HEAVY MESON SPECTROSCOPY AND PREDICTION OF BOTTOM MESONS

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CERTIFICATE

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ABSTRACT

The approximate symmetries of Quantum Chromo Dynamics in the infinite heavy quark $(Q = c, b)$ mass limit $(m_Q \to 0)$ and in the chiral limit for the light quarks $(m_q \to 0, q = u, d, s)$ can be used together to build up an effective chiral Lagrangian for heavy and light mesons describing strong interactions among effective meson fields. We studied the heavy mesons masses, where by the symmetries of heavy and light quarks are exploited to formulate a theory that describes the low energy interaction among heavy mesons. Within the framework of heavy quark effective theory and chiral perturbation theory, masses of even and odd parity heavy mesons are studied. Mass formulae are developed for ground state $J_p = 0^-$ and 1^- and first excited state, $J_p = 0^+$ and 1^+ for the heavy meson up to $O\left(1/m_Q\right)$ corrections.

We begin by constructing an effective Lagrangian which includes all possible terms that transform correctly under the symmetries, and is typically non-renormalizable with an infinite number of terms. These terms are organized in importance by power counting in a small parameter. Identifying a small expansion parameter usually depends on having scales which are widely separated. Further the hadronic mass terms are developed from the Lagrangian in terms of $\overline{\Lambda}$, λ_1 and λ_2, where λ_1 and λ_2 are non-peturbative parameters of QCD. Effective field theory approach has been used where different scales in the problem are separated, so as to concentrate on the interesting physics at a our 1GeV scale. Furthermore, the power counting gives us a way to estimate the uncertainty in working at this given order. Also we have calculated the bottom meson masses, by using the data of charm meson masses and their hyperfine splitting.

PHYSICAL CONSTANTS AND UNITS

Units

Quantity	High Energy Units	value of SI Units
Length	1 fm	10^{-15} m
Energy	1 GeV=10^9 eV	1.602×10^{-10} J
mass, E/c^2	1 GeV/c^2	1.78×10^{-27} Kg
$\hbar=h/(2\pi)$	6.588×10^{-25} GeVs	1.055×10^{-34} Js
c	2.988×10^{23} fm s^{-1}	2.998×10^8 ms^{-1}
$\hbar c$	0.1975 GeV fm	3.162×10^{-26} Jm

Natural Units	$\hbar=c=1$
Mass, Mc^2/c^2	1GeV
Length, $\hbar c/(Mc^2)$	1 GeV^{-1}=0.1975fm
Time, $\hbar c/(M c^3)$	1GeV^{-1}=6.59×10^{-25}s

Fine structure constant,

$$\alpha = e^2/(4\pi) = 1/137.06$$

Relation between energy Units

1MeV=106 eV , 1GeV= 103 eV, 1TeV=103 eV

Constants

• Fermi	fm	10-3 cm
• Electron charge	e	1.602….× 10^{-19} C
• Planck's constant, reduced	ħ=h/2π	6.582…..× 10^{-22}MeVs
• Conversion constant	ħc	197.3….. MeV fm
• Electron mass	m_e	0.511…. MeV
• Proton mass	m_p	938.3….MeV
• Fine structure constant	α	1/137.035
• Strong interaction coupling constant	g	1

CONTENTS

Motivation

To discover what the universe is made of and how it works is the challenge of particle physics. The landmark Quantum Universe report defines the quest of particle physicists to explain the universe in terms of nine key questions. In pursuit of answers to these questions, Fermi lab supports researchers, experiments and facilities that promise to revolutionize our understanding of the universe.

- *Are there undiscovered principles of nature: new symmetries, new physical laws?*
- *How can we solve the mystery of dark energy?*
- *Are there extra dimensions of space?*
- *Do all the forces become one?*
- *Why are there so many kinds of particles?*
- *What is dark matter? How can we make it in the laboratory?*
- *What are neutrinos telling us?*

How did the universe come to be? What happened to the antimatter? are some of the questions which motivated us. Accelerator-based experiments, for example, shed light on the fundamental particles and forces that have governed the evolution of the universe since the big bang. Supersensitive particle detectors, located deep underground, look for the elusive particles that make up dark matter. Powerful telescopes reveal images of galaxies that are many billions of years old and unveil the distribution of dark matter and dark energy. Neutrino experiments examine the role the elusive neutrino played in the formation of the universe, perhaps answering the question why the universe is made of matter instead of antimatter [1].

Theorists bring together the knowledge gathered from these experiments to find a better, more complete description of the laws of physics. The aim to find the fundamental symmetries and mathematical equations that describe how our universe works. Their ideas, calculations and predictions also guide the future direction of experimental programs. Experimental results, in turn, can confirm or rule out theoretical models, or they can lead to unexpected discoveries that stimulate the development of new theoretical ideas. Theorists and experimenters share the need for new measurements to advance our understanding of the universe.

Looking to the future

One of the primary goals for the new and upgraded facilities in Fermi lab near Chicago (the Tevatron) and CERN in Geneva Switzerland (the Large Hadrons Collider or LHC) is to find the Higgs boson, the one missing element of the Standard Model.

Evidence for super symmetric partners of the known particles is a goal in all experiments, as part of the search for the true particle theory beyond the Standard Model. Beyond that is the need to find anything that can point to a real Grand Unification with the gravitational force. A different kind of $e^+ e^-$ collider is being planned internationally, The International Linear Collider or ILC, a very high energy linear collider, with two opposing linear accelerators tens of kilometers long. The technical challenges are many and this is likely to be the first truly world-wide accelerator collaboration.

Chapter-1

Introduction

During the past two centuries, scientists have made great progress in understanding what we and the world about us are made of. First came the realization that matter consists of basic substances, or elements, with well defined physical and chemical properties. These elements range from hydrogen, the lightest, through to uranium and beyond.

Each element consists of building blocks atoms unique to the element, but the different atoms can combine to form an enormous variety of compounds from simple water to complex proteins. Yet, as scientists first discovered towards the end of the 19th century, atoms are not the simplest building bricks of matter.

We now know that most of the mass of an atom is concentrated in a small, dense, positively-charged nucleus. A cloud of tiny negatively-charged electrons envelopes the nucleus, but at a relatively large distance, so that much of the volume of an atom is empty space. In most atoms the nucleus contains two types of particle of almost equal mass: positively-charged protons and electrically neutral neutrons. To make the atom neutral overall, the number of protons exactly balances the number of electrons.

This picture of the atom stems largely from pioneering work at Cambridge and Manchester Universities. At Cambridge in the 1890s, two physicists began unwittingly to probe the world within the atom. One, Joseph ('J.J.') Thomson, discovered the first known subatomic particle, the electron, while one of his students, Ernest Rutherford, started to explore the new phenomenon of radioactivity, in which atoms change from one kind to another. This was to lead Rutherford eventually to the discovery of the atomic nucleus, in work with Hans Geiger (of Geiger counter fame) and Ernest Marsden at Manchester University in 1909-10. Later, Rutherford found that atoms contain positively-charged particles, identical to the nucleus of hydrogen. He called the particles protons. And at Cambridge in 1932, James Chadwick showed that the nucleus must also contain neutrons. By this time Rutherford and his colleague Universities had established much of the modern picture of the atom [2].

Protons, electrons, neutrons, neutrinos and even quarks are often featured in news of scientific discoveries. All of these, and a whole "zoo" of others, are tiny sub-atomic particles too small to be seen even in microscopes. While molecules and atoms are the basic elements of familiar substances that we can see and feel, we have to "look" within atoms in order to learn about the "elementary" subatomic particles and to understand the nature of our Universe. The science of this study is called Particle Physics, Elementary Particle Physics or sometimes High Energy Physics (HEP).

Atoms were postulated long ago by the Greek philosopher Democritus, and until the beginning of the 20th century, atoms were thought to be the fundamental indivisible building blocks of all forms of matter. Protons, neutrons and electrons came to be regarded as the fundamental particles of nature when we learned in the 1900's through the experiments of Rutherford and others that atoms consist of mostly empty space with electrons surrounding a dense central nucleus made up of protons and neutrons.

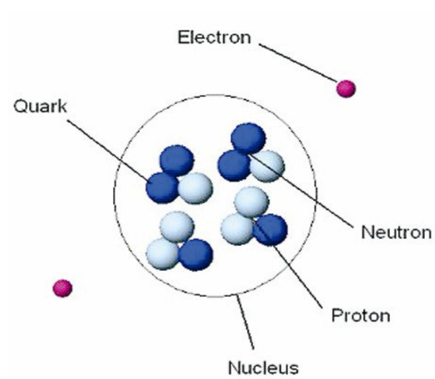


Fig:1.1 - The central nucleus contains protons and neutrons which inTurn contain quarks. Electron clouds surround the nucleus of an atom

1.1. High Energy Physics (HEP)

The science of particle physics surged forward with the invention of particle accelerators that could accelerate protons or electrons to high energies and smash them into nuclei to the surprise of scientists, a whole host of new particles were produced in these collisions.

By the early 1960s, as accelerators reached higher energies, a hundred or more types of particles were found. Could all of these then be the new fundamental particles? Confusion reigned until it became clear late in the last century, through a long series of experiments and theoretical studies, that there existed a very simple scheme of two basic sets of particles**:** the quarks and leptons (among the leptons are electrons and neutrinos), and a set of fundamental forces that allow these to interact with each other. By the way, these "forces" themselves can be regarded as being transmitted through the exchange of particles called Gauge Bosons. An example of these is the photon, the quantum of light and the transmitter of the electromagnetic force we experience every day.

Together these fundamental particles from various combinations that are observed today as protons, neutrons and the zoo of particles seen in accelerator experiments.

How do we get to study quarks and such, if they don't exist freely now?

Just as in the Big Bang, if we can manage to make high enough temperatures, we can create some pairs of quarks & anti-quarks, by the conversion of energy into matter. (Particles & anti-particles have to be created in pairs to balance charge, etc.)

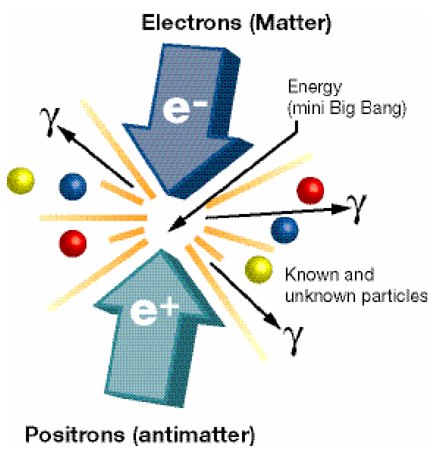


Fig1.2 -When particles of matter and antimatter collide they annihilate each other, creating conditions like those that might have existed in the first fractions of a second after the big bang.

When a quark-antiquark pair is produced in a head-on collision with excess energy (i.e., $E > 2m_q c^2$) the quark and antiquark fly off in opposite directions until "the string breaks into two" and each of the pair finds itself bound with another quark. What we actually observe is a pair of mesons being produced, each meson consisting of a quark and an antiquark bound together. With enough excess energy, larger clumps of quarks and antiquarks can be produced: protons, neutrons and heavier particles classed as baryons. These mesons and baryons make up the zoo of particles discovered earlier. What we have thus found is that to study quarks, one has to create them in high energy collisions, but they can only be observed clumped into mesons and

Baryons. We have to infer the properties of individual quarks through the study of the decay and interactions of these mesons and baryons.

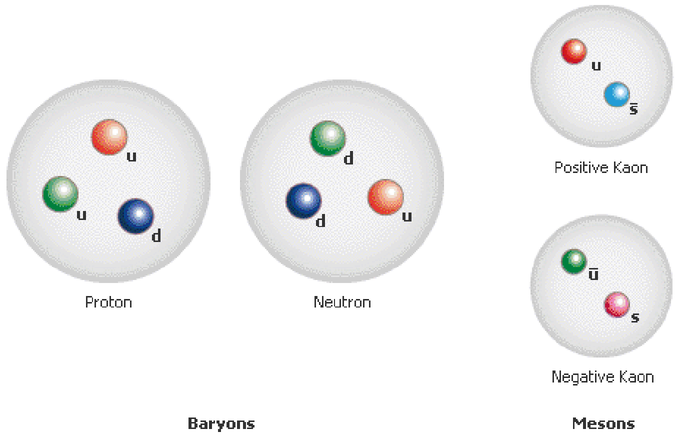


Fig.1.3 -Baryons and Mesons contain combinations of quarks and anti-quarks.

1.2. The Standard Model

Today, the Standard Model is the theory that describes the role of these fundamental particles and interactions between them. And the role of Particle Physics is to test this model in all conceivable ways, seeking to discover whether something more lies beyond it.

Particle physicists now believe they can describe the behavior of all known subatomic particles within a single theoretical framework called the Standard Model, incorporating quarks and leptons and their interactions through the strong, weak and electromagnetic forces. Gravity is the one force not described by the Standard Model. The Standard Model is the fruit of many years of international effort through experiments, theoretical ideas and discussions. We can summarize it this way:

All of the known matter in the Universe today is made up of quarks and leptons, Held together by fundamental forces which are represented by the exchange of particles known as gauge bosons.

Matter is composed of tiny particles called quarks. Quarks come in six varieties: up (u), down (d), charm (c), strange (s), top (t), and bottom (b). Quarks also have antimatter counterparts called antiquarks (designated by a line over the letter symbol). Quarks combine to form heavier particles called baryons, and quarks and antiquarks combine to form mesons. Protons and neutrons, particles that form the nuclei of atoms, are examples of baryons. Positive and negative kaons are examples of mesons.

One guiding principle that led to current ideas about the nature of elementary particles was the concept of Symmetry. Nature points the way to many of its underlying principles through the existence of various symmetries.

1.3. Quarks and Leptons

The quarks and leptons are the fundamental particles of the standard model. They are fermions having spin quantum number which is half integer and obey Fermi-Dirac statics.

Quarks come in 6 flavors: up (u), down (d), strange (s), charm (c), bottom (b) and top (t).

The quarks can be arranged in three generations of doublets, the properties of the generation being similar, but the masses of which are successively heavier.

$$\begin{pmatrix} u^{+2/3} \\ d^{-1/3} \end{pmatrix} \begin{pmatrix} c^{+2/3} \\ s^{1/3} \end{pmatrix} \begin{pmatrix} t^{+2/3} \\ b^{-1/3} \end{pmatrix}$$

Leptons come in 2 types and 3 flavor the three flavors are electron (e), moun (μ) and tau(τ). And the neutral leptons or neutrinos denoted by the symbols ν. The leptons can also be arranged in three generational doublets whose charged leptons masses from the electron which has a mass of 0.51 MeV to the tau which has the mass 1.8 GeV. The neutrinos have the zero mass.

$$\begin{pmatrix} \upsilon_e{}^0 \\ e^- \end{pmatrix} \begin{pmatrix} \upsilon_\mu{}^0 \\ \mu^- \end{pmatrix} \begin{pmatrix} \upsilon_\tau{}^0 \\ \tau^- \end{pmatrix}$$

The quarks and leptons carry quantum numbers. These quantum numbers explains the kind of interactions a particle can have. Some of this quantum number has to do with their type such as, baryon number which is $\frac{1}{3}$ of all the quarks nucleon and 0 for the leptons. The quantum s number, “lepton number” is 1 for all the leptons and 0 for all the quarks. Some of their quantum numbers determine how strongly they will couple to the Gauge Bosons. The quantum number for the electro-magnetic forces is the electric charge, for example, the charged leptons with charge ±e will couple more strongly to EM Forces than the quark with charge of $\pm\frac{1}{3}e$ or $\pm\frac{3}{2}e$, the quantum number for the weak forces is called the weak isospin, mainly because each of the quarks and leptons in the above doublets have weak isospin of $+\frac{1}{3}$ for the upper element and $+\frac{1}{3}$ for the lower element. In fact, the above doublets are called weak iso-doublets because of this the weak interaction has the eligibly to span different generations. The quantum number of the strong forces is called color. Each of the quarks comes in one of three color s called red, green and blue. The leptons do not carry the colors quantum number, and the neutrinos do not carry an EM quantum number either. Free quarks are never observed because the strong forces get stronger with distances so that the quarks become more tightly bound as they get further away from each other.

The quarks and leptons carry quantum numbers, some of which are conserved. The quantum numbers which are conserved in all interactions are baryon number and lepton flavor number, B-L, color and electric charge. Weak isospin is not conserved. These conservation laws are very important when it comes to drawing Feynman diagrams and calculating cross sections. All these fundamental particles are point like that is they have no discernable size. This is mainly what leads us to believe they are indeed in fundamentals and they themselves do not have substructure.

All of the quarks and leptons (the fermions) have antiparticles. The masses of particle and antiparticle are exactly the same but they have different charge, spin and magnetic moments. In the case of electrically charged particles, the anti-particle has the opposite electric charge. If the particle has baryon number $B = +\frac{1}{3}$ then the antiparticle has $B = -\frac{1}{3}$ If the particle has leptons number $L = 1$, the antiparticle has $L = -1$. The quantum number is all equal and opposite.

1.4. Quark Model

The quark model was developed to account for the regularities observed in the hadron spectrum, with hadrons interpreted as bound states of localized but essentially non-interacting quarks. It provides us a simple picture of internal structure of hadrons and an effective way to describe their dynamics at high energy. Much of the success of the model lies in the circumstance that to a reasonably good approximation we can regard quarks as free or weakly interacting particles (except for the confining mechanism).

The low-lying baryons were interpreted in the quark model as symmetric states of space, spin and SU $(3)_f$ flavor degrees of freedom. However, Fermi-Dirac statistics requires a total anti-symmetry of the wave function. The resolution of this dilemma comes through the introduction of color degree of freedom. The baryon wave functions are totally anti-symmetric in the color degree of freedom. Of course, the introduction of another degree of freedom would lead to a proliferation of states, so the color degree of freedom had to be supplemented by a requirement that only color singlet states exist in nature. Hence proton would be a bound state of (uud) and neutron would be a bound state of (udd) quarks which makes them color singlet. This model had great success in predicting new hadronic states, and in explaining the strength of electromagnetic and weak interaction transitions among different hadrons. In particular, it naturally incorporates the most important symmetry relations among hadrons.

1.5. Forces and Interactions

There are 3 types of interactions which the fundamental particles can undergo:

- Strong via Strong Forces(or color forces)
- Weak via Weak Forces
- Electro-Magnetic(EM) via Electromagnetic Forces

The quarks interact via all three of these forces, the leptons do not interact with the strong forces. The weak and electromagnetic forces are not independent but are related to each other by a factor. They are collectively referred to as the electroweak forces. Our present understanding is that the color forces is independent, although, the holy grail of particle physics is to unify all the forces(including gravity) such that they can all be written in the terms of one fundamental coupling constants.

The strong force binds quarks together and holds nucleons (protons & neutrons) in nuclei. The weak force is responsible for the radioactive decay of unstable nuclei and for interactions of neutrinos and other leptons with matter. The intrinsic strengths of the forces can be compared relative to the strong force, here considered to have unit strength (i.e., =1). In these terms, the electromagnetic force has an intrinsic strength of $(\frac{1}{137})$. The weak force is a billion times weaker than the strong force. The weakest of them all is the gravitational force. This may seem strange, since it is strong enough to hold the massive Earth & planets in orbit around the Sun. But we know that the gravitational force between two bodies a distance ***r*** apart is proportional to the product of the two masses (*M* & *m*) and inversely proportional to the distance ***r*** squared

$$F = \frac{GMm}{r^2}$$

We see now what is meant by intrinsic strength. It is given by the magnitude of the universal force constant, in this case, ***G***, independent of the masses or distances involved. In similar terms, the electromagnetic force between two particles is proportional to the product of the two charges (*Q* & *q*) and inversely to the distance ***r*** squared

$$F_{em} = \alpha \frac{Qq}{r^2}$$

Here the universal constant alpha, α gives the intrinsic strength. As we noted before, forces can be represented in the theory as arising from the exchange of specific particles called gauge bosons, the quanta of the "force field". Just as photons are real (i.e., quanta of light) and can be radiated (shaken off) when charged particles are accelerated or decelerated, the other gauge bosons can also be created and observed as real particles. All the bosons have 0 or integer spins.

The carriers of the strong force are called gluons, the glue that holds quarks together in protons and neutrons and also helps form nuclei. The carriers of the weak force come in three forms, and are called weak bosons: the $W^{\pm}$ and the Z^0. The carriers of the gravitational field are called gravitons and are unique in having a spin of 2.

The gauge bosons are the carrier of the forces. They interact always with the same strength to the forces multiplied by the appropriate quantum number carried by the fermion.

They have a quantum number called spin which is =1(hence boson) and therefore obey Bose-Einstein statistics. The photon is massless and chargeless although it carries the electromagnetic forces. Because it is massless, the range of the electromagnetic potential is infinite, falling off like 1/r, where r is the distance between the forces.

Force	EM	Strong	Weak
Gauge Boson	γ	8g,Π	W^+, W^+, Z^0
Mass	0	0,100 MeV	100 GeV
Quantum Number	electric charge	color	weak isospin
Range	∞	$\infty, 1fm$	$10^{-18}m$
Strength	$\alpha_{EM} = 0.008$	$\alpha_S \approx 1.2$	$\alpha_w = 0.03$
Type	Abelian	Non-Abilian	Non-Abelian
High Energy Dependence	stronger	Weaker	Weaker

Table : summary of characteristics of the forces

The most important difference between gluons and photons is that gluons carry color charge, but photons do not carry electric charge. The color forces is the mediated by colored gluons. These do not carry electric or weak charge, but do carry the color charge. These are 8 different gluons. They are massless and because of this their range is infinite. However, the color forces itself is somewhat different from the other two, in that the strength of the forces increases the further away you are from the source, as opposed to decreasing. Obviously, if the forces gets stronger with the distance. Then we could be very badly affected by quarks in another galaxy. ”Confinement” and “Asymptotic Freedom” comes to the rescue and ensure that gluons and quarks are bound inside about 1 fm but can behave like free particle at very small distance large energy scale. The strong forces, manifests itself out beyond 1fm to about 2fm in the form of bound states of quarks called pions which we shall come to later on. However, the color and the strong forces are just different manifestations of the same forces.

The coupling constant of the forces, which will be referred to as g_f are energy dependent and are therefore often referred to as running couplings. Fig shows the running coupling constants of the forces as a function of distance from the source. The color force has the most dramatic dependence. At about 1fm=1m. Whereas inside 10^{-16} m the strong coupling is weaker than the coupling constants.

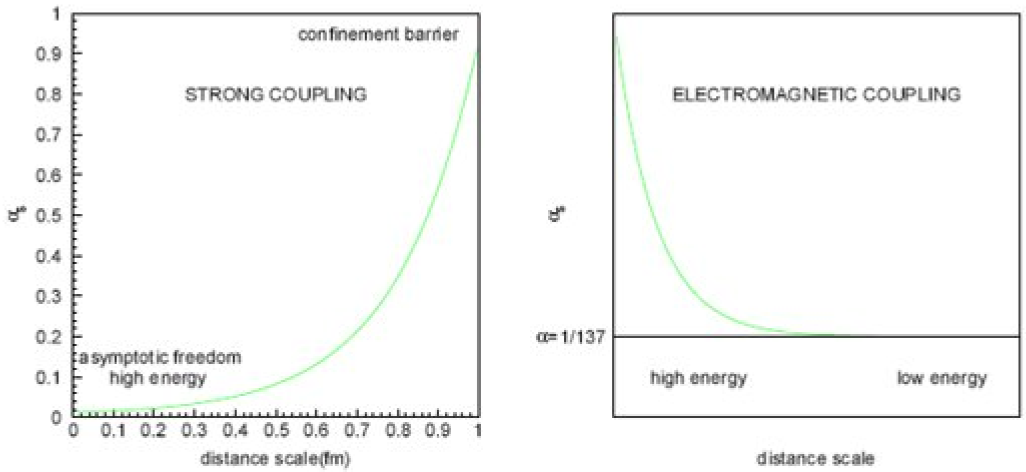


Fig 1.4- Behavior of coupling constants for EM and strong forces

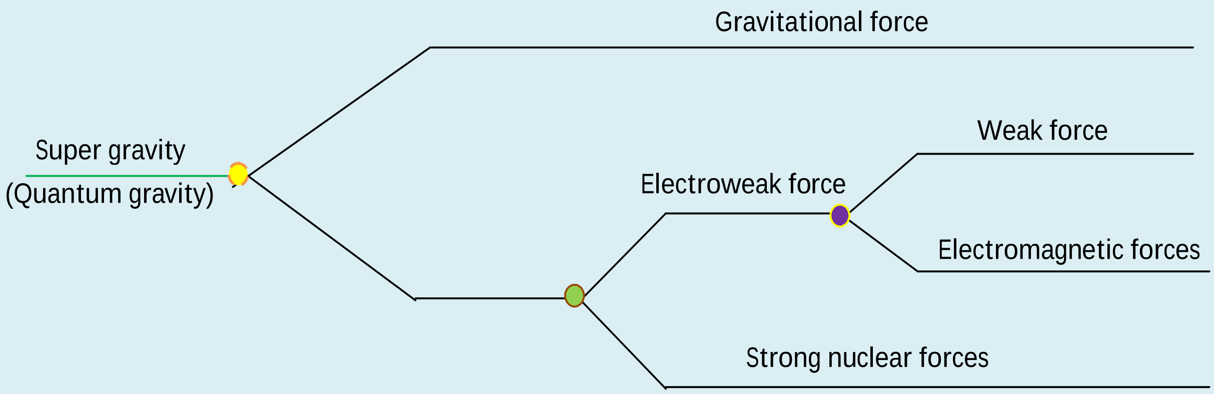
1.6. Unification Theory

All the forces of nature should be capable of being described by a single theory. but only at high energies(10^{14}GeV) should the behavior of all forces combine. This is called unification.

It is not quite satisfactory to have four different theories to account for these four forces. The electromagnetic interaction of particles is explained by a well established modern theory of Quantum Electrodynamics (QED). The weak interaction had its own theory but these two have now been combined as the Electroweak Theory in the Standard Model. The strong interaction between quarks and gluons has another theory called Quantum Chromo Dynamics (QCD), where the equivalent of electric charge is named "color". And Einstein's General Theory of Relativity explains how the gravity we know is a manifestation of the basic geometry of space-time.

Unification

All the forces of nature should be capable of being described by a single theory. but only at high energies(10^{14}GeV) should the behavior of all forces combine. This is called unification



Before the unification point, the forces are indistinguishable and have symmetry .but after the unification point, the forces act differently and the symmetry is broken.

1.7. Beyond the Standard Model

Theories, called "Grand Unification Theories" or GUTs, have been proposed to unify the electroweak force with the strong force. But so far no concrete evidence has been found for them. Beyond that, the holy grail of unification has long been the unification of gravity with all the other forces.

The theory of Super symmetry requires a whole new set of particles beyond the Standard Model complement: a heavy partner for each quark, lepton and gauge Boson of the old set, together all of them making up one great super-family of particles. The three forces strong, electromagnetic and weak all have exactly equal strengths in this theory at a very high energy. And of course, it gives experimentalists a whole new game of looking for new particles. It is just possible that one of these new super particles is a primordial relic of the Big Bang and makes up the Dark Matter in the Universe, a further incentive to discover these Super-partners.

Meanwhile theoretical studies range far and wide in a search for the Theory of Everything (TOE). Most familiar is String Theory, which pictures particles as infinitesimal little vibrating loops of strings in 10 dimensions. Further refinements lead to Membrane Theory, with the entire Universe regarded as existing on multidimensional sheets or membranes, with particles as loops anchored on "our" sheet and gravitons ranging into the continuum between sheets. We await predictions that can be tested.

1.8. Particle Physics Experiments

Throughout the history of Physics, experimental discoveries and theoretical ideas and explanations have moved forward together, sometimes playing leap-frog, but always drawing inspiration one from the other. Modern versions of Rutherford's table-top experiment on the scattering of alpha particles occupy many square kilometers of land, with massive and costly apparatus in underground tunnels tens of kilometers long. These are the particle accelerators that speed protons, antiprotons, electrons, or positrons to near the speed of light and then make them collide head-on with each other or with stationary targets.

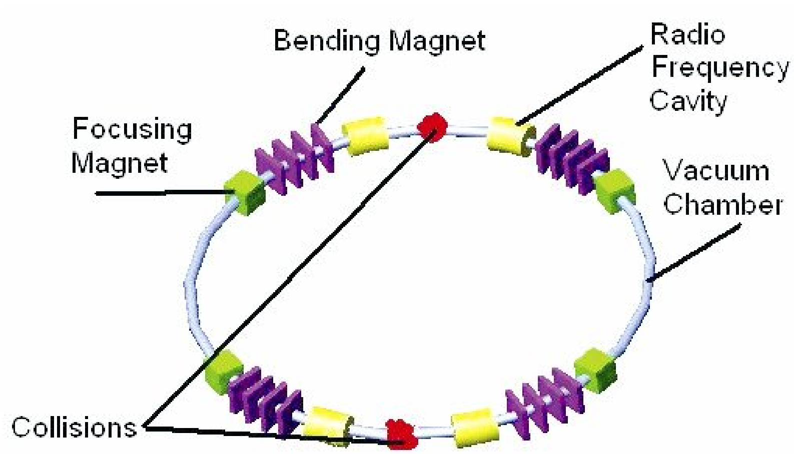


Fig. 2.5-In an accelerator, focusing magnets and bending magnets guide the beam of Particles around a ring. (Only a few of the bending magnets are shown here)High frequency microwave (RF) cavities accelerate the beams as they pass Through.

The quest has mostly been for higher and higher collision energies. To make a pair of massive new particles and observe them flying apart, one has to generate excess energy over and above the equivalent of the mass ($2m_X$) of the pair: $E_{collision} > 2m_Xc^2$. High energy is also needed to probe deeper and deeper to smaller length scales in studying the unknown this is the equivalent of using X rays of shorter wave-lengths to probe smaller crystal structures. On the other hand, to look for rare phenomena, it is necessary to increase the intensity of particle beams and the collision rates. So accelerators have proceeded along parallel paths of ever higher energies and ever higher intensities.

To observe and interpret the results of collisions, particle detectors have to be developed that can track and analyze the particles that fly apart and disappear in nanoseconds. The detector consists of many different types of complex apparatus and electronics, requiring a cadre of experts in every conceivable technology. Collider experiments use large detectors completely surrounding the "interaction point" where high energy particles and antiparticles collide head-on. Typical are electron-positron colliders, proton-antiproton colliders and massive detectors at the interaction points.

Other experiments study the collisions of intense beams with fixed (stationary) solid targets. Typical are several experiments with intense high energy neutrino beams and massive detectors in which neutrinos can interact. Many are studying the conversion of one type of neutrino (the muon-neutrino) into another (e.g., the tau-neutrino). Evidence for this is now pretty definite after decades of research, and precise measurements may pin down the non-zero mass of each neutrino. Relic neutrinos from the Big Bang populate the Universe, and even a tiny mass can explain some of the Dark Matter.

1.9. Groups and Symmetries

The simplest meaning of symmetry is that it is the nondistinguishability of an object or structure across a dividing line or a point. This implies that by doing something we wish to observe the changes in the object. If the operation does not bring about any changes in the object, the object is said to be symmetric under that particular operation. The transformation which when applied on the system, do not change it. Such transformations are called symmetric transformation and we say in other words that a system remain invariant under the application of symmetric transformations. If the system remains invariant on applying the translations or rotations, the system is said to be having translational or a rotational symmetry.

We have a set of symmetry transformations in which each transformation can be seen as an element of the set. Obviously, each transformation leaves the system invariant. It is found that the set of symmetry transformations obeys the properties of a group and we call such groups as symmetry groups. They obeys various properties i.e. closure, identity, inverse, associatively and also posses properties abelian/non-abelian groups, finite/infinite group, discrete/continuous groups, simple and semi-simple groups.

All the continuous groups appearing in particle physics are Lie Groups and generally are U(n), SU(n), O(n) and SO(n) group. We have U(n) as unitary matrices. A set of all n×n unitary matrices, obeying the defining properties of a group, forms the U(n) group. The unitary matrices, for which the inverse is equal to transpose conjugate $U^{-1}=U^{*}$, constitute the elements of the U(n) group. For n=1, the group is U (1), which is abilian. For n>1, U (n) is Non – Abilian. The groups SU (n) and O (n) are the sub groups of U (n).If the unitary matrices of U (n) group have their determinant equal to one, they are called special (det U=1) matrices, and the group whose elements are these special (S) matrices is called the SU (n) group. In the SU (n) family, we frequently come across with SU(2), SU(3), SU(4), SU(5) and SU (6) groups which are the special unitary groups respectively in two, three, four, five and six dimensions. SU (2) group is the important as it can describe both spin and isospin. SU(3) group is the generalization of SU(2) isospin group where we include hypercharge.

i. SU(2) Group

There are three group parameters. We write the 2×2 unitary unimodular matrices as

$$U\left(\varepsilon_1,\varepsilon_2,\varepsilon_3\right)=\exp\left\{i\varepsilon_a\sigma_a\right\}$$

Where the $\sigma_a's$ are 2×2 traceless hermitian matrices. We chose the basis to be the standard Pauli matrices.

$$\sigma_1=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\sigma_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix},\sigma_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

The generators defined by $J_i=\frac{\sigma_i}{2}$ will give the commutation relation

$$[J_a, J_b] = i\varepsilon_{abc} J_c$$

Where ε_{abc} is totally antisymmetric and $\varepsilon_{123} = 1$ we then abstract this as the general Lie algebra of SU(2) and all representations of the generators satisfy this set of commutation relations.

ii. SU(3) Group

There are eight group parameters. For the defining representation we write the 3×3 unitary matrices

$$U(\varepsilon_1, \varepsilon_2, \varepsilon_3 ....\varepsilon_8) = \exp\{i\varepsilon_a \lambda_a\}$$
$$a = 1, 2, 3 ....8$$

The $\lambda_a 's$ are 3×3 tracless hermitian matrices, which may be chosen to have the form (Gell-Mann 1962a)

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$
$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

They have the normalization

$$tr(\lambda_a, \lambda_b) = 2\delta_{ab}$$

And satisfy the commutation relation

$$\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right] = if_{abc} \frac{\lambda_c}{2}$$

f_{abc} is totally antisymmetric relation with nonvanishing member. [3]

1.10. Symmetry breaking

Symmetry breaking describes a phenomenon where (infinitesimally) small fluctuations acting on a system crossing a critical point decide a system's fate, by determining which branch of a bifurcation is taken. For an outside observer unaware of the fluctuations (the "noise"), the choice will appear arbitrary. This process is called symmetry "breaking", because such transitions usually bring the system from a disorderly state into one of two states. Since disorder is less symmetric in the sense that small variations to it don't change its overall appearance, the symmetry gets "broken".

In particular, we can distinguish between:

- Explicit Symmetry Breaking
- Spontaneous Symmetry Breaking

i. Explicit symmetry breaking

Explicit symmetry breaking indicates a situation where the dynamical equations are not manifestly invariant under the symmetry group considered. This means, in the Lagrangian (Hamiltonian) formulation, that the Lagrangian (Hamiltonian) of the system contains one or more terms explicitly breaking the symmetry.

ii. Spontaneous Symmetry Breaking

Spontaneous symmetry breaking (SSB) is the process by which a system described in a theoretically symmetrical way ends up in a non symmetrical state. It describes the case where the laws are invariant but it appears the system isn't because the background of the system, its vacuum, is no invariant. Such a symmetry breaking is parameterized by an order parameter.

For spontaneous symmetry breaking to occur, there must be a system in which there are several equally likely outcomes. The system as a whole is therefore symmetric with respect to these outcomes (if we consider any two outcomes, the probability is the same). However, if the system is sampled (i.e. if the system is actually used or interacted with in any way) a specific outcome must occur. Though we know the system as a whole is symmetric, we also know that it is never encountered with this symmetry, only in one specific state. Because one of the outcomes is always found with probability 1, and the others with probability 0, they are no longer symmetric. Hence, the symmetry is said to be spontaneously broken in that theory.

1.11. Quantum Field Theory

In the classical physics, classical mechanics describes a discrete system, a single particle or a system of finite number of p articles. A continuous system with an infinite number of degrees of freedom is described by classical field theory. In quantum mechanics, the dynamical variables like the coordinates of a particle become operators. In field theory, the field plays a role similar to a coordinate. It is natural, therefore, to ask what would happen if a field becomes an operator. Not unrelated to this question, is a problem associated with the interpretation of the relativistic quantum mechanics. Relativistic quantum mechanics gives a description of the dynamics of a relativistic particle. But the interpretation of the negative energy solutions of relativistic quantum mechanical equations necessitates the existence of an infinite number of particles filling all the negative energy states. The relativistic quantum mechanics, therefore, has to be a theory of infinitely many particles. We shall see that a quantized field describes a system of infinite number of identical particles. Quantization of a field, therefore, not only satisfies a natural curiosity of finding out what quantization of a field would lead to, but it also removes the essential difficulty in the understanding of relativistic quantum mechanics. The construction of this many particle theories is called field quantization and the theory is known as quantum field theory (QFT).

Quantum field theory describes the interaction between elementary particles or quanta's. Its method often is named second quantization [4]. Quantum field theory (QFT) is the application of Quantum Mechanics (QM) to the dynamical system of fields in the same sense that basic Quantum mechanics deals mainly with the quantization of dynamical system of particles.[5]

There are three types of Quantum Field Theory:

i. Quantum Electro Dynamics
ii. Quantum Electroweak Theory
iii. Quantum Chromo Dynamics

i. Quantum electrodynamics

Quantum Electro-Dynamics (QED) provides the rigorous theoretical foundations underlying atomic physics, allowing extraordinarily precise predictions of the spectra and properties of one and two electron atoms. The predictions for the Lamb shift, hyperfine splitting, fine structure, and the decay rates of hydrogen, muonium, positronium, and helium take into account not only radiative corrections due to quantum fluctuations of the electromagnetic field, but also subtle relativistic recoil and bound-state corrections. These high order calculations not only verify the applicability and consistency of the perturbative renormalization procedure of gauge theory, but they also are the forerunners of calculations for the non-Abelian extensions of QED, including the radiative corrections needed for precision tests of the unified theory of electro-weak interactions and the gauge theory of the strong and nuclear interactions, quantum chromodynamics. Much of the physics of quarkonium, heavy quark pairs (qQ) bound by gluonic interactions in quantum chromodynamics, has a direct counterpart with the physics of positronium (e^+e^-) in QED[6]

ii. Quantum Electroweak Theory

The electroweak theory presents a unified description of two of the four fundamental forces of nature: electromagnetism and the weak nuclear force. Although these two forces appear very different at everyday low energies, the theory models them as two different aspects of the same force. Above the unification energy, on the order of 10^2 GeV, they would merge into a single electroweak force.

The unification is accomplished under an $SU(2) \times U(1)$ gauge group. The corresponding gauge bosons are the photon of electromagnetism and the W and Z bosons of the weak force.

iii. QCD: The Gauge Theory of Strong Interactions

Quantum Chromo-Dynamics (QCD), the gauge field theory which describes the strong interactions of colored quarks and gluons, is one of the components of the $SU(3)\times SU(2)\times U(1)$ Standard Model. A quark of specific flavor (such as a charm quark) comes in 3 colors; gluons come in eight colors; hadrons are color-singlet combinations of quarks, anti-quarks, and gluons. [8].Physicists found that mesons could be classified in groups defined by symmetries under transformations of charge and a new quantum number called strangeness. Figure 4 shows this symmetry for spin 0 mesons. This classification scheme, known as the quark model and developed by Gell-Mann and Nishijima [7], explains the symmetry by postulating three types of quarks, u, d and s, that form the mesons in quark-antiquark pairs and baryons in triplets of quarks or antiquarks.

The presence of one baryon, the Δ^{++}, introduced a problem for the quark model since it is composed of three u quarks with parallel spin in seeming violation of the Pauli exclusion principle. This was solved by the introduction of an SU(3) gauge degree of freedom for quarks that would eventually be identified as the color symmetry of the strong force. [8] Associated with this symmetry is the octet of vector gauge bosons that are now called the gluons.

When high energy hadron collisions were interpreted to show point-like constituents within hadrons such as the proton, the quarks, till then a theoretical construct, were identified as fundamental particles that make up the mesons and baryons. The later discovery of the c, b and t quarks combined with u, d and s quarks as well as the postulated SU(3) color symmetry form the modern theory of QCD in the Standard Model.

In QCD, the quarks belong to color triplets of the SU(3) representation where the color charges are defined as red (r), yellow (y) and blue (b). The gauge bosons are the octet of gluons that mediate the color interaction and carry color charge as well. The interactions of the theory include gluon-gluon interactions since they carry color charge as well. The fact that color is not observed directly in nature means there must be an exact color symmetry for observable states in QCD. These color singlet states exist for qqq and $\bar{q}q$ combinations with the irreducible representations:

$$3\otimes 3\otimes 3 = 1\oplus 8\oplus 8\oplus 10$$

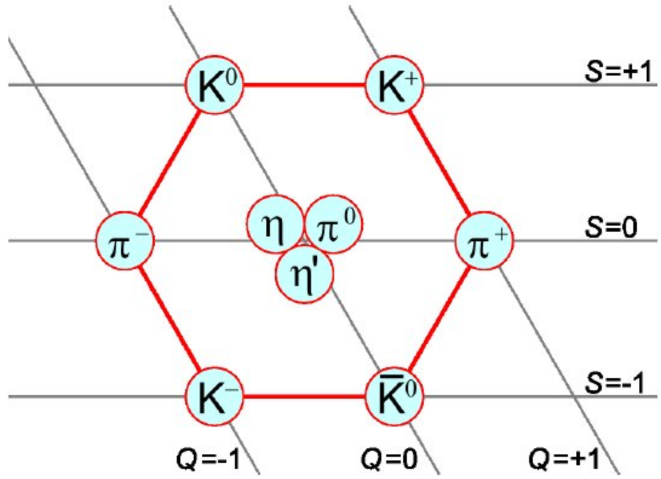


Fig. 1.6: Mesons in the spin 0 nonet.

$$3 \otimes 3^{*} = 1 \otimes 8$$

Here the antiquarks belong to the complex conjugate representation 3^{*}. There are no color singlet representations for qq or qqqq and these types of states have not been observed The limitation of QCD states to color singlets is referred to as quark confinement since it means quarks are always observed as constituents of qqq or $\bar{q}q$ bound states. An explanation of confinement as a dynamical consequence of QCD is beyond the reach of calculations but it is possible to postulate a qualitative explanation. Since the gluons carry charge, they interact with each other in addition to interacting with the quarks. Fig 1.6 shows an approximate description of field lines between two quarks interacting via the strong force.

One can see that the fields lines tend to bunch together compared to the electric interaction. This is due to the fact that the gluons carry charge and interact. If the quarks are separated, the amount of energy in the strong field increases until there is enough energy to produce a $\bar{q}q$ pair from the vacuum. Two new mesons form from the original $\bar{q}q$ pair and the new $\bar{q}q$ pair,

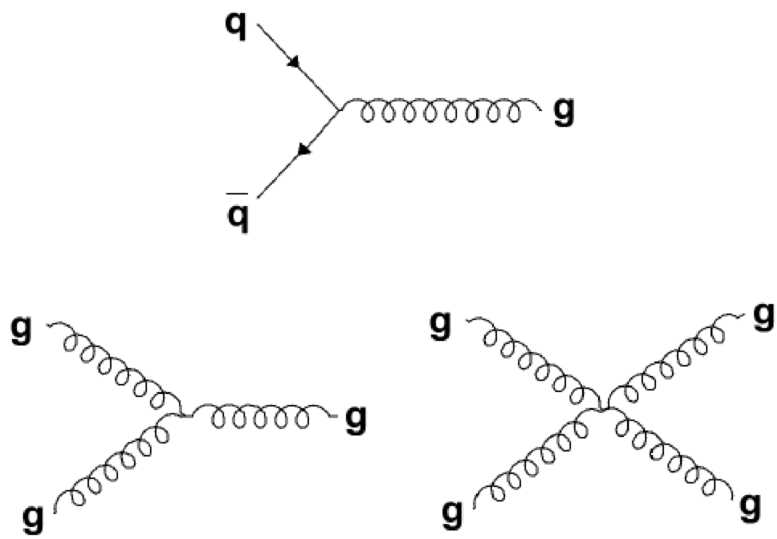


Fig. 1.7: Diagrams of the strong interactions via the gluons of QCD.

Notice the gluons interact with themselves since they also carry color charge. preserving confinement q q

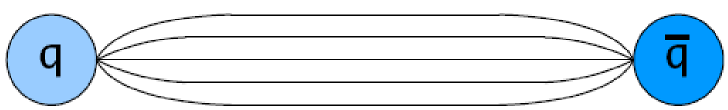


Fig.1.8: Qualitative picture of the strong field lines between two interacting quarks.

The confinement of quarks in color singlet states characterizes the theory at small energy (long distance) scales. The inability to derive a quantitative description of confinement stems from that fact that the coupling constant of the strong force s is of order 1 for the binding energy scales within hadrons. If one looks at higher energies however, one sees a running of the coupling constant towards lower values as shown in Fig1.7. This property, known as asymptotic freedom, means quarks will couple ”weakly” in high energy interactions and the theory can be treated with a perturbative expansion in powers of the coupling constants[9]

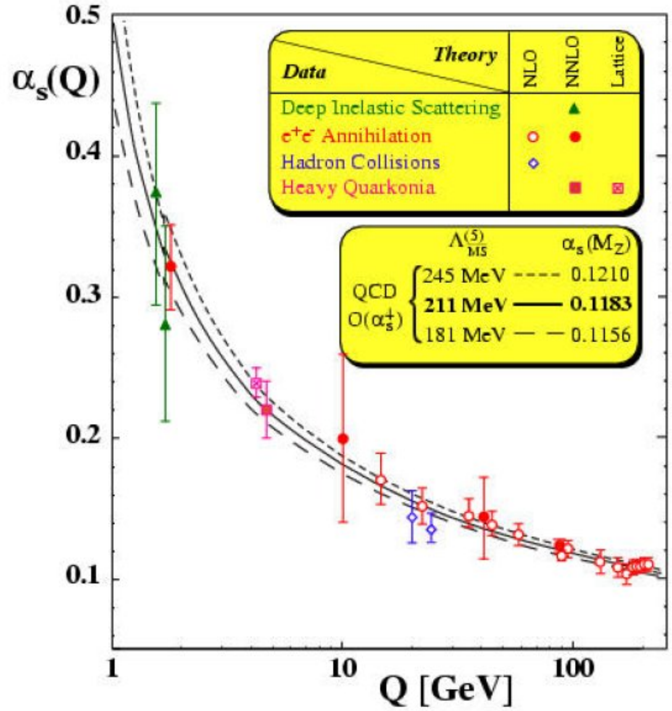


Fig.1.9: Running of the coupling constants.

The interplay between the short distance perturbative asymptotic freedom and the long distance, nonperturbative confinement of QCD plays an important role in understanding the phenomenology of the theory. While experiments often probe strongly interacting particles at energy scales where asymptotic freedom applies, these particles are still the color singlets of the theory and must be handled in the nonperturbative regime.

QCD has two peculiar properties:

- Asymptotic freedom, which means that in very high-energy reactions, quarks and gluons interact very weakly.
- Confinement, which means that the force between quarks does not diminish as they are separated. Because of this, it would take an infinite amount of energy to separate two quarks; they are forever bound into hadrons such as the proton and the neutron. Although analytically unproven, confinement is widely believed to be true because it explains the consistent failure of free quark searches.[10]

Chapter-2

An Effective Theory for Heavy Mesons

Effective Field theory (EFT) is important tool in theoretical physics [11].In the particle physics; it is often the case that the effects of a very heavy particle become irrelevant at low energies. It is then useful to construct a low energy effective theory in which this heavy particle no longer appears. Eventually, this effective theory will be easier to deal with than the full theory. It provides a systematic formalism for the analysis of multi-scale problems. This is particularly important in QCD, where the value of the running coupling $\alpha_s(\mu)$ can change significantly between different energy scales. As such, EFT greatly simplifies practical calculations in field theory; indeed, it often makes such calculations feasible. As we will discuss, EFT also provides a new, modern meaning to "renormalization".

Effective field theories have become a widely used tool in modern elementary particle physics. An effective theory treatment is convenient if the problem under consideration involves very disparate mass scales such that the physics that is to be described happens at much lower energies than the scale set by some heavy particles in the theory. In such a case it is useful to switch to an effective theory in which the heavy degrees of freedom do not appear explicitly; they only reappear in the effective theory as higher dimensional operators, which are multiplied by coupling constants with negative mass dimension. The scale of the coupling constants is set by the large mass and thus these contributions are small, if the scale of the physics described with the help of the effective theory is small compared to this large mass. An effective theory is always valid only in a limited region of scales, a natural cut-off is given by the mass of the particle which has been removed by switching from the full to the effective theory. As mentioned above an effective theory involves interactions which would lead to a non-renormalizable theory, if one would consider the theory to all orders in these higher dimensional operators. However, working to a definite order of the expansion in inverse powers of the large scale one does not face any problem concerning renormalization. Starting from the renormalizable dimension-4 piece of the effective theory Lagrangian we may use its renormalization group properties to study the cut-off dependence of the effective theory, which is determined by the short distance properties of the effective theory. Applying effective theory methods corresponds to an expansion of the Greens functions of the full theory in inverse powers of the large mass scale; such an expansion is only possible up to logarithmic dependences on this large scale. These logarithms may be accessed using a properly constructed effective theory, where these logarithms correspond to the renormalization group logarithms of the cut off. In this way one may even achieve a resummation of the logarithmic terms using renormalization group methods in the effective theory.

In the case at hand the large scale is the mass m_Q of the heavy quark, and the leading term of this expansion is the static limit. This effective theory (Heavy Quark Effective Theory, HQET) is a powerful tool, allowing for numerous purely QCD based calculations. Renormalization in this effective theory implies a factorization theorem for the Greens function of full QCD, which means that to any order in the $1/m_Q$ expansion one, may factorize the short distance physics from the long distance effects.

2.1. Heavy Quark Effective Theory

Heavy Quark Effective Field Theory (HQET) [12] has been established as the theoretical tool of choice for the description of heavy mesons (Qq) and baryons containing one heavy quark (Q). Q is heavy quark (c, b, t), q is light quark (u, d, s). More generally, the expansion in inverse powers of the heavy quark mass m_Q, has become a generally accepted and widely used tool in heavy quark physics. Based on the infinite mass limit $m_Q \to \infty$ of QCD it provides a model independent starting point for the description of weak transitions involving heavy quarks. HQET is an effective field theory which may be obtained from QCD by performing a $1/m_Q$ expansion.

The idea to exploit the fact that the mass of a heavy quark is large compared to the typical scale Λ of the light QCD degrees of freedom (e.g. the constituent mass of a light quark or the scale of the QCD coupling constant Λ_{QCD}) is in fact quite old. However, in the late eighties a breakthrough was achieved by mainly two observations. First, in the infinite mass limit QCD exhibits an additional flavor symmetry and a spin symmetry, the group theory of which allow model independent statements concerning weak decays of heavy hadrons. Second, it was noted that the $1/m_Q$ expansion of QCD can be formulated as an effective field theory [14], which allows accessing the corrections to the infinite mass limit in a systematic way.

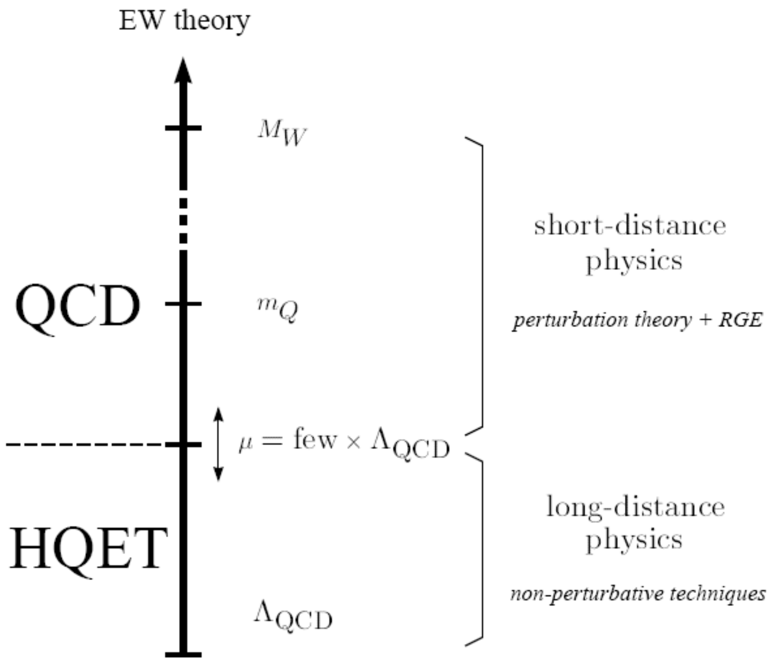


Fig.2.1: Philosophy of the heavy-quark effective theory.

The heavy-quark effective theory (HQET) is constructed to provide a simplified description of processes where a heavy quark interacts with light degrees of freedom predominantly by the exchange of soft gluons [13-23]. Clearly, M_q is the high-energy scale in this case, and Λ_{QCD} is the scale of the hadronic physics we are interested in. The situation is illustrated in Fig. 2.1. At short distances, i.e. for energy scales larger than the heavy-quark mass, the physics is perturbative and described by ordinary QCD. For mass scales much below the heavy-quark mass, the physics is complicated and non-perturbative because of confinement. Our goal is to obtain a simplified description in this region using an effective field theory. To separate short- and long-distance effects, we introduce a separation scale μ such that $\Lambda_{QCD} \ll \mu \ll m_Q$. The HQET will be constructed in such a way that it is identical to QCD in the long-distance region, i.e. for scales below μ. In the short-distance region, the effective theory is incomplete, however, since some high-momentum modes have been integrated out from the full theory. The fact that the physics must be independent of the arbitrary scale μ allows us to derive renormalization-group equations, which we shall employ to deal with the short-distance effects in an efficient way. Compared with most effective theories, in which the degrees of freedom of a heavy particle are removed completely from the low-energy theory, the HQET is special in that its purpose is to describe the properties and decays of hadrons which do contain a heavy quark. Hence, it is not possible to remove the heavy quark completely from the effective theory. What is possible is to integrate out the "small components" in the full heavy-quark spinor, which describe the fluctuations around the mass shell.

2.2. Chiral Symmetry and Heavy quark symmetry

The strong interaction of system containing heavy quarks is easier to understand than those of systems containing only light quark. One reason is the asymptotic freedom, i.e. the effective coupling constant of QCD becomes weak in process with large momentum transfer corresponding to interaction at short distance scales. At large distance coupling becomes strong, leading to non-perturbative phenomenon such as the confinement of quarks and gluons on a length scale $R_{had}\sim 1/\Lambda_{QCD}\sim 1$fm which determines the size of the hadrons. $\Lambda_{QCD}\sim 0.2$ GeV is the scale that separates the regime of large and small coupling constant. Masses much above this scale are called a heavy quark like c, b and t while u, d, and s are the light quarks.

The system become complicated for the heavy meson Qq which contain one heavy quark (Q) and one light quark(q) where $m_Q >> \Lambda_{QCD}$ and $m_q <<< \Lambda_{QCD}$. Such a heavy light meson has a typical size of the order of Λ_{QCD}^{-1} and typical momentum exchanged between heavy and light constituent is of the order Λ_{QCD}. In the rest frame of the heavy quark, it is in fact only the electric color field that is important; relativistic effects such as color magnetism vanish as $m_Q\rightarrow\infty$. Since the heavy-quark spin participates in interactions only through such relativistic effects, it decouples. Thus the heavy-quark mass becomes irrelevant can be seen as follows: As $m_Q \rightarrow \infty$, the heavy quark and the hadrons carrying it have the same velocity. And hence the configuration of light degrees of freedom in hadrons containing a single heavy quark with velocity v does not change if this quark is replaced by another heavy quark with different flavour or spin, but with the same velocity. In the rest frame of hadron, the heavy quark is at rest too. Both heavy quarks lead to the same static color field.[24]

Heavy-quark symmetry is an approximate symmetry, and corrections arise since the quark masses are not infinitely heavy. In many respects, it is complementary to chiral symmetry, which arises in the opposite limit of small quark masses. However, chiral symmetry is a symmetry of the QCD Lagrangian in the limit of vanishing quark masses, whereas heavy-quark symmetry is not a symmetry of the Lagrangian (not even an approximate one), but rather a symmetry of an effective theory, which is a good approximation of QCD in a certain kinematics region. It is realized in only in those systems in which a heavy quark interacts predominantly by the exchange of soft gluons. In such systems the heavy quark is almost on shell; its momentum fluctuates around the mass shell by an amount of order Λ_{QCD}. The corresponding fluctuations in the velocity of the heavy quark vanish as $\Lambda_{QCD}/m_Q \rightarrow 0$. The velocity becomes a conserved quantity and is no longer a dynamical degree of freedom. Nevertheless, results derived on the basis of heavy-quark symmetry are model-independent consequences of QCD in a well-defined limit. These symmetry breaking correction can be systematically studied in a convenient framework provided by HQET.

In heavy meson, in the limit of infinite quark mass the gluon decouples from the quark spin; in other words the effective Lagrangian is invariant under heavy quark spin transformations and has, therefore, a further SU(2) spin symmetry. In conclusion, the complete symmetry of the effective Lagrangian is a $SU(2N_f)$ of flavor (N_f is the number of heavy flavors) and spin for each value of the heavy quark velocity. From the point of view of HQET, it is natural to divide quarks into two classes by comparing their Lagrangian mass with Λ_{QCD}. In this partition $m_{u,}$ m_d and m_s are considered to be light quarks (m_q) where as m_b, m_c, and m_t are the heavy quarks ($m_{u,}$ m_d and $m_s << \Lambda_{QCD} << m_b$, m_c, and m_t). The light quark part of the QCD Lagrangian has a chiral symmetry which arises since the current masses of the light quarks are small compared to the intrinsic mass scale of the strong interaction. Thus in the limit m_u, m_d and $m_s \rightarrow 0$ the QCD Lagrangian for these quarks possess $SU(3)_L \otimes SU(3)_R \otimes U(1)_V$ approximate symmetry.

This approximate symmetry is spontaneously broken to vector $SU(3)_V \otimes U(1)_V$ subgroup. Associated with this spontaneous breaking of the approximate chiral symmetry are the pseudoscalars octet π, k, η. The interaction of this pseudoscalar Goldstone boson with the heavy meson at low momentum can be described by an effective chiral symmetric Lagrangian that contains the most general couplings consistent with chiral symmetry.

2.3. Renormalization

Renormalization provides a common framework to critical phenomena, where scale dependence plays an important role in describing Physics in a consistent way. When one considers elementary processes in Particle Physics, the usual way to proceed is to start form a classical description, based on equations of motion, and then look for the quantum corrections. We consider here an example taken from Quantum Electrodynamics (QED) which captures the essential features.

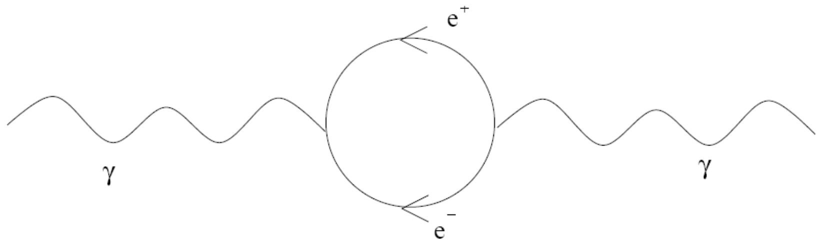


Fig.2.2: Creation/annihilation of a virtual pair electron/positron. Being antiparticles, the electron and positron have same mass, but opposite charge (and thus electric charge is conserved at a vertex photon-electron-positron).

In fig.2.2, a photon propagating creates a pair electron/positron, which finally annihilates to generate another photon. This process is possible as a consequence of two fundamental properties of relativistic quantum theory:

1. Relativistic aspect: the equivalence mass/energy enables the massless photon to ”split” into massive electron and positron. For this event to start playing a role, the energy of the photon should be at least twice the mass energy of the electron, i.e. roughly 106 eV.
2. Quantum aspect: the uncertainty energy/time allows the pair electron/positron to exist, during a time proportional to the inverse of their mass. This time is of the order of 10^{-21}s.

This electron/positron pair is said virtual since it is probabilistic and exists for a very short time only. Nevertheless, this quantum effect influences the strength of the interaction between electric charges, and therefore contributes to the value of the electron’s charge that is measured.[25]

The computation of the quantum correction to the photon propagation shown in fig.2.2

involves an integration over all the Fourier modes of the electron/positron. This integration happens to be divergent if it is done in a straightforward manner. The origin of this divergence is the point-like structure of the electron/positron. To avoid this divergence, one has to regularize the integration, and several techniques exist.

The idea is to start from a classical theory containing a very large mass scale, such that quantum fluctuations are frozen and thus can be neglected compared to the classical description of the system. As this mass decreases, quantum fluctuations progressively appear in the system and the parameters tend to their physical values. The interesting point is that it is possible to describe the appearance of quantum fluctuations by an exact equation. The advantage though is that this scheme is independent of any cut off procedure, since it does not deal with a classification of the degrees of freedom in terms of their Fourier modes, but in terms of their quantum fluctuations' amplitude. It is consistent with gauge invariance and reproduces the well known renormalization flows that are obtained by regularizing would-be divergences.

Note that the concept of renormalization is in principle independent of would-be divergences. It is the presence of these divergences which enforces the use of renormalization. The toy model QED in 2 space dimensions is an example where no divergences occur but the flow of the parameters with the amplitude of the quantum corrections can be studied [26]. In such a theory, physical quantities can be expressed in terms of the bare as well as the dressed parameters.

2.4. Chiral perturbation theory

Chiral perturbation theory provides a systematic method for discussing the consequences of the global flavor symmetries of QCD at low energies by means of an effective field theory. The effective Lagrangian is expressed in terms of those hadronic degrees of freedom which, at low energies, show up as observable asymptotic states. At very low energies these are just the members of the pseudoscalar octet (π, K, η) which are regarded as the Goldstone bosons of the spontaneous breaking of the chiral $SU(3)_L \times SU(3)_R$ symmetry down to $SU(3)_V$. The non-vanishing masses of the light pseudoscalar in the "real" world are related to the explicit symmetry breaking in QCD due to the light quark masses [27].

In this review recent developments in chiral perturbation theory (CHPT), which is the effective field theory of the standard model below the chiral symmetry breaking scale, are considered. The effective chiral Lagrangian formulated in terms of the pseudoscalar Goldstone bosons (pi, K, eta) is briefly discussed. It is shown how one can gain insight into the ratios of the light quark masses and to what extent these statements are model independent. A few selected topics concerning the dynamics and interactions of the Goldstone bosons are considered. These are pi pi and pi K scattering, some non-leptonic kaon decays and the problem of strong pionic final-stage interactions. CHPT also allows us to make precise statements about the temperature dependence of QCD Green functions and the finite-size effects related to the propagation of the (almost) massless pseudo scalar mesons. A central topic is the inclusion of matter fields, baryon CHPT. The relativistic and the heavy-fermion formulation of coupling the baryons to the Goldstone fields are discussed. As applications, photonucleon processes, the pi N Sigma term and nonleptonic hyperon decays are presented. Implications of the spontaneously broken chiral symmetry on the nuclear forces and meson exchange currents are also described. Finally, the use of effective field theory methods in the strongly coupled Higgs sector and in the calculation of oblique electroweak corrections is touched upon [28]

2.5. Hadrons with Heavy Quarks

The B meson is the hydrogen atom of quantum chromodynamics (QCD), the simplest non-trivial hadrons. In the leading approximation, the b quark in it just sites at rest at the origin and creates a chromoelectric field. Light constituents (gluons, light quarks, and antiquark) move in this external field. Their motion is relativistic; the number of gluons and light quark-antiquark pair in this light cloud is undetermined and varying. Therefore, there are no reasons to expect that a non-relativistic potential quark model would describe the B meson well enough (in contrast to the Y meson, where the non-relativistic two-particle picture gives a good starting point).

Similarly, the λ_b baron can be called the helium atom of QCD. Unlike in atomic physics, where the hydrogen atom is much simpler than helium, the B and λ_b are equally difficult. Both have a light cloud with a variable number of relativistic particles. The size of this cloud is the confinement radius $1/\Lambda_{QCD}$; its properties are determined by large- determined by large-distance nonperturbative QCD.

The analogy with atomic physics can tell us a lot about hadrons with a heavy quark. The usual hydrogen and tritium have identical chemical properties, despite the fact that the tritium nucleus is three times heavier than the proton. Both nuclei create identical electric fields, and both stay at rest. Similarly, the D and B mesons have identical ‘’hadro-chemical’’ properties, despite the fact that the b quark is three times heavier than c.

In the limit $m\rightarrow\infty$, the heavy quark spin does not interact with the gluon field. Therefore, it may be rotated at well, without changing the physics. Such rotation can transform the B and B^* into each other; they are degenerate and have identical properties in this limit. This heavy quark spin symmetry yield many useful relations among heavy hadron form factor [29]. Not only the orientation, but also the magnitude of the heavy quark spin is irrelevant in the infinite mass limit. We can switch off the heavy quark spin, making its spinless, without affecting the physics. The trick considerably simplifies counting independent form factors, and we shall use it often. Or, if we wish, we can make the heavy quark have spin 1. It does not matter.

This leads to a supersymmery group called super flavor symmetry [30]. It can be used to predict properties of hadrons containing a scalar or vector heavy quark. Such quarks exist in some extensions of the standard Model (for example, supersymmetric or composite extensions). This idea can also be applied to baryons with two heavy quarks. They form a small-size bound state (with a radius of order 1/m) which has spin 0 an1 and is ant triplet in color. Therefore, these baryons are similar to mesons with a heavy anti-quark that has spin 0 or 1. The accuracy of this picture cannot be high, however, because even the radius of the bb di-quark is only a few times smaller than the confinement radius.

Chapter-3

Heavy Meson Masses

B Mesons having one bottom quark or anti-quark. D mesons containing a charm quark or anti-quark (i.e. $(c\bar{d}, c\bar{u}, c\bar{s})$). The D mesons of lowest mass (D_0, D^+) are pseudoscalar (spin parity $J^P = 0^-$) and decay weakly into non-charmed (and predominantly strange) mesons. The more massive D^* mesons are broad resonance decaying strong interactions to lighter charm states, and are either vector or tensor mesons $(J^P = 1^-, 2^+)$

3.1 Mesons with Heavy Quarks

Let us consider mesons with the quark content $\bar{Q}q$, where Q is a heavy quark with mass m (c or b), and q is the light quark (u, d, or s). as discussed above ,the heavy quark spin is inessential in the limit m→∞, and may be switched off. In a world with a scalar heavy antiquark, S-wave mesons have angular momentum and parity $j^P = (1/2)^+$. P-wave mesons have $(1/2)^-$ and $(3/2)^-$. The energy difference between these two P-wave states (fine splitting) is a constant times Λ_{QCD} at m→∞, just like the splitting between these P-wave states and the ground state; however, this constant is likely to be small.

In the real world, the heavy anti quark $\bar{Q}$ has spin and parity $S_Q^P = (1/2)^-$. The quantum numbers in the above paragraph are those of the cloud of light field of a meson. Adding the heavy-antiquark spin, we obtain, in the limit m→∞, a degenerate doublet of S-wave mesons with spin and parity $s^P=0^-$ and 1^-, and two degenerate doublets of P- wave mesons, one with $s^P=0^+$ and 1^+, and the other with $s^P=1^+$ and 2^+. At a large but finite heavy quark mass m, these doublets are not exactly degenerate. Hyperfine splitting, equal to some dimensionless numbers times Λ^2_{QCD}/m, appear. It is natural to expect that hyperfine splitting in P-wave mesons are less than in the ground state S-wave doublet, because the characteristic distance between the quarks is large in the P-wave case. Note that the 1^+ mesons from the different doublets do not differ from each other by any exactly conserved quantum numbers and hence can mix. They differ by the angular momentum of the light fields, which is conserved up to *1/m* corrections; therefore, the mixing angle should be of the order of Λ_{QCD}/m.

Mesons with q=u and d form isodoublets ; together with isosinglet with q=s, they form SU(3) triplets. The experimentally observed [31] mesons contain the $\bar{c}$ antiquark are shown in fig. the energy scale at the left is in MeV, relative to the lowest mass meson. The meson $\overline{D_1}$ and $\overline{D_2}$ form a doublet, with the quantum numbers of the light fields $J^P = (3/2)^+$.The second P-wave doublet is suspiciously absent. It should be closed to the $(3/2)^+$ one ; it is not more difficult to produce these mesons than the $(3/2)^+$ one. The problem is that they are too wide, and cannot be cleanly separated from the continuum.

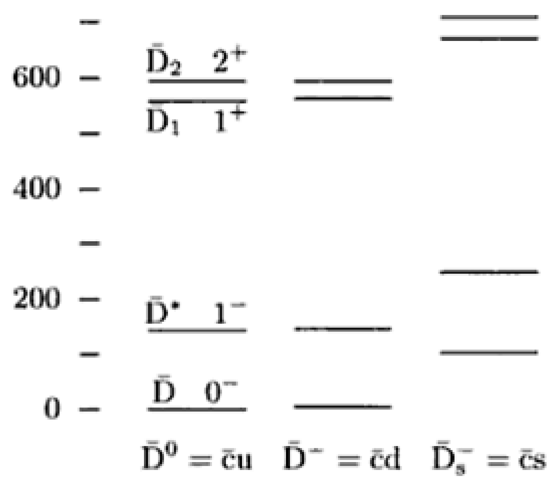


Fig.3.1: meson containing $\bar{c}$

In the leading approximating, the spectrum of $\bar{b}$ containing mesons is obtained from the spectrum of $\bar{c}$ containing mesons simply by a shift by m_b-m_c. the experimentally observed mesons containing the $\bar{b}$ antiquark are shown in fig; the spectrum of $\bar{c}$ containing mesns is shown by dashed lines for comparison. It is positioned in such a way that the weighted average energies of the ground states doublets coincide, where the 1^- meson has weight 3 and the 0^- meson has weight 1. The state B_1 and B_2 are not resolved, and are shown by single line.

Experimentally, it is difficult to measure their masses exactly enough. The hyper fine splitting of the ground state doublet is smaller for B mesons than for D mesons, as observed. [32]

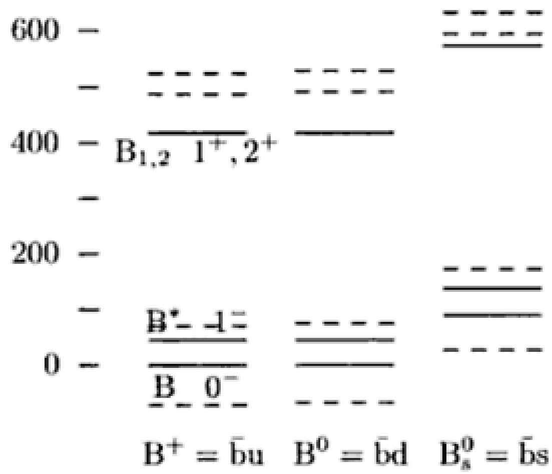


Fig.3.2: meson containing $\bar{b}$

i. <u>Masses of heavy mesons</u> $(Q\bar{q})$

$S_Q = \left(\frac{1}{2}\right)$	$S_1 = \left(\frac{1}{2}\right)^-$ **Ground state**		$S_2 = \left(\frac{1}{2}\right)^+$ **LL excited state**	
J^P	0^-	1^-	0^+	1^+
General states	D	D^*	D_0	D_1'
$c\bar{d}$	$\mathbf{D^0} = 1864.84$ ± 0.17MeV (m_{H_1})	$\mathbf{D^{*0}} = 2006.97$ ± 0.19MeV $(m^*_{H_1})$	$\mathbf{D}_0^0 = ?$ (m_{S_1})	$\mathbf{D}_1^{0'} = 2422.3$ ± 1.3MeV $(m^*_{S_1})$
$c\bar{u}$	$\mathbf{D}^+ = 1869$ ± 0.20MeV (m_{H_2})	$\mathbf{D}^{*+} = 2010.27$ ± 0.17MeV $(m^*_{H_2})$	$\mathbf{D}_0^+ = ?$ (m_{S_2})	$\mathbf{D_1}^{+'} = ?$ $(m^*_{S_2})$
$c\bar{s}$	$\mathbf{D}_S^+ = 1968.49$ ± 0.34MeV (m_{H_3})	$\mathbf{D}_S^{*+} = 2112.3$ ± 0.5 MeV $(m^*_{H_3})$	$\mathbf{D}_{0S}^+ = 2317.8$ ± 0.6MeV (m_{S_3})	$\mathbf{D}_{1S}^+ = 2535.35 \pm 0.34$ ± 0.5MeV $(m^*_{S_3})$
	Pseudo scalars	Vector	scalars	Axial vector

Table3.1:Masses of D-Mesons $(c\bar{d}, c\bar{u}, c\bar{s})$ [33]

ii. Mass spectra of Heavy Mesons

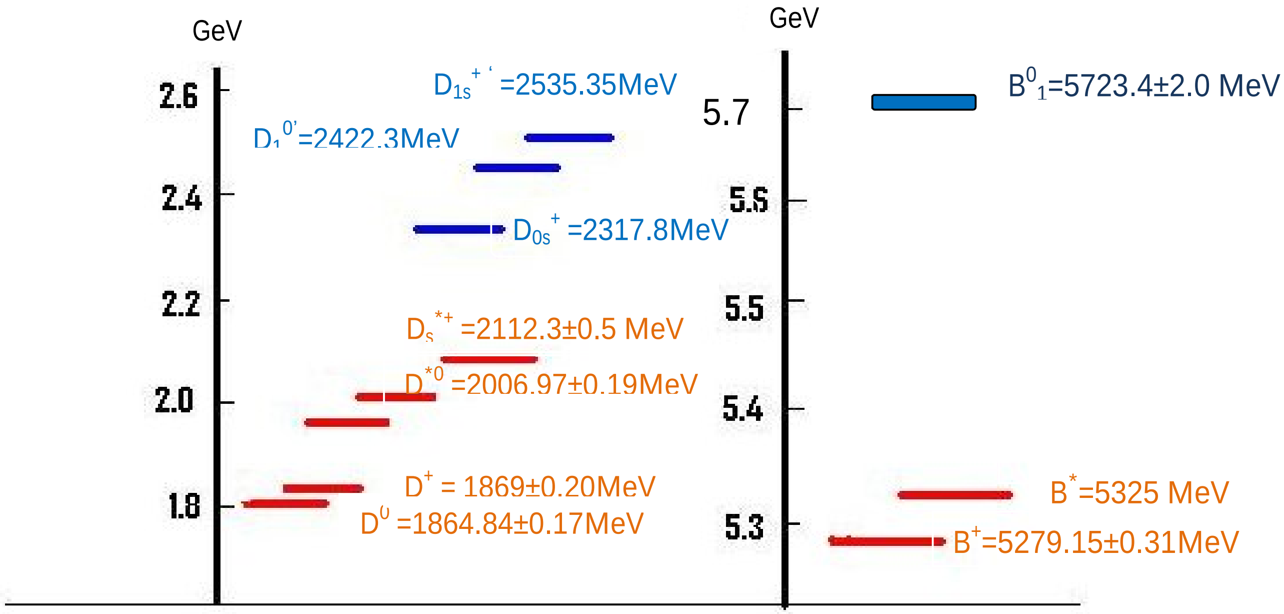


Table 3.2: Spectroscopy of heavy mesons

3.2 Effective Chiral Lagrangian

Since the heavy quark constitute most of the mass of heavy meson and the velocity of heavy quark is going to be that of meson itself. Hence, we can write

$$p_Q^\mu = m_Q v^\mu + k^\mu \tag{1}$$

where v^μ is four velocity and k^μ is residual momentum of heavy quark which is of order of Λ_{QCD}. We note that changes in heavy quark velocity vanish as $\Lambda_{QCD}/m_Q \to 0$. Now we will introduce a large and small component field h_v and H_v as below,

$$h_v(x) = e^{(im_Q v.x)} P_+ Q(x)$$

.

$$H_v(x) = e^{(im_Q v.x)} P_- Q(x) \tag{2}$$

Where $P_{\pm} = \frac{(1 \pm v)}{2}$ are the projection operators and they are such that $P_+P_+ = P_+, P_-P_- = P_-$ and $P_+ + P_- = 1$. Here we have used the relation $v.v = 1$. The exponential pre-factors in the field definitions is a rescaling precisely defined to get m_Q out of leading term in langrangian. From above equations we get,

$$Q(x) = e^{(-im_Q v.x)}[h_v(x) + H_v(x)] \tag{3}$$

From the definitions of projection operators, we can see that the large and small components fields h_v and H_v satisfy the constraints

$$\begin{aligned} vh_v &= e^{(im_Q v.x)} v\frac{(1+v)}{2}Q \\ &= e^{(im_Q v.x)} \frac{(1+v)}{2}Q \\ &= h_v \end{aligned}$$

and similarly, for $vH_v = -H_v$

Now, we see these fields to rewrite the QCD langrangian as follows

$$L_{QCD} = L_q + L_Q \tag{4}$$

where q corresponds to light and Q corresponds to heavy quarks.

$$\begin{aligned} L_Q &= \overline{Q}(i\not{D} - m_Q)Q \\ &= e^{im_Q v.x}[\overline{h_v} + \overline{H_v}](i\not{D} - m_Q)e^{-im_Q v.x}[h_v + H_v] \\ &= [\overline{h_v} + \overline{H_v}](i\not{D} - m_Q)[h_v + H_v] + [\overline{h_v} + \overline{H_v}](m_Q)[h_v + H_v] \\ &= \overline{h_v} i\not{D} h_v + \overline{H_v}(i\not{D} - 2m_Q)H_v + \overline{h_v}(i\not{D} - 2m_Q)H_v + \overline{H_v} i\not{D} h_v \end{aligned}$$

Using the standard γ matrix relations we obtain the relation,

$$\frac{1+\not{v}}{2}\gamma_\mu \frac{1+\not{v}}{2} = v^\mu \frac{1+\not{v}}{2} \tag{5}$$

With this relation and using the fact that $vh_v = h_v$ and $vH_v = -H_v$, we get,

$$L_Q = \overline{h_v}(iv.D)h_v - \overline{H_v}(iv.D + 2m_Q)H_v + \overline{h_v}(i\not{D}_\perp)H_v + \overline{H_v}(i\not{D}_\perp h_v)$$

Where $D_{\perp}^{\mu} = D^{\mu} + v^{\mu}(v.D)$ and is orthogonal to heavy quark velocity i.e. $v.D_{\perp} = 0$. From the above equation we can see that the h_v field corresponds to mass-less degree of freedom while H_v represents fluctuations with mass $2m_Q$ which corresponds to pair creation. These are the heavy degrees of freedom that will be eliminated in the process of constructing effective theory. Using the above Lagrangian, we get the equation of motion as

$$\frac{\partial L}{\partial H} = (iv.D + 2m_Q)H_v = i\not{D}_{\perp}h_v = 0 \tag{6}$$

By solving this equation we get,

$$H_v = \frac{1}{2m_Q + iv.D} i\not{D}_{\perp}h_v = 0 \tag{7}$$

which tells that the small component field is indeed goes as$1/m_Q$. Substituting this into the langrangian we obtain

$$L_{eff} = \overline{h_v}(iv.D)h_v + \overline{h_v}i\not{D}_{\perp}\frac{1}{2m_Q + iv.D}i\not{D}_{\perp}h_v \tag{8}$$

Let us rewrite this non-local langrangian in powers of $1/m_Q$ as below.

$$L_{eff} = \overline{h_v}(iv.D)h_v + \frac{1}{2m_Q}\overline{h_v}i\not{D}_{\perp}\frac{1}{1+\frac{iv.D}{2m_Q}}i\not{D}_{\perp}h_v$$

$$= \overline{h_v}(iv.D)h_v + \frac{1}{2m_Q}\sum_{n=0}^{\infty}\overline{h_v}i\not{D}_{\perp}(\frac{iv.D}{2m_Q})^n i\not{D}_{\perp}h_v$$

Using the identity

$$\frac{1+\not{v}}{2}\gamma_{\mu}\frac{1-\not{v}}{2}\gamma_{\nu}\frac{1+\not{v}}{2} = \frac{1+\not{v}}{2}(g_{\mu\nu} - v_{\mu}v_{\nu} - i\sigma_{\mu\nu})\frac{1+\not{v}}{2}, \tag{9}$$

Where $[iD^{\mu}, iD^{\nu}] = ig_{\alpha}G^{\mu\nu}$ we write the Lagrangian as

$$L_{eff} = \overline{h_v}v.Dh_v + \frac{1}{2m_Q}\overline{h_v}(iD_{\perp})^2 h_v + \frac{g_{\alpha}}{4m_Q}h_v\sigma_{\mu\nu}G^{\mu\nu}h_v + O(1/(m_Q^2) \tag{10}$$

In the limit $m_Q \to \infty$ the lagrangian becomes

$$L_{\infty} = \overline{h_v} iv.Dh_v \tag{11}$$

It is obvious from above lagrangian that it does has spin-flavor symmetry as $m_Q \rightarrow \infty$ (because it does not have both Dirac matrices and mass term). But if consider the lagrangian given by equation (10), which has two additional terms, breaks this symmetry. The first term in the lagrangian arising from the off-shell motion of the heavy quark, breaks the flavor symmetry while the second term arising from chromo-magnetic interaction of heavy quark spin with the gluon field, breaks the spin symmetry. If we include hard gluon exchange, then the coefficient in the lagrangian changes and it looks like [34]

$$L = \overline{h_v}(iv.D)h_v + \frac{a_1}{2m_Q}\overline{h_v}(iD)^2 h_v + \frac{g_\alpha a_2}{4m_Q} 4m_Q h_v G^{\alpha\beta} h_v. \tag{12}$$

3.3 Mass formula for Heavy Mesons

In HHχPT, the ground state doublet ($J^P = 0^-, 1^-$) consisting of pseudoscalar and vector meson is represented by the field

$$H_a = \frac{1+\not{v}}{2}(H^{\mu}{}_a \gamma_\mu - H_a \gamma_5) \tag{13}$$

where as the lowest lying excited state($J^P = 0^+, 1^+$) consisting of scalar and axial vector meson is represented by the field

$$S_a = \frac{1+\not{v}}{2}(S^{\mu}_a \gamma_\mu \gamma_5 - S_a) \tag{14}$$

The total Lagrangian consist of the kinetic, axial, and mass terms. The kinetic term of these above two fields are included in:

$$L_v^{kinetic} = -Tr\left[\overline{H_a}(iv.D_{ba} - \delta_H \delta_{ab})H_b\right] - Tr\left[\overline{S_a}(iv.D_{ba} - \delta_S \delta_{ab})S_b\right]$$

δ_H and δ_S are the residual masses of the H and S fields, respectively and D_{ba} is chirally covariant derivative. In the theory with only H fields one is free to set δ_H =0. Once S fields are added to the theory, there is another dimensionful quantity, δ_H - δ_S, which does not vanish as $m_q \to 0$ and $m_Q \to \infty$.

The fields have axial couplings to the pseudo-Goldstone bosons

$$L_v^{axial} = gTr\left[\overline{H_a} H_b A_{ba} \gamma_5\right] + g' Tr\left[\overline{S_a} S_b A_{ba} \gamma_5\right] + hTr\left[\overline{H_a} S_b A_{ba} \gamma_5 + h.c\right] \tag{15}$$

where g, g' and h are dimensionless constants to be determined from the experiments.

$$\begin{aligned} L_v^{mass} = & -\frac{\Delta_H}{8} Tr\left[\overline{H_a} \sigma^{\mu\nu} H_a \sigma_{\mu\nu}\right] + \frac{\Delta_S}{8} Tr\left[\overline{S_a} \sigma^{\mu\nu} S_a \sigma_{\mu\nu}\right] + a_H Tr\left[\overline{H_a} H_b\right] m_{ba}^{\xi} - \\ & a_S Tr\left[\overline{S_a} S_b\right] m_{ba}^{\xi} - \sigma_S Tr\left[\overline{S_a} S_a\right] m_{bb}^{\xi} + \sigma_H Tr\left[\overline{H_a} H_a\right] m_{bb}^{\xi} - \frac{\Delta_H^{(a)}}{8} Tr\left[\overline{H_a} \sigma^{\mu\nu} H_b \sigma_{\mu\nu}\right] m_{ba}^{\xi} \\ & + \frac{\Delta_S^{(a)}}{8} Tr\left[\overline{S_a} \sigma^{\mu\nu} S_b \sigma_{\mu\nu}\right] m_{ba}^{\xi} - \frac{\Delta_H^{(\sigma)}}{8} Tr\left[\overline{H_a} \sigma^{\mu\nu} H_a \sigma_{\mu\nu}\right] m_{bb}^{\xi} + \frac{\Delta_S^{(\sigma)}}{8} Tr\left[\overline{S_a} \sigma^{\mu\nu} S_a \sigma_{\mu\nu}\right] m_{bb}^{\xi} \end{aligned} \tag{16}$$

where $m_{ba}^{\xi} = \frac{1}{2}(\xi m_q \xi + \xi^+ m_q \xi^+)_{ba}$

Δ_H and Δ_S are the spin-flavor symmetry violating terms and is responsible for the hyperfine mass splitting terms at leading order. Where a and σ are functions of $1/m_Q$ which starts at O(1).[35] The term proportional to a results in $SU(3)_V$-violating mass splittings amongst the H_a^{μ} mesons. The term proportional to σ leads to a singlet contribution to the masses which depends linearly on the light quark masses, and is the heavy meson analog of the pion-nucleon sigma term.

Where the fields H_a describes the mesons made up by the heavy quark Q and the light anti-quark $\overline{q}_a$ (*a*=1,2,3). This H_a field transforms as a doublet under heavy quark spin symmetry and as $\overline{3}$ under flavor $SU(3)_V$. m_q is the diagonal light quark mass matrix.

$$m_q = \begin{bmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{bmatrix}$$

In order to construct invariant derivative couplings, one need covariant derivatives or gauge fields. This is provided by the vector and axial current

$$V^{\mu} = \frac{1}{2}\left(\xi\partial^{\mu}\xi^{+} + \xi^{+}\partial^{\mu}\xi\right) \qquad A^{\mu} = \frac{1}{2}\left(\xi\partial^{\mu}\xi^{+} - \xi^{+}\partial^{\mu}\xi\right)$$

Under SU(3) ⊗ SU(3) chiral symmetry these field transforms as

$$V^{\mu} \rightarrow UV^{\mu}U^{+} + U\partial^{\mu}U^{+}$$

$$A^{\mu} \rightarrow UA^{\mu}U^{+}$$

$$\xi \rightarrow L\xi U^{+}$$

Where $\xi = e^{i\pi/f}$, $\Sigma = \xi^{2} = e^{2i\pi/f}$ and f ≈132 MeV is the pion decay constant [40].

The pseudo-scalar Goldstone boson are incorporated in a 3 × 3 unitary matrix $\Sigma(x) \in SU(3)$ transforming under $SU(3)_L \otimes SU(3)_R$ as

$$\Sigma \rightarrow g_L \Sigma g_R^{+}$$

The meson octet is introduced via the exponential representation

$$\Sigma = \exp(\frac{2iM}{f})$$

where M is a 3 × 3 hermitian, traceless matrix:

$$M = \begin{bmatrix} \frac{1}{\sqrt{2}}\pi^{0} + \frac{1}{\sqrt{6}}\eta & \pi^{+} & K^{+} \\ \pi^{-} & -\frac{1}{\sqrt{2}}\pi^{0} + \frac{1}{\sqrt{6}}\eta & K^{0} \\ K^{-} & \overline{K}^{0} & -\frac{2}{\sqrt{6}} \end{bmatrix}$$

An effective chiral symmetric Lagrangian describing low momentum interactions of heavy mesons with the pseudo Goldstone bosons of the 0^- octet can be written as:

$$L=\frac{f^2}{8}Tr\left[\partial^\mu\Sigma\partial_\mu\Sigma^+\right]+\lambda_0 Tr\left[m_q\Sigma+m_q\Sigma^+\right]-iTr\left[\bar{H}_a v_\mu\partial^\mu H_a\right]+\frac{i}{2}Tr\left[\bar{H}_a v_\mu H_b(\xi^+\partial^\mu\xi+\xi\partial^\mu\xi^+)_{ba}\right]$$
$$+i\frac{g}{2}Tr\left[\bar{H}_a H_b\gamma^\mu\gamma_5(\xi^+\partial^\mu\xi-\xi\partial^\mu\xi^+)_{ba}\right]+\lambda_1 Tr\left[\bar{H}_a H_b(\xi\partial^\mu\xi+\xi^+\partial^\mu\xi^+)_{ba}\right]+$$
$$\lambda_1' Tr\left[\bar{H}_a H_a(m_q\Sigma+m_q\Sigma^+)_{bb}\right]+\frac{\lambda_2}{m_Q}Tr\left[\bar{H}_a\sigma_{\mu\nu}H_a\sigma^{\mu\nu}\right]+-----$$

The residual masses for the ground state and excited state charm meson is given by:

$$m^0_{R_a}=\delta_R+\frac{n_J}{4}(\Delta_R+\Delta_R^\sigma\bar{m}+\Delta_R^{(a)}m_a)+\sigma_R\bar{m}+a_R m_a$$
$$+\frac{g_R^2}{f^2}c^{R_a}K_1+\frac{h_R^2}{f^2}c^{R_a}K_2$$

where R is an index that labels the ground state (H) and excited state (S), each of the ground and excited states having members corresponding to J = 0, 1, with $n_0=-3, n_1=1$, the index a labels the light flavour and runs over u, d, s, the functions K_1 and K_2 are the chiral loop functions, and the cRa are coefficients listed in [42] and $g_H=g, g_S=g'$ in the notation therein.

At tree level the residual masses are

$$m^0_{H_a}=\delta_H-\frac{3}{4}\Delta_H+\sigma_H\bar{m}+a_H m_a-\frac{3}{4}\Delta_H^{(\sigma)}\bar{m}-\frac{3}{4}\Delta_H^{(\sigma)}m_a$$

$$m^0_{H_a^*}=\delta_H+\frac{1}{4}\Delta_H+\sigma_H\bar{m}+a_H m_a+\frac{1}{4}\Delta_H^{(\sigma)}\bar{m}+\frac{1}{4}\Delta_H^{(\sigma)}m_a$$

$$m^0_{S_a}=\delta_S-\frac{3}{4}\Delta_S+\sigma_S\bar{m}+a_S m_a-\frac{3}{4}\Delta_S^{(\sigma)}\bar{m}-\frac{3}{4}\Delta_S^{(\sigma)}m_a$$

$$m^0_{S_a}=\delta_S+\frac{1}{4}\Delta_S+\sigma_S\bar{m}+a_S m_a+\frac{1}{4}\Delta_S^{(\sigma)}\bar{m}+\frac{1}{4}\Delta_S^{(\sigma)}m_a \quad (17)$$

Where $m_a=(m_u,m_d,m_s)$, $m_q=diag(m_u,m_d,m_s)$ and $\bar{m}=m_u+m_d+m_s$

The experimentally measured residual masses relative to the non strange spin averaged H mass, $(m_{H_1}+3m_{H_1^*})/4=0$

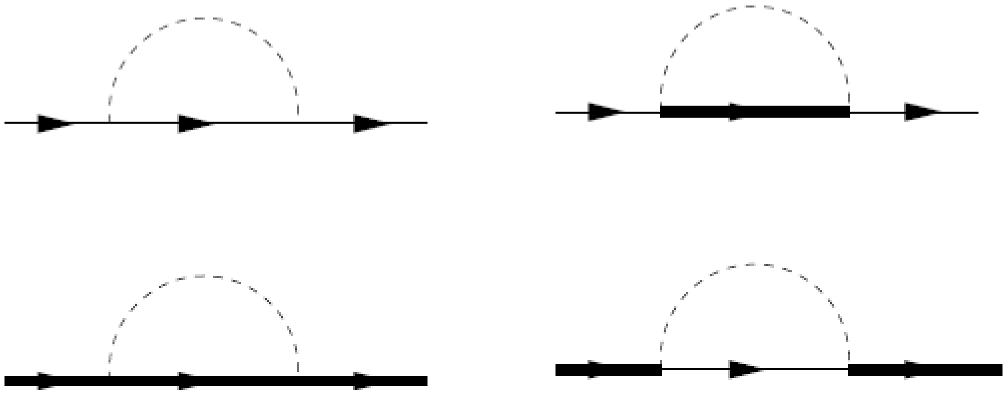


Fig: 3.3: One-loop self energy diagrams for the H and S fields. H fields are light lines, S fields are thick lines and goldstone bosons are dashed lines

The one loop calculation for the heavy mesons exchanging pion, kaon and eta are written with $1/m_Q$ correction. Hence we write our results in terms of the functions K_1 and F(x) as below

$$K_1(\eta, M) = \frac{1}{16\pi^2}\left[(-2\eta^3 + 3M^2\eta)\ln(\frac{M^2}{\mu^2}) + 2\eta(\eta^2 - M^2)F(\frac{\eta}{M}) + 4\eta^3 - 5\eta M^2\right]$$
$$K_2(\eta, M) = \frac{1}{16\pi^2}\left[(-2\eta^3 + M^2\eta)\ln(\frac{M^2}{\mu^2}) + 2\eta^3 F(\frac{\eta}{M}) + 4\eta^3 - \eta M^2\right] \qquad (18)$$

Where $$F(x) = 2\frac{\sqrt{1-x^2}}{x}\left[\frac{\pi}{2} - Tan^{-1}(\frac{x}{\sqrt{1-x^2}})\right] \qquad |x| < 1$$

$$F(x) = -2\frac{\sqrt{x^2-1}}{x}\ln(x + \sqrt{x^2-1}) \qquad |x| > 1 \qquad (19)$$

The function $K_1(\eta, M)$ appears whenever the virtual heavy meson inside the loop is in the same doublet as the external heavy meson, while $K_2(\eta, M)$ appears when the virtual heavy meson is from the opposite parity doublet.

In the limit $M \gg \eta$ these functions became

$$K_1(\eta,M)=\frac{1}{16\pi^2}\left[-2\eta^3\ln(\frac{4\eta^2}{\mu^2})+3\eta M^2\ln(\frac{4\eta^2}{\mu^2})+\frac{3M^4}{4\eta}\ln(\frac{M^2}{4\eta^2})+.....\right]$$
$$K_2(\eta,M)=\frac{1}{16\pi^2}\left[-2\eta^3\ln(\frac{4\eta^2}{\mu^2})+\eta M^2\ln(\frac{4\eta^2}{\mu^2})-\frac{M^4}{4\eta}\ln(\frac{M^2}{4\eta^2})+......\right] \quad (20)$$

In these equations we have dropped polynomials of η, M. The functions $K_1(\eta,M)$ and $K_2(\eta,M)$ have well-defined $M/\eta\to 0,$ so in this limit the S fields can be integrated out and their effect on the chiral corrections can be absorbed into local counterterms as expected. This limit is not relevant to the real world as $\eta \square M$. In the opposite limit, $\eta=0,$ which is relevant for loops in which external and virtual heavy mesons are the same.

$$K_1(\eta,M)=-\frac{M^3}{8\pi}+\frac{3}{16\pi^2}\eta M^2\ln(\frac{4\eta^2}{\mu^2})+O(n^3)$$
$$K_2(\eta,M)=\frac{1}{16\pi^2}\eta M^2\ln(\frac{4\eta^2}{\mu^2})+O(\eta^3)$$

The loop corrections to the masses are shown in fig.3.3 to light lines represent the H field and heavy lines represents the S fields. The loop corrections are regulated using dimensional regularization.

The one loop mass formulae are given in terms of the functions K_1, K_2 and F(x) defined above

$$m_{H_1}=m^0_{H_1}+\frac{g^2}{f^2}\left[\frac{3}{2}K_1(m^0_{H_1^*}-m^0_{H_1},m_\pi)+\frac{1}{6}K_1(m^0_{H_1^*}-m^0_{H_1},m_\eta)+K_1(m^0_{H_1^*}-m^0_{H_1},m_K)\right]$$
$$+\frac{h^2}{f^2}\left[\frac{3}{2}K_2(m^0_{S_1}-m^0_{H_1},m_\pi)+\frac{1}{6}K_2(m^0_{S_1}-m^0_{H_1},m_\eta)+K_1(m^0_{S_3}-m^0_{H_1},m_K)\right].$$

$$m_{H_3}=m^0_{H_3}+\frac{g^2}{f^2}\left[2K_1(m^0_{H_1^*}-m^0_{H_3},m_K)+\frac{2}{3}K_1(m^0_{H_3^*}-m^0_{H_3},m_\eta)\right]$$
$$+\frac{h^2}{f^2}\left[2K_2(m^0_{S_1}-m^0_{H_3},m_K)+\frac{2}{3}K_2(m^0_{S_1}-m^0_{H_1},m_\eta)\right].$$

$$m_{H_1^*} = m_{H_1^*}^0 + \frac{g^2}{f^2}\frac{1}{3}\left[\frac{3}{2}K_1(m_{H_1}^0 - m_{H_1^*}^0, m_\pi) + \frac{1}{6}K_1(m_{H_1}^0 - m_{H_1^*}^0, m_\eta) + K_1(m_{H_3}^0 - m_{H_1^*}^0, m_K)\right]$$
$$+\frac{g^2}{f^2}\frac{2}{3}\left[\frac{3}{2}K_1(0, m_\pi) + \frac{1}{6}K_1(0, m_\eta) + K_1(m_{H_3^*}^0 - m_{H_1^*}^0, m_K)\right]$$
$$+\frac{h^2}{f^2}\left[\frac{3}{2}K_2(m_{S_1^*}^0 - m_{H_1^*}^0, m_\pi) + \frac{1}{6}K_2(m_{S_1^*}^0 - m_{H_1^*}^0, m_\eta) + K_1(m_{S_3^*}^0 - m_{H_1^*}^0, m_K)\right]$$

$$m_{H_3^*} = m_{H_3^*}^0 + \frac{g^2}{f^2}\frac{1}{3}\left[2K_1(m_{H_1}^0 - m_{H_3^*}^0, m_K) + \frac{2}{3}K_1(m_{H_3}^0 - m_{H_3^*}^0, m_\eta)\right]$$
$$+\frac{g^2}{f^2}\frac{2}{3}\left[2K_1(m_{H_1}^0 - m_{H_3^*}^0, m_K) + \frac{2}{3}K_1(0, m_\eta)\right]$$
$$+\frac{h^2}{f^2}\left[2K_2(m_{S_1^*}^0 - m_{H_3^*}^0, m_K) + \frac{2}{3}K_2(m_{S_3^*}^0 - m_{H_3^*}^0, m_\eta)\right]$$

$$m_{S_1} = m_{S_1}^0 + \frac{g'^2}{f^2}\left[\frac{3}{2}K_1(m_{S_1^*}^0 - m_{S_1}^0, m_\pi) + \frac{1}{6}K_1(m_{S_1^*}^0 - m_{S_1}^0, m_\eta) + K_1(m_{S_3^*}^0 - m_{S_1}^0, m_K)\right]$$
$$+\frac{h^2}{f^2}\left[\frac{3}{2}K_2(m_{H_1}^0 - m_{S_1}^0, m_\pi) + \frac{1}{6}K_2(m_{H_1}^0 - m_{S_1}^0, m_\eta) + K_2(m_{H_3}^0 - m_{S_1}^0, m_K)\right]$$

$$m_{S_3} = m_{S_3}^0 + \frac{g'^2}{f^2}\left[2K_1(m_{S_1^*}^0 - m_{S_3}^0, m_K) + \frac{2}{3}K_1(m_{S_3^*}^0 - m_{S_3}^0, m_\eta)\right]$$
$$+\frac{h^2}{f^2}\left[2K_2(m_{H_1}^0 - m_{S_3}^0, m_K) + \frac{2}{3}K_2(m_{H_3}^0 - m_{S_3}^0, m_\eta)\right]$$

$$m_{S_1^*} = m_{S_1^*}^0 + \frac{g'^2}{f^2}\frac{1}{3}\left[\frac{3}{2}K_1(m_{S_1}^0 - m_{S_1^*}^0, m_\pi) + \frac{1}{6}K_1(m_{S_1}^0 - m_{S_1^*}^0, m_\eta) + K_1(m_{S_3}^0 - m_{S_1^*}^0, m_K)\right]$$
$$+\frac{g'^2}{f^2}\frac{2}{3}\left[\frac{3}{2}K_1(0, m_K) + \frac{1}{6}K_1(0, m_\eta) + K_1(m_{S_3^*}^0 - m_{S_1^*}^0, m_K)\right]$$
$$+\frac{h^2}{f^2}\left[\frac{3}{2}K_2(m_{H_1^*}^0 - m_{S_1^*}^0, m_\pi) + \frac{1}{6}K_2(m_{H_3^*}^0 - m_{S_1^*}^0, m_\eta) + K_2(m_{H_3^*}^0 - m_{S_1^*}^0, m_K)\right]$$

$$m_{S_3^*} = m_{S_3^*}^0 + \frac{g'^2}{f^2}\frac{1}{3}\left[2K_1(m_{S_1}^0 - m_{S_3^*}^0, m_K) + \frac{2}{3}K_1(m_{S_3}^0 - m_{S_3^*}^0, m_\eta)\right]$$
$$+\frac{g'^2}{f^2}\frac{2}{3}\left[2K_1(m_{S_1^*}^0 - m_{S_3^*}^0, m_K) + \frac{2}{3}K_1(0, m_\eta)\right]$$
$$+\frac{h^2}{f^2}\left[2K_2(m_{H_1^*}^0 - m_{S_3^*}^0, m_K) + \frac{2}{3}K_2(m_{H_3^*}^0 - m_{S_3^*}^0, m_\eta)\right]$$

We agree with ref.[36] for the H field in the limit where $m_\pi \to 0, m_\eta^2 \to \frac{4}{3}m_K^2$ And $\eta/M \ll 1$.

3.4 Mass Splitting

Heavy quark symmetry can be used to obtain relations between hadron masses.[37] .At order m_Q, all heavy hadrons containing Q are degenerate, and have the same mass m_Q. At the order of unity, the hadron masses get the contribution

$$\frac{1}{2}\left\langle H^{(Q)}\left|\mathrm{H}_0\right|H^{(Q)}\right\rangle \equiv \overline{\Lambda} \tag{20}$$

Where H_0 is the order $1/m_Q^0$ terms in the HQET Hamiltonian obtained from the Lagrangian term $\overline{Q}_v(iv.D)Q_v$, as well as the terms involving light quarks and gluons. In this section, the hadron states $\left|H^{(Q)}\right\rangle$ are in the effective theory with $v = v_r = (1,0)$. here $\overline{\Lambda}$ is a parameter of HQET and has the same value for all particles in a spin- flavor multiplet. The values will be denoted by $\overline{\Lambda}$ for the B, B^*, D and D^*. in the SU(3) limit, $\overline{\Lambda}$ does not depend on the light quark flavor. if SU(3)breaking is included, $\overline{\Lambda}$ is different for the $B_{u,d}$ and B_s mesons, and will be donated by $\overline{\Lambda}_{u,d}$ and $\overline{\Lambda}_S$,respectively.

At order $1/m_Q$,there is an additional contribution to the hadron masses given by the expectation values of the $1/m_Q$ correction to the Hamiltonian:

$$\mathrm{H}_1 = -L_1 = \overline{Q}_v \frac{D_\perp^2}{2m_Q} Q_v + a(\mu) g \overline{Q}_v \frac{\sigma_{\alpha\beta}}{4m_Q} Q_v. \tag{21}$$

The matrix element of the to terms in Eq.(21)define two nonperturtubative parameters, λ_1 and λ_2

$$\begin{aligned} 2\lambda_1 &= -\left\langle H^{(Q)}\left|\overline{Q}_{v_r} D_\perp^2 Q_{v_r}\right|H^{(Q)}\right\rangle, \\ 16(S_Q.S_l)\lambda_2(m_Q) &= a(\mu)\left\langle H^{(Q)}\left|\overline{Q}_{v_r} g\sigma_{\alpha\beta}G^{\alpha\beta} Q_{v_r}\right|H^{(Q)}\right\rangle. \end{aligned} \tag{22}$$

The term $\overrightarrow{S_Q}.\overrightarrow{S_l}$ can be written as $\overrightarrow{S_Q}.\overrightarrow{S_l} = 1/2(\overrightarrow{J^2} - \overrightarrow{S_Q}^2 - \overrightarrow{S_L}^2) = \frac{1}{2}[J(J+1) - 3/2]$

Here λ_1 is independent of m_Q, and λ_2 depends on m_q through the logarithmic m_Q dependence of in $a(\mu)$

$$a(\mu) = \left[\frac{\alpha_S(m_Q)}{\alpha_S(\mu)}\right]^{9/(33-2N_q)}$$

$\lambda_{1,2}$ have the same value for all states in a given spin-flavor multiplets and are expected to be of the order of Λ^2_{QCD}.

The native expectation that the heavy quark kinetic energy is positive suggests that λ_1 should be negative. The λ_2 matrix element transforms like $S_Q.S_l$ under the spin symmetry, since that is the transformation property of $\overline{Q}_{v_r}\sigma_{\alpha\beta}G^{\alpha\beta}Q_{v_r}$. Only the two upper components of Q_{v_r} are non zero, since $\gamma^0 Q_{v_r} = Q_{v_r}$ and $\overline{Q}_{v_r}\sigma_{\alpha\beta}G^{\alpha\beta}Q_{v_r}$, reduces to the matrix elements of $\overline{Q}_{v_r}\sigma.BQ_{v_r}$, where B is the chromodynamics field . The operator $\overline{Q}_{v_r}\sigma Q_{v_r}$ is the heavy quark spin, and the matrix element of B in the hadron must be proportional to the spin of the light degree of freedom, by rotational invariance and time–reversal invariance , so that the chromodynamics operator contribution is proportional to $S_Q.S_l$. Using $S_Q.S_l = (J^2 - S_Q^2 - S_l^2)/2$, one finds that

$$\begin{aligned} m_B &= m_b + \overline{\Lambda} - \frac{\lambda_1}{2m_b} - \frac{3\lambda_2(m_b)}{2m_b}, \\ m_{B^*} &= m_b + \overline{\Lambda} - \frac{\lambda_1}{2m_b} + \frac{\lambda_2(m_b)}{2m_b}, \\ m_D &= m_c + \overline{\Lambda} - \frac{\lambda_1}{2m_c} - \frac{3\lambda_2(m_c)}{2m_c}, \\ m_{D^*} &= m_c + \overline{\Lambda} - \frac{\lambda_1}{2m_c} - \frac{\lambda_2(m_c)}{2m_c}, \end{aligned} \tag{23}$$

The average mass of a heavy quark spin symmetry multiplets, eg. $(3m_{p^*} + m_p)/4$ For the meson multiplet, does not depend on λ_2. The magnetic interaction λ_2 is responsible for the $B^* - B$ and $D^* - D$ splitting. The observed value of the $B^* - B$ mass difference gives $\lambda_2(m_b) \square\ 0.12 GeV^2$.

Eq (23) gives the meson mass relation

$$0.49GeV^2 \approx m_{B^*}{}^2 - m_B^2 \approx 4\lambda_2 \approx m_{D^*}^2 - m_D^2 \approx 0.55GeV^2, \tag{24}$$

Upto correction of the order $1/m_{b,c}$ and ignoring the weak mq dependence of λ_2 .Similarly, one finds that

$$90 \pm 3MeV = m_{B_s} - m_{B_d} = \overline{\Lambda}_s - \overline{\Lambda}_{u,d} = m_{D_S} - m_{D_d} = 99 \pm 1MeV, \tag{25}$$

The parameter λ_1 and λ_2 are non-peturbative parameter of QCD and have not been computed from first principles, it might appear that very little has been gained by using eq (23) for the hadron masses in terms of $\overline{\Lambda}$, λ_1 and λ_2. However, the same hadronic matrix elements also occur in other quantities, such as form factors. One can then use the value of $\overline{\Lambda}$, λ_1 and λ_2. Obtained by fitting to the hadron masses to compute the form factors, without making any model dependent assumptions.

3.5 HQET formula for the prediction of excited B-meson masses

The commonly used definitions of the quark masses for heavy quarks are the pole mass, the potential model mass used in γ and φ spectroscopy, and HQET mass.[38] We use the heavy quark effective theory to estimate the excited state B meson masses. Physical quantity such as hadron masses can in principle be computed in HQET in terms of the HQET mass m_Q. Computation can not be done analytically in practice because of non-perturbative effects in QCD, which prevent the extraction of quark mass from QCD lagrangian. Nevertheless for heavy quarks it is possible to parameterize the non-perturbative affects to a given order in $1/m_Q$ expansion in terms of a few unknown constants that can be obtained from the experiments. B and D meson masses in the heavy quark effective theory are given in terms of a single non-perturbative parameter $\overline{\Lambda}$.

In general, the mass of a hadron H_Q containing a heavy quark Q obey an expansion of the form

$$m_X = m_Q + \overline{\Lambda} + \frac{\Delta m^2}{2m_Q} + O(\frac{1}{m_Q}) \tag{26}$$

where X is the hadron, it can be either a ground state (H) or an excited state(S), m_Q is the mass of the heavy quark. X=H, S

where as $\Delta m^2 = -\lambda_1 + 2[J(J+1) - \frac{3}{2}]\lambda_2$

J is the total spin of meson. The two parameters λ_1 and λ_2 are non perturbative parameters of QCD. Here λ_1 is independent of m_Q and λ_2 depends on m_Q logarithmically. λ_1 and λ_2 have same values for all states in a given spin-flavor multiplets and are expected to be of the order of Λ_{QCD}^2 . $\overline{\Lambda}$, λ_1 and λ_2 characterize the properties of light constituents. The term

-λ_1/m_Q arises from kinetic energy of the heavy quark inside hadron(meson). The magnetic interaction λ_2 describes the interaction of the heavy quark spin with the gluon field, and responsible for $B^* - B$ and $D^* - D$ splittings. Thus the formula for masses can be written in terms of $\overline{\Lambda}$, λ_1 and λ_2 as

$$m_X^{(Q)} = m_Q + \overline{\Lambda} - \frac{\lambda_1^X}{2m_Q} + n_j \frac{\lambda_2^X}{2m_Q} \tag{27}$$

where $n_j = +1$ for J=1, $n_j = -3$ for J=0

In particular for D and B mesons, the masses for the spin state (0 , 1) can be written as:

$$m_B = m_b + \overline{\Lambda} - \frac{\lambda_1^B}{2m_b} - 3\frac{\lambda_2^B}{m_b}; \qquad m_{B^*} = m_b + \overline{\Lambda} - \frac{\lambda_1^B}{2m_b} + \frac{\lambda_2^B}{m_b}$$

$$m_D = m_c + \overline{\Lambda} - \frac{\lambda_1^D}{2m_c} - 3\frac{\lambda_2^D}{m_c}; \qquad m_{D^*} = m_c + \overline{\Lambda} - \frac{\lambda_1^D}{2m_c} + \frac{\lambda_2^D}{m_c}$$

where $\overline{m}_H^{(Q)} = (3m_{H^*}^{(Q)} + m_H^{(Q)})/4$ and $\overline{m}_S^{(Q)} = (3m_{S^*}^{(Q)} + m_S^{(Q)})/4$

The difference between spin averaged masses of the $j^P = \left(\frac{1}{2}\right)^-$ and $j^P = \left(\frac{1}{2}\right)^+$ mesons

is given by

$$\overline{m}_S^{(Q)} - \overline{m}_H^{(Q)} = \overline{\Lambda}^S - \overline{\Lambda}^H - \frac{\lambda_1^S}{2m_Q} + \frac{\lambda_1^H}{2m_Q} \qquad (28)$$

It leads to the formulas for the splitting of the even and odd-parity states in the bottom sector

$$\overline{m}_S^{(b)} - \overline{m}_H^{(b)} = \overline{m}_S^{(c)} - \overline{m}_H^{(c)} - (\lambda_1^S - \lambda_1^H)(\frac{1}{2m_c} - \frac{1}{2m_b})$$

since $\overline{\Lambda}$ is a parameter of HQET and has the same value for all particles in a spin-flaovor multiplet. Hence ($\overline{\Lambda}^S - \overline{\Lambda}^H$) is same for B and D mesons. A recent global fit to B decay yields λ_1^S=0.20±0.06GeV^2. The parameter λ_1^S is unknown. From the spectroscopy of excited j^p=3/2+ D and B mesons extracts $\lambda_1^{3/2} - \lambda_1^H$=-0.23 GeV^2, where $\lambda_1^{3/2}$ is the λ_1 matrix element for the j^p=3/2+ doublet. The sign here indicates that the kinetic energy of the heavy quark in the excited heavy meson is larger than that in the ground state, which agrees with intuition. We expect the kinetic energy of the heavy quark in the j^p=1/2+ states to be comparable to that of j^p=3/2+ states. To estimate $\overline{m}_S^{(b)}$ with conservative errors, we take $\lambda_1^S - \lambda_1^H$=-0.2±0.1GeV^2, and m_c=1.4GeV, m_b=4.8GeV to find

$$\overline{m}_S^{(b)} - \overline{m}_H^{(b)} = \overline{m}_S^{(c)} - \overline{m}_H^{(c)} = 50 \pm 25 MeV \qquad (29)$$

In bottom strange sector, only the 0- state with mass $m_H^{(b)}$=5370 MeV has been observed. We need to estimate the masses of the excited bottom meson(1^-) with the help of the parameters given for the ground state and lowest lying excited charmed meson using the ratio

$$\frac{m_{H^*}^{(b)} - m_H^{(b)}}{m_{H^*}^{(c)} - m_H^{(c)}} = \frac{m_{S^*}^{(b)} - m_S^{(b)}}{m_{S^*}^{(c)} - m_S^{(c)}} = \frac{m_c}{m_b} \qquad (30)$$

upto O(1/m_Q) corrections.

Using the experimental values for the masses of the ground state and the excited state D meson we can calculate specifically their spin-averaged masses of the jp=1/2- and ½+ meson as

$$\begin{aligned}
\overline{m}^c_{H_1} &= (3m^{(c)}_{H_1^*} + m^{(c)}_{H_1})/4 = [3(35.4) - 106.1]/4 = 0.025 \\
\overline{m}^c_{H_3} &= (3m^{(c)}_{H_3^*} + m^{(c)}_{H_3})/4 = [3(139.1) - 4.75]/4 = 103.06 \\
\overline{m}^c_{S_1} &= (3m^{(c)}_{S_1^*} + m^{(c)}_{S_1})/4 = [3(465.0) + 335.0]/4 = 432.50 \\
\overline{m}^c_{S_3} &= (3m^{(c)}_{S_3^*} + m^{(c)}_{S_3})/4 = [3(486.3) + 344.4]/4 = 450.80
\end{aligned} \tag{31}$$

We can also calculate the hyperfine splitting of the B mesons using the splittings given for the D mesons and using the relation (30).

$$\begin{aligned}
m_c &= 1.4 GeV \\
m_b &= 4.8 GeV \\
\frac{m_c}{m_b} &= 0.2916
\end{aligned}$$

$$\begin{aligned}
m^{(b)}_{H_1^*} - m^{(b)}_{H_1} &= (\frac{m_c}{m_b})(m^{(c)}_{H_1^*} - m^{(c)}_{H_1}) = (0.2916)(141.5) = 41.26 \\
m^{(b)}_{H_3^*} - m^{(b)}_{H_3} &= (\frac{m_c}{m_b})(m^{(c)}_{H_3^*} - m^{(c)}_{H_3}) = (0.2916)(143.85) = 41.95 \\
m^{(b)}_{S_1^*} - m^{(b)}_{S_1} &= (\frac{m_c}{m_b})(m^{(c)}_{S_1^*} - m^{(c)}_{S_1}) = (0.2916)(130.0) = 37.90 \\
m^{(b)}_{S_3^*} - m^{(b)}_{S_3} &= (\frac{m_c}{m_b})(m^{(c)}_{S_3^*} - m^{(c)}_{S_3}) = (0.2916)(141.9) = 41.38
\end{aligned} \tag{32}$$

The value for $m^{(b)}_{H_3^*}$ will be:

$$m^{(b)}_{H_3^*} = (\frac{m_c}{m_b})(m^{(c)}_{H_3^*} - m^{(c)}_{H_3}) + m^{(b)}_{H_3} = 41.95 + 5370.0 = 5411.95 \text{ MeV} \tag{33}$$

Using the values of bottom non strange sector $m^{(b)}_{H_1} = 5279$ MeV and $m^{(b)}_{H_1^*} = 5325 MeV$, $\overline{m}^b_{H_1}$ is found out to be 5314 MeV. $m^{(b)}_{H_3} = 5370\ MeV$ given in particle data group and from the relation (29) we get the spin-averaged masses of excited B- mesons

$$
\begin{aligned}
\overline{m}_{S_1}^{(b)} - \overline{m}_{H_1}^{(b)} &= \overline{m}_{S_1}^{(c)} - \overline{m}_{H_1}^{(c)} - 50 \pm 25MeV \\
\overline{m}_{S_1}^{(b)} &= \overline{m}_{S_1}^{(c)} - \overline{m}_{H_1}^{(c)} + \overline{m}_{H_1}^{(b)} - 50 \pm 25MeV \\
\overline{m}_{S_1}^{(b)} &= 432.5 - 0.025 + 5314 - 50 \pm 25MeV \\
\overline{m}_{S_1}^{(b)} &= 5696.5 \pm 25MeV
\end{aligned} \tag{34}
$$

Similarly

$$
\begin{aligned}
\overline{m}_{S_3}^{(b)} - \overline{m}_{H_3}^{(b)} &= \overline{m}_{S_3}^{(c)} - \overline{m}_{H_3}^{(c)} - 50 \pm 25MeV \\
\overline{m}_{S_3}^{(b)} &= \overline{m}_{S_3}^{(c)} - \overline{m}_{H_3}^{(c)} + \overline{m}_{H_3}^{(b)} - 50 \pm 25MeV \\
\overline{m}_{S_3}^{(b)} &= 450.8 - 103.06 + 5404 - 50 \pm 25MeV \\
\overline{m}_{S_3}^{(b)} &= 5701.2 \pm 25MeV
\end{aligned} \tag{35}
$$

Thus the spin-averaged masses of the B-meson sector are found out to be:

$$
\begin{aligned}
\overline{m}_{H_1}^{b} &= (3m_{H_1^*}^{(b)} + m_{H_1}^{(b)})/4 = 5313.5MeV \\
\overline{m}_{H_3}^{b} &= (3m_{H_3^*}^{(b)} + m_{H_3}^{(b)})/4 = 5401.5MeV \\
\overline{m}_{S_1}^{b} &= (3m_{S_1^*}^{(b)} + m_{S_1}^{(b)})/4 = 5696.5 \pm 25MeV \\
\overline{m}_{S_3}^{b} &= (3m_{S_3^*}^{(b)} + m_{S_3}^{(b)})/4 = 5701.2 \pm 25MeV
\end{aligned}
$$

Equations can be solved to get the values for the masses of excited B mesons…….

$m_{S_1^*}^{(b)} = 5705.9MeV$; $m_{S_1}^{(b)} = 5668.3MeV$; $m_{S_3^*}^{(b)} = 5711.55MeV$; $m_{S_3}^{(b)} = 5670.17MeV$;

3.6 Spectroscopic Implications

The spin-flavor symmetry leads to many interesting relations between the properties of hadrons containing a heavy quark. The most direct consequences concern the spectroscopy of such states [39]. In the $m_Q \rightarrow \infty$ limit, the spin of the heavy quark and the total angular momentum j of the light degree of freedom are separated conserved by the strong interactions. Because of heavy quark symmetry, the dynamics is independent of the spin. And mass of the heavy quark. Hadronic states can thus be classified by the quantum numbers (flavor, spin, parity etc) of the light degrees of freedom. The spin symmetry predicts that, for fixed j ≠ 0, there is a doublet of degenerate states with total spin J= J± $\frac{1}{2}$.

The flavor symmetry relates the properties of states with different heavy-quark flavor. Consider, as an example, the ground- state mesons containing a heavy quark. In this case the light degree of freedom have the quantum numbers of a light antiquark, and the degenerate states are the pseudoscalar (J=0) and vector (J=1) mesons. In the charm and bottom systems, one knows experimentally

$$m_{B^*} - m_B \approx 46MeV,$$
$$m_{D^*} - m_D \approx 142MeV,$$
$$m_{D_S^*} - m_{D_S} \approx 142MeV,$$

These mass splitting are in fact reasonably small. To be more specific, at order 1/mQ one expects hyperfine corrections to resolve the degeneracy, for instance $m_{B^*} - m_B \propto 1/m_b,$. This leads to the refined prediction $m_{B^*}^2 - m_B^2 \approx m_D^2 - m_D^2 \approx const$. The data are compatible with this:

$$m_{B^*}^2 - m_B^2 \approx 0.49GeV^2, m_D^2 - m_D^2 \approx 0.55GeV^2$$

The spin symmetry also predicts that

$$m_{B_S^*}^2 - m_{B_S}^2 \approx m_{D_S^*}^2 - m_{D_S}^2 \approx const.$$

but this constant could in principle be different from that for nonstange mesons ,since the flavor quantum numbers of the light degree of freedom are different in both cases. Experimentally, however,

$$m_{D_S^*}^2 - m_{D_S}^2 \approx m_{D^*}^2 - m_D^2$$

indicating that to first approximation hyperfine corrections are independent of the flavor of the “brown muck”.

One can also study excited meson states, in which the light constituents carry orbital angular momentum. It is tempting to interpret $D_1(2420)$ with $J^p=1^+$ and $D_2(2460)$ with $J^p=2^+$ as the spin doublet corresponding to j=3/2. In fact, the small mass difference

$$m^2_{D_2^*} - m^2_{D_1} \approx 35 MeV$$

Supports this assertion. One then expects

$$m^2_{B_2^*} - m^2_{B_1} \approx m^2_{D_2^*} - m^2_{D_1} \approx 0.17 GeV^2$$

For the corresponding states in the bottom system. The fact that this mass splitting is smaller than for the ground-state mesons in above eq. is not unexpected. For instance, in the non-relativistic constituent quark model the light antiquark in these excited mesons is in a p-wave state, and its wave function at the location of the heavy quark vanishes. Hence, in this model hyperfine corrections are strongly suppressed.

A typical prediction of the flavor symmetry is that the "excitation energies" for states with different quantum numbers of the light degrees of freedom are approximately the same in the charm and bottom systems. For instance, one expects

$$\begin{aligned} m_{B_S} - m_B &\approx m_{D_S} - m_D \approx 100 MeV, \\ m_{B_1} - m_B &\approx m_{D_1} - m_D \approx 557 MeV, \\ m_{B_2^*} - m_B &\approx m_{D_2^*} - m_D \approx 593 MeV, \end{aligned}$$

Recently, the first relation has been confirmed very nicely by the discovery of the B_S meson by the ALEPH collaboration at LEP[40]. The observed mass,

$$m_{B_S} = 5.369 \pm 0.006 GeV \ ,$$

corresponds to an excitation energy

$$m_{B_S} - m_B = 90 \pm 6 MeV \ .$$

Chapter-4

Results

From the experimental data the residual masses for $j^P = \frac{1}{2}^-$ and $j^P = \frac{1}{2}^+$

$$m_{H_1} = -106.1 MeV \,, m_{S_1} = 335.0 MeV \;\; m_{H_3} = -4.75 MeV \;\; m_{S_3} = 344.4 MeV$$

$$m_{H_1^*} = 35.4 MeV \,, m_{S_1^*} = 465.0 MeV \,, m_{H_3^*} = 139.1 MeV \,, m_{S_3^*} = 486.3 MeV$$

Where we have used $(m_{H_1} + 3m_{H_1^*})/4$ as reference mass. Using the one loop calculations we fit these observed masses to get fundamental parameters such strong coupling constant (q),coupling of pions to heavy mesons(f), the mass and hyperfine splitting parameter etc, if we use these mass values in tree level expressions we get the following values:

$$\delta_S + \sigma_S \overline{m} = 431.6 \pm 26.09 MeV$$

$$\delta_H + \sigma_H \overline{m} = -4.8 \pm 0.65 MeV \qquad a_H = 1.2 \pm 0.01, \qquad a_S = 0.21 \pm 0.29$$

$$\Delta_H + \Delta_H^{(\sigma)} \overline{m} = 141.3 \pm 1.17 MeV \qquad \Delta_H^{(a)} = 0.028 \pm 0.02 \qquad \Delta_S^{(a)} = 0.14 \pm 0.55$$

$$\Delta_S + \Delta_S^{(\sigma)} \overline{m} = 129.4 \pm 49.72 MeV$$

For this and other fittings listed here,we have used a minimization programme. If we use the one loop mass formulae then we will have 11 parameters,

$$g, g', h, a_H, a_S, \Delta_H^{(a)}, \Delta_S^{(a)}, \delta_H + \sigma_H \overline{m}, \delta_S + \sigma_S \overline{m}$$
$$\Delta_H + \Delta_H^{(\sigma)} \overline{m}, \Delta_S + \Delta_S^{(\sigma)} \overline{m}$$

The parameters $a_H, a_S, \Delta_H^{(a)}, \Delta_S^{(a)}$ cannot be seperatly determined because they always appear in linear combination with the parameters $\delta_H, \delta_S, \Delta_H$ and Δ_S respectively. Hence we have absorbed the contribution of the parameters $\sigma_H, \sigma_S, \Delta_H^{(\sigma)}$ and $\Delta_S^{(\sigma)}$ into the measured values of $\delta_H, \delta_S, \Delta_H$ and Δ_S ,respectively.now,we have only eight mass data for 11 parameters, which gives different possibilities for the parameters. One such fit is given below:

$$g = 0.27 \pm 0.06$$
$$\delta_H = 261 \pm 0.74 MeV$$
$$\Delta_H = 629.8 \pm 1.79 MeV$$
$$a_H = 0.6929 \pm 0.01$$
$$\Delta_H^{(a)} = -2.13 \pm 0.03$$

$$h = 0.69 \pm 0.09$$
$$\delta_S = 368.1 \pm 2.93 MeV$$
$$\Delta_S = 672.4 \pm 8.5 MeV$$
$$a_S = -7.77 \pm 0.03$$
$$\Delta_S^{(a)} = 0.04 \pm 0.03$$
$$g' = 0.0$$

If we use the some of the new values for g,g',h from other experiments we can fix the remaining 8 parameters without ambiguity. Using the values

$$g = 0.66, h = 0.47, g' = -0.06 \quad [41]$$

we get

$$\delta_H = 227 \pm 0.65 MeV \qquad \delta_S = 356.1 \pm 15.14 MeV$$
$$\Delta_H = 460 \pm 1.69 MeV \qquad \Delta_S = -32.43 \pm 20.7 MeV$$
$$a_H = 0.675 \pm 0.01 \qquad a_S = -2.44 \pm 0.01$$
$$\Delta_H^{(a)} = -6.172 \pm 0.02 \qquad \Delta_S^{(a)} = 1.32 \pm 0.35$$

All these fits give mass values for 0 state to be in the range 2200-2250 MeV , and 1 to be in the range 2335-2375 MeV . Even though there are yet other regions of parameter space in which these mass values can change, here it is our main aim to point out that these values do not contradict any present day theory.

Chapter-5

Summary and Conclusion

Recently a lot of experiments started to look at charm and bottom mesons (which are the ideal systems for this theory), the old theories have been revived. It is found that the prediction is in well agreement with the experiments. Hence if we have reliable data on the any of the parameters, quoted in the one loop mass relations, from other experiments or theoretical models (like lattice calculations), we would be able fix the remaining parameters in the theory.

Within the framework of heavy quark effective theory and chiral perturbation theory, masses of even and odd parity charm and bottom mesons are studied. Heavy quark and chiral symmetries are exploited to formulate a theory that can explain the experimental data to a very precise extent. In this work we have analyzed the masses of lowest-lying even-parity excited states in the open charm system and bottom sector. We have used the framework of HHχPT including the $O(1/m_Q)$ corrections, namely the framework employed in Mehen paper.

Our main motivation is to find the masses of the non-strange excited states from the observed experimental values of all the ground states and excited strange mesons only. In the present work, we employ the expressions presented in the comprehensive work of Mehen and Springer.

Mass formulae is developed for ground state $J^P = 0^-$ and 1^- and first excited state, $J^P = 0^+$ and 1^+ charmed meson up to one loop chiral corrections. Heavy quark symmetry and chiral symmetry is used to obtain the relation between heavy meson masses. There are in total 12 equations for each states, and imposition on chiral symmetry ($m_u = m_d$) bring the number to just 8 equations in terms of symmetry breaking and conserving variables. The equations are thus fitted with the specific ranges to the variables constrained by the experimental data. The key differences is that we are now constraining the values of the parameters *g, g' ,h* to the values that are determined from the decays. Specifically we restricted them within a range of 0 to 1. Furthermore we have imposed the requirement that m_S~ 130 MeV and m_u, m_d~4 MeV at our energy scale of 1GeV, and have required the parameters determining the tree-level hyperfine structure to be in a range determined by the well-established states.

From this fit, we obtain for the non-strange 0^+ state the mass of 2155.7 MeV and for 1^+ 2395.1MeV. We find that the masses for the non-strange states that we determine can be lower than the numbers obtained by the Belle Collaboration. These values (ranges) will be useful for experimentalists, who are looking for these states, to tell them where to look for. These states have masses far below the theoretical predictions compared to the corresponding D_u states. There are also some contravesial issues about newly discovered states D_s = 2317 MeV and D_s = 2460 MeV. Also this heavy quark formalism can be used to analyse the decays of heavy meson accurately than any other existing theories.

Also this relation will be useful in predicting the masses of a few B mesons which are still awaiting experimental observations. In the bottom strange sector, only four B meson states have been observed. From the Heavy quark theory we have the relation

$$\frac{m_{H^*}^{(b)} - m_H^{(b)}}{m_{H^*}^{(c)} - m_H^{(c)}} = \frac{m_{S^*}^{(b)} - m_S^{(b)}}{m_{S^*}^{(c)} - m_S^{(c)}} = \frac{m_c}{m_b}$$

Up to O ($1/m_Q$) corrections. Thus, all the hyperfine splitting in the bottom sector is related to those in the charm sector by a universal factor. Combining this with the measured value of $m_{H_1^*}^{(a)} - m_{H_1}^{(a)}$ leads to the prediction that $m_{H_3^*}^{(b)} - m_{H_3}^{(b)} = m_{S_3^*}^{(b)} - m_{S_3}^{(b)} = 46MeV$ and $m_{S_1^*}^{(b)} - m_{S_1}^{(b)} = 42MeV$. Given these hyperfine splitting , one expects $\overline{m}_{H_3}^{(b)} = 5404MeV$.

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Programs using Mathematica software

1. Variation of K_1, K_2 function with η and M

```
x = η / M
```

$$\frac{\eta}{M}$$

```
F[x_] = ((2 (1 - x x) ^ 0.5) / x) (Pi / 2 - ArcTan[x / ((1 - x x) ^ 0.5)])
```

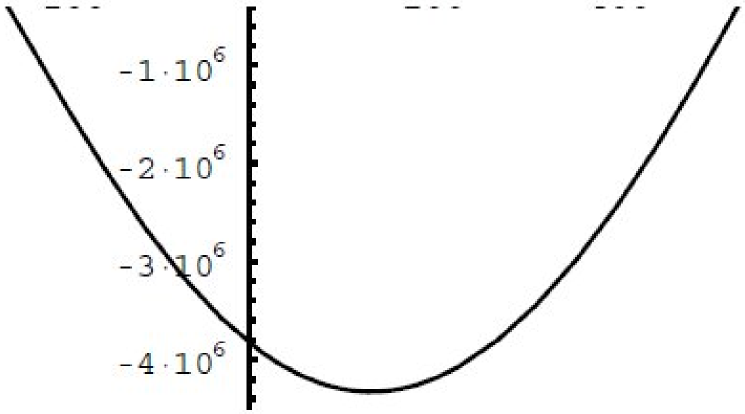
$$\frac{2\,M\left(1-\frac{\eta^2}{M^2}\right)^{0.5}\left(\frac{\pi}{2}-\mathrm{ArcTan}\left[\frac{\eta}{M\left(1-\frac{\eta^2}{M^2}\right)^{0.5}}\right]\right)}{\eta}$$

```
K10[η_, M_] =
 1 / (16 * Pi * Pi) ((-2 η ^ 3 + 3 M ^ 2 η) Log[M ^ 2 / μ ^ 2] + 2 η (η ^ 2 - M ^ 2) F[x] + 4 η ^ 3 - 5 η M ^ 2)
```

$$\frac{1}{16\,\pi^2}\left(-5\,M^2\,\eta+4\,\eta^3+4\,M\left(-M^2+\eta^2\right)\left(1-\frac{\eta^2}{M^2}\right)^{0.5}\left(\frac{\pi}{2}-\mathrm{ArcTan}\left[\frac{\eta}{M\left(1-\frac{\eta^2}{M^2}\right)^{0.5}}\right]\right)+\left(3\,M^2\,\eta-2\,\eta^3\right)\mathrm{Log}\left[\frac{M^2}{\mu^2}\right]\right)$$

```
μ = 1000.0;

Plot[K10[η, 458.0], {η, -458, 569}]
```



- Graphics -

K10[0, 458] // N

3.82258×10^{6}

K11[0, 458] // N

K11[0., 458.]

F1[x_] = - ((2 (x x - 1) ^ 0.5) / x) (Log[x + (x x - 1) ^ 0.5])

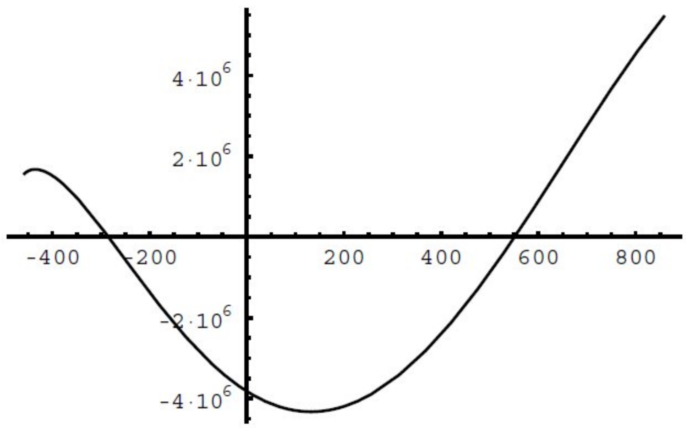
$$-\frac{2\,M\left(-1+\frac{\eta^2}{M^2}\right)^{0.5}\,\mathrm{Log}\left[\frac{\eta}{M}+\left(-1+\frac{\eta^2}{M^2}\right)^{0.5}\right]}{\eta}$$

K11[η_, M_] =
1 / (16 * Pi * Pi) ((-2 η ^ 3 + 3 M ^ 2 η) Log[M ^ 2 / μ ^ 2] + 2 η (η ^ 2 - M ^ 2) F1[x] + 4 η ^ 3 - 5 η M ^ 2)

$$\frac{1}{16\,\pi^2}\left(-5\,M^2\,\eta+4\,\eta^3+\left(3\,M^2\,\eta-2\,\eta^3\right)\,\mathrm{Log}\left[1.\times10^{-6}\,M^2\right]-4\,M\left(-M^2+\eta^2\right)\left(-1+\frac{\eta^2}{M^2}\right)^{0.5}\,\mathrm{Log}\left[\frac{\eta}{M}+\left(-1+\frac{\eta^2}{M^2}\right)^{0.5}\right]\right)$$

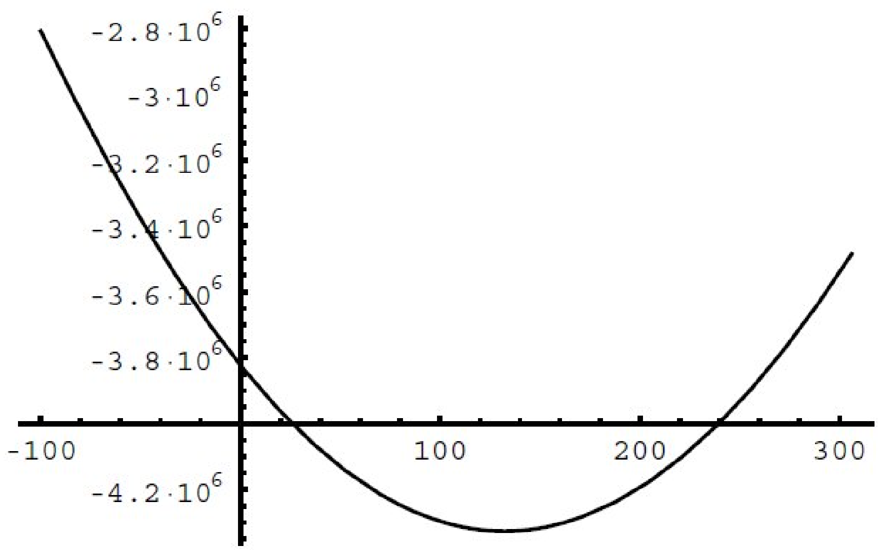
```
Plot[K11[η, 458.0], {η, -459, 858}]
```

Plot::plnr : K11[η, 458.] is not a machine-size real number at η = -459.. More…



- Graphics -

```
Plot[K11[η, 458.0], {η, -100, 306}]
```



- Graphics -

```
K20[η_, M_] = 1 / (16 * Pi * Pi) ((-2 η^3 + M^2 η) Log[M^2 / μ^2] + 2 η ^3 F[x] + 4 η^3 - η M^2)
```

$$\frac{1}{16\,\pi^2}\left(-\mathrm{M}^2\,\eta + 4\,\eta^3 + 4\,\mathrm{M}\,\eta^2\left(1-\frac{\eta^2}{\mathrm{M}^2}\right)^{0.5}\left(\frac{\pi}{2}-\mathrm{ArcTan}\left[\frac{\eta}{\mathrm{M}\left(1-\frac{\eta^2}{\mathrm{M}^2}\right)^{0.5}}\right]\right) + \left(\mathrm{M}^2\,\eta - 2\,\eta^3\right)\mathrm{Log}\left[1.\times 10^{-6}\,\mathrm{M}^2\right]\right)$$

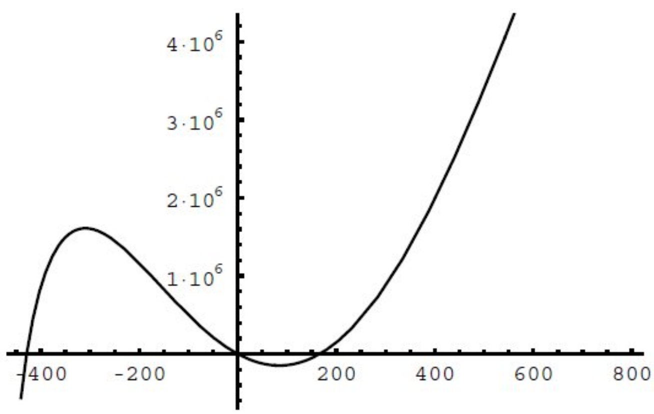
```
x = η / M
```

$$\frac{\eta}{\mathrm{M}}$$

```
F[x_] = ((2 (1 - x x) ^0.5) / x) (Pi / 2 - ArcTan[x / ((1 - x x) ^0.5)])
```

$$\frac{2\,\mathrm{M}\left(1-\frac{\eta^2}{\mathrm{M}^2}\right)^{0.5}\left(\frac{\pi}{2}-\mathrm{ArcTan}\left[\frac{\eta}{\mathrm{M}\left(1-\frac{\eta^2}{\mathrm{M}^2}\right)^{0.5}}\right]\right)}{\eta}$$

```
Plot[K20[η, 458.0], {η, -440, 789}]
```



- Graphics -

```
K21[η_, M_] = 1 / (16 * Pi * Pi) ((-2 η^3 + M^2 η) Log[M^2 / μ^2] + 2 η ^3 F1[x] + 4 η^3 - η M^2)
```

$$\frac{1}{16\,\pi^2}\left(-\mathrm{M}^2\,\eta + 4\,\eta^3 + \left(\mathrm{M}^2\,\eta - 2\,\eta^3\right)\mathrm{Log}\left[1.\times 10^{-6}\,\mathrm{M}^2\right] + 4\,\mathrm{M}\,\eta^2\left(-1+\frac{\eta^2}{\mathrm{M}^2}\right)^{0.5}\mathrm{Log}\left[\frac{\eta}{\mathrm{M}}+\left(-1+\frac{\eta^2}{\mathrm{M}^2}\right)^{0.5}\right]\right)$$

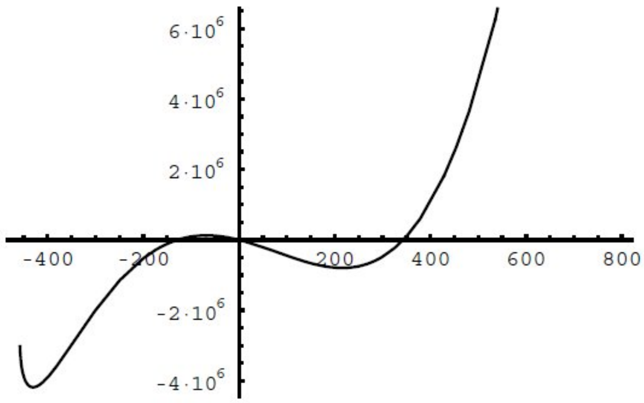
```
F1[x_] = ((2 (x x - 1) ^0.5) / x) (Log[x + (x x - 1) ^0.5])
```

$$\frac{2\,\mathrm{M}\left(-1+\frac{\eta^2}{\mathrm{M}^2}\right)^{0.5}\mathrm{Log}\left[\frac{\eta}{\mathrm{M}}+\left(-1+\frac{\eta^2}{\mathrm{M}^2}\right)^{0.5}\right]}{\eta}$$

```
Plot[K21[η, 458.0], {η, -460, 789}]
```

Plot::plnr : K21[η, 458.] is not a machine-size real number at η = -460.. More…

Plot::plnr : K21[η, 458.] is not a machine-size real number at η = -458.458. More…



- Graphics -

2. Calculation of masses using various parameter.

```
μ = 1000.0;

mη = 547.0;

mK = 495.0;

mπ = 140.0;

General::spell1 :
 Possible spelling error: new symbol name "mπ" is similar to existing symbol "mη". More…

m1 = 4.0;

m3 = 90.0;

g = 0.0;

g1 = 0.0;

h = 0.0;

f = 120.0;

δH = -4.77;

δS = 431.65;

General::spell1 :
 Possible spelling error: new symbol name "δS" is similar to existing symbol "δH". More…

ΔH = 140.4;

General::spell1 :
 Possible spelling error: new symbol name "ΔH" is similar to existing symbol "δH". More…

ΔS = 129.5;

General::spell : Possible spelling error: new
   symbol name "ΔS" is similar to existing symbols {ΔH, δS}. More…

aH = 1.20;

aS = 0.21;

ΔHa = 0.03;

General::spell1 : Possible spelling error:
   new symbol name "ΔHa" is similar to existing symbol "ΔH". More…
```

```
ΔSa = 0.14;

General::spell : Possible spelling error: new
   symbol name "ΔSa" is similar to existing symbols {ΔHa, ΔS}. More…

F1[x_] = If[Abs[x] < 1,
   2 Sqrt[1 - x^2] (Pi / 2 - ArcTan[x / Sqrt[1 - x^2]]), -2 Sqrt[x^2 - 1] Log[x + Sqrt[x^2 - 1]]];

K1[η_, M_] = 1 / (16 Pi^2) ((-2 η^3 + 3 M^2 η) Log[M^2 / μ^2] +
     2 M (η^2 - M^2) F1[η / M] + 4 η^3 - 5 η M^2);

K2[η_, M_] = 1 / (16 Pi^2) ((-2 η^3 + M^2 η) Log[M^2 / μ^2] +
     2 η^2 M F1[η / M] + 4 η^3 - η M^2);

mH10 = δH - 3 / 4 ΔH + aH m1 - 3 / 4 ΔHa m1;

mH30 = δH - 3 / 4 ΔH + aH m3 - 3 / 4 ΔHa m3;

mH10e = δH + 1 / 4 ΔH + aH m1 + 1 / 4 ΔHa m1;

General::spell1 : Possible spelling error: new
   symbol name "mH10e" is similar to existing symbol "mH10". More…

mH30e = δH + 1 / 4 ΔH + aH m3 + 1 / 4 ΔHa m3;

General::spell1 : Possible spelling error: new
   symbol name "mH30e" is similar to existing symbol "mH30". More…

mS10 = δS - 3 / 4 ΔS + aS m1 - 3 / 4 ΔSa m1;

General::spell1 : Possible spelling error: new
   symbol name "mS10" is similar to existing symbol "mH10". More…

mS30 = δS - 3 / 4 ΔS + aS m3 - 3 / 4 ΔSa m3;

General::spell1 : Possible spelling error: new
   symbol name "mS30" is similar to existing symbol "mH30". More…

mS10e = δS + 1 / 4 ΔS + aS m1 + 1 / 4 ΔSa m1;

General::spell : Possible spelling error: new symbol
   name "mS10e" is similar to existing symbols {mH10e, mS10}. More…
```

```
mS30e = δS + 1 / 4 ΔS + aS m3 + 1 / 4 ΔSa m3;

General::spell : Possible spelling error: new symbol
   name "mS30e" is similar to existing symbols {mH30e, mS30}. More…

mH1 = mH10 + (g^2 / f^2) (1.5 K1[mH10e - mH10, mπ] +
      (1 / 6) K1[mH10e - mH10, mη] + K1[mH30e - mH10, mK]) +
   (h^2 / f^2) (1.5 K2[mS10 - mH10, mπ] +
      (1 / 6) K2[mS10 - mH10, mη] + K2[mS30 - mH10, mK])

-106.11

mH3 = mH30 + (g^2 / ( f^2)) (2 K1[mH10e - mH30, mK] + (2 / 3) K1[mH30e - mH30, mη]) +
   (h^2) / ( f^2) (2 K2[mS10 - mH30, mK] + (2 / 3) K2[mS30 - mH30, mη])

-4.845

mH1e = mH10e +
   (g^2 / (3 f^2)) (1.5 K1[mH10 - mH10e, mπ] + (1 / 6) K1[mH10 - mH10e, mη] + K1[mH30 - mH10e, mK]) +
   ((2 g^2) / (3 f^2)) (1.5 K1[0, mπ] + (1 / 6) K1[0, mη] + K1[mH30e - mH10e, mK]) +
   (h^2 / f^2) (1.5 K2[mS10e - mH10e, mπ] + (1 / 6) K2[mS10e - mH10e, mη] + K2[mS30e - mH10e, mK])

General::spell : Possible spelling error: new symbol
   name "mH1e" is similar to existing symbols {mH1, mH10e}. More…

35.41 + 0. i

mH3e = mH30e + (g^2 / (3 f^2)) (2 K1[mH10 - mH30e, mK] + (2 / 3) K1[mH30 - mH30e, mη]) +
   ((2 g^2) / (3 f^2)) (2 K1[mH10e - mH30e, mK] + (2 / 3) K1[0, mη]) +
   (h^2 / f^2) (2 K2[mS10e - mH30e, mK] + (2 / 3) K2[mS30e - mH30e, mη])

General::spell : Possible spelling error: new symbol
   name "mH3e" is similar to existing symbols {mH3, mH30e}. More…

139.25500000000002
```

```
mS1 =
 mS10 + (g1^2 / f^2) (1.5 K1[mS10e - mS10, mπ] + (1 / 6) K1[mS10e - mS10, mη] + K1[mS30e - mS10, mK]) +
  (h^2 / f^2) (1.5 K2[mH10 - mS10, mπ] + (1 / 6) K2[mH10 - mS10, mη] + K2[mH30 - mS10, mK])

334.945 + 0. i

mS3 = mS30 + (g1^2 / f^2) (2 K1[mS10e - mS30, mK] + (2 / 3) K1[mS30e - mS30, mη]) +
  (h^2) / ( f^2) (2 K2[mH10 - mS30, mK] + (2 / 3) K2[mH30 - mS30, mη])

343.975

mS1e = mS10e +
  (g1^2 / (3 f^2)) (1.5 K1[mS10 - mS10e, mπ] + (1 / 6) K1[mS10 - mS10e, mη] + K1[mS30 - mS10e, mK]) +
  ((2 g1^2) / (3 f^2)) (1.5 K1[0, mπ] + (1 / 6) K1[0, mη] + K1[mS30e - mS10e, mK]) +
  (h^2 / f^2) (1.5 K2[mH10e - mS10e, mπ] + (1 / 6) K2[mH10e - mS10e, mη] + K2[mH30e - mS10e, mK])

General::spell : Possible spelling error: new symbol
   name "mS1e" is similar to existing symbols {mH1e, mS1, mS10e}. More…

465.005 + 0. i

mS3e = mS30e + (g1^2 / (3 f^2)) (2 K1[mS10 - mS30e, mK] + (2 / 3) K1[mS30 - mS30e, mη]) +
  ((2 g1^2) / (3 f^2)) (2 K1[mS10e - mS30e, mK] + (2 / 3) K1[0, mη]) +
  (h^2 / f^2) (2 K2[mH10e - mS30e, mK] + (2 / 3) K2[mH30e - mS30e, mη])

General::spell : Possible spelling error: new symbol
   name "mS3e" is similar to existing symbols {mH3e, mS3, mS30e}. More…

486.075
```