FUZZY PROGRAMMING TECHNIQUE FOR SOLVING DIFFERENT TYPES OF MULTI-OBJECTIVE TRANSPORTATION PROBLEM

Thesis submitted in partial fulfillment of the requirement for

The award of the degree of

Masters of Science

in

Mathematics and Computing

Submitted by

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JULY 2010

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TO

MY PARENTS, GOD AND MY SUPERVISOR

CERTIFICATE

I hereby certify that the work which is being presented in the thesis entitled "**Fuzzy programming technique for solving different types of multi-objective transportation problem**" in partial fulfillment of the requirements for the award of degree of Master of Science, School of Mathematics and Computer Applications (SMCA), Thapar University, Patiala is an authentic record of my own work carried out under the supervision of **Dr. Amit Kumar**.

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

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ACKNOWLEDGEMENT

First of all, I would like to thank the Almighty for granting perseverance. I would like to express my gratitude to **Dr. Amit Kumar, Assistant Professor, SMCA, Thapar University, Patiala**, for their patient guidance and support throughout this work. I was truly very fortunate to have the opportunity to work under him as a student. It was both an honor and a privilege to work with him. He also provides help in technical writing and presentation style and I found this guidance to be extremely valuable. I take this opportunity to express my sincere thanks to **Prof. S. S. Bhatia, Head SMCA, Thapar University, Patiala**, for their valuable support and help without which it would not have been possible for me to complete this work.

I am also thankful to **Mrs. Anila Gupta** and **Mr. Manjit Verma**, Research Scholars, SMCA and all my friends who devoted their valuable time and helped me in all possible ways towards successful completion of this work. I do not find enough words with which I can express my feeling of thanks to the entire research scholars, faculty and staff of SMCA, Thapar University, Patiala, for their help, inspiration and moral support which went a long way in successful completion of my work. I thank all those who have contributed directly or indirectly to this work.

Lastly, and more importantly, I would like to thank my parents for their years of unyielding love and encouragement. They have always wanted the best for me and I admire my parent's determination and sacrifice to put me through college.

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ABSTRACT

In today's highly competitive market, the pressure on organizations to find better ways to create and deliver value to customers become stronger. How and when to send the products to the customers in the quantities, they want in a cost-effective manner, become more challenging. Transportation models provide a powerful framework to meet this challenge. Transportation models ensure the efficient movement and timely availability of raw materials and finished goods.

This thesis is devoted to fuzzy programming technique for solving different types of multi-objective transportation problem. The main topics are multi-objective transportation problem, multi-objective capacitated transportation problem, multi-objective interval transportation problem.

The chapter-wise summary of the thesis is as follows:

Chapter 1 is introductory in nature. This chapter includes basic concepts used throughout the work.

Chapter 2 presents brief review of the work done in the area of multi-objective transportation problem.

In **Chapter 3**, fuzzy programming technique to solve the multi-objective transportation problems with different type of membership functions is presented. To illustrate the presented technique a numerical example is solved.

In **Chapter 4**, fuzzy programming technique to solve multi-objective capacitated transportation problem in which the supply and demand constraints are equality type, capacity restriction on each route are specified and the objectives are conflicting in nature, is presented. To illustrate the presented technique a numerical example is solved.

In **Chapter 5**, we focus on the solution procedure of the multi-objective transportation problem, where the cost coefficients of the objective functions, and the source and destination parameters are expressed as interval values by the decision maker. This problem is transformed into a classical multi-objective transportation problem. The equivalent transformed problem is solved by fuzzy programming technique.

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Chapter 1

INTRODUCTION

Decision making is the process of identifying and choosing alternatives based on the values and preferences of the decision maker. It is the process of sufficiently reducing uncertainty and doubt about alternatives to allow a reasonable choice to be made from among them. Decision making based solely on a single criterion appears insufficient as soon as the decision-making process deals with the complex organizational environment. So, one must acknowledge the presence of several criteria that lead to the development of multi-criteria decision making.

Optimization is a kind of the decision making, in which decisions have to be taken to optimize one or more objectives under some prescribed set of circumstances. These problems may be a single or multi-objective and are to be optimized (maximized or minimized) under a specified set of constraints. The constraints usually are in the form of inequalities or equalities. Such problems which often arise as a result of mathematical modeling of many real life situations are called optimization problems.

1.1 Single-objective optimization problem

In many real life situations optimization problems are modeled and solved as single-objective optimization problems in a deterministic and crisp environment. The general form of single-objective optimization problem is:

$$
\begin{aligned}
&\text{Minimize } (\text{or Maximize}) \ f(X), \qquad X = (x_1, x_2, ..., x_n) \\
&\text{subject to} \\
&g_j(X) \le 0, \quad j = 1, 2, ..., k \\
&l_j(X) \ge 0, \quad j = 1, 2, ..., r \\
&h_j(X) = 0, \quad j = 1, 2, ..., m
\end{aligned}
$$

where $f, g_1, g_2, ..., g_k, l_1, l_2, ..., l_r, h_1, h_2, ..., h_m$ are real valued functions defined on R^n. $X = (x_1, x_2, ..., x_n) \in R^n$ is called decision vector and $x_1, x_2, ..., x_n$ are called decision or unknown variables. In case all the functions (objective function and constraints) are linear then the above problem is called linear programming problem, otherwise it is called non-linear programming problem.

1.2 Transportation problem

It is a special type of linear programming problem which arises in many practical applications. In the beginning it was formulated for determining the optimal shipping pattern, so it is called transportation problem. The conventional and very well known transportation problem consists in transporting a certain product from each of m origins $i = 1, 2, ..., m$ to any of n destination $j = 1, 2, ..., n$. The origins are production facilities with respective capacities $a_1, a_2, ..., a_m$ and the destinations are warehouses with required levels of demand $b_1, b_2, ..., b_n$. For the transport of a unit of the given product from the i^{th} source to the j^{th} destination a cost c_{ij} is given for which, without loss of generality, we can assume $c_{ij} \ge 0, \forall i, j.$ Hence, one must determine the amounts x_{ij} to be transported from all the origins $i = 1, 2, ..., m$ to all the destinations $j = 1, 2, ..., n$ in such a way that the total cost is minimised. This problem can be suitably modeled as a linear programming problem. Thus the conventional transportation problem can be mathematically expressed as:

$$\text{Minimize } Z=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_{ij}$$

subject to

$$\sum_{j=1}^{n}x_{ij}\le a_i \quad i=1,2,...,m \quad (\text{row restrictions})$$

$$\sum_{i=1}^{m}x_{ij}\ge b_j \quad j=1,2,...,n \quad (\text{column restrictions})$$

$$x_{ij}\ge 0 \qquad \forall i \text{ and } j$$

$$\sum_{i=1}^{m}a_i=\sum_{j=1}^{n}b_j \quad (\text{balanced condition})$$

Definition 1.1

A set of non-negative allocations x_{ij} is said to be feasible solution of fuzzy linear programming problem if and only if it satisfies the row and column restrictions of the problem.

Definition 1.2

If there are m equations in $(m+n)$ variables then to solve these equations put any n variables equal to zero and find the solution of m equations in m variables. If the obtained solution is unique then it is called basic solution, otherwise, it is called non-basic solution. The zero valued variables are called non-basic variables and the remaining variables are called basic variables.

Definition 1.3

A feasible solution to a *m*-origin and *n*-destination problem is said to be basic feasible solution if the number of positive allocations are $(m+n-1)$.

Definition 1.4

If the number of allocations in a basic feasible solutions are less than $\left(m+n-1\right)$, it is called degenerate basic feasible solution (otherwise non-degenerate).

.**Definition 1.5**

A feasible solution (not necessarily basic) is said to be optimal solution if it minimizes the total transportation cost.

1.3 Multi-objective optimization problem

Many real life optimization problems are multi-objective in nature and are to be optimized simultaneously subject to a common set of constraints. The most general mathematical model of a multi-objective optimization problem is:

$$\begin{aligned}
&\text{Maximize } F\left(X\right)=\left[f_1\left(X\right), f_2\left(X\right),..., f_m\left(X\right)\right], \quad X=\left(x_1,x_2,...,x_\mathrm{n}\right)\\
&\text{subject to}\\
&g_j\left(X\right)\le 0, \quad j=1,2,...,k\\
&h_j\left(X\right)=0, \quad j=1,2,...,m\\
&l_j\left(X\right)\ge 0, \quad j=1,2,...,r
\end{aligned}$$

where $f_1, f_2,..., f_m$ are the objective functions. Variables $x_1, x_2,..., x_n$ are called decision variables and X is called decision vector. This problem is also called multi-objective programming problem.

1.4 Multi-objective transportation problem (MOTP)

In real life situations, all the transportation problems are not single-objective. The transportation problems which are characterized by multiple objective functions are considered here. A special type of linear programming problem in which constraints are of equality type and all the objectives are conflicting with each other, are called MOTP. Similar to a typical transportation problem, in a MOTP problem a product is to be transported from m sources to n destinations and their capacities are $a_1, a_2, ..., a_m$ and $b_1, b_2, ..., b_n$ respectively. In addition, there is a penalty c_{ij} associated with transporting a unit of product from i^{th} source to j^{th} destination. This penalty may be cost or delivery time or safety of delivery or etc. A variable x_{ij} represents the unknown quantity to be shipped from i^{th} source to j^{th} destination. A mathematical model of MOTP with r objectives, m sources and n destinations can be written as:

$$\text{Minimize } Z_r = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{r} x_{ij}, \quad r = 1, 2, ..., K$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \quad i = 1, 2, ..., m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad j = 1, 2, ..., n$$

$$x_{ij} \geq 0 \qquad \forall i, j$$

The subscript on Z_r and superscript on c_{ij}^{r} are related to the r^{th} penalty criterion. Without loss of generality, it may be assumed that $a_i \geq 0$ and $b_j \geq 0 \ \forall i, j$ and the equilibrium condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ is satisfied.

1.5 Different types of multi-objective transportation problem

Since different assumptions about the way of satisfying the constraints and some lack of precision in the statement of the problem can be supposed, several versions of the problem can arise.

1.5.1 MOTP with equality constraints

MOTP with equality constraint is of the form:

$$
\begin{aligned}
&\text{Minimize } Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij},\ k = 1,2,\ldots,K \\
&\text{subject to} \\
&\sum_{j=1}^{n} x_{ij} = a_i,\ i = 1,2,\ldots,m \\
&\sum_{i=1}^{m} x_{ij} = b_j,\ j = 1,2,\ldots,n \\
&x_{ij} \ge 0, \quad \forall\, i, j
\end{aligned}
$$

Above problem is feasible if and only if the condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ holds.

1.5.2 MOTP with inequality constraints

MOTP with inequalities both in supply and demand constraints can be presented as:

$$
\begin{aligned}
&\text{Minimize } Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij}, \quad k = 1,2,\ldots,K \\
&\text{subject to} \\
&\sum_{j=1}^{n} x_{ij} \le a_i,\ i = 1,2,\ldots,m \\
&\sum_{i=1}^{m} x_{ij} \ge b_j,\ j = 1,2,\ldots,n \\
&x_{ij} \ge 0, \quad \forall\, i, j
\end{aligned}
$$

This problem is feasible if and only if $\sum_{i=1}^{m} a_i \ge \sum_{j=1}^{n} b_j$.

1.5.3 MOTP with mixed constraints

(a) Transportation problem constraints, that is, with equality supply constraints and inequality demand constraints is as follows:

$$\begin{aligned}&\text{Minimize } Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij}, \quad k = 1,2,\ldots,K\\&\text{subject to}\\&\sum_{j=1}^{n} x_{ij} = a_i,\ i = 1,2,\ldots,m\\&\sum_{i=1}^{m} x_{ij} \ge b_j,\ j = 1,2,\ldots,n\\&x_{ij} \ge 0, \quad \forall\, i, j\end{aligned}$$

This problem is feasible if and only if $\sum_{i=1}^{m} a_i \ge \sum_{j=1}^{n} b_j$.

(b) Transportation problems with inequality in supply constraints and equality in demand constraints can be presented as:

$$\begin{aligned}&\text{Minimize } Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij}, \quad k = 1,2,\ldots,K\\&\text{subject to}\\&\sum_{j=1}^{n} x_{ij} \le a_i,\ i = 1,2,\ldots,m\\&\sum_{i=1}^{m} x_{ij} = b_j,\ j = 1,2,\ldots,n\\&x_{ij} \ge 0, \quad \forall\, i, j\end{aligned}$$

This type of problem is feasible if and only if $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.

1.6 Different approaches to solve MOTP

The linear MOTP is a special type of linear programming problem in which constraints are of equality type and the objectives are conflicting with each other. The existing solution procedure of this problem can be divided into two categories.

First consists those that are generating all the sets of efficient solutions and the second category represents the procedures that are seeking the best compromise solution among the set of efficient solutions. From a practical point of view the knowledge of the set of efficient solutions is not always necessary. In such a case, a procedure is needed to determine a compromise solution. As a result, different approaches are developed in the context of MOTP to find the compromise solution.

The various approaches to solve MOTP are:

- Fuzzy programming technique
- Fuzzy goal programming approach
- Interactive procedure
- Spanning tree based genetic algorithm
- Geometric programming approach
- Genetic algorithm
- Interactive fuzzy multi-objective linear programming

Each and every approach has its own drawbacks as well as its own strengths, and any rational choice as to which should be used is almost dependent on at least two vital considerations:

1. The type and size of the problem
2. The characteristics of the ultimate decision maker(s)

But in this thesis we will present only fuzzy programming technique to find an optimal compromise solution of a transportation problem with several objectives. Here both linear and non-linear membership functions are used and it is found that the linear membership function makes the given problem linear and easy to solve as compared to using non-linear membership function.

Definition 1.6

A feasible solution $\hat{x} = \{\hat{x}_{ij}\} \in X$ is said to be a non-dominated solution of the MOTP if there is no other feasible solution $x = \left\{x_{ij}\right\} \in X$ such that $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij} \leq \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} \hat{x}_{ij}$ for all k and $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij} < \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} \hat{x}_{ij}$ for at least one k.

Definition 1.7

An optimal compromise solution of the MOTP is a solution $\hat{x} = \left\{\hat{x}_{ij}\right\} \in X$ which is preferred by the decision maker to all other solutions, taking into consideration all criteria contained in the multi-objective functions. Hence, an optimal compromise solution has to be a non-dominated solution according to the definition of non-dominated solution.

1.7 Fuzzy set theory

In this section some basic definitions of fuzzy set theory are presented:

Definition 1.8

A crisp set or a classical set A is defined as a collection of well defined objects**.** The objects are called elements of A. A crisp set A, defined on the universal set X, can also be represented by

$$A = \left\{\left(x, \mu_A\left(x\right)\right); x \in X\right\}$$

where $\mu_A\left(x\right)$ is called characteristic function, that declares which element of universal set X are member of a set and which are not. Set A is defined by its characteristic function $\mu_A(x)$ as:

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$$

The characteristic function maps elements of set X to elements of set $\{0,1\}$, which is formally expressed as:

$$\mu_A : X \to \{0,1\}$$

Definition 1.9

The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range $[0,1]$ i.e. $\mu_{\tilde{A}} : X \to [0,1]$. The assigned values indicate the membership grade of the element in the set A.

The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set. $\mu_{\tilde{A}}(x)$ is the degree of membership of x in $\tilde{A}$. The closer the value of $\mu_{\tilde{A}}(x)$ is to 1, the more x belongs to A.

Definition 1.10

Let $\tilde{A}$ be a fuzzy set and α be a real number in the interval $[0,1]$. The crisp set A_α defined by $A_\alpha = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha\}$ is called $\alpha-$cut of $\tilde{A}$. The crisp set $A_{\alpha^+} = \{x \in X : \mu_{\tilde{A}}(x) > \alpha\}$ is called strong $\alpha-$cut of $\tilde{A}$.

Chapter 2

LITERATURE REVIEW

The basic transportation problem was originally developed by Hitchcock (1941). Efficient methods of solution derived from the simplex algorithm were developed in 1947. The transportation problem can be modeled as a standard linear programming problem, which can then be solved by the simplex method. However, because of its very special mathematical structure, it was recognized early that the simplex method applied to the transportation problem can be made quite efficient in terms of how to evaluate the necessary simplex-method information (variable to enter the basis, variable to leave the basis and optimality conditions).

Charnes and Cooper (1954) developed the Stepping Stone Method which provides an alternative way of determining the simplex-method information.

Dantzig (1963) used the simplex method to the transportation problem as the Primal simplex transportation method. An initial basic feasible solution for the transportation problem can be obtained by using the North west corner rule, Row minima, Column minima, Matrix minima, or the Vogel approximation method. The Modified distribution method is useful for finding the optimal solution for the transportation problem.

The linear Interactive and Discrete Optimization (LINDO), General Interactive Optimizer (GINO) and TORA packages as well as many other commercial and academic packages are useful to find the solution of the transportation problem.

Arsham and Kahn (1989) introduced a new algorithm for solving the transportation problem. The proposed method used only one operation, the Gauss Jordan pivoting used in simplex method. The final table can be used for the post-optimality analysis of transportation problem. This algorithm is faster than simplex, more general than stepping stone and simpler than both in solving general transportation problem.

Kikuchi (2000) suggested that in many problems of transportation engineering and planning, the observed or derived values of the variables are approximate, yet the variables themselves must satisfy a set of rigid relationships dictated by physical principle. They proposed a simple adjustment method that finds the most appropriate set of crisp numbers. The method assumes that each observed value is an approximate number (or a fuzzy number) and the true value is found in the support of the membership function. For each of many possible sets of values that satisfy the relationships, the lowest membership grade is checked and the set whose lowest membership grade is the highest, is chosen as the best set of values for the problem. This process is performed using the fuzzy linear programming method.

The multi-objective transportation problem refers to a special class of linear programming problem in which the constraints are of equality type and all objectives are conflicting with each other. All the proposed methods to solve multi-objective linear programming problem generate a set of non-dominated or compromise solution. A variety of approaches, such as lexicographic goal programming approach, interval goal programming approach, interactive algorithms, fuzzy programming approach, the step method, the utility function method have been developed by many researchers for the multi-objective linear programming problem.

Lee and Moore (1973) applied goal programming to find a solution of multi-objective transportation problem. Goal programming has widely applied to solve different problems which involve multiple objectives. Virtually, all models developed to solve transportation problems ignored the multiple conflicting objectives involved in the problem, the priority structure of these objectives, various environmental constraints, unique organizational values of the firm, and bureaucratic decision structures. However, in reality these are important factors which greatly influence the decision process of transportation problems. They studied the goal programming approach is utilized in order to allow for the optimization of multiple conflicting goals while permitting an explicit consideration of the existing decision environment.

Zimmermann (1978) first applied the fuzzy set theory concept with some suitable membership function to solve multi-objective transportation problems. He presented the application of fuzzy linear programming to approaches to linear vector maximum problem. It has been found that solutions obtained by fuzzy linear programming are always efficient.

Isermann (1979) developed an algorithm for identifying all the non-dominated solutions for a linear multi-objective transportation problem.

Leberling (1981) used hyperbolic membership function for multi-objective linear programming problem. He found that using the fuzzy min-operator together with linear as well as special nonlinear membership functions (hyperbolic) the obtained solutions are always compromise solutions of the original multi-criteria problem.

Slowinski (1986) presented a method for solving a multi-criteria linear program where the coefficients of the objective functions and the constraints are fuzzy numbers of the *L-R* type. He transformed the original problem into a multi-criteria linear fractional problem by assuming the aspiration levels for particular criteria to be fuzzy and basing on comparison of fuzzy numbers, and then solved the obtained problem by using an interactive technique involving a linear programming procedure in the calculation phase.

Ringuest and Rinks (1987) proposed interactive algorithms to find more than k non-dominated and dominated solutions if there are k objectives. Thus the decision maker has to determine a compromise solution from the set of non-dominated solutions. For the larger problem, it is not easy to find the compromise solution by using the algorithm developed by Ringuest and Rinks (1987) but, using the fuzzy programming method, one can easily find a compromise solution.

Sakawa et al. (1987) proposed an interactive fuzzy decision making model using linear and non-linear membership functions to solve the multi-objective linear programming problem.

Dhingra and Moskowitz (1991) defined some non-linear membership functions like exponential, quadratic and logarithmic, and applied them to an optimal design problem. This procedure is useful in engineering and management design situations where uncertainty or ambiguity arises about the preciseness of permissible parameters, degree of credibility, and correctness of statements and judgements.

Bit et al. (1992) considered a k- objective transportation problem fuzzified by fuzzy numbers and used α- cut to obtain a transportation problem in the fuzzy sense expressed in linear programming form.

Bit and Alam (1993) introduced an additive fuzzy programming model for the multi-objective transportation problem. The method aggregates the membership functions of the objectives to construct the relevant decision function. Weights and priorities for non-equivalent objectives are also incorporated in the method. This model gives a non-dominated solution which is nearer to the best compromise solution

Verma et al. (1997) proposed a special type of non-linear (hyperbolic and exponential) membership functions to solve the multi-objective transportation problem and compared the obtained result with the solution obtained by using a linear membership function and shown that the results found to be nearly same.

Hussien (1998) studied the complete set of α - possibly efficient solutions of multi-objective transportation problem with possibilistic coefficients of the objective functions.

Das et al. (1999) focused on the solution procedure of the multi-objective transportation problem where the cost coefficients of the objective functions, and the source and destination parameters are expressed as interval values by the decision maker. They transformed the problem into a classical multi-objective transportation problem where to minimize the interval objective function. They defined the order relations that represent the decision maker's preference between interval profits. They converted the constraints with interval source and destination parameters into deterministic ones. Finally, they solved equivalent transformed problem by fuzzy programming technique.

Li and Lai (2000) presented a fuzzy compromise programming approach to multi-objective transportation problems. A characteristic feature of the approach proposed is that various objectives are synthetically considered with the marginal evaluation for individual objectives and the global evaluation for all objectives. The decision-maker's preference is taken into account by assigning the weights of objectives. With the global evaluation for all objectives, a compromise programming model is formulated. Using ordinary optimization technique, fuzzy compromise programming model is solved to obtain a non-dominated compromise solution at which the synthetic membership degree of the global evaluation for all objectives is maximum.

Wahed and Sinna (2001) presented a fuzzy programming approach to determine the optimal compromise solution of a multi-objective transportation problem and tested the approach performance by measuring the degree of closeness of the compromise solution to the ideal solution using a family of distance functions.

Sakawa et al. (2001) dealed with actual problems on production and work force assignment in a housing material manufacturer and a subcontract firm. He formulated two kinds of two-level programming problems: one is a profit maximization problem of both the housing material manufacturer and the subcontract firm, and the other is a profitability maximization problem of them. Applying the interactive fuzzy programming for two-level linear and linear fractional programming problems, he obtained the satisfactory solutions to the problems.

Ammar and Youness (2005) investigated the efficient solutions and stability of multi-objective transportation problem with fuzzy coefficient and/or fuzzy supply quantities and/or fuzzy demands quantities.

Wahed and Lee (2006) proposed an interactive fuzzy goal programming approach to determine the preferred compromise solution for the multi-objective transportation problem. The proposed approach considers the imprecise nature of the input data by implementing the minimum operator and also assumes that each objective function has a fuzzy goal. The approach focuses on minimizing the worst upper bound to obtain an efficient solution which is close to the best lower bound of each objective function. The solution procedure controls the search direction via updating both the membership values and the aspiration levels.

Zangiabadi and Maleki (2007) presented a fuzzy goal programming approach to determine an optimal compromise solution for the multi-objective transportation problem by assuming that each objective function has a fuzzy goal. A special type of non-linear (hyperbolic) membership function is assigned to each objective function to describe each fuzzy goal. The approach focuses on minimizing the negative deviation variables from 1 to obtain a compromise solution of the multi-objective transportation problem.

Surapati and Roy (2008) presented a priority based fuzzy goal programming approach for solving a multi-objective transportation problem with fuzzy coefficients. Firstly, they defined the membership functions for the fuzzy goals. Subsequently, they transformed the membership functions into membership goals, by assigning the highest degree (unity) of a membership function as the aspiration level and introducing deviational variables to each of them. In the solution process, negative deviational variables are minimized to obtain the most satisfying solution.

Lau et al. (2009) presented an algorithm called the fuzzy logic guided non-dominated sorting genetic algorithm to solve the multi-objective transportation problem that deals with the optimization of the vehicle routing in which multiple depots, multiple customers, and multiple products are considered. Since the total travelling time is not always restrictive as a time constraint, the objective considered comprises not only the total travelling distance, but also the total travelling time.

Lohgaonkar and Bajaj (2010) used fuzzy programming technique with linear and non-linear membership function (hyperbolic, exponential) to find the optimal compromise solution of a multi-objective capacitated transportation problem.

Chapter 3

FUZZY PROGRAMMING TECHNIQUE TO SOLVE MULTI-OBJECTIVE TRANSPORTATION PROBLEMS

In this chapter, fuzzy programming technique to solve the MOTP, with different type of membership functions is presented. To illustrate the presented technique a numerical example is solved.

3.1 Linear programming formulation of MOTP

A MOTP can be stated as:

$$\text{Minimize } Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{k} X_{ij}, \quad k = 1, 2, ..., K$$

subject to

$$\sum_{j=1}^{n} X_{ij} = a_i, \quad i = 1, 2, ..., m$$

$$\sum_{i=1}^{m} X_{ij} = b_j, \quad j = 1, 2, ..., n$$

$$X_{ij} \geq 0 \quad \forall\ i \text{ and } \text{j}$$

The subscript on Z_k and superscript on C_{ij}^{k} denote the k^{th} penalty criterion.

We assume that $a_i \geq 0$ for all *i*, $b_j \geq 0$ for all *j*, $C_{ij} \geq 0$ for all i and j, and

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \text{ (equilibrium condition)}$$

a_i is the quantity of material available at source O_i $(i = 1, 2, ..., m)$

b_j is the quantity of material required at destination D_j $(j = 1, 2, \ldots, n)$ and

C_{ij} is fuzzy unit cost of transportation from source O_i to destination D_j .

3.2 Fuzzy programming technique to solve MOTP

In fuzzy programming technique, we first find the lower bound as L_k and the upper bound as U_k for the k^{th} objective function $Z_k,\ k = 1, 2, ..., K$ where U_k is the highest acceptable level of achievement for objective $k,\ L_k$ the aspired level of achievement for objective k and $d_k = U_k - L_k$ the degradation allowance for objective k.

When the aspiration levels for each of the objective have been specified, a fuzzy model is formed and then the fuzzy model is converted into a crisp model. The solution of MOTP can be obtained by the following steps:

Step 1. Solve the MOTP as a single-objective transportation problem K times by taking one of the objectives at a time

Step 2. From the above results, determine the corresponding values for every objective at each solution derived. According to each solution and value for every objective, we can find a pay-off matrix as follows:

	$Z_1(X)$	$Z_2(X)$	.	.	.	$Z_K(X)$
$X^{(1)}$	Z_{11}	Z_{12}	.	.	.	Z_{1K}
$X^{(2)}$	Z_{21}	Z_{22}	.	.	.	Z_{2K}
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
$X^{(K)}$	Z_{K1}	Z_{K2}	.	.	.	Z_{KK}

where $X^{(1)}, X^{(2)}, ..., X^{(K)}$ are the isolated optimal solutions of the K different transportation problems for K different objective functions, $Z_{ij} = Z_j(X^i)$, $i = 1,2,...,K; j = 1,2,...,K$ be the i^{th} row and j^{th} column element of the pay-off matrix

Step 3. From Step 2, find for each objective the U_k and the L_k corresponding to the set of solutions, where,

$$U_k = \text{maximum}(Z_{1K}, Z_{2K}, ..., Z_{KK}) \text{ and } L_k = \text{minimum}(Z_{1K}, Z_{2K}, ..., Z_{KK}),$$

$$k = 1,2,...,K$$

An initial fuzzy model of the problem can be

Find $X_{ij}, i = 1,2,...,m, \ j = 1,2,...,n$

$$Z_k \lesssim L_k, \qquad k = 1,2,...,K$$

subject to

$$\sum_{j=1}^{n} X_{ij} = a_i, \qquad i = 1,2,...,m$$

$$\sum_{i=1}^{m} X_{ij} = b_j, \qquad j = 1,2,...,n$$

$$X_{ij} \geq 0 \qquad \text{for all } i \text{ and } j$$

Step 4. Define a membership function $\mu(Z_k)$, for the k^{th} objective function

Step 5. Convert the fuzzy model of the problem, obtained in step 3, into the following crisp model

Maximize λ

subject to

$$\lambda \leq \mu(Z_k)$$

$$\sum_{j=1}^{n} X_{ij} = a_i, \qquad i = 1,2,...,m$$

$$\sum_{i=1}^{m} X_{ij} = b_j, \qquad j = 1,2,...,n$$

$$X_{ij} \geq 0 \qquad \forall i \text{ and j}$$

$$\lambda \geq 0$$

Step 6. Solve the crisp model by an appropriate mathematical programming algorithm

Step 7. The solution obtained in Step 6 will be the optimal compromise solution of the MOTP

3.2.1 Fuzzy programming technique with linear membership function

A linear membership function is defined as:

$$\mu^L(Z_k) = \begin{cases} 1, & \text{if } Z_k \le L_k \\ 1 - \dfrac{Z_k - L_k}{U_k - L_k}, & \text{if } L_k < Z_k < U_k \\ 0, & \text{if } Z_k \ge U_k \end{cases}$$

If we use a linear membership function, the crisp model can be simplified as:

Maximize λ

subject to

$$Z_k + \lambda(U_k - L_k) \le U_k,\ k = 1,2,\ldots,K$$
$$\sum_{j=1}^{n} X_{ij} = a_i,\ \ i = 1,2,\ldots,m$$
$$\sum_{i=1}^{m} X_{ij} = b_j,\ \ j = 1,2,\ldots,n$$
$$X_{ij} \ge 0 \quad \forall i,j$$
$$\lambda \ge 0$$

3.2.2 Fuzzy programming technique with exponential membership function

An exponential membership function is defined by

$$\mu^E(Z_k) = \begin{cases} 1, & \text{if } Z_k \le L_k \\ \dfrac{e^{-s\psi_k(X)} - e^{-s}}{1 - e^{-s}}, & \text{if } L_k < Z_k < U_k \\ 0, & \text{if } Z_k \ge U_k \end{cases}$$

where

$$\psi_k(X) = \frac{(Z_k - L_k)}{(U_k - L_k)}, \qquad k = 1, 2, \ldots, K$$

s is a non-zero parameter prescribed by the decision maker.

If we use the exponential membership function, an equivalent crisp model for the fuzzy model can be formulated as follows:

Maximize λ

subject to

$$\lambda \le \frac{e^{-s\psi_k(X)} - e^{-s}}{1 - e^{-s}}, \qquad k = 1, 2, \ldots, K$$

$$\sum_{j=1}^{n} X_{ij} = a_i, \qquad i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} X_{ij} = b_j, \qquad j = 1, 2, \ldots, n$$

$X_{ij} \ge 0$ for all i and j

$\lambda \ge 0$

The above problem can be further simplified as:

Maximize λ

subject to

$$e^{-s\psi_k(X)} - (1 - e^{-s})\lambda \ge e^{-s}, \qquad k = 1, 2, \ldots, K$$

$$\sum_{j=1}^{n} X_{ij} = a_i, \qquad i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} X_{ij} = b_j, \qquad j = 1, 2, \ldots, n$$

$X_{ij} \ge 0$ for all i and j

$\lambda \ge 0$

3.2.3. Fuzzy programming technique with hyperbolic membership function

. A hyperbolic membership function is defined by

$$\mu^H\left(Z_k\right)=\begin{cases} 1, & \text{if } Z_k \le L_k \\ \dfrac{1}{2}\dfrac{e^{\{(U_k+L_k)/2-Z_k(X)\}\alpha_k}-e^{-\{(U_k+L_k)/2-Z_k(X)\}\alpha_k}}{e^{\{(U_k+L_k)/2-Z_k(X)\}\alpha_k}+e^{-\{(U_k+L_k)/2-Z_k(X)\}\alpha_k}}+\dfrac{1}{2}, & \text{if } L_k < Z_k < U_k \\ 0, & \text{if } Z_k \ge U_k \end{cases}$$

where

$$\alpha_k = \frac{6}{(U_k - L_k)}$$

If we will use the hyperbolic membership function then an equivalent crisp model for the fuzzy model can be formulated as:

Maximize λ

subject to

$$\lambda \le \frac{1}{2}\frac{e^{\{(U_k+L_k)/2-Z_k(X)\}\alpha_k}-e^{-\{(U_k+L_k)/2-Z_k(X)\}\alpha_k}}{e^{\{(U_k+L_k)/2-Z_k(X)\}\alpha_k}+e^{-\{(U_k+L_k)/2-Z_k(X)\}\alpha_k}}+\frac{1}{2}, \qquad k=1,2,\ldots,K \tag{3.1}$$

$$\sum_{j=1}^{n} X_{ij} = a_i, \quad i=1,2,\ldots,m \tag{3.2}$$

$$\sum_{i=1}^{m} X_{ij} = b_j, \quad j=1,2,\ldots,n \tag{3.3}$$

$$X_{ij} \ge 0 \qquad \text{for all } i \text{ and } j$$

$$\lambda \ge 0$$

Constraint (3.1) can further be simplified as:

$$\lambda \le \frac{1}{2}\tanh\left[\left\{\frac{\left(U_k+L_k\right)}{2}-Z_k\left(X\right)\right\}\alpha_k\right]+\frac{1}{2}$$

$$2\lambda \le \tanh\left[\left\{\frac{\left(U_k+L_k\right)}{2}-Z_k\left(X\right)\right\}\alpha_k\right]+1$$

$$\tanh^{-1}\left(2\lambda-1\right)\leq\left\{\frac{\left(U_k+L_k\right)}{2}-Z_k\left(X\right)\right\}\alpha_k$$

$$\alpha_k Z_k+\tanh^{-1}\left(2\lambda-1\right)\leq\frac{\left(U_k+L_k\right)\alpha_k}{2}$$

Now, putting $\tanh^{-1}\left(2\lambda-1\right)=X_{mn+1}$, constraint (3.1) is converted to

$$\alpha_k Z_k\left(X\right)+X_{mn+1}\leq\frac{\alpha_k}{2}\left(U_k+L_k\right)$$

Hence, the given problem is simplified as

$$\text{Maximize } X_{mn+1}$$

subject to

$$\alpha_k Z_k(X)+X_{mn+1}\leq\frac{\alpha_k}{2}(U_k+L_k),\qquad k=1,2,\ldots,K$$

$$\sum_{j=1}^{n}X_{ij}=a_i,\qquad i=1,2,\ldots,m$$

$$\sum_{i=1}^{m}X_{ij}=b_j,\qquad j=1,2,\ldots,n$$

$$X_{ij}\geq 0\qquad \forall\ i,j$$

$$X_{mn+1}\geq 0$$

where

$$X_{mn+1}=\tanh^{-1}(2\lambda-1)$$

3.3 Numerical example

In this section, a MOTP is solved by using the method discussed in section 3.2, and the obtained results are compared.

Example 3.1 The data is collected by a person who supplies a product to different companies after taking it from different sources. There are three different suppliers named as S_1, S_2, S_3 and three destinations namely D_1, D_2, D_3. How much amount of material be supplied from different sources to all other destinations so that total cost of transportation and time of transportation is minimum. Data for time is:

Destinations → Sources ↓	D_1	D_2	D_3	Supply
S_1	16	19	12	14
S_2	22	13	19	16
S_3	14	28	8	12
Demand	10	15	17	42

Data for cost is:

Destinations → Sources ↓	D_1	D_2	D_3	Supply
S_1	9	14	12	14
S_2	16	10	14	16
S_3	8	20	6	12
Demand	10	15	17	42

Solution:

The given problem may be formulated as:

Minimize $Z_1 = 16X_{11} + 19X_{12} + 12X_{13} + 22X_{21} + 13X_{22} + 19X_{23} + 14X_{31} + 28X_{32} + 8X_{33}$
Minimize $Z_2 = 9X_{11} + 14X_{12} + 12X_{13} + 16X_{21} + 10X_{22} + 14X_{23} + 8X_{31} + 20X_{32} + 6X_{33}$
subject to

$$\sum_{j=1}^{3} X_{1j} = 14, \quad \sum_{j=1}^{3} X_{2j} = 16, \quad \sum_{j=1}^{3} X_{3j} = 12$$
$$\sum_{i=1}^{3} X_{i1} = 10, \quad \sum_{i=1}^{3} X_{i2} = 15, \quad \sum_{i=1}^{3} X_{i3} = 17$$
$$X_{ij} \geq 0, \quad i = 1,2,3; \quad j = 1,2,3.$$

where Z_1, Z_2 represents total cost and total time of transportation respectively.

Solving

Minimize $Z_1 = 16X_{11} + 19X_{12} + 12X_{13} + 22X_{21} + 13X_{22} + 19X_{23} + 14X_{31} + 28X_{32} + 8X_{33}$
subject to
$$\sum_{j=1}^{3} X_{1j} = 14, \quad \sum_{j=1}^{3} X_{2j} = 16, \quad \sum_{j=1}^{3} X_{3j} = 12$$
$$\sum_{i=1}^{3} X_{i1} = 10, \quad \sum_{i=1}^{3} X_{i2} = 15, \quad \sum_{i=1}^{3} X_{i3} = 17$$
$$X_{ij} \geq 0, \quad i = 1,2,3; \quad j = 1,2,3.$$

we find the optimal solution as

$$X^{(1)} = \begin{cases} X_{11} = 10, \ X_{13} = 4, X_{22} = 15, \\ X_{23} = 1, \quad X_{33} = 12, \\ \text{and rest all are zeros} \end{cases}$$

$$Z_1\left(X^{(1)}\right) = 517 \quad \text{and} \quad Z_2\left(X^{(1)}\right) = 379$$

Solving

Minimize $Z_2 = 9X_{11} + 14X_{12} + 12X_{13} + 16X_{21} + 10X_{22} + 14X_{23} + 8X_{31} + 20X_{32} + 6X_{33}$
subject to
$$\sum_{j=1}^{3} X_{1j} = 14, \quad \sum_{j=1}^{3} X_{2j} = 16, \quad \sum_{j=1}^{3} X_{3j} = 12$$
$$\sum_{i=1}^{3} X_{i1} = 10, \quad \sum_{i=1}^{3} X_{i2} = 15, \quad \sum_{i=1}^{3} X_{i3} = 17$$
$$X_{ij} \geq 0, \quad i = 1,2,3; \quad j = 1,2,3.$$

we find the optimal solution as

$$X^{(2)} = \begin{cases} X_{11} = 9,\ X_{13} = 5,\ X_{21} = 1, \\ X_{22} = 15,\ X_{33} = 12, \\ \text{and rest all } X_{ij} \text{ are zeros} \end{cases}$$

$$Z_2\left(X^{(2)}\right) = 374 \quad \text{and} \quad Z_1\left(X^{(1)}\right) = 518$$

using these results, obtained in step 1, the pay-off matrix is

	$Z_1(X)$	$Z_2(X)$
$X^{(1)}$	517	379
$X^{(2)}$	518	374

From the pay-off matrix we find

$$U_1 = \text{maximum}\{517,518\} = 518 \qquad U_2 = \text{maximum}\{379,374\} = 379$$

$$L_1 = \text{minimum}\{517,518\} = 517 \qquad L_2 = \text{minimum}\{379,374\} = 374$$

If we will use a linear membership function, the crisp model can be presented as follows:

$$\begin{aligned}
&\text{Maximize } \lambda \\
&\text{subject to} \\
&16X_{11} + 19X_{12} + 12X_{13} + 22X_{21} + 13X_{22} + 19X_{23} + 14X_{31} + 28X_{32} + 8X_{33} + \lambda \le 518 \\
&9X_{11} + 14X_{12} + 12X_{13} + 16X_{21} + 10X_{22} + 14X_{23} + 8X_{31} + 20X_{32} + 6X_{33} + 5\lambda \le 379 \\
&\sum_{j=1}^{3} X_{1j} = 14, \quad \sum_{j=1}^{3} X_{2j} = 16, \quad \sum_{j=1}^{3} X_{3j} = 12 \\
&\sum_{i=1}^{3} X_{i1} = 10, \quad \sum_{i=1}^{3} X_{i2} = 15, \quad \sum_{i=1}^{3} X_{i3} = 17 \\
&X_{ij} \ge 0, \quad i = 1,2,3; \quad j = 1,2,3 \ \text{and} \ \lambda \ge 0
\end{aligned}$$

The optimal solution of the problem is presented as:

$$X^* = \begin{cases} X_{11} = 9.5,\ X_{13} = 4.5,\ X_{21} = 0.5, \\ X_{22} = 15.0,\ X_{23} = 0.5,\ X_{33} = 12.0, \\ \text{and rest all } X_{ij} \text{ are zeros} \end{cases}$$

$Z_1^* = 517.5 \quad Z_2^* = 376.5$ and $\lambda^* = 0.50$

If we will use the exponential membership function with the parameter $s = 1$, an equivalent crisp model can be formulated as:

Maximize λ

subject to

$$\exp\{\frac{(-16X_{11} - 19X_{12} - 12X_{13} - 22X_{21} - 13X_{22} - 19X_{23} - 14X_{31} - 28X_{32} - 8X_{33} + 517)}{1}\} - 0.6321205\lambda \geq 0.3678794$$

$$\exp\{\frac{(-9X_{11} - 14X_{12} - 12X_{13} - 16X_{21} - 10X_{22} - 14X_{23} - 8X_{31} - 20X_{32} - 6X_{33} + 374)}{5}\} - 0.6321205\lambda \geq 0.3678794$$

$$\sum_{j=1}^{3} X_{1j} = 14, \quad \sum_{j=1}^{3} X_{2j} = 16, \quad \sum_{j=1}^{3} X_{3j} = 12$$

$$\sum_{i=1}^{3} X_{i1} = 10, \quad \sum_{i=1}^{3} X_{i2} = 15, \quad \sum_{i=1}^{3} X_{i3} = 17$$

$$X_{ij} \geq 0, \quad i = 1,2,3; \quad j = 1,2,3 \text{ and } \lambda \geq 0$$

Solving the above problem by Linear Interactive General Optimizer (LINGO) software. The optimal solution is presented as follows:

$$X^* = \begin{cases} X_{11} = 9.500013,\ X_{13} = 4.499987,\ X_{21} = 0.499987, \\ X_{22} = 15.0000,\ X_{23} = 0.500013,\ X_{33} = 12.00000, \\ \text{and rest all } X_{ij} \text{ are zeros} \end{cases}$$

$Z_1^* = 517.50001, \quad Z_2^* = 376.49994$

and

$\lambda^* = 0.377491$

However, if we use the hyperbolic membership function, an equivalent crisp model can be formulated as:

Maximize X_{10}

subject to

$$96X_{11}+114X_{12}+72X_{13}+132X_{21}+78X_{22}+114X_{23}+84X_{31}+ \\ 168X_{32}+48X_{33}+X_{10}\le 3105$$

$$54X_{11}+84X_{12}+72X_{13}+96X_{21}+60X_{22}+84X_{23}+48X_{31}+ \\ 120X_{32}+36X_{33}+X_{10}\le 2259$$

$$\sum_{j=1}^{3}X_{1j}=14,\quad \sum_{j=1}^{3}X_{2j}=16,\quad \sum_{j=1}^{3}X_{3j}=12$$

$$\sum_{i=1}^{3}X_{i1}=10,\quad \sum_{i=1}^{3}X_{i2}=15,\quad \sum_{i=1}^{3}X_{i3}=17$$

$$X_{ij}\ge 0,\ i=1,2,3;\ j=1,2,3 \text{ and } X_{10}\ge 0$$

Solving the above problem, the optimal solution is presented as follows:

$$X^{*} = \begin{cases} X_{11}=9.5,\ X_{13}=4.5,\ X_{21}=0.5, \\ X_{22}=15,\ X_{23}=0.5,\ X_{33}=12, \\ \text{and all rest } X_{ij} \text{ are zeros} \end{cases}$$

$X_{10}=0.00$

Therefore

$Z_1^{*}=517.5 \quad Z_2^{*}=376.5 \quad$ and $\quad \lambda^{*}=0.50$

3.4 Conclusion

In this chapter, two special type of membership functions linear and non-linear are used to solve the MOTP. If we use the hyperbolic membership function, then the crisp model becomes linear. The optimal compromise solution does not change if we compare with the solution obtained by the linear membership function. However, if we use the exponential type membership function, with different values of s then the crisp model becomes non-linear and the optimal compromise solution does not change significantly, if we compare with the solution obtained by the linear membership function. It is shown that the fuzzy optimal values does not depend on the chosen membership function whether we use linear or non-linear membership function.

Chapter 4

FUZZY PROGRAMMING TECHNIQUE TO SOLVE MULTI-OBJECTIVE CAPACITATED TRANSPORTATION PROBLEMS

In this chapter, fuzzy programming technique to solve multi-objective capacitated transportation problems in which the supply and demand constraints are equality type, capacity restriction on each route are specified and the objectives are conflicting in nature, is presented. To illustrate the presented technique a numerical example is solved.

4.1 Multi-objective capacitated transportation problem

Consider m origins and n destination. At each origin O_i, let a_i be the amount of a homogeneous product which we want to transport to n destinations D_j to satisfy the demand for b_j units of the product there. A penalty c_{ij}^k is associated with transportation of a unit of the product from source i to destination j for the k^{th} criterion. The penalty could represent transportation cost, delivery time, quantity of goods delivered, under used capacity. A variable X_{ij} represents the unknown quantity to be transported from origin O_i to destination D_j. Let r_{ij} be the capacity restrictions on route i, j for capacitated transportation problem.

A multi-objective capacitated transportation problem can be stated as:

$$\text{Minimize } Z_k = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^k X_{ij}, \quad k = 1, 2, ..., K$$

subject to

$$\sum_{j=1}^{n} X_{ij} = a_i, \quad i = 1, 2, ..., m$$
$$\sum_{i=1}^{m} X_{ij} = b_j, \quad j = 1, 2, ..., n$$
$$0 \le X_{ij} \le r_{ij} \quad \forall\ i \text{ and } j$$

The subscript on Z_k and superscript on c_{ij}^k denote the k^{th} penalty criterion. We assume that $a_i > 0$ for all i, $b_j > 0$ for all j, $r_{ij} \ge 0$ for all i and j, and

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \qquad (\text{balanced condition})$$

This balanced condition is necessary condition for the problem to have a feasible solution, however, this is not sufficient because of the capacity restriction on each route.

4.2 Fuzzy programming technique to solve multi-objective capacitated transportation problem

To solve the multi-objective capacitated transportation problem we will use the fuzzy programming technique used to solve the MOTP in chapter 3. Here, we will use three different types of membership functions namely linear, exponential, hyperbolic. The crisp model formed by all the above membership functions is same as that of the MOTP as explained in chapter 3, with the only difference that in this case we have capacity restriction on each route i.e.

(a) With linear membership function, we have the formulation as:

Maximize λ
subject to
$$Z_k + \lambda(U_k - L_k) \le U_k,\ k = 1, 2, ..., K$$

$$\sum_{j=1}^{n} X_{ij} = a_i, \quad i = 1, 2, \ldots, m$$
$$\sum_{i=1}^{m} X_{ij} = b_j, \quad j = 1, 2, \ldots, n$$
$$0 \le X_{ij} \le r_{ij} \qquad \forall\, i, j$$
$$\lambda \ge 0$$

(b) With exponential membership function the formulation of the equivalent crisp model is as follows:

Maximize λ
subject to
$$e^{-s\psi_k(X)} - (1 - e^{-s})\lambda \ge e^{-s}, \qquad k = 1, 2, \ldots, K$$
$$\sum_{j=1}^{n} X_{ij} = a_i, \quad i = 1, 2, \ldots, m$$
$$\sum_{i=1}^{m} X_{ij} = b_j, \quad j = 1, 2, \ldots, n$$
$$0 \le X_{ij} \le r_{ij} \qquad \forall\, i, j$$
$$\lambda \ge 0$$

where

$$\psi_k(X) = \frac{(Z_k - L_k)}{(U_k - L_k)}, \qquad k = 1, 2, \ldots, K$$

(c) With hyperbolic membership function the formulation of the equivalent crisp model is given by:

Maximize X_{mn+1}
subject to
$$\alpha_k Z_k(X) + X_{mn+1} \le \frac{\alpha_k}{2}(U_k + L_k), \quad k = 1, 2, \ldots, K$$
$$\sum_{j=1}^{n} X_{ij} = a_i, \quad i = 1, 2, \ldots, m$$
$$\sum_{i=1}^{m} X_{ij} = b_j, \quad j = 1, 2, \ldots, n$$

$$0 \le X_{ij} \le r_{ij} \quad \forall\, i, j$$

where

$$X_{mn+1} = \tanh^{-1}(2\lambda - 1)$$

4.3 Numerical example

In this section, a multi-objective capacitated transportation problem is solved by using the method explained in section 4.2 with all types of membership function.

Example 4.1

$$\text{Minimize } Z_1 = 5X_{11} + 3X_{12} + 2X_{13} + 6X_{21} + 4X_{22} + 7X_{23} + 2X_{31} + 8X_{32} + 6X_{33}$$
$$\text{Minimize } Z_2 = 4X_{11} + 6X_{12} + 5X_{13} + 7X_{21} + 8X_{22} + 6X_{23} + 5X_{31} + 2X_{32} + 3X_{33}$$
$$\text{Minimize } Z_3 = 9X_{11} + 9X_{12} + 7X_{13} + 3X_{21} + 9X_{22} + 3X_{23} + 7X_{31} + 9X_{32} + 10X_{33}$$

subject to

$$\sum_{j=1}^{3} X_{1j} = 120, \quad \sum_{j=1}^{3} X_{2j} = 145, \quad \sum_{j=1}^{3} X_{3j} = 95 \tag{4.1}$$

$$\sum_{i=1}^{3} X_{i1} = 80, \quad \sum_{i=1}^{3} X_{i2} = 100, \quad \sum_{i=1}^{3} X_{i3} = 180 \tag{4.2}$$

$$X_{ij} \ge 0, \quad i = 1,2,3; \quad j = 1,2,3$$

capacity restrictions of the route are given as:

$$\begin{aligned} &0 \le x_{11} \le 45, \quad 0 \le x_{12} \le 60, \quad 0 \le x_{13} \le 100 \\ &0 \le x_{21} \le 90, \quad 0 \le x_{22} \le 100, \quad 0 \le x_{23} \le 80 \\ &0 \le x_{31} \le 125, \quad 0 \le x_{32} \le 85, \quad 0 \le x_{33} \le 130 \end{aligned} \tag{4.3}$$

Solution: Solving

$$\text{Minimize } Z_1 = 5X_{11} + 3X_{12} + 2X_{13} + 6X_{21} + 4X_{22} + 7X_{23} + 2X_{31} + 8X_{32} + 6X_{33}$$

subject to

$$\sum_{i=1}^{3} X_{i1} = 80, \quad \sum_{i=1}^{3} X_{i2} = 100, \quad \sum_{i=1}^{3} X_{i3} = 180$$

$$X_{ij} \ge 0, \quad i = 1,2,3; \quad j = 1,2,3$$

capacity restrictions of the route are given as:

$$\begin{aligned} &0 \le x_{11} \le 45, \quad 0 \le x_{12} \le 60, \quad 0 \le x_{13} \le 100 \\ &0 \le x_{21} \le 90, \quad 0 \le x_{22} \le 100, \quad 0 \le x_{23} \le 80 \\ &0 \le x_{31} \le 125, \quad 0 \le x_{32} \le 85, \quad 0 \le x_{33} \le 130 \end{aligned}$$

The optimal solution is found to be

$$X^1 = \begin{cases} X_{11} = 20,\ X_{12} = 60, X_{13} = 40, X_{21} = 25 \\ X_{22} = 40,\ X_{23} = 80, X_{31} = 35, X_{33} = 60 \\ \text{and rest all are zeros} \end{cases}$$

$$Z_1\left(X^{(1)}\right) = 1660,\ Z_2\left(X^{(1)}\right) = 1570,\ Z_3\left(X^{(1)}\right) = 2670$$

Similarly, solving

$$\text{Minimize } Z_2 = 4X_{11} + 6X_{12} + 5X_{13} + 7X_{21} + 8X_{22} + 6X_{23} + 5X_{31} + 2X_{32} + 3X_{33}$$

under the same constraints (4.1),(4.2), (4.3), the optimal solution is found to be

$$X^2 = \begin{cases} X_{11} = 45,\ X_{12} = 35, X_{13} = 40, X_{21} = 35 \\ X_{22} = 30,\ X_{23} = 80, X_{31} = 35, X_{33} = 60 \\ \text{and rest all are zeros} \end{cases}$$

$$Z_1\left(X^{(2)}\right) = 1935,\ Z_2\left(X^{(2)}\right) = 1805,\ Z_3\left(X^{(2)}\right) = 2530$$

and solving

$$\text{Minimize } Z_3 = 9X_{11} + 9X_{12} + 7X_{13} + 3X_{21} + 9X_{22} + 3X_{23} + 7X_{31} + 9X_{32} + 10X_{33}$$

under the same constraints (4.1), (4.2), (4.3), the optimal solution is found to be

$$X^3 = \begin{cases} X_{11} = 20,\ X_{12} = 60, X_{13} = 40, X_{21} = 60 \\ X_{22} = 5,\ X_{23} = 80, X_{32} = 35, X_{33} = 60 \\ \text{and rest all are zeros} \end{cases}$$

$$Z_1\left(X^{(3)}\right) = 1940,\ Z_2\left(X^{(3)}\right) = 2190,\ Z_3\left(X^{(3)}\right) = 2380$$

using these results, the pay-off matrix is

	$Z_1(X)$	$Z_2(X)$	$Z_3(X)$
$X^{(1)}$	1660	1970	2520
$X^{(2)}$	1935	1805	2530
$X^{(3)}$	1940	2190	2380

$$\begin{array}{l}
U_1 = \text{maximum}\{1660, 1935, 1940\} = 1940, \quad L_1 = \text{minimum}\{1660, 1935, 1940\} = 1660 \\
U_2 = \text{maximum}\{1970, 1805, 2190\} = 2190, \; L_2 = \text{minimum}\{1970, 1805, 2190\} = 1805 \\
U_3 = \text{maximum}\{2520, 2530, 2380\} = 2530, \; L_3 = \text{minimum}\{2520, 2530, 2380\} = 2380
\end{array}$$

Now, to find the optimal solution of the given problem, equivalent crisp model can be formed by using different membership functions.

Case (a) Solving the problem using linear membership function

If we use the linear membership function, the equivalent crisp model can be formulaed as:

$$\begin{array}{l}
\text{Maximize } \lambda \\
\text{subject to} \\
5X_{11} + 3X_{12} + 2X_{13} + 6X_{21} + 4X_{22} + 7X_{23} + 2X_{31} + 8X_{32} + 6X_{33} + 280\lambda \le 1940 \\
4X_{11} + 6X_{12} + 5X_{13} + 7X_{21} + 8X_{22} + 6X_{23} + 5X_{31} + 2X_{32} + 3X_{33} + 385\lambda \le 2190 \\
9X_{11} + 9X_{12} + 7X_{13} + 3X_{21} + 9X_{22} + 3X_{23} + 7X_{31} + 9X_{32} + 10X_{33} + 150\lambda \le 2530 \\
\sum_{j=1}^{3} X_{1j} = 120, \quad \sum_{j=1}^{3} X_{2j} = 145, \quad \sum_{j=1}^{3} X_{3j} = 95 \\
\sum_{i=1}^{3} X_{i1} = 80, \quad \sum_{i=1}^{3} X_{i2} = 100, \quad \sum_{i=1}^{3} X_{i3} = 180 \\
X_{ij} \ge 0, \; i = 1, 2, 3; \; j = 1, 2, 3 \\
\text{capacity restrictions of the route are given as:} \\
0 \le x_{11} \le 45, \quad 0 \le x_{12} \le 60, \quad 0 \le x_{13} \le 100 \\
0 \le x_{21} \le 90, \quad 0 \le x_{22} \le 100, \quad 0 \le x_{23} \le 80 \\
0 \le x_{31} \le 125, \quad 0 \le x_{32} \le 85, \quad 0 \le x_{33} \le 130
\end{array}$$

Solving this problem, the optimal solution of the given problem is found to be:

$$X^* = \begin{cases} X_{11} = 20, X_{12} = 60, X_{13} = 40, X_{21} = 41.896553, X_{22} = 23.103449 \\ X_{23} = 80, X_{31} = 18.103449, X_{32} = 16.896551, X_{33} = 60 \end{cases}$$

$Z_1^* = 1789.3943 \quad Z_2^* = 1715.3103 \quad Z_3^* = 2448.7931$ and $\lambda = 0.5172$

Case (b) Solving the problem using exponential membership function

If we use the exponential membership function with value of $s = 1$, equivalent crisp model is as follows:

Maximize λ

subject to

$$\exp\{\frac{(-5X_{11} - 3X_{12} - 2X_{13} - 6X_{21} - 4X_{22} - 7X_{23} - 2X_{31} - 8X_{32} - 6X_{33} + 1660)}{280}\} - 0.6321205\lambda \geq 0.3678794$$

$$\exp\{\frac{(-4X_{11} - 6X_{12} - 5X_{13} - 7X_{21} - 8X_{22} - 6X_{23} - 5X_{31} - 2X_{32} - 3X_{33} + 1805)}{385}\} - 0.6321205\lambda \geq 0.3678794$$

$$\exp\{\frac{(-9X_{11} - 9X_{12} - 7X_{13} - 3X_{21} - 9X_{22} - 3X_{23} - 7X_{31} - 9X_{32} - 10X_{33} + 2380)}{150}\} - 0.6321205\lambda \geq 0.3678794$$

$$\sum_{j=1}^{3} X_{1j} = 120, \quad \sum_{j=1}^{3} X_{2j} = 145, \quad \sum_{j=1}^{3} X_{3j} = 95$$

$$\sum_{i=1}^{3} X_{i1} = 80, \quad \sum_{i=1}^{3} X_{i2} = 100, \quad \sum_{i=1}^{3} X_{i3} = 180$$

$X_{ij} \geq 0, \ i = 1,2,3; \ \ j = 1,2,3$

capacity restrictions of the route are given as:

$0 \leq x_{11} \leq 45, \quad 0 \leq x_{12} \leq 60, \quad 0 \leq x_{13} \leq 100$

$0 \leq x_{21} \leq 90, \quad 0 \leq x_{22} \leq 100, \quad 0 \leq x_{23} \leq 80$

$0 \leq x_{31} \leq 125, \quad 0 \leq x_{32} \leq 85, \quad 0 \leq x_{33} \leq 130$

Solving above problem by using LINGO software, optimal solution of the problem is found to be:

$$X^* = \begin{cases} X_{11} = 20.0000, X_{12} = 60.00000, X_{13} = 40.0000, \\ X_{21} = 43.353267, X_{22} = 22.303449, X_{23} = 79.343286, \\ X_{31} = 16.646733, X_{32} = 17.696551, X_{33} = 60.656714 \end{cases}$$

$Z_1^* = 1803.5425$, $Z_2^* = 1898.5570$, $Z_3^* = 2451.18393$ and $\lambda = .8070$

Case (c) Solving the problem using hyperbolic membership function

When hyperbolic membership function is used to find the optimal solution of the given problem, equivalent crisp model is presented as:

$$\begin{aligned}
&\text{Maximize } X_{3\times 3+1} \\
&\text{subject to} \\
&30X_{11} + 18X_{12} + 21X_{13} + 36X_{21} + 24X_{22} + 42X_{23} + 12X_{31} + 38X_{32} + 36X_{33} + 280X_{10} \le 10800 \\
&24X_{11} + 36X_{12} + 30X_{13} + 42X_{21} + 48X_{22} + 36X_{23} + 30X_{31} + 12X_{32} + 18X_{33} + 385X_{10} \le 11985 \\
&54X_{11} + 54X_{12} + 42X_{13} + 18X_{21} + 54X_{22} + 18X_{23} + 42X_{31} + 54X_{32} + 60X_{33} + 150X_{10} \le 14730 \\
&\sum_{j=1}^{3} X_{1j} = 120, \quad \sum_{j=1}^{3} X_{2j} = 145, \quad \sum_{j=1}^{3} X_{3j} = 95 \\
&\sum_{i=1}^{3} X_{i1} = 80, \quad \sum_{i=1}^{3} X_{i2} = 100, \quad \sum_{i=1}^{3} X_{i3} = 180 \\
&X_{ij} \ge 0, \ i = 1,2,3; \ \ j = 1,2,3 \\
&\text{capacity restrictions of the route are given as:} \\
&0 \le x_{11} \le 45, \quad 0 \le x_{12} \le 60, \quad 0 \le x_{13} \le 100 \\
&0 \le x_{21} \le 90, \quad 0 \le x_{22} \le 100, \quad 0 \le x_{23} \le 80 \\
&0 \le x_{31} \le 125, \quad 0 \le x_{32} \le 85, \quad 0 \le x_{33} \le 130
\end{aligned}$$

Solving the above problem by using the optimal solution is found to be:

$$X^* = \begin{cases} X_{11} = 20, X_{12} = 60, X_{13} = 40, X_{21} = 41.896553, X_{22} = 23.103449 \\ X_{23} = 80, X_{31} = 18.103449, X_{32} = 16.896551, X_{33} = 60 \end{cases}$$

$Z_1^* = 1789.3943$ $\quad Z_2^* = 1715.3103$ $\quad Z_3^* = 2448.7931$ and $X_{10} = 0.1034$

4.4 Conclusion

In this chapter, multi-objective capacitated transportation problem is solved by using fuzzy programming technique with linear as well as non-linear membership functions. It has been found that for a multi-objective capacitated transportation problem with k objective functions, this technique with all type of membership functions gives same k non-dominated solutions and an optimal compromise solution.

Chapter 5

FUZZY PROGRAMMING TECHNIQUE TO SOLVE MULTI-OBJECTIVE INTERVAL TRANSPORTATION PROBLEMS

In this chapter, we focus on the solution procedure of the MOTP where the cost coefficients of the objective functions, and the source and destination parameters are expressed as interval values by the decision maker. This problem is transformed into a classical MOTP where to minimize the interval objective function, the order relations that represent the decision maker's preference between interval profits is defined by the right limit, left limit, centre, and half-width of an interval. The constraints with interval source and destination parameters are converted into deterministic ones. Finally, the equivalent transformed problem is solved by fuzzy programming technique.

5.1 Interval in terms of its centre and width

An interval is defined by an ordered pair of brackets as:

$$A=[a_L, a_R]=\{a : a_L \le a \le a_R, a \in R\},$$

where a_L and a_R are, respectively, the left and right limits of A. The interval is also denoted by its centre and width as

$$A=\langle a_C, a_W \rangle=\{a : a_C - a_W \le a \le a_C + a_W, a \in R\},$$

where $a_C = \frac{(a_R + a_L)}{2}$ and $a_W = \frac{(a_R - a_L)}{2}$ are, respectively, the centre and half width of A.

Definition 5.1

Let $* \in (+,-,.,\%)$ be a binary operation on the set of real numbers. If A and B are closed intervals, then $A*B=\{a*b : a \in A, b \in B\}$ defines a binary operation on the set of closed intervals. In the case of division, it is assumed that $0 \notin B$.

According to above definition interval operations are defined as:

(1) $A+B=[a_L,a_R]+[b_L,b_R]=[a_L+b_L,a_R+b_R]$

(2) $A+B=\langle a_C,a_W\rangle+\langle b_C,b_W\rangle=\langle a_C+b_C,a_W+b_W\rangle$

(3) $kA=k[a_L,a_R]=\begin{cases}[ka_L,ka_R] & if\ k \geq 0\\ [ka_R,ka_L] & if\ k<0\end{cases}$

(4) $kA=k\langle a_C,a_W\rangle=\langle ka_C,|k|a_W\rangle$

where k is a real number.

Let the order relations represent the decision maker's preference between interval costs. Let the uncertain costs from two alternatives be represented by intervals A and B, respectively. It is assumed that the cost of each alternative is known only to lie in the corresponding interval.

Definition 5.2 The order relation $\leq_{LR}$ between $A=[a_L,a_R]$ and $B=[b_L,b_R]$ is defined as

$A \leq_{LR} B$ iff $a_L \leq b_L$ and $a_R \leq b_R$

$A <_{LR} B$ iff $A \leq_{LR} B$ and $A \neq B$

if $A \leq_{LR} B$; then A is preferred to B.

Definition 5.3 The order relation $\leq_{CW}$ between $A=\langle a_C,a_W\rangle$ and $B=\langle b_C,b_W\rangle$ is defined as

$A \leq_{CW} B$ iff $a_C \leq b_C$ and $a_W \leq b_W$

$A <_{CW} B$ iff $A \leq_{CW} B$ and $A \neq B$

The order relation $\leq_{CW}$ represents the decision maker's preference for the alternative with the lower expected cost and less uncertainty, i.e. if $A \leq_{CW} B$, then A is preferred to B.

5.2 Multi-objective interval transportation problem (MITP)

The formulation of MITP is the problem of minimizing K interval valued objective functions with interval source and interval destination parameters is given in the section 5.2.1 and an efficient algorithm is presented to find the optimal solution of MITP.

5.2.1 Linear programming formulation of MITP

The mathematical model of MITP when all the cost coefficient, supply and demand are interval-valued is given by:

$$\text{Minimize } Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n}[c^k_{L_{ij}}, c^k_{R_{ij}}]x_{ij} \text{ where } k = 1, 2, ..., K,$$

subject to

$$\sum_{j=1}^{n} x_{ij} = [a_{L_i}, a_{R_i}], \quad i = 1, 2, ..., m,$$

$$\sum_{i=1}^{m} x_{ij} = [b_{L_j}, b_{R_j}], \quad j = 1, 2, ..., n,$$

$$x_{ij} \geq 0, \quad i = 1, 2, ..., m, \quad j = 1, 2, ..., n,$$

with

$$\sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j} \quad \text{and} \quad \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j},$$

where $[c^k_{L_{ij}}, c^k_{R_{ij}}], (k = 1, 2, ..., K)$ is an interval representing the uncertain cost for the transportation problem; it can represent delivery time, quantity of goods delivered, under used capacity, etc. The source parameter lies between left limit a_{L_i} and right limit a_{R_i}. Similarly, destination parameter lies between left limit b_{L_j} and right limit b_{R_j}.

5.3 Formulation of the crisp objective function

Let S be the set of all feasible solution of the problem given in section 5.2.1.

Definition 5.4 $x^0 \in S$ is an optimal solution of the problem (section 5.2.1) iff there is no other solution $x \in S$ which satisfies $Z(x) <_{LR} Z(x^0)$ or $Z(x) <_{CW} Z(x^0)$

Definition 5.5 $A \leq_{RC} B$ iff $A \leq_{LR} B$ or $A \leq_{CW} B$

$A <_{RC} B$ iff $A <_{LR} B$ or $A <_{CW} B$

where the order relation $\leq_{RC}$ is defined as $A \leq_{RC} B$ iff $a_R \leq b_R$ and $a_c \leq b_c, A <_{RC} B$ iff $A \leq_{RC} B$ and $A \neq B.$

Definition 5.6 $x^0 \in S$ is an optimal solution of problem (section 5.2.1) iff there is no other solution $x \in S$ which satisfies $Z(x) <_{RC} Z(x^0).$

The right limit $Z_R^k(x)$ of the interval objective function $Z_R^k(x)$ in given problem may be elicited as:

$$Z_R^k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{C_{ij}}^k x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{W_{ij}}^k \left|x_{ij}\right|$$

where $c_{C_{ij}}^k$ is the centre and $c_{W_{ij}}^k$ is the half-width of the coefficients c_{ij}^k of $Z^k\left(x\right)$.

In this case when $x_{ij} \geq 0,\ i = 1,2,...,m,\ j = 1,2,...,n,\ Z_R^k\left(x\right)$ can be modified as:

$$Z_R^k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{C_{ij}}^k x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{W_{ij}}^k x_{ij.}$$

The centre of the objective function, Z_C^k can be elicited as:

$$Z_C^k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{C_{ij}}^k x_{ij}.$$

The solution set of the given equation, by Definition 5.6, can be taken as the optimal solution of the following multi-objective problem

Minimize $\{Z_R^k, Z_C^k\},\ k = 1,2,...,K$

subjected to given constraints.

5.4 Cases of MITP

Three major cases that may arise in a MITP can be described as:

I. The coefficients c_{ij}^{k} are in the form of interval, whereas source and destination parameters are deterministic.

II. The source and destination parameters, i.e., a_i and b_j, are in the form of intervals but the objective function's coefficients c_{ij}^{k} are deterministic.

III. All parameters, i.e., objective function's coefficients, the source (a_i) and destination (b_j) parameters are in the form of interval.

Case-I

When the objective function's coefficients c_{ij}^{k} are in the form interval, i.e., $c_{ij}^{k}=[c_{L_{ij}}^{k},c_{R_{ij}}^{k}]$; and the constraints are deterministic, i.e., the parameters a_i and b_j are deterministic, the MITP is as follows:

$$\text{Minimize } Z^k=\sum_{i=1}^{m}\sum_{j=1}^{n}\left[c_{L_{ij}}^{k},c_{R_{ij}}^{k}\right]x_{ij} \text{ where } k=1,2,...,K$$

subject to

$$\sum_{j=1}^{n}x_{ij}=a_i \tag{5.1}$$

$$\sum_{i=1}^{m}x_{ij}=b_j \tag{5.2}$$

$$x_{ij}\geq 0,\ i=1,2,...,m,\ j=1,2,...,n \tag{5.3}$$

$$\sum_{i=1}^{m}a_i=\sum_{j=1}^{n}b_j \tag{5.4}$$

The corresponding equivalent classical MOTP can be restated as:

$$\text{Minimize } Z_R^k(x)=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{C_{ij}}^{k}x_{ij}+\sum_{i=1}^{m}\sum_{j=1}^{n}c_{W_{ij}}^{k}x_{ij}$$

$$\text{Minimize } Z_C^k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{C_{ij}}^k x_{ij}$$

where $k = 1, 2, ..., K$ subject to the constraints (5.1), (5.2) non-negativity conditions (5.3) and balanced condition (5.4) defined above. Now, fuzzy programming technique, as described in the earlier section, is applied to obtain the optimal solution.

Example 5.1

$$\begin{aligned}
&\text{Minimize } Z^1 = \sum_{i=1}^{3}\sum_{j=1}^{4} [c_{L_{ij}}^1, c_{R_{ij}}^1] x_{ij} \\
&\text{Minimize } Z^2 = \sum_{i=1}^{3}\sum_{j=1}^{4} [c_{L_{ij}}^2, c_{R_{ij}}^2] x_{ij} \\
&\text{subject to} \\
&\sum_{j=1}^{4} x_{1j} = 8,\ \sum_{j=1}^{4} x_{2j} = 19,\ \sum_{j=1}^{4} x_{3j} = 17,\ \sum_{i=1}^{3} x_{i1} = 11 \\
&\sum_{i=1}^{3} x_{i2} = 3,\ \sum_{i=1}^{3} x_{i3} = 14,\ \sum_{i=1}^{3} x_{i4} = 16 \qquad (5.5) \\
&x_{ij} \geq 0,\ i = 1, 2, 3, \quad j = 1, 2, 3, 4
\end{aligned}$$

where

$$c^1 = \begin{bmatrix} [1,2] & [1,3] & [5,9] & [4,8] \\ [1,2] & [7,10] & [2,6] & [3,5] \\ [7,9] & [7,11] & [3,5] & [5,7] \end{bmatrix}, \qquad c^2 = \begin{bmatrix} [3,5] & [2,6] & [2,4] & [1,5] \\ [4,6] & [7,9] & [7,10] & [9,11] \\ [4,8] & [1,3] & [3,6] & [1,2] \end{bmatrix},$$

By above formulation, equivalent deterministic problem as

$$\text{Minimize } Z_R^1 = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{R_{ij}}^1 x_{ij}, \qquad \text{Minimize } Z_R^2 = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{R_{ij}}^2 x_{ij},$$

$$\text{Minimize } Z_C^1 = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{C_{ij}}^1 x_{ij}, \qquad \text{Minimize } Z_C^1 = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{C_{ij}}^1 x_{ij},$$

where

$$c_{R_{ij}}^1 = \begin{bmatrix} 2 & 3 & 9 & 8 \\ 2 & 10 & 6 & 5 \\ 9 & 11 & 5 & 7 \end{bmatrix}, \qquad c_{R_{ij}}^2 = \begin{bmatrix} 5 & 6 & 4 & 5 \\ 6 & 9 & 10 & 11 \\ 8 & 3 & 6 & 2 \end{bmatrix},$$

$$c_{C_{ij}}^1 = \begin{bmatrix} 1.5 & 2 & 7 & 6 \\ 1.5 & 8.5 & 4 & 4 \\ 8 & 9 & 4 & 6 \end{bmatrix}, \qquad c_{C_{ij}}^2 = \begin{bmatrix} 4 & 4 & 3 & 3 \\ 5 & 8 & 8.5 & 10 \\ 6 & 2 & 4.5 & 1.5 \end{bmatrix}$$

subject to above constraints (5.5). i.e. given problem is equivalent to

Minimize $Z_R^1 = 2x_{11} + 3x_{12} + 9x_{13} + 8x_{14} + 2x_{21} + 10x_{22} + 6x_{23} + 5x_{24} + 9x_{31} + 11x_{32} + 5x_{33} + 7x_{34}$

Minimize $Z_R^2 = 5x_{11} + 6x_{12} + 4x_{13} + 5x_{14} + 6x_{21} + 9x_{22} + 10x_{23} + 11x_{24} + 8x_{31} + 3x_{32} + 6x_{33} + 2x_{34}$

Minimize $Z_C^1 = 1.5x_{11} + 2x_{12} + 7x_{13} + 6x_{14} + 1.5x_{21} + 8.5x_{22} + 4x_{23} + 4x_{24} + 8x_{31} + 9x_{32} + 4x_{33} + 6x_{34}$

Minimize $Z_C^2 = 4x_{11} + 4x_{12} + 3x_{13} + 3x_{14} + 5x_{21} + 8x_{22} + 8.5x_{23} + 10x_{24} + 6x_{31} + 2x_{32} + 4.5x_{33} + 1.5x_{34}$

subject to

$$\sum_{j=1}^{4} x_{1j} = 8, \ \sum_{j=1}^{4} x_{2j} = 19, \ \sum_{j=1}^{4} x_{3j} = 17, \ \sum_{i=1}^{3} x_{i1} = 11$$

$$\sum_{i=1}^{3} x_{i2} = 3, \ \sum_{i=1}^{3} x_{i3} = 14, \ \sum_{i=1}^{3} x_{i4} = 16$$

$$x_{ij} \geq 0, \ i = 1,2,3, \quad j = 1,2,3,4$$

Solving this classical MOTP using fuzzy programming technique, we find

	Z_R^1	Z_R^2	Z_C^1	Z_C^2
X_R^1	187	311	148.5	259.5
X_R^2	273	211	218.5	172
X_C^1	187	312	148.5	259.5
X_C^2	273	211	218.5	172

From the above table, for each objective we find

L_R^1 = 187 $\qquad$ U_R^1 = 273

$L_R^2 = 211 \qquad U_R^2 = 312$

$L_C^1 = 148.5 \qquad U_C^1 = 218.5$

$L_C^2 = 172 \qquad U_C^2 = 259.5$

Case (a) Solving the problem by using linear membership function

If we use the linear membership function, as explained in chapter 3, the optimal solution of the problem is found by solving the following linear programming problem.

$$
\begin{aligned}
&\text{Maximize } \lambda \\
&\text{subject to} \\
&Z_k + \lambda\left(U_k - L_k\right) \le U_k \qquad k = 1,2,3,4 \\
&\sum_{j=1}^{4} x_{ij} = 8,\ \sum_{j=1}^{4} x_{ij} = 19,\ \sum_{j=1}^{4} x_{ij} = 17,\ \sum_{i=1}^{3} x_{ij} = 11 \\
&\sum_{i=1}^{3} x_{ij} = 3,\ \sum_{i=1}^{3} x_{ij} = 14,\ \sum_{i=1}^{3} x_{ij} = 16 \\
&x_{ij} \ge 0,\ i = 1,2,3,\ \ j = 1,2,3,4
\end{aligned}
$$

i.e.

$$
\begin{aligned}
&\text{Maximize } \lambda \\
&\text{subject to} \\
&Z_R^1 + 86\lambda \le 273 \\
&Z_R^2 + 101\lambda \le 312 \\
&Z_C^1 + 70\lambda \le 218.5 \\
&Z_C^1 + 87.5\lambda \le 259.5 \\
&\sum_{j=1}^{4} x_{1j} = 8,\ \sum_{j=1}^{4} x_{2j} = 19,\ \sum_{j=1}^{4} x_{3j} = 17,\ \sum_{i=1}^{3} x_{i1} = 11 \\
&\sum_{i=1}^{3} x_{i2} = 3,\ \sum_{i=1}^{3} x_{i3} = 14,\ \sum_{i=1}^{3} x_{i4} = 16 \\
&x_{ij} \ge 0,\ i = 1,2,3,\ \ j = 1,2,3,4
\end{aligned}
$$

i.e. the optimal solution is found by solving the following linear programming problem:

Maximize λ

subject to

$$2x_{11}+3x_{12}+9x_{13}+8x_{14}+2x_{21}+10x_{22}+6x_{23}+5x_{24}+9x_{31}+11x_{32}+5x_{33}+7x_{34}+86\lambda \le 273$$

$$5x_{11}+6x_{12}+4x_{13}+5x_{14}+6x_{21}+9x_{22}+10x_{23}+11x_{24}+8x_{31}+3x_{32}+6x_{33}+2x_{34}+101\lambda \le 312$$

$$1.5x_{11}+2x_{12}+7x_{13}+6x_{14}+1.5x_{21}+8.5x_{22}+4x_{23}+4x_{24}+8x_{31}+9x_{32}+4x_{33}+6x_{34}+70\lambda \le 218.5$$

$$4x_{11}+4x_{12}+3x_{13}+3x_{14}+5x_{21}+8x_{22}+8.5x_{23}+10x_{24}+6x_{31}+2x_{32}+4.5x_{33}+1.5x_{34}+87.5\lambda \le 259.5$$

$$\sum_{j=1}^{4} x_{1j}=8,\ \sum_{j=1}^{4} x_{2j}=19,\ \sum_{j=1}^{4} x_{3j}=17,\ \sum_{i=1}^{3} x_{i1}=11$$

$$\sum_{i=1}^{3} x_{i2}=3,\ \sum_{i=1}^{3} x_{i3}=14,\ \sum_{i=1}^{3} x_{i4}=16$$

$$x_{ij}\ge 0,\ i=1,2,3,\quad j=1,2,3,4$$

Solving this classical transportation problem, the optimal solution of the problem is found to be:

$$X^{*}=\begin{cases} x_{11}=5, x_{12}=3, \\ x_{21}=6,\ x_{23}=11.85,\ x_{24}=1.15, \\ x_{33}=2.15,\ x_{34}=14.85, \end{cases}$$

$$Z^{1}=[172.2, 222.55],\ \ Z^{2}=[206.1, 252.75] \text{ and } \lambda=0.59$$

Case (b) Solving the problem using exponential membership function

If we use the exponential membership function, the optimal solution of the problem 5.1, is given by solving the following linear programming problem:

Maximize λ

subject to

$$e^{-s\psi_k(X)}-\left(1-e^{-s}\right)\lambda \ge e^{-s},\quad k=1,2,3,4$$

$$\sum_{j=1}^{4} x_{1j}=8,\ \sum_{j=1}^{4} x_{2j}=19,\ \sum_{j=1}^{4} x_{3j}=17,\ \sum_{i=1}^{3} x_{i1}=11$$

$$\sum_{i=1}^{3} x_{i2} = 3,\ \sum_{i=1}^{3} x_{i3} = 14,\ \sum_{i=1}^{3} x_{i4} = 16$$

$$x_{ij} \geq 0,\ i = 1,2,3,\quad j = 1,2,3,4$$

where

$$\psi_k = \frac{(Z_k - L_k)}{(U_k - L_k)}$$

Here we use the exponential membership function defined in chapter 3 with the parameter $s = 1$, an equivalent crisp model can be formulated as:

Maximize λ

subject to

$$\exp\left\{\frac{-2x_{11} - 3x_{12} - 9x_{13} - 8x_{14} - 2x_{21} - 10x_{22} - 6x_{23} - 5x_{24} - 9x_{31} - 11x_{32} - 5x_{33} - 7x_{34} + 187}{86}\right\} - 0.6321205\lambda \geq 0.3678794$$

$$\exp\left\{\frac{-5x_{11} - 6x_{12} - 4x_{13} - 5x_{14} - 6x_{21} - 9x_{22} - 10x_{23} - 11x_{24} - 8x_{31} - 3x_{32} - 6x_{33} - 2x_{34} + 211}{101}\right\} - 0.6321205\lambda \geq 0.3678794$$

$$\exp\left\{\frac{-1.5x_{11} - 2x_{12} - 7x_{13} - 6x_{14} - 1.5x_{21} - 8.5x_{22} - 4x_{23} - 4x_{24} - 8x_{31} - 9x_{32} - 4x_{33} - 6x_{34} + 148.5}{70}\right\} - 0.6321205\lambda \geq 0.3678794$$

$$\exp\left\{\frac{-4x_{11} - 4x_{12} - 3x_{13} - 3x_{14} - 5x_{21} - 8x_{22} - 8.5x_{23} - 10x_{24} - 6x_{31} - 2x_{32} - 4.5x_{33} - 1.5x_{34} + 172}{87.5}\right\} - 0.6321205\lambda \geq 0.3678794$$

$$\sum_{j=1}^{4} x_{1j} = 8,\ \sum_{j=1}^{4} x_{2j} = 19,\ \sum_{j=1}^{4} x_{3j} = 17,\ \sum_{i=1}^{3} x_{i1} = 11$$

$$\sum_{i=1}^{3} x_{i2} = 3,\ \sum_{i=1}^{3} x_{i3} = 14,\ \sum_{i=1}^{3} x_{i4} = 16$$

$$x_{ij} \geq 0,\ i = 1,2,3,\quad j = 1,2,3,4$$

This problem is solved by LINGO software and the optimal solution of the problem is found to be

$$X^* = \begin{cases} x_{11} = 5.0000, x_{12} = 3.00000, \\ x_{21} = 6.00000, x_{23} = 11.63543, x_{24} = 1.36457, \\ x_{33} = 2.499987, x_{34} = 14.50001 \\ \text{rest variables are zeros.} \end{cases}$$

$Z^1 = [171.50, 221.63]$ $Z^2 = [207.54, 254.36]$ and $\lambda = .6831$

Case (c) Solving the problem using hyperbolic membership function

If we use the hyperbolic membership function, explained in chapter 3, the optimal solution of the problem is found as:

$$\alpha_1 = \frac{6}{U_1 - L_1} = \frac{6}{273-187} = \frac{3}{43}$$
$$\alpha_2 = \frac{6}{U_2 - L_2} = \frac{6}{312-211} = \frac{6}{101}$$
$$\alpha_3 = \frac{6}{U_3 - L_3} = \frac{6}{218.5-148.5} = \frac{3}{35}$$
$$\alpha_4 = \frac{6}{U_4 - L_4} = \frac{6}{259.5-172} = \frac{6}{87.5}$$

using these values of $\alpha's$ and technique explained in chapter 3, the optimal solution of given problem is found by solving following linear programming problem:

$$\begin{aligned} &\text{Maximize } x_{13} \\ &\text{subject to} \\ &\alpha_k Z_k + x_{13} \le \frac{\alpha_k}{2}(U_k + L_k) \qquad k = 1,2,3,4 \\ &\sum_{j=1}^{4} x_{ij} = 8, \sum_{j=1}^{4} x_{ij} = 19, \sum_{j=1}^{4} x_{ij} = 17, \sum_{i=1}^{3} x_{ij} = 11 \\ &\sum_{i=1}^{3} x_{ij} = 3, \sum_{i=1}^{3} x_{ij} = 14, \sum_{i=1}^{3} x_{ij} = 16 \\ &x_{ij} \ge 0, i = 1,2,3, \quad j = 1,2,3,4 \end{aligned}$$

i.e. Maximize x_{13}
subject to

$$\begin{aligned}
&6x_{11}+9x_{12}+27x_{13}+24x_{14}+6x_{21}+30x_{22}+18x_{23}+15x_{24}+27x_{31}+33x_{32}+ \\
&\qquad\qquad\qquad\qquad 15x_{33}+21x_{34}+43x_{13}\le 690 \\
&30x_{11}+36x_{12}+24x_{13}+30x_{14}+36x_{21}+54x_{22}+60x_{23}+66x_{24}+48x_{31}+18x_{32}+ \\
&\qquad\qquad\qquad\qquad 36x_{33}+12x_{34}+101x_{13}\le 1569 \\
&9x_{11}+12x_{12}+42x_{13}+36x_{14}+9x_{21}+51x_{22}+24x_{23}+24x_{24}+48x_{31}+54x_{32}+ \\
&\qquad\qquad\qquad\qquad 24x_{33}+36x_{34}+70x_{13}\le 1101 \\
&24x_{11}+24x_{12}+18x_{13}+18x_{14}+30x_{21}+48x_{22}+51x_{23}+60x_{24}+36x_{31}+12x_{32}+ \\
&\qquad\qquad\qquad\qquad 27x_{33}+9x_{34}+87.5x_{13}\le 1294.5 \\
&\sum_{j=1}^{4}x_{1j}=8,\ \sum_{j=1}^{4}x_{2j}=19,\ \sum_{j=1}^{4}x_{3j}=17,\ \sum_{i=1}^{3}x_{i1}=11 \\
&\sum_{i=1}^{3}x_{i2}=3,\ \sum_{i=1}^{3}x_{i3}=14,\ \sum_{i=1}^{3}x_{i4}=16 \\
&x_{ij}\ge 0,\ i=1,2,3,\quad j=1,2,3,4
\end{aligned}$$

Solving this classical transportation problem, the optimal solution of the problem is found to be

$$X^{*}=\begin{cases} x_{11}=5, x_{12}=3, \\ x_{21}=6,\ x_{23}=11.85,\ x_{24}=1.15, \\ x_{33}=2.15,\ x_{34}=14.85, \end{cases}$$

and $Z^1=[172.2,222.55],\ Z^2=[206.1,252.75],\ x_{13}=0.52$

Case-II

When the objective function's coefficients c_{ij}^k are crisp but the source parameters a_i and destination parameters b_j are in the form of interval, the MITP can be stated as follows:

Minimize $Z^k=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}^k x_{ij}$

subject to

$$\sum_{j=1}^{n} x_{ij} = [a_{L_i}, a_{R_i}]$$

$$\sum_{i=1}^{m} x_{ij} = [b_{L_j}, b_{R_j}]$$

$$x_{ij} \geq 0,\ i = 1, 2, ..., m,\ j = 1, 2, ..., n$$

$$\sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j} \text{ and } \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} a_{R_j}$$

5.5 Formulation of the crisp constraints

Consider the following MOTP:

$$\text{Minimize } Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} \leq a_{R_i},\ \sum_{j=1}^{n} x_{ij} \geq a_{L_i},$$

$$\sum_{i=1}^{m} x_{ij} \leq b_{R_j},\ \sum_{i=1}^{m} x_{ij} \geq b_{L_j},$$

with

$$\sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j},\ \sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j}$$

$$x_{ij} \geq 0,\ i = 1, 2, ..., m,\ j = 1, 2, ..., n$$

where $a_{L_i}, b_{L_j}, a_{R_i}$ and b_{R_j} are the left and right end points of the respective supply and demand values a_i and b_j, respectively.

We find that the given MITP with crisp objective function and source and destination as interval, is equivalent to above MOTP.

As defined above, the equivalent deterministic MOTP may be written as:

$$\text{Minimize } Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij} \text{ where } k = 1, 2, ...K$$

subject to,

$$\sum_{j=1}^{n} x_{ij} \leq a_{R_i}$$

$$\sum_{j=1}^{n} x_{ij} \ge a_{L_i}$$

$$\sum_{i=1}^{m} x_{ij} \le b_{R_j}$$

$$\sum_{i=1}^{m} x_{ij} \ge b_{L_j}$$

$$x_{ij} \ge 0,\ i = 1, 2, ..., m,\ j = 1, 2, ..., n$$

$$\sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j} \text{ and } \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j}$$

Example 5.2

$$\text{Minimize } Z^1 = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij}^1 x_{ij}$$

$$\text{Minimize } Z^2 = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij}^2 x_{ij}$$

subject to

$$\sum_{j=1}^{4} x_{1j} = [7,9],\ \sum_{j=1}^{4} x_{2j} = [17,21],\ \sum_{j=1}^{4} x_{3j} = [16,18],\ \sum_{i=1}^{3} x_{i1} = [10,12]$$

$$\sum_{i=1}^{3} x_{i2} = [2,4],\ \sum_{i=1}^{3} x_{i3} = [13,15],\ \sum_{i=1}^{3} x_{i4} = [15,17]$$

$$x_{ij} \ge 0,\ i = 1, 2, 3,\quad j = 1, 2, 3, 4$$

The equivalent deterministic transportation problem is as follows:

$$\text{Minimize } Z^1 = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij}^1 x_{ij}$$

$$\text{Minimize } Z^2 = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij}^2 x_{ij}$$

subject to

$$\sum_{j=1}^{4} x_{1j} \le 9,\ \sum_{j=1}^{4} x_{1j} \ge 7,\ \sum_{j=1}^{4} x_{2j} \le 21,\ \sum_{j=1}^{4} x_{2j} \ge 17, \sum_{j=1}^{4} x_{3j} \le 18,$$

$$\sum_{j=1}^{4} x_{3j} \ge 16, \sum_{i=1}^{3} x_{i1} \le 12,\ \sum_{i=1}^{3} x_{i1} \ge 10, \sum_{i=1}^{3} x_{i2} \le 4, \sum_{i=1}^{3} x_{i2} \ge 2,$$

$$\sum_{i=1}^{3} x_{i3} \le 15,\ \sum_{i=1}^{3} x_{i3} \ge 13, \sum_{i=1}^{3} x_{i4} \le 17, \sum_{i=1}^{3} x_{i4} \ge 15$$

$$x_{ij} \geq 0,\ i = 1,2,3,\quad j = 1,2,3,4$$

where

$$c^1 = \begin{bmatrix} 1 & 2 & 7 & 7 \\ 1 & 9 & 3 & 4 \\ 8 & 9 & 4 & 6 \end{bmatrix}, \qquad c^2 = \begin{bmatrix} 4 & 4 & 3 & 3 \\ 5 & 8 & 9 & 10 \\ 6 & 2 & 5 & 1 \end{bmatrix}.$$

The given problem is equivalent to

$$\begin{aligned}
&\text{Minimize } Z^1 = x_{11} + 2x_{12} + 7x_{13} + 7x_{14} + x_{21} + 9x_{22} + 3x_{23} + 4x_{24} + 8x_{31} + 9x_{32} + 4x_{33} + 6x_{34} \\
&\text{Minimize } Z^2 = 4x_{11} + 4x_{12} + 3x_{13} + 3x_{14} + 5x_{21} + 8x_{22} + 9x_{23} + 10x_{24} + 6x_{31} + 2x_{32} + 5x_{33} + x_{34} \\
&\text{subject to} \\
&\sum_{j=1}^{4} x_{1j} \leq 9,\ \sum_{j=1}^{4} x_{1j} \geq 7,\ \sum_{j=1}^{4} x_{2j} \leq 21,\ \sum_{j=1}^{4} x_{2j} \geq 17, \sum_{j=1}^{4} x_{3j} \leq 18, \\
&\sum_{j=1}^{4} x_{3j} \geq 16, \sum_{i=1}^{3} x_{i1} \leq 12,\ \sum_{i=1}^{3} x_{i1} \geq 10, \sum_{i=1}^{3} x_{i2} \leq 4, \sum_{i=1}^{3} x_{i2} \geq 2, \\
&\sum_{i=1}^{3} x_{i3} \leq 15,\ \sum_{i=1}^{3} x_{i3} \geq 13, \sum_{i=1}^{3} x_{i4} \leq 17, \sum_{i=1}^{3} x_{i4} \geq 15 \\
&x_{ij} \geq 0,\ i = 1,2,3,\quad j = 1,2,3,4
\end{aligned}$$

Solving this multi-objective classical transportation problem using fuzzy programming technique, we find the pay-off matrix as:

	Z^1	Z^2
X^1	132	241
X^2	195	148

From the above table, for each objective we find

L^1 = 132 $\qquad U^1$ = 195

L^2 = 148 $\qquad U^2$ = 241

Case (a) Solving the problem by using linear membership function

If we use the linear membership function, the optimal solution of the problem is found by solving the following linear programming problem:

$$\begin{aligned}
&\text{Maximize } \lambda\\
&\text{subject to}\\
&Z^1+63\lambda\le 195\\
&Z^2+93\lambda\le 241\\
&\sum_{j=1}^{4}x_{1j}\le 9,\ \sum_{j=1}^{4}x_{1j}\ge 7,\ \sum_{j=1}^{4}x_{2j}\le 21,\ \sum_{j=1}^{4}x_{2j}\ge 17,\sum_{j=1}^{4}x_{3j}\le 18,\\
&\sum_{j=1}^{4}x_{3j}\ge 16,\sum_{i=1}^{3}x_{i1}\le 12,\ \sum_{i=1}^{3}x_{i1}\ge 10,\sum_{i=1}^{3}x_{i2}\le 4,\sum_{i=1}^{3}x_{i2}\ge 2,\\
&\sum_{i=1}^{3}x_{i3}\le 15,\ \sum_{i=1}^{3}x_{i3}\ge 13,\sum_{i=1}^{3}x_{i4}\le 17,\sum_{i=1}^{3}x_{i4}\ge 15\\
&x_{ij}\ge 0,\ i=1,2,3,\quad j=1,2,3,4
\end{aligned}$$

i.e., the optimal solution of the problem is found by solving the following linear programming problem:

$$\begin{aligned}
&\text{Maximize } \lambda\\
&\text{subject to}\\
&x_{11}+2x_{12}+7x_{13}+7x_{14}+x_{21}+9x_{22}+3x_{23}+4x_{24}+8x_{31}+9x_{32}+\\
&\qquad\qquad 4x_{33}+6x_{34}+63\lambda\le 195\\
&4x_{11}+4x_{12}+3x_{13}+3x_{14}+5x_{21}+8x_{22}+9x_{23}+10x_{24}+6x_{31}+2x_{32}+\\
&\qquad\qquad 5x_{33}+x_{34}+93\lambda\le 241\\
&\sum_{j=1}^{4}x_{1j}\le 9,\sum_{j=1}^{4}x_{1j}\ge 7,\sum_{j=1}^{4}x_{2j}\le 21,\sum_{j=1}^{4}x_{2j}\ge 17,\\
&\sum_{j=1}^{4}x_{3j}\le 18,\sum_{j=1}^{4}x_{3j}\ge 16,\sum_{i=1}^{3}x_{i1}\le 12,\sum_{i=1}^{3}x_{i1}\ge 10,\\
&\sum_{i=1}^{3}x_{i2}\le 4,\sum_{i=1}^{3}x_{i2}\ge 2,\sum_{i=1}^{3}x_{i3}\le 15,\sum_{i=1}^{3}x_{i3}\ge 13,\\
&\sum_{i=1}^{3}x_{i4}\le 17,\sum_{i=1}^{3}x_{i4}\ge 15\\
&x_{ij}\ge 0,\ i=1,2,3,\quad j=1,2,3,4
\end{aligned}$$

Solving this classical transportation problem, the optimal solution of the problem is found to be

$$X^* = \begin{cases} x_{11} = 3.6, x_{12} = 2, x_{13} = 1.4, \\ x_{21} = 6.4, x_{23} = 10.6, \\ x_{33} = 1, x_{34} = 15, \end{cases}$$

$\lambda = 0.72,\ \ Z^1 = 149.6 \text{ and } Z^2 = 174$

Case (b) Solving the problem using exponential membership function

If we use the exponential membership function with $s = 1$, to solve the same problem, the optimal solution of the problem is found by solving the following linear programming problem:

Maximize λ
subject to

$$\exp\left\{\frac{-x_{11} - 2x_{12} - 7x_{13} - 7x_{14} - x_{21} - 9x_{22} - 3x_{23} - 4x_{24} - 8x_{31} - 9x_{32} - 4x_{33} - 6x_{34} + 132}{63}\right\} - 0.6321205\lambda \geq 0.3678794$$

$$\exp\left\{\frac{-4x_{11} - 4x_{12} - 3x_{13} - 3x_{14} - 5x_{21} - 8x_{22} - 9x_{23} - 10x_{24} - 6x_{31} - 2x_{32} - 5x_{33} - x_{34} + 148}{93}\right\} - 0.6321205\lambda \geq 0.3678794$$

$$\sum_{j=1}^{4} x_{1j} \leq 9,\ \sum_{j=1}^{4} x_{1j} \geq 7,\ \sum_{j=1}^{4} x_{2j} \leq 21,\ \sum_{j=1}^{4} x_{2j} \geq 17, \sum_{j=1}^{4} x_{3j} \leq 18,$$

$$\sum_{j=1}^{4} x_{3j} \geq 16, \sum_{i=1}^{3} x_{i1} \leq 12,\ \sum_{i=1}^{3} x_{i1} \geq 10, \sum_{i=1}^{3} x_{i2} \leq 4, \sum_{i=1}^{3} x_{i2} \geq 2,$$

$$\sum_{i=1}^{3} x_{i3} \leq 15,\ \sum_{i=1}^{3} x_{i3} \geq 13, \sum_{i=1}^{3} x_{i4} \leq 17, \sum_{i=1}^{3} x_{i4} \geq 15$$

$$x_{ij} \geq 0,\ i = 1,2,3,\ \ j = 1,2,3,4$$

This problem is solved by using LINGO software. The optimal solution of the given problem is found to be

$$X^* = \begin{cases} x_{11} = 3.600013, x_{12} = 2.0000, x_{13} = 1.39986, \\ x_{21} = 6.39986, x_{23} = 10.600013, \\ x_{33} = 1.0000, x_{34} = 15.0000 \end{cases}$$

$Z^1 = 149.59,\ Z^2 = 173.99$ and $\lambda = 0.55$

Case (c) Solving the problem by using hyperbolic membership function

Again if we use the hyperbolic membership function, the optimal solution is found by solving the following linear programming problem:

$$\begin{aligned}
&\text{Maximize } x_{13} \\
&\text{subject to} \\
&2x_{11} + 4x_{12} + 14x_{13} + 14x_{14} + 2x_{21} + 18x_{22} + 6x_{23} + 8x_{24} + 16x_{31} + 18x_{32} + \\
&\qquad\qquad\qquad\qquad 8x_{33} + 12x_{34} + 21x_{13} \le 327 \\
&8x_{11} + 8x_{12} + 6x_{13} + 6x_{14} + 10x_{21} + 16x_{22} + 18x_{23} + 20x_{24} + 12x_{31} + 4x_{32} + \\
&\qquad\qquad\qquad\qquad 10x_{33} + 2x_{34} + 31x_{13} \le 389 \\
&\sum_{j=1}^{4} x_{1j} \le 9,\ \sum_{j=1}^{4} x_{1j} \ge 7,\ \sum_{j=1}^{4} x_{2j} \le 21,\ \sum_{j=1}^{4} x_{2j} \ge 17, \sum_{j=1}^{4} x_{3j} \le 18, \\
&\sum_{j=1}^{4} x_{3j} \ge 16, \sum_{i=1}^{3} x_{i1} \le 12,\ \sum_{i=1}^{3} x_{i1} \ge 10, \sum_{i=1}^{3} x_{i2} \le 4, \sum_{i=1}^{3} x_{i2} \ge 2, \\
&\sum_{i=1}^{3} x_{i3} \le 15,\ \sum_{i=1}^{3} x_{i3} \ge 13, \sum_{i=1}^{3} x_{i4} \le 17, \sum_{i=1}^{3} x_{i4} \ge 15 \\
&x_{ij} \ge 0,\ i = 1,2,3,\quad j = 1,2,3,4
\end{aligned}$$

Hence the optimal solution is found to be

$$X^* = \begin{cases} x_{11} = 3.6, x_{12} = 2, x_{13} = 1.4, \\ x_{21} = 6.4, x_{23} = 10.6, \\ x_{33} = 1, x_{34} = 15 \end{cases}$$

$Z^1 = 149.6,\ \mathrm{Z}^2 = 174$ and $x_{13} = 0.69.$

Case-III

When the objective function's coefficients c_{ij}^k, the source parameters a_i, and destination parameters b_j are in the form of interval, the MITP can be formulated as:

$$
\begin{aligned}
&\text{Minimize } Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[c_{L_{ij}}^k, c_{R_{ij}}^k\right]x_{ij},\ k = 1,2,...,K\\
&\text{subject to}\\
&\sum_{j=1}^{n} x_{ij} = \left[a_{L_i}, a_{R_i}\right],\ \sum_{i=1}^{m} x_{ij} = \left[b_{L_j}, b_{R_j}\right]\\
&x_{ij} \geq 0,\ i = 1,2,...,m,\ j = 1,2,...,n,\\
&\text{with } \sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j} \text{ and } \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j}
\end{aligned}
$$

Equivalent multi-objective deterministic transportation problem may be written as:

$$
\begin{aligned}
&\text{Minimize } Z_R^k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{C_{ij}}^k x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{W_{ij}}^k x_{ij}\\
&\text{Minimize } Z_C^k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{C_{ij}}^k x_{ij},\ \ k = 1,2,...,K\\
&\text{subject to}\\
&\sum_{j=1}^{n} x_{ij} \leq a_{R_i}, \sum_{j=1}^{n} x_{ij} \geq a_{L_i}, \sum_{j=1}^{n} x_{ij} \leq b_{R_j}, \sum_{j=1}^{n} x_{ij} \geq b_{L_j}\\
&x_{ij} \geq 0,\ i = 1,2,...,m,\ j = 1,2,...,n,\\
&\text{with } \sum_{i=1}^{\mathrm{m}} \mathrm{a}_{L_i} = \sum_{j=1}^{n} b_{L_j} \text{ and } \sum_{i=1}^{m} \mathrm{a}_{R_i} = \sum_{j=1}^{n} b_{R_j}
\end{aligned}
$$

Example 5.3

$$
\begin{aligned}
&\text{Minimize } \mathrm{Z}^1 = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}^1 x_{ij}, \text{Minimize } \mathrm{Z}^2 = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}^2 x_{ij},\\
&\text{subject to}\\
&\sum_{j=1}^{4} x_{1j} = [7,9], \sum_{j=1}^{4} x_{2j} = [17,21], \sum_{j=1}^{4} x_{3j} = [16,18], \sum_{i=1}^{3} x_{i1} = [10,12],\\
&\sum_{i=1}^{3} x_{i2} = [2,4], \sum_{i=1}^{3} x_{i3} = [13,15], \sum_{i=1}^{3} x_{i4} = [15,17],\\
&x_{ij} \geq 0,\ i = 1,2,3,\ \ j = 1,2,3,4.
\end{aligned}
$$

By above formulation, equivalent multi-objective deterministic transportation problem is:

$$\text{Minimize } Z_R^1 = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{R_{ij}}^1 x_{ij}, \qquad \text{Minimize } Z_R^2 = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{R_{ij}}^2 x_{ij},$$

$$\text{Minimize } Z_C^1 = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{C_{ij}}^1 x_{ij}, \qquad \text{Minimize } Z_C^2 = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{C_{ij}}^1 x_{ij}$$

subject to

$$\sum_{j=1}^{4} x_{1j} \le 9, \sum_{j=1}^{4} x_{1j} \ge 7, \sum_{j=1}^{4} x_{2j} \le 21, \sum_{j=1}^{4} x_{2j} \ge 17,$$

$$\sum_{j=1}^{4} x_{3j} \le 18, \sum_{j=1}^{4} x_{3j} \ge 16, \sum_{i=1}^{3} x_{i1} \le 12, \sum_{i=1}^{3} x_{i1} \ge 10,$$

$$\sum_{i=1}^{3} x_{i2} \le 4, \sum_{i=1}^{3} x_{i2} \ge 2, \sum_{i=1}^{3} x_{i3} \le 15, \sum_{i=1}^{3} x_{i3} \ge 13,$$

$$\sum_{i=1}^{3} x_{i4} \le 17, \sum_{i=1}^{3} x_{i4} \ge 15$$

$$x_{ij} \ge 0,\ i = 1,2,3, \quad j = 1,2,3,4$$

with

$$\sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j} \text{ and } \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j}$$

where

$$c_{R_{ij}}^1 = \begin{bmatrix} 2 & 3 & 9 & 8 \\ 2 & 10 & 6 & 5 \\ 9 & 11 & 5 & 7 \end{bmatrix}, \qquad c_{R_{ij}}^2 = \begin{bmatrix} 5 & 6 & 4 & 5 \\ 6 & 9 & 10 & 11 \\ 8 & 3 & 6 & 2 \end{bmatrix},$$

$$c_{C_{ij}}^1 = \begin{bmatrix} 1.5 & 2 & 7 & 6 \\ 1.5 & 8.5 & 4 & 4 \\ 8 & 9 & 4 & 6 \end{bmatrix}, \qquad c_{C_{ij}}^2 = \begin{bmatrix} 4 & 4 & 3 & 3 \\ 5 & 8 & 8.5 & 10 \\ 6 & 2 & 4.5 & 1.5 \end{bmatrix}$$

Solving this multi-objective classical transportation problem using fuzzy programming technique, we find

	Z_R^1	Z_R^2	Z_C^1	Z_C^2
X_R^1	172	283	137	236
X_R^2	245	190	195.5	154.5
X_C^1	172	283	137	236
X_C^2	253	190	202	153

From the above table, for each objective we find

$L_R^1 = 172$ $\quad$ $U_R^1 = 253$

$L_R^2 = 190$ $\quad$ $U_R^2 = 283$

$L_C^1 = 137$ $\quad$ $U_C^1 = 202$

$L_C^2 = 153$ $\quad$ $U_C^2 = 236$

Case (a) Solving the problem using linear membership function

By using the linear membership function, the optimal solution of the problem is found by solving the following linear programming problem:

Maximize λ
subject to

$$Z_R^1 + 81\lambda \le 253$$
$$Z_R^2 + 93\lambda \le 283$$
$$Z_C^1 + 65\lambda \le 202$$
$$Z_C^2 + 83\lambda \le 236$$
$$\sum_{j=1}^{4} x_{1j} \le 9, \sum_{j=1}^{4} x_{1j} \ge 7, \sum_{j=1}^{4} x_{2j} \le 21, \sum_{j=1}^{4} x_{2j} \ge 17,$$
$$\sum_{j=1}^{4} x_{3j} \le 18, \sum_{j=1}^{4} x_{3j} \ge 16, \sum_{i=1}^{3} x_{i1} \le 12, \sum_{i=1}^{3} x_{i1} \ge 10,$$
$$\sum_{i=1}^{3} x_{i2} \le 4, \sum_{i=1}^{3} x_{i2} \ge 2, \sum_{i=1}^{3} x_{i3} \le 15, \sum_{i=1}^{3} x_{i3} \ge 13,$$

$$\sum_{i=1}^{3} x_{i4} \le 17, \sum_{i=1}^{3} x_{i4} \ge 15$$

$$x_{ij} \ge 0,\ i = 1,2,3, \quad j = 1,2,3,4$$

$$\text{with } \sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j} \text{ and } \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j}$$

i.e. the optimal solution of the problem is found by solving the following linear programming problem:

Maximize λ

subject to

$$\begin{aligned}
&2x_{11} + 3x_{12} + 9x_{13} + 8x_{14} + 2x_{21} + 10x_{22} + 6x_{23} + 5x_{24} + 9x_{31} + 11x_{32} + \\
&\qquad\qquad 5x_{33} + 7x_{34} + 81\lambda \le 253 \\
&5x_{11} + 6x_{12} + 4x_{13} + 5x_{14} + 6x_{21} + 9x_{22} + 10x_{23} + 11x_{24} + 8x_{31} + 3x_{32} + \\
&\qquad\qquad 6x_{33} + 2x_{34} + 93\lambda \le 283 \\
&1.5x_{11} + 2x_{12} + 7x_{13} + 6x_{14} + 1.5x_{21} + 8.5x_{22} + 4x_{23} + 4x_{24} + 8x_{31} + 9x_{32} + \\
&\qquad\qquad 4x_{33} + 6x_{34} + 65\lambda \le 202 \\
&4x_{11} + 4x_{12} + 3x_{13} + 3x_{14} + 5x_{21} + 8x_{22} + 8.5x_{23} + 10x_{24} + 6x_{31} + 2x_{32} + \\
&\qquad\qquad 4.5x_{33} + 1.5x_{34} + 83\lambda \le 236
\end{aligned}$$

$$\sum_{j=1}^{4} x_{1j} \le 9, \sum_{j=1}^{4} x_{1j} \ge 7, \sum_{j=1}^{4} x_{2j} \le 21, \sum_{j=1}^{4} x_{2j} \ge 17,$$

$$\sum_{j=1}^{4} x_{3j} \le 18, \sum_{j=1}^{4} x_{3j} \ge 16, \sum_{i=1}^{3} x_{i1} \le 12, \sum_{i=1}^{3} x_{i1} \ge 10,$$

$$\sum_{i=1}^{3} x_{i2} \le 4, \sum_{i=1}^{3} x_{i2} \ge 2, \sum_{i=1}^{3} x_{i3} \le 15, \sum_{i=1}^{3} x_{i3} \ge 13,$$

$$\sum_{i=1}^{3} x_{i4} \le 17, \sum_{i=1}^{3} x_{i4} \ge 15$$

$$x_{ij} \ge 0,\ i = 1,2,3, \quad j = 1,2,3,4$$

$$\text{with } \sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j} \text{ and } \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j}$$

Solving this classical transportation problem, the optimal solution of the problem is found to be:

$$X^{*}=\begin{cases} x_{11}=5, x_{12}=2, \\ x_{21}=5, x_{23}=11.01, x_{24}=0.99, \\ x_{33}=1.99, x_{34}=14.01 \end{cases}$$

$$Z^{1}=\left[159.02,205.03\right], Z^{2}=\left[176.54,227.95\right] \text{ and } \lambda=0.59$$

Case (b) Solving the problem using exponential membership function

Solving the given problem by using exponential membership function with $s=1$ requires to solve the following linear programming problem:

Maximize λ
subject to

$$\exp\left\{\frac{-2x_{11}-3x_{12}-9x_{13}-8x_{14}-2x_{21}-10x_{22}-6x_{23}-5x_{24}-9x_{31}-11x_{32}-5x_{33}-7x_{34}+172}{81}\right\}$$
$$-0.6321205\lambda \geq 0.3678794$$

$$\exp\left\{\frac{-5x_{11}-6x_{12}-4x_{13}-5x_{14}-6x_{21}-9x_{22}-10x_{23}-11x_{24}-8x_{31}-3x_{32}-6x_{33}-2x_{34}+190}{93}\right\}$$
$$-0.6321205\lambda \geq 0.3678794$$

$$\exp\left\{\frac{-1.5x_{11}-2x_{12}-7x_{13}-6x_{14}-1.5x_{21}-8.5x_{22}-4x_{23}-4x_{24}-8x_{31}-9x_{32}-4x_{33}-6x_{34}+137}{65}\right\}$$
$$-0.6321205\lambda \geq 0.3678794$$

$$\exp\left\{\frac{-4x_{11}-4x_{12}-3x_{13}-3x_{14}-5x_{21}-8x_{22}-8.5x_{23}-10x_{24}-6x_{31}-2x_{32}-4.5x_{33}-1.5x_{34}+153}{83}\right\}$$
$$-0.6321205\lambda \geq 0.3678794$$

$$\sum_{j=1}^{4} x_{1j} \leq 9, \sum_{j=1}^{4} x_{1j} \geq 7, \sum_{j=1}^{4} x_{2j} \leq 21, \sum_{j=1}^{4} x_{2j} \geq 17,$$

$$\sum_{j=1}^{4} x_{3j} \leq 18, \sum_{j=1}^{4} x_{3j} \geq 16, \sum_{i=1}^{3} x_{i1} \leq 12, \sum_{i=1}^{3} x_{i1} \geq 10,$$

$$\sum_{i=1}^{3} x_{i2} \leq 4, \sum_{i=1}^{3} x_{i2} \geq 2, \sum_{i=1}^{3} x_{i3} \leq 15, \sum_{i=1}^{3} x_{i3} \geq 13,$$

$$\sum_{i=1}^{3} x_{i4} \leq 17, \sum_{i=1}^{3} x_{i4} \geq 15$$

$$x_{ij} \geq 0,\ i=1,2,3, \quad j=1,2,3,4$$

with $\sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j}$ and $\sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j}$

Hence solving above problem by using LINGO, the optimal solution is found to be:

$$X^* = \begin{cases} x_{11} = 5.000, x_{12} = 2.00000, \\ x_{21} = 5.0000, x_{23} = 10.97, x_{24} = 1.03, \\ x_{33} = 1.9875, x_{34} = 14.0000 \end{cases}$$

$$Z^1 = [158.95, 204.90],\ Z^2 = \left[186.48, 227.95\right] \text{ and } \lambda = 0.62$$

Case (c) Solving the problem using hyperbolic membership function

Using the hyperbolic membership function, applying the same technique as explained in chapter 3 the optimal solution of the given problem is found by solving following equations:

Maximize x_{13}
subject to

$$\begin{aligned} &4x_{11} + 6x_{12} + 18x_{13} + 16x_{14} + 4x_{21} + 20x_{22} + 12x_{23} + 10x_{24} + 18x_{31} + 22x_{32} + 10x_{33} + \\ &\qquad\qquad 14x_{34} + 27x_{13} \le 425 \end{aligned}$$

$$\begin{aligned} &10x_{11} + 12x_{12} + 8x_{13} + 10x_{14} + 12x_{21} + 18x_{22} + 20x_{23} + 22x_{24} + 16x_{31} + 6x_{32} + 12x_{33} + \\ &\qquad\qquad 4x_{34} + 31x_{13} \le 473 \end{aligned}$$

$$\begin{aligned} &9x_{11} + 12x_{12} + 42x_{13} + 36x_{14} + 9x_{21} + 51x_{22} + 24x_{23} + 24x_{24} + 48x_{31} + 54x_{32} + 24x_{33} + \\ &\qquad\qquad 36x_{34} + 65x_{13} \le 1017 \end{aligned}$$

$$\begin{aligned} &24x_{11} + 24x_{12} + 18x_{13} + 18x_{14} + 30x_{21} + 48x_{22} + 51x_{23} + 60x_{24} + 36x_{31} + 12x_{32} + 27x_{33} + \\ &\qquad\qquad 9x_{34} + 83x_{13} \le 1167 \end{aligned}$$

$$\sum_{j=1}^{4} x_{1j} \le 9, \sum_{j=1}^{4} x_{1j} \ge 7, \sum_{j=1}^{4} x_{2j} \le 21, \sum_{j=1}^{4} x_{2j} \ge 17,$$

$$\sum_{j=1}^{4} x_{3j} \le 18, \sum_{j=1}^{4} x_{3j} \ge 16, \sum_{i=1}^{3} x_{i1} \le 12, \sum_{i=1}^{3} x_{i1} \ge 10,$$

$$\sum_{i=1}^{3} x_{i2} \le 4, \sum_{i=1}^{3} x_{i2} \ge 2, \sum_{i=1}^{3} x_{i3} \le 15, \sum_{i=1}^{3} x_{i3} \ge 13,$$

$$\sum_{i=1}^{3} x_{i4} \le 17, \sum_{i=1}^{3} x_{i4} \ge 15$$

$x_{ij} \geq 0,\ i = 1,2,3, \quad j = 1,2,3,4$

with $\sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j}$ and $\sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j}$

Solving this, optimal solution is found to be

$$X^* = \begin{cases} x_{11} = 5, x_{12} = 2, \\ x_{21} = 5, x_{23} = 11.01, x_{24} = 0.99, \\ x_{33} = 1.99, x_{34} = 14.01 \end{cases}$$

$Z^1 = [159.02, 205.03], Z^2 = [176.54, 227.95]$ and $x_{13} = 0.55.$

5.6 Conclusion

In this chapter we have presented a solution procedure of the MITP, where the coefficients of the objective functions and the source and destination parameters are considered as interval. Initially, the problem is converted into a classical MOTP where the objectives which are the right limit and centre of the interval objective function are minimized. These objective functions can be considered as the minimization of the worst case and the average case.

To obtain the solution of the transformed classical MOTP, the fuzzy programming technique is used. Here three different types of membership functions are used namely linear, hyperbolic, exponential membership functions to solve the transformed classical transportation problem. It is found that all the membership functions give almost the same solution. But with linear membership function, the problem is solved in lesser steps, so it’s better to use the linear membership function rather than hyperbolic and exponential one.

The advantage of fuzzy programming technique is that, for a MOTP with K objective functions, this technique leads to K non-dominated solutions and one optimal compromise solution, whereas other algorithms like those of Ringuest and Rinks (1987) lead to more than K non-dominated and dominated solutions which force the decision maker to determine a compromise solution from the set of non-dominated solutions.

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