Cost-Benefit Analysis of Single Unit Systems Using Matlab

Thesis submitted in partial fulfillment of the requirements of the award of degree of

Master of Technology

in

Computer Science and Applications

Submitted By

Chetan Jindal

(Roll No. 601003005)

Under the supervision of

Dr. Jitender Kumar

School of Mathematics and Computer Applications

Thapar University

Patiala – 147004

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CERTIFICATE

I hereby certify that the work which is being presented in thesis entitled **"Cost-Benefit Analysis of Single Unit Systems Using Matlab"**, in the partial fulfillment of the requirements for the award of degree of Master of Technology in Computer Science and Applications (*CSA*) submitted in School of Mathematics and Computer Applications (SMCA), Thapar University Patiala is an authentic record of my own work carried out under the supervision of Dr. Jitender Kumar and refers other researcher's work which are duly listed in reference section.

The material presented in this thesis has not been submitted for the award of any other degree of this or any other university.

(Chetan Jindal)
Roll No. 601003005
M.Tech (CSA)

This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

14/06/2012
(Dr. Jitender Kumar)
Lecturer
SMCA

Countersigned by

(Dr. S.S. Bhatia)
Head
SMCA
Thapar University
Patiala

(Dr. S. K. Mohapatra)
Dean (Academic Affairs)
Thapar University
Patiala

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Chetan Jindal

Roll No. 601003005

M.Tech (CSA)

LIST OF ABBREVIATIONS

CDF	Cumulative Density Function
MOT	Maximum Operation Time
MTBF	Mean Time Between Failures
MTSF	Mean Time to System Failure
PDF	Probability Density Function
PFO	Partially Failed and Operative
PFS	Partially Fail Stage
PM	Preventive Maintenance
TPM	Transition Probability Matrix

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ABSTRACT

The whole range of our study work in this thesis is covered by five chapters and the chapter wise summary as follows

CHAPTER-1

This chapter is introductory in nature and contains basic ideas, origin, development and concept of reliability, stochastic processes, system configuration, transforms, some distributions and the profit function.

CHAPTER-2

In this chapter, we study single unit with two-dissimilar components reliability model arranged in parallel configuration. There is a single server who attends the system immediately whenever needed. Upon failure each component can be replaced with a similar component with both the component (when failed) can also be replaced simultaneously. The system remains operative even if a single component operates. The system is analyzed under the assumption that the failure, repair and replacement times of the units are assumed to be arbitrarily distributed. By using regenerative point technique various reliability characteristic such as mean time to system failure (MTSF), steady state availability and expected no. of visits by the server are evaluated for obtaining the profit function of the system.

CHAPTER-3

Here we study the extension of the work reported in previous chapter with the separate assumption that preventive maintenance of the unit is carried out before the failure stage. By using regenerative point technique some reliability characteristic of interest are also obtained.

CHAPTER-4

The present chapter focuses on the development of a stochastic model for a one-unit system in which unit fails completely either directly from normal mode or via partial failure. The partially failed operating unit is shutdown after a maximum operation time for preventive maintenance (PM). There is a single server who attends the system immediately whenever needed. The unit becomes degraded after repair. The server inspects the degraded unit at its failure to see the feasibility of repair. If the repair of the degraded unit is not feasible, it is replaced by new unit. The distributions of failure, maximum rate of operation times, preventive maintenance, inspection and repair time of the unit are arbitrary distributed. By using regenerative point technique some reliability characteristic of interest are also obtained.

CHAPTER-1

INTRODUCTION

The tremendous development in the field of modern technology has played a key role in our society. The failure of such systems may be costly, dangerous and confusive to a nation. It is, therefore, of great importance to operate such systems with high reliabilities. Reliability means probability of no failure. However, no system can work forever, because of many unforeseen conditions such as environmental changes, earth quake, etc. But this does not mean that system cannot be made considerably reliable. This can be done by providing component of low failure rates, introduction of redundancy, rectification of faults at proper time, replacement of units when its repair is not feasible, etc.

The subject reliability has attracted the attention of many scholars, scientists and engineers due to its importance in day to day problems. Since 1950's many research laboratories initiated studies on failure of equipments and components under different type of assumptions other studies like preventive maintenance, imperfect switching, delayed repair etc. have been carried out by Barlow and Hunter [1960]. Branson and Shah [1971] applied semi-Markov process method to analyze reliability models when repair time distribution was general and exponential failure time

So, in view of common problems pertaining to operation of systems the issue of reliability has invited the attention of a good number of scholars of various disciplines. In the late 1950's the subject of the reliability is debated vehemently by eminent scholars belonging to the faculty of science and technology. Subsequently several studies have been under taken while having a focused attention on hypothetical aspects. In [1953], Epstein and Sobel began work in the field of life testing. After [1956], system maintainability problems were also considered besides reliability. Gaver [1963] was the first who generalized repair time distribution to analyses his model. An excellent account of the early development of the mathematical theory of reliability has been given by Barlow and Proschan [1965].

Branson and Shah [1971] applied semi-Markov method when repair time distribution was general with exponential failure time. Srinivasan and Gopalan [1973] highlighted the regenerative point technique for analyzing a two-unit system with warm standby and single repair facility. Nakagawa [1976] analyzed the system with replacement of the unit at certain level of damage. Gopalan and Marathe [1978] evaluated the availability of one server two dissimilar unit systems with slow switch.

Ramamurthy and Jaiswal [1982] analyzed a two dissimilar unit cold standby system with allowed down time. Murari and Goyal [1984] made a comparison of two-unit cold standby reliability models with three types of repair facility. Gopalan and Ramesh [1986] analyzed one-server two-unit parallel system subject to degradation. Singh [1989] evaluated the profit of two-unit cold standby system with random appearance and disappearance time of the service facility. Gupta and Bansal [1991] studied profit analysis of a two-unit priority standby system subject to degradation. Gupta et al. [1993] discussed profit analysis of a two-unit priority standby system subject to degradation and random shocks. Yang and Dhillon [1995] analyzed a general standby system with constant human error and arbitrary system repair rates. Mokaddis et al. [1997] discussed a two-unit warm standby system subject to degradation. The concept of on-line repair of single-unit system has been introduced by Ahlawat [1998]. Arora and Kumar [2000] discussed the system analysis and maintenance management for the coal handling system in a paper plant.

Kadyan et al. [2004] has made a stochastic analysis of non-identical units reliability models with priority and different modes of failure. Chander [2005] investigated reliability models with priority for operation and repair with arrival time of server. Chander and Singh [2005] have evaluated profit and reliability of an electric supply system. Khaled and Salah [2006] compared reliability characteristics between two different systems. Chand et al. [2007] discussed availability analysis of cotton mill. Malik and Nandal [2008] carried out MTSF and Profit of a system with the provision of a spare unit. Malik et al. [2008] discussed stochastic analysis of an operating system with two types of inspection subject to degradation. Chander and Singh [2009] developed reliability model for a 2-out-of-3 redundant system subject to degradation. Jitender Kumar [2010] has proposed reliability models of redundant systems subject to degradation and inspection. Kumar and Kadyan [2012] analyzed a system of non-identical units with degradation and replacement.

However, these studies are not exhaustive in nature and further investigations are required to fill up the gap.

1.1 Reliability Engineering

Reliability engineering is the discipline of ensuring that a system (or a device in general) will perform its intended function (s) when operated in a specified manner for a specified length of time. Reliability engineering is performed throughout the entire life cycle of a system, including development, test, production and operation.

Reliability may be defined in several ways

- The idea that something is fit for purpose with respect to time;
- The capacity of a device or system to perform as designed;
- The resistance to failure of a device or system;
- The ability of a device or system to perform a required function under stated conditions for a specified period to time;
- The probability that a functional unit will perform its required function for a specified interval under stated conditions.

Reliability engineers rely heavily on statistics, probability theory, and reliability theory. Many engineering techniques are used in reliability engineering, such as reliability prediction, Weibull analysis, thermal management, reliability testing and accelerated life testing. Because of the large number or reliability techniques, their expense, and the varying degrees of reliability required for different

Situations, most projects develop a reliability program plan to specify the reliability tasks that will be performed for that specific system.

The function of reliability engineering is to develop the reliability requirements for the product, establish an adequate reliability program, and perform appropriate analyses and tasks to ensure the product will meet its requirements. These tasks are managed by a reliability engineer, who has additional reliability specific education and training. Reliability engineering is closely associated with maintainability engineering and logistics engineering. Many problems from other fields, such as security engineering, can also be approached using reliability engineering techniques.

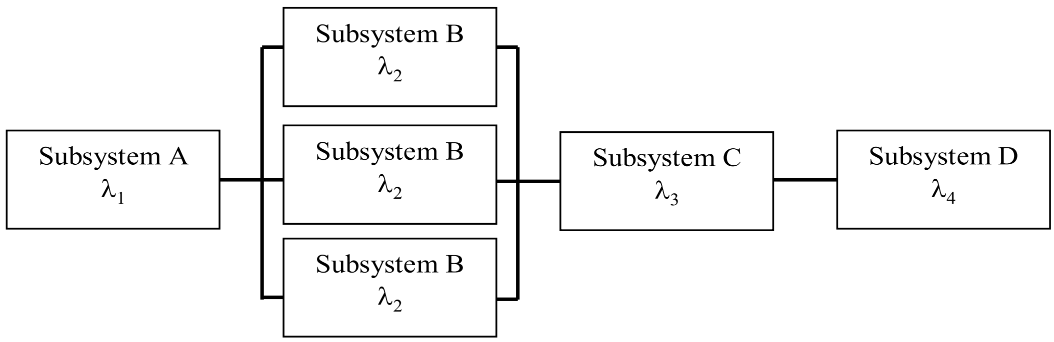


Fig. 1.1: A Reliability Block Diagram

Reliability theory is the foundation of reliability engineering. For engineering purposes, reliability is defined as

The probability that a device (unit) will perform its intended function adequately for a given period of time under stated conditions or environment.

Mathematically, if T is time till the failure of a unit occurs, this may be expressed as,

$$R(t) = \Pr(T>t) = \int_t^{\infty} f(x)dx$$

where

$f(x)$ is the failure probability density function and t is the length of the period of time (which is assumed to start from time zero).

Reliability engineering is concerned with four key elements of this definition

- First, reliability is a probability. This means that there is always some chance for failure. Reliability engineering is concerned with meeting the specified probability of success, at a specified statistical confidence level. Since it is a probability, its numerical value is always between one and zero, i.e.

  $$R(0)=1, \quad R(\infty)=0$$

  And R(t) is a non-increasing function between these limits.
- Second, reliability is predicated on "intended function" Generally, this is taken to mean operation without failure. However, even if no individual part of the system fails, but the system as a whole does not do what was intended, then it is still charged against the system reliability. The system requirements specification is the criterion against which reliability is measured.

- Third, reliability applies to a specified period of time. In practical terms, this means that a system has a specified chance that it will operate without failure before time t. Reliability engineering ensures that components and materials will meet the requirements during the specified time. Units other than time may sometimes be used. The automotive industry might specify reliability in terms of miles, the military might specify reliability of a gun for a certain number of rounds fired. A piece of mechanical equipment may have a reliability rating value in terms of cycles of use.
- Fourth, reliability is restricted to operation under stated conditions. This constraint is necessary because it is impossible to design a system for unlimited conditions.

1.1.1 Reliability Program Plan

Many tasks, methods, and tools can be used to achieve reliability. Every system requires a different level of reliability. A commercial airliner must operate under a wide range of conditions, the consequences of failure are grave, but there is a correspondingly higher budget. A pencil sharpener may be more reliable than an airliner, but has a much different set of operational conditions, mild consequences of failure, and correspondingly lower budget.

A reliability program plan is used to document exactly what tasks, methods, tools, analyses and tests are required for a particular system. For complex systems, the reliability program plan is a separate document. For simple systems, it may be combined with the systems engineering management plan. The reliability program plan is essential for a successful reliability program and is developed early during system development. It specifies not only what the reliability engineer does, but also the tasks performed by others. The reliability program plan is approved by top program management.

1.2 Reliability Requirements

1.2.1 System Reliability Parameters

Requirements are specified using reliability parameters. The most common reliability parameter is the Mean Time Between Failures (MTBF), which can also be specified as the failure rate or the number of failures during a given period. These parameters are very useful for systems that are operated on a regular basis, such as most vehicles, machinery and electronic equipment. Reliability increases as the MTBF increases. The MTBF is usually specified in hours; but can also be used with any unit of duration such as miles or cycles.

In other cases, reliability is specified as the probability of mission success. For example, reliability of a scheduled aircraft flight can be specified as a dimensionless probability or a percentage.

A special case of mission success is the single-shot device or system. These are devices or systems that remain relatively dormant and only operate once. Examples include automobile airbags, thermal batteries and missiles. Single-shot reliability is specified as a probability of success, or is subsumed into a related parameter. Single-shot missile reliability may be incorporated into a requirement for the probability of hit.

In addition to system level requirements, reliability requirements may be specified for critical subsystems. In all cases, reliability parameters are specified with appropriate statistical confidence intervals.

It is a general praxis to model the early failure rate with an exponential distribution. This less complex model for the failure distribution has only one parameter, the constant failure rate.

1.3 Failure Rate

Failure rate is the frequency with which a system or component fails, expressed for example in failures per hour. It is often denoted by the Greek letter λ (lambda) and is important in reliability theory. In practice, the reciprocal rate MTBF is more commonly expressed and used for high quality components or systems.

Failure rate is usually time dependent, and an intuitive corollary is that both rates change over time versus the expected life cycle of a system. For example, as an automobile grows older, the failure rate in its fifth year of service may be many times greater than its failure rate during its first year of service – one simply does not expect to replace an exhaust pipe, overhaul the brakes, or have major power plant-transmission problems in a new vehicle. So in the special case when the likelihood of failure remains constant with respect to time (for example, in some product like a brick or protected steel beam), failure rate is simply the inverse of the mean time between failure (MTBF), expressed for example in hours per failure. MTBF is an important specification parameter in all aspects of high importance engineering design – such as naval architecture, aerospace engineering, automotive design, etc. – in short, any task where failure in a key part or the whole of a system needs be minimized and severely curtailed, particularly where lives might be lost if such factors are not taken into account. These factors account for many safety and maintenance practices in engineering and industry practices and government regulations, such as how often certain inspections and overhauls are required on an aircraft. A similar ratio used in the transport industries, especially in railways and trucking is 'Mean Distance Between Failure', a variation which attempts to correlate actual loaded distances to similar reliability needs and practices. Failure rates and their projective manifestations are important factors in insurance, business, and regulation practices as well as fundamental to design of safe systems throughout a national or international economy.

1.3.1 Failure Rate in the Discrete Sense

In words appearing in an experiment, the failure rate can be defined as **"The total number of failures within an item population, divided by the total time expended by that population, during a particular measurement interval under stated conditions".**

Here failure rate λ (t) can be thought of as the probability that a failure occurs in a specified interval, given no failure before time t. It can be defined with the aid of the reliability function or survival function R(t), the probability of no failure before time t, as

$$\lambda = \frac{R(t_1) - R(t_2)}{(t_2 - t_1)R(t_1)} = \frac{R(t) - R(t + \Delta t)}{\Delta t.R(t)}$$

where t_1 (or t) and t_2 are respectively the beginning and ending of a specified interval of time spanning Δt. Note that this is a conditional probability, hence the R(t) in the denominator.

1.3.2 Failure Rate in the Continuous Sense (Instantaneous Hazard Rate)

By calculating the failure rate for smaller intervals of time Δt, the interval becomes infinitesimally small. This results in the hazard function, which is the instantaneous failure rate at any point in time

$$r(t) = \lim_{\Delta t \to 0} \frac{R(t) - R(t + \Delta t)}{\Delta t.R(t)}$$

Continuous failure rate depends on a failure distribution, F(t), which is a cumulative distribution function that describes the probability of failure prior to time t,

$$P\ (T \leq t) = F(t) = 1 - R(t),\ t \geq 0.$$

where T is the failure time. The failure distribution function is the integral of the failure density function, f(x),

$$F(t) = \int_0^t f(x)dx.$$

Now, the hazard function can be defined as

$$r(t) = \lim_{\Delta t \to 0} \frac{R(t + \Delta t) - R(t)}{R(t).\Delta t} = \frac{-R'(t)}{R(t)}$$

$$r(t) = \frac{f(t)}{R(t)}$$

There are many failure distributions. A common failure distribution is the exponential failure distribution.

$$F(t) = \int_0^t \lambda e^{-\lambda x} dx = 1 - e^{-\lambda t}\ ,$$

which is based on the exponential density function. This leads to a constant hazard rate. For other distributions, such as the Weibull distribution, log-normal distribution or bathtub curve, the hazard function is not constant, which means that the failure rate varies with time.

1.3.3 Units of Failure Rate

Failure rates can be expressed using any measure of time, but hours is the most common unit in practice. Other units, such as miles, revolutions, etc., can also be used in place of "time" units.

Failure rates are often expressed in engineering notation as failures per million, or 10^6, especially for individual components, since their failure rates are often very low.

1.4 System Configurations

By system, we mean an arbitrary device having several units/sub systems/components assuming that their reliabilities are known which help us to predict the reliability of whole system. It is now important that the system structure be known. Various system structures have been considered as follows

1.4.1 Series Configuration

A system having n-units is said to have series configuration if the failure of an arbitrary unit, say i^{th} unit causes the entire system failure. The examples of the series configurations are

- The aircraft electronic system consists of a sensor sub system, a guidance subsystem, computer subsystem and the fire control subsystem. These systems can only operate successfully if all these operate simultaneously.
- Deepawali or Christmas glow bulb, where if one bulb fails the whole lead fails. The block diagram of a series system is shown in Fig. 1.2.

Let $R_i(t)$ be the reliability of i^{th} component, then the system reliability is given by

$$R(t) = Pr[T > t] = Pr\,[\min(T_1, T_2, T_3 \ldots\ldots\ldots, T_n) > t]$$

$$= \prod_{i=1}^{n} P[T_i > t] = \prod_{i=1}^{n} R_i(t)$$

where

T_i is the life time of the i^{th} unit of the system. The system hazard rate, therefore is

$$r(t) = \sum_{i=1}^{n} r_i(t)$$

where

$r_i(t)$ is the instantaneous failure rate of i^{th} unit.

1.4.2 Parallel Configuration

In this configuration, all the units are connected in parallel i.e. the failure of the system occurs only when all the units of system fail. For example, four engine aircraft which is still able to fly with only two engines working. Block diagram representing a parallel configuration is shown in Fig. 1.3.

Suppose $R_i(t)$ and T_i be the reliability of i^{th} components and the life time of the i^{th} unit in time t respectively, then the system reliability is given by

$$R(t) = Pr\,[T > t]$$
$$= Pr\,[\max.\,(T_1, T_2, T_3,\ldots\ldots, T_n) > t]$$
$$= 1 - P[T_1 \le t, T_2 \le t, T_3 \le t,\ldots\ldots\ldots T_n \le t]$$

If the units function independently, then

$$R(t) = 1 - [1\text{-}R_1(t)]\,[1\text{-}R_2(t)]\,[1\text{-}R_3(t)]\ldots\ldots[1\text{-}R_n(t)]$$
$$= 1 - \prod_{i=1}^{n}[1 - R_i(t)]$$

1.4.3 Standby Redundant Configuration

To assure high reliability of a system, redundancy is incorporated. In redundant system more units than the required are used so that when failures occur in a system, it does not stop functioning. In standby redundant system with n units, only one unit is on-line at a time. When it fails, it is replaced manually or automatically by a standby unit. This process continues until all (n-1) standby units have been exhausted. For example, consider a cinema hall in a city where power supply is irregular. In order to ensure uninterrupted supply of power apart from the regular source of supply, a generator is kept as standby. The generator is switched on as and when the main supply is resumed. A block diagram of such system is shown in Fig. 1.4.

Gnedenko et al. [1969] classified the standby units as follows

- If the off-line unit can fail and is loaded in exactly the same way as the operating unit. It is called the hot standby unit.
- If the off-line unit can fail and can diminish the load, it is called warm standby unit. The probability of failure for a warm standby is less than the failure of operative unit.
- If the off-line unit cannot fail and is completely unloaded, it is called cold standby.

Reliability R(t) of an n-unit standby system at any time instant t is given as

$$R(t) = Pr\left[\sum_{i=1}^{n} T_i > t\right]$$

where

T_i is the life time of i^{th} unit and all the n units are independent.

1.4.4 k-out-of-n-Configuration

In many problems, the system operates if at least k-out-of-n-units function e.g. a bridge supported by n cables, k of which are necessary to support the maximum load. If each of n-units are identical with the same reliability $R_0(t)$ (say), then the system reliability becomes

$$R(t) = \sum_{i=k}^{n} {}^{n}C_i R_o^i(t)[1 - R_o]^{n-1}$$

Series (k = n) and parallel (k = 1) system are subclasses of k-out-of-n structure. There are many other configurations as series-parallel, parallel series, mixed-parallel, etc. some of them are depicted in Fig. 1.5 to Fig. 1.7.

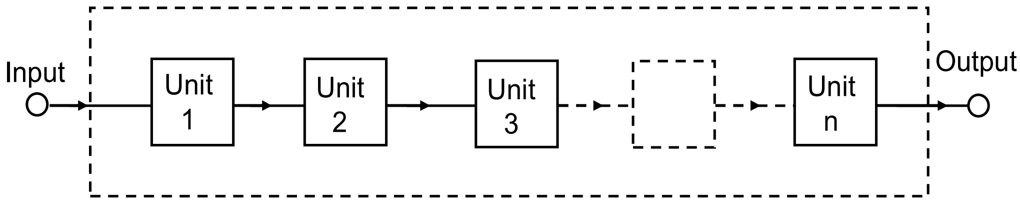


Fig. 1.2: Series Configuration

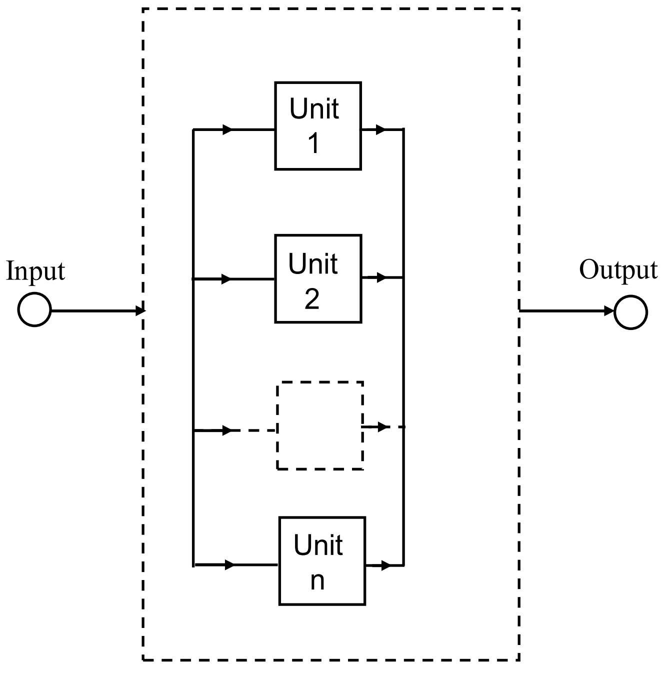


Fig. 1.3: Parallel Configuration

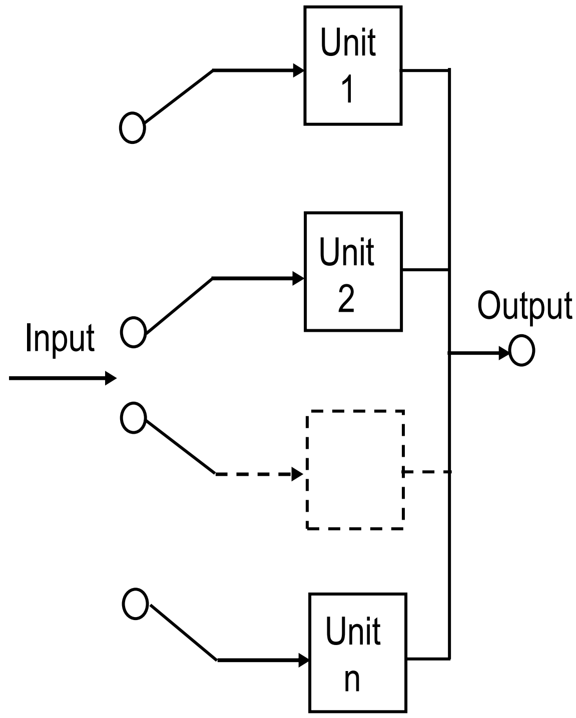


Fig. 1.4: Standby Redundant Configuration

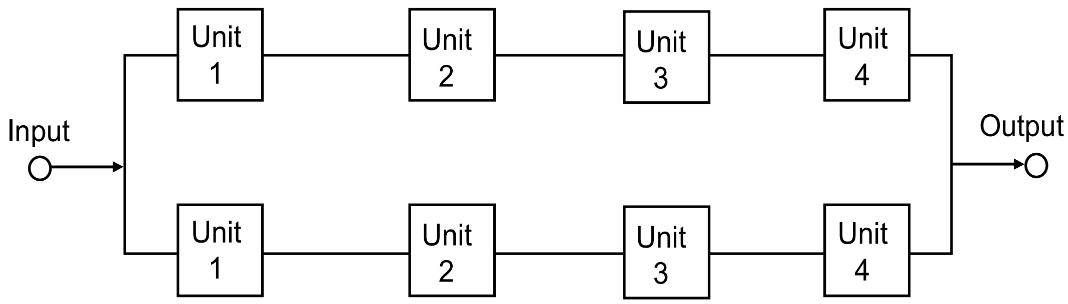


Fig. 1.5: Series Parallel Configuration

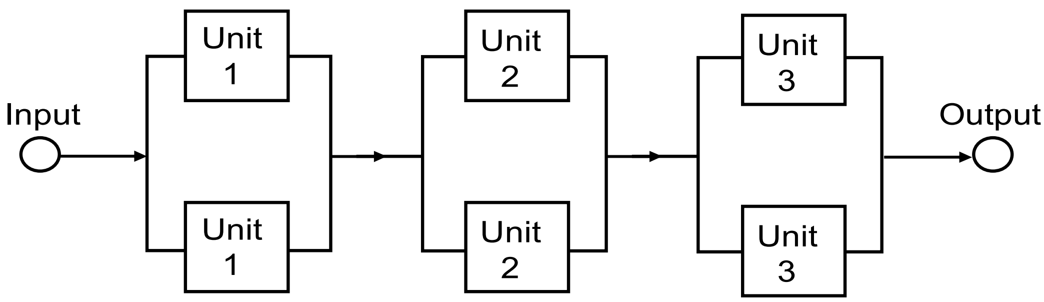


Fig. 1.6: Parallel Series Configuration

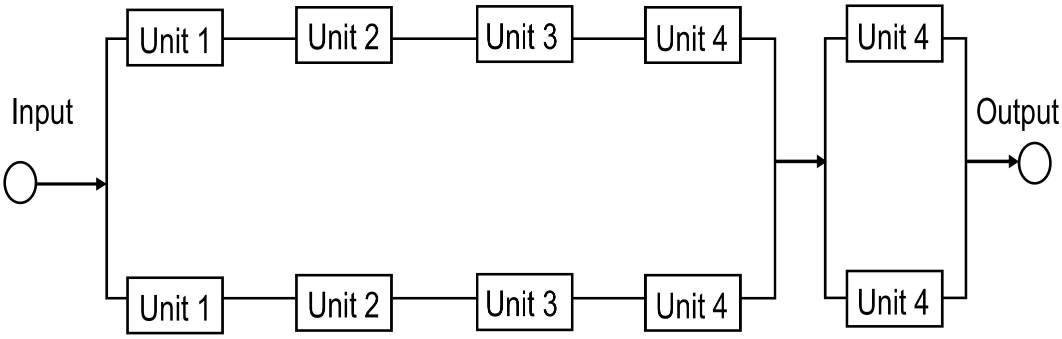


Fig. 1.7: Mixed Parallel Configuration

1.5 Mean Sojourn Time in a State

The expected time taken by the system in a particular state before transiting to any other state in known as mean sojourn time or mean survival time in that state. If T_i be the sojourn time in state i, then the mean sojourn time in state i is

$$\mu_i = \int_0^\infty \Pr(T_i > t)dt$$

1.6 First Passage Time

Suppose that a system starts with the state j, and then time taken to reach a given state k for the first time from state j is called first passage time. In general, first passage time is a measure of how long it takes to reach a given state from another state.

1.7 Mean Time to System Failure (MTSF)

It is defined as the expected time for which the system is in operation before it completely fails.

Let f(t) be the probability density function of life time of the system, then we have

$$\text{MTSF} = E(T) = \int_0^\infty tf(t)dt = \int_0^\infty R(t)dt$$

Also

$$\lim_{s\to 0} R^*(s) = \int_0^\infty R(t)dt$$

$$\Rightarrow \qquad \text{MTSF} = \lim_{s\to 0} R^*(s)$$

Let $\phi_0(t)$ be the cumulative distribution function of the first passage time from initial state to a failed state, then

$$R^*(s) = \frac{1 - \phi_0^{**}(s)}{s}$$

from above equations, we have

$$\text{MTSF} = \lim_{s \to 0} R^*(s) = \lim_{s \to 0} \frac{1 - \phi_0^{**}(s)}{s}$$

where

$R^*(s)$ and $\phi_0^{**}(s)$ are respectively the Laplace transform and Laplace Stieltjes transform of $R(t)$ and $\phi_0(t)$.

1.8 Availability

Availability is well established in the literature of stochastic modeling and optimal maintenance. Barlow and Proschan [1975] define availability of a repairable system as "the probability that the system is operating at a specified time t" and in reliability theory, the term availability has the following meanings

The degree to which a system, subsystem or equipment is operable and in a committable state at the start of a mission, when the mission is called for at an unknown, i.e. a random time. Simply put, availability is the proportion of time a system is in a functioning condition. In general, we may categorize this measure as

1.8.1 Instantaneous Availability

This is the probability that the system will be able to operate within the tolerances at a given instant of time t(say). Let this probability be denoted by $A(t)$.

Let $X(t) = 1$ if the system is operable at time t and $X(t) = 0$ when it is not operable. The availability $A(t)$ of the system at time t is given by

$$A(t) = \Pr[X(t) = 1 \mid X(0) = 1]$$

1.8.2 Average (Interval) Availability

It is the expected fraction of a given interval of time that the system will be able to operate within tolerances.

Suppose the given interval of time is (0, t] then interval availability H(0, t] of this interval is given by

$$H(0, t] = \frac{1}{t}\int_0^t A(u)du = \frac{\mu_{up}(t)}{t}$$

when $\mu_{up}(t)$ = expected up time of the system during (0, t].

1.8.3 Steady-State (Limited Interval) Availability

The long run or steady-state availability is defined as the proportion of the time during which equipment is available for use.

Mathematically, it is the limiting value of the point wise availability when t becomes finitely large i.e.

$$A = \lim_{t\to\infty} A(t)$$

1.9 Reliability and Availability

The availability function A(t) is defined as the probability that the equipment is operating at time t. Although this definition appears to be very similar to the reliability function R(t), the two have different meaning. While reliability places emphasis on failure – free operation up to time t, availability is concerned with the status of the equipment at time t. The availability function does not say anything about the number of failures that occurred during time t. This means that two equipments A and B can have different number of failures in a given time interval and can still have the same availability.

1.10 Maintainability

Maintainability is the probability that the system will be restored to operational effectiveness within a specified time when the maintenance action is taken in accordance with prescribed conditions. Maintenance is one of the effective ways of increasing the reliability of a system. Maintenance action can be classified in several categories: preventive maintenance, corrective maintenance and repair maintenance.

Preventive maintenance includes actions such as lubrication, replacement of a nut or screw of some part of the system, refueling, cleaning, etc. It is designed to minimize the limit that the system will spend in degraded states, it is a sort of repair that is done before a unit actually fails. Corrective maintenance deals with the system performance when it gives wrong result and it involves minor repairs that may creep up between inspections.

Repair maintenance is also concerned with increasing the system availability. In order to increase availability, failed unit upon failure is returned to operation by sending it to a repair facility if available, otherwise waits for repair. There may be two types of repair policies

1.10.1 Repeat Repair Policy

Due to certain reasons the repair of a failed unit has to be stopped. When the repair is begun again, it is started all over again.

1.10.2 Resume Repair Policy

The repair of failed component is terminated before completion due to one reason or the other. When it begins again, it is started from the stage where it was prior to the termination of repair.

1.11 Busy Period of the Repairman/Server with the System

Let B(t) be the probability that a repairman/server is busy with the system in the interval (0, t], then in the long run the total fraction of time for which a repairman is busy is given by

$$B = \lim_{t \to \infty} B(t)$$

1.12 Expected Number of Visits by the Server

Let N(t) be a random variable representing the number of times, the repairman has visited the system in the interval (0, t], then the expected number of visits by the repairman to the system in (0, t], is E[N(t)] and in the long run this number per unit time is given by

$$N = \lim_{t \to \infty} \frac{E[N(t)]}{t}$$

1.13 Profit Analysis

Any manufacturing industry is basically a profit making organization and no organization can survive for long without minimum financial returns for its investment. There must be an optimal balance between the reliability aspect of a product and its cost. The major items contributing to the total cost are research and development, production, spares and maintenance. How the cost of these individual items varies with reliability shown in Fig.1.8. In order to increase the reliability of the products, we would require a correspondingly high investment in the research and development activities. The production cost also would increase with the requirement of greater reliability.

The revenue and cost function lead to the profit function of a firm, as the profit is excess of revenue over the cost of production. The profit function in time t is given by

P(t) = Expected revenue in (0, t] – Expected total cost in (0, t]

In general, the optimal policies can more easily be derived for an infinite time span or compared to a finite time span. The profit per unit time, in infinite time span is expressed as

$$\lim_{t \to \infty} \frac{P(t)}{t}$$

i.e. profit per unit time = total revenue per unit time – total cost per unit time. Considering the various costs, the profit equation is given as

$$P = K_1A_0 - K_2B_0 - K_3N_0$$

where

P = Profit per unit time incurred to the system

K_1 = Revenue per unit up time of the system

A_0 = Total fraction of time for which the system is up

K_2 = Cost per unit time for which server is busy

B_0 = Total fraction of time for which the server is busy

K_3 = Cost per visit by the server

N_0 = Expected number of visits per unit time for the server

1.14 Exponential Distribution

The Probability Density Function (pdf) of an exponential distribution has the form

$$f(x; \lambda) = \begin{cases} \lambda e^{-\lambda x} & , \ x \geq 0, \\ 0 & , \ x < 0. \end{cases}$$

where $\lambda > 0$ a parameter of the distribution is, often called the rate parameter. The distribution is supported on the interval $[0,\infty)$. If a random variable X has this distribution, we write $X \sim \text{Exp.}(\lambda)$.

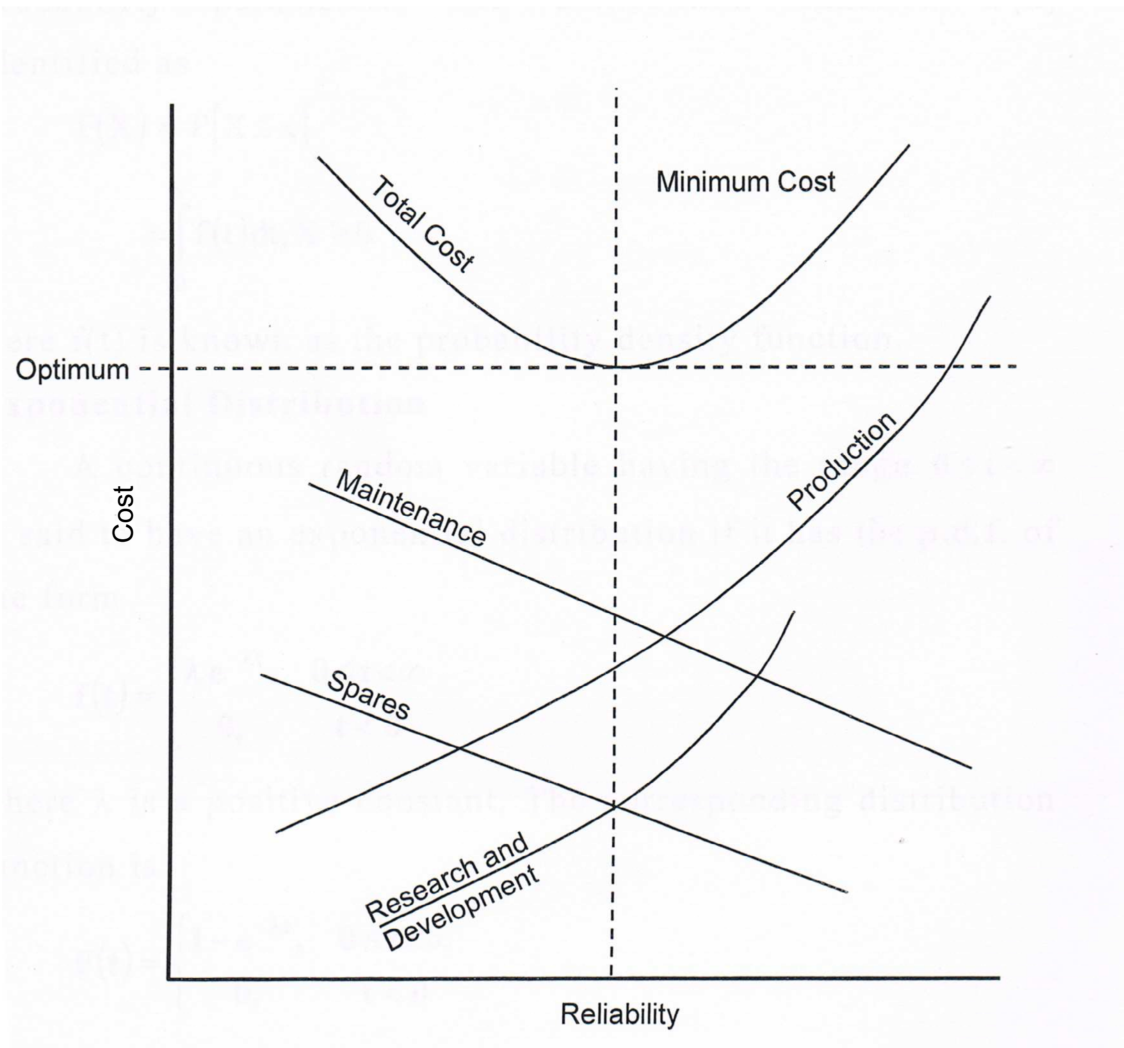


Fig. 1.8: Reliability v/s Cost

1.15 Weibull Distribution

The Weibull distribution (named after Waloddi Weibull) is a continuous probability distribution with the probability density function

$$f(x; k, \lambda) = \begin{cases} \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}, & \text{for } x \geq 0 \\ 0 & \text{for } x < 0. \end{cases}$$

where $k > 0$ is the shape parameter and $\lambda > 0$ is the scale parameter of the distribution.

1.16 Stochastic Process

A stochastic process is a family of random variables $\{X(t) \mid t \in T\}$, defined on a given probability space, indexed by the parameter t, where t varies over an index set T. Both the parametric set and state space can be independently either discrete or continuous.

In stochastic process $\{X(t), t\in T\}$, where X(t), t and T respectively, the state space, parameter (generally taken to be time) and the index set if T is a countable set as T={0,1,2,3………..}, then the stochastic process is said to be a discrete parameter process and if $T=\{t : -\infty < t < \infty\}$, the stochastic process is said to be a continuous parametric process. The state space is classified as discrete if it is finite or countable and continuous if it consists of an interval on the real line. In the present study, we have only dealt with discrete state space continuous time parameter stochastic processes.

1.17 Markov Process

If $\{X(t), t\in T\}$ is a stochastic process such that given the value of X(s), the value of X(t), t>s do not depend on the values of X(u), u<s i.e. for t>s, $i\in s$.

$$\Pr\{X(t)=i|X(u), 0\leq u\leq s\} = \Pr\{X(t) = i|X(s)\}$$

Then the process $\{X(t), t\in T\}$ is a Markov process.

1.18 Markov Chain

A discrete parameter Markov process is known as a Markov Chain. The stochastic process $\{X_n, n = 0, 1, 2,\ldots\}$ is called a Markov chain, if, for j, k, j_1, j_2…………, $j_{n-1}\in N$,

$$\Pr[X_n = k/X_{n-1} = j, X_{n-2} = j_1,\ldots\ldots, X_0 = j_{n-1}]$$
$$= \Pr\{ X_n = k \mid x_{n-1} = j\}$$
$$= p_{jk} \text{ (say)}$$

The conditional probability p_{jk} is called transition probability from the state j at(n-1)th trial to the state k at nth trial. If the transition probability p_{jk} is independent of n, the Markov Chain is said to be homogenous; and if it is dependent of n, the chain is said to be non-homogeneous.

1.19 Semi-Markov Process

In the above, assume that the process is time homogeneous, i.e.

$\Pr\{X_{n+1} = j, t_{n+1} - t_n \leq t \mid X_n = i\} = Q_{ij}(t)$, $i, j \in s$, is independent of n, then there exist limiting transition probabilities.

$$p_{ij} = \lim_{t\to\infty} Q_{ij}(t) = \Pr\{X_{n+1} = j \mid Xn = i\}$$

then $\{X_n, n = 0, 1, 2,\ldots\ldots)$ constitute a Markov Chain with state space E={0,1,2,…..} and transition probability matrix (t.p.m.)

$$P = [p_{ij}]$$

The continuous parameter stochastic process Y(t) with state space E defined by

$$Y(t) = X_n, t_n < t < t_{n+1}$$

is called a semi-Markov process. The Markov Chain X_n is said to be an embedded Markov chain of the semi-Markov process.

In other words, we define the semi-Markov process is a process in which transition from one state to another is governed by the transition probabilities of a Markov process but the time spent in each state before a transition occurs, is a random variable depending upon the last transition made. Thus at transition instants the semi-Markov process behaves just like a Markov process. However, the times at which transitions occur are governed by a different probability mechanism.

1.20 Regenerative Process

Regenerative stochastic process was defined by Smith [1955] and has been crucial in the analysis of complex system. In this, we take time points at which the system history prior to the time points is irrelevant to the system conditions. These points are called regenerative points. Let X(t) be the state of the system of epoch. If t_1, $t_{2,}$… are the epochs at which the process probabilistically restarts, then these epochs are called regenerative epochs and the process $\{X(t), t = t_1, t_2 \ldots\ldots\ldots\}$ is called regenerative process.

CHAPTER-2

RELIABILITY MODELING AND ANALYSIS OF SINGLE UNIT WITH TWO DISSIMILAR COMPONENTS SYSTEM WITH REPLACEMENT OF THE UNITS AT ITS FAILURE

Several authors such as Goel and Gupta (1984), Goel and Sharma (1986), Gopalan and Nagarwalia have studied a two similar unit standby redundant system with preventive maintenance, inspection and two types of repair. Researchers in reliability have shown a keen interest in the analysis of two (or more) component parallel systems owing to their practical utility in modern industrial and technological set-ups. For example we can consider engine failure in two engine planes, wear of two pens on an executive's desk or the performance of an individual's eyes, ears, kidneys and other paired physical organs.
Here we study a reliability model developed by Khaled and Salah (2006) in which there is single unit with two dissimilar components operate in parallel configuration. Each component has two modes- normal and total failure. There is single server which is immediately available.

By using regenerative point technique the following system characteristics are obtained

- Transition probabilities and mean sojourn times of the system.
- Mean Time to System Failure(MTSF)
- Availability analysis of system.
- Expected number of replacements of the system.
- Profit analysis of the system.

2.1 System Descriptions and Assumptions

- The system consists of a single unit having two dissimilar parallel components, say A and B.
- The system remains operative even if a single component operates.
- The failure of a component changes the life time parameter of the other.
- Upon failure each component can be replaced with a similar component with both the component (when failed) can also be replaced simultaneously.
- After replacement of each component, the system is as good as new.

2.2 Notations

E_0	State of the system at t =0.
E	Set of regenerative state.
$\bar{E}$	Set of non-regenerative state.
$F_1(t), f_1(t)$	cdf and pdf of failure time of component A.
$F_2(t), f_2(t)$	cdf and pdf of failure time of component B.
$F_3(t), f_3(t)$	cdf and pdf of failure time of component A when B has already failed.
$F_4(t), f_4(t)$	cdf and pdf of failure time of component B when A has already failed.
$G_1(t), g_1(t)$	cdf and pdf of replacement time of component A.
$G_2(t), g_2(t)$	cdf and pdf of replacement time of component B.
$G_3(t), g_3(t)$	cdf and pdf of replacement time of component A,B.
$q_{ij}(t), Q_{ij}(t)$	pdf and cdf of transition from state S_i to S_j .
P_{ij}	Steady state probability of transition from state S_i to S_j (i.e. $t\to\infty$).
μ_i	Mean sojourn time in state S_i.
$M_i(t)$	The probability that the system, having started from S_i, is up at time t without making any transition into any other regenerative state.
$u(t), U(t)$	pdf and cdf of time for taking a unit into preventive maintenance.
$v(t), V(t)$	pdf and cdf of preventive maintenance time.
$N_{ip}(t)$	Expected frequency of preventive maintenance in (0,t] $E_0=S_i \in E$.
A_N	Component A in normal mode and operative.
B_N	Component B in normal mode and operative
A_F	Component A in failure mode and needs replacement.
B_F	Component B in failure mode and needs replacement.
A_{NP}	Component A in normal mode and under preventive maintenance.
B_{NP}	Component B in normal mode and under preventive maintenance.

The system can have any one of the following states (see Fig. 2.1)

- Up states: $S_0 = (A_N, B_N)$, $S_1 = (A_F, B_N)$,
  $S_2 = (A_N, B_F)$.
- Down states: $S_3 = (A_F, B_F)$.

2.3 The Transition and Mean Sojourn Times

The steady-state transition probabilities are

$$P_{01} = \int_0^\infty f_1(t)\,\bar{F}_2(t)dt\,, \qquad P_{02} = \int_0^\infty f_2(t)\,\bar{F}_1(t)dt\,,$$

$$P_{10} = \int_0^\infty g_1(t)\,\bar{F}_4(t)dt, \qquad P_{13} = \int_0^\infty f_4(t)\,\bar{G}_1(t)dt,$$

$$P_{20} = \int_0^\infty g_2(t)\,\bar{F}_3(t)dt, \qquad P_{23} = \int_0^\infty f_3(t)\,\bar{G}_2(t)dt,$$

$$P_{30} = \int_0^\infty g_3(t)\,dt. \qquad (2.1\text{-}2.7)$$

The mean sojourn times μ_i in various states are

$$\mu_i = E(T) = \int P\,(T > t)dt.$$

Using this we can find the following expression

$$\mu_0 = \int_0^\infty \bar{F}_1(t)\,.\,\bar{F}_2(t)dt, \qquad \mu_1 = \int_0^\infty \bar{G}_1(t)\,.\,\bar{F}_4(t)dt,$$

$$\mu_2 = \int_0^\infty \bar{G}_2(t)\,.\,\bar{F}_3(t)dt, \qquad \mu_4 = \int_0^\infty \bar{G}_3(t)\,dt \qquad (2.8\text{-}2.11)$$

It can be easily verified that

$$P_{01} + P_{02} = P_{13} + P_{10} = P_{23} + P_{20} = 1.$$

2.4 Mean Time to System Failure (MTSF)

Time to system failure can be regarded as the first passage to any of the failed state S_3which is considered as absorbing. Employing the arguments used for regenerative process, the following recursive relations for $x_i(t)$(i=0,1,2) are obtained

$$x_i(t)=\sum_{j=1}^{2} Q_{ij}(t)(s)x_j(t), \quad \text{where i=0}$$

$$x_i(t)=Q_{ij}(t)(s)x_j(t) + Q_{ik}(t) \quad \forall\text{i=1,2 at j=0, k=3.} \qquad (2.12\text{-}2.13)$$

Taking Laplace–Stiltjes transforms of Eq. (2.12–2.13) and solving $\tilde{x}_0(s)$ (dropping the arguments for brevity), then

$$\tilde{x}_0(s) = \frac{\tilde{Q}_{01}\tilde{Q}_{13}+\tilde{Q}_{02}\tilde{Q}_{23}}{1-\tilde{Q}_{01}\tilde{Q}_{10}-\tilde{Q}_{02}\tilde{Q}_{20}} \qquad (2.14)$$

The Mean Time to System Failure (MTSF) of the system is given by

$$\text{MTSF}=\frac{\mu_0+\mu_1 P_{01}+\mu_2 P_{02}}{1-P_{01}P_{10}-P_{02}P_{20}}$$

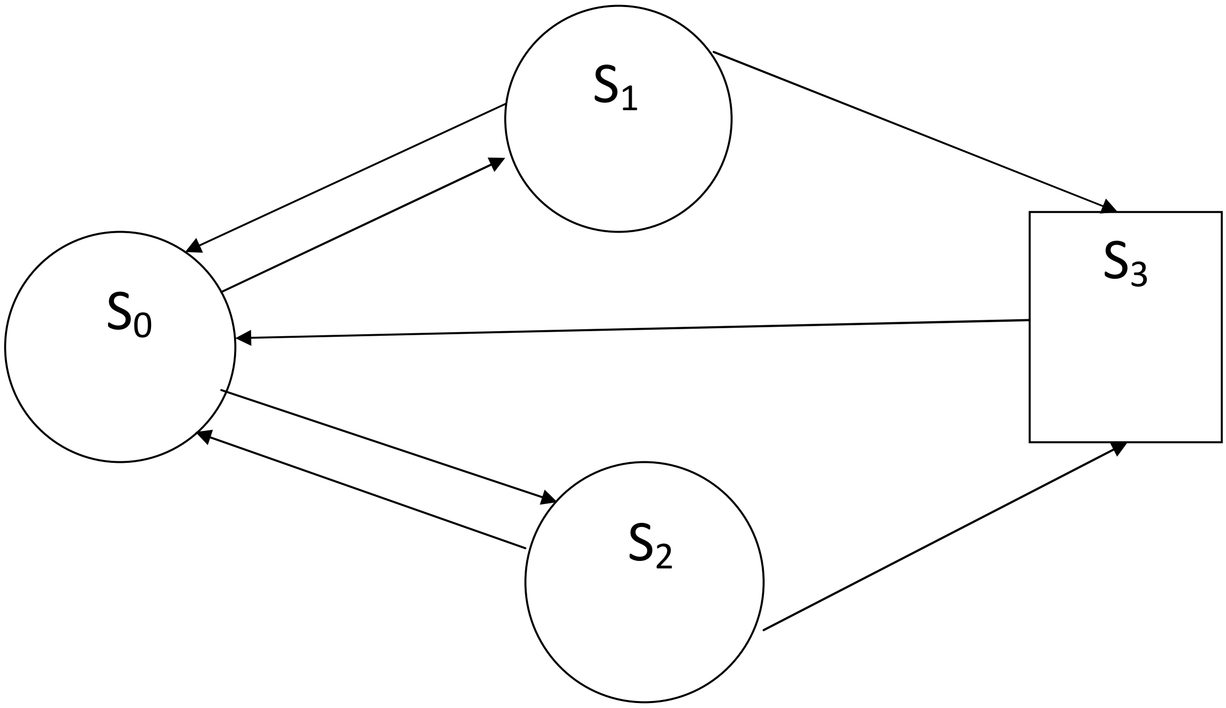


Fig. 2.1: State Transition Diagram for the System

Up State

Down State

2.5 Availability Analysis

Let

$$M_0(t) = \bar{F}_1(t)\bar{F}_2(t),\ M_1(t) = \bar{F}_4(t)\bar{G}_2(t),\ \ M_2(t) = \bar{F}_3(t)\bar{G}_2(t).$$

The point-wise availabilities of the system which starts from a given regenerative point are as follows

$$A_i(t) = M_k(t) + \textstyle\sum_{j=1}^{2} q_{ij}(t)(c)A_j(t), \quad \text{where i=k=0,}$$

$$A_i(t) = M_k(t) + \textstyle\sum_{j=0,3} q_{ij}(t)(c)A_j(t), \quad \forall \text{i=k=1, 2,}$$

$$A_i(t) = q_{ij}(t)(c)A_i(t), \quad \text{where i =3,j=0.} \qquad (2.15\text{-}2.17)$$

Taking Laplace-transforms of the above equations, then

$$A_0^* = \frac{M_0^*+q_{01}^*M_1^*+q_{02}^*M_2^*}{1-q_{01}^*(q_{10}^*+q_{30}^*q_{13}^*)-q_{02}^*(q_{20}^*+q_{23}^*q_{30}^*)}.$$

The steady-state availability of the system can be obtained as

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} sA_0^*(s),$$

$$A_0 = \left[1 + \frac{\mu_3(P_{01}P_{13}+P_{02}P_{23})}{\mu_0+P_{01}\mu_1+P_{02}\mu_2}\right]^{-1}. \quad (2.18)$$

2.6 Expected Number of Replacements

The expected number of replacements of component A in (0, t] by the definition of $N_{iA}(t)$ it follows that

$$N_{iA}(\mathrm{t}) = \sum_{j=1}^{2} Q_{ij}(t)(s)N_{jA}(t), \qquad \text{where } i=0,$$

$$N_{iA}(\mathrm{t}) = \sum_{j=0,3} Q_{ij}(t)(s)N_{jA}(t) + Q_{ik}(t) \qquad \forall i=1,\ k=0,$$

$$N_{iA}(\mathrm{t}) = \sum_{j=0,3} Q_{ij}(t)(s)N_{jA}(t), \qquad \text{where } i=2,$$

$$N_{iA}(\mathrm{t}) = Q_{ij}(t)(s)N_{jA}(t) + Q_{ik}(t) \qquad \forall i=3 \text{ at } k=j=0. \quad (2.19\text{-}2.22)$$

The Laplace–Stiltjes transforms of (2.19–2.22) and solving for $\widetilde{N}_{0A}(s)$, we obtain

$$\widetilde{N}_{0A}(s) = \frac{\tilde{Q}_{01}(\tilde{Q}_{10}+\tilde{Q}_{13}\tilde{Q}_{30})+\tilde{Q}_{30}\tilde{Q}_{02}\tilde{Q}_{23}}{1-\tilde{Q}_{01}(\tilde{Q}_{10}+\tilde{Q}_{13}\tilde{Q}_{30})-\tilde{Q}_{02}(\tilde{Q}_{20}+\tilde{Q}_{23}\tilde{Q}_{30})}. \quad (2.23)$$

In the steady-state, the expected number of replacements of component A, per unit time is

$$N_{0A} = \lim_{t\to\infty}\left(\frac{\widetilde{N}_{0A}(t)}{t}\right) = \lim_{s\to 0} s\widetilde{N}_{0A}(s). \quad (2.24)$$

$$N_{0A} = \frac{[P_{01}+P_{02}P_{23}]}{[\mu_0+P_{01}\mu_0+P_{02}\mu_2+\mu_3(P_{01}P_{13}+P_{02}P_{23})]}.$$

Similarly for the expected number of replacements of component B in (0, t], we have

$$N_{iB}(\mathrm{t}) = \sum_{j=1}^{2} Q_{ij}(t)(s)N_{jB}(t), \qquad \text{where } i=0,$$

$$N_{iB}(\mathrm{t}) = \sum_{j=0,3} Q_{ij}(t)(s)N_{jB}(t), \qquad \text{where } i=1,$$

$$N_{iB}(\mathrm{t}) = \sum_{j=0,3} Q_{ij}(t)(s)N_{jB}(t) + Q_{ik}(t) \qquad \forall i=3 \text{ at } j=k=0,$$

$$N_{iB}(\mathrm{t}) = Q_{ij}(t)(s)N_{jB}(t) + Q_{ik}(t) \qquad \forall i=3 \text{ at } j=k=0. \quad (2.25\text{-}2.28)$$

The Laplace–Stiltjes transforms of (2.25–2.28), on simplification yield

$$\widetilde{N}_{0B}(s) = \frac{\tilde{Q}_{02}(\tilde{Q}_{20}+\tilde{Q}_{23}\tilde{Q}_{30})+\tilde{Q}_{01}\tilde{Q}_{30}\tilde{Q}_{13}}{1-\tilde{Q}_{01}(\tilde{Q}_{10}+\tilde{Q}_{13}\tilde{Q}_{30})-\tilde{Q}_{02}(\tilde{Q}_{20}+\tilde{Q}_{23}\tilde{Q}_{30})}. \quad (2.29)$$

So that $\widetilde{N}_{0B}$, the expected number of replacements of component B, per unit time, in the long run, is

$$N_{0B} = \frac{[P_{02}+P_{01}P_{13}]}{[\mu_0+P_{01}\mu_0+P_{02}\mu_2+\mu_3(P_{01}P_{13}+P_{02}P_{23})]}. \quad (2.30)$$

Hence the expected number of total replacements per unit time, in the steady state, is given by

$$N_0 = N_{0A} + N_{0B} = \frac{[1+P_{02}P_{23}+P_{01}P_{13}]}{[\mu_0+P_{01}\mu_0+P_{02}\mu_2+\mu_3(P_{01}P_{13}+P_{02}P_{23})]}. \quad (2.31)$$

2.7 Profit Analysis

To compute the profit incurred to this system, we have expected total profit in (0, t] equal expected total revenue in (0, t] minus expected total cost in (0, t].

In steady-state

Profit = Total revenue - Total cost

$$P(t)=C_0A_0(t) - C_AN_{0A}(t) - C_BN_{0B}(t). \quad (2.32)$$

where

P is the profit incurred to the first system;

C_0 is revenue per unit up-time;

C_A is cost of replacement of type A component;

C_B is cost of replacement of type B component.

2.8 Special Case

We assume that the failure and replacement times are Weibull distribution with two parameters. But the density function of Weibull distribution with two parameters is given by

$$f(t) = at^b exp[-at^{b+1}/b + 1].$$

Using the Weibull distribution to study two cases when b = 0, it become the exponential distribution and when b = 1, it become the Rayleigh distribution.

Let

$F_1(t) = 1 - exp[-\alpha t^{b+1}/b + 1]$, $F_2(t) = 1 - exp[-\beta t^{b+1}/b + 1]$,

$F_3(t) = 1 - exp[-\alpha' t^{b+1}/b + 1]$, $F_4(t) = 1 - exp[-\beta' t^{b+1}/b + 1]$,

$G_1(t) = 1 - exp[-\gamma t^{b+1}/b + 1]$, $G_2(t) = 1 - exp[-\delta t^{b+1}/b + 1]$,

$G_3(t) = 1 - exp[-\theta t^{b+1}/b + 1]$.

Transition probabilities and mean sojourn times. The steady-state transition probabilities are

$$P_{01} = \frac{\alpha}{\alpha+\beta}\ , \qquad P_{02} = \frac{\beta}{\alpha+\beta}\,, \qquad P_{10} = \frac{\gamma}{\gamma+\beta'}\,,$$

$$P_{13} = \frac{\beta'}{\gamma+\beta'}\,, \qquad P_{23} = \frac{\alpha'}{\delta+\alpha'}\,, \qquad P_{20} = \frac{\beta}{\delta+\alpha'}\,,$$

Obviously

$$P_{01}+P_{02}=P_{10}+P_{13}=P_{20}+P_{23}=P_{30}=1.$$

The mean sojourn times in various states are

$$\mu_0 = \frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)\left[\frac{\alpha+\beta}{b+1}\right]^{\frac{1}{b+1}}}\ , \qquad \mu_1 = \frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)\left[\frac{\gamma+\beta'}{b+1}\right]^{\frac{1}{b+1}}}\,,$$

$$\mu_2 = \frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)\left[\frac{\alpha'+\delta}{b+1}\right]^{\frac{1}{b+1}}}\ , \qquad \mu_3 = \frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)\left[\frac{\theta}{b+1}\right]^{\frac{1}{b+1}}}\ .$$

Mean Time to System Failure (MTSF)

$$\text{MTSF} = \frac{\left[\frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)^{\frac{1}{b+1}}}\right]\left[\frac{\alpha+\beta}{(\alpha+\beta)^{\frac{1}{b+1}}} + \frac{\alpha}{(\gamma+\beta')^{\frac{1}{b+1}}} + \frac{\beta}{(\alpha'+\delta)^{\frac{1}{b+1}}}\right]}{(\alpha+\beta) - \frac{\gamma\delta}{\gamma+\beta'} - \frac{\beta\delta}{\delta+\alpha'}}.$$

The availability in the steady-state is

$$\hat{A}_0 = \left[1 + \frac{[\theta]^{\frac{-1}{b+1}}\left[\frac{\alpha\beta'}{\gamma+\beta'} + \frac{\beta\alpha'}{\alpha'+\delta}\right]}{(\alpha+\beta)^{\frac{b}{b+1}} + \frac{\alpha}{\left(\gamma+\beta'\right)^{\frac{1}{b+1}}} + \frac{\beta}{(\alpha'+\delta)^{\frac{1}{b+1}}}}\right]^{-1}\ .$$

In steady-state the expected number of total replacements per unit time is

$$\widehat{N}_0 = \frac{\left[(\alpha+\beta) + \frac{\beta\alpha'}{\alpha'+\delta} + \frac{\alpha\beta'}{\gamma+\beta'}\right]}{\left[\frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)^{\frac{1}{b+1}}}\right]\left[(\alpha+\beta)^{\frac{b}{b+1}} + \frac{\alpha}{\left(\gamma+\beta'\right)^{\frac{1}{b+1}}} + \frac{\beta}{(\alpha'+\delta)^{\frac{1}{b+1}}} + \frac{1}{[\theta]^{\frac{1}{b+1}}}\left[\frac{\alpha\beta'}{\gamma+\beta'} + \frac{\beta\alpha'}{\alpha'+\delta}\right]\right]}\ .$$

In steady-state the profit function is

$$\hat{P} = C_0\hat{A}_0 - C_A\widehat{N}_{0A} - C_B\widehat{N}_{0B}.$$

Table 2.1: Comparison Between the Effects of the Exponential and Rayleigh Distributions on the System

∝	MTSF in the exponential distribution	MTSF in the Rayleigh distribution	A_0 in the exponential distribution	A_0 in the Rayleigh distribution	Profit in the exponential distribution	Profit in the Rayleigh distribution
0.2	7.678571	8.727692	0.504563	0.676238	490.2477	651.3621
0.3	6.456044	8.066916	0.458288	0.658767	442.4919	632.3086
0.4	5.691964	7.604849	0.425394	0.645386	408.5449	617.7368
0.5	5.169173	7.258674	0.40081	0.634653	383.1741	606.06
0.6	4.788961	6.986958	0.38174	0.625762	363.4939	596.3946
0.7	4.5	6.766397	0.366516	0.618221	347.7828	588.2005
0.8	4.272959	6.582755	0.354081	0.611706	334.9501	581.1248
0.9	4.089862	6.426783	0.343733	0.605995	324.2711	574.9251

CHAPTER-3

RELIABILITY MODELING AND ANALYSIS OF SINGLE UNIT WITH TWO DISSIMILAR COMPONENTS SYSTEM WITH PREVENTIVE MAINTENANCE

In Chapter 2, we have studied a single unit having two dissimilar components reliability model with continuous operation. But the continued operation and ageing of the systems gradually reduce their performance, reliability, and safety. It can be seen literature that Preventive Maintenance (PM) can slow the deterioration process of a repairable system and restore the system to a younger age or state. Therefore, preventive maintenance of the systems is necessary after a pre-specific period of time not only to maintain the operational power but may also reduce the failure.

There is single server who attends the system immediately whenever needed to conduct preventive maintenance and replace at completely failure of the unit. The unit works as new after preventive Maintenance (PM).

Keeping the above consideration in mind here we study a reliability model investigated by Khaled and Salah (2006) with the concept of Preventive Maintenance (PM). This model consists of as single unit having two dissimilar components.
Using the regenerative point technique the following measures are obtained.

- Transition probabilities and mean sojourn times of the system.
- Mean Time to System Failure(MTSF)
- Availability analysis of system.
- Expected number of replacements of the system.
- Profit analysis of the system.

3.1 System Descriptions and Assumptions

- The system consists of a single unit having two dissimilar parallel components, say A and B.
- The system remains operative even if a single component operates.
- The failure of a component changes the life time parameter of the other.
- Upon failure each component can be replaced with a similar component with both the component (when failed) can also be replaced simultaneously.

- After replacement of each component, the system is as good as new.
- Preventive maintenance (e.g., overhaul, inspection, minor repairs, etc.) is provided to the system at random epochs when the system is in the state S_0 where both the components are normal.

3.2 Notations

E_0	State of the system at t =0.
E	Set of regenerative state.
$\bar{E}$	Set of non-regenerative state.
$F_1(t), f_1(t)$	cdf and pdf of failure time of component A.
$F_2(t), f_2(t)$	cdf and pdf of failure time of component B.
$F_3(t), f_3(t)$	cdf and pdf of failure time of component A when B has already failed.
$F_4(t), f_4(t)$	cdf and pdf of failure time of component B when A has already failed.
$G_1(t), g_1(t)$	cdf and pdf of replacement time of component A.
$G_2(t), g_2(t)$	cdf and pdf of replacement time of component B.
$G_3(t), g_3(t)$	cdf and pdf of replacement time of component A,B.
$q_{ij}(t), Q_{ij}(t)$	pdf and cdf of transition from state S_i to S_j .
P_{ij}	Steady state probability of transition from state S_i to S_j (i.e. $t\rightarrow\infty$).
μ_i	Mean sojourn time in state S_i.
$M_i(t)$	The probability that the system, having started from S_i, is up at time t without making any transition into any other regenerative state.
$u(t), U(t)$	pdf and cdf of time for taking a unit into preventive maintenance.
$v(t), V(t)$	pdf and cdf of preventive maintenance time.
$N_{ip}(t)$	Expected frequency of preventive maintenance in (0,t] $E_0=S_i\in E$.
A_N	Component A in normal mode and operative.
B_N	Component B in normal mode and operative
A_F	Component A in failure mode and needs replacement.
B_F	Component B in failure mode and needs replacement.
A_{NP}	Component A in normal mode and under preventive maintenance.
B_{NP}	Component B in normal mode and under preventive maintenance.

The system can have any one of the following states (see Fig. 3.1)

- Up states: $S_0 = (A_N, B_N)$, $S_1= (A_F,B_N)$, $S_2= (A_N,B_F)$.

- Down states: $S_3 = (A_F, B_F)$.

3.3 The Transition and Mean Sojourn Times

The steady-state transition probabilities are

$$P_{01} = \int_0^\infty f_1(t)\,\bar{F}_2(t)\bar{U}(t)dt, \qquad P_{02} = \int_0^\infty f_2(t)\,\bar{F}_1(t)\bar{U}(t)dt,$$

$$P_{04} = \int_0^\infty u(t)\,\bar{F}_2(t)\bar{F}_1(t)dt, \qquad P_{10} = \int_0^\infty g_1(t)\,\bar{F}_4(t)dt,$$

$$P_{13} = \int_0^\infty f_4(t)\,\bar{G}_1(t)dt, \qquad P_{20} = \int_0^\infty g_2(t)\,\bar{F}_3(t)dt,$$

$$P_{23} = \int_0^\infty f_3(t)\,\bar{G}_2(t)dt, \qquad P_{30} = \int_0^\infty g_3(t)\,dt,$$

$$P_{40} = \int_0^\infty u(t)\,dt. \qquad (3.1\text{-}3.9)$$

The mean sojourn times in various states are

$$\mu_0 = \int_0^\infty \bar{F}_1(t)\,.\,\bar{F}_2(t)\bar{U}(t)dt, \qquad \mu_1 = \int_0^\infty \bar{G}_1(t)\,.\,\bar{F}_4(t)dt,$$

$$\mu_2 = \int_0^\infty \bar{G}_2(t)\,.\,\bar{F}_3(t)dt, \qquad \mu_3 = \int_0^\infty \bar{G}_3(t)\,dt,$$

$$\mu_4 = \int_0^\infty \bar{V}(t)\,dt. \qquad (3.10\text{-}3.14)$$

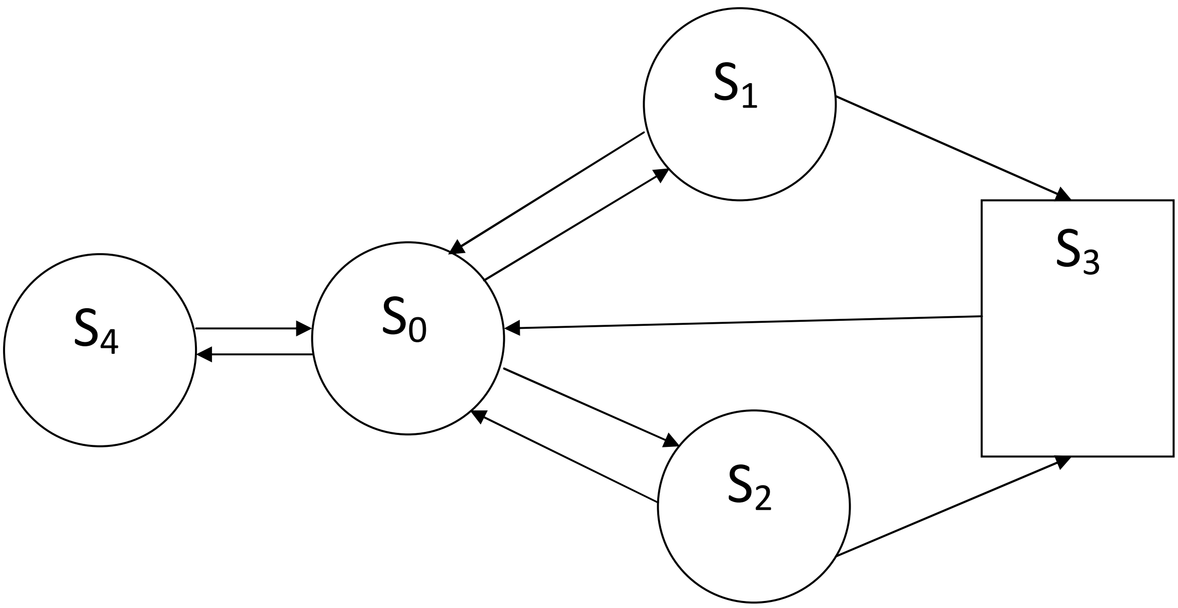
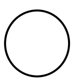


Fig. 3.1: State Transition Diagram for the System



Up State

Down State

3.4 Mean time to System Failure (MTSF)

Time to system failure can be regarded as the first passage to any of the failed state S_3 which is considered as absorbing. Employing the arguments used for regenerative process, the following recursive relations for $x_i(t)$(i =0,1,2) are obtained.

$x_i(t)=\sum_{j=1,2,4}^{2} Q_{ij}(t)(s)x_j(t),$ where i=0

$x_i(t)=\sum_{j=0} Q_{ij}(t)(s)x_j(t) + Q_{ik}(t)$ ∀i=1,2 at k=3.

$x_i(t)=Q_{ij}(t)(s)x_j(t),$ where i=4 at j=0. (3.15-3.17)

Taking Laplace–Stiltjes transforms of Eq. (3.15–3.17) and solving $\overline{x}_0(s)$ (dropping the arguments for brevity), then

$$\tilde{x}_0(s) = \frac{\tilde{Q}_{01}\tilde{Q}_{13}+\tilde{Q}_{02}\tilde{Q}_{23}}{1-\tilde{Q}_{01}\tilde{Q}_{10}-\tilde{Q}_{02}\tilde{Q}_{20}-\tilde{Q}_{04}\tilde{Q}_{40}} \quad (3.18)$$

The Mean Time to System Failure (MSTF) of the system is given by

$$\text{MTSF}=\frac{\mu_0+\mu_1P_{01}+\mu_2P_{02}+\mu_4P_{04}}{1-P_{01}P_{10}-P_{02}P_{20}-P_{04}P_{40}}$$

3.5 Availability Analysis

Let

$$M_0(t) = \bar{F}_1(t)\bar{F}_2(t),\ M_1(t) = \bar{F}_4(t)\bar{G}_2(t),$$
$$M_2(t) = \bar{F}_3(t)\bar{G}_2(t),\ M_4(t) = \bar{V}(t)$$

The point-wise availabilities of the system which starts from a given regenerative point are as follows

$A_i'(t) = M_k(t) + \sum_{j=1,2,4} q_{ij}(t)(c)A_j'(t).$ where i=0 at k=0,

$A_i'(t) = M_k(t) + \sum_{j=0,3} q_{ij}(t)(c)A_j'(t),$ ∀i=1,2 at k=1,2,

$A_i'(t) = q_{ij}(t)(c)A_j'(t),$ where i=3 at j=0,

$A_i'(t) = M_k(t)+q_{ij}(t)(c)A_j'(t),$ ∀i=4 at k=4,j=0. (3.19-3.22)

Taking Laplace-transforms of the above equations, then

$$A_0'^*(s) = \frac{M_0^*+q_{01}^*M_1^*+q_{02}^*M_2^*+q_{04}^*M_4^*}{1-q_{01}^*q_{10}^*-q_{02}^*q_{20}^*-q_{04}^*q_{40}^*-q_{30}^*(q_{01}^*q_{13}^*+q_{23}^*q_{02}^*)}.$$

Thus, the steady-state system availability

$$A_0'^{*} = \lim_{t\to\infty} A_0'(t) = \lim_{s\to 0} sA_0'^{*}(s)$$

$$A_0 = \left[1 + \frac{\mu_3(P_{01}P_{13}+P_{02}P_{23})}{\mu_0+P_{01}\mu_1+P_{02}\mu_2+P_{04}\mu_4}\right]^{-1}. \quad (3.23)$$

3.6 Expected Number of Replacements

The expected number of replacements of component A in (0, t] by the definition $N_{iA}^{0'}(t)$it follows that

$$N_{iA}'(\mathrm{t}) = \textstyle\sum_{j=1,2,4} Q_{ij}(t)(s)N_{jA}'(\mathrm{t}), \qquad \text{where i=0,}$$

$$N_{iA}'(\mathrm{t}) = \textstyle\sum_{j=0,3} Q_{ij}(t)(s)N_{jA}'(\mathrm{t}) + Q_{ik}(t), \qquad \forall i = 1 \text{ at k=0,}$$

$$N_{iA}'(\mathrm{t}) = \textstyle\sum_{j=0,3} Q_{ij}(t)(s)N_{jA}'(\mathrm{t}), \qquad \text{where i=2,}$$

$$N_{iA}'(\mathrm{t}) = Q_{ij}(t)(s)N_{jA}'(\mathrm{t}) + Q_{ik}(t), \qquad \text{where i=3 at j=k=0,}$$

$$N_{iA}'(\mathrm{t}) = Q_{ij}(t)(s)N_{jA}'(\mathrm{t}), \qquad \text{where i=4 at j=0.} \quad (3.24\text{-}3.28)$$

The Laplace–Stiltjes transforms of (3.24–3.28) and solving for $\widetilde{N}_{0A}'(s)$,we obtain

$$\widetilde{N}_{0A}'(s) = \frac{\tilde{Q}_{30}(\tilde{Q}_{01}\tilde{Q}_{13}+\tilde{Q}_{23}\tilde{Q}_{02})+\tilde{Q}_{01}\tilde{Q}_{10}}{1-\tilde{Q}_{01}\tilde{Q}_{10}-\tilde{Q}_{02}\tilde{Q}_{20}-\tilde{Q}_{04}\tilde{Q}_{40}-\tilde{Q}_{30}(\tilde{Q}_{01}\tilde{Q}_{13}+\tilde{Q}_{23}\tilde{Q}_{02})}. \quad (3.29)$$

In the steady state, the expected number of replacements of component A, per unit time is

$$N_{0A}' = \frac{P_{01}+P_{02}P_{23}}{\mu_0+P_{01}\mu_1+P_{02}\mu_2+\mu_3(P_{01}P_{13}+P_{02}P_{23})+P_{04}\mu_4}. \quad (3.30)$$

Similarly for the expected number of replacements of component B in (0, t], we have

$$N_{iB}'(t) = \textstyle\sum_{j=1,2,4} Q_{ij}(t)(s)N_{jB}'(t), \qquad \text{where i=0,}$$

$$N_{iB}'(t) = \textstyle\sum_{j=0,3} Q_{ij}(t)(s)N_{jB}'(t), \qquad \text{where i=1,}$$

$$N_{iB}'(t) = \textstyle\sum_{j=0,3} Q_{ij}(t)(s)N_{jB}'(t) + Q_{ik}(t), \qquad \text{i=2 at k=0,}$$

$$N_{iB}'(t) = \textstyle\sum_{j=0,3} Q_{ij}(t)(s)N_{jB}'(t), \qquad \text{where i=2,}$$

$$N_{iB}'(t) = Q_{ij}(t)(s)N_{jB}'(t) + Q_{ik}(t), \qquad \text{where i=3 at j=k=0,}$$

$$N_{iB}'(t) = Q_{ij}(t)(s)N_{jB}'(t), \qquad \text{where i=4 at j=0} \quad (3.31\text{-}3.36)$$

The Laplace–Stiltjes transforms of (3.31–3.36), on simplification yield

$$N_{0B}'(s) = \frac{\tilde{Q}_{30}(\tilde{Q}_{01}\tilde{Q}_{13}+\tilde{Q}_{23}\tilde{Q}_{02})+\tilde{Q}_{02}\tilde{Q}_{20}}{1-\tilde{Q}_{01}\tilde{Q}_{10}-\tilde{Q}_{02}\tilde{Q}_{20}-\tilde{Q}_{04}\tilde{Q}_{40}-\tilde{Q}_{30}(\tilde{Q}_{01}\tilde{Q}_{13}+\tilde{Q}_{23}\tilde{Q}_{02})}. \quad (3.37)$$

So that, N'_{0B} the expected number of replacements of component B, per unit time, in the long run, is

$$N'_{0B} = \frac{P_{02}+P_{01}P_{13}}{\mu_0+P_{01}\mu_1+P_{02}\mu_2+\mu_3(P_{01}P_{13}+P_{02}P_{23})+P_{04}\mu_4}. \tag{3.38}$$

Hence the expected number of total replacements per unit time, in the steady-state, is given by

$$N'_0 = N'_{0A} + N'_{0B}.$$

$$= \frac{[1+P_{01}P_{13}+P_{02}P_{23}]}{[\mu_0+P_{01}\mu_1+P_{02}\mu_2+\mu_3(P_{01}P_{13}+P_{02}P_{23})+P_{04}\mu_4]}. \tag{3.39}$$

The expected frequency of preventive maintenance

$N'_{ip}(t) = \sum_{j=1,2,4} Q_{ij}(t)(s)N^{\mu}_{jp}(t),$ where i=0,

$N'_{ip}(t) = \sum_{j=0,3} Q_{ij}(t)(s)N'_{jp}(t),$ ∀i=1, 2,

$N'_{ip}(t) = Q_{ij}(t)(s)N'_{jp}(t),$ where i=3 at j=0,

$N'_{ip}(t) = Q_{ij}(t)(s)N'_{jp}(t) + Q_{ik},$ where i=4 at j=k=0 (3.40-3.43)

The Laplace–Stiltjes transforms of (3.40–3.43), on simplification yield

$$\tilde{N}'_{0p}(s) = \frac{\tilde{Q}_{04}\tilde{Q}_{40}}{1-\tilde{Q}_{01}\tilde{Q}_{10}-\tilde{Q}_{02}\tilde{Q}_{20}-\tilde{Q}_{04}\tilde{Q}_{40}-\tilde{Q}_{30}(\tilde{Q}_{01}\tilde{Q}_{13}+\tilde{Q}_{23}\tilde{Q}_{02})}. \tag{3.44}$$

In the steady-state, the expected frequency of preventive maintenance per unit time is

$$N'_{0p} = \frac{[P_{04}]}{[\mu_0+P_{01}\mu_1+P_{02}\mu_2+\mu_3(P_{01}P_{13}+P_{02}P_{23})+P_{04}\mu_4]}. \tag{3.45}$$

3.7 Profit Analysis

To compute the profit incurred to this system, we have expected total profit in (0, t] equal expected total revenue in (0, t] minus expected total cost in (0, t].

In steady-state

Profit = Total revenue - Total cost

$$P(t) = C_0A'_0(t) - C_AN'_{0A}(t) - C_BN'_{0B}(t) - C_MN_{0p}(t). \tag{3.46}$$

where

P is the profit incurred to the first system;

C_0 is revenue per unit up-time;

C_A is cost of replacement of type A component;

C_B is cost of replacement of type B component;

C_M is cost per preventive maintenance.

3.8 Special Case

We assume that the failure and replacement times are Weibull distribution with two parameters. But the density function of Weibull distribution with two parameters is given by

$$f(t) = at^b exp[-at^{b+1}/b + 1].$$

Using the Weibull distribution to study two cases when b = 0, it become the exponential distribution and when b = 1, it become the Rayleigh distribution.

Let

$$F_1(t) = 1 - exp[-\alpha t^{b+1}/b + 1], \qquad F_2(t) = 1 - exp[-\beta t^{b+1}/b + 1],$$
$$F_3(t) = 1 - exp[-\alpha' t^{b+1}/b + 1], \qquad F_4(t) = 1 - exp[-\beta' t^{b+1}/b + 1],$$
$$G_1(t) = 1 - exp[-\gamma t^{b+1}/b + 1], \qquad G_2(t) = 1 - exp[-\delta t^{b+1}/b + 1],$$
$$G_3(t) = 1 - exp[-\theta t^{b+1}/b + 1], \qquad U(t) = 1 - exp[-\lambda t^{b+1}/(b + 1)],$$
$$V(t) = 1 - exp[-\mu t^{b+1}/b + 1].$$

Transition probabilities and mean sojourn times. The steady-state transition probabilities are

$$P_{01} = \frac{\alpha}{\alpha+\beta+\lambda}, \qquad P_{02} = \frac{\beta}{\alpha+\beta+\lambda}, \qquad P_{04} = \frac{\lambda}{\alpha+\beta+\lambda},$$
$$P_{10} = \frac{\gamma}{\gamma+\beta'}, \qquad P_{13} = \frac{\beta'}{\gamma+\beta'}, \qquad P_{23} = \frac{\alpha'}{\alpha'+\delta},$$
$$P_{20} = \frac{\delta}{\alpha'+\delta}.$$

Obviously

$$P_{01} + P_{02} + P_{04} = P_{10} + P_{13} = P_{23} + P_{20} = P_{30} = P_{40} = 1.$$

The mean sojourn times in various states are

$$\mu_0 = \frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)\left[\frac{\alpha+\beta+\lambda}{b+1}\right]^{\frac{1}{b+1}}}, \qquad \mu_1 = \frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)\left[\frac{\gamma+\beta^{\mu}}{b+1}\right]^{\frac{1}{b+1}}},$$
$$\mu_2 = \frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)\left[\frac{\alpha'+\delta}{b+1}\right]^{\frac{1}{b+1}}}, \qquad \mu_3 = \frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)\left[\frac{\theta}{b+1}\right]^{\frac{1}{b+1}}},$$

$$\mu_4 = \frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)\left[\frac{\mu}{b+1}\right]^{\frac{1}{b+1}}}.$$

Mean Time to System Failure (MTSF) is

$$\text{MTSF} = \frac{\left[\frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)^{\frac{1}{b+1}}}\right]\left[\frac{\alpha+\beta+\lambda}{(\alpha+\beta+\lambda)^{\frac{1}{b+1}}} + \frac{\alpha}{(\gamma+\beta')^{\frac{1}{b+1}}} + \frac{\beta}{(\alpha'+\delta)^{\frac{1}{b+1}}} + \frac{\lambda}{(b+1)\left[\frac{\mu}{b+1}\right]^{\frac{1}{b+1}}}\right]}{(\alpha+\beta+\lambda) - \frac{\gamma\delta}{\gamma+\beta'} - \frac{\beta\delta}{\delta+\alpha'} - \lambda}.$$

The availability in the steady-state is

$$\hat{\hat{A}}_0 = \left[1 + \frac{[\theta]^{\frac{-1}{b+1}}\left[\frac{\alpha\beta'}{\gamma+\beta'} + \frac{\beta\alpha'}{\alpha'+\delta}\right]}{(\alpha+\beta+\lambda)^{\frac{b}{b+1}} + \frac{\alpha}{(\gamma+\beta')^{\frac{1}{b+1}}} + \frac{\beta}{(\alpha'+\delta)^{\frac{1}{b+1}}} + \frac{\lambda}{(\mu)^{\frac{1}{b+1}}}}\right]^{-1}.$$

In steady-state the expected number of total replacements per unit time is

$$\hat{\hat{N}}_0 = \frac{\left[(\alpha+\beta+\lambda) + \frac{\beta\alpha'}{\alpha'+\delta} + \frac{\alpha\beta'}{\gamma+\beta'}\right]}{\left[\frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)^{\frac{1}{b+1}}}\right]\left[(\alpha+\beta+\lambda)^{\frac{b}{b+1}} + \frac{\alpha}{\left(\gamma+\beta'\right)^{\frac{1}{b+1}}} + \frac{\beta}{\left(\alpha'+\delta\right)^{\frac{1}{b+1}}} + \frac{1}{[\theta]^{\frac{1}{b+1}}}\left[\frac{\alpha\beta'}{\gamma+\beta'} + \frac{\beta\alpha'}{\alpha'+\delta}\right] + \frac{\lambda}{(\mu)^{\frac{1}{b+1}}}\right]}.$$

In steady-state the expected number of preventive maintenance per unit time is

$$\hat{\hat{N}}_{0p} = \frac{\lambda}{\left[\frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)^{\frac{1}{b+1}}}\right]\left[(\alpha+\beta+\lambda)^{\frac{b}{b+1}} + \frac{\alpha}{\left(\gamma+\beta'\right)^{\frac{1}{b+1}}} + \frac{\beta}{\left(\alpha'+\delta\right)^{\frac{1}{b+1}}} + \frac{1}{[\theta]^{\frac{1}{b+1}}}\left[\frac{\alpha\beta'}{\gamma+\beta'} + \frac{\beta\alpha'}{\alpha'+\delta}\right] + \frac{\lambda}{(\mu)^{\frac{1}{b+1}}}\right]}.$$

In steady-state the profit function is

$$\hat{\hat{P}} = C_0\hat{\hat{A}}_0 - C_A\hat{\hat{N}}_{0A} - C_B\hat{\hat{N}}_{0B} - C_M\hat{\hat{N}}_{0P}(\text{t}).$$

Table 3.1: Comparison Between the Effects of the Exponential and Rayleigh Distributions on the System

$\propto$	MTSF in the exponential distribution	MTSF in the Rayleigh distribution	A_0 in the exponential distribution	A_0 in the Rayleigh distribution	Profit in the exponential distribution	Profit in the Rayleigh distribution
0.2	17.45614	17.89247	0.611054	0.810677	586.9191	780.8533
0.3	14.06667	14.8182	0.558694	0.780038	534.7043	749.3425
0.4	11.98925	12.91733	0.519007	0.755581	495.128	724.1928
0.5	10.58559	11.62106	0.487889	0.735528	464.0969	703.5717
0.6	9.573643	10.67768	0.462836	0.718734	439.1131	686.2995
0.7	8.809524	9.958498	0.442231	0.704425	418.5657	671.5807
0.8	8.212121	9.390839	0.424987	0.69206	401.3696	658.8579
0.9	7.73224	8.930502	0.410343	0.681247	386.7666	664.6145

CHAPTER-4

RELIABILITY MODELING AND ANALYSIS OF SINGLE UNIT SYSTEM SUBJECT TO DEGRADATION, INSPECTION AND PREVENTIVE MAINTENANCE

Recently, the reliability models of single-unit systems operating under different assumptions have been proposed by Nakagawa and Osaki [1976], Chander and Singh [2005], and Renbin and Zaiming [2011] under these assumptions that unit works as new after repair and there is no need to give preventive maintenance to a unit. In fact, these assumptions cannot be imposed on every system due to different operating and repair characteristics. And, the unit may have increased failure rate after its repair by an ordinary server. In such a situation the unit becomes degraded after repair. Also, on some cases, the repair of the degraded unit is neither possible nor economical to the system due to its excessive use. Under such conditions, the degraded failed unit may be replaced by new one and this can be revealed by inspection. Malik et al. [2008] analyzed a system with two types of inspection subject to degradation. Preventive maintenance of the systems is necessary after a pre-specific period of time not only to maintain the operational power but may also reduce the failure and the degradation rate. Singh and Agarafiotis [1995] have discussed a system under preventive maintenance subject to maximum operation and repair times.

While considering above facts and to fill up the gap, the main purpose of the present chapter is to develop and analyze a stochastic model for a one-unit system introducing the concepts of Preventive Maintenance (PM) at Partially Fail Stage (PFS), Maximum Operation Time (MOT) and degradation. The partially failed operating unit is shutdown after a maximum operation time for Preventive Maintenance (PM).There is a single server who attends the system immediately whenever needed to conduct required action. The unit becomes degraded after repair.

4.1 Model Description and Assumptions

- The system consists of a single-unit reliability model in which unit may fail totally either directly from normal mode or via. partial failure.
- The unit works as new after preventive maintenance.
- The partially failed operating unit is shutdown after a maximum operation time for preventive maintenance (PM).

- There is a single server who attends the system immediately whenever needed to conduct preventive maintenance at partial failure stage and repair at completely failure of the unit.
- The unit becomes degraded after repair.
- The server inspects the degraded unit at its failure to see the feasibility of repair.
- Switches are perfect.
- The distribution of failure, maximum rate of operation times, preventive maintenance, inspection and repair times are taken as arbitrary distribution.
- All the random variables are mutually independent.

4.2 Notations

E_0	State of the system at t =0.
E	Set of regenerative state.
O	The unit is operative and in normal mode.
$f(t)/f_1(t)/f_2(t)$	Failure rate of unit from normal mode to complete failure/Normal mode to partial failure/ partial failure to complete failure.
$f_3(t)$	Failure rate of degraded system.
Z(t)	Maximum constant rate of operation after partial failure.
PFP_m	The unit is partially failed and under preventive maintenance.
FUr	The unit is completely failed and under repair.
$g(t)/G(t)$	pdf /cdf of the time for preventive maintenance of the unit.
$g_1(t)/G_1(t)$	pdf/cdf of the time for repair of a failed new unit.
$g_2(t)/G_2(t)$	pdf/cdf of the time has repair of a failed degraded unit.
$h(t)/H(t)$	pdf/cdf of the inspection time of degraded system.
p/q	Probability that repair of degraded system is feasible/not feasible.
DFUi	Degraded unit is failed and under inspection.
DFUr	Degraded unit is failed and under repair.
D_0	The unit is degraded and operative.
PFO	The unit is partially failed and operative.

The system can have any one of the following states (see Fig. 4.1)

Up state: S_0= (O), S_1= (PFO),
S_4= (D_0).

Down state: S_2= (FUr), S_3= (PFP_m),
S_5= (DFUi), S_6= (DFUr).

4.3 The Transition and Mean Sojourn Times

The steady-state transition probabilities are

$P_{01} = \int_0^\infty f_1(t)\bar{F}\ (t)dt,$ $\quad$ $P_{02} = \int_0^\infty f(t)\overline{F_1}\ (t)dt,$

$P_{12} = \int_0^\infty f_2(t)\bar{Z}\ (t)dt,$ $\quad$ $P_{13} = \int_0^\infty z(t)\overline{F_2}\ (t)dt,$

$P_{24} = \int_0^\infty g_1(t)\, dt,$ $\quad$ $P_{30} = \int_0^\infty g(t)\, dt,$

$P_{45} = \int_0^\infty f_3(t)\, dt,$ $\quad$ $P_{50} = \int_0^\infty q\, h(t)dt,$

$P_{56} = \int_0^\infty p\, h(t)dt,$ $\quad$ $P_{64} = \int_0^\infty g_2(t)\, dt.$ $\quad$ (4.1-4.10)

The mean sojourn times μ_i in various states are

$\mu_0 = \int_0^\infty \bar{F}(t)\,.\,\bar{F}_1(t)dt,$ $\quad$ $\mu_1 = \int_0^\infty \bar{Z}(t)\,.\,\bar{F}_2(t)dt,$

$\mu_2 = \int_0^\infty \bar{G}_1\ (t)dt,$ $\quad$ $\mu_3 = \int_0^\infty \bar{G}\ (t)dt,$

$\mu_4 = \int_0^\infty \bar{F}_3\ (t)dt,$ $\quad$ $\mu_5 = \int_0^\infty \bar{H}(t),$

$\mu_6 = \int_0^\infty \bar{G}_2\ (t)dt.$ $\quad$ (4.11-4.17)

4.4 Mean Time to System Failure (MTSF)

Let Φ_i be the cdf of the first passage time from regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$

$\Phi_0(t) = Q_{01}(t) ® \Phi_1(t) + Q_{02}(t),$

$\Phi_1(t) = Q_{12}(t)\ + Q_{13}(t).$ $\quad$ (4.18-4.19)

Taking LST of above relations (4.18-4.19) and solving for $\bar{\Phi}_0(s)$, we have

R*(s)= $\lim_{s\to 0} \frac{1-\bar{\Phi}_0(s)}{s}$ $\quad$ (4.20)

The reliability R(t) of the model can be obtained by taking Laplace inverse transform of MTSF $=\mu_0 + P_{01}\mu_1$.

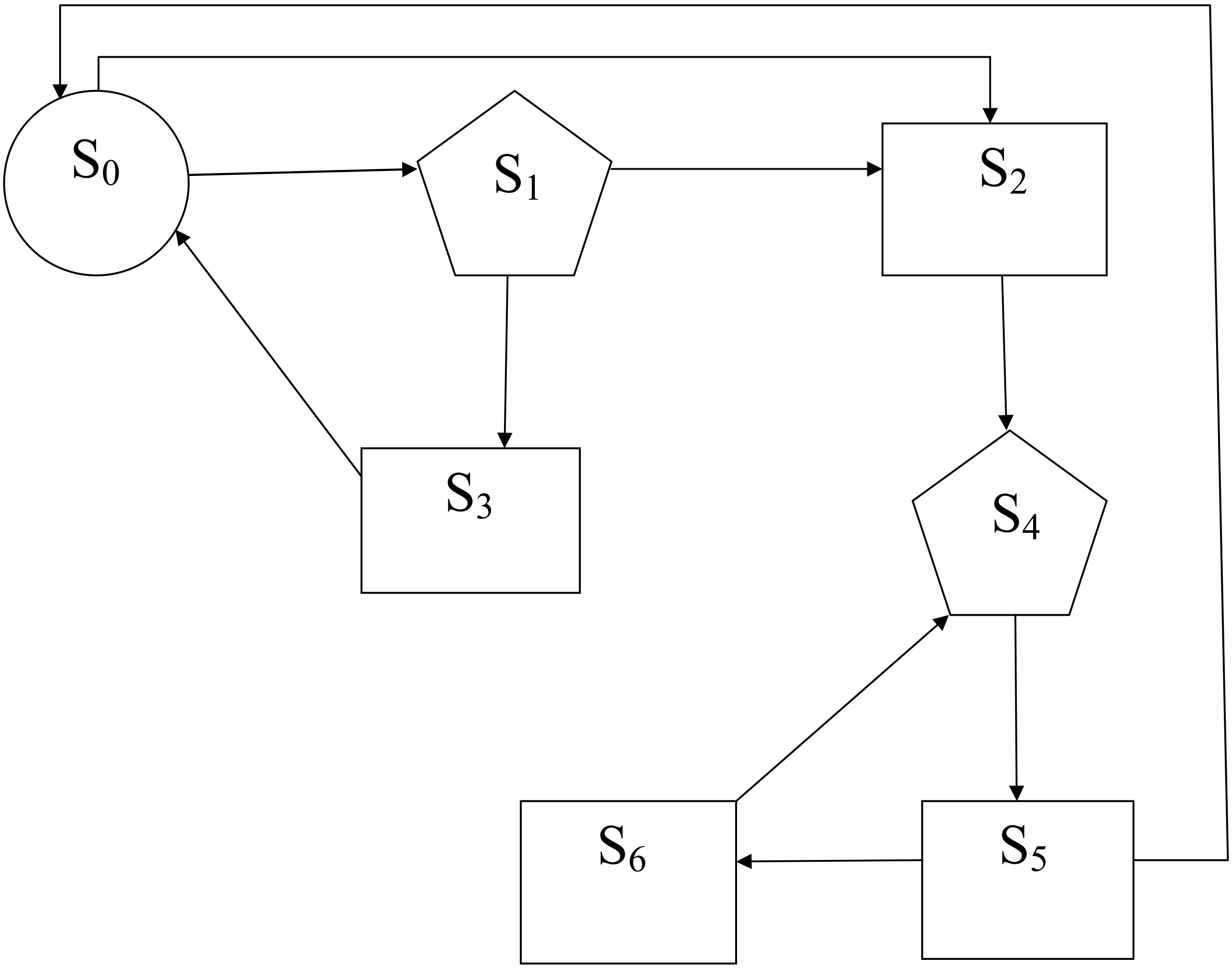
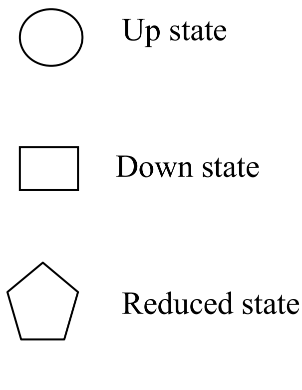


Fig. 4.1: State Transition Diagram for the System



4.5 Availability Analysis

Let

$M_0 = \mu_0$, $M_1 = \mu_1$, $M_4 = \mu_4$.

Let $A_i(t)$ be the probability that the system is in upstate at instant 't' given that the system entered regenerative state i at t = 0. The recursive relations for $A_i(t)$ are given as

$A_0(t) = M_0(t) + q_{01}(t) ©A_1(t) + q_{02}(t) ©A_2(t)$,

$A_1(t) = M_1(t) + q_{12}(t) ©A_2(t) + q_{13}(t) ©A_3(t)$,

$A_2(t) = q_{24}(t) ©A_4(t)$,

$A_3(t) = q_{30}(t) ©A_0(t)$,

$A_4(t) = M_4(t) + q_{45}(t) ©A_5(t)$,

$A_5(t) = q_{56}(t) ©A_6(t) + q_{50}(t) ©A_0(t)$,

$A_6(t) = q_{64}(t) ©A_4(t)$. (4.21-4.27)

Taking LT of above relations (4.21-4.27) and solving for $A_0(s)$. Using this, the steady-state availability is given as

$A_0 = \lim_{s\to 0} s.A_0(s) = \frac{N_2}{D_2}$.

where

$D_2 = P_{50}[\mu_0 + P_{01}(\mu_1 + P_{13}\mu_3)] + (1 - P_{01}P_{13})[P_{50}\mu_2 + \mu_4 + \mu_5 + P_{56}\mu_6]$,

$N_2 = P_{50}(\mu_0 + P_{01}\mu_1) + \mu_4[1 - P_{01}P_{13}]$.

4.6 Busy Period Analysis Due to Repair

Let $BR_i(t)$ be the probability that the server is busy in repairing the unit at an instant 't' given that the system entered regenerative state i at t = 0. The recursive relations for $BR_i(t)$ are given as

$BR_0(t) = q_{01}(t) © BR_1(t) + q_{02}(t) © BR_2(t)$,

$BR_1(t) = q_{12}(t) © BR_2(t) + q_{13}(t) © BR_3(t)$,

$BR_2(t) = W_2(t) + q_{24}(t) ©BR_4(t)$,

$BR_3(t) = q_{30}(t) ©BR_0(t)$,

$BR_4(t) = q_{45}(t) ©BR_5(t)$,

$BR_5(t) = q_{56}(t) © BR_6(t) + q_{50}(t) © BR_0(t)$,

$BR_6(t) = W_6(t) + q_{64}(t) ©BR_4(t)$. (4.28-4.34) where

$W_2 = \mu_2$, $W_6 = \mu_6$.

Taking LT of above relations (4.28-4.34) and solving for$BR_0(s)$, we get in the long run the time for which the system is under repair as

$BR_0 = \lim_{s\to 0} s.BR_0(s) = \frac{N_3}{D_2}$

where

$N_3 = (1 - P_{01}P_{13})(P_{56}\mu_6 + P_{50}\mu_2)$,

D_2 is already mentioned.

4.7 Busy Period Analysis Due to Preventive Maintenance

Let $BP_i(t)$ be the probability that the server is busy for preventive maintenance at an instant't' given that the system entered regenerative state i at t = 0. The recursive relations for $BP_i(t)$ are given as

$BP_0(t) = q_{01}(t) ©BP_1(t) + q_{02}(t) ©BP_2(t)$,
$BP_1(t) = q_{12}(t) ©BP_2(t) + q_{13}(t) ©BP_3(t)$,
$BP_2(t) = q_{24}(t) ©BP_4(t)$,
$BP_3(t) = W_3(t) + q_{30}(t) ©BP_0(t)$,
$BP_4(t) = q_{45}(t) ©BP_5(t)$,
$BP_5(t) = q_{56}(t) ©BP_6(t) + q_{50}(t) ©BP_0(t)$,
$BP_6(t) = q_{64}(t) ©BP_4(t)$. (4.35-4.41)

where

$W_3 = \mu_3$.

Taking LT of above relations (4.35-4.41) and solving for $BP_0(s)$, we get in the long run time for which the system is under preventive maintenance as

$BP_0 = \lim_{s\to 0} s.BP_0(s) = \frac{N_4}{D_2}$

Where

$N_4 = P_{01}P_{13}P_{50}\mu_3$,

D_2 is already mentioned.

4.8 Busy Period Analysis Due to Inspection

Let $BI_i(t)$ be the probability that the server is busy for inspection at an instant 't' given that the system entered regenerative state i at t = 0. The recursive relations for $BI_i(t)$ are given as

$BI_0(t)$= q_{01} (t) ©$BI_1(t)$ + q_{02} (t) ©$BI_2(t)$,
$BI_1(t)$ = q_{12} (t) ©$BI_2(t)$ + q_{13} (t) ©$BI_3(t)$,
$BI_2(t)$ = q_{24} (t) ©$BI_4(t)$,
$BI_3(t)$ = q_{30} (t) ©$BI_0(t)$,
$BI_4(t)$ = q_{45} (t) ©$BI_5(t)$,
$BI_5(t)$ = W_5 (t) + q_{56} (t) ©$BI_6(t)$ + q_{50} (t) ©$BI_0(t)$,
$BI_6(t)$ = q_{64} (t) ©$BI_4(t)$. (4.42-4.48)

where
W_5= μ_5.

Taking LT of above relations (4.42-4.48) and solving for$BI_0(s)$, we get in the long run time for which the system is under preventive maintenance as

BI_0 =$\lim_{s\to 0} s.BI_0(s) = \frac{N_5}{D_2}$

where

$N_5 = (1 - P_{01}P_{13})\mu_5$,

D_2 is already mentioned.

4.9 Expected Number of Visit by Server Due to Repair

Let $NR_i(t)$be the expected number of visits by the server due to repair in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $NR_i(t)$ are given as

$NR_0(t)$ = Q_{01} (t) ® $NR_1(t)$ + Q_{02} (t) ® [1+ $NR_2(t)$],
$NR_1(t)$ = Q_{12} (t) ®$NR_2(t)$ + Q_{13} (t) ®$NR_3(t)$,
$NR_2(t)$ = Q_{24} (t) ®$NR_4(t)$,
$NR_3(t)$ = Q_{30} (t) ®$NR_0(t)$,
$NR_4(t)$ = Q_{45} (t) ®$NR_5(t)$,

$NR_5(t)$ = Q_{56} (t) ®$NR_6(t)$ + Q_{50} (t) ®$NR_0(t)$,

$NR_6(t)$ = Q_{64} (t) ®$NR_4(t)$ (4.49-4.55)

Taking LST of above relations (4.49-4.55) and solving for$NR_0(s)$, we get the expected number of visits by server for repair per unit time as

$NR_0 = \lim_{s\to 0} s.NR_0(s) = \frac{N_6}{D_2}$

$N_6 = P_{02}P_{50}$,

D_2 is already mentioned.

4.10 Expected Number of Visit Due to Inspection

Let $NI_i(t)$be the expected number of visits by the server due to inspection in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $NI_i(t)$ are given as

$NI_0(t)$= Q_{01} (t) ®$NI_1(t)$ + Q_{02} (t) ®$NI_2(t)$,

$NI_1(t)$= Q_{12} (t) ®$NI_2(t)$ + Q_{13} (t) ®$NI_3(t)$,

$NI_2(t)$= Q_{24} (t) ® $NI_4(t)$,

$NI_3(t)$= Q_{30} (t) ®$NI_0(t)$,

$NI_4(t)$= Q_{45} (t) ® [1+ $NI_5(t)$],

$NI_5(t)$= Q_{56} (t) ®$NI_6(t)$ + Q_{50} (t) ®$NI_0(t)$,

$NI_6(t)$= Q_{64} (t) ®$NI_4(t)$. (4.56-4.62)

Taking LST of above relations (4.56-4.62) and solving for$NR_0(s)$, we get the expected number of visits by server for inspection per unit time as

$NI_0 = \lim_{s\to 0} s.NI_0(s) = \frac{N_7}{D_2}$

$N_7 = (1 - P_{01}P_{13})$,

D_2 is already mentioned.

4.11 Profit Analysis

Profit incurred to the system model in steady state is given by

$$P = K_1A_0 - K_2BR_0 - K_3BP_0 - K_4BI_0 - K_5NR_0 - K_6NI_0. \quad (4.63)$$

where

K_1: Revenue per unit up-time of the system,

K_2: Cost per unit time for which server is busy in repair,

K_3: Cost per unit time for which server is busy in preventive maintenance,

K_4: Cost per unit time for which server is busy in inspection,

K_5: Cost per unit visit by the server for repair,

K_6: Cost per unit visit by the server for inspection.

4.12 Special Case

We assume that the failure and replacement times are Weibull distribution with two parameters. But the density function of Weibull distribution with two parameters is given by

$$f(t) = \lambda t exp[-\lambda t^{b+1}/b + 1].$$

Using the Weibull distribution to study two cases when b = 0, it become the exponential distribution and when b = 1, it become the Rayleigh distribution.

Let

$F_1(t) = 1 - exp\left[\frac{-\lambda_1 t^{b+1}}{b+1}\right]$, $F_2(t) = 1 - exp\left[\frac{-\lambda_2 t^{b+1}}{b+1}\right]$,

$F_3(t) = 1 - exp\left[\frac{-\lambda_3 t^{b+1}}{b+1}\right]$, $G(t) = 1 - exp\left[\frac{-Q t^{b+1}}{b+1}\right]$,

$G_1(t) = 1 - exp\left[\frac{-Q_1 t^{b+1}}{b+1}\right]$, $G_2(t) = 1 - exp\left[\frac{-Q_2 t^{b+1}}{b+1}\right]$,

$Z(t) = 1 - exp\left[\frac{-\alpha t^{b+1}}{b+1}\right]$, $H(t) = 1 - exp\left[\frac{-\alpha_1 t^{b+1}}{b+1}\right]$,

$F(t) = 1 - exp\left[\frac{-\lambda t^{b+1}}{b+1}\right]$.

Transition probabilities and mean sojourn times. The steady-state transition probabilities are

$P_{01} = \frac{\lambda_1}{\lambda+\lambda_1}$, $P_{02} = \frac{\lambda}{\lambda+\lambda_1}$,

$P_{12} = \frac{\lambda_2}{\lambda+\lambda_1}$, $P_{13} = \frac{\alpha}{\alpha+\lambda_2}$,

$P_{24} = 1$, $P_{30} = 1$,

$P_{45} = 1$, $P_{64} = 1$,

$P_{56} = p$, $P_{50} = q$.

Obviously

$P_{01}+P_{02} = P_{12}+P_{13} = P_{24} = P_{30} = P_{45} = P_{64} = P_{56}+P_{50} = 1$

The mean sojourn times in various states are

$$\mu_0 = \frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)\left[\frac{\lambda+\lambda_1}{b+1}\right]^{\frac{1}{b+1}}} , \qquad \mu_1 = \frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)\left[\frac{\lambda_2+\alpha}{b+1}\right]^{\frac{1}{b+1}}},$$

$$\mu_2 = \frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)\left[\frac{Q_1}{b+1}\right]^{\frac{1}{b+1}}}, \qquad \mu_3 = \frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)\left[\frac{Q}{b+1}\right]^{\frac{1}{b+1}}},$$

$$\mu_4 = \frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)\left[\frac{\lambda_3}{b+1}\right]^{\frac{1}{b+1}}}, \qquad \mu_5 = \frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)\left[\frac{\alpha_1}{b+1}\right]^{\frac{1}{b+1}}},$$

$$\mu_6 = \frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)\left[\frac{Q_2}{b+1}\right]^{\frac{1}{b+1}}}.$$

Table 4.1: Comparison Between the Effects of the Exponential and Rayleigh Distributions on the System

∝	MTSF in the exponential distribution	MTSF in the Rayleigh distribution	A_0 in the exponential distribution	A_0 in the Rayleigh distribution	Profit in the exponential distribution	Profit in the Rayleigh distribution
5	3.2270	2.5994	0.9065	0.7438	4515.6	3683.6
10	3.1770	2.3437	0.9061	0.7360	4513.8	3643.6
15	3.1599	2.3255	0.9060	0.7354	4513.1	3641.1
20	3.1513	2.3163	0.9059	0.7352	4512.8	3639.8
25	3.1461	2.3107	0.9058	0.7350	4512.6	3639.0
30	3.1426	2.3070	0.9058	0.7349	4512.4	3638.4
35	3.1401	2.3043	0.9058	0.7348	4512.4	3638.1
40	3.1382	2.3023	0.9058	0.7348	4512.3	3637.8
45	3.1368	2.3008	0.9058	0.7347	4512.2	3637.6

CHAPTER-5
CONCLUSION

The behavior of mean time to system failure (MTSF), availability and profit with respect to failure rate of component A (α) for fixed values of other parameters is shown in Table 2.1 and Table 3.1. It is observed that MTSF, availability and profit decrease with the increase of failure rate of component A (α). So, we have the comparison between two distributions in two systems and we notes the Rayleigh distribution is better than the Exponential distribution in two systems. We have showed that our results in Table 2.1 and Table 3.1. Under general assumptions and from the comparison of two systems we conclude that, model of chapter-3 is better than the first system when additional preventive maintenance is provided when both the components are normal and used Rayleigh distribution.

Table 4.1 in chapter-4, reflect that mean time to system failure (MTSF), availability and profit of the system model decrease with the increase of maximum rate of operation (α_0) for fixed values of other parameters. In this chapter we get different trend between Exponential distribution and Rayleigh distribution as compare to chapter-2 and chapter-3. We have the comparison between two distributions and we notes the Exponential distribution is better than the Rayleigh distribution in system under stated conditions. The study also reveals that a system which undergoes preventive maintenance at partially fail stage can be made more profitable if the degraded unit is replaced by new one when it fails.

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