

PHENOMENOLOGICAL STUDY OF BARYONS

IN $J^P = \frac{1}{2}^+$ STATE

A Thesis

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DEDICATED TO MY PARENTS

All life is a struggle of dark
This dread and darkness of mind cannot
be dispelled by the sunbeams, the shining shaft
of the day, but only by an understanding of the
outward from and inner working of nature.
And now to work I will explain....

Lucretius "On the Nature of Universe"

CERTIFICATE

This is certified that dissertation entitled "PHENOMENOLOGICAL STUDY OF BARYONS IN $J^P = \frac{1}{2}^+$ STATE" being submitted by Mr. Ashish Sharma Regn. No. 301204001 for the fulfilment of the requirements for the award of Degree of Master of Science in the School of Physics and Material Science, Thapar University, Patiala, is a the record of the candidate's own work carried out by him under my supervision. The dissertation is not substantially the same as any that have submitted for a degree or diploma or other qualification at other university. All the information in this dissertation has been presented in accordance with the academic rule and ethical conduct.


This is to further state that, no part of dissertation has already been or is being concurrently submitted for such degree or diploma or other qualification. Except where explicit references are made to the work of others, this dissertation is the result of candidate's own works and includes nothing, which is the outcome of the work done in the collaboration.

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ABSTRACT

With the upcoming accelerator and detectors, lot of information about the hadrons and their properties are coming. In addition to hadron, lots of unknown particles are also seen with different properties and specific quantum number. In the present thesis, our approach is to study through phenomenological models, such properties of the hadrons. The thesis describes the properties of hadrons (for maximum of cases) especially of baryons octet in $J^P = \frac{1}{2}^+$ state.

With the number of theoretical approaches available, we made the approach to study these baryon in non-relativistic model such as SU(6) simple quark model with the valence quarks and combination of the sea quarks. Here we assume the quarks and gluon of the specific baryon to behave as static candidates with their quantum number giving total wave function to the baryon.

Our motivation to choose the present model is to justify the results at the experiments to check the validity of the available theoretical approaches and models developed so far in literature. For this purpose we use the effective mass and effective charge of the quarks combing to form the hadron. The effective mass and screening charge have been studied, using constituent quark model in past to show, how baryon magnetic moments varies in the presence of these effects. We studied the baryon octet with such combined effects of effective mass and screened charges in addition to the modification in the hyperfine splitting value which is taken to be 200 MeV. Besides that we made an attempt to study the baryon wave function in the presence of the orbiting valence quarks, allowing the study of the non-relativistic model with the spin structure and give the expression for the spin in term of the coefficients of the baryon wave function. We further use it to calculate the magnetic moment of baryon in the presence of spin and orbital motion of quarks i.e. valence quarks and calculated the best fit value of the parameters.

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DEDICATION
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 ABSTRACT
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TABLE OF CONTENTS

	Page No.
Chapter 1 Introduction to High Energy Physics	11
1.1 Motivation and Introduction	11
Chapter 2 Group Theory	15
2.1 Concept of Group	15
2.2 Group Representation	15
2.3 Lie Group and Lie Algebra	16
2.3.1 Lie Group	16
2.3.2 Lie Algebra	17
2.4 Representation of Specific group	18
2.4.1 U(1) Group	18
2.4.2 SU(2) Group	19
2.4.3 SU(3) Group	19
2.5 Symmetries	20
2.6 SU(2) Isospin symmetry	21
2.7 SU(3) flavor symmetry	23
Chapter 3 Construction of QCD Lagrangian	25
3.1 Natural Units	26
3.2 Lagrangian formalism of QCD	26
3.2.1 Quarks and their Masses	26
3.2.2 Color Charge	27
3.2.3 Gluon and QCD	27
3.2.4 Confinement and Asymptotic Freedom	29
3.3 Chiral Symmetry	30
3.4 Chiral Symmetry Breaking	32
Chapter 4 Quarks, Hadrons and Their properties	34
4.1 Quarks	35
4.1.1 Evidence of Quarks	35
4.2 Bound State of Quarks and Properties	36
4.2.1 Hadron Quantum number	36
4.2.2 Flavor and Charge Independence	36
4.2.3 Color	37
4.2.4 Gluons	38
4.3 Hadrons	39

4.3.1	Mesons	39
4.3.2	Baryons	40
4.4	SU(6) Quark Model	41
4.5	Baryon Octet $J^P = \frac{1}{2}^+$ state representation	42
4.6	Anomalies of the Static Quark Model	43
4.7	Baryon wave function in ground state (valence and sea quarks)	44
4.8	Baryon wave function with orbiting valence quarks	45
4.9	Baryon Wave Function in Excited State	46
Chapter 5	Phenomenological Study of Baryons Properties	49
5.1	Effective Mass	49
5.2	Screening Charge	54
5.3	Magnetic Moment	56
5.4	Angular Momentum	57
Chapter 6	Calculations, Results and Conclusion	59
6.1	Magnetic Moment	59
6.2	Magnetic Moment with effective mass	60
6.3	Magnetic moment with screening charge effect	62
6.4	Magnetic Moment with effective charge and screening effect	63
6.5	Magnetic moment with spin and orbital motion of valence quarks	64
6.6	Conclusion	65
Appendix A		67
	Baryon wave function in spin and flavor	
	List of Tables and Figures	
	Standard Units	

LIST OF TABLES AND FIGURES

TABLES:

4.1	Classifications of the Quarks on the basis of their mass charge and spin.	35
4.2	Quantum number of the quarks	36
4.3	Value of the color charge for the color states of the quarks and anti-quarks	38
4.4	List of baryons	40
5.1	Effective mass formula of baryons in $J^P = \frac{1}{2}^+$ state	53
5.2	Effective mass formula for baryons with heavy quark	54
6.1	Predicted and Observed magnetic moment of stable baryons octet state	60
6.2	Effective masses of the constituent of baryons	61
6.3	Magnetic moment of baryons with effective mass	61
6.4	Magnetic moment of baryons with screen charge	63
6.5	Magnetic moment of baryons with the combine effect of effective mass and screen charge	64
6.6	magnetic moment of baryon in spin orbit motion of valence quarks	65
6.7	Comparisons of magnetic moment of baryons in different schemes	66
A.1	SU(6) representation in form of SU(3) SU(2) sub group	67
A.2	Symmetric and Antisymmetric combinations of spin for baryons octet $J^P = \frac{1}{2}^+$ state	68
A.3	Symmetric and Antisymmetric wave functions of baryon in flavor	68
A.4	Spin-Flavor combined wave function of baryon	69

FIGURES:

4.1	Representations of strong interaction of quarks though gluons	38
4.1	Representations of the baryon octet $J^P = \frac{1}{2}^+$	43

STANDERD UNITS

Quantity	Symbol	Value
Speed of light in vacuum	C	$2.99 \times 10^8 ms^{-1}$
Planck's constant	h	$4.136 \times 10^{-24} GeVs$
	\hbar	$6.582 \times 10^{-25} GeVs$
Electron charge	e	$1.602 \times 10^{-19} C$
Boltzmann constant	k_B	$8.617 \times 10^{-11} MeVK^{-1}$
Electron mass	m_e	$0.511 MeV/c^2$
Proton mass	m_p	$0.9383 GeV/c^2$
Neutron mass	m_n	$0.9396 GeV/c^2$
W boson mass	M_W	$80.43 GeV/c^2$
Z boson mass	M_Z	$91.19 GeV/c^2$
atomic mass unit	u	$931.494 MeV/c^2$
Bohr magneton	μ_B	$5.788 \times 10^{-11} MeVT^{-1}$
Nuclear magneton	μ_N	$3.152 \times 10^{-14} MeVT^{-1}$
Fine structure constant	α	1/134.04
strong coupling constant	α_s	0.119

Introduction to High Energy Physics

1.1 Motivation and Introduction

“But why are such terrific efforts made just to find the new particles?” asked Mr. Tompkins, “Well, this is science,” replied by the professor, “the attempt of the human mind to understand everything around us, be it giant stellar galaxies or these elementary particles. It is interesting and exciting, and that is why we are doing it.

Mr. Tompkins in paperback, Cambridge University Press (1965), p.186

What is world made up of? Present day particle physics researcher’s most organised efforts gives the answer to the man’s most ambitious and curious question. We all know that matter, a simple unchanging elementary constituent comes into different qualities, shapes and forms. However there are many recurring properties such that many properties remain unchanged or if changed there are some type of conditions to be satisfied. This concept was taken as the origin of elementary particles. The search of elementary constituents of all matter has occupied the generations of human being over the speculation philosophy of different philosopher from different parts of the world. The so called “Elementary Particles” are undoubtedly the focus of interest and of research for the both experimental and theoretical physicist exploring and understanding the properties of the ultimate and basic elementary constituent of matter. While talking about particles the first question comes into the mind is, what is the ultimate constituents of matter? What are elementary particles and the nature of elementary particles? How are they categorised? How do they interact with each other? These question might be dominant the physicist for more than two century and is an exciting question in present time because the physicist now have the tools which can provides the answers to these questions. Now we know that everything is made from a few basic elements known as elementary particles, the idea is basically correct.

Classical elementary particle is nothing but an indivisible atom, mass point of the classical mechanics. The properties of these classical particles are described by the famous rules and laws of the Newtonian mechanics. The result that was obtained is considerable, but the study or investigation of the electromagnetic phenomenon was not described by this mechanics. As a result to study these properties open a new door to research field in particle field. It became evident that atom is no longer indivisible but has some internal structure and the properties. The dualism of particles and fields had just appeared in physics. To restore the unity or for the justification, to prove the phenomenon occurring inside the particle a need of new physics was required. Then the development of quantum mechanics (1915-1938) came into existence, a new period of the continuum theory helps to solve the problem of dualism of particles and fields i.e. nature of matter.

This situation occurs before a one and around half century ago, while during the study of electricity in 1897 J.J. Thomson discovered the electron. The electron is still thought to be a structure less point particle, and one of the elementary particles of Nature. At that time only three elementary particles were known i.e. electron proton and neutron. After that the discoveries increases rapidly and so the number of elementary particles goes on. Particles like lepton, neutrino's, mesons, heavy mesons and various hyperons and hadrons other particles were discovered and considered as the elementary particles, have a complex structure. Elementary particles represent the lightest physical system and strongly affected by the quantum mechanical consideration. Quantum mechanics as name suggest is much of system whose variables are quantized. The quantization appears very strongly in the angular momentum of atoms atomic nuclei and elementary particles.

The great advancement of the empirical knowledge is due to the advancement in the experimental technology. The construction of vast particles accelerators at like CERN, LBNL SLAC, Tevatron Fermi lab, KEK, HERA etc. are capable of producing particle up to energy of order 10^4GeV . The experimental efforts over a decade were a sequence of very important discoveries in the last decade that directly guided us to Quark Leptons and Gluon Bosons. To explore and understand the picture of the ultimate or the basic elementary constituent of matters forms natural pic of nature.

Today a very large number of particles are known and some of them are stable and others are not, because of the phenomenon of the mutual interaction happening between these particles with different life time is most important nature of these particles. These are further divided into subgroups in accordance to the type of interactions in which they participate. All the electronic charge particles interact electromagnetically. Whereas some particles responds only to weak forces such type of particles are collectively known as leptons for e.g. electron(e^-), muon(μ), taon(τ), neutrino's (ν_e, ν_μ, ν_τ). All leptons have intrinsic spin $\frac{1}{2}$ and are called fermions. While those participate in strong interactions are known as hadrons. Hadrons can be either fermion or boson unlike leptons. The hadrons spin $\frac{1}{2}$ known as baryons amongst of them are proton and neutron. The mesons named because they had mass intermediate between the light or zero mass lepton and the heavier baryon are boson.

For hadrons at low energy, the structure of hadrons in form of quarks was first put forward by Gell-Mann [4] and Zweig [5] and stated that hadrons known at that time could be built up of as a composite system of three quarks (u, d, s) and with fractional charge so as to obey SU(3) symmetry. Later on the picture of colored quarks was formulated by Gell-Mann and observed hadrons are singlet of SU(3) group but was unable to explain the results observed at experiments. The first historical step in understanding of quark-gluon nature was in the constituent quark model. Later on SU(3) states got combined with the SU(2) states of spin and led to six fold symmetry known as SU(6) (discussed in chapter 4). The model uses six quark states to constitute and classify the hadrons into mesons and baryons and known as SU(6) quark model. SU(6) model explained some of the low energy properties of nucleonic

system like magnetic moment ratio, still for some parameters, for which extension of SU(6) Quark model is needed.

Since we are studying about the baryons, we know that baryons are the composite system of three quarks (valence quark) and sea [1]. Sea is considered as a cluster of Fock states which can consist of the gluon-gluon, gluon-anti gluon, quark-antiquark pairs [2]. SU(6) quark model provides a good description of hadrons consist of confined colored quark and vector gluons: Baryons are color singlet of three quark in appropriate flavor–spin combinations. The space-time part of a hadron wave function can be determined by using a specific model of confinement, e.g. bag model, simple harmonic oscillator model, or other phenomenological models [1]. Within the framework of QCD, quarks interact through color forces mediated by vector gluons. The spin dependent forces also cause different space-time distributions for different quark favours and provide a good description of baryon magnetic moments and form factors.

In the low energy limit, QCD has a challenging behaviour (discussed in chapter 3). The non-perturbative and relativistic nature of quarks and gluons makes the hadronic structures difficult to understand [6]. The constituent quark model explains the static properties of a baryonic system; still there is a need of some new degrees of freedom, which can explain all the problems [9]. A variety of approaches have been developed to understand the behaviour of baryons with three valence quarks and sea consisting of gluons and quark-antiquark pair. These models help to make a deeper understanding to the various properties like weak decay coupling ratios, magnetic moments, masses of hadrons and flavor asymmetry [1]. Latest studies, dedicated towards the study of quark-antiquark pairs and gluon condensates includes statistical balance model [7], lattice QCD, effective QCD approaches [8] etc.

A nonrelativistic constituent quark model has been remarkably successful in describing many phenomena in hadron physics [9]. However there are other areas where the model has been spectacularly unsuccessful. George Zweig used to say that the quark model gives an excellent description of half the world. The model has no sound theoretical basis. The early days there was no clue to the underlying theory. However the equations of QCD are so complicated that no one has been able to solve them to derive hadron spectroscopy and dynamics [10]. The constituent quark model can now be considered as an intermediate phenomenological model which fits the experimental data and will hopefully be derived from QCD. The simple non relativistic quark model was first introduced to explain the quantum number occurring in the low lying mesons and baryons spectrum. The simple nonrelativistic constituent quark model has remarkable success in describing the low mass spectroscopy of quark- antiquark and three quark system. However for the detail properties of the multiple quark system this model has failed almost completely and given no prediction which has been verified by the experiments. We tried to understand how the model can be so successful in the quark – antiquark. The constituent quark model divided into three stages: the simple non relativistic quark model, color non relativistic quark model, the QCD motivated non relativistic quark model.

There are several problems of the quark model and low energy regime some of that are: The statistics problem, the saturation problem, the meson-baryon problem, the free quark problem. The solution of the statistics problem was found in 1964. The additional internal degree of freedom now called color enabled quarks to satisfy Fermi statistics with wave functions symmetric in all the previously known degrees of freedom and antisymmetric in color. The color degree of freedom was explicitly introduced into the phenomenological dynamical quark models 1973 to solve the saturation and meson-baryon problems. After the introduction of the asymptotic freedom and confinement in non-abelian gauge theories led to the development of QCD, De Rujula, Georgi and Dashow [11] introduced the idea from the QCD to the non-relativistic quark model. They attributed the spin dependence of the two body's interaction to the spin dependent part of a gluon exchange interaction. With this model it was easier to obtain the two independent relations between the strange and non-strange quark masses in terms of the experimental hadrons masses and use them to predict the magnetic moment. The original prediction by DGC in 1975 is particularly impressive because it was made before the magnetic moment had been measured precisely.

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Group Theory

It is not possible to study any physical system for very long time before finding the regularities or the symmetries in the system which demands the explanation at both experimental and theoretical research level. Even if the system is complex one can expect that regularities have simple explanations. Symmetries play an impotent role in the physics and increasing its importance with the modern development. Without the symmetries there would be no clear understanding of the particles. The existence of symmetries leads to the variety of the physical simplicities both in classical and quantum level although physical system are so different from each other, in spite of that the same theory of symmetry may be applied to all of them. The study of symmetry therefore helps us to unify the physics by emphasising the similarities between different fields. As it is true, symmetry plays an important role in both classical and quantum physics. The key mathematical framework for the symmetry is group theory because symmetry transformations form group under compositions. Although symmetry of a physical system are not sufficient to fully describe its behaviour. Therefore one requires a full dynamical theory with the help of symmetry; it is possible to find the useful constraint. For a physical system group elements depend smoothly on a finite number of parameters called coordinate. These groups are lie groups. The properties of these groups encoded in infinitesimal transformations which are closed in some sense to identity. So to investigate a physical system or consequences of the symmetry of a system one must learn about the transformations, which leaves some functions like potential invariant. The theory of the set of transformations is known as “Group Theory” which is an appropriate tool used by physicist to study the symmetries.

2.1 Concept of Group

We start by giving a brief overview of the basic notion from the group theory that will be important throughout this thesis. The section based on information given in [8]. Any standard work on basic (lie) group theory could in principle have been used. Since the main aim of study is to investigate the connection between the quantum theory and the group theory. It is important to establish a precise relation with the mathematical entities called group.

2.2 Group Representations

As we have seen groups ranges in the type from very abstract to very concrete. A good way to representing a group elements is needed, that is where matrix representation comes in. Each element in a group can be assigned to a $n \times n$ matrix to represent it, provided it meets one condition. If each element $A \in G$ is represented by $\Gamma(A)$ we must have

$$\Gamma(AA') = \Gamma(A) \Gamma(A') \tag{2.1}$$

for each $A, A' \in G$. If this condition meets, then Γ is a d - dimensional representation of group G or simply a unitary operator. The mapping that satisfies the equation (2.1) is simply known as representations of a group G . The matrix multiplication of a group of elements is always used as the group multiplication operations. The multiplication of the linear transformation on a linear vector space is, in general associative, but not necessarily commutative. Hence it is basically a group multiplication. Using matrices to represent the group elements has some clear advantages, as they are matrices are like non-abelian group elements. The existence of an inverse group element requires that all group elements be represented by the square non-singular matrices. For χ a fixed operation (i.e. a constant matrix) and $A \in G$

$$\Gamma'(A) = \chi \Gamma(A) \chi^{-1} \quad (2.2)$$

and $\Gamma'(A)$ and $\Gamma(A)$ are equivalent representation. By using simple transformation we can change one representation into a potentially more useful one. The much more important concept in the transformation or representation is the idea of reducibility. By using such transformations, a group can be converted to the block diagonal form, which is a reducible form. A representation is reducible if it is equivalent to representation Γ' with block diagram then from equation (2.2)

$$\Gamma'(A) = \chi \Gamma(A) \chi^{-1} = \begin{vmatrix} \Gamma'(A_1) & 0 \\ 0 & \Gamma'(A_2) \end{vmatrix} \quad (2.3)$$

If there are no possibilities for such type of transformation then it is an irreducible transformation. So can't be put into the block diagram of similar transformations [6].

The concept of group formalism brings one more new concept i.e. the direct product of groups. The direct product of group is not absolute in the realm representation, on the contrary; it is the property of a group itself. Essentially two groups are combined in a direct product to produce elements which have a component from each group. These elements operate as; for $A_i \in G$ and $B_j \in H$, where G and H are group of elements.

$$U_{ij} = A_i \otimes B_j \in G \otimes H \quad (2.4)$$

$$U_{ij} \circ U_{kl} = (A_i \circ A_j) \otimes (B_k \circ B_l) \quad (2.5)$$

The representation for a direct product group is a block diagonal matrix with each block as an irreducible representation. Since different blocks in the block diagonal matrices do not interact through multiplication and multiply themselves normally this is a good representation [5].

2.3 Lie Group and Lie Algebra

2.3.1 Lie Group

We have kept the concept of the group very general. However it is not hard to see that it is possible to identify the class of groups, for e.g. one can discriminate between the discrete and continues group. Our focus will be on group of latter kind, or to be more specific, on lie group.

A lie group is defined as “A group in which all the elements are labelled by a set of continuous parameters with the multiplication law that smoothly depends on the parameters” [6]. The lie group can describe much continuous symmetry of particles and can also describe some discrete phenomenon too after the formalism of these groups, because symmetry operation can be discrete and continuous [4]. Lie group describe the major properties of the groups. It satisfies the group axioms discussed in article 2.1. The elements of the group form a type of space, which have a set of points and satisfies the set of postulates. A lie group can be defined in several ways depending upon the degree of special importance. Every lie group have a special importance in solving the physical problem and therefore known as “Linear Lie Group”. The word linear means that the entire lie group have at least one faithful finite dimension representation and each element is parameterized in an analytic ways [3]. The most familiar tool or object of these groups is matrices, which has a basic feature that it has non countable number of elements lying near the region of identity. The structure of this region determines the structure of whole group and corresponding to real lie algebra.

Let us consider if we have any n dimensional lie group G , which can be parameterized in such a way that it can be described in term of n subgroups each of which is labelled as one parameter. If group g is n dimensional stating that element $g \in G$, can be labelled by n parameter

$$g = g(\alpha_1 \dots \dots \alpha_n), \quad (2.6)$$

We can choose the parameterization such that the set of elements of the form

$$g_k(t) = g(0, \dots, 0, t, 0 \dots \dots 0), \quad (2.7)$$

are the parameter subgroup of group G . where t is the k^{th} position and $1 \leq k \leq n$. The condition that $\{g_k(t); t \in R\}$ form a subgroup of G , as follow.

$$g_k(t)g_k(s) = g_k(t + s), \quad (2.8)$$

We will now discuss the some topological properties of the lie group G , namely as compactness. Since lie group carry the structure of real and complex analytic manifold. So we can talk about such topological properties. A lie group is said to be compact if the set of parameter used to label the group is compact and vice versa. Compactness of group becomes more important when we talk about the representation.

2.3.2 Lie Algebra

Lie algebras are probably the favourite group theoretical tool for physicist, because they are much easier to work with the group themselves. We come to the classification of the unitary irreducible representations of specific group. We mainly work with the corresponding lie algebra.

If we have a compact lie group G , group with the unitary operators in which every element is labelled by certain set of parameters. The volume of the parameter space for a compact lie group is finite. Any group with continuous change in parameters can be written as

$$\exp(i\alpha_a X_a) \quad (2.9)$$

where $\alpha_a (a = 1 \text{ to } N)$ are real parameters and X_a are linearly dependent hermitian operator or also known as group generators(interchangeably). The set of all linear combination $\alpha_a X_a$

is a vector space and X_a are the basis in the vector space. For a lie group we can always choose the space on which generators act to be finite dimensional. By the group properties the group elements can be written as

$$\lambda^2[X_a X_b] = i\beta_c X_c \quad (2.10)$$

$$\beta_c = \lambda^2 f_{abc} \quad (2.11)$$

so $[X_a X_b] = i f_{abc} X_c \quad (2.12)$

where parameters f_{abc} are called structure constant of the group, which determines the multiplication law for the elements that are continuously connected to the identity as follow

$$\exp(\alpha_a X_a) \exp(i\beta_b X_b) = \exp(i\delta_c X_c) \quad (2.13)$$

where δ_c is determined by the α, β and f parameters. The structure constant depends upon the basis we choose in vector space. The generators satisfy the Jacobi's identity. The generator and commutation relation defines the lie algebra associated with lie group [9-10]. For a lie group G to be lie algebra $\mathcal{L}(G)$, it must satisfy the properties given below.

1) Skew-symmetric: $[x, y] = -[y, x] \forall x, y \in \mathcal{L}(G). \quad (2.14.1)$

2) Linear: $[\alpha x_1 + \beta x_2, y] = \alpha[x_1, y] + \beta[x_2, y] \forall \alpha, \beta \text{ constants } x_1, x_2, y \in \mathcal{L}(G), \quad (2.14.2)$

3) Jacobi's identity: $[[x, y], z] + [[z, x], y] + [[y, z], x] = 0, \forall x, y \in \mathcal{L}(G). \quad (2.14.3)$

The generators in the representation, when exponentiated, give the operator of the group representation. For a set of the matrix T_a in term of structure constant

$$(T_a)_{bc} = -i f_{abc} \quad (2.15)$$

so $[T_a T_b] = -i f_{abc} T_c \quad (2.16)$

which is an adjoint representation and represent the hermitian matrices.

The invariant algebra is a set of generators which under goes into itself under communication with every element. Algebra with on non-trivial invariant algebra is called simple algebra and generates a simple group. The generators in an abelian invariant algebra commute with each elements. Each of these generators associated with what we call a $U(1)$ factor of the group. The group $U(1)$ is the group of phase transformations and do not show up in the structure constant. If X_a is a $U(1)$ generator, $f_{abc} = 0$ for all b and c and generator $k_a = 0$.

2.4 Representations of Specific Groups

In this section we introduce the unitary group and their irreducible representations we will defining uncertainty in term of invariance and proceed toward the conc. of the irreducible representations of these groups in N dimensions.

2.4.1 $U(1)$ Group

Each group element of $U(1)$ can be represented by the pure phase factor $e^{i\alpha}$. The parameter α is real and continuous which indicates that $U(1)$ has an infinite set of group elements and is continuous. But we know that

$$e^{i\alpha} = \cos(\alpha) + i \sin(\alpha), \quad (2.17)$$

$U(1)$ is isomorphic to 2 by 2 rotation matrices. The equation (2.17) can be represented in the matrix form as given below

$$e^{i\alpha} = \cos(\alpha) + i \sin(\alpha) \sim \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}, \quad (2.18)$$

2.4.2 SU(2) Group

The special unitary group SU(2) is the group of all 2×2 unitary matrices with determinant equal to one. If $\det U_1 = 1$ and $\det U_2 = 1$ then $\det(U_1 U_2) = \det U_1 \det U_2 = 1$. This shows that SU(2) is a group of U(2). Every element of U(2) is a product of phase factor $e^{i\alpha}$, which is element of both U(1) and SU(2).

$$U = \exp(iH), \quad (2.19)$$

where H is the Hamiltonian of matrix. 2×2 Hamiltonian matrix can be written as

$$H = \alpha^0 I + \alpha^k \sigma^k \quad (2.20)$$

where α^0 represent the energy or time space co-ordinate and α^k ($k = 1, 2, 3$) represent the momentum space and σ^k represent the Pauli's spin matrices given as

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (2.21)$$

$$(\sigma_1)^2 = (\sigma_2)^2 = (\sigma_3)^2 = I, \quad (2.22)$$

from 2.5.2 we get another fact is for any SU(N) group is, the rank will be N-1. Therefore for SU(2) group we have

$$U = \exp(i \sum_{a=1}^3 \varepsilon_a \sigma_a), \quad (2.23)$$

above equation is similar to the equation (2.16). Here σ_a represent the Pauli's spin matrices and T_a represent the generators. Since SU(2) is an exact symmetry group the generators are defined as

$$J_i = \frac{\sigma_i}{2} \quad (i = a, b, c) \quad (2.24)$$

$$[J_a J_b] = i \Sigma_{abc} J_c$$

Where Σ_{abc} is totally antisymmetric levi-civita symbol and $\Sigma_{abc} = 1$.

2.4.3 SU(3) Group

The special unitary group SU(3) is the group of 3×3 unitary matrices with determinant equal to one, so an element of SU(3) group can be represented as in equation (2.25).

Since the SU(3) is generalised by $3^2=9$ parameters but $\det U=1$ or equivalent $\text{tr} H=0$ reduces this value to 8. Therefore for SU(3) group we have

$$U = \exp(\sum_{a=1}^8 \varepsilon_a \lambda_a) \quad (2.25)$$

So we have 8 traceless hermitian matrices introduce by Gell-Mann

$$\lambda_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \lambda_2 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \lambda_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \lambda_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\lambda_5 = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix}, \lambda_6 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \lambda_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix}, \lambda_8 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \quad (2.26)$$

here λ_3 and λ_8 are non-diagonal matrices these are higher order Pauli's spin matrices. These matrices also have the property

$$\text{tr}(\lambda_a \lambda_b) = 2\delta_{ab} \quad (2.27)$$

T_1, T_2 and T_3 generate an SU(2) subgroup of SU(3) which is called isospin subgroup, because in the physical application of SU(3) as an internal symmetry, it is isospin. It satisfy the commutation relation for SU(3) as

$$\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2} \right] = \sum_{c=1}^8 i f_{abc} \lambda_{\frac{c}{2}} \quad (2.28)$$

with SU(3) generators represented as

$$\lambda_i = \frac{\lambda_a}{2} \quad (i = a, b, c) \quad (2.29)$$

where f_{abc} are the structure constant.

2.5 Symmetries

In this section we discuss about the symmetries and how they are related to transformation in quantum mechanics. Symmetries play an important role in particle physics. The key factor of extraction for symmetries is that they give rise to conserved quantities which was discussed by Emmy Noether in 1917.

Any physical system is said to have symmetry, if there is no change in the system when some transformation properties are applied to the system. So we can say that symmetry is due to the absence of the absolute references and corresponds to the concept of indistinguishability. That is why it is turns out that symmetry are associated with the physical quantities of the physical system. Transformations associated with the symmetry are divided into two types

1. Active transformation- To move the object for e.g. changes the co-ordinates in the co-ordinate frame.
2. Passive transformations- Change the description for e.g. change the co-ordinate frame instead of the co-ordinates.

The transformation are further classified into three types on the basis of symmetry

1. Space-time symmetry

- a. Translation transformations in (x, t)
- b. Rotation transformations
- c. Parity (reflection) in (x, t)

2. Internal symmetry

These transformations are associated with the quantum number of the physical system. These relate processes involving different types of particles. The primary use of the internal symmetry is recognised in particle classifications, where mesons and baryons (discussed in chapter 4) were grouped into the degenerate state multiplets of a symmetric group for which SU(2) and SU(3) groups have particular importance. For example, isospin relates u and d quarks. Conservation laws can be found for particular types of interaction. These symmetries are also approximate; isospin is not exact because there is a (small) mass difference between m_u and m_d .

3. Gauge symmetry

The term gauge simply associated with the degree of freedoms of a physical system in term of their Lagrangian form. These lead to specific types of dynamical theories describing types of particles, and give rise to conserved charges. Gauge symmetries if present, appear to be exact. The term gauge is sometime used for all continuous internal symmetries

We are mainly concern about the symmetries under different co-ordinates transformations of space-time. Such symmetries are often called external symmetry. There are also some symmetries which when undergo the transformation do not act directly on space-time itself but on the field define in the space-time. These types of symmetry are often called internal symmetry.

2.6 Isospin Symmetry

W. Heisenberg in 1932 conjectured that neutron and protons are the states of the single entity i.e. nucleons and the concept of the isospin were given after the discovery of neutron. This concept was originated on the basis that their masses are approximately same. At the same time if we eliminate the small mass difference between the proton and neutron all forces between the proton and neutron become same and having almost same energy. According to the special theory of relativity

$$E = mc^2 \quad (2.30)$$

This mass equivalence can be seen as energy degeneracy of the underlying interactions. This approximate degeneracy led to the idea of the existence of the approximate symmetry obeyed by the interaction taking place between the particles.

There are three forces of interactions taking place between the particles i.e. strong, EM, weak forces. Followed by this speculate, Heisenberg and E.U. Condon proposed that strong interactions are invariant under transformations in an internal space in which nucleon is a spinor. Thus nucleon is an isospin doublet with $I_Z(p) = \frac{1}{2}$, $I_Z(n) = -\frac{1}{2}$ and isospin is conserved.

$$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.31)$$

Proton and neutron form a doublet $\begin{pmatrix} p \\ n \end{pmatrix}$ means

$$T_3|p\rangle = \frac{1}{2}|p\rangle, \quad T_3|n\rangle = -\frac{1}{2}|n\rangle \quad (2.32)$$

In analogy to the concept of the rotation in the space the isospin symmetry can also be governed by SU(2) group under the operation of rotation, rotating the components of (2.31) with each other in abstract isospin space.

Here we are much curious about the strong forces so we omit other forces. The strong forces between the nucleons are independent of particle type i.e. it is same for p-n, n-p, n-n and p-p, there is only a small difference between the charges of the content. As a result Heisenberg introduce the concept of SU(2) isospin symmetry to synthesis this concept. The strong interactions are not able to distinguish between the proton and neutron. Therefore the Hamiltonian of strong interactions has the property

$$[T_a + H_s] = 0; (a = 1, 2, 3). \quad (2.33)$$

for the exact symmetry the generators T_a of the group should commute with Hamiltonian in such a way

$$[T_a + H] = 0 \quad (2.34)$$

So all members of the isospin multiplets would be strictly generate in mass. The concept of isosymmetry can be used for the other hadron (isosinglet, isodoublet, isotriplet). Particles in the same isospin multiplet have the same baryon number, spin, parity same content of the non- light quarks, and almost same mass. Isospin is conserved in all knowing strong interactions and related to the quark content is by the relations

$$I_3 = \frac{1}{2} [N(u) - N(\bar{u}) - (N(d) - N(\bar{d}))] \quad (2.35)$$

The mass difference between the proton and neutron could be attributed to the charge content of the latter. The mass difference (or the energy difference) was to be purely electrostatic in nature; the proton had to be heavier. However the proton is the lighter of the two. If it were, the proton would be unstable and decay into the neutron, spelling the disaster for the stability of matter.

Therefore isospin symmetry is not exact symmetry of strong interactions, albeit it is a good approximate symmetry. So remain useful in case of the strong interactions. In terms of group theory, contention is that the strong interactions are invariant under the transformation of group SU(2) and particles are transformed as irreducible representations. The expression of charge of nucleons, anti-nucleons is

$$Q = I_z + \frac{B}{2} \quad (2.36)$$

equation (2.36) shows that, charge choose out a particular direction in internal space. After the discovery of pions, having mass remarkably equal to that was predicted by the H. Yukawa (1935), while explaining nuclear forces in terms of exchange of massive quanta of mesonic field. As a consequence, concept of isospin acquires a new significance and naturally can be visualise in the small $\pi^+ - \pi^0$ mass differences. The invariance of pion-nucleon interactions under isospin transformations led to the number of predictions, all are of which were confined.

$$\frac{m_n - m_p}{m_n + m_p} = 0.7 \times 10^{-3}, \quad \frac{m_{\pi^+} - m_{\pi^0}}{m_{\pi^+} + m_{\pi^0}} = 1.7 \times 10^{-2} \quad (2.37)$$

In the quark model the isospin symmetry correspond to the doublet that contains up and down quarks (proton and neutron)

$$u = \left| \frac{1}{2}, \frac{1}{2} \right\rangle, \quad d = \left| \frac{1}{2}, \frac{1}{2} \right\rangle \quad (2.38)$$

The mass scale of the up and down quarks is not same but they are both of order of few MeV/c² which is minuscule compared to the energy scale of the hadrons (only for the stronger interacting particles) which is about GeV/c². That is why isospin is such a good symmetry [1].

The SU(2) invariance is described by defining generators obeying the lie algebra. Isospin symmetry, the symmetry transformations are same at all space time points and are not associated with gauge boson or forces. Later on M. Gell-Mann and K. Nishijima extended the concept of Isospin conservation in strong interaction of the new particles classified them and then assign a new quantum number S (strangeness) to each isospin multiplets.

$$Q = I_z + \frac{B+S}{2} \quad (2.39)$$

including all quarks except c, b and t quarks. After the strangeness there is an immediate search for the higher symmetry that would include the isospin and strangeness of hypercharge.

$$Y = B + S \quad (2.40)$$

this search was ended by M. Gell-Mann and Y. Ne'eman with the discovery of the lie group SU(3) and represent the global symmetry. The first three generators T_a ($a=1, 2, 3$) of the SU(3) group are identified as isospin generators T_a ($a=8$) is related to the hypercharge and remaining generators represent the isospin and strangeness. SU(3) is only appropriate symmetry of strong interactions.

2.7 SU(3) Flavor Symmetry

The discovery of the concept of strangeness, introduced in the previous section was motivated by the existence of the particles that are produced strongly but decay weakly. Gell-Mann and Nishijima postulated the existence of the quantum number strangeness (S). It was assumed that strong interactions conserved the strangeness; therefore in the light quark model it appears to pronounced generalizations to add another component for the extra quark to the isospin vector space

$$u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad s = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (2.41)$$

The transformations that rotate the components of equation (2.41) into each other while preserving the standard; have to be the elements of the group SU(3). The first particle multiplets were identified as the representations of the SU(3) group. In this interpretation the flavor SU(3) symmetry may appear to be of prior interest. However the SU(3) symmetry appears to be another and much more fundamental context in strong interactions. The quarks possesses another quantum number, called color, which also form the representation of the SU(3) group. The SU(3) believed to be an exact symmetry of the strong interactions, in fact the modern theory of the strong interaction is gauge theory of this color group called QCD. The flavor SU(N_f) symmetry, where N_f is the number of the quarks flavor, become increasingly inaccurate for $N_f > 3$. The reason for this discrepancy is that the other quarks namely charm (c), bottom (b) , top (t) are significantly heavier than the hadronic energy scale.(the bare mass of the charm quark is heavier than the mass of two nucleons which sets the hadronic energy scale, whereas the bottom and top quarks are even heavier than then charm quark.).

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Formalism of QCD and Symmetry

The standard model of the elementary particles and their interactions has two basic components: the spontaneously broken $SU(2)\times U(1)$ electroweak theory and unbroken $SU(3)$ color gauge theory known as QCD. When QCD comes into the existence it was centred on the general principle of the scattering amplitude that could be developed without information on elementary constituents [1] and that the strong interactions theories were not be described by the renormalization field theory of point particles, which had been successful in QED. The study of strong interactions was transformed with the extremely importance of the accelerators in the multi GeV energy range. The famous SLAC experiment of the 1960's and 1970's were the first to show the substructure of the hadrons [2-4]. The Parton model shows the elementary constituents, interacting weakly. In the same period the quark model [5-6] rationalized hadron spectroscopy out of which the idea of the color grew up [7-8]. A new quantum number postulated in the first case to avoid the apparent paradox that the quark model seems to require spin $\frac{1}{2}$ quarks with the bosonic statistics.

In addition to this, as consequences, the field of QCD is currently experiences some short of revolution on perturbative and non-perturbative scale. On the perturbative side, new methods to compute scattering amplitude with very high particle multiplicities are being developed, together with advance technology for combining all order re-summation framework. In absence of the feasible theory of strongly interacting elementary particles, it was necessary to rely on general properties of the scattering matrix. So here perturbative theory utilized at all and could be employed to illustrate and explore the properties [9]. On the non-perturbative side the wealth of data on soft physics processes from the LHC is forcing to re-consider the reliability of standard models and heavy ion collisions providing new insight into the behaviour of hadronic matter. The non-perturbative QCD also address the issue like the mechanism for confinement and chiral symmetry breaking, role of topology and equilibrium properties of QCD at finite temperature [10].

The growth of technology of the renormalization group and operator product expansion made it clear that any field theory of strong interactions would have to have an energy dependent coupling strength, to balance the low energy nature of the strong interactions, with their weakness at high energy(at short distances). QCD is a non-abelian gauge theory having unusual property that the effective interactions between the quarks decreases as the energy momentum transfer increases. These variables increase asymptotically to infinity. This theory tends to free field theory and property is called asymptotic freedom. The concept of the asymptotic freedom which is almost satisfied almost by QCD filled all these requirements [11-12]. As the asymptotic freedom is perturbative concept, the S matrix and cross-section for isolated quark and gluons in QCD all vanishes and replaced by the bound state dynamics [1]. This is the hypothesis of the confinement. Although the asymptotic freedom is the perturbative prediction, confinement is not. So the use of perturbative QCD is "pQCD"

developed slowly. The short distances predictions are the area of the perturbative of QCD. QCD essentially is a theory of the Partons i.e. quarks and gluons, when they are probed at smaller wavelength. But at large wavelength of order of proton size $\approx 1\text{fm} = 10^{-15}$, one can see strongly bound resonances. A tremendous progress has been made in order to understand and test the QCD since it was proposed. Due to our inability to perform the analytic calculations in the strongly coupled theories, QCD is therefore still partially solved. Its entire feature across the distance scale is believed to encode in a single one line formula of mysterious simplicity i.e. “The Lagrangian of QCD”. Moreover QCD is the largest Gauge theory we have so far encountered, its emergent phenomenon, unitary properties color structure, non perturbative dynamics quantum vs. classical limit, interplay between scale invariant and scale dependent properties and its wide range of phenomenological properties are still very much interesting topic of active discussion.

3.1 Natural Units

It is necessary to introduce the appropriate units to the particle physics. The two fundamental constant of the relativistic quantum mechanics are planks constants and velocity of light.

$$\hbar = \frac{h}{2\pi} = 1.05 \times 10^{-34} \text{Jsec} \quad (3.1)$$

$$c = 2.998 \times 10^8 \approx 3 \times 10^8 \quad (3.2)$$

since any system in particle physics defined totally in terms of energy. Therefore it is important to measure physical quantities in the units of energy i.e. GeV. A choice motivated by the fact that rest energy of photon is 1Gev. By choosing $\hbar = c = 1$, saving a lot of time and help in to drive the system towards the simplicity. In usual practise it is necessary to speak of mass (m), momentum (p) and energy (E) all in terms of GeV and length and time in terms of GeV^{-1} .

3.2 Lagrangian Formalism of QCD

3.2.1 Quarks and their Masses

Quarks are the most fundamental particles participating in the strong interactions. They are simply structure less (as far we know) spin $\frac{1}{2}$ particles. Therefore in the relativistic theory they are described by the Dirac spinners $\psi_\alpha(x)$ with four components where $\alpha = 1,2,3,4$ are the function of space time coordinate $x^\mu (t, x, y, z)$. If quarks did not interact with other particles or field they obey the free Dirac equation [13]

$$(i\partial - m)\psi(x) = 0, \quad (3.3)$$

m is the “free” mass. The simple state of these quarks in term of plane wave is

$$\psi_{k\lambda}(x) = U(k, \lambda) \exp(-i(Et - x \cdot k)), \quad (3.4)$$

Where $k^\mu = (E, k)$ and λ denotes the four momentum polarisation and $U(k, \lambda)$ is the spin dependent momentum space wave function so the Lagrangian density is given as

$$\mathcal{L}(x) = \bar{\psi}(x)(i\partial - m)\psi(x), \quad (3.5)$$

In reality free quarks have never been observed in laboratory till yet. Therefore called quark confinement and is an important consequence of the low energy dynamics of the strong interactions. Quarks are not observed in Free State so as their masses. Therefore the meaning of mass requires some explanations. The mass of the quark is a parameter in Lagrangian which describe the self-interactions of the quarks. The mass parameter is much like a coupling constant in quantum field theory and depends upon the momentum scale and renormalization scheme and is scale dependent. According to the standard model the mass of quarks are generated through a symmetry breaking phase transition of the electroweak interactions. The detail aspect of the symmetry breaking is as suggest by the Higgs and others [14] and clarified with the discovery of the Higgs boson in present year (March 2013) in experiment at high energy collider.

3.2.2 Color Charge

Quarks participate in the strong interactions in the strong interactions because they carry color charges. The color charges are the analogue to electron charge in QED, with important differences. Unlike electron charges which are scalar quantity in the sense that total electric charge of a system is algebraic sum of the individual charges whereas the color charge is quantum vector charge similar to angular momentum in quantum mechanics. The total charge of the system must be obtained by the combining individual charges of the constituent according to the group theory rule. The quarks have basic three basic color charges state which are labelled as Red, Green, Blue, the three fundamental colors. The three color states form a basis in the 3-dimensional complex vector space. As a conclusion of this the color state of a quark is then a vector in this space. The color state can be represented by the matrix

$$3 \times 3 = 1 + 8,$$

where one correspond to the color singlet state and 8 correspond to gluon fields. All such unitary transformations with unit determinant form a lie group SU(3). The three dimensional color space form a fundamental representation of SU(3). The rule of the adding of the color charge follows the adding representation space of the SU(3) group. The quarks like electron have antiparticles called antiquark and denoted by the \bar{q} . The antiquarks have the same spin and mass as the quark, but with the opposite electric charge and color charge is denoted by $\bar{3}$. The important view of hypothesis is to visualize the quark confinement as the color confinement: as strong interactions do not allow the sates other than color singlet state or color neutral appear in nature, which is the evidence of the lattice QCD (non-perturbative).

3.2.3 Gluons and QCD

Quarks do not interact with each other directly; they do so through the intermediate agents called gluons. Gluons are massless spin 1 particles with two polarisation states i.e. left handed and right handed. They are represented by the four component vector potential $A^\mu(x)$ with the Lorentz index $\mu = 0,1,2,3$. Therefore the condition must be imposed on $A^\mu(x)$ to select only the physical degree of freedom. Different $A^\mu(x)$ can also give the same degree of freedom. These conditions are called gauge conditions for this region $A^\mu(x)$ is called gauge

potential and gluons are called gauge particles. There are 8 types of gluons mediating the strong interactions. Since the Lagrangian is invariant under the global SU(3) transformation.

$$\psi'(x) = U\psi(x), \quad (3.6)$$

where U is a 3×3 unitary matrix acting on color index. Global means that field at different space time is transformed in exactly the same way. So the SU(3) matrix require 8 real parameters written as

$$U = \exp(i \sum_a \theta^a \frac{\lambda^a}{2}), \quad (3.7)$$

The exponential in the form of $\exp(iax)$ is a phase factor. The local gauge invariance is independent of the space time dependent phase factor [15]. Here $\frac{\lambda^a}{2}$ (a=1.....8) are 3×3 hermitian matrices and are called generators of SU(3) rotation.

if we introduce 8 real independent gluon gauge fields A_a^μ then the associated covariant derivative

$$D^\mu = \partial^\mu + igA^\mu, \quad (3.8)$$

here A^μ is simply taken to be Hermitian and traceless. The parameter appears in (3.8) is the strong coupling constant. So the free quark Lagrangian

$$\mathcal{L}_q = \bar{\psi}(iD - m_q)\psi, \quad (3.9)$$

This new Lagrangian has the gauge symmetry. It is invariant under a space-time dependent SU(3) rotation U(x) of the quark field. So the gauge field transformed according to

$$A^\mu = U(A^\mu + \frac{i}{g}\partial^\mu), \quad (3.10)$$

We introduce the 3×3 gluon field matrix and can be expressed in term of λ^a as below

$$A^\mu = \sum_{a=1}^8 A_a^\mu \frac{\lambda^a}{2}, \quad (3.11)$$

The factor $\frac{1}{2}$ is conventional in (3.11). This transformation is generalization of gauge transformation in classical electromagnetism and reflects the number of physical degree of freedom associated with each gauge field is 2 for the massless spin 1 particle. By using local color symmetry we find that quarks are no longer free particles but they interact through each other by new gauge field. As a result of this gauge symmetry has generated a well define dynamics of color charge and gluons are related to 8 parameters of the SU(3) transformations. The color interactions in term of Lagrangian is given as

$$\mathcal{L}_{int} = -g\bar{\psi}_j A_a^\mu \frac{\lambda_{ji}^a}{2} \gamma_\mu \psi_i, \quad (3.12)$$

where g is a coupling constant. The new feature of this eq. (3.12) is that quark can change their color from i to j by emitting or absorbing a gluon of any color, coupling through the SU(3) generator define as

$$t_{ij}^a = \frac{\lambda_{ij}^a}{2}, \quad (3.13)$$

As gluon are the physical degree of freedom discussed above, therefore carry energy and momentum themselves as a result these terms must be includes in the Lagrangian to describe the physical feature. Following the Maxwell electromagnetism theory [16] we introduce an asymmetric field tensor $F_{\mu\nu}^a$ and K.E. terms for gluons.

$$\mathcal{L}_g = -\frac{1}{4} F^{\mu\nu a} F_{\mu\nu a}, \quad (3.14)$$

Where summation over color is inherent. to ensure the new terms added in the gauge invariant the field strength tensor required to transform accordingly to Yang Mills

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - g[A^\mu, A^\nu], \quad (3.15)$$

lead to the results, that under SU(3) transformation of form

$$F^{\mu\nu} = UF^{\mu\nu}U, \quad (3.16)$$

Gluon Lagrangian density is taken as

$$\mathcal{L}_g = \frac{1}{2}Tr[A_{\mu\nu}A^{\mu\nu}] \quad (3.17)$$

There is a commutation relation term $[A^\mu, A^\nu]$ which is non-linear in term of gauge field. Therefore as a result gluon does not come in a simple repetition but they have three or four gluon interactions [17]. So the full QCD Lagrangian density in term of quark and gluon and ghost/virtual/ Fock states

$$\mathcal{L}_{eff}^{QCD}[\psi_f(x)\bar{\psi}_f(x)A(x)C(x)\bar{C}(x); n, m_f] = \mathcal{L}_{invar} + \mathcal{L}_{gauge} + \mathcal{L}_{ghost}, \quad (3.18)$$

Which is the function of field ψ_f quarks, A gluons C ghost state and n, m_f are the parameters. Here f labels for the distinct quark fields. \mathcal{L}_{invar} is the classical density, invariant under local SU(N_c) gauge transformations with $N_c=3$ for QCD. It was originally written by Yang and Mills [18] i.e.

$$\mathcal{L}_{invar} = \sum_f \bar{\psi}_f [iD[A] - m_f] \psi_f - \frac{1}{4}F^2[A], \quad (3.19)$$

So

$$\mathcal{L}_{QCD} = \sum_{f=1}^{n_f} \sum_{\alpha, \beta=1}^4 \sum_{i,j}^{N_c} \bar{\psi}_{f\beta j} (i(\gamma)_{\beta\alpha}^\mu D_{\mu ji}[A] - m_f \delta_{\beta\alpha} \delta_{ji}) \psi_{f\alpha i} - \frac{1}{4} \sum_{\mu, \nu=0}^3 \sum_{\alpha=1}^{N_c^2-1} F_{\mu\nu}[A]_a F^{\mu\nu}[A]_a \quad (3.20)$$

Where $F^{\mu\nu}[A]_a$ is the gluon field strength tensor and can be define as in equation (3.16). N_c is often called number of colors but here represents the group components and are real number known as structure constant.

3.2.4 Confinement and Asymptotic Freedom

The structure constant denoted by SU(2)

$$[T_a T_b] = if^{abc} T^c, \quad (3.21)$$

Where f^{abc} is levi-civita tensor such that

$$f^{abc} = 1, \quad (3.22)$$

Now the QCD normalization factor can be calculated in order g^2 therefore

$$Z_m = 1 - 6 \left(\frac{e^2}{16\pi^2 \epsilon} \right), \quad (3.23)$$

$$= 1 - 6 \left(\frac{g^2 T^A T^A}{16\pi^2 \epsilon} \right), \quad (3.24)$$

$T^A T^A = \frac{4}{3}$ for quarks in QCD. The β function for QCD is

$$\beta(g) = -\frac{g^3}{\pi^2} \left(11 - \frac{2}{3} N_q \right) + Other(g^5), \quad (3.25)$$

Where N_q represent the quark flavor and by replacing the $e^2 \rightarrow N_q \frac{g^2}{2}$. Other terms in the β function form the gluon interactions. QCD β function is negative as long as number of the quark flavor N_q is less than 16. So QCD fine structure constant is given by

$$\alpha_s(\mu) = \frac{g^2(u)}{4\pi}, \quad (3.26)$$

α_s is smaller at larger u and known as asymptotic freedom. The u dependence of α_s is as

$$\alpha_s(\mu_2) = \frac{1}{\left[\frac{1}{\alpha_s(u_1)} + \beta_0 \ln\left(\frac{\mu_2^2}{\mu_1^2}\right) \right]} \quad (3.27)$$

Above equation is valid as long as μ_1 and μ_2 are large enough. Therefore it is convenient to introduce an independent constant (Λ_{QCD}) with dimension mass

$$\Lambda_{QCD} = \mu \exp\left(-\frac{1}{2\beta_0\alpha_s(\mu)}\right), \quad (3.28)$$

So at last strong interaction fine structure constant become

$$\alpha_s(\mu) = \frac{12\pi}{(33-2N_q) \ln\left(\frac{\mu^2}{\Lambda_{QCD}^2}\right)}, \quad (3.29)$$

QCD coupling constant diverges as $\mu \rightarrow \Lambda_{QCD}$ and expression for the α_s ceases to be valid, when α_s get large. In spite of that one can see $\Lambda_{QCD}(200 \text{ MeV})$ as the scale at which QCD become strongly coupled so that perturbation theory breaks down and non perturbative effect becomes important [19]. Since change in the charge in quarks is not strictly constant and varies with distance because of quantum effect hence fine structure constant is also variable.

3.3 Chiral Symmetry

We know that fermions (i.e. quarks) have helicity $\pm \frac{1}{2}$, which represents their spin orientations either in the direction of the motion or opposite to the direction of the motion. If we consider that fermions to be massless then it will move with the velocity of light and its helicity is along the direction of motion. Therefore fermions have a definite handedness i.e. positive or negative and become decoupled. That's why called as chiral fermions. From the Dirac equation (3.13) if we put mass equal to zero then equation transformed as

$$i\partial\psi = 0 \quad (3.30)$$

But we know that (from section 3.3) γ_5 anti-commute with γ_μ . Therefore equation represented as

$$i\gamma_5\partial\psi = -i\partial\gamma_5\psi = 0 \quad (3.31)$$

Therefore the linear combination of the equation can be represented as

$$\psi_L(x) = P_L\psi(x) = \frac{1}{2}(1 - \gamma_5)\psi \quad (3.32)$$

$$\psi_R(x) = P_R\psi(x) = \frac{1}{2}(1 + \gamma_5)\psi \quad (3.33)$$

Where P_L and P_R are chirality operator and 1 represent the 4×4 unitary matrix. The terms $\psi_L(x)$ and $\psi_R(x)$ are left handed and right handed projection of the $\psi(x)$. The right handed fermions means that massless fermions has helicity $+\frac{1}{2}$ and left handed fermions has helicity $-\frac{1}{2}$. Therefore the Lagrangian for the massless fermions is given as

$$\mathcal{L}_0 = i\bar{\psi}\partial\psi \quad (3.34)$$

$$\mathcal{L}_0 = i\bar{\psi}_L\partial\psi + i\bar{\psi}_R\partial\psi \quad (3.35)$$

Above equation (3.35) represent Lagrangian depend either left handed or right handed but not on both simultaneously. As a result in QCD the chiral field do not couple to each other. Therefore the Lagrangian is seen to be invariant under a global chiral phase change.

$$\psi_L(x) = e^{i\theta_L}\psi_L(x) \quad (3.36)$$

$$\psi_R(x) = e^{i\theta_R}\psi_R(x) \quad (3.37)$$

Where θ_L and θ_R are the real phase constants named as $U_L(1) \times U_R(1)$. If we consider the case of the up (u) and down (d) quark and assumed them to be massless then they can be regarded as the two independent states of same object forming a two component spinor (as discussed in chapter 2) an isospin doublet.

$$\text{i.e.} \quad \begin{pmatrix} u_{L,R} \\ d_{L,R} \end{pmatrix} = U_{L,R} \begin{pmatrix} u'_{L,R} \\ d'_{L,R} \end{pmatrix} \quad (3.38)$$

in equation (3.38) $U_{L,R}$ represents the 2×2 unitary matrices i.e. the quark part of Lagrangian is invariant. Therefore Lagrangian has a chiral symmetry $U_L(2) \times U_R(2)$. Two unitary matrices can be decomposed into the product of the phase (equation (3.36) and (3.37)) and a special unitary group matrix with unit determinant. As a result the chiral symmetry decomposed as

$$U(2)_L U(2)_R = SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R \quad (3.39)$$

So the Lagrangian density is

$$\mathcal{L}_{m=0} = i\bar{\psi}\partial\psi; \quad \text{or} \quad \mathcal{L}_0 = i\bar{\psi}^f\partial\psi^f \quad (3.40)$$

$$\mathcal{L}_{m=0} = i\bar{\psi}^u\partial\psi^u + i\bar{\psi}^d\partial\psi^d \quad (3.41)$$

where $\partial = \partial_\mu \gamma^\mu$ and f represents the number of flavor. If we consider the transformation such that

$$\psi_L = \psi_L' = e^{-i\frac{\sigma}{2}\vec{\theta}_L}\psi_L \quad (3.42)$$

$$\psi_R = \psi_R' = e^{-i\frac{\sigma}{2}\vec{\theta}_R}\psi_R \quad (3.43)$$

where ψ_L and ψ_R are helicity eigenstates.

$$\psi = \frac{1}{2}(1 \pm \gamma_5)\psi \quad (3.44)$$

σ represents the Pauli's spin matrices and $\vec{\theta}_L$ and $\vec{\theta}_R$ are the arbitrary parameters and symmetry group is $SU(2)_L \times SU(2)_R$. On separating $\psi\bar{\psi}$ in the form of left and right hand components

$$\bar{\psi}^f\partial\psi^f = \bar{\psi}_L^f\partial\psi_L^f + \bar{\psi}_R^f\partial\psi_R^f \quad (3.45)$$

$$\bar{\psi}^f\partial\psi^f = \bar{u}_L\partial u_L + \bar{d}_L\partial d_L + \bar{u}_R\partial u_R + \bar{d}_R\partial d_R \quad (3.46)$$

The matrices like γ_μ and σ act on the different space so they commute. This symmetry is called chiral symmetry. The symmetric and anti-symmetric combinations of the states of opposite parities are transferred independently under rotation in isospin space. These combinations have the property of the left and right hand to transfer into each under space reflection.

From equation (3.37) the symmetry group $SU(2)_L \times SU(2)_R$ has not been seen clearly in the particle spectrum or in the scattering matrix elements. It is a hidden symmetry because it breaks simultaneously into the isospin subgroup $SU(2)_L \times SU(2)_R$ corresponding to the $U(2)_L \times U(2)_R$ group. Similarly we can also calculate the chiral symmetry for the higher order flavor terms like $SU(3)_L \times SU(3)_R$.

3.4 Chiral Symmetry Breaking

In the previous section we discussed about the chiral symmetry when the masses of the fermion are assumed to be zero. If we introduce the symmetry term then the change in Lagrangian is given by equation

$$\delta\mathcal{L} = m(\psi\bar{\psi}) \quad (3.47)$$

$\delta\mathcal{L}$ is invariant under the vector and axial transformations because these transformations measure the path integral known as axial anomaly. Hence the axial U(1) is not a symmetry of QCD.

$$\Lambda_A = m(\psi\bar{\psi}) \quad (3.48)$$

Λ_A is not a good symmetry if the fermions have a finite mass. The mass term in a Lagrangian always mix chiral terms.

$$m\psi\bar{\psi} = m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) \quad (3.49)$$

If the masses are considered to be very then Λ_A is an approximate symmetry. In QCD, we know masses of quarks are about 5-10 MeV. The relevant energy scale is much larger; therefore we consider Λ_A being approximate. The chiral $SU(2)_L \times SU(2)_R$ symmetry of massless flavor QCD is spontaneously broken by the vacuum expectation value of quarks

$$\langle\bar{\psi}_L^f\psi_R^f\rangle = v\delta^f \quad (3.50)$$

where $v = \Lambda_{QCD}$. The transformation in $SU(2)_L \times SU(2)_R$ is

$$\langle\bar{\psi}_L^f\psi_R^f\rangle = v\langle LR^\dagger\rangle^f \quad (3.51)$$

Transformation of L=R leave the vacuum expectation value changed. Thus the non-perturbative strong interactions dynamics breaks the $SU(2)_L \times SU(2)_R$ chiral symmetry to its diagonal subgroup.

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Quarks, Hadron and Their Properties

From the starting of twentieth century, the particle physics was very confusing. The elementary particles photon electron muon neutrinos were known, but addition to this many other particles come into the existence. There are several exciting new discoveries has been made in the experimental particle physics which added more and more particles to this collection of particles. As a result it become very difficult to classified them. The main property that these particles have in common is that they interact strongly with proton and neutron. But at the same time proton and neutron are also mysterious as well and complicated compound states. Therefore there was a requirement of the classification theory for all particles. At that time theoretical particle physicist M. Gell-Mann contribute to the process of systematization and clarification of the all elementary particles. The result of this massive amount of theoretical and experimental work introduces the quarks and help to understand the new particles as well as proton and neutron.

In the mid of century during 1960's various experiments are performed to know about the spectra of elementary particles. One of them is at SLAC. Very high energetic beams of electrons are bombarded on the proton to reveal the spectra [1]. The electron beams were found to be scattered with large transformations of momentum. These observations then suggested that proton contained the discreet scattering centre inside it called "quarks". The scattering centre had no internal structure of their own as suggested by Bjorken (1967), Feynman (1969), Paschos (1969). These scattering centres are thought to be point like and then referred as the constituents. Here constituents mean that these are not resolved yet or may have to be.

With the arrival of high energy lepton beam at Fermi Lab, SLAC, CERN, HERA enable physicist to uncover the structure and various experiments was performed during last few decades. The most recent experiment at HERA Accelerator Hamburg Germany resulting in the high resolution image of the proton have given the fundamental insight into the nature of forces binding the quark to one another. This gives rise to theory of quarks as known as QCD discussed in chapter 3. As a result new kind of particles has been discovered and we learn about that proton and neutrons are not the elementary particles but instead of them someone else. These particles are known as Partons. These newly discovered particles then divided in two categories: 1. electrically neutral particles-gluons, massless and vector particles. 2. Half integral spin fermion fields called quarks. Since every discovered particle has not fit into the classification scheme postulated by Gell-Mann, Ne'eman [2, 3] based on symmetry group SU(3). A three quark model proposed by Gell-Mann and Zweig [4, 5] corresponding to SU(3). Later on many models have been proposed and discussed and each one shows that quarks have half integral spin [6]. In the starting, we have only three known quarks: up (u),

down (d), strange (s), but now we have six known quarks i.e. up (u), down (d), strange (s), charm (c), top (t), and bottom (b) [7]

4.1 Quarks

There are six type of flavor [3, 4] of spin $\frac{1}{2}$ [8] quarks are known to exist till now. These quarks are occur in the pair or generation according to their mode of interactions and represented as

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

Quarks in the first family constitute the basic component of the existing matter whereas the quarks of the other families have unstable particles. All these quarks have spin $\frac{1}{2}$ and exist in the spinorial state. Each generation consist of a quark with the charge $+2/3$ (u, c, t) together with a quark of charge $-1/3$ (d, s, b) in the units of electronic charge. The anti-particles of these six flavors spin $\frac{1}{2}$ quarks are same in mass and spin and also having same charge but with the reversed sigh of the quark charges.

Table 4.1: Classification of quarks on basis of their mass, charge and spin

Quark	Symbol	Free mass (in GeV/c^2)	Constituent mass (in GeV/c^2)	Charge	Spin
Up	u	0.00230 ± 0.7	0.363	$+2/3$	$\frac{1}{2}$
Down	d	0.00480 ± 0.5	0.363	$-1/3$	$\frac{1}{2}$
Charm	c	1.275 ± 0.025	1.7	$+2/3$	$\frac{1}{2}$
Strange	s	0.0950 ± 5	0.5	$-1/3$	$\frac{1}{2}$
Top	t	173.07 ± 0.6	172	$+2/3$	$\frac{1}{2}$
Bottom	b	4.66 ± 0.03	4.5	$-1/3$	$\frac{1}{2}$

4.1.1 Evidence for Quarks

The strong evidence for the existence of the quark inside the nucleon is provided by the experiments known as deep inelastic scattering i.e. lepton hadron interactions. There is a large amount of momentum transfer to the nucleon. Several hundred hadrons have been observed since pions were introduced in the laboratory in 1950. Hadrons are bound state of the fundamental spin $\frac{1}{2}$ quarks, who's electric charge can be $+2/3$ or $-1/3$ and anti-quark with opposite charge of that of quarks. Since quarks are never found in the observable free particle states the evidence of the existence of the quarks comes from three main areas [9] hadron spectroscopy, lepton scattering and jet production. The failure to detect the free quark is not an experimental problem. The free quarks would be easily distinguishable from other particles by their fraction charge. The electronic charge conservation implies that a fractionally charged particle cannot decay into the final state composed entirely of the particle with larger integer electronic charge. Hence light quarks are stable and would be easy to observe. Some of the quarks are not very massive and because they interact through the

strong interactions, so one can expect free quarks produces abundantly. However with too much careful and profound search in ordinary matter in cosmic rays in high energy collision, free quarks have never been found. The conclusion is that quarks exist solely in the bound state and is the region we are force to study the properties of the bound states of the quarks.

4.2 Bound States of Quarks and Properties

4.2.1 Hadron Quantum Number

Hadrons includes a set of the quantum number having the total internal symmetry that is, symmetry under transformations do not change the space time. The known quantum numbers corresponding to the internal symmetry are electronic charge, baryon number, isospin, strangeness, charm, parity, charge conjugation etc. The internal quantum numbers are used to describe the hadrons are independent to each other. For e.g. charge Q , z -component of Isospin I_z , Baryon number B , and strangeness S are related to each other by the formula proposed by Gell-Mann, Nishijima and Nakano [10, 11]

$$Q = I_z + \frac{1}{2}[B + S + C + b] \quad (4.1)$$

All the quantum number represented by the equation (4.1) are additive in nature. The quantum numbers can also be written as in the form of hypercharge as

$$Y = B + S - C + b \quad (4.2)$$

Table 4.2 Quantum number of the quarks

Quarks	Symbol	Spin (I)	Isospin (I_z)	Baryon number (B)	Charm (C)	Strangeness (S)	Bottom (b)	Top (T)
Up	u	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	0	0	0	0
down	d	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{3}$	0	0	0	0
charm	c	$\frac{1}{2}$	0	$\frac{1}{3}$	+1	0	0	0
strange	s	$\frac{1}{2}$	0	$\frac{1}{3}$	0	-1	0	0
Top	t	$\frac{1}{2}$	0	$\frac{1}{3}$	0	0	0	+1
bottom	b	$\frac{1}{2}$	0	$\frac{1}{3}$	0	0	-1	0

4.2.2 Flavor and charge independence

All quarks are distinguished by a term usually known as flavor. Gell-Mann (1964) and Zweig (1964) [3, 4] originally proposed that there are different kinds of the quarks exist between because SU(3) group was considered as the symmetry group of the strong interactions. The strong interactions between the two quarks at affixed distance are independent of the quark flavor u, d, s, t, c, b are involved, for e.g. strong interaction between the quark pair us and ds are identical. The same principle applied to the quark and anti-quark forces which are however not identical to quark-quark forces because in this case the annihilation can occur. On applying the flavor independence one must also account for the mass difference, which

can be non-trivial. The consequence of the flavor independence is that hadrons occur in the family of the particles with the approximately same mass called charge multiplets with in a given family. All particles have same spin parity and the same baryon number, Strangeness, Charm and Beauty quantum number but different electronic charge. The close behaviour reflects the approximate symmetry between the u and d quarks. This is because these two quarks only have a very small difference.

$$m_d - m_u = (3 \pm 1)MeV/c^2 \quad (4.3)$$

The flavor independence of the strong interactions also leads to the charge independence of the forces e.g. the equality force between any pair of the quarks provided two identical states which are in the same spin state giving the idea of charge symmetry [12] of the two states.

4.2.3 Color

In addition to the flavor symmetry quarks also exhibits a new degree of freedom known as color. Any quark u, d, s... can exist in three different color states. There is a direct experimental evidence from the data of the electron positron annihilation [13] that just three such states are exist we denote r, g, b for red, green, blue respectively. Each of these states is characterised by the value of the two conserve color charges denoted by I_3^C and Y_3^C , which are strong interactions analogue to the electric charge in electromagnetic interactions. These states depend upon the color states not on the flavor state. The particular value of the quark anti-quark are a consequence of a fundamental interaction called SU(3) color symmetry (discussed in chapter 2). For multi-particle state the color charge of the individual state are simply added. Only the state with the total color charge zero value is observed as free particle till yet. These states are called color singlets. This is the hypotheses of the color confinement. For a three quark state both Y_3^C and I_3^C can be equal to zero, if we have one quark in red state, one in green state and one quark in blue state. This implies that all the three quarks in different states and Pauli's exclusion principle can be satisfied. So the total wave function is product of spatial part $\psi_{spatial}$, spin part ψ_{spin} and color part ψ_{color} i.e.

$$\psi = \psi_{spatial}(x)\psi_{spin}\psi_{color} \quad (4.4)$$

The Pauli's principle now interrupted as applying to the total wave function including the color part ψ_{color} . Since quarks are supposed to be spin half particle according to the field theory, the wave function of the particles of the half integral spin is antisymmetric under the exchange of the two particles and combined space and spin wave function is thus symmetric under the exchange of the two flavor quark. Thus the ψ_{color} is given as

$$\psi_{color} = \frac{1}{\sqrt{6}} [R_1G_2B_3 + G_1B_2R_3 + B_1R_2G_3 - R_1B_2G_3 - B_1G_2R_3 - G_1R_2B_3] \quad (4.5)$$

Where R, B, G represent the quarks with red, blue, green color respectively, If we consider that any quark can have color degree of freedom, so the wave function is antisymmetric in color degree of freedom [6]. Baryons are bound state of three quarks, corresponding to that they have three colors i.e. red, blue and green. Each quark with a color degree of freedom (either be same or different) is the combination of antisymmetric wave function. Such a combination represents the color singlet state and said to be colorless. It has been proposed that quarks are themselves representing the composite system [14]. On the other hand it was

stated that a quark is composed of two objects one carry color and other carry flavor [15]. It is still a mystery till now that, what is the nature of color degree of freedom because all hadrons are color singlet.

Han and Nambu proposed that each quark flavor should come in three types of color [16]. They suggested that, whenever the number of quarks becomes greater than three, there is no need for the quarks to have the fraction charge or baryon number individually. The color symmetry with fractional charge quarks is called Gell-Mann Zweig color, and symmetry with integral charged quarks is called Han-Nambu color. It is assumed that Gell-Mann Zweig color symmetry is exact. However Han-Nambu color symmetry must be broken. This is because of the fact that quarks with same flavor and different color have the same charge and therefore they must interact differently with the fields.

Table: 4.3 Value of color charge for the color state of quark and anti-quark

Quarks color charge	I_3^C	Y_3^C	Anti-quarks color charge	I_3^C	Y_3^C
Red (r)	$\frac{1}{2}$	$\frac{1}{3}$	Anti-red (\bar{r})	$-\frac{1}{2}$	$-\frac{1}{3}$
Green (g)	$-\frac{1}{2}$	$\frac{1}{3}$	Anti-green (\bar{g})	$\frac{1}{2}$	$-\frac{1}{3}$
Blue (b)	0	$-\frac{2}{3}$	Anti-blue (\bar{b})	0	$\frac{2}{3}$

4.2.4 Gluons

Gluons are the bosons or the field quanta that are responsible for the strong interactions between the quarks. Gauge field theory states that SU(3) symmetry is an exact symmetry and gluons form the SU(3) color octet. The important aspect of this theory is that, gluons do not interact with themselves. Therefore theory is said to be non-abelian gauge field theory associated to Yang-Mills field [13]. The particles which are not SU(3) color singlet such as quark, gluon, bound state of two quarks etc. are called colored while SU(3) singlet are known as colorless. All the observed hadrons are color singlet.

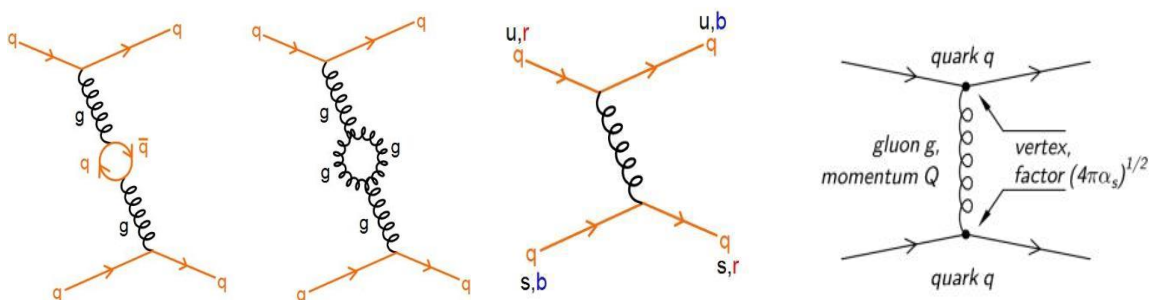


Fig: 4.1 Representation of strong interaction between the Quarks through gluons.

4.3 Hadrons

The hadron spectroscopy can be explained by using the static quark model of the hadrons. The constituent quarks i.e. valence quarks explain the regularities of the hadrons. Evidence of the existence of the quarks occurred via the lepton hadron collision with the high momentum transfer (deep inelastic scattering) [17]. Where as in the also contains the other particles namely gluon and virtual quark anti quark pairs rapidly created and annihilated. These virtual quark anti-quark pairs are called sea quarks. Before the quarks were discovered the increasing number of the hadronic particles leads to the existence of the some inner symmetry that could explain the rapid increase in the particles. The quark model [3, 4] solves the problem of the classification of hadrons. As we know the quarks have the fractional charge w.r.t. that of nucleons having same spin and parity. According to the standard model quarks are group in order to form a particle with integral charge in unit of the charge of electron. Since hadrons are considered as the eigen state of the quarks interacting via strong interactions. We know that properties of the atoms and nucleus can be calculated in terms of their constituent's proton neutron and electron. Although in practise it is not possible to calculate their properties very accurately. However the properties of these particles can be determined by studying them directly as free particle in the laboratory. In case of hadrons the situation is much more complicated, because there properties are explained in terms of fundamental constituents i.e. quarks. But the properties of the quarks themselves can only be studied experimentally by approximate measurement of hadrons. As a result the study of hadrons is not possible without quarks. In the experimental investigations of quarks an isolated quarks has never been observed. Quarks are always confined in compound system in the range of 1fm distance. Therefore quarks are now considered as the most fundamental particles. The particles whose constituents are quarks are known as hadrons. Hadrons further classified into two categories depending upon their quark content and spin: 1.Baryons 2.Mesons. The most elementary quark system, Baryons have net three quarks for e.g. proton and neutron etc. and Mesons have net zero quarks essentially a quark and anti-quark pair for e.g. pion($\pi\pi^{0,\pm}$) etc. the quarks are bounded by strong interaction (discussed in chapter 3).

4.3.1 Mesons

The bound states with quark-antiquark (flavor can be different) combination are called Mesons. Quarks have intrinsic half integral spin ($\frac{1}{2}$) [6], so the total intrinsic spin of the mesons can be either 0 or 1 etc. , i.e. they are bosonic in nature and follow the Bose-Einstein statistics. If we consider the three quarks u, d, s there will be nine possible states. Color charge plays no role as bound states are colorless. All the particles are classified in a particular manner given in the ref. [18]. The particles are arranged by the strangeness and charge, the strangeness of particles is determined by the number of strange quarks in that particle. The quarks in the meson can have the spin either in the opposite direction ($\downarrow\uparrow$) or in the same direction ($\uparrow\uparrow$) with respect to each other. So in the first case meson have zero spin and negative parity and in the second case the meson have spin one and negative parity. If the

orbital angular momentum of quark anti-quark state is represented by L then the parity of the quark anti-quark state is given by

$$P = (-1)^{L+1} \quad (4.6)$$

Since a quark anti-quark is also an Eigen states of charge conjugation with

$$C = (-1)^{L+S} \quad (4.7)$$

where the spin S is 0 or 1. L=0 states are pseudoscalar, $J^P = 0^-$, and vector in $J^P = 1^-$. Mesons in the normal spin parity is given as

$$P = (-1)^J \quad (4.8)$$

4.3.2 Baryons

Baryons are considered to be bound states of the three quarks (valence) plus any number of quark antiquark pairs (sea). Here we are interested only in the static quark model therefore we will omit the option of the sea quarks. There are two possible combinations for the spin of baryon constituents to be aligned 1) spin of the quark content aligned parallel ($\uparrow\uparrow\uparrow$) to each other 2) spin of one quark is aligned in opposite direction ($\uparrow\uparrow\downarrow$) to that of other two quarks.

Table 4.4 List of Baryons [40]

Particle	Symbol	Content	Charge	Rest mass MeV/c ²	Spin	Isospin	Baryon number	Strangeness
Proton	p	uud	+1	938.27	1/2	1/2	+1	0
Neutron	n	ddu	0	939.56	1/2	-1/2	+1	0
Lambda	Λ^0	uds	0	1115.683	1/2	0	+1	-1
Sigma	Σ^0	uds	0	1192.64	1/2	0	+1	-1
Sigma	Σ^+	uus	+1	1189.32	1/2	+1	+1	-1
Sigma	Σ^-	dds	-1	1197.44	1/2	-1	+1	-1
Cascade	Ξ^0	ssu	0	1314.86	1/2	1/2	+1	-2
Cascade	Ξ^-	ssd	-1	1321.71	1/2	-1/2	+1	-2
Omega	Ω^-	sss	-1	1672.45	3/2	0	+1	-3

So in the first case baryon has spin 3/2 with positive parity represents the $J^P = \frac{3}{2}^+$ baryon decuplet state and in the second case the baryons have the spin 1/2 with positive parity represents the $J^P = \frac{1}{2}^+$ baryon octet. Each baryon state is an SU(3) color singlet, a completely antisymmetric state of three possible colors. So there are three possible states for each quark i.e. up (u), down (d), strange (s). For a combination of quark there are $3 \times 3 \times 3 = 27$ states. The 27 states of quark form a singlet, two octets and a decuplet having spin 1/2 and 3/2 respectively. A baryon is a fermions with spin 1/2 and Baryon number (B) =1. Since the quarks are fermions, the state function must be antisymmetric under interchange of any two equal-mass quarks. Thus the state function can be written as

$$|qqq\rangle_A = |color\rangle_A \times |space, spin, flavour\rangle_S \quad (4.9)$$

Where the subscripts S and A indicate symmetry or antisymmetry under interchange of any two equal-mass quarks. The three quark flavour implies an approximation flavour SU(3) which requires that baryons are made of these quarks belongs to multiplets.

$$3 \otimes 3 \otimes 3 = 10_S \oplus 8_{MS} \oplus 8_{MA} \oplus 1_A \quad (4.10)$$

Here the subscripts indicate symmetric, mixed-symmetry, or antisymmetric states under interchange of any two quarks. If 1 and 8 has same spin and parity then they can mix. The same mechanism is applied for mesons. In ground state multiplets, the SU(3) flavour singlet is forbidden by F-D statistics[19, 20]. The baryons (no c and b quarks) flavor and spin may be combined in an approximate flavor spin SU(6). Then the baryon multiplets are represented as

$$6 \otimes 6 \otimes 6 = 56_S \oplus 70_{MS} \oplus 70_{MA} \oplus 20_A \quad (4.11)$$

SU(6) multiplets decompose into the SU(3) flavor as

$$56 = {}^4 10 \oplus {}^2 8 \quad (4.12)$$

$$70 = {}^2 10 \oplus {}^4 8 \oplus {}^2 8 \oplus {}^2 1 \quad (4.13)$$

$$20 = {}^2 8 \oplus {}^4 1 \quad (4.14)$$

Where the subscript (2S+1) gives the net spin S of the quarks for each particle in the SU(3) multiplet. The $J^P = \frac{1}{2}^+$ octet containing the nucleon and $J^P = \frac{3}{2}^+$ decuplet containing the Δ together make up the ground state in which the orbital angular momentum between the quark pairs is zero. The 70 and 20 requires some excitation of the spatial part of the state function in order to make the overall state function symmetric. State with the nonzero orbital angular momentum are classified in SU(6) \otimes O(3) supermultiplet. Baryons with the same quantum numbers do not belong to the single supermultiplet, since SU(6) is broken by the spin dependent interactions difference in the quark masses etc. the SU(6) \otimes O(3) basis provide the suitable frame work for the baryon state wave functions.

4.4 SU(6) Baryon Quark Model

The first step in understanding the quark structure of hadrons were made in the frame work of non-relativistic quark model, when SU(6) symmetry was introduced. The naive non-relativistic quark model provided the qualitative description of light hadrons but according to the [21] each quark has color degree of freedom and known simple quark model. The introduction of quarks [1] as the constituents of hadrons has given a new unifying point of view to particle physics. The fundamental triplet achieved the striking success when combine with the idea of approximate symmetry of the interaction relevant to the low lying hadronic states. This SU(6) theory [3] led to a number of striking results which obtained in the elementary way using the quark model [4]. One of the most surprising feature that comes from the study of baryons is that SU(6) wave function of the baryons is totally symmetric [2].

Consider the octet of spin $\frac{1}{2}$ and decuplet of the spin $\frac{3}{2}$ baryon. The splitting between the octet and decuplet is much similar to the SU(3) representation [22]. Therefore as a result the two representations can be combine to form a large group with irreducible representations greater than the SU(3) group. Since the representation in the group larger than the SU(3) have different spin so there will be mixing of the internal symmetry and the space time symmetry. Form equation (4.11) 56 state is purely symmetric [23] and represents the

combination of the three spin half quarks, each of which comes into the $2 \times 3 = 6$ states. From equation (4.12) the approximate degeneracy shows that 8 and 10 suggest that forces that bind quarks together are not the approximate SU(3) invariant but the spin independent as well [24]. All above discussion leads to the group known as SU(6) where 6 dimensional representation consist of the 6 quark states. The group SU(6) is the product representation of the SU(3) and SU(2) group [25]. So the generators of the SU(6) groups are the product of the 3×3 matrices of SU(3) space and 2×2 matrices in spin space. The SU(3) generators commute with the SU(2) generators. Therefore we can say that SU(6) has an SU(3) and SU(2) sub algebra. The SU(3) generators are form equation (2.38) are

$$\frac{1}{2} \lambda_a \quad (4.15)$$

and SU(2) generators are

$$\frac{1}{2} \sigma_i \quad (4.16)$$

Therefore the SU(6) generators are represented as

$$\frac{1}{2} \lambda_a \sigma_i \quad (4.17)$$

So the total number of the generators for the SU(6) group is $8+3+24 = 35$. For the low lying baryons consisting of three quarks transform like the 56 dimensional representation of SU(6) [22]. Equation (4.12) represents the symmetric representation of the 56. In baryons the decuplet states are much easier and simple to represent in SU(6) multiplet because both SU(2) and SU(3) indices are symmetric, for e.g.

$$|\Delta^{++}, \frac{3}{2}\rangle = |uuu\rangle |\uparrow\uparrow\uparrow\rangle \quad (4.18)$$

whereas the octet states are much complicate and difficult to get a symmetric wave function state. To get a symmetric wave function we multiply the SU(3) antisymmetric state with the SU(2) antisymmetric state and add cyclic permutation to obtain a complete symmetric wave function i.e.

$$|\Lambda, \frac{1}{2}\rangle = \frac{1}{2\sqrt{3}} [(|uds\rangle - |dus\rangle)(|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle) + (|sud\rangle - |sdu\rangle)(\uparrow\uparrow\downarrow) - \uparrow\downarrow\uparrow) + (|dsu\rangle - |usd\rangle)(\downarrow\uparrow\uparrow) - \uparrow\uparrow\downarrow)] \quad (4.19)$$

in other approach we can multiply the symmetric SU(3) state with the SU(2) symmetric state

$$|p, \frac{1}{2}\rangle = \frac{1}{3\sqrt{2}} [(|uud\rangle)(2|\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle)] \quad (4.20)$$

4.5 $J^P = \frac{1}{2}^+$ Baryon Octet Representation

Since we are interested only in the $J^P = \frac{1}{2}^+$ baryon state therefore we will neglect the $J^P = \frac{3}{2}^+$ baryon decuplet state. For $J^P = \frac{1}{2}^+$ baryon octet state we consider all the possible states of the u, d, s quarks with the spin i.e. one spin is not aligned with other two spin as mention in (4.3.2). Therefore the resulting spin is half. The spin $\frac{1}{2}$ baryons includes the

- 1) An isospin doublet: proton and neutron
- 2) An isospin singlet: Λ^0 (antisymmetric under the exchange of two quarks)

3) An isospin triplet: $\Sigma^0, \Sigma^+, \Sigma^-$ (symmetric under the exchange of two quarks)

4) Another isospin doublet: Ξ^0, Ξ^-

In the $J^P = \frac{1}{2}^+$ baryon octet state, all the states are symmetric under the simultaneous exchange of the spin and flavor of each pair but all are antisymmetric under the exchange of the spin and flavor only. The eight fold way arranged the baryons into the geometrical pattern according to their charge and strangeness. The eight lightest baryons fit into the hexagonal array with two particles at centre. The group is known as baryon octet. The particles with the like charges lie along the downward sloping diagonal lines. For the quark structure of the $J^P = \frac{1}{2}^+$ state baryon octet, we assume that spin and flavor are antisymmetric combination but globally are symmetric combination. Whereas the color wave functions is totally antisymmetric combination.

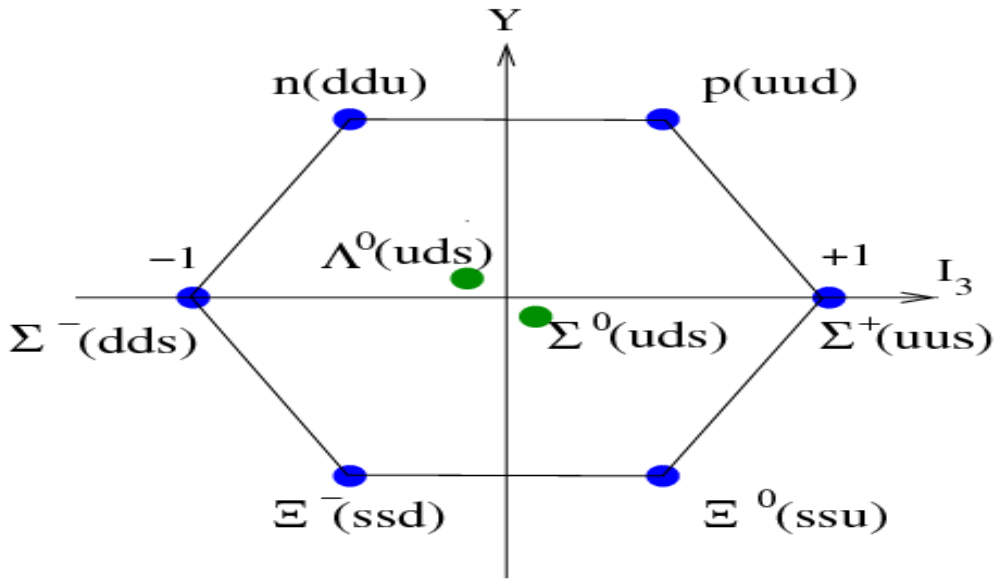


Fig: 4.2 represent the baryon octet state for $J^P = \frac{1}{2}^+$ state.

If we consider the case of proton the spin wave function with the two quarks can be placed in the antisymmetric spin singlet state by

$$\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \quad (4.21)$$

Correspondingly antisymmetric flavor state with the two light quarks

$$\frac{1}{\sqrt{2}}(ud - du) \quad (4.22)$$

Therefore on combining the two equations (4.21) and (4.22) in order to have a symmetric wave function under the simultaneous exchange of the spin and flavor is represented as

$$A = u(\uparrow)d(\downarrow) - u(\downarrow)d(\uparrow) - d(\uparrow)u(\downarrow) + d(\downarrow)u(\uparrow) \quad (4.23)$$

at last on adding the third quark in combination $A u(\uparrow)$. The expression in equation (4.23) since A is already symmetric under the exchange of the spin and flavor therefore the three quarks system become symmetric through the cyclic permutation as a result

$$\left(p, J^P = \frac{1}{2}^+\right) = (2u(\uparrow)u(\uparrow)d(\downarrow) + 2d(\downarrow)u(\uparrow)u(\uparrow) + 2u(\uparrow)d(\downarrow)u(\uparrow) - u(\downarrow)d(\uparrow)u(\uparrow) - u(\uparrow)u(\downarrow)d(\uparrow) - u(\downarrow)u(\uparrow)d(\uparrow) - d(\uparrow)u(\downarrow)u(\uparrow) - u(\uparrow)d(\uparrow)u(\downarrow) - d(\uparrow)u(\uparrow)u(\downarrow)) \quad (4.44)$$

The more detail of the wave functions of baryons octet $J^P = \frac{1}{2}^+$ state given in the appendix A.

4.6 Anomalies of the Static Quark Model

Since baryons have the three color degree of freedom in QCD, which neutralize into the colorless bound state we still do not know how to describe baryons in term of the fundamental quark and gluon degree of freedom. In the low energy limit QCD has an odd behaviour. The structure of the baryons is also complicated due to the non-perturbative and relativistic nature of the quarks and gluons. The complications also occur from the sea quarks in the baryons. Since the constituent quark model (CQM) tends to explain the static properties of the baryons, but still there I need of some new degree of freedoms. Still there are many possible direct and indirect tests on the static quark model and on the hadron composition in terms of quarks. The static model is appropriate for the explaining the hadrons classification, but it is not satisfactory for representing the quarks dynamics, which needs the contribution of the gluon and quark antiquark pairs (sea quarks) inside the hadrons. The introduction of valence part to the sea part (\bar{q} pairs) helps to make the deeper understanding of the various properties like the magnetic moment, mass of hadrons and flavor asymmetry. The hadronic physics describes the hadrons in term of their fundamental quarks and gluon degree of freedom (as discussed above). Therefore the structures of the hadrons are found to be more non-perturbative or relativistic nature of the quarks and gluons motion inside the hadrons. There have been many surprising discoveries concerning the structure of nucleon and the sea content inside the nucleons. The sea content found to be in more abundant than natural expectations [26]. As a result, there is a large asymmetry between \bar{u} and \bar{d} quarks distribution of the proton has been observed during the experiment [deep inelastic scattering]. Many theoretical attempts have been made to understand the sea flavor symmetry. And it is believed that mesons inside the nucleons can account for such asymmetry. There has been a new attempt to understand the sea flavor symmetry of proton from pure statistical considerations [27]. The sea quark anti-quark $u\bar{u}$ and $d\bar{d}$ pairs can be produced by the gluon splitting.

4.7 Baryon Wave Function with Valence and Sea Quarks in Ground State

From the section 4.3.2 we can write the wave function for the baryons as

$$\Psi = \Phi(|\phi\rangle|\chi\rangle|\psi\rangle)(|\xi\rangle) \quad (4.45)$$

Where $|\phi\rangle$, $|\chi\rangle$, $|\psi\rangle$, $|\xi\rangle$ denote flavor, spin, color and space-time q^3 wave functions. For the lowest-lying hadrons, quarks appear to be in S-wave states and the space-time wave function $|\xi\rangle$ is totally symmetric under permutation of any two quarks. Hence the flavor-spin-color part F should be totally antisymmetric under $q_i \leftrightarrow q_j$. In the conventional quark model, the color wave function ψ is taken to be total antisymmetric, i.e. a color singlet. Let superscripts S and A denotes total permutation symmetry and antisymmetry, and λ , ρ denote

symmetric and antisymmetric under quark permutation $q_i \leftrightarrow q_j$. Then the wave function for flavor octet baryons [29]

$$\begin{aligned}
\Phi_1^{\left(\frac{1}{2}\right)} &\equiv \Phi\left(8, \frac{1}{2}, 1_c\right) = F_S \psi_1^A; F_S = \frac{1}{\sqrt{2}}(\phi^\lambda \chi^\lambda + \phi^\rho \chi^\rho) \\
\Phi_8^{\left(\frac{1}{2}\right)} &\equiv \Phi\left(8, \frac{1}{2}, 8_c\right) = \frac{1}{\sqrt{2}}(F_{MS} \psi_8^\rho - F_{MA} \psi_8^\lambda); \\
F_{MS} &= \frac{1}{\sqrt{2}}(\phi^\rho \chi^\rho - \phi^\rho \chi^\rho); F_{MA} = \frac{1}{\sqrt{2}}(\phi^\rho \chi^\lambda + \phi^\rho \chi^\lambda) \\
\Phi_{10}^{\left(\frac{1}{2}\right)} &\equiv \Phi\left(8, \frac{1}{2}, 10_c\right) = F_A \psi_{10}^S; F_A = \frac{1}{\sqrt{2}}(\phi^\lambda \chi^\rho - \phi^\rho \chi^\lambda) \\
\Phi_8^{\left(\frac{3}{2}\right)} &\equiv \Phi\left(8, \frac{3}{2}, 8_c\right) = F_A' \chi^{\frac{3}{2}}; F_A' = \frac{1}{\sqrt{2}}(\phi^\lambda \psi_8^\rho - \phi^\rho \psi_8^\lambda)
\end{aligned} \tag{4.46}$$

The possible combinations of q^3 and sea wave function can give a spin 1/2 flavor octet, color singlet state. $H_{0,1,2}$ and $G_{1,8,10}$ [30, 31, 32, 33] denotes the spin and color possibilities of sea with spin and color combination of 0, 1, 2 and 1, 8, 10 respectively. The total flavor spin color wave function of a spin up baryon which consist of three valence quark and sea components can be written as [29, 31, 32]

$$\begin{aligned}
\langle \phi_{1/2}^\uparrow | o | \phi_{1/2}^\uparrow \rangle &= \frac{1}{N^2} [a \sum_i [\langle o_f^i \rangle^{\lambda\lambda} \langle \sigma_z^i \rangle^{\lambda\uparrow\lambda\uparrow} + \langle o_f^i \rangle^{\rho\rho} \langle \sigma_z^i \rangle^{\rho\uparrow\rho\uparrow} + 2\langle o_f^i \rangle^{\lambda\rho} \langle \sigma_z^i \rangle^{\lambda\uparrow\rho\uparrow}] + \\
&b \sum_i [\langle o_f^i \rangle^{\lambda\lambda} + \langle o_f^i \rangle^{\rho\rho}] [\langle \sigma_z^i \rangle^{\lambda\uparrow\lambda\uparrow} + \langle \sigma_z^i \rangle^{\rho\uparrow\rho\uparrow}] + c \sum_i [\langle o_f^i \rangle^{\lambda\lambda} \langle \sigma_z^i \rangle^{\rho\uparrow\rho\uparrow} + \langle o_f^i \rangle^{\rho\rho} \langle \sigma_z^i \rangle^{\lambda\uparrow\lambda\uparrow} - \\
&2\langle o_f^i \rangle^{\lambda\rho} \langle \sigma_z^i \rangle^{\lambda\uparrow\rho\uparrow}] + d \sum_i [\langle o_f^i \rangle^{\lambda\lambda} + \langle o_f^i \rangle^{\rho\rho}] + e \sum_i [\langle o_f^i \rangle^{\rho\rho} - \langle o_f^i \rangle^{\lambda\lambda}] \langle \sigma_z^i \rangle^{\lambda\uparrow 3/2\uparrow} + \\
&2 \sum_i \langle o_f^i \rangle^{\lambda\rho} \langle \sigma_z^i \rangle^{\rho\uparrow 3/2\uparrow}]
\end{aligned} \tag{4.47}$$

where $N^2 = 1 + a_8^2 + a_{10}^2 + b_1^2 + b_8^2 + b_{10}^2 + c_8^2 + d_8^2$ and $\langle o_f^i \rangle^{\lambda\lambda} = \langle \phi^\lambda | o_f^i | \phi^\lambda \rangle$, $\langle \sigma_z^i \rangle^{\lambda\uparrow\lambda\uparrow} \equiv \langle \chi^{\lambda\uparrow} | \sigma_z^i | \chi^{\lambda\uparrow} \rangle$.

The properties like magnetic moment spin distribution, weak decay coupling constant ratios are calculated by defining suitable operators for flavor and spin part. Suitable expression is obtained from the eigenvalues coming from the above defined operators. For instance, substituting the spin operator in spin 1/2 proton wave-function when it operates on symmetric part of wave function gives 2/3 and operating on antisymmetric wave-function gives 0.

4.8 Baryon Wave Function with Orbiting Valence Quark

The experimental and theoretical evidences shows that the orbital motion of the valence particles has to be taken into account as these particles behave as the Dirac particles. We know that the quark model successfully describe the magnetic moment of the baryons in the absence of the orbital motion of the valence quarks. But now the orbital motion of the valence of the quarks had been also introduced to study the properties of the baryons. Here we made an attempt to present a wave function including the orbital motion of the three valence quark. There is a relativistic modification of excited state of the baryons up to v^2/c^2 . Its advantage however is its simplicity to study the validity of non-relativistic model to spin structure and give the analytic expression for the spin coefficient of the baryon wave function [34]. The three valence wav function for baryon can be written as from equation (4.45)

$$\Psi = \Gamma(|\phi \rangle |\chi \rangle |\psi \rangle)(|\xi \rangle)$$

Where $|\phi\rangle$, $|\chi\rangle$, $|\psi\rangle$, $|\xi\rangle$ denotes flavor spin space and color wave functions. Due to color confinement ξ is taken to be totally antisymmetric i.e. color singlet. As a result of this flavor spin space part should be symmetric under any permutation of two quarks. Since baryons are spin $\frac{1}{2}$ particles, conservation of angular momentum restricted the orbital angular momentum L of the valence quark can only be 0,1,2 coupled to quark spin wave function part to give a spin $\frac{1}{2}$ baryon. Let superscripts S and A denote total permutation symmetry and antisymmetry, and λ , ρ denote symmetric and antisymmetric under quark permutation $q_i \leftrightarrow q_j$. Then the flavor spin orbit wave function of a spin up baryon can be [35, 36]

$$\Gamma_{\frac{1}{2}}^{(\uparrow)} = a_S \Gamma_S^{(0)(\uparrow)} + a_{S'} \Gamma_{S'}^{(0)(\uparrow)} + a_M \Gamma_M^{(0)(\uparrow)} + a_A \Gamma_A^{(1)(\uparrow)} + a_D \Gamma_M^{(2)(\uparrow)}, \quad (4.48)$$

where

$$\begin{aligned} \Gamma_S^{(0)(\uparrow)} &= F_S \psi_{00}^S, \quad \Gamma_{S'}^{(0)(\uparrow)} = F_{S'} \psi_{20}^S, \quad \Gamma_M^{(0)(\uparrow)} = \frac{1}{\sqrt{2}} (F_\lambda \otimes \psi_{20}^\lambda + F_\rho \otimes \psi_{20}^\rho), \quad \Gamma_A^{(1)(\uparrow)} = F_A \otimes \\ &\psi_{21}^A, \quad \Gamma_M^{(2)(\uparrow)} = \frac{1}{\sqrt{2}} (\phi_\lambda \psi_{22}^\lambda + \phi_\rho \psi_{22}^\rho) \otimes \chi^{3/2} \end{aligned} \quad (4.49)$$

where F_α ($\alpha = S, A, \lambda, \rho$) are the symmetric antisymmetric or the mixing combination of symmetric flavor and spin wave functions.

4.9 Baryon Wave Function in Excited State

The dipole magnetic moment encodes information about the leading-order response of a bound system to a soft external magnetic field. In particular, baryon magnetic moments are dynamical characteristics which provide valuable insights into baryon internal structure in terms of quark and gluon degrees of freedom.

In case of baryons, the non-relativistic quark model involves an interesting three body problem where the constituents are bound by a confining potential. There are various possible methods which can be used for solving the three-quark problem accurately and discuss selected applications to baryon spectroscopy. The non-relativistic quantum three-body problem, as it appears in baryon spectroscopy, is reasonably easy to handle. Three quarks in a baryon have an antisymmetric colour wave function and thus behave as bosons bound by a symmetric potential which does not depend much on spin. The aim is to discuss elaborate methods for solving the three-body problem; the very simple harmonic-oscillator model deserves some presentation. First, it has played an essential role in the development of the quark model, with, in particular, the pioneering works of Dalitz and Greenberg and the very complete and convincing studies of the baryon spectrum by Dalitz and Horgan, Isgur and Karl, Gromes, Cutkosky and Hendrick, Hey and many others. The harmonic-oscillator model may also serve as a basis or a starting point for accurate variation methods. Its understanding is anyhow necessary for getting some insight into the three-body problem and, in particular, into the difficulties associated with the symmetrisation of the wave functions [37].

The purpose of the present study of this problem is to extend the quark model described previously to low lying positive parity excitations of the three quark system. There are many excited positive-parity orbital modes for a three-particle system [38]. These modes differ

from each other in orbital angular momentum L and their behaviour under permutations. Each type of permutation behaviour corresponds to an $SU(6)$ multiplet (from 4.4) if forces between quarks are $SU(6)$ invariant. Thus for harmonic forces between quarks one would expect five degenerate $SU(6)$ multiplets at $N=2$: 56 ($L=0$ and 2), 70 ($L=0$ and 2), and 20 ($L=1$). These states are not, however, all found to be degenerate experimentally. There are two simple reasons for the departure from this ideal limit: Forces between quarks are not harmonic and, secondly, quark masses are not identical. Because forces between quarks are not harmonic in nature therefore different orbital modes become separated from each other in mass [39]. Therefore the wave function for the lowest lying nucleons is given as

$$|N^2P_{1/2}\rangle = \frac{1}{\sqrt{2}} \sum_{m_l m_s} \left\langle 1 \frac{1}{2} m_l m_s \left| \frac{1}{2} \frac{1}{2} \right. \right\rangle \times \left\{ \psi_{1m_l}^\rho \left[\frac{1}{\sqrt{2}} (\chi_{m_s}^\lambda \phi^\rho + \chi_{m_s}^\rho \phi^\lambda) \right] + \psi_{1m_l}^\lambda \left[\frac{1}{\sqrt{2}} (\chi_{m_s}^\rho \phi^\rho - \chi_{m_s}^\lambda \phi^\lambda) \right] \right\} \quad (4.50)$$

where ψ , χ , and ϕ denote the spatial, spin, and flavor wave functions. The superscripts ρ and λ of these wave functions indicate that they are totally symmetric among three quarks, or odd (even) under the exchange of the first two quarks.

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Phenomenological Study of Baryons Property

In this chapter we will discuss the various properties of the hadrons basically for the baryons in $J^P = \frac{1}{2}^+$ state.

5.1 Effective Mass

Since the magnetic moments of baryon are studied very effectively till now and are known to good precision. The constituent quark masses explain the magnetic moment ratio of the proton to the neutron i.e. μ_p/μ_n with excellent argument. Sogami and Oh'yamaguchi [7] proposed a thought on the calculation of magnetic moment of the baryons. They suggested that whenever a quark is probed by a soft proton, the mass of proton get modified because of the quarks interactions with the spectator quarks inside the baryons. They proposed that the measurement of the magnetic moment of the baryons involves the photon carrying momentum, transfer as probe. So the parameter m_q defines the effective mass of quarks inside the baryons. Since quarks are deeply confined inside the baryon and soft proton see a coherent internal structure of baryon. Thus the mass parameter appearing in the magnetic moment must reflect the effect of confinement [7]. The electromagnetic contribution of the baryon mass M_B arises mainly due to the virtual exchange of photons with the momentum of order of strong interactions mass scale. Therefore the effective mass observed from the outside of the baryons by the soft photon should be distinguishable from the effective mass, with which quarks are assumed to interact with each other.

The mass relation between the different member of SU(3) multiplet are flavor dependent. But there is the mass variation between the particles of the multiplet. Since the spin of baryon octet coupled to give total spin either $J^P = \frac{1}{2}^+$ or $J^P = \frac{3}{2}^+$. Now if we consider that hadrons to be the bound state of the interacting quarks the mass difference between the multiplets and the real member of the same multiplet can be attributed to a spin-spin interaction. But we know that the current theory of the interacting quarks is QCD in which the colored quarks interact through the exchange of colored gluons. However the role of the QCD theory in the CQM, there is no complete connection given. Therefore as a result the mass splitting [1] of light hadrons can be described in the frame work of non- relativistic quark model by spin-spin interactions [2]. The hadron mass splitting is only because of the short range part of interactions which is connected to the gluon exchange. So the effective potential for the interaction of two quarks is obtained as

$$V_{ij} = \pm\alpha_s \left(\frac{\lambda_i \lambda_j}{2}\right) \left(-\frac{2\pi}{3} \frac{\sigma_i \sigma_j}{m_i m_j} \delta(r_{ij})\right) \quad (5.1)$$

Where α_s represents the gluon quark coupling constant or strong coupling constant, and λ represents the Gell-Mann matrices acting on the color indices of ith and jth quark, whereas \pm

sign represents the interaction of two quarks or quark and anti-quark respectively. If there is no color charge then the potential V_{ij} between two quarks have negative value.

The remaining part of the interaction which is due to gluon exchange is average and gives the contribution to the potential which confines the quarks. The spin-spin interactions give rise to the hyperfine splitting as

$$\Delta E_{hfs} = \frac{2}{3} \mu_1 \mu_2 |\psi(0)|^2 \quad (5.2)$$

The magnetic moment of the particles with the charge e_i spin s_i and mass m_i is given by

$$\mu_i = \frac{e_i}{m_i} s_i \quad (5.3)$$

using the relation

$$e_1 e_2 = e^2 = 4\pi\alpha \quad (5.4)$$

we get that

$$\Delta E_{hfs} = \frac{8\pi\alpha}{3} |\psi(0)|^2 \frac{s_i \cdot s_j}{m_i m_j} \quad (5.5)$$

The term $|\psi(0)|^2$ in equation (5.5) considered as the phenomenological constant, which obtained from the comparison of the masses of baryons multiplets (or the meson) and also represents the value of the wave function $\psi(r_1, r_2)$ at origin with zero separation and having same value for all states. The term $|\psi(0)|^2$ assumed to vanish in the chiral limit $m_q \rightarrow 0$. Here α represents the fine structure constant. Consequently the current masses are not same as the current algebra or dynamic quark masses. The some kind of vacuum condensate adds a constant 200MeV to 300MeV to the dynamical quark masses to give the constituent quark masses. To get the result in the QCD on changing the electric charge with color charge we conclude that

$$\alpha = \frac{4}{3} \alpha_s ; \quad \text{for mesons,} \quad (5.6)$$

$$\alpha = \frac{2}{3} \alpha_s ; \quad \text{for baryons.} \quad (5.7)$$

where $\alpha_s = 4\pi$. Therefore the hyperfine splitting value for the baryons is given as

$$\Delta E_{hfs} = \frac{16\pi}{9} \alpha_s |\psi(0)|^2 \sum_{i<j} \frac{s_i \cdot s_j}{m_i m_j} \quad (5.8)$$

In this simple model of hadrons masses are supposed to arise from the sum of the constituent quark masses and hyperfine interactions. Thus for the baryons the masses of the S-wave 56-plet [3, 4] the expression is given as

$$M_B = \sum_{i=1}^3 m_i + \sum_{i<j} b_{ij} s_i \cdot s_j \quad (5.9)$$

$$\Delta M = \sum_{i<j} b_{ij} s_i \cdot s_j \quad (5.10)$$

and

$$b_{ij} = \frac{16\pi}{9m_i m_j} \alpha_s |\psi(0)|^2 \quad (5.11)$$

In equation (5.10) $s_i \cdot s_j$ represents the effective spin of the three quarks state for baryons and term $m_i m_j$ represents the effective mass. Where α and b_{ij} and quark masses are free parameters and explains the hadron masses spectrum with a constituent set of values as suggested by Gell-Mann Okubo mass formula [5]. Therefore for the baryons in constituent quark model spin term can be solved as

$$S^2 = (s_1 + s_2 + s_3)^2 \quad (5.12)$$

$$S^2 = s_1^2 + s_2^2 + s_3^2 + 2s_1 \cdot s_2 + 2s_2 \cdot s_3 + 2s_1 \cdot s_3 \quad (5.13)$$

Since the baryons are spin half particles as well as quarks are also spin half fundamental particles therefore the total spin is given by S and s_1 and s_2 are the spin of interacting particles ($s_1 = s_2 = \frac{1}{2}$). The value of S lies in the range between the values given by (5.14)

$$S = |s_1 - s_2| \dots \dots |s_1 + s_2| \quad (5.14)$$

therefore S has two possible set of values given as $S = 0, 1$. The general form of equation (5.13) can be written as

$$\sum_{i<j} s_i \cdot s_j = \frac{1}{2} \left[S(S+1) - \frac{9}{4} \right] \quad (5.15)$$

on substituting the values of the $S = \frac{1}{2}, \frac{3}{2}$ we get that for baryon octet $S = \frac{1}{2}$ equation (5.15) yields

$$\sum_{i<j} s_i \cdot s_j = -\frac{3}{4} \quad (5.16)$$

and for baryon decuplet $S = \frac{3}{2}$ equation (5.15) yield

$$\sum_{i<j} s_i \cdot s_j = \frac{3}{4} \quad (5.17)$$

On considering the case of baryon octet we will pursue with equation (5.16). Therefore from equation (5.9) we get that [6]

$$M_B = m_1 + m_2 + m_3 + \frac{1}{2} \sum_{i<j} b_{ij} s_i \cdot s_j \quad (5.18)$$

The factor $\frac{1}{2}$ in equation (5.18) introduces or reflects the reduction in the strength of the gluon exchange between the quarks in a color anti- triplet state relative to that between a quark and anti-quark in color singlet state.

$$M_B = m_1 + m_2 + m_3 + \frac{1}{2} [b_{12} s_1 \cdot s_2 + b_{13} s_1 \cdot s_3 + b_{23} s_2 \cdot s_3] \quad (5.19)$$

If we consider the flavor symmetry then $m_1 = m_2$ and $b_{23} = b_{13}$. Then equation (5.19) reduces to the form

$$M_B = 2m_1 + m_3 + \frac{1}{2} [b_{12} s_1 \cdot s_2 + b_{13} s_1 \cdot s_3 + b_{13} s_2 \cdot s_3] \quad (5.20)$$

$$M_B = 2m_1 + m_3 + \frac{1}{2} [b_{12} s_{12} + b_{13} [s_1 \cdot s_3 + s_2 \cdot s_3]] \quad (5.21)$$

From equation (5.12) and (5.13) we get that

$$[S^2 - s_1^2 - s_2^2 - s_3^2] = 2s_1 \cdot s_2 + 2s_2 \cdot s_3 + 2s_1 \cdot s_3 \quad (5.22)$$

$$\frac{1}{2} [S^2 - s_1^2 - s_2^2 - s_3^2] = s_1 \cdot s_2 + s_2 \cdot s_3 + s_1 \cdot s_3 \quad (5.23)$$

on substituting the value of the S , s_1, s_2, s_3 we get that

$$-\frac{3}{4} = s_1 \cdot s_2 + s_2 \cdot s_3 + s_1 \cdot s_3 \quad (5.24)$$

according to the Pauli's spin matrices

$$s_1 = \frac{1}{2} \hbar \sigma_1 \text{ and } s_2 = \frac{1}{2} \hbar \sigma_2 \quad (5.25)$$

where σ_1 and σ_2 are Pauli's spin matrices are defined in chapter 2. Therefore from equation (5.25) we get that

$$s_1 \cdot s_2 = \frac{1}{4} \quad (5.26)$$

so equation (5.24) gives

$$-\frac{3}{4} - \frac{1}{4} = s_2 \cdot s_3 + s_1 \cdot s_3 \quad (5.27)$$

but $b_{23} = b_{13}$ so $s_2 \cdot s_3 = s_1 \cdot s_3$ and equation (5.27) reduces to the form

$$s_1 \cdot s_3 = -\frac{1}{2} \quad (5.28)$$

So the effective mass formula for the baryons is given by

$$m_1^{eff} = m_2^{eff} = m + \alpha b_{12} + \beta b_{13} \quad (5.29)$$

$$m_3^{eff} = m + \beta b_{23} + \beta b_{13} \quad (5.30)$$

but $b_{23} = b_{13}$ so equation(5.30) becomes

$$m_3^{eff} = m + 2\beta b_{13} \quad (5.31)$$

where α and β can be calculated from the equations (5.21) (5.26) and (5.28) respectively and values comes out to be

$$\alpha = \frac{1}{8}, \beta = -\frac{1}{4} \quad (5.32)$$

Therefore equation (5.21) gives the final expression in the form of

$$M_B = 2m_1 + m_3 + \frac{1}{8}b_{12} - \frac{1}{4}b_{13} \quad (5.33)$$

form equation (5.19) we get that

$$M_B = m_1 + m_2 + m_3 + \frac{1}{4}b_{12} - \frac{1}{2}b_{13} - \frac{1}{2}b_{23} \quad (5.34)$$

where 1, 2, 3 represent the flavor of corresponding quark related to the baryon. In other way we can proceed as from equation (5.16) and from equation (5.9) we get effective mass for nucleons as

$$M_N = m_1 + m_2 + m_3 - \frac{1}{4}b_{12} - \frac{1}{4}b_{23} - \frac{1}{4}b_{13} \quad (5.35)$$

for $m_1 = m_2 = m_3$ and $b_{13} = b_{23}$, we get final result as

$$M_N = 3m - \frac{3}{4}b_{11} \quad (5.36)$$

But for the strange quark in Λ and Σ we have account the fact that $m_s > m_u, m_d$ quarks in the calculation of the hyperfine splitting terms. Since both Λ and Σ have the common isospin i.e. $I = 0, 1$ respectively. Since strange quark has $I = 0$, so u and d quarks must be in the $I = 1$ combination (symmetric) in case of Σ and $I = 0$ (antisymmetric) in case of Λ . In order to satisfy these conditions the spin and flavor wave function may be symmetric or anti-symmetric but the color wave function must be symmetric. Therefore for Σ we have

$$S_{ud}^2 = (s_u + s_d)^2 \quad (5.37)$$

$$S_{ud}^2 = s_u^2 + s_d^2 + 2s_u \cdot s_d \quad (5.38)$$

Therefore

$$s_u \cdot s_d = \frac{1}{2} [S_{ud}(S_{ud} + 1) - \frac{3}{2}] \quad (5.39)$$

So for $S_{ud} = 0$ from equation (5.39) we get that (for Λ baryon)

$$s_u \cdot s_d = -\frac{3}{4} \quad (5.40)$$

and for $S_{ud} = 1$ we get (for Σ baryon)

$$s_u \cdot s_d = \frac{1}{4} \quad (5.41)$$

Hence from equation (5.9) we get that

$$M_\Lambda = m_u + m_d + m_s + b_{ud}s_u \cdot s_d + b_{us}s_u \cdot s_s + b_{ds}s_d \cdot s_s \quad (5.42)$$

but for Λ , $m_u = m_d$ and $b_{us} = b_{ds}$. So

$$M_\Lambda = 2m_u + m_s + b_{ud}s_u \cdot s_d + b_{us}[s_u \cdot s_s + s_d \cdot s_s] \quad (5.43)$$

$$M_\Lambda = 2m_u + m_s + [b_{ud}s_u \cdot s_d + b_{us}[s_1 \cdot s_2 + s_2 \cdot s_3 + s_1 \cdot s_3 - s_u s_d]] \quad (5.44)$$

from equation (5.24) the term $[s_1 \cdot s_2 + s_2 \cdot s_3 + s_1 \cdot s_3 - s_u s_d] = 0$ as $s_u \cdot s_d = -\frac{3}{4}$ therefore as a result we get

$$M_\Lambda = 2m_u + m_s - \frac{3}{4}b_{ud} \quad (5.45)$$

Similarly calculating for Σ yields that

$$M_\Sigma = m_u + m_d + m_s + [b_{ud}s_u \cdot s_d + b_{us}[s_1 \cdot s_3 + s_2 \cdot s_3]] \quad (5.46)$$

$$M_\Sigma = m_u + m_d + m_s + \left[\frac{1}{4}b_{ud} + b_{us}[s_1 \cdot s_3 + s_2 \cdot s_3] \right]$$

On calculating the value of term comes out to be $[s_1 \cdot s_3 + s_2 \cdot s_3] = -1$ and using (5.41) we get (for $m_1 = m_2$)

$$M_\Sigma = 2m_u + m_s + \left[\frac{1}{4}b_{ud} - b_{us} \right] \quad (5.47)$$

In the similar way we can write the effective mass terms for all baryons in the ground state.

Table 5.1: Effective masses formula of baryon octet in $J^P = \frac{1}{2}^+$ state

Baryon	Effective mass term
Nucleon	$3m_u - \frac{3}{4}b_{uu}$
Λ	$2m_u + m_s - \frac{3}{4}b_{uu}$
Σ	$2m_u + m_s + \left[\frac{1}{4}b_{uu} - b_{us} \right]$
Ξ	$m_u + 2m_s + \left[\frac{1}{4}b_{ss} - b_{us} \right]$
Ω	$3m_s + \frac{3}{4}b_{ss}$

The values of the m_u , m_s and b_{uu} , b_{ss} , b_{us} are obtained from the physical baryon masses[6] and obtained as

$$m_u = m_d = 363\text{MeV}, \quad m_s = 538\text{MeV}, \quad (5.48)$$

$$\frac{\alpha}{m_u^2} = b_{uu} = b_{dd} = b_{ud} = 200\text{MeV} \quad (5.49)$$

Now if we consider there is one heavy quark in the baryon octet with the remaining two light quarks then the effective mass formula can be given as [9, 10] (in case of nucleons there is no heavy quark) i. e. For the quarks (aab) type we get that from equation (5.33)

$$m_1^{eff} = m_2^{eff} = m + \frac{b_{12}}{8} - \frac{b_{13}}{4} \quad (5.50)$$

here in equation (5.50) and $m = m_1 = m_2$

$$m_3^{eff} = m_3 + \frac{b_{12}}{8} - \frac{b_{13}}{4} \quad (5.51)$$

For the quarks (abc) type we conclude that

a) Λ type baryons (usd)

$$m_1^{eff} = m_1 - \frac{3}{8}b_{12} \quad (5.52)$$

$$m_2^{eff} = m_2 - \frac{3}{8}b_{21} \quad (5.53)$$

$$m_3^{eff} = m_3 \text{ (for } 1 \neq 2 \neq 3) \quad (5.54)$$

b) Σ type baryons (uds, uus, dds)

$$m_1^{eff} = m_1 + \frac{b_{12}}{8} - \frac{b_{13}}{4} \quad (5.55)$$

$$m_2^{eff} = m_2 + \frac{b_{12}}{8} - \frac{b_{23}}{4} \quad (5.56)$$

$$m_3^{eff} = m_3 - \frac{b_{13}}{4} - \frac{b_{23}}{4} \quad (\text{for } 1 \neq 2 \neq 3) \quad (5.57)$$

Therefore for the Ω quark we have from equation (5.36)

$$m_\Omega = 3m_s + \frac{3}{4}b_{ss} \quad (5.58)$$

Now the effective mass term for one heavy quark is changes to

$$m_1^{eff} = m_2^{eff} = m_1 + \frac{b_{12}}{8} - \frac{b_{13}}{4} \quad (5.59)$$

Table 5.2: Effective mass of baryons octet $J^P = \frac{1}{2}^+$ state with heavy quark scheme

Baryon	Effective mass (with heavy quarks)		
	m_1^{eff}	m_2^{eff}	m_3^{eff}
Λ	$m_1 - \frac{3}{8}b_{12}$	$m_2 - \frac{3}{8}b_{21}$	$m_3 (1 \neq 2 \neq 3)$
Σ	$m_1 + \frac{b_{12}}{8} - \frac{b_{13}}{4}$	$m_2 + \frac{b_{12}}{8} - \frac{b_{23}}{4}$	$m_3 - \frac{b_{13}}{4} - \frac{b_{23}}{4}$ ($1 \neq 2 \neq 3$)
Ξ	$m_1 + \frac{b_{12}}{8} - \frac{b_{13}}{4}$	$m_2 + \frac{b_{12}}{8} - \frac{b_{23}}{4}$	$m_3 - \frac{b_{13}}{4} - \frac{b_{23}}{4}$
Ω	$m_1 + \frac{b_{12}}{8} - \frac{b_{13}}{4}$	$m_2 + \frac{b_{12}}{8} - \frac{b_{13}}{4}$	$m_3 - \frac{b_{13}}{2}$ ($1=2 \neq 3$)

5.2 Screened Charge

QCD in phenomenological models deals only with the effective interaction terms taking place between the particles i.e. quarks and gluons. In QCD color plays an important role charge. Gluons are the quanta of the color filled that binds the quarks into the hadrons. Gluon themselves carry a color charge, so that there is the process of the self-coupling between the gluons with at least 3 to 4 gluon vertices. The process of scattering of light quark by light quark consists of virtual gluons and also involves the real photons. This direct coupling of the gluon has substantial implication that become evident for the effect of the charge screening in QCD.

Due to the gluon self-coupling interactions, the vacuum will be filled with the virtual particles i.e. virtual gluon pairs. Because the gluon carries the color charge this color charge seems to be leak out in the vacuum [11]. As a consequence penetrating probe will feel an effective charge that become smaller with the smaller distance. Therefore the effective small coupling becomes small at smaller distances. This property of the running coupling constant leads to the property, called asymptotic freedom and allows use of the perturbation theory with the larger distances. Whereas with the increase in the distance, coupling become so strong that it is impossible to isolate a quark from hadrons. This mechanism is called confinement and is non perturbative mechanism.

When a quark is place in the vacuum and probed by the soft proton [12] it was surrounded by the virtual particles namely gluons, pairs of quarks and their antiquarks. But because of the

strong nuclear forces the virtual gluon likes photon, do have color charge. This means that virtual gluon cloud and the overlapping cloud of the quarks and antiquarks pair interferes with the forces away from the core of the quarks. Since there are three color charges red, blue and green in QCD. If quarks core carried red charge then in the surrounding cloud of the virtual gluons pairs, the virtual anti-red quark pair attracted toward the red quark core. Whereas the virtual red color quark will be repelled to a greater average distance. This effect shield the color charge of the quark core, but the overlapping cloud if the virtual gluon has color charge and has opposite spin. This overall effect diffuses the color charge of the quarks core over the volume of the surrounding virtual particle clouds. As a result the net color charge decreases as we penetrate through the virtual cloud. Thus if the quarks are make to collide at very high energies they will penetrate further the virtual cloud of the color charge and experience a reduction in the color charge.

In the symmetric constituent quark model, the baryon magnetic moment in term of the individual quark moment is given by

$$\mu_B = \frac{1}{3}(2\mu_a + 2\mu_b - \mu_c) \quad \text{if } s_{12} = 1 \quad (5.70)$$

and

$$\mu_B = \mu_c \quad \text{if } s_{12} = 0 \quad (5.71)$$

but the magnetic moment operator is given as

$$\mu_i = \frac{e_i}{2m_i} \quad i = a, b, c \quad (5.72)$$

For baryons B (aab) containing two quarks of the same type flavor, the magnetic moment thus given as

$$\mu_B(aab) = \frac{4}{3} \frac{e_a}{2m_a} - \frac{1}{3} \frac{e_b}{2m_b} \quad (5.73)$$

As we know that Λ and Σ both have three quarks with different flavor. Because Λ is an isospin singlet therefore $s_{12} = 0$. So

$$\mu_\Lambda = \frac{e_s}{2m_s} \quad (5.74)$$

The modification in the magnetic moment of the quark is due to the occurrence of the effective charge. As we discussed above, shielding of the quarks charges due to the interaction with the neighbouring quarks much similar to the shielding of the electronic charge. Therefore the effective charge of the quark a is represented by the e_a^B i.e.

$$e_a^B = e_a + \alpha_{ab}e_b + \alpha_{ac}e_c \quad (5.75)$$

on taking

$$\alpha_{ab} = \alpha_{ba} \quad (5.78)$$

Equation (5.78) appeal for the isospin symmetry such that

$$\alpha_{uu} = \alpha_{ud} = \alpha_{dd} = \beta \quad (5.79)$$

$$\alpha_{us} = \alpha_{ds} = \alpha \quad (5.80)$$

and also considering that

$$\alpha_{ss} \approx \alpha \quad (5.81)$$

So screened charge of quarks for various baryons is given as from equation (5.75) we have

$$e_u^p = \frac{2}{3} + \frac{2}{3}\alpha_{uu} - \frac{1}{3}\alpha_{ud} \quad (5.82)$$

$$e_u^p = \frac{2}{3} \left(1 + \beta - \frac{1}{2}\beta \right) \quad (5.83)$$

$$e_u^p = \frac{2}{3} \left(1 + \frac{1}{2}\beta \right) \quad (5.84)$$

Similarly

$$e_d^p = -\frac{1}{3} + \frac{2}{3}\alpha_{ud} + \frac{2}{3}\alpha_{ud} \quad (5.85)$$

$$e_d^p = -\frac{1}{3}(1 - 2\beta - 2\beta) \quad (5.86)$$

$$e_d^p = -\frac{1}{3}(1 - 4\beta) \quad (5.87)$$

In the same manner we can write the screen charge equation for all baryons in $J^P = \frac{1}{2}^+$ octet state. i.e.

$$\begin{aligned} e_u^p &= \frac{2}{3}(1 + \frac{\beta}{2}) & e_d^p &= -\frac{1}{3}(1 - 4\beta) \\ e_u^n &= \frac{2}{3}(1 - \beta) & e_d^n &= -\frac{1}{3}(1 - \beta) \\ e_s^{\Lambda^0} &= e_s^{\Sigma^0} = -\frac{1}{3}(1 - \alpha) \\ e_u^{\Sigma^+} &= \frac{2}{3}(1 - \frac{1}{2}\alpha + \beta) & e_s^{\Sigma^+} &= -\frac{1}{3}(1 - 4\alpha) \\ e_d^{\Sigma^-} &= -\frac{1}{3}(1 + \alpha + \beta) & e_s^{\Sigma^-} &= -\frac{1}{3}(1 + 2\alpha) \\ e_u^{\Xi^0} &= \frac{2}{3}(1 - \alpha) & e_s^{\Xi^0} &= -\frac{1}{3}(1 - \alpha) \\ e_u^{\Xi^-} &= -\frac{1}{3}(1 + 2\alpha) & e_s^{\Xi^-} &= -\frac{1}{3}(1 + 2\alpha) \\ e_s^{\Omega^-} &= -\frac{1}{3}(1 + 2\alpha) \end{aligned} \quad (5.88)$$

5.3 Magnetic Moments

Here we will discuss about the magnetic moment of the light baryons using SU(6) quark model and using effective mass and screen charge. The magnetic moments are measured by the energy difference of the baryons at rest with spin aligned or anti-aligned with the magnetic field. This is a low energy and long distance phenomenon. Since all the constituents of the baryons experience the same magnetic field and contribute coherently to the baryon magnetic moment in terms of the interaction energy which is small as compared as to the baryon masses. The prediction of the baryon magnetic moment is the striking success of the quark model. As we know that quarks behave like a point Dirac particle so the magnetic moment can be represented as

$$\mu_i = \left(\frac{e_i}{2m_i}\right)\sigma_i \quad (5.89)$$

where e_i , m_i and σ_i are the charge, mass and the spin (in units $\hbar = c = 1$). From the symmetry properties of the three quark wave function in baryons, we can compute their magnetic moment as the vector sum of the constituent quarks. So the magnetic moment of the individual quark is given as

$$\mu_q = Z_q \left(\frac{e_q}{2m_q}\right)\sigma_q \quad (5.90)$$

and for nucleons

$$\mu_i = \left(\frac{e_i}{2m_i}\right) \quad (5.91)$$

Magnetic moment operator in term of effective mass and screen charge is given as
For shielding charge scheme we have

$$\mu_i = \sum_i \frac{e_i^B}{2m_i} \sigma_i \quad (5.92)$$

For the effective mass and shielding charge scheme

$$\mu = \sum_i \frac{e_i^B}{2m_i^{eff}} \sigma_i \quad \text{where } i = 1, 2, 3 \quad (5.93)$$

5.4 Angular Momentum

Since we know that the total angular momentum of the system is defined as the sum of the total orbital angular momentum L and the spin angular momentum S i.e. $J = L + S = S$. In high energy physics, the lowest lying negative parity nucleons resonance in the nonrelativistic SU(6) quark model is represented by the notation $|N^2P_{1/2}\rangle$ where the notation $^2P_{1/2}$ represents the total quark content spin of baryon $S = \frac{1}{2}$ ($2S + 1 = 2$), orbital angular momentum $L = 1$ (P -wave), and total angular momentum $J = \frac{1}{2}$. In the ground state, the baryon octet has the same S -wave spatial wave function with $L = 0$, and thus the total angular momentum is $J = L + S = S$.

Or in terms of magnetic moment operator total angular momentum is represented by the terms

$$\mu = \mu_L + \mu_S \quad (5.94)$$

$$\mu_S = \sum_i \mu_S^i = \sum_i \frac{Q_i}{m_i} S_i \quad (5.95)$$

$$\mu_L = \sum_i \mu_L^i = \sum_i \frac{Q_i}{m_i} l_i \quad (5.96)$$

where index i represents the sum over three quarks. Therefore from the chapter 4 (4.9 equation (4.50)) we get that

$$|N^2P_{1/2}\rangle = \frac{1}{\sqrt{2}} \sum_{m_l m_s} \left(1 \frac{1}{2} m_l m_s \left| \frac{1}{2} \frac{1}{2} \right. \right) \times \left\{ \psi_{1m_l}^\rho \left[\frac{1}{\sqrt{2}} (\chi_{m_s}^\lambda \phi^\rho + \chi_{m_s}^\rho \phi^\lambda) \right] + \psi_{1m_l}^\lambda \left[\frac{1}{\sqrt{2}} (\chi_{m_s}^\rho \phi^\rho - \chi_{m_s}^\lambda \phi^\lambda) \right] \right\} \quad (5.97)$$

So therefore on using equation (5.93) (5.95) (5.96) in equation (5.97) we get

$$\mu |N^2P_{1/2}\rangle = \langle N^2P_{1/2} | \mu_L | N^2P_{1/2} \rangle \quad (5.98)$$

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Calculations, Results and Conclusion

6.1 Magnetic Moments

from chapter 5 (5.4) the baryon magnetic moment can be calculated as by using the wave function given in appendix A (Table A.4) w get that

$$\mu_i = \left(\frac{e_i}{2m_i} \right) \sigma_i \quad (6.1)$$

$$\mu_B = \langle \psi_B | \sum_{i=1}^3 \hat{\mu}_q(i) \hat{\sigma}_z(i) | \psi_B \rangle, \quad (6.2)$$

where $\hat{\mu}_q$ is the operator for the magnetic moment of quarks, and $\hat{\sigma}_z(i)$ is the Pauli's spin operator and i runs over {u, d, s}. Therefore for proton we get

$$\left\langle \frac{1}{\sqrt{18}} \begin{bmatrix} 2u \uparrow d \downarrow u \uparrow + 2u \uparrow u \uparrow d \downarrow + \\ 2d \downarrow u \uparrow u \uparrow \\ -u \uparrow u \downarrow d \uparrow - u \uparrow d \uparrow u \downarrow - \\ u \downarrow d \uparrow u \uparrow - d \uparrow u \downarrow u \uparrow - \\ d \uparrow u \uparrow u \downarrow - u \downarrow u \uparrow d \uparrow \end{bmatrix} \right| \sum_{i=1}^3 \hat{\mu}_q(i) \hat{\sigma}_z(i) \left| \frac{1}{\sqrt{18}} \begin{bmatrix} 2u \uparrow d \downarrow u \uparrow + 2u \uparrow u \uparrow d \downarrow + \\ 2d \downarrow u \uparrow u \uparrow \\ -u \uparrow u \downarrow d \uparrow - u \uparrow d \uparrow u \downarrow - \\ u \downarrow d \uparrow u \uparrow - d \uparrow u \downarrow u \uparrow - \\ d \uparrow u \uparrow u \downarrow - u \downarrow u \uparrow d \uparrow \end{bmatrix} \right\rangle \quad \mu_p = \quad (6.3)$$

Where $\mu_u = \frac{q_u}{2m_u}; \quad q_u = qe \quad (6.4)$

$$\mu_d = \frac{q_d}{2m_d}; \quad q_d = -2qe \quad (6.5)$$

$$\mu_u = -2\mu_d \quad (6.6)$$

where q is the charge of the quark and e is the electronic charge. Therefore from equation (6.3) we get that

$$\left\langle \frac{1}{\sqrt{18}} [u \uparrow u \downarrow d \uparrow] \right| \sum_{i=1}^3 \hat{\mu}_q(i) \hat{\sigma}_z(i) \left| \frac{1}{\sqrt{18}} [u \uparrow u \downarrow d \uparrow] \right\rangle \quad (6.7)$$

i.e. $= \frac{1}{18} \{6 \times [\mu_u - \mu_u + \mu_d]\} \quad (6.8)$

$$= \frac{1}{3} \mu_d \quad (6.9)$$

and

$$\left\langle \frac{1}{\sqrt{18}} [2u \uparrow d \downarrow u \uparrow] \right| \sum_{i=1}^3 \hat{\mu}_q(i) \hat{\sigma}_z(i) \left| \frac{1}{\sqrt{18}} [2u \uparrow d \downarrow u \uparrow] \right\rangle \quad (6.10)$$

So $= \frac{1}{18} \{3 \times 4[\mu_u + \mu_u - \mu_d]\} \quad (6.11)$

$$= \frac{1}{18} \{12 \times [2\mu_u - \mu_d]\} \quad (6.12)$$

$$= \frac{2}{3} [2\mu_u - \mu_d] + \frac{1}{3} \mu_d \quad (6.13)$$

$$= \frac{4}{3} \mu_u - \frac{1}{3} \mu_d \quad (6.14)$$

Similarly the stable baryon magnetic moment for $J^P = \frac{1}{2}^+$ state is in table 6.1 below

Table 6.1 Predicted and observed magnetic moment of the stable baryon of the $J^P = \frac{1}{2}^+$ octet state (in units of nuclear magneton).

Particle	Formula	Calculated (μ_N)	Observed (Experimental) (μ_N) [6]
P	$\frac{1}{3}(4\mu_u - \mu_d)$	2.80	2.79
N	$\frac{1}{3}(4\mu_d - \mu_u)$	-1.91	-1.91
Λ^0	μ_s	-0.613	-0.613
Σ^0	$\frac{2}{3}(\mu_d + \mu_u) - \frac{1}{3}\mu_s$	0.797	
Σ^+	$\frac{1}{3}(4\mu_u - \mu_s)$	2.68	2.458
Σ^-	$\frac{1}{3}(4\mu_d - \mu_s)$	-1.09	-1.160
Ξ^0	$\frac{1}{3}(4\mu_s - \mu_u)$	-1.44	-1.250
Ξ^-	$\frac{1}{3}(4\mu_s - \mu_d)$	-0.493	-0.6507

These results are in good agreement (in constituent quark masses) with that of the experimental values but are not exact or outstanding results. The magnetic moment values are contradictory to that of experimental values in term of the bare mass of the quarks. The experimental magnetic moment value of the Σ^0 is not observed till yet because of its short life time due to which it decay to Λ^0 .

Here

$$m_u = 336 \text{ MeV}, m_d = 322 \text{ MeV}, m_s = 510 \text{ MeV}$$

and

$$\mu_N = \frac{e\hbar}{2m_p c} = 3.152 \times 10^{-18} \text{ MeV/Gauss} (\hbar = c = 1)$$

$$\mu_u = 1.862\mu_N, \mu_d = -0.972\mu_N, \mu_s = -0.613\mu_N$$

6.2 Effective Masses of Quarks and Baryon Magnetic Moment

From chapter 5 (art. 5.1) we can use those expressions to calculate the effective mass of the baryons. From table 5.1 we have

$$m_u = m_d = 363 \text{ MeV}, m_s = 538 \text{ MeV}$$

$$\frac{\alpha}{m_u^2} = b_{uu} = b_{ud} = b_{dd} = 200 \text{ MeV}$$

$$b_{us} = b_{ds} = \left(\frac{m_u}{m_s}\right) b_{uu} = 134.9 \text{ MeV} \quad (6.15)$$

$$b_{ss} = \left(\frac{m_u}{m_s}\right)^2 b_{uu} = 91.04 \text{ MeV} \quad (6.16)$$

So the values of the effective masses of the constituents are given as in table

Table 6.2 Effective masses of the constituent of the baryons in $J^P = \frac{1}{2}^+$ octet state

Particle constituent	Calculated (MeV)	Ref. (MeV) [1]
m_u^p	338	338
m_d^p	263	265
m_u^n	263	265
m_d^n	338	338
$m_u^{\Lambda^0} = m_d^{\Lambda^0}$	288	290
m_s^Λ	538	538
$m_u^{\Sigma^0} = m_d^{\Sigma^0}$	288	
$m_s^{\Sigma^0}$	470.55	516
$m_u^{\Sigma^+}$	357.27	356
$m_s^{\Sigma^+}$	468.05	473
$m_d^{\Sigma^-}$	357.27	356
$m_s^{\Sigma^-}$	468.05	473
$m_u^{\Xi^0}$	295.55	297
$m_s^{\Xi^0}$	515.65	516
$m_d^{\Xi^-}$	295.5	297
$m_s^{\Xi^-}$	515.65	516

Therefore the magnetic moment calculate as by using the magnetic moment operator from equation (6.2) and (6.4) we get that

$$\mu_B = \sum_i \frac{e_i}{2m_i^{eff}} \sigma_i \quad (6.17)$$

Now by using the wave functions given in the appendix (A.4) and proceeding as article 6.1 and on using the value of the effective mass given in (Table 6.2) instead of the constituent mass we get the magnetic moment of the baryons as

Table 6.3 Magnetic moment values of the baryons with effective masses

Particle	μ_u	μ_d	μ_s	Calculated values (μ_N)	Ref. (μ_N) [1]	Experimental values (μ_N)[6]
p	+1.85	-1.189		2.863	2.86	2.79
n	1.189	-1.85		-2.20	-2.01	-1.91
Λ^0	2.172	-1.086	-0.58	-0.58	-0.58	-0.613
Σ^0	2.172	-1.086	-0.664	0.936		
Σ^+	1.75		-0.668	2.556	2.56	2.458
Σ^-		-0.875	-0.668	-0.944	-0.96	-1.160
Ξ^0	2.11		-0.606	-1.511	-1.50	-1.250
Ξ^-		-1.058	-0.606	-0.455	-0.46	-0.6507

6.3 Screening Charge and Baryon Magnetic Moment

From chapter 5 (article 5.3) using the screen charged equation parameters from equation (5.88) and on modifying the baryon magnetic momentum operator for shielding charge magnetic moment operator again on using the baryon wave functions from appendix A we get

$$\mu_p = \left\langle \frac{1}{\sqrt{18}} [u \uparrow u \downarrow d \uparrow] \left| \sum_{i=1}^3 \hat{\mu}_q(i) \hat{\sigma}_z(i) \right| \frac{1}{\sqrt{18}} [u \uparrow u \downarrow d \uparrow] \right\rangle \quad (6.18)$$

where

$$\hat{\mu}_q(i) = q \frac{e_i^{screen}}{2m_i} \quad (6.19)$$

here q represents the charge of quarks. Therefore we conclude that

$$\mu_p = \left\langle \frac{1}{\sqrt{18}} [u \uparrow u \downarrow d \uparrow] \left| \sum_{i=1}^3 \frac{e_i^{screen}}{2m_i} \hat{\sigma}_z(i) \right| \frac{1}{\sqrt{18}} [u \uparrow u \downarrow d \uparrow] \right\rangle \quad (6.20)$$

$$= \frac{1}{18} \left[\frac{2}{3} \left(1 + \frac{\beta}{2} \right) \frac{e_u}{2m_u} (1) + \frac{2}{3} \left(1 + \frac{\beta}{2} \right) \frac{e_u}{2m_u} (-1) + \left(-\frac{1}{3} \right) (1 - 4\beta) \frac{e_d}{2m_d} (1) \right] \quad (6.21)$$

$$= \frac{1}{18} \left[\frac{2}{3} \left(1 + \frac{\beta}{2} \right) \frac{e_u}{2m_u} - \frac{2}{3} \left(1 + \frac{\beta}{2} \right) \frac{e_u}{2m_u} + \left(-\frac{1}{3} \right) (1 - 4\beta) \frac{e_d}{2m_d} \right] \quad (6.22)$$

$$= \frac{1}{18} \left[-\frac{1}{3} (1 - 4\beta) \frac{e_d}{2m_d} \right] \quad (6.23)$$

Now for the SU(6) symmetry to be hold we will consider β equal to zero in units of $\frac{e_i}{2m_i}$. So from equation (6.23) we get that

$$= \frac{1}{18} \left[-\frac{1}{3} \right] \quad (6.24)$$

i.e.

$$\mu_p = -\frac{1}{18} \times 6 \times \frac{1}{3} \quad (6.25)$$

$$\mu_p = -\frac{1}{9} \quad (6.26)$$

and for second term we get

$$\mu_p = \left\langle \frac{1}{\sqrt{18}} [2u \uparrow d \downarrow u \uparrow] \left| \sum_{i=1}^3 \hat{\mu}_q(i) \hat{\sigma}_z(i) \right| \frac{1}{\sqrt{18}} [2u \uparrow d \downarrow u \uparrow] \right\rangle \quad (6.27)$$

$$\mu_p = \left\langle \frac{1}{\sqrt{18}} [2u \uparrow d \downarrow u \uparrow] \left| \sum_{i=1}^3 \frac{e_i^{screen}}{2m_i} \hat{\sigma}_z(i) \right| \frac{1}{\sqrt{18}} [2u \uparrow d \downarrow u \uparrow] \right\rangle \quad (6.28)$$

$$= \frac{1}{18} \times 4 \left[\frac{2}{3} \left(1 + \frac{\beta}{2} \right) \frac{e_u}{2m_u} (1) + \frac{2}{3} \left(1 + \frac{\beta}{2} \right) \frac{e_u}{2m_u} (1) + \left(-\frac{1}{3} \right) (1 - 4\beta) \frac{e_d}{2m_d} (-1) \right] \quad (6.29)$$

again on substituting β zero therefore from equation (6.29) we get that

$$= \frac{1}{18} \times 4 \left[\frac{2}{3} + \frac{2}{3} + \frac{1}{3} \right] \quad (6.30)$$

$$\mu_p = \frac{20}{18} \quad (6.31)$$

Therefore the total magnetic moment of proton in case of the shielding quark charges from equation (6.26) and (6.31) we get that

$$\mu_p = \frac{20}{18} - \frac{1}{9} \quad (6.32)$$

i.e.

$$\mu_p = 1 \quad (6.33)$$

Similarly we can calculate the magnetic moment of the baryons in terms of the shielding quark charge [2]

$$\mu_p = 1, \quad \mu_n = -\frac{2}{3}, \quad \mu_{\Lambda^0} = -\frac{y}{3} (1 - \alpha)$$

$$\mu_{\Sigma^+} = \frac{1}{9} [(8 + y) - 4\alpha(1 + y)], \quad \mu_{\Sigma^-} = \frac{1}{9} [(y - 4) + 2\alpha(y - 2)],$$

$$\mu_{\Xi^-} = \frac{1}{9}(1 - 4y)(1 + 2\alpha), \quad \mu_{\Xi^0} = -\frac{1}{9}(2 + 4y)(1 - \alpha) \quad (6.34)$$

All the magnetic moments are in units of $\frac{e_i}{2m_i}$ and y represents the term $\frac{m_u}{m_s}$ because in case of strange baryons $\frac{e}{2m_u} = \frac{e}{2m_s} = \frac{e}{2m_u} \frac{m_u}{m_s}$. As a result the magnetic moment of the baryon in term of effective charge scheme are given in table below

Table 6.4: Magnetic moment of baryons with screen charge effect

Particle	Calculate (μ_N)	Ref. (μ_N) [2]	Experimental (μ_N)[6]
P	2.79	2.79	2.79
N	-1.91	-1.86	-1.91
Λ^0	-0.53	-0.54	-0.613
Σ^+	1.982	2.45	2.458
Σ^-	-0.78	-1.11	-1.160
Ξ^0	-1.237	-1.25	-1.250
Ξ^-	-0.68	-0.66	-0.6507

6.4 Baryon Magnetic Moment with Effective Mass and Shielding Charge

From chapter 5 article 5.4 equation 5.93 we have the magnetic moment operator in terms of the both effective mass and charge scheme i.e.

$$\mu = \sum_i \frac{e_i^B}{2m_i^{eff}} \sigma_i \quad \text{where } i = 1, 2, 3 \quad (6.35)$$

Now on using the baryon wave function from appendix A (table A.4) we have the magnetic moment terms for various baryon particles given as [3]

$$\begin{aligned} \mu_P &= -\frac{[(-1+4\beta)m_n]}{9\left(-\frac{b_{ud}}{2}+m_d\right)} + \frac{8\left(1+\frac{\beta}{2}\right)m_n}{9\left(-\frac{b_{ud}}{4}+\frac{b_{uu}}{8}+m_u\right)}, \quad \mu_n = \frac{4(-1+\beta)m_n}{9\left(\frac{b_{dd}}{8}-\frac{b_{ud}}{4}+m_d\right)} - \frac{2(1-\beta)m_n}{9\left(-\frac{b_{ud}}{2}+m_u\right)}, \quad \mu_{\Lambda^0} = \frac{(-1+\alpha)m_n}{3m_s} \\ \mu_{\Sigma^+} &= -\frac{[(-1+4\alpha)m_n]}{9\left(-\frac{b_{us}}{2}+m_s\right)} + \frac{8\left(1-\frac{\alpha}{2}+\beta\right)m_n}{9\left(-\frac{b_{us}}{4}+\frac{b_{uu}}{8}+m_u\right)}, \quad \mu_{\Sigma^-} = \frac{4(-1-\alpha-\beta)m_n}{9\left(\frac{b_{dd}}{8}-\frac{b_{ds}}{4}+m_d\right)} - \frac{(-1-2\alpha)m_n}{9\left(-\frac{b_{ds}}{2}+m_s\right)} \\ \mu_{\Xi^0} &= \frac{4(-1+2\alpha-\gamma)m_n}{9\left(\frac{b_{ss}}{8}-\frac{b_{us}}{4}+m_s\right)} - \frac{2(1-\alpha)m_n}{9\left(-\frac{b_{us}}{2}+m_u\right)}, \quad \mu_{\Xi^-} = -\frac{[(-1-2\alpha)m_n]}{9\left(-\frac{b_{ds}}{2}+m_d\right)} + \frac{4(-1-\alpha-\gamma)m_n}{9\left(-\frac{b_{ds}}{4}+\frac{b_{ss}}{8}+m_s\right)} \end{aligned} \quad (6.36)$$

In above equation (6.36) α, β and γ are the parameters. We calculate the value of these parameters from the magnetic moment value of the can be calculated by using μ_P and μ_n using as the input value. Whereas the other parameters $b_{uu}, b_{ud}, b_{dd}, b_{us}, b_{ds}$ and b_{ss} represents the hyperfine splitting constant and can be calculated as given below. To reduce the parameters and simply the calculations we consider the SU(3) flavor symmetry so we have $\alpha = \beta = \gamma$ and $u = d$ [3]. Therefore we have

$$\begin{aligned} m_u &= m_d = m_n = 336 \text{ MeV}, \quad m_s = 450 \text{ MeV} \\ b_{uu} &= b_{ud} = b_{dd} = 200 \text{ MeV} \\ b_{us} &= b_{ds} = \frac{m_u}{m_s} b_{uu} = 148 \text{ MeV}, \\ b_{ss} &= \left(\frac{m_u}{m_s}\right)^2 b_{uu} = 108 \text{ MeV} \end{aligned} \quad (6.37)$$

The value of shielding effect comes out to be $\alpha = 0.15$. Therefore from equation (6.36) and (6.37) we have the values of the magnetic moment are as

Table 6.5: Magnetic moment of baryons with combine effect of effective mass and shielding charge

Baryon Particle	Calculated (μ_N)	Experimental (μ_N)[6]
p	1.1	2.79
n	-0.64	-1.91
Λ^0	-0.42	-0.613
Σ^+	1.22	2.458
Σ^-	-0.484	-1.160
Ξ^0	-0.485	-1.250
Ξ^-	-0.26	-0.6507

6.5 Magnetic Moment of the Baryons in Spin Orbiting Valence Quarks

From chapter 4 article number 4.8 we have the wave function of the baryons in the orbiting valence quarks. Therefore from appendix B using Jacobi's co-ordinate we have the magnetic moment as the combination of spin and orbital angular momentum expressed as

$$\mu_B = \left\langle B \left| \sum_i \frac{e_i^2}{2m_i} (\sigma_3^Z + L_i^Z) \right| B \right\rangle$$

In equation L_i^Z represents the angular momentum along the z-axis corresponding to the Jacobi's co-ordinates and B represents the baryon wave function. By using the baryon wave functions from the appendix A (table A.4) the magnetic moment of the baryon in $J^P = \frac{1}{2}^+$ octet state can be represented in terms of the quark magnetic moment $\mu_q = \frac{e_q}{2m}$ ($q = u, d, s$) and parameters α, β as [4]

$$\begin{aligned} \mu_p &= 3(2\mu_u\alpha - \mu_d\beta), & \mu_n &= 3(2\mu_d\alpha - \mu_u\beta), \\ \mu_{\Sigma^+} &= 3(2\mu_u\alpha - \mu_s\beta), & \mu_{\Sigma^-} &= 3(2\mu_d\alpha - \mu_s\beta), & \mu_{\Sigma^0} &= \frac{1}{2}(\mu_{\Sigma^+} + \mu_{\Sigma^-}) \\ \mu_{\Xi^0} &= 3(2\mu_s\alpha - \mu_u\beta), & \mu_{\Xi^-} &= 3(2\mu_s\alpha - \mu_d\beta), \end{aligned} \quad (6.38)$$

To obtain the results we first calculate the parameters i.e. μ_q ($q = u, d, s$), α and β . For the magnetic moment of the quarks on the valence system of quarks we take the values from [5]. After that we calculate the value of α and β taking value of the μ_p, μ_n and $\mu_{\Sigma\Lambda}$. Therefore

$$\mu_u = 2.47\mu_N, \quad \mu_d = -1.30\mu_N, \quad \mu_s = -0.72\mu_N,$$

and $\alpha = 0.166, \quad \beta = 0.082$

So the magnetic moment values obtained are mention as, in table given below

Table 6.6 Magnetic moment values of the baryons in spin orbiting valence quark system

Baryon particles	Calculated (μ_N)	Ref (μ_N)[4]	Experimental (μ_N)[6]
p	2.779	2.79	2.79
n	-1.90	-1.91	-1.91
Λ^0	-0.609	-0.613	-0.613
Σ^+	2.66	2.678	2.458
Σ^0	0.785	0.761	
Σ^-	-1.09	-1.156	-1.160
Ξ^0	-1.424	-1.463	-1.250
Ξ^-	-0.496	-0.537	-0.6507

6.6 Conclusions

The magnetic moment of baryons are computed as listed below in table 6.7 by considering the effective interactions of the bound state quarks with in the baryon as well as using the spin flavor wave function of baryons .

The motivation for taking the effective mass and charge is that, the calculation of the magnetic moment does depend precisely on the quark mass and charge parameters. Thus as an effective degree of freedom, one must include the confinement effect on the mass parameters of the quarks for the bound quarks with in the baryonic state. The special feature of the present study is to compute the magnetic moment of the baryons in the consideration of the effective interactions of the bound state of quarks by defining an effective bound state mass to quarks, which varies accordingly to the variation in the different inter quark potential as well as quark composition.

We compare the magnetic moment of the baryons in $J^P = \frac{1}{2}^+$ octet state in different schemes i.e. effective mass, shielding or screen charge, combine effect of effective mass and shielding charge and magnetic moment with spin and orbital contribution from valence quark system (given in table 6.7). From this comparison we conclude that, in each case the magnetic moment comes out to be approximately equal to that of the experimental values but in case of the combined effect of the effective mass and shielding charge, the magnetic moments are not corresponding to the experimental values. The possible justification for this mismatch could be because of the simultaneous application of both effects, and can better be studies individually to see their effects on the baryonic system. With a system having quarks and gluons, there may be present various interactions between the quarks through the exchange of gluons, that may lead to the mismatch between experiments and theory. Hence we speculate some other effects to be dominant which need to be studied to nullify the effects to match the result well.

Table 6.7 Comparisons of magnetic moment of baryons in the different schemes

Baryon particles	Effective mass(μ_N)	Shielding charge(μ_N)	Effective mass + Shielding charge(μ_N)	Spin+ Orbit (μ_N)	Experimental values(μ_N)[6]
p	2.863	2.79	1.1	2.779	2.79
N	-2.20	-1.91	-0.64	-1.90	-1.91
Λ^0	-0.58	-0.53	-0.42	-0.609	-0.613
Σ^+	0.936	1.982	1.22	2.66	2.458
Σ^0	2.556			0.785	
Σ^-	-0.944	-0.78	-0.484	-1.09	-1.160
Ξ^0	-1.511	1.237	-0.485	-1.424	-1.250
Ξ^-	-0.455	-0.68	-0.26	-0.496	-0.6507

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APPENDIX A

Baryons Wave Functions in terms of Spin and Flavor

In SU(6) quarks model

$$6 \otimes 6 \otimes 6 = 56 \oplus 70 \oplus 70 \oplus 20 \quad (\text{A.1})$$

The representation 56 is completely symmetric irreducible representation of the SU(6). The six quark states $(u \uparrow u \downarrow d \uparrow d \downarrow s \uparrow s \downarrow)$ can be put in the fundamental representation 6. We denote such state with $\psi_{i\alpha}$ where $\alpha = 1, 2$ and $i = 1, 2, 3$ therefore

$$\psi = \begin{pmatrix} u \uparrow d \uparrow s \uparrow \\ u \downarrow d \downarrow s \downarrow \end{pmatrix} \quad (\text{A.2})$$

Now SU(3), SU(2) and SU(3)×SU(2) are the subgroup of the SU(6) so the representation of SU(6) splits in the form under these subgroup as

Table A.1: SU(6) representation in the form of SU(3) and SU(2) subgroup

Subgroup of SU(6)	Quark representation	Generators
SU(3)	$(u \uparrow d \uparrow s \uparrow)(u \downarrow d \downarrow s \downarrow)$ (3, 1) (3, 1)	$\frac{1}{2} \lambda_A \otimes 1$ ($A = 1 \dots 8$)
SU(2)	$(u \uparrow u \downarrow)(d \uparrow d \downarrow)(s \uparrow s \downarrow)$ (1, 2) (1, 2) (1, 2)	$1 \otimes \frac{1}{2} \sigma_n$ ($n = 1, 2, 3$)
SU(3)×SU(2)	(3, 2)	$\frac{1}{2} \lambda_A \otimes \frac{1}{2} \sigma_n$

Thus we can say that SU(6) has generators

$$6 \otimes \bar{6} = 35 \oplus \quad (\text{A.3})$$

Hence the adjoint representation of SU(6) has dimension 35. The representation 56 and 35 splits under the subgroup SU(3)×SU(2) as follows

$$56 = [(3, 2) \otimes (3, 2) \otimes (3, 2)]_{\text{symmetric}} \quad (\text{A.4})$$

$$56 = (10, 4) + (8, 2) \quad (\text{A.5})$$

Where (10, 4) represents the baryon decuplet and (8, 2) represents the baryon octet. Since the lowest lying baryons are made up of the three quarks with spin half each. It is convenient to combine first two spin. There we get $S = 0, 1$ and spin wave functions χ_A and χ_S respectively. The subscripts A and S mention asymmetric and symmetric representation of the spin part of baryons. Now we combine the third spin half with the spin states $S = 0, 1$ to give overall spin half, thus the wave function obtained is as

$S = 0$ with $\frac{1}{2}$ there are two values of spin $\frac{1}{2}$ and $-\frac{1}{2}$.

$$\chi_{MA}^{\frac{1}{2}} = \frac{1}{\sqrt{2}} [(\uparrow\downarrow - \downarrow\uparrow) \uparrow] \quad (\text{A.6})$$

$$\chi_{MA}^{-\frac{1}{2}} = \frac{1}{\sqrt{2}} [(\uparrow\downarrow - \downarrow\uparrow) \downarrow] \quad (\text{A.7})$$

Now on combining spin $S = 1$ with remaining spin $\frac{1}{2}$, we get $S = \frac{1}{2}, \frac{3}{2}$. Thus the spin wave function is

Table A.2: Symmetric and Antisymmetric combination of the spin

$S_z = \frac{3}{2}$	$S_z = \frac{1}{2}$	$S_z = -\frac{1}{2}$	$S_z = -\frac{3}{2}$
$\chi_S^{\frac{3}{2}}$ symmetric $ \uparrow\uparrow\uparrow\rangle$	$\frac{1}{\sqrt{3}} \uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow\rangle$	$\frac{1}{\sqrt{3}} \downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow\rangle$	$ \downarrow\downarrow\downarrow\rangle$
$\chi_{MS}^{\frac{1}{2}}$ (Mixed symmetric)	$\frac{1}{\sqrt{6}} -(\uparrow\downarrow + \downarrow\uparrow) \uparrow + 2 \uparrow\uparrow\downarrow\rangle$	$\frac{1}{\sqrt{6}} (\uparrow\downarrow + \downarrow\uparrow) \uparrow - 2 \uparrow\uparrow\downarrow\rangle$	

The numerical co-efficient with the terms in table are the C.G. coefficient on combining spin = 0, 1, $\frac{1}{2}$. Therefore the complete symmetric representation 56 of the SU(6) can be written as

$$\Phi_s \chi_s + \frac{1}{\sqrt{2}} [\Phi_{MS} \chi_{MS} + \Phi_{MA} \chi_{MA}] \quad (\text{A.8})$$

We know that symmetric and antisymmetric wave functions of the baryons octet are given as

Table A.3: Symmetric and Antisymmetric wave functions of baryons in terms of flavor

$J^P = \frac{1}{2}^+$	Antisymmetric	Symmetric
$ p\rangle$	$\frac{1}{\sqrt{2}} [ud + du]u\rangle$	$\frac{1}{\sqrt{6}} [[ud + du]u - 2uud]$
$ n\rangle$	$\frac{1}{\sqrt{2}} [ud + du]d\rangle$	$-\frac{1}{\sqrt{6}} [[ud + du]d - 2udd]$
$ \Lambda^0\rangle$	$\frac{1}{\sqrt{12}} [2[ud + du]s - [ds + sd]u - [su + us]d]$	$-\frac{1}{2} [sd + ds]u - [su + us]d]$
$ \Sigma^0\rangle$	$\frac{1}{2} [ds + sd]u + [us + su]d\rangle$	$\frac{1}{\sqrt{12}} [-2[ud + du]s + [us + su]d + [ds + sd]u]$
$ \Sigma^+\rangle$	$\frac{1}{\sqrt{2}} [us + su]u\rangle$	$\frac{1}{\sqrt{6}} [[us + su]u - 2uud]$
$ \Sigma^-\rangle$	$\frac{1}{\sqrt{2}} [ds + sd]d\rangle$	$\frac{1}{\sqrt{6}} [[ds + sd]d - 2dds]$
$ \Xi^0\rangle$	$\frac{1}{\sqrt{2}} [su + us]s\rangle$	$\frac{1}{\sqrt{6}} [[su + us]s - 2ssu]$
$ \Xi^-\rangle$	$\frac{1}{\sqrt{2}} [ds + sd]s\rangle$	$\frac{1}{\sqrt{6}} [-[ds + sd]s + 2ssd]$

The wave function for the baryon $J^P = \frac{1}{2}^+$ state in the explicit state is given below. Here we take the example of proton to solve the wave functions of the baryons.

$$|P, S_z = \frac{1}{2}\rangle = \frac{1}{\sqrt{2}} \left\{ \left(-\frac{1}{6}\right) [(ud + du)u - 2uud] \left(-\frac{1}{6}\right) ([\uparrow\downarrow + \downarrow\uparrow] \uparrow - 2 \uparrow\uparrow\downarrow) + \frac{1}{\sqrt{2}} [(ud - du)u] \frac{1}{\sqrt{2}} [(\uparrow\downarrow - \downarrow\uparrow) \uparrow] \right\} \quad (\text{A.9})$$

$$= \frac{1}{6\sqrt{2}} [(u \uparrow d \downarrow + u \downarrow d \uparrow)u \uparrow - 2u \uparrow d \uparrow u \downarrow + (d \uparrow u \downarrow + d \downarrow u \uparrow)u \uparrow - 2d \uparrow u \uparrow u \downarrow - 2u \uparrow u \uparrow d \downarrow - 2u \downarrow u \uparrow d \uparrow + 4u \uparrow u \uparrow d \downarrow + 3u \uparrow d \downarrow u \uparrow - 3u \downarrow d \uparrow u \uparrow - 3d \uparrow u \downarrow u \uparrow + 3d \downarrow u \uparrow u \uparrow] \quad (\text{A.10})$$

$$= \frac{1}{\sqrt{18}} [2u \uparrow d \downarrow u \uparrow + 2u \uparrow u \uparrow d \downarrow + 2d \downarrow u \uparrow u \uparrow - u \uparrow u \downarrow d \uparrow - u \uparrow d \uparrow u \downarrow - u \downarrow d \uparrow u \uparrow - d \uparrow u \downarrow u \uparrow - d \uparrow u \uparrow u \downarrow - u \downarrow u \uparrow d \uparrow] \quad (\text{A.11})$$

In the similar way the wave function of the other baryons are given in table below

Table A.4: Spin-Flavor combined wave function of baryons

Particle state	Wave function
$ P, S_z = \frac{1}{2}\rangle$	$\frac{1}{\sqrt{18}} [2u \uparrow d \downarrow u \uparrow + 2u \uparrow u \uparrow d \downarrow + 2d \downarrow u \uparrow u \uparrow - u \uparrow u \downarrow d \uparrow - u \uparrow d \uparrow u \downarrow - u \downarrow d \uparrow u \uparrow - d \uparrow u \downarrow u \uparrow - d \uparrow u \uparrow u \downarrow - u \downarrow u \uparrow d \uparrow]$
$ n, S_z = \frac{1}{2}\rangle$	$\frac{1}{\sqrt{18}} [-2d \uparrow u \downarrow d \uparrow - 2d \uparrow d \uparrow u \downarrow - 2u \downarrow d \uparrow d \uparrow + d \uparrow d \downarrow u \uparrow + d \uparrow u \uparrow d \downarrow + d \downarrow u \uparrow d \uparrow + u \uparrow d \downarrow d \uparrow + u \uparrow d \uparrow d \downarrow + d \downarrow d \uparrow u \uparrow]$
$ \Sigma^+, S_z = \frac{1}{2}\rangle$	$-\frac{1}{\sqrt{18}} [2u \uparrow s \downarrow u \uparrow + 2u \uparrow u \uparrow s \downarrow + 2s \downarrow u \uparrow u \uparrow - u \uparrow u \downarrow s \uparrow - u \uparrow s \uparrow u \downarrow - u \downarrow s \uparrow u \uparrow - s \uparrow u \downarrow u \uparrow - s \uparrow u \uparrow u \downarrow - u \downarrow u \uparrow s \uparrow]$
$ \Sigma^0, S_z = \frac{1}{2}\rangle$	$-\frac{1}{\sqrt{36}} [2d \uparrow s \downarrow u \uparrow - d \downarrow s \uparrow u \uparrow - s \uparrow d \downarrow u \uparrow + 2s \downarrow d \uparrow u \uparrow + 2u \uparrow d \uparrow s \downarrow - u \downarrow d \downarrow s \uparrow - u \uparrow s \uparrow d \downarrow + 2u \uparrow s \downarrow d \uparrow + 2s \downarrow u \uparrow d \uparrow - s \uparrow u \uparrow d \downarrow + -d \downarrow u \uparrow s \uparrow + 2d \uparrow u \uparrow s \downarrow - s \uparrow u \downarrow d \uparrow - u \uparrow s \uparrow d \downarrow - d \uparrow s \uparrow u \downarrow - d \uparrow u \downarrow s \uparrow - u \downarrow d \uparrow s \uparrow - s \uparrow d \uparrow u \downarrow]$
$ \Sigma^-, S_z = \frac{1}{2}\rangle$	$\frac{1}{\sqrt{18}} [2d \uparrow s \downarrow d \uparrow + 2d \uparrow d \uparrow s \downarrow + 2s \downarrow d \uparrow d \uparrow - d \uparrow d \downarrow s \uparrow - d \uparrow s \uparrow d \downarrow - d \downarrow s \uparrow d \uparrow - s \uparrow d \downarrow d \uparrow - s \uparrow d \uparrow d \downarrow - d \downarrow d \uparrow s \uparrow]$
$ \Lambda^0, S_z = \frac{1}{2}\rangle$	$-\frac{1}{2\sqrt{12}} [2u \uparrow d \downarrow s \uparrow - 2u \downarrow d \uparrow s \uparrow - 2d \uparrow u \downarrow s \uparrow + 2d \downarrow u \uparrow s \uparrow + 2s \uparrow u \uparrow d \downarrow - 2s \uparrow u \downarrow d \uparrow - 2s \uparrow d \uparrow u \downarrow + 2s \uparrow d \downarrow u \uparrow + 2d \downarrow s \uparrow u \uparrow - 2d \uparrow s \uparrow u \downarrow - 2u \downarrow s \uparrow d \uparrow + 2u \uparrow s \uparrow d \downarrow]$

$ \Xi^0, S_z = \frac{1}{2}\rangle$	$\frac{1}{\sqrt{18}}[-2s \uparrow u \downarrow s \uparrow - 2s \uparrow s \uparrow u \downarrow - 2u \downarrow s \uparrow s \uparrow + s \uparrow s \downarrow u \uparrow + s \uparrow u \uparrow s \downarrow + s \downarrow u \uparrow s \uparrow + u \uparrow s \downarrow s \uparrow + u \uparrow s \uparrow s \downarrow + s \downarrow s \uparrow u \uparrow]$
$ \Xi^-, S_z = \frac{1}{2}\rangle$	$\frac{1}{\sqrt{18}}[-2s \uparrow d \downarrow s \uparrow - 2s \uparrow s \uparrow d \downarrow - 2d \downarrow s \uparrow s \uparrow + s \uparrow s \downarrow d \uparrow + s \uparrow d \uparrow s \downarrow + s \downarrow d \uparrow s \uparrow + d \uparrow s \downarrow s \uparrow + d \uparrow s \uparrow s \downarrow + s \downarrow s \uparrow d \uparrow]$

The magnetic moment operator consists of contributions from both intrinsic spins of the constituent quarks that make up the bound state μ_s and angular momentum of the three-quark system μ_l with the centre of mass motion removed.

i.e.
$$\mu = \mu_l + \mu_s \quad (\text{A.12})$$

Taking the constituent quarks to be Dirac point particles the spin contribution and orbital contribution in equation (34) is determined by the effective quark masses ω_i

$$\mu_s = \sum_a \frac{e_a \sigma_a}{2\omega_a} \quad , \quad (\text{A.13})$$

$$\mu_L = \sum_a \frac{e_a}{2\omega_a} r_a \times p_a \quad , \quad (\text{A.14})$$

The three quark basis state diagonalizes the confinement problem with eigen functions that correspond to separate excitations of the quark 3 (ρ and λ excitations, respectively). In particular, excitation of the λ variable unlike excitation in ρ involves the excitation of the ‘‘odd’’ quark (d for uud or u for ddu). The physical P-wave states are not pure ρ or λ excitations but linear combinations of all states with a given total momentum J. Most physical states are, however, close to pure ρ or λ states

In terms of Jacobi variables,

$$\rho = r_2 - r_1, \quad \lambda = \frac{1}{\sqrt{3}}(2r_3 - r_1 - r_2), \quad R = \frac{1}{3}(r_1 + r_2 + r_3),$$

On solving eq. (A.12) on substituting the value of $p_a = p_a - e_a A_a$ we get that on solving

$$\mu_L = \frac{1}{2}(\mu_1 + \mu_2)l_\rho + \frac{1}{6}(\mu_1 + \mu_2 + 4\mu_3)l_\lambda + \frac{\mu_1 - \mu_2}{2\sqrt{3}}(\rho \times p_\lambda + \lambda \times p_\rho) \quad , \quad (\text{A.15})$$

In terms of charge Q we get above equation as

$$\frac{(Q_1 + Q_2)}{4m}l_\rho + \left(\frac{Q_1 + Q_2}{12m} + \frac{Q_3}{3m}\right)l_\lambda + \frac{Q_1 - Q_2}{4\sqrt{3}}(\rho \times p_\lambda + \lambda \times p_\rho) \quad (\text{A.16})$$

Since the contribution of last term vanishes for the pure λ and ρ excitations which on further calculations gives us

$$\mu_L = \frac{1}{2}\left[l_\rho + \frac{1}{3}l_\lambda\right](\mu_1 + \mu_2) + \frac{2}{3}\mu_3 \quad , \quad (\text{A.17})$$

where

$$l_\rho = \rho \times p_\rho, \quad l_\lambda = \lambda \times p_\lambda \quad (\text{A.18})$$

The calculation for the quark spin part of baryon MDMs (magnetic dipole moment) is straightforward. But the calculation for the orbital angular momentum part is much less familiar, since it vanishes for ground state octet and decuplet baryons, which have no orbital excitations. The angular operators in the above equation act as special wave function $\psi_{1m}^{\rho, \lambda}$ as follows

$$\begin{aligned}
l_{\rho z}\psi_{1m}^{\rho} &= m\psi_{1m}^{\rho}, & l_{\rho z}\psi_{1m}^{\lambda} &= 0, \\
l_{\lambda z}\psi_{1m}^{\lambda} &= m\psi_{1m}^{\lambda}, & l_{\lambda z}\psi_{1m}^{\rho} &= 0, \\
(\rho \times p_{\lambda})_z\psi_{1m}^{\lambda} &= m\psi_{1m}^{\rho}, & (\rho \times p_{\lambda})_z\psi_{1m}^{\rho} &= 0 \\
(\rho \times p_{\rho})_z\psi_{1m}^{\rho} &= m\psi_{1m}^{\lambda}, & (\rho \times p_{\rho})_z\psi_{1m}^{\lambda} &= 0
\end{aligned} \tag{A.19}$$

Now on solving for the eigen values in term of the $l_{\lambda z}$ and $l_{\rho z}$ we get

$$\begin{aligned}
&\left\langle N^2 P_{\frac{1}{2}} \left| l_{\lambda z} \right| N^2 P_{\frac{1}{2}} \right\rangle \\
&= m\psi_{1m}^{\lambda} \left[\frac{1}{2} \langle \chi_{m_s}^{\rho} \phi^{\rho} | l_{\lambda z} | \chi_{m_s}^{\rho} \phi^{\rho} \rangle - \frac{1}{2} \langle \chi_{m_s}^{\rho} \phi^{\rho} | l_{\lambda z} | \chi_{m_s}^{\lambda} \phi^{\lambda} \rangle \right. \\
&\quad \left. - \frac{1}{2} \langle \chi_{m_s}^{\lambda} \phi^{\lambda} | l_{\lambda z} | \chi_{m_s}^{\rho} \phi^{\rho} \rangle + \frac{1}{2} \langle \chi_{m_s}^{\lambda} \phi^{\lambda} | l_{\lambda z} | \chi_{m_s}^{\lambda} \phi^{\lambda} \rangle \right], \tag{A.20}
\end{aligned}$$

or

$$\left\langle N^2 P_{\frac{1}{2}} \left| l_{\lambda z} \right| N^2 P_{\frac{1}{2}} \right\rangle = m\psi_{1m}^{\lambda},$$

Similarly

$$\left\langle N^2 P_{\frac{1}{2}} \left| l_{\rho z} \right| N^2 P_{\frac{1}{2}} \right\rangle = m\psi_{1m}^{\rho},$$

From above equation we calculate the value of m depending upon the value of N then we further solve the wave function to calculate for the angular momentum of the nucleons. The value of angular momentum comes out to be

$$\left\langle N^2 P_{\frac{1}{2}} \left| \mu_z \right| N^2 P_{\frac{1}{2}} \right\rangle = \frac{2}{9} \frac{Q_u}{m_u} + \frac{1}{9} \frac{Q_d}{m_d}$$