Flavor Oscillations in Neutrinos

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In

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Submitted by

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Certificate

This is to certify that the thesis entitled " Flavor Oscillation in Neutrino" submitted by Miss Reema Garg, Roll No. 301104001 in the partial fulfilment of the requirement for award of the degree of MASTERS OF PHYSICS from the School of Physics and Material Science, Thapar University, Patiala. It is to certify that the matter embodied in this report is of the candidates own record and has not been submitted in part or full to any other University or Institute for the award of any degree.

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Abstract

Neutrino Oscillations are sign for small masses of neutrinos. In Standard Model neutrinos are considered to be massless, hence this observation points to the possibility of physics beyond standard model. Thus the precise determination of neutrino oscillation are important for formulating theories beyond standard model. In this work, we study the phenomenology of two and three-flavor neutrino oscillations, focusing in particular on the prospects of recent experiments. We analysed various experiments, which are initially going on, for their potential to determine these parameters precisely. After discussing the details about the detectors and reactors working on neutrino oscillations, we have given a theoretical formalism on solar and atmospheric neutrino oscillation probabilities for three generation case. We also discuss the three – flavour effects appearing in $\nu_e \rightarrow \nu_e$, $\nu_e \rightarrow \nu_\mu$, and $\nu_e \rightarrow \nu_\tau$ oscillation probabilities through their plots. The probability curves have been compared with the theoretical and experimental paper's curves and found to be consistant.

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Chapter:1 Introduction:-

1.1 High Energy physics:-

Particle physics is a branch of physics that studies the nature of particles that are the constituents of what is usually referred to as matter and radiation. In current understanding, particles are excitations of quantum fields and interact following their dynamics. Most of the interest in this area is in fundamental fields, each of which cannot be defined as a bound state of other fields. The current set of fundamental fields and their dynamics are summarized in a theory called the Standard Model , therefore particle physics is largely the study of the Standard Model's particle content and its possible extensions.

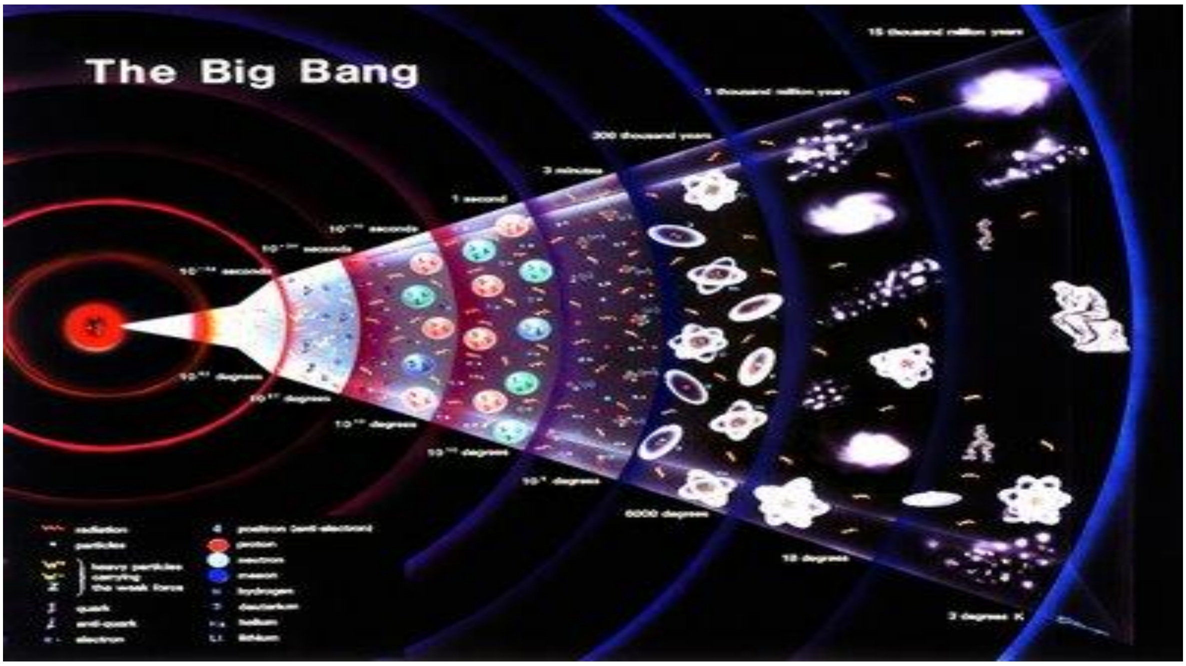

Fig.1.1: High Energy Physics

1.2 Standard Model:-

The Standard Model of particle physics is a theory concerning the electromagnetic, weak, and strong nuclear interactions, which mediate the dynamics of the know subatomic particles. Developed throughout the mid to late 20th century, the Standard Model is truly “a tapestry woven by many hands”, sometimes driven forward by new experimental discoveries, sometimes by theoretical advances. It was a collaborative effort in the largest sense, spanning continents and decades. The current formulation was finalized in the mid-1970s upon experimental confirmation of the existence of quarks. Since then, discoveries of the bottom quark (1977), the top quark (1995), and the tau neutrino (2000) have given further credence to the Standard Model. More recently (2011–2012), the possible detection of the Higgs boson would complete the set of predicted particles upon its verification. Because of its success in explaining a wide variety of experimental results, the Standard Model is sometimes regarded as a "theory of almost everything"

Standard model consists of elementary particles divided into two classes:

Bosons & Fermions:

<u>Bosons:-</u> Bosons are the particles that obey Bose- Einstein Statistics and when we interchange the two bosons the wavefunction of the system is unchanged.Several bosons can occupy the same quantum state. So, Bosons are having spin either 0 or 1 or 2

<u>Fermions:-</u> Fermions are the particles that obey Fermi Dirac statistics & Pauli Exclusion Principle and when we interchange the two fermions the wavefunction of the system is changed. Two fermions can't occupy the same quantum state. So fermions are having spin ½. Neutrinos are also having existence in the well known standard model but they have mass less than fermions.

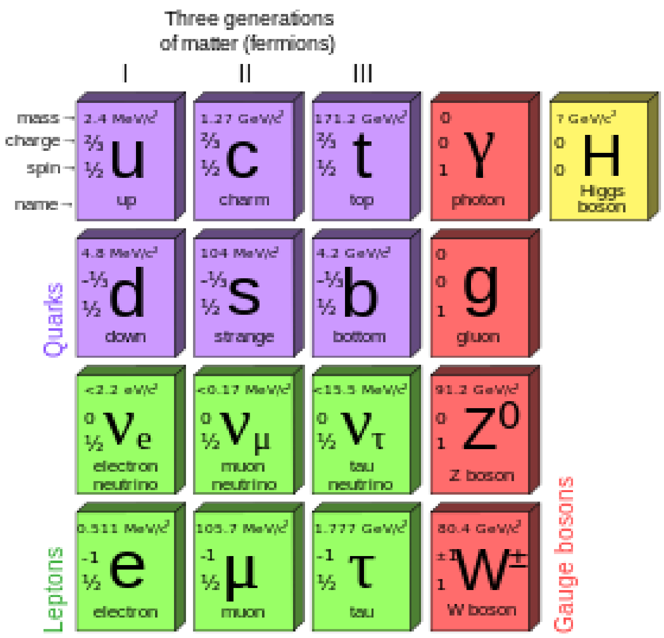

Fig.1.2 : Six Leptons, Six Quarks and Six Force Carriers.

1.3 Fundamental Particles:-

Quarks and Leptons are fundamental particles. Quarks are the building blocks of protons and neutrons but not electrons.

There are six types of quarks and antiquarks

Quarks	Spin	Charge
Up	½	2/3
Down	½	-1/3
Strange	½	-1/3
Charm	½	2/3
Top	½	2/3
Bottom	½	-1/3

Table:1.1: Quarks with spin and charge

Leptons:- A lepton is an elementary particle with no fundamental constituent into it.

Leptons	symbol	Generation	Spin	Charge
Electron	e^-	Ist	½	-1
Electron neutrino	v_e	Ist	½	0
Muon	μ^-	2^{nd}	½	-1
Muon neutrino	v_μ	2^{nd}	½	0
Tau	τ^-	3^{rd}	½	-1
Tau neutrino	v_τ	3^{rd}	½	0

Table:1.2: Generation Of Leptons

1.4 Fundamental Interactions:-

There are four basic fundamental interactions:-

1) Electromagnetic Interaction:-

The electromagnetic force is one of the four fundamental interactions in nature. The electromagnetic force is the interaction responsible for almost all the phenomena encountered in daily life, with the exception of gravity. The electromagnetic force is a force of infinite range which obey the inverse square law. The electromagnetic force holds the atoms and molecules together.

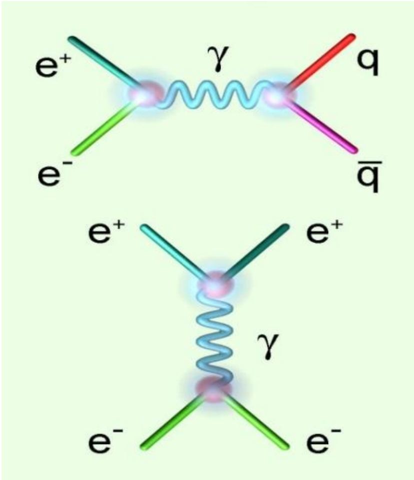

Fig.1.3: Electromagnetic Interaction

2) Strong Interaction:-

The Strong Interaction is one of the most fundamental interaction out of the four interactions.The strong interaction is observable in two areas: on a larger scale (ab out 1 to 3 femtometers (fm)), it is the force that binds protons and neutrons (nucleons) together to form the nucleus of an atom - in this form, it is often referred to as the nuclear force. On the smaller scale (less than about 0.8 fm, the radius of a nucleon), it is the force (carried by gluons) that holds quarks together to form protons, neutrons and other hadron particles.

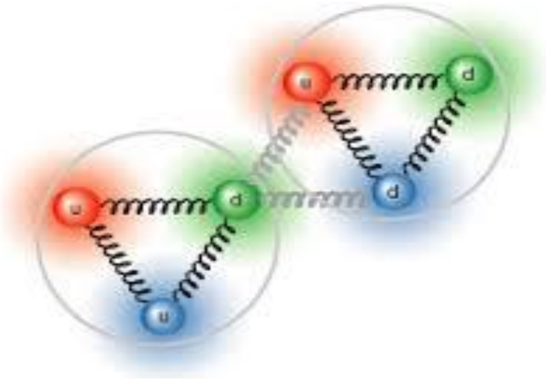

Fig.1.4: Strong Interactions

3) Weak Interactions:-

Weak interaction is one of the four fundamental forces of nature, alongside the strong nuclear force, electromagnetism, and gravitation. It is responsible for the radioactive decay of subatomic particles and initiates the process known as hydrogen fusion in stars. Weak interactions affect all known fermions; that is, particles whose spin (a property of all particles) is a half-integer.

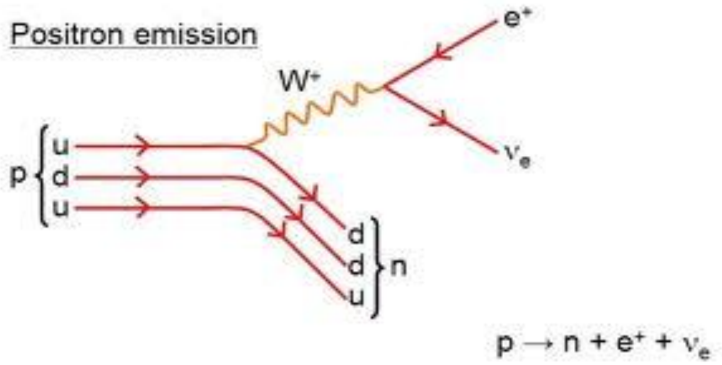

Fig.1.5: Positron emission through Weak Interaction

4) Gravitational Interaction:-

Gravitation is one of the four fundamental interactions of nature, along with electromagnetism, and the nuclear strong force and weak force. The simpler Newton's law of universal gravitation provides an accurate approximation for most physical situations including calculations as critical as spacecraft trajectory. The Gravitational force between masses m_1 and m_2 is given by:

$F_{gravity} = Gm_1m_2/r^2$

Where $G = 6.67\times10^{-11} Nm^2 / Kg^2$

This is often called the "Universal law of gravitation" and G is universal gravitation constant. It is an example of inverse square law force. This force is always attractive & acts along the line joining the centre of mass of two masses.

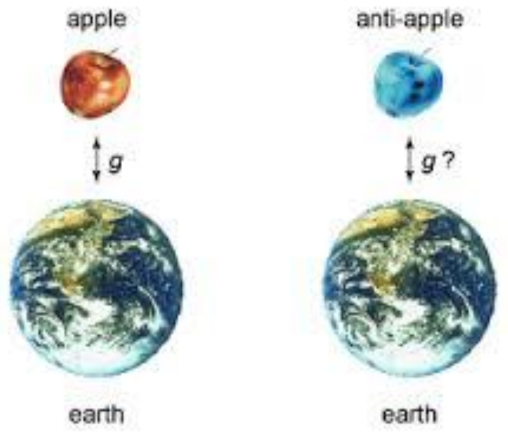

Fig.1.6: Gravitational Interaction

1.5 Leptons:-

A lepton is an elementary, spin-1⁄2 particle that does not undergo strong interactions, but is subject to the Pauli exclusion principle. The best known of all leptons is the electron, Two main classes of leptons exist: charged leptons (also known as the electron-like leptons), and neutral leptons (better known as neutrinos). Charged leptons can combine with other particles to form various composite particles such as atoms and positronium, while neutrinos rarely interact with anything.

There are six types of leptons, known as flavours, forming three generations. The first generation is the electronic leptons, comprising the electron (e^-) and electron neutrino (ν_e); the second is the muonic leptons, comprising the muon (μ^-) and muon neutrino (ν_μ); and the third is the tauonic leptons, comprising the tau (τ^-) and the tau neutrino (ν_τ). Electrons have the least mass of all the charged leptons. The heavier muons and taus will rapidly change into electrons through a process of particle decay: the transformation from a higher mass state to a lower mass state. Thus electrons are stable and the most common charged lepton in the universe, whereas muons and taus can only be produced in high energy collisions (such as those involving cosmic rays and those carried out in particle accelerators).

Leptons have various intrinsic properties, including electric charge, spin, and mass.

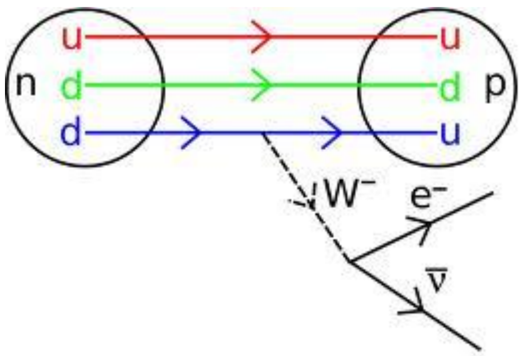

Fig.1.7: Beta Decay

It was first verified in Cowan- Reines neutrino experiment conducted by Clyele Cowan and Frederick Reines in 1956 by the inverse beta decay.

$$\overline{\nu_e} + p \rightarrow n + e^-$$

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2.1 Neutrinos:-

A neutrino is an electrically neutral, weakly interacting elementary subatomic particle with half-integer spin. The neutrino (meaning "small neutral one" in Italian) is denoted by the Greek letter ν (nu). All evidence suggests that neutrinos have mass but that their mass is tiny even by the standards of subatomic particles. Their mass has never been measured accurately.

Neutrinos do not carry electric charge, which means that they are not affected by the electromagnetic forces that act on charged particles such as electrons and protons. Neutrinos are affected only by the weak sub-atomic force, of much shorter range than electromagnetism, and gravity, which is relatively weak on the subatomic scale. Therefore a typical neutrino passes through normal matter unimpeded.

Neutrinos are created as a result of certain types of radioactive decay, or nuclear reactions such as those that take place in the Sun, in nuclear reactors, or when cosmic rays hit atoms. There are three types, or "flavors", of neutrinos: electron neutrinos, muon neutrinos and tau neutrinos. Each type is associated with an antiparticle, called an "antineutrino", which also has neutral electric charge and half-integer spin. Whether or not the neutrino and its corresponding antineutrino are identical particles has not yet been resolved, even though the antineutrino has an opposite chirality to the neutrino.

Most neutrinos passing through the Earth emanate from the Sun. About 65 billion (6.5×10^{10}) solar neutrinos per second pass through every square centimeter perpendicular to the direction of the Sun in the region of the Earth.

2.1.1 History of Neutrinos:-

In 1931 a hypothetical particle is predicted by the theorist Wolfgang Pauli. Pauli based his prediction on the fact that energy and momentum did not appear to be conserved in certain radioactive decays. Pauli suggested that this missing energy might be carried off, unseen, by a neutral particle which was escaping detection. After that in 1934 Enrico Fermi develops a comprehensive theory including the Pauli’s hypothetical particles, which Fermi coins the “neutrino.” In 1942 Wang Ganchang first proposed the use of beta-capture to experimentally detect neutrinos.

In the July 20, 1956 issue of Science, Clyde Cowan, Frederick Reines, F. B. Harrison, H. W. Kruse, and A. D. McGuire published confirmation that they had detected the neutrino, a result that was rewarded almost forty years later with the 1995 Nobel Prize.

1962 - Experiments at Brookhaven National Laboratory and CERN, the European Laboratory for Nuclear Physics make a surprising discovery: neutrinos produced in association with muons do not behave the same as those produced in association with electrons. They have, in fact, discovered a second type of neutrino (the muon neutrino). In 1968 “solar neutrinos” were observed.

In this experiment, now known as the Cowan–Reines neutrino experiment, antineutrinos created in a nuclear reactor by beta decay reacted with protons producing neutrons and positrons:

$\nu_e^- + p^+ \rightarrow n^0 + e^+$

The positron quickly finds an electron, and they annihilate each other. The two resulting gamma rays (γ) are detectable. The neutron can be detected by its capture on an appropriate nucleus, releasing a gamma ray. The coincidence of both events – positron annihilation and neutron capture – gives a unique signature of an antineutrino interaction.

2.1.2 Flavours of Neutrinos:-

There are three main flavours of neutrino:

1) Electron neutrino (ν_e)
2) Muon neutrino (ν_μ)
3) Tau neutrino (ν_T)

Similarly there are also three anti neutrino ($\overline{\nu_e}, \overline{\nu_\mu}, \overline{\nu_\tau}$). Here electron neutrino means it consists of some proportion of mass of electron, mass of muon & mass of tauon, but mass of electron is more than the mass of muon & mass of tauon so for that it was given a name electron neutrino similar for muon and tauon neutrino. The distinction between neutrinos and antineutrinos lies in the spin of these particles that is spin of neutrinos is opposite in the direction to the direction of its motion.

The possibility of sterile neutrinos relatively light neutrinos which don’t participate in the weak interaction but which could be created through flavour oscillations is unaffected by these Z-bosons based measurements, and the existence of such particles is in fact hinted by experimental data from the LSND experiment. A Sterile neutrino is a hypothetical neutrinos, which don't interact via any of the fundamental interactions. It is a right handed neutrino or left handed neutrino.

2.1.3 Properties of Neutrinos:-

The neutrino has half-integer spin (½ħ) and is therefore a fermion. Neutrinos interact primarily through the weak force. The discovery of neutrino flavor oscillations implies that neutrinos have mass. The existence of a neutrino mass strongly suggests the existence of a tiny neutrino magnetic moment of the order of 10^{-19} μB, allowing the possibility that neutrinos may interact electromagnetically as well.

Some of the properties of neutrinos are as follows:

SPEED:-

Before the idea of neutrino oscillations came up, it was generally assumed that neutrinos travel at the speed of light. The question of neutrino velocity is closely related to their mass. According to relativity, if neutrinos are mass less, they must travel at the speed of light.

In the early 1980's, first measurement of neutrino speed were done using pulsed pion beams. The pions decayed producing neutrinos and the neutrino interactions observed with in a time window in a detector at a distance were consistent with speed of light. This measurement has been repeated using MINOS detectors, which found the speed of 3 GeV neutrinos to be (1-(5.1+/-2.9)* 10^{-5}) times speed of light. This measurement set an upper bound on mass of muon neutrino of 50 MeV at 99% confidence. While the central value is lower than the speed of light, the uncertainty is great enough that it is very likely that the true velocity is too close to the speed of light to see the difference.

MASS:-

The Standard Model of particle physics assumed that neutrinos are mass less, although adding massive neutrinos to the basic framework is not difficult. Indeed, the experimentally established phenomenon of neutrino oscillation requires neutrinos to have nonzero masses.This was originally conceived by Bruno Pontecorvo in the 1950s. The strongest upper limit on the masses of neutrinos comes from cosmology: the Big Bang model predicts that there is a fixed ratio between the number of neutrinos and the number of photons in the cosmic microwave background. If the total energy of all three types of neutrinos exceeded an average of 50 electron volts per neutrino, there would be so much mass in the universe that it would collapse. This limit can be circumvented by assuming that the neutrino is unstable; however, there are limits within the Standard Model that make this difficult. A much more stringent constraint comes from a careful analysis of cosmological data, such as the cosmic microwave background radiation, galaxy surveys and the Lyman-alpha forest. These indicate that the sum of the neutrino masses must be less than 0.3 electron Volt.

In 1998, research results at the super-kamiokande neutrino detector determined that neutrinos do indeed flavor Oscillate, and therefore have mass. While this shows that neutrinos have mass, the absolute neutrino mass scale is still not known. This is because neutrino oscillations are sensitive only to the difference in the squares of the masses. The best estimate of the difference in the squares of the masses of mass Eigen states 1 and 2 was published by KamLAND in 2005: $\Delta m_{21}^2 = 0.000079\ \text{eV}^2$.

In 2006, the MINOS experiment measured oscillations from an intense muon neutrino beam, determining the difference in the squares of the masses between neutrino mass Eigen states 2 and 3. The initial results indicate $|\Delta m_{32}^2| = 0.0027\ \text{eV}^2$, consistent with previous results from Super-Kamiokande. Since $|\Delta m_{32}^2|$ is the difference of two squared masses, at least one of them has to have a value which is at least the square root of this value. Thus, there exists at least one neutrino mass Eigen state with a mass of at least 0.04 eV.

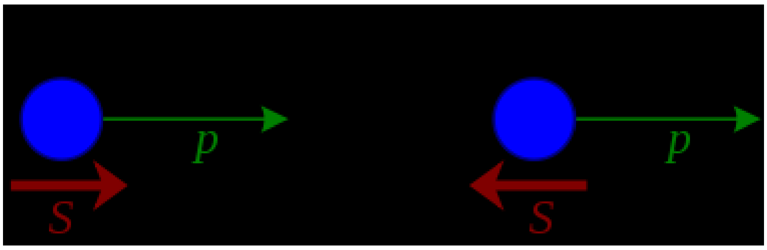
Size:-

The physical size of neutrinos can be defined using their electroweak radius (apparent size in electroweak interaction). The average electroweak characteristic size is $\langle r^2 \rangle = n \times 10^{-33}\ \text{cm}^2$ ($n \times 1$ nanobarn), where n = 3.2 for electron neutrino, n = 1.7 for muon neutrino and 1.0 for tau neutrino; it depends on no other properties than mass.

Handedness:-

Experimental results show that (nearly) all produced and observed neutrinos have left-handed helicities (spins antiparallel to momenta), and all antineutrinos have right-handed helicities, within the margin of error.

Helicity showing direction and momentum-



Right handed Left handed

Fig.2.1: Showing Right and Left handed Helicity

Antineutrinos:-

Antineutrinos are the antiparticles of neutrinos, which are neutral particles produced in nuclear beta decay. These are emitted in beta particle emissions, where a neutron turns into a proton. They have a spin of ½, and are part of the lepton family of particles. The antineutrinos observed so far all have right-handed helicity (i.e. only one of the two possible spin states has ever been seen), while the neutrinos are left-handed. Antineutrinos, like neutrinos, interact with other matter only through the gravitational and weak forces, making them very difficult to detect experimentally. Neutrino oscillation experiments indicate that antineutrinos have mass, but beta decay experiments constrain that mass to be very small. A neutrino-antineutrino interaction has been suggested in attempts to form a composite photon with the neutrino theory of light.

Because antineutrinos and neutrinos are neutral particles it is possible that they are actually the same particle. Particles which have this property are known as Majorana particles. If neutrinos are indeed Majorana particles then the neutrinoless double beta decay process is allowed. Several experiments have been proposed to search for this process.

Researchers around the world have begun to investigate the possibility of using antineutrinos for reactor monitoring in the context of preventing the proliferation of nuclear weapons.

Antineutrinos were first detected as a result of their interaction with protons in a large tank of water. This was installed next to a nuclear reactor as a controllable source of the antineutrinos.

2.1.4 Sources of Neutrinos:

1. Natural sources.
2. Artificial sources.

1. Natural sources:

- ❖ Atmospheric neutrinos
- ❖ Solar neutrinos
- ❖ Supernova neutrinos
- ❖ High energy neutrino sources
- ❖ Cosmic neutrino background

2. Artificial sources:

- ❖ Accelerator neutrinos
- ❖ Reactor neutrinos

Atmospheric neutrinos:-

Atmospheric neutrinos result from the interaction of cosmic rays with atomic nuclei in the Earth's atmosphere, creating showers of particles, many of which are unstable and produce neutrinos when they decay. A collaboration of particle physicists from Tata Institute of Fundamental Research (India), Osaka City University (Japan) and Durham University (UK) recorded the first cosmic ray neutrino interaction in an underground laboratory in Kolar Gold Fields in India in 1965.

Large detectors such as IMB, MACRO, and Kamiokande II observed a deficit in the ratio of the flux of muon to electron flavor atmospheric neutrinos (see muon decay). The Super Kamiokande experiment provided a very precise measurement of neutrino oscillation in an energy range of hundreds of MeV to a few TeV, and with a baseline of the diameter of the Earth: the first experimental evidence for atmospheric neutrino oscillations was announced in 1998.

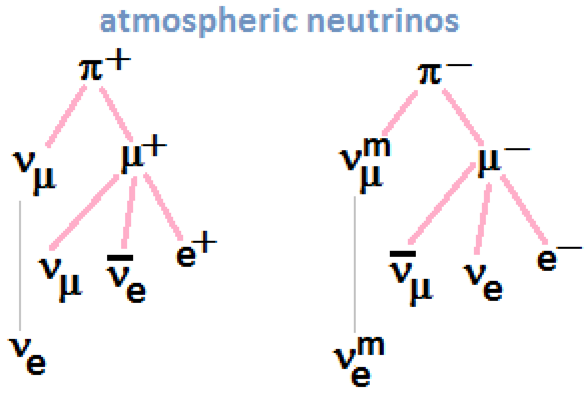


Fig. 2.2: Atmospheric Neutrinos

Solar neutrinos:-

Electron neutrinos are produced in the Sun as a product of nuclear fusion. By far the largest fraction of neutrinos passing through the Earth are Solar neutrinos. Solar neutrinos originate from the nuclear fusion powering the Sun and other stars. The details of the operation of the Sun are explained by the Standard Solar Model. In short: when four protons fuse to become one helium nucleus, two of them have to convert into neutrons, and each such conversion releases one electron neutrino.

The Sun sends enormous numbers of neutrinos in all directions. Every second, about 65 billion (6.5×10^{10}) solar neutrinos pass through every square centimeter on the part of the Earth that faces the Sun. Since neutrinos are insignificantly absorbed by the mass of the Earth, the surface area on the side of the Earth opposite the Sun receives about the same number of neutrinos as the side facing the Sun.

As we have seen, the evidence is strong that the overall reaction is "burning" hydrogen to make helium:

4 1H + 2 e^- --> 4He + 2 neutrinos + 6 photons

In this reaction, the final particles have less internal energy than the starting particles. Since energy is conserved, the extra energy is released as energy of motion of the nuclei in the solar gas and the production of lots of photons and, finally, the energy of the neutrinos. That is the gas gets hotter and has lots of photons. The neutrinos just zip right out of the Sun. The amount of energy involved is

26 MeV = 4.3×10^{-12} J

for each time the reaction above happens.

For each reaction, two neutrinos are made. The number is 2 because to turn 4 protons into 2 protons + 2 neutrons (a 4He) we need to change 2 protons into neutrons using the weak interaction.

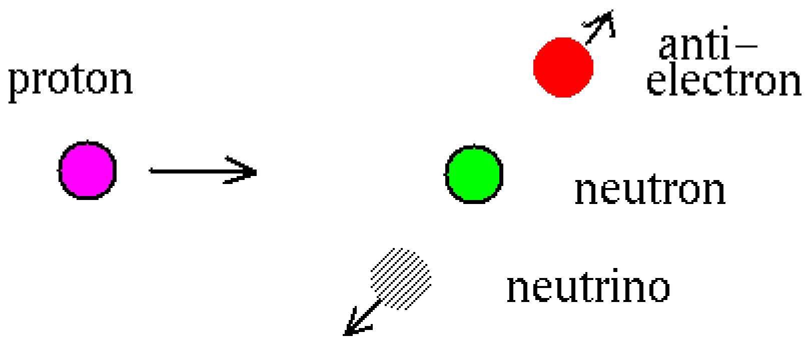


Fig.2.3: Reaction involving inside the sun

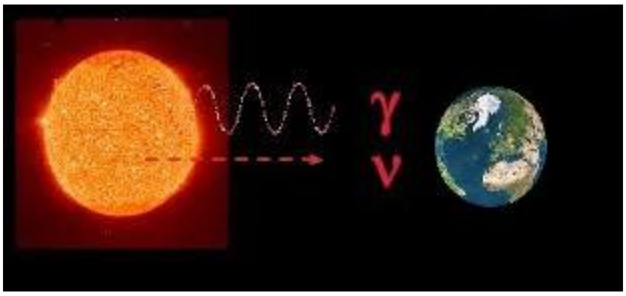


Fig.2.4: Solar Neutrinos

Supernova neutrinos:-

When a massive star at the end of its life collapses to a neutron star, it radiates almost all of its binding energy in the form of neutrinos, most of which have energies in the range 10-30 MeV. These neutrinos come in all flavors, and are emitted over a timescale of several tens of seconds. The neutrino luminosity of a gravitational collapse-driven supernova is typically 100 times its optical luminosity.

The neutrino signal emerges from the core of a star promptly after core collapse, whereas the photon signal may take hours or days to emerge from the stellar envelope. The neutrino signal can therefore give information about the very early stages of core collapse, which is inaccessible to other kinds of astronomy. In fact, an optical supernova display may never be seen at all for a given core collapse: some collapsing stars may never blow up into supernovae, or the star may live in an obscured region of the galaxy.

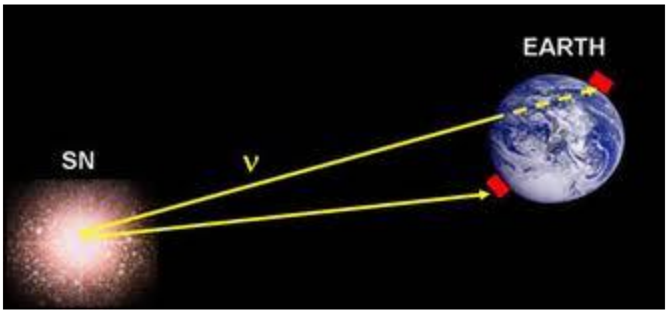


Fig.2.5: Supernova Neutrinos

High Energy Neutrino Sources:-

High energy cosmic neutrinos can be produced by protons and nuclei accelerated in cosmic sources as well as by relic Big Bang particles, cosmic strings etc. The most promising accelerating sources of supernovae in our galaxy and active galactic nuclei. High energy neutrinos can also be produced due to the decays or annihilation of massive particles. The postulated examples include annihilation of neutrinos in the earth and the sun, decays of neutralinos due to weak R-parity breaking, decays of heavy and superheavy particles of cosmological origin, and the production and decay of superheavy particles from cosmic strings.

Cosmic Neutrino Background:-

The cosmic neutrino background is the universe's background particle radiation composed of neutrinos. They are sometimes known as relic neutrinos.

Like the cosmic microwave background radiation left over the big bang, there is a background of low energy neutrinos in our universe. In 1980s it was proposed that these may be the explanation of dark matter thought to exit in the universe. From particle experiments, it was found that neutrinos are very light, this means that they moves as the same velocity as the velocity of light. Thus dark matter made from neutrinos is termed “hot dark matter”.

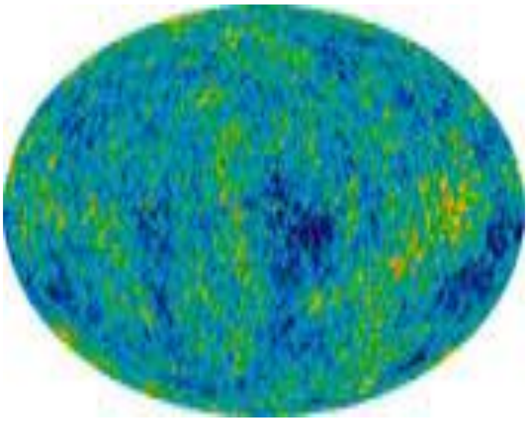


Fig.2.6: Cosmologic Neutrinos

Accelerator neutrinos:-

Now neutrino physics has entered into precision age from discovery era. Only in a well tuned, fully optimized environment will it be possible to perform precision measurements of neutrino oscillation parameters. A high intensity neutrino source with known spectrum is most desirable for precision measurements, the consensus direction for the future. Man-made accelerator based neutrino beams of the energy ranging typically between 30 MeV to 30 GeV are the novel, intense sources of neutrinos, an important tool for studying neutrino properties. Beam shape parameters play a very key role for the measurement of oscillation length, while the absolute normalization is crucial for the determination of the mixing angle. One of the most important features of the neutrino beam is that one can control the flux of the produced neutrinos and can tune the main parameters that govern the systematic uncertainties on the neutrino fluxes. Currently, there are three widely different schemes for producing neutrino beams and they mainly differ from each other on the issue of what particle is decaying (pion decay, muon decay and radioactive ion decay) to give rise to the neutrinos.

Reactor neutrinos:-

Electron antineutrinos with E ∼ MeV are produced copiously in the process of generating electrical power in nuclear power plants using controlled fission technique. A 3 GW plant releases about 7.7×10^{20} $\overline{\nu_e}$ per second and creates a flux of ∼ 6×10^{11} $cm^{-2}s^{-1}$ at 100 m. Due to the low energy, e's are the only charged leptons which can be produced in the neutrino CC interaction. If the $\overline{\nu_e}$ oscillated to another flavour, its CC interaction would not be observed. Therefore only disappearance experiments can be performed with reactors. The KamLAND experiment , a 1000 ton liquid scintillation detector, is currently in operation in the Kamioka mine in Japan. This underground site is located at an average distance of 150-210 km from several Japanese nuclear power stations. This experiment has played a crucial 6 role to establish the fact that the solar neutrino puzzle can be explained by the so-called LMA-MSW solution. Gosgen , Krasnoyarsk , Bugey , CHOOZ and Palo Verde are the examples of reactor experiments which are performed at relatively short or intermediate baselines. It is worth-while to mention here that none of these experiments find a positive evidence of flavour mixing.

2.1.5 Parameters Of Neutrinos:-

- **The angle θ:-** this is the so-called mixing angle. It defines how different the flavour states are from the mass states. If θ=0, the flavour states are identical to the mass states (that is, the ν_x will propagate from source to detector as a ν_x with definite momentum. Clearly in this case, oscillations cannot happen). If $\theta = \pi/4$ then the oscillations are said to be maximal and at some point along the path between source and detector all of the ν_x we started with will oscillate to ν_y.
- **The mass squared difference, Δm^2:-** If there are 2 flavours there will be 2 mass states. This parameter is the difference in squared masses of each of these states $\Delta m^2 = m_1^2 - m_2^2$. For neutrino oscillations to occur, at least one of the mass states must be non-zero. This simple statement has huge implications - for oscillations to happen, the neutrino must have mass. Furthur the masses of the mass states must be different, else $\Delta m^2 = 0$ and $P(\nu_x \rightarrow \nu_y) = 0$.
- **L/E:-** This is the parameter we, as experimentalists control. L is the distance between the source and the detector, and E is the energy of the neutrino. For a given Δm^2, the probability of oscillation will change as one moves away from the detector, or scans over different neutrino energy. Experimentally, if we suspect that Δm^2 has a particular value, then we should build our experiment to be maximally sensitive to the oscillation probability. That is, we want to build it such that

$$1.27\Delta m^2 L / E = \Pi / 2$$

or

$$L / E = \Pi / 2.54\Delta m^2$$

We are free then to either change the beam energy, or the baseline (L), or both. Ideally we want to maximise L and minimise E.

Neutrino Oscillations:-

Neutrino oscillations are a quantum mechanical phenomenon predicted by Bruno Pontecorvo whereby a neutrino created with a specific lepton flavor (electron, muon or tau) can later be measured to have a different flavor. The probability of measuring a particular flavor for a neutrino varies periodically as it propagates. Neutrino oscillation is of theoretical and experimental interest since observation of the phenomenon implies that the neutrino has a non-zero mass, which is not part of the original Standard Model of particle physics.

A zero mass particle would have to travel at the speed of light. At the speed of light, time stands still so no change (and, therefore, no oscillation) is possible. Therefore if particles change, they must have mass. Now we start discussing oscillations in vacuum.

Neutrino Oscillations in Vaccum:-

One-particle quantum mechanics is appropriate for describing neutrino oscillations. In all cases of practical interest neutrino fluxes are sufficiently weak that multi-particle Fermi-Dirac effects can be neglected. Concerning this aspect, a neutrino beam is simpler than an electro-magnetic field, That can be composed by in equivalent configurations of many photons. Therefore, one should

1) Build a neutrino wave-packet, taking into account the dynamics of the specific process that produces it, For example, atmospheric and beam neutrinos are mostly produced in $\prod$ and μ decays. Solar νe are produced in collisions and decays of light nuclei inside the sun. Reactor $\bar{\nu}_e$ in decays of fragments of fissioned heavy radioactive nuclei. Supernova neutrinos are produced mostly thermally.
2) Study its evolution. Different mass Eigen states acquire different phases, giving rise to oscillations. The mass difference also generates other effects. The lighter mass Eigen state moves faster than the heavier one: at some point their wave packets no longer overlap, destroying oscillations. While in neutrinos this effect is usually negligible, the mass differences between quarks are so large that there are no oscillations between quarks: e.g. the down type quark q produced in decays of charmed hadrons.
3) Compute the observable to be measured, taking into account what the detector is really doing. Oscillations are a quantum interference effect. The necessary coherence is destroyed if the

neutrino mass is measured (for example by measuring the neutrino energy and momentum) with enough precision to distinguish which one of the different neutrino mass eigen values has been detected.

2.2 Experiments Based On Solar Neutrinos:-

Abbreviation	Full name	Location	Operation	Type	Type of detector
BOREXINO	Boron Experiment	Gran Sasso,Italy	May 2007	ν_e	Scintillation
Double Chooz	Double Chooz reactor neutrino experiment	Chooz,France	November 2011	$\overline{\nu_e}$	Scintillation
SNO	Sudbury Neutrino Observatory	Creighton Mine, Ontario	1999-2006	ν_e, ν_μ, ν_τ	Cherenkov
HOMESTAKE CHLORINE	Homestake Chlorine Experiment	Homestake Mine, South Dakota	1967-1998	ν_e	Radiochemical
GALLEX	Gallium Experiment	Gran Sasso,Italy	1991-1997	ν_e	Radiochemical

Table: 2.1: Experiments for Solar Neutrions

2.3 Experiments Based On Atmospheric Neutrinos:-

Abbreviation	Full Name	Location	Operation	Type	Type of detector
MINOS	Main Injection Neutrino Oscillation Search	Illinois and Minnisota, United States	2005-	ν_e, ν_μ	Scintillation
Kamiokande	Kamioka Neucleon Decay Experiment	Kamioka, Japan	1986-1995	ν_e	Cherenkov
Super-K	Super- Kamiokande	Kamioka, Japan	1996-	ν_e, ν_μ, ν_τ	Cherenkov
IceCube	IceCube Neutrino Detector	South Pole, Antarctica	2006-	ν_e, ν_μ, ν_τ	Cherenkov
NEVOD	Cherenkov water detector , NEVOD	Moscow, Russia	1993-	ν_μ	Cherenkov

Table: 2.2: Experiments for Atmospheric Neutrinos

2.4 Hierarchy:-

The measurement of decay width of Z^0 boson at LEP constrains the number of neutrinos to three and only three light, active neutrinos. These three active neutrinos mix to form three mass eigen states. The mixing is parameterized by three mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$ and one CP -violating phase δ_{CP}. Survival and oscillation probabilities depends on the above four mixing parameters as well as on the two independent mass- squared differences: $\Delta m^2_{21} = m^2_2 - m^2_1$ and $\Delta m^2_{31} = m^2_3 - m^2_1$. Without loss of generality, we take Δm^2_{21} to be the smaller mass- squared differences (known as solar mass-squared differences, since it governs the oscillations of solar neutrinos) and Δm^2_{31} to be the larger mass-squared difference (known as atmospheric mass squared differences, since it governs the oscillations of atmospheric neutrinos). Solar neutrino data require Δm^2_{21} to be positive. However, data from atmospheric neutrinos as well as accelerator neutrino experiments (K2K and MINOS) constrain only the magnitude of

Δm_{31}^2 but not its sign. Determination of sign (Δm_{31}^2) is also called mass hierarchy determination, in the limit where the lightest mass eigen state is essentially massless.

There are two types of hierarchies:

1) <u>Normal Hierarchy(NH):-</u>

   If sign (Δm_{31}^2) is positive, then we have the following mass pattern $m_3 \succ\succ m_2 \succ\succ m_1$, which is similar to the charged lepton mass pattern. This is called normal hierarchy.

2) <u>Inverted Hierarchy(IH):-</u>

   If sign (Δm_{31}^2) is negative, then the mass pattern is $m_2 \geq m_1 \succ\succ m_3$. This is referred to as Inverted hierarchy.

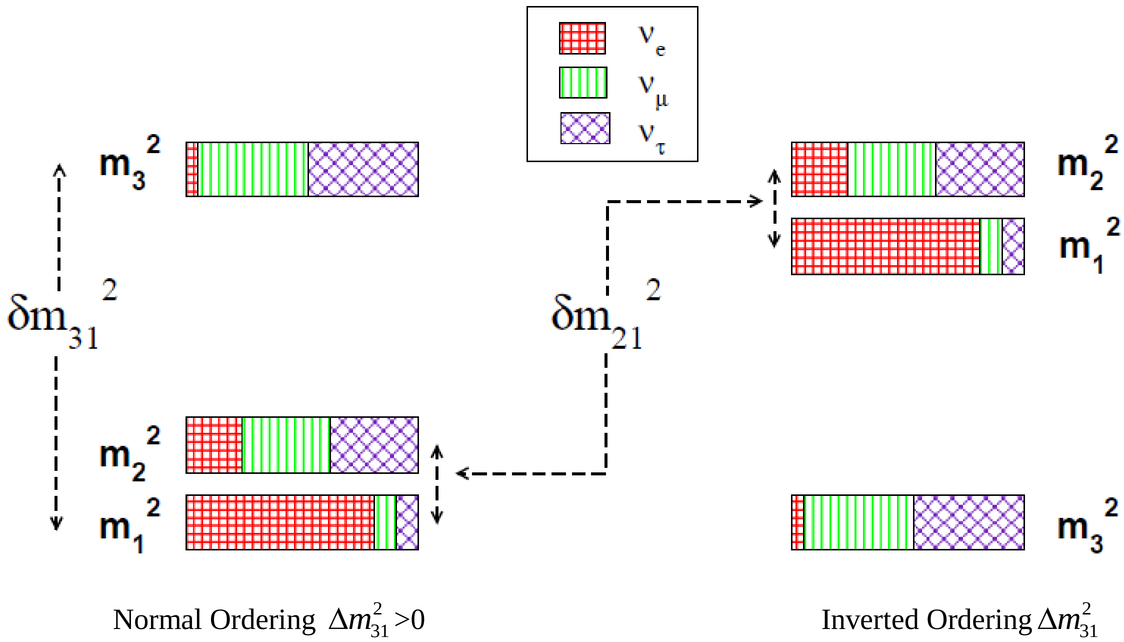


Fig. 2.7: The two possible hierarchies of neutrino mass eigen states normal hierarchy(left) and Inverted hierarchy(right). The flavour composition of the states in vaccum is also depicted for a standard set of parameters.

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3.1 Significance of L/E:-

As we know $L/E = \Pi / 2.54\Delta m^2$

Neutrino beams diverge like an electric field from a point source, so the surface area of a detector placed at a distance L has to grow by L^2, and so does the cost. At the same time, the neutrino cross-section decreases as the neutrino energy decreases and so the running time to collect a useful number of events increases linearly (and so does the cost). On the flip-side, if L/E is fixed for us by nature (as it is, for example, in solar neutrinos), then we can only probe parameters will yield too small a probability of oscillation for observation to be feasible (we may have to wait decades to get enough events). There are two types of neutrino oscillation experiments one could think of doing. The first is to start with a pure beam of known flavour ν_x, and look to see how many have disappeared. This is a "disappearance" experiment and measures the survival probability : $P(\nu_x \rightarrow \nu_x) = 1 - \sin^2(2\theta)\sin^2(1.27\Delta m^2\ L(km)/\ E(GeV\ )\ )$. The second type of experiment is an "appearance" experiment, in which one starts with a pure beam of known flavour νx and looks to see how many neutrinos of a different flavour ν_y are detected.

3.2 Unitary matrix:-

In mathematics, a complex square matrix U is unitary if

$$U^*U = UU^*$$

where I is the identity matrix and U * is the conjugate transpose of U.

The real analogue of a unitary matrix is an orthogonal matrix.

Properties:-

For any unitary matrix U, the following hold:

- Given two complex vectors x and y, multiplication by U preserves their inner product; that is,

$$\langle U_x U_y \rangle = \langle x, y \rangle$$

- U is normal
- U is diagonalizable; that is, U is unitarily similar to a diagonal matrix, as a consequence of the spectral theorem. Thus U has a decomposition of the form

$$U = VDV^{*}$$

  where V is unitary and D is diagonal and unitary.

- $|\det(\mathrm{U})| = 1$
- Its eigen spaces are orthogonal.
- For any positive integer n, the set of all n by n unitary matrices with matrix multiplication forms a group, called the unitary group U(n).
- Any square matrix with unit Euclidean norm is the average of two unitary matrices.

3.3 Dirac Delta Function:-

Since the smallest leptonic mixing angle θ_{13} has been measured to be relatively large, it is quite promising to constrain or determine the leptonic Dirac CP-violating phase δ in future neutrino oscillation experiments. Given some typical values of δ = π/2, π, and 3π/2 at the low-energy scale, as well as current experimental results of the other neutrino parameters, we perform a systematic study of the radiative corrections to δ by using the one-loop renormalization group equations in the minimal supersymmetric standard model and the universal extra-dimensional model. It turns out that δ is rather stable against radiative corrections in both models, except for the minimal supersymmetric standard model with a very large value of tan β. Both cases of Majorana and Dirac neutrinos are discussed. In addition, we use the preliminary indication of $\delta = (1.08^{+0.28}_{-0.31})\pi$ or $\delta = (1.67^{+0.37}_{-0.77})\pi$ from the latest global-fit analyses of data from neutrino oscillation experiments to illustrate how it will be modified by radiative corrections

3.4 Types Of Baseline:-

a) Long baseline:-

1. Explicitly observe the energy dependence in the survival probability P($(\nu_\mu \rightarrow \nu_\mu)$

2. Determine $|\Delta_{32}|$ and $\sin^2 2\theta_{23}$ as accurately as possible.

3. Measure the value of θ_{13} or improve the upper limit.

4. Obtain evidence for the modification of $\nu_\mu \rightarrow \nu_e$ oscillation probability due to matter effect and determine the sign of Δ_{32}, and

5. Search for CP violation in the lepton sector.

Experiments based on large baseline:-

➢ T2K:-

T2K (Tokai to Kamioka, Japan) is a particle physics experiment that is a collaboration between several countries, including Japan, Canada, France, Germany, Italy, South Korea, Poland, Russia, Spain, Switzerland, the United States, and the United Kingdom. It is the second generation follow up to the K2K experiment, a similar long baseline neutrino oscillation experiment. The main goal of T2K is to measure the oscillation of $\nu_\mu \rightarrow \nu_e$ and to measure the value of θ_{13}, one of the parameters of the Pontecorvo–Maki–Nakagawa–Sakata matrix

The goal of the T2K experiment is to gain a more complete understanding of neutrino oscillation parameters. Precise measurements of the other neutrino mixing parameter Δm^2_{23} and θ_{23} are another aim of the experiment. Future upgrades to T2K could yield measurement of the CP violation phase δ by comparing oscillations of neutrinos to those of antineutrinos.

➢ Super-Kamiokande:-

Super-Kamiokande (full name: Super-Kamioka Neutrino Detection Experiment, abbreviated to Super-K or SK) is a neutrino observatory which is under Mount Kamioka near the city of Hida, Gifu Prefecture, Japan. The observatory was designed to search for proton decay, study solar and atmospheric neutrinos, and keep watch for supernovae in the Milky Way Galaxy.

MicroBooNE:-

The Booster Neutrino Beamline (BNB) is a conventional neutrino beam at Fermilab which supplies a family of experiments with neutrinos of peak energy around 1GeV. The MiniBooNE experiment in the BNB has reported an excess of ν_e like events at low energy in both neutrino and antineutrino mode , and at intermediate energies in antineutrino mode . Many explanations have been proposed, including new oscillation physics, photon production in neutrino scattering events and the presence of misunderstood instrumental effects.

Neutrino interactions in MicroBooNE produce zero or more charged particles with a momentum in the beam direction.

Short Baseline:-

Recent results from short-baseline (SBL) neutrino oscillation studies show indications of oscillations in the $\Delta m^2 \sim 1eV^2$ region and, if corroborated by future more definitive experiments, may point towards the existence of sterile neutrinos. The strongest indication comes from the LSND experiment , which has reported a 3.8σ excess of $\overline{\nu_e}$ events in a beam of $\overline{\nu_\mu}$. The associated oscillation Δm^2 is too large to be explained with only three active neutrinos, and so oscillations involving sterile neutrinos are invoked. The LSND result is supported by an apparent excess of $\overline{\nu_e}$ events in a beam of $\overline{\nu_\mu}$ beam observed in MiniBooNE . This result is consistent with two-neutrino $\overline{\nu_\mu} \rightarrow \overline{\nu_e}$ oscillations at >99.4% confidence level.

3.5 Energy:-

a) High Energetic Neutrinos
b) Low Energetic Neutrinos

a) High Energetic Neutrinos:-

The high energy neutrinos are mainly produced in high energy particle collisions producing short lived mesons, decaying to neutrinos and other particles. The high energy neutrinos have energies above 10^{th} of GeV (Giga electron Volts).

High-energy (>100 MeV) neutrino astrophysics enters an era of opportunity and discovery as the sensitivity of detectors approaches astrophysically relevant flux levels. We review the major challenges for this emerging field, among which the nature of dark matter, the origin of cosmic rays, and the physics of extreme objects such as active galactic nuclei, gamma-ray bursts, pulsars, and supernova remnants are of prime importance. Variable sources at cosmological distances allow the probing of neutrino propagation properties over baselines up to about 20 orders of magnitude larger than those probed by terrestrial long-baseline experiments. We review the possible astrophysical sources of high-energy neutrinos, which also act as an irreducible background to searches for phenomena at the electroweak and grand-unified-theory symmetry-breaking scales related to possible supersymmetric dark matter and topological defects.

b) Low Energetic Neutrinos:-

The low energy neutrinos are mainly produced in nuclear processes, like the fusion reactions in the sun or in the center of an exploding Supernova. In a particle physics scale the low energy neutrinos have energies in the 10^{th} of MeV (Mega electron Volts).

3.6 Expected precision for θ_{13} :-

The improvement on value of $\sin^2 2\theta_{13}$ is expected to be about a factor of 2 over current bound from global data from MINOS, ICARUS and OPERA combined. Superbeam experiments T2K and NOvA separately will improve $\sin^2 2\theta_{13}$ by a factor of 6 over current bound from global data. D-CHOOZ aims to improve $\sin^2 2\theta_{13}$ by a factor of 4 over global current value including CHOOZ.

A non-zero value of $\sin^2 2\theta_{13}$ with an uncertainty of about 0.02 (D-CHOOZ + Daya Bay + Triple-CHOOZ).

3.7 Neutrino Oscillation Probability

3.7.1 Two Flavor Case:

Here ν_e, ν_μ be the flavour eigen states and ν_1, ν_2 be the mass eigen states with masses m_1 and m_2 respectively and both are having momentum p. States in the flavour and mass bases are related by a mixing matrix U. Where u is the orthogonal transformation for two dimensions.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$Where\, U = \begin{pmatrix} Cos\theta & Sin\theta \\ -Sin\theta & Cos\theta \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

At time t=0;

$$|\nu_e\rangle = Cos\theta\,|\nu_1\rangle + Sin\theta\,|\nu_2\rangle$$

$$|\nu_\mu\rangle = -Sin\theta\,|\nu_1\rangle + Cos\theta\,|\nu_2\rangle$$

At time t=t;

$$|\nu_e\rangle = Cos\theta\,|\nu_1\rangle e^{-iE_1 t/\hbar} + Sin\theta\,|\nu_2\rangle e^{-iE_2 t/\hbar} \cdots\cdots\cdots\cdots\cdots\cdots (3.1)$$

$$|\nu_\mu\rangle = -Sin\theta\,|\nu_1\rangle e^{-iE_1 t/\hbar} + Cos\theta\,|\nu_2\rangle e^{-iE_2 t/\hbar} \cdots\cdots\cdots\cdots\cdots (3.2)$$

And,

$$E_1^2 = p^2 + m_1^2 c^4$$

$$E_2^2 = p^2 + m_2^2 c^4$$

$$E_1 = p(1 + \mathrm{m}_1^2/\mathrm{p}^2)^{1/2}$$
$$E_2 = p(1 + \mathrm{m}_2^2/\mathrm{p}^2)^{1/2}$$

Now apply the bionomial theorem,

$$E_1 = p(1 + \mathrm{m}_1^2/2\,\mathrm{p}^2)$$
$$\mathrm{E}_2 = p(1 + \mathrm{m}_2^2/2\,\mathrm{p}^2)$$

Put the value of E_1 *and* E_2 *in eq.* (3.1) and (3.2);

$$|\nu_e\rangle = Cos\theta\,|\nu_1\rangle e^{-i(\mathrm{p}+\mathrm{m}_1^2/2\mathrm{p})\mathrm{t}} + Sin\theta\,|\nu_2\rangle e^{-i(\mathrm{p}+\mathrm{m}_2^2/2\mathrm{p})\mathrm{t}} \cdots\cdots\cdots\cdots\cdots (3.3)$$

$$|\nu_\mu\rangle = -Sin\theta\,|\nu_1\rangle e^{-i(\mathrm{p}+\mathrm{m}_1^2/2\mathrm{p})\mathrm{t}} + Cos\theta\,|\nu_2\rangle e^{-i(\mathrm{p}+\mathrm{m}_2^2/2\mathrm{p})\mathrm{t}} \cdots\cdots\cdots\cdots(3.4)$$

$$E^2 = p^2 + m^2c^4$$

$$E = p(1+\mathrm{m}^2/\mathrm{p}^2)^{1/2}$$

$$E = p(1+\mathrm{m}^2/2\,\mathrm{p}^2)$$

$$\mathrm{E} = \mathrm{p}+\mathrm{m}^2/2\,p$$

Now,

$$E^2 = p^2 + m^2$$

$$E \approx p$$

$$E \approx p + m^2/2E$$

$$|\nu_e\rangle = Cos\theta\,|\nu_1\rangle e^{-i(\mathrm{p}+\mathrm{m}_1^2/2\mathrm{p})\mathrm{t}} + Sin\theta\,|\nu_2\rangle e^{-i(\mathrm{p}+\mathrm{m}_2^2/2\mathrm{p})\mathrm{t}}$$

$$|\nu_e\rangle = e^{-i(\mathrm{p}+\mathrm{m}_1^2/2\mathrm{p})\mathrm{t}}(\mathrm{Cos}\,\theta\,|\nu_1\rangle + \mathrm{Sin}\,\theta\,|\nu_2\rangle\,\mathrm{e}^{-i(\mathrm{p}+\mathrm{m}_2^2/2\mathrm{p})\mathrm{t}}\mathrm{e}^{i(\mathrm{p}+\mathrm{m}_1^2/2\mathrm{p})\mathrm{t}})$$

$$|\nu_e\rangle = e^{-i(\mathrm{p}+\mathrm{m}_1^2/2\mathrm{p})\mathrm{t}}(\mathrm{Cos}\,\theta\,|\nu_1\rangle + \mathrm{Sin}\,\theta\,|\nu_2\rangle\,\mathrm{e}^{-i(\mathrm{m}_2^2-\mathrm{m}_1^2/2\mathrm{p})\mathrm{t}})$$

$$|\nu_e\rangle = e^{-i(\mathrm{p}+\mathrm{m}_1^2/2\mathrm{p})\mathrm{t}}(\mathrm{Cos}\,\theta\,|\nu_1\rangle + \mathrm{Sin}\,\theta\,|\nu_2\rangle\,\mathrm{e}^{-i(\Delta\mathrm{m}^2/2\mathrm{p})\mathrm{t}})$$

$$\Delta m^2 = m_2^2 - m_1^2\ ;$$

$$Put\ z = (\mathrm{p}+\mathrm{m}_1^2/2\,\mathrm{p})\,\mathrm{t}$$

$$|\nu_e\rangle = e^{-iz}(\mathrm{Cos}\,\theta\,|\nu_1\rangle + \mathrm{Sin}\,\theta\,|\nu_2\rangle\,\mathrm{e}^{-i(\Delta\mathrm{m}^2/2\mathrm{p})\mathrm{t}})\cdots\cdots\cdots\cdots(3.5)$$

$$|\nu_\mu\rangle = -Sin\theta\,|\nu_1\rangle e^{-i(\mathrm{p}+\mathrm{m}_1^2/2\mathrm{p})\mathrm{t}} + Cos\theta\,|\nu_2\rangle e^{-i(\mathrm{p}+\mathrm{m}_2^2/2\mathrm{p})\mathrm{t}}$$

$$|\nu_\mu\rangle = e^{-i(\mathrm{p}+\mathrm{m}_1^2/2\mathrm{p})\mathrm{t}}(-\mathrm{Sin}\,\theta\,|\nu_1\rangle + \mathrm{Cos}\,\theta\,|\nu_2\rangle\,\mathrm{e}^{-i(\mathrm{p}+\mathrm{m}_2^2/2\mathrm{p})\mathrm{t}}\mathrm{e}^{i(\mathrm{p}+\mathrm{m}_1^2/2\mathrm{p})\mathrm{t}})$$

$$|\nu_\mu\rangle = e^{-i(\mathrm{p}+\mathrm{m}_1^2/2\mathrm{p})\mathrm{t}}(-\mathrm{Sin}\,\theta\,|\nu_1\rangle + \mathrm{Cos}\,\theta\,|\nu_2\rangle\,\mathrm{e}^{-i(\mathrm{m}_2^2-\mathrm{m}_1^2/2\mathrm{p})\mathrm{t}})$$

$$|\nu_\mu\rangle = e^{-i(\mathrm{p}+\mathrm{m}_1^2/2\mathrm{p})\mathrm{t}}(-\mathrm{Sin}\,\theta\,|\nu_1\rangle + \mathrm{Cos}\,\theta\,|\nu_2\rangle\,\mathrm{e}^{-i(\Delta\mathrm{m}^2/2\mathrm{p})\mathrm{t}})$$

$$\Delta m^2 = m_2^2 - m_1^2\ ;$$

$$Put\ z = (\mathrm{p}+\mathrm{m}_1^2/2\,\mathrm{p})\,\mathrm{t}$$

$$|\nu_\mu\rangle = e^{-iz}(-\text{Sin}\,\theta\,|\nu_1\rangle + \text{Cos}\,\theta\,|\nu_2\rangle\,\text{e}^{-i(\Delta\text{m}^2/2\text{p})\text{t}})\cdots\cdots\cdots\cdots\cdots\cdots\cdots(3.6)$$

Now, Probability of neutrinos oscillation:-

(i) Probability of ν_e-------------->ν_e

P=|amplitude|^2

$$|\Psi|^2 = |\Psi^*\Psi|$$

$$|\Psi|^2 = |\langle\nu_e|\nu_e\rangle(\text{t})|^2$$

from equation (3.5) & (3.6)

$$|\nu_e\rangle = e^{-iz}(\cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle\,\text{e}^{-i(\Delta\text{m}^2/2\text{p})\text{t}})$$

$$|\nu_\mu\rangle = e^{-iz}(-\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle\,\text{e}^{-i(\Delta\text{m}^2/2\text{p})\text{t}})$$

$$\langle\nu_e|\nu_e\rangle = e^{-iz}(\cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle\,\text{e}^{-i(\Delta\text{m}^2/2\text{p})\text{t}})\,\text{e}^{-iz}(\cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle\,\text{e}^{-i(\Delta\text{m}^2/2\text{p})\text{t}})$$

$$\langle\nu_e|\nu_e\rangle = e^{-2iz}(\cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle\,\text{e}^{-i(\Delta\text{m}^2/2\text{p})\text{t}})(\cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle\,\text{e}^{-i(\Delta\text{m}^2/2\text{p})\text{t}})$$

$$\delta_{ij} = 1 \quad if\ i = j$$
$$\delta_{ij} = 0 \quad if\ i \neq j$$

$$\langle\nu_e|\nu_e\rangle = e^{-2iz}(\cos^2\theta + \sin^2\theta\,\text{e}^{-i(\Delta 2\text{m}^2/2\text{p})\text{t}})$$

$$\langle\nu_e|\nu_e\rangle = e^{-2iz}(\cos^2\theta + \sin^2\theta\,\text{e}^{-i(\Delta\text{m}^2/\text{p})\text{t}}) = \psi$$

$$\langle\nu_e|\nu_e\rangle = e^{2iz}(\cos^2\theta + \sin^2\theta\,\text{e}^{i(\Delta\text{m}^2/\text{p})\text{t}}) = \psi^*$$

$$|\psi\psi^*| = |\psi|^2 = e^{-2iz}e^{2iz}(\cos^4\theta + \sin^4\theta + \cos^2\theta\sin^2\theta\,\text{e}^{-i(\Delta\text{m}^2/\text{p})\text{t}} + \cos^2\theta\sin^2\theta\,\text{e}^{i(\Delta\text{m}^2/\text{p})\text{t}})$$

$$= (\cos^2\theta + \sin^2\theta)^2 - 2\sin^2\theta\cos^2\theta + \cos^2\theta\sin^2\theta(\text{e}^{-i(\Delta\text{m}^2/\text{p})\text{t}} + \text{e}^{i(\Delta\text{m}^2/\text{p})\text{t}})$$

$$= 1 - 2sin^2\theta\cos^2\theta + 2\cos^2\theta\sin^2\theta\left[e^{-i(\Delta\text{m}^2/\text{p})\text{t}} + e^{i(\Delta\text{m}^2/\text{p})\text{t}}\right]/2$$

$$= 1 - 2\sin^2\theta\cos^2\theta\left[1 - \cos(\Delta\,\text{m}^2/\,\text{p})\,\text{t}\right]$$

$$=1-4\sin^2\theta\cos^2\theta.\sin^2(\Delta m^2/4p)t$$
$$=1-\sin^2 2\theta.\sin^2(\Delta\,\mathrm{m}^2/4\,\mathrm{p})\,\mathrm{t}$$

(ii) Probability of ν_e-------------->ν_μ

P=|amplitude|^2

$$|\Psi|^2=|\Psi^*\Psi|$$

$$|\Psi|^2=|\langle\nu_e|\nu_\mu\rangle(\mathrm{t})|^2$$

from equation (3.5);

$$|\nu_e\rangle=e^{-iz}(\cos\theta|\nu_1\rangle+\sin\theta|\nu_2\rangle\,\mathrm{e}^{-i(\Delta\mathrm{m}^2/2\mathrm{p})\mathrm{t}})$$

$$\langle\nu_e|\nu_\mu\rangle=e^{-iz}(\cos\theta|\nu_1\rangle+\sin\theta|\nu_2\rangle\,\mathrm{e}^{-i(\Delta\mathrm{m}^2/2\mathrm{p})\mathrm{t}})\,\mathrm{e}^{-iz}(-\sin\theta|\nu_1\rangle+\cos\theta|\nu_2\rangle\,\mathrm{e}^{-i(\Delta\mathrm{m}^2/2\mathrm{p})\mathrm{t}})$$

$$\langle\nu_e|\nu_\mu\rangle=e^{-2iz}(\cos\theta|\nu_1\rangle+\sin\theta|\nu_2\rangle\,\mathrm{e}^{-i(\Delta\mathrm{m}^2/2\mathrm{p})\mathrm{t}})(\sin\theta|\nu_1\rangle+\cos\theta|\nu_2\rangle\,\mathrm{e}^{-i(\Delta\mathrm{m}^2/2\mathrm{p})\mathrm{t}})$$

$$\delta_{i,j}=1\quad if\ i=j$$
$$\delta_{i,j}=0\quad if\ i\neq j$$

$$\langle\nu_e|\nu_\mu\rangle=e^{-2iz}(-\cos\theta\sin\theta+\cos\theta\sin\theta e^{-i(\Delta 2\mathrm{m}^2/2\mathrm{p})\mathrm{t}})$$

$$\langle\nu_e|\nu_\mu\rangle=e^{-2iz}(-\sin\theta\cos\theta+\sin\theta\cos\theta\,\mathrm{e}^{-i(\Delta\mathrm{m}^2/\mathrm{p})\mathrm{t}})=\Psi$$

$$\langle\nu_e|\nu_\mu\rangle=e^{2iz}(-\cos\theta\sin\theta+\cos\theta\sin\theta\,\mathrm{e}^{i(\Delta\mathrm{m}^2/\mathrm{p})\mathrm{t}})=\Psi^*$$

$$|\Psi\Psi^*|=|\Psi|^2=e^{-2iz}e^{2iz}(\sin^2\theta\cos^2\theta-\cos^2\theta\sin^2\theta\,\mathrm{e}^{i(\Delta\mathrm{m}^2/\mathrm{p})\mathrm{t}}-\cos^2\theta\sin^2\theta\,\mathrm{e}^{-i(\Delta\mathrm{m}^2/\mathrm{p})\mathrm{t}}+\sin^2\theta\cos^2\theta)$$

$$=(2\sin^2\theta\cos^2\theta-\cos^2\theta\sin^2\theta)(\mathrm{e}^{i(\Delta\mathrm{m}^2/\mathrm{p})\mathrm{t}}+\mathrm{e}^{-i(\Delta\mathrm{m}^2/\mathrm{p})\mathrm{t}})$$

$$=2\sin^2\theta\cos^2\theta\left[1-(\mathrm{e}^{-i(\Delta\mathrm{m}^2/\mathrm{p})\mathrm{t}}+\mathrm{e}^{i(\Delta\mathrm{m}^2/\mathrm{p})\mathrm{t}})/2\right]$$

$$=2\sin^2\theta\cos^2\theta(1-\cos(\Delta\,\mathrm{m}^2/\mathrm{p})\,\mathrm{t})$$

$$= 2\sin^2\theta\cos^2\theta . 2\sin^2(\Delta m^2/4p)t$$
$$= \sin^2 2\theta . \sin^2(\Delta m^2/4p)t$$

3.7.2 Three Flavor Case:-

Neutrinos interact according to their three flavor states ν_e, ν_μ, ν_τ . These flavors form a useful basis for describing the state of a neutrino. An equally good basis for describing states is the mass of the neutrino. Since mass and flavor are incompatible observables, they are not simultaneously good quantum labels. However, we know that the two bases are related by a unitary transformation.

$$|\nu_\alpha\rangle = \sum_{K=1}^{2} U^*_{\alpha K} |\nu_K\rangle \cdots\cdots\cdots(3.7)$$

$$|\nu_\beta\rangle = \sum_{K=1}^{2} U^*_{\beta K} |\nu_K\rangle \cdots\cdots\cdots(3.8)$$

Where α and β are flavour states and K is the mass state.

$$|\nu_K\rangle = \sum_{K=1}^{2} U_{\beta K} |\nu_\beta\rangle \cdots\cdots\cdots(3.9)$$

Put (3.9) in (3.7) we get ,

$$|\nu_\alpha\rangle = \sum_{K=1}^{2} U^*_{\alpha K} U_{\beta K} |\nu_\beta\rangle e^{-iE_K t} \cdots\cdots\cdots(3.10)$$

$$\sum_{K=1}^{2} U^*_{\alpha K} U_{\beta K} e^{-iE_K t} = A_{\nu_\alpha \to \nu_\beta}(t) \cdots\cdots\cdots(3.11)$$

From (3.11), (3.10) becomes,

$$|\nu_\alpha\rangle = A_{\nu_\alpha \to \nu_\beta}(t).|\nu_\beta\rangle$$

Now Probability of changing $\nu_\alpha \to \nu_\beta$ is $P_{\nu_\alpha \to \nu_\beta}$

So, $P_{\nu_\alpha \to \nu_\beta} = |\Psi^* \Psi|^2 \cdots\cdots\cdots (3.12)$

$P_{\nu_\alpha \to \nu_\beta} = |A_{\nu_\alpha \to \nu_\beta}|^2 \cdots\cdots\cdots (3.13)$

Put (3.11) in (3.13) we get,

$$P_{\nu_\alpha \to \nu_\beta} = |\sum_{K=1}^{2} U_{\alpha K}^* U_{\beta K} e^{-iE_K t}|^2 \cdots\cdots\cdots (3.14)$$

Use (3.12) in (3.14) it becomes,

$$\Psi\Psi^* = (\sum_{K=1}^{2} U_{\alpha K}^* U_{\beta K} e^{-iE_K t})(\sum_{K=1}^{2} U_{\alpha K} U_{\beta K}^* e^{iE_K t}) \cdots\cdots\cdots (3.15)$$

Here $\begin{matrix} \alpha = e \\ \beta = \mu \end{matrix}$

Put in (3.15) we get,

$$\Psi\Psi^* = (\sum_{K=1}^{2} U_{eK}^* U_{\mu K} . e^{-iE_K t})(\sum_{K=1}^{2} U_{eK} . U_{\mu K}^* e^{iE_K t})$$

Now,

$$= (U_{e1}^* U_{\mu 1} e^{-iE_1 t} + U_{e2}^* U_{\mu 2} e^{-iE_2 t})(U_{e1} U_{\mu 1}^* e^{iE_1 t} + U_{e2} U_{\mu 2}^* e^{iE_2 t})$$

$$= U_{e1}^* U_{\mu 1} . U_{\mu 1}^* U_{e1} + U_{e1}^* U_{e2} U_{\mu 1} U_{\mu 2}^* e^{-it(E_1 - E_2)} + U_{e2}^* U_{e1} U_{\mu 2} U_{\mu 1}^* e^{-it(E_2 - E_1)} + U_{\mu 2}^* U_{\mu 2} U_{e2}^* U_{e2}$$

$$= 2 + U_{e1}^* U_{e2} U_{\mu 1} U_{\mu 2}^* e^{it(E_2 - E_1)} + U_{e2}^* U_{e1} U_{\mu 2} U_{\mu 1}^* e^{-it(E_2 - E_1)} \cdots\cdots\cdots (3.16)$$

Let $U_{e1}^* U_{e2} U_{\mu 1} U_{\mu 2}^* e^{it(E_2 - E_1)} = a$

Equation (3.16) becomes,

$$= 2 + (a + a^*)$$

Take $\begin{matrix} a = x + iy \\ a^* = x - iy \end{matrix}$

And,

Real part is $a+a^{*}=2x$

Imaginary part is $a-a^{*}=2iy$

$$=2+2\,\mathrm{Re}[U^{*}_{e1}U^{*}_{\mu 2}U_{e2}U_{\mu 1}e^{it(E_2-E_1)}+U_{e1}U_{\mu 2}U^{*}_{e2}U^{*}_{\mu 1}e^{-it(E_2-E_1)}]$$

$$=2+2\,\mathrm{Re}[U^{*}_{e1}U^{*}_{\mu 2}U_{e2}U_{\mu 1}\cos(E_2-E_1)\,t+U_{e1}\,U_{\mu 2}U^{*}_{e2}U^{*}_{\mu 1}\cos(E_2-E_1)\,t]$$

$$=2+2\,\mathrm{Re}[U^{*}_{e1}U^{*}_{\mu 2}U_{e2}U_{\mu 1}+U_{e1}U_{\mu 2}U^{*}_{e2}U^{*}_{\mu 1}]\cos(E_2-E_1)\,t$$

$$P_{\nu_\alpha\to\nu_\beta}=|\sum_{K=1}^{n}U_{\beta K}\,U^{*}_{\alpha K}e^{-iE_K t}\,|^2$$

$$\sum(U_{\beta K}U^{*}_{\alpha K}e^{-iE_K t})\sum(U^{*}_{\beta K'}U_{\alpha K'}e^{iE_{K'}t})$$

$$\sum_{K=1}^{n}U_{\beta K}U^{*}_{\alpha K}\sum_{K'=1}^{n}U^{*}_{\beta K'}U_{\alpha K'}\sum e^{-iE_K t}.e^{iE_{K'}t}$$

Or,

$$\sum_{K=1}^{n}U_{\beta K}U^{*}_{\alpha K}\sum_{K'=1}^{n}U^{*}_{\beta K'}U_{\alpha K'}\sum e^{-i(E_K-E_{K'})t}$$

$$E_{K'}-E_K=\Delta m^2_{K,K'}/2E$$

And $t\approx L$

$$\sum U_{\beta K}U^{*}_{\alpha K}\sum U^{*}_{\beta K'}U_{\alpha K'}\sum_{K,K'=1}^{n}\exp(-i\,\Delta\,m^2_{K,K'=1}L)/2E$$

$$\sum|U_{\beta K}|^2|U_{\alpha K}|^2+2\,\mathrm{Re}\sum_{K\succ K'}(U_{\beta K}U^{*}_{\alpha K}U^{*}_{\beta K'}U_{\alpha K'}e^{-i\Delta m^2_{K,K'}L/2E})$$

Further, from the unitary relation we can easily obtain the following relation,

$$\sum|U_{\beta K}|^2|U_{\alpha K}|^2=\delta_{\alpha\beta}-2\,\mathrm{Re}\sum_{K\succ K'}(U_{\beta K}U^{*}_{\alpha K}U^{*}_{\beta K'}U_{\alpha K'})\cdots\cdots\cdots\cdots\cdots\cdots(3.17)$$

$$=\delta_{\alpha\beta}-2\,\mathrm{Re}\sum_{K\succ K'}(U_{\beta K}U^{*}_{\alpha K}U^{*}_{\beta K'}U_{\alpha K'})+2\,\mathrm{Re}\sum_{K\succ K'}U_{\beta K}U^{*}_{\alpha K}U^{*}_{\beta K'}U_{\alpha K'}e^{-i\Delta m^2_{K,K'}L/2E}$$

$$= \delta_{\alpha\beta} - 2\,\text{Re} \sum_{K \succ K'} U_{\beta K} U^{*}_{\alpha K} U^{*}_{\beta K'} U_{\alpha K'} (1 - e^{-i\Delta m^2_{K \succ K'} L/2E})$$

Finally for any complex a and b, Re(ab)= Re(a)Re(b)−Im(a)Im(b),

$$= \delta_{\alpha\beta} - 2\,\text{Re} \sum_{K \succ K'} (U_{\beta K} U^{*}_{\alpha K} U^{*}_{\beta K'} U_{\alpha K'})(1 - \cos\Delta m^2_{K \succ K'} L/2E + i\sin\Delta m^2_{K \succ K'} L/2E)$$

$$= \delta_{\alpha\beta} - 2\,\text{Re} \sum_{K \succ K'} (U_{\beta K} U^{*}_{\alpha K} U^{*}_{\beta K'} U_{\alpha K'})(1 - \cos\Delta m^2_{K \succ K'} L/2E) - 2 \sum_{K \succ K'} \text{Im}(U_{\beta K} U^{*}_{\alpha K} U^{*}_{\beta K'} U_{\alpha K'})(\sin\Delta m^2_{K \succ K'} L/2E)$$

Here $U = \begin{matrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta_{13}} \\ -S_{12}C_{23} - C_{12}S_{23}S_{13}e^{i\delta_{13}} & C_{12}C_{23} - S_{12}S_{23}S_{13}e^{i\delta_{13}} & S_{23}C_{13} \\ S_{12} - C_{12}C_{23}S_{13}e^{i\delta_{13}} & -C_{12}S_{23} - S_{12}C_{23}S_{13}e^{i\delta_{13}} & C_{23}C_{13} \end{matrix}$

The phase δ is responsible for effects of the CP violation which can take values from 0 to 2 Π. The mixing angles are parameters which can take values in the ranges $0 \le \theta_{12} \le \pi$, $0 \le \theta_{13} \le \pi$, $0 \le \theta_{23} \le \pi$. For three neutrino oscillation in vacuum, all real parts of the quadratic products of elements of the mixing matrix entering in the three-neutrino oscillation probabilities is given as:. The individual probability expression for three neutrino flavors changing into others can be achieved by solving for individual matrix elements. For electron to muon transition: $P(\nu_e \to \nu_\mu)$ can be calculated as:

$$\text{Re}(U_{\beta K} U^{*}_{\alpha K} U^{*}_{\beta K'} U_{\alpha K'}) = (U_{22} U^{*}_{12} U^{*}_{21} U_{11}) \;\; for\; \beta = 2, \alpha = 1, K = 2, K' = 1$$

$$= (C_{12}C_{23} - S_{12}S_{23}S_{13}e^{i\delta_{13}})(-S_{12}C_{23} - C_{12}S_{23}S_{13}e^{-i\delta_{13}})(S_{12}C_{13})(C_{12}C_{13})$$

$$= (-C_{12}S_{12}C^2_{23} - C^2_{12}C_{23}S_{13}S_{23}e^{-i\delta_{13}} + S^2_{12}S_{13}C_{23}S_{23}e^{i\delta_{13}} + S_{12}C_{12}S^2_{13}S^2_{23})(C^2_{13}S_{12}C_{12})$$

$$= -1/4C^2_{13}Sin2\theta_{12}(C^2_{23}\text{Sin}\,2\theta_{12} + C^2_{12}S_{13}\text{Sin}\,2\theta_{23}\,e^{-i\delta_{13}} - S^2_{12}S_{13}\text{Sin}\,2\theta_{23}\,e^{i\delta_{13}} - S^2_{23}S^2_{13}\text{Sin}\,2\theta_{12})$$

$$= -1/4C^2_{13}Sin2\theta_{12}(C^2_{23}\text{Sin}\,2\theta_{12} + C^2_{12}S_{13}\text{Sin}\,2\theta_{23}\,\text{Cos}\,\delta_{13} - S^2_{12}S_{13}\text{Sin}\,2\theta_{23}\,\text{Cos}\,\delta_{13} - S^2_{23}S^2_{13}\text{Sin}\,2\theta_{12})$$

$$= -1/4C^2_{13}Sin2\theta_{12}[\text{Sin}\,2\theta_{12}(C^2_{23} - S^2_{23}S^2_{13}) + S_{13}\text{Sin}\,2\theta_{23}\,\text{Cos}\,\delta_{13}(\text{Cos}^2\theta_{12} - \text{Sin}^2\theta_{12})]$$

$$\text{Re}(U_{\beta K} U^{*}_{\alpha K} U^{*}_{\beta K'} U_{\alpha K'}) = (U_{23} U^{*}_{13} U^{*}_{21} U_{11}) \;\; for\; \beta = 2, \alpha = 1, K = 3, K' = 1$$

$$= (S_{23}C_{13})(S_{13}e^{i\delta_{13}})(-S_{12}C_{23} - C_{12}S_{23}S_{13}e^{-i\delta_{13}})(C_{12}C_{13})$$

$$= S_{23}C^2_{13}S_{13}C_{12}e^{i\delta_{13}}(-S_{12}C_{23} - C_{12}S_{23}S_{13}e^{-i\delta_{13}})$$

$$= -C_{13}^2 S_{13} C_{12} S_{23} (\mathrm{C_{23} S_{12} e^{i\delta_{13}} + S_{13} S_{23} C_{12}})$$

$$\mathrm{Re}(\mathrm{U_{\beta K} U^*_{\alpha K} U^*_{\beta K'} U_{\alpha K'}}) = (U_{23} U^*_{13} U^*_{22} U_{12}) \ \ for\ \beta = 2, \alpha = 1, K = 3, K' = 2$$

$$= (\mathrm{S_{23} C_{13}})(\mathrm{S_{13} e^{i\delta_{13}}})(\mathrm{C_{12} C_{23} - S_{12} S_{23} S_{13} e^{-i\delta_{13}}})(\mathrm{S_{12} C_{13}})$$

$$= (\mathrm{S_{23} C_{13}^2 S_{12} S_{13} e^{i\delta_{13}}})(\mathrm{C_{12} C_{23} - S_{12} S_{23} S_{13} e^{-i\delta_{13}}})$$

$$= -S_{23} C_{13}^2 S_{12} S_{13} (\mathrm{S_{12} S_{23} S_{13} - C_{12} C_{23} e^{i\delta_{13}}})$$

$$= -S_{23} C_{13}^2 S_{12} S_{13} (\mathrm{S_{12} S_{23} S_{13} - C_{12} C_{23} Cos\, \delta_{13}})$$

Now, $P(\nu_e \to \nu_\mu)$

$$= \delta_{21} - 4[-1/4\, \mathrm{C_{13}^2 Sin\, 2\theta_{12} (Sin\, 2\theta_{12} (C_{23}^2 - S_{23}^2 S_{13}^2) + Cos\, 2\theta_{12}\ Sin\, 2\theta_{23}\ S_{13} Cos\, \delta_{13})] Sin\, \Delta\, m_{21}^2}\ L/2E$$

$$+\delta_{31} - 4[-\mathrm{C_{13}^2 S_{13} C_{12} S_{23} (C_{23} S_{12} e^{i\delta_{13}} + S_{13} S_{23} C_{12})] Sin\, \Delta\, m_{31}^2}\ L/4E$$

$$+\delta_{32} - 4[-\mathrm{S_{23} C_{13}^2 S_{12} S_{13} (S_{12} S_{23} S_{13} - C_{12} C_{23} Cos\, \delta_{13})] Sin}\, \Delta m_{32}^2 L/4E$$

For atmospheric neutrinos,

$$\Delta m_{32}^2 \cong \Delta m_{31}^2 \cong \Delta m_{atm}^2$$
$$\Delta m_{21}^2 \cong \Delta m_{sol}^2 \cong 0$$

$$P(\nu_e \to \nu_\mu)$$
$$= -4[-\mathrm{C_{13}^2 S_{13}^2 S_{12}^2 S_{23}^2 + S_{12} S_{23} S_{13} C_{23} C_{12} C_{13}^2 Cos\, \delta_{13}} - S_{12}\ \mathrm{S_{23} S_{13} C_{23} C_{12} C_{13}^2 Cos\, \delta_{13} - C_{13}^2 S_{13}^2 C_{12}^2 S_{23}^2] Sin\, \Delta\, m_{atm}^2}\ L/4E$$
$$= -4[-\mathrm{C_{13}^2 S_{23}^2 S_{13}^2 (S_{12}^2 + C_{12}^2)] sin\, \Delta\, m_{atm}^2}\ L/4E$$

$$= 4 \sin^2 \theta_{23} \cos^2 \theta_{13} \sin^2 \theta_{13} \sin \Delta m_{atm}^2 L/4E$$

$$P(\nu_e \to \nu_\mu) = \sin^2(2\theta_{13}) \sin^2 \theta_{23} \sin \Delta\, \mathrm{m}_{atm}^2\ L/4E \cdots\cdots\cdots\cdots\cdots (3.18)$$

$$P(\nu_e \to \nu_\tau) = \sin^2(2\theta_{13}) \cos^2 \theta_{23} \sin \Delta m_{atm}^2 L/4E \cdots\cdots\cdots\cdots\cdots (3.19)$$

The survival probability of electron neutrinos is,

$$P(\nu_e \to \nu_e) = 1 - [\mathrm{P}(\nu_e \to \nu_\mu) + \mathrm{P}(\nu_e \to \nu_\tau)]$$

$$= 1 - [\sin^2(2\theta_{13}) \sin^2(\theta_{23}) \sin \Delta\, \mathrm{m}_{atm}^2 \mathrm{L}/4\mathrm{E} + \sin^2(2\theta_{13}) \cos^2(\theta_{23}) \sin \Delta\, \mathrm{m}_{atm}^2 \mathrm{L}/4\mathrm{E}]$$

$$= 1 - [\sin^2(2\theta_{13}) \sin \Delta\, \mathrm{m}_{atm}^2 \mathrm{L}/4\mathrm{E} (\sin^2 \theta_{23} + \cos^2 \theta_{23})]$$

$$P(\nu_e \to \nu_e) \;\; = 1 - \sin^2(2\theta_{13}) \sin \Delta \mathrm{m}^2_{atm} \, L/4E \cdots\cdots\cdots\cdots\cdots (3.20)$$

In the case of oscillation in vacuum, electron neutrinos going into muon neutrinos and vice-versa probability remains the same.

Therefore in case of vacuum,

$$P(\nu_e \to \nu_\mu) \to \mathrm{P}(\nu_\mu \to \nu_e)$$

$$P(\nu_\mu \to \nu_e) = \sin^2(2\theta_{13}) \sin^2\theta_{23} \sin \Delta \mathrm{m}^2_{atm} \, L/4E \cdots\cdots\cdots\cdots\cdots (3.21)$$

Similarly we can calculate the probability for muon neutrinos changing into tau neutrinos,

$$P(\nu_\mu \to \nu_\tau) = \sin^2(2\theta_{23}) \cos^4(\theta_{13}) \sin \Delta \mathrm{m}^2_{atm} \, L/4E \cdots\cdots\cdots\cdots\cdots (3.22)$$

Now we will calculate the survival probability of muon neutrinos i.e.

$$P(\nu_\mu \to \nu_\mu) = 1 - [\mathrm{P}(\nu_\mu \to \nu_e) + \mathrm{P}(\nu_\mu \to \nu_\tau)]$$

$$= 1 - [\sin^2(2\theta_{13}) \sin^2(\theta_{23}) \sin \Delta \mathrm{m}^2_{atm} \mathrm{L}/4\mathrm{E} + \sin^2(2\theta_{23}) \cos^4(\theta_{13}) \sin \Delta \mathrm{m}^2_{atm} \mathrm{L}/4\mathrm{E}]$$

$$= 1 - [\sin^2(2\theta_{13}) \sin^2(\theta_{23}) + \sin^2(2\theta_{23}) \cos^4(\theta_{13})] \sin \Delta \mathrm{m}^2_{atm} \, L/4E \cdots\cdots\cdots (3.23)$$

Probability	**Expression**
$P(\nu_e \to \nu_\mu)$	$\sin^2(2\theta_{13}) \sin^2\theta_{23} \sin \Delta \mathrm{m}^2_{atm} \, L/4E$
$P(\nu_e \to \nu_\tau)$	$\sin^2(2\theta_{13}) \cos^2\theta_{23} \sin \Delta m^2_{atm} L/4E$
$P(\nu_e \to \nu_e)$	$1 - \sin^2(2\theta_{13}) \sin \Delta \mathrm{m}^2_{atm} \, L/4E$
$P(\nu_\mu \to \nu_e)$	$\sin^2(2\theta_{13}) \sin^2\theta_{23} \sin \Delta \mathrm{m}^2_{atm} \, L/4E$
$P(\nu_\mu \to \nu_\tau)$	$\sin^2(2\theta_{23}) \cos^4(\theta_{13}) \sin \Delta \mathrm{m}^2_{atm} \, L/4E$
$P(\nu_\mu \to \nu_\mu)$	$= 1 - [\sin^2(2\theta_{13}) \sin^2(\theta_{23}) + \sin^2(2\theta_{23}) \cos^4(\theta_{13})] \sin \Delta \mathrm{m}^2_{atm} \, L/4E$

Table: 3.1: Showing the expression for different probabilities of atmospheric neutrinos.

Solar Neutrinos in case of vaccum:-

The fact that neutrino oscillations being observed in atmospheric neutrino, further strengthens the case that oscillations occur for solar neutrinos too. Solar neutrino experimental data constrains that mass squared difference Δm_{21}^2 is only taken where other mass differences are neglected.

$$P(\nu_e \to \nu_\mu)$$
$$= \delta_{21} - 4[-1/4\,C_{13}^2 \text{Sin}\,2\theta_{12}(\text{Sin}\,2\theta_{12}(C_{23}^2 - S_{23}^2 S_{13}^2) + \text{Cos}\,2\theta_{12}\,\text{Sin}\,2\theta_{23}\,S_{13}\text{Cos}\,\delta_{13})]\text{Sin}\,\Delta m_{21}^2\,L/2E$$

As
$$\delta_{ij} = 1 \quad for\ i = j$$
$$\delta_{ij} = 0 \quad for\ i \neq j$$

Therefore, $\delta_{21} = 0$

$$= -4[-1/4\,C_{13}^2 \sin 2\theta_{12}(\sin 2\theta_{12}(C_{23}^2 - S_{23}^2 S_{13}^2) + \cos 2\theta_{12}\sin 2\theta_{23}\,S_{13}\cos\delta_{13})]\sin\Delta m_{21}^2\,L/2E$$

$$for\ oscillation\ in\ vaccum\ \delta = 0 \quad so, \cos\delta_{13} = 1$$

$$= [\sin^2 2\theta_{12}\cos^2\theta_{13}(\cos^2\theta_{23} - \sin^2\theta_{23}\sin^2\theta_{13}) + 1/4\sin 4\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}]\sin\Delta m_{21}^2 L/2E \cdots\cdots (3.24)$$

Similarly,

$$P(\nu_e \to \nu_\tau)$$
$$= [-\sin^2 2\theta_{12}\cos^2\theta_{13}(\cos^2\theta_{23}\sin^2\theta_{13} - \sin^2\theta_{23}) - 1/4\sin 4\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}\cos\theta_{13}]\sin\Delta m_{21}^2\,L/2E \cdots\cdots (3.25)$$

Now, Survival Probability is,

$$P(\nu_e \to \nu_e) = 1 - [P(\nu_e \to \nu_\mu) + P(\nu_e \to \nu_\tau)]$$

$$= 1 - [\sin^2 2\theta_{12}\cos^2\theta_{13}(\cos^2\theta_{23} - \sin^2\theta_{23}\sin^2\theta_{13}) - \sin^2 2\theta_{12}\cos^2\theta_{13}(\cos^2 2\theta_{23}\sin^2\theta_{13} - \sin^2\theta_{23})]\sin\Delta m_{21}^2\,L/2E$$

$$= 1 - [\sin^2 2\theta_{12}\cos^2\theta_{13}(\cos^2\theta_{23} - \sin^2\theta_{23}\sin^2\theta_{13} - \cos^2\theta_{23}\sin^2\theta_{13} + \sin^2\theta_{23})]\sin\Delta m_{21}^2\,L/2E$$

$$= 1 - [\sin^2 2\theta_{12}\cos^2\theta_{13}\{\cos^2\theta_{23}(1 - \sin^2\theta_{13}) + \sin^2\theta_{23}(1 - \sin^2\theta_{13})\}]\sin\Delta m_{21}^2\,L/2E$$

$$= 1 - [\sin^2 2\theta_{12}\cos^2\theta_{13}\{\cos^2\theta_{23}\cos^2\theta_{13} + \sin^2\theta_{23}\cos^2\theta_{13}\}]\sin\Delta m_{21}^2 L/2E$$

$$=1-[\sin^2 2\theta_{12}\cos^2\theta_{13}\cos^2\theta_{13}(\cos^2\theta_{23}+\sin^2\theta_{23})]\sin\Delta \mathrm{m}_{21}^2 L/2E$$

$$=1-[\sin^2 2\theta_{12}\cos^2\theta_{13}\cos^2\theta_{13}]\sin\Delta \mathrm{m}_{21}^2 L/2E$$

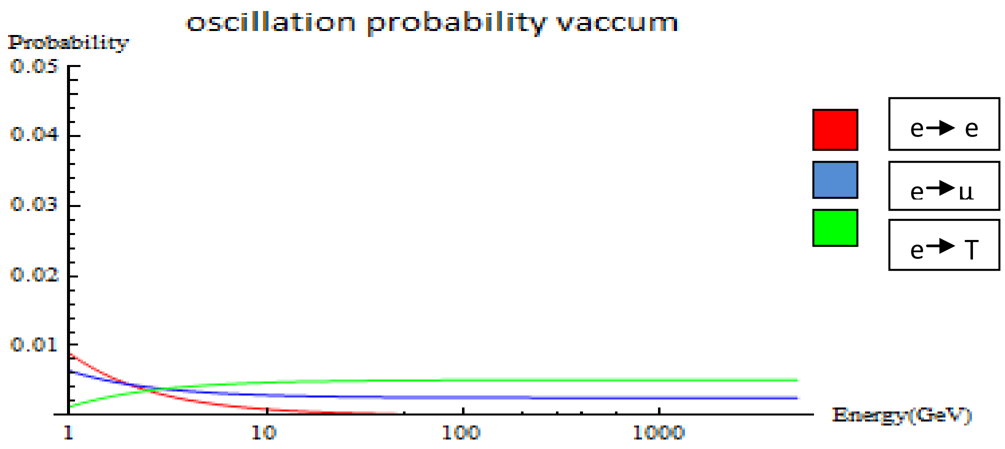
$$=1-(\sin^2 2\theta_{12}\cos^4\theta_{13})\sin\Delta \mathrm{m}_{21}^2 L/2E\cdots\cdots\cdots\cdots\cdots(3.26)$$

Probability	Expression
$P(\nu_e\to\nu_\mu)$	$[\sin^2 2\theta_{12}\cos^2\theta_{13}(\cos^2\theta_{23}-\sin^2\theta_{23}\sin^2\theta_{13})+1/4\sin 4\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}]\sin\Delta m_{21}^2 L/2E$
$P(\nu_e\to\nu_\tau)$	$[-\sin^2 2\theta_{12}\cos^2\theta_{13}(\cos^2\theta_{23}\sin^2\theta_{13}-\sin^2\theta_{23})-1/4\sin 4\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}\cos\theta_{13}]\sin\Delta \mathrm{m}_{21}^2 L/2E$
$P(\nu_e\to\nu_e)$	$1-(\sin^2 2\theta_{12}\cos^4\theta_{13})\sin\Delta \mathrm{m}_{21}^2 L/2E$

Table: 3.2: Showing the expression for different probabilities in solar neutrinos

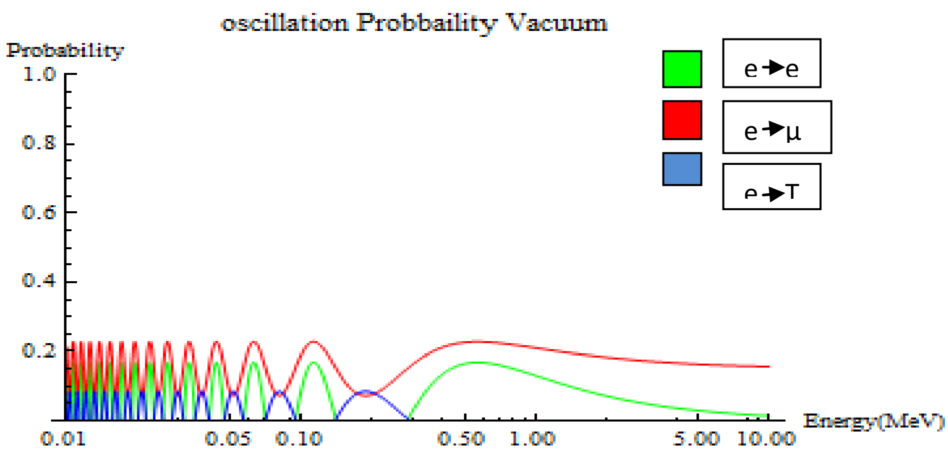
3.8 Oscillation Plots and Analysis:-

For studying the Probability for three neutrino oscillations, we are taking the parameters from Daya Bay Experiment. In this experiment we are taking the length L= 1650 m , mass squared difference as $\Delta m_{21}^2 \approx 7.6\times10^{-5} eV^2$, the mixing angle $\theta_{12}=35^\circ$, $\theta_{13}=9^\circ$, $\theta_{23}=40^\circ$ and the energy is in GeV. We are plotting the graph between Probability and energy using expression (3.24), (3.25), (3.26). Probability taken along Y –axis and Energy along X- axis. In this plot three probabilities are shown electron to electron , electron to muon , and electron to tauon



Plot 3.1: Probability vs Energy for Solar vaccum oscillation.

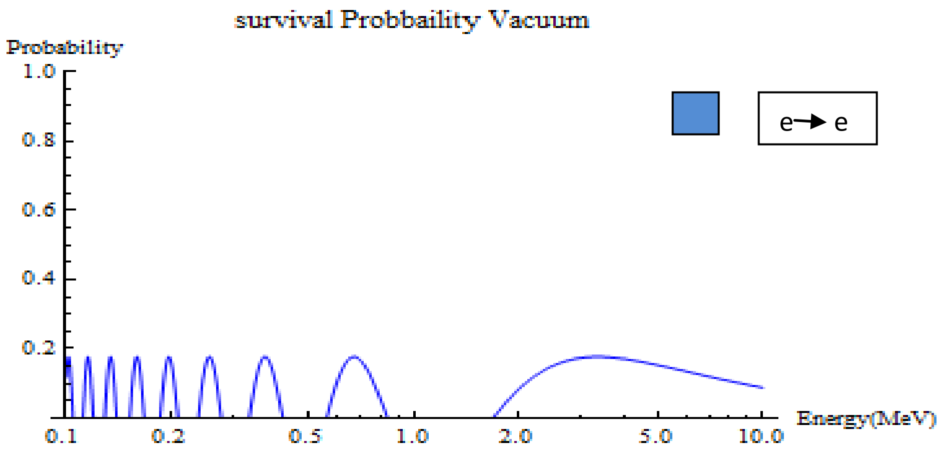
Now we are taking the parameters of KamLAND Experiment. In this experiment we are taking the length of L=17500 m , the mass squared difference $\Delta m^2_{21} = 8\times10^{-5}$ the mixing angle $\theta_{12} = 32.2^{\circ}$, $\theta_{13} = 12.7^{\circ}$, and $\theta_{23} = 35.9^{\circ}$ and the energy is in MeV. We are plotting the graph between Probability and energy using expression (3.24), (3.25), (3.26). Probability taken along Y –axis and Energy along X- axis. In this plot three probabilities are shown electron to electron , electron to muon , and electron to tauon.



Plot 3.2: Probability vs Energy for solar vaccum oscillation for LBL

Now we are taking the parameters of Double CHOOZ Experiment. In this experiment we are taking the length of L=105000 m , the mass squared difference $\Delta m^2_{21} = 7.9\times10^{-5} eV^2$ the mixing angle $\theta_{12} = 32^{\circ}$, $\theta_{13} = 11.5^{\circ}$ and the energy is in

MeV. We are plotting the graph between Probability and energy using expression (3.26).



Plot 3.3: Probability vs Energy for solar vaccum survival oscillation for LBL

3.8.1 Conclusion:-

For these plots we are using the three parameters Length (L), Energy (E) and the mixing angles θ_{12} , θ_{13} , θ_{23}.

From Daya Bay Experiment the probability of the survival oscillation goes on decreasing as we increase the energy & the probability of $e \rightarrow \mu$ goes on decreasing as we increase energy but not goes to zero & the probability of $e \rightarrow \tau$ starts from origin and goes on increasing as we increase the energy.

From the KamLAND Experiment, if we increase the length from few km. To hundreds of kms. the oscillation come into existence for this energy range. KamLAND was the first experiment to provide direct evidence for this oscillation. Still major part of flavor transitions is from electron to muon, not from electron to tauon.

From Double CHOOZ Experiment the survival oscillation probability goes on increasing as we increase energy. i.e. if we increase the energy number of oscillations also goes on increasing.

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