

**STUDY OF LOW ENERGY PROPERTIES
OF HADRONS USING
PHENOMENOLOGICAL MODELS AND
EFFECTIVE THEORIES**

A THESIS

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by

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“The web of our life is of a Mingled Yarn, good and ill together. Our virtues would be proud if our faults whipped them not, and our crimes would despair if they were not cherished by our virtues”- William Shakespeare

All's well that ends well.


Dedicated to
My Beloved Family

Certificate

I hereby certify that the work which is being presented in this thesis entitled "STUDY OF LOW ENERGY PROPERTIES OF HADRONS USING PHENOMENOLOGICAL MODELS AND EFFECTIVE THEORIES" being submitted by **Meenakshi Batra** for the fulfillment of the requirements for the award of Degree of Doctor of Philosophy in the School of Physics and Materials Science, Thapar University, Patiala, is a record of the candidate's own work carried out by her under my supervision. The matter presented in this thesis has not been submitted in part or full for the award of any degree in any university or institute.



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Abstract

The work presented here is concerned about the theoretical study of static properties of different hadronic systems using statistical models and effective theories. The various aspects of spin $\frac{1}{2}^+$ baryonic systems and heavy-light mesons are analyzed deeply. This thesis is dedicated to the some of the basic questions which are yet unanswered. For example, proton spin crisis and presence of strange quark content inside the nucleonic system. Therefore, the strange quark contributions are explicitly added in the present work by modifying the principle of detailed balance. Statistical model is used to find the answers of some of the queries by using the statistical techniques. The hyperons are also checked against the impact of $SU(3)$ breaking on the different properties of baryons. In addition to this, the latest resonances in the heavy-light mesonic sector are analyzed and few basic properties like masses, splittings and decay widths of some of the charm and bottom mesons are calculated using heavy hadron chiral perturbation theory. The thesis is organized into the following seven chapters.

***Chapter I** gives a brief account of the underlying principles of QCD and its Lagrangian. It also includes the summary of various experimental investigations regarding the structure of proton and experimental status of very famous anomaly regarding the spin of proton. Chapter I presents a brief idea of perturbative and*

non-perturbative aspects of QCD and how the running coupling constant leads to interesting features of QCD like quark confinement and asymptotic freedom. The basic symmetries of QCD and their breakings is discussed in brief. One of the importance of breaking lies in the fact that spontaneous breaking of well known chiral symmetry leads to the formulation of chiral perturbation theory. Similarly, the limit $m_Q \rightarrow \infty$ leads to heavy quark flavor and spin symmetry. These symmetries help to analyze the basic properties of a heavy hadronic system using effective theory which is famous as heavy quark effective theory. Heavy quark effective theory and chiral perturbation theory can be combined to form heavy hadron chiral perturbation theory. A brief introduction to heavy hadron chiral perturbation theory is also included.

Chapter II *gives relevant details of the statistical model applied to study the static properties of spin $\frac{1}{2}^+$ baryonic systems. The statistical model is based on the assumption that hadronic structure can be assumed to be made up of quark-gluon Fock states. The Fock states can be single gluon and multi-gluon states along with quark-antiquark pairs. The hadronic structure consists of two parts, one is q^3 core and other is sea containing quark-gluon Fock states multi-connected non-perturbatively through gluons. Since, the baryon should be colorless, a q^3 can have the color quantum numbers $1_c, 8_c, 10_c$ so as to make suitable combinations for color quantum numbers of sea. The statistical model is aimed at finding the relative probabilities in flavor, spin and color space. The probabilities in flavor are computed by using principle of detailed balance while the probabilities in spin and color space is estimated using relative multiplicities in the form of ratios. The present work uses three different types of statistical models named as Model C, P(Model assuming sea with pseudoscalars) and D(Model assuming sea with suppressed multiplicity). Chap-*

ter II also discusses the various low energy properties in brief.

In Chapter III, the strange and non-strange quark-gluon Fock states are analyzed for the low energy properties of nucleonic system. The low energy properties include magnetic moments, distribution of spin among quarks, axial-vector form factors etc. To add contribution of strange quark-gluon Fock states, principle of detailed balance is modified. The strange sea contribution comes out to be negligible but it affects the spin distribution of quarks inside the nucleons. Thus, the spin polarization densities of three quarks has been studied with strange and without strange sea. On the basis of statistical modeling, the contribution from various parts of sea has been analyzed. We name the sea as scalar, vector and tensor sea with Fock states having spin 0, 1 and 2 respectively. The statistically determined properties of a nucleonic system favors a vector dominated sea where the sea also includes strange quark anti-quark pairs. The obtained results for all the properties are compared with different theoretical models and experimental data. Model D comes with more authentic results when strange sea is taken into consideration.

In Chapter IV, statistical model is applied to a system with strange quarks in the valence part i.e. lambda and other hyperons. The role of strange quark mass in the structure of hyperon is to break the $SU(3)$ symmetry in the baryon spectrum. The effects of symmetry breaking in sea as well as valence part on all the properties of hyperons is checked in the present work. The strange mass corrections are parameterized through the correction factor "r". Here "r" depends on the ratio of mass of s and u quark. Symmetry breaking corrections are applied and effects of symmetry breaking lead to modified values of spin distribution among quarks and axial vector form factors. The results are compared with the available theoretical and experimen-

tal data. The strange mass corrections also help in finding the corrected values of axial vector coupling constants F and D . The ratio $\frac{F}{D}$ is found to be deviating by 17% from the experimental data.

Chapter V deals with the heavy light mesons using heavy hadron chiral perturbation theory. Masses, splittings, decays and branching ratios are the fundamental properties of heavy mesons. Heavy hadron chiral perturbation theory uses both chiral as well as heavy quark spin and flavor symmetry to form an effective Lagrangian to calculate masses in this theory and it describes the interplay of these two symmetries in the form of low energy gradients. This Lagrangian produces a residual mass formula in terms of the light quark masses along with some unknown parameters. These parameters are constrained using experimental data and masses of the ground as well as low lying excited states of heavy charm meson are calculated. Similarly, B meson masses in the heavy quark effective theory are given in terms of a single non-perturbative parameter $\bar{\Lambda}$ and non-perturbative QCD parameters λ_1 and λ_2 . The best fit of λ_1 and λ_2 helps to find out masses of excited bottom mesons masses. Thus, both charm and bottom meson masses and their splittings have been found to be matching well with the experimental results.

In **Chapter VI**, we study the spectra of several newly observed resonances by different collaborations. Using an effective Lagrangian approach based on heavy quark symmetry and chiral dynamics, we explore the strong decay widths and branching ratios of various resonances and suggest their J^P values. We try to fit the experimental data to find the coupling constants involved in the strong decays through pseudoscalar

mesons. The present work also discusses about the possible spin-parity assignments of recently observed states by LHCb collaboration. The tentative assignment of newly discovered state $D_J^*(3000)$ can be natural parity states $(0^+, 1^-, 2^+, 3^-, \dots)$ while $D_J(3000)$ can be identified with unnatural parity states like $(0^-, 1^+, 2^-, 3^+, \dots)$. Therefore, the missing doublets $2S, 2D, 1F, 2P$ and $3S$ can be thought of filled up with these states. We study the two-body strong decay widths and branching ratios of missing doublets and plot branching ratios vs mass of decaying particle. These plots are used to analyze all assignments to $D_J(3000)$ deeply and various possibilities for J^P values.

Finally, in **chapter VII**, conclusions are summarized and an outlook of our work is presented.

Chapter 1

Introduction

The study of modern physics is focussed on achieving the answers of basic questions of nature and an understanding of fundamental constituents of matter and interactions. Most of the matter in this universe is, however, made up of nucleons that form the nuclei of atoms around us. The study of these building blocks of matter has been found to be carrying utmost importance to us. Protons and neutrons have been studied in detail since their discovery in the beginning of twentieth century. The fundamental structure of these nucleons still lacks proper understanding. Rutherford discovery of structure of atom that is atoms consist of nucleons bound in the nucleus surrounded by the clouds of electrons, led to the prolonged deep analysis of structure of basic units of atom through experimental discoveries. After more than 50 years of Rutherford's discovery, the experimentalists at SLAC [1] provided evidence for sub-structure of proton and neutrons. The first progress in nuclear forces came with the work of H. Yukawa in 1935 who proposed a scalar meson exchange model for the interactions between nucleons. These scalar mesons were later identified as neutral pions. This concept led to the development of the initial ideas. Feynman [2]

also favored the above situation by suggesting that proton can be thought of made up of small particles named as partons and the now a days the model is famous as quark-parton model. The quark model was given by Gellmann [3] and Zweig [4] in 1964. The strange particles brought a new additive quantum number to the hadron physics. This new quantum number was strangeness, S and was proposed independently by M. Gell-Mann and Nishijima. The grounds for this proposal were the fact that these particles appeared in pairs in pion-proton collisions and that some of these particles had unexpectedly large lifetimes when compared with the time scale of the strong interactions, in spite of the fact that their expected decays did not violate either charge nor baryon number conservation. However, this thesis is concerned with the theoretical and phenomenological studies of hadrons. Hadrons are bound state of quarks. The first part of thesis is devoted to the study of nucleons and other spin $\frac{1}{2}$ hyperons whereas the second part of this thesis is dedicated to the heavy mesons and their interactions. The present theory of elementary particle physics is known as standard model. This theory is so called gauge theory and underlying gauge symmetry group is $SU(3)_{color} \otimes SU(2)_L \otimes U(1)$. The $SU(3)_{color}$ part of the group is gauge group of strong interactions and quantum gauge group is known as quantum chromodynamics. QCD describes these composite particles in terms of their constituent particle known as quarks and the carriers of strong force as gluons. This chapter is dedicated to a survey of particles having strong interactions. These particles are now a days famous as hadrons. Hadrons are divided into mesons and baryons. Mesons are bosons with integral spin and baryons are fermions having spin $\frac{1}{2}$ and $\frac{3}{2}$. As for today, we are still not in a situation to describe completely the structure of nucleon at low energy and high energy momentum transfers. The structure of nucleon has

been investigated since many years using lepton beams. These experiments have provided a useful data which helps to explore more and more the nucleonic structure. Despite having been investigated by various experimental collaborations [5], [6], the spin structure of nucleon remain a mystery. The first anomaly regarding spin structure of nucleons was reported by European Muon Collaboration [7] and later on confirmed by Spin Muon Collaboration [8]. The well-known "spin crisis" came into existence. At small Q^2 , the hadronic properties can be explained by quark model in terms of massive constituent quarks but at higher Q^2 and at comparably smaller distances, hadronic structure reveals a pattern of weakly interacting gluons, nearly mass-less current quarks and a sea of quark-antiquark pairs. The present report is devoted to the deep analysis of various aspects of hadronic properties at low energy. In the first part of thesis, several properties of spin $\frac{1}{2}^+$ baryons have been studied in various statistical models whereas the second part is devoted to the study of low energy properties like masses, splittings for charm and bottom mesonic systems using an effective approach. The thesis begins with introductory review of all phenomenon related to the statistical models and effective theories.

1.1 Special Unitary Groups(SU(2) and SU(3))

Group theory is the branch of physics which underlies the treatment of symmetry and symmetry is defined as the operation which do not change the state of system. The special unitary group $SU(2)$ is the group of all 2×2 unitary matrices with determinant equal to one. Each group element of $U(1)$ is represented by the pure phase factor $e^{i\alpha}$. Every element of $U(2)$ is a product of phase factor $e^{i\alpha}$, which is

element of both $U(1)$ and $SU(2)$. Under a symmetry operation R , the Hamiltonian of the system is unchanged.

$$\langle \phi | H | \psi \rangle = \langle \phi | U^\dagger H U | \psi \rangle = \langle \phi | H | \psi \rangle \quad (1.1)$$

All the group properties follow from the infinitesimal rotations in the neighborhood of the identity. It is written as $U = 1 - \epsilon J_3$ where J_3 is the generator of rotations. In $SU(2)$ group, the simplest example of a multiplet of $SU(2)$ is the fundamental or two-dimensional representation which is an object that can occur in two states labeled up and down or (u, d) . The proton and neutron form an example of such a representation of the $SU(2)$ isospin group. In the lowest-dimension non-trivial representation of the group ($j = \frac{1}{2}$), the generators are written as $J_i = \frac{1}{2}\sigma_i$ with $i=1,2,3$ and σ_i are the Pauli-spin matrices. The Pauli-spin matrices are hermitian and transformation matrices $U_i = e^{-i\theta_i\sigma_i/2}$ are unitary. This set of unitary traceless matrices form a subgroup $SU(2)$. The set of unitary 3×3 matrices with $\det U=1$ form a subgroup of $SU(3)$. The generators are taken to be 8 hermitian traceless matrices. The fundamental representation of $SU(3)$ is a triplet. The generators are taken as 3×3 matrices denoted as λ_i for $i=1$ to 8. The matrix representation of the λ_i are $[\lambda_i, \lambda_j] = i \sum_k f_{ijk} \frac{\lambda_k}{2}$. In the quark picture of hadrons, internal symmetries refer to the fact that particle come in families known as multiplet having nearly degenerate masses. Among the hadrons, hypercharge is defined as the sum of baryon number and strangeness i.e $Y=B+S$ and electric charge, Q is related to third component of isospin(I_3) and electric charge Q as $Q = I_3 + \frac{Y}{2}$ which is called as GellMann-Nishijima [9] [10] formula. $SU(3)$ flavor group of hadrons was formulated with the

existence of additional quantum number called "S" and it groups $n, p, \lambda, \Sigma, \Xi$ baryons in $SU(3)$ octet representation. Another group is formed as a doublet with 10 baryons in the lowest lying state.

1.2 Non-relativistic Quark Model for Hadrons

The basic well-known $SU(6)$ model along with its necessary details is given below. The idea of quark structure of hadrons was first put forward by GellMann [3] and Zweig [4] which assume hadrons as composed of basic elementary particles known as quarks with certain internal symmetries. The basis for their hypothesis was eight-fold $SU(3)$ symmetry. In the quark picture of hadrons, the mesons consist of quark-antiquark pair whereas baryons consist of three quarks. Each multiplet can be realized as irreducible representation of internal symmetry group. In the non-relativistic model, light quarks realize the fundamental representation of larger symmetric group which acts in the space of spin and flavors. $SU(6)$ interpretation can be fulfilled in terms of composite system of quarks which require introduction of a new quantum number which is color quantum number. The idea of color was introduced in mid 1960's by MooYoung Han and Yoichiro Nambu [11] as well as Oscar W Greenburg [12]. Later on, it was shown by Kenneth G Williams [13] that for sufficiently strong coupling, quarks exhibit confinement of color. The color group is $SU(3)_c$ where the indices make the difference between flavor $SU(3)$ and color $SU(3)$. Also, it was postulated that hadrons are singlets of color group $SU(3)_c$ group. Neglecting the mass-difference between strange and non-strange quarks leads to fundamental representation of $SU(3)$ as $SU(3)_{flavor}$ where antiquarks belong to the conjugate

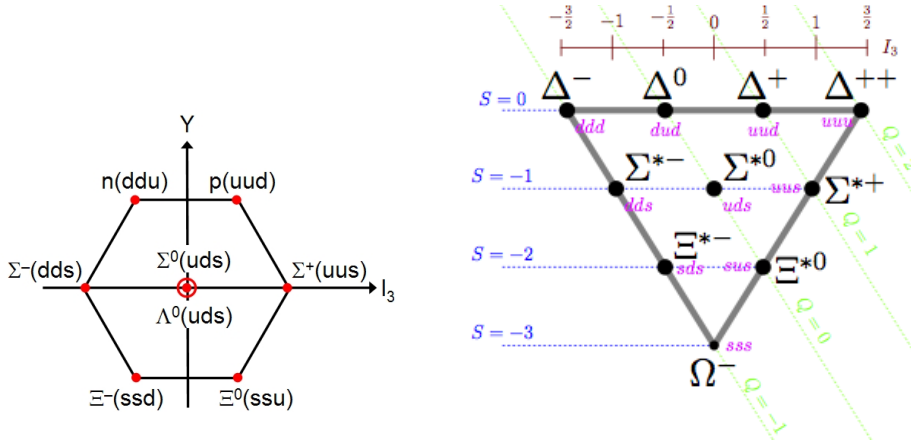


Figure 1.1 The $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ baryon octet and decuplet

representation 3^* . Baryons and mesons involve the following group presentation products:

$$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10 \tag{1.2}$$

Thus, baryons appear as octets and decuplets whereas mesons appear as singlets and octets.

$$3 \otimes 3^* = 8 \oplus 1 \tag{1.3}$$

$1 \oplus 8$ represents $0^-, 1^-, 1^+, 2^+$ mesons and the masses of particles can be predicted by observing the breaking of $SU(3)$ symmetry. In terms of weight diagram, the baryon octet and decuplet are represented in the 1.1 The present thesis is concerned in the studies of lowest lying baryon octet states and heavy light mesons. The wavefunctions are often labeled with N , the number of quanta and l^P represents the total angular momentum and parity with the dimension of $SU(6)$ representation. $SU(6)$ becomes exact in the limit of equal light quark masses and when hyperfine interactions are neglected. In non-relativistic quark model, hadrons are considered as bound colorless system of three quarks which are fermionic particle with flavor, spin and

color degrees of freedom. Taking into account the relevant degrees of freedom, the wave-functions can be written as;

$$\Psi_h = |\phi\rangle |\psi\rangle |\chi\rangle |\xi\rangle \quad (1.4)$$

where $|\phi\rangle$ denotes the flavor part of wave-function, $|\chi\rangle$ represents spin of valence part in the wave-function, $|\psi\rangle$ is for color and $|\xi\rangle$ is for space contribution to the whole wave-function. From direct product of $q \otimes \bar{q}$ states, we get $6 \otimes \bar{6} = 1 \oplus 35$ where 1 and 35 are two irreducible representations in $SU(6)$. The subgroup of $SU(6)$ is $SU(3) \otimes SU(2)$ therefore 35 meson states can be described in terms of their $[SU(3), SU(2)]$ content. $SU(2)$ refers to the ordinary spin group. The decomposition of 35 can be described as: $35 = (8, 1) \oplus (1, 3) \oplus (8, 3)$.

Out of 35, the vector ($J=1$) nonet has 27 meson states, three states form vector meson singlet and 24 states form vector meson octet. Other 8 states form the pseudoscalar octet. Considering baryon as qqq state, flavor and spin can be combined with an approximate $SU(6)$ symmetry in which six quark states are $u \uparrow, u \downarrow, d \uparrow, d \downarrow, s \uparrow, s \downarrow$. The baryon states in $SU(6)$ can be grouped in multiplet whose dimension and symmetry properties are:

$$SU(6) : 6 \otimes 6 \otimes 6 = 20 \oplus 70 \oplus 70 \oplus 56 \quad (1.5)$$

In terms of decomposition of $SU(6)$, it can be written as: $6 \otimes 6 \otimes 6 = (3, 2) \otimes (3, 2) \otimes (3, 2)$. $SU(2)$ decomposition of spin states that $2 \otimes 2 \otimes 2 = 4_s + 2_{ms} + 2_{ma}$. Similarly, $3 \otimes 3 \otimes 3 = 10_s \otimes 8_{ms} \otimes 8_{ma} \otimes 1_a$ for three flavor u, d, s . The spin-flavor content of each $SU(6)$ state can be easily reconstructed. For instance, proton wave-function

can be written as:

$$|p \uparrow\rangle = \frac{1}{\sqrt{18}} \epsilon_{ijk} (\{u_i^\downarrow d_j^\uparrow - u_i^\uparrow d_j^\downarrow\}) u_k^\uparrow |0\rangle \quad (1.6)$$

The simplified $SU(6)$ wave-function in terms of permutation among the flavor can be written as:

$$\begin{aligned} |p \uparrow\rangle = & -\frac{1}{\sqrt{18}} [u \uparrow u \downarrow d \uparrow + u \downarrow u \uparrow d \uparrow - 2u \uparrow u \uparrow d \downarrow \\ & + u \uparrow d \uparrow u \downarrow + u \downarrow d \uparrow u \uparrow - 2u \uparrow d \downarrow u \uparrow \\ & + d \uparrow u \downarrow u \uparrow + d \uparrow u \uparrow u \downarrow - 2d \downarrow u \uparrow u \uparrow] \end{aligned}$$

From this wave-function, one can count the average number of quark flavors with spin parallel and anti-parallel to the spin of proton. The general form of $SU(6)$ wave-function for all baryons in baryon octet is represented as:

$$\begin{aligned} |B \uparrow(p, q, q)\rangle = & \frac{1}{\sqrt{18}} [q \uparrow q \downarrow p \uparrow + q \downarrow q \uparrow p \uparrow - 2q \uparrow q \uparrow p \downarrow \\ & + q \uparrow p \uparrow q \downarrow + q \downarrow p \uparrow q \uparrow - 2q \uparrow p \downarrow q \uparrow \\ & + q \uparrow p \downarrow p \uparrow + q \uparrow p \uparrow q \downarrow - 2q \downarrow q \uparrow p \uparrow] \\ |B \uparrow(p, q, r)\rangle = & \frac{1}{6} [p \uparrow q \downarrow r \uparrow + q \downarrow r \uparrow p \uparrow + r \uparrow p \uparrow q \downarrow \\ & + q \downarrow p \uparrow r \uparrow + p \uparrow r \uparrow q \downarrow + r \uparrow q \downarrow p \uparrow - 2(q \uparrow r \downarrow p \uparrow \\ & + p \uparrow r \downarrow q \uparrow + p \uparrow q \uparrow r \downarrow + r \downarrow q \uparrow p \uparrow + r \downarrow p \uparrow q \uparrow + q \uparrow p \uparrow r \downarrow)] \end{aligned}$$

1.3 Parton Model

The quark parton model was invented by Feynman [2] and Bjorken [14] and it was born to describe DIS on a nucleon as sum of elastic scattering amplitudes on non-interacting point like partons. These partons are assumed to carry a four momentum of fraction x of the nucleon in infinite momentum frame such that rest masses of the

partons as well as transverse momenta to the direction of motion can be neglected. In the limit $Q^2 \rightarrow \infty$ and $\nu \rightarrow \infty$, in which quark parton model is applicable, the structure function does not depend upon Q^2 , but become a function of x^2 only. The structure functions are written in quark parton model.

$$F_1(x) = \frac{1}{2} \sum_f e_f^2 (q_f(x) + \bar{q}_f(x)) \quad (1.7)$$

$$F_2(x) = x \sum_f e_f^2 (q_f(x) + \bar{q}_f(x)) \quad (1.8)$$

$$g_1(x) = \frac{1}{2} \sum_f e_f^2 (\Delta q_f(x) + \Delta \bar{q}_f(x)) \quad (1.9)$$

$$g_2(x) = \frac{1}{2} \sum_f e_f^2 (\Delta q_f(x) + \Delta \bar{q}_f(x)) \quad (1.10)$$

where e_f is the fractional charge of quark flavor f . The distribution $q(x)$ are the unpolarized parton density function (PDF) or quark distribution function and $\Delta q(x)$ are the polarized quark distribution function.

$$q(x) = q^{\uparrow\uparrow}(x) + q^{\uparrow\downarrow}(x) \quad (1.11)$$

$$\Delta q(x) = q^{\uparrow\uparrow}(x) - q^{\uparrow\downarrow}(x) \quad (1.12)$$

If the constituents of nucleon behave like free and are point like then structure functions should not depend upon Q^2 . This phenomenon is called Bjorken scaling. For small value of $x < 0.05$ and $x > 0.3$, the structure function $F_2^p(x, Q^2)$ shows a dependence on Q^2 . This observation is not expected in naive quark model. The structure function $F_2^p(x, Q^2)$ decreases with increasing Q^2 for large x values while for small x values, the structure functions increases with increasing Q^2 . This can be explained by

the existence of sea quarks arising from gluon splitting, which carry a small fraction x of the nucleon momentum. For higher Q^2 values, the sea quarks can be resolved in the scattering process resulting in an increase of the structure function F_2 . The structure functions F_2 and g_1 vary with Q^2 .

1.4 Basic Concepts in QCD

QCD(Quantum Chromodynamics) is quantum field theory of strong interactions, with quark-gluon being its elementary degree of freedom. A non-abelian color $SU(3)$ gauge theory which is represented by quark-gluon as degrees of freedom. Quarks are structureless and point like fermions and can be characterized by their color quantum numbers. Gluons are neutral and massless bosons mediating the strong interaction among the quarks, analogous to photons in quantum electrodynamics. The short distance physics when calculated in QCD, agrees well with the deep inelastic scattering experiments. With increasing momentum transfer, decreasing distances, the quark-gluon effective coupling is reduced with the tendency to converge to the zero value. This phenomenon is called "asymptotic freedom". Thus, QCD can be referred as a well tested theory in high energy regime but at larger distances, or at low Q^2 the effective coupling constant becomes too large for the perturbative expansion to converge. The lack of knowledge of non-perturbative nature of interaction of quarks with gluons makes the internal structure of nucleons a mystery. There are many approaches that emerged in past decades to describe the interaction among the fundamental particles. These approaches are either based upon certain assumptions or some theories applicable at certain scale. For instance, quark models have been applied successfully to

describe many of the properties of hadrons. These days most of the available models make the assumption to reproduce the experimental data. Strong interactions in standard model are well described by the following terms of the Lagrangian which is famous as QCD Lagrangian. The Lagrangian is based on invariance under a local gauge group i.e. $SU(3)_{color}$ and to explain the fundamental interactions among quarks and gluons; each quark can exist in three color states while each gluon can exist in eight color states. Under an $SU(3)_{color}$ group, quarks and gluons form a triplet and octet. The Lagrangian in terms of quark, gluon degrees can be written as:

$$L_{QCD} = -\frac{1}{4}G_a^{\mu\nu}G_{\mu\nu}^a + \bar{q}(i\not{D} - M)q \quad (1.13)$$

where q is N_f multiplet containing N_f flavors of spin $\frac{1}{2}$ quark fields and $M = \text{diag}\{M_u, M_d, M_s\}$ the corresponding mass matrix. The operator D_μ is $SU(3)$ color covariant derivative and given by:

$$(D_\mu)_{ab} = (\delta_{ab})_\mu + ig_s(t^c A_\mu^c)_{ab} \quad (1.14)$$

$G_a^{\mu\nu}$ is quark-gluon field strength tensor. t^a are the color matrices ($t^c = \frac{\lambda_c}{2}$) and A_μ^c are the gauge fields. The gluonic field tensor gets the term due to non-abelian property of the QCD.

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s[t_a, t_b]A_\mu^a A_\nu^b \quad (1.15)$$

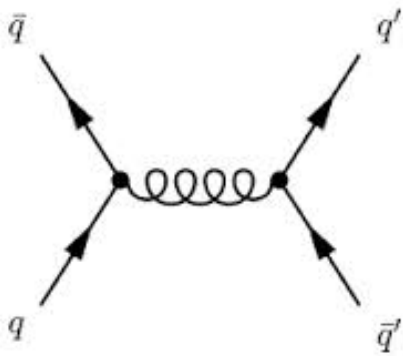
Extra term in $G_{\mu\nu}^a$ makes it invariant under non-abelian gauge transformations. QCD can be said to possess similar structure as QED except the difference that

gauge group is non-abelian $SU(3)$ and self-interacting gluons. Three quarks (u, d, s) are having mass less than 1GeV and therefore known as light quarks whereas other three (c, b, t) are known as heavy quarks. The scale which separates these two regimes is called as Λ_{QCD} . QCD exhibits two important properties that is quark confinement and asymptotic freedom. QCD has a peculiar property according to which coupling strength decreases at short distance. This occurs due to presence of gluons which carry color charge. Because of the asymptotic freedom, the strong interaction physics can be calculated when momentum transfer is large. At high energies, we are at the asymptotic freedom regime. In this regime, quarks interact weakly and the perturbative calculations are possible. The discovery of asymptotic freedom was a breakthrough for non-abelian gauge theories in the development of quantum field theories. In QED, the shielding of a trial charge source due to polarization by virtual pairs created in the field of this trial charge source. A similar kind of shielding takes place in QCD as well. However, in this case, gluons themselves carry the color charge and therefore spread out the color charge of the source. This can be elaborated in the manner as follows: In QCD, in addition to the processes similar to QED, the additional processes come into foreplay due to couplings among three gluons. The emission of a gluon leaks away the color charge of the heavy particle into the cloud of virtual particles. The anti-screening of QCD expects $\frac{d}{dt_0}(\alpha_s(t_0)) < 0$. If we define $\mu^2 = -t_0, \mu \frac{d}{d\alpha_s(\mu)} = \beta(\alpha_s(\mu))$ where α_s is the coupling constant in QCD and μ is the renormalization scale. The lowest order $\alpha_s(\mu^2)$ is written in terms of a single variable $\alpha_s(\mu^2) = \frac{4\pi}{\beta_1 \ln \frac{\mu^2}{\Lambda^2}}$. This Λ sets the scale of running coupling constant. This new phenomenon, leads to the asymptotic freedom in QCD. The asymptotic freedom in QCD i.e. the logarithmic decrease of QCD coupling constant α_s at large momen-

tum transfers $Q^2 \rightarrow \infty$ allows one to perform reliable computation of hard processes in QCD using perturbation theory. However, the same property of QCD implies an increase in the running coupling constant in QCD at small momentum transfers i.e at larger distances. This leads to the remarkable property of QCD which is known as quark confinement due to which quarks and gluons can not leave the region of their strong interactions and cannot be observed as the real physical objects, only baryons and mesons are observed. Hadron spectra are very well described by the quark model, but quarks have never been seen in isolation. Any effort in scattering experiment leads only to the production of the familiar mesons and baryons. Evidently, the forces between quarks are strong. For studying the system of quarks, baryon or meson, the two approaches in our hand are perturbative and non perturbative approach. The faith in QCD as a true physics theory is founded, on the successes of perturbative quantum chromodynamics(PQCD) where its wide-ranging predictions are compared to the experimental data on high energy processes, at variety of experimental facilities, covering numerous physical processes in lepton-lepton, lepton-hadron, and hadron-hadron collisions.

1.5 Perturbative and Non-perturbative QCD studies at Low Energy

Quantum Chromodynamics (QCD) is the universally accepted theory of strong interaction physics. The theory of QCD has a remarkable simplicity and elegance at the classical level, with its under-lying non-abelian $SU(3)$ color symmetry and unmatched richness after quantization, as revealed by a whole spectrum of contrasting

Figure 1.2 Feynman Diagram

behaviors over a wide range of energy scales, from confinement to asymptotic freedom. As said earlier when the coupling is small, the interactions can be defined as perturbations to the free field solution.

The first step is to divide the Lagrangian into a free term and interaction pieces. The action principle then calculates the Green function which is called a propagator for the free term using Feynman diagrams. The second one is the interaction Lagrangian which is proportional to the coupling constant. The Green function are now called as vertices. The transition matrix elements involve addition of external lines for fermions that is the Dirac spinors $u(p)$, $v(p)$ and polarization vector $e^\mu(k)$ for the gluons. The perturbative QCD leads to computation of reaction amplitudes from the fundamental vertices and propagators using the Feynman rules. The QCD Feynman diagram for quark-antiquark scattering amplitude is shown in figure 1.2. For studying the system of quarks, baryon or meson, the two approaches in our hand are perturbative and non perturbative approach. Due to asymptotic freedom, the perturbation series expansion of QCD breaks down at low energy. The expansion parameter, the coupling constant, becomes too large and therefore we cannot rely on results anymore. The value of the coupling constant approaches the order of 1

at an energy of several hundred MeV, a scale referred to as Λ_{QCD} . Perturbation theory becomes reliable at several GeV above this scale. There is a variety of interesting phenomena at a relatively low energy scale. This means that a perturbative approach in the conventional sense (expansion in the coupling constant) is not sufficient to gain a complete picture of QCD. On the one hand, there are traditional approaches, for example lattice QCD, where it is assumed that dynamics takes place on a discrete lattice and solve problems numerically. On the other hand, there are novel approaches like the so-called AdS/QCD duality, which is a manifestation of the AdS/CFT correspondence. It applies the holographic principle and maps a strongly coupled field theory (which is similar to QCD) to a string theory in its weak-coupling limit which allows for the treatment of problems which would be otherwise inaccessible. Also experiments such as ALICE at LHC in CERN and RHIC are studying the collisions that should, theoretically, produce quark-gluon plasma. A state, theoretically, where quarks and gluons are decoupled: since in nature, quarks are never on their own, they are either in meson 'state' - two coupled quarks, or baryon 'state' - three coupled quarks (both part of hadrons - family of coupled quarks). But it is still not clear how this happens and what could be the mechanism behind it. As is well known, the unique feature of the underlying quantum field theory which makes the perturbative approach useful in QCD, is asymptotic freedom. Equally important are the crucial concepts of infrared safety and factorization, without which it would not be possible to apply the results of perturbative calculations on partons (quark, gluons, vector bosons etc.) to the world of observed hadrons, electro-weak bosons, Higgs and other new physics particles. In fact, the proof of factorization establishes the theoretical foundation of the QCD parton model which provides the basic language

and picture for describing all high energy interactions involving hadrons nowadays in particle physics. Due to asymptotic freedom, the perturbation series expansion of QCD breaks down at low energy. The expansion parameter, the coupling constant becomes too large and therefore we cannot rely on results anymore. The value of the coupling constant approaches the order of 1 at an energy of several hundred MeV, a scale referred to as Λ_{QCD} . Perturbation theory becomes reliable several GeV above this scale. The perturbative QCD approach helps to construct the scattering cross-sections for hadrons. There is a variety of interesting phenomena at a relatively low energy scale. This means that a perturbative approach in the conventional sense (expansion in the coupling constant) is not sufficient to gain a complete picture of QCD. Phenomena like the transition to quark-gluon plasma, color-superconductivity and color-flavor-locking are out of reach. And quark-gluon plasma is theoretically assumed to have been existed microseconds after the universe was created (because of high temperatures at the beginning of the 'Big Bang').

Non-perturbative QCD refers to the study of interactions among the hadrons in a regime where the direct theoretical approaches are hard to be applied. The value of running coupling constant $\alpha(Q^2)$ increases at small momentum transfers, reaching a value comparable equal to 1, at the momenta value of around $Q \sim 500\text{MeV}$. This leads to the general problem in QCD that it becomes non-perturbative at small momenta or energies $E \leq 1\text{GeV}$. For example, the properties of hadrons at low energy cannot be studied via simple perturbative methods. One has to look for other approaches to study the properties of hadrons. Various non-perturbative techniques are lattice gauge theory, sum rules, phenomenological models and effective theories. Some situations arise when the symmetries and dynamics of QCD gives birth to

new constants other than coupling constants in the form of expansion parameters. Thus non-perturbative effects arise in the form of low energy parameters. A lot of experimental data can help to extract these low energy parameters. Effective field theories becomes useful technique to implement the above said idea. Another way of simplifying the hadron structure is to chose phenomenological model and to study the low energy parameters associated with it. The first part of present thesis is concerned with application of phenomenological models to find different low energy signatures of spin $\frac{1}{2}$ baryonic systems. Another category of particles of our interest is a heavy-light system. This interest is growing rapidly with advancements in the experimental facilities and discoveries of new resonances at different energy levels. To explore the decay widths and masses of newly observed resonances in heavy-light sector, an effective theory is the most helpful.

1.6 Phenomenological Models

Particle physics phenomenology is a part of theoretical particle physics which deals with their application of theory to the calculation of quantities that in principle can be directly compared with observations. Our currently observed theories of particle physics and their interactions is represented by standard model of particle physics. In a few recent years, some important limitations of standard model have come into picture and some of the extended models were defined to explain the theories beyond the standard model. These extended models usually have a large number of unknown parameters and in order to predict their accurate predictions and to distinguish from one-another, the phenomenologist's work play an important part. These models aim

towards computing, for a given model, the predictions for observable quantities and compare them with measured data. The main focus while framing a phenomenological model is to keep in mind all the basic interactions of the system and a phenomenologist work in order to find the answers of some of the basic questions like- Which measurement has a larger sensitivity to unknown model parameter? Which is the best way to measure an unknown signatures of particle? How can one distinguish one model from others using experimental predictions?

1.6.1 Survey of Models

The most common among all ranges of quark models are constituent quark models. These models assume baryons to be made up of three constituent quarks bound in a confining potential. One of the most important successes of these models is that these models help to explain the anomalous magnetic moment of proton and deuteron. Isgur and Karl Model [16] suggested that the interaction between quarks lies in terms of harmonic oscillator combined with anharmonic perturbation and hyperfine interactions. Soliton models view the nucleon as a localized lump of energy density formed out of mesons and quarks. Bag Model [17] considers the region of space with hadronic fields and having constant energy per unit volume. This region of space is assumed as a "Bag". A bag model is based on the assumption that quarks are assumed to be moving freely with in a cavity of radius R . The radius is determined by the condition that the quark field 'pressure' on the surfaces of the cavity are balanced by a universal pressure B which comes by adding a term to the stress energy tensor for hadronic constituents. The MIT bag model was constructed by dividing the space into two regions, one is the interior of bag in which quarks had very small masses

and felt only weak forces and exterior in which quarks were not allowed to propagate and having different vacuum energy. In the chiral quark model [18] quarks move along with $\bar{q}q$ condensates and an octet of Goldstone bosons is said to be generated as a result of spontaneous broken global symmetry. Integration over meson fields in baryon wave-function gives rise to simple QQQ Fock component with constituent quark condensates. The chiral quark soliton model are based on the observation that there exists an energetically favored configuration of chiral fields which binds valence quarks in a well localized object, called as soliton. Another renowned approach where the visualization of hadronic structure occurs fundamental quarks (as valence part) interacting through gluons and quark-antiquark pairs. Relativistic models requires modification in the form of confining potential. The model with relativistic correction uses generally Bethe-Salpeter equation.

1.7 Effective Theories

In case of processes where the energy scales are widely separated and several different scales are involved, effective field theories play an important role. An effective field theory in particle physics is a field theory where low energy physics is described by a simplified approach. The low energy physics refers to the low compared to some cut-off scale or Λ . Therefore, an effective field theory is a theory which only describes the physics below some scale Λ , as opposed to a fundamental field theory which should be valid at arbitrary higher scale of energies. For example, an effective theory is applicable to a heavy hadron decaying into lighter mesons because its low energy behavior can be looked upon independent of higher energy. Effective field theories are

approximations by nature. In view of particle physics, an effective theory can be implemented to make it applicable in a convenient way. An effective theory, if applicable to some scale Λ , only finite number of parameters can describe the physical process. An EFT is also useful because of the necessity of ultra-violet regularization. This makes the process of constructing the effective theory nontrivial because the limit in which the small distance scales are taken to zero, must be handled carefully. One consequence of the ultraviolet behavior is the renormalization group running of coupling constants with the renormalization scale, μ . Going to the effective theory actually changes the running of coupling constants by trading logarithmic dependence on heavy particle masses for scale dependence. The theoretical basis of effective field theories can be formulated [19] as a theorem: for a given set of asymptotic states, perturbation theory with the most general Lagrangian containing all terms allowed by the assumed symmetries will yield the most general S-matrix elements consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetries. Thus, an effective field theory can be implemented by writing down fields for all the relevant degrees of freedom and constructing an effective Lagrangian. The Lagrangian includes all possible terms that transform correctly under the symmetries. Number of terms can be infinite which leads to typically non-renormalizable in nature. Various terms in the Lagrangian are of the type $C_\mu O_\mu$ which encodes short distance and long distance physics respectively. μ is the renormalization scale separating the two regimes. This is essentially known as operator product expansion.

1.7.1 Operator Product Expansion

An effective field theory Lagrangian can be written on operator expansion form like

$$L = \Sigma_i C_\mu O_\mu \quad (1.16)$$

where O_μ are the operators constructed out of the light quark fields and C_μ are the couplings that contains all the heavy degrees of freedom. These co-efficients are referred to as Wilson co-efficients. The operator-product expansion is a technique in which the singularities of the operator products are expressed as a sum of nonsingular operators with the coefficients being singular c -number functions. The physical basis for operator product expansion is that all the operators product of local operators at distances small compared to the characteristic length of the system should look like a local operator. It also enables us to extract a short distance piece in the scattering cross section which is calculable through the QCD Lagrangian by using renormalization group method. OPE can be defined using proper renormalization scale which is used to separate hard and soft momenta. Wilson hypothesized that the singular part of the product $A(x)B(y)$ of two operators is given by a sum over other local operators O_i .

$$A(x)B(y) = C_{ij}(x-y)O_i(\frac{1}{2}(x-y)) \quad (1.17)$$

where $C_{ij}(x-y)$ are the co-efficient functions which are singular in the limit of $x \rightarrow y$. Such expansion were proven to be held in renormalizable field theories. According to Wilson's picture, all the parameters in a renormalizable theory can be thought of being the scale dependent quantities. A renormalizable theory may depend upon the cut-off parameter scale Λ . By introducing a cut-off, Lagrangian L_{eff} can be rewritten. The

terms in L_{eff} is a set of local operators, O_i , that can be added as perturbations to full scale Lagrangian L_0 . We begin with a theory where the low energy dynamics are separated from the high energy by a chosen energy scale. Depending on what scale we are working, the different techniques can be used for the calculations; at high energy we can put the low energy scales to zero, and when working at low scales we put the heavy scales to infinity. The non-local heavy interactions are then replaced with local non-renormalizable interactions. As mentioned before, we still have the infrared behavior at low scales but the ultraviolet behavior will be changed as these levels and the high energy dynamics will be imbedded in the low energy couplings. To understand the low and high energy dynamics of elementary particles, symmetries and anomalies play an important role in particle physics.

1.8 Fundamental Symmetries and their Breaking

Transformations which do not change the physics of a system are symmetry transformations. In classical physics, this means that the equations of motion are unchanged. Symmetries of a physical system provide useful constraints on the possible forms of its Lagrangian. In a path integral formalism, a symmetry is given if the Lagrangian and the path integral measure are invariant under the respective transformations. The relationship between symmetries and conservation laws is expressed via the Noether theorem which says that for every continuous transformation that leaves the action invariant then there exists a time independent classical charge Q and a corresponding conserved current $\partial_\mu J^\mu = 0$.

(1) Symmetries can be internal symmetries and can be space-time symmetries. The

symmetry transformations related to space-time connect fields at different space-time points. For instance, a Lorentz invariant quantity remains invariant under Lorentz transformation whereas transformation connecting different fields, or connecting different components of the field at the same space-time point can be internal symmetries. Gauge transformation in QED is an example of internal symmetry.

(2) One more classification is related to discrete and continuous symmetries. Symmetries are discrete when parameters under symmetry transformation can take only discrete values. Examples of such symmetries are parity, time reversal, charge conjugation etc. On the other hand, symmetries are of continuous type if the parameters can take continuous values. Examples of such symmetries are invariance of various free Lagrangian under phase rotations of the field involved.

(3) Symmetry is said to be exact in nature if a number of fields transform like a multiplet under a given symmetry, the quanta of those fields should have the same mass. For example, if proton and neutron form a doublet under an isospin symmetric group, proton and neutron would have the same mass under exact symmetry. But in reality, masses of proton and neutron vary by a little amount thus isospin symmetry is an approximate symmetry.

1.9 Chiral Symmetry

Quarks are the fundamental building block for hadrons. Chiral Symmetry is an internal symmetry of right and left handed spinors. Its spontaneous breaking generates Goldstone bosons with negative parity, zero spin, unit isospin and zero baryon number called pions. Thus, a broken approximate chiral symmetry entails the existence

of pions where u and d quarks have small but non-zero masses whereby spontaneous breaking of a symmetry is expressed as the non-vanishing of the vacuum when operated by the charge Q . In the low energy limit, where masses of light quarks approaches to zero, the left handed and right handed quark fields can be separated from each other in QCD Lagrangian. The gluons couple identically to all quark flavors. If all the masses are equal for n_F flavors we have a $SU(n_F)_V \times U(1)_V$ symmetry. The $U(1)_V$ symmetry changes the phase of all quark fields simultaneously. The $SU(n_F)_V$ symmetry acts as $q(x) \rightarrow Uq(x)$ where $U \in SU(n_F)_V$

Considering QCD Lagrangian in the form:

$$L = \sum_{i=u,d,s} i\bar{q}_i \not{D} q_{iL} + i\bar{q}_i \not{D} q_{iR} - m_i \bar{q}_{iR} q_{iL} - m_i \bar{q}_{iL} q_{iR} \quad (1.18)$$

Here D is the co-variant derivative with the gluon field and the dots indicate the purely gluonic terms. The left and right handed quark fields are given by

$$q_R = \frac{1}{2}(1 + \gamma_5)q$$

$$q_L = \frac{1}{2}(1 - \gamma_5)q$$

Left and right-handed column vectors q_L and q_R modifies the QCD Lagrangian as:

$$L_{QCD} = i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - \bar{q}_R M q_L - \bar{q}_L M q_R \quad (1.19)$$

The above form of equation exhibits large symmetry whenever quark masses are equal to 0. Lagrangian in QCD is:

$$L_{QCD} = \bar{q}_i i \not{D} q_L + \bar{q}_R i \not{D} q_R \quad (1.20)$$

would be invariant under left and right handed rotations. Defining the chiral symmetry group, $G = SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A$ with $q_R \rightarrow \exp(i\Sigma_j \lambda_j \beta_j) q_R$ and $q_L \rightarrow \exp(i\Sigma_j \lambda_j \alpha_j) q_L$. Continuing to neglect quark masses, chiral symmetry would lead to sixteen conserved quark currents, $\bar{q}_L \gamma_\mu \frac{1}{2} \lambda_i q_L$ and $\bar{q}_R \gamma_\mu \frac{1}{2} \lambda_i q_R$ which leads to eight vector and eight axial conserved currents $V_\mu^i = \bar{q} \gamma_\mu \frac{1}{2} \lambda_i q$ and $A_\mu^i = \bar{q} \gamma_\mu \gamma_5 \frac{1}{2} \lambda_i q$. Under the condition of masslessness, vector and axial currents are conserved. The same is true for a vector singlet current but the axial singlet current has an anomaly. Under $U(1)_V$ all quarks have the same change in phase while under $U(1)_A$ the right and left-handed quarks have opposite change in phase.

1.10 Spontaneous Symmetry Breaking

A symmetry is said to be spontaneously broken if its ground state of the system is no longer invariant under the full group of the Hamiltonian. Quantum field theory and statistical mechanics have some analogies between them for example, a scalar field theory can be seen as a continuum description of a system that allows second order phase transitions. If we continue this analogue, the QFT system, like the statistical system, might therefore have a field that has non-zero global value. As this global field might also have a direction, it will then violate the symmetry of the Lagrangian. The spontaneous symmetry breaking can be considered as:

$$L = \frac{1}{2} m (\dot{R}^2 + \dot{\theta}^2 + \omega^2 R^2 \sin^2 \theta) + mgR \cos \theta \quad (1.21)$$

where θ measures the angular displacement of the bead from the nadir. L is symmetric under the angular parity transformation $L(\theta) = L(-\theta)$, but for the equilibrium

condition is found to be more complex than the displaced oscillator. In equilibrium condition, the equation $\frac{\partial \mathcal{L}}{\partial \theta} = m\omega^2 R^2 \sin(\theta)(\cos(\theta) - \frac{g}{\omega R})$, leads to equilibrium position at higher angular velocities when $\theta_E = \pm \arcsin(\frac{g}{\omega R})$ and here the choice of + vs. - is not determined by the physics but rather by the history of motion of the system as the critical angular velocity was reached. Neither of these equilibrium positions exhibits the symmetry of the underlying potential, which is invariant under the exchange of θ and $-\theta$. This is an example of spontaneous symmetry breaking, wherein the Lagrangian of a system possess a symmetry, but this symmetry is broken by the ground (equilibrium) state of the system. In particle physics, the force carrier particles are normally specified by field equations with gauge symmetry; their equations predict that certain measurements will be the same at any point in the field. The symmetry of the equations is not reflected by the individual solutions but it is reflected by the range of solutions. An actual measurement reflects only one solution representing a breakdown in the symmetry of the underlying theory. This is so called as spontaneous symmetry breaking. Examples are breaking of chiral symmetry and Higgs mechanism. The Lagrangian used in motivating the phenomenon of spontaneous symmetry breakdown are typically constructed in such a manner that the degeneracy of the ground state is built into potential at the classical level(i. e. Mexican Hat Potential)

1.10.1 Chiral Symmetry Breaking and Goldstone Theorem

Consider the case of weak decays of pions. In the simple Fermi theory, weak interaction Hamiltonian is represented by sum of axial and vector currents. Because of the parity, weak decay of pion is controlled by the axial current between pion and

vacuum i.e. $\langle 0 | A_\mu^a(x) | \pi^b f_q \rangle = i f_\pi q_\mu \delta^{ab} \exp^{-iq \cdot x}$. The smallness of pions is directly related to partial conservation of axial current. On the one hand, meson mass spectrum does not reflect the axial-vector symmetry and on the other side, weak decay of pion seems to be consistent with the partial conservation of axial current. This leads to the conclusion that axial current is spontaneously broken. The finite value of u -, d - and s - quark masses in the QCD lagrangian explicitly break the chiral symmetry, resulting in divergences of symmetry currents. Recall the quark mass term, $L_M = -\bar{q}Mq = -\bar{q}_L M q_R + \bar{q}_R M q_L$ mixes the right-handed and left-handed fields. The symmetry breaking group transforms under $SU(3)_L \times SU(3)_R$ as a member of $(3, 3^*)$ and $(3^*, 3)$ representation. Thus, it can also be stated that axial symmetry is spontaneously broken and Goldstone theorem states that when a continuous symmetry is spontaneously broken, there must also be generated massless bosons having the quantum numbers of broken generator, especially in this case, a pseudoscalar boson. Also, when the axial-charge acts on a single particle eigen state of opposite parity in return, thus one or more pseudo-scalar bosons are generated. The number of Goldstone bosons depend upon the structure of the symmetrical group. According to this, one would expect eight pseudo-scalar mass states which are called as Goldstone bosons in QCD.

1.11 Chiral Perturbation Theory

Chiral perturbation theory is the low energy effective theory of QCD where the degrees of freedom, taken into account are the Goldstone bosons from the spontaneous breakdown of chiral symmetry and their interactions. It is a kind of low energy

effective theory and is valid in terms of expansion of momenta. The Lagrangian is represented as a series of terms with increasing powers of momenta. In case of chiral symmetry with its parity transformation, Goldstone fields can be collected in a unitary matrix field $U(\phi)$ transforming as: $U(\phi) \rightarrow g_R U(\phi) g_L^{-1}, (g_L, g_R) \in G$ under chiral rotations. The exponential parametrization of $U(\phi)$ for $N_f = 3$, can be written as:

$$\Sigma = \frac{\exp(2i\lambda_a \phi^a)}{f} \quad (1.22)$$

$$\text{and } \Pi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & \frac{-2}{\sqrt{6}}\eta \end{pmatrix}$$

In the above equation $f \sim f_\pi = 131\text{MeV}$, where f_π is the pion decay constant, $\langle 0 | \bar{u}\gamma^\mu\gamma_5 d | \pi^-(p) \rangle = -if_\pi p^\mu$. Chiral perturbation theory is the low energy effective theory of standard model. The non-zero quark masses $m_q = \text{diag}(m_u, m_d, m_s)$ break the chiral symmetry. To include m_q in the low energy Lagrangian term is required that transforms the same way as the light quark mass term in L_{QCD} . The Lagrangian with the fewest derivatives and powers of m_q that satisfies the symmetry constrains is:

$$L_\chi^{(2)} = \frac{f^2}{8} (\text{Tr} \partial^\mu \Sigma \partial_\mu \Sigma^\dagger + \frac{f^2 B_0}{4} \text{Tr} (m_q \Sigma + m_q \Sigma^\dagger)) \quad (1.23)$$

Expanding the Σ fields in terms of Π gives a canonically normalized kinetic term and an infinite number of interaction terms with determined co-efficients. The Lagrangian is not the most general one that is invariant under the desired symmetries. An operator with dimension m and involving more derivatives or powers of m_q and a coupling of dimension $4 - m$ can be added. Along with higher dimension operators,

loop corrections must be considered. The chiral power counting is applied to loop diagrams.

1.12 Heavy Quark Symmetry

For quarks with masses m_Q greater than the non-perturbative scale of QCD, it is a good approximation to take the limit $m_Q \rightarrow \infty$. At this approximation, the heavy quark possess a special kind of symmetry which is known as heavy quark symmetry. For a system containing heavy quarks, the effective coupling constant α_s becomes weak which implies that on length scales comparable to Compton wave-length $\lambda_Q \sim \frac{1}{m_Q}$ the strong interactions becomes perturbative in nature. Thus, a charmonium system can be compared to the system like that of hydrogen atom. This symmetry has some important implications for the hadrons containing a heavy quark and a light quark. The QCD Lagrangian describes the strong interaction of quarks and gluons and a non-perturbative scale Λ_{QCD} is dynamically generated by QCD. Consider a $Q\bar{q}$ meson that contains a heavy quark with mass $m_Q \gg \Lambda_{QCD}$ and a light quark with mass $m_q \ll \Lambda_{QCD}$. In the infinite limit of heavy quark mass, the heavy quark can be labeled by four velocity vector v_μ . The size of such systems is determined by R_{had} and typical momenta of such systems are of the order of Λ_{QCD} . The heavy quark is surrounded by a system of light degrees of freedom consisting of light quarks interacting with gluons. In this case, $\lambda_Q \ll R_{had}$ and a hard probe is needed to resolve the quantum numbers of heavy quark. Thus, light degrees of freedom are blind to spin and flavor of heavy quark. They can experience only its color field. It follows that hadronic systems containing a quark with mass $m_Q \gg 0$ and which only

differ in their flavor and spin quantum numbers, will have the same constituents of light degrees of freedom. The configuration of light degrees of freedom in a hadronic system with a single heavy quark does not change if this heavy quark with velocity "v" does not change if the heavy quark is replaced by another heavy quark due to static velocity and flavor-spin symmetries. The heavy quark $SU(2)$ spin symmetry and $U(N_h)$ flavor symmetries can be embedded into a larger group $U(2N_h)$ spin-flavor symmetry in the $m_Q \rightarrow \infty$.

1.13 Heavy Quark Effective Theory

The QCD Lagrangian does not manifest heavy quark spin and flavor symmetry when $m_Q \rightarrow \infty$ therefore it is convenient to use an effective field theory which manifests such kind of symmetries in the $m_Q \rightarrow \infty$. This effective field theory is famous as heavy quark effective field theory. HQET is helpful in explaining the dynamics of heavy hadrons containing a light quark. The theory is constructed in such a way that inverse powers of m_Q appears in the effective Lagrangian leading to the independence on mass of heavy quark in the $m_Q \rightarrow \infty$ regime.

1.13.1 Quantum Numbers of Heavy Hadronic System

A heavy-light hadronic system contains heavy hadron and light quarks-antiquarks and gluons. All the degrees of freedom other than heavy quark are known as light degrees of freedom. The total angular momentum of the hadron J is conserved and in the limit $m_Q \rightarrow \infty$, S_Q is also a conserved quantity, then the spin of light degrees of freedom can be related to other angular momentum operators as : $S_l = J - S_Q$. J ,

S_Q and S_l . In the infinite heavy quark mass limit, a heavy light system $Q\bar{q}$ can be classified into doublets depending upon their quantum numbers. A heavy hadronic system containing heavy quark with spin quantum number S_Q and light degrees of freedom where light degrees of freedom include light quark and gluons interacting through quark-anti quark pairs. It should have the quantum number of light quark that is S_l in order to have total conserved quantum number J where $J = S_Q + S_l$. Defining J as $J^2 = j(j+1)$ and $S_Q^2 = (s_Q)(s_Q+1)$ and $S_l^2 = (s_l)(s_l+1)$, the total spin $j \pm \doteq s_l \pm \frac{1}{2}$ can be obtained by combining the spin of heavy quark with spin of light degrees of freedom. The ground state heavy mesonic system form a degenerate doublet with $j \doteq 0 \pm 1$ and negative parity denoted as D and D^* for charm meson. The first excited states 0^+ and 1^+ heavy mesons are the quantum numbers of the $j^P \doteq \frac{1}{2}^+$ doublet. There also exists an excited doublet of heavy mesons with $J^P \doteq 1^+$ and 2^+ . Latest discoveries of charm and bottom mesons include higher and radially excited states too.

1.13.2 Covariant Representation of Fields and Effective Lagrangian

The ground-state $Q\bar{q}$ hadrons can be represented by fields $H_v^{(Q)}$. Since heavy quark symmetry implies that the heavy quark doublets may be treated as a single object transforming linearly under heavy quark symmetries. Thus, the field $H_v^{(Q)}$ is a combination of pseudoscalar field and vector field.

$$H_v^{(Q)} = \frac{1 + \not{v}}{2} [\not{P}_v^{*Q} + iP_v^{(Q)} \gamma_5] \quad (1.24)$$

where $\psi H_v^Q = H_v^Q \psi$ and $H_v^Q \psi = -H_v^Q \psi$. A single heavy quark with velocity v interacting with external fields, momentum of an on-shell quark is defined by $p = m_Q v + k$, k is the residual momentum of the order of Λ_{QCD} and determines the amount by which the quark is off-shell. In heavy quark effective theory, usual quark field can be written as:

$$Q(x) = e^{-im_Q v \cdot x} [Q_v(x) + \Omega_v(x)] \quad (1.25)$$

where $Q_v(x) = \exp^{im_Q v \cdot x} \frac{1+\not{v}}{2} Q(x)$, $\Omega_v(x) = \exp^{im_Q v \cdot x} \frac{1-\not{v}}{2} Q(x)$ are the large component and small component fields respectively. One can use these fields to rewrite QCD Lagrangian as follows:

$$L_{QCD} = L_Q + L_q \quad (1.26)$$

where Q and q corresponds to heavy and light quarks respectively. This leads to

$$\begin{aligned} L_Q &= \bar{Q}(i\not{D} - m_Q)Q \\ &= e^{-im_Q v \cdot x} (Q_v(x) + \Omega_v(x)) (i\not{D} - m_Q) e^{im_Q v \cdot x} (Q_v(x) + \Omega_v(x)) \\ &= \bar{Q}_v(x) i\not{D} Q_v(x) + \bar{\Omega}_v(x) (i\not{D} - 2m_Q) \Omega_v(x) + \bar{Q}_v(x) (i\not{D} - 2m_Q) \Omega_v(x) + \bar{\Omega}_v(x) i\not{D} Q_v(x) \end{aligned}$$

Using the relation: $\Omega_v(x) = \frac{1}{2m_Q + iv \cdot D} i\not{D}_\perp Q_v(x)$ where $D_\perp^\mu = D^\mu + v^\mu v \cdot D$. The $Q_v(x)$ produces effects at the leading order whereas effects of $\Omega_v(x)$ are suppressed by the powers of m_Q . The effective Lagrangian in the limit $m_Q \rightarrow \infty$ can be rewritten as:

$$L_{eff} = \sum_{i=1}^{N_h} \bar{Q}_v(i v \cdot D) Q_v \quad (1.27)$$

The above Lagrangian exhibits the spin and flavor symmetry and N_h is the number of heavy quark flavors all having the same four-velocity v .

1.14 Heavy Hadron Chiral Perturbation Theory

Heavy Hadron Chiral Perturbation theory is a theory which is well applied to a system with one heavy and other light quark. This theory unites two important theories, one is chiral perturbation theory and other is heavy quark effective theory. Heavy-light mesons come in triplets under the $SU(3)$ symmetry, (D^0, D^+, D_s) . Two velocity dependent fields $P_a^Q(v)$ and $P^{(Q)\mu}$ destroys a meson with velocity v can be included in a 4×4 matrix as:*

$$H_a = \frac{1 + \not{v}}{2} (P_a^{*\mu} \gamma_\mu - P_a \gamma_5) \quad (1.28)$$

*Here a is the $SU(3)$ index. In charm mesons sector H_a consists of the D^0, D^+, D_s^+ pseudo-scalar mesons and D^{*0}, D^{*+}, D_s^{*+} vector mesons. H_a^Q transforms linearly under a heavy quark spin and flavor transformation. The pion octet is introduced by the vector and axial combinations $V^\mu = \frac{1}{2} \xi \partial_\mu \xi^\dagger + \xi^\dagger \partial^\mu \xi$ and $A^\mu = \frac{1}{2} \xi \partial_\mu \xi^\dagger - \xi^\dagger \partial^\mu \xi$. The lowest lying excited states are the $J^P = 0^+$ and 1^+ i.e. $s_l^P = \frac{1}{2}^+$ doublet and represented by the fields S_a . The fields for excited spin doublets are mentioned below:*

$$S_a = \frac{1 + \not{v}}{2} (P_{1a}^\mu \gamma_\mu \gamma_5 - P_{0a}) \quad (1.29)$$

The lowest order Lagrangian invariant under these symmetries is:

$$\mathcal{L}_H^{(1)} = -Tr \bar{H}_a i v \cdot D_{ba} H_b + g Tr \bar{H}_a H_b \gamma_\mu \gamma_5 A_{ba}^\mu \quad (1.30)$$

where the chiral co-variant derivative is $D_{ab}^\mu = \delta_{ab} \partial^\mu - V_{ab}^\mu$. Terms in \mathcal{L}_H involves expansion in powers of $\frac{1}{m_Q}$, where terms at order $\frac{1}{m_Q}$ break heavy quark symmetry.

The propagators for heavy pseudo-scalar meson is $\frac{i\delta_{ab}}{2(v \cdot k + i\epsilon)}$ and of the order is $\frac{1}{p}$.

1.15 Organization of Thesis

The thesis is organized in the following manner.

Chapter II describes the detailed methodology used, that is statistical model which assumes the hadronic structure to be made up of fundamental quarks (known as valence quarks) interacting with sea quark-antiquark pairs through gluons. Thus, a hadronic system consists of a number of quark-gluon Fock states multi-connected non-perturbatively through gluons. These Fock states are in agreement with the total anti-symmetric spin $\frac{1}{2}$ colorless nature of the baryonic systems. Moreover, sea is assumed to be in relative *s*-wave. A suitable wave-function [20] can be framed with suitable quantum numbers in such a way that each term in the wave-function should produce an anti-symmetric spin $\frac{1}{2}$ wave-function. The main feature of the statistical model is that sea also act as an active participant and contributes to all the low energy properties of hadrons. A detailed methodology for calculation of relevant probabilities for all possible Fock states has been included in chapter II. Three different versions of statistical model named as C, P and D are discussed in detail. A brief introduction to all the static properties is also discussed in this chapter.

Chapter III describes the application of statistical model to a nucleonic system without and with strange sea to find the low energy properties. The low energy properties include magnetic moments, distribution of spin among quarks, axial-vector form factors etc. The calculated results are compared with the available experimental data [21] and other theoretical models [20, 22]. The results have been computed in the three different cases. The importance of all low energy parameters have been analyzed and it has been found that vector sea is the most dominant contributor to the most of the low energy properties of baryons. Based on the statistical approach

and using principle of detailed balance, strange sea contributions are calculated for nucleon's low energy properties. The comparison of our data in different cases with experimental results show that although strange contribution in sea is negligible, yet its effects can be seen in the data of the static properties. Inclusion of strange sea gives the data more close to experiments. The principle of detailed balance finds the values of strange quark content ratio to be $\frac{2\bar{s}}{(\bar{u}+\bar{d})} = 0.37$; $\frac{-2\bar{s}}{(u+d)} = 0.03$ which are found to be agree well with results of the NuTeV Collaboration [24].

In **Chapter IV** statistical model is applied to a system with strange quarks i.e. lambda and other hyperons. The suitable modifications in the principle of detailed balance leads to inclusion of the strange quark-antiquark pairs in the sea. The non-negligible mass of strange quark affects the probability of all the Fock states. It also restricts the total number of partons in the sea. The probabilities are helpful in finding the low energy properties for these hyperons. In this chapter, all the low energy observables include only the symmetry breaking effects in the sea part only. The strange sea seems to favor the experimental values [21], there by suggesting the gluons in the sea to be generating strange quark condensates with similar quantum numbers. $SU(3)$ breaking corrections in valence part were applied to the all strange baryons. Symmetry breaking corrections in valence part is applied and effects of symmetry breaking leads to modified values of magnetic moment, spin distribution among quarks and axial vector form factors. The strange mass corrections also helps in finding the corrected values of axial vector coupling constants F and D . The ratio $\frac{F}{D}$ comes out to be 0.676 and found to be deviating about 17% from the experimental value 0.575 ± 0.016 [25]. In case of $SU(3)$ symmetry, we find $\Delta u = 0.91$, $\Delta d = -0.24$ and $\Delta s = 0$ respectively. Also, when m_s corrections are applied, the spin

polarized densities change to $\Delta u = 0.76$, $\Delta d = -0.18$ and $\Delta s = -0.019$ respectively. This leads to the conclusion that even in $SU(3)$ symmetry breaking, strange quark contribution to the spin of proton is very small.

Chapter V deals with the detailed formalism of heavy hadron chiral perturbation theory. Here an effective Lagrangian is framed using approximate symmetries. The two global approximate symmetries of QCD can be employed to explain the various properties of a hadronic bound state having one heavy quark interacting via the exchange of light pseudoscalar meson like pions, kaons and eta mesons using systematic expansions in light quark masses m_q and heavy quark mass expansion as $\frac{1}{m_Q}$. Effective Lagrangian here describes the interplay between the chiral symmetry and heavy quark symmetry in the form of low energy gradients with heavy and light fields as operators. At tree level, the residual masses are given by a generalized formula [26, 27] where the residual masses are defined to be the difference between the real mass and an arbitrarily chosen reference mass. The residual mass formula involves simultaneous fitting of 8 parameters which is performed by using a fitting program in Mathematica. Also, the values of the mass splittings are available at various experiments; we also try to fit these mass splittings over a range of the parameters. The effect of variation in the values of the parameters on various splittings through graphical interpretations is also studied. The possible range of some parameters is available from various experiments. Moreover, the importance of non-perturbative QCD parameters is included by looking at the masses of bottom meson masses of different excited states. In the same chapter, we also try to find the masses of the non-strange excited states for bottom mesons from the observed experimental values of all the ground states and excited strange mesons. The excited

meson spectra has thus been found to be matching well with other models [28, 29].

Chapter VI describes the latest resonances discovered in the heavy meson sector which are $D(2550)$, $D(2600)$, $D(2750)$ and $D(2760)$ in the decay channels $D^0(2550) \rightarrow D^{*+}\pi^-$, $D^0(2600) \rightarrow D^{*+}\pi^-$, $D^+\pi^-$, $D^0(2750) \rightarrow D^+\pi^-$, $D^+(2600) \rightarrow D^0\pi^+$ and $D^+(2760) \rightarrow D^0\pi^+$ in the inclusive $e^+e^- \rightarrow c\bar{c}$ interactions by BaBar Collaboration [30]. Using heavy hadron chiral perturbation theory, the decay widths of these recent states are computed and using a MINUIT fitting program, the relevant coupling constants are found out. The most suitable spin-parity assignments for $D(2750)$, $D(2760)$ is $(2^-, 3^-)$ or $1D$ and for $D(2550)$, $D(2600)$ are $(0^-, 1^-)$ i.e $2S$ state respectively. The present work estimates the coupling constants for all these recently observed states by using a Chi-Square minimization technique in MINUIT and finds the suitable J^P values for all the states. Similarly, for the $D_J(3000)$ state, various possibilities include 3^1S_0 , $2P(1^+)$ and $2D(2^-)$ and $1F$ etc.. However, if we consider the $D_J(3000)$ as the spin partner of $D_J^*(3000)$ then the only possibility can be that $2P(1^+)$ in $(0^+, 1^+)$ doublet overlaps with that with 1^+ in $(1^+, 2^+)$ doublet.

In **Chapter VII**, the summary and significance of the work of this thesis and the scope for possible extension of present work is discussed.

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Chapter 2

Methodology

The structure of hadrons in form of quarks was first put forward by Gell-Mann [1] and Zweig [2]. They stated that hadrons could be built up of as a composite system of three quarks (u , d , s) and with fractional charge so as to obey $SU(3)$ symmetry. The picture of colored quarks was formulated by Gell-Mann [3]. It was postulated that hadrons are singlet of $SU(3)_{color}$ group. The first historical step in understanding of quark gluon nature is the constituent quark model [4]. $SU(3)$ flavor states are combined with the $SU(2)$ states of spin and leads to the six fold symmetry known as $SU(6)$. The model uses six quark states to constitute and classify the hadrons into mesons and baryons and known as $SU(6)$ quark model. $SU(6)$ model explains some of the low energy properties of nucleonic system like magnetic moment ratio of proton and neutron is found to be matching with experimental data, still there are some low energy parameters for which extension of $SU(6)$ quark model is needed. One such big question in the mind of researchers is the proton spin problem. Recent experiments at CERN, RHIC, HERMES and COMPASS collaborations [5] [6] [7] measured the spin structure of proton and deuteron system which elaborates the concept of missing

spin inside the nucleon. Phenomenologist also tries to propose different models in more reliable ways so as to satisfy the experimental data. As a result, a vast range of models are available in hand.

2.1 Wave-Function For Spin $\frac{1}{2}$ Baryons

The hadronic structure is assumed to be made up of two parts, one is valence part and other is sea-part where sea has the structure of quark-antiquark pair multi-connected non-perturbatively through gluons. The three core quarks in hadronic state can be thought of as embedded by the quark-antiquark pair through gluons which is referred as sea-stuff. Since the hadron should be colorless and a q^3 state can be in color state 1, 8 and 10 respectively. This restricts sea to be in specific color state to make hadrons as a colorless entity. Similarly, if sea is assumed to be in more general form like quark-antiquark pair and gluon or a mixture of both then in S-wave state, sea can be of spin 0, 1 or 2. Sea-part is initially assumed to be in S-wave and of non-relativistic nature. The q^3 core part is also assumed to be in ground state initially but in future we may also try to include orbital angular momentum corrections for a nucleonic system. Therefore, a total flavor-spin-color-space wave-function with all possibilities of quark-gluon Fock states is required to be constructed. The wave-function for the valence part is denoted by $\psi = \Phi[| \phi \rangle | \psi \rangle | \chi \rangle] | \xi \rangle$ where $| \phi \rangle$ denotes the flavor part of q^3 wave-function for baryons, $| \chi \rangle$ represents spin of valence part in the wave-function, $| \psi \rangle$ is for color and $| \xi \rangle$ is for space contribution to the whole wave-function [8]. We now write wave-function for baryon-octet. Let λ denotes symmetric under interchange of any two quarks in q^3 wave-function whereas ρ denotes anti-

symmetric for $q_1 \rightarrow q_2$. Then $\Phi[| \phi \rangle | \chi \rangle | \psi \rangle]$ denotes flavor-color-spin part of q^3 wave-function and ξ denotes the space-part in complete wave-function which is symmetric under exchange of any two quarks for lowest lying baryon states. This makes $\Phi[| \phi \rangle | \xi \rangle | \psi \rangle]$ antisymmetric in nature in order for complete function to be antisymmetric. In order to maintain the antisymmetry property of the baryonic system, all possible combinations of flavor, spin and color are included. Since, we are assuming a more general situation where the color quantum number has the freedom to acquire values other than one for the valence part, it also frees the sea quarks to have possibilities of color quantum numbers as $8, \bar{10}$ in addition to one.

$$\phi_1^{\frac{1}{2}} = \phi(8, \frac{1}{2}, 1_c) = F_S \psi_1^A \quad (2.1)$$

where

$$F_S = \frac{1}{\sqrt{2}}(\phi^\lambda \chi^\lambda + \phi^\rho \chi^\rho) \quad (2.2)$$

$\phi_1^{\frac{1}{2}}$ function is for spin $\frac{1}{2}$ color singlet and 8 represents the flavor part and it can be written as product of two functions F_S and ψ_1^A (contributing for color of baryons being anti-symmetric in nature) and F_S denotes flavor and spin of the q^3 wave-function. The subscripts S and A represents the symmetry and anti-symmetry of wave-function. For F_S to be symmetric, ϕ and χ should either be both symmetric or antisymmetric in nature.

$$\phi_8^{\frac{1}{2}} = \phi(8, \frac{1}{2}, 8_c) = \frac{1}{\sqrt{2}}(F_{MS}\psi_8^\rho - F_{MA}\psi_8^\lambda) \quad (2.3)$$

where

$$F_{MS} = \frac{1}{\sqrt{2}}(\phi^\rho \chi^\rho - \phi^\lambda \chi^\lambda) \quad (2.4)$$

$$\phi_{10}^{\frac{1}{2}} = \phi(8, \frac{1}{2}, 10_c) = F_A \psi_{10}^S \quad (2.5)$$

where

$$F_A = \frac{1}{\sqrt{(2)}}(\phi^\lambda \chi^\rho - \phi^\rho \chi^\lambda) \quad (2.6)$$

$$\phi_8^{\frac{3}{2}} = \phi(8, \frac{3}{2}, 8_c) = F_A^\lambda \chi^{\frac{3}{2}} \quad (2.7)$$

$$F_A^\lambda = \frac{1}{\sqrt{(2)}}(\phi^\lambda \psi^\rho - \phi^\rho \psi^\lambda) \quad (2.8)$$

In above equations, $\phi_1^{\frac{1}{2}}$ represents wave-function for $J^P = \frac{1}{2}^+$ baryons in such a way that it gives total spin $\frac{1}{2}$ -color singlet state. Similarly $\phi_8^{\frac{1}{2}}$, $\phi_{10}^{\frac{1}{2}}$ denotes the wave-function with spin $\frac{1}{2}$, color quantum numbers 8 and 10 whereas $\phi_8^{\frac{3}{2}}$ is for spin $\frac{3}{2}$ respectively. Each wave-function is a composition of two parts such that one is symmetric and other is anti-symmetric. Further, if sea is assumed to be flavorless and in S-wave then spin can be 0, 1, 2 and color can be $(1_c, 8_c, (\overline{10}_c))$. Let $H_{0,1,2}$ and $G(1, 8, \overline{10}_c)$ and denote spin and color sea wave function, which satisfy $\langle H_i | H_j \rangle = \delta_{ij}$, $\langle G_k | G_l \rangle = \delta_{kl}$. Now, two gluons each with spin 1 have the following possibilities. Similar kind of possibilities are expected for sea containing two $q\bar{q}$ pairs. This is due to the assumption in our model that a gluon and $q\bar{q}$ carry the same quantum numbers.

$$\text{Spin} : 1 \otimes 1 = 0_s \oplus 1_a \oplus 2_s \quad (2.9)$$

The color space includes the following symmetric and anti-symmetric states because of the gluon being a color octet.

$$\text{Color} : 8 \otimes 8 = 1_s \oplus 8_s \oplus 8_a \oplus \bar{10}_a \oplus 10_a \oplus 27_s \quad (2.10)$$

The subscripts s and a represents the symmetry and antisymmetry combinations of the states respectively. The possible combinations of q^3 and sea which can give a spin $\frac{1}{2}$, flavor octet, color singlet state are $\phi_1^{\frac{1}{2}} H_0 G_1$, $\phi_8^{\frac{1}{2}} H_0 G_8$, $\phi_{10}^{\frac{1}{2}} H_0 G_{\bar{10}}$, $\phi_1^{\frac{1}{2}} H_1 G_1$, $\phi_8^{\frac{1}{2}} H_1 G_8$, $\phi_{10}^{\frac{1}{2}} H_1 G_{\bar{10}}$ and $\phi_8^{\frac{3}{2}} H_1 G_8$, $\phi_8^{\frac{3}{2}} H_2 G_8$, Here $\phi_8^{\frac{3}{2}}$ wave-function can only give spin resultant $\frac{1}{2}$ only if sea-part is having spin either one or two. All other possibilities like $H_2 G_1, H_2 G_{\bar{10}}$ are excluded because of their failure in giving rise to color singlet states. Contributions from states like $H_0 G_{27}, H_2 G_{27}$ are ignored due to higher multiplicities and less contribution. The total flavor-spin-color wave function of a spin up baryon which consist of three-valence quarks and sea in the form of various components can be written as:

$$\begin{aligned} |\phi_1^\uparrow\rangle &= \frac{1}{N} [\phi_1^{(\frac{1}{2})^\uparrow} H_0 G_1 + a_8 \phi_8^{(\frac{1}{2})^\uparrow} H_0 G_8 + a_{10} \phi_{10}^{(\frac{1}{2})^\uparrow} H_0 G_{\bar{10}} \\ &+ b_1 [\phi_1^{\frac{1}{2}} \otimes H_1]^\uparrow G_1 + b_8 (\phi_8^{\frac{1}{2}} \otimes H_1)^\uparrow G_{\bar{10}} \\ &+ c_8 (\phi_8^{\frac{3}{2}} \otimes H_1)^\uparrow G_8 + d_8 (\phi_8^{\frac{3}{2}} \otimes H_2)^\uparrow G_8] \end{aligned} \quad (2.11)$$

where $N^2 = 1 + a_8^2 + a_{10}^2 + b_1^2 + b_8^2 + b_{10}^2 + c_8^2 + d_8^2$

The three terms in above equation are obtained by combining q^3 wave-function with spin 0 (scalar sea) and other three are as a result of coupling with spin 1(vector sea)

such as

$$(\phi_1^{\frac{1}{2}} \otimes H_1)^\uparrow = \phi_{b_1}^{(\frac{1}{2})\uparrow} \psi_1^A \quad (2.12)$$

$$\phi_8^{\frac{1}{2}} \otimes H_1^\uparrow = \phi_{b_8}^{(\frac{1}{2})} \quad (2.13)$$

$$\phi_{10}^{\frac{1}{2}} \otimes H_1^\uparrow = \phi_{b_{10}}^{(\frac{1}{2})\uparrow} \psi_{10}^S \quad (2.14)$$

where

$$\phi_{b_1}^{(\frac{1}{2})\uparrow} = \sqrt{\frac{2}{3}} H_{1,1} F_S^{(\frac{1}{2})\downarrow} - \sqrt{\frac{1}{3}} H_{1,0} F_S^{(\frac{1}{2})\uparrow} \quad (2.15)$$

$$\phi_{b_8}^{(\frac{1}{2})\uparrow} = \sqrt{\frac{1}{2}} [\phi_{b_{8S}}^{(\frac{1}{2})\uparrow} \psi_8^\rho - \phi_{b_{8A}}^{(\frac{1}{2})\uparrow} \psi_8^\lambda] \quad (2.16)$$

$$\phi_{b_{10}}^{(\frac{1}{2})\uparrow} = \sqrt{\frac{2}{3}} H_{1,1} F_A^{(\frac{1}{2})\downarrow} - \sqrt{\frac{1}{3}} H_{1,0} F_A^{(\frac{1}{2})\uparrow} \quad (2.17)$$

The final two c_8, d_8 terms come from $\text{spin } (\frac{3}{2})(q^3) \otimes \text{spin } 1(\text{vector sea})$ and $\text{spin } (\frac{3}{2})(q^3) \text{ spin } 2(\text{tensor sea})$ respectively. Their expressions are

$$\phi_8^{(\frac{3}{2})} \otimes H_1^\uparrow = \phi_{c_8}^{(\frac{1}{2})\uparrow} \quad (2.18)$$

$$\phi_8^{(\frac{3}{2})} \otimes H_2^\uparrow = \phi_{d_8}^{(\frac{1}{2})\uparrow} \quad (2.19)$$

where

$$\phi_{c_8}^{(\frac{1}{2})\uparrow} = [\sqrt{\frac{1}{2}} H_{1,-1} \chi_{\frac{3}{2}}^{\frac{3}{2}} - \sqrt{\frac{1}{3}} H_{1,0} \chi_{\frac{3}{2}}^{\frac{1}{2}} + \frac{1}{6} H_{1,1} \chi_{\frac{-1}{2}}^{\frac{3}{2}}] F'_A \quad (2.20)$$

$$\phi_{d_8}^{(\frac{1}{2})\uparrow} = [\sqrt{\frac{2}{3}} H_{2,2} \chi_{\frac{-3}{2}}^{\frac{3}{2}} - \sqrt{\frac{3}{10}} H_{2,1} \chi_{\frac{-1}{2}}^{\frac{3}{2}} + \frac{1}{5} H_{2,-1} \chi_{\frac{3}{2}}^{\frac{3}{2}}] F'_A \quad (2.21)$$

Wave-functions $\phi_{b_1}^{(\frac{1}{2})\uparrow}$, $\phi_{b_8}^{(\frac{1}{2})\uparrow}$, $\phi_{b_{10}}^{(\frac{1}{2})\uparrow}$, $\phi_{c_8}^{(\frac{1}{2})\uparrow}$, $\phi_{d_8}^{(\frac{1}{2})\uparrow}$ are written by taking coupling between spins of sea part and flavor-spin part of q^3 wave-function along with suitable normalization constant. For example, in $\phi_{b_1}^{(\frac{1}{2})\uparrow}$ wave-function, spin 1 of sea content can

result in spin $\frac{1}{2}$ only by coupling with valence part with spin $\frac{1}{2}$. Also, spin 0 sea-parts can give rise to total spin $\frac{1}{2}$ only by combining with valence part having spin $\frac{1}{2}$. Properties of all baryonic systems are studied using flavor-spin-color wave-function of baryonic system with possible combination of q^3 and sea such as to give spin $\frac{1}{2}$ and flavor-octet and color-singlet state. The wave-function implies that the coefficients associated with each possible combination need to be found out statistically. The first three terms in the wave-function describe a spin $\frac{1}{2}q^3$ coupled to a spin(0) scalar sea and the other terms with co-efficients $(b_1, b_8, b_{\bar{10}}, c_8)$ coupled to spin 1(vector sea) where d_8 is signifying a tensor contribution. The above mentioned wave-function can also be rewritten in the form of $\phi_{val}\phi_{sea}$ and the co-efficients $a_0, a_8, a_{\bar{10}}, b_1, b_8, b_{\bar{10}}, d_8$ by a factor $\sum n_{\mu\nu}^* c_{sea}$ in the wave-function $|\phi_{\frac{1}{2}}^\uparrow\rangle = \sum_{\mu,\nu} (n_{\mu\nu}^* c_{sea}) \phi_{val} \phi_{sea}$. Here $\mu = 2$ goes only with $\nu = 8$ in a view to maintain the anti-symmetrization of the wave-function. This means that, in spite of nine terms we are left with seven terms in the wave-function with seven coefficients. The last co-efficient c_8 comes as a result of coupling between $\phi_{val}^{(\frac{3}{2})}$ and spin one color octet sea. A table for $\sum n_{\mu\nu} c_{sea}$ can be constructed for strange as well as non-strange baryon which produces all the co-efficients. Each co-efficient will have a particular value of "nc" combination depending upon the Fock component.

$$a_8 = (n_{08} c_{sea})_{|gg\rangle} + (n_{08} c_{sea})_{|\bar{u}ug\rangle} + (n_{08} c_{sea})_{|\bar{d}dg\rangle} + \dots \quad (2.22)$$

$$b_1 = (n_{11} c_{sea})_{|gg\rangle} + (n_{11} c_{sea})_{|\bar{u}ug\rangle} + (n_{11} c_{sea})_{|\bar{d}dg\rangle} + \dots \quad (2.23)$$

$$b_{\bar{10}} = (n_{1\bar{1}0} c_{sea})_{|gg\rangle} + (n_{1\bar{1}0} c_{sea})_{|\bar{u}ug\rangle} + (n_{1\bar{1}0} c_{sea})_{|\bar{d}dg\rangle} + \dots \quad (2.24)$$

The vector contributions are found to be dominant over scalar and tensor part for low energy properties of non-strange baryons [10] [8]. To calculate the static properties of the baryonic systems, the above mentioned co-efficients should be calculated. For this, first task is the computation of c_{sea} and other is to find all $n'_{\mu\nu}$ s. Each baryon octet contains set of different possible Fock states $|gg\rangle, |\bar{u}ug\rangle, |ggg\rangle, |\bar{d}dg\rangle$ etc. The generalized expressions in terms of two parameters α and β are useful for studying the above said low energy properties of a hadronic system.

$$\alpha = \frac{1}{N^2} \frac{4}{9} (2a + 2b + 3d + \sqrt{2}e) \quad (2.25)$$

$$\beta = \frac{1}{N^2} \frac{1}{9} (2a - 4b - 6c - 6d + 4\sqrt{2}e) \quad (2.26)$$

where the following coefficients are defined as

$a = \frac{1}{2} \frac{(1-b_1^2)}{3}$, $b = \frac{1}{4}(a_8^2) - \frac{(b_8^2)}{3}$, $c = \frac{1}{2}(a_{10}^2 - \frac{(b_{10}^2)}{3})$, $d = \frac{1}{18}(5c_8^2 - 3d_8^2)$, $e = \frac{\sqrt{2}}{3}b_8c_8$. Thus, it can be stated as:

$$\alpha = \frac{2}{27} \left(\frac{6 + 3a_8^2 - 2b_1^2 - b_8^2 + 4b_8c_8 + 5c_8^2 - 3d_8^2}{1 + a_{10}^2 + a_8^2 + b_1^2 + b_8^2 + c_8^2 + d_8^2} \right) \quad (2.27)$$

$$\beta = \frac{1}{27} \left(\frac{3 - 9a_{10}^2 - 3a_8^2 - b_1^2 + b_8^2 + 8b_8c_8 - 5c_8^2 + 3d_8^2 + 3b_{10}^2}{1 + a_{10}^2 + a_8^2 + b_1^2 + b_8^2 + c_8^2 + d_8^2} \right) \quad (2.28)$$

The importance of these two parameters lies in the fact that the parameters are directly related to number of spin up ($n(q \uparrow)$) and spin-down ($n(q \downarrow)$) baryons. If $\Delta q = n(q \uparrow) + n(\bar{q} \uparrow) - n(q \downarrow) - n(\bar{q} \downarrow)$ where ($q=u,d,s$) then the quark spin polarizabilities Δu and Δd can be directly related to the parameters α and β . It is estimated from the above wave-function that $\Delta u = 3\alpha$ and $\Delta d = -3\beta$. The other important low energy properties are directly related to the polarized quark spin distributions.

Thus, it becomes important to define all the properties in terms of α and β . To find the suitable expression in terms of α and β , suitable operator is defined depending upon flavor and spin and assuming the wave-function as color symmetric then

$$\begin{aligned} \langle \phi_{\frac{1}{2}}^{\uparrow} | \hat{o} | \phi_{\frac{1}{2}}^{\uparrow} \rangle &= \frac{1}{N^2} \langle \phi_{\frac{1}{2}}^{\uparrow} | \hat{o} | \phi_{\frac{1}{2}}^{\uparrow} \rangle + \sum_{i=8,10} a_i^2 \langle \phi_{\frac{1}{2}}^{\uparrow} | \hat{o} | \phi_{\frac{1}{2}}^{\uparrow} \rangle + \sum_{i=1,8,10} b_i^2 \langle \phi_{\frac{1}{2}}^{\uparrow} | \hat{o} | \phi_{\frac{1}{2}}^{\uparrow} \rangle \\ &+ 2b_8 c_8 \langle \phi_{\frac{1}{2}}^{\uparrow} | \hat{o} | \phi_{\frac{1}{2}}^{\uparrow} \rangle + c_8^2 \langle \phi_{\frac{1}{2}}^{\uparrow} | \hat{o} | \phi_{\frac{1}{2}}^{\uparrow} \rangle + d_8^2 \langle \phi_{\frac{1}{2}}^{\uparrow} | \hat{o} | \phi_{\frac{1}{2}}^{\uparrow} \rangle \end{aligned}$$

Any operator $\hat{o} = \sum_i \hat{o}_f^i \hat{\sigma}_z^i$ where \hat{o}_f^i depends upon on the flavor of i^{th} quark and $\hat{\sigma}_z^i$ is the spin projection operator of i^{th} quark where $\langle \hat{o}_f^i \rangle^{\lambda\lambda} = \langle \phi^\lambda | \hat{o}_f^i | \phi^\lambda \rangle$ and $\langle \hat{\sigma}_z^i \rangle^{\rho\uparrow\rho\uparrow} = \langle \chi^{\rho\uparrow} | \hat{\sigma}_z^i | \chi^{\rho\uparrow} \rangle$, $\langle \hat{o}_f^i \rangle^{\lambda\rho} = \langle \phi^\lambda | \hat{o}_f^i | \phi^\rho \rangle$ where λ denotes the symmetric wave-function and ρ denotes the antisymmetry of the wave-function. The wave-function mentioned above are rewritten in terms of parameters a , b , c , d and e .

$$\begin{aligned} \langle \phi_{\frac{1}{2}}^{\uparrow} | \hat{o} | \phi_{\frac{1}{2}}^{\uparrow} \rangle &= \frac{1}{N^2} [a \sum_i [\langle \hat{o}_f^i \rangle^{\lambda\lambda} \langle \hat{\sigma}_z^i \rangle^{\lambda\uparrow\lambda\uparrow} + \langle \hat{o}_f^i \rangle^{\rho\rho} \langle \hat{\sigma}_z^i \rangle^{\rho\uparrow\rho\uparrow} + 2 \langle \hat{o}_f^i \rangle^{\lambda\rho} \langle \hat{\sigma}_z^i \rangle^{\lambda\uparrow\rho\uparrow}] \\ + b \sum_i [\langle \hat{o}_f^i \rangle^{\lambda\lambda} + \langle \hat{o}_f^i \rangle^{\rho\rho} \langle \hat{\sigma}_z^i \rangle^{\lambda\uparrow\lambda\uparrow} + \langle \hat{\sigma}_z^i \rangle^{\rho\uparrow\rho\uparrow}] + c \sum_i [\langle \hat{o}_f^i \rangle^{\lambda\lambda} \langle \hat{\sigma}_z^i \rangle^{\lambda\uparrow\lambda\uparrow}] + d \sum_i [\langle \hat{o}_f^i \rangle^{\lambda\lambda} + \sum_i \langle \hat{o}_f^i \rangle^{\rho\rho}] \\ + e \sum_i [(\langle \hat{o}_f^i \rangle^{\rho\rho} - \langle \hat{o}_f^i \rangle^{\lambda\lambda}) \langle \hat{\sigma}_z^i \rangle^{\lambda\uparrow\frac{3}{2}\uparrow} + 2 \sum_i \langle \hat{o}_f^i \rangle^{\lambda\rho} \langle \hat{\sigma}_z^i \rangle^{\rho\uparrow\frac{3}{2}\uparrow}] \end{aligned}$$

Suitable expressions are obtained from the eigen values coming from the above defined operators. For instance, substituting the spin operator in spin $\frac{1}{2}$ baryon wave-function when it operates on symmetric part of wave-function gives $\frac{2}{3}$ and operating on antisymmetric wave-function gives 0.

$$\langle \chi^{\lambda\uparrow} | \sigma_z^i | \chi^{\lambda\uparrow} \rangle = \langle \frac{1}{\sqrt{6}}(\uparrow\downarrow + \downarrow\uparrow) \uparrow - 2 \uparrow\uparrow\downarrow | \sigma_z^{(1)} | \frac{1}{\sqrt{6}}(\uparrow\downarrow + \downarrow\uparrow) \uparrow - 2 \uparrow\uparrow\downarrow | \sigma_z^{(1)} \rangle = \frac{2}{3}$$

$$\langle \chi^{\rho\uparrow} | \sigma_z^i | \chi^{\rho\uparrow} \rangle = \langle \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \uparrow | \sigma_z^{(1)} | \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \uparrow \rangle = 0$$

For static properties like magnetic moments, the suitable operator is defined as $O_f^i = \frac{e^i}{2m_i}$, "i" refers to (u,d,s). In this way, by defining a suitable operator, the magnetic moments and other static properties at low energy for all lowest lying baryonic states with spin $\frac{1}{2}$ are expressed in the form of α and β .

2.2 Hadronic Properties at Low Energy

Low energy properties of hadrons are an important tool to understand the structure of hadrons. The properties at low energy can be magnetic moments, masses, distribution of spin among quarks, axial vector form factors and coupling constant ratios etc. One of the important question for particle physicists since last 20 years is: How is the proton's spin built from constituent quarks and gluons? A historical discovery in this context regarding the measurement of spin structure function g_1 was that of European Muon Collaboration [EMC] [11] and flavor singlet axial currents through Deep Inelastic Scattering Experiments. The results predicted that the quark's intrinsic spin contributes very little to the total spin of proton. The results were found contrary to the non-relativistic quark model predictions which assumes that whole of the proton spin comes from the constituent quarks. This led the phenomenologist and experimentalists to think beyond the non-relativistic quark model. The story of proton's spin dates from the discovery by Dennison(1927) that proton is a fermion of spin $\frac{1}{2}$. Six years later Estermann and Stern in 1933 measured the proton's anomalous magnetic moment, $\kappa_p = 1.79$ Bohr Magneton recalling that proton is not point like and has internal structure. The first polarized deep inelas-

tic scattering experiments were performed by SLAC-Yale Collaboration by Baum et al. [12] in the early 1980's using an electron beam and proton target and measured g_1 down to $x=0.1$. The EMC experiment at CERN used a muon beam and extended the inclusive measurement to $x=0.01$. These findings triggered a whole program of DIS with electron beams at SLAC [13], muon beams at CERN[SMC] [14] and the electron ring at HERA collider on an internal target(HERMES Collaboration) [Ackerstaff et al.(1997) [15]; Airapetian et al. (1998)] [7] at DESY. A direct measurement of gluon polarization Δg in the region $x \sim 0.1$ has been measured by COMPASS collaborations [17]. One of the possible solution to proton spin puzzle can be from anomalous gluon effect arising from the axial anomaly. Thus, in the low-energy processes, the proton behaves like a system of three massive constituent-quark quasi-particles interacting self-consistently with a cloud of virtual pions and condensates generated through spontaneous breaking of the chiral symmetry between left and right handed quarks. When probed at high resolution the proton looks like three valence "current" quarks plus a sea of quark-antiquark pairs and gluons. The current quarks probed in high-resolution processes are almost massless whereas the constituent quarks are the quasi-particles of low-energy QCD such that each have a mass about one third of the mass of the proton. Nucleonic spin can be thought of distributed among the gluons, valence and sea quarks and also their angular momenta. The same spin crisis can be imagined for other particles too. Another interesting point is to note that the quark contribution of spin among quarks can be directly related to other low energy properties of hadrons. Thus, it gives us motivation to look into details of all the low energy observables using either effective theories or phenomenological models.

2.2.1 Spin Structure of Hadron

The $SU(6)$ wave-function for hadron can be helpful in counting the average number of particle with spin parallel and anti-parallel to the spin of the baryon. For instance, for a protonic system a non-relativistic model predicts that the number $n(u \uparrow) = \frac{5}{3}$, $n(u \downarrow) = \frac{1}{3}$, $n(d \uparrow) = \frac{1}{3}$ and $n(d \downarrow) = \frac{2}{3}$. The differences are: $\Delta u = \frac{4}{3}$, $\Delta d = -\frac{1}{3}$ and $\Delta s = 0$. The sum of these differences is $\Delta\Sigma = \Delta u + \Delta d + \Delta s = 1$. In 1988, very firstly EMC collaboration [11, 18] measured the total spin carried by the quarks and it was claimed that $\Delta\Sigma \simeq 0$ and $\Delta s \neq 0$. The experimental results [19] may be summarized as: $\Delta\Sigma = 0.31 \pm 0.07$, $\Delta u = 0.83 \pm 0.03$, $\Delta d = -0.43 \pm 0.03$, $\Delta s = -0.10 \pm 0.03$. The E143 collaboration [13] also found the total contribution from all quarks to be $\Delta\Sigma = 0.30 \pm 0.06$ and the contributions from the strange sea in a nucleonic system has been found to be $\Delta s = -0.09 \pm 0.02$. The nucleon spin can be decomposed into contributions from different components and can be written as:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_g + L_q \quad (2.29)$$

where $\Delta\Sigma$ is contribution from quark and anti-quarks, ΔG is contribution from gluons. L_q and L_g are the angular momenta of quarks and gluons respectively. In the modified \overline{MS} , ΔG does not contribute to first moment in quark parton model. There are many other schemes such as Adler and Bardeen [20], Carlitz, Collins and Mueller schemes [21] and Jet scheme [22]. In these approaches, $\Delta\Sigma$ is independent of Q^2 and this leads to the suggestion that polarized quark structure function $\Delta\Sigma$ should be modified so as to include anomalous gluonic contribution. Lots of experiments are taking data to measure gluonic contribution to the spin of nucleon. A first

measurement of the gluon polarization was reported by the E581/704 experiment at Fermilab [23] measuring p_0 production with high transverse momentum p_T . Most recently, COMPASS [46] has announced a new determination via the asymmetry in high p_T hadron-pair production, which yields $\frac{\Delta G}{G} = 0.024 \pm 0.089(\text{stat}) \pm 0.057(\text{syst})$ at an average $\langle x \rangle = 0.095$. However, all measurements of the gluon polarization have still large uncertainties both systematic and statistical. Therefore, more additional information about ΔG with greater accuracy is necessary for conclusive statements. Phenomenologically, the quark spin polarizations, Δq^B , where $q=u,d,s$, is defined as:

$$\Delta q^B = \langle B \uparrow | \sigma_z^i | B \uparrow \rangle = n_{q\uparrow}(B) - n_{q\downarrow}(B) \quad (2.30)$$

where σ_z^i is the third Pauli spin matrix of quark q . The quark spin polarizations are: $\Delta q^B = \Delta q^B + \Delta \bar{q}^B$. Thus, quark spin polarizations can be defined as sum of quark and anti-quark polarization densities. To investigate the spin structure of proton experimentally, longitudinal polarized leptonic beams are scattered off targets by longitudinal or transversely polarized nucleons. Polarized deep inelastic scattering has been the key tool for probing the internal structure of nucleon. The exchange of a virtual vector bosons between the leptons and one of the partons inside the target nucleon exchanges a high four-momentum that breaks up the nucleon and forms a final hadronic state. In our model, the polarized quark and anti-quark densities have been computed for spin $\frac{1}{2}$ baryons using $SU(3)$ flavor symmetry. Later on the impact of strange mass corrections have also been checked on the polarized spin densities using the same model.

2.2.2 Total Distribution of Spin

The total contribution of spin among quarks for a baryonic system is given by: $\Delta\Sigma = \Delta u + \Delta d + \Delta s$. The total contribution of spin of quarks among the nucleon can also be derived from the integration of g_1 over x , which is called first moment of g_1 over x , which is called first moment. The first moment of g_1 is given by:

$$\Gamma_1 = \int_0^1 g_1(x) dx = \frac{1}{2} \int_0^1 \sum_f e_f^2 \Delta q_f(x) dx \quad (2.31)$$

Under the $SU(3)_f$ symmetry, flavor symmetry of the axial current in the spin $\frac{1}{2}$ baryon octet, the matrix elements can be defined as:

$$\Gamma_1^{p,n} = \pm \frac{1}{12}(\Delta u - \Delta d) + \frac{1}{36}(\Delta u + \Delta d - 2\Delta s) + \frac{1}{9}(\Delta u + \Delta d + \Delta s) \quad (2.32)$$

The matrix elements a_0 , a_3 and a_8 are linked with polarized quark and anti-quark distributions. The matrix elements a_0 , a_3 and a_8 are related to the polarized distributions.

$$a_0 = \Delta u + \Delta d + \Delta s + \Delta \bar{u} + \Delta \bar{d} + \Delta \bar{s} \quad (2.33)$$

$$a_3 = \Delta u - \Delta d + \Delta \bar{u} - \Delta \bar{d} \quad (2.34)$$

$$a_8 = \Delta u + \Delta d + \Delta \bar{u} + \Delta \bar{d} - 2(\Delta s + \Delta \bar{s}) \quad (2.35)$$

The matrix elements a_3 and a_8 are related to weak decay constants. The matrix elements a_3 and a_8 are calculated from neutron decay constant and hyperon decay constants. However, QCD corrections induce a Q^2 dependence which breaks Bjorken scaling. This implies that $\Delta\Sigma$ should be corrected due to an anomalous gluon con-

tribution.

$$\Delta\Sigma^1 = \Delta\Sigma - n_f \frac{\alpha_s}{2\pi} \Delta G(Q^2) \quad (2.36)$$

Inclusive Deep Inelastic Experiments on polarized proton targets with polarized electrons at SLAC [25] have already measured Γ_1 . Very first result for Γ_1 was reported at EMC [11] and later on confirmed by some other experiments. The quark contribution $\Delta\Sigma$ to the nucleonic spin was found to be $\Delta\Sigma^{DIS} = \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s} = 0.29 \pm 0.06$ at $\langle Q^2 \rangle = 5 \frac{\text{GeV}^2}{c^2}$ [23]. To calculate the first moment of Γ_1 and Δq for all member of spin structure of all spin $\frac{1}{2}$ baryon octet, matrix elements need to be calculated. The $SU(3)$ flavor symmetry is used in terms of their current operators and baryon state. The current operators of the axial vector can be written as:

$$j_\mu^i = \bar{\psi} \gamma_\mu \gamma_5 \left(\frac{\lambda_i}{2} \right) \psi, i = 0, 1, 2 \dots 8. \quad (2.37)$$

ψ is quark field triplet and the conjugate quark field $\bar{\psi}$ is constructed to find the wave-function of baryon octets.

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

$$\bar{\psi} = (ud - du, ds - sd, us - su)$$

λ_i are the hermitian traceless Gellmann matrices and act as the generators of the elements of the group $SU(3)$. The hadronic matrix elements from a_0 to a_8 for a specific baryon with spin S , mass m_B and momentum P are $\langle P, S | J_\mu^0 | P, S \rangle = 2M_B S_\mu a_0, \langle P, S | J_\mu^i | P, S \rangle = 2M_B S_\mu a_i, i = 1, 2 \dots 8$ with the flavor-singlet axial-

vector currents. The axial singlet a_0 corresponds to the expectation value of the z -component of all quark spins in naive QPM. The baryon matrix B sums up each 3×3 matrix of baryon octet fields.

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & P \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & N \\ \Sigma^- & \Sigma^0 & \frac{-2}{\sqrt{6}}\Lambda \end{pmatrix} \quad \text{The baryon octets are constructed}$$

with each matrix element of the octet axial current for two baryon states B_j and B_k . Three matrices can be combined to form an $SU(3)$ flavor singlet involving commutator and anti-commutator of two of the matrices and defining F and D such that $\langle B_j; P, S | J_\mu^i | B_k; P, S \rangle = D \text{tr}(J_\mu^i \{B_k, \overline{B_j}\}) + F \text{tr}(J_\mu^i [B_k, \overline{B_j}])$. All the spinors, momentum dependence and the rest of the matrix elements are contained in F and D . Thus, this enables us to relate the invariants in one matrix elements to those in another. From the definition of B_j and J_μ^i , one can see the similarity to the λ -matrices because $B_i \sim \lambda_i$ and $J^i \sim \frac{\lambda_i}{2}$, which leads to:

$$\text{tr}(J_\mu^i \{B_k, \overline{B_j}\}) \sim \frac{1}{2} \text{tr}(\lambda_i, [\lambda_k, \lambda_j]) = -if_{ijk}, \quad \text{tr}(J_\mu^i [B_k, \overline{B_j}]) \sim \frac{1}{2} \text{tr}(\lambda_i, \{\lambda_k, \lambda_j\}) = d_{ijk} \quad (2.38)$$

where d_{ijk} and f_{ijk} are the usual $SU(3)$ group structure constants. The F coupling is proportional to structure constant f_{ijk} which is fully antisymmetric under interchange of any pair of indices, while D coupling is proportional to the symmetric invariant tensor d_{ijk} . It can be found out that:

$$\langle B_j; P, S | J_\mu^i | B_k; P, S \rangle = 2M_B S_\mu (D d_{ijk} - iF f_{ijk}) \quad (2.39)$$

F and D have been determined experimentally from the hyperon β -decay because of the involvement of the same currents where $F=0.463\pm 0.08$ and $D=0.804\pm 0.008$ [26]. The calculation of matrix elements can lead to $\langle | J_\mu^i | \rangle = M_B S_\mu (D d_{ijk} - i F f_{ijk})$, ($i = 1\dots 8$), to calculate the Γ_1^B , one needs to create the axial current representation. From the above discussion, it is concluded that the polarized quark distributions, total spin distribution and matrix elements F and D are related. The calculation of matrix elements result in relevant expressions for the first moment and polarized quark densities respectively. The table 2.2.2 presents the polarized quark densities and first moment of g_1 in terms of F, D and $\Delta\Sigma$ under $SU(3)_f$ symmetry.

Table 2.1

Baryon	Γ_1	Δu	Δd	Δs
p	$\frac{1}{\Delta\Sigma}(2\Delta\Sigma + D + 3F)$	$\frac{1}{3}(\Delta\Sigma + D + 3F)$	$\frac{1}{3}(\Delta\Sigma - 2D)$	$\frac{1}{3}(\Delta\Sigma + D - 3F)$
n	$\frac{1}{9}(\Delta\Sigma - D)$	$\frac{1}{3}(\Delta\Sigma - 2D)$	$\frac{1}{3}(\Delta\Sigma + D + 3F)$	$\frac{1}{3}(\Delta\Sigma + D - 3F)$
Λ	$\frac{1}{18}(2\Delta\Sigma - D)$	$\frac{1}{3}(\Delta\Sigma - D)$	$\frac{1}{3}(\Delta\Sigma - D)$	$\frac{1}{3}(\Delta\Sigma + 2D)$
Σ^+	$\frac{1}{18}(2\Delta\Sigma + D + 3F)$	$\frac{1}{3}(\Delta\Sigma + D + 3F)$	$\frac{1}{3}(\Delta\Sigma + D - 3F)$	$\frac{1}{3}(\Delta\Sigma - 2D)$
Σ^0	$\frac{1}{18}(2\Delta\Sigma + D)$	$\frac{1}{3}(\Delta\Sigma + D)$	$\frac{1}{3}(\Delta\Sigma + 3D)$	$\frac{1}{3}(\Delta\Sigma - 2D)$
Σ^-	$\frac{1}{18}(2\Delta\Sigma + D - 3F)$	$\frac{1}{18}(2\Delta\Sigma + D - 3F)$	$\frac{1}{3}(\Delta\Sigma + D + 3F)$	$\frac{1}{3}(\Delta\Sigma - 2D)$
Ξ^0	$\frac{1}{9}(2\Delta\Sigma - D)$	$\frac{1}{3}(\Delta\Sigma - 2D)$	$\frac{1}{3}(\Delta\Sigma + D - 3F)$	$\frac{1}{3}(\Delta\Sigma + D + 3F)$
Ξ^-	$\frac{1}{18}(2\Delta\Sigma + D - 3F)$	$\frac{1}{3}(\Delta\Sigma - 2D)$	$\frac{1}{3}(2\Delta\Sigma - 2D)$	$\frac{1}{3}(\Delta\Sigma + D + 3F)$

Table 2.1 Polarized Quark Densities and their First moments in $SU(3)$ symmetry

In our approach, total spin distributions is determined using the operator $\frac{1}{2}e_i^2\sigma_z^i$ where e_i is the charge of quark and σ_z^i is the spin projection operator. This produces the values in terms of α and β for all the baryons. F and D are the two universal parameters which have their own significance. These parameters are proportional to structure constant f_{ijk} and antisymmetric invariant tensor d_{ijk} . We find $F = \frac{3(\alpha)}{2}$ and $D = \frac{3(\alpha+2\beta)}{2}$ in our model.

2.2.3 Axial Vector form factors for Baryon

Another fundamental property of a nucleonic system is its axial vector form factors which determines the matrix element for beta-decay. In quark model g_A is expressed in terms of the axial current contributions from each quark. The axial current is $A_\mu^a = \bar{\psi}\gamma_\mu\gamma_5\frac{\lambda_a}{2}\psi$ with $a = 3, 8$ and along with this, flavor singlet current ($A_\mu^0 = \bar{\psi}\gamma_\mu\gamma_5\psi$). For the semileptonic decays of $B \rightarrow B'l^-\bar{\nu}_l$, the matrix element of weak V-A current for lowest order can be written as:

$$\langle B' | J_{weak}^\mu | B \rangle = \bar{\psi}'(f_1\gamma_\mu - g_1\gamma_\mu\gamma_5)\psi \quad (2.40)$$

where $\psi(\psi')$ and $B(B')$ are the Dirac spinor and baryon state(internal and external), f_1 and g_1 are the vector and axial form factors of $SU(6)$ quark-wave-function for baryon. The axial-vector form factor $g_A = \frac{g_1}{f_1}$ and all axial vector form factors can

be explained in the form of F and D . They can be expressed as for $\Delta s = 0$ decays:

$$\begin{aligned}
 g_A(n \rightarrow p) &= F + D = \Delta u - \Delta d \\
 g_A(\Sigma^- \rightarrow \Sigma^0) &= F = \frac{1}{2}(\Delta u - \Delta s) \\
 g_A(\Sigma^+ \rightarrow \Lambda) &= \frac{2}{3}D = \frac{1}{\sqrt{6}}(\Delta u - 2\Delta d + \Delta s) \\
 g_A(\Xi^- \rightarrow \Xi^0) &= F - D = \Delta d - \Delta s
 \end{aligned}$$

For $\Delta s = 1$ decays:

$$\begin{aligned}
 g_A(\Sigma^- \rightarrow n) &= F - D = \Delta d - \Delta s \\
 g_A(\Xi^- \rightarrow \Lambda) &= F - \frac{D}{3} = \frac{1}{3}(\Delta u + \Delta d - 2\Delta s) \\
 g_A(\Xi^- \rightarrow \Sigma^0) &= F + D = \Delta u - \Delta d \\
 g_A(\Lambda \rightarrow p) &= F + \frac{D}{3} = \frac{1}{3}(2\Delta u - \Delta d - \Delta s) \\
 g_A(\Xi^0 \rightarrow \Sigma^+) &= F + D = \Delta u - \Delta d
 \end{aligned}$$

In all the expressions above F and D has already been defined in terms of α and β . The quark spin polarization densities can be directly related to matrix elements F and D by the relation as: $F + D = \Delta u - \Delta d$ and $3F - D = \Delta u + \Delta d - 2\Delta s$. Thus, the beauty of statistical model lies in the fact that once the information about the two key parameters α and β is in hand, it becomes very easy to compute all the static properties easily. Moreover, by looking at the individual probabilities, the contributions from each Fock state can be analyzed and a more viable picture of

hadronic structures can be achieved.

2.2.4 $\bar{u} - \bar{d}$ Asymmetry

The $\bar{u} - \bar{d}$ asymmetry is called as quark-anti-quark flavor asymmetry and it is described as difference and ratio between anti-quarks $\bar{u} - \bar{d}$ and $\frac{\bar{u}}{\bar{d}}$. The idea of flavor asymmetry comes from the presence of sea quarks inside the baryons. The excess of \bar{d} over \bar{u} is due to gluon splitting into $\bar{u}u$ and $\bar{d}d$ pairs. The very first experiment for the measurement of experimental data was at SLAC [27] in 1975 and a significant deviation was observed from that of expected vanishing value. The next experiment was E288 collaboration with the Drell-Yan experiment at Fermilab in 1981 [28] to measure $\bar{u} - \bar{d}$ asymmetry. The more accurate x -dependent prediction of $\frac{\bar{u}}{\bar{d}}$ were obtained by FermiLab/E866 collaboration [29–31] using 800 GeV protons interacting with liquid hydrogen and deuterium targets. The HERMES collaboration at DESY obtained results through semi-inclusive DIS experiments and results were found to be consistent with that of NMC, NuSea collaborations. The basic mechanism involves generation of sea by splittings into quark-antiquark pairs. Field and Feymann [32] suggested that the presence of extra $u\bar{u}$ in the sea can lead to suppression of extra $g \rightarrow u\bar{u}$ via Pauli blocking. There are several phenomenological models to find flavor symmetry like meson cloud and chiral quark model. One of the recent approach suggested by Zhang et al. [34] to calculate flavor asymmetry in sea using principle of detailed balance. This principle was successful in explaining the $\bar{u} - \bar{d}$ asymmetry and predicted the value of 0.118 [33] which matches with experimental value of 0.124 [31].

2.2.5 Baryon Magnetic Moments

The magnetic moment can be defined as expectation value of z-component of magnetic moment operator with maximal spin projection along z-axis.

$$\mu(B) = \langle B; J; J_z = J | \mu_z | B; J; J_z = J \rangle \quad (2.41)$$

where μ_z is z-component of magnetic moment operator. In the absence of orbital motion of quarks, for a spin $\frac{1}{2}$ charged particle, the magnetic moment operator can be defined as: $\mu_q = \frac{e_q \sigma_z^i}{2m_q}$. The parameters e_q, m_q are the electric charge and mass of the quarks respectively. For example, the magnetic moment operator over the proton (\uparrow) state is $\mu_p = \langle p(\uparrow) | \sum_{i=1}^3 \mu_i (\sigma_3)_i | p(\uparrow) \rangle$ and the wave-function for a proton up state in $SU(6)$ model is: $| p \uparrow \rangle = \frac{1}{\sqrt{2}}(p_{ms} \chi_{ms} + p_{ma} \chi_{ma})$ where p_{ms} and p_{ma} are mixed symmetric and mixed anti-symmetric wave-function for proton.

$$p_{ms} \chi_{ms} = \frac{1}{6\sqrt{2}}(u \uparrow d \downarrow u \uparrow + d \downarrow u \uparrow u \uparrow - 2u \uparrow u \uparrow d \downarrow)$$

$$p_{ma} \chi_{ma} = \frac{1}{2\sqrt{2}}(u \uparrow d \downarrow u \uparrow - d \downarrow u \uparrow u \uparrow)$$

Thus, the magnetic moment of baryonic system can be parameterized as:

$$\mu_B = \sum_{i=u,d,s} \langle B | \frac{e_i \sigma_z^i}{2m} | B \rangle$$

Magnetic moment of a baryonic system can also be defined in terms of quark polarizations:

$$\mu(B) = \sum_{q=u,d,s} \Delta q^B \mu_q = \Delta u \mu_u + \Delta d \mu_d + \Delta s \mu_s \quad (2.42)$$

The magnetic moment of all baryons lead to verification of the famous Coleman-Glashow sum rule [47].

$$\mu(p) - \mu(n) + \mu(\Sigma^-) - \mu(\Sigma^+) + \mu(\Xi^0) - \mu(\Xi^-) = 0 \quad (2.43)$$

The expressions for magnetic moment is written in terms of α and β statistically. In all the expressions, μ_u, μ_d, μ_s are the magnetic moments of u, d, s quark respectively.

Magnetic Moments of Baryon with $j^p = \frac{1}{2}^+$

Baryon	Expression
p	$\mu_p = 3(\mu_u \alpha - \mu_d \beta)$
n	$\mu_n = 3(\mu_d \alpha - \mu_u \beta)$
Σ^+	$\mu_{\Sigma^+} = 3(\mu_u \alpha - \mu_s \beta)$
Σ^-	$\mu_{\Sigma^-} = 3(\mu_d \alpha - \mu_s \beta)$
Σ^0	$\mu_{\Sigma^0} = \frac{1}{2}(\mu_{\Sigma^+} + \mu_{\Sigma^-})$
Ξ^0	$\mu_{\Xi^0} = 3(\mu_s \alpha - \mu_u \beta)$
Ξ^-	$\mu_{\Xi^-} = 3(\mu_s \alpha - \mu_d \beta)$

Table 2.2 Magnetic Moment Expressions

In the forthcoming chapters, magnetic moments of all lowest lying baryonic strange as well as non-strange states have been calculated with in a statistical approach.

2.3 Statistical Model

Quarks obey the Fermi-Dirac distribution functions whereas gluons obey Bose-Einstein statistics. Different statistical approaches [37] are applied to study parton distribution functions. Statistical models provide intuitive appeal and physical simplicity that have made amazing success in describing the parton structure functions for nucleons. It can be speculated that wee partons being bound inside the nucleon, does possess some statistical properties. Bhalero et al. [38] introduced finite size corrections to the PDF's in an infinite momentum frame. The main advantage of using statistical models is that no additional input parameters are required, the quantities can be expressed in terms of the most common statistical quantities. Recently, a new statistical model has been used by Zhang et al. [33, 34] in which a nucleon is taken as an ensemble of quark gluon Fock states. In this model, principle of detailed balance [33] is applied to find the probability of all Fock states. Principle of detailed balance assumes that every Fock state should be balanced with nearby Fock state. The model provides a better explanation of flavor asymmetry of the sea and provides results matching with the experimental results. The quarks and gluons in the Fock states are the intrinsic partons of the baryon and they are multi-connected non-perturbatively to the gluons. Such partons are different from the extrinsic partons generated from QCD hard bremsstrahlung and gluon splitting as a part of lepton scattering interaction. Thus, the statistical model is expected to work at scale of intrinsic partons which is $Q^2 \simeq 1\text{GeV}^2$. This explains the feasibility of statistical model at low energy. Also, the basic property of ensemble assumption of the baryon is that the probability of finding the baryon in a Fock state should not change during the time. The same assumption for the ensemble can be assumed only at low energy.

Thus, a more general description of above model can be imagined where in addition to flavor, each Fock state has a definite spin and color quantum number and a particular symmetry property [9]. The resulting formalism is based purely on a statistical formalism to explore the light quark spin content of nucleons, magnetic moments and semi-leptonic decay constant of neutron etc. The most interesting point to note here is that all the properties are directly linked with the probabilities associated with each Fock state in definite spin-color-flavor space. In this model, all $n'_{\mu,\nu}$ s (defined in the previous wave-function) are calculated from multiplicities of each Fock state in spin and color space. These multiplicities are expressed in the form of $\rho_{j_1 j_2}$ where the core quark part has spin j_1 and the sea quark-gluons carry spin j_2 so that resultant spin is $\frac{1}{2}$. The method is based on the counting of multiplicities in spin and color space for all possible sets of Fock states in valence as well as sea and defining these multiplicities in the form of suitable ratios. Such a comparison of multiplicities in Fock states serves two purposes. The first one is to find a common multiplier ("c") for each particular combination of valence and sea to realize a baryon. The second is to calculate probability of each substate with specified spin and color quantum numbers. For instance, a simple two gluon sea can have spin (0, 1, 2) and color (1, 8, $\bar{10}$) and similar is the case for higher number of gluons. A general way to present comparison of such probabilities can be written as:

$$\frac{\rho_{j_1 j_2}}{\rho_{j'_1 j'_2}} = \frac{x(r_1)y(r_2)z(r_3)}{x'(r'_1)y'(r'_2)z'(r'_3)} \quad (2.44)$$

Coefficients are nothing but the sum of product of multiplicities with common factor for each feasible combination of the sea. A detailed comparison of the all such

probabilities on a purely statistical basis is given. The model given below is named as "Model C". Different Fock states involve computation of relative multiplicities.

2.3.1 Model C

Consider the decomposition of the state $|gg\rangle$ sea then on the basis of decomposition in spin and color space:

$$\begin{aligned} \frac{\rho_{\frac{1}{2}0_s}}{\rho_{\frac{1}{2}1_a}} &= \frac{(\frac{4}{8}) \cdot (\frac{1}{9}) \cdot 1}{(\frac{4}{8}) (\frac{3}{9}) (\frac{2}{6})} = 1 \\ \frac{\rho_{\frac{1}{2}0_s}}{\rho_{\frac{3}{2}2_s}} &= 2 \\ \frac{\rho_{\frac{3}{2}1_a}}{\rho_{\frac{3}{2}2_s}} &= 1 \\ \frac{\rho_{\frac{1}{2}1_a}}{\rho_{\frac{3}{2}1_a}} &= 2 \end{aligned}$$

Similarly, we can compare the probabilities for the q^3 core and gg to be in different color substates which finally give a color singlet hadron.

$$\begin{aligned} \frac{\rho_{11_s}}{\rho_{88_s}} &= \frac{1}{2} = \frac{\rho_{11_s}}{\rho_{88_a}} \\ \frac{\rho_{11_s}}{\rho_{10\bar{10}}} &= \frac{(\frac{1}{27}) \cdot (\frac{1}{64}) \cdot 1}{(\frac{10}{27}) (\frac{10}{64}) (\frac{1}{100})} = 1 \end{aligned}$$

The product of probabilities in spin and color space can be written in terms of common parameter "c" as

$$\begin{aligned} \rho_{\frac{1}{2}0_s} [\rho_{11_s}, \rho_{8,8_s}] &= 2c(1, 2) \\ \rho_{\frac{1}{2}1_a} [\rho_{88_a}, \rho_{10, \bar{10}}] &= 2c(1, 2) \\ \rho_{\frac{3}{2}1_a} [\rho_{88_a}] &= 2c, \rho_{\frac{3}{2}2_s} [\rho_{8,8_s}] = 2c \end{aligned}$$

There is no contribution from two gluon states to $H_0G_{\bar{1}0}$. Similar decomposition holds good for all other possibilities of sea. Proceeding in a similar way for decomposition of $|u\bar{u}d\bar{d}\rangle$:

$$\rho_{\frac{1}{2}0_s}[\rho_{11_s}, \rho_{88_s}]; \rho_{\frac{1}{2}1_a}[\rho_{88_a}, \rho_{10\bar{1}0}]; \rho_{\frac{3}{2}2_s}[\rho_{88_s}] = 2c(1, 2; 2, 1; 2; 2)$$

If we consider the decomposition of a state $|u, d, s, 1, 0, 1, 0\rangle$ or $|u\bar{u}, s\bar{s}\rangle$: Different cases of probabilities in spin and color space include:

$$\begin{aligned} \frac{\rho_{\frac{1}{2},0}}{\rho_{\frac{1}{2},1}} &= 1, \frac{\rho_{\frac{1}{2},0}}{\rho_{\frac{3}{2},1}} = 2, \\ \frac{\rho_{\frac{1}{2},0}}{\rho_{\frac{3}{2},2}} &= 2, \frac{\rho_{1,1}}{\rho_{8,8}} = \frac{1}{4}, \frac{\rho_{1,1}}{\rho_{10,\bar{1}0}} = 1 \end{aligned}$$

These are the probabilities of finding the three core quarks in spin $\frac{1}{2}$ and color singlet states with sea in a specific combination of spin and color quantum numbers. Here $q\bar{q}$ carries the quantum number of a gluon due to sub processes $g \Leftrightarrow q\bar{q}$ therefore no symmetry conditions can be applied. For the computation of a common factor 'c', relative probabilities are represented as given below:

$$\rho_{\frac{1}{2},0}[\rho_{1,1}, \rho_{8,8}, \rho_{10,\bar{1}0}]; \rho_{\frac{1}{2},1}[\rho_{1,1}, \rho_{8,8}, \rho_{10,\bar{1}0}]; \rho_{\frac{3}{2},1}[\rho_{8,8}]; \rho_{\frac{3}{2},2}[\rho_{8,8}] = 2c[1, 2, 1; 1, 2, 1; 2; 2]$$

For decomposition of the $|gq\bar{q}$ and $|\bar{u}u\bar{d}d\rangle$, symmetry considerations is not needed. We have considered that $q\bar{q}$ carries the quantum numbers of a gluon due to the subprocesses $g \Leftrightarrow q\bar{q}$. The products of densities in spin and color space comes out to be:

$$\rho_{\frac{1}{2},0}[\rho_{1,1}, \rho_{8,8}, \rho_{10,\bar{1}0}]; \rho_{\frac{1}{2},1}[\rho_{1,1}, \rho_{8,8}, \rho_{10,\bar{1}0}]; \rho_{\frac{3}{2},1}[\rho_{8,8}]; \rho_{\frac{3}{2},2}[\rho_{8,8}] = 2c[1, 4, 1; 1, 4, 1; 2; 2]$$

For a three gluon sea: The following are the possibilities.

$$\text{Spin} : (1 \otimes 1) \otimes 1 = (0 \oplus 1 \oplus 2) \otimes 1$$

$$\text{and } 1 \otimes 2 = 1 \oplus 2 \oplus 3$$

$$\text{Color} : (8 \otimes 8 \otimes) = (1 \oplus 8 \oplus 8 + 10 \oplus \bar{10} \oplus 27) \otimes 8$$

$$10 \otimes 8 = 8 \oplus 10 \oplus 27 \oplus 35$$

$$27 \otimes 8 = 8 \oplus 10 \oplus \bar{10} \oplus 2(27) \oplus 2(35) \oplus 64$$

The relative probability densities in spin and color space becomes:

$$\begin{aligned} \frac{\rho_{\frac{1}{2}0}}{\rho_{\frac{1}{2}1_a}} &= \frac{(\frac{1}{27}) \cdot 1}{(\frac{3}{27}) \cdot (\frac{2}{6})} = 1, \\ \frac{\rho_{\frac{1}{2}0_a}}{\rho_{\frac{1}{2}1_a}} &= \frac{(\frac{1}{27}) \cdot 1}{(\frac{6}{27}) \cdot (\frac{4}{12})} = \frac{1}{2}, \\ \frac{\rho_{\frac{1}{2}1_a}}{\rho_{\frac{3}{2}1_s}} &= \frac{(\frac{3}{27}) \cdot (\frac{1}{3})}{(\frac{3}{27}) \cdot (\frac{1}{6})} = 2 = \frac{\rho_{\frac{1}{2}1_a}}{\rho_{\frac{3}{2}1_a}}, \\ \frac{\rho_{\frac{3}{2}1_a}}{\rho_{\frac{3}{2}2_a}} &= \frac{(\frac{3}{27}) \cdot \frac{2}{12}}{(\frac{5}{27}) \cdot (\frac{2}{20})} = 1, \\ \frac{\rho_{\frac{3}{2}1_a}}{\rho_{\frac{3}{2}2_s}} &= \frac{(\frac{6}{27}) \cdot (\frac{4}{24})}{(\frac{5}{27}) \cdot (\frac{2}{20})} = 2 \\ \frac{\rho_{11_s}}{\rho_{88_s}} &= \frac{(\frac{1}{27}) \cdot (\frac{1}{512}) \cdot (1)}{(\frac{16}{27}) \cdot (\frac{32}{512}) \cdot (\frac{1}{64})} = \frac{1}{8}, \\ \frac{\rho_{11_s}}{\rho_{10\bar{10}_a}} &= \frac{(\frac{1}{27}) \cdot (\frac{1}{512}) \cdot (1)}{(\frac{16}{27}) \cdot (\frac{32}{512}) \cdot (\frac{1}{64})} = \frac{1}{8} \end{aligned}$$

The combined probabilities in spin and color space for $u\bar{u}gg$ etc. are written as:

$$\rho_{\frac{1}{2}0_a} [\rho_{11_a}, \rho_{88_a}, \rho_{10, \bar{10}_a}]; \rho_{\frac{1}{2}1_a} [\rho_{11_a}, \rho_{88_a}, \rho_{10, \bar{10}_a}]; \rho_{\frac{1}{2}1_s} [\rho_{11_s}, \rho_{88_s}, \rho_{10, \bar{10}_s}]; \rho_{\frac{3}{2}1_a} [\rho_{88_a}];$$

$\rho_{\frac{3}{2}1_s}[\rho_{88_s}]; \rho_{\frac{3}{2}2_a}[\rho_{88_a}] = 2c(1, 8, 2; 1, 8, 2; 2, 16, 4; 4; 8; 4)$ Next, considering the case of $|u\bar{u}d\bar{d}g\rangle$ sea: There is no symmetry requirement. The ratios of probability densities in spin and color space are:

$$\rho_{\frac{1}{2}0}[\rho_{11}, \rho_{88}, \rho_{10, \bar{10}}]; \rho_{\frac{1}{2}1}[\rho_{11}, \rho_{88}, \rho_{10, \bar{10}}]; \rho_{\frac{3}{2}1}[\rho_{88}]; \rho_{\frac{3}{2}2}[\rho_{88}] = 2c(1, 8, 2; 3, 24, 6; 12; 8)$$

$|ggg\rangle$ sea: The wave-function for this sea should be completely symmetric under the exchange of any two gluons. Among the product color functions, there is one color singlet state and one color octet state which are completely antisymmetric, there is no color singlet state and one color octet state which are completely symmetric. This gives us the product of probabilities in spin and color space as:

$$\rho_{\frac{1}{2}0_a}[\rho_{11_a}, \rho_{88_a}]; \rho_{\frac{1}{2}1_s}[\rho_{11_s}, \rho_{88_s}]; \rho_{\frac{3}{2}1_s}[\rho_{88_s}] = 2c(1, 2; 1, 2; 1)$$

The Fock states with a single gluon in the sea may be considered as consisting of TE gluon [36]. A gluon in the sea will contribute to H_1G_8 component of the sea. Taking the sum of all probabilities, common factor is obtained. On the R.H.S, values present in the indices are the multiplicities for the particular Fock state. States like $|u\bar{u}g\rangle, |d\bar{d}g\rangle, |s\bar{s}g\rangle, |d\bar{d}s\bar{s}\rangle$ and $|u\bar{u}d\bar{d}\rangle$ will follow the same symmetry conditions. But for $|g\bar{g}\rangle$ the symmetry conditions should be taken into account:

$\rho_{\frac{1}{2}0_s}[\rho_{11_s}, \rho_{8,8_s}]; \rho_{\frac{1}{2}a}[\rho_{8,8_a}, \rho_{10, \bar{10}}]; \rho_{\frac{3}{2}a}[\rho_{8,8_a}]; \rho_{\frac{3}{2}2_s}[\rho_{8,8_s}] = 2c[1, 2; 2, 1; 1, 1]$ The treatment of single $q\bar{q}$ pair in the sea requires special attention which may require the special knowledge of the space wave-function. In this case, to keep the parity of the system positive, one of the parton from five partons must be in p -wave state. This may require the knowledge of space wave-function in detail but the contribution of

a single $q\bar{q}$ in flavor space is calculated using the principle of detailed balance and scaled it to give the same probability. The computed value of multiplicities of various Fock states is given in table 2.3.1.

Table 2.3

States	Value of n for different values of c							
	H_0G_1	H_0G_8	H_0G_{10}	H_1G_1	H_1G_8	H_1G_{10}	H_2G_8	$H_1G_8^{\frac{3}{2}}$
$ gg\rangle$	2	4	0	0	4	2	2	2
$ u\bar{u}g\rangle$	2	8	2	2	8	2	4	4
$ d\bar{d}g\rangle$	2	8	2	2	8	2	4	4
$ s\bar{s}g\rangle$	2	8	2	2	8	2	4	4
$ u\bar{u}d\bar{d}\rangle$	2	8	2	2	8	2	4	4
$ d\bar{d}d\bar{d}\rangle$	2	8	2	2	8	2	4	4
$ ggg\rangle$	1	4	0	1	2	0	0	0
$ u\bar{u}s\bar{s}g\rangle$	1	2	2	3	24	6	8	12
$ d\bar{d}s\bar{s}g\rangle$	1	2	2	3	24	6	8	12
$ d\bar{d}d\bar{d}g\rangle$	2	16	4	6	48	12	8	12
$ u\bar{u}gg\rangle$	2	16	4	6	48	12	8	24
$ d\bar{d}gg\rangle$	2	16	4	6	48	12	8	24
$ s\bar{s}gg\rangle$	2	16	4	6	48	12	8	24
$ u\bar{u}u\bar{u}\rangle$	2	4	0	0	4	2	2	2
$ u\bar{u}u\bar{u}g\rangle$	2	16	4	6	48	12	8	12
$ u\bar{u}d\bar{d}s\bar{s}\rangle$	1	16	0	3	48	0	24	0

Table 2.3 Table showing different values of "n" in Model "C"

Thus, statistical approach is one of the successful approaches to compute the low energy hadronic observables. In addition to this, confining forces also play a crucial role in contributing to low energy properties therefore it becomes plausible that some changes may be applied in order to check the stability of results. On the basis of above point of view, some modifications may be applied via the following two models.

2.3.2 Model P

In the statistical formulation, quark-antiquark pair created from gluon, naturally carries the quantum numbers of gluon but a situation may arise where the pair on exchange of a soft gluon with the rest of the system, also possibly on a spin-flip may evolve to a colorless pseudo-scalar form, well known as internal Goldstone bosons. Therefore, it is worth to say that a model is having sea in the form of pseudoscalar bosons. If all the $q\bar{q}$ pairs are in one or the other pseudoscalar form then no contribution can be expected for spin and color charge of baryon. With this assumption, the quark-gluon Fock states can be decomposed as:

(1). If a quark-antiquark pair is in one or other pseudo-scalar form then the only possibility for a $|q\bar{q}\rangle$ to have spin and color quantum numbers equal to one thus only possibility for $q\bar{q} \sim H_1 G_1$.

(2). Similarly, $|u\bar{u}d\bar{d}\rangle$ and $|q\bar{q}q\bar{q}\rangle$ sea can have the only possibility $H_0 G_1$.

(3). In the case of $|g\bar{u}u\bar{d}d\rangle$, $|q\bar{q}g\rangle$, $|q\bar{q}g\rangle$, $|q\bar{q}q\bar{q}\rangle$ and $|g\rangle$, both share the same quantum number.

(4). For $|q\bar{q}gg\rangle$: The decomposition of Fock states in spin and color space can be written as:

$$\rho_{\frac{1}{2}0_a}[\rho_{11}, \rho_{88}, \rho_{10, \overline{10}}]; \rho_{\frac{1}{2}1}[\rho_{11}, \rho_{88}, \rho_{10, \overline{10}}]; \rho_{\frac{3}{2}1}[\rho_{88}]; \rho_{\frac{3}{2}2}[\rho_{88}] = 2c(1, 4, 1; 1, 4, 1; 2, 2)$$

Thus, the sea will not actively contribute to static properties of baryons.

2.3.3 Model D

Another modification to the basic model, has been proposed in which the contribution to the states with higher multiplicities can be suppressed. The solid ground for such an assumption arises from the fact that when a "set" of intrinsic gluons exist in a hadronic system, their preference is to remain in the similar state. Moreover, the larger is the color multiplicity of the sea, larger will be the probability of interaction and smaller is the probability of survival. The parametrization can be made to achieve by assuming that probability of a system is inversely proportional to multiplicity both in spin and color of the state. The new probability factor is additional to the previously incorporated factors.

(i) Consider the decomposition of the state $|gg\rangle$ sea then on the basis of decomposition in spin and color space:

$$\rho_{\frac{1}{2}0_s}[\rho_{11_s}, \rho_{8,8_s}]; \rho_{\frac{1}{2}1_a}[\rho_{11_a}, \rho_{8,8_a}]; \rho_{\frac{3}{2}1_a}[\rho_{8,8_a}]; \rho_{\frac{3}{2}2_a}[\rho_{8,8_s}] = 2c(2, \frac{1}{16}, \frac{1}{48}, \frac{1}{50}, \frac{1}{192}, \frac{1}{320}) \text{ for } |gg\rangle, \text{ for } |u\bar{u}u\bar{u}\rangle, \text{ for } |d\bar{d}d\bar{d}\rangle$$

(ii) $|\bar{q}q\rangle, |\bar{u}u\bar{d}d\rangle$:

$$\rho_{\frac{1}{2}0}[\rho_{11}, \rho_{8,8}, \rho_{10,\bar{10}}]; \rho_{\frac{1}{2}1}[\rho_{11}, \rho_{8,8}, \rho_{10,\bar{10}}]; \rho_{\frac{3}{2}1}[\rho_{8,8}]; \rho_{\frac{3}{2}2}[\rho_{8,8}] = 2c(2, \frac{1}{8}, \frac{1}{50}, \frac{1}{24}, \frac{1}{150}, \frac{1}{96}, \frac{1}{160})$$

(iii) $|gg\bar{q}q\rangle, |\bar{q}q\bar{q}q\rangle$:

$$\rho_{\frac{1}{2}0_s}[\rho_{11_a}, \rho_{8,8_a}, \rho_{10,\bar{10}_a}]; \rho_{\frac{1}{2}1_s}[\rho_{11_s}, \rho_{8,8_a}, \rho_{10,\bar{10}_a}];$$

$$\rho_{\frac{3}{2}1_a}[\rho_{8,8_s}]; \rho_{\frac{3}{2}1_s}[\rho_{8,8_s}]; \rho_{\frac{3}{2}2_a}[\rho_{8,8_a}] = 2c(1, \frac{1}{8}, \frac{1}{50}, 1, \frac{1}{8}, \frac{1}{50}, \frac{1}{32}, \frac{1}{32}, \frac{1}{160})$$

(iv) $|g\bar{u}u\bar{d}d\rangle$:

$$\rho_{\frac{1}{2}0_s}[\rho_{11}, \rho_{8,8}, \rho_{10,\bar{10}}]; \rho_{\frac{1}{2}1}[\rho_{11}, \rho_{8,8}, \rho_{10,\bar{10}}]; \rho_{\frac{3}{2}2}[\rho_{8,8}] = 2c(\frac{1}{2}, \frac{1}{16}, \frac{1}{100}, \frac{1}{2}, \frac{1}{16}, \frac{1}{100}, \frac{1}{160})$$

$$(v) |ggg\rangle: \rho_{\frac{1}{2}0_a}[\rho_{11_a}, \rho_{8,8_a}]; \rho_{\frac{1}{2}1_s}[\rho_{11_s}, \rho_{8,8_s}]; \rho_{\frac{3}{2}1_s}[\rho_{8,8_s}] = 2c(1, \frac{1}{32}, 1, \frac{1}{3}, \frac{1}{96}, \frac{1}{384})$$

The various Fock states and their multiplicities is shown in 2.3.3.

Table 2.4

States	Value of n for different values of c							
	H_0G_1	H_0G_8	H_0G_{10}	H_1G_1	H_1G_8	H_1G_{10}	H_2G_8	$H_1G_{\frac{3}{2}8}$
$ gg\rangle$	2	$\frac{1}{16}$	0	0	$\frac{1}{48}$	$\frac{1}{150}$	$\frac{1}{192}$	$\frac{1}{320}$
$ u\bar{u}g\rangle$	2	$\frac{1}{8}$	$\frac{1}{150}$	$\frac{2}{3}$	$\frac{1}{24}$	$\frac{1}{150}$	$\frac{1}{96}$	$\frac{1}{160}$
$ d\bar{d}g\rangle$	2	$\frac{1}{8}$	$\frac{1}{150}$	$\frac{2}{3}$	$\frac{1}{24}$	$\frac{1}{150}$	$\frac{1}{96}$	$\frac{1}{160}$
$ s\bar{s}g\rangle$	2	$\frac{1}{8}$	$\frac{1}{150}$	$\frac{2}{3}$	$\frac{1}{24}$	$\frac{1}{150}$	$\frac{1}{96}$	$\frac{1}{160}$
$ u\bar{u}d\bar{d}\rangle$	2	$\frac{1}{8}$	$\frac{1}{150}$	$\frac{2}{3}$	$\frac{1}{24}$	$\frac{1}{150}$	$\frac{1}{96}$	$\frac{1}{160}$
$ d\bar{d}d\bar{d}\rangle$	2	$\frac{1}{16}$	0	0	$\frac{1}{48}$	$\frac{1}{150}$	$\frac{1}{192}$	$\frac{1}{320}$
$ ggg\rangle$	1	$\frac{1}{32}$	0	$\frac{1}{3}$	$\frac{1}{96}$	0	0	$\frac{1}{192}$
$ u\bar{u}s\bar{s}g\rangle$	1	$\frac{1}{8}$	$\frac{2}{100}$	1	$\frac{24}{192}$	$\frac{6}{300}$	$\frac{8}{320}$	$\frac{12}{192}$
$ d\bar{d}s\bar{s}g\rangle$	1	$\frac{1}{8}$	$\frac{2}{100}$	1	$\frac{24}{192}$	$\frac{6}{300}$	$\frac{8}{320}$	$\frac{12}{192}$
$ d\bar{d}d\bar{d}g\rangle$	2	$\frac{2}{8}$	$\frac{2}{50}$	1	$\frac{1}{8}$	$\frac{1}{50}$	$\frac{1}{32}$	$\frac{1}{160}$
$ u\bar{u}gg\rangle$	1	$\frac{1}{8}$	$\frac{1}{50}$	1	$\frac{1}{8}$	$\frac{1}{50}$	$\frac{1}{32}$	$\frac{1}{160}$
$ d\bar{d}gg\rangle$	1	$\frac{1}{8}$	$\frac{1}{50}$	1	$\frac{1}{8}$	$\frac{1}{50}$	$\frac{1}{32}$	$\frac{1}{160}$
$ s\bar{s}gg\rangle$	1	$\frac{1}{8}$	$\frac{1}{50}$	1	$\frac{1}{8}$	$\frac{1}{50}$	$\frac{1}{32}$	$\frac{1}{160}$
$ u\bar{u}u\bar{u}\rangle$	2	$\frac{1}{16}$	0	0	$\frac{1}{48}$	$\frac{1}{150}$	$\frac{1}{192}$	$\frac{1}{320}$
$ u\bar{u}u\bar{u}g\rangle$	2	$\frac{2}{8}$	$\frac{2}{50}$	1	$\frac{1}{8}$	$\frac{1}{50}$	$\frac{1}{32}$	$\frac{1}{160}$

Table 2.4 Table with differnt values of "n" in model "D"

Thus, we have different forms of statistical models, sufficient to analyze the contributions of sea quarks to several static properties. The different forms of the statis-

tical model also helps to cross-check the role of dynamism in the case of sea-quarks. In the coming chapters, we apply the above said methodology with certain modifications to all the baryonic systems having spin $\frac{1}{2}^+$. The above mentioned wave-function and statistical model is found to be the suitable for all lowest lying states of baryon octet. In the forthcoming chapters, we will look at the application of above said models to nucleons as well as hyperons.

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Chapter 3

Strange and Non-strange Sea

Quark-gluon Effects for Nucleon

3.1 Introduction

The structure of nucleons is the most well known hadronic structure till now due to the reason that a variety of parton distribution functions have been generated from the available data from different experiments. The latest studies predict that only 30% of total spin of proton is carried by quarks itself. The spin structure of the proton is one of the key examples that confirm the presence of gluons which provide non-negligible contribution to spin of proton. The non-perturbative QCD dynamics is reflected in nucleon's intrinsic sea of quarks and gluons. The sea in the form of quark-antiquark pairs and gluons plays a crucial role in visualizing the hadronic structure. The different measurement of g_1 [1, 2] sum rule indicates that quark sea is strongly polarized in a nucleon. In recent years, there lies a great deal of interest in measuring and understanding the strangeness content of the nucleon. There are

some experimentally observed phenomenon like Gottfried Sum Rule [3] and flavor asymmetry [4] which can only be described in the non-perturbative regime of QCD. Some predictions (experimental as well as theoretical) confirm the presence of hidden strange and anti-strange components in the nucleonic wave-function. CERN and SLAC observed large violation of Ellis and Jaffe sum rules [5] which shows that only a small fraction of proton's helicity is carried by the quarks itself. The discovery of the large and negative polarization of the strange quark in polarized deep inelastic scattering [6] which led to a small flavor-singlet g_A^0 about $\frac{1}{4} - \frac{1}{3}$ of that predicted by the non-relativistic quark model. This discrepancy is not satisfactorily explained in any hadronic valence quark model. Thus, the distribution of non-strange and strange sea is another non-trivial aspect of protonic structure. The evidences for $s - \bar{s}$ asymmetry has also been found in the literature [7]. In fact, there is no fundamental symmetry which suggests that $s(x) = \bar{s}(x)$. The evidence for hidden strangeness are also obtained from pion-nucleon sigma terms [8] and deep polarized scattering experiments. Another possibility to explore the strange components of the sea in a nucleonic system is by looking at their strange spin. The strange spin distribution is calculated via the quantities $\Delta s = s \uparrow + \bar{s} \uparrow - s \downarrow - \bar{s} \downarrow$. However, HERMES collaboration have measured Δs as $0.03 \pm 0.03 \pm 0.01$ [9]. NuTeV collaboration [10] at FermiLab predicted the non-zero value of the quark contribution to the spin of nucleon via the strange-quark content ratio which is the fraction of nucleon momentum that is carried by strange quarks to that carried by non-strange quarks $\frac{2(s+\bar{s})}{u+\bar{u}+d+\bar{d}} = 0.477 \pm 0.063 \pm 0.053$ [11]. This ratio implies the existence of strange quarks in the sea. Thus, in the present chapter, we investigate the strange sea of the nucleonic system in detail. R. Bijker et al. [12] have concluded that a strange quark-antiquark pair contributes

very slightly to the magnetic moment of the nucleon and also have suggested that the inclusion of higher Fock components is very small. Despite much experimental and theoretical data, the spin content of the nucleon is still not clearly understood. At present there are phenomenological models available in hand, whose domains of validity must be checked using the available experimental data. It is a well-known fact that hadrons can be assumed as an ensemble of quark-gluon Fock states. The composition of nucleons in terms of quark-gluon degrees of freedom has been modeled in chapter II in statistical way to gather information about the sub-structure of nucleons and various low energy properties. The effect of sea on nucleon properties such as weak decay coupling ratios, magnetic moments and masses can be studied via these models. Our goal is to extend the basic statistical model described in chapter II so as to include the strange and non-strange Fock states and calculate the relevant probabilities in flavor, spin and color. As described earlier in chapter II, Zhang et al. [13] calculated probabilities in flavor space based on statistical approach without introducing any external parameter to find the $\bar{u} - \bar{d}$ asymmetry of the sea. Thus, we can think of an extension of approach defined by Zhang et al. [13] in such a way so as to include the Fock states with strange quark-antiquark pair in the sea. The modified expressions for calculating the relevant probabilities using principle of detailed balance are mentioned in the next section. The nucleonic state is expanded in a complete set of quark-gluon Fock states as:

$$|p\rangle = \sum_{i,j,k,l} C_{i,j,k,l} |uud, i, j, k, l\rangle \quad (3.1)$$

where i is the number of $u\bar{u}$ pairs, j is the number of $d\bar{d}$ pairs, l is the number of $s\bar{s}$ pairs and k represents the number of gluons. The probability of finding the proton in the Fock states $|uud, i, j, k, l\rangle$ is $\rho_{i,j,k,l} = |C_{i,j,k,l}|^2$ and satisfies the following normalization condition $\sum_{i,j,k,l} \rho_{i,j,k,l} = 1$. The complete version of the above wavefunction for spin-up proton consisting of three quarks in the core and quark-antiquark pairs and gluons in the sea in such a way that it maintains the total antisymmetry of the proton which is mentioned in equation(2.11) in section II of chapter II. The basic statistical model and its relevant details are already described in chapter II. This approach is applied to explore the structure of proton in terms of quark-gluon Fock states. The basic model is named as Model "C". The confining forces among the constituents play a significant role in determining the low energy observables and this fact forces us to check the stability of statistical approach with respect to certain modifications. Model "P" is based on the constrain that a quark-antiquark pair can reside in the form of colorless pseudo-scalar Goldstone bosons which may appear because of soft gluonic exchange interactions and spin-flip processes. These internal pseudoscalar bosons contribute to some additional symmetry in the quark-gluon Fock states and the sea is no-longer active participant now. To compensate for the odd parity of $q\bar{q}$ pair, one of the gluons is assumed to be in TE(Transverse Electric) mode while the other is assumed to be in TM(Transverse Magnetic) mode. In this model, decomposition of Fock states leads to equivalence among a single gluon Fock state and states with any number of $(q\bar{q})$ with g . The assumption for two gluon states is similar. A state with a single quark-antiquark pair is assumed to be a color-singlet state carrying spin 1 and states with two $q\bar{q}$ pairs have zero spin and color singlet states. Model D assumes the suppression of Fock states with higher multiplicities. A

sea with greater color multiplicity has a lower probability of survival because of the larger likelihood of interaction. This model can be assumed to be a special case of model "C" in which suppression can be achieved by assuming that the probability of a system in a spin and color sub-state is inversely proportional to the multiplicity of the state.

3.2 Principle of Detailed Balance

Assuming hadrons as a complete set of quark-gluon Fock states, principle of detailed balance is concerned with calculation of probability of every Fock state inside the hadrons. It assumes that equal probability for arriving in from one state is equal to probability of leaving out. The composition of hadrons is expanded by Fock state description in terms of complete set of quark-gluon Fock states as

$$|h\rangle = \sum_{i,j,l,k} c_{i,j,l,k} | \{q\}, \{i, j, l\}, \{k\} \rangle \quad (3.2)$$

where $\{q\}$ represents the valence quarks of the baryon, i is the number of quark-antiquark $u\bar{u}$ pairs, j is the number of quark-antiquark $d\bar{d}$, k is the number of gluons and l can be the number of $s\bar{s}$ pairs. The density operator of the ensemble is:

$$\hat{\rho} = \sum_{i,j,l,k} \rho_{i,j,l,k} |qqq, i, j, l, k\rangle \langle qqq, i, j, l, k|$$

Taking the hadron system as an ensemble of Fock states where $\rho_{i,j,l,k}$ is the probability associated to find the quark-gluon Fock states

$$\rho_{i,j,l,k} = |c_{i,j,l,k}|^2 \quad (3.3)$$

Also, $\rho_{i,j,l,k}$ satisfy the normalization condition $\sum_{i,j,l,k} \rho_{i,j,l,k} = 1$. We apply the principle of detailed balance to find the probability distributions where the balancing of any two ensembles with each other can be expressed in the form

$$\rho_{i,j,l,k} | \{q\}, \{i, j, l, k\} \rangle \iff \rho_{i',j',l',k'} | \{q\}, \{i', j', l', k'\} \rangle$$

The transition between two states involve two kinds of processes: splitting and recombination. In the splitting process, the transition probability is proportional to number of partons splitting in that state. This leads to $R_{q \implies qg} \propto N_q$ and $R_{g \implies q\bar{q}} \propto N_g$. The recombination processes involves the transition probability depending upon the number of two kinds of partons that recombine to form a final state. Thus, it can be stated that $R_{qg \implies q} \propto N_q N_g$ and $R_{q\bar{q} \implies g} \propto N_q N_{\bar{q}}$. These probabilities in flavor space is further needed to calculate probabilities in spin and color space [14] [15]. The decomposition of baryonic states in various quark-gluon Fock states with relevant operators acting on the valence and sea part is used to find probabilities for all possible spin and color sub-states so as to produce spin $\frac{1}{2}$ and color singlet state of the baryon. The procedure can be applied to states like $|\bar{u}ug\rangle, |\bar{d}dg\rangle, |\bar{s}sg\rangle, |\bar{u}udd\rangle, |gg\rangle$ etc. Moreover, non-zero mass of s-quark limits the free energy of gluon and hence the states with strange quark-antiquark pairs are assumed to be less probable. To accommodate the strange quark and to allow processes such as $g \iff s\bar{s}$, we should

have a system with energy greater than twice that of the strange quark mass. For this, we first extend the principle of detailed balance to obtain the probability of each Fock state that include $s\bar{s}$ content with due consideration of the mass of the strange quark. Three types of processes must require balancing.

(i) When $q \Leftrightarrow qg$ is considered: The addition of $s \Leftrightarrow sg$ gives the general expression of probability as follows:

$$\frac{\rho_{i,j,l,k}}{\rho_{i,j,l,k-1}} = \frac{1}{k} \quad (3.4)$$

(ii) When both the processes $q \Leftrightarrow qg$ and $g \Leftrightarrow gg$ are included. We can write

$$\frac{\rho_{i,j,l,k}}{\rho_{i,j,l,k-1}} = \frac{3 + 2i + 2j + 2l + k - 1}{(3 + 2i + 2j + 2l)k + \frac{k(k-1)}{2}} \quad (3.5)$$

(iii) When the processes $g \Leftrightarrow q\bar{q}$ are involved: The transition probabilities involving $g \Leftrightarrow q\bar{q}$ depend upon the valence quark content and differ in all baryons. The generation of $s\bar{s}$ pair from gluons is restricted due to non-negligible mass of s -quark. The constraint on the free energy of gluon comes in the form of factor $k(1 - C_l)^{n-1}$ [16] where k is the number of gluons and n represents the total number of partons present in that state. Therefore, the splitting and recombination for the processes involving $g \Leftrightarrow q\bar{q}$ undergoes $SU(3)$ symmetric breaking in sea where q is for some heavier quark flavor. In general, $n = 3 + 2i + 2j + 2l + k$ and $C_{l-1} = \frac{2M_s}{M_B - sM_s - 2(l-1)M_s}$, M_s is the mass of s -quark and M_B is the mass of the baryon.

$$| \{q\}, g \rangle \xrightleftharpoons[2.1]{1(1-C_0^3)} | q, \bar{s}s |$$

$$| \{q\}, u\bar{u}g \rangle \xrightleftharpoons[1.2]{1(1-C_0^5)} | q, u\bar{u}\bar{s}s |$$

Generalizing to a number of gluons "k" and numbers of quark-antiquark pairs "i", "j" and "l" for proton.

$$\{ | \{q\}, i, j, l-1, k \rangle \} \xrightleftharpoons[l.(l+1)]{k(1-C_{l-1}^{n-1})} \{ | \{q\}, i, j, l, k \rangle \}$$

$$\frac{\rho_{i,j,l,k}}{\rho_{i,j,l-1,k}} = \frac{k(1-C_{l-1}^{n-1})}{l(l+1)}$$

Suppose, initially no $s\bar{s}$ is present and that the generation of one $s\bar{s}$ pair require gluon to have sufficient energy then the condition becomes:

$$| \{q\}, i, j, 0, k \rangle \xrightleftharpoons[1(1+1)]{k(1-C_0^{n-2l-1})} | \{q\}, i, j, 1, k-1 \rangle$$

$$\frac{\rho_{ij1k-1}}{\rho_{ij0k}} = \frac{k(1-C_0)^{n-2l-1}}{1(1+1)}$$

$$| \{q\}, i, j, 1, k-1 \rangle \xrightleftharpoons[2(2+1)]{(k-1)(1-C_1^{n-2l})} | \{q\}, i, j, 2, k-2 \rangle$$

The go-out probability depends upon the number of partons that are present at that time. The similar treatment can be extended until all gluons have been converted into strange quark-antiquark pairs.

$$| \{q\}, i, j, k-1, 1 \rangle \xrightleftharpoons[k(k+1)]{1(1-C_{k-1}^{n-k-2})} | \{q\}, i, j, k, 0 \rangle$$

Thus,

$$\frac{\rho_{i,j,l,0}}{\rho_{i,j,0,l}} = \frac{(k(k-1)(k-2)\dots 1)(1(1-C_0^{n-2l-1})(1-C_1^{n-2l})\dots(1-C_{l-1}^{n-k-2}))}{k!(k+1)!} \quad (3.6)$$

Generalizing it to "k" number of gluons, the ratio becomes:

$$\frac{\rho_{i,j,l,k}}{\rho_{i,j,l+k,0}} = \frac{(k)(k-1) - \dots - 1(1-C_0)^{n+k-2}}{(l+1)(l+2) - \dots - (l+k)(l+k+1)} \quad (3.7)$$

The condition of normalization, $\sum_{i,j,k,l} \rho_{i,j,k,l} = 1$ is used to determine the individual probabilities which requires $\rho_{i,j,k,l}$ to be expressed in terms of $\rho_{0,0,0,0}$.

$$\frac{\rho_{i,j,l+k,0}}{\rho_{0,0,0,0}} = \frac{2}{i!(i+2)!j!(j+1)!(l+k)!(l+k)!} \quad (3.8)$$

Using the condition of normalization, the sets of individual probabilities of all Fock states can be computed. The contribution to various Fock states in terms of probabilities in flavor space using principle of detailed balance is mentioned below. The entire table for all $\rho_{i,j,l,k}$ in terms of $\rho_{0,0,0,0}$ is shown in table 3.2. From the table 3.2, it is clear that the presence of gluon in the sea gives rise to positive finite probability for strange sea. Hidden strangeness can be said to be generated via the process $quarks \rightarrow g \rightarrow s\bar{s}$. The non-negligible mass of strange quark leads to constraint on total number of strange quark-antiquark pair equal to one. Thus, we have Fock states with number of strange quark-antiquark limited to one. The flavor probabilities in the form of table given above is used to show graphically the variation of number of $s\bar{s}$ pairs with the mass of strange quark. The graph showing the variation of mass of strange quark with the number of $s\bar{s}$ pairs is shown in figure 3.1. Within the mass range of 100-150 MeV, the number of $s\bar{s}$ pairs lies in the range of 0.6–0.8 only. The probabilities in spin and color space is calculated using the relevant multiplicities in statistical model, the relevant operators act on the sea-part as well and the probabilities in spin and color space are the outcomes of the active participation of sea quarks

as well. Various multiplicities for all quark-gluon Fock states are mentioned in section II of chapter II. The statistical model and several approaches to it are used to identify the separate contributions from the scalar, vector and tensor sea. The table given below presents the probabilities in flavor space and the common parameters in different forms of statistical model. One table presents the "common" parameters in model "C" and other for model "D". In case of strange sea, Fock states with a single gluon Fock state shows a remarkable decrease in probability due to possibility of a gluon into strange quark anti-quark pair. The Fock states with more than one gluon shows an increasing trend because of the restriction of single strange-antiquark pair can be accommodated in the sea. The common factors are calculated from the spin and color multiplicities and depends indirectly on flavor probabilities too.

Table 3.1

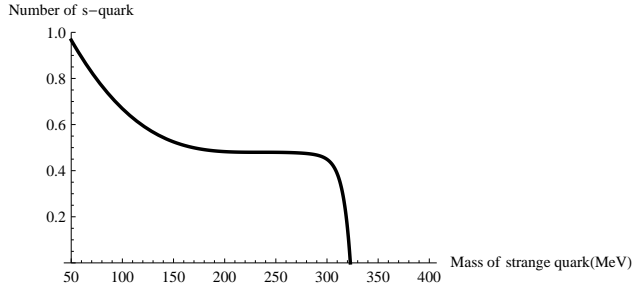


Figure 3.1 The number of $s\bar{s}$ pair Vs mass of strange quark(MeV)

i	j	l	$\rho_{i,j,l}$	$k=0$	$k=1$
0	0	0	$\rho_{0,0,0}$	0.095	0.045
0	0	1	$\rho_{0,0,1}$	0.095
0	1	0	$\rho_{0,1,0}$	0.047	0.014
1	0	0	$\rho_{1,0,0}$	0.032	0.009
0	1	1	$\rho_{0,1,1}$	0.014	0.047
1	0	1	$\rho_{1,0,1}$	0.032	0.032
1	1	0	$\rho_{1,1,0}$	0.029	0.03
1	1	1	$\rho_{1,1,1}$	0.015
0	2	0	$\rho_{0,2,0}$	0.032	0.014
2	0	0	$\rho_{2,0,0}$	0.039	0.007

Table 3.1 Table for flavor probabilities with strange sea

Computed values of co-efficients in the three models and in two forms (with $s\bar{s}$ and without $s\bar{s}$) in the model C, P and D are given below where model "P" assumes sea

as an inactive contributor, so the inclusion of the strange quark does not modify the results. These co-efficients carry the utmost importance in themselves because these co-efficients are identified with over-all probability of Fock states.

Table 3.2

States	Without $s\bar{s}$			With $s\bar{s}$		
	Probability	Value of c	Value of d	Probability	Value of c	Value of d
$ gg\rangle$	0.082	0.005	0.039	0.070	0.004	0.034
$ u\bar{u}g\rangle$	0.055	0.002	0.020	0.009	0.003	0.003
$ d\bar{d}g\rangle$	0.083	0.003	0.028	0.014	0.0004	0.005
$ s\bar{s}g\rangle$	0.096	0.0099	0.0047
$ u\bar{u}d\bar{d}\rangle$	0.029	0.0009	0.010	0.016	0.0005	0.0056
$ d\bar{d}d\bar{d}\rangle$	0.015	0.0009	0.007	0.032	0.014	0.004
$ ggg\rangle$	0.032	0.0046	0.024	0.045	0.0064	0.0038
$ u\bar{u}s\bar{s}g\rangle$	0.032	0.00049	0.020
$ d\bar{d}s\bar{s}g\rangle$	0.048	0.0007	0.0016
$ d\bar{d}d\bar{d}g\rangle$	0.015	0.0013	0.003	0.014	0.00011	0.0068
$ u\bar{u}gg\rangle$	0.03	0.0002	0.019	0.014	0.0001	0.03
$ d\bar{d}gg\rangle$	0.045	0.00038	0.0096	0.020	0.00017	0.045
$ s\bar{s}gg\rangle$	0.052	0.0004	0.011
$ u\bar{u}u\bar{u}\rangle$	0.007	0.00045	0.0034	0.004	0.009	0.002
$ u\bar{u}d\bar{d}s\bar{s}\rangle$	0.015	0.0014	0.055
$ g\rangle + q\bar{q}\rangle$	0.350	0.220

Table 3.2 Multiplicities of various Fock states

Table 3.3

Sr. No.	Co-efficients	Statistical Model without $s\bar{s}$			Statistical Model with $s\bar{s}$	
		C	P	D	C	D
1.	a_0	1	1	1	1	1
2.	a_8	0.520	0.470	0.200	0.970	0.230
3.	a_{10}	0.082	0.150	0.076	0.400	0.090
4.	b_1	0.120	0.200	0.470	0.490	0.610
5.	b_8	1.760	1.120	0.050	1.580	0.059
6.	b_{10}	0.190	0.260	0.064	0.590	0.073
7.	d_8	0.850	0.790	0.055	0.680	0.074
8.	c_8	0.240	0.300	0.350	1.090	0.410

Table 3.3 The computed values of the co-efficients

3.3 Nucleonic Parameters at Low Energy

A nucleon is described as composed of three valence quarks surrounded by quark-gluon sea multi-connected to each other. This sea plays an important role at long distance scales. This sea is filled with quark-antiquark pairs ($u\bar{u}$, $d\bar{d}$, $s\bar{s}$) and the importance of sea is realized from the fact that half of the nucleon spin can be carried by gluons. The overall spin of nucleon also receives contribution from quark-antiquark pairs in the sea. The strange quark-antiquark pairs present in the sea also provides contribution to the total spin of nucleon. Polarized quark distribution function in

the nucleon when integrated over x yields the fraction of spin carried by the quarks i.e. $\Delta q = \int_0^1 \Delta q(x) dx$ where $q(x)$ and $\Delta q(x)$ are interpreted as probability of finding the unpolarized and polarized quark with a certain flavor in the nucleon. $\Delta q + \Delta \bar{q}$ is called the axial charge of the nucleon. The fraction of spin carried by quarks is predicted in the form of three flavors with u , d and s quark. Although the overall strangeness to the nucleon is zero, but there can be a non-uniform distribution of strangeness due to strange sea. The strange quark contribution to sea is studied using polarized deep-inelastic scattering experiments of the electrons or muons from nucleons. The spin polarization measurement can be made to archive by two structure functions $g_1(x, Q^2)$, $g_2(x, Q^2)$ respectively. As mentioned in the chapter II, the first moment of proton is defined as:

$$\Gamma_1 = \int_0^1 g_1(x) dx \quad (3.9)$$

Under the assumption of $SU(3)$ symmetry, the axial vector current in spin $\frac{1}{2}$ baryon octet can be rewritten in terms of matrix elements a_0, a_3 and a_8 as follows:

$$\begin{aligned} \Gamma^1(p) &= \frac{1}{12}(\pm a_3 + \frac{1}{\sqrt{3}}a_8) + \frac{1}{9}a_0 \\ &= \pm(\Delta u - \Delta d) + \frac{1}{36}(\Delta u + \Delta d - 2\Delta s) + \frac{1}{9}(\Delta u + \Delta d + \Delta s) \end{aligned}$$

The above expression highlights the importance of polarized spin quark densities $\Delta u, \Delta d$ and Δs respectively. The spin structure function $\Gamma^1(p)$ is also important, as it could further be utilized to obtain the three matrix elements a_0, a_3 and a_8 under $SU(3)$ flavor symmetry. It is also written in terms of the polarized quark and anti-quark densities. The matrix element a_0 gives the contribution $\Delta\Sigma = \Delta u + \Delta d + \Delta s$.

An $SU(6)$ model predicts the value of $\Delta\Sigma$ to be 0.75 but the polarized DIS scattering experiments find this flavor singlet axial current value to be 0.27 [17]. This chapter is dedicated to the analysis of the spin polarized densities from the three quarks individually. The strange quark spin distribution is an open challenge from recent years for phenomenologist as well as experimentalists. Thus, we check the validity of the different models C , P and D to find the distribution of spin among the constituent quarks. In first case, the flavor singlet axial current coupling constant is calculated assuming that there is no sea content. In this case, the flavor singlet axial current coupling constant reads the values same as in conventional quark model results. In addition to this, the individual spin polarized densities for u and d quark in first case also produces the same results matching with the non-relativistic quark model results. Further, to check the contributions from strange and non-strange quark polarized spin densities, strange sea is being added. Table 3.3 provides the results for axial coupling constants and quark spin content of the nucleon and their comparison with the experimental results and non-relativistic quark model. We see in table 3.3 that strange quark contributions are although very negligible but the value comes out to be different from zero when the strange sea is included in the sea. Non-strange quark spin content is also giving closer results to the experimental data. The flavor singlet axial current coupling constant is the quark spin content of the nucleon and DIS scattering experiments read its value very less than that of non-relativistic quark model. This anomaly is better known as proton spin crisis. When the strange sea is added in the sea, this value is found to be closer to the experimental value. This comes as a surprise to previous quark model predictions and it can be claimed that strange sea is an important part of the nucleonic structure.

Table 3.4

	<i>Without strange sea</i>	<i>With strange sea</i>	<i>NRQM</i>	<i>Experiments</i>
Δu	0.65	0.78	$\frac{4}{3}$	0.82 [6]
Δd	-0.21	-0.37	$\frac{-1}{3}$	-0.44 [6]
Δs	0	-0.03	0	-0.10 [6]
$g_A^0 = \Delta u + \Delta d + \Delta s$	0.44	0.38	1	0.28 [17]
$g_A^8 = \Delta u + \Delta d - 2\Delta s$	0.44	0.47	1	0.754 [18]

Table 3.4 Polarized quark spin densities

The polarized s -quark distribution at $Q^2 = 2.5(\frac{\text{GeV}}{c})^2$ is compatible with $\Delta s = 0.03 \pm 0.03 \pm 0.01$ [19]. But the results are said to be affected by a large amount of corrections due to di-quark fragmentation.

3.3.1 Importance of Sea Contributions to Nucleons

The statistical model provides us the feature to check the effect of individual contributions of the different Fock states of sea. Each Fock state has associated probability which helps to analyze the importance of various states of sea contributions. This section also highlights the effect of suppressing the contribution from scalar and vector sea in statistical model and vice-versa. The table 3.5 shows the comparison of the calculated magnetic moment ratios, spin distribution and weak decay coupling constant for nucleons in different approaches (C,P,D) of the statistical model and the simple quark model.

Table-3.5

Para meters	Stat Model (all sea)			Without scalar and tensor sea			SQM (all sea)	Without scalar and tensor	Stat Model with strange sea		Exp. Results
	C	P	D	C	P	D			C	D	
α	0.216	0.297	0.299	0.255	0.285	0.378	0.214	0.341	0.19	0.26	
β	0.07	0.08	0.081	0.092	0.10	0.11	0.039	0.074	0.061	0.073	
$\frac{\mu_p}{\mu_n}$	-1.4	-1.47	-1.46	-1.37	-1.38	-1.46	-1.59	-1.53	-1.41	-1.46	-1.46 [20]
$\frac{g_A}{g_V}$	0.86	1.13	1.14	1.04	1.16	1.45	0.76	1.26	0.75	1.02	1.25 ± 0.03 [20] [21]
$\frac{F}{D}$	0.61	0.65	0.65	0.58	0.59	0.64	0.73	0.68	0.61	0.59	0.575 ± 0.016 [20] [22]
I_1^p	0.132	0.184	0.185	0.155	0.173	0.233	0.136	0.215	0.115	0.16	0.127 ± 0.004 [23]
I_{1^n}	-0.011	-0.043	-0.047	-0.018	-0.019	-0.008	0.009	0.005	-0.01	-0.006	-0.030 [23]

Table 3.5 Table showing static properties in various cases

The static properties (obtained without $s\bar{s}$ and with the strange quark condensates) in all of these approaches are shown in table 3.5. To check the contributions from the scalar sea alone, we suppress the vector and tensor sea contributions and we use a similar approach to find the individual contributions from the vector and tensor sea. As the sea part is dominated by the emission of virtual gluons, we can expect b_8 and c_8 to be more dominant. If only the vector sea is assumed to contribute, then nucleonic properties such as the coupling constant ratios and $\frac{F}{D}$ ratios are primarily affected by parameters b_8 and c_8 for sea without strange quarks. It can be seen from table 3.5 that for the simple quark model, if we consider non-zero scalar and tensor sea contributions from the available statistical data then the percentage error increases to 7 – 8% except in case of the spin distribution of the nucleons for

which deviation decreases from $\sim 58\%$ to $6 - 7\%$ approximately. The tensor sea appears to be less dominating because of quark spin-flip processes but cannot be neglected in all cases. The vector sea plays an important role in determining values that are close to those found in experiments. Some of the properties such as the spin distribution and $\frac{g_A}{g_V}$ ratio seem to be more strongly affected by changes in the values of these coefficients. The extent, to which the sea contribution affects the nucleon properties, is illustrated in table 3.5. For instance, when the scalar and tensor sea are neglected in statistical model (C, P, D) and the simple quark model, the magnetic moment ratio deviates by 6% from experimental data [20] shown in table 3.5. The weak decay matrix element ratio deviates by $30 - 40\%$ from the original value when the sea contribution is included in statistical model as well as in simple quark model. However in this case, when the sea is excluded, the ratio is fairly similar to the experimental results, the deviation is $0 - 17\%$. The $\frac{F}{D}$ ratio is closer to the experimental data when the sea is excluded in all the approaches but when the sea is taken into account, the statistical approach yields much better results, the C model in particular performs better than the other. As we go from the simple quark model to statistical, the sea becomes the dominant contributor to the spin distribution, therefore, the results are better matched to the experimental results in the latter case. A similar observation holds when we move from the exclusion of the sea to the inclusion of the sea in C model. The inclusion of strange quark condensates in the sea produces results that are more similar to experimental observations. The extension of the principle of detailed balance can be compared to flavor asymmetry with a value 0.71 and 0.124 for $\frac{u}{d}$ and $\bar{u} - \bar{d}$, respectively [24] which have been found to be agreed well with the experiments and with other theoretical models. Also, Gao

and Ma [25] showed that neutrino induced DIS experiments can be more sensitive towards strange quark-antiquark asymmetry. The principle of detailed balance finds the values of strange quark content ratio to be $\frac{2\bar{s}}{(\bar{u}+\bar{d})} = 0.37$; $\frac{2\bar{s}}{(u+d)} = 0.03$ which are found to be agree well with results of the NuTeV Collaboration [11].

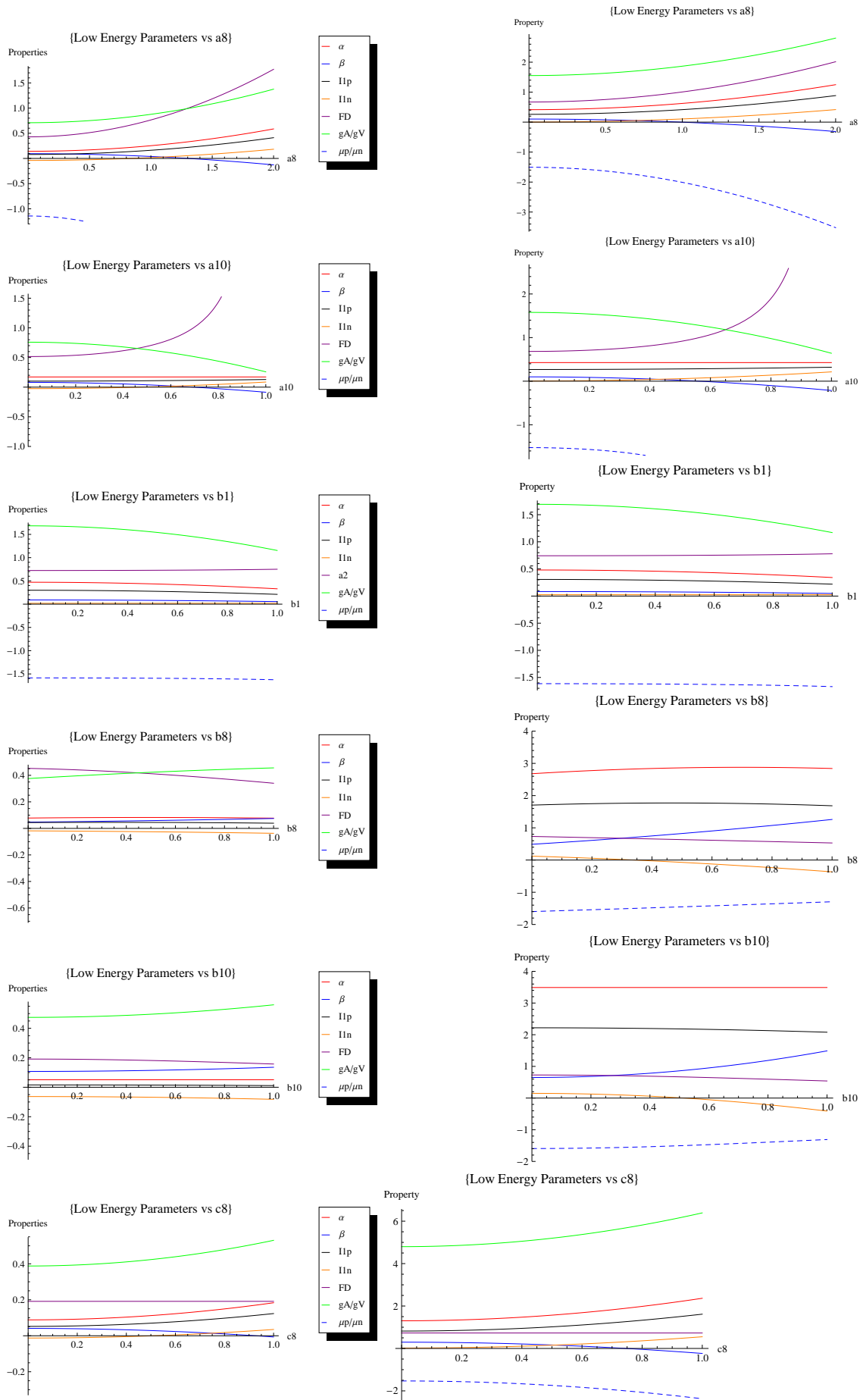
3.4 Graphical Analysis of Low Energy Observables

Moreover, a graphical analysis of low energy properties with respect to different parameters like a_8 , a_{10} , b_1 , b_8 , b_{10} is shown in figure 3.2 for strange as well as non-strange sea. The importance of these parameters lie in the fact that they are directly related with the various probabilities of quark-gluon Fock states. Here to check the contributions from the scalar sea, we suppress the vector and tensor sea contributions and apply the similar approach to find the individual contributions from vector and tensor sea. The graphical analysis for sea with strange and non-strange quarks shows that weak decay coupling constant ratio decrease to a large extent for the parameter a_{10} but axial vector matrix element shows a sufficient increase with increasing value in a_{10} . This shows the significance of parameter a_{10} . The graphical interpretation shows that maximum variation of parameters can be seen either due to suppression of vector sea or the parameters a_8 and a_{10} . As the sea part is dominated by emission of virtual gluons so we can expect b_8 and c_8 to be more varying in nature for strange as well as non-strange sea. If only vector sea is assumed to be dominating, nucleonic properties like coupling constant and $\frac{F}{D}$ ratios are mainly affected with parameters b_8 and c_8 for sea constituents. This may be due to the reason that the probability

for $g \Leftrightarrow s\bar{s}$ affects the emission of virtual gluon. Tensor sea appears to be less dominating due to quark-spin flip process but cannot be neglected in all cases. Some of the properties like $\frac{g_A}{g_V}$ are the most affected by the change in the values of these coefficients. Thus, it can be stated that the distribution of spin among the quarks get influenced by the change in probability of various quark-gluon Fock states .

3.5 Conclusion

Thus, we study a statistical model that assumes that the sea contains an admixture of gluons and quark antiquark pairs in addition to the three valence quarks. This statistical approach is based on the principle of detailed balance [13]. The calculation for strange quark is based on application of constraint with a mass of 110 MeV and analysis is summarized in for three different versions of statistical model C, P and D. Our calculations hold for the scale of order 1GeV^2 . The comparison of our data in different cases to the corresponding experimental results demonstrates that although the strange contribution in sea is negligible, yet its effects can still be seen in the data of the static properties. The inclusion of strange sea yields results that are closer to the experimental observations. The strange contribution to various static properties is also mentioned in the table. The sum of the probabilities of Fock states in flavor, spin and color space(model D) with the strange sea is $\sum_{\bar{s}s g + \bar{s}s + \bar{s}s g g} P(\text{flavor, spin, color}) = 0.099$. This small value suits the recent experimental data [30], [32] that indicates the strange sea contribution is very small. Moreover, the suppression of higher Fock components in the sea is applied in model D. Model D includes more dynamism than the other considered models and thus produces data in



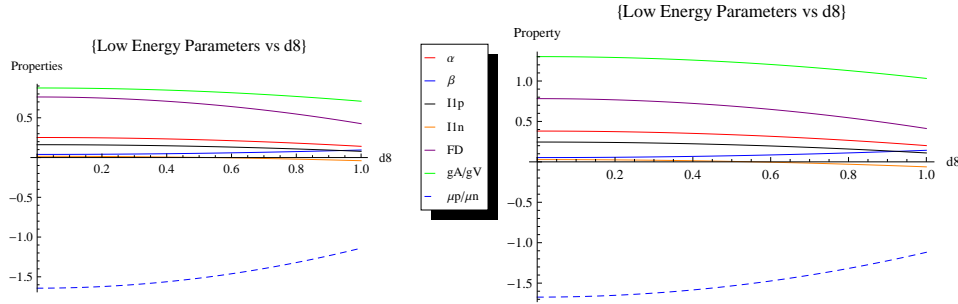


Figure 3.2 Low energy parameters for nucleons vs coefficients contributing to various types of sea

a more authentic manner when strange sea is being considered. This may be due to the reason that states with larger number of gluons are having corresponding smaller probabilities and approximate with saturated gluons. The saturated gluons possess a color neutrality scale known as saturation scale. The uniqueness of the statistical framework lies in the fact that the same model is expected to work well for all spin $\frac{1}{2}$ strange baryons. Additionally, the individual probabilities from all Fock states can be obtained allowing to check the impact of the presence of various possibilities of sea configurations to be investigated individually.

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Chapter 4

Hyperonic Properties in Statistical Model

4.1 Introduction

The discovery of strange particles brought a new additive quantum number to the hadronic physics. These particles appeared in pairs during proton-proton collisions and had an unexpected large life-time. In order to have a deeper understanding of strange hadrons at low energy, it becomes substantially important to have a detailed knowledge of the structure. The properties and interactions of ground state baryons are essential to understand those of the atomic nuclei or of more exotic kinds of systems like the strange matter, which is believed to play a key role in the macroscopic properties of astrophysical objects, e.g. neutron stars. Besides that, baryon phenomenology allows to study the non-perturbative regime of Quantum Chromodynamics (QCD). In spite of hard efforts to study baryon spectra, we do not have complete information about several resonances observed at different experiments. The unstable

nature of these particles and the non-perturbative aspects of QCD put constraints on a detailed view of the structure. Recently, cascade physics and its related researches are getting their importance and the interest is increasing for the experimentalists. KTeV Collaboration [1] and NA48/1 Collaboration [2] reported the new activities in weak decays of Ξ^0 hyperons. Also, recent research activities on Ξ resonances in relativistic heavy-ion collisions can be found in Ref. [3]. In addition, the cascade physics program of the CLAS Collaboration at the Thomas Jefferson National Accelerator Facility has been launched and some preliminary results has already been reported [4] [5] [6]. The purpose of this program includes searching for and confirming Ξ resonances as well as understanding their properties through electromagnetic production processes. Thus, many of the models are predicting various resonances in strange baryonic spectrum. It is a well known fact that hadrons are a composite system of quarks and gluons with three core quarks and a cluster of different flavors of quark-antiquark and gluons. The sea content of hadrons has always been of special interest for particle physicists since last 20 years. Presently, there are various ways (experimental as well as theoretical) to measure or calculate the quantities related to spin and flavor structure of nucleons. One of the most suitable ways is through the study of lambda. This is the lightest particle with one strange quark in valence part. A detailed knowledge of Λ and $\bar{\Lambda}$ polarizations are potentially a powerful tool for probing s and \bar{s} quark densities of nucleons. The non-zero polarizations of strange quark in nucleons comes from violation of Ellis and Jaffe sum rule [7]. Moreover, experimental results and $SU(3)$ flavor symmetry tell us that the contribution from non-strange and strange components to the spin of lambda is in the ratio 4:6. u and d quarks have negative polarizations but in several models, it has also been claimed

that $SU(3)$ breaking at large x can also lead to change of sign of polarizations [8–10]. Λ differs from that of proton due to presence of strange quark in valence part which leads to $SU(3)$ breaking corrections. This motivates us to throw some light on the spin and flavor structure of Λ . The importance of spin structure of Λ also lies in the fact that it acts as a frontier for studies strange quark-antiquark symmetry of nucleon sea which can be justified from decays of Λ . Various phenomenologist have suggested different kinds of models [11, 12] to study the details of spin and quark content for Λ and other hyperons. B. Q. Ma et al. suggested that the charged baryon such as Σ^+ may be used as a beam to measure the structure if the structure of target is completely known. Thus, flavor asymmetries for the valence and sea quarks of the Σ^\pm can be obtained from Drell-Yan experiments using charged hyperon beams on proton and deuteron targets. Mary Alberg [13] predicted that flavor asymmetry for Σ^+ is expected to be larger than the proton. It is very interesting to note that flavor asymmetry for Σ^+ baryons in a model based on meson cloud, the excess of \bar{u} over \bar{d} has been found to be in comparison to $SU(3)$ predictions. From the above observations, a substantial interest is generated in studying the flavor asymmetry and spin structure of hyperons.

4.2 Principle of Detailed Balance Revisited for Hyperons

The statistical approach finds the individual contributions from strange and non-strange components of all strange baryonic systems. This approach assumes that the hadrons can be expanded in terms of the quark-gluon Fock states. Principle of

detailed balance [14] assumes balancing of splitting and recombination of all Fock states inside the hadrons. This principle can provide the probability associated with each Fock states where each Fock state can be written as $|\{q\}, \{i, j, l, k\}\rangle$. $\{q\}$ represents the valence quarks and $\{i, j, l, k\}$ refers to number of quark-antiquark pairs and gluons. The details of the procedure for calculation of probabilities of various Fock states is already mentioned in the section II of chapter III. The generation of $s\bar{s}$ pair from gluons is restricted to either one or two due to non-negligible mass of s -quark. The constraint on the free energy of gluon comes in the form of factor $k(1 - C_l)^{n-1}$ [14] where k is the number of gluons and n represents the total number of partons present in that state. Therefore, the splitting and recombination for the processes involving $g \rightleftharpoons q\bar{q}$ undergoes $SU(3)$ symmetric breaking in sea where q is for some heavier quark flavor. In general, $n = 3 + 2i + 2j + 2l + k$ and $C_{l-1} = \frac{2M_s}{M_B - sM_s - 2(l-1)M_s}$, M_s is the mass of s -quark and M_B is the mass of the baryon.

$$\frac{\rho_{i,j,l,k}}{\rho_{i,j,l+k,0}} = \frac{(k)(k-1) - -1(1 - C_0)^{n+k-2}}{(l+1)(l+2) - - - (l+k)(l+k+1)} \quad (4.1)$$

The above expression produces all the probabilities in the form of ratios which can be expressed as:

<i>Baryon</i>	<i>Expression</i>
$\Sigma^0 \text{ or } \lambda^0(uds)$	$\frac{\rho_{i,j,l+k,0}}{\rho_{0,0,0,0}} = \frac{1}{(i)!(i+1)!(j)!(j+1)!(l+k)!(l+k+1)!}$
$\Sigma^-(dds)$	$\frac{\rho_{i,j,l+k,0}}{\rho_{0,0,0,0}} = \frac{2}{(i)!(i)!(j)!(j+2)!(l+k)!(l+k+1)!}$
$\Sigma^+(uus)$	$\frac{\rho_{i,j,l+k,0}}{\rho_{0,0,0,0}} = \frac{2}{(i)!(i+2)!(j)!(j)!(l+k)!(l+k+1)!}$
$\Xi^0(uss)$	$\frac{\rho_{i,j,l+k,0}}{\rho_{0,0,0,0}} = \frac{2}{(i)!(i+1)!(j)!(j)!(l+k)!(l+k+2)!}$
$\Xi^-(dss)$	$\frac{\rho_{i,j,l+k}}{\rho_{0,0,0,0}} = \frac{2}{(i)!(i)!(j)!(j+1)!(l+k)!(l+k+2)!}$

Table 4.1 Probability Expressions

These probabilities in flavor space is further needed to calculate probabilities in spin and color space [15,16]. Our aim is to calculate individual contributions from strange as well as non-strange components inside the $J^P = (\frac{1}{2})^+$ baryonic states. The non-negligible mass of strange quark enters while calculating the probability of all the Fock states. Chiral symmetry for strong interactions is almost exact in the light of u and d flavor sector, but it becomes approximate when strangeness is included in the sea. The role of strange quark mass in the structure of baryon is to break the $SU(3)$ symmetry in the baryon spectrum. Thus, we are motivated to explore the sea content and its contribution to the various properties of baryon octet. Several models present the effect of symmetry breakings via various ways. Yang et al. [17] studied the hyperon polarization effects and claimed that the experimental data on hyperon polarization seems to favor the theoretical predictions of $SU(3)$ symmetry breaking. The Chiral quark soliton model studied breaking in $SU(3)$ by describing the baryons as rotating solitons adiabatically in flavor space. This model uses a collective Hamiltonian term which includes the flavor rotation degrees of freedom and strange mass correction linear in M_s [18]. On the other hand, chiral constituent quark model in ref. [19] adds the strange quark mass contribution to the weak decay ratios by considering the Goldstone boson masses as non-degenerate. Various phenomenologist have suggested different models [20] [21] [22] [23] [24] to extract V_{us} from semi-leptonic decays but different models have their own limitations. A high-precision measurement of axial-vector coupling ratios like $\Sigma^- \rightarrow n$ [30] and $\Lambda \rightarrow p$ [31] demands an approach for the analysis of $SU(3)$ breaking in detail. In fact, the ratio $\frac{g_1}{f_1}$ in the limit of $SU(3)$ breaking matches with the latest data of the NA48/1 Collaboration [32]. Various stud-

ies [18], [19], [20], [21], [22], [23], [24], [28] [29], [25], [26], [27] on $SU(3)$ breaking for the matrix elements F and D , suggest the importance of strange mass corrections in the static properties of baryon. Avenarius [28] suggested that $SU(3)$ symmetry breaking in constituent quark level gives $\frac{F}{D} = 0.73 \pm 0.09$, which is larger than that of $SU(3)$ symmetric value 0.59 ± 0.02 . Ratcliffe [29] using weak decay ratios compared the values of F and D in both the cases and claimed that the difference of these two values is found to be 0.012. In order to have an accurate prediction of matrix elements F and D , elaborated data related to the individual contribution of the valence and the sea quark gluon Fock state is needed. In light of the above investigations, we studied the $SU(3)$ symmetry breaking effects using statistical approach. The discussion given below is associated with both $\Delta s = 0$ and $\Delta s = 1$ decays. To study $SU(3)$ breaking in detail, we need to incorporate the effects of strange quark mass into the probabilities associated with these Fock states. Thus, we choose a framework where the constrains due to non-negligible mass of strange quark in terms of the state densities is calculated from principle of detailed balance [14]. The unique combination of possible states forming baryon wave function with strange mass corrections in the valence and the sea part leads to justified analysis of the quark dynamics.

4.3 $SU(3)$ Symmetry for Static Properties of Strange Baryons

The importance of spin structure of baryons also lies in the fact that it acts as a frontier for the studies related to strange quark-antiquark asymmetry of nucleon sea which can be justified from decays of lambda. Baryons with spin $\frac{1}{2}$ are assigned

to a $SU(3)$ flavor octet to deduce the relations between weak matrix elements and spin distribution densities. Naive quark model predicts that all the spin for lambda baryon is carried by s -quark, the contribution from u , d quarks are zero but the recent studies are in favor of non-zero contribution from u and d quark too. Under $SU(3)$ flavor symmetry, the matrix elements can be calculated using current algebra, already discussed in chapter II. The current operator for axial vector is $J_\mu^\sigma = \bar{\psi}\gamma_\mu\gamma_5\lambda^{\frac{\sigma}{2}}\psi$, where $\sigma = 0$ to 8 with ψ as the quark-field triplet and $\bar{\psi}$ as the conjugate quark field. The baryon states are constructed with each matrix element of the octet axial currents for B_j and B_k so that

$$\langle B_j | J_\mu^\sigma | B_k \rangle = D \text{tr}(J_\mu^\sigma \{B_k, \bar{B}_j\}) + F \text{tr}(J_\mu^\sigma [B_k, \bar{B}_j]).$$

F and D couplings are proportional to structure constant f_{ijk} and symmetric invariant tensor d_{ijk} where f_{ijk} and d_{ijk} are usual $SU(3)$ group structure constants. The experimental values for F and D come out to be 0.46 and 0.80 respectively [33]. The two reduced matrix elements F and D can also be related to polarized quark distributions Δq . F and D can be related to (α, β) in the statistical model via the relation $F = \frac{3\alpha}{2}$ and $D = \frac{3(\alpha+2\beta)}{2}$. From the semi-leptonic decays like $B \rightarrow B' e \nu$, all baryonic vector and axial coupling constant ratios can be related to F and D and also the parameters of the statistical model. For each baryon, the $SU(3)$ symmetry analysis provides the relation between the spin-polarized densities and the axial-vector matrix elements. The axial vector coupling constant ratio $\frac{g_A}{g_V}(\Sigma^- \rightarrow n) = F - D = \Delta d - \Delta s$, $\frac{g_A}{g_V}(n \rightarrow p) = F + D = \Delta u - \Delta d$ and other decay ratios are also defined in terms of the F and D . For each baryon, the $SU(3)$ symmetry analysis provides the rela-

tion between the spin-polarized densities and the axial-vector matrix elements. The relative expressions for $SU(3)$ symmetric case are defined in section-IV in chapter II.

4.4 $SU(3)$ Symmetry Breaking in Sea and Valence

The importance of studying symmetry breaking has already been mentioned previously. In our case, strange sea being an active contributor to the static properties of baryon, it becomes essential for us to check the impact of symmetry breaking both in sea as well as valence quarks. The table 4.3 and 4.4 depicts the effect of strange mass corrections in two different cases.

- (i) When strange mass corrections are applied to sea containing strange quark-antiquark pair,
- (ii) When strange mass corrections are applied to both sea and valence.

Flavor breaking in sea is said to be broken by mass difference between u - and s -quark. Need for broken sea in $SU(3)$ comes from the sufficient higher value of mass of strange quark. Strange quark mass limits the exchange of gluon into strange quark and antiquark pair. We, therefore, propose to include $SU(3)$ breaking in sea for spin $\frac{1}{2}$ strange baryons like lambda and other hyperons. In order to include sea contribution, the probability in flavor space was calculated using principle of detailed balance [14]. The details of the processes have been included in the section II of chapter III. The general expression for calculating the probability from this principle includes breaking effects for $SU(3)$ while calculating free energy of gluons. Free energy of gluon is restricted by factors such as k and C_1 which shows dependence of

mass of strange quark. Thus, the statistical method with broken sea calculates new values of α and β and information about the spin and semileptonic decay constants can be obtained from these two universal parameters. Several static properties are calculated in both the cases. One of the most peculiar feature of nuclear structure is that sea exhibits a kind of flavor asymmetry. The excess of \bar{d} over \bar{u} in case of proton is predicted experimentally [34] and phenomenologically [14]. The evidence for this kind of asymmetry comes from Deep-Inelastic scattering and Drell-Yan experiments. Similar kind of asymmetry can be expected for sigma hyperons composed of uus and dds quarks. Thus, it becomes interesting to check above said argument by using principle of detailed balance. Principle of detailed balance [14] is very much successful in explaining the flavor asymmetry of proton. The same principle with addition of strange quark-antiquark pairs in the sea exclusively calculates the $\bar{u} - \bar{d}$ asymmetry. Another constraint is applied here by limiting the maximum number of $s\bar{s}$ pairs as $1 - 2$ due to momenta and mass of s -quark which limits the gluons to have larger free energy. The table given below presents the results for flavor asymmetry of the sea for sigma and lambda baryons with spin $\frac{1}{2}$.

Baryon	\bar{u}	\bar{d}	$\bar{u} - \bar{d}$
Σ^+	0.315	0.723	0.408
Σ^-	0.712	0.316	0.396
Σ^0 or Λ	0.439	0.439	0

Table 4.2 Flavor Asymmetry

Table 4.3 lists all the calculated values of above mentioned properties and compares with the results from other phenomenological models.

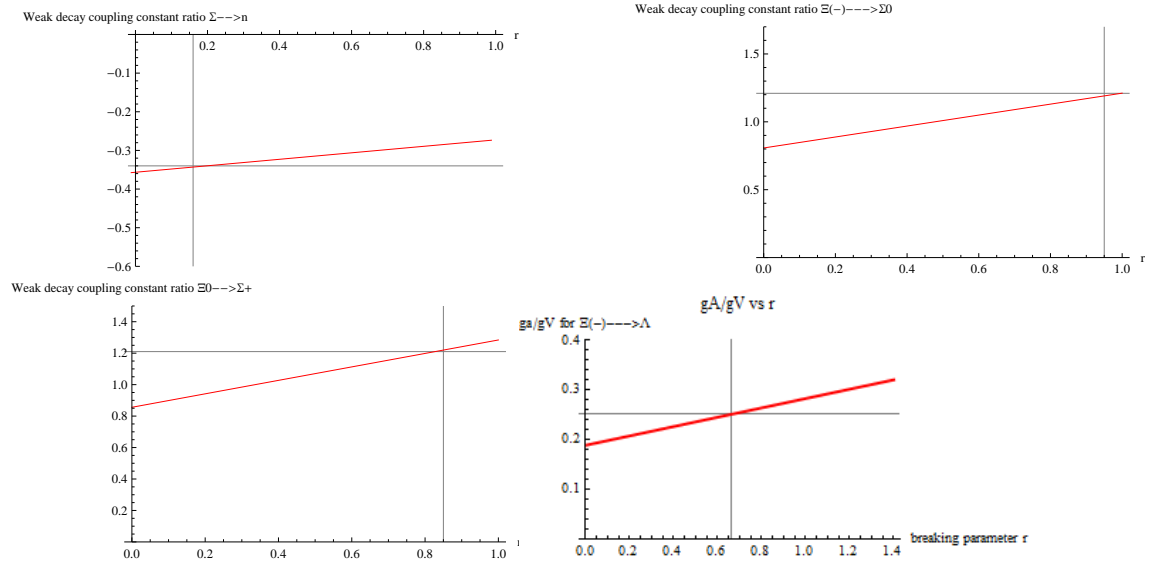
Table 4.3

<i>Co-efficients</i>	a_8	a_{10}	b_1	b_8	b_{10}	c_8	d_8	F	D
Σ^-	0.20	0.09	0.54	0.51	0.06	0.32	0.06	0.41	0.65
	0.18	0.06	0.36	0.58	0.04	0.41	0.04	0.44	0.72
Σ^+	0.22	0.09	0.56	0.66	0.07	0.46	0.07	0.40	0.63
	0.17	0.06	0.35	0.74	0.04	0.52	0.04	0.46	0.75
Σ^0	0.20	0.09	0.54	0.29	0.07	0.18	0.06	0.41	0.64
	0.17	0.06	0.35	0.74	0.04	0.52	0.04	0.46	0.75
Ξ^0	0.20	0.09	0.53	0.16	0.06	0.16	0.06	0.41	0.63
	0.19	0.07	0.42	0.30	0.05	0.21	0.05	0.51	0.77
Ξ^-	0.23	0.1	0.16	0.52	0.08	0.52	0.07	0.41	0.64
	0.19	0.07	0.42	0.5	0.05	0.50	0.046	0.47	0.64

Table 4.3 Parameters for spin $\frac{1}{2}$ strange baryons in $SU(3)$ symmetric sea and its breaking

It is clear that from the table 4.3 that the contributions from various parts of the sea get reduced except for case of the spin-1 and color-octet sea. Thus, $SU(3)$ breaking in sea favors spin-1 and color-octet sea. This may be due to the lesser splitting and recombination between gluon and a strange quark-antiquark pair. The effect of breaking in the sea for axial-vector matrix elements and spin distribution has been shown in tables 4.4 and 4.5. Conventionally, axial-vector coupling ratios $\frac{g_A}{g_V}$ are directly expressed in terms of the weak decay matrix elements F and D . It is worth to express the symmetry breaking effects in valence part in terms of conventional parameters F and D and symmetry breaking parameter r . Here r can be stated as the symmetry breaking parameter and defined as the $r = \frac{\mu_s}{\mu_d}$ [35] where μ_s is the

magnetic moment of strange quark and μ_d is the magnetic moment of the d quark. Clearly, "r" involves a direct dependence of m_s and m_1 along with some constant of proportionality. Here $m_1 = m_u$ or m_d under isospin symmetry. An operator $\hat{o} = \sum_i O_f^i \sigma_z^i$ is defined to find a suitable dependence on "r" where o_f^i depends upon the flavor of i^{th} quark and σ_z^i is the spin projection of i^{th} quark. For the case of weak decay coupling constant ratios, the operator for $\frac{g_A}{g_V}$ must include an isospin raising or lowering operator in either of T, U and V space in case of SU(3)symmetry. But, for the case of symmetry breaking corrections in valence part, the same isospin raising or lowering operator must be accompanied by symmetry breaking operator "r". In the case of breaking in SU(3), the eigenvalues obtained have the form of seven coefficients and symmetry breaking operator "r", due to which the expressions modify in terms of F, D and r. Once the eigenvalues are obtained for all the properties, it can be expressed in the form of α and β . But we find it more useful to denote the semi-leptonic decay ratios and the spin distribution of quarks in terms of more general parameters, that is the weak decay matrix elements (F and D). $\frac{g_A}{g_V}$ is plotted against "r" for each decay. The estimated value of "r" from experimental values [37] for semileptonic decays is depicted in figure 4.1. To find the unknown "r", the graph for $\frac{g_A}{g_V}$ versus the parameter "r" has been plotted for different strange baryons. The experimental values for each decay is taken from Particle Data Group and values of "r" are fitted from these experimental values. The best-fit value of "r" is obtained by using a suitable fitting algorithm and found to be "r"=0.859. The best-fit value of "r" will give the theoretical results for spin distribution and weak decay constants, provided that we know the axial-vector matrix elements calculated from the statistical approach using the principle of the detailed balance. The results are discussed on the



basis of the $SU(3)$ broken sea. This study makes a platform for the analysis of the low-energy properties of the strange baryons.

$$\begin{aligned}
 \frac{g_A}{g_V \Xi^- \rightarrow \Sigma^0} &= \left(\frac{2}{3} + \frac{r}{3}\right)F + \left(\frac{2}{3} + \frac{r}{3}\right)D \\
 \frac{g_A}{g_V \Sigma^- \rightarrow \Lambda} &= \frac{1}{9}(6 + 5r)F + (2 + r)D \\
 \frac{g_A}{g_V \Sigma^- \rightarrow n} &= -\frac{1}{3}(2 + r)F - rD \\
 \frac{g_A}{g_V \Xi^0 \rightarrow \Sigma^+} &= \left(\frac{2}{3} + \frac{r}{3}\right)F + \left(\frac{2}{3} + \frac{r}{3}\right)D \\
 \frac{g_A}{g_V \Lambda \rightarrow p} &= \left(\frac{3}{2}F + \frac{1}{6}D\right)r
 \end{aligned}$$

Figure 4.1 Graph between weak decay coupling constant Vs the parameter "r"

Hyperons	Spin polarized Densities	Statistical model			Chiral Quark Soliton Model	
		Exact SU(3)symmetry	SU(3)breaking (sea)	SU(3)breaking (valence+sea)	Exact SU(3) symmetry	Symmetry Breaking
Σ^+	$\Delta u = \frac{1}{3}F(r+6) - \frac{1}{9}Dr$	0.81	0.87	0.99	$+0.98 \pm 0.023$	$+0.73 \pm 0.17$
	$\Delta d = -\frac{1}{9}(D-3F)$	0	0	-0.07	-0.02 ± 0.09	-0.37 ± 0.019
	$\Delta s = \frac{1}{3}F(r+3) - \frac{1}{9}D(r+9)$	-0.23	-0.26	-0.185	-0.29 ± 0.13	-0.18 ± 0.39
Σ^-	$\Delta u = -\frac{1}{9}r(D-3F)$	0	0.042	0.072	-0.02 ± 0.09	-0.18 ± 0.39
	$\Delta d = \frac{1}{3}F(r+6) - \frac{1}{9}Dr$	0.88	0.91	1.07	0.98 ± 0.23	$+0.73 \pm 0.17$
	$\Delta s = \frac{1}{3}F(r+3) - \frac{1}{9}D(r+9)$	-0.22	-0.28	-0.18	-0.29 ± 0.13	-0.18 ± 0.39
Σ^0	$\Delta u = -\frac{1}{9}r(D) + \frac{Fr}{3} + F$	0.41	0.46	0.44	$+0.48 \pm 0.16$	$+0.18 \pm 0.08$
	$\Delta d = -\frac{1}{9}(Dr) + \frac{1}{3}Dr + F$	0.41	0.46	0.44	$+0.48 \pm 0.16$	$+0.18 \pm 0.08$
	$\Delta s = \frac{1}{3}F(r+3) - \frac{1}{9}D(r+9)$	-0.23	-0.28	-0.27	-0.29 ± 0.13	-0.18 ± 0.39
Ξ^0	$\Delta u = \frac{1}{3}F(r+3) - \frac{1}{9}D(r+9)$	-0.21	-0.25	-0.15	-0.29 ± 0.13	-0.14 ± 0.21
	$\Delta d = -\frac{1}{9}r(D-3F)$	0	0	0.008	-0.02 ± 0.09	-0.37 ± 0.19
	$\Delta s = \frac{1}{9}(3(D+F(r+6)) - D(r+3))$	0.89	1.03	1.06	0.98 ± 0.23	1.50 ± 0.60
Ξ^-	$\Delta u = -\frac{1}{9}r(D-3F)$	0	-0.06	-0.07	-0.02 ± 0.09	-0.37 ± 0.19
	$\Delta d = \frac{1}{3}F(r+3) - \frac{1}{9}D(r+9)$	-0.19	-0.18	-0.17	-0.29 ± 0.13	-0.14 ± 0.21
	$\Delta s = \frac{1}{9}(3F(r+6) - D(r-3))$	0.77	1.19	1.2	0.98 ± 0.23	1.50 ± 0.60

Table 4.4 The spin polarized densities and modified expressions for SU(3) symmetry breaking

4.5 Results and Discussions

Various static properties for $J^P = \frac{1}{2}^+$ baryons are calculated in three different cases and compared with other theoretical models. In the present work, we focus on the semileptonic decays and axial vector coupling ratios in SU(3) symmetry and its break-

ing using statistical approach. The sea acts as an active participant with direct inclusion of strange mass corrections and one body operator directly involves the strange quark mass in the form of parameter "r". We provide a best fit for the axial vector matrix elements F and D and find the contribution of strange quark to the spin of proton. Table shows the axial-vector coupling constant ratios for $\Delta s = 1$ decays and mention the calculated values in different theoretical models.

Table 4.5

Ratio	Statistical Model	Symmetry Breaking in sea	Symmetry Breaking in valence+sea	Theoretical [35] Values	Chiral [18] [36] Solitan Model	Exp. [37] Results	Chiral Constituent Quark Model	[19] [24]
$\frac{g_A}{g_V} \Xi^- \rightarrow \Sigma^0$	1.03	1.14	1.15	1.29	...	0.95	1.27
$\frac{g_A}{g_V} \Xi^- \rightarrow \Lambda$	0.20	0.22	0.23	0.33	0.21	0.25 ± 0.5	0.21	0.21
$\frac{g_A}{g_V} \Sigma^- \rightarrow n$	-0.24	-0.28	-0.27	...	-0.31	-0.34 ± 0.17	-0.16	-0.31
$\frac{g_A}{g_V} \Xi^0 \rightarrow \Sigma^+$	1.04	1.28	1.22	1.21 ± 0.05	0.95	1.27
$\frac{g_A}{g_V} \Xi^- \rightarrow \Xi^0$	-0.21	-0.27	-0.27	-0.333	-0.31	...	-0.16	-0.31
$\frac{g_A}{g_V} \Lambda \rightarrow p$	0.764	0.783	0.744	0.78	0.718 ± 0.0015	0.58	0.74
$\frac{g_A}{g_V} \Sigma^+ \rightarrow \Lambda$	0.52	0.61	0.61	0.45	0.65

Table 4.5 $\frac{g_A}{g_V}$ computed for strange baryons in various cases

It is quite interesting to mention here that strange mass corrections when applied in sea, shows a remarkable increase in the value of polarized quark-antiquark densities. Our data is found to be consistent with ref. [18] for the polarization densities of strange quark Δs but it shows deviation for $\Delta u, \Delta d$ in few cases. However, there is lack of available data for spin polarized densities for comparison. As far as the available data for weak decay ratios is considered, our results match the experimental value in most of the cases. However, decays like $\Sigma^- \rightarrow n$, $\Xi^- \rightarrow \Lambda$ produce results

closer to experimental data, when breaking in sea part is concerned. The other decay channels are in favor of the corrections in the valence part too. The deviation in our results from experimental values [37] is below 1% for $\Xi^0 \rightarrow \Sigma^+$ and is up to 20% for $\Sigma^- \rightarrow n$. The statistical model produces favorable results for decays involving lambda baryon too. For $\Lambda \rightarrow p$, the calculation of weak decay coupling constant ratio gives an error of 3.5%. The results go well with other theoretical models for most of the decays and provide even better matching with $\Xi^0 \rightarrow \Sigma^+$. Quarks and gluons in the statistical model are considered as intrinsic and the principle of detailed balance provides complete details of the intrinsic structure of the baryon. This leads to the statement that the statistical model can be chosen as a strong base to study hadronic properties. Additionally, our results match with those of ref. [28] where a remarkable increase in the value of $\frac{F}{D}$ is observed for broken symmetry. Ehrnsperger and Schafer [38] led to the conclusion that the effects of symmetry breaking lead to a reduction in the values of axial matrix elements. Philip G. Ratcliffe [29] re-examined the data of hyperonic decays and parameterized the matrix parameters F and D . He calculated the deviation due to symmetry breaking to be just 2%. Different authors suggest either increase or decrease in the new fitted values of F and D , but Cheng and Li [25] mentioned that there exists no a priori reason to expect the corrections to either increase or decrease the ratio. The best-fitted values of F and D are found to be 0.48 and 0.71, respectively. The ratio $\frac{F}{D}$ comes out to be 0.676 and found to be deviating about 17% from the experimental value of 0.575 ± 0.016 [36]. It is worth to mention that authors of ref. [26] suggested a 20% contribution in $SU(3)$ breakings in octet axial-charge currents. The extracted values of F and D are further useful to find the total spin distribution of nucleon, especially Γ_1^p and $\Delta\Sigma$ of a nucleonic

system. In case of $SU(3)$ symmetry, we find $\Delta u = 0.91, \Delta d = -0.24$ and $\Delta s = 0$ respectively. Also, when m_s corrections are applied, the spin polarized densities change to $\Delta u = 0.76, \Delta d = -0.18$ and $\Delta s = -0.019$ respectively. This leads to the conclusion that even in $SU(3)$ symmetry breaking, strange quark contribution to the spin of proton is very small.

4.6 Conclusion

The validity of the statistical model in the hadronic sector has already been proved for various cases. In our work, the corrections in $SU(3)$ symmetry comes in the form of single parameter. The results provide a deeper understanding for the baryon structure thereby motivating experiments for further inspection, especially the spin distribution among the quarks and gluons. It can be mentioned here that $\Delta s, \Delta \Sigma$ get more affected by the symmetry breaking. The spin distribution due to strange quarks in strange baryons matches with the chiral soliton model, both for $SU(3)$ and for its breaking. The strange sea seems favoring the experimental data, thereby, seems to the proof of the fact that gluons undergo quark-antiquark pair annihilation and construction. The data shifts by 7 to 46% approaching the experimental value favoring the strangeness in the sea. Here the vector strange sea dominates and favors the results. The present framework suggests a stronger base to choose the model with the suggested cases to verify the experimental and other theoretical values and hence provide a deeper understanding to the strange baryon structure. Our calculation work holds good for energy scale $\sim 1\text{GeV}^2$ and may hold good for higher energy scales too. We have tried to construct a sea wave function in the baryonic rest frame but it may

be further modified to include the transverse motion of quarks inside the nucleons. We conclude that the strange sea provides a better $SU(3)$ analysis for semileptonic decays and the spin distribution in terms of quark polarization in the strange baryon sector. Its contribution plays a significant role in determining the validity of the present approach.

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Chapter 5

Heavy-Light Mesons in Effective Theories

5.1 Introduction

It has been discussed earlier that hadrons are the bound particles and confined as color singlets. It is worth to mention that lightest quark masses m_u, m_d, m_s are small compared to non-perturbative scale Λ_{QCD} which is of the order of 0.2 GeV. The QCD gauge coupling constant is not small for distance and energy scales involved in the structural properties of these hadrons. The non-perturbative QCD dynamics is reflected in nucleon's intrinsic sea of quarks and gluons. The mass of heavy quark is $m_Q > \Lambda_{QCD}$. Taking limit $m_Q \rightarrow \infty$ as a good approximation, heavy quark spin and flavor symmetries are manifested in QCD Lagrangian. The interactions of a heavy and light quark system are also of the order of Λ_{QCD} and governed by non-perturbative QCD. A deep understanding of non-perturbative QCD can lead to the understanding of the most common anomalies regarding the structure of hadronic

systems. The question like "What carries the proton spin?" is still an alarming problem for researchers in particle physics. For this, we may rely upon non-perturbative QCD approaches like phenomenological models, effective theories and lattice QCD etc. We have discussed in the previous chapters the use of statistical ideas to find the quark contribution to the spin of a nucleonic system. Further, the presence of $s\bar{s}$ in the nucleonic system is also confirmed in the sea. The $s\bar{s}$ in the sea is said to be generated via the basic quark mechanism but suppressed by the strange quark mass factor $m_s > m_{u,d}$. On the similar grounds, the impact of presence of sea is checked for the low energy properties of hyperons. These low energy properties are weak decay coupling constant ratios, axial vector form factors and magnetic moments. The present thesis is aimed towards exploring the low energy properties of different hadronic systems. Another hadronic system of our interest is the study of heavy light mesons. The heavy meson spectrum is one of the basic motivations to search for the various particles at different resonances and energies. Various models like quark models, potential models and lattice studies were used to calculate meson masses earlier times but the calculated charm masses were found to be of higher values as compared with experiments [1] [2]. The approximate symmetries of QCD for heavy quarks can be incorporated to get information about the heavy-light system of mesons like charm and bottom mesons. Heavy meson spectroscopy can be analyzed by electron collider experiments but observations of such a long range of resonances lead to different puzzles. The one of the most important evidence of $c\bar{s}$ broad states was provided by CLEO Collaboration [3], which observed a state of mass 2460 MeV and width 290 MeV. The BaBar Collaboration observed another narrow peak in the $D_s^+\pi^0$ invariant mass distribution, corresponding to a state of mass 2.317 GeV [4].

Both of these states are confirmed by FOCUS and Belle Collaborations [5]. Recent experimental evidence lies in $D_{sJ}(2860)$ which was observed by BaBar Collaborations [6] with mass $2856.6 \pm 1.5 \pm 50 \text{ MeV}$ and Belle Collaboration measured another peak in B^+ decays with mass as $M(D_s^j(2715)) = 2715 \pm 11_{-14}^{+11} \text{ MeV}$. Moreover, in the DK mass distribution, BaBar Collaboration noticed a broad structure with mass, $M = 2688 \pm 4 \pm 3 \text{ MeV}$ [7] and width $\Gamma = 112 \pm 7 \pm 36 \text{ MeV}$, the same resonance ($D_{sJ}(2700)$) was found by Belle. Most recent state in the charm strange meson sector was (announced in 2009) $D_{sJ}(3040)$ with mass $M \doteq 3044 \pm 8(\text{stat})_{-5}^{+30}(\text{sys}) \text{ MeV}$ [8]. This state was observed in D^*K channel mode not DK bound state. Recently, the charm meson states in the non-strange sector with higher quantum numbers have been discovered and verified by both BaBar [9] and LHCb collaboration [10]. Therefore in the spirit of this work, we include strange and non-strange mesons both to calculate mass and mass splittings. The status of bottom meson spectroscopy on experimental grounds is slightly at a lower position than charm mesons. Some p -wave bottom and their strange counterpart mesons have been discovered earlier by collaborations like DELPHI [11] and ALEPH [12]. Some excited measurement of excited bottom mesons have been reported by D0 and CDF in 2005 [14]. D0 has also observed evidence for the B_{2s}^* meson at a mass of (5839.1 MeV) [15]. But there are other measurements of excited B meson masses $B_{s1}(5829.4 \pm 0.7)$ and B_{s2}^* reported by D0 [15] and CDF [18] which differ significantly and more data are needed to get precise masses and widths. V. M. Abazov (D0 Collaboration) [16] in 2008 presented first strong evidence for resolution of excited B mesons B_1 and B_2^* . The mass of B_1 is measured to be $5720 \pm 1.4 \frac{\text{MeV}}{c^2}$ and $B_2^* = 5746.8 \pm 2.4 \pm 1.7 \frac{\text{MeV}}{c^2}$. Very recently, the new states have been predicted for orbitally excited bottom mesons by

CDF collaboration. The mass of this new resonance is found to be $5978 \pm 12 \frac{\text{MeV}}{c^2}$ for neutral states and $5961 \pm 5 \pm 12 \frac{\text{MeV}}{c^2}$ for charged state [17]. A lot of experiments investigated for charm and bottom meson spectrum and the old theories have also been revived. One of these theories is heavy quark effective theory 'HQET' [19] [20] to study heavy-light hadrons. To summarize, the experimental determinations of heavy-light mesons remain inconsistent, although, the variety of states have been predicted. We are interested in a situation where the interaction of a heavy quark with mass m_Q occurs with light degrees of freedom where light degrees carry a momentum much less than m_Q . In such a situation, the limit of QCD is taken where $m_Q \rightarrow \infty$ provided four velocity of heavy quark remains fixed. The interactions of heavy quarks are independent of the heavy quark mass. Thus, there exists a $SU(N)$ flavor symmetry for N heavy flavors. The effective theory for the interaction of the light degrees of freedom with heavy quark has $SU(2)$ spin symmetry generated by the spin of heavy quark. These two symmetries help to determine the hadronic matrix elements involved in various types of decays and masses.

5.1.1 Charm and Bottom Mesons and their Spectrum

In the infinite heavy quark mass limit, a heavy light system $Q\bar{q}$ can be classified into doublets depending upon their quantum numbers. A heavy hadronic system containing heavy quark with spin quantum number S_Q and light degrees of freedom where light degrees of freedom include light quark and gluons interacting through quark-anti quark pairs. It should have the quantum number of light quark that is S_l in order to have total conserved quantum number J where $J = S_Q + S_l$. Defining J as $J^2 = j(j+1)$ and $S_Q^2 = (s_Q)(s_Q+1)$ and $S_l^2 = (s_l)(s_l+1)$, the total spin

$j_{\pm} \doteq s_l \pm \frac{1}{2}$ can be obtained by combining the spin of heavy quark spin $\frac{1}{2}$ with spin of light degrees of freedom. The ground state heavy mesonic system form a degenerate doublet with $j \doteq 0 \pm 1$ and negative parity denoted as D and D^* for charm meson. The first excited states 0^+ and 1^+ heavy mesons are the quantum numbers of the $j^P \doteq 1/2^+$ doublet. The two doublets D_0^*, D_1 and D_1, D_2^* have spin parity $J^P = (0^+, 1^+)$ and $J^P = (1^+, 2^+)$ respectively for $L=1$; the two doublets D_1, D_2^* and D_2^*, D_3 have spin parity $J^P = (1^-, 2^-)$ and $J^P = (2^-, 3^-)$ for $L=2$ mesons. States with radial excitations i.e. $n=2, 3, 4$ have spin and parity analogous to $n=1$. Similar states are expected for bottom mesons.

Table 5.1

Sr. No.	Mesonic State	J^P
1.	$D^{0,+}$	0^-
2.	D_s^0	0^-
3.	D_0^0	0^+
4.	D_{s0}^0	0^+
5.	D^{*0+}	1^-
6.	D_s^{*+}	1^-
7.	D_1^0	1^+
8.	D_{s1}^+	1^+

Table 5.1 Masses of strange and non-strange charm mesonic states

Masses, splittings, decay widths and branching ratios are the fundamental parameters in describing a hadronic system. The experimental data for heavy light

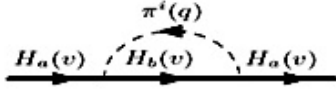
systems comes in the form of these parameters. The properties of hadrons with a heavy quark coupled with light degrees of freedom can be explained on the basis of symmetry occurring in heavy quark limit which takes a particular simpler form in the limit m_Q . It is also well known that the interaction among Goldstone bosons can be described by the low momentum expansion in terms of momenta and meson masses. Thus, the effective Lagrangian must include terms exhibiting the chiral symmetry which is based on $SU(3)_L \times SU(3)_R$ global symmetry and other heavy quark symmetry assuming charm, bottom and top quark masses as infinite as compared to intrinsic scale $\Lambda_{QCD} \sim 0.2$ GeV. This separates two classes of quarks (u, d, s) as light quarks whose masses are very less than this scale and considered as zero, on the other hand, heavy quarks whose masses are greater than Λ_{QCD} , considered as infinite. Therefore, heavy degrees of freedom can be removed from the system. Two theories related to two symmetries, chiral symmetry for light quarks u, d, s and heavy quark spin and flavor symmetry for heavy quarks c and b can be exploited to explain a system with one heavy quark and other light one. The two global approximate symmetries of QCD can be employed to explain the various properties of a hadronic bound states having one heavy quark interacting via the exchange of light pseudoscalar meson like pions, kaons and eta mesons using systematic expansions in light quark masses m_q and heavy quark mass expansion as $1/m_Q$. Heavy light mesons can be studied by implementing both the chiral symmetry and heavy quark spin and flavor symmetry in the form of an effective Lagrangian. Effective Lagrangian describes the interplay between the chiral symmetry and heavy quark symmetry in the form of low energy gradients with heavy and light fields as operators. Thus, chiral Lagrangian for heavy mesons incorporating the heavy quark and chiral symmetry

can be written by including a kinematic term and all the possible interactions with the Goldstone bosons and fields in which both the symmetry breaking and conserving terms are expected. In this chapter, we use heavy hadron chiral perturbation theory to analyze masses of the low lying states with total angular momentum $J=0$ and 1. The idea of synthesis of two symmetries has become more important as the low energy theorems can be more useful to define the matrix elements in the decay of $D^* \rightarrow D\pi$ and other soft pion or electromagnetic decays. We first review the construction of leading order Lagrangian, then $O(1/m_Q)$ corrections to Lagrangian are incorporated to develop a generalized mass formula. The main purpose of the present work is to find significance of several parameters involved in relation to the masses of the charm mesons.

5.2 The Interaction Lagrangian for Heavy Light Mesons

In heavy meson chiral perturbation theory, it is required to deal with two types of mesons, the heavy mesons interacting through light pseudo-scalar mesons and they appear in spin doublets. Therefore, it becomes convenient to express the two fields in a single term H_a where it collects two mesons in doublets, one represents the vector mesons and other is the scalar meson. A single field H_a where it annihilates the $s_l = \frac{1}{2}^-$ meson doublet, scalar and vector mesons can be mentioned as [1].

$$H_a = \frac{1 + \psi}{2} (P_a^{*\mu} \gamma_\mu - P_a \gamma_5) \quad (5.1)$$



Here a is the $SU(3)$ index. In charm mesons sector P_a consists of the D^0 , D^+ , D_s^+ pseudo-scalar mesons and P_a^μ are the D^{*0} , D^{*+} , D_s^{*+} vector mesons. The lowest lying excited states are the $J^P = 0^+$ and 1^+ i.e. $j^P = \frac{1}{2}^+$ doublet and represented by the fields [21].

$$S_a = \frac{1 + \not{v}}{2} (P_{1a}^\mu \gamma_\mu \gamma_5 - P_{0a}) \quad (5.2)$$

The interaction of heavy light mesons through exchange of pseudo-Goldstone bosons is shown through one loop diagram

The low energy interactions of heavy light meson interacting with pions are described by chiral Lagrangian which respects heavy quark spin flavor symmetry [22] and it is formulated using the velocity dependent field and a loop expansion in terms of $1/m_Q$. The total Lagrangian involves various terms like kinetic term, axial term, mass term, terms preserving symmetries (like heavy quark spin symmetry), symmetry violating terms etc. The interactions among heavy and light mesons are ob-

tained by expanding the field $\xi = e^{(i\pi)/f}$ in terms of π, η, K fields. The pion octet is here introduced by the vector and axial combinations $V^\mu = \frac{1}{2}\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial^\mu\xi$ and

$$A^\mu = \frac{1}{2}\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial^\mu\xi. \quad \Pi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & \frac{-2}{\sqrt{6}} \end{pmatrix}$$

The leading order chiral Lagrangian for heavy light mesons interacting with pions is given by:

$$L_v^0 = -i\text{Tr}\bar{H}_v(v.\partial)H_v + i\text{Tr}\bar{H}_vH_v(v.V) + 2g\text{Tr}\bar{H}_vH_v(S_{lv}.A) \quad (5.3)$$

Here all traces are taken over Dirac spinor indices, light quark $SU(3)_V$ flavor indices $a = u, d, s$ and heavy quark flavor indices $Q = c, b$ and the Dirac structure of chiral Lagrangian has been replaced by velocity vector v .

Higher order terms in chiral Lagrangian break heavy quark spin and flavor symmetry involve factors of $\frac{1}{m_Q}$ or insertions of light mass matrix. The total Lagrangian involves various terms like kinetic term, axial term, mass term, terms preserving symmetries (like heavy quark spin symmetry), symmetry violating terms etc.

$$L_v^{kinetic} = -\text{Tr}[\bar{H}_a(iv.D_{ba} - \delta_H\delta_{ab})H_b] + \text{Tr}[\bar{S}_a(iv.D_{ba} - \delta_S\delta_{ab})H_b] \quad (5.4)$$

where δ_H and δ_S are the residual masses of the H and S fields respectively and D_{ba} is the chiral covariant derivative. The axial couplings are included in the term given below:

$$L_v^{axial} = g\text{Tr}[\bar{H}_aH_bA_{ba}\gamma_\mu\gamma_5] + g'\text{Tr}[\bar{S}_aS_bA_{ba}\gamma_\mu\gamma_5] + h\text{Tr}[\bar{H}_aS_bA_{ba}\gamma_\mu\gamma_5] \quad (5.5)$$

Where g, g' and h are the dimensionless constants. The other terms in the Lagrangian are the higher order terms. Higher dimensional operators of the chiral Lagrangian which break heavy quark spin-flavor symmetry and chiral symmetry involve factors of $\frac{1}{m_Q}$ or the insertion of the light quark mass matrix $M = \text{diag}(m_u, m_d, m_s)$. For the calculation of the heavy light meson masses, it is useful to classify the symmetry-violating operators by the number of insertions of the quark mass matrix and to check whether they violate the heavy quark spin symmetry. Operators which respect heavy quark spin symmetry have coefficients which start at $O(1)$ in the $\frac{1}{m_Q}$ expansion, whereas operators which violate heavy quark spin symmetry have coefficient which start at $\frac{1}{m_Q}$. Counter terms to the one loop calculations of the heavy light meson masses are proportional to the power of the light quark masses. Thus, the chiral Lagrangian contributing to mass include the following terms:

$$\begin{aligned}
L_v^{mass} = & -\frac{\Delta_H}{8} \text{Tr}[\overline{H}_a \sigma^{\mu\nu} H_a \sigma_{\mu\nu}] + \frac{\Delta_S}{8} \text{Tr}[\overline{S}_a \sigma^{\mu\nu} S_a \sigma_{\mu\nu}] + a_H \text{Tr}[\overline{H}_a H_b] m_{ba}^\xi \\
& - a_S \text{Tr}[\overline{S}_a S_b] m_{ba}^\xi - \sigma_S \text{Tr}[\overline{S}_a S_a] m_{bb}^\xi + \sigma_H \text{Tr}[\overline{H}_a H_a] m_{bb}^\xi - \frac{\Delta_H^{(a)}}{8} \text{Tr}[\overline{H}_a \sigma^{\mu\nu} H_b \sigma_{\mu\nu}] m_{ba}^\xi \\
& + \frac{\Delta_S^{(a)}}{8} \text{Tr}[\overline{S}_a \sigma^{\mu\nu} S_b \sigma_{\mu\nu}] m_{ba}^\xi - \frac{\Delta_H^{(\sigma)}}{8} \text{Tr}[\overline{H}_a \sigma^{\mu\nu} H_a \sigma_{\mu\nu}] m_{bb}^\xi + \frac{\Delta_S^{(\sigma)}}{8} \text{Tr}[\overline{S}_a \sigma^{\mu\nu} S_a \sigma_{\mu\nu}] m_{bb}^\xi
\end{aligned}$$

Where the terms which do not contain the light quark mass matrix is

$$L_v^1 = -\frac{\Delta_H}{8} \text{Tr}[\overline{H}_a \sigma^{\mu\nu} H_a \sigma_{\mu\nu}] + \frac{\Delta_S}{8} \text{Tr}[\overline{S}_a \sigma^{\mu\nu} S_a \sigma_{\mu\nu}] \quad (5.6)$$

This term violates the heavy quark spin symmetry and is responsible for the hyperfine mass splitting at the leading order. The parameter Δ is a function of $\frac{1}{m_Q}$ which results in hyperfine splitting $P^* - P = \Delta$ at tree level. The terms depending upon

the light quark masses and respect the heavy quark spin symmetry are given by

$$L_v^2 = a_H \text{Tr}[\overline{H}_a H_b] m_{ba}^\xi - a_S \text{Tr}[\overline{S}_a S_b] m_{ba}^\xi - \sigma_S \text{Tr}[\overline{S}_a S_a] m_{bb}^\xi + \sigma_H \text{Tr}[\overline{H}_a H_a] m_{bb}^\xi \quad (5.7)$$

Where "a" and σ are the functions of $\frac{1}{m_Q}$ which start at $O(1)$. The term proportional to "a" results in $SU(3)_v$ violating mass splitting amongst the vector mesons. The term proportional to σ leads to a singlet contribution to the masses which depends linearly on the light quark masses.

5.3 Mass Formula for Charm Mesons

In the heavy quark limit, the heavy quark spin and angular momentum for light degrees of freedom are totally conserved. As an application of chiral perturbation theory to masses of charm mesons, independent contribution to all the terms of effective Lagrangian are computed. In the framework of heavy hadron chiral perturbation theory, chiral corrections and corrections due to heavy quark symmetry are encountered and at tree level, the residual masses can be given by a generalized formula. The residual masses are defined to be the difference between the real mass and an arbitrarily chosen reference mass of $O(m_q)$.

$$m_{R_a}^0 = \delta_R + \frac{n_J}{4} (\Delta_R + \Delta_R^\sigma \overline{m} + \Delta_R^{(a)} m_a) + \sigma_R \overline{m} + a_R m_a + \frac{g_R^2}{f^2} c^{R_a} K_1 + \frac{h^2}{f^2} c^{R_a} K_2 \quad (5.8)$$

Where R is an index that labels the ground state (H) and excited state (S), each of the ground state and excited states having members corresponding to $J=0,1$ where $n_J = n_0 = -3$ and $n_1 = 1$. These coefficients come from S_Q . S_l operator and

gives $\frac{-3}{4}$ for pseudoscalar mesons and $\frac{1}{4}$ for vector mesons. Here $a = u, d, s$ and $\bar{m} = m_u + m_d + m_s$. Thus in total, we obtain 12 equations representing the residual masses at tree level for low lying doublets. In isospin symmetry, the equations reduce to eight in number. The index (a) labels the light flavor and runs over u, d, s and K_1 and K_2 are the chiral loop functions defined as, and c^{R_a} are the coefficients. The experimentally measured residual masses with reference to the non-strange spin averaged ($m_{H1} + \frac{3}{4}m_{H1}^*$) are:

$$m_{H1} = -106.1\text{MeV}, m_{H3} = -4.75\text{MeV}, m_{H1}^* = 35.4\text{MeV}, m_{H3} = 139.1\text{MeV}, \\ m_{S1} = 335.0\text{MeV}, m_{S3} = 344.4\text{MeV}, m_{S1}^* = 465.0\text{MeV}, m_{S3}^* = 486.3\text{MeV}$$

The calculations here depend on eleven parameters $g, g', h, a_H, a_S, \Delta H^a, \Delta S^a, \delta H + \sigma H \bar{m}, \delta S + \sigma S \bar{m}, \Delta H + \Delta H^{(\sigma)} \bar{m}, \Delta S + \Delta S^{(\sigma)} \bar{m}$. The parameters $\sigma H, \sigma S, \delta\sigma^{(H)}, \delta\sigma^{(S)}$ cannot be separately determined because they always appear in linear combination with the parameters $\delta H, \delta S, \Delta H, \Delta S$ respectively. So the contribution of these four parameters is absorbed into measured values of $\delta H, \delta S, \Delta H$ and ΔS respectively. The values of residual masses given above can be fitted by varying the values of these parameters. Also, the values of the mass splitting are available at various experiments; we also try to fit these mass splitting over a range of the parameters. The possible range of some parameters is available from various experiments. Different predictions from relativistic and non-relativistic quark model restrict the value of coupling constants g and h to lie between 0 and 1 but $g' \in [-1, 1]$ [22]. Taking an indication from here, in our work, we vary these values over the range 0-1 so as to include all possible values. We used $f=120$ MeV extracted in ref. [19]. We set $m_u = m_d = 4\text{MeV}$ and $m_s = 90\text{MeV}$ while other parameters are unknown. Using Mathematica 7.0 as a programming language to fit the values, a number of sets can

be obtained. The parameter set that we used for calculating the mass and mass splitting are:

$g = 0.1, g' = 0.1, h = 0.07, \delta_H = 4, \delta_S = 431, \Delta H = 144, \Delta S = 126, a_H = 1.1, a_S = 0.21, \Delta H^a = -0.04, \Delta S^a = 0.14$. Below is given the table of masses calculated in our work and compared to experimentally available masses. All the observed masses are taken from Particle Data Group [24]. D_0^0 mass has also been measured at Belle [7] and is equal to 2308 ± 36 similarly other masses are also available but our results are matching more closely with that given in PDG [24] and the fitted masses are given in table 5.2:

Table 5.2

Sr. No.	State	Mesonic State(J^P)	Residual Mass(MeV)	Real Mass(MeV)	Exp. Mass(MeV)
1.	m_{H1}	$D^{0+}(0^-)$	-105.97	1867.04	1867.21
2.	m_{H3}	$D_s^0(0^-)$	-5.27	1967.74	1968.47 ± 0.33
3.	m_{S1}	$D_0^0(0^+)$	329.27	2314.08	2318 ± 29
4.	m_{S3}	$D_{s0}^0(0^+)$	341.07	2314.55	2317.8 ± 0.6
5.	m_{H1}^*	$D^{*0+}(1^-)$	44.46	2017.24	2008.6
6.	m_{H3}^*	$D_s^{*+}(1^-)$	136.48	2109.49	2112.1
7.	m_{S1}^*	$D_1^0(1^+)$	460.51	2433.52	2438 ± 22
8.	m_{S3}^*	$D_{s1}^+(1^+)$	483.28	2456.29	2459.5 ± 0.6

Table 5.2 Masses of strange and non-strange charm mesonic states

Similarly, the hyperfine and mass splittings for low lying charm mesonic states are found using mass values and compared with experimental values on mass splittings. The table 5.3 with different states and their mass and hyperfine splittings is shown.

Table 5.3

Spin Splittings			Mass Splittings		
State	Experimental Value(MeV)	Calculated Value(MeV)	State	Experimental Value(MeV)	Calculated Value(MeV)
$D_s^* - D_s(1^- - 0^-)$	143.8	141.4	$D_s^* - D_u^*(1^- - 1^-)$	105.4	94.4
$D_{1s}^* - D_{0s}(1^+ - 0^+)$	141.9	138.7	$D_{0s}^* - D_{0u}^*(0^+ - 0^+)$	98.8	96.6
$D_u^* - D_u(1^- - 0^-)$	142.2	143.9	$D_{1s}^* - D_{1u}^*(1^+ - 1^+)$	21.3	20.3
$D_{1u}^* - D_{0u}(1^+ - 0^+)$	130	126.5	$D_s - D_u(0^- - 0^-)$	9.4	8.1

Table 5.3 Masses and hyperfine splittings

The residual mass difference $\delta S - \delta H$ represents the shift between the center of mass of even and odd parity mesons. The terms with ΔH and ΔS is with no insertion of light quark mass matrix and is responsible for hyperfine splitting therefore from experimental data of hyperfine splitting for low as well as excited states, we fit the above mentioned parameters between 100-150 MeV. The terms proportional to 'aH' and ' σ ' respects heavy quark spin symmetry and the terms proportional to 'a' result in $SU(3)_V$ mass splitting among the vector mesons therefore sufficient variation for mass splitting is observed for this term. The same behavior is expected for the term aS. Moreover, from mass splitting, $s - u \sim 100$ therefore aH, aS and other chiral contributions are taken of order unity. The coupling constants which provides the strength of interaction between the heavy mesons and pseudoscalar light mesons, is one of the basic parameters in heavy hadron chiral Lagrangian. The most important parameter affecting the higher order corrections are the coupling constants between different low and excited states respectively. Various models propose different values of constants g , g' and h . Coupling constants have been estimated by using

several quark models [25] [26] [27]. QCD sum rule approach gives an estimate of $g = 0.44 \pm 0.16$ [28]. The combined fit from various relativistic and non relativistic quark model along with QCD sum rule provide a best fit of $g \sim 0.38$ with an uncertainty around $\pm 20\%$ [28] [29]. The radiative and pion decay width of vector mesons constrain $g^2 < 0.5$ [29]. One more approach is to imbibe chiral corrections to decays of heavy mesons and inclusion to counter terms provides the following best fit data as $g = +0.66_{-0.06}^{+0.08}, g' = -0.06_{-0.04}^{+0.03}$ and $h = +0.47_{-0.06}^{0.07}$ [27] where all these parameters are computed through a renormalization scheme with a renormalization scale set to $\mu = 1\text{GeV}$. In the present work, we try to constrain these bare couplings by analyzing the existing data available in literatures. In spite of these, various mass and hyperfine splittings for different levels verifies that all the hyperfine splittings are within range of 140-145 MeV and they can be unrelated by heavy quark symmetry. The mass difference between strange and non-strange mesons whose quantum numbers are identical is expected to be of the order of 100 MeV. It is indeed the similar case for the ground state, but for excited case the value decreases to small extent. For heavy quarks, it is also possible to parameterize in HQET, the non-perturbative affects to a given order in $\frac{1}{m_Q}$ expansion in terms of a few unknown constants where these unknown constants can be obtained from the experiments. These unknown constants are actually the non-perturbative QCD parameters and once their values in hand, it is possible to calculate masses of various excited states in the heavy meson spectrum. We find it more suitable to obtain bottom meson spectrum due to lesser values of mass splitting for bottom mesons than that of charm mesons. B meson masses in the heavy quark effective theory are given in terms of a single non-perturbative parameter $\bar{\Lambda}$ and non-perturbative parameters of QCD, λ_1 and λ_2 .

5.4 Bottom Meson Masses

An interesting class of non-perturbative quantum chromodynamics, now a day, is the study of mesons containing a heavy quark with a light meson using heavy quark effective theory [HQET]. In the light of heavy quark effective theory, spin and parity of the heavy quark decouples from that of light quark. Thus, the properties of heavy hadrons are independent of spin and flavor of heavy meson and HQET provides a basis for estimation of several properties of heavy mesons. HQET provides a systematic expansion of mass of heavy quark in terms of QCD parameters in terms of a few unknown constants where these unknown constants can be obtained from the experiments. These un-known constants are actually the non-perturbative QCD parameters and once their values in hand, it is possible to calculate masses of various excited states in the heavy meson spectrum. We find it more suitable to obtain bottom meson spectrum due to lesser values of mass splitting for bottom mesons than that of charm mesons. B meson masses in the heavy quark effective theory are given in terms of a single non-perturbative parameter $\bar{\Lambda}$ and non-perturbative QCD parameters λ_1 and λ_2 . In general, the mass of a hadron containing a heavy quark Q obey an expansion of the form:

$$m_X = m_Q + \bar{\Lambda} + \frac{\Delta m^2}{2m_Q} + O\left(\frac{1}{m_Q}\right) \quad (5.9)$$

where X is the hadron either in the ground state(H) and an excited state(S), m_Q is the mass of heavy quark whereas $\Delta m^2 = -\lambda_1 + 2[J(J+1) - \frac{3}{2}]\lambda_2$. J is the total spin of meson. The two parameters λ_1 and λ_2 are non-perturbative parameters of QCD and can be estimated by fitting the theoretical and experimental data and

their uncertainties [30] [31]. A good estimation of these parameters may reduce theoretical errors and uncertainties up to significant level. Although there exists several predicted values in literature [32] [33] for $\bar{\Lambda}$ and λ_1 . In all the cases, the value for λ_1 lie close to 1.0 GeV. The lowest and highest bounds on the parameters set can be found by using different values from the literature [34]. $\bar{\Lambda}$ and λ_1 can't be measured by simple mass measurements on dimensional grounds [35]. λ_1 is independent of m_Q and λ_2 depends on m_Q logarithmically. λ_1 and λ_2 are considered to possess same values for all states in a given spin-flavor multiplet and of the order of Λ_{QCD} [35]. The term $\frac{\lambda_1}{m_Q}$ arises from kinetic energy of the heavy quark inside hadrons. The magnetic interaction term λ_2 describes the interaction of the heavy quark spin with the gluon field and responsible for $B^* - B$ and $D^* - D$ splitting [35]. The parameters are here allowed to vary within their allowed values and then some of the sets that reproduce the masses with minimum error are chosen. One such set is shown as $\bar{\Lambda} = 0.6\text{GeV}$ which is close to global fitted value 0.57 ± 0.06 given by [32] and $\lambda_1 = -0.18 + 0.06\text{GeV}^2$ for u and d light quarks. Assuming $SU(3)$ breaking, $\lambda_s = 0.7\text{GeV}$ and $\lambda_{1s} = -0.18 \pm 0.06\text{GeV}^2$. For the first excited and ground state doublet of bottom and charm mesons, the formula for the difference of spin-averaged masses of $J^P = 0^-, 1^-$ and $J^P = 0^+, 1^+$ states in the bottom sector is:

$$\bar{m}_S(Q) - \bar{m}_H(Q) = \bar{\Lambda}^S - \bar{\Lambda}^H - \frac{\lambda_1^S}{2m_Q} + \frac{\lambda_1^H}{2m_Q} \quad (5.10)$$

We use charm meson results to find the masses of higher bottom meson states, the hyperfine operators should be rescaled by $\frac{m_c}{m_b}$. This leads to the formula for splitting

of the even and odd-parity states in the bottom sector.

$$\bar{m}_S^b - \bar{m}_H^b = \bar{m}_S^c - \bar{m}_H^c - (\lambda_1^S - \lambda_1^H) \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right) \quad (5.11)$$

where $\bar{m}_H^{(Q)} = \frac{3(m_{H^*}^{(Q)} + m_H^{(Q)})}{4}$ and $\bar{m}_S^{(Q)} = \frac{3(m_{S^*}^{(Q)} + m_S^{(Q)})}{4}$

Using $\lambda_1^H = -0.18 \pm 0.06 \text{GeV}^2$, $\lambda_1^{\frac{3}{2}} - \lambda_1^H = -0.23 \text{GeV}^2$ where $\lambda_1^{\frac{3}{2}}$ is the λ_1 matrix element for the $j^P = \frac{3}{2}^+$ doublet. We expect that the kinetic energy of the heavy quark in the $j^P = \frac{1}{2}^+$ state to be comparable to $j^P = \frac{3}{2}^+$ state. We take $\lambda_1^s - \lambda_1^H = -0.2 \pm 0.1 \text{GeV}^2$. The mass of charm and bottom quarks, $m_c = 1.29_{-0.11}^{+0.05} \text{GeV}$, $m_b = 4.67 \pm 0.18 \text{GeV}$ to find

$$\bar{m}_S^b - \bar{m}_H^b = \bar{m}_S^c - \bar{m}_H^c - 56.1 \pm 25 \text{MeV} \quad (5.12)$$

$$\frac{m_H^{*(b)} - m_H^b}{m_H^{*(c)} - m_H^c} = \frac{m_S^{*(b)} - m_S^b}{m_S^{*(c)} - m_S^c} = \frac{m_c}{m_b} \quad (5.13)$$

Using Particle Data Group [24], we can calculate specifically their spin-averaged masses of the $j^P = \frac{1}{2}^-$ and $j^P = \frac{1}{2}^+$ meson as,

$$\begin{aligned} m_{H1}^{*(b)} - m_{H1}^{(b)} &= \frac{m_c}{m_b} (m_{H1}^{*(c)} - m_{H1}^{(c)}) = 39.03 \pm 0.12 \text{MeV} \\ m_{H3}^{*(b)} - m_{H3}^{(b)} &= \frac{m_c}{m_b} (m_{H3}^{*(c)} - m_{H3}^{(c)}) = 39.03 \pm 0.12 \text{MeV} \\ m_{S1}^{*(b)} - m_{S1}^{(b)} &= \frac{m_c}{m_b} (m_{S1}^{*(c)} - m_{S1}^{(c)}) = 39.72 \pm 0.5 \text{MeV} \\ m_{S3}^{*(b)} - m_{S3}^{(b)} &= \frac{m_c}{m_b} (m_{S3}^{*(c)} - m_{S3}^{(c)}) = 39.08 \pm 0.5 \text{MeV} \end{aligned}$$

Using the values from the bottom non-strange sector $m_{H1}^{(b)} = 5279.1 \pm 0.4 \text{MeV}$ and $m_{H1}^{*(b)} = 5325.1 \pm 0.5 \text{MeV}$, $m_{H1}^{(b)}$ is found out to be $5313.62 \pm 0.03 \text{MeV}$, $m_{H3}^b =$

$5366.3 \pm 0.6 \text{ MeV}$ is given in particle data group [24]. The value for $m_{H3^*}^{(b)}$ will be:

$$m_{H3}^{*(b)} = \frac{m_c}{m_b} (m_{H3}^{*(c)} - m_{H3}^{(c)}) + m_{H3}^{(b)} = 5404.82 \pm 0.7 \text{ MeV} \quad (5.14)$$

From the relation, we get the spin-averaged masses of excited B-mesons:

$$m_{S1}^{- (b)} = (m_{S1}^{- (c)} - m_{H1}^{- (c)} - 56.1 \pm 25 \text{ MeV}) + m_{H1}^{- (b)} = 5705.44 \pm 28 \text{ MeV} \quad (5.15)$$

Similarly, for the strange bottom and charm mesons, the strange bottom meson relation leads to:

$$m_{S3}^{- (b)} = (m_{S3}^{- (c)} - m_{H3}^{- (c)} - 56.1 \pm 25 \text{ MeV}) + m_{H1}^{- (b)} = 5691.8 \pm 27 \text{ MeV} \quad (5.16)$$

Equations are solved to get the values for the masses of excited B mesons as shown in table 5.4:

Table 5.4

Sr. No.	State	Calculated Mass[MeV]	Experimental [24] Value[MeV]	Potential [36] Model[MeV]	Relativistic [37] Model[MeV]
1.	$m_{S1}^{(b)}$	5691.6 ± 345	5366.7 ± 0.24	5697	5738
2.	$m_{S1}^{*(b)}$	5709.02 ± 50	5415.8 ± 1.8	5740	5757
3.	$m_{S3}^{(b)}$	5662.48 ± 27	5716	5841
4.	$m_{S3}^{*(b)}$	5701.57 ± 27	5828.7 ± 0.4	5760	5859

Table 5.4 Bottom meson masses

Comparison of our results with other models predicts the results to be matching well. In the charm and bottom systems, one knows experimentally [24]

$$m_{B^*} - m_B \sim 46 \text{ MeV},$$

$$m_{D^*} - m_D \sim 142 \text{ MeV},$$

$$m_{D_{s^*}} - m_{D_s} \sim 142 \text{ MeV}$$

These mass splitting are in fact reasonably small. To be more specific, at order $\frac{1}{m_Q}$ one expects hyperfine corrections to resolve the degeneracy, for instance $m_{B^} - m_B \propto \frac{1}{m_b}$. This leads to the refined prediction:*

$$m_{B^*}^2 - m_B^2 \approx m_{D^*}^2 - m_D^2 \approx \text{const.}$$

$$m_{B^*}^2 - m_B^2 \approx 0.8 \text{ GeV}^2$$

$$m_{D^*}^2 - m_D^2 \approx 0.8 \text{ GeV}^2$$

The spin symmetry also predicts that for strange mesons

$$m_{B_{s^*}}^2 - m_{B_s}^2 \approx m_{D_{s^*}}^2 - m_{D_s}^2 \approx \text{constt.} \quad (5.17)$$

But this constant could in principle be different from that for non strange mesons, since the flavor quantum numbers of the light degree of freedom are different in both cases. Experimentally, however,

$$m_{D_{s^*}}^2 - m_{D_s}^2 \approx m_{D^*}^2 - m_D^2 \quad (5.18)$$

Indicating that to first approximation, hyperfine corrections are independent of the flavor of the "brown muck". One then expects the corresponding states in the bottom sector is

$$m_{B_{2*}}^2 - m_{B_1}^2 \approx m_{D_{2*}}^2 - m_{D_1}^2 \approx 0.17\text{GeV}^2 \quad (5.19)$$

The fact that above mass splitting is smaller for the ground-state mesons is not unexpected. For instance, in the non-relativistic constituent quark model, the light antiquark in these excited mesons is in a p-wave state and its wave function at the location of the heavy quark vanishes. Hence, in this model hyperfine corrections are strongly suppressed. A typical prediction of the flavor symmetry is that the "excitation energies" for states with different quantum numbers of the light degrees of freedom are approximately the same in the charm and bottom systems. For instance, one expects

$$m_{B_s} - m_B \approx m_{D_s} - m_D \approx 100\text{MeV} \quad (5.20)$$

$$m_{B_1} - m_B \approx m_{D_1} - m_D \approx 557\text{MeV} \quad (5.21)$$

$$m_{B_2}^* - m_B \approx m_{D_2}^* - m_D \approx 593\text{MeV} \quad (5.22)$$

The first relation has been confirmed very nicely by the discovery of the B_s meson by the ALEPH collaboration at Large Electron Positron Collider [13]. The observed mass, $m_{B_s} = 5.369 \pm 0.006\text{GeV}$, corresponds to an excitation energy $m_{B_s} - m_B = 90 \pm 6\text{MeV}$.

5.5 Conclusion

The spin-flavor symmetry in HQET leads to many interesting relations between the properties of hadrons containing a heavy quark. The most direct consequences concern the spectroscopy of such states. In the $m_Q \rightarrow \infty$ limit, the spin of the heavy quark and the total angular momentum of the light degree of freedom are separately conserved by the strong interactions. Because of heavy quark symmetry, the dynamics is independent of the spin and mass of the heavy quark. Hadronic state can thus be classified by the quantum numbers (flavor, spin, parity) of the light degrees of freedom. The importance of QCD parameters lies in the fact that it becomes comparatively easier to find using data of mass splitting and hyperfine splitting of heavy mesons. Since, $\bar{\Lambda}$ has the same value for all particles in a spin-flavor multiplet, then $(\bar{\Lambda}^s - \bar{\Lambda}^H)$ can be taken to possess the same values for B and D mesons. The spin symmetry predicts that for fixed $j \neq 0$, there is a doublet of degenerate states with total spin $J \pm \frac{1}{2}$ and the flavor symmetry relates the properties of states with different heavy-quark flavor. This leads to the prediction that mass splitting among the various doublets are independent of heavy quark flavor. Our main purpose, in this work is to find the masses of the non-strange excited states from the observed experimental values of all the ground states and excited strange mesons using the consequences of spin-flavor symmetry. Experimental data of ground state charmed meson is used to constrain the symmetry conserving and symmetry breaking parameter in the effective Lagrangian to predict the masses for non strange low lying excited states for even and odd parities. The coupling constants and the mass parameters in effective Lagrangian reproduce QCD in specific limits and represent important input parameters for the description of the hadrons properties like masses and mass splitting of charm

meson have been calculated in HQET and matched with the available results. The excited meson spectra calculated in the present chapter for bottom mesons is thus found to be matching well with other models [13] [37]. Moreover, spin and flavor symmetry leads to various predictions for mass and hyperfine splitting which also has been discussed. We also discuss briefly the predictions that can be made related to hyperfine splitting for strange as well as non-strange mesons. Since, $\bar{\Lambda}$ has the same value for all particles in a spin-flavor multiplet, then $(\bar{\Lambda}^S - \bar{\Lambda}^H)$ can be taken to possess the same values for B and D mesons. To summarize, it can also be said that the extraction of various coupling constants from the masses of these mesons can be further helpful to find other physical parameters like decay widths and form factors. An improved measurement of branching ratios can be utilized to find accurate values of coupling constants. Thus we are motivated to study the branching ratios of latest resonances observed at BaBar and LHC collaborations.

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Chapter 6

Strong Decays of Charm Mesons

6.1 Introduction

In the previous chapter, the hadrons containing a single heavy quark were analyzed in a framework which is formulated for N_f heavy quarks with mass $m_Q \gg \Lambda_{QCD}$ using heavy quark effective theory [1] to find the masses of some of the charm and bottom mesonic states. The heavy light meson spectrum is one of the recent interest to place for the various particles at different resonances and energies. In the limit $m_Q \rightarrow \infty$, heavy quark exhibits spin and flavor symmetry. The heavy quark spin-flavor symmetry is exploited further to explore several hadronic properties like in calculation of decay widths, branching ratios, masses etc. The typical energy scale of QCD is Λ_{QCD} . The light quarks have masses very smaller than Λ_{QCD} whereas heavy quarks have mass very much greater than the energy scale of QCD. The symmetry exhibited by heavy quarks can be synthesized with symmetry of light quarks and can leads to the calculation of various signatures like decays and form-factors for a heavy-light system. In other words, spin and flavor symmetry of HQET is used together

with chiral $SU(3)_L X SU(3)_R$ symmetry to form an effective Lagrangian whose basic fields are heavy and light quark operators. The basic Lagrangian terms include kinetic energy arising from the off-shell motion of heavy quark and chromo-magnetic interaction of heavy quark spin with the gluon field. The effective Lagrangian describing strong interaction of heavy mesons with pionic fields by expanding the fields $\xi(x)$ and taking the traces. The heavy meson chiral Lagrangian at the leading order is described in details in the next section. The motivation for present work arises due to recently observed charm and bottom meson states by experimental collaborations like BaBar, LHCb and CLEO [4]. Recently, some excited charm meson states were observed which are $D(2550), D(2600), D(2750)$ and $D(2760)$ in the decay channels $D^0(2550) \rightarrow D^{*+}\pi^-, D^0(2600) \rightarrow D^{*+}\pi^-, D^+\pi^-, D^0(2750) \rightarrow D^+\pi^-, D^+(2600) \rightarrow D^0\pi^+$ and $D^+(2760) \rightarrow D^0\pi^+$ in the inclusive $e^+e^- \rightarrow c\bar{c}$ interactions by BaBar Collaboration [2]. The most suitable spin-parity assignments for $D(2750), D(2760)$ is $(2^-, 3^-)$ or $1D$ and for $D(2550), D(2600)$ is $(0^-, 1^-)$ i.e. $2S$ state respectively. LHCb collaboration [3] observed some new resonances in addition to above i.e. $D_J(3000)^{+,0}$ and $D_J^*(3000)^0$ around 3GeV in association with $D_2^*(2460)^0$ and $D_J^*(2760)^0$ and exists in the $D^+\pi^-$ invariant mass spectrum. The states, $D_J^*(3000)^+, D_2^*(2460)^+$ and $D_J^*(2760)^+$ were observed in $D_0\pi^+$ spectrum whereas the states measured in $D^{*+}\pi^-$ spectrum were $D_1(2420)^0, D_2^*(2460)^0, D_J^*(2760)^0, D_J(2580)^0, D_J(2740)^0$ and $D_J(3000)^0$ respectively. A table for the properties of recently observed states has been shown in table 6.4. Similar is the case with beauty sector. Very recently CDF Collaboration [4] has found evidence for a new resonance $B(5970)$ simultaneously in $B^0\pi^+$ and $B^+\pi^-$ mass distributions with a significance of 4.4σ standard deviations and further reported the first study of resonances with orbitally excited B^+ mesons

and updated measurement of orbitally excited B^0 and B_s^0 mesons. The branching ratio for $B_{s2}^{*0} \rightarrow B^{*+} K^-$ decays is also measured. The masses of new $B(5970)$ [4] measured resonances are $5978 \pm 5(\text{stat}) \pm 12(\text{syst}) \text{MeV}/c^2$ for neutral state and $5961 \pm 5(\text{stat}) \pm 12(\text{syst}) \text{MeV}/c^2$ for charged asymmetry into $B\pi$ states. This state may be proposed as belonging to radially excited bottom meson family. Therefore, in past decades, we faced several ground as well as excited states of charm meson family such as discovery of $D_{sJ}(2460), D_{sJ}(2632)$ and $D_{sJ}(3040)$ etc. In bottom meson family, we witness some new states such as $B(5279), B^*(5325)$ for $n=1$ family in $(0^-, 1^-)$ doublet. In the infinite heavy quark mass limit, a heavy light system $Q\bar{q}$ can be classified into doublets depending upon their quantum numbers. The heavy light mesonic system form a degenerate doublet of ground state with $J = 0, 1$ and negative parity denoted as D and D^* for charm meson. The first excited states 0^+ and 1^+ heavy light mesons are the quantum numbers of the $s_l^p = \frac{1}{2}^+$ doublet. There is also an excited doublet of heavy meson with $J^P = 1^+$ and 2^+ . Similarly, other excited mesonic states have their J^P states. In this article, we identify the recent charmed meson states $D_J(2550), D_J^*(2600), D_J(2740), D_J^*(2760), D_J(3000)$ and $D_J^*(3000)$ with their J^P assignment. These states were observed by LHCb collaboration and predicting their decay widths and masses. We study strong decays of these charmed mesons to ground state mesons along with the emission of pseudo-scalar pions in heavy quark effective theory in the leading order approximations. Although the same work has been studied by [16] but we extend their work by fitting the experimental data to find the coupling constants in various strong decays. Also, we include two additional possibilities for assignment of J^P states to $D_J^*(3000)$ and $D_J(3000)$. In the end, we also try to justify all the possible assignments to $D_J^*(3000)$ and $D_J(3000)$ by analyzing

their branching ratios graphically.

6.2 The Lagrangian for Strong Decays to Heavy Mesons

A single field H_a where it annihilates the $s_l = \frac{1}{2}^-$ meson doublet, pseudoscalar and vector mesons can be mentioned as:

$$H_a = \frac{1 + \not{\psi}}{2} (P_a^{*\mu} \gamma_\mu - P_a \gamma_5) \quad (6.1)$$

Here a is the $SU(3)$ index. In charm mesons sector, H_a consists of the D^0, D^+, D_s^+ pseudo-scalar mesons and D^{*0}, D^{*+}, D_s^{*+} vector mesons. The lowest lying excited states are the $J^P = 0^+$ and 1^+ i.e. $s_l^P = \frac{1}{2}^+$ doublet and represented by the fields S_a [5]. The fields for excited spin doublets are mentioned below:

$$S_a = \frac{1 + \not{\psi}}{2} (P_{1a}^\mu \gamma_\mu \gamma_5 - P_{0a}^*) \quad (6.2)$$

$$T_a^\mu = \frac{1 + \not{\psi}}{2} (P^{*\mu\nu} \gamma_\nu - P_{1a\nu} \sqrt{\frac{3}{2}} \gamma_5 [g^{\mu\nu} - \frac{1}{3} \gamma^\nu (\gamma^\mu - v^\mu)]) \quad (6.3)$$

$$X_a^\mu = \frac{1 + \not{\psi}}{2} (P_{2a}^{*\mu\nu} \gamma_5 \gamma_\nu - P_{1a\mu}^* \sqrt{\frac{3}{2}} [g^{\mu\nu} - \frac{1}{3} \gamma^\nu (\gamma^\mu + v_\mu)]) \quad (6.4)$$

$$Y_a^{\mu\nu} = \frac{1 + \not{\psi}}{2} (P_{3a}^{*\mu\nu\sigma} \gamma_\sigma - P_{2a}^{\alpha\beta} \sqrt{\frac{5}{3}} \gamma_5 [g_\alpha^\mu g_\beta^\nu - \frac{g_\beta^\nu \gamma_\alpha (\gamma^\mu - v^\mu)}{5} - \frac{g_\alpha^\mu \gamma_\beta (\gamma^\nu - v^\nu)}{5}]) \quad (6.5)$$

$$Z_a^{\mu\nu} = \frac{1 + \not{\psi}}{2} (P_{3a}^{*\mu\nu\sigma} \gamma_5 \gamma_\sigma - P_{2a}^{*\alpha\beta} \sqrt{\frac{5}{3}} [g_\alpha^\mu g_\beta^\nu - \frac{g_\beta^\nu \gamma_\alpha (\gamma^\mu + v^\mu)}{5} - \frac{g_\alpha^\mu \gamma_\beta (\gamma^\nu + v^\nu)}{5}]) \quad (6.6)$$

$$R_a^{\mu\nu\rho} = \frac{1+\psi}{2} (P_{4a}^{*\mu\nu\rho\sigma} \gamma_5 \gamma_\sigma - P_{3a}^{\alpha\beta\tau} \sqrt{\frac{7}{4}} [g_\alpha^\mu g_\beta^\nu g_\tau^\rho - \frac{g_\beta^\nu g_\tau^\rho \gamma_\alpha (\gamma^\mu - v^\mu)}{7} - \frac{g_\alpha^\nu g_\tau^\rho \gamma_\beta (\gamma^\mu - v^\mu)}{7} - \frac{g_\alpha^\mu g_\beta^\nu \gamma_\tau (\gamma^\rho - v^\rho)}{7}]) \quad (6.7)$$

The super fields H_a contain s -wave mesons whereas S_a, T_a contain the p -wave mesons. The light pseudoscalar mesons are described by the fields $\xi = \exp \frac{iM}{f}$. The pion octet is introduced by the vector and axial combinations $V^\mu = \frac{1}{2} \xi \partial^\mu \xi^\dagger + \xi^\dagger \partial^\mu \xi$ and $A^\mu = \frac{1}{2} \xi \partial^\mu \xi^\dagger - \xi^\dagger \partial^\mu \xi$. We choose $f_\pi = 130 \text{ MeV}$. Here, all traces are taken over Dirac spinor indices, light quark $SU(3)_V$ flavor indices $a = u, d, s$ and heavy quark flavor indices $Q = c, b$. The Dirac structure of chiral Lagrangian has been replaced by velocity vector v . At the leading order, the heavy meson chiral Lagrangian terms $L_H, L_S, L_T, L_X, L_Y, L_Z, L_R$ for the strong decays to the $D^{(*)}\pi, D^{(*)}\eta$ and $D_s^{(*)}K$ states can be written as:

$$L_H = g_H \text{Tr} \{ \bar{H}_a H_b \gamma_\mu \gamma_5 A_{ba}^\mu \} \quad (6.8)$$

$$L_S = g_S \text{Tr} \{ \bar{H}_a S_b \gamma_\mu \gamma_5 A_{ba}^\mu \} + h.c., \quad (6.9)$$

$$L_T = \frac{g_T}{\Lambda} \text{Tr} (\bar{H}_a T_b^\mu (D_\mu \not{A} + i \not{D} A_\mu)_{ba} \gamma_5) + h.c., \quad (6.10)$$

$$L_X = \frac{g_X}{\Lambda} \text{Tr} (H_a X_b^\mu (i D_\mu \not{A} + i \not{D} A_\mu)_{ba} \gamma_5) + h.c., \quad (6.11)$$

$$L_Y = \frac{1}{\Lambda^2} \text{Tr} (\bar{H}_a Y_b^{\mu\nu} [k_1^Y \{D_\mu, D_\nu\} A_\lambda + k_2^Y (D_\mu D_\lambda A_\nu + D_\nu D_\lambda A_\mu)]_{ba} \gamma^\lambda \gamma_5) + h.c., \quad (6.12)$$

$$L_Z = \frac{1}{\Lambda^2} \text{Tr} (\bar{H}_a Z_b^{\mu\nu} [k_1^Z \{D_\mu, D_\nu\}, D_\nu\} A_\lambda + k_2^Z (D_\mu, D_\lambda A_\nu + D_\nu D_\lambda A_\mu)]_{ba} \gamma^\lambda \gamma_5) + h.c., \quad (6.13)$$

where $D_\mu = \partial_\mu + \nu_\mu$, $\{D_\mu, D_\nu\} = D_\mu D_\nu + D_\nu D_\mu$, $\{D_\mu, D_\nu, D_\rho\} = D_\mu D_\nu D_\rho + D_\mu D_\rho D_\nu + D_\nu D_\rho D_\mu + D_\rho D_\mu D_\nu + D_\rho D_\nu D_\mu$, These terms describe the transitions of positive and negative parity mesons with the emission of light pseudo-scalar mesons. The mixing

angle between two states are determined by including spin symmetry violating corrections in the Lagrangian. The term should respect parity and time reversal and may be of generic form as written below.

$$L_{d1} = \frac{h_1}{2m\Lambda} Tr[\bar{H}\sigma^{\mu\nu}T^\alpha\sigma_{\mu\nu}\gamma^k\gamma^5(iD_\alpha A_\kappa + iD_\kappa A_\alpha)] + h.c. \quad (6.14)$$

The corresponding operator for the mixing of 1^+ in $2S$ and $1D$ respectively is due to spin symmetry violating effect and can be written as: $L_{mix} = g_1 Tr[\bar{H}\phi_s^{\mu\nu}X_\mu\sigma_{\nu\alpha}V^\alpha] + h.c..$

6.3 Strong Decay Width Formula and Coupling Constants

From the chiral Lagrangian terms, we can obtain the decay widths Γ for strong decays to final states $D^{(*)}\pi$, $D^{(*)}\eta$, $D_s^{(*)}K$ where the symmetry breaking scale $\Lambda_X = 1\text{GeV}$. The expression for decay widths if we consider various doublets which the decaying meson belongs to are as follows where M denotes the emission of pseudoscalar mesons i.e. π , K and η fields.

$(0^-, 1^-)$ to $(0^-, 1^-) + M$

$$\Gamma(1^- \rightarrow 0^-) = C_M \frac{g_H^2}{6\pi f_\pi^2} \frac{M_f}{M_i} |p_M^\rightarrow|^3 \quad (6.15)$$

$$\Gamma(1^- \rightarrow 1^-) = C_M \frac{g_H^2}{3\pi f_\pi^2} \frac{M_f}{M_i} |p_M^\rightarrow|^3 \quad (6.16)$$

$(0^+, 1^+) \text{ to } (0^-, 1^-) + M$

$$\Gamma(1^+ \rightarrow 1^-) = C_M \frac{g_S^2}{2\pi f_\pi^2} \frac{M_f}{M_i} |p_M^\rightarrow| [m_M^2 + |p_M^\rightarrow|^2] \quad (6.17)$$

$$\Gamma(1^+ \rightarrow 0^-) = C_M \frac{g_S^2}{2\pi f_\pi^2} \frac{M_f}{M_i} |p_M^\rightarrow| [m_M^2 + |p_M^\rightarrow|^2] \quad (6.18)$$

$(1^+, 2^+) \text{ to } (0^-, 1^-) + M$

$$\Gamma(1^+ \rightarrow 1^-) = C_M \frac{2g_T^2}{3\pi f_\pi^2 \Lambda^2} \frac{M_f}{M_i} |p_M^\rightarrow|^5 \quad (6.19)$$

$$\Gamma(2^+ \rightarrow 0^-) = C_M \frac{4g_T^2}{15\pi f_\pi^2 \Lambda^2} \frac{M_f}{M_i} |p_M^\rightarrow|^5 \quad (6.20)$$

$$\Gamma(2^+ \rightarrow 1^-) = C_M \frac{2g_T^2}{5\pi f_\pi^2 \Lambda^2} \frac{M_f}{M_i} |p_M^\rightarrow|^5 \quad (6.21)$$

$(1^-, 2^-) \text{ to } (0^-, 1^-) + M$

$$\Gamma(1^- \rightarrow 0^-) = C_M \frac{4g_X^2}{9\pi f_\pi^2 \Lambda^2} \frac{M_f}{M_i} |p_M^\rightarrow|^3 |m_M^2 + |p_M^\rightarrow|^2| \quad (6.22)$$

$$\Gamma(1^- \rightarrow 1^-) = C_M \frac{2g_X^2}{9\pi f_\pi^2 \Lambda^2} \frac{M_f}{M_i} |p_M^\rightarrow|^3 |m_M^2 + |p_M^\rightarrow|^2| \quad (6.23)$$

$$\Gamma(2^- \rightarrow 1^-) = C_M \frac{2g_X^2}{3\pi f_\pi^2 \Lambda^2} \frac{M_f}{M_i} |p_M^\rightarrow|^5 |m_M^2 + |p_M^\rightarrow|^2| \quad (6.24)$$

$(2^-, 3^-)$ to $(0^-, 1^-) + M$

$$\Gamma(2^- \rightarrow 1^-) = C_M \frac{4g_Y^2}{15\pi f_\pi^2 \Lambda^4} \frac{M_f}{M_i} |p_M^7| \quad (6.25)$$

$$\Gamma(3^- \rightarrow 0^-) = C_M \frac{4g_Y^2}{35\pi f_\pi^2 \Lambda^4} \frac{M_f}{M_i} |p_M^7| \quad (6.26)$$

$$\Gamma(3^- \rightarrow 1^-) = C_M \frac{16g_Y^2}{105\pi f_\pi^2 \Lambda^4} \frac{M_f}{M_i} |p_M^7| \quad (6.27)$$

The coefficients C_M are different for the various light pseudoscalar mesons: $C_{\pi^+} = C_{K^+} = 1, C_{\pi^0} = C_{K_s} = \frac{1}{2}, C_\eta = \frac{1}{6}$. p_M is the three momentum of M . The higher order corrections to heavy quark limit can also be considered by adding terms of the order $\frac{1}{m_Q}$ with some un-known constants. The decay rates depend upon effective coupling constants. The decay rates depend upon effective coupling constants. The parameters used in the above expressions for decay widths are taken from the particle data group [6]. Thus the numerical values of decay widths comes out in terms of coupling constants g_H, g_Y, g_X etc. Here the first radial excitation of D^* is represented as \tilde{D}^* . The first radially excited state of H is governed by the decay constant \tilde{g}_H which can be fitted to experimental data within mass range of 2600-2700 MeV. Coupling constants can either be determined theoretically or by fitting the experimental data. However, various quark models [7, 8] and sum rule (eg. QCD sum rules) [9, 11, 13] techniques predict the coupling constants. Another possible method is to use lattice QCD [12] which incorporate QCD in their first principle. Using experimental data of decay widths and branching ratios as input, one can fit the experimental data to find the effective coupling constants. The coupling constants play an important role

in heavy quark phenomenology. They are directly related to charm meson strong decays and are further useful to explore other decays of charm mesons involving pionic emissions.

6.4 Spin-parity Analysis for Non-strange Charm Meson States

The recent experimental data of charm meson states from LHCb and BaBar collaboration motivates us to find the best fit values of coupling constants in strong decays.

The table 6.4 mentions the recent experimental data of non-strange charm mesons.

Sr.No	Charm Meson state	Mass[MeV][LHCb]	Mass[MeV][BaBar]	Width[MeV][LHCb]	Width[MeV][BaBar]	Decay Channel
1	$D_J^*(2650)^0$	$2649.2 \pm 3.5 \pm 3.5$	$2608.7 \pm 2.4 \pm 2.5$	$140.2 \pm 17.1 \pm 18.6$	$93 \pm 6 \pm 13$	$D^{*+}\pi^-$
2	$D_J^*(2760)^0$	$2761.1 \pm 5.1 \pm 6.5$	$2763.3 \pm 2.3 \pm 2.3$	$74.4 \pm 3.4 \pm 37.0$	$60.9 \pm 5.1 \pm 3.6$	$D^{*+}\pi^-$
3	$D_J(2580)^0$	$2579.5 \pm 3.4 \pm 5.5$	$2539.4 \pm 4.5 \pm 6.8$	$177.5 \pm 17.8 \pm 46.0$	$130 \pm 12 \pm 13$	$D^{*+}\pi^-$
4	$D_J(2740)^0$	$2737.0 \pm 3.5 \pm 11.2$	$2752.4 \pm 1.7 \pm 2.7$	$73.2 \pm 13.4 \pm 25.0$	$71 \pm 6 \pm 11$	$D^{*+}\pi^-$
5	$D_J(3000)^0$	2971.8 ± 8.7		188.1 ± 44.8		$D^{*+}\pi^-$
6	$D_J^*(2760)^0$	$2760.1 \pm 1.1 \pm 3.7$		$74.4 \pm 3.4 \pm 19.1$		$D^+\pi^-$
7	$D_J^*(3000)^0$	3008.1 ± 4.0		110.5 ± 11.5		$D^+\pi^-$
8	$D_J^*(2760)^+$	$2771.7 \pm 1.7 \pm 3.8$	$2769.7 \pm 3.8 \pm 1.5$	$66.7 \pm 6.6 \pm 10.5$	60.9	$D^0\pi^+$
9	$D_J^*(3000)^+$	3008.1		110.5		$D^0\pi^+$

Table 6.1 Experimental status of latest charm mesons and their masses

The states with $J^P = (0^-, 1^-, 0^+, 1^+, 1^+, 2^+)$ are well known. The doublets having spin-parity assignments $J_P = \frac{3}{2}^+$, consists of $D_1(2420)$ and $D_2^*(2460)$ in non-strange sector. The states ($D_0^*(2400)$, $D_1'(2430)$) belong to $s_i^P = \frac{1}{2}^+$ charm doublet [2]. The experimental data on decay widths suggest that states $(0^+, 1^+)$ are quite broad, ex-

pecting to decay via s -wave whereas the states belonging to $(1^+, 2^+)$ doublets are quite narrow and decay via d -wave. The measured branching ratio by BaBar Collaboration is given as:

$$\frac{BR(D_2^{*0}(2460) \rightarrow D^+\pi^-)}{BR(D_2^{*0}(2460) \rightarrow D^{*+}\pi^-)} = 1.47 \pm 0.03 \pm 0.16 \quad (6.28)$$

There are few more recent states whose branching ratios as measured by BaBar is mentioned below.

$$\frac{BR(D^0(2600) \rightarrow D^+\pi^-)}{BR(D^0(2600) \rightarrow D^{*+}\pi^-)} = 0.32 \pm 0.02 \pm 0.09 \quad (6.29)$$

$$\frac{BR(D^0(2760) \rightarrow D^+\pi^-)}{BR(D^0(2750) \rightarrow D^{*+}\pi^-)} = 0.42 \pm 0.05 \pm 0.11 \quad (6.30)$$

The information from the BaBar Collaboration and the quark model suggests that $D^0(2550)$ state lies in 0^- state. The $D^0(2600)$ corresponds to 1^- state either in the $2S$ or $1D$ spectrum respectively because this state was observed in both $D\pi$ and $D^*\pi$ channels. If we find the mass of these particular states using heavy quark symmetry and other theoretical models [14], [15], it can be suggested that the state $D(2600)$ can be identified as either radial excitation of heavy quark doublet H or as $1D$. The branching ratios for $D\pi$ and $D^*\pi$ for both the decay states are calculated as $\frac{BR(D^0(2600) \rightarrow D^+\pi^-)}{BR(D^0(2600) \rightarrow D^{*+}\pi^-)} = 0.82$ for $2S$ and $\frac{BR(D^0(2600) \rightarrow D^+\pi^-)}{BR(D^0(2600) \rightarrow D^{*+}\pi^-)} = 0.38$ for $1D$ respectively. The comparison with the experimental data results in favor of $1D$ assignment. Therefore, the possible assignment for this particular state can be $1D$ respectively. The theoretical estimation of coupling constant for the strong decay width of mesons in this particular state is 0.53 ± 0.01 . The theoretical estimation of branching ratios

from the heavy quark effective theory [17] leads to the conclusion that there may be possibility of violations in flavor and spin symmetry. In ref. [19], Sun et al. used the 3^3P_0 model to examine the strong decays of these states and they concluded that $D^0(2600)$ state can be identified as the mixture of 2^3S_1 and 1^3D_1 state. Therefore, the other possibility is that $D(2600)$ may be considered as a mixing state of $2S$ and $1D$ respectively. The other two states $D(2750)$ and $D(2760)$ can be identified with $J^P = (2^-, 3^-)$ assignment. It is very interesting to point out that non-strange partner of $D_{sJ}(2860)$ can be associated with $D(2760)$ due to mass gap which is about 150 MeV. There are also several references like [16], [17] which suggest possible assignment for $D(2750)$ and $D^*(2760)$ state with the $l=2, n=1$ state. Moreover, the branching ratio measurement $\frac{BR(D^0(2760) \rightarrow D^+\pi^-)}{BR(D^0(2750) \rightarrow D^{*+}\pi^-)}$ gives value 0.80 from the leading order effective theory which is found to be matched with the experimental data. Saturating the total decay width with the ground state to two body decays, we can fit the experimental data of LHCb and BaBar to estimate the coupling constants. We take the experimental data of decay widths of recent states as mentioned in table 6.4 to find the hadronic coupling constants. The decay widths are calculated using the decay formulae given above. We can fit the experimental data of BaBar and LHCb collaborations to estimate the coupling constants. The observed radially excited non-strange charm meson states in the heavy meson spectrum are the two resonances ($D(2550), D^*(2600)$). The values of coupling constant is obtained from the measured width of ($D(2550)$) and the computed value is 0.35 ± 0.03 [17]. The calculated value of coupling constants in our fitting program for $D(2550)$ comes out to be 0.40 ± 0.05 . The best fit value of these coupling constants is estimated with in experimental error using chi-square minimization technique. The errors are clearly dominated by

the statistical and systematic uncertainties. To check the consistency of our fitting algorithm, the coupling constant estimation is being carried out for the decays of $J^P = (0^+, 1^+)$ and the fitted value comes out to be 0.53 ± 0.04 . This value has been found to be matching well with predictions from other theoretical approaches [17]. The value of coupling constants for $D(2750)$ and $D^*(2760)$ states are also estimated in our fitting program which are 0.61 ± 0.01 and 0.79 ± 0.03 . Let us consider new states observed by LHCb in the $D\pi$ and $D^*\pi$ spectrum and from the strong decays, the states are labeled as $D_J^*(3000)^0 \rightarrow D^+\pi^-$ and $D_J^*(3000)^+ \rightarrow D^0\pi^+$. On the basis of LHCb data [2], the angular distribution of $D_J(3000) \rightarrow D^*\pi$ is found to be consistent with unnatural parity. The possible spin-parity assignment for $D_J(3000)^0$ can be $J^P = 0^-, 1^+, 2^-, 3^+ \dots$ and $D_J^*(3000)$ have possible spin-parity assignments $J^P = 0^+, 1^-, 2^+, 3^-, 4^+ \dots$. Thus, it can be stated that the two states can be higher radial excitations or can belong to $1F$ states in the meson spectrum. Some possible indications about the possible assignment of J^P quantum numbers can be realized from the masses of these states. One of the well known potential models [14] calculated the masses of all possible excited mesonic states and we can suggest the possible spin and parities assignments of these newly discovered states. To extract the detailed information about the newly observed states, we present the summary of all D meson states and various possibilities of $D_J^*(3000)$ and $D_J^*(3000)^0$ in the table 6.2 below.

s_l^p	$\frac{1}{2}^-$	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$	$\frac{7}{2}^+$	$\frac{5}{2}^-$
$n=1$	$D(1869)(J^P = 0^-)$	$D_0^*(2400)(J^P = 0^+)$	$D_1(2460)(J^P = 1^+)$	$D(2750)(J^P = 1^-)$	$D_J^*(3000)(J^P = 2^+)*$	$D_J(3000)(J_P = 3^+)*$	$D_J(3000)(J^P = 2^-)*$
	$D^*(2010)(J^P = 1^-)$	$D_1^*(2430)(J^P = 1^+)$	$D_2^*(2430)(J^P = 2^+)$	$D(2760)(J^P = 2^-)$	$D_J(3000)(J^P = 3^+)*$	$D_J^*(3000)(J_P = 4^+)*$	$D_J^*(3000)(J^P = 3^-)*$
$n=2$	$D(2550)(J^P = 0^-)$	$D_J^*(3000)(J^P = 0^+)*$	$D_J(3000)(J^P = 1^+)*$	$D_J^*(3000)(J^P = 1^-)*$			
	$D(2600)(J^P = 1^-)$	$D_J(3000)(J^P = 1^+)*$	$D_J^*(3000)(J^P = 2^+)*$	$D_J(3000)(J^P = 2^-)*$			
$n=3$	$D_J(3000)(J^P = 0^-)*$	$D_J^*(3000)(J^P = 1^-)*$					

Table 6.2 Table showing all non-strange charm meson states

Wang et al. [16] also suggested the various possibilities of $D_J^*(3000)$ states and calculated the decay widths of these states in terms of relevant coupling constants. We also suggest the same but we add the two more possibilities i.e. $(1^-, 2^-)$ and $(2^-, 3^-)$ lying in $1D$ spectrum. To analyze the spectrum of above J^P states, we study the two body decay behavior and calculate the branching ratios for the states for which decay to PM and P^*M both are allowed. Considering the branching ratios for strong decays $\frac{BR(D_2^* \rightarrow D^{*+} \pi^-)}{BR(D_2^* \rightarrow D^+ \pi^-)}$ and the decay occurs via relative f -wave and similarly for other decay channels, the branching ratio and the relative wave is mentioned in the table given below.

Table 6.3 Ratios of decay widths for $D_J^*(3000)$ state in all possible assignments.

$D_J^*(3000)$	$D_J^*(3000) \rightarrow D^*\pi$	$\frac{BR(D_J^* \rightarrow D^{*+}\pi^-)}{BR(D_J^* \rightarrow D^+\pi^-)}$
$s_l^p = \frac{5}{2}^+, J^P = 2^+$	f -wave	0.343
$s_l^p = \frac{7}{2}^+, J^P = 4^+$	f -wave	0.52
$s_l^p = \frac{1}{2}^+, J^P = 0^+$	s -wave	0
$s_l^p = \frac{3}{2}^+, J^P = 2^+$	d -wave	0.955
$s_l^p = \frac{1}{2}^-, J^P = 1^-$	p -wave	1.57
$s_l^p = \frac{5}{2}^-, J^P = 3^-$	f -wave	0.68
$s_l^p = \frac{3}{2}^-, J^P = 1^-$	p -wave	0.32

The table 6.3 collects the ratios of decay width for $D_J^*(3000)$ state. Predicted ratios along with graphs can be analyzed to exclude some of the assignments. The graphs show the variation of branching ratios Vs mass of decaying particle. From the graphs, it can be stated that the states lying in $3S$ doublet produce ratios more than one around the mass values of 3000 MeV. The decay of $D_J^*(3000)$ into $D^{*+}\pi^-$ is the dominant mode for $3S$ doublet which is inconsistent with the experimental observations. $D_J(3000)$ is observed to decay via $D^+\pi^-$ channel. However, if we assume the $D_J(3000)$ as the spin partner of $D_J^*(3000)$ then $D^*\pi$ is the most dominant decay mode. The other possible decay modes are DK and $D\eta$. If $D_J(3000)$ and $D_J^*(3000)$ belongs to $2P(0^+, 1^+)$ doublet then the allowed decay mode for $D_J^*(3000)$ forbids $D^{*+}\pi^-$ decay channels which agrees well with the experimental observation. In all other cases, decays to both the channels $D\pi$ and $D^*\pi$ are allowed. Moreover, the theoretical calculation of branching ratios $(\frac{D_J^*(3000) \rightarrow D^+\pi^-}{D_J^*(3000) \rightarrow D^{*+}\pi^-})$ for all possible doublets

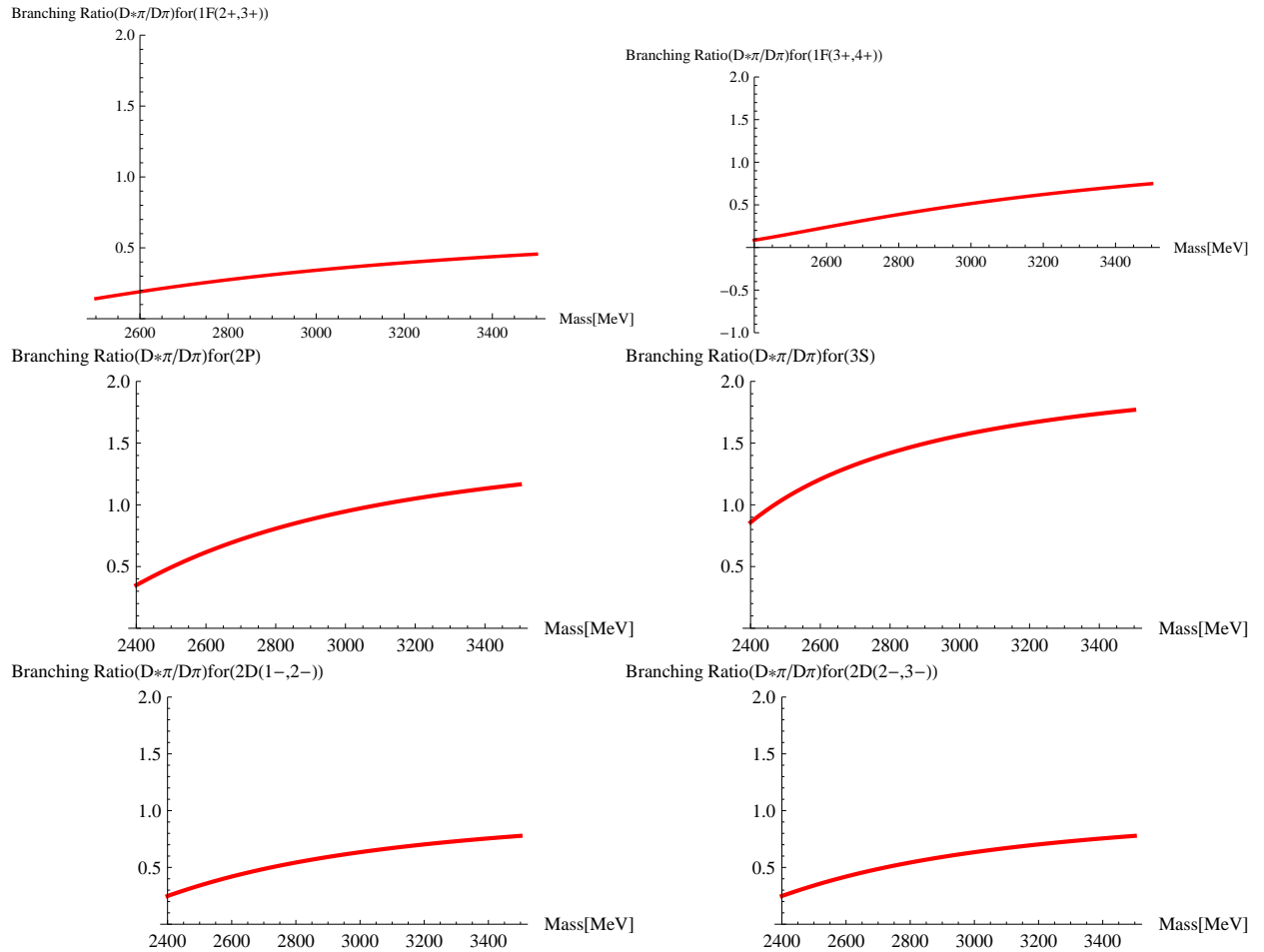


Figure 6.1 Graphs showing branching ratios $\left(\frac{BR(D_J^* \rightarrow D^* + \pi^-)}{BR(D_J^* \rightarrow D + \pi^-)}\right)$ Vs mass of decaying particle

and comparison with the mass spectrum observed by LHCb collaboration suggests that the possible assignment of these two states can be either ($J^P = (1^-, 2^-)$) or ($J^P = (2^+, 3^+)$) for $D_J^*(3000)$ and $D_J(3000)$ mesons. The state 3^3S_1 state decays to $D^*\pi, D\pi, D_s^*K, D_sK$ and $D\eta$ and the coupling constant for this particular state lies near the value of $\simeq 0.1$. Also, the state 3^3S_1 appears to be a narrow D meson state. But its partial decay width to $D^*\pi$ appears to be greater than that of $D\pi$ mode which is completely inconsistent with observations by LHCb collaboration. If we assign $2P(J^P = (1^+, 2^+))$ doublet to $D_J^*(3000)$ and $D_J(3000)$ mesons than the coupling constant for strong decays to π, K, η should lie between 0.12-0.15. For (2^3D_3) state, ($D\pi, DK, D\eta$) are the other allowed decay modes. Additional decay channels may include $D(2460)\pi$ and $D(2420)\pi$. The additional information about these decay channels may help to estimate coupling constant more precisely. In addition to this, for the 2^3D_1 state, the most prominent decay mode is $D\pi$ therefore the ideal decay mode to search for this state is $D\pi$. The ratio of total decay widths for $D_J^*(3000)$ and $D_J(3000)$ can be calculated from the experimental data of LHCb collaboration.

$$\frac{\Gamma(D_J^*(3000))}{\Gamma(D_J(3000))} = 0.587 \pm 0.083 \quad (6.31)$$

In our work, $\frac{\Gamma(D_J^*(3000))}{\Gamma(D_J(3000))}$ is calculated in heavy quark effective theory. The comparison of theoretical value with experimental data favors in $(1^-, 2^-)$ assignment. Similar analysis can be carried out for the $D_J(3000)$ mesons. The various possibilities include $3^1S_0, 2P(1^+)$ and $2D(2^-)$ and $1F$ etc. A closer look at the decay width of all the above states in the $D^*\pi$ spectrum suggests that the most possible assignment for $D_J^*(3000)$ and $D_J(3000)$ belongs to either $2P$ or $2D$ state. The most prominent

decay mode for $2P(1^+)$ in $(1^+, 2^+)$ doublet is found to be $D^*\pi$ which matches well with the experimental data on decay width for $D_J(3000)$. However, if we consider the $D_J(3000)$ as the spin partner of $D_J^*(3000)$ then the another possibility can be that $2P(1^+)$ in $(0^+, 1^+)$ overlaps with that with 1^+ in $(1^+, 2^+)$ doublet.

6.5 Conclusion

We study the heavy meson decay width in the framework of heavy quark effective theory that represents heavy quark and chiral symmetry at chiral symmetry breaking scale $\Lambda_\chi \simeq 1\text{GeV}$ [5]. We studied the recent charm meson states with their J^P assignment. The upcoming results at collaborations like LHCb produces the data for branching ratios that is used to calculate the decay width, coupling constants and suitable J^P states. The coupling constants and their studies are important to study heavy meson phenomenology. The accurate estimation of coupling constants help to study the detailed interaction of heavy mesons. The present work calculates the coupling constant for strong decays of non-strange charm meson states ($D(2550), D(2600), D(2750)$ and $D(2760)$) by using Chi-square minimization techniques. The numerical values of decay widths from the collaborations like LHCb, BaBar and CDF are used to extract the values of coupling constants. The various assignments of J^P values to the above mentioned states are also analyzed. The J^P assignment for $D^0(2550)$ state is 0^- while $D^0(2600)$ is identified as mixture of 2^3S_1 and 1^3D_1 state with $J^P = 1^-$. The states $D(2750)$ and $D(2760)$ are identified with $J^P = (2^-, 3^-)$ assignment. All assignments to $D_J(3000)$ are analyzed deeply and various possibilities for J^P states has been checked. Two more possibilities i.e. $(1^-, 2^-)$ and $(2^-, 3^-)$ lying in $2D$ spectrum

have also been included for analysis. Most possible assignment in the present work is favored in $2P(1^+)$ for $D_J(3000)$ state. While investigating for decays, it is concluded that the results on decay widths are further helpful to search for un-known resonances so that the excited meson spectrum for D meson family is clear to theorists as well as experimentalists.

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Chapter 7

Summary and Outlook

In this thesis, various low energy properties of hadronic systems have been studied either using the statistical model or effective theories. Quantum Chromodynamics (QCD) is the universally accepted theory of strong interaction physics. The theory of QCD has a remarkable simplicity and elegance at the classical level, with its underlying non-abelian $SU(3)$ color symmetry. The faith in QCD as a true physics theory is founded, on the successes of perturbative quantum chromodynamics (PQCD) where its wide-ranging predictions are compared to the experimental data on high energy processes, at variety of experimental facilities, covering numerous physical processes in lepton-lepton, lepton-hadron and hadron-hadron collisions. Due to asymptotic freedom, the perturbation series expansion of QCD breaks down at low energy. For studying the system of quarks, baryons or mesons, the two approaches in our hand, are the perturbative and non-perturbative approach. The value of the coupling constant approaches the order of 1 at an energy of 1GeV , a scale referred to as Λ_{QCD} . There is variety of interesting phenomena at relatively low energy scale. This means that a perturbative approach in conventional sense (expansion in the coupling con-

stant) is not sufficient to gain a complete picture of QCD. Non-perturbative QCD refers to the study of interactions among the hadrons in a regime where the direct theoretical approaches are hard to be applied. The value of running coupling constant $\alpha(Q^2)$ increases at small momentum transfers, reaching a value equal to 1, at the momenta transfer of around $Q^2 \sim 500\text{MeV}^2$. This leads to the general problem in QCD that it becomes non-perturbative at small momenta or energies $E \leq 1\text{GeV}$. For example, the properties of hadrons at low energy cannot be studied via simple perturbative QCD methods. One has to look for other approaches to calculate the properties of hadrons. Various non-perturbative techniques are lattice gauge theory, sum rules, phenomenological models and effective theories. Some situations arise when the symmetries and dynamics of QCD gives birth to new constants other than coupling constants in the form of expansion parameters. Thus, non-perturbative effects arise in the form of low energy parameters. A lot of experimental data can help to find constraints on these low energy parameters. Effective field theories can be very useful to implement the above said idea. Another situation can be the use of phenomenological models to find the low energy parameters. The first part of present thesis focus on low energy properties. These properties can be calculated in various relativistic and non-relativistic methods. Our concern is to find out these properties using statistical models where strange and non-strange effects on sea can be studied. Nucleons and hyperons are studied in a statistical framework using principle of detailed balance. Another category of particles of our interest is a heavy-light D and B system. This interest is due to rapid growth and advancements in the experimental facilities and discoveries of new resonances at different energy levels. To explore the decay widths and masses of newly observed resonances, an effective theory is the

most helpful. An overview of basic information about the phenomenological models and effective theories is discussed in chapter-I. The basic symmetries and their breaking also play an important role in explaining the structure and dynamics of hadronic systems. The chapter I includes the discussion of such symmetries existing in nature and their breakings .

The first historical step in the understanding of quark gluon nature was the constituent quark model. $SU(3)$ flavor states when combined with the $SU(2)$ states of spin, leads to the six fold symmetry known as $SU(6)$. The well known $SU(6)$ model uses six quark states to constitute and classify the hadrons into mesons and baryons. $SU(6)$ model explains some of the low energy properties of nucleonic system like the magnetic moment ratio of proton and neutron but there are some low energy parameters for which extension of $SU(6)$ quark model is needed. One such big question in the mind of researchers is the proton spin problem. It is speculated that partons being bound inside the nucleon, does possess some statistical properties. Statistical models provide intuitive appeal and physical simplicity, that have made success in describing the parton structure functions for nucleons. Statistical model in our work is based on the renowned approach where the visualization of hadronic structure occurs through fundamental quarks (as valence part) interacting through gluons and quark-antiquark pairs. The model assumes the hadronic structure to be made up of two parts, one is valence part and another is sea-part, where sea has the structure of quark-antiquark pair multi-connected non-perturbatively through gluons. Since the hadron should be colorless and a q^3 state can be in color state 1, 8 and 10 respectively. This restricts sea to be in specific color state to make hadrons as a colorless entity. Similarly, if sea is assumed to be in more general form like quark-antiquark

pair and gluon or a mixture of both then in S -wave state, sea can be of spin 0, 1 or 2. The basic instructions about the statistical model and an overview of current researches related to low energy properties of hadronic systems is given in chapter II.

A new statistical model has been used by Zhang et al. in which a nucleon is taken as an ensemble of quark gluon Fock states. In this model, principle of detailed balance is applied to find the probability of all Fock states. Principle of detailed balance assumes that every Fock state should be balanced with nearby Fock state. The model explains flavor asymmetry of the sea and provides results matching with the experimental results. Thus, a more general description of above model is proposed in chapter II where in addition to flavor, each Fock state have a definite spin and color quantum number and a particular symmetry property. The resulting formalism is based purely on a statistical formalism to explore the light quark spin content of nucleons, magnetic moments and semi-leptonic decay constant of neutron etc. The most interesting point to note here is that, all the properties are directly linked with the probabilities associated with each Fock state in definite spin-color-flavor spaces. Therefore, a total flavor-spin-color-space wave-function with all possibilities of quark-gluon Fock states is constructed. Properties of all baryonic systems are calculated using flavor-spin-color wave-function of baryonic system with possible combination of q^3 and sea such as to give spin $\frac{1}{2}$, flavor-octet and color-singlet state. The wave-function implies that the coefficients associated with each possible combination need to be evaluated statistically. These coefficients are determined by using the probability of each Fock state.

To check the effectiveness of statistical model, we have extended the principle of

detailed balance to include the strange quark-antiquark pairs in the sea. Assuming hadrons as a complete set of quark-gluon Fock states, principle of detailed balance is concerned with calculation of probability of every Fock state inside the hadrons. It assumes that probability for arriving in from one state is equal to probability of leaving out from that state. The balancing of any two ensembles with each other can be expressed in the form

$$\rho_{i,j,l,k} | \{q\}, \{i, j, l, k\} \rangle \iff \rho_{i',j',l',k'} | \{q\}, \{i, j, l, k\} \rangle$$

Moreover, non-zero mass of s -quark limits the free energy of gluon and hence the states with strange quark-antiquark pairs are assumed to be less probable. To accommodate the strange quark in sea and to allow processes such as $g \iff s\bar{s}$, a system must have energy greater than twice that of the strange quark mass. Thus, we have Fock states with number of strange quark-antiquark limited to one for a nucleonic system. The probabilities in spin and color space is calculated using the relevant multiplicities in statistical model, the relevant operators act on the sea-part and the probabilities in spin and color space suggest the active participation of sea quarks. These probabilities are further helpful to provide information about the static properties of hadrons like magnetic moments of baryons, spin distribution of quarks inside the baryon, total spin content, axial vector form factors etc. The results in our model is found to be matching with the experimental data and other theoretical models. The present work shows that although the strange quark contribution is negligible but if the strange quark is added in sea then the quark spin content of the nucleon is found to be matching with the experimental data of Spin Muon Col-

laboration. The polarized s -quark distribution at $Q^2 = 2.5(\frac{\text{GeV}}{c})^2$ is compatible with $\Delta s = 0.03 \pm 0.03 \pm 0.01$. To highlight the presence of strange sea quarks inside the nucleonic system and to analyze the importance of various contribution of sea components, statistical model provides us the framework to check the effect of individual contributions from the different Fock states of sea. The details of the results are included in chapter III.

The uniqueness of the statistical framework lies in the fact that the same model is expected to work well for all spin $\frac{1}{2}$ strange baryons. The $SU(3)$ breaking is expected to be in valence as well as sea part. $SU(3)$ breaking in sea comes due to strange mass corrections of strange sea. To study $SU(3)$ breaking for hyperons in detail, we need to incorporate the effects of strange quark mass into the probabilities associated with these Fock states. Here we define r as the symmetry breaking parameter and $r = \frac{\mu_s}{\mu_d}$ where μ_s is the magnetic moment of strange quark and μ_d is the magnetic moment of the d quark. It is quite interesting to mention here that strange mass corrections when applied in sea, shows a remarkable increase in the value of polarized quark-antiquark densities and our data is found to be consistent for the polarization densities of strange quark but it shows deviation for Δu , Δd in few cases. As far as the available data for weak decay ratios is concerned, our results match the experimental value in most of the cases. The results go well with other theoretical models for most of the decays and provide even better matching for decays like $\Xi^0 \rightarrow \Sigma^+$. The ratio $\frac{F}{D}$ comes out to be 0.676 and found to be deviating about 17% from the experimental value 0.575 ± 0.016 . In case of $SU(3)$ symmetry, we find $\Delta u = 0.91$, $\Delta d = -0.24$ and $\Delta s = 0$ respectively. Also, when m_s corrections are applied, the spin polarized densities change to $\Delta u = 0.76$, $\Delta d = -0.18$ and $\Delta s = -0.019$ respectively.

This leads to the conclusion that even in $SU(3)$ symmetry breaking, strange quark contribution to the spin of proton is very small.

Mass, splittings, decay widths and branching ratios are the fundamental parameters in describing a hadronic system. The experimental data for heavy light systems comes in the form of above said parameters. The properties of hadrons in which a heavy quark is coupled with light degrees of freedom, is explained using the heavy quark symmetry and chiral symmetry. The effective Lagrangian is based on $SU(3)_L \times SU(3)_R$ global symmetry and other heavy quark symmetry assuming charm, bottom and top quark masses as infinite and light quark masses as approaching to zero. Thus, chiral Lagrangian for heavy light mesons incorporating the heavy quark symmetry is written by including a kinematic term and all the possible interactions with the Goldstone bosons. The fields include both the symmetry breaking and conserving terms are taken. In the framework of heavy hadron chiral perturbation theory, chiral corrections and corrections due to heavy quark symmetry are encountered and at tree level the residual masses is given by a generalized formula given in chapter V. The residual masses are defined to be the difference between the real mass and an arbitrarily chosen reference mass of $O(m_q)$. The formula is written in terms of various non-perturbative QCD parameters. These parameters can be fitted to produce the masses of various heavy-light mesonic states. Taking the hyperfine and mass splitting data, available at experiments, the symmetry conserving and violating parameter's ranges are predicted. The spin symmetry predicts that, for fixed $j \neq 0$, there is a doublet of degenerate states with total spin $J \pm \frac{1}{2}$ and the flavor symmetry relates the properties of states with different heavy-quark flavor. This concludes that mass splitting among the various doublets are independent of heavy quark flavor.

Chapter 6 discusses the most recent study of charm non-strange mesonic states. The latest upcoming results from the experimental collaboration motivates us to study strong decays of $D(2550), D(2600), D(2750), D(2760)$ and $D(3000)$. From the heavy meson chiral Lagrangian terms, we can obtain the strong decay widths Γ to final states $D^{(*)}\pi, D^{(*)}\eta, D_s^{(*)}K$ where the symmetry breaking scale $\Lambda_X = 1\text{GeV}$. The study of strong decays of mesons is the key to understand the importance of coupling constant. These effective coupling constants can either be determined theoretically or by fitting the experimental data. The recent experimental data of above mentioned states is used to estimate the effective strong coupling constants. The value of coupling constants obtained for $D(2750)$ and $D^*(2760)$ are 0.61 ± 0.01 and 0.79 ± 0.03 . The tentative assignment of newly discovered state $D_J^*(3000)$ can be natural parity states ($0^+, 1^-, 2^+, 3^- \dots$) while $D_J(3000)$ can be identified with unnatural parity states like ($0^-, 1^+, 2^-, 3^+ \dots$). Therefore, the missing doublets $2S, 1D, 1F, 2P$ and $3S$ can be thought of filled up with these states. We study the two-body strong decay widths and branching ratios of missing doublets and plot branching ratios vs mass of decaying particle. These plots are used to analyze all assignments to $D_J(3000)$ deeply and various possibilities for J^P values.

Therefore, the above mentioned work is concerned with the properties of different hadronic systems. These properties help to get into details of the structure of these hadrons. Moreover, to match the experimental data with the theoretical predictions, these properties need to be calculated. The present work concludes that at low energy, the fundamental symmetries of QCD plays a crucial role to find the structural details of the hadrons. The statistical model here is applied to spin $\frac{1}{2}$ baryons but this model can be extended to a system with spin $\frac{3}{2}$ baryons. In addition to this, the orbital

angular momentum of quarks can also be taken into account so as to check the validity of the statistical techniques.

There are lots of upcoming data for radially excited strange and non-strange charm and bottom meson states. The masses and decays of heavy-light systems can be explored in details by using an effective Lagrangian including symmetry breaking corrections and higher order correction terms so that more accurate predictions of coupling constants can be obtained. We can also include some additional channels like decays to vector mesons and decays to the states like 2420π and 2430π in our work by introducing additional phenomenological Lagrangian terms. This will include more un-known constants and the experimental data of decay widths can be confronted to fit these un-known constants. The heavy quark effective theory with higher order terms and chiral loop corrections can be applied to verify the masses of excited charm meson spectrum.