An Improved Grunwald-Letnikov Fractional Differential Mask for Video Enhancement

A Thesis Submitted in Partial Fulfillment of the Requirement for the Award of the Degree of

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DECLARATION

I, Arpit Kapil hereby declare that the work presented in this thesis entitled "**An Improved Grunwald-Letnikov Fractional Differential Mask for Video Enhancement**" in partial fulfillment of the requirement for the award of degree of Master of Engineering (ECE) submitted at Electronics and Communication Engineering Department, Thapar Institute of Engineering and Technology (Deemed to be University), Patiala is an authentic record of work carried out under supervision of **Dr. Kulbir Singh (Professor)** Electronics and Communication Engineering Department of Thapar Institute of Engineering and Technology, (Deemed to be University), Patiala. The matter presented in this thesis has not been submitted either in part or full to any other university or institute for the award of any other degree.

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It is certified that the above statement made by the student is correct to the best of my knowledge and belief.

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ABSTRACT

The texture of images and videos is utilized to segregate or classify between segmented regions. But at the time of image and video acquisition, textural features lose its contrast. During image/video processing, these textural features get blurred due to some natural or artificial effects. Therefore, image/video enhancement techniques are required to emphasize and sharpen these textural features. Generally, the problem of oversaturation arises in the conventional enhancement methods. However, the fact is that the conventional methods enhance the images/video frames, but at the same time introduces significant unnatural noises in images/video frames. Moreover, the brightness of dark region in images/frames increases. In the proposed enhancement approach, improved G-L method is applied on the images for benchmarking. In case of videos, first of all frames are excerpted from input video and then, non-linear mask which is created using improved G-L method is applied on the video frames. For enhancing the textural information and to achieve recognition accuracies in images/video frames, a fractional differential operator is realized (two dimensional) which is an improved version of Grunwald-Letnikov (G-L) based differential operator.

Lagrange's method of 3-point interpolation is applied to simple G-L equation for creation of mask. Experimental results demonstrate that, the proposed mathematical fractional differential method efficiently enhances the images/video frames without the oversaturation problem. Moreover, this approach provides high degree of enhancement for low light and bright light images/videos. Enhancement method is applied on various videos and it demonstrates that the presented method enhances the quality of frames and apprizes the accuracy. The enhancement approach is compared to standard and existing enhancement approaches and the results show that this model outperforms the existing models. To figure out the enhancement criteria, average gradient, peak signal to noise ratio (PSNR), structural similarity index and information entropy values are calculated.

It is also shown that the proposed mask gives better performance in terms of Information Entropy and Average Gradient than Grunwald-Letnikov and Riemann-Liouville Fractional Differential Masks.

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LIST OF ABBREVIATIONS

2D	Two-Dimensional
AIV	Adjust Intensity Values
AG	Average Gradient
AGC	Adaptive Gamma Correction
AHE	Adaptive Histogram Equalization
BHE	Bi- Histogram Equalization
CHE	Clipped- Histogram Equalization
CLAHE	Contrast-Limited Adaptive Histogram Equalization
DCT	Direct Cosine Transform
DHE	Dynamic Histogram Equalization
DR	Dynamic Range
FD	Fractional Differential
FIR	Finite Impulse Response
FODs	Fractional Order Differentiators
FPS	Frames Per Second
G-L	Grunwald- Letnikov
GLCM	Grey Level Co-Occurrence matrix
HE	Histogram Equalization
HMM	Hidden Markov Model
HSV	Hue Saturation Value
IE	Information Entropy
IIR	Infinite Impulse Response
LCA	Linear Contrast Adjustment
LOG	Laplacian of Gaussian Filter
MSE	Mean Square Error
NLM	Non Local Mean
PSNR	Peak Signal to Noise Ratio
RGB	Red Green Blue
R-L	Riemann-Liouville
RMSHE	Recursive Mean Separate Histogram Equalization
SSIM	Structure Similarity
VTR	Video Tape Recorder

CHAPTER 1

INTRODUCTION

The introduction defines important terms, critical issues and prepares the mind for research to investigate in the new territory of science and innovation. The introduction of basics of videos, image and video texture, Fractional Differential and its applications in image and video processing are given in this chapter.

1.1 PREAMBLE

Over the last few decades, there have been modern advancements in digital world of image processing which enables users to take video shoots and images of immense quality using inexpensive, compact digital camcorders including high resolutions and great sensitivity. Images and videos provide visual representation of something that is to be examined. The amateur or beginners can now easily click images, capture videos, save, store and edit these pictures or videos as well as share these images and videos. Research scholars and professional users rely on them to identify areas of interest, scrutinize and inspect details and consider their findings effectively.

However, dynamic range (DR) is still limited in digital camera technology [1]. Due to this limited dynamic range, a video frame having dark objects and bright backgrounds loses textural informative content. Their background region becomes over saturated i.e. low light videos have poor dynamic range. There is another problem of low value of peak signal-to-noise ratio (PSNR) for low light video frames [2]. The problem of low PSNR exists even when frames are enhanced with the conventional methods. So, an effective enhancement approach is appropriate to enhance the texture and information of video frames.

The procedure of digital video/image adjustment in order to deliver the outcomes which are more reasonable for further video/image investigation is called as enhancement. For example: removal of unnatural noise, sharpen the frame and enhance contrast of the frame of a video / image and making it simpler to recognize key features are all parts of enhancement. It is a major and essential instrument for experts in wide assortment of fields as well as restorative imaging, legal sciences and barometrical terms, workmanship contemplates, forensics and furthermore, atmospheric terms and also art studies. Enhancement methods are application specific, so these methods are reasonable for one issue may be insufficient for another. Low resolution problems in medical and forensic images/videos can be resolved with enhancement techniques. Our aim is to enhance the video, to provide better representation and visual appearance of the video. Enhancement processes are required for the various analysis, text detection, segmentation, recognition, CCTV footages, underwater videos, surveillance and video processing for future automation.

Motion blur problem can also be resolved by enhancement techniques [3]. The low resolution of CCTV videos/images and underwater images/videos can be resolved by increasing their resolution and employing different methods for texture enhancement [3].

1.2 DIGITAL IMAGE

An image is 2D signal representation of an object. A digital picture or image is made out of a fixed number of elements. Each element has a distinct region. Video is nothing, but a set of moving images. Image is a rectangular matrix or grid which contains definite width, length and height measured in form of pixels [4]. Scientifically, it is characterized as a function of $f\ (a,b)$ where a,b are horizontal and vertical correlates and f at that point, is the amplitude of any correlates (a,b) is known as gray level or intensity value of picture or image. They are essentially inspected from a simple analog signal at specific set focuses and further mapped together as a lattice of pixels or grid of dots [4]. The acquisition of image process has been exemplified in Figure 1.1.

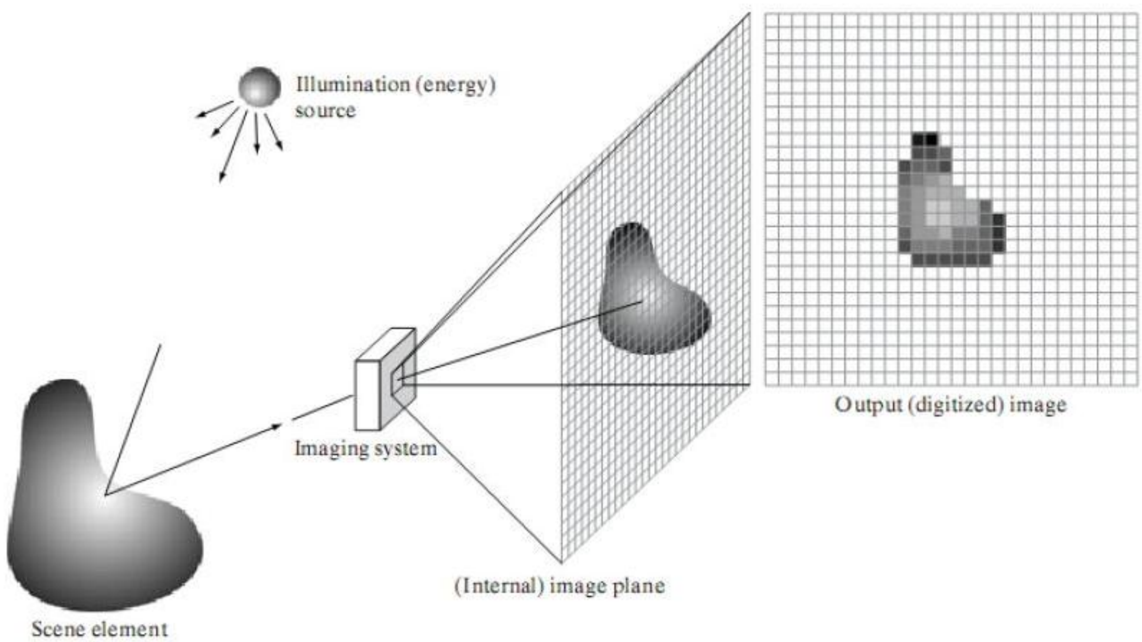


Figure 1.1: Acquisition process of the image [4]

1.3 VIDEOS AND VIDEO FRAMES

Every video is made of up of many frames or images. We can relate video frames to video, same as that of a book. Book contains many pages. Similarly, video is like a book which is made up of many frames. Camera does not record motion, it records the frames at different frame rates e.g. 3fps, 24fps etc, where, fps stands for frames per second. 24fps means that our camera is recording 24 frames per second. These frames are then combined and compressed to make the video. There are many cameras which are discovered now and can capture 1000 frames per second. These are generally used for slow motion videos. Each frame has its own resolution. Bigger the size of the video frame/image, greater the value of resolution present in it. The technology is improving day by day, so does the resolution. These days we are talking about the 2k videos, 4k videos and moreover 8k videos [5]. These are sometimes called as ultra HD videos. We always see the term p in resolution like 1080p, 720p etc. Here p stands for progressive scan. In 1080p it contains 1080 horizontal lines of vertical resolution. Moreover these days even latest phones can capture higher resolution videos and images. Old technology of video based on tapes used the same idea i.e. frames which are moving to give the idea of motion. Enhancement of the frames of the video is an important task.

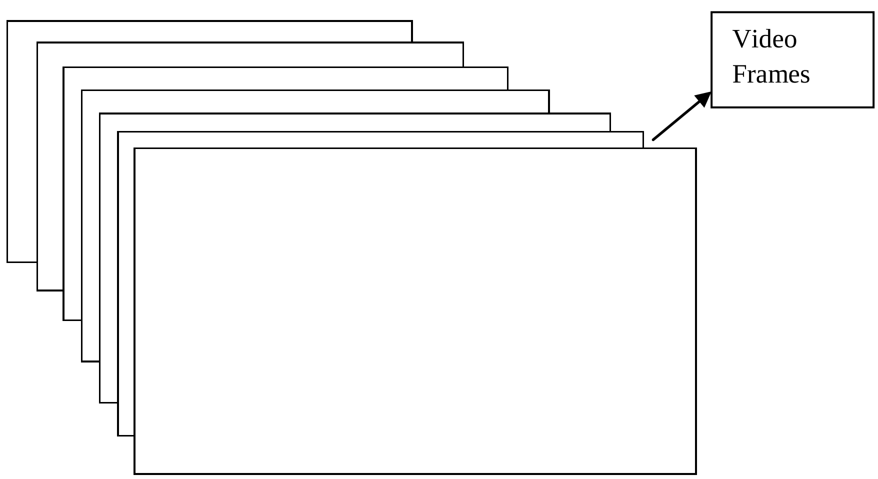


Figure 1.2: Frames of the video which define the video [5]

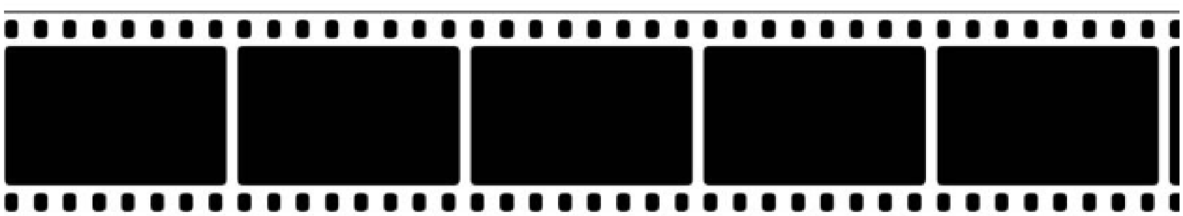


Figure 1.3: Video tape formed from various images [6]

Figure 1.4: Frames of video having different frame rates [6]

Images presented in Figure 1.2, 1.3, and 1.4 clarify about the definition of the video, different frame rates and it shows that a video is built up of many frames.

Video development was first made for mechanical TV systems, which were quickly supplanted by cathode beam tube (CRT) TV structures, however a couple of new advances for video display devices have been spontaneous. Video was at first exclusively and only a live innovation. Charles Ginsburg drove an Ampex explore group making one of the guideline valuable for video recording gadgets (VTR) [6]. In 1951, the first and main video recording device (VTR) caught live pictures from TV cameras by transforming electrical impulses of the cameras and sparing the data onto video tapes which are magnetic.[6] As of 2015, with the expanding utilization of high-determination camcorders with enhanced powerful range and shading extents, and high-dynamic-go advanced transitional information positions with enhanced shading profundity, modern computerized video innovation is merging with digital film innovation.

1.4 VIDEO FRAME TEXTURE

Texture is one of the significant and necessary features that help to segment images, video frames which are excerpted from the videos and to classify those regions of interest in the particular image. Identifying objects in video frame is also a part of texture characteristics, whether the video frame is underwater frame, or aerial frame, or medical/forensic frame. Texture gives us knowledge and instruction of the spatial arrangement of the intensities or colors of the image and video frames. Texture is an instinctive and essential property of virtually all the surfaces - the weave of textile and fiber, grain of wood/timber, pattern of crop etc. [7]. An image and video frame texture is a course of action of measurements ascertained in frame handling expected to measure the apparent texture of the frame of the video. Image textures are intricate visual examples made out of substances with sub-designs having highlights of splendor, shape, shading, measure and so on If the characteristics of the images are constant, very slow change occurs or approximately periodic then image region has constant texture. Texture refers to structure, appearance, and arrangement of object parts in the image and same in case of frame [8]. Texture can be classified and seen as being fine, smooth or coarse, lineated or unregulated and rippled. It can be seen in pictures , images and videos of all kind i.e. from multispectral images of the scanner obtained from satellite or aircraft (that are analyzed by the community of remote sensing) to images and frames which are microscopic of the cells, tissue cultures, and other samples (which are analyzed by the forensic community) [9].

1.4.1 Texture Enhancement

Textual properties of pictures and videos appear to carry useful data and information for purpose of discrimination. So, for differentiating between different kind of images and video datasets, these features are useful and thus, extracting textual features is an important task. In order to extract textural features efficiently, methods of image enhancement are required [10]. Some of the applications where we use enhancement for better picture quality are; medical/forensic image data, in the field of robotics, remote sensing, recognition of various patterns, restorations of the images and interpretation of image data and so on. The aim of enhancement is to increase the visibility and to expand the visual impact of the video frames/images with the help of purposefully emphasizing whole or local features of image and impairing the characteristics. With the enhancement methods, quality is increased and useful data/information would be enriched. There are only a few reliable methods for enhancement which are developed over last some decades. Various multi-resolution [11] methods are also available for the enhancement of images/frames. Image/frame sharpening is one of the methods of implementation of texture enhancement, which enhanced the images, edges and contrast. Blurriness can be decreased by image sharpening methods. Various common image and frame sharpening techniques deployed are [11-12]:

a) Unsharp masking
b) High boost filtering
c) Derivative filters: Two types of derivative operators used are:
    - Derivative filters of first order: derivative filters of first order are Roberts, Sobel, and Prewitt.
    - Derivative filters of second order: derivative filters of second order are Laplacian and Laplacian of Gaussian filter.

There are two methods to enhance video frames which are region enhancement and whole frame enhancement. The difference between the above two enhancements, region and whole frame enhancement is shown in Figure 1.5.

Both the methods shown in the Figure1.5 are used for enhancing the videos. One method is for enhancing the small part of the video frame and other is for whole frame. John Canny in 1986 proposed and formulated an operator which is improved edge detection operator [13], known as canny operator. Another non-linear edge detector for feature extraction is Kirsch operator.

Two regular sorts of contrast enhancement strategies are described in [14] are:

- Linear Contrast Enhancement: It includes Min-Max Linear Contrast Stretch, Percentage Linear Contrast Stretch, and Piecewise Linear Contrast Stretch.
- Non-Linear Contrast Enhancement: It involves Histogram Equalization, Adaptive Histogram Equalization, and Homomorphic Filtering.

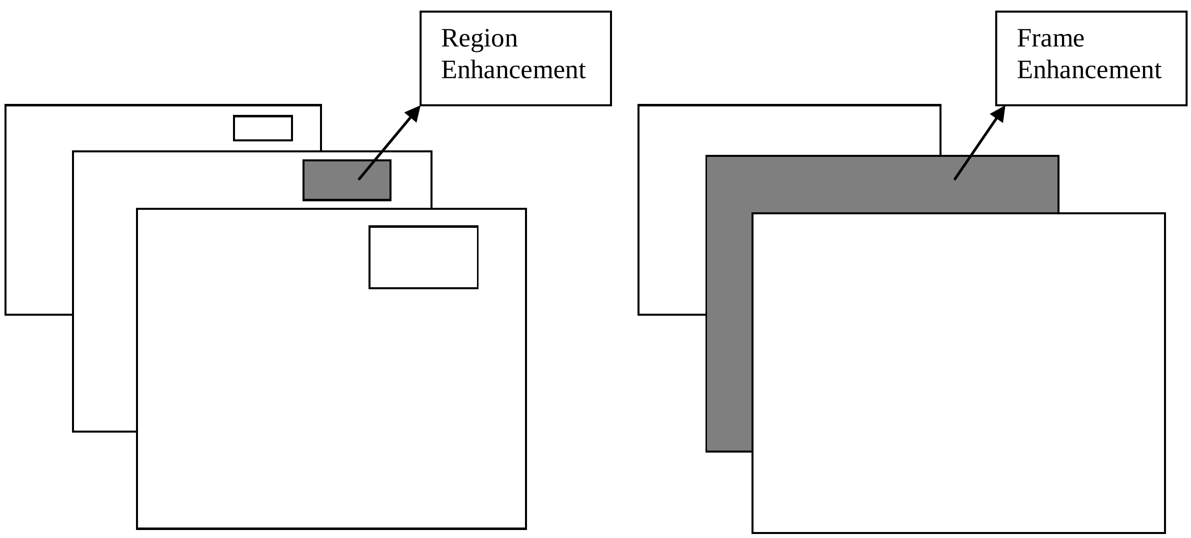


Figure 1.5: Difference between region and frame enhancement of the frames taken from video

1.5 FRACTIONAL CALCULUS

Differentiation and integration are regularly viewed as discrete tasks. In these calculations, separation or coordination of any function once, twice or any number of times is easily done [15]. The fractional order mathematics is 300 year-old subject [15]; the fractional order mathematics is more common after nineteenth century and has many applications in engineering fields. Applications of control system can be handled by fractional order of integration and differentiation. Fractional calculus is very old concept and came into knowledge in 1965 with discussion between L'Hospital and Leibniz. Fourier, Laplace and Euler are among the numerous that dallied with fractional calculus and the numerical results [16]. Numerous scientists discovered, utilizing their own documentation and approach, definitions that fit the idea of a fractional order of the integral and fractional order derivative. Three important fractional concepts are Grunwald-Letnikov (G-L), Caputo and Riemann-Liouville (R-L) [17-19]. In Fractional order mathematics, it is general concept of standard integration and differentiation to request of non-integer order of fractional derivative i.e. taking differentiation operators having real number of powers [15, 16]:

$$D^{v}f(x) = \frac{d^{v}f(x)}{dx^{v}} \tag{1.1}$$

In 1695, L'Hospital conveys to Leibniz getting some information about a specific documentation for the linear function's nth-derivative and he was the first person to give the following equation:

$$f(x) \;=\; x\frac{D^{n}f(x)}{D^{n}x} \tag{1.2}$$

If v is the number presents that is written in the above equation then it is called higher order differentiation. Real numbers that are positive correlates, to fractional order differentiation.

The historical development in the field of higher order differentiators is given by two researchers, one by Riemann and Liouville and other by Grunwald and Letnikov. Riemann and Liouville integral is able to get fractional equation in canonical form. The equation for this canonical form is as follows [15]:

$$_{\beta}D_{t}^{-\beta}f(t) = \;_{\beta}I_{t}^{\beta}f(t) = \frac{1}{\Gamma(\beta)}\int_{\beta}^{t}(t-\tau)^{(\beta-1)}f(\tau)d\tau \tag{1.3}$$

The calculation of identical or parallel derivative is done with the help of Lagrange's method for creation of differential operators.

Calculating the n^{th} order derivative over the integral of order $(n-\beta)$, we obtained the β order derivative. Here, $n > \beta$ [15]:

$$ {}_{\beta}D_t^{\beta} f(t) = \frac{d^n}{dt^n}\, {}_{\beta}D_t^{-(n-\beta)} f(t) = \frac{d^n}{dt^n}\, {}_{\beta}I_t^{(n-\beta)} f(t) \tag{1.4} $$

There is another definition given by Caputo which reformulated the definition of the (R-L) Riemann-Liouville derivative [16]:

$$ {}_{\beta}D_t^{\beta} f(t) = \frac{1}{\Gamma(1-\delta)} \int_{\beta}^{t} (t-\tau)^{(-\delta)} f^{m+1}(\tau) d\tau \tag{1.5} $$

where, $\beta = m + \delta$ and $0 < \beta \leq 1$, m is an integer.

Kolowankar in 1996 has given another definition of the Riemann-Liouville fractional derivative and it has given good result as compared to R-L. The revelation of calculus turned out to be remarkable to the point that in its acknowledgment J. V. Neumann (1903-1957) cited: '*The calculus was the first achievement of modern mathematics and it is highly difficult to overestimate its importance'*.

There is a long and developing rundown of reasonable applications for the expanded power of the fractional calculus. Some of these are as follows [20-22]:

- Control Engineering
- Thermal Engineering
- Robotics
- Remote Sensing and Under Water Image Applications and Coastline of seas.
- Electromagnetism
- Acoustics
- Edge Detection
- Viscous-elasticity

1.6 DIFFERENT TYPES OF VIDEO FRAMES

As we know, effect of light is texture defining factor in digital image and video processing because some videos are captured in low light and some videos are captured in bright light. Different types of videos are processed in this digital world and frames are extracted from these videos for further processing like compression of video, decompression and enhancement etc. Some of the types of video frame are shown below in the Figure 1.6, 1.7 and 1.8.

Figure 1.6: Extremely low light video frames

Figure 1.7: Moderately low light video frames

Figure 1.8: Bright light video frames

1.7 CONTRIBUTION

This research gives a remarkable contribution and commitment to objectives by advancing our understanding of the concept and methodology used for goal fulfillment and satisfaction of objectives. Contribution to information implies making new learning in view of the past accessible information by doing broad and imaginative research.

In this thesis, work is related to image and video enhancement. Thesis contributes to various image/video enhancements like low light image/video, bright light image/video and normal light images/videos. Increasing resolution of low light images and videos is a tedious task. Underwater video enhancement is also presented in this dissertation. CCTV footage enhancement is very important for the security reasons. Faces of thieves can be enhanced by improving resolutions in these theft videos. CCTV video enhancement is also a part of this work. Comparison with different research papers is also presented in the results section which provides improved and exceptionally better results than those research papers.

1.8 ORGANIZATION OF THESIS

The dissertation involves the accompanying sections which are outlined underneath as an overview of the detailed work. The chapters are organized as follows:

Chapter 1: Introduction

This chapter gives an overview of essential terms and definitions identified with the hypothesis on which the proposition work is based. A brief introduction about video and video frames is also provided in it. It comprises of theory on basics of a video frame, image, fractional operators and fundamentals of fractional calculus.

Chapter 2: **Literature Review and Problem Detection**

This chapter reviews the work which has been done regarding 'Image and Video Enhancement' using various classical methods and Fractional approaches. A brief writing about the different related techniques in the field of enhancement is also presented in this section. It gives a concise data with respect to the different methodologies proposed by the scientists for development in video and image enhancement approach. The gaps found in the study are also presented in this chapter. Objectives of the thesis are also presented.

Chapter 3: **Texture enhancement and Fractional differentiation**

This chapter includes the various methods for texture enhancement in detail. It also consists of details about fractional differentiation and various operators of fractional differentiation.

Chapter 4: **Video enhancement methodology using Fractional approach**

This chapter contains detail of the proposed work for video enhancement using improved G-L definition of fractional differentiation. Improved G-L equation is discussed in this chapter and mask which is created for the video enhancement methodology is also analyzed in this chapter. In the last part of this chapter, how the mask is convoluted on the frames of the video is presented.

Chapter 5: Results and Discussion

This chapter comprises of three parts. First part is of image texture enhancement on various images datasets and second part is of video enhancement on various video datasets. The non-linear mask created from the improved G-L equation is applied on SVT and MSRA image datasets. The proposed enhancement mask is realized on SULFA video dataset and also on videos of ICDAR 2013 dataset. All results are obtained in MATLAB R2016a. Third part is of the qualitative analysis of enhancement criteria on the comparison basis of PSNR, IE, AG and SSIM. Results are also compared with the existing techniques.

Chapter 6: **Conclusion and Future scope**

This chapter confers the conclusion of whole work which is based on the obtained practical results. Further, the future extent of execution of the proposed procedure is presented.

CHAPTER 2

LITERATURE REVIEW AND PROBLEM DEFINITION

2.1 INTRODUCTION

This chapter reviews the work which has been done regarding 'Image and Video Enhancement' using various classical methods and fractional approaches. Various existing methods and techniques of HE, which are realized from time to time is presented along with various fractional order masks which has been already proposed. Methods and techniques have been discussed, which provide an insight into the area of image enhancement and video enhancement.

2.2 FRACTIONAL DIFFERENTIAL METHODS

Hemalatha *et al.* [22] presented a method, in order to enhance the images, based on the fractional modified/improved version of G-L differential definition of mask. It is modified using autocorrelation between the intensity values or pixels of the neighborhood. Gray Level Co-occurrence matrix (GLCM) is the criteria used in this research work for texture enhancement. On comparing the obtained results with the Histogram Equalization (HE), it gives more promising and better results than the existing techniques.

Jalab *et al*. [23] applied the method of fractional differential masks on the medical based images. Most of the medical diagnosis requires texture enhanced version of the medical images. Fractional differential mask (FD) on the basis of Srivastwa-Owa operator is used for texture enhancement. With the help of vision science and by the use of Sobel/prewitt/canny edge theory, performance and enhancement parameters are compared. Edges are enhanced by the proposed method accurately and smooth texture is sustained.

Qiu *et al*. [24] in 2016 presented a novel method to overcome the prone of classical edge based techniques. As there is effect of noise in the present methods of edge detection, and results given by these are unclear, we are not able to get ideal edges. A new method is given which is the combination of sobel operator and fractional differential. According to his theory, method is very appropriate because it leads to detailed edge information when compared to classical edge methods with better anti noise capability.

Li *et al.* [25] presented a new technique, on the basis of image complexity and it has been applied on the images, with the use of adaptive fractional differential. With this method, the low frequency information is maintained in the picture while high frequency components are also improved and strengthened in the same picture. Adaptive complexity of the image is basically used for defining the fractional order and this will provide image enhancement algorithm. In order to avoid the edges which are fake, the optimized fractional differential order is taken into account and it is selected on the basis image complexity to achieve high enhancement degree and optimal effect.

Zhao *et al.* [26] concluded a new approach for the designing of differentiator on basis of fractional order differentiation. First of all, a fractional order differentiator (FOD) of digital signal is determined in the frequency domain. After that, FIR filter is chosen to approximate to the ideal digital FOD under the weighting mean square error (MSE) in frequency response. Design example and fractional derivative in frequency domain is described in this paper gives better results than other existing methods of designing FOD.

Cafagna [27] in 2007 concluded a fractional calculus method, by considering three definitions based on G-L, Riemann-Liouville and Caputo. An important expression for fractional derivative and fractional integral is derived. Fractional calculus application in field of biomedical and control engineering is also discussed in this paper. There are two fundamental kinds of techniques which are utilized for determining equations of fractional differential i.e. time-domain technique and frequency-domain technique.

Zhang *et al.* [28] uses the Riemann-Liouville (R-L) equation of fractional calculus for one-dimensional signal. This equation is extended for digital images. Fractional differential mask with a particular order is implemented on eight symmetric directions to obtain eight fractional differential masks. For gray scale digital image, filter is then convoluted respectively on eight directions using eight fractional differential masks. For colored images, fractional differential on each component is done individually and then combined. It is observed that grayscale image processed with integral differential operator obtain edges clearly but the texture features are abandoned. But, image processed with fractional differential mask have more clear edges. The textural features of images are also enhanced.

Pu *et al*. [29] proposed a texture segmentation approach which is based on fractional differential. Using the G-L definition of fractional differential, a fractional differential mask is obtained and its parameters and structures are presented in eight directions. The G-L Fractional Differential is applied on various signals to show better performance of fractional differential than integral differential. This G-L based

Fractional Differential Mask is then applied on texture enriched images for values of fractional order. The texture-segmentation performance using this mask is compared with other classical algorithms for texture segmentation. It is observed that multiscale-texture segmentation of texture-enriched images using fractional differential mask is more efficient than the classical texture segmentation algorithms. This approach is additionally executed by for low differentiation and hazy/murky images.

Changyun *et al.* [30] proposed an enhanced fractional differential calculation which is utilized to improve images influenced with poisson noise proficiently. With the assistance of Taylor expansion, a distinction articulation on a one-dimensional (1D) signal is acquired and contrast coefficients of fractional order differential are derived from the articulation. A fractional differential layout on x and y headings are acquired utilizing fractional differential coefficients. The fractional differential layout is actualized on picture influenced with poisson noise. It is seen that under impact of noise integral differential method, results missed some edge data. However the proposed approach and calculations performed better for edge detection and smoothen noise.

Poldubny *et al.* [31] used a lattice shaped portrayal of discrete analogs of different types of differential and integral fractional order. The integer order differentiation of numeric request and the n-overlay integration is brought together utilizing the supposed matrices of triangular strips. When applied to mathematical and algebraic arrangement of differential equations, it likewise binds together the arrangement of conventional integral and differential fractional order in partial equations. The proposed approach prompts noteworthy improvement of the numerical arrangement of equation of fractional order integrals and differentials. It also leads to simplification of equations.

2.3 IMAGE ENHANCEMENT METHODS

Blaschke *et al.* [8] described an approach for image analysis on the basis of objects based GEOBIA. This method is a new paradigm presented after analyzing the old method presented by Kuhn. This GEOBIA is the new paradigm which a mixture of Kuhn's method and methods of peer-reviewed studied in the literature. In the applications of remote sensing, this described technique is very useful. In the analysis of Arial images this presented paradigm plays an important role.

Liu *et al*. [21] proposed an advance and unique technique which is utilized on the basis of fractional differential, which is applied for texture enhancement of remote sensing images, especially texture details of these images. Fractional differential algorithm is applied and by this numerical algorithm an operator is constructed. With the help of image convolution, above numerical algorithm of fractional differential can be implemented. There is an advantage of edge enhancement by using this methodology as well as advantage of keeping the texture details. There are very less chances of loss of textual information using this method.

Rahman *et al.* [32] firstly classified the images on the basis of statistical information and after that the images that are classified, divided into several classes. AGC method is suggested by the author known as adaptive gamma correction, in which on the basis of image information, parameters of AGC are set dynamically. The fundamental objective is to upgrade the image by maximization of the detailed information. On comparison with other techniques, results of AGC are better on qualitative analysis. This approach is also applied on the color images. If the image is colored, then RGB values are changed to HSV. Image is processed for increasing texture enhancement in which c and γ are two parameters that control the enhancement degree. Value of γ is set, using logarithmic function and exponential function. Value of c is also defined because both parameters are enhancement controlling factors.

Kansal *et al.* [33] proposed a simple approach in which histogram equalization (HE) is combined with unsharp mask filtering. Entropy is calculated for qualitative analysis which is improved when compared with the previous traditional and classical methods. In this approach, value of histogram of the image is clipped so as to control the degree of under and over enhancement. Method is applied on standard datasets and after applying this method it is observed that, in enhanced image information is highest i.e. information entropy (IE) is highest and brightness is also increased. MOS (Mean Opinion Score) is another parameter which is also calculated and it demonstrates that nature of the image is superior to other methods.

Jalab *et al.* [34] presented a new methodology in 2017. In this proposed approach, the accuracy of medical pictures is mainly affected by low contrast is shown. To overcome this low contrast problem, a research based analysis is provided by these researchers. Edges are enhanced by the proposed method accurately and smooth texture is sustained. The results are compared by calculating the PSNR of the images. Image entropy is also calculated and comparison with existing methods is also provided. This proposed method is better for gray scale images.

Kaur *et al*. [35] presented a generalized method for improving the quality of the videos and images taken from the foggy road scenes, airplane scenes, foggy mornings of the winter and dust storms. Enhancement depends upon different weather conditions. This method is the combination of two techniques i.e. gamma correction and RSWHE (Recursively Separated and Weighted Histogram Equalization). Dark channel priority, HE, preserving brightness and RSWHE are main components of the proposed model. This approach gives exceptional results. In any case, the precision of this strategy is not high; however, its handiness cannot be eclipsed.

The fundamental thought of **Raj** *et al.* [36] in the method of AHE (adaptive histogram equalization) is to discover the mapping for every pixel in view of it local (neighborhood) low contrast scale conveyance. In this approach, enhancement is mapped to a particular pixel which is a function of pixel values immediately to the values on which operation is applied. Computation requirement is extensive in this method. By this CLE method, unnatural noise is removed. This method is implemented in FPGA. Xilinx Spartan 3AN is the board (NB3000 Altium Nanoboard) for algorithm implementation.

Pu *et al.* [37] described a fractional calculus method for the texture enhancement of digital image. This paper realized G-L definition for obtaining the fractional differential mask. The mask obtained from the G-L definition is implemented in various directions. This mask which is generalized from the equations of fractional differential and then it is applied on an image for the enhancement of texture of image. The investigation of the proposed strategy demonstrates that it is relevant for computing the FOD of both noisy and natural images. The author contemplated the surface division, texture segmentation and enhancement of texture of FOD multi scale filters. A conclusion is realized that the presented calculation is predominant and more compelling than the old integral and differential-based methodologies.

Kim *et al.* [38] concluded that this proposed method is an early method which is an upgraded version of HE known as BBHE i.e. bi-histogram equalization. In this approach, an input image is divided into two sub images and it is an initial attempt utilizing the mean power as the limit in the brilliance safeguarding the two sub-pictures were then handled autonomously utilizing Histogram Equalization with reference to an objective uniform histogram. Brightness is preserved using this method of histogram equalization. Brightness of output and input pictures is not changed generally using this approach. By the implementation of this method, there is clear difference between output and input image.

Huading *et al.* [39] proposed an approach for digital watermarking. In this methodology, author described that with the use of fractional calculus digital watermarking can be achieved. By the examining deviation of fractional order calculus, sinusoidal signal usage of partial analytics pseudo-irregular succession is considered. A watermark implanting calculations with the use of fractional calculus is examined which gives security based on fractional mathematics. A watermark implanting calculation utilizing fractional calculus is examined which gives security based on the fractional mathematics. The power of the calculation is seen by executing calculation on advanced picture and watching that watermark extraction arrangement totally relies upon learning of fractional order and beginning expression and initial phrase.

In 2003 **Wan** *et al.* [40] presented an algorithm named as DSIHE (Dualistic Sub Image Histogram Equalization) and presented that it is superior to BBHE as far as it is concerned. This approach is also improved for data textural information content (entropy) of the picture or image. In this algorithm, DSIHE median value is taken which separates histogram instead of mean. It implies that histogram of the image contains equal number of intensity values or pixels. When the entropy is calculated and compared to BBHE, results are exceptional and more promising.

Garg *et al.* [41] modeled a method based on the non-linear operator of 2D FOD for enhancement of image. On basis of this approach, a non-linear mask is realized which is convoluted on the image, which is needed to be enhanced. 2D operator is an improved version of simple G-L based differential equation definition. The scheme modeled by the author is divided into two parts. In first part, fractional differential order is varied from $0 < v < 1$. Value of the mask which is to be convoluted on the image is taken from the improved G-L equation. Results are taken for every fractional order and it is seen that first IE increases up to some fractional order and then, it starts decreasing. Image starts distorting for higher fractional order. Values of information entropy for different images are higher for different fractional order. It depends upon the textural information present in the image. Average gradient (AG) is another qualitative measure which is used as an enhancement parameter. AG is directly proportional to fractional order. As we increase the order of fractional differential, the value of AG also starts increasing. In second part, intensity factor is varied and fractional order is not varied. Enhancement degree is controlled by intensity factor. But, on increasing intensity value, after some point the textural information of the image start losing its texture.

2.4 VIDEO ENHANCEMENT METHODS

Kim *et al.* [2] presented a method for the enhancement of extremely low light video sequences. The overall frame work of the author depends on four steps. These steps include the input of video which is extremely low light, temporal noise reduction, tone mapping and spatial noise reduction. Spatial temporal filtering is the main step to remove most of the noise from the video. Tone mapping is generally used for increasing the level of noise from already denoised portion of the sequences. NLM denoising filter is used for the removal of residual noise. The experimentation and examination of the calculation demonstrates that this strategy has a decent criticism to differentiate the improvement of dark pictures.

Kansal *et al*. [33] generalized an approach in which histogram equalization (HE) is combined with unsharp mask filtering. Entropy is calculated for qualitative analysis. In this method, value of histogram of the image is clipped and combined with the sharpened image. The image sharpening is done with the help of high pass Gaussian filter. Clipping of the image is done which is taken after Gaussian filtering. Enhancement of the sharpened image is done by histogram equalization. This enhanced image is again sharpened with the help of Gaussian filter. There is enhancement in the output image on implementing this method on the input image and results are improved as compared to other methods.

Hung *et al*. [42] designed an example based method for the enhancement of frames of degraded video. It is a general enhancement approach. Initially, the method depends upon checking the non-degraded section of the video frame. On the basis of this checking, degraded section of the frame is enhanced. The image degradation needs to start from a repeatable process, so the word reference image patches (blocks) are similarly degraded. In this way, there is beginning of a reference image with degraded blocks and their deposits (contrasts in the middle of degraded and unique blocks). If there is any match found between a degraded patch of video and a degraded patch of dictionary, the related buildup of the last is delicately added to the block of the previous. The approach is the realization of the example-based enhancement method for super-resolution.

Roy *et al.* [43] generalized a model of enhancement on the basis of fractional calculus for increasing the quality of frames taken from the input videos. Laplacian operation is initially applied and after that fractional method of "poisson fractional" is implemented. The described technique considers edges and the information of their neighbors to determine a scientific method for enhancement of low values of contrast information in frames of video and scene pictures. This method is realized for the text detections of various different datasets of ICDAR, MSRA, SVT (street view data). Text detection is easy as compared to other classical text detection methods.

Paik *et al*. [44] implemented a method for dark region detection in this paper. This described method of enhancement has very low computation complexity. The proposed method is very useful for the enhancement of those videos which are captured from the low cost devices. Over-saturation issue and the dull area increments are the real problems with those conventional enhancement techniques. To resolve this issue, the proposed strategy first partitions an information image or a frame into dark object and splendid background regions utilizing adaptively divided squares. Computational complexity is very low in the proposed approach. Using this, very low light video frames can be easily enhanced in the video surveillance system. Results of this method showed that enhanced videos have no oversaturation problem and these results are better than the existing methods.

Quevedo *et al.* [45] concluded that image / video spatial resolution is basic feature for some of fields like medication, correspondences or satellite communications, and submerged applications. While an expansive assortment of systems for picture restoration and enhancement methods have been realized in the literature, this paper centers around a unique super-resolution fusion calculation. On the basis of this multi-camera condition, it grants to upgrade the nature of submerged video successions in underwater without essentially expanding computations. This method is analyzed and qualitatively measured with existing methods using PSNR and SSIM. Under water video frames obtained from the applied algorithm are the enhanced version of input video sequences.

Sain *et al.* [46] used a method of text detection in videos using Laplacian Fourier filtering which is in frequency domain. This method includes a technique of verification using HMM known as Hidden Markov Model. The presented method manages the text region showing up in horizontal or vertical bearings, as well as in some other diagonal or curved orientation in the image/video frame. Up to this point, just a couple of techniques have been suggested that investigate bended text detection in frames of the video. Initially, in this approach, Fourier-Laplacian transform is implemented and then Laplacian-Gaussian filtering is followed. Next, the acquired associated segments are skeletonized to recognize different content strings. Complex parts are degenerated into less complex ones as indicated by an intersection calculation took after by a connection performed on conceivable mix of the disjoint skeletons to acquire the relating text content. HMM based text detection methods are very useful for the text detection.

Soumya *et al.* [47] described a night video enhancement method. Night video enhancement methods are generally utilized for distinguishing suspicious actions caught by visuals which are captured during night's surveillance systems or reconnaissance frameworks. However, unnatural light sources active during nights degraded the quality of the videos captured during nights. This non-uniform light lessens the identification of an ongoing visual security framework. This proposed method improves the neighboring pixels of dull and light areas in each edge. In this approach, two-level self-sorting out neural networks was progressively organized to amass comparative image elements presented in the dim light video sequences and frames.

2.5 MOTIVATION

An extensive variety of methodologies of image and video enhancement have been discussed in the literature, which take advantage of the inconsistencies. This can be observed by identifying the research gaps and formulating the research question as the starting point for further analysis. It can also include the identification of opportunities that are yet to be explored in a research study.

The research is done to identify, investigate, explore, improve, and discover something. Research is generally the systematic approach to achieve goal. The gaps that are found during the literature survey till the day, and the objectives of the study are defined in the below section.

2.6 GAPS IN STUDY

A research gap is the missing element in the existing research literature. These are as follows:

1. In most of the papers, only the improved versions of the already present equations like R-L equation, G-L equation and Caputo equation is discussed. These definitions are very old. No new definition or equation is derived since a long time.
2. The existing enhancement methods are mainly applied to images. Video enhancement methods are not discussed by most of the authors. Video frames can also be enhanced using these fractional differential masks.
3. In literature review, it is found that noise removal techniques in enhancement process are not discussed in most of the papers.
4. FODs are also present in many literature surveys but analytical discovery and interpretation of FODs is still a monotonous or we can say a tedious task.

2.7 OBJECTIVES

The objective of the thesis is as follows:

1. To apply this improved G-L fractional differential definition on images and video frames and enhance the video/image texture quality.

2. To enhance the textural features of images and video frames using this G-L Fractional Differential Mask and analyze texture enhancing capability.
3. To compare the results of enhanced video frames and images with various existing methods.

2.8 RESEARCH METHODOLOGY

Video and image enhancement is important for different engineering applications. In this research work, various enhancement methods will be discussed for image and video enhancement. An improved G-L mask will be considered in greater detail for frame/image enhancement. Consequently, the realization of the non-linear filter will be analyzed in depth. The order selection of fractional differential will be examined broadly in this research. The convolution of the fractional mask is done with each video frame for enhancement purpose. Then the benefits that occur in video enhancement from non-linear filter will be revealed in different applications like low light video enhancement, bright light video enhancement, under water video enhancement and CCTV footages. For comparison purpose, various datasets of images (SVT, MSRA) and videos (ICDAR 2013, SULFA) are chosen. The proposed approach of video enhancement using improved G-L fractional differential mask has been simulated on the platform of MATLAB (2016a software) on the system having configurations Intel(R) Core(TM), i5, CPU 2.50 GHz and processor having RAM 4 GB. The performance of proposed mask is evaluated on the basis of various parameters like Peak Signal to Noise Ratio (PSNR), Information Entropy (IE), and Average Gradient.

CHAPTER 3

ENHANCEMENT METHODS AND FRACTIONAL DIFFERENTIATION

Texture variation has a vital role in distinguishing surface anomalies. Image, picture and video enhancement is a technology in which image/frame quality is mainly enhanced and featured the image/frame highlights to get an outwardly more pleasant, more point by point and less noisy image/frame. The motive of video enhancement is to improve the quality of data and information in videos, frames and pictures for humans or to give better contribution to computerized and digital image/video processing. Because of some common and artificial relics and artifacts, the textural highlights of a video frames/images get corrupted. So, enhancement methods and approaches for video and image enhancing is employed that can upgrade the textural highlights of pictures, images and video frames.

3.1 ENHANCEMENT TECHNIQUES

Contrast and texture enhancement, in recent days specified as one of the most significant topics of concern in video and image processing. Enhancement, as of late indicated as a standout amongst the most critical points of worry in image handling. Contrast enhancement is built up by the distinction in brilliance imitated from two surfaces which are neighboring [14]. In visual concept, it is defined by the transformation in color and brightness of an entity with other entities.

Sometimes during acquisition of images, due to the following reasons low contrast may be resulted: poor brightness/illumination, deficiency of dynamic range in the image sensor and wrong position of camera focal point gap. The purpose for contrast enhancement is to expand dark level's dynamic range in the photos, videos and frames. Various techniques are presented below.

3.1.1 Histogram Equalization (HE)

HE is the most commonly used method for the image and video enhancement. It is a non-linear contrast enhancement method. HE is computationally expeditious, fast and very easy to implement on video frames and images. It is a contrast enhancement method that flattens and distributes the dynamic range of intensity values uniformly among all histogram bins.

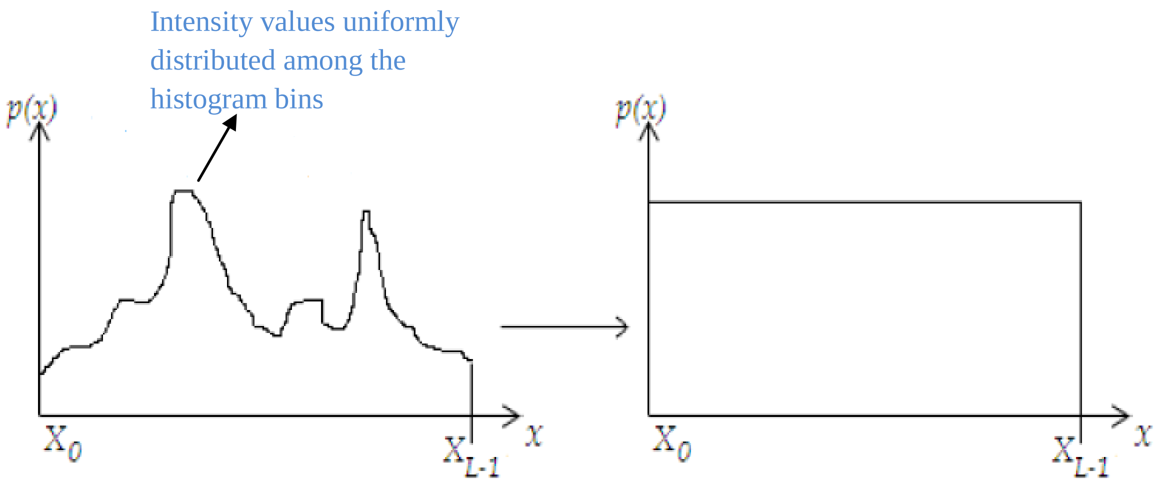
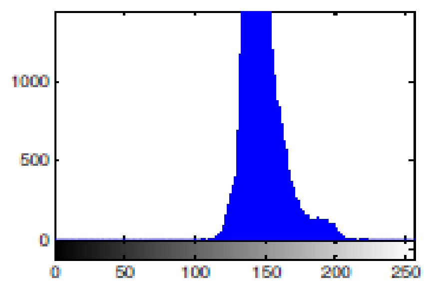


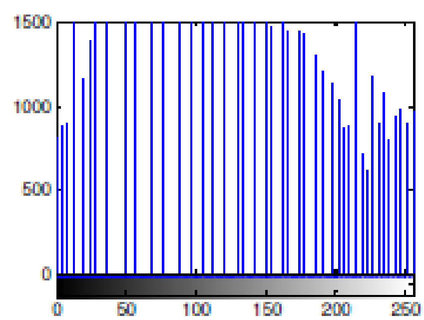
Figure 3.1: Representation of HE Method

a) Original Image



b) Histogram of original image

c) Transformed Image



d) Histogram of transformed image

Figure 3.2: Histogram Equalization effect on low contrast image

Let the normalized histogram of image/frame (f) is denoted by p with a bin for each gray level which is possible. So,

$$p_n = \frac{number\ of\ pixels\ with\ intensity\ n}{total\ number\ of\ pixels}\ where\ n = 0,1,2 \ldots\ldots L-1 \tag{3.1}$$

The histogram equalized image/frame will be defined by:

$$g_{a,b} = floor\left((L-1)\sum_{n=0}^{f_{a,b}} p_n\right) \tag{3.2}$$

Where, g is represented as image and $(L-1)$ is represented as the number of intensity values. Therefore, HE highlights borders and edges and at the same time reduces the local details of the video frame and image. Another disadvantage is that it produces over enhancement and a saturation artifact throughout the frame/image.

3.1.2 Bi - Histogram Equalization

Bi-Histogram Equalization (BHE) methods are presented to overthrown the disadvantages of HE. A method proposed by Kim [38] for brightness preserving Bi- histogram equalization is known as BBHE. In this method, the image's histogram is divided into two sub images as shown in the Figure 3.3; and after dividing into two sub images, histogram of both images is calculated independently. BBHE conserves the brightness of video frame/ image extent superior than HE [38].

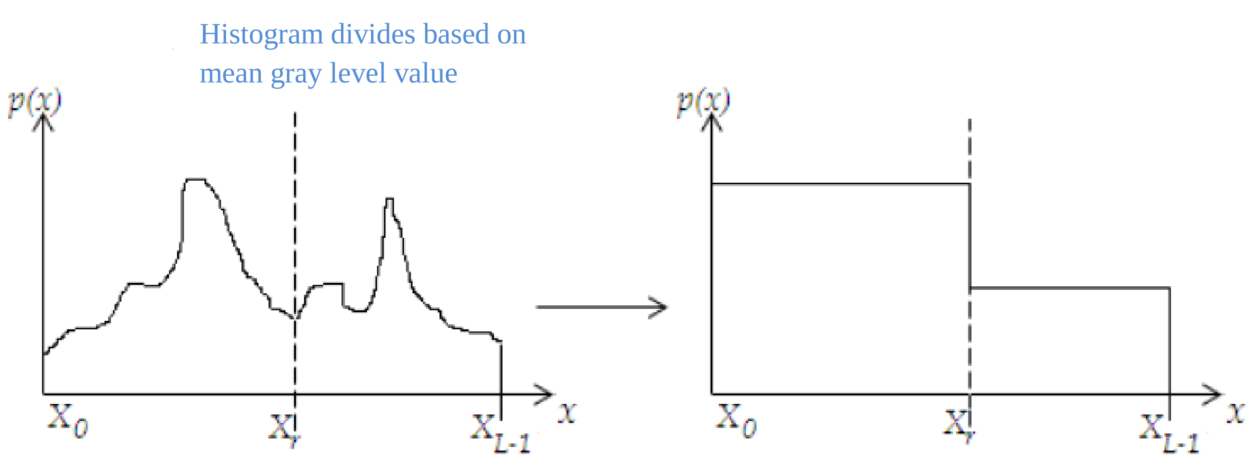


Figure 3.3: Representation of Bi-Histogram Equalization (BBHE) method [38]

There are many other Bi-HE methods such as Dualistic Sub-Image Histogram Equalization - DSIHE proposed by Wang [48], Minimum Mean Brightness Error Bi-Histogram Equalization - MMBEBHE proposed by Chen [49] and so on were proposed in quite a long while, which improved the contrast fundamentally and saved brightness to some degree. Sometimes, these methods introduced artifacts that are undesirable.

3.1.3 Multi - Histogram Equalization

Multi - HE methods were given for enhancing contrast and preserving brightness at same time. It divides the image histogram recursively into several sub-images. After dividing into sub images, it evens out the sub-histograms autonomously, to get effective and better equalization. Chen [50] proposed a Recursive Mean-Separate Histogram Equalization (RMSHE) method which is an improved version of BBHE. In this method, they separated each new histogram recursively in a view of their particular means and then equalized all the histograms as shown in Figure 3.4.

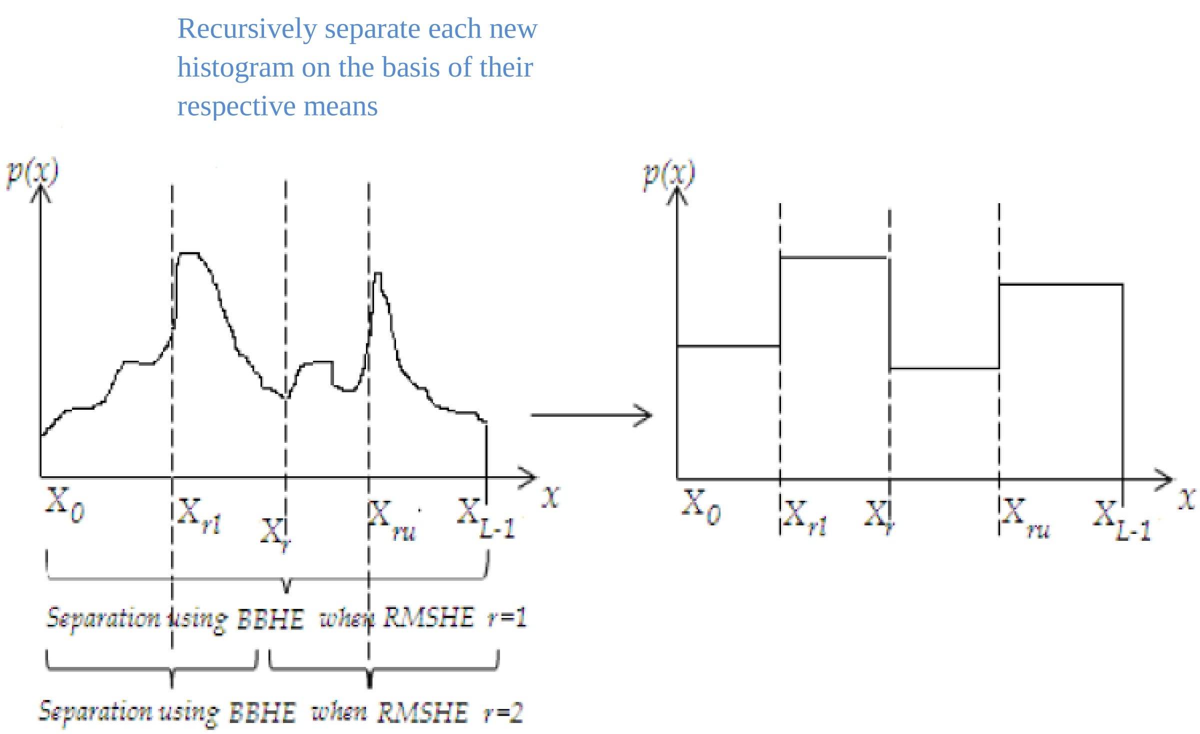


Figure 3.4: Representation of Recursive Mean Separate Histogram Equalization (RMSHE) method [50]

Dynamic Histogram Equalization (DHE) [51] is the technique that discards the domination of higher components of histogram on lower components of histogram in the image. This approach controls the extending of dark levels for better enhancement of the image, by utilizing neighborhood minima division of histogram. DHE takes limitations over the results of conventional HE. This technique of DHE performs the enhancement of an image without making any loss of details [51]. This method is shown below in the given Figure 3.5.

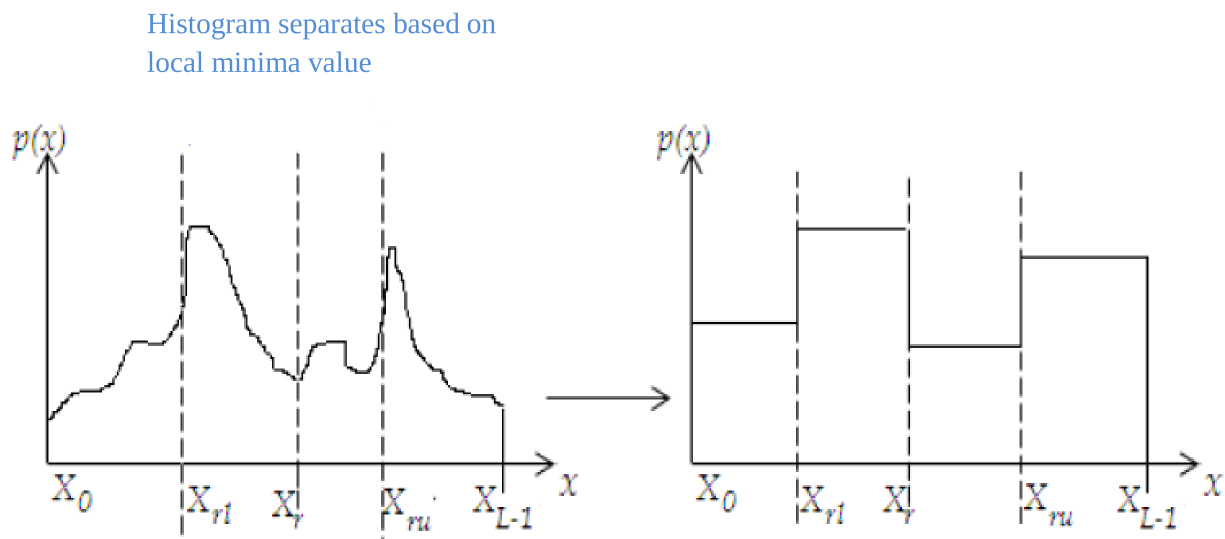


Figure 3.5: Representation of Dynamic Histogram Equalization (DHE) Method

Other methods of Multi-HE such as Recursive Sub-Image Histogram Equalization-(RSIHE) [52], Recursively Separated and Weighted Histogram Equalization-(RSWHE) [53] were also proposed in the last few years that remove the artifacts and oversaturation problems up to some extent.

3.1.4 Clipped - Histogram Equalization

This method is useful for stretching the contrast of high histogram regions without affecting the contrast of dark histogram regions. On the basis of decreasing or increasing the value in the histogram bin's clipping limit (i.e. level farthest point), Clipped Histogram Equalization (CHE) approach alters the shape of the input histogram. In and afterward equalization takes place inside every histogram, it confines the over enhancement. AHE is an excellent enhancement method for both videos and images. By redistributing the low contrast values, this method computes several histograms. The histogram representation of the CHE method is shown in Figure 3.6.

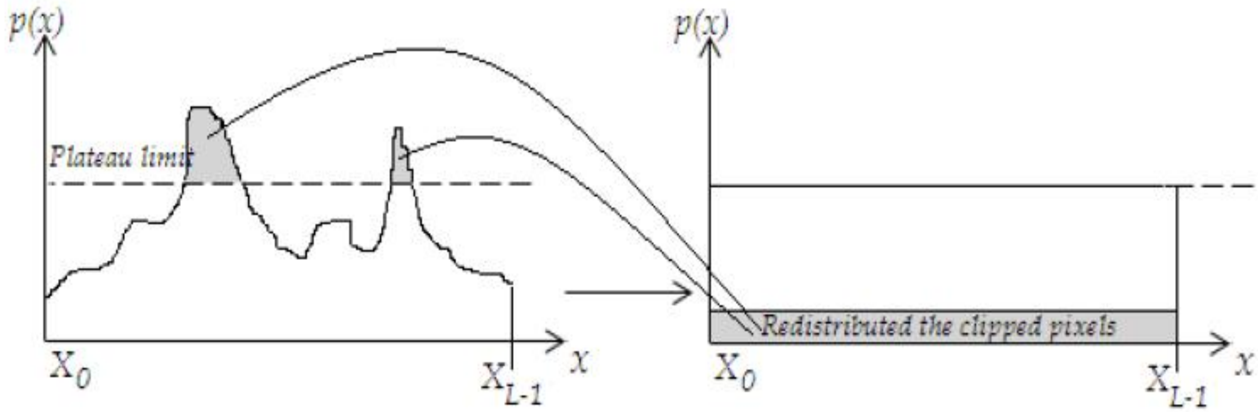


Figure 3.6: Representation of Adaptive Histogram Equalization (AHE) Method

However, this method also amplifies noise in some regions of the videos and images. CLHAE [36] (Contrast Limited Adaptive Histogram Equalization method) is a CHE method which is recently used for many video and image enhancement processes.

3.1.5 Conventional Operators

These operators are mainly used for image/video frame sharpening. Nowadays, images in present world and in this informative society are becoming more and more prominent. Various technologies of image processing are changing life of every person. Digital video and image processing is an important area of innovation. Edge detection and enhancement are important part of digital video/ image processing, which is the key step in image analysis, pattern recognition, and other deep-level processing. Integer-order edge detection is often used in image processing, especially first-order differential operation of the gradient operator and the second-order Laplacian, then find the local maximum (or crossing point).

3.1.6 Unsharp Filtering

The unsharp filter enhances the edges. It is used for sharpening and called as sharpening operator. It is a simple method in which subtraction of the original frame of the video/image is done from the unsharp or smoothed version of the video/image. In the procedure of unsharp filtering, a highpass separated, scaled rendition of a image is added to the image/video frame [54].

$$h(a,b) = I(a,b) - I_{smooth}(a,b) \tag{3.3}$$

where, $h(a,b)$, an edge image is delivered from an information image $I(a,b)$ and this $I(a,b)$ is smoothed version of input image. High frequency segments are extracted simply because all low frequency parts are subtracted from the primary image. It requires an operator that is able to give back original image, also known as sharpening operator [54]. For achieving this, some portion of gradient is added back into the original image. Let the scaling constant factor be k and we get the enhanced image in the form [54]:

$$f(a,b) = I(a,b) + k * h(a,b) \tag{3.4}$$

Original image   Image after unsharp filtering

Figure 3.7: Effect of unsharp filtering on an image

3.1.7 First Order Derivative Operators

First order derivative filters are basically used for the edge detection and improvement of edges. First order derivative edge gradient is obtained by making difference of intensity values along columns and rows of the image. There are many first order derivative operators which are as follows:

- Prewitt operator
- Sobel operator
- Roberts cross gradient operator

3.1.7.1 Prewitt Operator

The Prewitt operator calculates the gradient of the image intensity at each point. The two masks shown in the Figure 3.8 are convolved with the original image to calculate the approximations of the derivatives. From these two masks which are given in Figure 3.8, one is used for the changes in horizontal direction and one is used for changes in vertical direction. The prewitt 3 x 3 filter mask is given as [5]:

$$\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

Gx Gy

Figure 3.8: Prewitt Gradient in x and y directions

3.1.7.2 Sobel Operator

In a Sobel operator, there is slight variation from Prewitt operator in weight of central coefficient is done i.e. weight of 2 is used in center coefficient [5]. The measurement of 2 is utilized to accomplish some smoothing by giving more significance to central point. The 3×3 Sobel mask is used for x and y directions are: [5]

$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \qquad \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

Gx Gy

Figure 3.9: Sobel Gradient in x and y directions

Original Image Sobel Operator

Figure 3.10: Effect of Sobel operator on an Image

3.1.8 Second Order Derivative Operators

Second order derivatives are employed when only edge magnitudes are of interest without regard to their orientations. There are two types of operators which are second order derivatives and these are given below [5]:

3.1.8.1 Laplacian Filter

Laplacian have same properties in all the directions and therefore, it is rotation invariant in an image. This Laplacian operator is a very popular operator relative to the second derivative. The 3 × 3 masks for the four and eight neighborhoods used are [5]:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Gx Gy

Figure 3.11: Laplacian filter in x and y directions

Original Image

Laplacian Filtering

Figure 3.12: Effect of Laplacian Filter operator on an Image

3.1.8.2 Laplacian of Gaussian (LOG) Filter

Marr-Hildreth has proposed the Laplacian of Gaussian Edge Detector. The steps involved in LOG filtering are:

- First step is to smoothing the image by Gaussian Filter.
- Second is to use Laplacian Operator for enhancing the edges
- Then to use the zero crossing for denoting edge locations.
- Final step is to use linear interpolation. This determines the position of sub-pixel edge.

It is defined by the following equation: [5]

$$LoG(x,y) = \frac{1}{\pi\sigma^4}\ [1 - \frac{x^2+y^2}{2\sigma^2}]\ e^{\frac{x^2+y^2}{2\sigma^2}} \tag{3.5}$$

Greater the value of σ, broader is the plot for Gaussian filter and more is the smoothing. But, if there is too much smoothing, it is difficult job to detect the edges. On using Laplacian Filter there is occurrence of noise in the image. This noise is removed by the use Laplacian of Gaussian Filter (LOG) [5].

Original Image

Laplacian Filtering

Figure 3.13: Effect of Laplacian Gaussian Filter operator to Lena Image

It can be noticed that LOG masking is less sensitive to noise as compared to Laplacian. So, image sharpening is of higher quality with LOG filtering.

3.2 FRACTIONAL DIFFERENTIATION

The previous couple of decades have seen an expanding enthusiasm for fractional differentials, because of its numerous applications. Fractional procedures characterized by utilizing fractional calculus are advantageous for depicting various issues seeming regularly in applications, particularly in material science, climatology, meteorology, geophysics, hydrology and economy etc. Fractional order differentiation is the subsidiary of calculus that generalizes the derivative of a function to non-integer order. The integral order differentiation is contemplated as the unique instance of fractional differentiation in which conversion of fractional order values to integer order values is done. On taking Gaussian signal φ (t) in Figure 3.14 [37] for instance to talk about the numerical execute of any order derivative, isn't losing all abstraction.

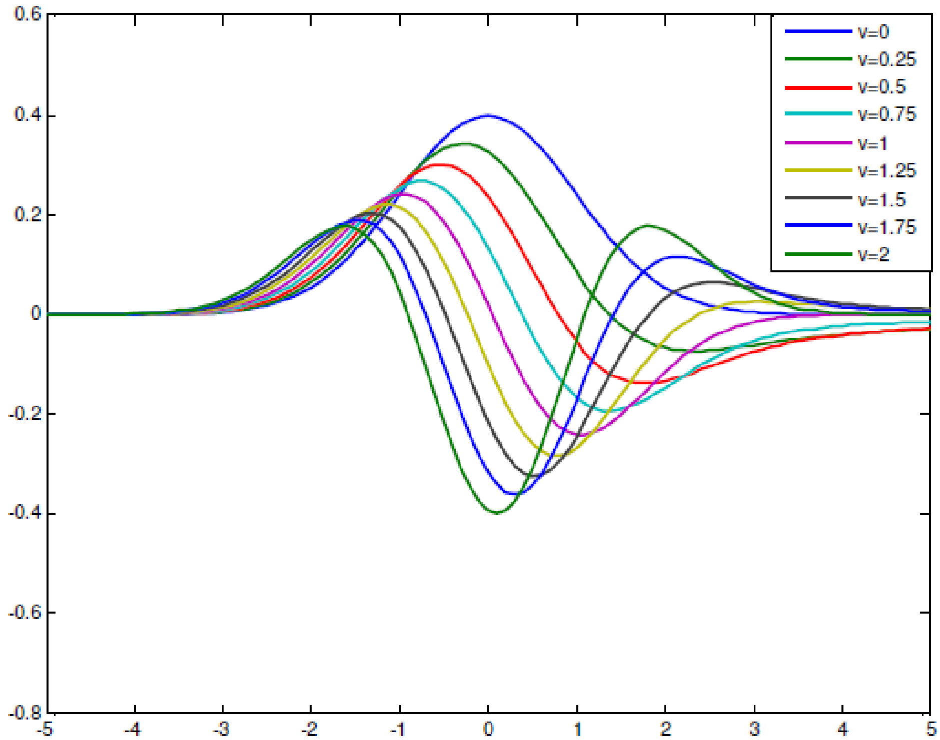


Figure 3.14: Fractional differential plot at different fractional order of simple Gaussian signal [37]

From the above shown Figure 3.14, it is observed that the first order fractional differential at stationary point is not equal to zero in general, and the first order stationary points and fractional stationary points are non-identity. Similarly, in case of two dimensional (2D), a first order differential stationary point curve surface of first order differential is zero, but its fractional differential cannot be zero [37].

3.2.1 Detection of Fractional Differential and its relation with Texture Enhancement

The filter/mask function of fractional order differential filter is defined by the equation [55]:

$$\hat{d}_{v_{ord}}(\omega) = (j\omega)^{v_{ord}} = |\omega|^{v_{ord}} = \exp(j\theta_{v_{ord}}(\omega)) \tag{3.6}$$

The phase characteristics are odd function while amplitude characteristics are even function. The analytical analysis of fractional order filter at values of $\omega > 0$ can be seen by the frequency response of the filter as shown in the Figure 3.5.

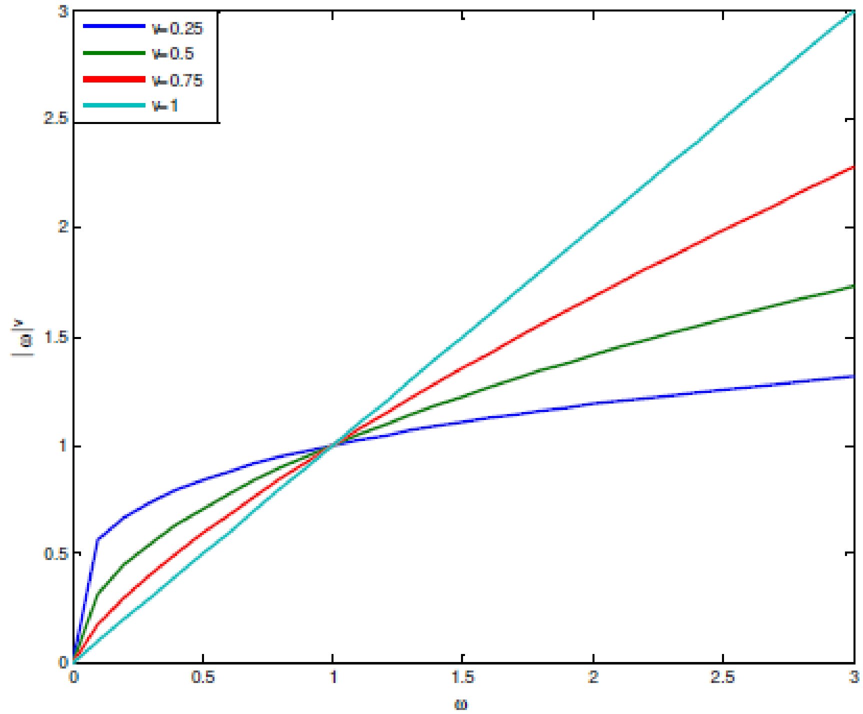


Figure 3.15: Plot of frequency response of Fractional Differential Mask [55]

From the graph, it is seen that fractional differential's frequency response is a non-linear filter when $v_{ord} = 0$, v_{ord} - order is all pass fractional differential filter and its frequency response is observed by the following equation [55]:

$$\hat{d}_{v_{ord}}(\omega) = 0 => d_{v_{ord}}(t) = \delta(t) \tag{3.7}$$

It is observed that, when $v_{ord} < 0$ it is low pass-integral filter, singular and fractional integrator. When $v_{ord} > 0$, it is fractional differential filter and $\hat{d}_{v_{ord}}(\omega)$ is a high pass filter. It is clearly seen from the Figure 3.15 that if v_{ord} is increasing, the transmission bands of $\hat{d}_{v_{ord}}(\omega)$ is becoming narrower and high-pass characteristic is becoming stronger [55]. In simple words, for $v_{ord} > 0$, $\hat{d}_{v_{ord}}(\omega)$ non-linearly constrains low frequency components of the signal and non-linearly improve high frequency components.

From previously mentioned analysis, one might say that the images having smooth territory whose gray level values does not convert comprehensively, the textural highlights in smooth area might be greatly attenuating and its differential outcome is approximately zero, when image is filtered by first order derivative filters like Prewitt, Sobel or Robert's gradient operator. Similar is the case of second order filters like Laplacian of Gaussian (LOG) filter. Also, fractional differential-based filters protect textural features included in smooth region to the greatest degree. This is the reason for using the fractional differential masks for enhancing the video frames and images of different textures.

3.2.2 Introduction to Grunwald and Letnikov (G-L) equation

Theoretical background of fractional differentiation of G-L (Grunwald - Letnikov) is taken into account in this section. The approach of G-L fractional calculus arises from integral differentiation's classical definition in which the integer order of the differentiation is changed into fractional differentiation. Assuming, $\forall\, v_{ord}$ belongs to R (where R is real set, v_{ord} is its integral part) and signal $F\,(t) \in\, [a, t]$, $a\, <\, t,\ a\, \in\, R,\ t\, \in\, R, m\ \ (m\, \in\, Z,\ Z$ is integer set) is the order of continuous differentiation. When $v_{ord} >\ 0$, so v_{ord} - order fractional [18-19]:

$$Df_t^{v_{ord}} F(t) = \lim_{h\to 0} F_h^{v_{ord}}(t) = \lim_{h\to 0} h^{-v_{ord}} \sum_{m=0}^{n-1} \binom{-v_{ord}}{m} F(t-mh) \tag{3.6}$$

where,

$$\binom{-v_{ord}}{m} = \frac{(-v_{ord})(-v_{ord}+1)\,\dots\dots(-v_{ord}+m-1)}{m!} \tag{3.7}$$

Thus,

$$Df_t^{v_{ord}} = \frac{h^{-v_{ord}}}{\Gamma(-v_{ord})} \sum_{m=0}^{n-1} \frac{\Gamma(m-v_{ord})}{\Gamma(m+1)} F(t-mh) \tag{3.8}$$

One dimensional fractional differential is defined by taking the duration of signal $h = 1$ as follows:

$$\frac{d^{v_{ord}} F(t)}{dt^{v_{ord}}} \approx F(t) + (-v_{ord})F(t-1) + \frac{(-v_{ord})(-v_{ord}+1)}{2} F(t-2) + \frac{-v_{ord}(-v_{ord}+1)(-v_{ord}+2)}{6}\, F(t-3)\,\dots\dots + \frac{\Gamma(n-v_{ord})}{\Gamma(-v_{ord})\Gamma(n+1)} F(t-n) \tag{3.9}$$

It can be observed that $(n-1)$ non-zero coefficients are the functions with respect to fractional order v_{ord} except the first term [18-19]. Also sum of n non-zero coefficients is non-zero, which is significant difference between fractional and integral differential.

3.3 SUMMARY

This chapter describes the various methods and techniques involved in the image enhancement and edge detection. Various techniques whether they are HE or advanced version of HE like BBHE [38], RMSHE [50], RSIHE [52], RSWHE [53], and CLAHE [36] are explored. Different methods described in [4, 5] are studied for detection of edges. First order and second order derivative filters are used for the edge detection [5] are also described. Conventional and unsharp filters are also explored and discussed in this chapter. Fractional order differentiation is also studied. Fractional order differentiation is the subsidiary of calculus that generalizes the derivative of a function to non-integer order [41]. Detection of fractional differential and its relation with texture enhancement is very important part of this chapter [37]. Next, introduction to improved G-L equation and fractional differential equation is presented [18, 19] in this chapter. Theoretical framework of fractional differentiation of G-L (Grunwald - Letnikov) is taken into account. Basically, this chapter summarizes the introduction of the methodology used in the thesis work.

CHAPTER 4

VIDEO ENHANCEMENT USING IMPROVED G-L FRACTIONAL DIFFERENTIAL

4.1 INTRODUCTION

Fractional Differentiation is an effective and efficient method for texture enhancement of the images and videos as compared to the present existing classical methods and different integral differential methods. These fractional differential methods not only performs enhancement of high frequency components, but also preserves low frequency components. There are various types of fractional definitions, but the most commonly available are G-L and R-L and Caputo based definitions. Based on these equations, mask of fractional order filter are convoluted on the images and video frames for enhancement of texture. Improved and modified G-L equation and mask created from that equation is discussed in next section.

4.2 PROPOSED VIDEO ENHANCEMENT APPROACH

In proposed video enhancement approach, initially frames are extracted from the input video. Using fractional differential approach, G-L equation is taken and improved version of this G-L equation is realized. This improved G-L equation is used for the non-linear mask creation with the help of different fractional order and intensity values. Non-linear fractional mask is applied on the excerpted video frames. Non-linear mask is shifted about the each frame of video. The order of the pixels in the windowed section of the each frame is rearranged, and the output pixel is generated from these rearranged input pixels. This improves the visual quality of the each video frame, as measured objectively in real-video experiments. The non-linearity of fractional differential mask maintains the high frequency marginal features and it enhanced the low frequency details. This motivates to utilize the fractional calculus to increase quality of video. Video enhancement approach using an improved G-L mask is described in flow diagram as presented in Figure 4.1. The proposed approach is also compared with existing techniques and the qualitative comparison of the results is shown in the experimental results section in chapter 5.

4.2.1 Flow Chart of proposed methodology

The flow diagram of the proposed approach is given in Figure 4.1 as shown below:

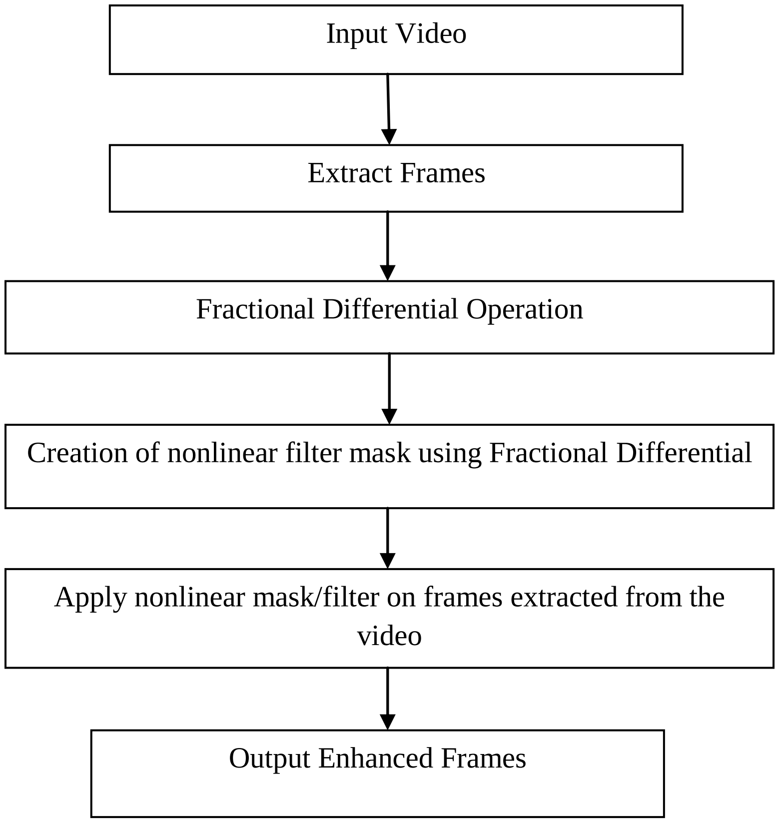


Figure 4.1: Flow diagram of the proposed work for video enhancement

4.3 IMPROVED G-L FRACTIONAL DIFFERENTIAL MASK

For making fractional more precise and accurate fractional differential operator fractional equation of simple G-L can be rewritten as [41, 55]:

$$Df_t^{v_{ord}} = \frac{h^{-v_{ord}}}{\Gamma(-v_{ord})} \sum_{m=0}^{n-1} \frac{\Gamma(m - v_{ord})}{\Gamma(m+1)} F\left(t + \frac{v_{ord}h}{2} - mh\right) \tag{4.1}$$

Comparing Equation (4.1) and simple fractional differential G-L equation of previous chapter, signal values of $F(t)$ is introduced on nodes besides supposing $v_{ord} = 0, \pm 2, \pm 4$, thus by taking three nodes

$F(t + h - mh), F(t - mh),\ F(t - h - mh)$ and with the help of 3 -point Lagrange's interpolation method it implies [41, 55]:

$$
\begin{aligned}
F(\xi) = & \frac{(\xi - t + mh)(\xi - t + h + mh)}{2h^2} F(t + h - mh) \\
& - \frac{(\xi - t - h + mh)(\xi - t + h + mh)}{h^2} F(t - mh) \\
& + \frac{(\xi - t + mh)(\xi - t - h + mh)}{2h^2} F(t - h - mh)
\end{aligned}
\tag{4.2}
$$

Here, assume that $\xi = t + (v_{ord}h/2) - mh$ and with interpolation of fractional, we obtain:

$$
\begin{aligned}
F\left(t + \frac{v_{ord}h}{2} - mh\right) \cong & \left(\frac{v_{ord}}{4} + \frac{{v_{ord}}^2}{8}\right) F(t + h - mh) + \left(1 - \frac{{v_{ord}}^2}{4}\right) F(t - mh) \\
& + \left(\frac{{v_{ord}}^2}{8} - \frac{v_{ord}}{4}\right) F(t + h - mh)
\end{aligned}
\tag{4.3}
$$

From Equations (4.1) and (4.3), we obtained the improved G-L fractional differential [41, 55]:

$$
\frac{\partial^{v_{ord}}}{\partial t} F(t) = \frac{h^{-v_{ord}}}{\Gamma(-v_{ord})} \sum_{m=0}^{n-1} \frac{\Gamma(m - v_{ord})}{\Gamma(m+1)} \begin{bmatrix} F_m + \frac{v_{ord}}{4}(F_{M-1} - F_{M+1}) + \\ \frac{{v_{ord}}^2}{8}(F_{M-1} - 2F_m + F_{M+1}) \end{bmatrix}
\tag{4.4}
$$

where, $F_M = F(t + h - mh),\ F_M - 1 = F(t + h - mh),\ F_M + 1 = F(t - h - mh)$.

4.4 REALIZATION OF A MODIFIED G-L FRACTIONAL DIFFERENTIAL MASK

In general, if the picture or video frame or image is required to be processed with the use of non-linear filtering mask, the intensity values of frames are convolved with a $m \times n$ size mask. It is described by the following equation [41]:

$$
h(a, b) = \sum_{F=-x}^{x} \sum_{t=-y}^{y} w(F, t)\, f(a + F, b + t)
\tag{4.5}
$$

where, $f(a,b)$ is the value of picture element and $w(F,t)$ is the value of filter. Considering the direction of gradient, the filter is designed into a $p \times p$-size matrix T which has p layers. (p is odd natural number). The eight directions of matrix T are as follows: 0, 22.5^0, 45^0, 67.5^0, 90^0, 112.5^0, 135^0 and 157.5^0, respectively [41]. Here, video enhancement specifies to the modification of the original values of digitals pixels of the frames and to obtain better textural and contrast features for the background of the targets.

In the proposed scheme, non-linear mask is created by using the following equation [41]:

$$T_i = \frac{1}{\Gamma(-v_{ord})}\left[\frac{\Gamma(i-v_{ord}+1)}{(i+1)!}\left(\frac{v_{ord}}{4}+\frac{{v_{ord}}^2}{8}\right)+\frac{\Gamma(i-v_{ord})}{(i)!}(1-\frac{{v_{ord}}^2}{4}) + \frac{\Gamma(i-v_{ord}-1)}{(i-1)!}\left(-\frac{v_{ord}}{4}+\frac{{v_{ord}}^2}{8}\right)\right] \tag{4.6}$$

where, T_i is the estimation of i^{th} layer/ covering of mask, $v_{ord} \in S^+$ and S^+ is real number. Specifically, in order to approach the summation value of matrix T to 0, Equation (4.6) can be rewritten as:

$$T_0 = -1 * \sum_{i=1}^{n} 8 * i * T_i \tag{4.7}$$

Clearly, the results of convoluting with T are that, the sharpening edges of the frames of video are obtained. For the intent of enhancement of texture, the sharpening corners or edges should enumerate to the value of original picture element or intensity value. So, the value of T should change to S and we finally get the value of mask by following:

$$\begin{cases} S_i = \gamma T_i & ;\ (i > 0) \\ S_0 = (1+\gamma)T_0 & ;\ (i = 0) \end{cases} \tag{4.8}$$

where, γ is the intensity factor.

When $v_{ord} \in (0,1)$, from Equation (4.7) and Equation (4.8):

$$\begin{cases} S_i = \gamma T_i & ;\ (i > 0) \\ S_0 = 1 - \sum_{i=1}^{n} 8 * i * T_i & ;\ (i = 0) \end{cases} \tag{4.9}$$

The mask realized using proposed enhancement method is presented in Figure 4.2. The dimension of S is odd natural number.

S1	S1	S1
S1	S0	S1
S1	S1	S1

S2	S2	S2	S2	S2
S2	S1	S1	S1	S2
S1	S1	S0	S1	S1
S2	S1	S1	S1	S2
S2	S2	S2	S2	S2

Figure 4.2: The mask of an improved G-L Fractional Differential, 3 x 3 size and 5 x 5 size [41]

-0.3661	-0.3661	-0.3661
-0.3661	3.9285	-0.3661
-0.3661	-0.3661	-0.3661

Figure 4.3: A 3 x 3 size mask at v_{ord} = 0.3 and intensity factor γ=1

There is a prominent difference between integral and fractional differential which is seen from the Figure 4.3, that sum of coefficients of the mask is not equal to zero in differential mask. Fractional order (v_{ord}) is optimized to 0.3 and intensity factor (γ) is 1. Enhancement degree decreases after increasing the value of fractional order further beyond 0.3. The value is chosen to be 0.3, because on comparing with HE and LCA techniques, all parameters i.e. IE, AG and PSNR have more qualitative values as compared to these basic approaches. It will be more cleared in the bar graphs shown in the next section, which are plotted for the benchmark images taken in the analysis. Intensity factor, γ is also enhancement degree controlling factor. When the value of γ is 1, maximum enhancement degree is achieved, as IE, AG and PSNR is improved. On changing intensity factor γ beyond 1, distortion of the frame or image takes place. This way, non-linear mask enhances the textural feature.

4.5 BRIEF DESCRIPTION OF PROPOSED APPROACH FOR VIDEO ENHANCEMENT

- Extract the frames from the input video.
- Realization of non-linear filter using improved G-L fractional order approach.
- Non-linear Filter is convoluted on whole frame which is extracted from the video.
- In the proposed mask of video enhancement, fractional order v_{ord} is selected 0.3 and the value of intensity factor γ selected is 1.
- For qualitative comparison, the enhancement criteria involves the parameters i.e. IE (information entropy), AG (average gradient), PSNR (peak signal to noise ratio).
- Comparative analysis is done for basic techniques like HE (histogram equalization) and LCA (linear contrast adjustment).
- Comparative analysis is also done for latest video and image enhancement research papers.

4.5.1 Selection Criteria of Fractional Differential Order in proposed method

The fractional differential filter is presented and realized from the improved G-L equation in the Figure 4.2, and it can be used for video and image enhancement. The generalized mask can improve edges and forms, and additionally holding the surface preserving texture details. But, selection of the fractional differential order is also a major problem. On the basis of definition of entropy, a new method is discussed in [21], by which the differential order can be chosen naturally or automatically. Entropy is a parameter used for measuring randomness which is utilized to characterize the texture of the input image.

- **Information Entropy:** The entropy indicates measure of magnitude of image data or frame data. If a picture/frame has no texture, its entropy is nearly zero. Generally, if image or video frame have great surface texture elements, its entropy is more noteworthy. Entropy is defined as [21]:

$$IE = sum\ (p.* \ log_2(p)) \tag{4.10}$$

where, p is the probability distribution function of image intensity also p contains the normalized histogram counts returned. Image/frame entropy is a measure for evaluation of the amount of information presented in the video frame or image. Larger the value of IE, the greater amount of information carried by image [21].

Mutual entropy between two different frames or images I and M can be defined as:

$$IE(I,M) = IE(I) + IE(M) - \sum_{i=0}^{i=255} \sum_{j=0}^{j=255} P(I,M) log \frac{1}{P(I,M)} \tag{4.11}$$

where, $P(i,j)$ is the two-dimensional joint distribution function of I and M . The mutual entropy reflects the similarity and comparability of two frames or images. So, a reasonable determination standard of differential order v_{ord} can also be established as:

$$v_{ord}{}^{*} = max_{v_{ord} \in (0,1)} (IE(I, I^{v_{ord}}) \tag{4.12}$$

where, $I^{v_{ord}}$ is the enhanced image by v_{ord} fractional order differential.

- **Average Gradient:** Average gradient is another criterion for selecting fractional order v_{ord}. It reflects the capability of contrast expression, used to evaluate image and frame clarity. The image/frame will be clearer if the value of average gradient is high. Spatial resolution of the image and video frame can be measured by average gradient. So, high resolution means greater value of average gradient.

  Gradient of an image $I(x,y)$ is given as [21, 56]:

$$AG = \frac{1}{PQ} \times \sum_{x=1}^{P} \sum_{y=1}^{Q} \sqrt{\frac{\left(\frac{\partial I}{\partial x}(x,y)\right)^2 + \left(\frac{\partial I}{\partial y}(x,y)\right)^2}{2}} \tag{4.13}$$

The graphs shown in Figure 4.4 are plotted for the Baboon image using 3 x 3 mask size and 5 x 5 mask size for different v_{ord} fractional orders. It is cleared from Figure 4.4, about the selection of different fractional orders. In the chapter 5, bar graph is also plotted for various images and compared with existing classical methods. The graphs shown in next chapter is also plotted for

HE and LCA techniques. The comparison shown in chapter 5 is further helpful for mask or filter selection.

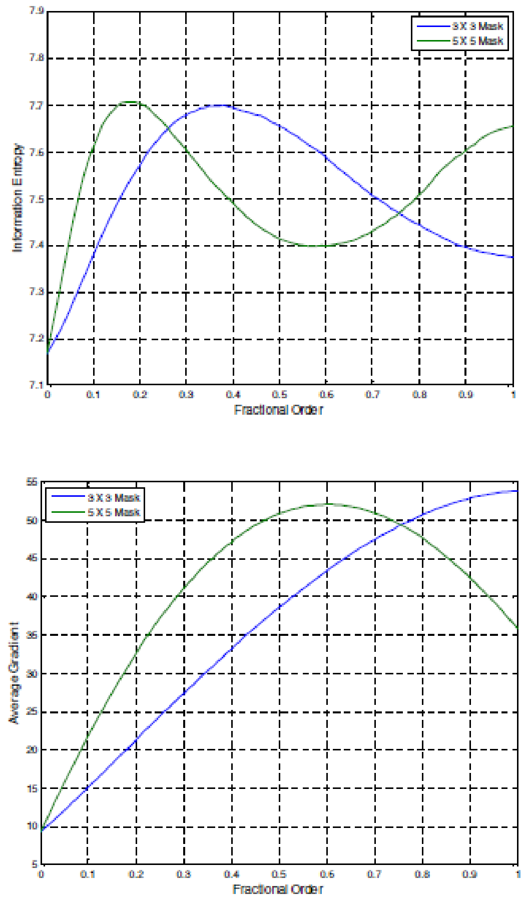


Figure 4.4: Information Entropy (IE) and Average Gradient (AG) of Baboon for varying v_{ord} fractional order through different size of mask

While enhancing the texture of an image and video using any enhancement technique, the information in the enhanced image and video frame might get decreased or increased. So, the capability of any texture enhancement technique can be analyzed using some other criterions as discussed below:

- **SSIM:** In the comparative evaluation, SSIM is also taken into account which is defined by the following Equation [43]:

$$SSIM(m,n) = [lf(m,n)]^a . [cf(m,n)]^b . [sf(m,n]^c \tag{4.14}$$

SSIM is used for the comparison of the structure of the processed frame/image with the original frame/image. In the above Equation lf, cf, sf is defined as the luminescence function, contrast comparison function and structure comparison function respectively and a, b, c are utilized for adjusting relative importance of these functions.

- **PSNR:** PSNR is determined by mean square error between the original video frame (A) / image (A) and the corresponding video frame (B)/image (B) which is extracted after the enhancement methods [43]:

$$PSNR = 10log\frac{max(B,A)^2}{MSE} \tag{4.15}$$

$$MSE = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(B(i,j) - A(i,j))^2 \tag{4.16}$$

where, max is the maximum possible value of the pixels in a particular video frame/image. In grayscale video frame/image, the max possible value of the pixel is 255. PSNR is inversely proportional to the MSE. PSNR is calculated in decibels. More the value of PSNR, more will be similarity between the original video frame and video frame after enhancement.

4.6 SUMMARY

In this chapter, the methodology used for the video enhancement purpose is presented. An improved version of G-L mask is described which is used for the video enhancement procedure. Mask/Filter which is created from the improved G-L equation is non-linear. First of all, realization of mask from the improved G-L equation is taken into account. Selection of the order of fractional mask is an important task. Mask selected for video enhancement purpose is of the order v_{ord} = 0.3. The generalized mask in this chapter, can improve edges and forms, and additionally holding the surface preserving texture details. Various criteria's on which mask/filter selection depends are also explored and discussed in this chapter. This chapter is basically the core part of the thesis on which the whole methodology used in the work is based.

CHAPTER 5

RESULTS AND DISCUSSION

In this part of thesis, the capability of video and image enhancement with the proposed mask is validated. The enhancement approach of video and picture is examined utilizing the conventional and classical strategies and with the use of our recently proposed procedure of non-linearly improved G-L mask as depicted in the previous section. Qualitative and comparative analysis is also discussed in this chapter.

5.1 DATASET AND SETTINGS

The proposed enhancement methodology is applied on various image and video datasets. All test cases on which proposed mask is applied are various types of datasets. This mask is also applied on the standard images (Lena, Cameraman and Baboon image). For bright light image benchmarking, office_6 image is chosen and for low light image benchmarking, random low light image from UCID dataset is chosen. Video enhancement mask is applied on the SULFA dataset [57], ICDAR 2013 video dataset [58], low light videos [61], bright light videos and under water video [62]. Thirty results of random videos are presented from the SULFA dataset. Five random videos are tested from ICDAR 2013 video dataset. Similarly, enhancement algorithm is also applied on the SVT [59] and MSRA [60] image dataset.

This chapter contains three parts. In first part, results of different fractional masks are discussed for standard images. In second part of this chapter, results of various images are discussed including low light and bright light images. In the last part of the chapter, various enhanced results of low light video, bright light video, CCTV video and under water video are presented. Results for the qualitative analysis and comparison with various existing methods are also discussed in the end of each part.

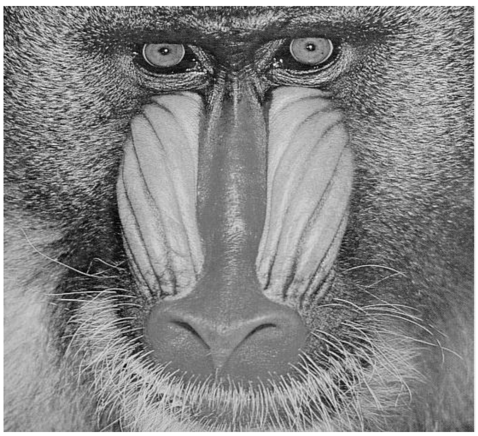
5.2 EXPERIMENTAL RESULTS

In this section, experimental results of the proposed mask based on improved G-L fractional order are presented. The mask created by fractional differential of G-L equation discussed in the previous chapter is applied on some datasets. Texture enhancement is analyzed by comparing with HE and LCA. To measure the nature of improved frames, the standard measures used are, Average Gradient (AG), Information

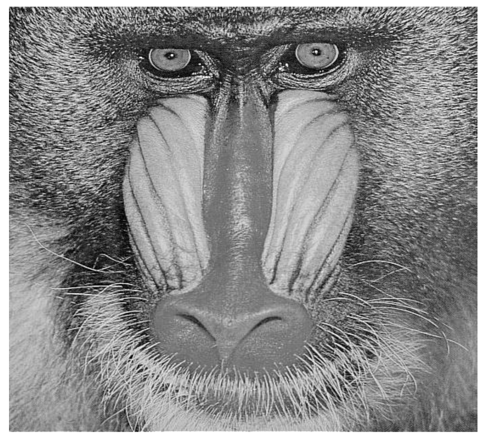
Entropy (IE), Peak Signal to Noise Ratio (PSNR) and Structural Similarity Index (SSIM). The reason to take these techniques for comparison is because most of the enhancement methods in literature use the HE and LCA techniques as a basis directly or indirectly.

5.2.1 Comparative Analysis Using Different Fractional Masks

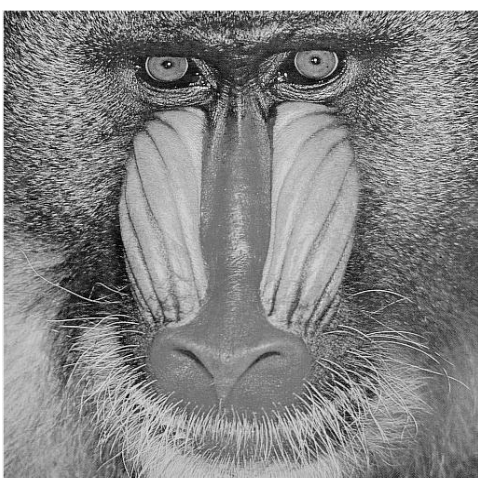
The R-L and G-L masks have been applied to standard images of Baboon and Barbara. The comparison is done on the basis of different fractional orders ranging from 0 < v_{ord} < 1 with intensity factor γ =1. The results are shown below:



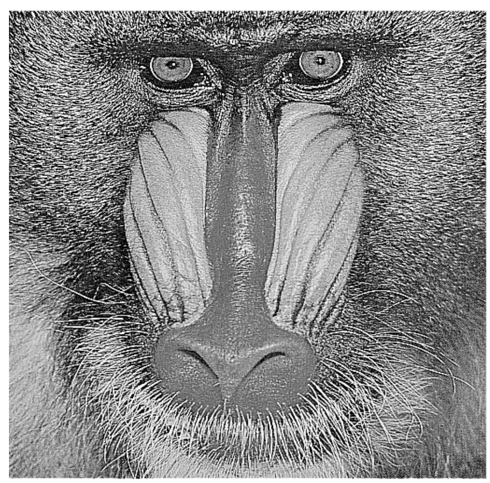
a) v_{ord}= 0.1 R-L FD



b) v_{ord} = 0.1 Improved G-L FD

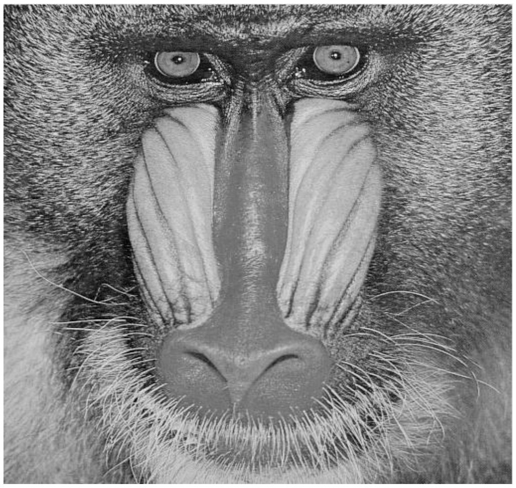


c) v_{ord} = 0.5 R-L FD

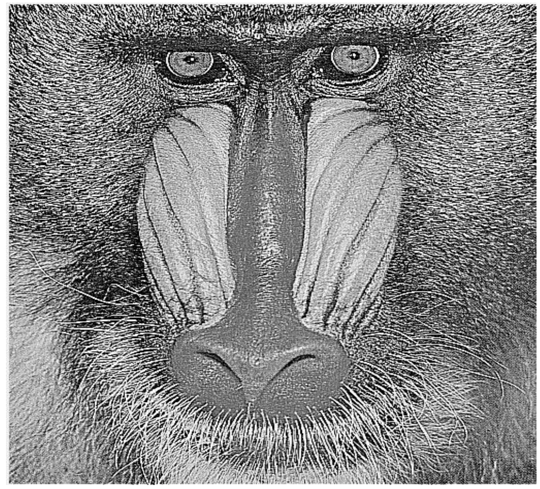


d) v_{ord} = 0.5 Improved G-L FD

Figure 5.1: Results of enhancement at different fractional orders using R-L and improved G-L FD (cont.)



e) v_{ord} = 0.9 R-L FD



f) v_{ord} = 0.9 Improved G-L FD

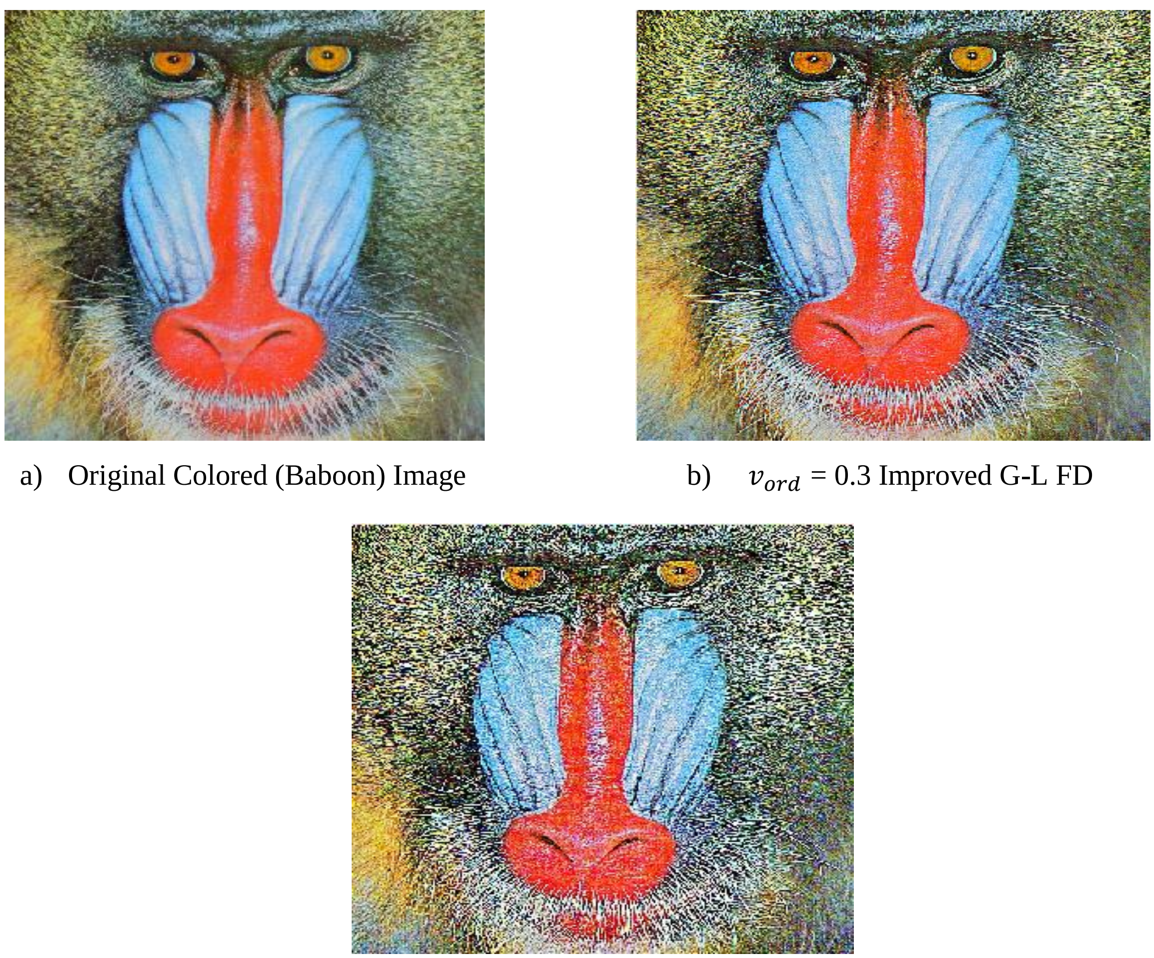
Figure 5.1: Results of enhancement at different fractional orders using R-L and improved G-L FD

Table 5.1 IE and AG for Baboon Image using different fractional masks

Parameter / **FD- Order**	**Information Entropy**		**Average Gradient**	
	R-L Mask [28]	Improved G-L Mask	R-L Mask [28]	Improved G-L Mask
0	7.3583	7.3583	12.9678	12.9678
0.1	7.4883	**7.7189**	16.2425	22.4618
0.2	7.5898	7.7046	18.6377	30.6113
0.3	7.6479	7.6823	20.1189	36.4609
0.4	**7.6693**	7.2485	**20.7280**	40.7344
0.5	7.6640	7.0393	20.5614	43.9485
0.6	7.6346	6.8616	19.7564	46.3842
0.7	7.5809	6.7186	18.4268	48.2053
0.8	7.50	6.6083	16.7421	49.5011
0.9	7.4303	6.5379	14.8538	**50.3258**
1	7.3583	6.5028	12.9678	50.2936

Following observations are made from above Figures and Table 5.1:

- When intensity factor (γ) is 1 and fractional order varies from 0< v_{ord}< 0.1, the textural feature of enhancement increases on increasing the order of fractional differential.
- The information entropy of image increases on increasing the fractional differential order and the maximum value obtained at order v_{ord} = 0.1 in case of improved G-L. Increasing the fractional order beyond 0.1, causes the loss in the information, thus resulting in decrease of IE.
- From the comparative analysis of improved G-L and R-L, it is observed that for G-L texture first enhances with increase in value of v_{ord} (fractional order) but above some value, there is occurrence of distortion in image. In case of R-L, there is minor enhancement in texture on increasing value of v_{ord}, but enhancement degree decreases beyond some value with increase in value of v_{ord}.



a) Original Colored (Baboon) Image

b) v_{ord} = 0.3 Improved G-L FD

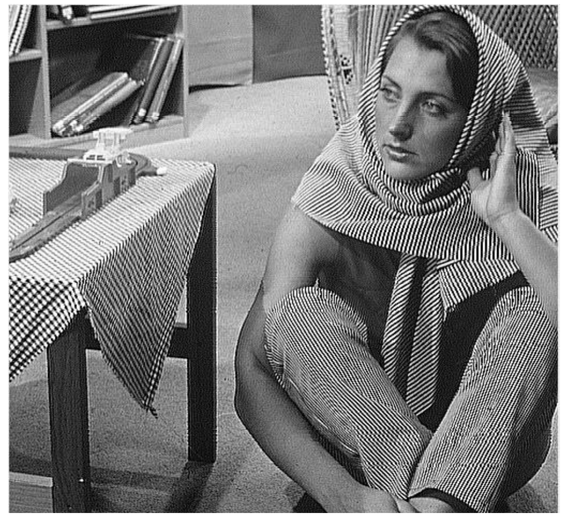
c) v_{ord} = 0.8 Improved G-L FD

Figure 5.2: Results of enhancement at different fractional orders (Baboon) using improved G-L FD

It is observed from Figure 5.2 that texture of Baboon image is enhanced on applying improved G-L fractional differential mask up to fractional order 0.3. Texture first enhances with increase in value of v_{ord} (fractional order) but beyond the fractional order of 0.3, there is distortion in image.

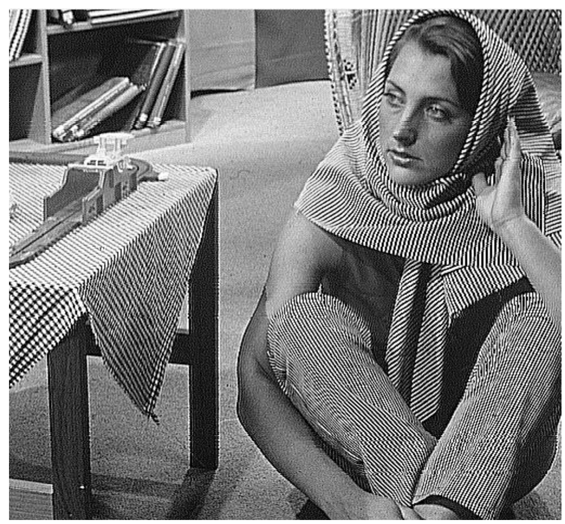
Barbara image

a) v_{ord} = 0.3 R-L FD



b) v_{ord} = 0.3 improved G-L FD

c) v_{ord} = 0.9 R-L FD



d) v_{ord} = 0.9 improved G-L FD

Figure 5.3: Results of enhancement at different fractional orders using improved G-L and R-L FD

It is observed from Figure 5.3, that when intensity factor (γ) is 1 and fractional order varies from 0< v_{ord}< 0.2, the textural features enhancement increases on increasing the order of fractional differential. Table 5.2 shows information entropy and average gradient of Barbara image for R-L and improved G-L mask.

Table 5.2 IE and AG for Barbara Image using different fractional masks

Parameter / FD-Order	Information Entropy		Average Gradient	
	R-L Mask [28]	Improved G-L Mask	R-L Mask [28]	Improved G-L Mask
0	7.6321	7.6321	9.2402	9.2402
0.1	7.6935	**7.7882**	11.4801	15.5342
0.2	7.7376	7.7763	13.0739	20.8157
0.3	7.7610	7.6829	14.0439	24.8363
0.4	**7.7694**	7.5809	**14.4368**	27.9275
0.5	7.7669	7.4821	14.3190	30.3539
0.6	7.7555	7.3908	13.8061	32.2580
0.7	7.7339	7.3143	12.9420	33.7380
0.8	7.7032	7.2530	11.8213	34.7870
0.9	7.6657	7.2127	10.5368	35.7742
1	7.6321	7.1950	9.2402	**35.7888**

It is observed from Table 5.2 that the IE of image increases on increasing the fractional differential order and the maximum value is obtained at order v_{ord} = 0.2 in case of improved G-L. Increasing the value of fractional order above 0.2, results in decrease in the value IE because results of fractional differential becomes close to integral differential. In case of R-L, there is minor enhancement in texture on increasing value of v_{ord}, but the measure of enhancement decreases after a particular value with increase in value of v_{ord} i.e. fractional order.

From the above shown figures and tables, it is observed that improved G-L mask is better than the simple R-L mask as entropy is better in case of G-L mask and average gradient is also better in G-L mask. So, improved G-L mask is applied for the enhancement of images and various video datasets. In the next part, qualitative analysis of image enhancement on standard images and video enhancement on different datasets using improved G-L fractional differential mask presented.

5.3 IMAGE ENHANCEMENT

For image enhancement using improved G-L fractional differential, following images are chosen and compared with different existing methods for testing purposes.


(a) (b) (c) (d)

Figure 5.4: Comparison enhancement results for standard Lena image; a) Original Image b) Histogram Equalization method c) LCA method d) Proposed Mask

The above shown figures are of Lena standard image and enhanced Lena images. These figures are the enhancement images using Histogram Equalization [4], LCA [5] and the proposed mask of fractional differential approach.

(a)

(b)

(c)

(d)

Figure 5.5: Comparison enhancement results for standard cameraman image; a) Original Image b) Histogram Equalization method c) LCA method d) Proposed Mask

In the above shown figure, when original Cameraman image is compared to the image after enhancement using proposed mask, it enhances the textural features. In Figure 5.5, enhancement is clearly seen in the tripod stand, background buildings and on the grass. In the Table 5.3, qualitative analysis of HE, LCA and proposed mask is presented.

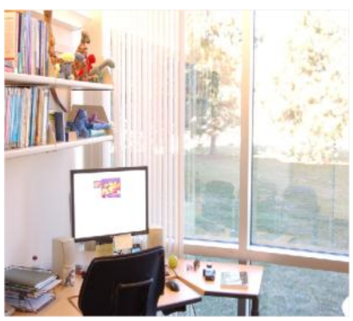
Table 5.3 Qualitative evaluation using IE, AG and PSNR for Lena and Cameraman images

	Techniques / Parameter	Original	Histogram Equalization (HE) [4]	Linear Contrast Adjustment (LCA) [5]	Improved G-L Mask at fractional order (0.3)
Lena	Information Entropy (IE)	7.7502	5.9863	6.1184	**7.8540**
	Average Gradient (AG)	5.0868	8.9323	5.7825	**13.7747**
	PSNR	-------	14.2271	16.4390	**22.9039**
Cameraman	Information Entropy (IE)	7.0097	5.9106	6.9307	**7.1534**
	Average Gradient (AG)	7.1606	10.1115	9.1272	**17.3948**
	PSNR	-------	19.0970	18.5525	**19.7128**

From the Table 5.3, it is observed that the proposed mask has more value of IE, AG and PSNR than the HE and LCA.

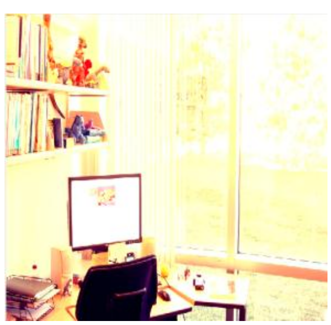
5.3.1 Bright light image

For benchmarking of bright light image, office_6 standard image is taken.

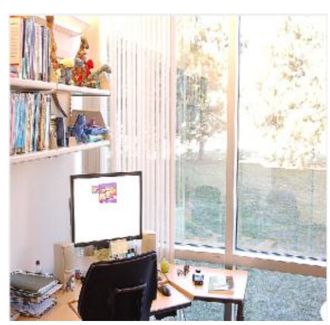


(a)

(b)



(c)



(d)

Figure 5.6: (a) Input office_6 Image (b) Image enhanced by HE method (c) Image enhanced by LCA method (d) Image enhanced by proposed mask (improved G-L at 0.3 fractional differential order)

Original image and image after applying various existing enhancement methods and proposed mask is shown in Figure 5.6. In Table 5.4 shown below, qualitative results are presented.

Table 5.4 Qualitative evaluation using IE, AG and PSNR (office_6) of Bright light image

Techniques / Parameter	Original Image	Histogram Equalization (HE) [4]	Linear Contrast Adjustment (LCA) [5]	Improved G-L Mask at fractional order (0.3)
Information Entropy (IE)	6.4344	5.4513	4.1030	**6.5071**
Average Gradient (AG)	4.8722	9.1298	3.8870	**10.571**
PSNR	-------	10.1970	17.1802	**23.0226**

It is clearly observed that values of all three parameters are better in proposed mask than existing methods because all the three parameters presented in the table have more values in case of proposed mask as compared to existing methods.

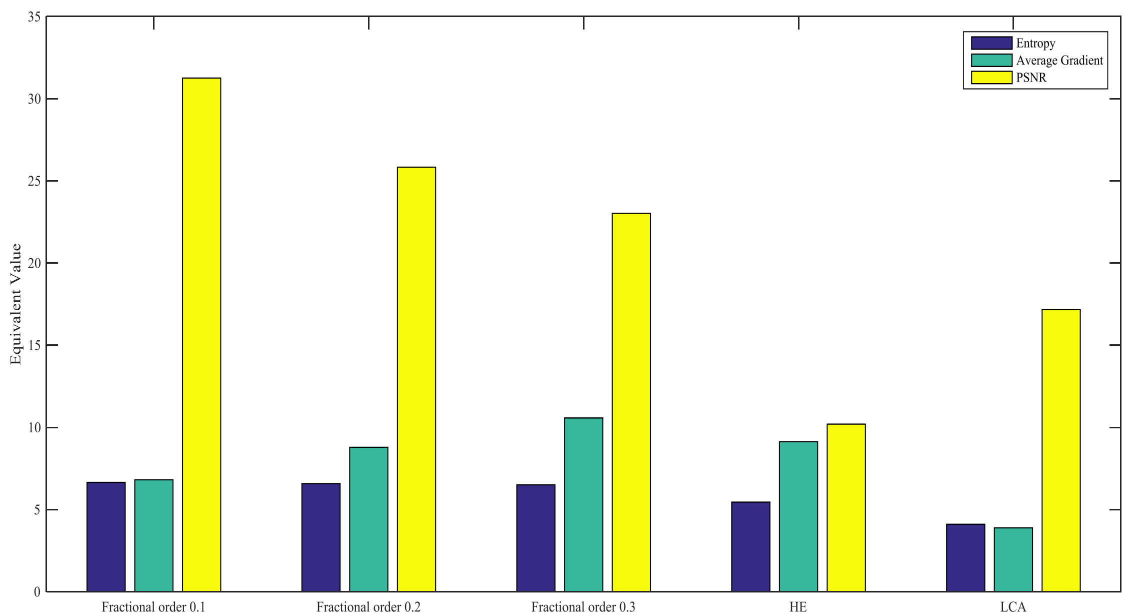


Figure 5.7: Comparison of existing image enhancement methods with equivalent values of IE, AG and PSNR for office_6 (bright light image)

The graph shown in Figure 5.7, is the comparison graph of various existing methods and improved G-L fractional order at orders 0.1, 0.2 and 0.3. In case of order 0.3, all three parameters have more value than HE and LCA. However, in case of improved G-L fractional order 0.1 and 0.2 PSNR is more, but the value of IE and AG is less than the HE and LCA techniques.

5.3.2 Low light image

For low light image, a random low light image is taken from UCID dataset (ucid_00754). HE, LCA and proposed mask are applied on this image. Figures are shown below using various methods:

(a) (b) (c) (d)

Figure 5.8: (a) Input low light Image (b) Image enhanced by HE method (c) Image enhanced by LCA method (d) Image enhanced by proposed mask (improved G-L at 0.3 fractional differential order)

Table 5.5 shows the qualitative analysis for low light image in terms of information entropy, average gradient and peak signal to noise ratio.

Table 5.5 Qualitative evaluation using IE, AG and PSNR (UCID dataset's random low light image ucid_00744)

Techniques / Parameter	Original Image	Histogram Equalization (HE) [4]	Linear Contrast Adjustment (LCA) [5]	Improved G-L Mask at fractional order (0.3)
Information Entropy (IE)	6.8929	5.9454	6.1091	**6.9780**
Average Gradient (AG)	5.1694	13.3571	7.0027	**14.0442**
PSNR	------	8.8243	19.6539	**22.4211**

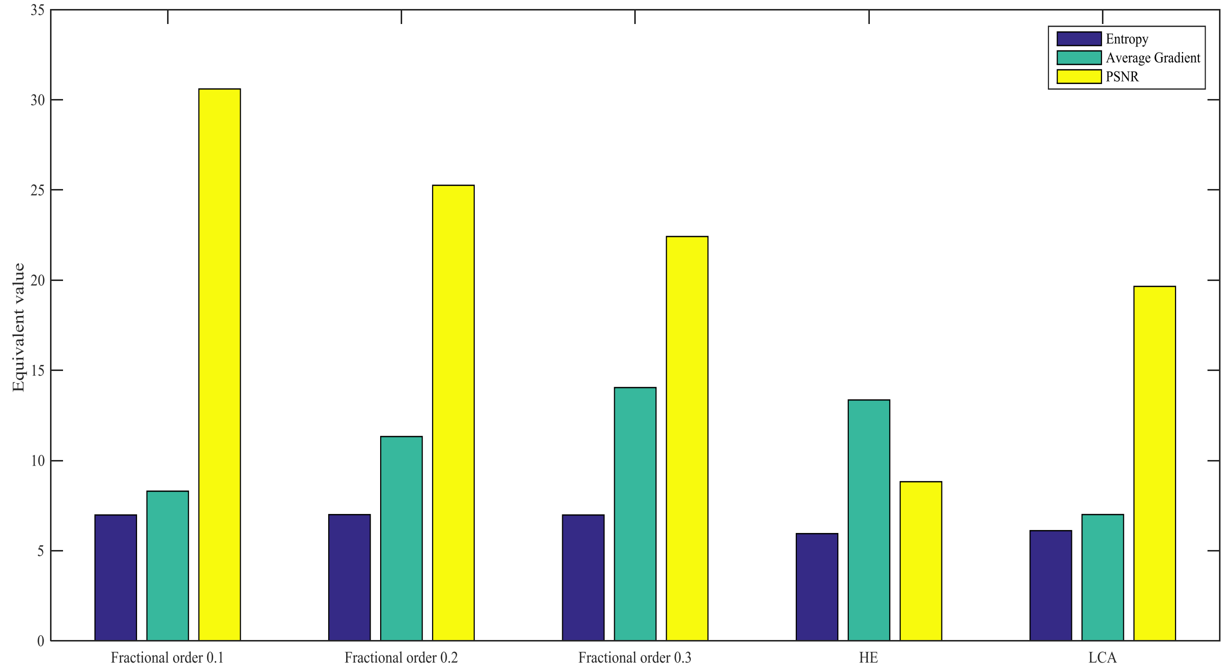


Figure 5.9: Comparison of existing image enhancement methods with equivalent values of IE, AG and PSNR for UCID dataset random image (low light image ucid00754)

These bar graphs show the comparison of the various methods and enhancement criteria. Bar graph is plotted for both the bright light image and low light image. The order chosen is 0.3 because in both, the bar graphs as shown in Figure 5.7, and Figure 5.9, it is clearly seen that at fractional order 0.3, the values of IE, AG, and PSNR are more than the equivalent values of HE and LCA. At fractional order 0.1, PSNR is highest but the value of AG is less than the equivalent value of HE and same in the case of fractional order 0.2. So, to improve all three parameters, value of fractional order is taken 0.3 and the value of IE, AG and PSNR is improved in both the bar graphs.

Improvement is clearly seen when compared to other methods. Generally, if image/video frame have great surface texture elements, its entropy is more noteworthy. The image/frame will be clearer if the value of average gradient is high. High resolution means greater value of average gradient. More the value of PSNR, more similar is the video frame after enhancing to the original video frame. So, aim is to improve all three parameters. The information entropy of image increases on increasing the value of fractional differential order and we obtain the maximum value at order v_{ord} = 0.3 in case of improved G-L. On increasing the value of fractional order above 0.3, resulted in decrease the value of IE because of loss in texture information.

5.3.1 Comparative Analysis of Image Enhancement

The comparison table for Lena image for different histogram equalizations method is presented below. Information Entropy is calculated for different methods.

Table 5.6 Qualitative evaluation of enhancement using IE of Lena image with proposed mask and comparison with existing methods

Methods	IE (information Entropy)
LCA [5]	6.12
BBHE [38]	7.34
DSIHE [48]	7.34
RSIHE [52]	7.35
RSWHE-D [53]	5.80
MMBEBHE [49]	7.34
ESIHE [63]	7.41
CEDHE [64]	6.98
UMHE [33]	7.44
Proposed Mask	**7.85**

The table shown above is the qualitative comparison of information entropy of Lena image between proposed mask and various existing methods [33]. Entropy is better for Lena image as compared to those methods. Also comparison is done for the fractional poisson method. AIV stands for adjust intensity values to specified range presented in [43]. Fraction poisson [43] is the method used for enhancement, as shown in Table 5.7. For comparison, standard datasets are taken namely (i) SVT (street view data) [59] (ii) MSRA data [60]. These results are shown below in Table 5.7.

Table 5.7 Qualitative evaluation of enhancement using PSNR and SSIM of various images (random) of two different datasets

Various Dataset	HE [4]		CLAHE [36]		AIV [5]		Fractional Poisson [43]		Proposed Mask	
	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	**PSNR**	**SSIM**
SVT	15.8	0.72	15.8	0.72	15.8	0.72	24.5	0.96	**30.55**	**0.97**
MSRA	16.6	0.75	16.6	0.75	16.6	0.75	26.9	0.91	**33.98**	**0.94**

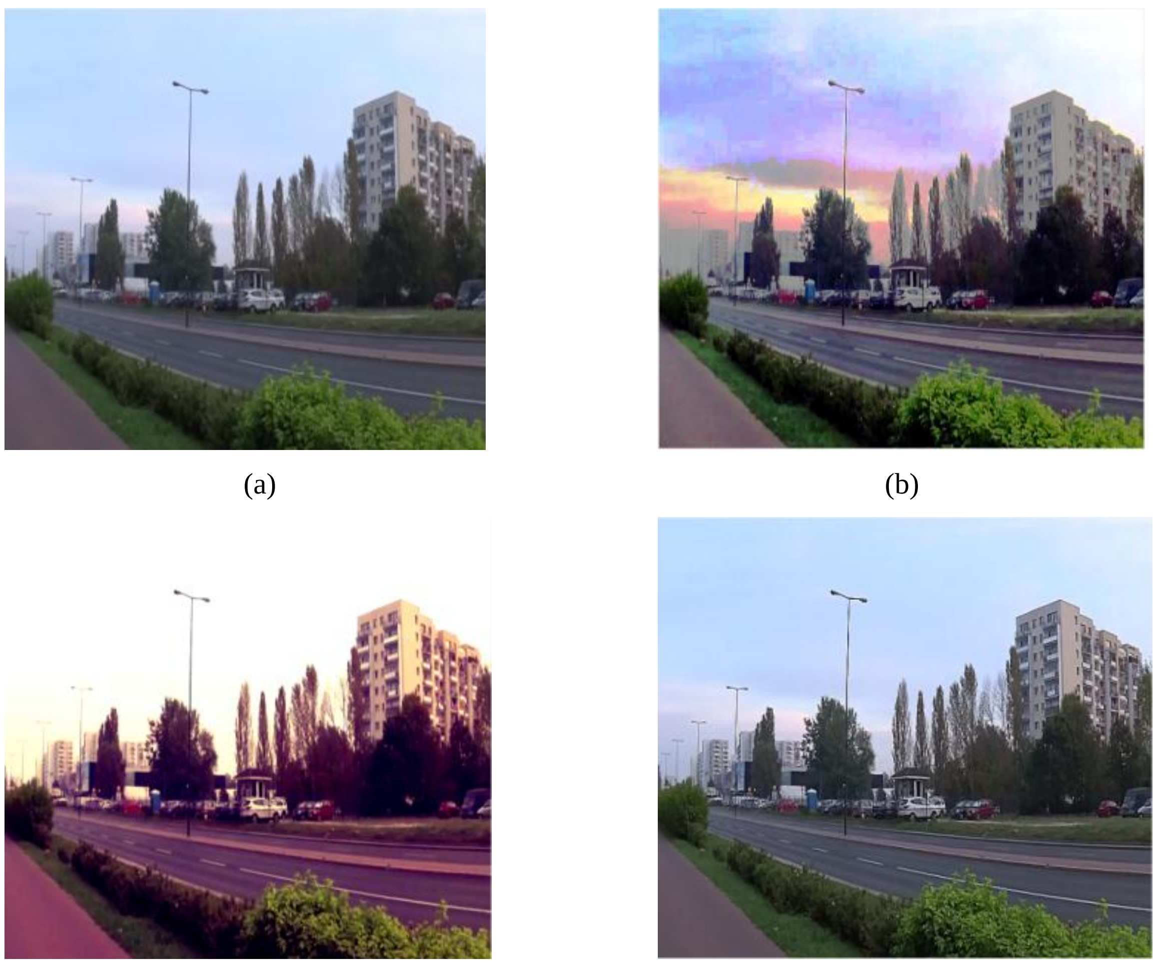
From the above shown Table 5.6, it is observed that when proposed model is compared to the fractional poisson method [36], the value of PSNR and SSIM is improved as compared to other methods. It shows that images after enhancement are more similar to original image using proposed mask as compared to other existing methods.

5.4 VIDEO ENHANCEMENT

In this section, the proposed mask is applied on the bright light videos and low light videos. The comparison of the proposed mask with existing techniques is also presented.

5.4.1 Bright Light Video

In case of video enhancement, first video for enhancement is taken **Bright Light Video** and results shown in the figure below are the results of 1^{st} Frame of bright light video.

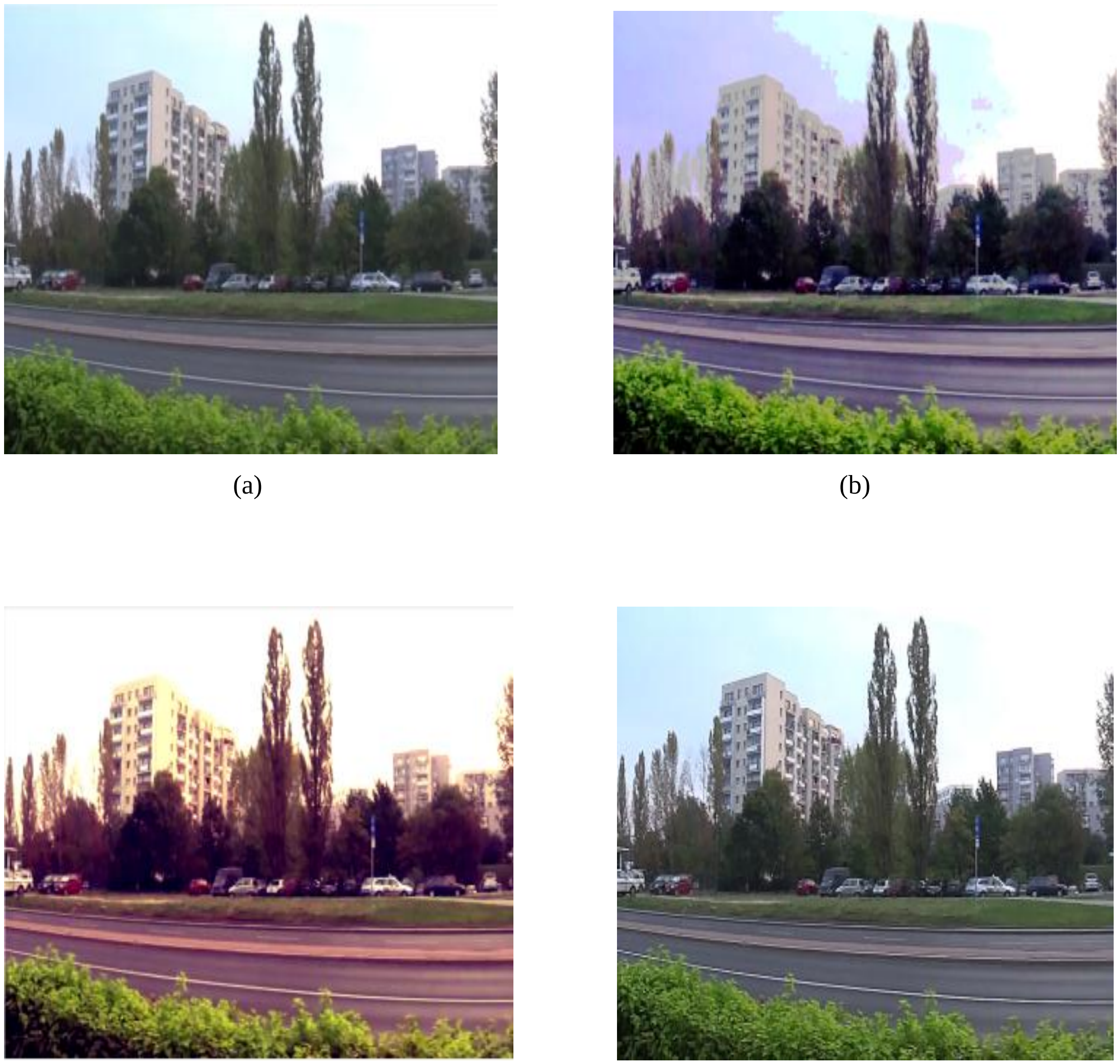


(a) (b)

(c) (d)

Figure 5.10: (a) Input video frame 1 (b) Enhanced frame 1 by HE method (c) Enhanced frame 1 by LCA method (d) Enhanced frame 1 by proposed mask (improved G-L at 0.3 fractional differential order)

Following results shown in the Figure 5.11 are the results of 4^{th} Frame of bright light video.



(a) (b)

(c) (d)

Figure 5.11: (a) Input video (4^{th} frame) (b) Enhanced 4^{th} frame by HE (c) Enhanced 4^{th} frame by LCA (d) Enhanced 4^{th} frame by proposed mask (improved G-L at 0.3 fractional differential order)

It is clearly observed that in case of fractional differential enhancement, distortion in the frames is less as compared to HE and LCA. In frame 4, after enhancement using proposed mask, textural features of the frames are increased. It is seen on the buildings and trees present in the background in the frame when compared to other two methods.

Table 5.8 Qualitative evaluation using IE, AG and PSNR (Frame 1) of Bright light video

Techniques / Parameters	Original Frame	Histogram Equalization (HE) [4]	Linear Contrast Adjustment (LCA) [5]	Improved G-L Mask at fractional order (0.3)
Information Entropy (IE)	7.2229	5.7313	5.4818	**7.2366**
Average Gradient (AG)	3.3825	4.2102	4.8193	**7.9859**
PSNR	------	3.9943	0.0341	**24.4084**

Table 5.9 Qualitative evaluation using IE, AG and PSNR (Frame 2) of Bright light video

Techniques / Parameters	Original Frame	Histogram Equalization (HE) [4]	Linear Contrast Adjustment (LCA) [5]	Improved G-L Mask at fractional order (0.3)
Information Entropy (IE)	7.2449	5.7549	5.5427	**7.2610**
Average Gradient (AG)	3.5039	4.4104	4.9775	**8.1036**
PSNR	------	4.9636	0.9341	**24.8400**

Table 5.10 Qualitative evaluation using IE, AG and PSNR (Frame 3) of Bright light video

Techniques / Parameters	Original Frame	Histogram Equalization (HE) [4]	Linear Contrast Adjustment (LCA) [5]	Improved G-L Mask at fractional order (0.3)
Information Entropy (IE)	7.2757	5.7909	5.8944	**7.2951**
Average Gradient (AG)	3.6649	4.8231	5.1709	**7.6465**
PSNR	------	6.3372	1.9382	**26.0957**

In the above shown Tables, qualitative observations are presented. It is observed that in maximum frames the values of IE, AG and PSNR is better than the HE and LCA methods. So, frames are clearer, have more textural features and more similar to the original frames. In case of HE and LCA, there is loss of more textural features and information.

5.4.2 Low Light video

For the testing of **Low Light Video**, a video is taken from [61]. Figure 5.12 shows, enhanced 1st frame after applying HE, LCA and proposed mask.

Figure 5.12: (a) Input video (1st frame) (b) Enhanced 1st frame by HE (c) Enhanced 1st frame by LCA (d) Enhanced 1st frame by proposed mask (improved G-L at 0.3 fractional differential order)

From the Figure 5.12, it is observed that in case of HE, distortion of the frame takes place and decrease in textural information in case of LCA.

(a)

(b)

(c)

(d)

Figure 5.13: (a) Input video (2nd frame) (b) Enhanced 2nd frame by HE (c) Enhanced 2nd frame by LCA (d) Enhanced 2nd frame by proposed mask (improved G-L at 0.3 fractional differential order)

In the above shown Figure 5.13, frame 2 is extracted from the input video after applying HE, LCA and proposed mask of fractional differential on input low light video [61]. Enhancement is more in case of proposed mask and it is observed in the Figure 5.13 d). The resolution of the cars, background building and tree become clearer in the frame. Distortion is less as compared to HE. Frame is clearer as compared to LCA method. Enhancement is more due to less distortion.

(a) (b) (c) (d)

Figure 5.14: (a) Input video (6th frame) (b) Enhanced (6th frame) by HE (c) Enhanced 6th frame by LCA (d) Enhanced 6th frame by proposed mask (improved G-L at 0.3 fractional differential order)

In the above shown Figure 5.14, frame 6 is extracted from the input video after applying HE, LCA and proposed mask of fractional differential on input low light video [61]. Enhancement in more in case of proposed mask and it is observed in the Figure 5.14 d). It is observed that frames are clearer, more similar to original frames, and have more textural features. In the Tables 5.11, 5.12 and 5.13, qualitative analysis of three parameters for three methods is presented.

Table 5.11 Qualitative evaluation using IE, AG and PSNR (Frame 1) of Low light video

Techniques / Parameters	Original Frame	Histogram Equalization (HE) [4]	Linear Contrast Adjustment (LCA) [5]	Improved G-L Mask at fractional order (0.3)
Information Entropy (IE)	7.0173	5.8815	4.6928	**6.9752**
Average Gradient (AG)	4.6264	5.4162	5.5051	**6.0982**
PSNR	----	12.5678	48.1308	**31.0562**

Table 5.12 Qualitative evaluation using IE, AG and PSNR (Frame 2) of Low light video

Techniques / Parameters	Original Frame	Histogram Equalization (HE) [4]	Linear Contrast Adjustment (LCA) [5]	Improved G-L Mask at fractional order (0.3)
Information Entropy (IE)	7.0750	5.9530	4.8225	**7.0394**
Average Gradient (AG)	4.7872	5.7895	5.5718	**6.3152**
PSNR	------	20.8962	19.8313	**31.4642**

Table 5.13 Qualitative evaluation using IE, AG and PSNR (Frame 3) of Low light video

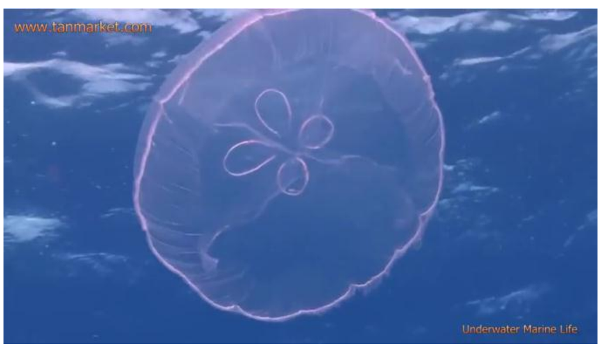
Techniques / Parameters	Original Frame	Histogram Equalization (HE) [4]	Linear Contrast Adjustment (LCA) [5]	Improved G-L Mask at fractional order (0.3)
Information Entropy (IE)	7.0677	5.9509	4.6714	**7.0967**
Average Gradient (AG)	4.3029	4.5025	4.8143	**5.6227**
PSNR	------	4.3242	3.0737	**32.1006**

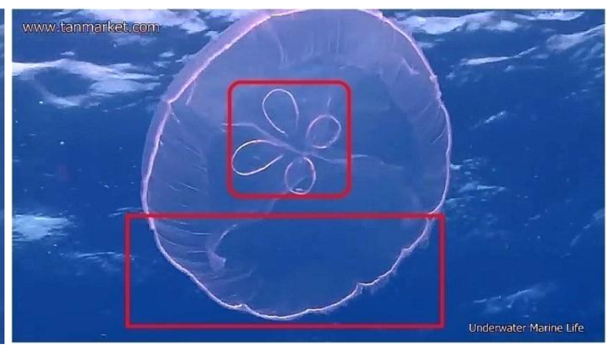
From above Tables, it is observed that the values of IE, AG and PSNR are more in each frame, excerpted from the proposed enhancement mask. There is no occurrence of unnatural noises and artifacts in the proposed enhancement mask as compared to existing methods. It is also observed that distortion is less in case of proposed mask.

5.5 UNDERWATER VIDEO ENHANCEMENT

There is requirement of high resolution for underwater videos and images. Enhancement of underwater videos is an important task. Figure 5.15 shows the enhanced frames of underwater video.

a) Original 1st frame b) Enhanced 1st frame by proposed mask





a) Original 3rd frame b) Enhanced 3rd frame by proposed mask

a) Original 8th frame b) Enhanced 8th frame by proposed mask

Figure 5.15: a) Original frames of underwater video [62]; b) Enhanced frames using proposed mask of improved G-L at fractional order 0.3 (cont.)

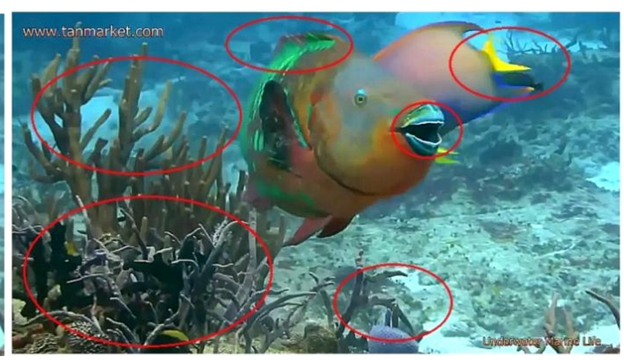
There are two frames, one is original frame and other is enhanced frame from the proposed mask. In the enhanced frames, there are red marked circles. Resolution is increased, which is clearly seen inside the circles. In all the frames shown in Figure 5.15, blurriness is decreased as compared to original frame.

a) Original 10th frame

b) Enhanced 10th frame by proposed mask

a) Original 12th frame

b) Enhanced 12th frame by proposed mask



a) Original 14th frame

b) Enhanced 14th frame by proposed mask

Figure 5.15: a) Original frames of underwater video [62]; b) Enhanced frames using proposed mask of improved G-L at fractional order 0.3 (cont.)

Texture of frames is increased. Edges of the fishes which are seen inside the circle have become clearer using proposed mask of improved G-L fractional differential. So, frames extracted after applying the proposed mask on the input under water video, have more resolution.

5.6 CCTV VIDEO ENHANCEMENT

These days, enhancement of textural features in digital security is an important issue. So, proposed mask is also tested for random theft CCTV video. Enhanced frame using proposed mask is shown below in Figure 5.16

a) Original 1st frame

b) Enhanced 1st frame by proposed mask

a) Original 2nd frame

b) Enhanced 2nd frame by proposed mask

a) Original 3rd frame

b) Enhanced 3rd frame by proposed mask

Figure 5.16: a) Original frames; b) Enhanced frames using proposed mask

On applying proposed mask on the CCTV video, it is observed that frames become clearer than the original frames extracted from the video. In the Figure shown in 5.16, the red marked circles show the clear enhancement as compared to original frame. In Table 5.14, comparison of proposed mask is given in terms of IE, AG and PSNR with the existing methods of HE, LCA and CLAHE.

Table 5.14 Qualitative evaluation using IE, AG and PSNR (Frame 1) of CCTV video

Techniques / Parameters	Original Frame	Histogram Equalization (HE) [4]	Linear Contrast Adjustment (LCA) [5]	CLAHE[36]	Improved G-L Mask at fractional order (0.3)
Information Entropy (IE)	7.5287	7.4874	5.9828	7.6360	**7.7009**
Average Gradient (AG)	6.4106	6.7723	7.7618	10.0847	**14.9566**
PSNR	--------	19.1876	13.8107	7.6247	**20.8059**

Table 5.15 Qualitative evaluation using IE, AG and PSNR (Frame 2) of CCTV video

Techniques / Parameters	Original Frame	Histogram Equalization (HE) [4]	Linear Contrast Adjustment (LCA) [5]	CLAHE[36]	Improved G-L Mask at fractional order (0.3)
Information Entropy (IE)	7.5265	7.4926	5.9824	7.6376	**7.7015**
Average Gradient (AG)	6.4144	6.7769	7.8003	10.1545	**14.9422**
PSNR	-------	38.5884	19.5035	9.7492	**20.8240**

Table 5.16 Qualitative evaluation using IE, AG and PSNR (Frame 3) of CCTV video

Techniques / Parameters	Original Frame	Histogram Equalization (HE) [4]	Linear Contrast Adjustment (LCA) [5]	CLAHE[36]	Improved G-L Mask at fractional order (0.3)
Information Entropy (IE)	7.5288	7.4904	5.9799	7.6389	**7.7010**
Average Gradient (AG)	6.4609	6.8244	7.8554	10.2506	**15.1000**
PSNR	--------	21.2823	22.1102	13.0133	**20.7897**

From the Tables 5.14, 5.15 and 5.16, it is observed that, the value of all three parameters is improved and better as compared to HE, LCA and CLAHE method. The enhancement of CCTV footages is an important issue in present day scenario. It is observed that after applying proposed mask on the CCTV video, frame become clearer as compared to other methods. As value of entropy in fractional differential mask is better than existing methods, amount of information present in the frames is enhanced. Textural features of the frames are also increased.

5.7 RESULTS OF SULFA (DATASET) VIDEO ENHANCEMENT

The proposed mask of video enhancement is applied on the videos of SULFA dataset. The results of some random SULFA videos dataset is presented below in the Table 5.17. Three techniques are compared and results observed are promising and exceptional.

Table 5.17 Qualitative evaluation using IE, AG and PSNR (random frames) of various videos (SULFA video dataset [57]) (C stands for canon_220 camera and F stands for Fuji camera in the Table)

SULFA VIDEO DATASET[57]		**HE [4]**			**LCA [5]**			**Improved G-L Mask of Fractional order v_{ord} = 0.3**		
S. No.	**Videos Name**	**IE**	**AG**	**PSNR**	**IE**	**AG**	**PSNR**	**IE**	**AG**	**PSNR**
1.	C_ball(2)	5.9116	4.5387	16.3095	5.9092	4.2701	19.1831	**7.3945**	**6.8124**	**24.7049**
2.	C_ball(3)	5.9795	13.9676	13.4829	5.3739	4.8589	1.9382	**7.3530**	**17.0493**	**21.1984**
3.	C_ball(4)	5.8666	4.7922	16.0896	5.6667	4.0855	16.3095	**7.4598**	**7.2744**	**24.3780**
4.	C_board(1)	5.9087	3.7051	12.2830	5.0378	2.9231	3.0737	**6.9677**	**4.4422**	**28.7259**
5.	C_book	5.9848	9.5641	13.4829	6.3315	10.7347	12.0072	**7.3829**	**18.0506**	**19.0511**
6.	C_busstop	5.0960	9.1766	3.0737	6.2114	7.4358	3.0737	**6.1907**	**13.7663**	**20.9401**
7.	C_carpark	5.1309	7.4012	3.0737	5.9552	8.8404	3.0737	**6.6507**	**16.7693**	**20.1072**
8.	C_garden	5.9288	17.3989	10.7462	6.9669	17.0393	16.7668	**7.0456**	**29.0155**	**17.4200**
9.	C_outdoor	5.7086	10.6943	4.1029	6.1418	12.4756	3.0737	**6.6332**	**21.5056**	**18.9621**
10.	C_road	4.7068	6.1166	3.0737	5.8034	7.1885	3.0737	**5.9013**	**11.9538**	**22.1078**
11.	C_room	5.9632	7.8713	20.8962	6.4344	8.2562	15.0666	**7.3585**	**13.1608**	**20.7406**
12.	C_street(2)	5.7390	8.0154	31.2288	5.1617	8.0022	22.5557	**7.1401**	**18.0009**	**29.3671**
13.	F_bridge_2	5.9534	9.1919	4.3242	6.8385	10.1928	3.0737	**7.4540**	**15.6928**	**21.6154**
14.	F_Lift	5.9891	6.7028	18.8828	7.3125	6.0381	9.1430	**7.5863**	**9.3616**	**23.5021**
15.	F_street(2)	5.6420	8.2666	14.3264	6.4444	8.6597	20.8962	**6.9365**	**14.9464**	**21.0075**

SULFA VIDEO DATASET[57]		HE [4]			LCA [5]			Improved G-L Mask of Fractional order v_{ord} = 0.3		
S. No.	**Videos Name**	**IE**	**AG**	**PSNR**	**IE**	**AG**	**PSNR**	**IE**	**AG**	**PSNR**
16.	C_flap(1)	5.9339	5.3796	3.0737	5.7809	2.9427	3.0737	**7.4133**	**5.7096**	**25.7244**
17.	C_street(1)	5.4700	4.6717	5.0241	4.8283	4.2211	3.0737	**6.7981**	**8.3860**	**24.7327**
18.	F_ball(3)	5.8860	15.5036	10.5145	6.9969	4.8903	23.5218	**6.3820**	**9.2047**	**27.5236**
19.	F_bridge_1	5.7628	12.3247	3.0737	6.3848	13.9301	4.5438	**6.9753**	**20.6512**	**20.9806**
20.	F_bstop_1	5.9726	6.3979	10.7462	6.3245	6.0914	7.7070	**7.5366**	**11.9453**	**22.4249**
21.	F_bstop_4	5.8392	6.5350	15.2618	6.3782	4.3464	15.8751	**7.1612**	**8.5312**	**24.7327**
22.	F_crpark_1	5.9813	6.2935	13.4829	5.8095	5.1229	13.9794	**7.5620**	**9.9607**	**23.3535**
23.	F_door	5.9928	18.3635	16.7668	7.4218	15.2485	7.8741	**7.6581**	**24.6769**	**17.9515**
24.	F_man(1)	5.9786	6.7482	17.0048	6.8245	5.0038	11.3538	**7.3047**	**8.1351**	**25.0471**
25.	F_man(2)	5.9616	4.4935	19.0460	6.7415	4.7201	15.4614	**7.0709**	**7.9270**	**23.5062**
26.	F_otdoor2	5.9859	10.5689	10.7462	7.1663	8.2426	3.5731	**7.6044**	**13.1225**	**23.4667**
27.	F_otdoor3	5.9697	11.8003	18.3036	7.0325	12.5976	14.3269	**7.3923**	**23.8441**	**18.1578**
28.	F_otdoor4	5.9875	13.2160	10.9842	7.4672	11.9764	10.5145	**7.4877**	**20.8264**	**18.7554**
29.	F_road(1)	5.8792	5.7778	8.6682	5.6580	5.9718	3.0737	**7.0131**	**10.5686**	**22.7491**
30.	F_road(5)	5.4323	5.3211	3.0737	4.3973	2.5024	3.0737	**6.2194**	**6.4153**	**26.9779**

Following observations are made from the Table 5.17:

- In the above table, the results of video enhancement on various videos taken from SULFA video dataset [57] are shown. The results are better than existing two techniques and more promising.
- Distortion in the frames is very less using the proposed mask, as compared to the basic existing approaches.
- Values of IE, AG and PSNR are better in the frames extracted from the input videos on applying proposed mask.
- So, it is observed that textural features of frames after applying the proposed methodology have more resolution and frames are clearer than the original frames.

5. 7. 1 Video Enhancement Comparisons

Comparison is also done for video enhancement of various frames using proposed mask and latest fractional poisson approach [43], and it shows that results are improved. Dataset taken for the comparison is ICDAR 2013 [58] video dataset.

Table 5.18 Qualitative evaluation of enhancement using PSNR and SSIM of various frames (random) of ICDAR 2013 video dataset

Dataset	**HE [4]**		**CLAHE [36]**		**AIV [5]**		**Fractional Poisson [43]**		**Proposed Mask**	
	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	**PSNR**	**SSIM**
ICDAR 2013 VIDEO[58]	18.1	0.73	18.1	0.78	17.7	0.78	24.3	0.95	**26.50**	**0.96**

In ICDAR 2013 video set, value of PSNR and SSIM is improved using proposed methods compared to HE, CLAHE, AIV and Fractional Poisson approach. More the value of PSNR, more similar is the frame after enhancement to the original frame. SSIM is also improved as compared to the other methods. So, there is more SSIM in case of proposed mask as compared to other approaches.

5.8 SUMMARY

Textural features of image and videos are enhanced by employing the fractional differential mask. The realization of improved G-L fractional order mask on various videos, different datasets and on low light, bright light images/videos is simulated in testing software. Image and video enhancement is done using the fractional differential mask. It is examined that the proposed methodology for video enhancement gives us flexibility to enhance various kinds of videos i.e. low light video, bright light video, CCTV footages and under water videos. On comparing the results with various other classical and existing methods, results are exceptionally better and more promising. Sharpening obtained in texture of video frames and images is greater than the HE and its improved versions, contrast adjusting values and other fractional methods like fractional poisson. So, this improved G-L mask is proved to be effective tool for enhancement purpose.

CHAPTER 6

CONCLUDING REMARKS AND FUTURE SCOPE

This chapter describes the contributions presented in previous chapters. The study examines the enhancement of images and videos. Conclusion and future scope has been summarized in this chapter.

6.1 CONCLUSION

In spite of the fact that it is difficult to compute fractional derivatives and integrals, still fractional order calculus is playing an imperative part in numerous designing and research fields. The proposed work is oriented towards the enhancement of images and videos using improved G-L fractional differential. In this thesis, firstly introduction of videos, image texture and texture enhancement of images/video frames is presented. The various techniques of image and video enhancement are discussed in the literature. Realization of improved version of G-L mask is described. However, there are other techniques of enhancement like HE, LCA and R-L fractional mask which enhance the textural features but there are various drawbacks of these existing techniques. The proposed improved G-L mask is efficient and better than existing classical methods. This improved version of the G-L mask is convoluted on the various images and frames excerpted from different types of videos.

When the proposed mask is implemented on different datasets of images and videos by retaining the value of intensity factor γ=1 and increasing fractional order of filter mask, from $0 < v_{ord} < 0.4$, the information entropy gets improved. IE is increased by different factor for distinctive videos and images, thus enhancing textural features of image. Also for large size filter mask, information entropy is more as compared to the case when filter mask of small size is implemented. Increasing fractional order v_{ord} from 0.4 to 1, average gradient increases but, there is loss of information entropy.

The conventional methods have problems of distortion, noises and have oversaturation of colors. The mask which is described is very effective for solving the above problems. In the proposed method, video enhancement is done by the improved G-L fractional Differential Mask. It enhances both the texture and brightness of the videos. The proposed fractional mask provides optimal results for the performance measures such as IE, AG and PSNR at v_{ord} 0.3 when compared to HE and LCA. Moreover, there is minimization of oversaturation. Therefore, it is concluded that the proposed model is exceptionally better and more promising. The information entropy with improved Fractional Differential Mask is more than both HE and LCA techniques, hence providing more texture enhancement. Average gradient has more qualitative value by proposed mask. It is increased by factor 3 to 5. PSNR is improved by 7dB to 12dB in enhanced videos.

6.2 FUTURE ASPECTS

This work can be further extended to outline a Fractional Differential Filter of Digital image utilizing the proposed Fractional Differential Mask. This Fractional Differential mask can be implemented on unique finger impression images and videos, satellite images and videos and for enhancement of foggy videos. Video enhancement methods can be used for enhancing the resolution of video games made for play station PS3, PS4 etc. for better gaming experiences.

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