

DESIGN AND DEVELOPMENT OF FRICTION
COMPENSATOR ALGORITHM FOR ONE LINK ROBOT

*Thesis submitted in the partial fulfilment of the requirement for the
award of degree of*

**Master Of Engineering
in
Electronics Instrumentation and Control Engineering**



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CERTIFICATE

This is to certify that my work presented in this thesis entitled “**Design and Development of Friction Compensator Algorithm for one link robot**” submitted by **Ms. Malika Arora** in partial fulfillment of the requirement for the award of the degree of **Master of Engineering in Electronics Instrumentation and Control Engineering at Thapar University (Deemed University), Patiala**, is an original record under supervision and guidance of **Dr Yaduvir Singh** . The matter embodied in this report has not been submitted anywhere for the award of any degree.

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LIST OF ABBREVIATIONS

- RIA Robotics Institute of America
- JIRA Japanese Industrial Robot Association
- NASA National Aeronautics and Space Administration
- PLC Programmable Logic Controller
- DOF Degree Of Freedom
- PD Proportional Derivative

ABSTRACT

Many robots excel at their positioning and trajectory tracking using software control and most successful robotic application utilize this ability – examples include CNC machining, robotic welding, painting and pick-and-place board assembly. The mechanical design of these robots focuses on rigid transmissions and minimizing compliance in the structure, so the software controller can accurately track a desired position with respect to time, regardless of any disturbance forces. However, there is class of tasks, for which rigid actuation is not ideal.

For dynamic behaviors such as running, the performance limitations of robot are often due to limitations of friction. A robot is an integrated system of electronics, software, and mechanical, and each part of the system limits or enables the behavior of the whole. Friction effects are particularly critical for industrial robots. Friction deteriorates the performance of industrial robots causes disturbances such as positioning errors, tracking errors and stick-slip errors in robotics. Friction effects in robotics depend on many factors, such as displacement and relative velocity of the robot, properties of surface materials, presence of lubrication. Robotic systems are equipped with position sensors and friction is generally velocity dependent, so friction requires velocity estimation techniques

The effects of friction in robotics are greatly compensated by friction models. The friction compensation model are of two types i.e. kinetic friction model and dynamic friction model.

To achieve high accuracy the negative effect of friction must be taken into account. The objective of this thesis is to derive a friction compensation strategy for a one link robot, which reduces steady-state errors. Because friction generally has a velocity dependency and in this case no velocity sensors are available an estimate of the velocities must be made; this is done by means of a reduced-order observer. Finally simulation and experimental results of the observer based friction compensation strategy are presented. This thesis is organized as follows:

Chapter 1 is the introduction chapter. The introduction regarding robotics, components, characteristics and literature survey is discussed. Applications of robot, and need for stabilization are also given.

Chapter 2 gives an introduction to Friction. Brief description of friction phenomenon and various friction models.

Chapter 3 gives an introduction to various friction compensation strategies used in the field of robotics.

Chapter 4 includes the problem formulation and provides the mathematical model for the friction compensator for one link robot system.

Literature Survey

Pierre E. Dupont, in his paper discusses the importance of appropriately modeling friction for the simulation and control of high performance robot systems. Incorporating Coulomb type friction in the dynamic equations introduces two difficulties in the forward dynamic solution. We show that the differential equations become discontinuous in the highest order derivative terms. In addition, the load dependency of this type of friction generally causes the equations to be implicit in the joint accelerations. For the important case of load-dependent transmission friction, the equations are shown to be explicit. Techniques for the forward solution are described through the example of a roller screw transmission. Experimental and simulation results are used to show the importance of load-dependent friction in a particular robot. Implementation issues are discussed as well as implications for robot control. [1]

Armstrong-Helouvry, B, proposed that the implications of Stribeck friction (a nonlinear low-velocity friction effect that contributes to and perhaps dominates stick-slip motion) for feedback control are explored. Through the use of dimensional analysis, the following are examined: (1) the minimum velocity below which stick-slip will occur; (2) the accuracy of sensing required to eliminate stick-slip by feedback control; (3) the slip distance during stick-slip motion; and (4) scaling of Stribeck friction and stick-slip. A seven-parameter friction-plus-control model is proposed; dimensional analysis is employed to reduce the degrees of freedom of this model to three. [2]

C. Canudas de Wit , P. Noël , A. Aubin , B. Brogliato , analyzes the problem of modeling and compensation of friction at velocities close to zero. A new model, linear in parameters, which captures the downward bends at low velocity, is used to adaptively compensate for friction. The need for this type of model is mainly motivated by instability phenomena that can be caused by overcompensation when

simple models (such as Coulomb friction models) are used as a basis for the friction compensation. This model, in combination with an adaptive computed torque method, was tested experimentally in a robot manipulator. [3]

Kiguchi, K. Fukuda, T. , Position and force controls are important and fundamental tasks of robot manipulators. In order to control position of the robot which simultaneously applies force to the environment, the friction between the robot and the environment has to be compensated. However, the friction force varies according to the applied force to the environment. Therefore, it is difficult to compensate the friction effectively with conventional controllers if we do not know the friction coefficient. Many researches have been done on fuzzy neural control, the combination of neural networks and fuzzy control, in order to make up for each other's weak points. The fuzzy neural control is expected to perform more sophisticated control than conventional control in an unknown environment. In this paper, we propose a new friction compensation method using the fuzzy neural network which contains a specialized neuron for friction compensation and a switch-learning. Simulation has done using a 3DOF planar robot manipulator to confirm the effectiveness of the proposed method [4]

Guillaume Morel, Karl Iagnemma, Steven Dubowsky , in their paper mentioned that Joint friction is a major problem in accurate robot position control, particularly during low-speed, small-amplitude tasks. This paper proposes a simple, practical, and elective method to compensate for joint friction, using a six-axis force/torque sensor mounted under the manipulator. From these measurements, joint torques are estimated and used in a torque controller, which virtually eliminates friction and gravity effects, providing high-precision motion control even for small motions at low speed. The method does not require complex analytical friction models. The method also does not require expensive and unreliable internal joint-torque sensors. Experimental results demonstrate the effectiveness and practicality of the method for an electrical PUMA and hydraulic Schilling Titan II manipulator. [5]

Der-Cherng Liaw and Jeng-Tze Huang, presented the issues of contact friction compensation for constrained robots. The proposed design consists of two loops. The inner loop is for the inverse dynamics control which linearizes the system by canceling nonlinear dynamics, while the outer loop is for friction compensation. Although various models of friction have been proposed in many engineering applications, frictional force can be modeled by the Coulomb friction plus the viscous force. Based on such a model, an on-line genetic algorithm is proposed to learn the friction coefficients for friction model. The friction compensation control input is also implemented in terms of the friction coefficients to cancel the effect of unknown friction. By the guidance of the fitness function, the genetic learning algorithm searches for the best-fit value in a way like the natural surviving laws. Simulation results demonstrate that the proposed on-line genetic algorithm can achieve good friction compensation even under the conditions of measurement noise and system uncertainty. Moreover, the proposed control scheme is also found to be feasible for friction compensation of friction model with Stribeck effect and position-dependent friction model. [6]

Dusko Stajic, in his research paper contribute that, Friction appears in bearings and reduction gear of controlled electrical drives and affects the quality of the position, speed or force control. The servo system performance can be improved by implementation of acceleration feedback. For the reason that acceleration measurement is often either not possible or practical, the implementation of acceleration estimation by observer or differentiation of speed is proposed. Another approach to friction influence compensation is based on the disturbance observer. Its advantage is that it can be applied generally for the estimation of various disturbances. This paper deals with these friction compensation methods which are not based on friction model, but on the compensation of friction as disturbance. The efficiency of these methods in a servo drive with Stribeck friction is compared by computer simulation and experimentally on a servo system laboratory model. [7]

Lischinsky, P.; Canudas-de-Wit, C.; Morel, G, presented a model based friction compensation scheme using a novel dynamical friction model was implemented on an

industrial Schilling Titan II hydraulic robot. Off-line estimation of parameters was carried out, using the results of two kinds of experiments. These experiments were done independently at each joint. A nonlinear PI type controller was used in the inner torque loop to improve its performance. The complete control scheme has shown to substantially improve the position precision in regulation and tracking. Higher precision applications can be performed by the hydraulic robot with this controller. [8]

Milos R. Popovic, Guangjun Liu, and Andrew A. Goldenberg , proposed that stick-slip phenomenon is often associated with the control of low velocity motion because of the positional dependency of friction and negative damping friction that decreases as the motion speed increases. In this paper, smooth low velocity tracking control of a commercial robot joint is demonstrated experimentally using a combination of high gain PID control, fast sampling rate and high position sensor resolution. The experimental results also reveal that the main source of instability is not negative damping friction, but position dependant friction that has been widely neglected. The short sampling period and a high resolution encoder have allowed us to compensate for the position dependant friction with a PID controller with sufficiently high gains. [9]

MEI Zhi – gian, YANG Ru-ging, ZHANG Guang-li, MAO Tai-Xaing, proposed that, in order to alleviate the steady state position error and the destabilizing effect of the non-linear friction, a novel compensation method is presented, which modified the traditional Southward's compensation method. Estimated the nonlinear friction model using an identification method, the effect caused by its nonlinear component can be compensated, and an enhanced tracking performance is verified on a selectively compliant articulated robot arm (SCARA) robot. [10]

Takashi Maeno , Tomoyuki Kawamura and Sen-chieh Cheng, proposed a method to estimate the friction coefficient between a planar surface and an elastic finger-shaped sensor by only pressing a sensor against the surface of an object. The contact condition between a planar surface and a half-cylindrical finger is considered using

finite element analysis. The deformation of the elastic finger, contact forces, and strain distribution inside the elastic finger are calculated for various friction coefficients between the finger and the surface. Results show that the shear strain differs when the friction coefficient differs. In addition, in the present study, an elastic finger-shaped sensor made of silicone rubber is designed and constructed. In an experiment using this newly designed sensor, the friction coefficient between the finger and the planar surface is estimated using the strain inside the finger. [11]

Evangelos G. Papadopoulos, Georgios C. Chasparis, Friction is responsible for several servomechanism limitations, and their elimination is always a challenge for control engineers. In this paper, model-based feedback compensation is studied for servomechanism tracking tasks. Several kinetic friction models are employed and their parameters identified experimentally. The effects of friction compensation on system response are examined using describing function analysis. A number of control laws including classical laws, rigid body motion models, and friction compensation are compared experimentally in large-displacement tasks. Results show that the best response is obtained using a controller that incorporates a rigid body model and a friction model based on an accurate description of identified kinetic friction effects. [12]

G. Liu, A. A. Goldenberg and Y. Zhang, proposed that Precise low speed motion control of a robot manipulator calls for precise position and velocity measurement and joint friction compensation, as well as robustness and adaptability of the control scheme. However, precise velocity measurement and friction compensation remain challenging research tasks, especially for very slow motions. In the present work, a simple and efficient method is proposed to estimate velocity from a sampled incremental encoder pulse train, which is then utilized in the experimental investigation on a proposed robust decomposition-based friction compensation method. The experimental results on a direct drive robot arm have demonstrated precise motion control at very slow speeds and in the presence of significant joint friction. [13]

RobWaiboer, Ronald Aarts and Ben Jonker, In their paper, analyses the problem of modelling joint friction in robotic manipulators with gear transmissions at joint velocities varying from close to zero until their maximum appearing values. It is shown that commonly used friction models that incorporate Coulomb, (linear) viscous and Stribeck components are inadequate to describe the friction behaviour for the full velocity range. A new friction model is proposed that relies on insights from tribological models. The basic friction model of two lubricated discs in rolling-sliding contact is used to analyse viscous friction and friction caused by asperity contacts inside gears and roller bearings of robot joint transmissions. The analysis shows different viscous friction behaviour for gears and pre-stressed bearings. The sub-models describing the viscous friction and the friction due to the asperity contacts are combined into two friction models; one for gears and one for the pre-stressed roller bearings. In this way, a new friction model [1] is developed that accurately describes the friction behaviour in the sliding regime with a minimal and physically sound parametrisation. The model is linear in the parameters that are temperature dependent, which allows to estimate these parameters during the inertia parameter identification experiments. The model, in which the Coulomb friction effect has disappeared, has the same number of parameters as the commonly used Stribeck model [2]. The model parameters are identified experimentally on a Staubli RX90 industrial robot. [14]

Basilio Bona and Marina Indri, Friction effects are particularly critical for industrial robots, since they can induce large positioning errors, stick-slip motions, and limit cycles. This paper offers a reasoned overview of the main friction compensation techniques that have been developed in the last years, regrouping them according to the adopted kind of control strategy. Some experimental results are reported, to show how the control performances can be affected not only by the chosen method, but also by the characteristics of the available robotic architecture and of the executed task. [15]

Basilio Bona and Marina Indri, in this paper mentioned that joint friction is a major problem in accurate robot position control, particularly during low-speed, small-

amplitude tasks. This paper proposes a simple, practical, and elective method to compensate for joint friction, using a six-axis force/torque sensor mounted under the manipulator. From these measurements, joint torques are estimated and used in a torque controller, which virtually eliminates friction and gravity effects, providing high-precision motion control even for small motions at low speed. The method does not require complex analytical friction models. The method also does not require expensive and unreliable internal joint-torque sensors. [16]

M. Kemal Ciliz , In this paper a framework is proposed for the adaptive control of robotic manipulators which combines parametric adaptive control with Artificial Neural Network (ANN)-based compensation of dynamic uncertainties like friction. The proposed method utilizes a passivity-based parametric adaptive control approach and makes use of the ANN models as generic identifiers to compensate for unmodelled friction effects. Unlike many approaches for ANN based control in the literature, parameter update equations for the ANN model and for the parametric adaptive model are driven by both the tracking error and the system identification error. A stability analysis is given based on the passivity properties of the manipulator dynamics. The methodology is successfully tested for the control of a Direct Drive SCARA arm and performance is compared with standard adaptive control schemes. [17]

Nathan van de Wouw, Niels Mallon, Henk Nijmeijer, In his paper studied the friction compensation in a controlled one-link robot using a reduced order observer. Since friction is generally velocity-dependent and controlled mechanical systems are often only equipped with position sensors, friction compensation requires velocity estimation. Here, a reduced-order linear observer is used for this purpose. For exact friction compensation, design criteria in terms of the controller and observer parameter settings guaranteeing global exponential stability of the set-point are proposed. Moreover, for non-exact friction compensation it is shown that under compensation leads to the existence of an equilibrium set and overcompensation leads to limit cycling. These results are obtained both numerically and experimentally. [18]

J.Wang, S. S. Ge and T. H. Lee , proposed that In motion control, two major sources of uncertainties are friction and inertia. In this paper, an adaptive neural network based compensation scheme is proposed to cope with both friction and inertia uncertainties. The dynamic LuGre friction model proposed in which captures most of the friction behaviors is used as a basis for this study. Neural networks are used to parameterize the unknown system nonlinear function and the unknown dynamic friction bounding function respectively. Based on Lyapunov synthesis, the adaptive control algorithms are designed to achieve asymptotic tracking of the desired trajectory and guarantee the bounded ness of all the signals in the closed-loop system. Simulations on the tracking control of a 2-link planar robot arm are carried out to verify the effectiveness of the proposed method. [19]

Kouichi Mitsunaga, Takami Matsuo, In his paper, design an adaptive controller to compensate the nonlinear friction model when the output is the position. First, we present an adaptive differential filter to estimate the velocity. Secondly, the dynamic friction force is compensated by a fuzzy adaptive controller with position measurements. Finally, a simulation result for the proposed controller is demonstrated. [20]

INTRODUCTION TO ROBOTICS

1.1 Robotics

“Robotics, computer-controlled machine that is programmed to move, manipulate objects, and accomplish work while interacting with its environment. Robots are able to perform repetitive tasks more quickly, cheaply, and accurately than humans. The term robot originates from the Czech word robota, meaning “compulsory labor.” The word robot has been used since to refer to a machine that performs work to assist people or work that human find difficult and undesirable.”

A Robot can be defined as a programmable, self-controlled device consisting of electronic, electrical, or mechanical units. More generally, it is a machine that functions in place of a living agent. Robots are especially desirable for certain work functions because, unlike humans, they never get tired; they can endure physical conditions that are uncomfortable or even dangerous; they can operate in airless conditions; they do not get bored by repetition; and they cannot be distracted from the task at hand. Robot is a system that contains sensors, control systems, manipulators, power supplies and software all working together to perform a task.



Fig 1.1 Material Handling Robot

1.2 History of Robots

In the early 1800's mechanical puppets were first built in Europe, just for entertainment value. And these were called robots since their parts were driven by linkage and cams and controlled by rotating drum selectors. In 1801 Joseph Maria Jacquard made the next great change and invented the automatic draw loom. The draw loom would punch cards and was used to control the lifting of thread in fabric factories. After that there were many small changes in robotics but we were slowly moving forward.

The first industrial robots were Unimates developed by George Devol and Joe Engelberger in the late 50's and early 60's. The first patents were by Devol but Engelberger formed Unimation which was the first market robots. So Engelberger has been called the "father of robotics". For a while the economic viability of these robots proved disastrous and things slowed down for robotics. But the industry recovered and by the mid-80's robotics was back on track.

Mobile Robotics moved into its own in 1983 when Odettec introduced this six-legged vehicle which was capable of climbing over objects. This robot could lift over 5.6 times its own weight parked and 2.3 times its weight moving.

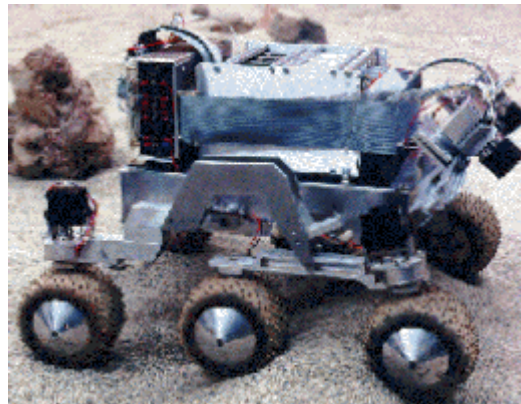


Fig 1.2 six – legged robot

1.3 Classification of Robots

Robotics Institute of America

Countries have different definitions of what it means to be a robot. For example, the Robotics Institute of America (RIA) defines a robot as:

A re-programmable multi-functional manipulator designed to move materials, parts, tools, or specialized devices through variable programmed motions for the performance of a variety of tasks. It also recognizes four classes of robot:

- A: Handling devices with manual control
- B: Automated handling devices with predetermined cycles
- C: Programmable, servo-controlled robots with continuous or point-to-point trajectories
- D: Robots capable of Type C specifications which also acquire information from the environment for intelligent motion

Japanese Industrial Robot Association

In contrast, the Japanese Industrial Robot Association (JIRA) recognizes as many as six classes:

- Class 1: Manual - Handling Devices : A device with multiple degrees of freedom that is actuated by an operator
- Class 2: Fixed Sequence Robot : A device that performs the successive stages of a task according to a predetermined, unchanging method and is hard to modify.
- Class 3: Variable-Sequence Robot : A device that performs the successive stages of a task according to a predetermined, unchanging method with easily modified sequence of control
- Class 4: Playback Robot : A human operator performs the task manually by leading the robot, which records the motion for later playback.
- Class 5: Numerical Control Robots : The operator supplies the robot with a movement program rather than teaching it the task manually
- Class 6: Intelligent robot : A device that can understand its environment and able to complete the task despite changes in the operation conditions

1.4 How Robot works

“The inspiration for the design of a robot manipulator is the human arm, but with some differences. For example, a robot arm can extend by telescoping—that is, by sliding cylindrical sections one over another to lengthen the arm. Robot arms also can be constructed so that they bend like an elephant trunk. Grippers, or end effectors, are designed to mimic the function and structure of the human hand. Many robots are equipped with special purpose grippers to grasp particular devices such as a rack of test tubes or an arc-welder. The joints of a robotic arm are usually driven by electric motors. In most robots, the gripper is moved from one position to another, changing its orientation. A computer calculates the joint angles needed to move the gripper to the desired position in a process known as inverse kinematics. Some multi-jointed arms are equipped with servo, or feedback, controllers that receive input from a computer. Each joint in the arm has a device to measure its angle and send that value to the controller. If the actual angle of the arm does not equal the computed angle for the desired position, the servo controller moves the joint until the arm's angle matches the computed angle. Controllers and associated computers also must process sensor information collected from cameras that locate objects to be grasped, or they must touch sensors on grippers that regulate the grasping force. Any robot designed to move in an unstructured or unknown environment will require multiple sensors and controls, such as ultrasonic or infrared sensors, to avoid obstacles. Robots, such as the National Aeronautics and Space Administration (NASA) planetary rovers, require a multitude of sensors and powerful onboard computers to process the complex information that allows them mobility. This is particularly true for robots designed to work in close proximity with human beings, such as robots that assist persons with disabilities and robots that deliver meals in a hospital. Safety must be integral to the design of human service robots.”

1.5 Components of Robot

A robot, as a system consists of the following components, which are integrated together to form a whole:

1.5.1 Manipulator

An industrial robot is comprised of a robot manipulator, power supply, and controllers. The robot manipulator can be divided into two sections, each with a different function:

- **Arm and Body** - The arm and body of a robot are used to move and position parts or tools within a work envelope. They are formed from three joints connected by large links.
- **Wrist** - The wrist is used to orient the parts or tools at the work location. It consists of two or three compact joints.

1.5.2 End Effectors

End-effectors are the tools attached to the end of the robot arm that enable it to do useful work. Most robot manufacturers either do not include end-effectors with their robots or include a general-purpose gripper to allow you to do simple tasks.

The action of the end effector is either controlled by the robot's controller, or the controller communicates with the end effector's controlling device (such as PLC)

1.5.3 Actuators

An actuator is a mechanical device for moving or controlling a mechanism or system. They convert electrical energy into mechanical energy. Actuators are controlled by the controller. Common type of actuators are : DC Motor , AC Motor, Servo Motor and Stepper Motor.

- **DC-Motors** : DC-motors are very easy to use, but like most other motors their usefulness for robotics is very dependant on the gearing available. DC-motors are made much more effective if they have an efficient gear ratio for a

particular task. If your priority is to have a fast spinning motor and torque is of little concern a low gearing or even no gearing may be what you need; however, most motors used in robots need torque over top speed so a motor with a high gear ratio could be more useful.

The control of a DC motor can be split into two parts: speed and direction.

- **AC-motor:** There are several different types of AC-motors, but their use is limited to high power stationary industrial robots. They are harder to use than DC-motors.
- **Servo-Motor :** Servos are used in robotics for different uses: e.g. to move a sensor around, or to move the legs of a robot. Some users modify the servo so they can use it as a DC-motor with a gearbox. Controlling a servo is done with Pulse –width modulation. The length of the pulse is relative to the position the servo will turn to. The length of this pulses is usually located between 1ms and 2ms, if so 1.5ms would be the center position. This pulse needs to be repeated with small intervals (otherwise the servo might turn to a "save" position or it might simply stay at its current position. This depends on the type of servo used).
- **Stepper Motor :** In robotics stepper motors are primarily used in stationary robots as they tend to consume quite a lot of power. They are ideal for movements that have to be accurate and are larger than 180°.

1.5.4 Sensors

Sensors are used to collect information about the internal state of the robot or to communicate it with outside environment. The sensors of the system allow the robot to get feedback on its position and in smarter systems data from the environment. Sensors integrated into the robot send information about each joint or link to the controller, which determines the configuration of robot. Sensors that a robot uses generally fall into three different categories:

- **Environmental sensors** : Environment sensors are sensors that detect and interpret things in the outside world (such as obstacles, sounds, and signs) or about the outside world (such as temperature, pressure, and viscosity)
 1. Thermal sensors
 2. Pressure sensors
- **Feedback sensors** : Feedback sensors are used to detect the actuator's output so that the control system can correct for external factors.
 3. Encoders
 4. Tachometers
 5. Accelerometers
 6. Rate Gyroscopes
- **Communication Sensors:** These sensors allow a human or computer to provide a robot other information. They are used to transfer to and from the robot.

1.5.5 Controller

Controllers interpret all the input from sensors and decide how to act in response. The controller is the brain of the system processing information and changing the actuators and mechanical parts based on this information. The controller receives its data from the computer, controls the motion of the actuators, and coordinates the motion with the sensory feedback information.

1.5.6 Processor

The processor calculates the motions of the robot's joints, determines how much and how fast each joint must move to achieve the desired location and speeds, and oversees the coordinated actions of the controller and the sensors. The processor is generally a computer, which works like all other computers, but it is dedicated to

robot task only. It requires an operating system, programs, peripheral equipment such as monitors, and has many of the same limitations and capabilities of a PC processor.

1.5.7 Software

Robot software is the coded commands that tell a mechanical device (known as a robot) what tasks to perform and control its actions. Robot software is used to perform tasks and automate tasks to be performed. Programming robots is a non-trivial task. Many software systems and frameworks have been proposed to make programming robots easier. Most robot manufacturers keep their software hidden. It is impossible to find out how most robots are programmed. It is almost as if they had no software in many cases. This is because these companies do not intend their customers to do their own programming and they sell complete proprietary application packages rather than general purpose software.

1.6 Characteristics of Robot:

The following definitions are used to characterize robot specifications:

- **Speed :**
The amount of distance per unit time at which the robot can move, usually specified in inches per second or meters per second. The speed is usually specified at a specific load or assuming that the robot is carrying a fixed weight. Actual speed may vary depending upon the weight carried by the robot.
- **Load Bearing Capacity**
The maximum weight-carrying capacity of the robot. Robots that carry large weights, but must still be precise are expensive.
- **Accuracy**
The ability of a robot to go to the specified position without making a

mistake. It is impossible to position a machine exactly. Accuracy is therefore defined as the ability of the robot to position itself to the desired location with the minimal error (usually 0.001 inch).

- **Repeatability**

The ability of a robot to repeatedly position itself when asked to perform a task multiple times. Accuracy is an absolute concept, repeatability is relative. Note that a robot that is repeatable may not be very accurate. Likewise, an accurate robot may not be repeatable.

- **Work Envelope**

The maximum reach, or volume within which a robot can operate. This is usually specified as a combination of the limits of each of the robot's parts. Work Envelope is a function of the robot's joint lengths and its configuration.

- **Precision**

Precision is defined as how accurately a specified point can be reached. This is a function of the resolution of the actuators, as well as its feedback devices. Most industrial robots can have precision of 0.001 inch or better

- **Degree of Freedom**

Degrees of Freedom (DOF) are the set of independent displacements that specify completely the displaced or deformed position of the body or system. A particle that moves in three dimensional space has three translational displacement components as DOFs, while a rigid body would have at most six DOFs including three rotations. Translation is the ability to move without rotating, while rotation is angular motion about some axis.

1.7 Kinematics and Dynamics of Robot

The study of motion of robot can be divided into [kinematics](#) and [dynamics](#).

Kinematics: Robot kinematics is the study of the motion of robots. In a kinematic analysis the position, velocity and acceleration of all the links are calculated without considering the forces that cause this motion. While dealing with the kinematics used in the robots we deal each part of the robot by assigning a frame of reference to it and hence a robot with many parts may have many individual frames assigned to each movable part. For simplicity we deal with the single manipulator arm of the robot. Each frame is named systematically with numbers, for example the immovable base part of the manipulator is numbered 0, and the first link joined to the base is numbered 1, and the next link 2 and similarly till n for the last nth link.

Robot kinematics are mainly of the following two types: Forward kinematics and Inverse kinematics

Forward kinematics is also known as Direct kinematics. Direct kinematics refers to the calculation of end effector position, orientation, [velocity](#) and acceleration when the corresponding joint values are known.

Inverse Kinematics refers to the opposite case in which required joint values are calculated for given end effector values, as done in path planning. Some special aspects of kinematics include handling of redundancy (different possibilities of performing the same movement), collision avoidance and singularity avoidance. Once all relevant positions, velocities and accelerations have been calculated using kinematics, methods from the field of dynamics are used to study the effect of forces upon these movements.

Dynamics: Dynamics of robot studies the relationship between motion, and the associated forces and torques. Robot Dynamics are mainly of the following two types: Forward Dynamics and Inverse Dynamics.

Forward Dynamics is used for simulation, and as feedforward in the robot's motion controller, i.e., the Forward Dynamics calculates what the robot does when specific joint torques are applied, and under the assumption that all physical parameters in the dynamics model are accurate. Direct dynamics refers to the calculation of accelerations in the robot once the applied forces are known. Direct dynamics is used in computer simulations of the robot.

Inverse Dynamics refers to the calculation of the actuator forces necessary to create a prescribed end effector acceleration. This information can be used to improve the control algorithms of a robot.

1.9 Robot Design Factors

An industrial robot is a complex system where the dynamics and kinematics aspects are very important to its general performance. Therefore, there is a demand for realistic models of robots to use in simulation, control design and diagnosis. However, it is not an easy task to design an accurate global model due to a lot of non-linearities and phenomena that are not fully understood. There are some known aspects of a robot that should be considered while designing a model.

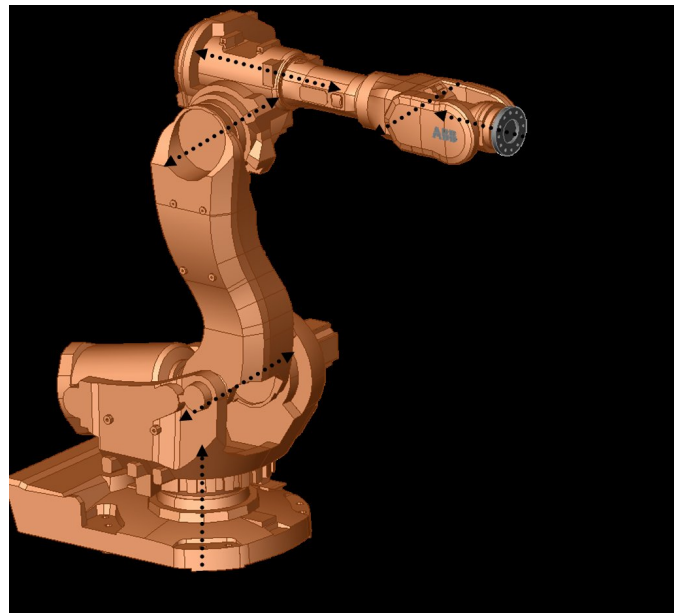


Fig 1.3 An Industrial Robot

Flexibilities In general, robot links can be considered as rigid-bodies (no flexibilities) but nowadays there is a special demand on reducing production costs which generally means reducing weight. Making lighter links reduces the production costs of the mechanical part of a robot (which represents more than 50% of its total price) but on the other hand, the flexible modes of the links get more evident. Also, the gearboxes present in a joint, specially the harmonic drive type, introduces flexibilities due to elastic deformation of bearings and gears. Such flexibilities are nonlinear which makes the system more challenging to control, model and estimate.

Friction Friction affects any mechanical moving parts and has extensively been studied due to its importance in mechanical systems. Even though there is no analytical model for friction, there are some well-known friction phenomena and models based on empirical experiments to describe it. An usual way of modeling friction is to consider only its static effects like Coulomb, viscous and Stribeck friction.

Backlash Backlash is present in all mechanical system where the motor is not directly coupled to the load. It can be described as the clearance between mating components when movement is reversed and contact is re-established. For a gear for example, the backlash is the amount of clearance between mated gear teeth. When the backlash gap is opened, the movement of the load is autonomous and the moment generated by the motors drives only the motor itself and not the load.

Torque Ripple In general, electric AC permanent magnet motors are used as joint actuators. These motors are compact, fast and robust. A drawback is that the generated torque changes periodically with the rotor position. Distortion of the stator flux linkage distribution and variable magnetic reluctance at the stator slots are the main causes of the resulting torque ripple.

1.9 Advantages of Robot

Robot offers following advantages :

- Robot increases productivity, safety, efficiency , quality , and consistency of products.
- Robot can work in hazardous conditions without the need for life support, comfort
- Robot needs no environmental comfort, such as lighting, air conditioning, ventilation.
- Robot can work continuously without experiencing fatigue
- Robot have repeatable precision at all times
- Robots are much more accurate than humans.
- Robots have capability beyond that of human
- Robot can process multiple stimuli or task simultaneously.

1.10 Disadvantages of Robot

Robots are superior in certain senses, have limited capabilities in:

- Robots lack capability to respond in emergencies
- A lack of decision making power
- Can injure human operator
- A loss of power
- Requires training sessions
- Requires the knowledge of programming
-

1.11 Robot Applications

Robot has wide applications in different field of science. The following is list of applications where robots are useful. The list is not complete by any set of imagination. There are many other uses as well, and other applications find their way into the industry and the society all the time :

Machine Loading

Machine loading robots work efficiently, tirelessly, and accurately. Instead of workers having to load and unload parts, an integrated machine loading robot handles the loading application effectively and saves on labor costs. Machine loading robots increase productivity, which in turn increases revenue.

Robotic Machine Loading Benefits:

Speed

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Precision
Tirelessness
Accuracy
Flexibility

Automobile Industry

A new generation of versatile robots are being introduced on the shop floor. A single robot can typically be fitted with devices equipping it with the skill to pick up parts, reposition, lubricate, drill, weld, or paint. In the production process, robots now unload molded car parts once they are cast, position parts to be forged, and paint car bodies. Welding commands the largest number of robots. In Japan, nearly 90 percent of body welding is done by automated processes; in North American firms the level exceeds 50 percent.

Assembly Operations

A method of assembling a part to a workpiece with a combination tool supported on a robot, where a mounting surface of the part is assembled on a target surface of the workpiece including the following steps:

- (a) Indexing to the assembly position.
- (b) Picking a part with the part gripper on the part delivery arm from a part supply station.
- (c) Indexing part delivery and workpiece preparation arms to the preparation position.
- (d) Operating the workpiece target surface scanning module and communicating workpiece target surface position data to the robot.
- (e) Moving the robot to set the tool operating axis and calculate tool offsets relative to the position and orientation of the scanned target surface.
- (f) Simultaneously: engaging the part mounting surface with the part surface preparation module; and engaging the workpiece target surface preparation module with the target surface.
- (g) Indexing to the assembly position.
- (h) Using the previously set tool operating axis and calculated tool offsets, engaging the prepared part mounting surface with the prepared workpiece target surface.
- (i) Releasing the part gripper and withdrawing the tool from the part/workpiece assembly.

Manufacturing

Robots are responsible for the success of many manufacturing companies. There are many benefits to integrating robotic equipment. Manufacturing robots bring about faster, more efficient production. The chief advantages of switching to manufacturing automation are cost cuts and quality increases. Robot in manufacturing includes many operations such as material removal, drilling, deburring, laying glue, cutting etc.

Medical Science

Robots have wide applications in the field of medical science. Some of the applications are:

1 Cardiac surgery

Endoscopic coronary bypass surgery and mitral valve replacement have been performed. Totally closed chest, endoscopic mitral valve surgeries are being performed now with the robot.

2 Gastrointestinal surgery

Multiple types of procedures have been performed with either the *Zeus* or *da Vinci robot* systems, including bariatric surgery.

3 Gynecology

Reproductive surgery and ablative surgery including hysterec have been performed.

4 Neurosurgery

Several systems for stereoactive intervention are currently on the market.

5 Orthopedics

The ROBODOC system was released in 1992 by the Integrated Surgical Systems

6 Urology

The da Vinci robot is commonly used to remove the prostate gland for cancer, repair obstructed kidneys, repair bladder abnormalities and removed diseased kidneys.

Assisting Disabled Individuals

Robots can help the disabled in their daily lives. A small robot was programmed to communicate with a disabled person and to perform simple tasks such as placing a foodplate into the microwave oven, removing the plate from oven, and placing the plate in front of the disabled person to eat.

CHAPTER –2

INTRODUCTION TO FRICTION

2.1 Friction

When a body moves or tends to move on another body, the force, which appears between the surfaces in contact and resists the motion or tendency towards motion, of one body relative to other is called as friction or frictional force or force of friction. It is due to friction that ball after being hit, comes to rest. Friction plays a vital role in braking systems or in the motion of wheels. Lubricants such as grease or oil are used to avoid the effect of friction.

Friction is undesirable in the performance of controlled electromechanical motion systems. The positioning performance of robots is greatly influenced by friction. The effect of friction on robotic system depends on many factors such as lubrication, temperature, properties of surface material, displacement and relative velocity of robot. To achieve the maximum accuracy of robot friction should be reduced as much as possible.

Friction is very important for the control engineer, for example in design of drive systems, high-precision servo mechanisms, robots, pneumatic and hydraulic systems and anti-lock brakes for cars. Friction is highly nonlinear and result in steady state errors, limit cycles, and poor performance. It is therefore important for control engineers to understand friction phenomena and to know how to deal with them.

2.2 Friction Phenomenon

The Friction phenomenon is divided in two groups as static friction Phenomena and dynamic friction phenomena:

2.2.1 Static friction Phenomena

- **Static Friction :**

The Force required to an object sliding across another is usually greater than the force required to keep them moving. Static friction is the force that holds back a stationary object up to the point that it just starts moving. The static coefficient of friction is the force that restricts the sliding movement of an object that is stationary on a relatively smooth, hard surface. Static friction is the force between two objects that are not moving relative to each other. The coefficient of static friction, denoted as μ_s , is greater than the Coulomb friction. The initial force to get an object moving is often dominated by static friction. The important example of static friction is the force that prevents a car wheel from slipping as it rolls on the ground.

- **Coulomb Friction :**

Coulomb friction is independent of velocity and depends only on direction of motion. The magnitude of coulomb friction depends on the properties of surfaces in contact and the normal force. Coulomb friction is also called as kinetic friction or dynamic friction. The Coulomb friction is given as :

$$F_f = \mu F_n$$

Where F_f is the force exerted by friction, μ is the coefficient of friction, which is an empirical property of the contacting materials, F_n is the normal force exerted between the surfaces .

- **Viscous Friction :**

Viscous friction depends upon velocity. At zero velocity, viscous friction is zero and the component of viscous friction increases with increase of velocity.

- **Stribeck Friction :**

The stribeck friction component is inversely proportional to velocity. Stribeck friction arises due to the presence of fluid lubrication.

2.2.2 Dynamic Friction Phenomena

- **Kinetic friction :**

kinetic friction is the force holding back regular motion. This, kinetic friction coefficient of friction is one of the forces restricting the movement of an object that is sliding on a relatively smooth, hard surface. Kinetic (or dynamic) friction occurs when two objects are moving relative to each other and rub together (like a sled on the ground). The coefficient of kinetic friction is denoted as μ_k , and is less than the coefficient of static friction

- **Pre sliding Displacement :**

When a frictional contact is in static friction there may none the less be relative motion. This arises with tangential compliance and, because there is no true sliding, is called pre sliding displacement. A friction model renders pre sliding displacement of variations in applied force below the breakaway force produce elastic deformation and movement. This friction phenomena arises from the elastic deformation of bonding sites between two surfaces when applied force exceeds the breakaway force. Pre-sliding displacement is also called as Dahl effect.

- **Varying Break-away Force :**

In this friction phenomenon, breakaway force will depends on the rate of increase of the applied force.

- **Frictional Lag :**

Frictional lag is the delay in the change of the friction force as the function of a Change in the velocity.

2.3 Friction Models

To compensate the effect of friction in robotics friction models are used. Each

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friction model incorporates one or more friction phenomena. The models are divided in two groups, static models and dynamic models. Static friction model incorporates only static friction phenomena and are a static function of velocity. Dynamic friction models contains behind static friction phenomena and also some dynamic friction phenomena.

2.3.1 Static Friction Model

Static friction models have a static dependency on velocity. These models are used in systems where the applied force is very large. The major problem with static friction model is their discontinuity at zero velocity. At zero velocity, friction force is not a function of velocity but a function of applied force.

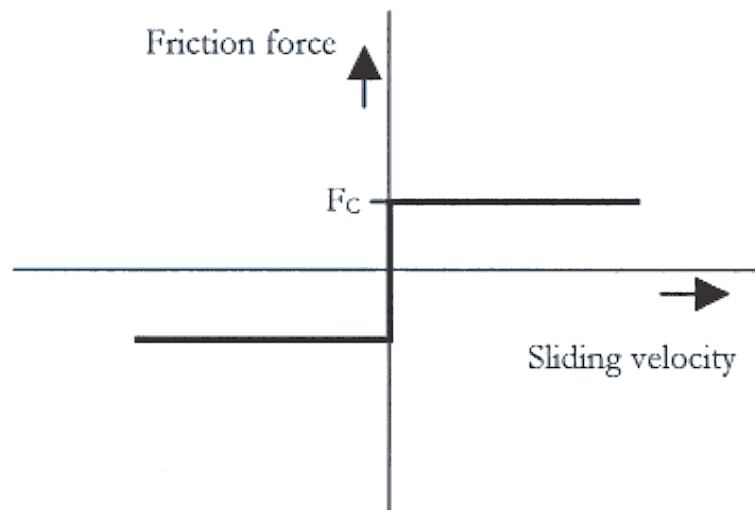


Fig 2.1 Columb Friction Model

Columb plus viscous Friction Model

The Coulomb+viscous friction model is commonly used because of its simplicity, and may be written:

$$f_f = f_c \operatorname{sgn}(v) + f_v v$$

where f_f is the friction force, v the relative velocity of two bodies in contact, f_c the Coulomb friction level, and f_v the coefficient of dynamic friction. The model is not well suited for implementation because of the discontinuous $\operatorname{sgn}()$ function.

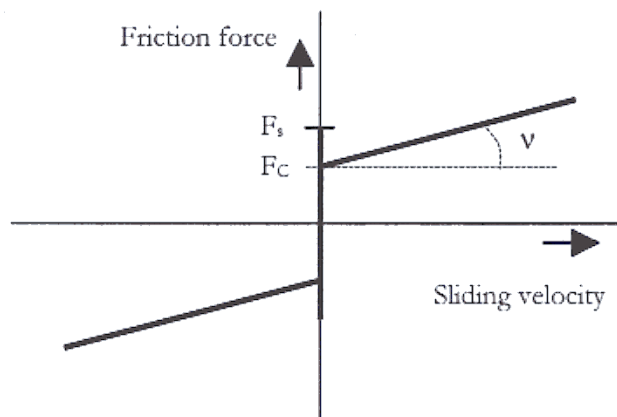


Fig 2.2 Columb plus viscous Friction Model

Coulomb plus viscous plus stribeck friction model:

Stribeck (1902) observed that for low velocities, the friction force is decreasing continuously with increasing velocities. This phenomenon of a decreasing friction at low, increasing velocities is called the Stribeck friction. This model includes Static, Coulomb, Viscous and Stribeck friction is given below.

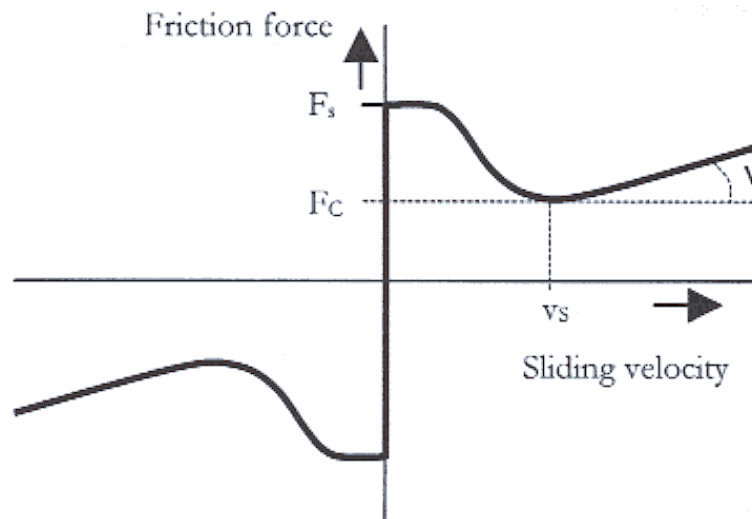


Fig 2.3 Columb plus viscous plus stribeck Friction Model

The Karnopp Model

Karnopp model is developed to overcome the problems with zero velocity detection and to avoid switching between different state equations for sticking and sliding. The model defines a zero velocity interval, $(v) < DV$. For velocities within this interval the internal state of the system (the velocity) may change and be non-zero but the output of the block is maintained at zero by a dead-zone. Depending on if $(v) < DV$ or not, the friction force is either a saturated version of the external force or an arbitrary static function of velocity. The interval $\pm DV$ can be quite coarse and still promote so called stick-slip behavior.

The drawback with the model is that it is so strongly coupled with the rest of the system. The external force is an input to the model and this force is not always explicitly given. The model therefore has to be tailored for each configuration. Variations of the Karnopp model are widely used since they allow efficient simulations. The zero velocity interval does, however, not agree with real friction. The friction models presented so far have considered friction only for steady velocities. No attention is paid to the behavior of friction as the velocity is varied.

Limitation of Static Friction Models

A major problem of the static friction models during simulation is the discontinuity at zero velocity. At zero velocity the friction force is not a function of the velocity, but a function of the applied force. The friction force has the same magnitude as the applied force, but in the opposite direction. So the effective force is zero. When the applied force exceeds the level of Coulomb or static friction (depends on the choice of the model) the friction force becomes either the Coulomb friction level or the static friction level. The detection of the zero velocity discontinuity in simulations with the static friction models make the simulations less efficient

2.3.2 Dynamic Friction Models

Dahl Model : Dahl is an example of dynamic Friction model. Dahl model is used for the purpose of simulating control systems with friction. According to this model, friction depends only on displacement. Dahl model accounts for the zero slip displacement and hysteresis effects of friction. It does not capture stribek effect and stick-slip motion effect of friction.

The starting point for Dahl's model is the stress-strain curve as shown in figure .Dahl modeled the stress-strain curve by a differential equation. Let x be the displacement, F the friction force, and F_c the Coulomb friction force. Then Dahl model is given by:

$$\frac{dF}{dx} = \sigma \left(1 - \frac{F}{F_c} \operatorname{sgn} v \right)^\alpha$$

Where σ is the stiffness constant and α is the parameter that is used to determine the shape of the stress- strain curve. The value of α is taken as 1.

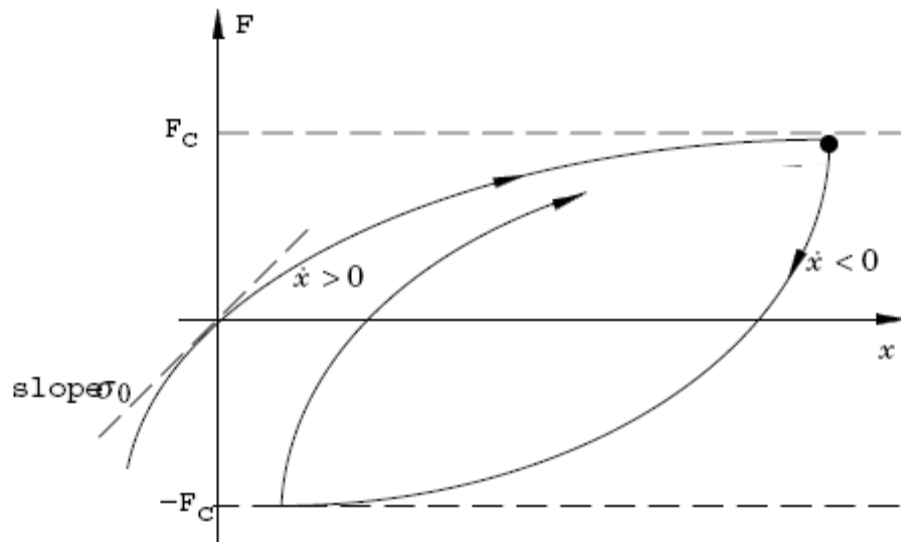


Fig 2.4 Friction force as a function of displacement for Dahl's model

In this model the friction force is only a function of the displacement and the sign of the velocity. This implies that the friction force is only position dependent. This so called rate independence is an important property of the Dahl model.

Bliman-Sorine Model: According to Bliman-Sorine Model friction depends only on displacement and it is independent of velocity. The Bliman-Sorine model is a second order model (4 parameters) that can be seen as a parallel connection of two Dahl models. It models static, viscous and Coulomb friction of the static friction phenomena and only pre-sliding displacement of the dynamic friction phenomena.

LuGre Model : LuGre Model is the modified form of Dahl model. The LuGre model is inspired by the bristle interpretation of friction, in combination with lubricant effects. Friction is modeled as the average deflection force of elastic springs. When a tangential force is applied the bristles will deflect. If the deflection is sufficiently large, the bristles start to slip. The average deflection at steady state slip is

determined by the velocity, so that for low velocities the steady state deflection, and therefore the friction force, decreases with increasing velocity. This corresponds to more lubricant being forced into the interface, and pushing the surfaces apart, as the velocity increases. This produces the Stribeck effect. The model also includes rate dependent friction phenomena such as varying break-away force and frictional lag. LuGre friction model can be written as :

$$F_t = \sigma z + \sigma \frac{dz}{dt} + g(v)$$

$$\frac{dz}{dt} = z - r \frac{|v|}{g(v)} z$$

where variable z is introducing a small displacement of the bristles. The function $g(v)$ is representing a stribeck friction. The function $f(v)$ is representing viscose friction, it can be nonlinear too. LuGre Friction curve near zero velocity is shown in figure 2.5 . This curve is acquired by presenting a sine shape velocity signal to the LuGre model. The curve may differ in other simulations.

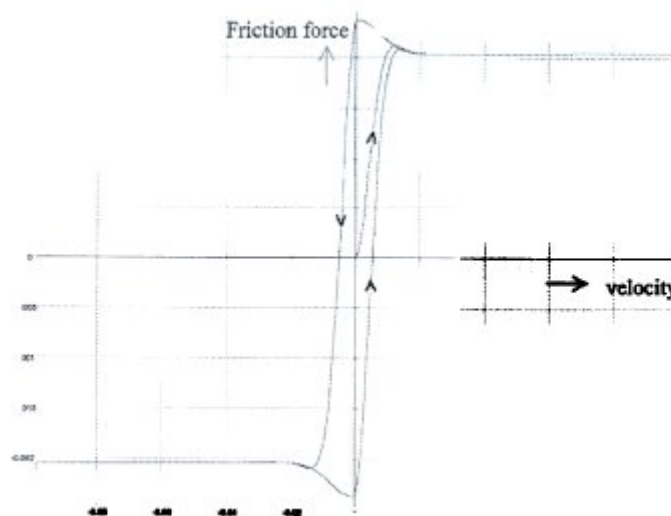


Fig 2 .5 LuGre Friction Curve

FRICTION COMPENSATION

3.1 Friction Compensation

The performance of robots is greatly affected by the presence of Friction. Friction can cause steady state and tracking errors, stick-slip motion and limit cycles. The negative influence of friction can be decreased by applying various friction compensation techniques. Friction compensation based on friction models is usually adopted to eliminate the nonlinear effect of friction.

3.2 Friction Compensation Techniques

There are various friction compensation techniques based on different types of friction. Mainly the friction compensation techniques are divided in four classes : friction avoidance, non model- based compensation, model-based compensation and model-free compensation.

3.2.1 Friction Avoidance

The first method to reduce the influence of friction on a system is the friction avoidance. This is usually done in the design and development phase of a robot. The different strategies available in this are :

- **Selection of Appropriate Lubricant**

By selecting an appropriate lubricant, the effects of friction effects can be Reduced to a great extent. Some of the good lubricants used are: grease, graphite, Molybdenum Disulphide, Boron Nitride, PTFE etc.

- **Increase of Stiffness and reduction of inertia**

The effect of stick-slip error can be eliminated by making the system more stiff and by reducing the inertia to a suitable amount.

- **Replacement of sliding contacts with rolling contacts**

Replacing sliding contacts with rolling contacts can reduce the friction.

3.2.2 Non-model-based Friction Compensation

The main advantage of using Non-model-based Compensation technique that they don't require friction model and can be applied generally for the compensation of various disturbances. The non model based friction compensators are :

- **Dither Signal**

A very simple way to eliminate some effects of friction is to use a dither signal that is a high frequency signal introduced into the system by adding it to the error signal in the feedback loop to smoothen the discontinuity of friction at low velocities added to the control signal. Dither can be introduced electronically or mechanically by a vibrator. If the frequency is chosen to be higher than the cut-off frequency of the system, the high-frequency behavior is filtered out mostly, leaving only the low-frequency "average" response. The effect of the dither is that it introduces extra forces that make the system move before the stiction level is reached. The Dither will help to remove the effect of stiction.

- **High gain PD/PI/PID control**

A common way of friction compensation is the use of a high gain PD, PI or PID controller. The bounded non-linear friction can be seen as disturbances that

must be cancelled out by the controller. For this compensation technique no

Characterization of friction is needed, but the use of high gains makes the Controller sensitive for high-frequency measurement noise and controller Saturation.

- **Fuzzy PI control**

This technique uses different gains for the different regions of the friction curve. So for low velocities other gains are used than for high velocities. The gains are computed on the basis of the input and output of the system and on the basis of the reference signal.

- **Impulsive control**

With this method a pulse is applied to a system in rest, which result in a small Displacement . By making the impulses of great magnitude but short duration, the static friction is overcome and the sensitivity to friction is reduce.

3.2.3 Model-based Friction Compensation

In this compensation techniques a friction model is utilized for the design of the friction compensation.. The friction component is calculated and added to the controller signal. The friction compensator has as an input signal either the reference, measured or estimated velocity.

- **Adaptive Friction Compensation**

Adaptive friction compensation method is used to compensate for nonlinear friction in a variety of mechanisms but they are usually based on the linearized model or a model which is linear in the parameters. Adaptive friction compensation technique includes static as well as dynamic friction phenomenon.

- **Learning Friction Compensation**

Learning model-based friction compensation is a friction compensation method in which the parameters of the friction model are learned. Learning friction compensation technique includes only the static friction Phenomenon. Learning friction compensation requires more memory as compare to adaptive friction compensation .

- **Robust Compensation**

With a robust model-based friction compensator the friction parameters do not have to be known exactly. It is assumed that the friction parameters of the friction model used are known within certain bounds. Robust compensation is used to compensate the uncancelled exponential and position-dependent friction.

- **Fixed Compensation**

Fixed model-based friction compensation uses a compensator with the parameters of the friction model fixed at one value. Once the parameters are estimated through experiments, they are used in the compensator.

3.2.4 Model-free Compensation

Model-free compensation is model-based compensation without using a Parametric friction model but where, after a certain learning period, a non-Parametric friction model is implicitly present within the friction compensator.

- **Repetitive compensation**

Whenever a machine has to carry out a repetitive task, repetitive friction compensation can be applied. A table of corrections for the friction effects is learned during the execution.

- **Learning compensation**

A learning friction compensator uses a-priori knowledge to determine the Friction curve. There are several learning algorithms to learn the friction curve.

PROBLEM FORMULATION

In this chapter, experimental set-up of controlled one-link robot is proposed. Friction is generally velocity dependent so model based friction model depending on velocity is used for friction compensation strategy. Linear proportional derivative controller is used to avoid the non-zero steady state positioning errors caused due to discontinuities in the friction and friction compensation strategy. The effects for both the exact and non exact friction compensation strategies are also find out.

4.1 Experimental Set – Up

The experimental set up of system involving one – link robot is shown in Fig 4.1. The link is driven by a control torque t which is supplied by induction motor. The angular position q is measured by a position encoder.

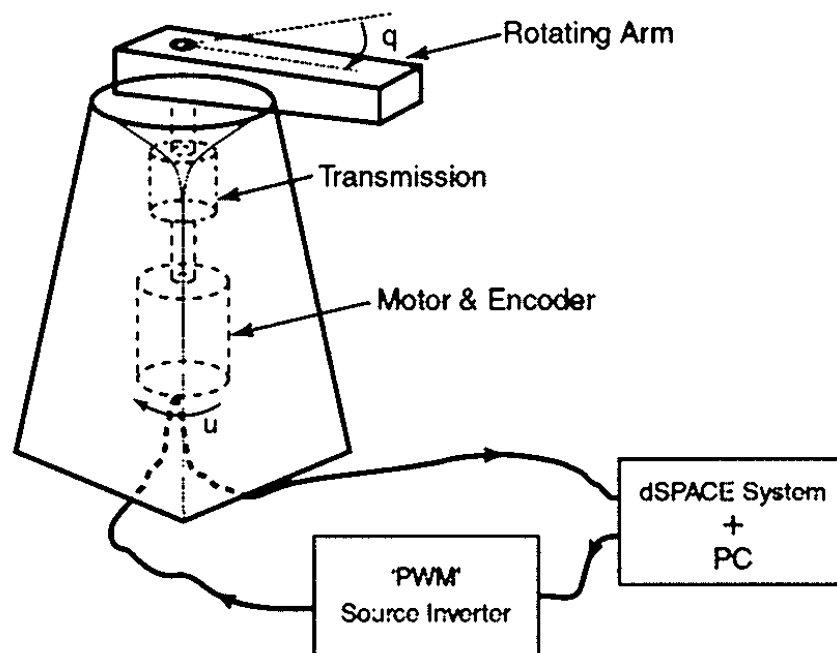


Fig 4.1 Experimental set up

The robot is modelled as a single inertia Y (modeling the inertia of the link and the driveline) subject to a viscous friction torque $-v\dot{\phi}$, a dry friction torque $-F_f(\dot{\phi})$ and a control torque t , which leads to the following model:

$$Y\ddot{\phi} + v\dot{\phi} = t - F_f(\dot{\phi}). \quad (1)$$

Using a frequency-domain identification technique, the total inertia of the system is identified to be $Y = 0.026 \text{ kgm}^2/\text{rad}$.

In order to identify the dry friction model, breakaway experiments are performed to measure the static friction torque and constant velocity experiments are performed to measure the friction torque at non-zero (constant) velocities. According to set valued force law the following algebraic equation is given as:

$$F_f(\dot{\phi}) \in g(\dot{\phi}) \text{Sign}(\dot{\phi}), \quad (2)$$

in which $g(\dot{\phi})\text{Sign}(\dot{\phi})$ represents the Stribeck curve including the modelling of stiction, with

$$g(\dot{\phi}) = F_c + \delta F_e - \left(\frac{|\dot{\phi}|}{v_s} \right)^\beta \quad (3)$$

and $\text{Sign}(x)$ the set-valued sign-function:

$$\begin{aligned} \text{Sign}(x) &= \{-1\} & x < 0 \\ \text{Sign}(x) &= [-1, 1] & x = 0 \\ \text{Sign}(x) &= \{1\} & x > 0 \end{aligned} \quad (4)$$

Herein, F_c is the Coulomb friction force, δF the difference between the static and Coulomb friction force ($\delta F = F_s - F_c$), v_s the Stribeck velocity and β the Stribeck shape parameter. The measurement results and the friction model (including both viscous and dry friction) are shown in figure 4.2. The resulting friction parameter estimates are given in table 4.1, where different parameter estimates are obtained for positive and negative velocities indicating an asymmetric friction model. The friction parameters used in the symmetric model are the mean values of those for positive and negative velocity, as given in table 4.1.

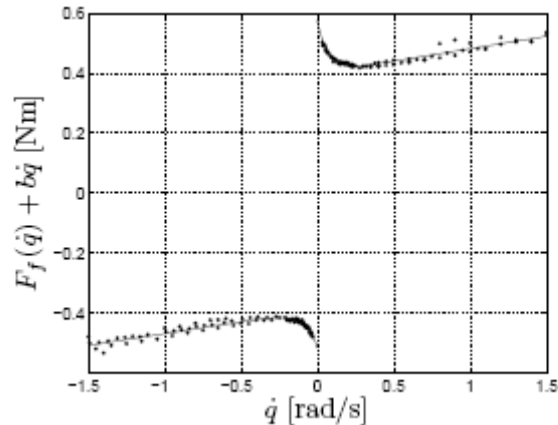


Fig 4.2 Friction measurements (dots) and friction model (solid line)

parameter	$\dot{q} > 0$	$\dot{q} < 0$	mean value
β [-]	1	1	1
F_s [Nm]	0.5735	0.5123	0.5429
F_c [Nm]	0.3990	0.3887	0.3939
v_s [rad/s]	0.0688	0.0817	0.0753
b [Nms/rad]	0.0828	0.0790	0.0809

Table 4.1 Estimation of Friction Parameters

4.2 Controller Design, Observer Design and Friction compensation Strategy

The friction compensation strategy incorporating the reduced-order linear observer and a proportional-derivative controller is shown in fig 4.3. The total control action u is composed by the feedback control u_c and the friction compensation u_{fc} :

$$u = u_c + u_{fc}.$$

Where,

$$u_c = n_1(q_r - q) - n_2 \dot{q} \quad (5)$$

Where q_r is the desired reference position, which will be assumed to equal zero, \hat{q} is the velocity estimate provided by the observer. n_1, n_2 are proportional and derivative gains of PD controller. The value of n_1, n_2 are taken as greater than zero,

$$n_1, n_2 > 0$$

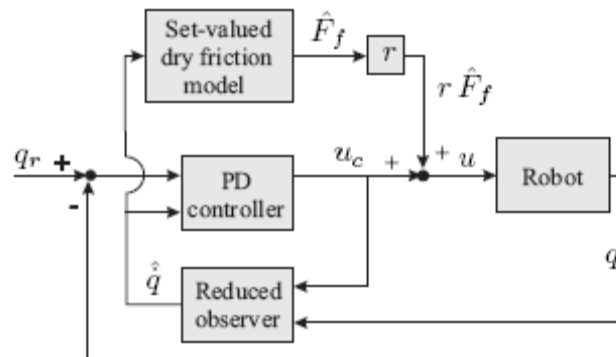


Fig 4.3 Friction Compensation strategy

According to set- valued friction compensation law

$$u_{fc} = rF_f(\hat{q}) \text{sgn}(\hat{q}), \quad (6)$$

r is a scaling factor of the friction compensation. Eq. (6) reflects a feedback compensation strategy where the estimated velocity is provided by an observer. When $r = 1$, exact friction compensation is attained and, when $r \neq 1$, non-exact friction compensation is attained.

The reduced-order observer is designed as

$$\dot{\hat{q}} = \frac{-b}{Y} \hat{q} + \frac{1}{Y} (u - u_{fc}) + L(q - \hat{q}) \quad (7)$$

Where \hat{q} is the observer state (the velocity estimate) and $L > 0$ is the observer gain.

The observer error is defined as

$$e = \hat{q} - q \quad (8)$$

The state coordinates are $x = [q \ \dot{q}]^T$. The dynamics of the closed-loop system, shown in figure 3, can be formulated in terms of these states by the following differential Equations :

$$\dot{x}_1 = x_2 + x_3 \quad (9)$$

$$\dot{x}_2 = \frac{-n_1}{Y} x_1 - \frac{b+n_2}{Y} x_2 + Lx_3 \quad (10)$$

$$\dot{x}_3 = -\frac{b+LY}{Y} x_3 + \frac{1}{Y} [rFf(x_2) - Ff(x_2 + x_3)] \quad (11)$$

4.3 Exact Friction compensation

The behaviour of the closed-loop system is determined for the case of exact friction compensation. i.e. $r = 1$ in eq(6). Firstly, the existence of an equilibrium set depending on the system (and control) parameters is find out and then the stability of the set-point (the origin) is determined.

4.3.1 Equilibrium set Parameter

The equilibrium of eq (9), (10), (11) satisfy the following equations :

$$x_2 = -x_3$$

$$x_1 = -\frac{LY + b + n_2}{n_1} x_2 \quad (12)$$

$$G(x_2) \in [-F_s, F_s]$$

Where $G(x) = (b+LY) x + Ff(x) \quad (13)$

Let us denote this equilibrium set by x^* . The origin is always equilibrium. However, depending on the observer gain L an equilibrium set exists. In figure 4.4, the equilibrium of the system with exact friction compensation are compared to those of the system with no compensation. In this figure, the effect of the existence the equilibrium set on the steady-state positioning error x_1 is depicted, for $n_1 = 0.4$ and $n_2 = 0.02$ and for varying L . Clearly, the equilibrium set can induce a non-zero steady-

state positioning error. However, friction compensation ensures a large decrease in the size of the equilibrium set. Moreover, in case of exact compensation the equilibrium set shrinks to an isolated equilibrium point for increasing observer gain at some critical value of the observer gain.

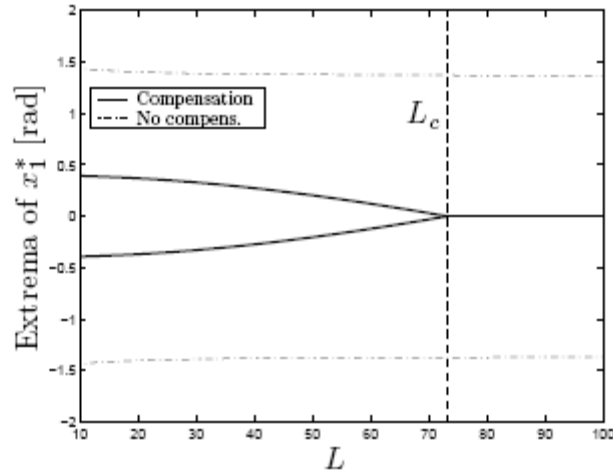


Fig 4.4 Extrema for steady- state error in x_1 for $n_1 = 0.4$ and $n_2 = 0.02$

To derive the condition for L so that a single equilibrium point exist there are two conditions that are :

$$\lim_{x \rightarrow 0} G(x) = F_s \text{ and } \lim_{x \leftarrow 0} G(x) = -F_s.$$

The sufficient and necessary condition, under which no equilibrium set can exist, is that the function $G(x)$ is strictly increasing for all $x \neq 0$ (as given in (12)).

This is attained if $\frac{\partial}{\partial x} G(x) > 0 \forall x \neq 0$ and, consequently, if $L > L_c$

Where

$$L_c = \frac{1}{Y} (-\lambda - b) \quad (14)$$

$$\lambda = -\frac{n\delta F}{vs} = \min_{x \in R \setminus \{0\}} \left(\frac{\partial g(x)}{\partial x} \right) \quad (15)$$

$$\eta = \{1 \text{ if } \beta=1$$

$$\eta = \begin{cases} \frac{(\beta-1)e^{-\frac{\beta-1}{\beta}}}{\sqrt[\beta]{\frac{\beta-1}{\beta}}} & \text{if } \beta > 1 \end{cases} \quad (16)$$

For the parameters of the model of the one-link robot (using mean values for the friction parameters) the critical observer gain is $L_c = 73.07$ as shown in fig 4.4. This value corresponds to the value for which the equilibrium set merges into an isolated equilibrium point in figure 4.4. The size of the equilibrium set (and thus the maximum steady state positioning error) can also be influenced by the controller parameters; if n_1 is increased the size of the equilibrium set decreases and if n_2 is increased the size of the equilibrium set increases, as given in equation (12).

4.3.2 Stability

In order to determine the stability of the origin of equation (9),(10),(11) we represent the closed loop system in the form of a cascade of a subsystem S_I and a subsystem S_{II} as shown in figure 4.5.

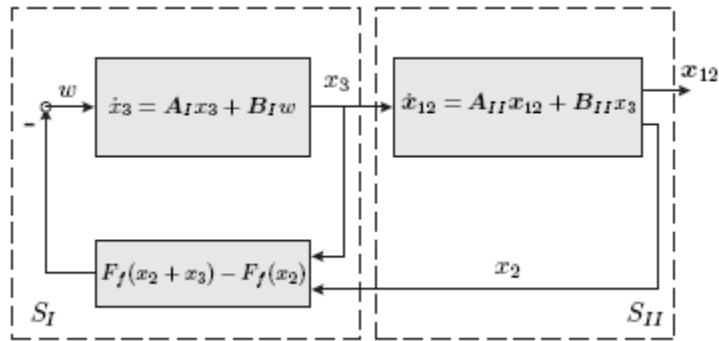


Fig 4.5 Cascade representation of closed loop system

In fig 4.5 $x_{12} = [x_1 \ x_2]^T$ and the system and input matrices of these subsystems are given by :

$$A_1 = -\left(\frac{b}{Y} + L\right), \quad B_1 = \frac{1}{Y} \quad (17)$$

$$A_{11} = \begin{bmatrix} 0 & 1 \\ -\frac{n1}{Y} & -\frac{b+n2}{Y} \end{bmatrix}, \quad B_{11} = \begin{bmatrix} 1 \\ L \end{bmatrix}$$

The S_I describes the observer error dynamics. To prove the global exponential stability (GES) of the origin of equation (9),(10),(11) we adopt the following reasoning. If the following three conditions are fulfilled:

- (a) $x_3 = 0$ is a globally exponentially stable equilibrium point of system S_I for all x_2 ;
- (b) $x_{12} = 0$ is a globally exponentially stable equilibrium point of system S_{II} for zero input x_3 ;
- (c) System S_{II} is input-to-state stable (ISS),

Then $x = 0$ is a globally exponentially stable equilibrium point of equations(9),(10),(11).

4.4 Non – Exact – Friction Compensation

In the experimental set up of the one-link robot as shown in fig 4.1, the friction model will never be exact, due to inevitable modelling errors. Due to these inevitable modelling errors we introduce a scaled friction compensation law, as shown in figure 4.3 and equation (6) with $r \neq 1$. Obviously, in practice modelling errors will not be of this form but this type of scaling of the friction compensation law allows to investigate the effects of both under compensation and overcompensation of the friction. Figure 4.6 shows the equilibrium set (in terms of x_1) is shown for different values of r .

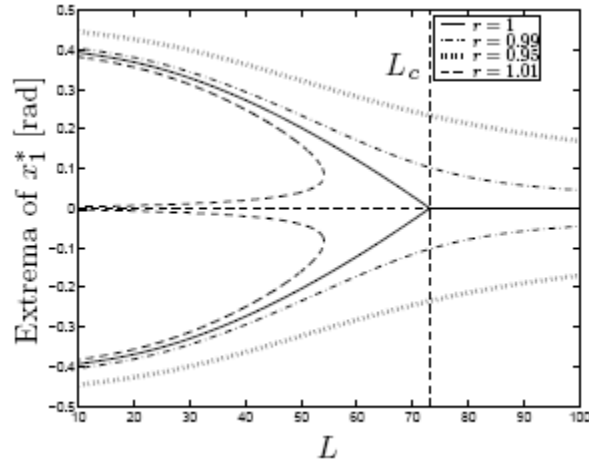


Fig 4.6 Extrema for steady – state error in x_1 for $n_1 = 0.4$, $n_2 = 0.02$ for various values of r

In the case of under compensation ($r < 1$), an equilibrium set will exist irrespective of the value of L . The value of L , however, influences the magnitude of the equilibrium set. The figure 4.6 indicates that friction compensation (even in the case of under compensation) ensures a smaller equilibrium set than exists without compensation, as shown in figure 4.4. In the case of the overcompensation ($r > 1$), an equilibrium set only exists for r very close to one; the equilibrium set rapidly shrinks to an isolated (unstable) equilibrium point for increasing the value of r .

Figure 4.7 represents a bifurcation diagram with bifurcation parameter r is depicted in terms of x_1 for a supercritical observer gain $L = 73.5 > L_c$.

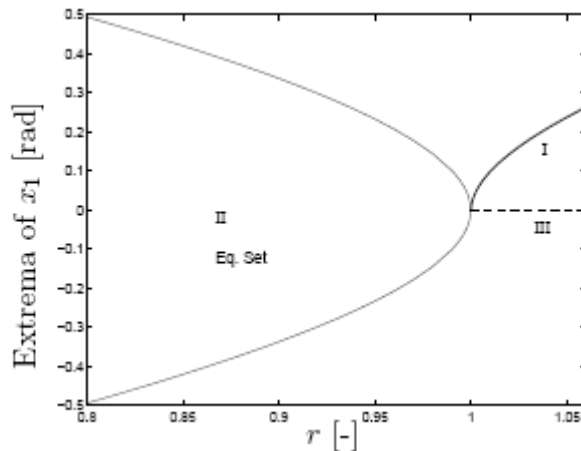


Fig 4.7 Bifurcation diagram for $n_1 = 0.4$, $n_2 = 0.02$, $L = 73.5$ ((I): Stable limit

cycle, (II): Equilibrium set and (III): Unstable equilibrium point.

For the limit cycle, $\max(\text{abs}(x_1))$ over a period of the limit cycle is plotted. This bifurcation diagram clearly shows that an equilibrium set exists when the friction is under compensated and a stable limit cycle exists in case of overcompensation. A corresponding bifurcation diagram involving experimental results is depicted in figure 4.8. Herein, the stars (*) indicated equilibria and the circles (◊) indicate limit cycles. Comparison of figures 4.7 and 4.8 reveals a clear qualitative correspondence. In figure 4.8 the bifurcation point is not located exactly at $r = 1$ since not only the friction compensation law is scaled but the real friction deviates from the friction model as well.

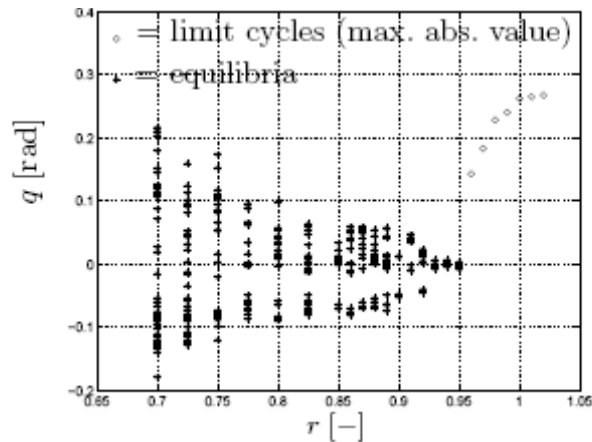


Fig 4.8 Experimental bifurcation diagram with bifurcation parameter r.

Moreover, the difference between the real friction and the friction model is not of the form of a mere scaling. Nevertheless, the theoretical and experimental results agree to the extent that under compensation leads to non-zero steady-state errors (due to the existence of an equilibrium set) and overcompensation leads to limit cycling.

SIMULATION

5.1 Friction Compensator algorithm for one link robot

Step 1 The mathematical equation for the system is

$$Y \ddot{\phi} + v \dot{\phi} = t - F_f(\phi).$$

From experimental results Y; Inertia = 0.026Kgm²/rad

F_f , Dry friction Torque

$$\begin{aligned} F_f &= F_s - F_c \\ F_f &= 0.5429 - 0.3939 \\ &= 0.149 \text{ Nm} \end{aligned}$$

Step 2 : According to Friction compensation Strategy

$$\text{The total control action, } u = u_1 + u_2 \tag{1}$$

$$U_2 = n_1(0 - q) - n_2 v \tag{2}$$

$$U_1 = r F v \tag{3}$$

$r = 1$ scaling factor

where n_1, n_2 are proportional and derivative gains

$n_1 = 0.4$ and $n_2 = 0.02$

The specified range for $q = -2$ to $+2$

The specified range for $v = -2$ to $+2$

For different values of q and v the value of u can be find out

Step 3: The state space coordinates for the robot can be find our based on above 3 equations

The constant values are $Y = 0.026\text{Kgm}^2/\text{rad}$
 $b = 0.0809\text{ Nms/rad}$
 $L = 80$

The specified range for a = -3 to +3

Step 4 : The three state space coordinates are [q a e]

The value of these coordinates can be find out with the following equations

$$\dot{x}_1 = x_2 + x_3 \quad (4)$$

$$\dot{x}_2 = \frac{-n_1}{Y} x_1 - \frac{b+n_2}{Y} x_2 + Lx_3 \quad (5)$$

$$\dot{x}_3 \in -\frac{b+LY}{Y} x_3 + \frac{1}{Y} [rFf(x_2) - Ff(x_2 + x_3)] \quad (6)$$

Where $x_1 = q$;
 $x_2 = a$;
 $x_3 = e$

Based on equation (4),(5),(6) the state space coordinates of robot can be find out.

The above algorithm is simulated in c code.

RESULT AND DISCUSSION

The aim of this thesis is to design a friction compensator algorithm for a controlled one link robot using a set valued friction model. We focused on the problem to find out the equilibrium position for robot origin. The friction compensator with a reduced order observer method for velocity estimates are proposed, implemented and assessed.

The potential of friction compensator methods for the purpose of reducing the effect of friction on robot performance has been investigated in detail and proposed and implemented. A new friction compensator algorithm based on velocity estimation was developed. The main advantage of proposed algorithm is to find out the equilibrium points for the robot origin. The algorithm uses friction compensation strategy and reduced order observer for velocity estimation. Friction is based on velocity measurements but the system is equipped with position sensors so as to obtain the velocity parameters reduced order observer is used. The algorithm for friction compensation is simulated in c code and the robot equilibrium points for origin in terms of state space coordinates are easily find out.

CONCLUSION AND FUTURE SCOPE

A friction compensation algorithm for a controlled one-link robot using a reduced-order observer is proposed. A set-valued friction model is identified to support a model-based friction compensation algorithm. Since only position measurements are available and the friction model depends on velocity, a reduced-order observer is used to provide velocity estimates. The combination of the reduced-order observer and a PD-controller exhibits only three design parameters (two controller gains and one observer gain). This allows for a simplified analysis of the effect of these parameters on the behavior of the closed loop system. The friction compensator algorithm is designed for different values of velocity in the specified range to obtain the equilibrium points for the exact robot origin position. The results are simulated in c-code.

The future scope of this thesis is that to find out the stability for this one-link robot we can use model free or non model based friction compensation strategy by using different friction models.

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ANNEXURE 1

```
/*Program to find state space co-ordinates A, B and C using 3 variables  
x1, x2 and x3 and building 3 equations based on which the values are  
evaluated.*/
```

```
#include<iostream.h>  
#include<conio.h>
```

```
//Some values are defined global since their use is in both the functions  
//out of the main()
```

```
float x1;  
const int r = 1;  
const float n1 = 0.4;  
const float n2 = 0.02;  
float q, v;  
float F;
```

```
// Function to calculate value of u based on u1 and u2
```

```
void calculate_u()  
{  
    const float Fs = 0.5429;  
    const float Fc = 0.3939;  
    float u1, u2, u;  
    F = Fs - Fc;  
    while(1)  
    {  
        cout<<"\nEnter the value of v between -2 and +2 :- ";  
        cin>>v;  
        if (v < -2 || v > 2)  
            cout<<"Wrong Value entered. Enter again :-\n";  
        else  
            break;  
    }  
    while(1)  
    {  
        cout<<"\nEnter the value of q between -2 and +2 :- ";  
        cin>>q;  
        if (q < -2 || q > 2)  
            cout<<"Wrong Value entered. Enter again :-\n";
```

```

        else
            break;
    }
    x1 = q;
    u1 = r * F * v;
    u2 = n1 * (0 - q) - n2 * v;
    u = u1 + u2;
    cout<<"\nValues of u1, u2 and u are as follows:-";
    cout<<"\n\n    u1 = "<<u1;
    cout<<"\n    u2 = "<<u2;
    cout<<"\n\n    u = u1 + u2 = "<<u<<endl;
}

```

//Function to calculate state space co-ordinates based on 3 equations
//The constant values are defined and 3 inputs are taken from user q, a
and v
//value of q and v are evaluated in the above function. a is evaluated in
//this function. Rest values are fixed.

```

void stt_spc_coordinates()
{
    const float Y = 0.026;
    const float b = 0.0809;
    const int L = 80;
    float a;
    float x2, x3, e;
    float A, B, C;
    while(1)
    {
        cout<<"\nEnter the value of a between -3 and +3 :- ";
        cin>>a;
        if (a < -3 || a > 3)
            cout<<"Wrong Value entered. Enter again :-\n";
        else
            break;
    }
    e = v-a;
    x3 = e;
    x2 = a;
    x1 = q;
    A = x2 + x3;
    B = (-1)*(n1/Y)*x1 - (b+n2)*x2/Y + L*x3;
    C = (-1)*(b+L*Y)*x3/Y + (r*F*x2 - F*(x2+x3))/Y;
}

```

```

    cout<<"\nValues of variables x1, x2 and x3 are as follows :-\n";
    cout<<"\n    x1 = "<<x1;
    cout<<"\n    x2 = "<<x2;
    cout<<"\n    x3 = "<<x3<<endl;
    cout<<"\nCalculating the values of A, B and C using above
equations, we get :-\n";
    cout<<"\n    A = "<<A;
    cout<<"\n    B = "<<B;
    cout<<"\n    C = "<<C;
}

void main()
{
    clrscr();
    cout<<"\nTo calculate value of 'u' we have to find out 'u1' and
'u2'."
        <<"\nSince 'u1' and 'u2' are :\n"
        <<"    u1 = r*f*v\n"
        <<"    u2 = n1*(0-q) - n2*v\n"
        <<"\nSo we hae to enter the value/range of all parameters on
which"
        <<"\n'u1' and 'u2' depends."
        <<"\nValues of parameters are as follows:-"
        <<"\n    r = 1"
        <<"\n    f = fs-fc = 0.5429 - 0.3939 = 0.149"
        <<"\n    n1 = 0.4"
        <<"\n    n2 = 0.02"
        <<"\n    v = -2 to +2"
        <<"\n    q = -2 to +2\n\n";

    calculate_u();
    cout<<"\nPress any key to continue...";
    getch();
    clrscr();

    cout<<"\nTo calculate state space co-ordinates q,a and e that are"
        <<"\n    representing x1, x2 and x3 in the following
equations :-\n"
        <<"\n    A = x2 + x3"
        <<"\n    B = -(n1/Y)*x1 - ((b+n2)/Y)*x2 + L*x3"
        <<"\n    C = -(b+LY)*x3/Y + (r*f*x2 - f*(x2+x3))/Y\n"
        <<"\n    To calculate A,"
        <<"\n    x2 = a"

```

```
<<"\n    x3 = e"
<<"\n    To calculate e,"
<<"\n    e = v - a"
<<"\n    Now, enter value/range of v and a"
<<"\n    v = -2 to +2 (already entered by user)"
<<"\n    L = 80"
<<"\n    Y = 0.026"
<<"\n    b = 0.0809"
<<"\n    x1 = q = -2 to +2 (already entered by user)"
<<"\n    a = -3 to +3\n";
```

```
stt_spc_coordinates();
cout<<"\n\nPress any key to continue...";
getch();
```

```
}
```




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