

“Architectural Acoustics: Numerical and Experimental study of a Classroom”

*A Dissertation submitted
in partial fulfillment in requirement
for the award of the degree*

MASTER OF ENGINEERING

In

CAD/CAM

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July-2017

Declaration

This is to certify that the work done in this dissertation title “**Architectural Acoustics: Numerical and Experimental study of Classroom**” submitted in partial fulfillment of requirement for the award of Master of Engineering degree in CAD /CAM in the Mechanical Engineering department of Thapar University, Patiala, is an authentic record of work carried out by me under the guidance of **Dr. Ashish Purohit**, Mechanical Engineering Department, Thapar University, Patiala. The matter embodied in this work has not been submitted in any part or full to any other university or institute for the award of any degree.


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This is to certify that above declaration made by the student concerned is correct to the best of my knowledge & belief.



Dr. ASHISH PUROHIT

Date: 27/7/2017

Acknowledgement

Through this, first of all I would like to acknowledge my parents who gifted me the ability to achieve things in my personal and professional life and in all hardships they stand beside me. They always sees the best in me, though I am not best, With the biggest contribution to this completion of work, I would like to thank **Dr. Ashish Purohit**, Mechanical Engineering Department, had given me his full support in guiding and encouraged to go ahead in all time during this dissertation. It is because of his guidance that I am able to complete this work. He believes that, if you join any institute then try to give something to that institute. His guidance and believe worked for me and surely will work in future. There are few friends that I would like to specially acknowledge, **Mr. Ishan Chawla, Mr. Jyotindra Narayan and Mr. Akash Soni**. They stand with me as my good friends during all time of masters, without them I feel there is no fun is this course work.

Lastly, I would like to thank to all my fellows and teachers, their contributions are sincerely appreciated and gratefully acknowledged.

Thank you all.

Abstract:

The problem of room acoustics has been studied from several decades, however it has always been complicated to understand sound propagation in an enclosed space and, also, its control for better hearing. In the present work, numerical and experimental investigation of sound propagation within a three-dimensional closed space and corresponding reverberation time are carried out. The numerical study is also performed for one and two dimensional domains. For the numerical study, basic hyperbolic sound wave equation is discretized by finite difference technique in both time and space. Different boundary conditions for room surfaces are implemented. The developed methodology is validated for the sound propagation from fundamental sources as point source, line source, and pulsating cylinder. Reverberation time is calculated for both two dimensional and three dimensional domains and compared with the results computed from the analytical formulation. Finally, reverberation time of a classroom is measured experimentally and a comparative analysis of experimental and numerical results is discussed, which shows a reasonable matching of numerical results with the measurement.

Key words: Reverberation time (RT_{60}), wave equation, finite difference schemes, reflecting surface, absorbing boundary condition, Sound source

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Nomenclature

c	Speed of sound
Z	Wall impedance
Z_c	Characteristic impedance
RT_{60}	Reverberation Time
k	Wave number
r	Radius

Greek Symbols:

α	Absorption coefficient
\mathfrak{R}	Reflection coefficient
γ	Attenuation coefficient
ζ	Specific impedance
λ	Courant number
Δ	Step size
μ	Viscosity
ρ	Density of medium
θ	Incident angle
σ	Flow resistivity
ω	Angular velocity

Acronyms

SPL	Sound Pressure Level
dB	Decibels
SWR	Standing Wave Ratio
RMS	Root Mean Square
NRC	Noise Reduction Coefficient
SAA	Sound Average Absorption
STC	Sound Transmission Class
FDM	Finite Difference Method
FDTD	Finite Difference Time Domain
LPS	Leapfrog Scheme

Chapter 1. Introduction

1.1 Overview

Acoustics is the science, which deals with the properties of sound i.e. how sound behave and propagate in any medium (solid, liquid or gas). Acoustical wave is the pressure perturbation in medium. It is a longitudinal wave propagates in medium by forming compression and rarefaction region. Wave propagation is generally a sound energy propagating phenomenon which is generated through vibration of particle in medium. For example, air particle vibrates (to and fro motion) which forming the compression and rarefaction regions, are responsible for the sound generation in air. Oscillation of particles causes the pressure fluctuation (over ambient pressure), hence, there is continuous pressure drop near these regions. This pressure difference (or pressure fluctuation) in a medium is easily senses by human ear. Hence, wave propagation in any medium is entirely a physical phenomenon. According to medium specification Acoustics divided into different categories like Architectural Acoustics or Room Acoustics, Underwater Acoustics, Aero Acoustics, Electro-acoustics, Psychoacoustics.

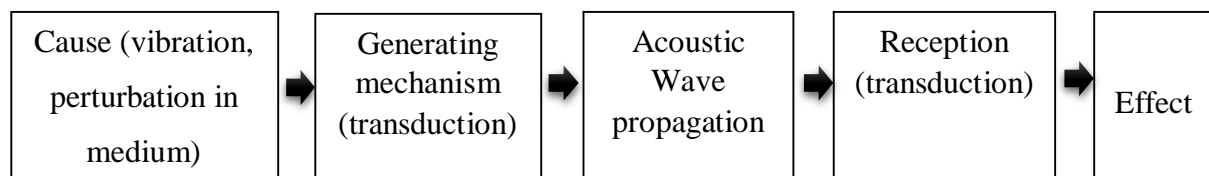


Figure 1. General propagation of acoustic wave

Architectural acoustics is one of the important area in the field of Acoustics which deals with the propagation of sound in closed spaces i.e. room, buildings, theatres etc. The architectural acoustics involves the physics of sound that includes sound reflection, sound absorption, sound transmission and sound dispersion in space and time. Engineers and researchers working on the acoustics from very long time. Design of classroom, living room, theater, concert hall architectural etc. have great importance for easy understanding of spoken words and played instruments. The history of architectural acoustics is very old, Romans and Greeks known for their designs of music hall, concert hall, theater. Their design of large theatre and auditoriums have steep path between the extreme walls and high stage for the speaker, to have better sound intelligibility [1, 2] is still use to have better building acoustics. Sometime design of rooms and buildings fulfills the aesthetic

criterion however, their design can be differ in acoustical aspects. Example large good looking rooms may have coloration effects (interference of wave after reflection) [1] , even spaces which are generally considered insignificant aesthetically like staircases, factories & concourses at railway station or airport may exhibit satisfactory acoustical effect as one can hear and understand the announcement easily.

Echoing and high reverberation time [1, 3] (time required for the decay of sound to 60 dB) causes the room, theatre etc. to be acoustical treated, otherwise it is hard to understand the speech at that place. Sound reproduction in rooms is determined by the room resonances often called room modes. Resonances are damped by sound absorption material and their decay determines the reverb time. Reverberation time is one of the important parameter in designing of class room, living room, theater hall, etc. In classroom less reverberation time required and in theatres and concert hall more reverberation time is required for the better clarity of sound [1]. They are accordingly designed to have respective reverberation time requirement. Hence, acoustical consultant has to propose all measures which will results in the favors of listener. Building acoustics treatment helps in better sound intelligibility, better sound transmission, better sound diffusion and better sound absorption or reflection.

1.2 Reverberation Time RT_{60}

Reverberation time defined as the time taken by the sound to reduce by 60 decibels (dB). Basically in simple way it is defines how long a sound persist at the place. It is measure by turning off the sound source after t sec. then calculate in how long it take to reduce by 60 decibels refer Figure 2. Class room should have low reverberation time for the good hearing other hand music hall have little long reverberation time for significant hearing of music. After reverb time range, one thing should be kept in mind that reverb time of place must not be zero. If it is zero value then speaker has to do more effort to make push the sound to listener's ear. Sometime RT_{60} is difficult to calculate because system require at least 30 dB noise level to calculate RT_{60} in that case, first calculate 30 dB or 20 dB drop and then extrapolate it to 60 dB for reverberation time.

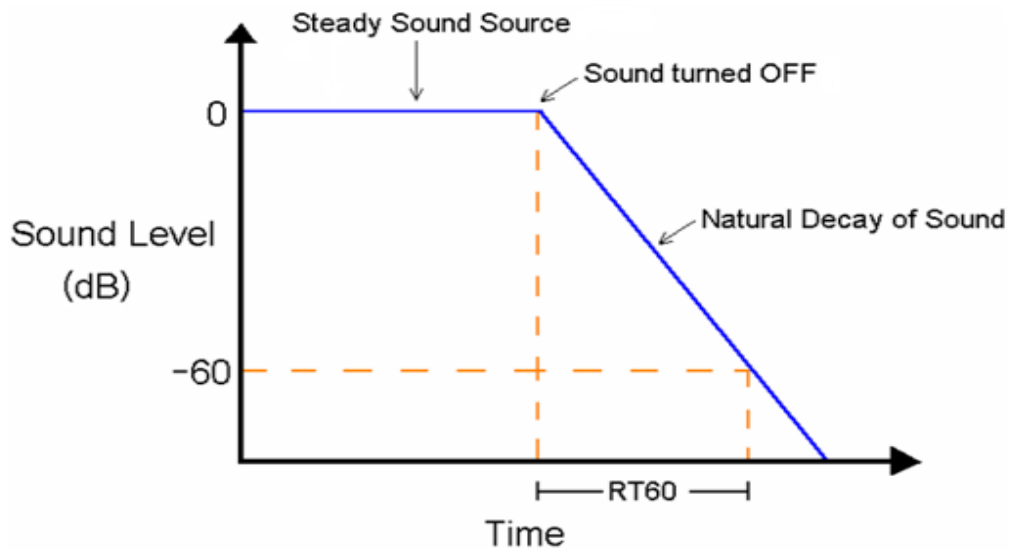


Figure 2.Reverberation time (RT_{60}), [W1]

Some optimize reverberation time of famous places is given in Table 1. It is clear that various place intentionally designed to have low or high reverberation time as required by human ear.

Table 1.Required reverberation time of some places, [W2]

	0.5-0.7(sec)	1.4-2.0 (sec)	2.1-3.0 (sec)	Optimum
Speech	Good	Fair-poor	Unacceptable	0.6-0.7 (sec)
Contemporary music	Fair-good	Fair	Poor	1.2-1.4 (sec)
Choral music	Poor-fair	Fair-good	Good-fair	1.8-2.0 (sec)

1.3 Room Modes

Room modes are the collection of resonances that exist in a room when the room excited by acoustical source such as a loudspeaker. Most rooms have their fundamental resonances in the speaking frequency range of 20 Hz to 200 Hz, each frequency being related to one or more of the room's dimension or a multiple. These resonances affect the low-frequency or mid-frequency response of a sound system in the room and are one of the biggest obstacles to accurate sound

reproduction. The input of acoustic energy to the room at the modal frequencies and multiples there causes standing waves. The nodes and antinodes of these standing waves results in the loudness of the particular resonant frequency being different at different locations of the room. The amplitude of each mode depend upon the frequency difference between driving frequency & resonating frequency of the standing wave and also depends on the location at which the source is placed. For 1D wave mode frequency and length of the wave given below:

$$f = \frac{nc}{2l} \qquad l = n\lambda/2 \qquad (1.1)$$

f , is natural frequency of room, c is velocity of sound, n is the integer and l is the length of room.

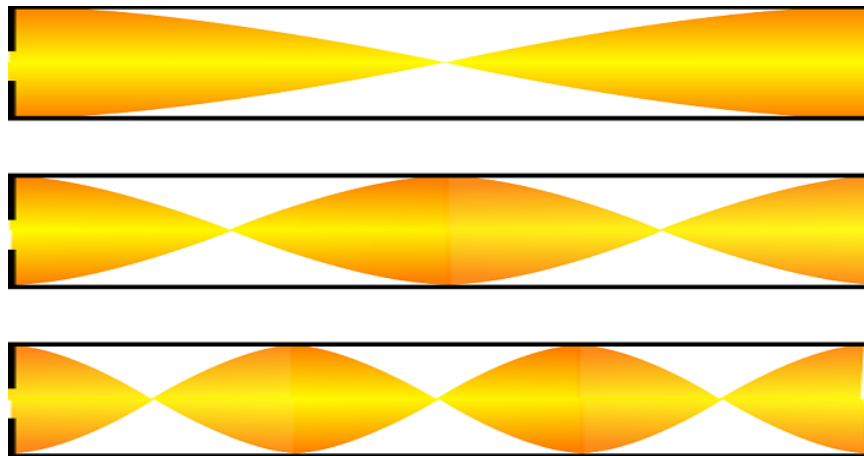


Figure 3. Room Modes, [W3]

First part of Figure 3 shows that when $n = 1$ then $l = \lambda/2$, next part of fig shows that when $n = 2$ then $l = 3\lambda/2$ and when $n = 4$ then $l = 2\lambda$. Low and mid frequency responses dominated by these room modes.

Room modes are critically effected at very high frequencies. As frequency become very high no of modes increases at very high rates and spacing between modes gets decreases. If spacing between modes is too low as compare to our ear spacing then one ear listen different than other.

1.4 Measurements of sound

Sound propagation in a medium follows Doppler criteria. Sound energy intensifies when source is near to object and it gets decrease when the source is away from the object. Sound energy dissipates

very quickly as there is exponential decay of amplitude of acoustic wave. Hence it is convenient to measure pressure drop log scale named decibel (dB) scale. Decibel is the logarithmic unit used to express gain or ratio between two values one of which is reference value. In sound measurement output generally consider as pressure over ambient pressure. Therefore the sound pressure level in dB is defined by equation:

$$SPL = 20 \log_{10} \left(\frac{p_{rms}}{p_{ref}} \right) dB \quad (1.2)$$

Where p_{ref} is $20 \mu Pa$ and p_{rms} is pressure fluctuation over ambient pressure. Reference pressure is the minimum acoustic pressure that audible to young human ear in good health. Some typical sound pressure level is describe in Table 2:

Table 2. Sound pressure level in Pa and corresponding dB level [W4]

Source at 1m	Sound pressure (Pa)	Sound pressure level (dB)
Rifle	200 Pa	140 dB
Threshold of pain	20 Pa	120 dB
Pneumatic hammer	2 Pa	100 dB
Newspaper press	1 Pa	94 dB
Street traffic	0.2 Pa	80 dB
Talking	0.02 Pa	60 dB
Library	0.002 Pa	40 dB
TV studio	0.0002 Pa	20 dB
Threshold of talking	0.00002 Pa	0 dB

1.5 Directivity of sound

It is the manner (patterns) in which predicted or measured sound at a fixed position vary with the angular position [4]. These patterns describes where sound radiates well. Generally sound sources are categorize in three categories a) Monopole, b) Dipole, and c) Quadrupole. Each source have different directivity pattern shown in Figure 4:

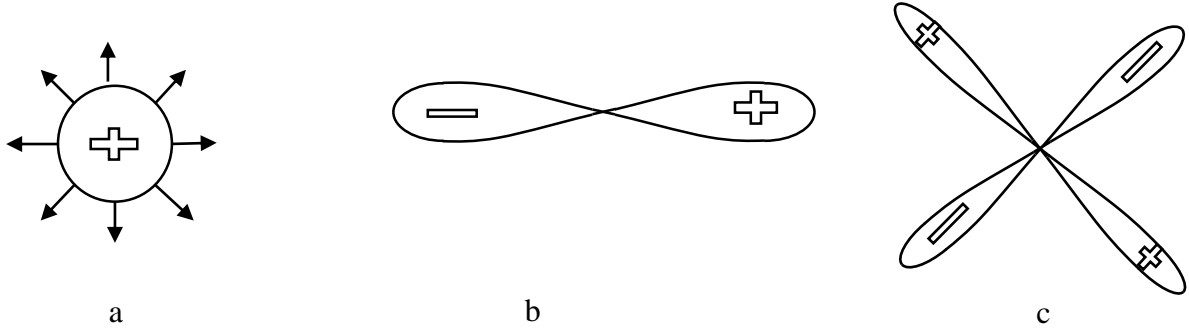


Figure 4. Directivity patterns of sources a) monopole, b) dipole and c) quadrupole

A simple point source or monopole has sound radiation in all direction equally does not depend on angular position [4]. A dipole radiates good sound in longitudinal direction however, in lateral direction sound is very poor as sound gets cancelled due to interference and quadrupole radiates well sound is four direction and other four direction sound propagation is very poor.

1.6 Sound absorption system

Sound energy absorption is determined by the absorption/reflection coefficient. Reflection coefficient is determined by short tube impedance method [5]. Standing Wave Ratio (SWR) is calculated i.e. ratio between maximum amplitude to minimum amplitude. Reflection coefficient is related with the SWR as:

$$r = \frac{SWR - 1}{SWR + 1} \quad (1.3)$$

Relationship between absorption coefficient and reflection coefficient (\mathfrak{R}) is defined by the power law as:

$$\alpha = 1 - \mathfrak{R}^2 \quad (1.4)$$

Sound propagation and reflection/absorption is very complex phenomenon therefore digital methods are used which make it simpler. It involves the computational and numerical methods to understand the propagation in different media. Numerical study uses the discretization of wave equation in uniform grid structure to analyze the sound behavior (sound absorption & reflection) in rooms, building etc. Numerical discretization gives the approximate results as it have high degrees of freedom. Modern world keep working on the old designs and try to modify it to

get better performance of that place. Hence architectural acoustics is purely an art and can be understood by practicing it to real world example because empirical and theoretical things will fail sooner or later. Therefore acoustician has to face two problem, first he has to apply the relation between structural feature (room's shape and geometry) and sound field and second he has to consider the measurable parameter of sound field (reflection or absorption) and their effect to listener. Architectural acoustics also involves the study of instruments required in propagation of sound like loud speakers (generator of sound) and microphone (receiver of sound) etc. Installation of loudspeaker and their performance plays a very important role to have better sound intelligibility index or the quality of sound that we usually hearing at places like theater auditorium hall etc.[6]

From the literature review, the following objective have been finalized as:

- Numerical study of reverberation time for classroom for one dimensional, two dimensional and three dimensional domain.
- An experimental measurement of reverberation time of classroom.
- Comparison of reverberation time computed by numerical and experimental study of classroom.

In this present work, a numerically discretized model is developed by using finite difference method (FDM) and leapfrog scheme. Higher frequency wave can easily be absorbed/reflect by the material at the boundaries because of their lower wavelength. However low frequency waves are not easily reflect due to their high wavelength. Speaking frequency between 20 Hz-200 Hz contains low and mid frequency element is studied in this dissertation work. Phenomenon of sound propagation and absorption/reflection in closed space is analyzed along with the various source excitation. Point source and cylindrical source are used for one dimensional and two dimensional model respectively, which is discussed in details later in this work. After validating the wave propagation and boundary of the model reverberation time of 2D diffuse field is validated with the numerically developed solver, results are discussed in chapter 4. Then this work is extended in three dimensional domain where a classroom is analyzed experimentally with source excitation and reverberation time of class room is deduced. Two case data is analyzed, in first case a balloon is burst and in second case an un-baffled loudspeaker is act as a source. Experimental data is compared with numerical model, having similar dimensional space and boundary condition. Also the data is compared with the result from empirical relation. Comparison gives the satisfactory results for 3D room. At last concluding remarks are presented.

Chapter 2. Literature Review

Sabine [2] carried out the work of Joseph Henry on reverberation time. He did a lot of experiments and been able to give an analytical relation between reverberation time and volume of room and absolute absorption by material and person in closed space. An experiment in lecture room at Harvard University conducted by him, before this experiment reverb time of hall is 5.62 sec and in experiment the cushions (absorber) were brought on the seats and curtains at the boundary was used. By covering the aisle, floor, ceiling, and in the presence of audience the reverberation time is reduced to 1.14 sec. Figure 5 shows the interior of lecture room and corresponding CATT model.

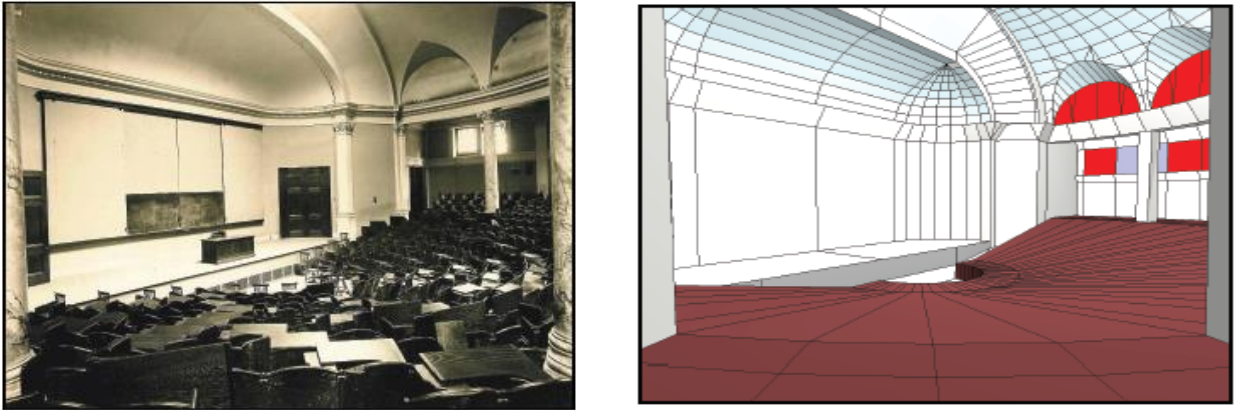


Figure 5. Interior of lecture hall at Fogg Art Museum

After a lot of experiment, Sabine gave the formulation of reverberation time which is called as a Sabine formula for reverberation time:

$$RT_{60} = \frac{0.049V}{\alpha} \quad (2.1)$$

Where, V is volume of room and α is the absorption coefficient. Two simple assumptions were made, (a) sound travels inside room in straight line similar to the propagation of light ray and (b) under steady state condition sound is perfectly diffuse it means after reflection sound travels in all directions with equal probability and sound pressure of reflected waves is equal everywhere [7].

Delany and Bézely [8] discussed about the acoustical properties of fibrous absorbent material. Authors consider the plane wave propagation in one dimension and they suggested that characteristic impedance $Z_c = X + iY$ and propagation constant $\gamma = a + ib$ can either be considered as the function of flow resistance or flow resistance per unit thickness (σ). In propagation of sound,

any material is determined by “characteristics impedance” and “propagation coefficient” and both are complex quantities with some real and imaginary parts. Normalizing real and imaginary component of characteristic impedance and propagation coefficients are the function of dimensional variable that is ratio of frequency and flow resistance $\left(\frac{f}{\sigma}\right)$. Simply the relation determine by the power law as follow:

$$\frac{X}{\rho_a c_o} = -11.9 \left(\frac{f}{\sigma}\right)^{-0.73} \quad (2.2)$$

Where X be the real component of characteristic impedance. Similarly other component is also relate by the ratio $\left(\frac{f}{\sigma}\right)$. Furthermore, the flow resistance per unit thickness depends upon the bulk density and fiber size. After taking above assumption an experiment is performed over the wide range of σ values and frequencies to determine the acoustical properties of material that is reflection coefficient or absorption coefficient. Two types of impedance tubes were used one have 4.5 cm diameter and 1 m long and other have 8.9 cm diameter and 2 m long. Measurement were made first with rigid backing in tube and then with sample backed by air filled space of determinable acoustic impedance. By doing all measure, absorption or reflection coefficient were found as shown in Figure 6:

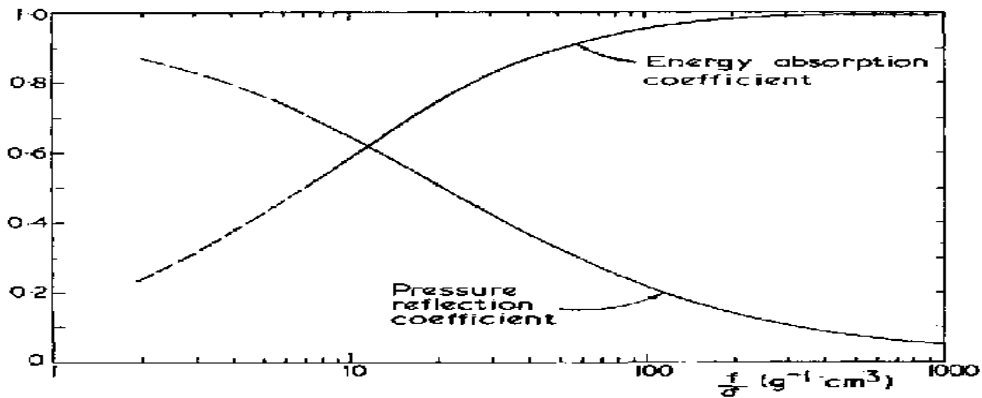


Figure 6. Normal incidence reflection & absorption coefficient as a function of f/σ [8]

For the higher value of $\left(\frac{f}{\sigma}\right)$ above than $1000g^{-1}cm^3$ are not advisable. To verify this experimental measure absorption coefficient of 2.5 cm and 5 cm thick rigidly backed material is calculated by the analytical formula as follow:

$$\alpha = 1 - \left| \frac{Z_c - \rho_a c_o}{Z_c + \rho_a c_o} \right|^2 \quad (2.3)$$

Another fibrous property, porosity factor is kept unity for the entire experiment though absorption coefficient depends on this factor. For ease of calculation it is kept unity.

Bies and Hansen [9], described the systematic use of flow resistance and flow resistivity in the calculation of acoustic properties. Flow resistance information a) controls the reverberant sound field b) attenuation of sound propagation in ducts. Dependency of propagation constant and flow resistance are discussed by Delany and Bezley. The main purpose of the author is to differentiate between the normal incidence absorption coefficient and statistical absorption coefficient [10], or absorption coefficient at random incidence. Flow resistance is defined by ratio of pressure and normal velocity component at particular point in porous media. Flow resistance is nothing but same as impedance when there is no porous material. It is experienced that the flow resistance is linearly related to the thickness (between porous layer and boundary surface) for small velocity, if induced velocity is high then relation become quadratic given by equation:

$$\frac{F_n}{\rho_c} = f_1 \left(\frac{\rho_b}{\rho_f} \right) \left(\frac{\mu l d^2}{\sigma} \right) + f_2 \left(\frac{\rho_b}{\rho_f} \right) \left(\frac{\rho l v^2}{d} \right) \quad (2.4)$$

If induced normal velocity tends to zero then:

$$\frac{F_n}{\rho_c} = f_1 \left(\frac{\rho_b}{\rho_f} \right) \left(\frac{\mu l v}{d^2} \right) \quad (2.5)$$

Where F_n is flow resistance, ρ_b is the bulk density & ρ_f is fiber density, μ is viscosity, l is thickness and d is fiber diameter. Pressure distribution was assumed to be steady however in reality pressure is unsteady or having fluctuating source. Author measured the range of the flow resistivity with varying thickness and density of fiber material. Sound propagation in porous media is complex phenomenon depends upon the complex density ρ_1 and compressibility (k is reciprocal

of bulk modulus of elasticity). Analytical characteristic impedance (Z_c) and propagation constant γ is given by:

$$Z_c = \rho_a c_o = \sqrt{k\rho_1} \quad (2.6)$$

$$\gamma = \frac{\omega}{c_m} + ia_n \quad (2.7)$$

where, real part is wave number of plane wave propagation and imaginary part is attenuation constant of propagating wave. Two boundary condition is used to calculate acoustic properties that is rigid wall and soft wall surface. For rigid wall equation (2.6) and for soft wall equation (2.7) is applicable.

Davern and Dubout [10], compared the absolute absorption coefficient given by Sabine, [2] and statistical absorption coefficient by Morse and Bolt [11], and from Sabine formula maximum absorption coefficient is 1 (for fibrous available materials) and statistical absorption coefficient is 0.92. This discrepancy occur due to the random incidence. Finally critical value of flow resistance is calculated i.e., absorption coefficient start decreases with the increase of flow resistance which is $4\rho c$.

Wassilieff [12], presented the study of sound absorption through wood based material rather than through glass fiber and mineral fiber. Plane sound wave propagation through slit apertures of wood and introduction to porosity and tortuosity in wood structure also discussed by the author i.e., how pore size and pore shape effects the sound absorption of the material. Wood fibers are of hollow shapes and glass and mineral fibers mostly are of solid cylinder. The sound propagation in hollow structure of wood based fibers simply follows Helmholtz resonance mechanism [13] and quarter wavelength resonances to increase the sound absorption. Glass and mineral fibers used for high quality sound absorption having diameter around 6-10 μm and length about 10-30 mm whereas wood fibers have diameter 30 μm and average length of fiber is 3 mm which make it fat & short and less cheaper than glass fiber. Sound absorption is more complex because absorption depends upon air flow resistivity, porosity, tortuosity (deviation of pore from normal) and pore shape factor (for other than cylindrical pores). Delany and Bezley assumed porosity factor unity for ease of calculation. Cremer and Muller [14] also studied the parallel slits in fibers rather than cylindrical

pores. Attenborough [15] gave formulation of sound propagation from slit shaped pore considering the flow resistivity (σ), porosity (θ), tortuosity (d) complex density (ρ) and angular velocity (ω)

$$\rho(\omega) = \rho_a q^2 \left(1 - \frac{\tanh(\sqrt{i\lambda})}{\sqrt{i\lambda}} \right)^{-1} \quad (2.8)$$

$$\lambda = \left(\frac{3\omega\rho_a q^2}{\theta\sigma} \right)^{1/2} \quad (2.9)$$

Empirical relation of tortuosity is given by:

$$q^2 = \left(\frac{c_m}{c_o} \right)^2 \quad (2.10)$$

Where c_m is velocity of sound in porous media and c_o is velocity of sound in air. An experiment performed on mild density fiberboard with consideration of different thickness of wood fibers, sound absorption coefficient are calculated. Figure 7 shows the result of normal incidence sound absorption with 50 mm and 70 mm thick wood fibers:

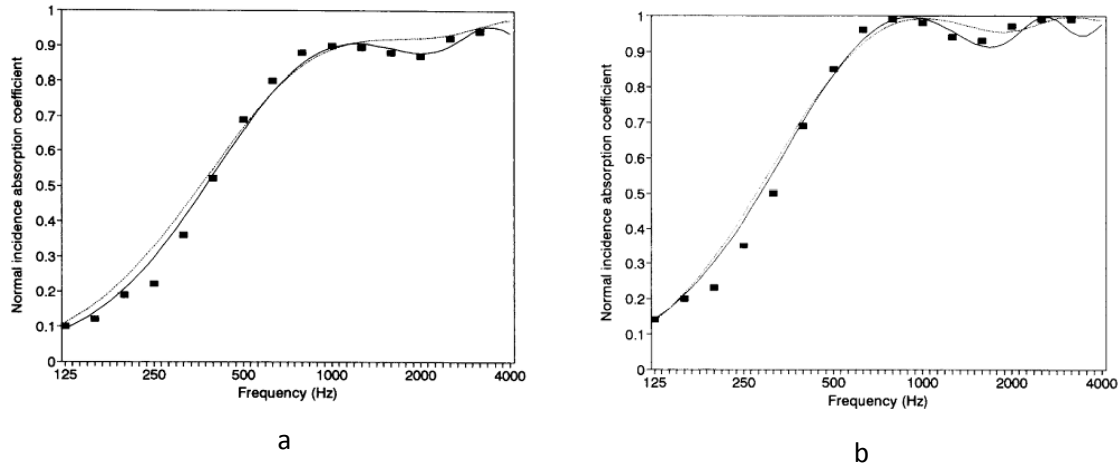


Figure 7. Normal incidence sound absorption of a) 50 mm and b) 70 mm thick wood fibers, with $\sigma= 31000$, $q^2=1.7$ and $\theta=0.85$ [15]

Sound sources are generally differentiate into three types of source Monopole, Dipole and quadrupole [16]:

a) Monopole: It is a point source which radiates sound spherically in all direction. Spherical wave front is generated around the source. It is extremely small in size (or source region is very small compared to its wavelength) and radiates equal sound in all direction. Variation of sound in

azimuthal and zenith direction is zero therefore, sound only vary in radial direction. Wave equation for 1D monopole radiates sound energy is given as:

$$\frac{\partial^2(pr)}{\partial r^2} = \frac{1}{c^2} \frac{\partial^2(pr)}{\partial t^2} \quad (2.11)$$

Here p is the pressure and r is the radius (distance from source). Solution to this partial double derivative equation is given as:

$$p(r,t) = \frac{f(t - \frac{r}{c})}{r} \quad (2.12)$$

As there is no reflection in free field only forward travelling component of wave is consider in the solution. A spherical radiation is inversely proportional distance from the source. Baffled loud speaker, vibrating panel, pulsating balloon are some examples of the monopole. It radiates in all direction hence human receive sound at every point.

b) Dipole: A dipole consist of two monopole with phase difference of 180 degree i.e.; if one monopole contracts other monopole expands resulting no net mass flow through sphere around source. It is used to represent the force applied to fluid. Dipole is not good sources of sound as human finds no sound in space where sound interferes (have opposite phase) from two sources. Un-baffled loudspeaker (open loudspeaker), tuning fork and fan blade are some common example of dipole. Figure 8 shows the common dipole:

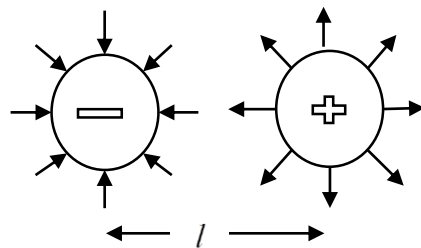


Figure 8. Dipole source

c) Quadrupole: It is combination of four monopoles as shown in Figure 9. Contraction and expansion of source happens simultaneously.

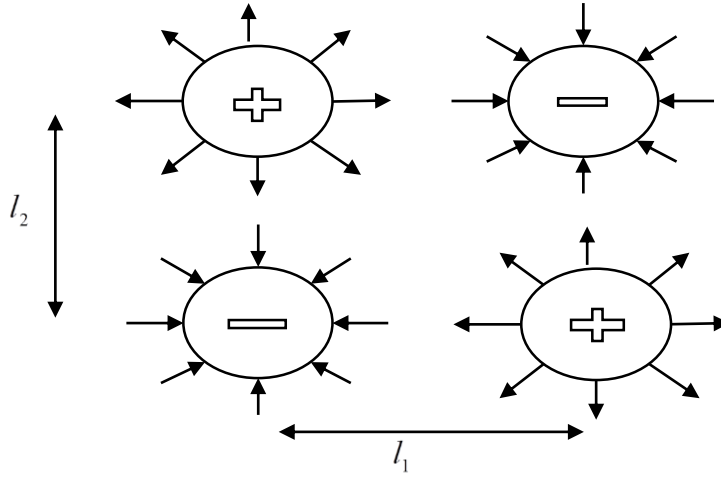


Figure 9. Lateral quadrupole

In this type of source, no net mass flow and no net force is around source but there is couple acting around the source. It represents any kind of source mechanism which results from rotational event. Hence this type of source is used where sound generation is from turbulence vortices in fluid. Jet mixing and turbulence in fluid are some common example of quadrupole.

Daniel A. Russel *et al.* [4] demonstrates the directivity patterns of simple monopole, dipole, longitudinal quadrupole and lateral quadrupole. Mayer and Neumann [17] studied that baffled loudspeaker acts as monopole and un-baffled loudspeakers acts as dipole. Authors revised this study with an experiment using four 4 inch baffled loudspeakers and switched box. Switch box helps to driven any combination of phase which give directivity patterns. For monopole sound pressure describe as:

$$|p(r, \theta, t)| = \frac{Q\rho ck}{4\pi r} \quad (2.13)$$

where, ρ fluid density, c is speed of sound k is wave number and r is the distance from source to fixed point Q is source strength. For a dipole pressure amplitude at a fixed point given as:

$$|p(r, \theta, t)| = \frac{Q\rho ck}{4\pi r} kd \cos\theta \quad (2.14)$$

Here d is the distance between two opposite phase source and θ defines the directivity of sound power as dipole not radiates equal sound in all direction. In a similar way quadrupole has pressure amplitude which describe as:

$$|p(r, \theta, t)| = \frac{Q\rho ck}{4\pi r} k^2 dD \cos\theta \sin\theta \quad (2.15)$$

Authors performed a simple experiment in which four loudspeaker placed at rotating stool in an enclosed space and obtained data through sound level meter and with the help of oscilloscope sound. Four cases is studied in which loudspeaker have frequency of 250 Hz sound level meter is placed at a distance of 2m:

- a) Loud speaker were driven with same polarity they acted in phase like a monopole source
- b) Loudspeakers driven in such way first two have same polarity and other two have same polarity they acted as dipole source
- c) Loudspeakers driven in such way that adjacent speakers has different polarity they acted as lateral quadrupole.
- d) Loud speakers is equally spaced on table they acted as longitudinal quadrupole.

Due to different combination of phases four different directivity patterns are obtained, shown in figure 10

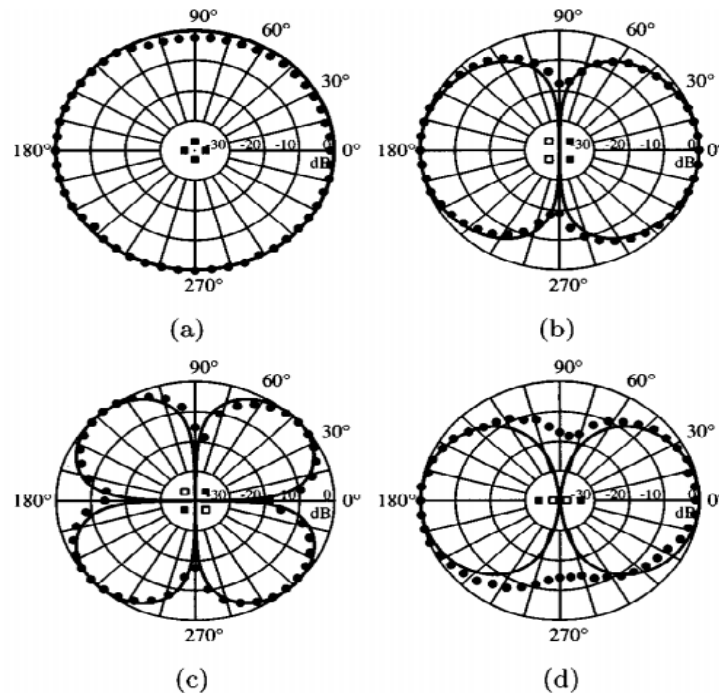


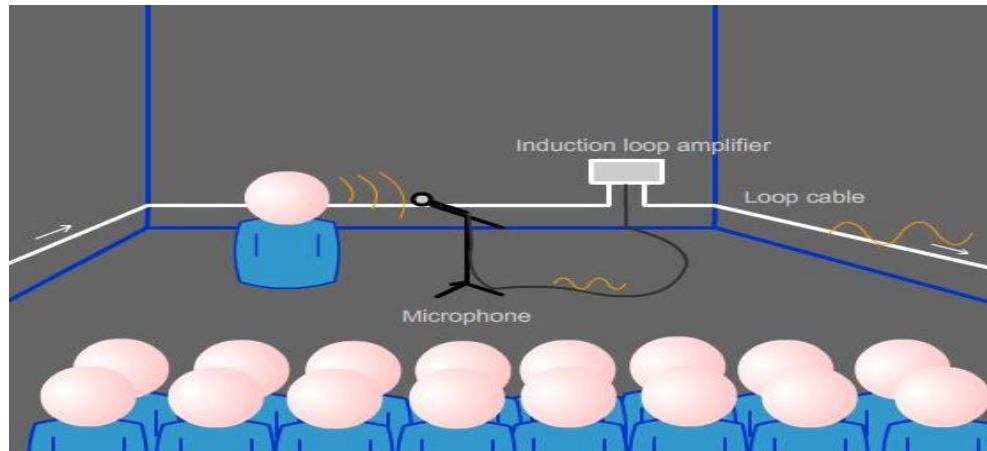
Figure 10. Directivity patterns for sound radiation from a) monopole, b) dipole, c) lateral quadrupole & d) longitudinal quadrupole [4]

Directivity pattern for longitudinal quadrupole differs from the theoretical one which is due to enclosed space. Reflection/absorption take place and more important sound meter is placed in the near field.

Kurt Eggenschwiler [6] presented the experience with lecture halls with regard to the room acoustics, sound reinforcement and audio frequency induction loops for persons with hearing impairment. As we know the value of optimum reverberation time for lecture halls, but we need the shorter reverberation time in sometime in lecture halls. All lecture halls can't provide the shorter reverberation time. Hence reinforcement of sound is necessary. If these reinforcement is fulfilled then desired reverberation time and speech intelligibility index can easily be obtained. Human impairment for hearing also taken into account when there is talk about intelligibility in the room. Good acoustics are also important in lecture halls. The background noise must be minimized and the room form and materials must be designed so as to support the acoustics in order to provide high speech intelligibility. Since the number of seats in a lecture hall is generally higher than in a small classroom because the achievement of good speech acoustics is more complex. A sound reinforcement fulfilling this criteria is usually required. Reinforcement of sound can be done in following methods:

Localization and Tone Quality: The correct localization of the sound source and a natural tone quality contribute to the speech intelligibility. To produce the correct localization the loudspeakers have to be provided with a suitable time delay. For the transmission of speech it is desirable to attenuate the low frequencies. This is especially important if pressure gradient microphones (for example, cardioid microphones) are employed since these accentuate the low frequencies when the microphone is placed close to the talker's lips.

Audio frequency induction loop systems (AFILS): The inductive loop transmission is usually preferred as the hearing impaired persons bring the receiver themselves. Most hearing aids are provided with a switch (T). Other ways are by frequency modulation and infrared sound which enhance the sound power and clarity and make it more intelligible. Figure 11 shows the reinforcement system with loudspeaker. Author experimentally found the Sound Transmission Index (STI) of 12 different lecture hall. Some room which have lower reverb time has better STI and other room which have higher value of reverb time have low STI value



*Figure 11.*Reinforcement system [6]

Acoustical rating of materials is standardized through different coefficients which give the single number that defines the amount of absorption by the material. Generally, absorbent material used in office, house, classroom to attenuate sound uses the rating system of NRC, SAA and STC [18]

a) Noise reduction coefficient (NRC): NRC is most commonly used to rate general acoustical properties of acoustic ceiling tiles, baffles, banners and acoustic wall panels. It is a scalar number which represents the amount of sound energy absorbed when striking the boundary or wall. It ranges between 0 and 1. For a perfect reflector NRC is 0 and for a perfect absorber NRC is 1. NRC is the average value of sound absorption coefficient of four frequencies: 250 Hz, 500 Hz, 1000 Hz and 2000 Hz and rounded off to the nearest multiple of 0.05. These frequencies encircle the fundamental frequencies, therefore NRC provides the notion of how good the surface absorbs human voices.

b) Sound Absorption Average (SAA): It is also a single number value which determines the sound absorption at a particular surface. It is the average of absorption coefficients of twelve 1/3 octave bands from 200 Hz to 2000 Hz and rounded off to the nearest multiple of 0.01.

c) Sound Transmission Class (STC): STC is the integral value that is used to rate the interior partition of an enclosed space and ceiling tiles, floor, etc. Generally, it tells how well the room partitions attenuate airborne sound. STC number is depicted from the sound attenuation value at 16 different frequencies (harmonics) between 125 Hz and 4000 Hz. These transmission loss values are plotted on an SPL graph and then compared to the standard contour. An acoustical engineer fits these curves to transmission loss curves to give an STC rating. In the USA, STC is widely used to rate acoustical bodies and outside the USA, Sound Reduction Index (SRI) is used.

Tohyama & Suzuki [19] derived the formula for reverberation time (RT_{60}) based on two dimensional diffuse field. Reverberation time formula uses the wave theory and is applicable in the region where two dimensional waves are dominating. Three cases are studied for the reverberation time, a) when, walls, floor (plastic tiles), and ceiling are reflected , b) when walls, floors and ceiling all are absorptive using mats and carpets and c) when walls are reflected but floor and ceilings is absorbing sound energy using mats and carpets. Reverberation time using condition (a) at 500 Hz frequency is 0.65 sec and when using condition (b) when all boundaries are absorptive then reverberation time decreases to 0.007 sec. When using condition (c) which is floor and ceiling absorptive the reverberation time 0.2 at 125 Hz and 0.4 at 500 Hz and increases with increase in frequency. Variation of RT_{60} with frequency is illustrated in Figure 12:

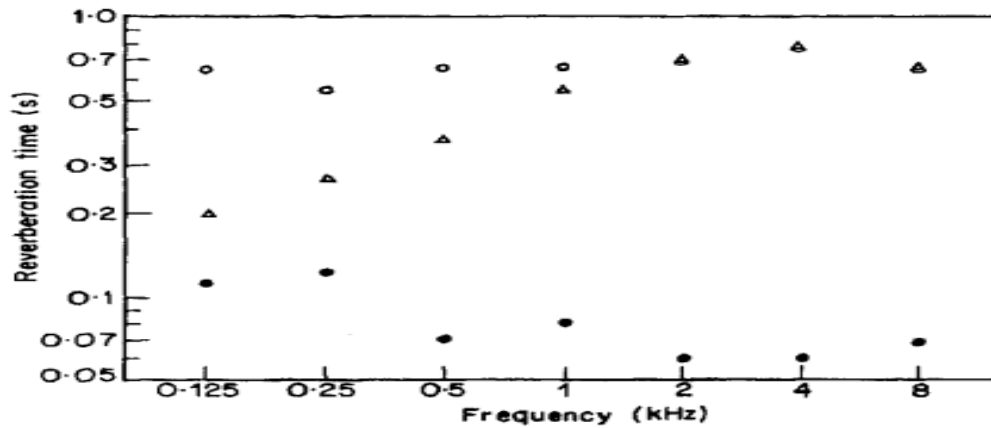


Figure 12. Reverberation Time under three condition ○) all sides reflective, ●) all sides Absorptive and Δ) walls reflective but ceiling and floor are absorptive [19]

Reverberation time of almost 2D diffuse field is derived as:

$$RT_{60} = \frac{0.128S}{\alpha L} \quad (2.16)$$

S , is the surface area occupied by the diffused field L is the perimeter of 2D diffuse field and $\alpha = -\ln(1 - \bar{\alpha})$ where, $\bar{\alpha}$ is the sound average absorption coefficient of walls in two dimensional diffusive field

Botteldooren [20] discussed the use of numerical simulation based on finite difference time domain (FDTD) method to study the low and middle frequency of sound propagation in room or

auditorium. This technique (FDTD) has an advantage over other technique like ray tracing, finite volume technique when simulation of large spaces is required and it is easy to use and implement. Author also illustrates the frequency dependent boundary condition and use of FDTD formulation to study the combined effect of staggered and non-staggered grids in sound propagation. All calculation directly done in time domain but there is some problem when it comes to frequency dependent boundary condition. It is found by the calculation that FDTD technique has no effect on amplitude of wave but there is phase change due to discretization. Low frequency boundary generally are of two type i.e., a) thin absorbing layer on hard or rigid surface for example walls and b) non-stiff walls or soft walls. For first material boundary condition complex impedance (Z) is defined as:

$$Z = Z_0 + Z_{-1}i\omega \quad (2.17)$$

And for second material complex impedance is:

$$Z = Z_0 + i\omega M \quad (2.18)$$

Where M is mass per unit area (inertial component). General frequency ω dependent complex impedance is defined by:

$$Z(\omega) = Z_{-1}i\omega + Z_0 + i\omega Z_1 \quad (2.19)$$

Here Z_{-1} be the integral coefficient of compliance component of boundary, Z_0 is integral coefficient of acoustic resistance and Z_1 is the integral coefficient of inertial element of boundary surface. Low frequency and high frequency wave simulation by FDTD is combined or compared by experiment. Predefined absorption coefficient at boundaries are taken as calculated by Sabine [3] and values of impedances (Z_{-1}, Z_0 & Z_1) are found from the above equation.

Table 3.a) Sabine absorption coefficient of material, b) Impedance approximation of material [20]

	Parquet	Persons	Curtain	Plaster	Carpet
Z_1	6.0	0.0	5.0	6.0	0.0
$Z_0 (\times 10^3)$	20.0	4.3	15.5	15.0	40.6
$Z_{-1} (\times 10^6)$	8.0	0.8	0.0	16.0	0.0

a

f (Hz)	Parquet	Persons	Curtain	Plaster	Carpet
63	0.03	0.16	0.05	0.02	0.02
125	0.04	0.17	0.05	0.03	0.02
250	0.04	0.24	0.03	0.05	0.06
500	0.05	0.56	0.035	0.04	0.13

b

The simulation region is discretized by the rough (non-staggered) and fine (staggered) grids. The rough grid have 27750 cells with average diameter of 50 cm and fine grid have 232560 grids with average diameter of 25 cm. The time step ∂t is taken 1/10 of the grid size. For rough grid ∂t is 0.5 ms and ∂t for fine grid is 0.25 ms. Sound pressure level (SPL) or reverberation response to the impulse function is shown in graph in comparison between low and high frequency wave.

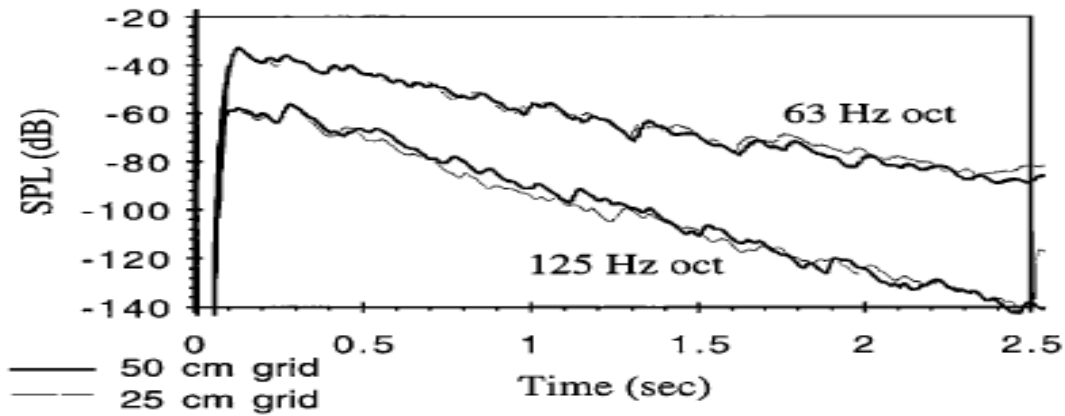


Figure 13. SPL in the center of room calculated in auditorium with grid steps 50 (thick line) and 25 cm (thin line) [20]

Results shows that at low frequency both grid sizes has almost same behavior but at high frequencies it is clear to find out the error between staggered and non-staggered grids. This discrepancy is due to the spatial discretization. Due to frequency wavelength changes and rough grids may not able to fulfill the 1/10 of relevant wavelength. Hence the evaluation of acoustic quality at very low frequencies is quiet difficult task.

Kowalczyk and Walstijn [21] had illustrated the new technique of numerical methods that can be applicable in the use of finite difference time domain (FDTD) simulation for analyzing the locally reacting surface. In this technique full analysis of corner edges and boundary edges are discussed. The boundary formulation is designed according to the leap frog scheme based on staggered and rectilinear grid. Numerical boundary analysis and experiments are analyzed in time and frequency domain in terms of pressure wave reflectance for different angle of incidences. Furthermore frequency dependent boundary analysis is done considering the complex boundaries with inertial, restoring and resistive parameters as discussed by the Botteldooren [20]. It is also the improved work of Botteldooren because he not introduced the condition of wave stability. Here, author

described the stability of wave in 2D and 3D simulation as per CFL theory [22]. For 2D wave courant number is $1/\sqrt{2}$ and for 3D wave it's value $1/\sqrt{3}$. Here author shows that reflectance magnitude of 1D wave boundary model is more than that of theoretical reflectance for the high angle of incidence these inaccuracies are greater when there is 2D or 3D simulation of wave. In addition the 2D and 3D boundary formulation are very useful for the formulation of corner points and all point on the boundary edges. Formulation use by the author is quiet simple as discretization of 1D, 2D and 3D wave equation by finite difference method by using the leapfrog scheme (LFS). Figure 14 shows the schematic of boundary mesh structure with leap frog scheme.

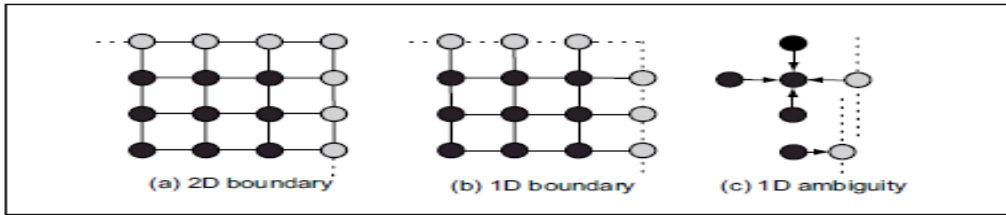


Figure 14. 2D and 1D termination boundary of mesh structure [21]

The grey colored circles are the boundary nodes or corner nodes. They are specifically referred as ghost points. Their formulation is described by the author by using (LFS) for example consider 1D wave equation it is discretized by including the right boundary impedance condition for the elimination ghost point as:

$$\frac{\partial p}{\partial t} = -c\zeta_w \frac{\partial p}{\partial x} \quad (2.20)$$

Since then the value of ghost point is given by:

$$p_{x+1}^{ts} = p_{x-1}^{ts} + \frac{1}{\lambda\zeta_w} (p_x^{ts-1} - p_x^{ts+1}) \quad (2.21)$$

This formulation is also done in the formulation of corner points (taking both side impedances at the corner ghost points). To verify the properties of numerical boundaries formulation, a 2D simulation procedure is designed and executed with the grid consistence of 1800×1400 nodes. It is found that at very low value of reflectance coefficient occur at high angle of incidence. Results shown in figure 15 for the incidence angles $15^\circ, 45^\circ, 75^\circ$ are as follow:

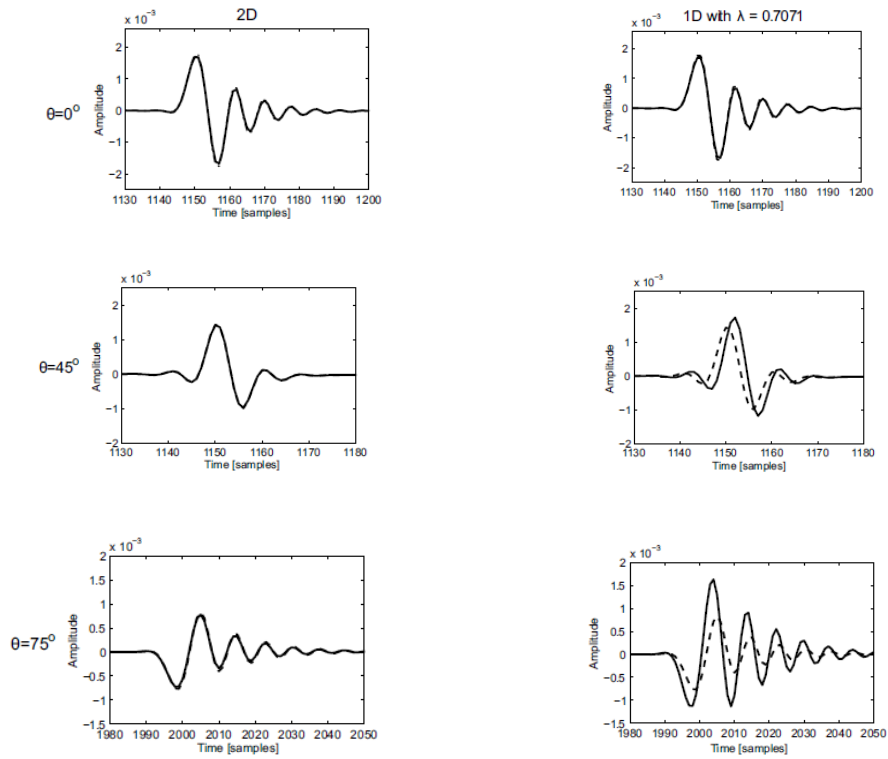


Figure 15. Reflected signal (solid lines) and theoretical signal (dashed lines) with normal incidence of $0^\circ, 45^\circ, 75^\circ$ and $\zeta_w = 9$ [21]

At 0° & 45° there 1D and 2D boundary yields the same phase but at 75° the phase with the theoretical reference is quite different. Similar 3D simulation is done with lesser nodal point because more nodal point (more space) causes more dispersion error.

In most of the work, the focus of the study is either on the development of an advanced numerical method [23-25] or only experimental investigations on the reverberation time have been performed.

Chapter 3. Theory & Methodology

3.1 Theory:

Wave propagation is governed by wave equation derived from the hydrodynamics and adiabatic relationship between pressure and density. One dimension model is equivalent to the very long duct where y & z direction are not consider for sound wave propagation. General free field propagation of sound wave in one dimension is given as:

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2} + F \quad (3.1)$$

This equation signifies how curvature of wave-front changes with the time derivative of pressure, here p is pressure over ambient pressure, x is space, t is time, F is the sound source & c is the speed of sound. Figure 16 shows how wave intensity decreases as distance increases from the source.

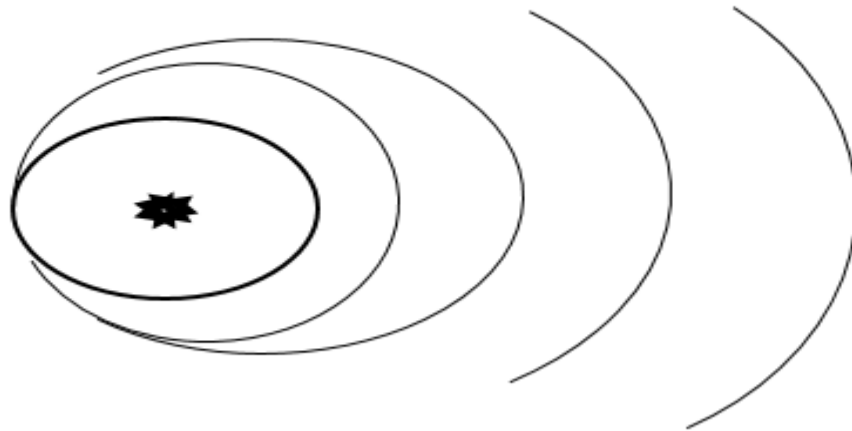


Figure 16. Propagation of spherical wave front

Mass and Momentum conservation are assumed for this model. The 1D wave equation is the combination of forward travelling and backward travelling waves. General solution of the wave equation is given as:

$$p(x,t) = f\left(c - \frac{x}{t}\right) + g\left(c + \frac{x}{t}\right) \quad (3.2)$$

$f\left(c - \frac{x}{t}\right)$, explains the forward motion of wave in positive x direction and $g\left(c + \frac{x}{t}\right)$ explains the motion in negative x direction. Euler defined that any harmonic oscillatory function represent as complex entity such that real part containing cosine term and imaginary part containing sine term. Hence in complex form the solution of wave equation becomes:

$$p(x,t) = p(e^{i(\omega t - kx)}) \quad (3.3)$$

Here $\omega = \frac{2\pi}{\lambda} = \frac{\omega}{c}$ signifies how phase changes with the wavelength (λ) and when the pattern of full wave repeats. Any wave motion requires two information to completely know the physical characteristics. First one is wave number and second is frequency of wave. It is assumed that pressure amplitude remain constant in wave medium but in reality this is not the case pressure amplitude decreases exponentially as there is damping in the medium. Hence Eq. (3.3) is modified as:

$$p(x,t) = p(e^{-\phi x/2(\omega t - kx)}) \quad (3.4)$$

ϕ , is the attenuation constant and if we described wave number in complex form and attenuation constant is in imaginary part as:

$$k = \frac{\omega}{c} + i \frac{\phi}{2} \quad (3.5)$$

The wave equation is a linear acoustic system however non-linearity induces when dealing with high amplitude. Finite difference time domain (FDTD) is one of the few techniques that strongly used for the acoustic system. Unlike Finite Element Methods (FEM), FDTD is straight forward approach and use for uniform grids and more suitable for virtual spaces with moving sound source and receiver. Within the finite difference techniques, Leapfrog scheme is incorporated. Leapfrog scheme is one of the most suited scheme among Lax-Wanderoff, Lax-Fredreich, and Forward Time Center Space (FTCS) schemes. The main advantage of this scheme is negligible diffusion [24]. FDTD with leapfrog scheme for linear acoustic system have negligible numerical error on wave amplitude. Undisturbed amplitude is appropriate for studying the long transient in sound pressure level. Figure 17 shows the leap frog scheme for $t+1$ time step:

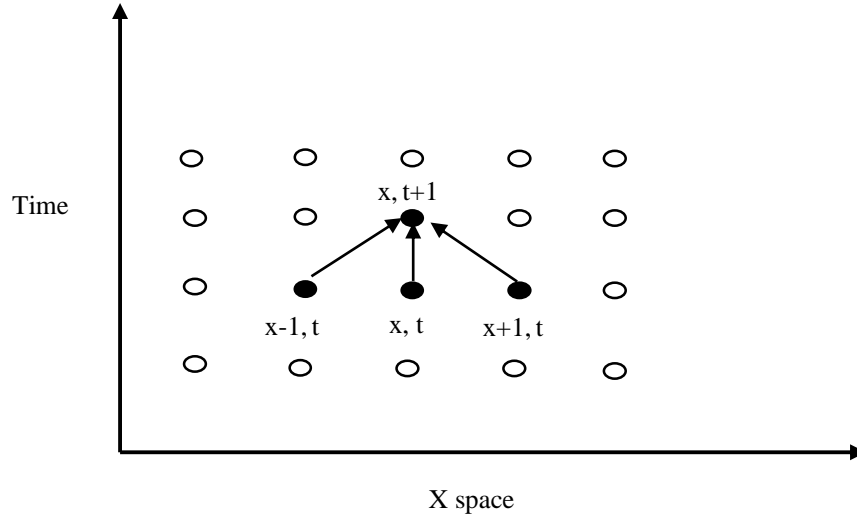


Figure 17. Leapfrog scheme

3.2 Methodology

3.2.1 Basic FDTD one dimensional formulation

Finite difference method (FDM) transforms the partial derivatives in the algebraic form. This technique involves forward, backward and central difference approaches. For an acoustic wave equation, a grid structure is formed and pressure/velocity distribution is calculated at adjacent grid point after applying initial and boundary conditions. A 1D plane wave equation is a second order partial differential hyperbolic equation have tendency to move in both direction and can be discretized by second order forward difference method. General one dimensional linear wave propagation (without source condition) equation given as:

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2} \quad (3.6)$$

Spatial & temporal discretization of this partial derivative equation with second order FDTD technique is given as:

$$\frac{p_x^{ts+1} - 2p_x^{ts} + p_x^{ts-1}}{\Delta t^2} = c^2 \frac{p_{x+1}^{ts} - 2p_x^{ts} + p_{x-1}^{ts}}{\Delta x^2} \quad (3.7)$$

Explicit value of pressure obtained at time step ts+1 from Eq. (3.7) such as:

$$p_x^{ts+1} = \lambda^2(p_{x+1}^{ts} + p_{x-1}^{ts}) + 2p_x^{ts}(1 - \lambda^2) - p_x^{ts-1} \quad (3.8)$$

where, Δx and Δt are the step size in space and time respectively & $\lambda = c \frac{\Delta t}{\Delta x}$ is the constant term

called Courant number, which decides the wave stability. For 1D wave propagation $\lambda \leq 1$ [22]

3.2.2 Acoustic Boundary Condition

Free field propagation have no reflection/absorption phenomenon, therefore Eq. (3.1) is suffice to understand the behavior of sound propagation. The notion of wave propagation become difficult when wave propagates in closed spaces and sound energy gets absorb/reflect simultaneously and field no longer remain free whereas, it become reverberant. Amount of absorption/reflection is determined by the impedance (resistance) of boundaries. Hence, in case of enclosed space, interaction of sound with the boundary must be defined by incorporating proper boundary condition. It may be noted that, to solve the wave equation at the boundary, pressure for the points lying beyond the boundary is not available, e.g. in Eq. (3.8), value at p_{x+1}^{ts} and p_{x-1}^{ts} will not be available for solving right and left boundaries respectively. These points are called ghost points. One of the ways to solve wave equation at the boundary (at the ghost point) is through the advection equation. A propagating wave can be presented by the combination of a set of advection equations, indicating propagation of a perturbation in forward and backward direction. Boundary condition is depicted by the linear advection equation as:

$$\frac{\partial p}{\partial t} \pm c \frac{\partial p}{\partial x} = 0 \quad (3.9)$$

Eq. (3.9) discretize with finite central difference technique for positive x direction as follow:

$$\frac{p_x^{ts+1} - p_x^{ts-1}}{2\Delta t} + c \frac{p_{x+1}^{ts} - p_{x-1}^{ts}}{2\Delta x} = 0 \quad (3.10)$$

Generally, there is material at the boundary which partially absorb, transmit and reflect the sound energy which is describe by the impedance condition. Proper model of wave propagation in 1D includes the impedance boundary condition. In case of 1D forward traveling wave, which is interacting with right boundary, the advection term can be expressed

$$\frac{\partial p}{\partial t} = -\zeta c \frac{\partial p}{\partial x} \quad (3.11)$$

$$\frac{p_x^{ts+1} - p_x^{ts-1}}{2\Delta t} + c\zeta \frac{p_{x+1}^{ts} - p_{x-1}^{ts}}{2\Delta x} = 0 \quad (3.12)$$

Eq. (3.12) shows the FDTD of Eq. (3.11), where ζ is specific impedance. The specific impedance is the ratio of wall impedance ($Z = \frac{P}{u_x}$) and characteristic impedance (ρc), ρ is density of the medium and c is speed of sound. Reflection/absorption depends upon specific impedance as: $\zeta = \frac{R+1}{R-1}$ where, R is reflection coefficient that determine by the material, it can be evaluated with standard short tube impedance method [5] and standing wave ratio (SWR). Through this SWR absorption of material is determine which is used in the industry acoustical rated as NRC, STC, etc. [18]

The ghost point is first calculated for modelling of correct boundary and then incorporating in the wave equation for appropriate model of wave propagation. Ghost point p_{x+1}^{ts} is calculated using central difference approach given as:

$$\frac{\partial p}{\partial x} = \frac{p_{x+1}^{ts} - p_{x-1}^{ts}}{2\Delta x} \quad (3.13)$$

Using equation (3.12) and (3.13), incorporating the value of p_{x+1}^{ts} in equation (3.8) and pressure at $ts+1$ time for the right boundary can be obtained by:

$$p_x^{ts+1} = \frac{2\lambda^2 p_{x-1}^{ts} + 2p_x^{ts}(1-\lambda^2) + p_x^{ts-1}(\lambda/\zeta - 1)}{\lambda/\zeta + 1} \quad (3.14)$$

Similarly we get the left boundary system equation by using left moving advection equation as:

$$p_x^{ts+1} = \frac{2\lambda^2 p_{x+1}^{ts} + 2p_x^{ts}(1-\lambda^2) + p_x^{ts-1}(\lambda/\zeta - 1)}{\lambda/\zeta + 1} \quad (3.15)$$

3.2.3 Two dimensional formulation with boundary condition

In two dimensional model, propagation in z direction is arrested and propagation in only x & y direction is allowed. A free field wave equation propagation in 2D is described as:

$$\frac{\partial^2 p}{\partial t^2} = c^2 \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) \quad (3.16)$$

Discretization of above equation with forward difference finite technique is described as:

$$p_{x,y}^{ts+1} = \lambda^2 (p_{x+1,y}^{ts} + p_{x-1,y}^{ts} + p_{x,y+1}^{ts} + p_{x,y-1}^{ts}) + 2p_{x,y}^{ts} (1 - 2\lambda^2) - p_{x,y}^{ts-1} \quad (3.17)$$

Here $\lambda=0.707$ [22] and again for the boundary we need to discretize the right/left moving wave with boundary impedance condition and determine the ghost point as done earlier then incorporate with the equation (3.17). Here two impedance condition for x & y direction is required for modelling the boundary of a plane which is describe as:

$$\frac{\partial p}{\partial t} = -\zeta_c \frac{\partial p}{\partial x} \quad , \quad \frac{\partial p}{\partial t} = -\zeta_c \frac{\partial p}{\partial y} \quad (3.18)$$

These conditions are applicable only for positive x and positive y direction. Explicit value of pressure at time step $ts+1$ for ghost point in positive x direction is given as:

$$p_{x,y}^{ts+1} = \frac{\lambda^2 (2p_{x-1,y}^{ts} + p_{x,y+1}^{ts} + p_{x,y-1}^{ts}) + 2p_{x,y}^{ts} (1 - 2\lambda^2) - p_{x,y}^{ts-1} \left(\frac{\lambda}{\zeta_x} - 1 \right)}{\left(\frac{\lambda}{\zeta_x} + 1 \right)} \quad (3.19)$$

Similarly for positive y direction the ghost point equation becomes:

$$p_{x,y}^{ts+1} = \frac{\lambda^2 (p_{x+1,y}^{ts} + p_{x-1,y}^{ts} + 2p_{x,y-1}^{ts}) + 2p_{x,y}^{ts} (1 - 2\lambda^2) - p_{x,y}^{ts-1} \left(\frac{\lambda}{\zeta_y} - 1 \right)}{\left(\frac{\lambda}{\zeta_y} + 1 \right)} \quad (3.20)$$

Equations for the other boundaries can be describe in a similar way. Here ζ_x & ζ_y are specific impedance of right boundary in positive x & y direction respectively. In this case, specific impedance also depend on the sound incidence angle, this is described by the equation as:

$$\zeta = \frac{R+1}{R \cos(\theta) - 1} \quad (3.21)$$

θ , indicates the incident angle in Cartesian system however, in this work effect of incident angle is neglected. The critical aspect of discretization of boundaries is discretization of corners. Corner points shares the impedances from both dimensions.

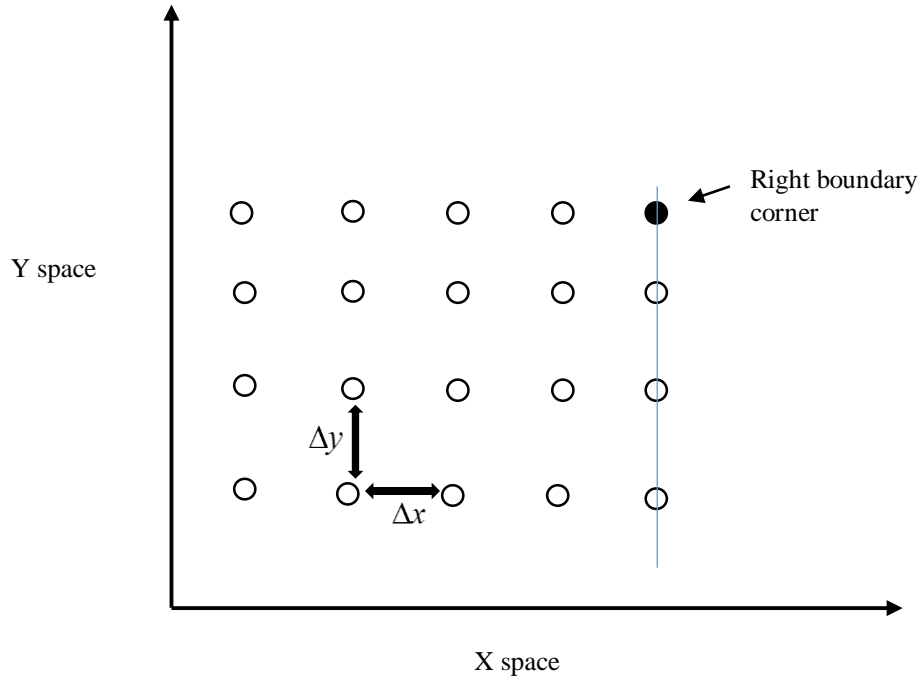


Figure 18. Corner at right boundary

The explicit value of pressure for ghost point (corner point) shown in Figure 18 is given by:

$$p_{x,y}^{ts+1} = \frac{\lambda^2(2p_{x-1,y}^{ts} + 2p_{x,y-1}^{ts}) + 2p_{x,y}^{ts}(1 - 2\lambda^2) - p_{x,y}^{ts-1}\left(\frac{\lambda}{\zeta_x} + \frac{\lambda}{\zeta_y} - 1\right)}{\left(\frac{\lambda}{\zeta_x} + \frac{\lambda}{\zeta_y} + 1\right)} \quad (3.22)$$

In similar way the remaining three corners are modelled by using advection equation and finite central difference technique as:

When $x = 0$ & $y = n$

$$p_{x,y}^{ts+1} = \frac{\lambda^2(2p_{x+1,y}^{ts} + 2p_{x,y-1}^{ts}) + 2p_{x,y}^{ts}(1 - 2\lambda^2) - p_{x,y}^{ts-1}\left(\frac{\lambda}{\zeta_x} + \frac{\lambda}{\zeta_y} - 1\right)}{\left(\frac{\lambda}{\zeta_x} + \frac{\lambda}{\zeta_y} + 1\right)} \quad (3.23)$$

When $x = 0$ & $y = 0$

$$p_{x,y}^{ts+1} = \frac{\lambda^2(2p_{x+1,y}^{ts} + 2p_{x,y+1}^{ts}) + 2p_{x,y}^{ts}(1 - 2\lambda^2) - p_{x,y}^{ts-1}\left(\frac{\lambda}{\zeta_x} + \frac{\lambda}{\zeta_y} - 1\right)}{\left(\frac{\lambda}{\zeta_x} + \frac{\lambda}{\zeta_y} + 1\right)} \quad (3.24)$$

When $x = n$ & $y = 0$

$$p_{x,y}^{ts+1} = \frac{\lambda^2(2p_{x-1,y}^{ts} + 2p_{x,y+1}^{ts}) + 2p_{x,y}^{ts}(1 - 2\lambda^2) - p_{x,y}^{ts-1}\left(\frac{\lambda}{\zeta_x} + \frac{\lambda}{\zeta_y} - 1\right)}{\left(\frac{\lambda}{\zeta_x} + \frac{\lambda}{\zeta_y} + 1\right)} \quad (3.25)$$

Here $\lambda = \frac{1}{\sqrt{2}}$ defines the wave stability as per CFL condition [22]. After discretization for 1D and 2D wave, proposed methodology is coded in house using MATLAB and validated for various benchmark problems.

Chapter 4. Validation study of numerical solver

4.1 One-dimensional propagation of sound wave

In this chapter, a case study of one dimensional plane sound wave propagation and its reflection/absorption from the boundary is discussed. For this purpose, a point source with frequency 100 Hz and amplitude of 5 Pa is considered. The computational space is discretized with a spatial step size, $\Delta x = 0.2$ m. Time step is calculate by using CFL criterion ($\Delta t = \lambda \frac{\Delta x}{c}$), which yield $\Delta t = 0.00041$ seconds for considered special step and velocity of sound 340 m/s. Four different reflection coefficient, viz. 0.3, 0.5, 0.7 and 1 are considered. The pressure is measured at a fixed distance from the boundary at point P and RMS value of pressure is compared for both incident and reflected waves. Schematic of 1D wave propagation and its reflection from wall is shown in Figure 19. It may be noted that the reflected wave is superimposed on the incident wave and the according to wave theory [26], the resultant wave has increased amplitude. Figure 20 shows the time history of the pressure for various reflections coefficient measured at a point P near to the right boundary.

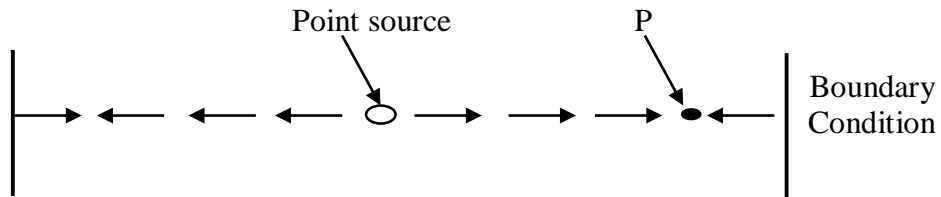


Figure 19. Instantaneous point P in 1D tube where pressure amplitude is measured

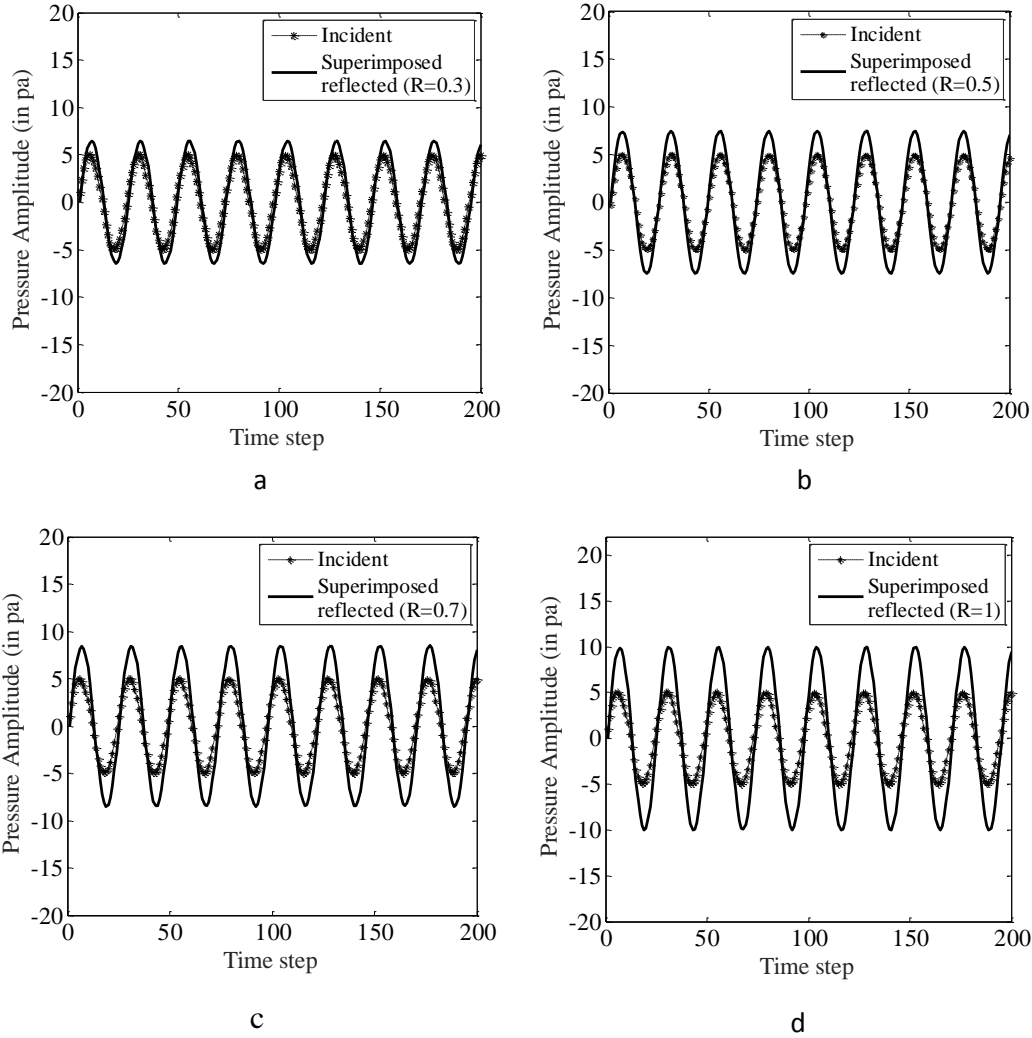


Figure 20. Comparison of incident and superimposed reflected wave using reflection coefficient of a) 0.3, b) 0.5, c) 0.7 & d) 1

In Figure 20, the pressure of incident wave is shown by dotted line and reflected wave (superimposed wave) is presented by solid line. Table 4 shows the comparison of RMS value of incident wave, superimposed wave, reflected wave (calculated by subtracting column 4 and 3). It is observed that the amount of reflection is in accordance to the amount of reflection provided to the wall, e.g. for a reflection coefficient 0.3, the amplitude of the reflected wave is close to 30 percent of the incident wave. It is noted that the reflected wave has a minor phase shift, which may be due to the discretization error in the FDTD scheme. However, the RMS value is independent of the phase shift, therefore, it is suffice to compare RMS values.

Table 4. Comparison of RMS values corresponds to reflection coefficient

Sr. No	Reflection Coefficient (R)	RMS Value of incident wave	RMS Value of superimposed-reflected wave	RMS value of reflected wave	Percentage change in RMS value
1	0.3	3.5276	4.5674	1.0392	29.48
2	0.5	3.5276	5.2743	1.7467	49.52
3	0.7	3.5276	5.9825	2.4549	69.59
5	1	3.5276	7.0473	3.5197	99.78

4.2 Two-dimensional sound radiation from pulsating cylinder

For validation of 2D wave propagation, a cylinder of very long length with radius, $a = 1$ unit is considered. Available theoretical formulation of cylindrical waves is used for the comparison with the numerical results. The pressure of pulsating cylinder, surface velocity $U_o = \sin(\omega t)$, at distance r from the center of the cylinder (as shown in Figure 21) can be deduce by [27]:

$$p(r,t) = i\rho c U_o \frac{H_o^{(2)}(kr)}{H_1^{(2)}(ka)} e^{i\omega t} \quad (4.1)$$

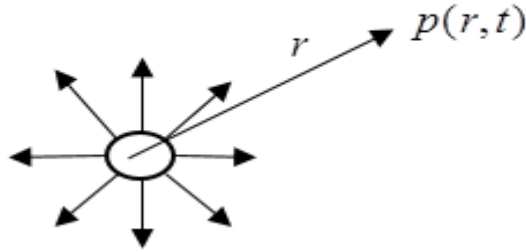


Figure 21. Pressure at distance r from the source

In Eq. (4.1), ρ is the density of medium (generally air), c is the speed of sound, U_o is the amplitude of surface velocity of the cylinder. $H_o^{(2)}$ & $H_1^{(2)}$, represent the *Hankel* function of second kind of order zero and first respectively.

To validate the acoustic radiation from the numerical solver, first the analytical pressure distribution is computed using Eq. (4.1). The time history of the resultant pressure is, then, recorded at radius, $r = 20$ units and used as input in the numerical simulation as source, shown in figure 22. For the numerical simulation, a pulsating cylinder is kept in the middle of square domain of 200×200 grid points with uniform grid step size of 0.2 in both x and y direction. The results obtained from analytical and numerical approach are compared for two points at a radial distance $r=40$ and $r=60$ (point P1 and P2 in Figure 22)

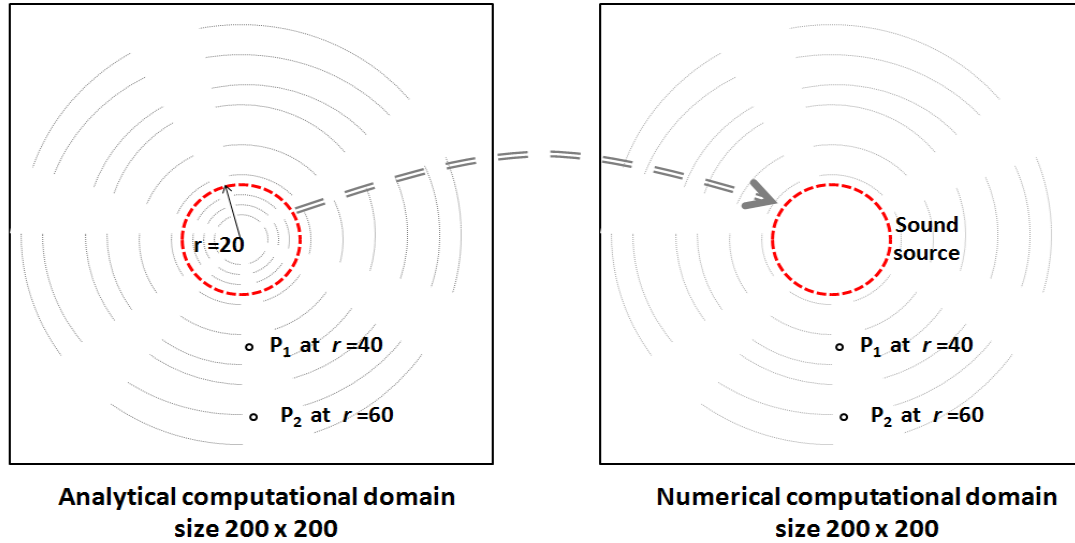


Figure 22. Schematic of consideration of source from analytical solution into numerical domain (figure not to scale)

Figure 23 shows the spatial distribution of pressure contours calculated by numerical solver. Figure 24 (a, b) shows time history of the measured sound pressure at the point P1 and P2 respectively. In Figure 24, the dotted line is corresponding to analytical pressure and solid line represents numerical pressure. The figure indicates that numerical results are in good agreement with analytical results at $r = 40$ & $r = 60$.

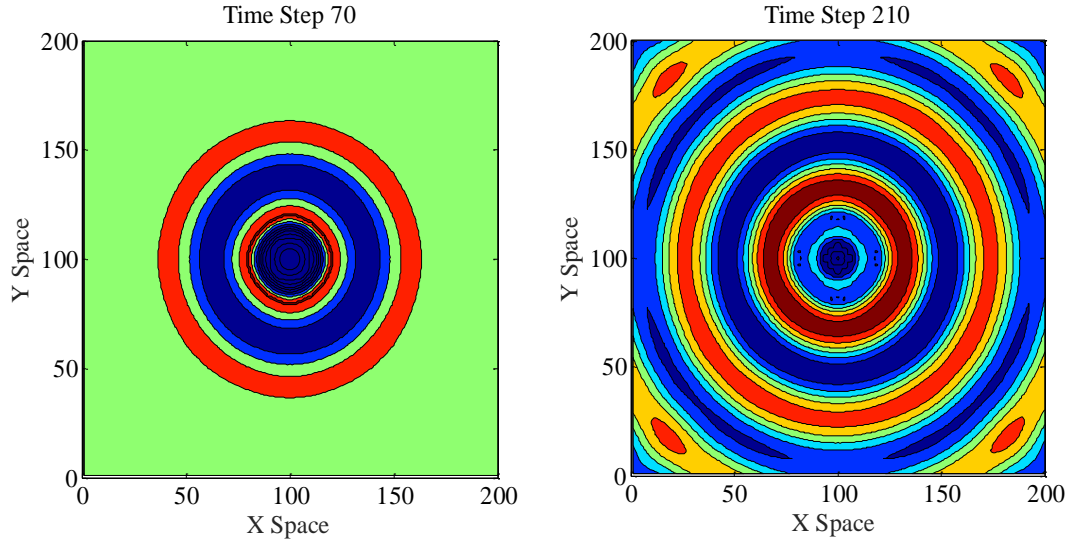


Figure 23. Spatial distribution of pressure at time step of 70 and 210

Now analytical distribution obtained from above equation is compared with the numerical pressure distribution.

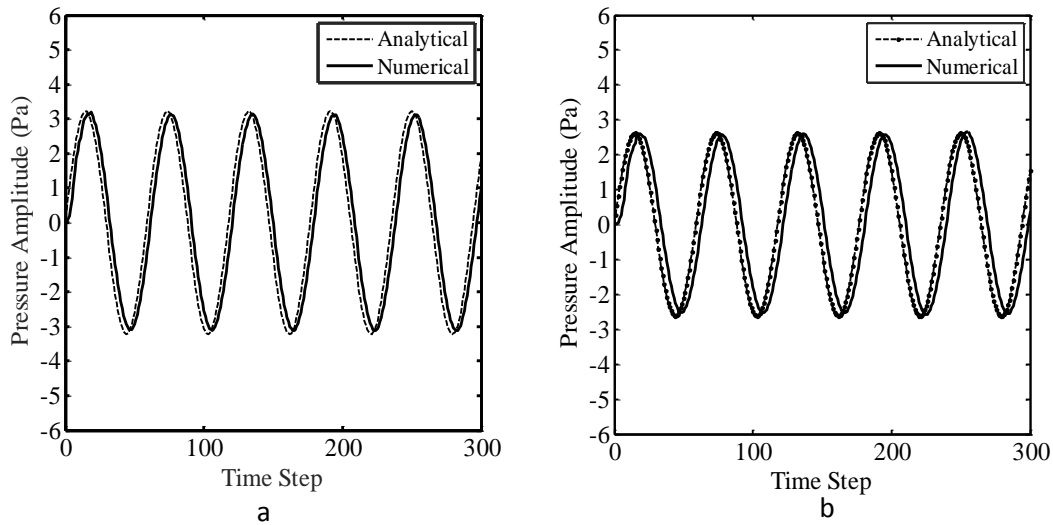


Figure 24. Comparison of analytical and numerical results of propagation of sound at distance r , a) 40 units and b) 60 units

Now, effect of various boundary conditions are analyzed for two dimensional wave propagation. A line source instead of point source is used as sound source. A rectangular duct of size 200×50 grid points with uniform grid spacing of 0.2 in x & y direction is considered as computational domain. Figure 25 shows the line AB as the source of impulse sound and pressure field distribution

at time step $t=70$ and at $t=145$. In the Figure 25, it can be noted that the forward wave is reflected at time step $t=145$, however the backward front is moving towards the left boundary.

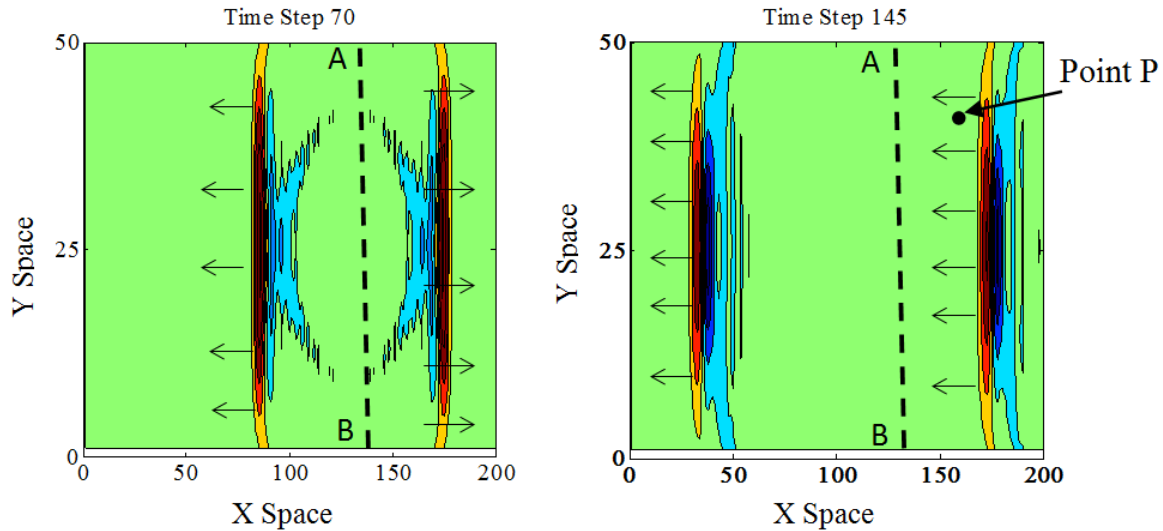


Figure 25. Spatial distribution of pressure with impulse source at time step of 70 & 145 (arrow indicates direction of motion of wave)

Figure 26 shows the time history of incident and reflected (superimposed) sound wave at point P (as shown in above figure) for various reflection coefficient of the boundary ($r=1$). The figure is plotted for a harmonic line source of amplitude 5 Pa and frequency 100 Hz place at line AB. The RMS pressure value observed at point P for different reflection condition is summarized in Table 5 from column 2 and column 6. It can be noted that the amplitude of the reflected sound reflects the level of reflection considered in the wall surface.

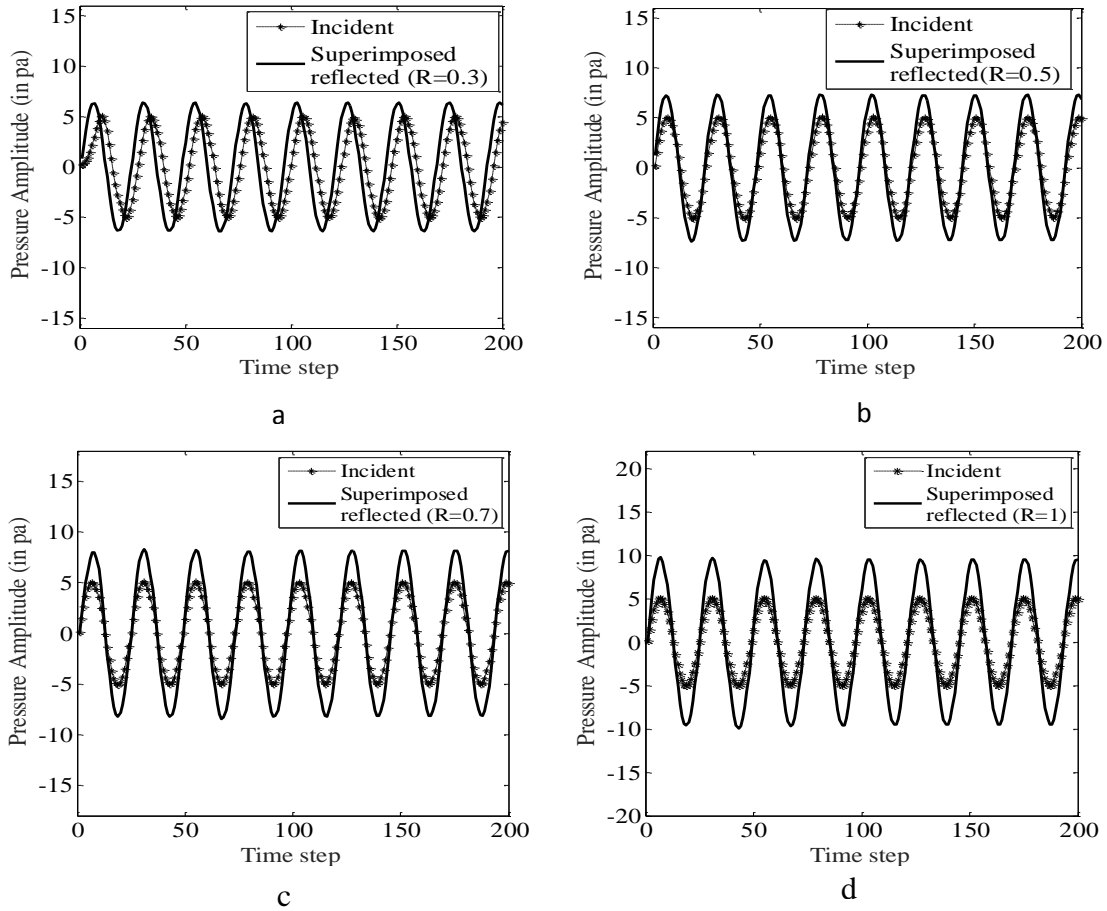


Figure 26. Comparison of incident and superimposed reflected wave in rectangular domain at point P using reflection coefficient of a) 0.3, b) 0.5, c) 0.7 & d) 1

Table 5. Comparison of RMS values corresponds to reflection coefficient for 2D

Sr. No	Reflection Coefficient (R)	RMS Value of incident wave	RMS Value of superimposed-reflected wave	RMS value of reflected wave	Percentage change in RMS value
1	0.3	3.5049	4.5366	1.0317	29.43
2	0.5	3.5049	5.2094	1.7045	48.52
3	0.7	3.5049	5.8240	2.3191	66.17
5	1	3.5049	6.8260	3.3211	94.76

4.3 Comparison of 2D reverberation time with the numerical value

In this section, a reverberation time of numerical 2D domain is compared with analytical value. Available theoretical formula to calculate the reverberation of two dimensional diffused field is given as:

$$RT_{60} = \frac{0.127F}{L \ln(1 - \alpha)} \quad (4.2)$$

Eq. (4.2) gives the equivalent reverberation time of 2D space where 2D waves have dominance and is derived through wave theory [19]. Here F is the surface area of domain, L is the length of perimeter of domain and α , is the average absorption coefficient of space. A $20 \times 20 \text{ ft}^2$ square domain with average absorption coefficient of 0.5 yield RT_{60} as 0.9234 sec. For comparison of reverberation time, a 2D numerical square domain of 100×100 grids is formed with spatial step size of 0.2 m in both direction. Time step is calculated by CFL criterion as explained earlier which yield $\Delta t = 0.0004 \text{ sec}$. A point source with combination of speaking frequency (100 Hz, 125 Hz and 150 Hz) is excited at the center and reverberation time is calculated. For calculation of RT_{60} curve fitting approach to the peak points of numerically obtained data is used. An exponential curve of second order is comes out to be the best curve fit for the numerical data. Results are explained by plotting the Sound Pressure Level (SPL) against Time. Figure 27 shows the numerical decay of SPL level and curve fitted of obtained data points.

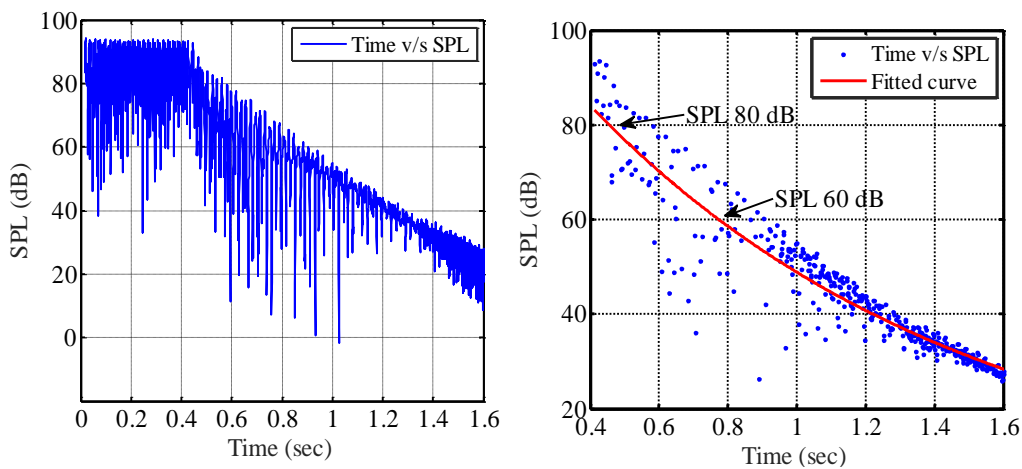


Figure 27. SPL decay with curve fitting and numerical data

A point source is shut down after few time steps, and decay of sound level is recorded. For point source with combination of different speaking frequencies, RT_{20} yields 0.32 sec. Furthermore, RT_{60} is calculated by extrapolating 30 dB drop to 60 dB drop with linear assumption which yields 0.96 sec which is close to the theoretical value.

Same reverberation time result is compared with having impulse source rather point source. An impulse of sinusoidal source with frequency of 100 Hz is excited at the center of the domain and then sound pressure level (SPL) is plotted against time as. Figure 28 shows the impulse response of room with fitted curve data.

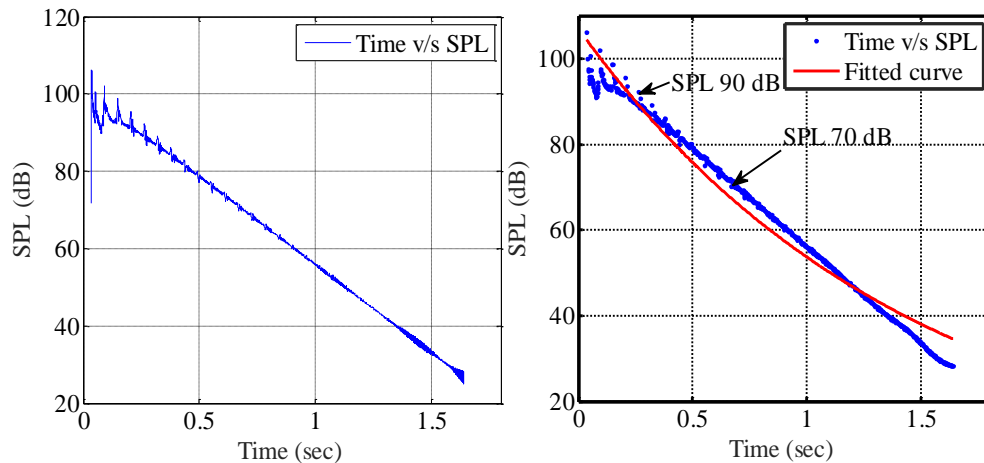


Figure 28. SPL decay with curve fitting and numerical data of an impulse

SPL decay at a point for different time is evaluated by curve fitting. In figure 28 red line shows the exponential curve which is best fit for the numerical data. 20 dB decay of sound is measured and extrapolate it for 60 dB drop for reverberation time, which yields RT_{60} is 1.09 sec. Again there is discrepancy with theoretical results due to approximation error.

Chapter 5. Three dimensional analysis

After the validation of wave propagation and boundary condition for one and three dimensional spaces, the investigations are carried out for 3D domain. Sound originates from the sound source and gets distributed all over the enclosed space in infinitely small fraction of time. When the field become reverberant there is multiple reflection from the boundary and human receives the sound after multiple reflection. Sometime direct and late reflection combines, which gives different perception. Interestingly, in some of the conditions, the interference happens in such a way that it results almost a quiet environment.

In this section, a problem of sound propagation in a 3D room is carried out. For this purpose, three dimensional wave equation has been discretized using FDTD and sound propagation in the room and reverberation time are calculated. The computational domain, boundary conditions and sound source considered in simulation are decided from the experimental study, discussed in next sections

5.1 3D Discretization of sound wave

3D space is discretize with FDTD technique likewise 2D plane. Hyperbolic sound propagation equation is describe as:

$$\frac{\partial^2 p}{\partial t^2} = c^2 \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \right) \quad (5.1)$$

The explicit value of pressure $p_{x,y,z}^{ts+1}$ is by FDTD is describe as:

$$p_{x,y,z}^{ts+1} = p_{x+1,y,z}^{ts} + p_{x-1,y,z}^{ts} + p_{x,y+1,z}^{ts} + p_{x,y-1,z}^{ts} + p_{x,y,z+1}^{ts} + p_{x,y,z-1}^{ts} + 2p_{x,y,z}^{ts} (1 - 3\lambda^2) - p_{x,y,z}^{ts-1} \quad (5.2)$$

Figure 29 shows the contour plot of propagation of sound using discretization method in 3D space.

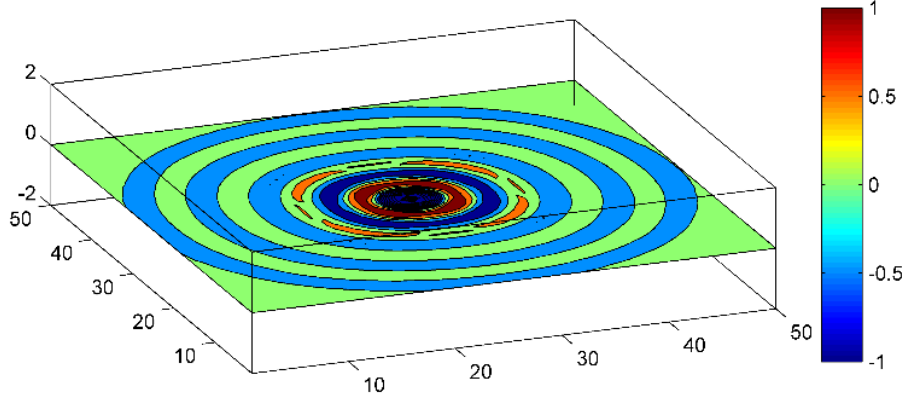


Figure 29. Contour of 3D propagation of sound wave from a point source.

Intensity of sound energy is more near the center where the source is placed and it decreases as wave travel forward. The sound intensity in free field decreases with the inverse square law:

$$I = \frac{P^2}{\rho c} \quad (5.3)$$

P , is pressure, ρ is density of air and c is speed of light is medium.

5.2 Numerical Boundary Analysis

As discuss earlier, for boundary of 3D space impedance in z direction is also considered for the analysis.

$$\frac{\partial p}{\partial t} = -\zeta_x c \frac{\partial p}{\partial x}, \quad \frac{\partial p}{\partial t} = -\zeta_y c \frac{\partial p}{\partial y} \quad \& \quad \frac{\partial p}{\partial t} = -\zeta_z c \frac{\partial p}{\partial z} \quad (5.4)$$

Explicit value of pressure at boundary $p_{x,y,z}^{ts+1}$ is again derived from the separate advection equation in all direction. Hence total of 26 boundary condition is needed to model the 3D space (6 for faces, 12 for edges and 8 for corners). Boundary for the wave moving toward positive direction in x, y & z direction is describe by following:

$$p_{x,y,z}^{ts+1} = \frac{\lambda^2 (2p_{x-1,y,z}^{ts} + p_{x,y+1,z}^{ts} + p_{x,y-1,z}^{ts} + p_{x,y,z+1}^{ts+1} + p_{x,y,z-1}^{ts+1}) + 2p_{x,y,z}^{ts} (1 - 3\lambda^2) - p_{x,y,z}^{ts-1} \left(\frac{\lambda}{\zeta_x} - 1 \right)}{\left(\frac{\lambda}{\zeta_x} + 1 \right)} \quad (5.5)$$

$$p_{x,y,z}^{ts+1} = \frac{\lambda^2(p_{x+1,y,z}^{ts} + p_{x-1,y,z}^{ts} + 2p_{x,y-1,z}^{ts} + p_{x,y,z+1}^{ts} + p_{x,y,z-1}^{ts}) + 2p_{x,y,z}^{ts}(1-3\lambda^2) - p_{x,y,z}^{ts-1}\left(\frac{\lambda}{\zeta_y} - 1\right)}{\left(\frac{\lambda}{\zeta_y} + 1\right)} \quad (5.6)$$

$$p_{x,y,z}^{ts+1} = \frac{\lambda^2(p_{x+1,y,z}^{ts} + p_{x-1,y,z}^{ts} + p_{x,y+1,z}^{ts} + p_{x,y-1,z}^{ts} + 2p_{x,y,z-1}^{ts}) + 2p_{x,y,z}^{ts}(1-3\lambda^2) - p_{x,y,z}^{ts-1}\left(\frac{\lambda}{\zeta_z} - 1\right)}{\left(\frac{\lambda}{\zeta_z} + 1\right)} \quad (5.7)$$

Here $\lambda = \frac{1}{\sqrt{3}}$ is Courant number [22]. Likewise boundary for sound travel in negative direction in x, y & z direction is modelled. These equation are required to model the face of the cuboidal/cubical space. The edges of space shared by two faces, hence two impedances is needed to model the edge of the boundary. Boundary edge shared by right and top face ($x = n1, y = n2, z > 1$ & $z < n3$) is describe as:

$$p_{x,y,z}^{ts+1} = \frac{\lambda^2(2p_{x-1,y,z}^{ts} + 2p_{x,y-1,z}^{ts} + p_{x,y,z+1}^{ts+1} + p_{x,y,z-1}^{ts+1}) + 2p_{x,y,z}^{ts}(1-3\lambda^2) - p_{x,y,z}^{ts-1}\left(\frac{\lambda}{\zeta_x} + \frac{\lambda}{\zeta_y} - 1\right)}{\left(\frac{\lambda}{\zeta_x} + \frac{\lambda}{\zeta_y} + 1\right)} \quad (5.8)$$

Similarly other edges can be modelled. Critical aspect of modeling is to modelled corners where they share three impedances at a time. Reflection/absorption from the corners is depend on the combination of three impedance. Top right corner ($x = n1, y = n2, \& z = n3$) is modelled as:

$$p_{x,y,z}^{ts+1} = \frac{\lambda^2(2p_{x-1,y,z}^{ts} + 2p_{x,y-1,z}^{ts} + 2p_{x,y,z-1}^{ts}) + 2p_{x,y,z}^{ts}(1-3\lambda^2) - p_{x,y,z}^{ts-1}\left(\frac{\lambda}{\zeta_x} + \frac{\lambda}{\zeta_y} + \frac{\lambda}{\zeta_z} - 1\right)}{\left(\frac{\lambda}{\zeta_x} + \frac{\lambda}{\zeta_y} + \frac{\lambda}{\zeta_z} + 1\right)} \quad (5.9)$$

Remaining seven corners are modelled in similar way.

5.3 Numerical Simulation

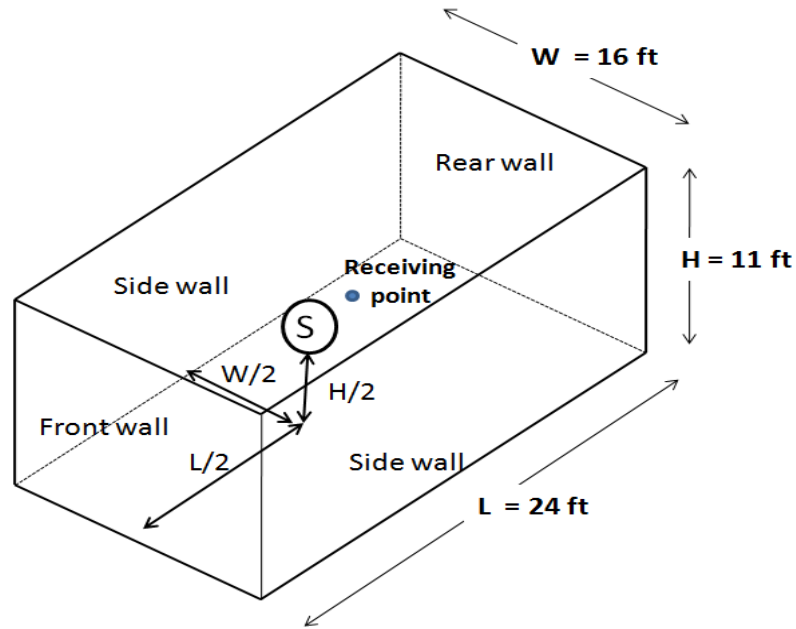


Figure 30. Schematic of computational domain

In the simulation a cuboidal computational domain equivalent to a room considered for the experiments, Figure 30 shows the schematic of the computational domain and placement of the source. The numerical domain is discretized by considering a uniform grid size ($\Delta x = \Delta y = \Delta z$) 0.2. An optimal temporal step size (Δt) is 0.00022 is used, which is smaller than the step computed by the CFL condition 0.00033 sec. A sound source of frequencies of 116 Hz, 133 Hz and 216 Hz is considered at the center of the domain. The considered relative amplitude corresponding to the three frequencies are in same proportion as observed in the Fourier transformation of measured data (Figure 33). Table 6 shows the reflection coefficient of various surfaces of the numerical domain that is equivalent to actual classroom.

Table 6. Reflection coefficient of material used at the wall of classroom [W5]

Faces	Reflection coefficients
Rear (glass)	0.9
Side faces (concrete)	0.8
Floor (concrete)	0.8
Ceiling (concrete)	0.8
Front face (combination of door, chalk board)	0.7

To simulate the reverberation effect in the room, the sound source has been shut down after few time steps and then decay of sound is recorded and used to measure the reverberation time. Figure 31 (a) indicates the time history of the SPL of sound measured at receiving point placed at some distance from source (Figure 30). In Figure 31 (a), point C shows the cut-off point on which the sound source is stopped. Figure 31 (b) is plotted to identify decay of sound using an exponential curve fitting approach. In this figure, the dots point out the peak points of the SPL and a continuous line indicates an exponential curve fitted using the peak points. It is observed that the sound pressure decays very fast in initially phase and reaches to steady state level. It may be noted that the sound level decays up to 69 dB (approximately) and remains steady further. It is expected that this level remains due to the numerical noise generated by the employed FDTD scheme. However, a decay of 20 dB is conveniently measured from the fitted curve and used to evaluate RT by extrapolating it for 60 dB drop (RT_{60}) with linear assumption. A drop between 89 dB to 69 dB results a reverberation time of 0.94 sec.

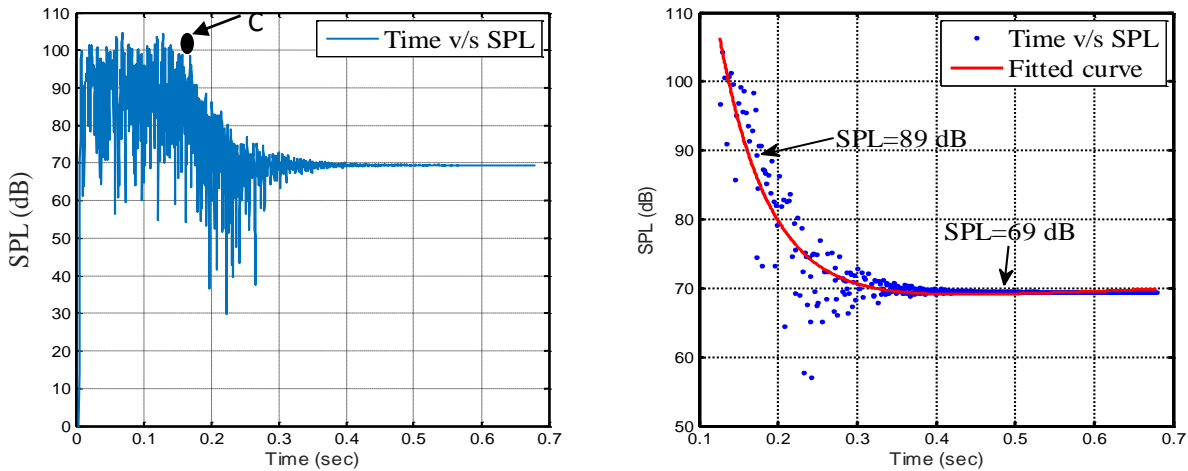


Figure 31. SPL decay of 3D computational domain

5.4 Experimental measurement

In this section, experimental measurement of reverberation time of a classroom of size $24 \times 16 \times 11 \text{ ft}^3$ is discussed. The measurements are carried out for two different sources (1) sound radiated by pricking a balloon, and (2) using a harmonic sound source. The source is placed at the center of the room and the acoustic data is recorded using a microphone placed at 1 m away from the source. Figure 32 (a) and 32 (b) show the actual picture of the classroom and a schematic

indicating source and measurement location (figure not to scale). For the data acquisition, an open source measurement tool, software name Audacity with sampling rate of 44100 Hz is used.

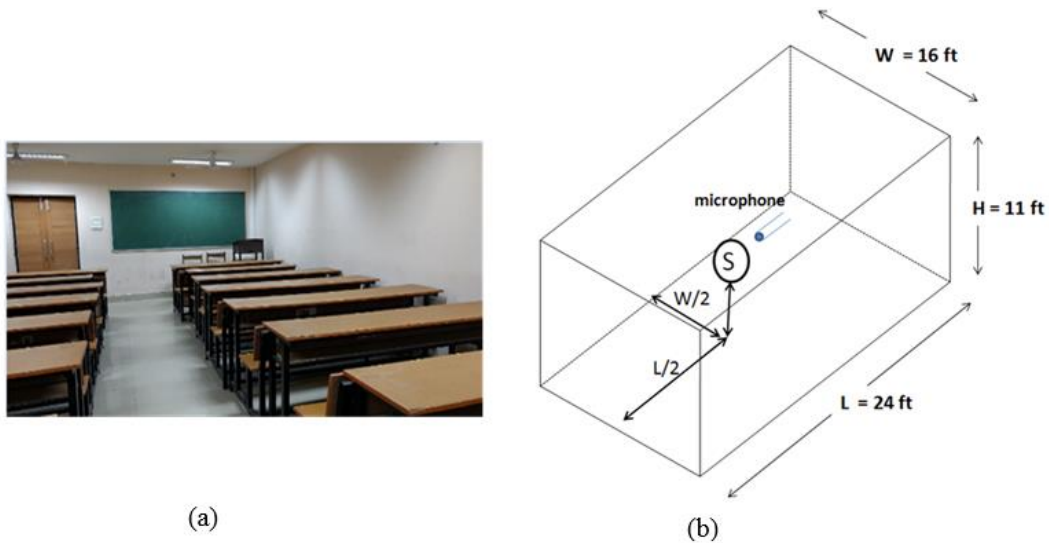


Figure 32. Actual picture of classroom and schematic of source and receiver location

In the first case, a balloon is burst and the sound pressure level is recorded. Figure 33 (a) shows the time history of the SPL measured, where in the solid line shows an exponential curve fitted using the measured data points. Figure 33 (b) illustrates the frequency spectrum of the measured sound signal. The frequency spectrum indicates that the sound generated by the bursting has a dominating component corresponding to 216 Hz. In addition to that, a number of frequency components are presented in the spectrum. It is noted that the sound decay very fast and reaches to a steady state level of 60 dB (approximately). It may be considered as background noise presented in the room. For the calculation of reverberation time (RT_{60}), time, an attenuation of 20 dB (sound pressure level decay from 98 dB to 78 dB) is measured and extrapolated for a 60 dB drop assuming linear variation. In this case, the observed RT_{60} is 0.84 seconds.

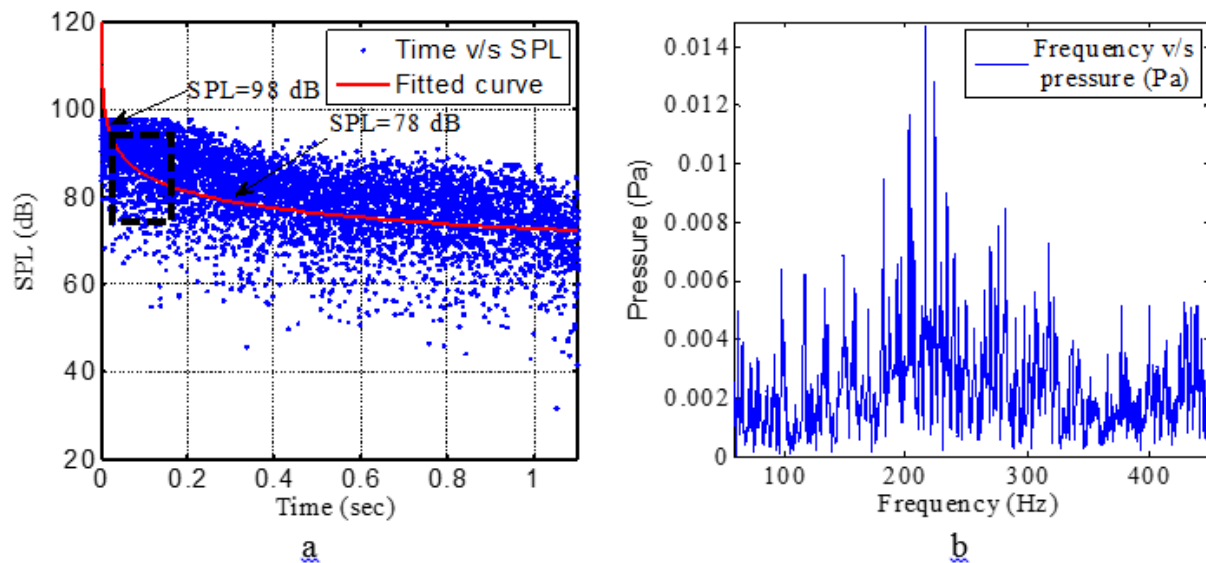


Figure 33. SPL decay of experimental data with balloon as a source and FFT of rectangular highlighted region

In second case, an un-baffled loudspeaker is used as a source, which emits harmonic sound waves. The frequency of the emitting sound is selected based of the frequency generated by the normal speaking in the classroom. In the present investigation, combination of two different frequencies as 125, Hz, 280 Hz, which falls in the range of normal speaking are considered. Relative amplitude of both the frequency components are equal. Before the measurement of reverberation time, the sound source is kept on for an adequate time (30 seconds), thus a diffused field can be created within the room. After that, the source is switched off and corresponding sound is recorded. Figure 34 (a) shows the time history of the SPL measured. In Figure 34 (b), the solid line shows an exponential curve fitted using the measured data points. From the figure, a 20 dB drop ranging from 89 dB to 69 dB is measured, which is extrapolated to give RT_{60} as 0.91 seconds.

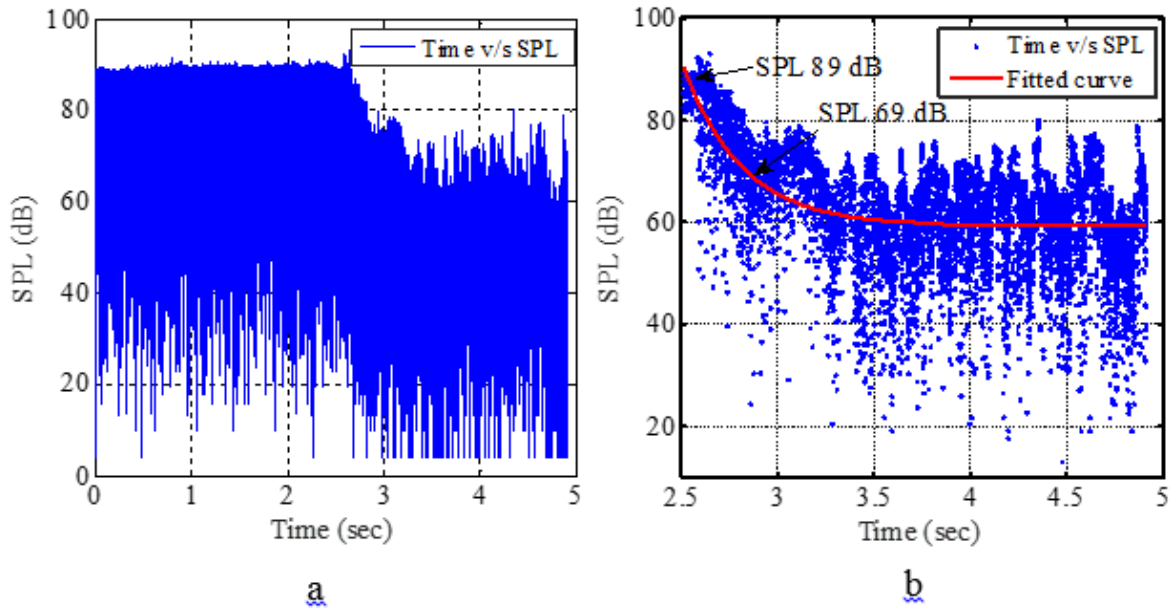


Figure 34. SPL decay of experimental data with an unbaffled loudspeaker

Table 7 shows the reverberation time measured by two different approaches. For the comparison, a reverberation time calculated from the empirical formulation, given by Sabine, is also added. The Sabine equation shows

$$RT_{60} = \frac{0.16V}{\Sigma\alpha S} \text{sec} \quad (5.10)$$

RT_{60} , Reverberation time, V is volume of enclosed space, $\Sigma\alpha S$ effective absorbing area, α is the average absorption of space and $S = S_1 + S_2 + S_3 \dots$ is the total surface area of space as there is multiple reflection. In addition, the table also shows RT_{60} , obtained from the numerical study carried out for an equivalent three dimensional space.

Table 7. Comparison of experimental, empirical and numerical reverberation time

	BY balloon pricking	Using harmonic source	Numerical methodology	Using Sabine equation
RT_{60} of a room size 24 x 16 x 11 ft^3	0.84 sec	0.91sec	0.94 sec	1.1 sec

The Table 7 shows that, in the experimental measurements (column 1 and 2), the reverberation time obtained by bursting a balloon is little lower than the case of using a harmonic source. It may be assumed that this difference is due to some measurement error and can be ignored. However, there is a significant difference between the experimental results and results obtained from the analytical approach (Sabine's equation). It may be expected as the empirical formulation involves a number of assumptions and primarily developed to give an approximation of the reverberation time of an enclosed space. In the analytical formulation, absorption or attenuation of sound within the volume (due to the medium) is not considered. In addition, in the actual classroom, some furniture, students & teachers are also presented, which may absorb the sound the leads to a smaller reverberation time.

The order of the reverberation time predicted by the numerical approach is comparable with the experimental results, however, in the numerical method, the effect of the medium (dispersion, dissipation of sound) is considered, but in the modeling of the enclosed space, other objects as furniture etc. has been ignored. Nevertheless, the numerical results are encouraging and may be utilize to predict reverberation time of an enclosed space.

Chapter 6. Conclusion

Study of sound propagation and its reflection, absorption from the boundary is an important aspect in the architectural acoustics. Even though, sound propagation has been researched from several decades, there are many fronts where numerical estimation of behaviour of sound in a closed space is a challenging task. In the presented work, a numerical solver has been developed to investigate the problem of room acoustics. The solver is developed using finite difference time domain (FDTD) technique with leapfrog scheme. Various boundary models are employed to illustrate reflection/absorption of sound from the boundaries. Investigations are carried out for all one, two and three dimensional spaces. The developed methodology is also validated for benchmark cases as sound radiation from a point source, pulsating cylinder, and line source. Various conditions are employed on the boundary and change in the amplitude is analyzed. Amplitude rises when wave get superimposed after reflection. The effect of boundary is modeled by considering reflection coefficient of the wall surfaces. It is noted that for a reflection coefficient of 0.3, 0.5 & 0.7, the amplitude of the reflected wave is close to 30, 50 and 70 percent of the incident wave. In comparison of numerical results with analytical results, slight phase shift is observed. This may be due to the discretization error with FDTD technique.

In the three dimensional numerical investigation, reverberation time of the closed space is simulated by radiating the sound for a short periods and then by measuring the decay of sound at a point in the space. The attenuation of sound is plotted on dB scale and a drop of 20 dB is calculated, which is then extrapolated to estimate RT_{60} . Experiments are performed to measure the reverberation time of a classroom. Two different cases are investigated for the calculation of RT_{60} . In first case, a pricking of balloon is used as a source of sound and in second case, a loudspeaker is used as source. Measured data is analyzed by plotting time history, frequency spectrum of sound pressure level (SPL). Finally, the experimentally measured reverberation time is compared with the numerical results and reasonably good matching is noted. It is interpreted that the difference between the two results may be due to the presence of furniture and other objects in the classroom, which have not been modeled in the numerical investigation. Nevertheless, the results are encouraging and suffice the application of the developed numerical tool to estimate the reverberation time of an enclosed room.

One of the outcomes of the study is measurement of the RT of classroom, RT_{60} (0.94 sec), which is not ideal for a classroom and motivates to study further to optimize its RT. It is understood that interference which aids echoing in the room should be less. To make it more acoustically suitable, good absorbing materials should be used in the classroom. Suspending ceiling with absorbing tiles can also be useful to reduce the interference in signals to construct a quieter classroom.

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