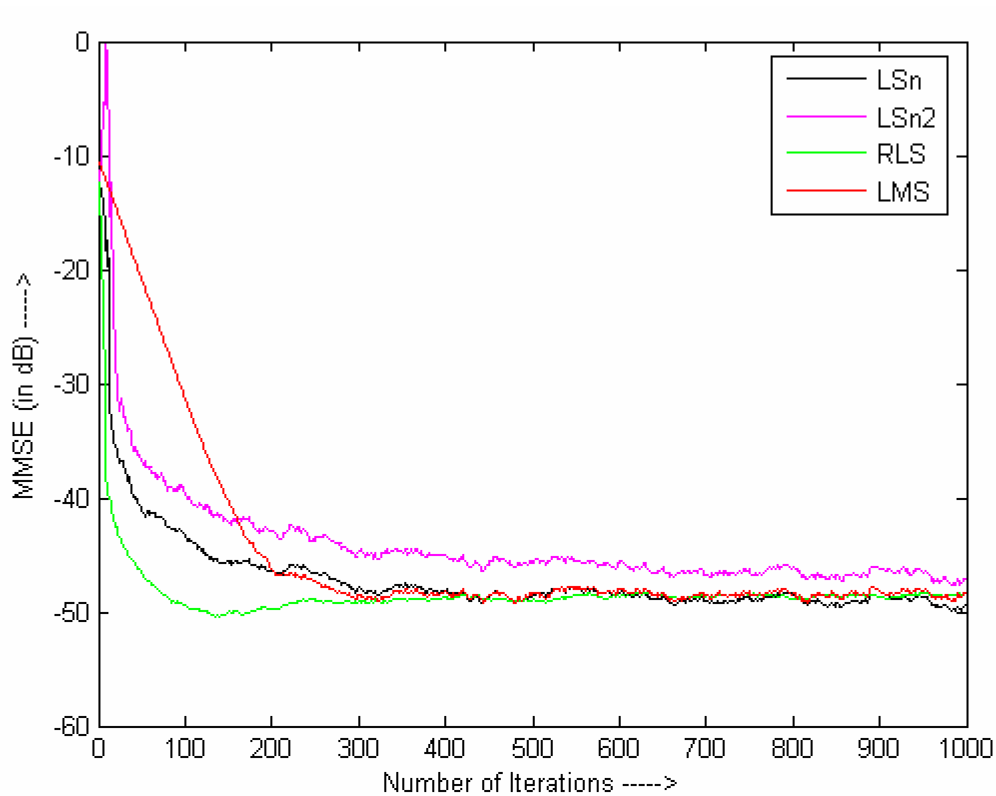


Fig. 4.1.1.2 MMSE Performance

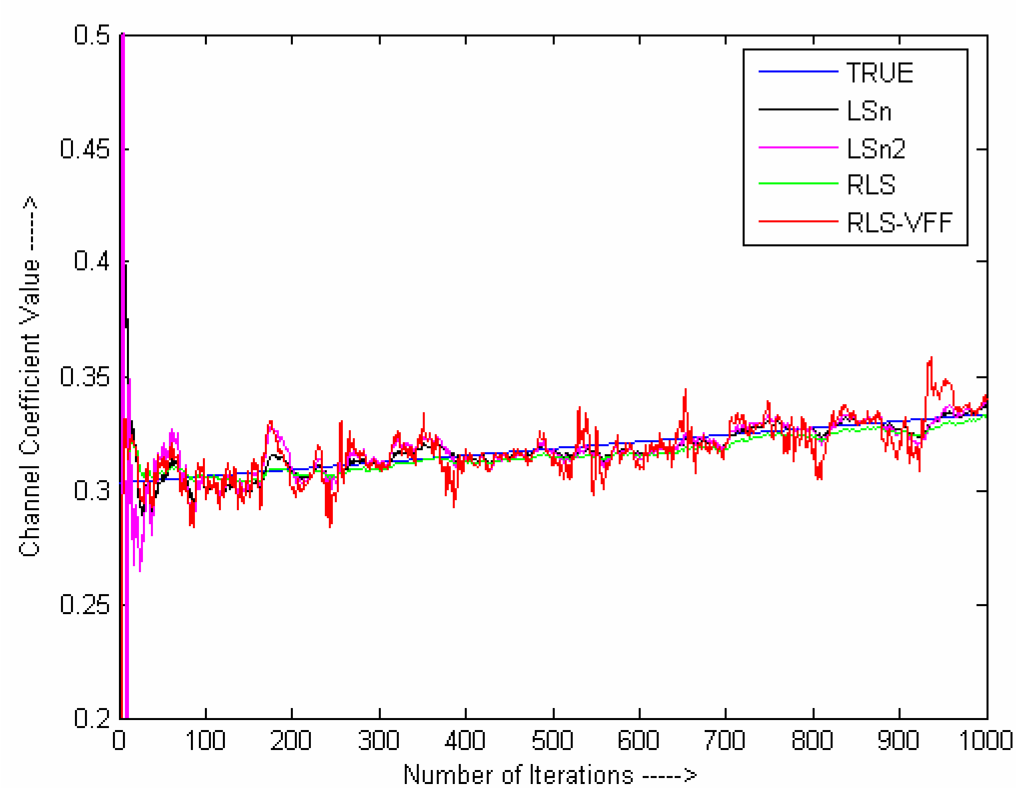


Fig. 4.1.1.3 Channel Tracking Performance

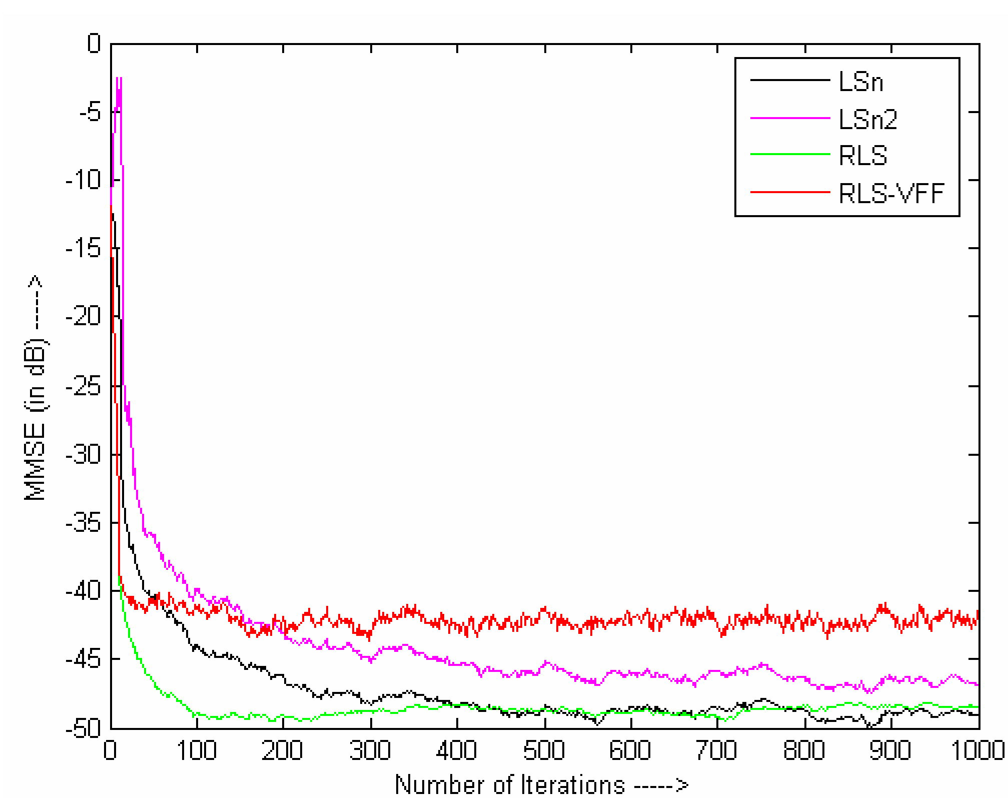


Fig. 4.1.1.4 MMSE Performance

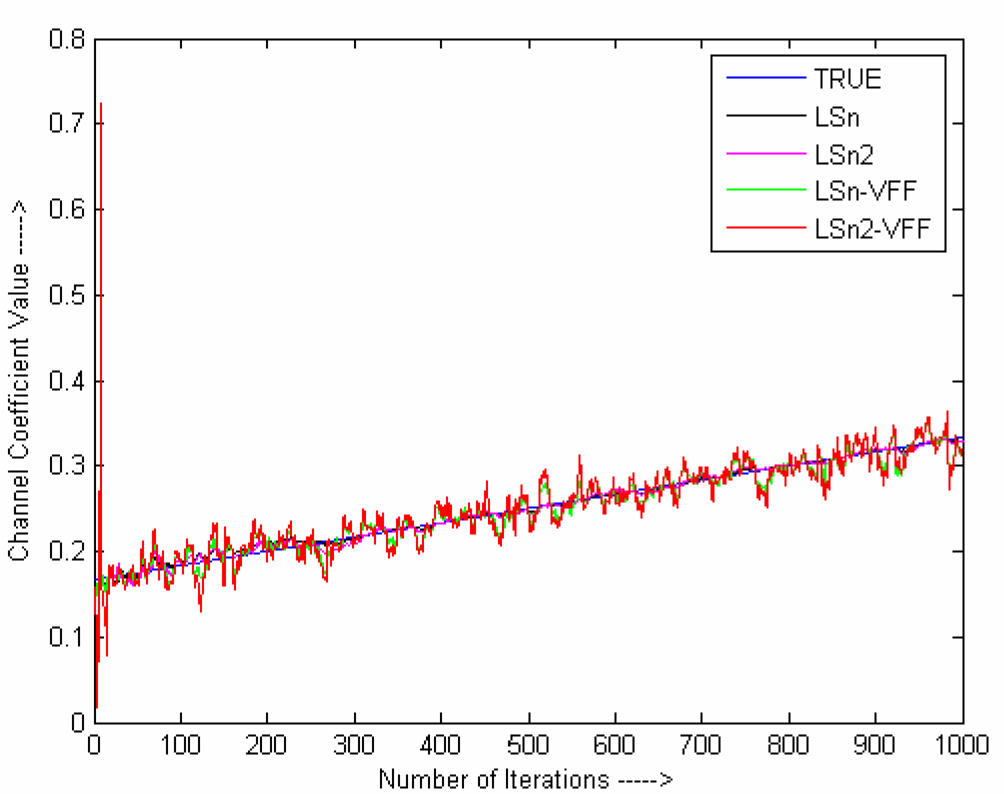


Fig. 4.1.1.5 Channel Tracking Performance

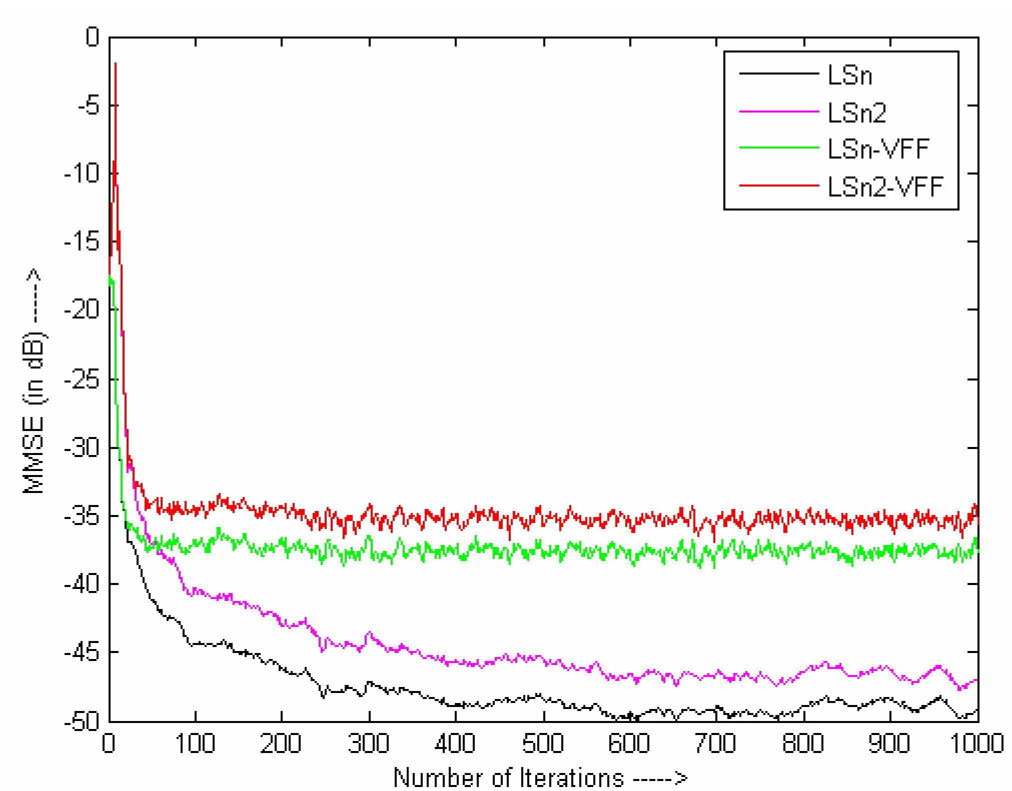


Fig. 4.1.1.6 MMSE Performance

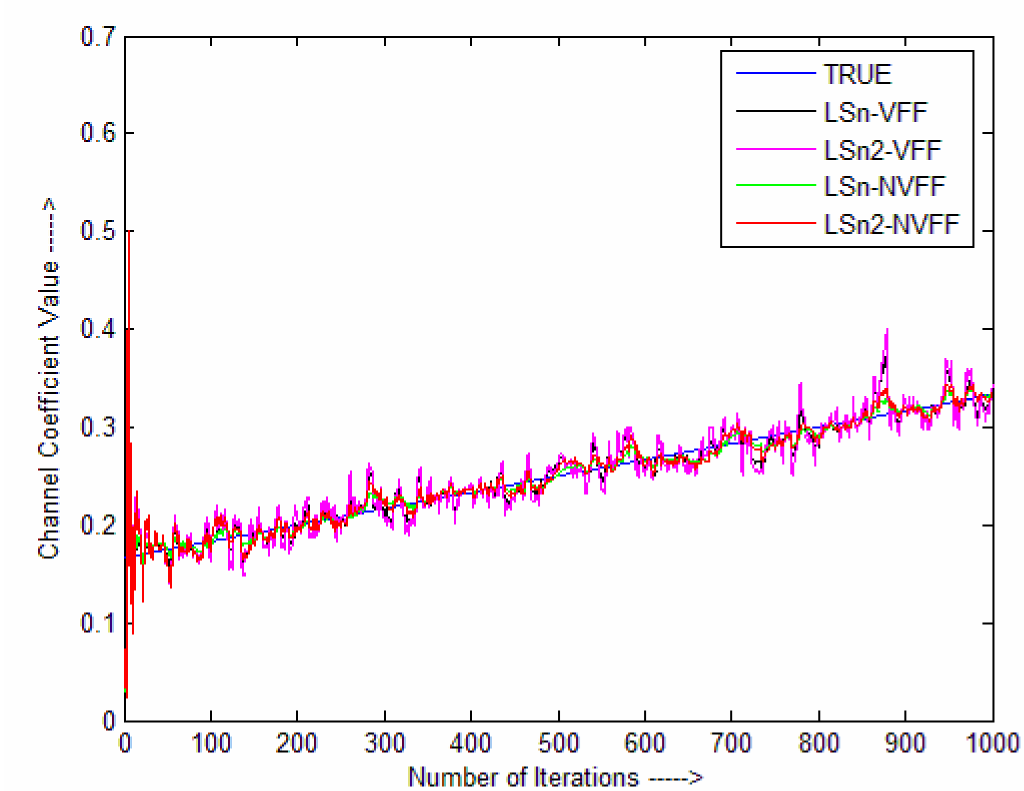


Fig. 4.1.1.7 Channel Tracking Performance

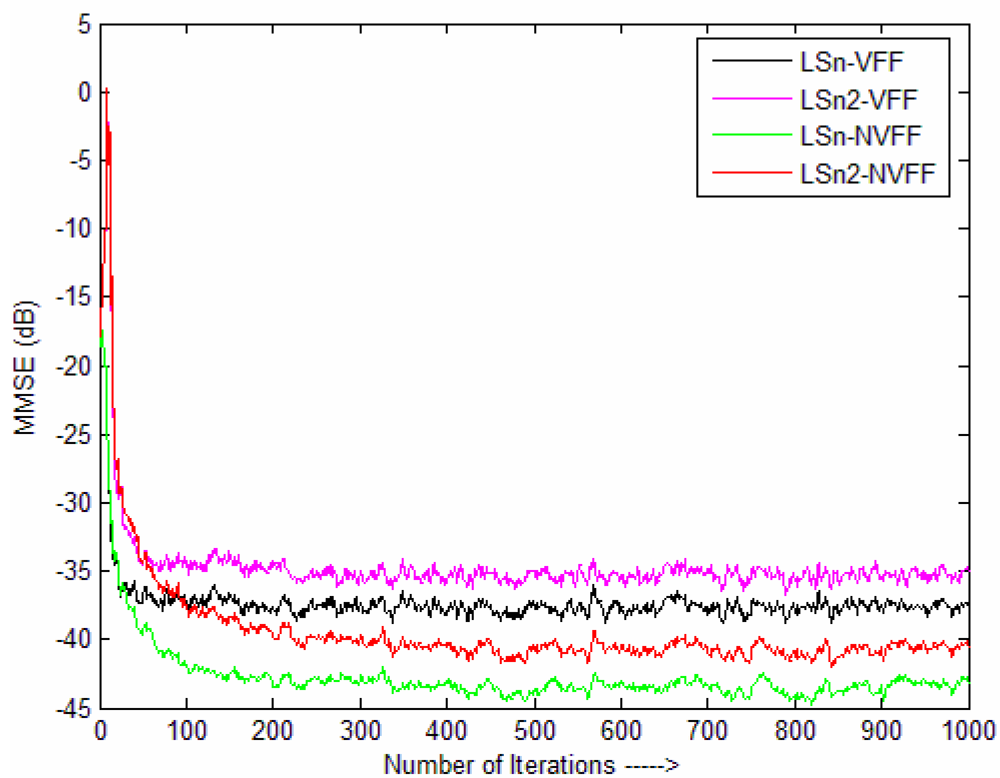


Fig. 4.1.1.8 MMSE Performance

4.1.1.2. Channel with Sinusoidal Variation

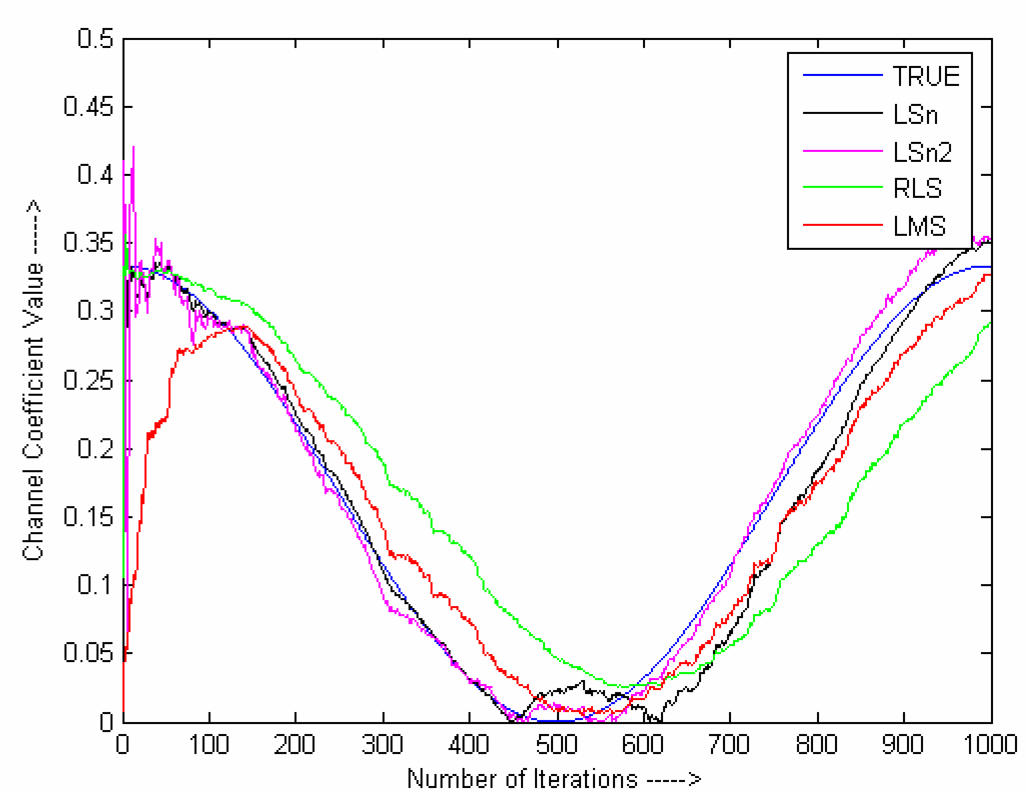


Fig. 4.1.1.9 Channel Tracking Performance

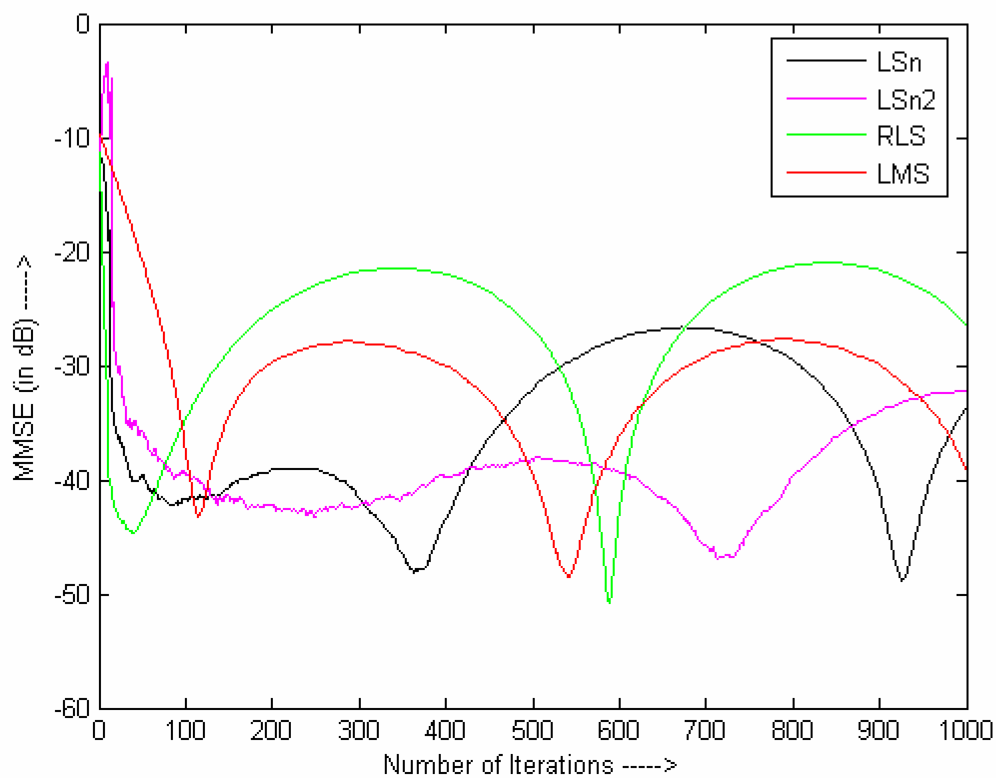


Fig. 4.1.1.10 MMSE Performance

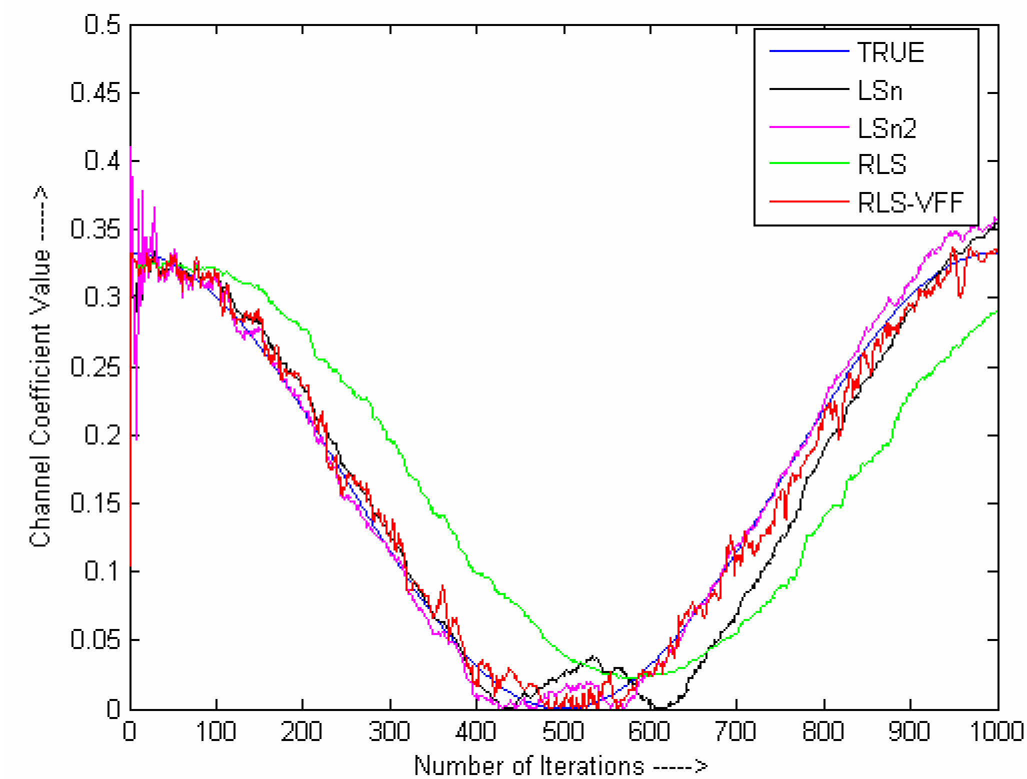


Fig. 4.1.1.11 Channel Tracking Performance

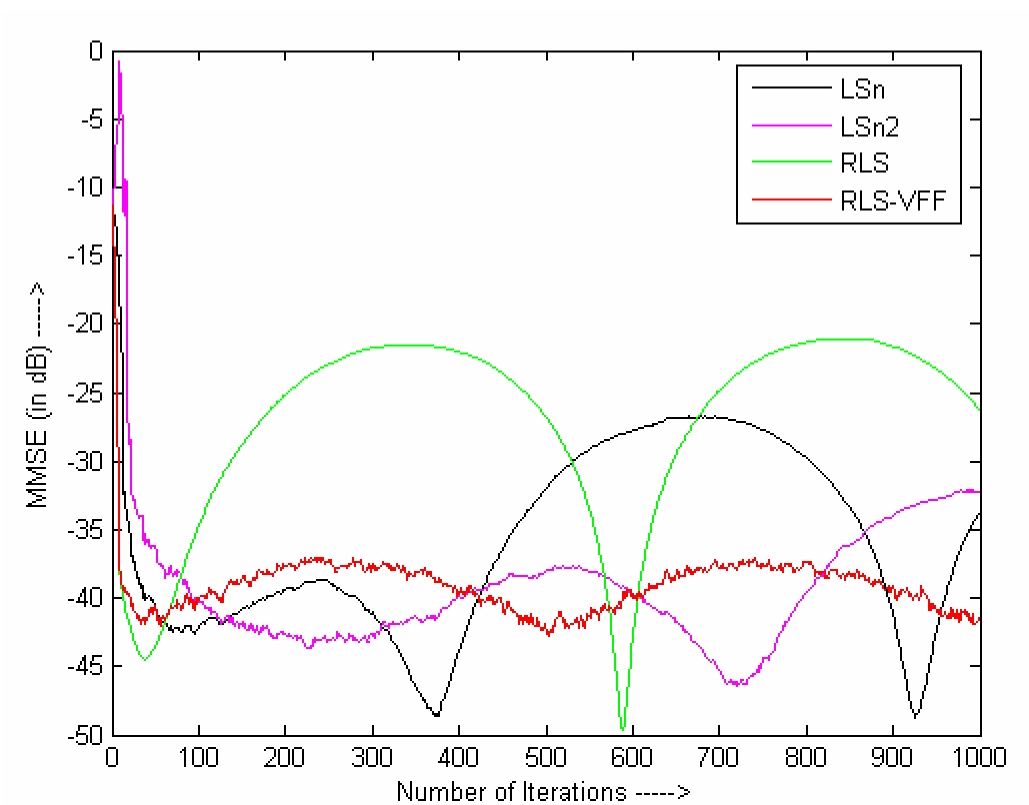


Fig. 4.1.1.12 MMSE Performance

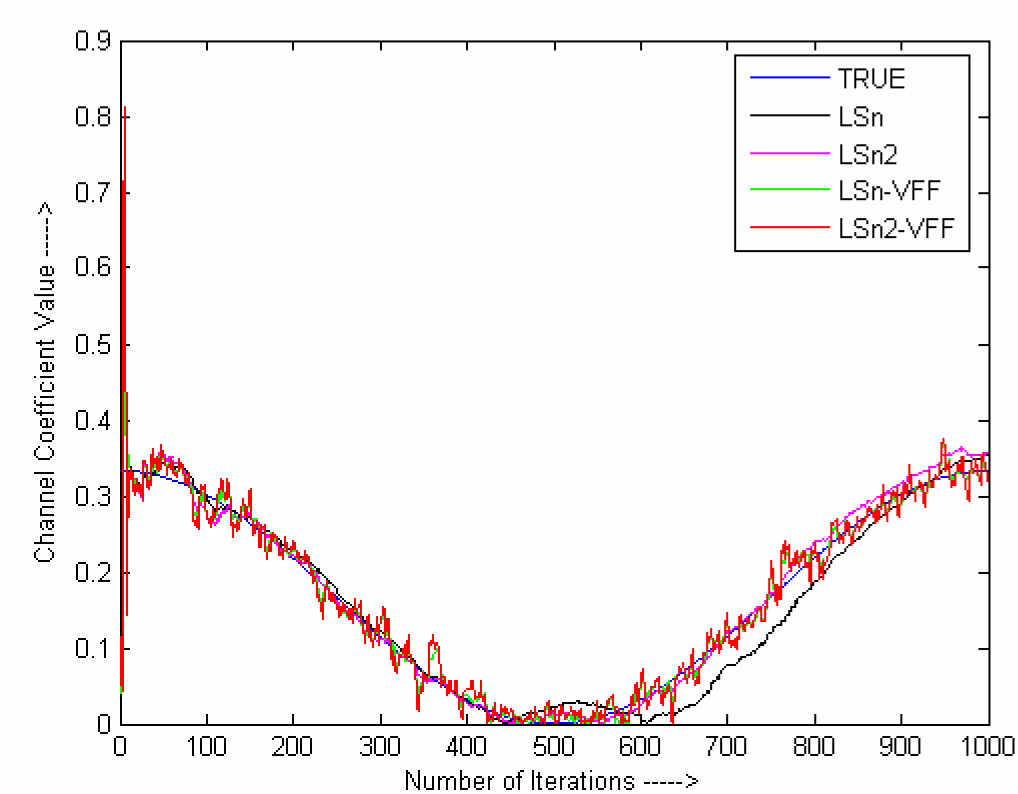


Fig. 4.1.1.13 Channel Tracking Performance

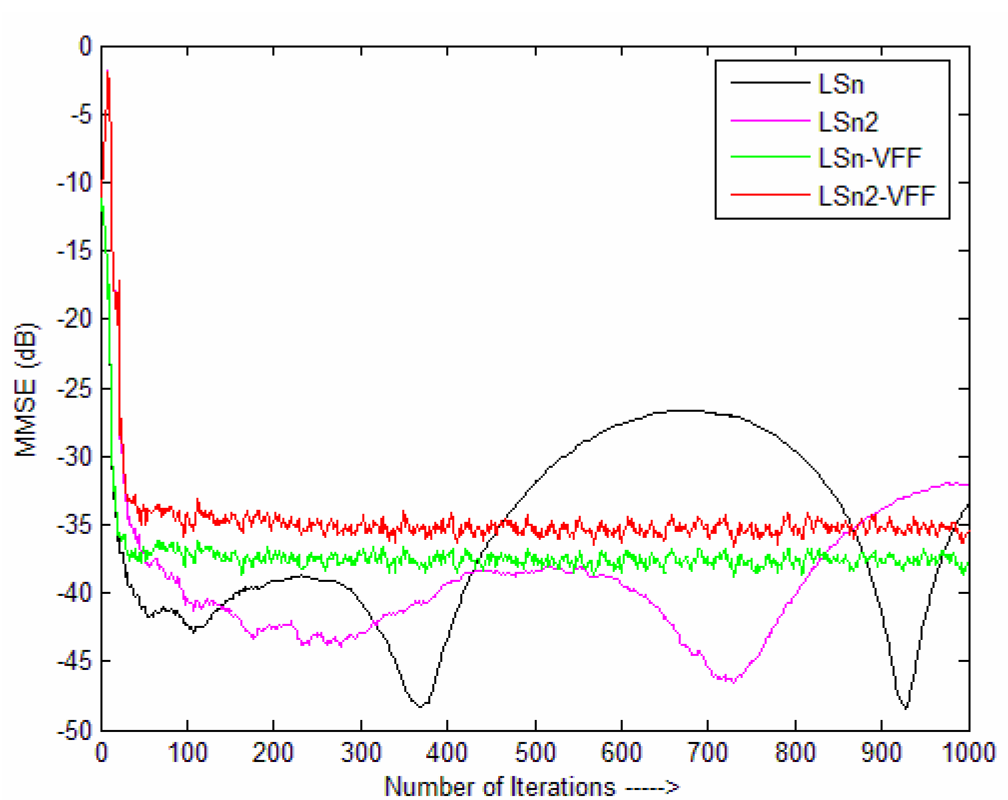


Fig. 4.1.1.14 MMSE Performance

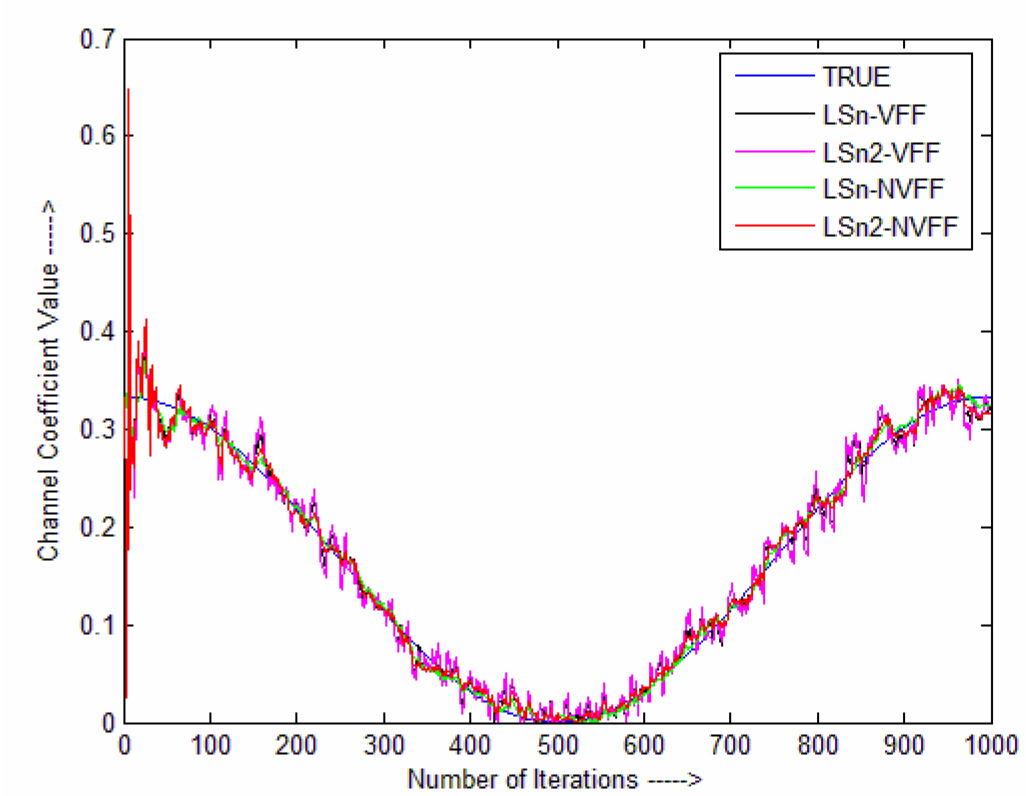


Fig. 4.1.1.15 Channel Tracking Performance

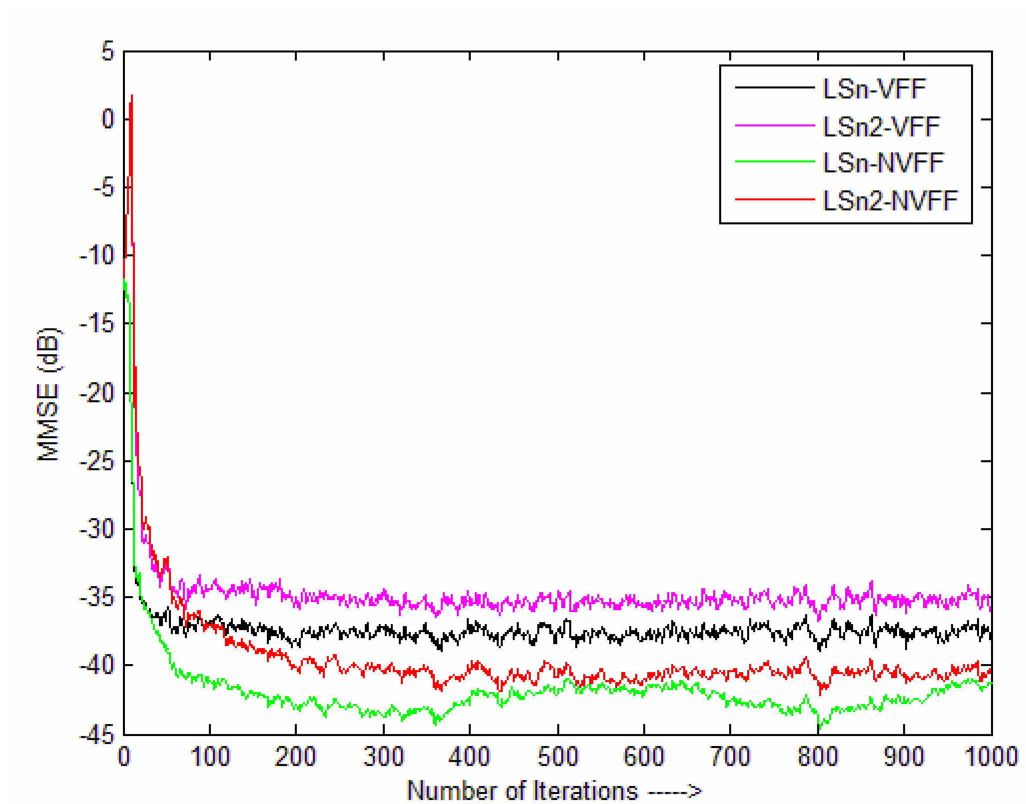


Fig. 4.1.1.16 MMSE Performance

4.1.1.3. Channel with Complex Variation

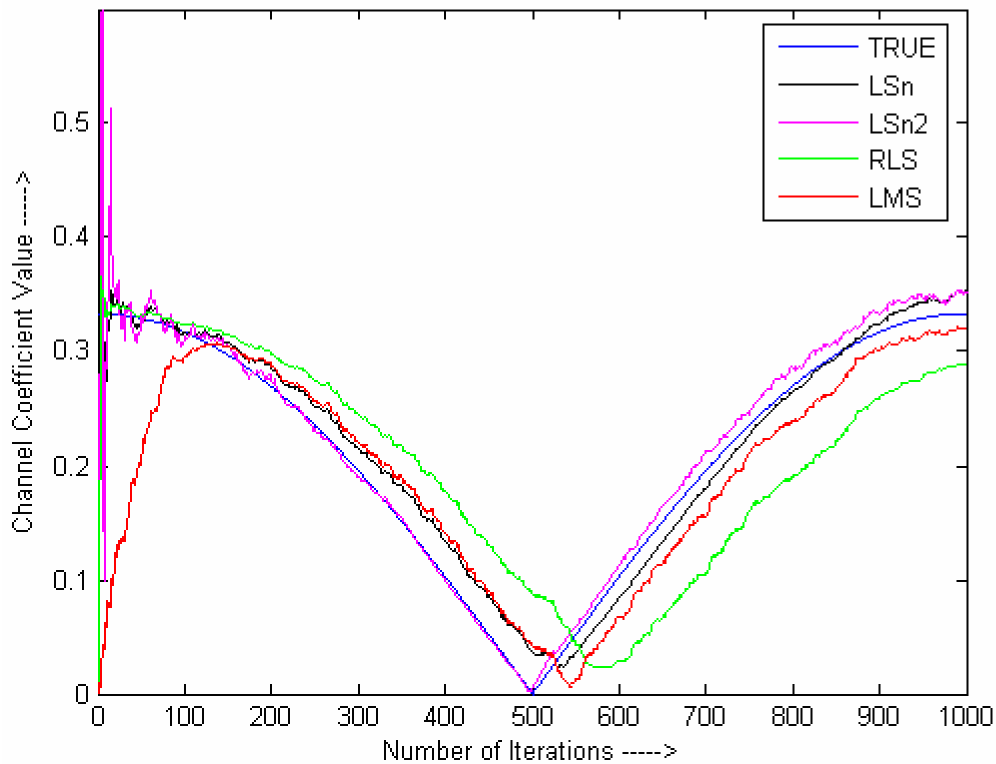


Fig. 4.1.1.17 Channel Tracking Performance

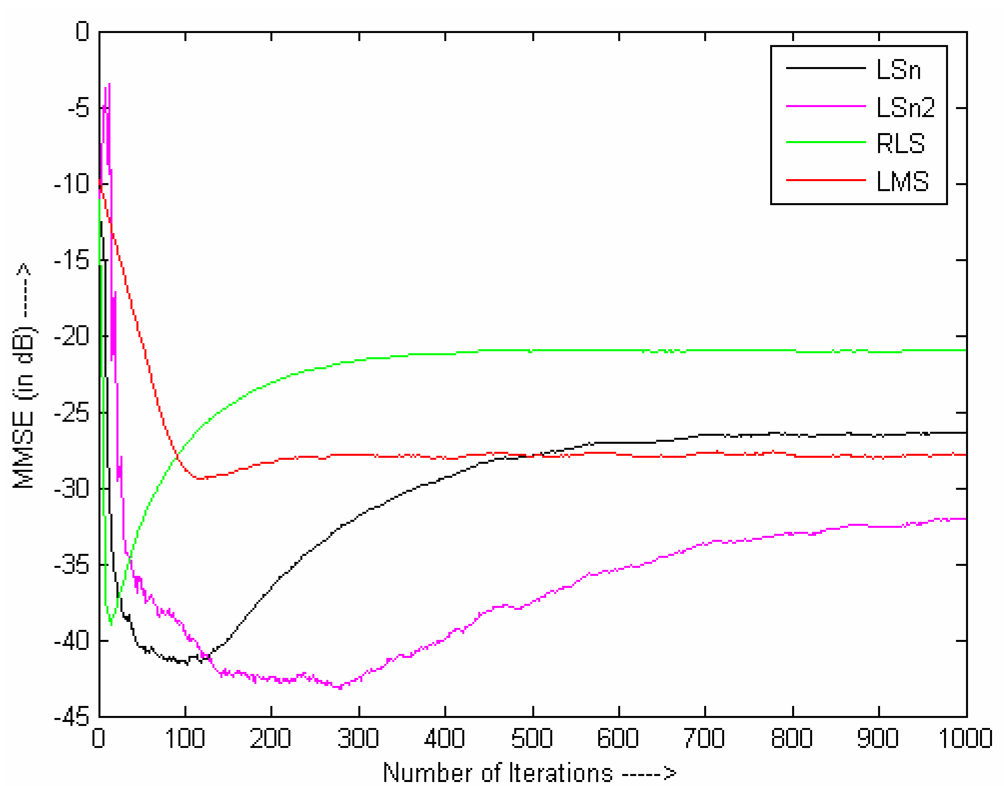


Fig. 4.1.1.18 MMSE Performance

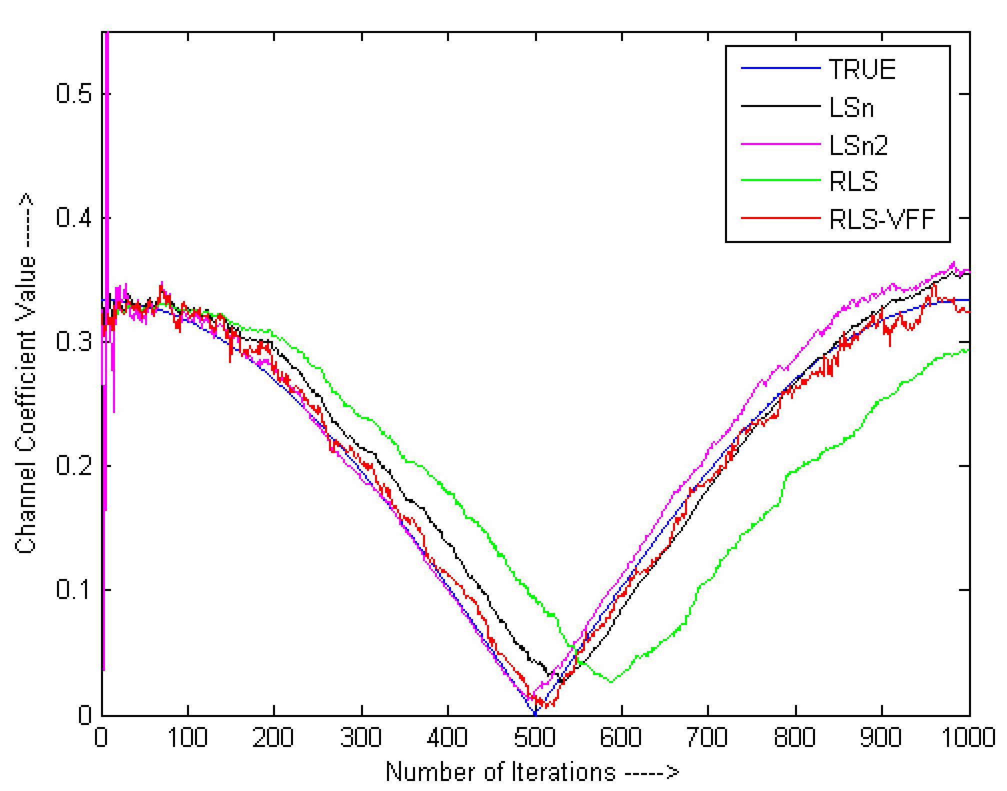


Fig. 4.1.1.19 Channel Tracking Performance

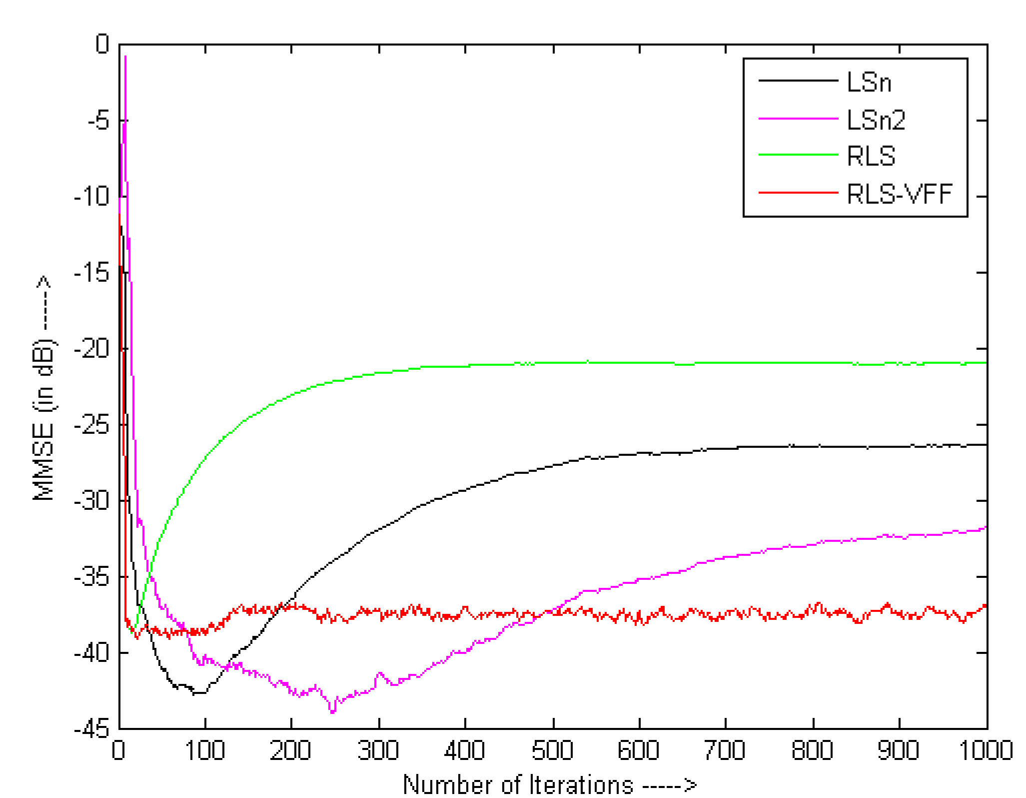


Fig. 4.1.1.20 MMSE Performance

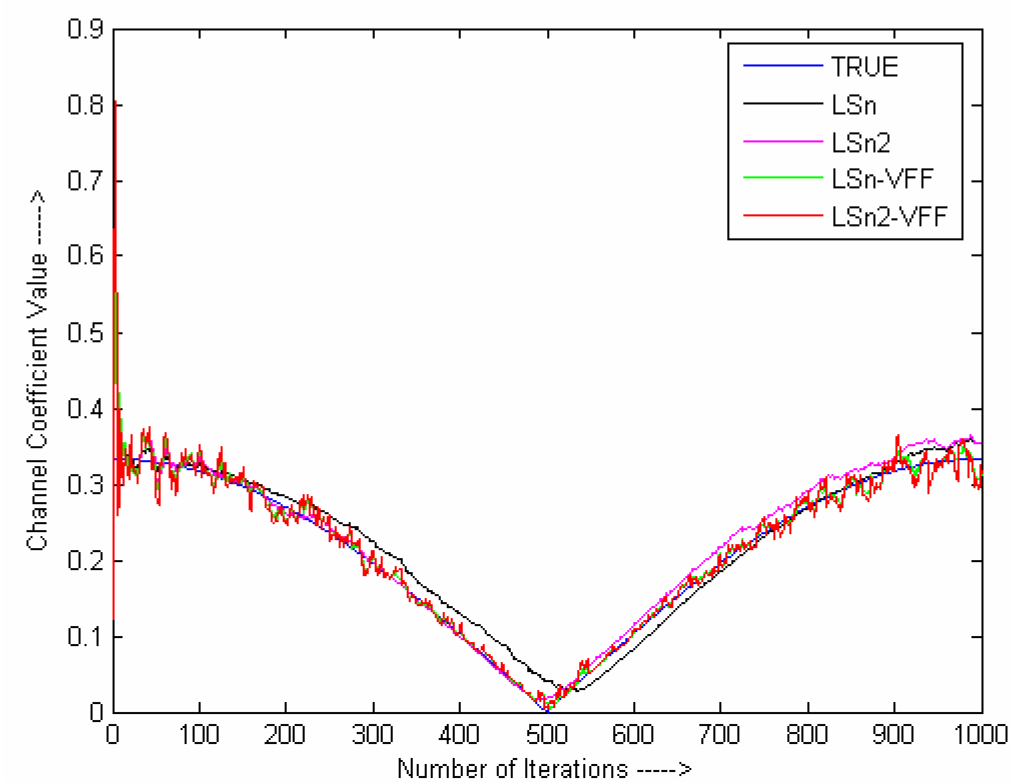


Fig. 4.1.1.21 Channel Tracking Performance

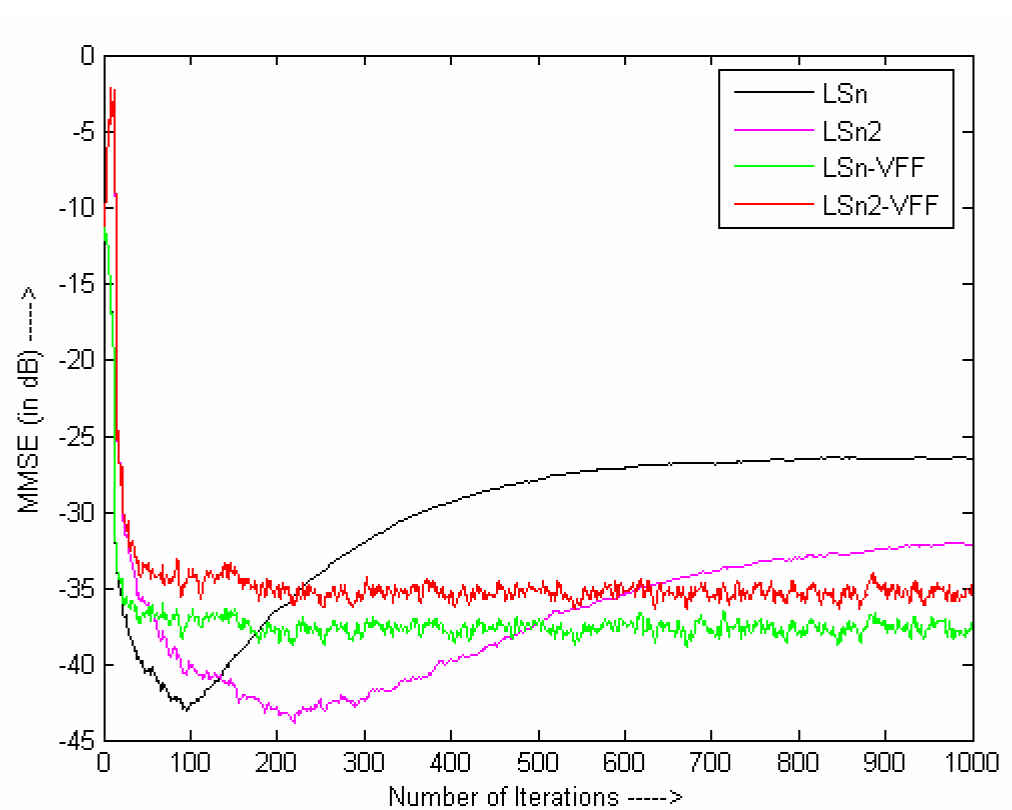


Fig. 4.1.1.22 MMSE Performance

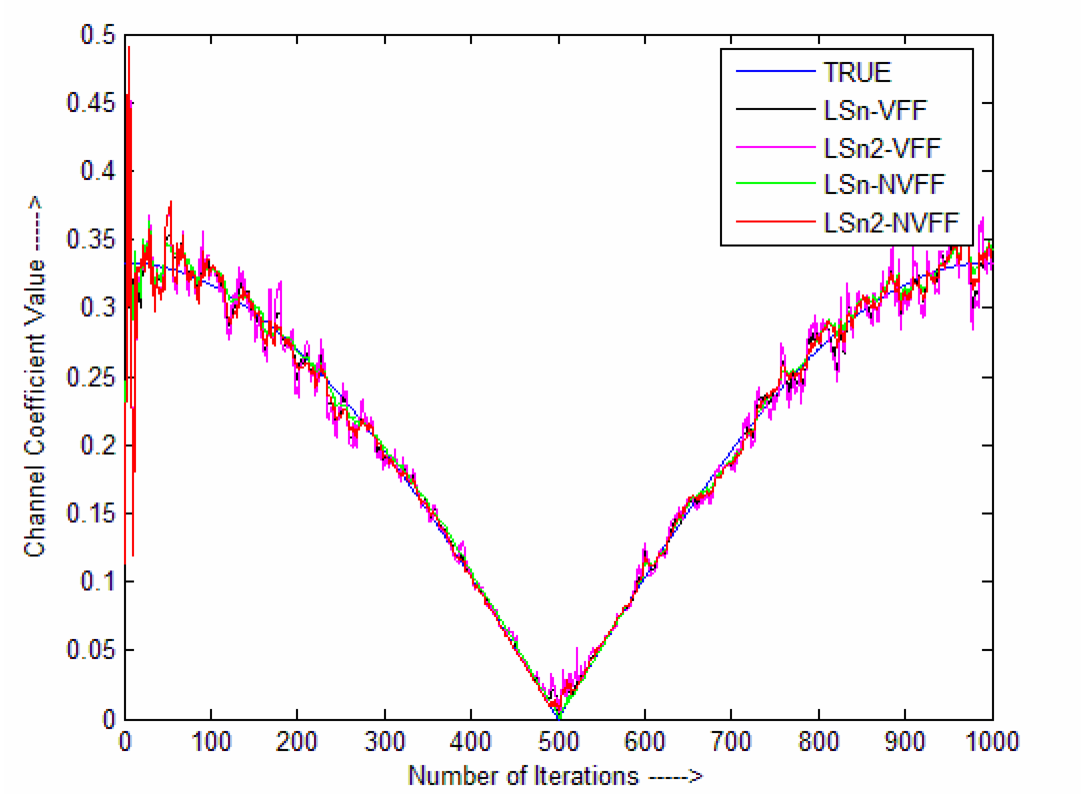


Fig. 4.1.1.23 Channel Tracking Performance

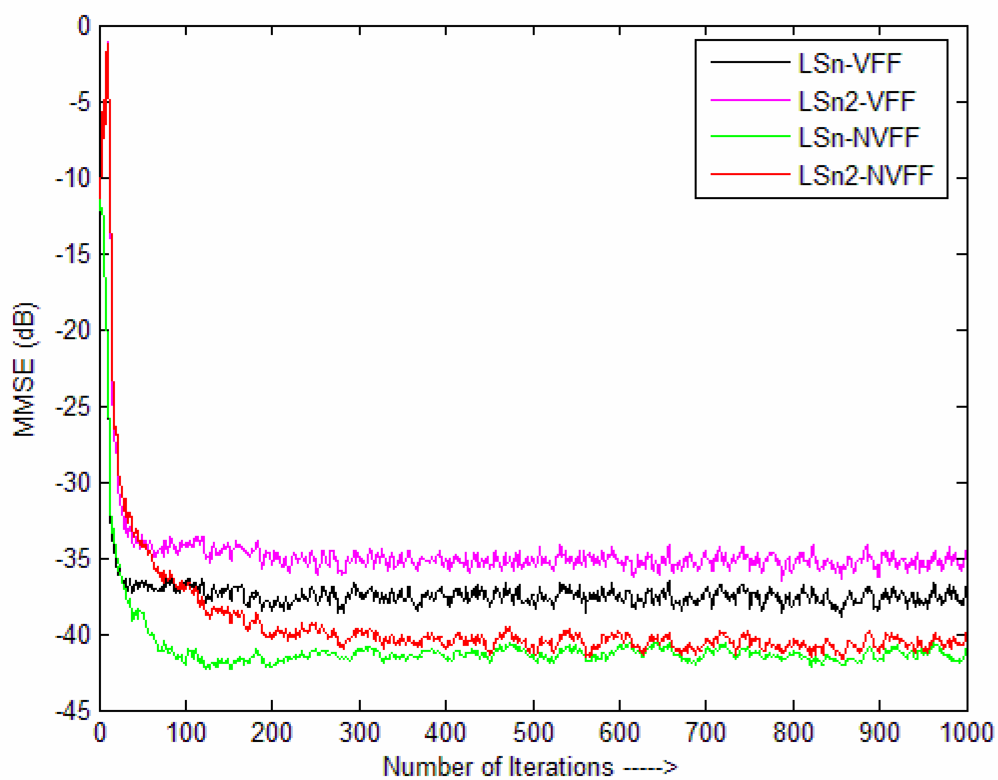


Fig. 4.1.1.24 MMSE Performance

4.1.1.4 General Model for Time-varying Channel (AR1 Model)

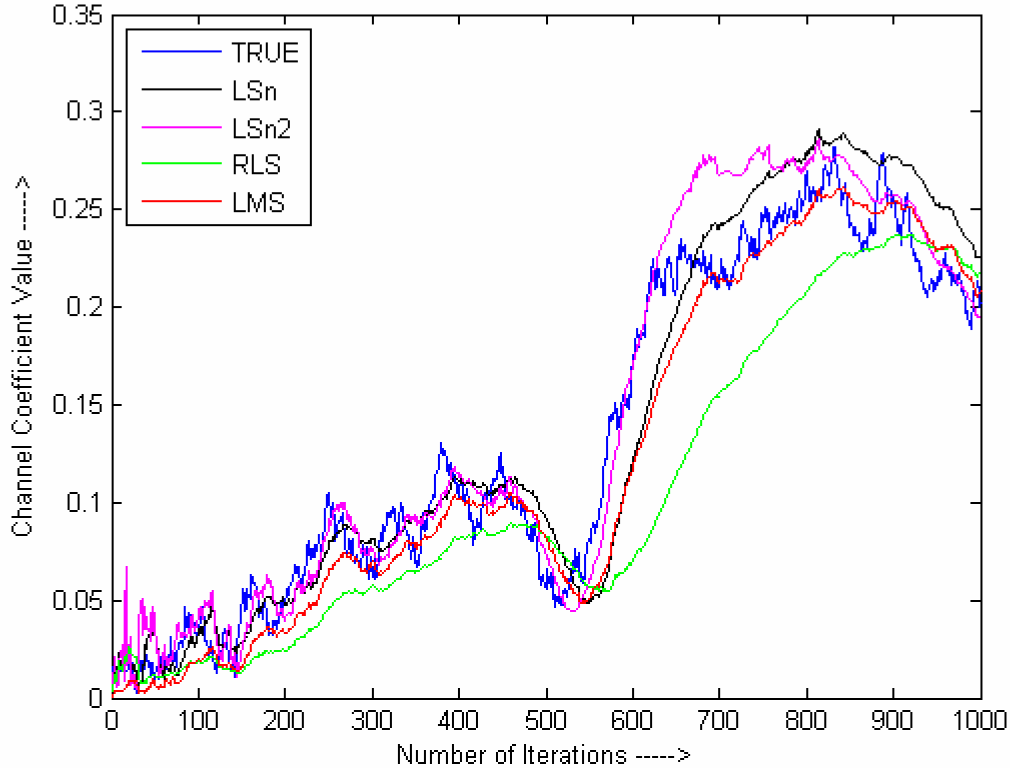


Fig. 4.1.1.25 Channel Tracking Performance

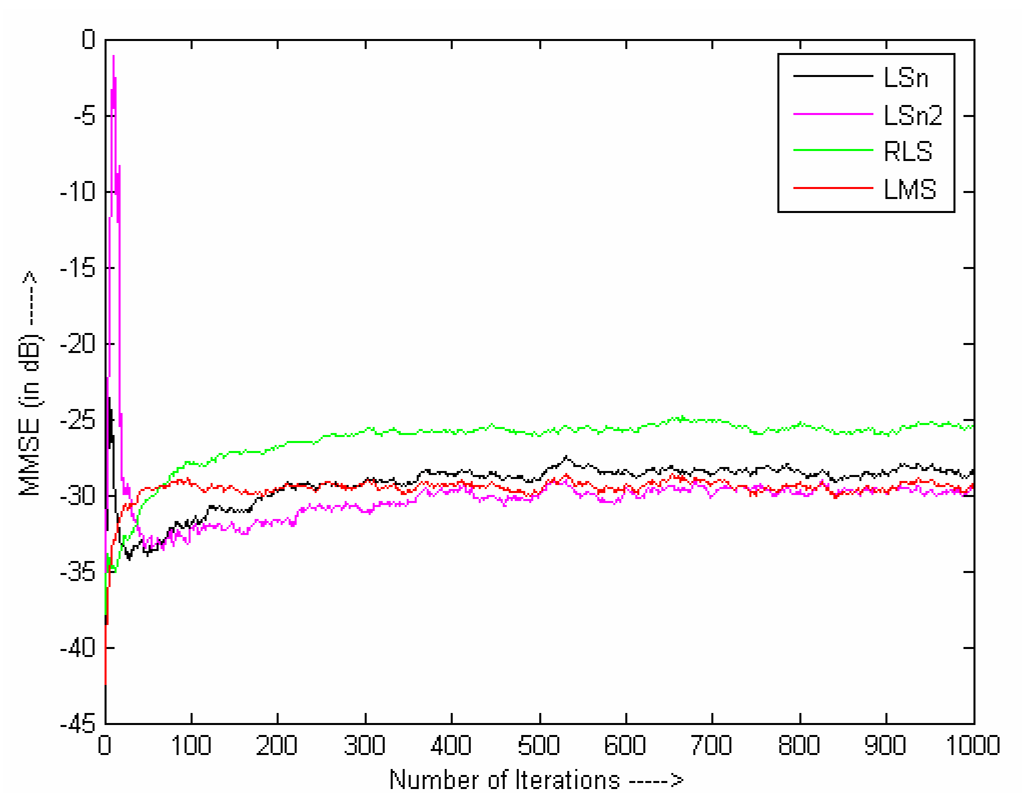


Fig. 4.1.1.26 MMSE Performance

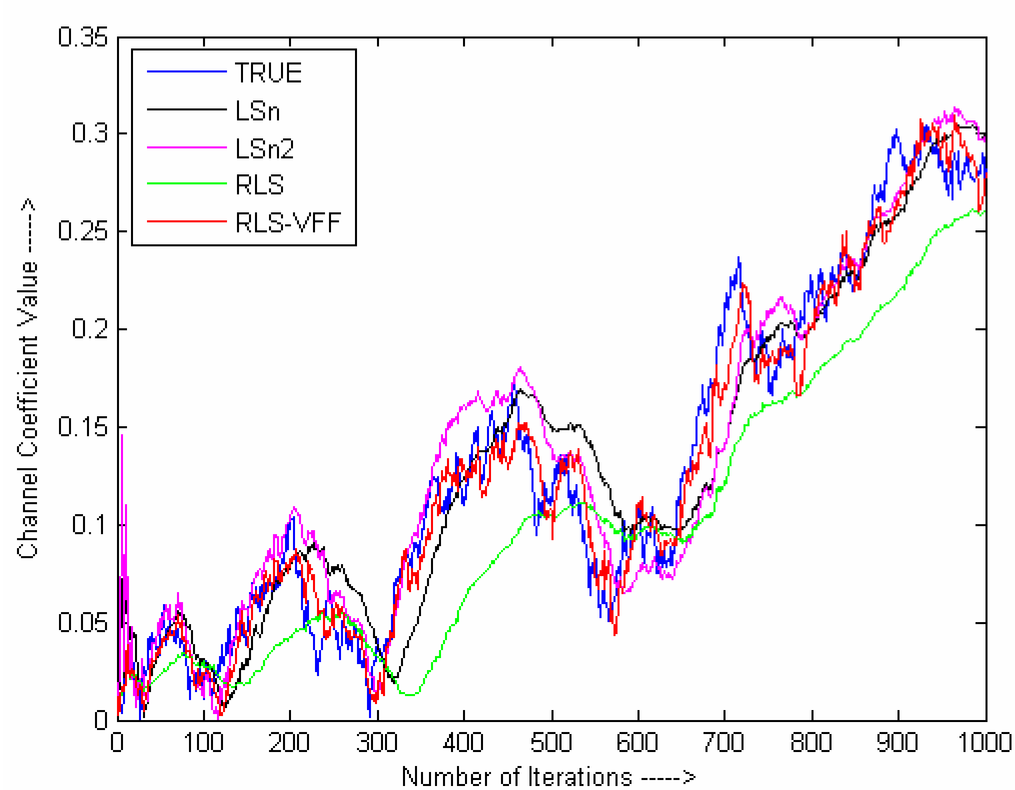


Fig. 4.1.1.27 Channel Tracking Performance

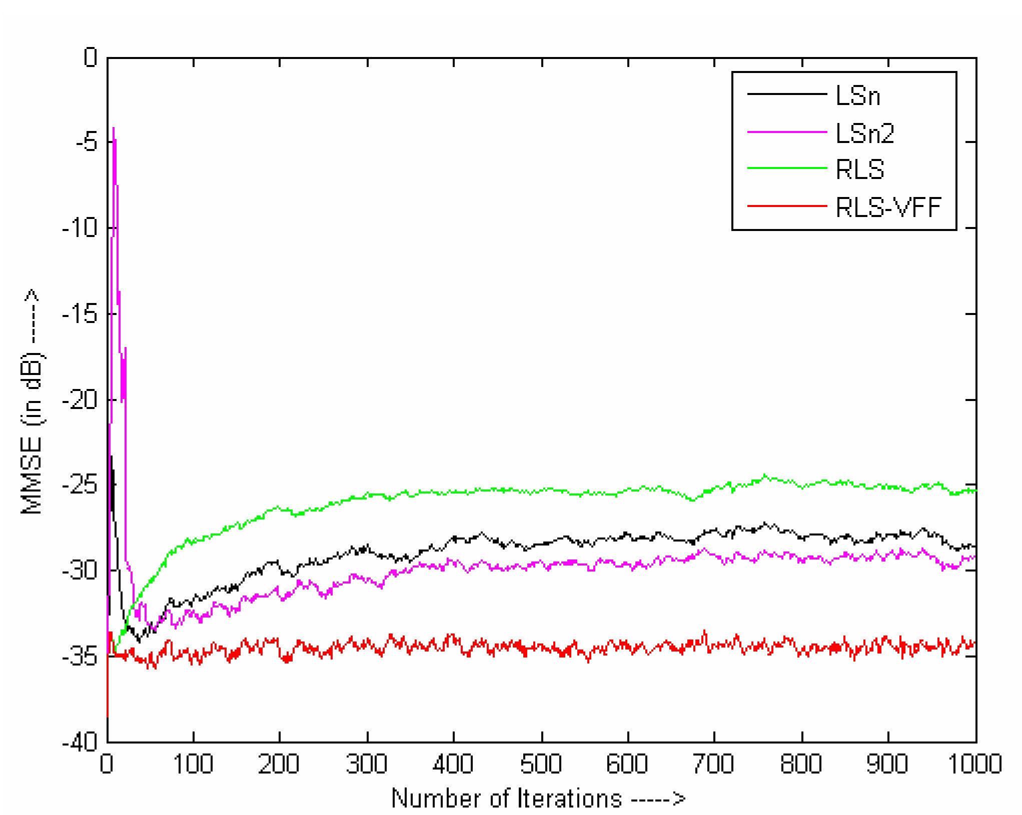


Fig. 4.1.1.28 MMSE Performance

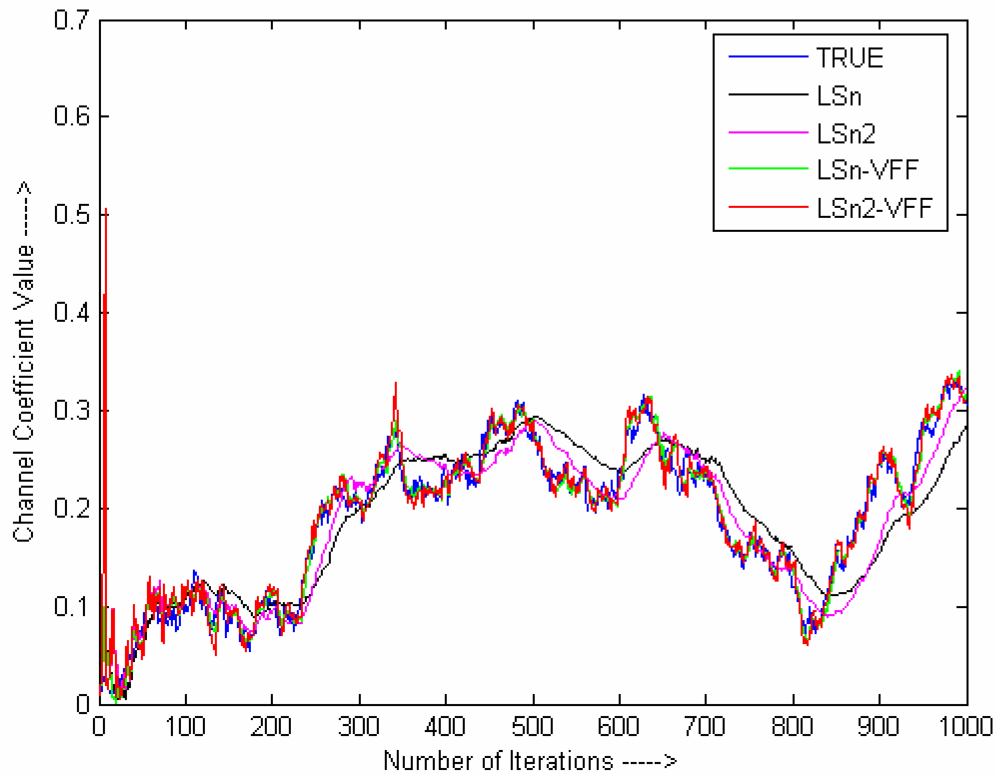


Fig. 4.1.1.29 Channel Tracking Performance

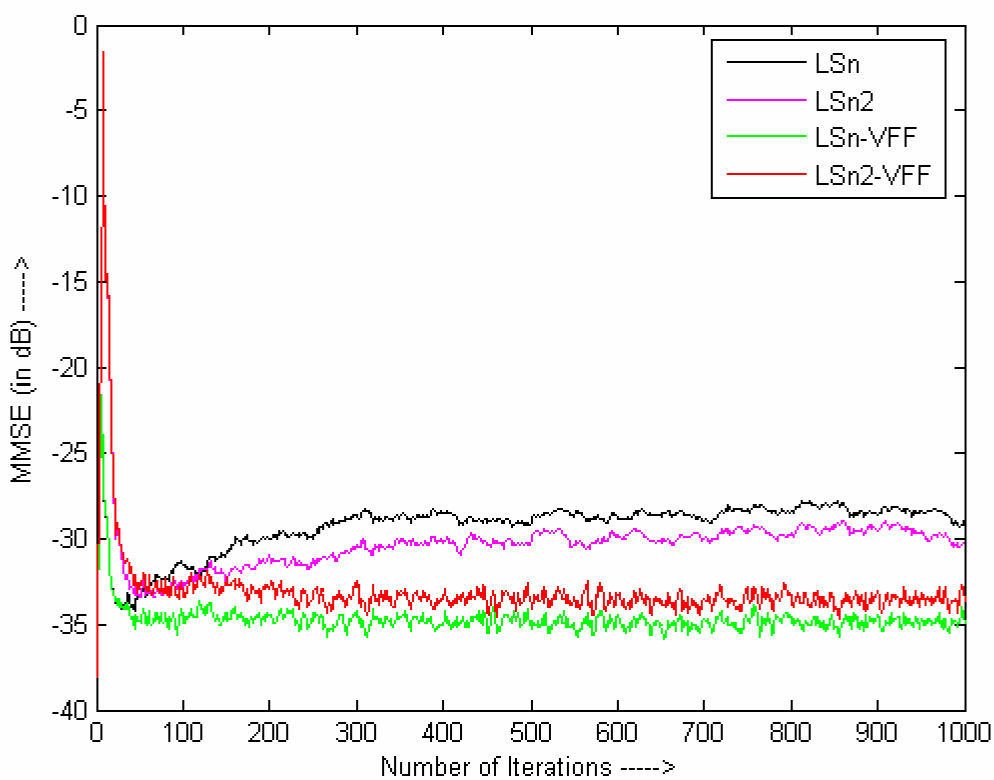


Fig. 4.1.1.30 MMSE Performance

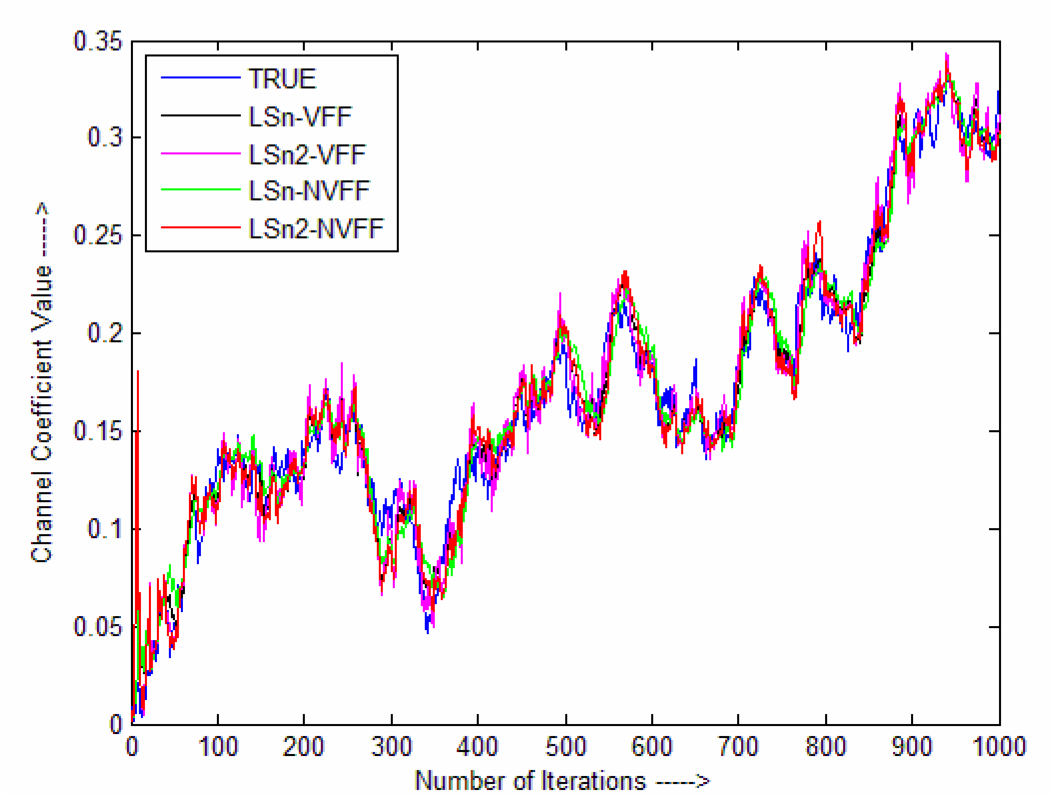


Fig. 4.1.1.31 Channel Tracking Performance

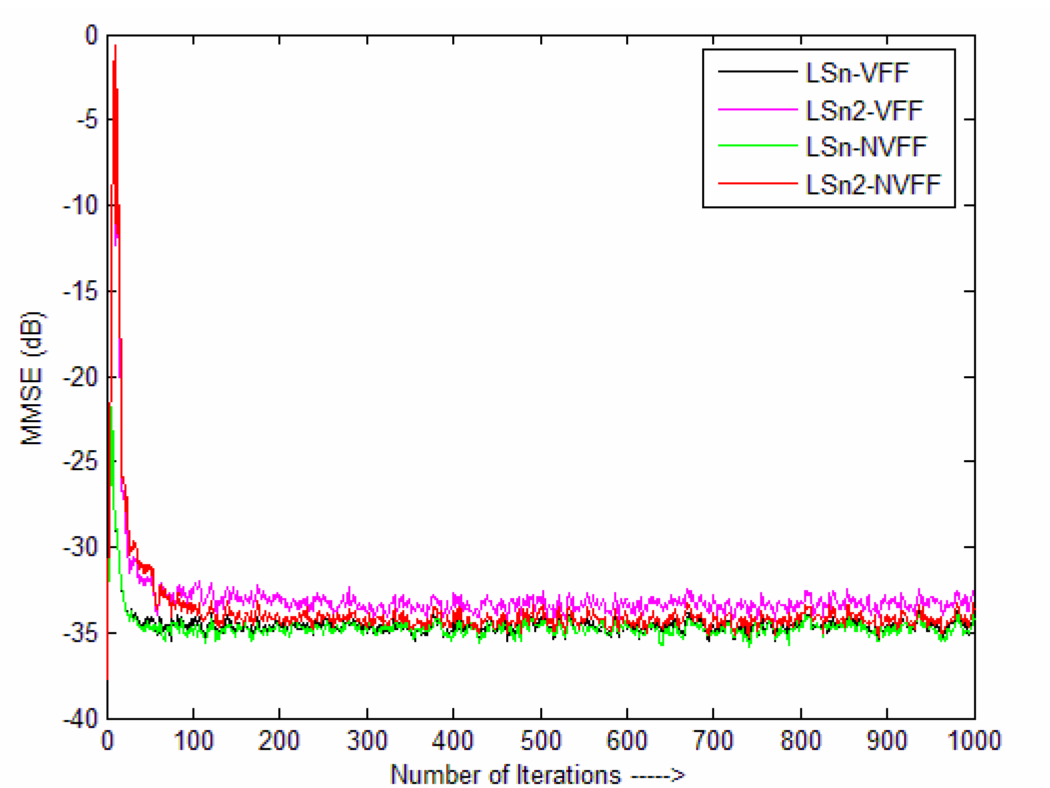


Fig. 4.1.1.32 MMSE Performance

4.1.2 MMSE Plots for Different fdT Conditions

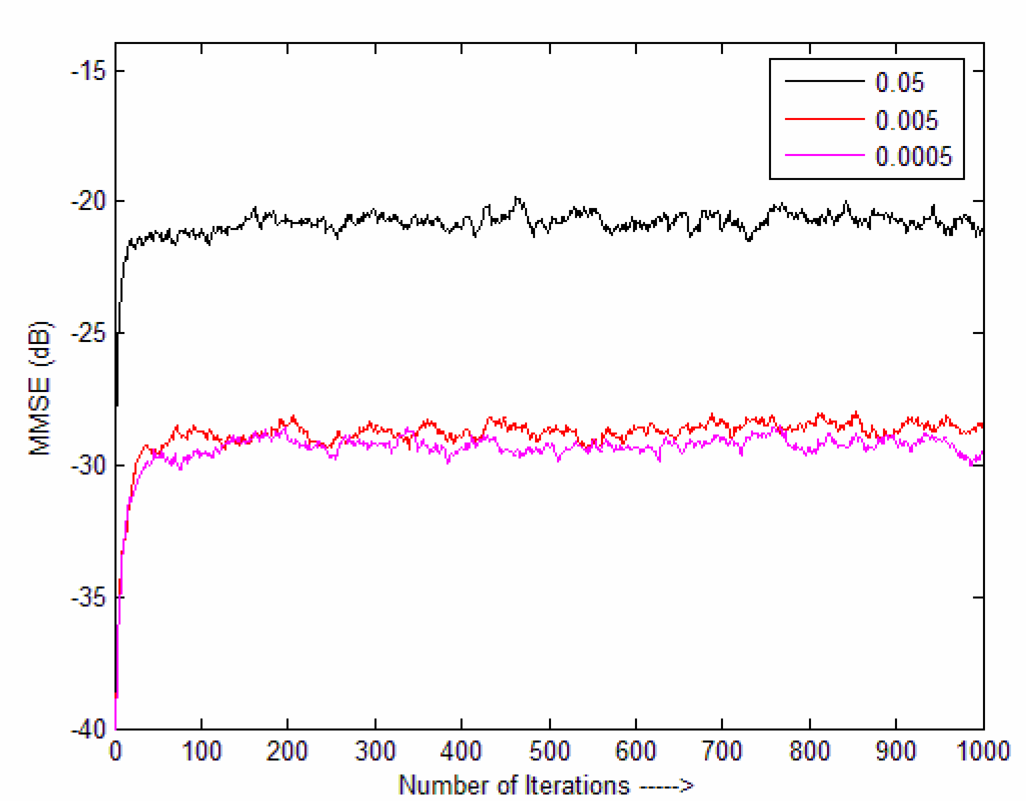


Fig. 4.1.2.1 LMS

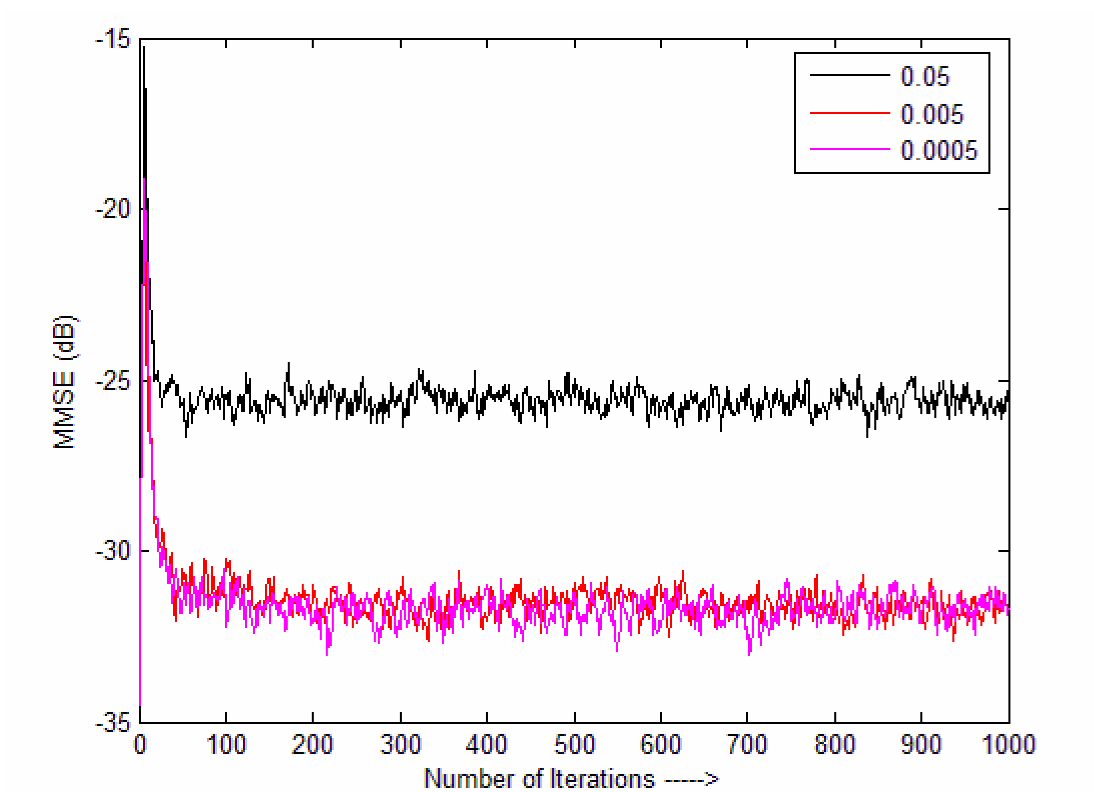


Fig. 4.1.2.2 LSn-VFF

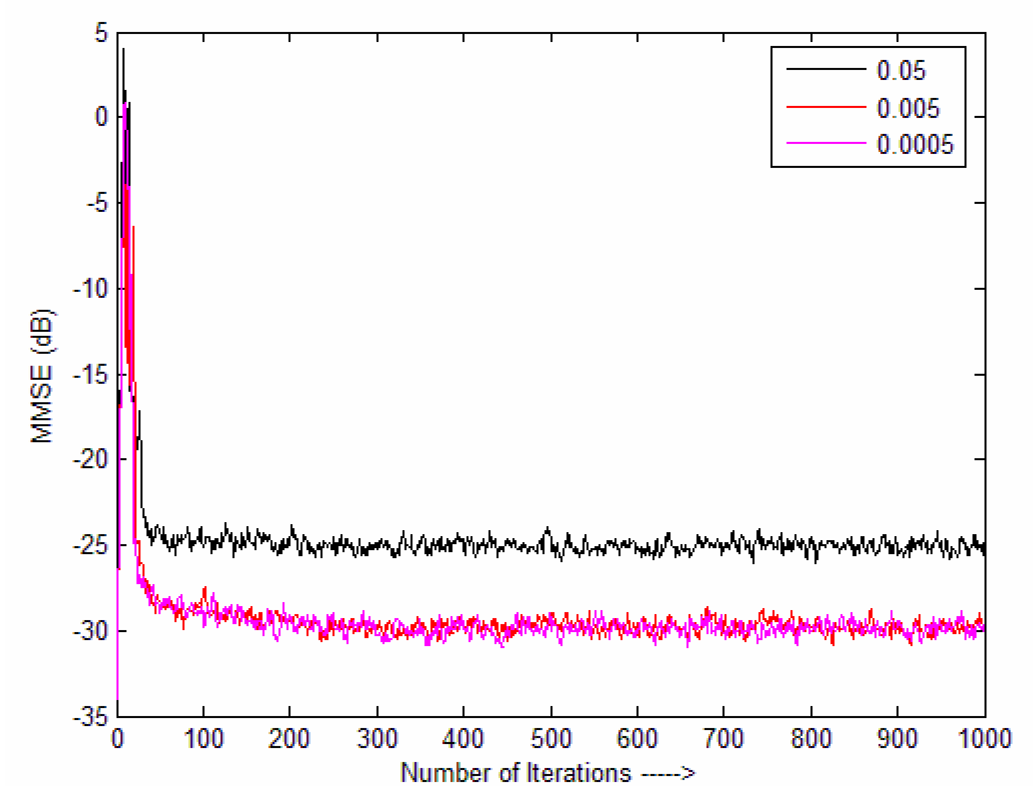


Fig. 4.1.2.3 LSn2-VFF

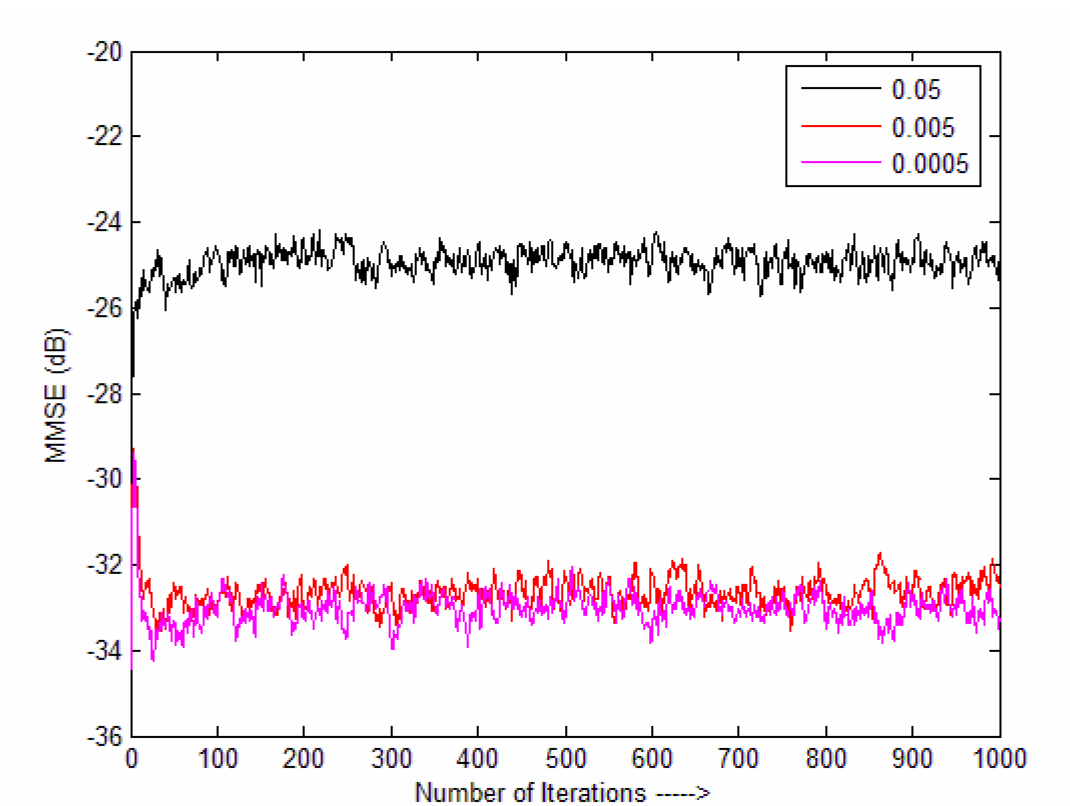


Fig. 4.1.2.4 RLS-VFF

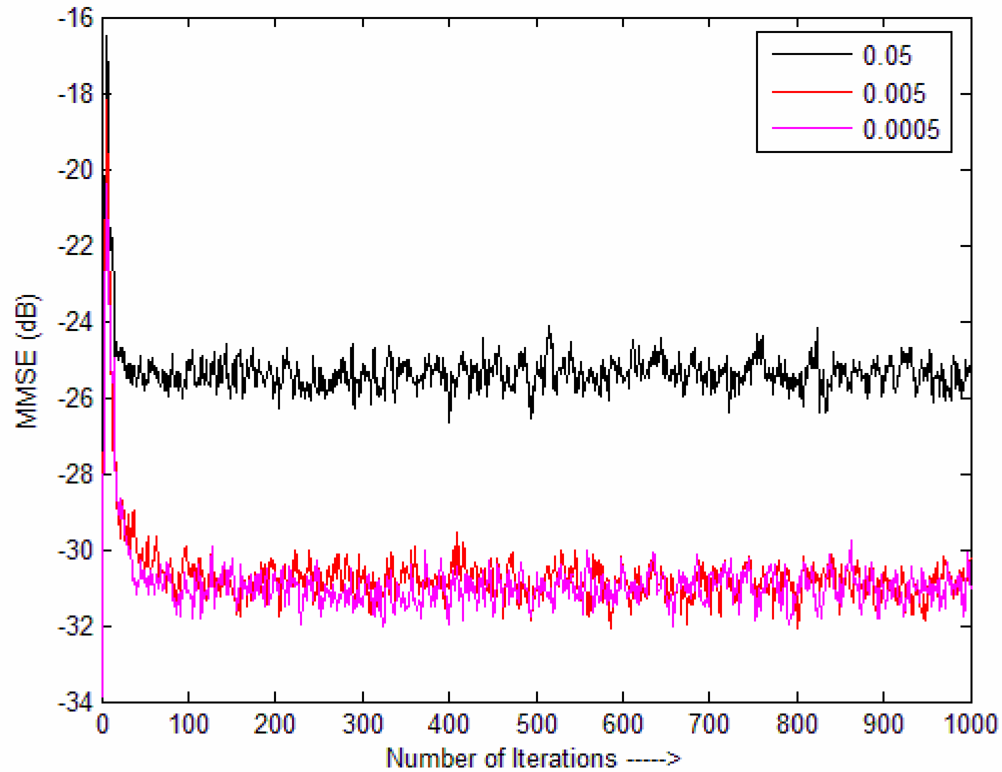


Fig. 4.1.2.5 LSn-NVFF

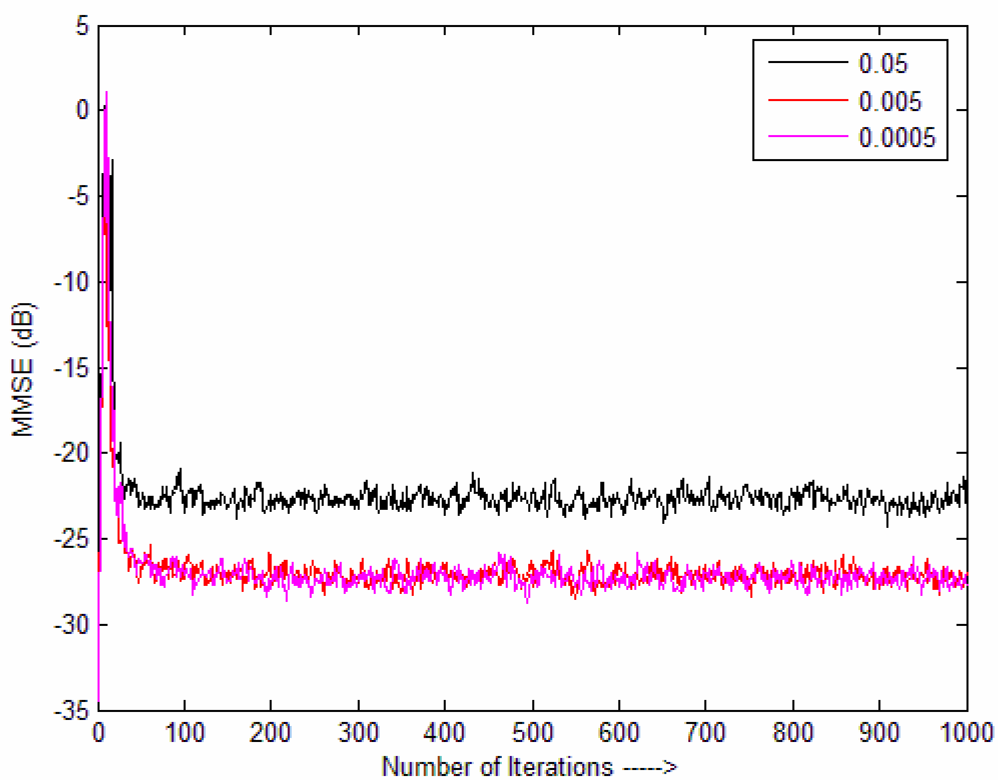


Fig. 4.1.2.6 LSn2-NVFF

4.1.3 MMSE Plots for Different SNR Conditions

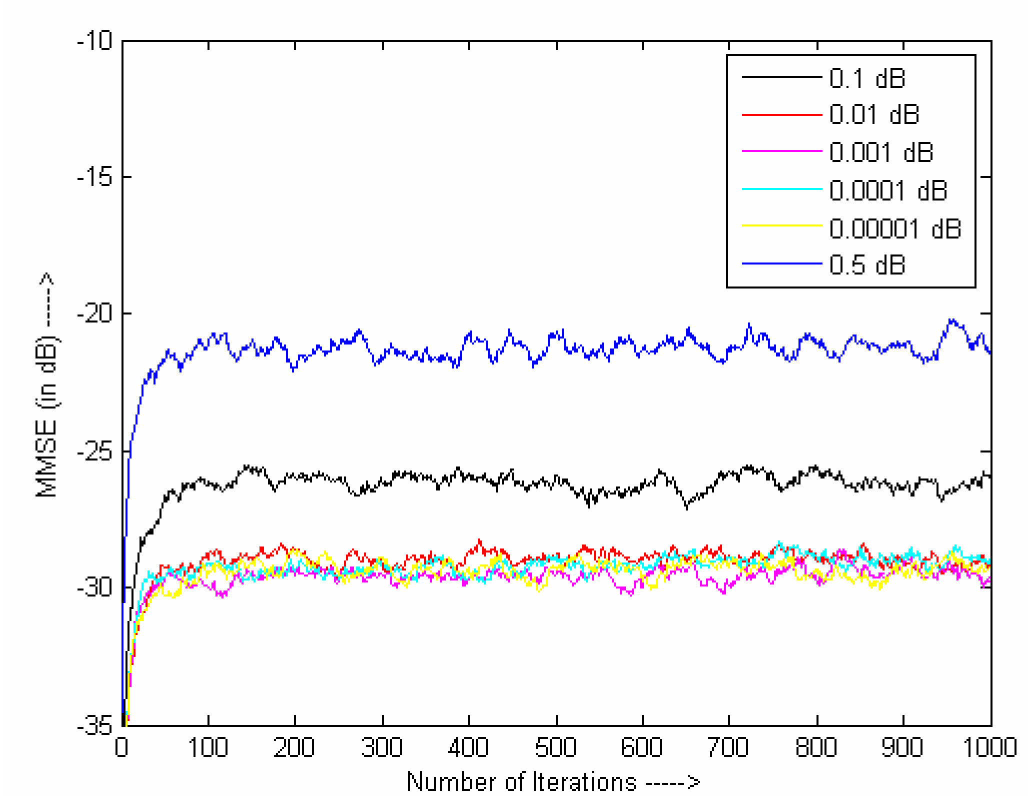


Fig. 4.1.3.1 LMS

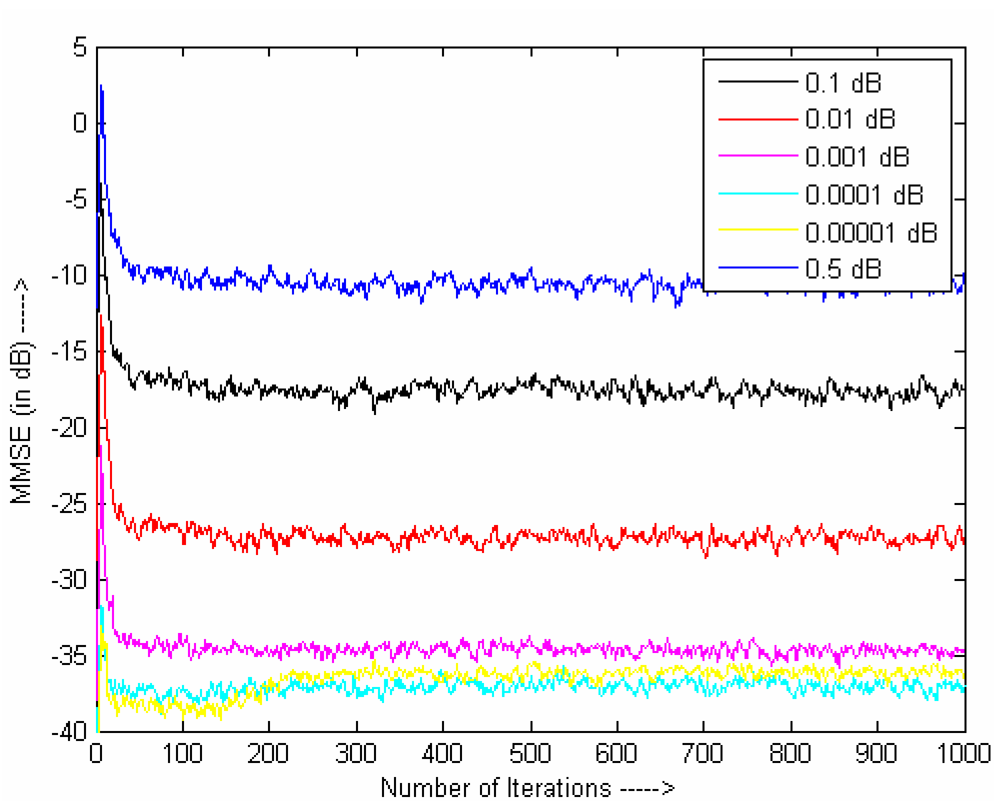


Fig. 4.1.3.2 LSn-VFF

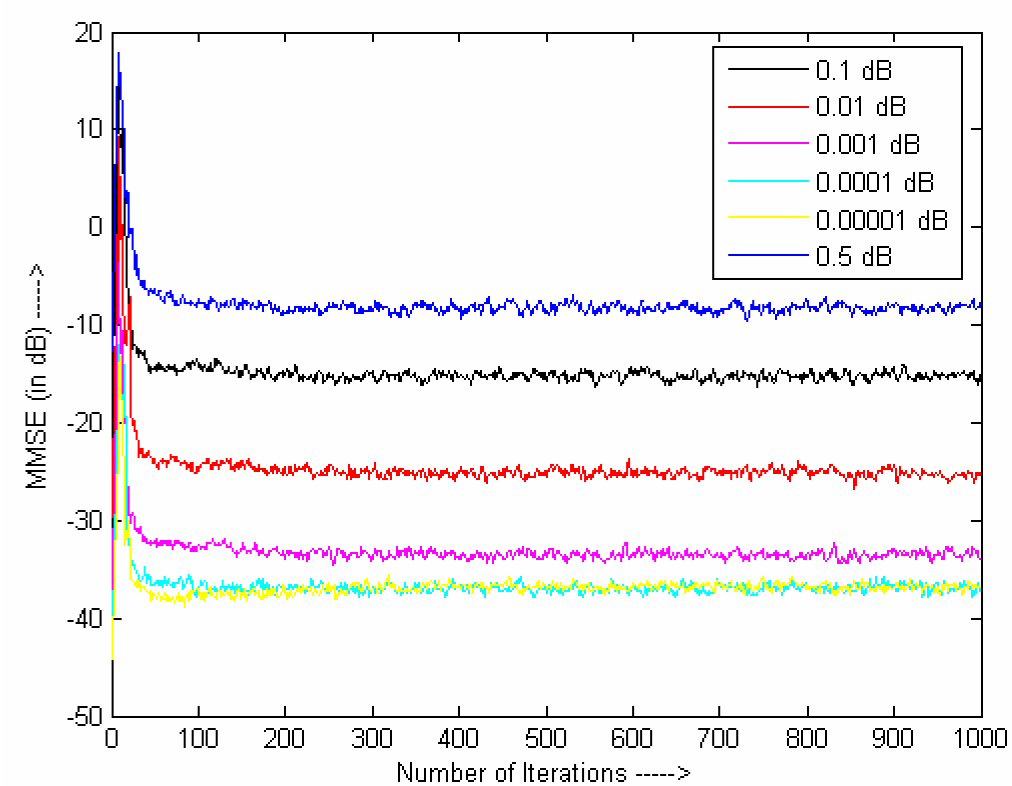


Fig. 4.1.3.3 LSn2-VFF

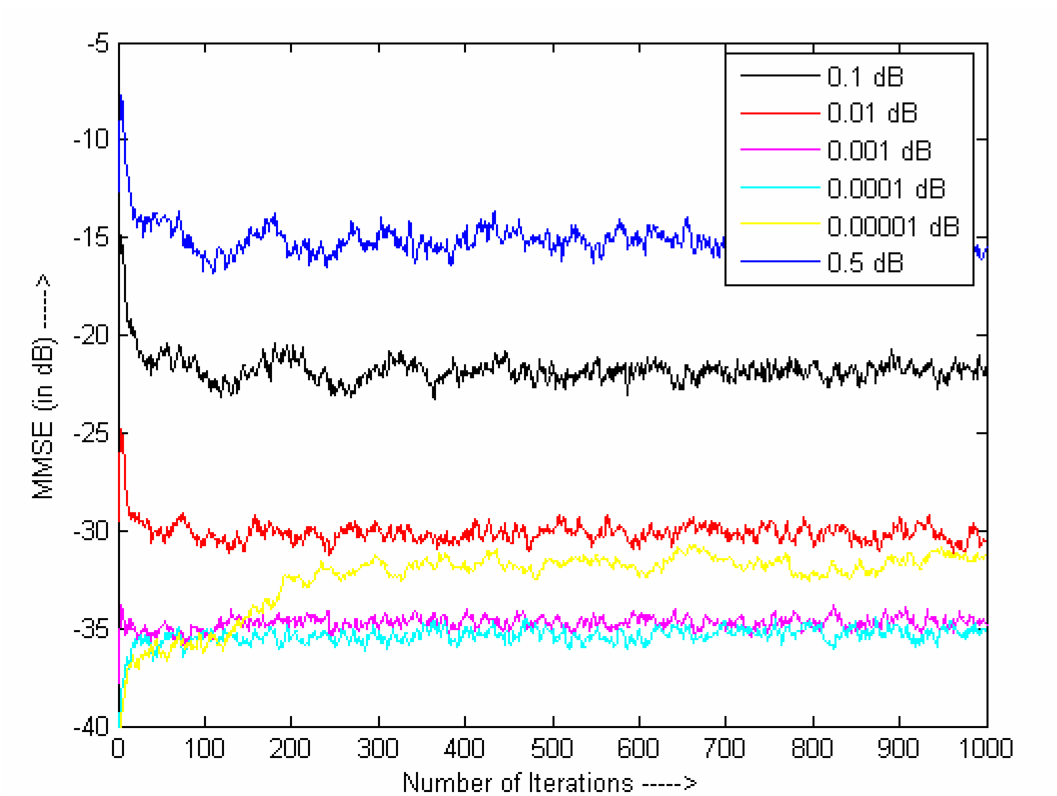


Fig. 4.1.3.4 RLS-VFF

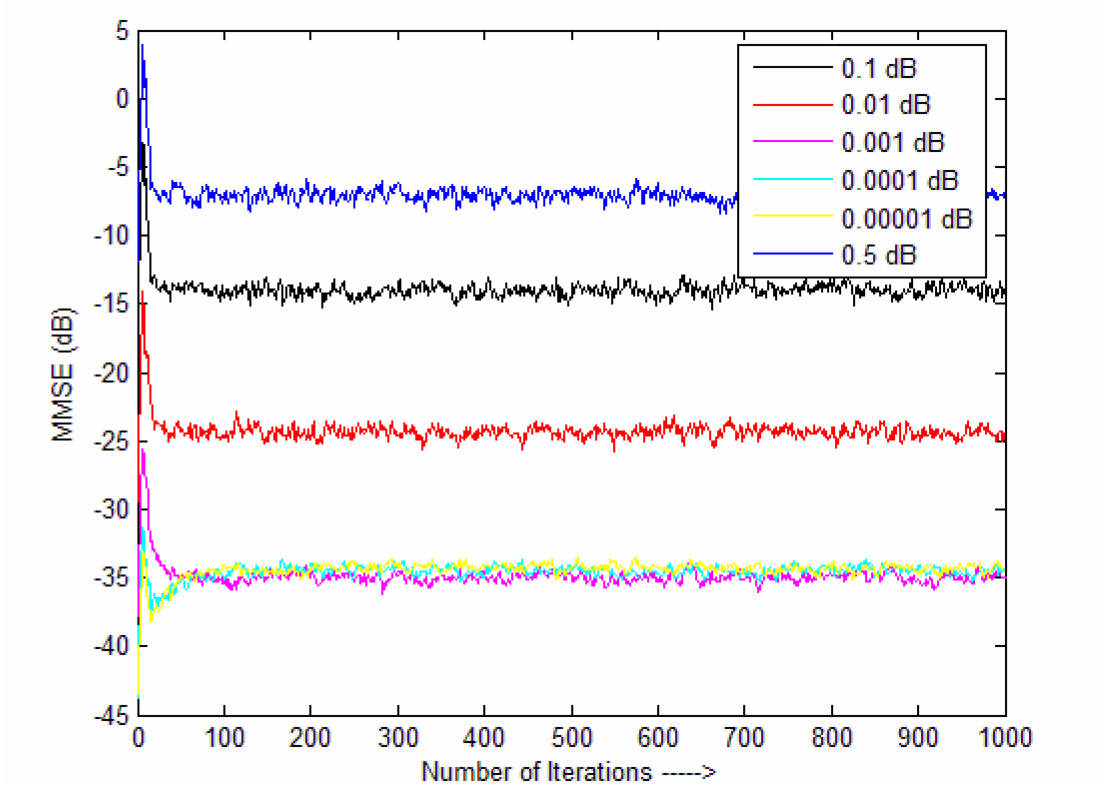


Fig. 4.1.3.5 LSn-NVFF

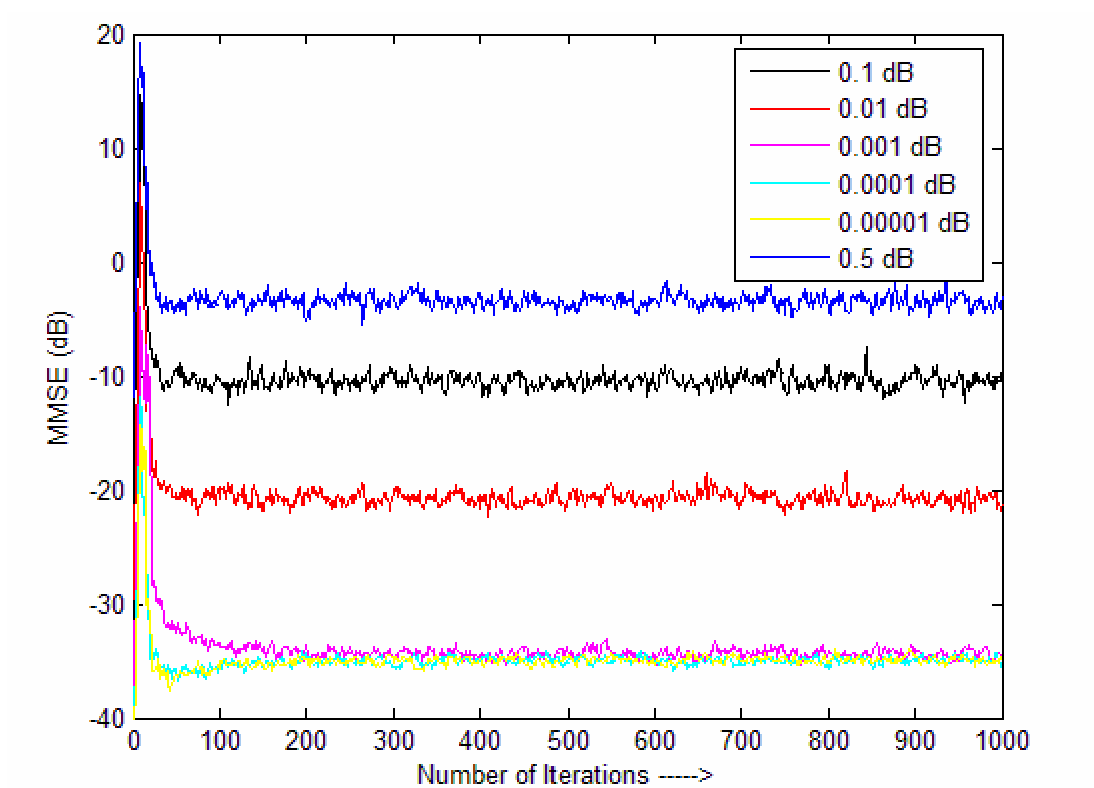


Fig. 4.1.3.6 LSn2-NVFF

4.2 BER Performance of STBC System

We assume that receiver with a single receiver antenna. We also denote the received signal over two consecutive symbol period as r_0 and r_1. The received signal can be expressed as

$$\mathbf{r} = \mathbf{H} \bullet \mathbf{s} + \mathbf{n} \quad (4.2.1)$$

where received signal $\mathbf{r} = \begin{pmatrix} r_0 \\ r_1 \end{pmatrix}$

channel matrix $\mathbf{H} = \begin{pmatrix} h_0 & h_1 \\ -h_1^* & h_0^* \end{pmatrix}$

AWGN with zero mean $\mathbf{n} = \begin{pmatrix} n_0 \\ n_1 \end{pmatrix}$

The channel is estimated by using pilot symbol. Then conjugate of this estimated channel matrix is multiplied to receive signal. After that maximum likelihood criterion is applied, which give the symbol actually transmitted. This complete process is shown in following figure.

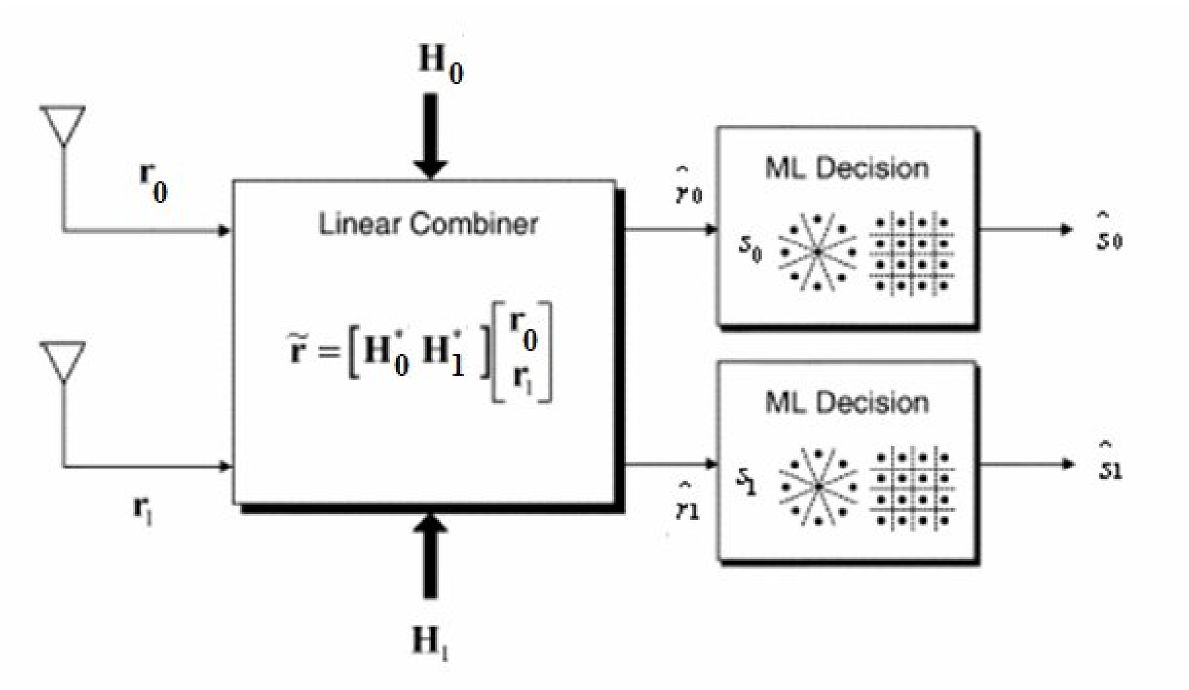


Fig 4.2.1 STBC Receiver Block Diagram [28]

In this section, Bit Error Rate performance of STBC system is shown, using different channel estimation algorithms. Simulation for BER is done for different STBCs of following dimension:

1) Dimension 2×2, having full diversity and full rate given by Alamouti [2].
2) Dimension 4×4, having full rate but not full diversity [59].
3) Dimension 8×8, having neither full rate nor full diversity [59].

For the simulation of different STBC codes process noise variance is set to -30 dB. For the simulation of algorithms with variable forgetting factor (LSn-VFF and LSn2-VFF), value of λ_max and λ_min is set to 0.99 and 0.75. The simulation is started with a initial forgetting factor value λ = 0.99. fdT product is kept 0.001. Value of step size parameter of LMS is empirically set to 0.0005. Value of M for New variable forgetting factor algorithms (LSn-NVFF and LSn2-NVFF) is 2 for all simulations.

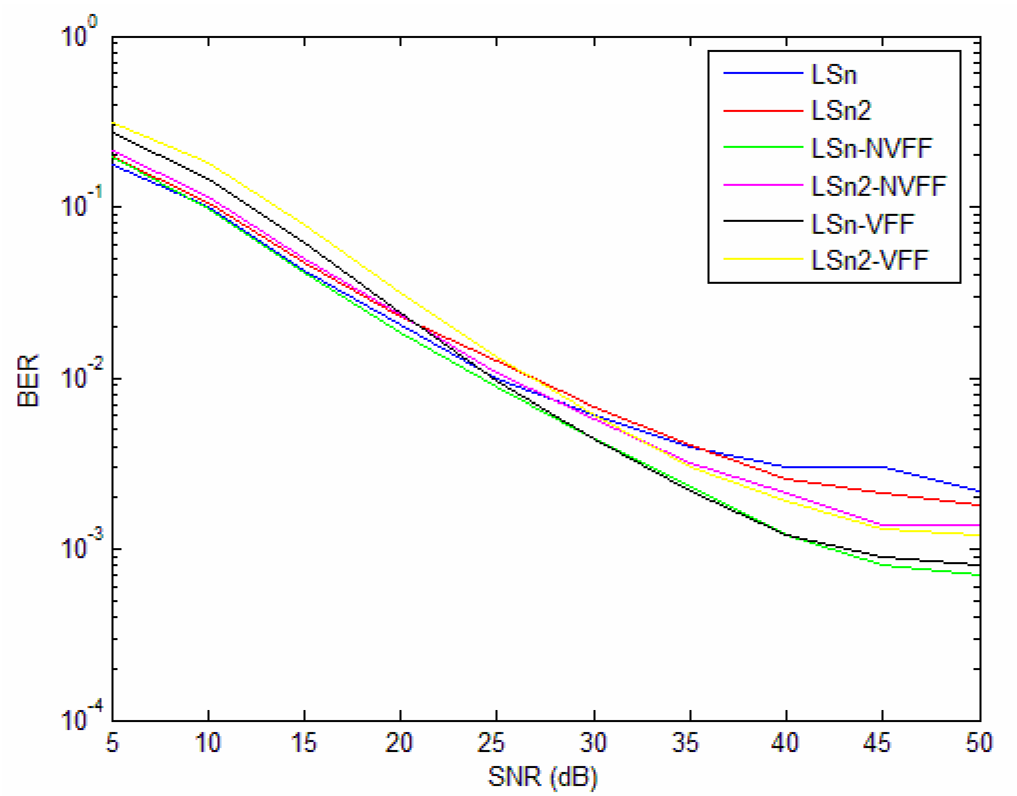


Fig. 4.2.2 Orthogonal (2×2) STBC with full rate

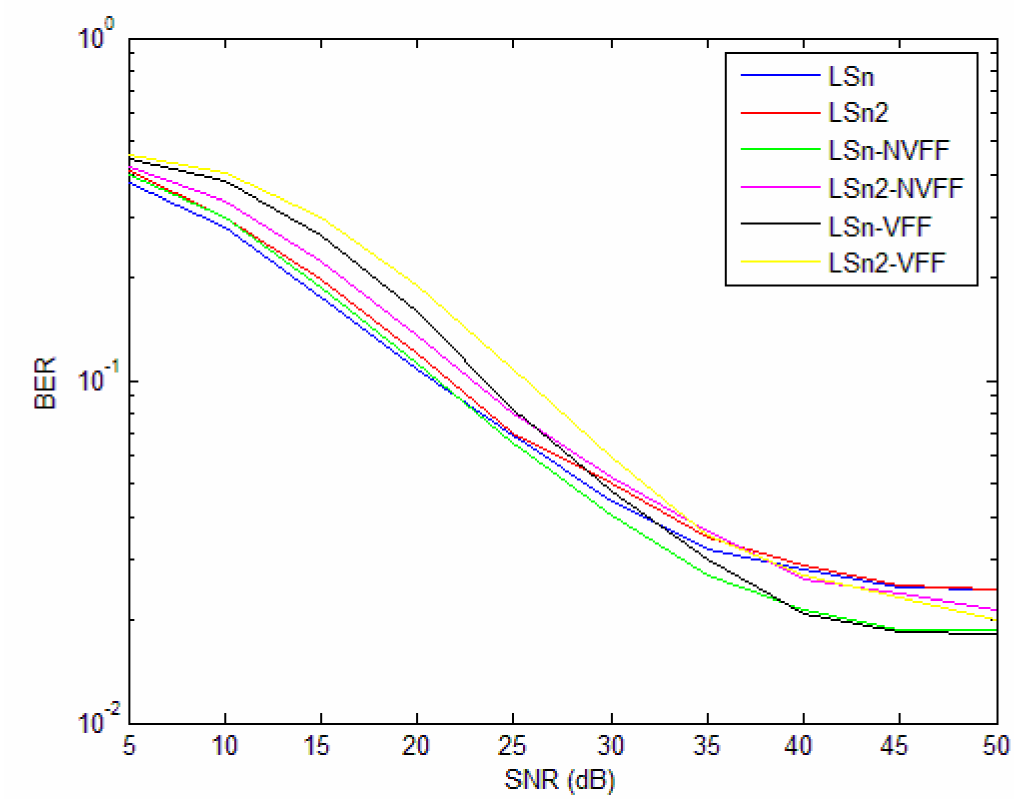


Fig. 4.2.3 Quasi-Orthogonal (4×4) STBC with full rate

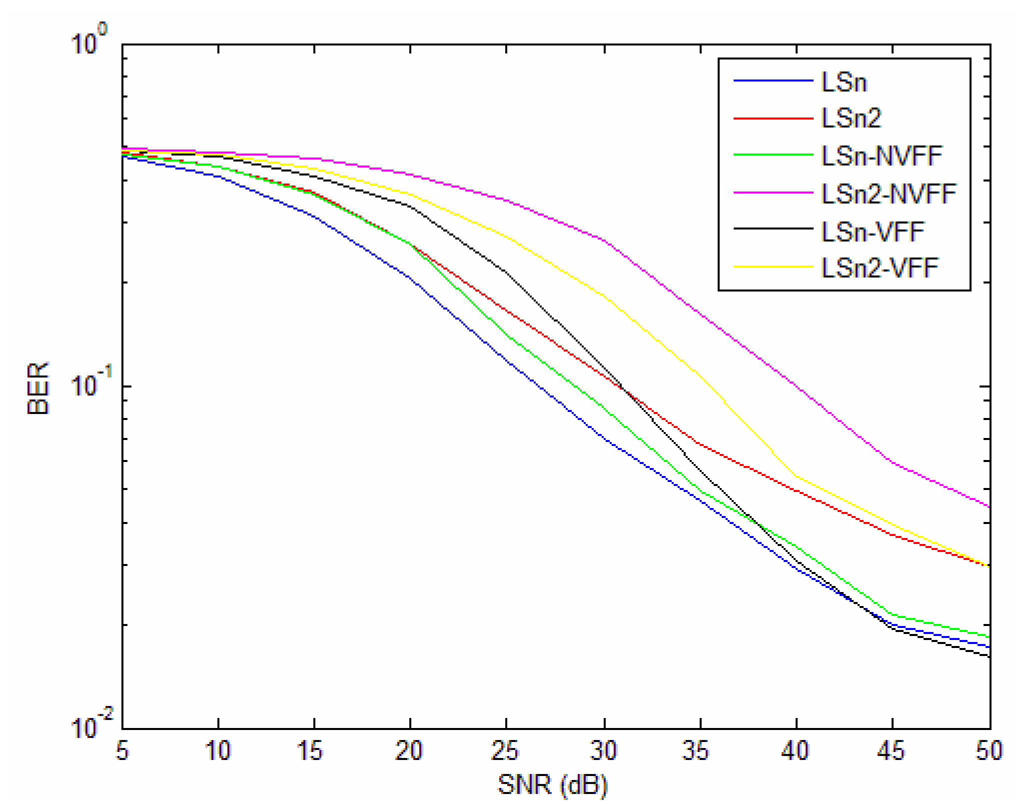


Fig.4.2.4 Quasi-Orthogonal (8×8) STBC with 3/4 rate

CHAPTER-5

CONCLUDING REMARKS

In this thesis, the convergence and tracking performance of LMS, RLS, LSn, LSn2, RLS-VFF, LSn-VFF, LSn2-VFF, LSn-NVFF and LSn2-NVFF algorithms are analyzed and compared for channel estimation under time-varying environment. From the simulations, following results are inferred for the channel estimation algorithms.

- LMS algorithm fails to perform well under non-stationary conditions.
- Polynomial model algorithm (LSn) outperforms RLS algorithm.
- Adaptation of forgetting factor increases the tracking capability of the algorithms i.e., RLS-VFF and LSn-VFF have better performance as compared to simple RLS and LSn algorithms respectively.
- Computational complexity of LSn-NVFF is lower than LSn-VFF.
- LSn2 channel tracking performance is better than LSn algorithm.
- But, LSn2-VFF and LSn2-VFF do not have performance as good as LSn-VFF and LSn-NVFF respectively, because of increase in computation complexity.
- The best performance is given by LSn-VFF and LSn-NVFF for all the time-varying channel models.

Space-time block coding technique for wireless communication systems utilizes the transmission antenna diversity, and requires perfect channel estimates at the receiver for optimum combining of the received signals. Therefore, the performance of channel estimator at the receiver has significant effect on the performance of wireless system, which is more prominent under the time-varying environment when channel coefficients change with time. We incorporated variable forgetting factor algorithm based channel estimator i.e. LSn-VFF, LSn2-VFF, LSn-NVFF and LSn2-NVFF in the space-time block code receiver to obtain better performance by using more accurate estimated channel coefficients.

Following results are obtained for different STBC codes using different channel estimation algorithms.

- 2×2 dimensional STBC system has least BER for different channel estimation algorithms, because of orthogonal STBC.
- For 2×2 STBC, BER performance of LSn-NVFF is best for all SNR condition.
- LSn-VFF's BER performance is good only in high SNR conditions for 2×2 STBC.

- BER for 4×4 and 8×8 dimensional STBC increases because these are Quasi-orthogonal STBC (due to coupling between symbols in Quasi-orthogonal STBC).
- Again, for 4×4 STBC, BER performance of LSn-NVFF is best for all SNR conditions.

- For 8×8 STBC system, LSn algorithm based approach outperforms the other algorithms because LSn-NVFF and LSn-VFF fail to track eight channels simultaneously due to increase in computation complexity of the algorithms and increase in number of parameter adjusted by single VFF algorithms for all channels.

Chapter 6

FUTURE SCOPE

STBCs are designed for MIMO flat fading channels [2], [57]. Enhanced data rates for GSM evolution (EDGE) standardization efforts aim at the providing a smooth transition from the time-division multiple-access second-generation digital cellular system (IS-136 and GSM) to third generation systems supporting higher data rates and enhanced services [61]. The uses of multiple transmit and receive antennas over the EDGE physical layer results in MIMO fading multipath channel. Therefore, implementation STBC over EDGE while preserving their linear decoding complexity feature requires effective MIMO equalization scheme that satisfy the linear receiver processing complexity requirement of STBC. This precludes the use of trellis based techniques whose complexity increases exponentially in the MIMO channel memory. We can apply the FIR MIMO minimum mean square error decision feedback equalizer (MMSE-DFE) developed in [62] and [63] to STBC in a typical urban (TU) EDGE environment [64]. Here we shows that how Alamouti's STBC with two transmit and two receive antenna can be combined with a two input two output FIR (finite impulse response) MMSE-DFE to combat frequency-selective fading under the constraint of linear processing only at the receiver. Block diagram of MMSE-DFE is shown Fig. 6.1

Here we consider a two input two output multipath fading channel model corrupted by additive white Gaussian noise. $\mathbf{y}_k^{(1)}$ and $\mathbf{y}_k^{(2)}$ are the signal received by the receiver antenna. The FIR MIMO MMSE-DFE filters are designed to minimized the mean square value of 2×1 error vector at time k is defined by

$$\mathbf{E}_k = \tilde{\mathbf{B}}^*_{opt}\mathbf{x} - \mathbf{W}^*_{opt}\mathbf{y} \tag{6.1}$$

where $\mathbf{W}_{opt}$ is the optimum MIMO feedforward filter matrix and $\mathbf{B}_{opt}$ is the optimum MIMO feedback filter matrix.

If we assume that input correlation matrix is a identity matrix and noise autocorrelation matrix is $\sigma_n^2 \mathbf{I}$ [64].

Then, feedforward matrix tap is represented by

$$\mathbf{W}_{opt}^* = \tilde{\mathbf{B}}_{opt}^* \mathbf{H}^* (\mathbf{H}\mathbf{H}^* + \sigma_n^2 \mathbf{I})^{-1} \qquad (6.2)$$

H is the estimated channel and σ_n^2 is the noise variance. **I** is the identity matrix.

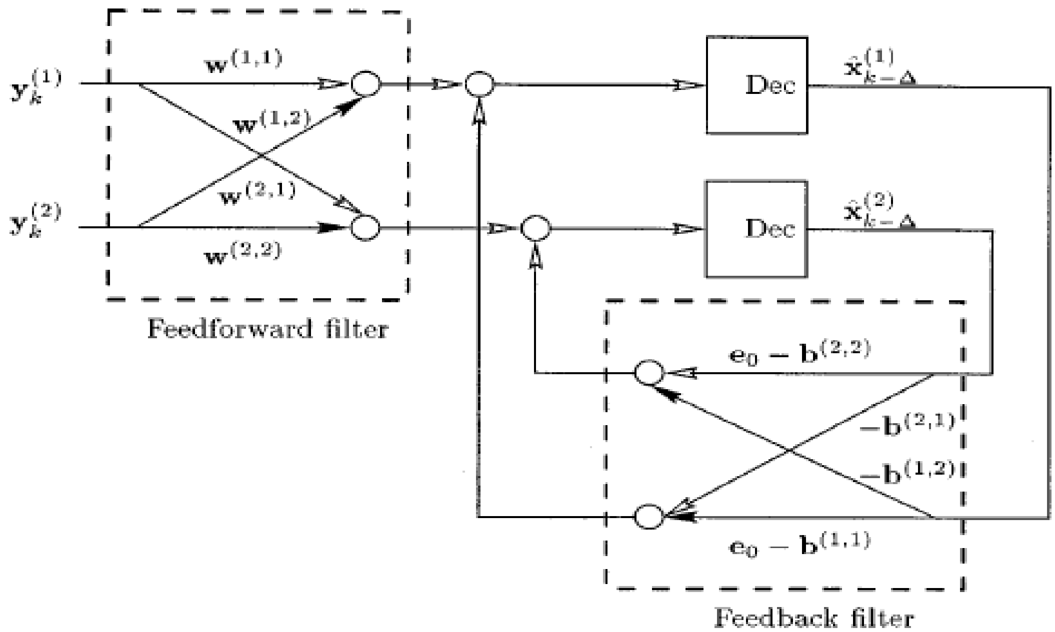


Fig. 6.1 Filter Connection for the MIMO MMSE-DFE with n_i=n_o=2 [64]

Optimum value of $\tilde{\mathbf{B}}_{opt}^*$ can be obtained as follows:

First we define the error autocorrelation matrix

$$\mathbf{R}_{ee} = E[\mathbf{E}_k \mathbf{E}_k^*] = \tilde{\mathbf{B}}^* \mathbf{R}^{-1} \tilde{\mathbf{B}} \qquad (6.3)$$

where $$\mathbf{R} = (\mathbf{I} - \mathbf{H}^* \sigma_n^2 \mathbf{I}\mathbf{H}) \qquad (6.4)$$

To determine the optimum matrix feedback filter coefficients, we need to solve the following constrained optimization problem

$$\min \text{trace}\,(\mathbf{R}_{ee}) = \min \text{trace}\,(\tilde{\mathbf{B}}^{*}\mathbf{R}^{-1}\tilde{\mathbf{B}}) \tag{6.5}$$

if we define the partitioning $\mathbf{R} = \begin{pmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} \\ \mathbf{R}_{12}^{*} & \mathbf{R}_{22} \end{pmatrix}$, (6.6)

then $$\tilde{\mathbf{B}}_{opt} = \begin{pmatrix} \mathbf{R}_{11} \\ \mathbf{R}_{12}^{*} \end{pmatrix} \mathbf{R}_{11}\mathbf{C} \tag{6.7}$$

The size of all parameter used and matrix **C** in this section are defined in [62].

We can combine a MIMO MMSE-DFE with space-time block codes to combat frequency selective fading. For Alamouti's space-time block code with two transmit and two receive antennas, the MIMO DFE consist of feedforward finite impulse response (FIR) filters that perform diversity combining and combat MIMO precursor intersymbol interference and feedback filters that combat MIMO postcursor intersymbol interference. From equation (6.2), (6.4), (6.6) and (6.7), it is clear that the performance of this system is dependent on channel estimation algorithm. Thus, this thesis work can be further used for MIMO MMSE-DFE with space-time block codes.

APPENDIX A
MAXIMAL RATIO RECEIVE COMBINING SCHEME

Fig. A.1 shows the baseband representation of the classical two branch MRRC.

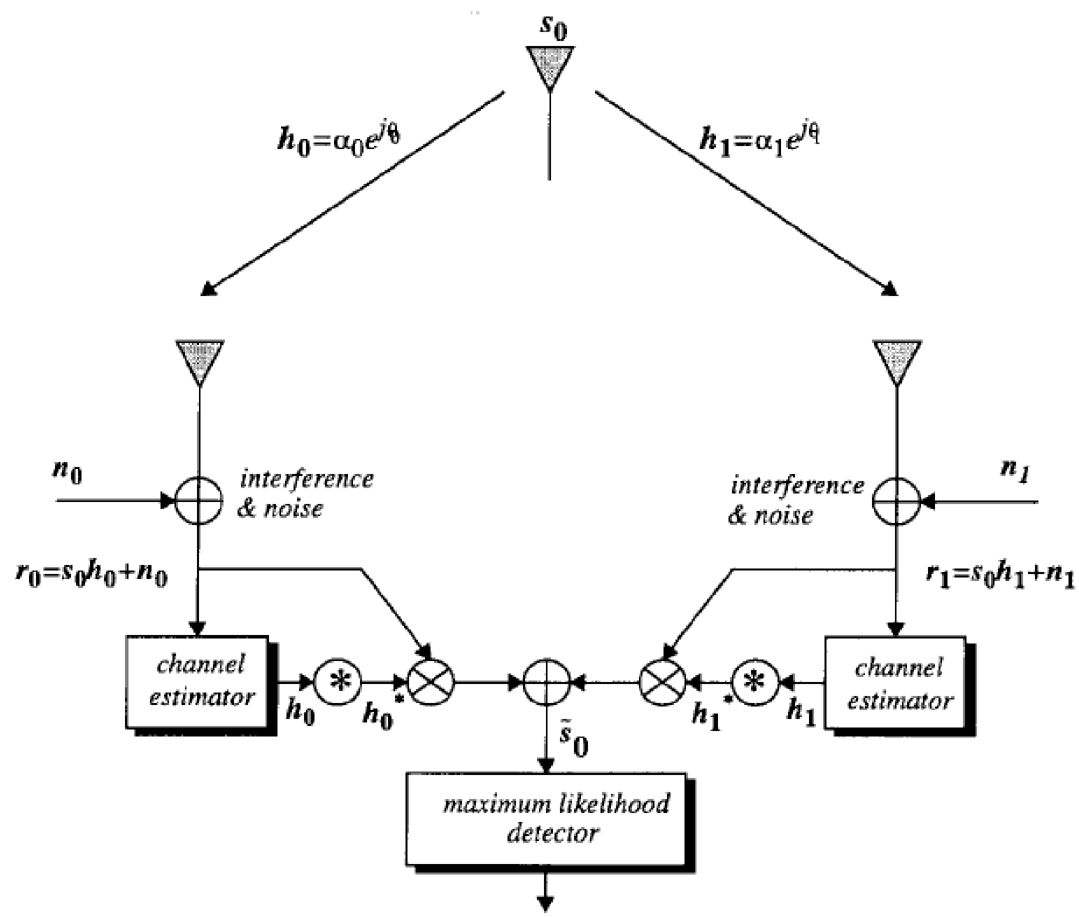


Figure A.1 Two Branch Diversity [2]

Signal s_0 is transmitted from the transmitter and received by the two receiver antenna. The channel between transmit antenna and receive antenna zero is denoted by h_0 and between the transmit antenna and receive antenna one is denoted by h_1. Assuming that fading is constant across the two consecutive symbols, then we can write

$$h_0 = \alpha_0 e^{j\theta_0}$$

$$h_1 = \alpha_1 e^{j\theta_1} \quad \text{(A.1)}$$

Received signal along with noise can be denoted as:

$$r_0 = h_0 s_0 + n_0$$
$$r_1 = h_1 s_1 + n_1 \quad \text{(A.2)}$$

Where n_0 and n_1 represent complex noise and interference.

Assuming n_0 and n_1 are Gaussian distributed, the maximum likelihood decision rule at the receiver for these received signals is to choose signal s_i if and only if

$$d^2(r_0, h_0 s_i) + d^2(r_1, h_1 s_i) <= d^2(r_0, h_0 s_k) + d^2(r_1, h_1 s_k)\ , \quad i \neq k \quad \text{(A.3)}$$

where $d^2(x, y)$ is the squared Euclidean distance between signals x and y calculated by the following expression:

$$d^2(x, y) = (x - y)(x^* - y^*) \quad \text{(A.4)}$$

* is the complex conjugate operation.

The receiver combining scheme for two branch MRRC is as follows:

$$\tilde{s}_0 = h_0^* r_0 + h_1^* r_1$$
$$= h_0^* (h_0 s_0 + n_0) + h_1^* (h_1 s_0 + n_1)$$
$$= (\alpha_0^2 + \alpha_1^2) + h_0^* n_0 + h_1^* n_1 \quad \text{(A.5)}$$

Expanding equation (A.3) using equation (A.4) and substituting the value from equation (A.5), we have

$$(\alpha_0^2 + \alpha_1^2 - 1)|s_i|^2 + d^2(\tilde{s}_0, s_i) \leq (\alpha_0^2 + \alpha_1^2 - 1)|s_k|^2 + d^2(\tilde{s}_0, s_k) \quad \text{(A.6)}$$

If we consider a PSK signal (equal energy constellations), then

$$|s_i|^2 = |s_k|^2 = E_s \quad \text{(A.7)}$$

Where E_s is the energy of the signal.

Solving equation (A.6) and (A.7),

Choose s_i if and only if

$$d^2(\tilde{s}_0, s_i) \leq (d^2(\tilde{s}_0, s_k)) \quad \text{(A.8)}$$

The maximal-ratio combiner may then construct the signal, as shown in Fig. A.1, so that the maximum likelihood detector may produce $\hat{s}_0$, which is a maximum likelihood estimate of s_0.

APPENDIX B

RECURSIVE CALCULATION OF $\mathbf{D}_n$ AND $\mathbf{M}_n$

1. Recursive Calculation for $\mathbf{D}_n$:

Substitute the value of $\mathbf{C}_n$ from equation 3.1.47 into equation 3.1.41,

$$\mathbf{D}_n = \frac{\partial(\mathbf{C}_{n-1} + \mathbf{k}_n e_n^*)}{\partial\lambda} \tag{B.1}$$

$$\mathbf{D}_n = \mathbf{D}_{n-1} + \frac{\partial(\mathbf{k}_n e_n^*)}{\partial\lambda} \tag{B.2}$$

Substitute the value of $\mathbf{K}_n$ from equation 3.1.24,

$$\mathbf{D}_n = \mathbf{D}_{n-1} + \frac{1}{\sigma_n^2}\frac{\partial(\mathbf{P}_n\mathbf{S}_n e_n^*)}{\partial\lambda} \tag{B.3}$$

$$\frac{\partial\mathbf{P}_n}{\partial\lambda} = \mathbf{M}_n \tag{B.4}$$

from equation 3.1.43 and 3.1.24,

$$\mathbf{D}_n = \mathbf{D}_{n-1} + \frac{1}{\sigma_n^2}\mathbf{M}_n\mathbf{S}_n e_n^* - \mathbf{k}_n\mathbf{S}_n^H\mathbf{D}_{n-1} \tag{B.5}$$

$$\mathbf{D}_n = [\mathbf{I} - \mathbf{k}_n\mathbf{S}_n^H]\mathbf{D}_{n-1} + \frac{1}{\sigma_n^2}\mathbf{M}_n\mathbf{S}_n e_n^* \tag{B.6}$$

2. Recursive Calculation for $\mathbf{M}_n$:

from equation 3.1.23 and 3.1.24,

$$\mathbf{P}_n = \lambda^{-1}[\mathbf{I} - \frac{\mathbf{P}_n\mathbf{S}_n\mathbf{S}_n^H}{\sigma_n^2}]\mathbf{P}_{n-1} \tag{B.7}$$

$$\lambda\sigma_n^2\mathbf{P}_n = [\sigma_n^2\mathbf{I} - \mathbf{P}_n\mathbf{S}_n\mathbf{S}_n^H]\mathbf{P}_{n-1} \tag{B.8}$$

differentiating with respect to λ

$$\lambda\sigma_n^2\frac{\partial\mathbf{P}_n}{\partial\lambda} + \sigma_n^2\mathbf{P}_n = -\frac{\partial\mathbf{P}_n}{\partial\lambda}\mathbf{S}_n\mathbf{S}_n^H\mathbf{P}_{n-1} + [\sigma_n^2\mathbf{I} - \mathbf{P}_n\mathbf{S}_n\mathbf{S}_n^H]\frac{\partial\mathbf{P}_{n-1}}{\partial\lambda} \tag{B.9}$$

$$\mathbf{M}_n[\lambda\sigma_n^2\mathbf{I} + \mathbf{S}_n\mathbf{S}_n^H\mathbf{P}_{n-1}] = [\sigma_n^2\mathbf{I} - \mathbf{P}_n\mathbf{S}_n\mathbf{S}_n^H]\mathbf{M}_{n-1} - \sigma_n^2\mathbf{P}_n \tag{B.10}$$

Calculation of $[\lambda\sigma_n^2\mathbf{I}+\mathbf{S}_n\mathbf{S}_n^H\mathbf{P}_{n-1}]^{-1}$

from Matrix Inverse Lemma:

$$(\mathbf{A}+\mathbf{x}\mathbf{y}^H)^{-1}=\mathbf{A}^{-1}+\frac{\mathbf{A}^{-1}\mathbf{x}\mathbf{y}^H\mathbf{A}^{-1}}{1+\mathbf{y}^H\mathbf{A}^{-1}\mathbf{x}} \tag{B.11}$$

Substituting

$$\begin{aligned}\mathbf{A}&=\lambda\sigma_n^2\mathbf{I}\\ \mathbf{x}&=\mathbf{S}_n\\ \mathbf{y}&=\mathbf{P}_{n-1}^H\mathbf{S}_n=\mathbf{P}_{n-1}\mathbf{S}_n\end{aligned} \tag{B.12}$$

$\{\because \mathbf{P}_n=\mathbf{P}_n^H\}$ (Correlation matrix is Hermitian always.)

$$[\lambda\sigma_n^2\mathbf{I}+\mathbf{S}_n\mathbf{S}_n^H\mathbf{P}_{n-1}]^{-1}=(\lambda\sigma_n^2\mathbf{I})^{-1}-\frac{(\lambda\sigma_n^2\mathbf{I})^{-1}\mathbf{S}_n\mathbf{S}_n^H\mathbf{P}_{n-1}(\lambda\sigma_n^2\mathbf{I})^{-1}}{1+\mathbf{S}_n^H\mathbf{P}_{n-1}^H(\lambda\sigma_n^2\mathbf{I})^{-1}\mathbf{S}_n} \tag{B.13}$$

$$[\lambda\sigma_n^2\mathbf{I}+\mathbf{S}_n\mathbf{S}_n^H\mathbf{P}_{n-1}]^{-1}=(\lambda\sigma_n^2\mathbf{I})^{-1}-\frac{(\lambda\sigma_n^2\mathbf{I})^{-1}\mathbf{S}_n\mathbf{S}_n^H\mathbf{P}_{n-1}^H}{\lambda\sigma_n^2\mathbf{I}+\mathbf{S}_n^H\mathbf{P}_{n-1}^H\mathbf{S}_n} \tag{B.14}$$

from 3.1.21 and 3.1.22

$$\mathbf{k}_n^H=\frac{\mathbf{S}_n^H\mathbf{P}_{n-1}^H}{\lambda\sigma_n^2\mathbf{I}+\mathbf{S}_n^H\mathbf{P}_{n-1}^H\mathbf{S}_n} \tag{B.15}$$

equation B.13 becomes

$$[\lambda\sigma_n^2\mathbf{I}+\mathbf{S}_n\mathbf{S}_n^H\mathbf{P}_{n-1}]^{-1}=(\lambda\sigma_n^2\mathbf{I})^{-1}[\mathbf{I}-\mathbf{S}_n\mathbf{k}_n^H] \tag{B.16}$$

Substituting this in equation B.10,

$$\mathbf{M}_n=\lambda^{-1}[\mathbf{I}-\mathbf{k}_n\mathbf{S}_n^H]\mathbf{M}_{n-1}[\mathbf{I}-\mathbf{S}_n\mathbf{k}_n^H]+\lambda^{-1}\mathbf{P}_n+\sigma_n^2\lambda^{-1}\mathbf{k}_n\mathbf{k}_n^H \tag{B.17}$$

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