TRANSPORTATION PROBLEMS IN INTUITIONISTIC FUZZY ENVIRONMENT

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in

MATHEMATICS

by

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CERTIFICATE

This is to certify that the thesis entitled, "**Transportation Problems in Intuitionistic Fuzzy Environment**" submitted by **Gourav Gupta** in the fulfillment of the requirement for the award of the degree of Doctor of Philosophy in the School of Mathematics, Thapar University, Patiala, is a record of candidate's own work carried out by his under our supervision and guidance.

The matter presented in this thesis has not been submitted in part or full for the award of any degree in any other University or Institute.

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It is certified that the thesis is entirely my own. The ideas and references cited herein have been duly acknowledged.

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DEDICATED

TO

MY PARENTS,

MY SUPERVISORS

&

GOD

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Abstract

In the last few years, several methods have been proposed for solving different types of intuitionistic fuzzy transportation problems. In this thesis, limitations and flaws of these existing methods are pointed out. Also, to resolve the flaws as well as to overcome the limitations of the existing method, new methods are proposed.

The thesis comprises seven chapters. A brief outline of the chapters is as follows:

Chapter 1 Introduction

Chapter 1 is introductory in nature. In this chapter, a need of intuitionistic fuzzy transportation problem as well as different types of intuitionistic fuzzy transportation problems are discussed. Furthermore, the existing methods [131,132] for solving intuitionistic fuzzy transportation problems are presented in a detailed manner.

Chapter 2 A simplified method for solving intuitionistic fuzzy transportation problems of type – I

In this chapter, an alternative method for solving intuitionistic fuzzy transportation problems of type – I is proposed. Also, the advantages of proposed method over the existing methods [9,46,48,64-66,118,132] are discussed. Furthermore, to illustrate the proposed method, the numerical examples, considered by Singh and Yadav [132], are solved by the proposed method.

Chapter 3 A simplified method for solving intuitionistic fuzzy transportation problems of type – II

In this chapter, an alternative method for solving intuitionistic fuzzy transportation prob-

lems of type – II is proposed. Also, the advantages of proposed method over the existing methods [1,3,119,131], are discussed. Furthermore, to illustrate the proposed method, the numerical examples, considered by Singh and Yadav [131], are solved by the proposed method.

Chapter 4 Modified approach for solving intuitionistic fully fuzzy transportation problems

Kumar and Hussain [84-86] proposed methods for solving intuitionistic fully fuzzy transportation problems. In this chapter, it is pointed out that for the ranking function, used by Kumar and Hussain [84-86], the linearity property is not satisfying. However, in the existing method [84-86], this property is used. Therefore, the existing methods [84-86] are not valid. Hence, the result of numerical problems, obtained by Kumar and Hussain [84-86] by their proposed method, is also not correct. Furthermore, it is shown that for the ranking function, used by Singh and Yadav [132], linearity property is satisfying. Hence, the existing methods [84-86], will be valid if the ranking function, used by Kumar and Hussain [84-86], is replaced with the ranking function, used by Singh and Yadav [132]. Also, the exact results of numerical problems, considered by Kumar and Hussain [86], are obtained.

Chapter 5 A new method for solving intuitionistic fully fuzzy transportation problems

In this chapter, flaws of the existing methods [18,29,45,49,112,118,123,133,134,138] for solving intuitionistic fully fuzzy transportation problems are pointed out. Also, a new method is proposed for solving intuitionistic fully fuzzy transportation problems. To illustrate the proposed method, the intuitionistic fully fuzzy transportation problem, considered by Roseline and Amirtharaj [123], is solved by proposed method.

Chapter 6 A new method for solving generalized intuitionistic fully fuzzy transportation problems

Chakraborty et al. [18] proposed the arithmetic operations on generalized trapezoidal intuitionistic fuzzy numbers and used these arithmetic operations to find the solution of such intuitionistic fully fuzzy transportation problems in which cost, availability and demand all are represented by generalized trapezoidal intuitionistic fuzzy numbers. In this chapter, it is shown that Chakraborty et al. [18] have used the property $R\left(\tilde{A}^I \otimes \tilde{B}^I\right) = R\left(\tilde{A}^I\right) \times R\left(\tilde{B}^I\right)$ in their proposed method. While, for the ranking function *R*, considered by Chakraborty et al. [18], this property is not satisfying. Hence, it is not genuine to used the method, proposed by Chakraborty et al. [18], to find the solution of generalized intuitionistic fully fuzzy transportation problem. Furthermore, a new method is proposed to resolve the flaws of the existing method [18]. To illustrate the proposed method, the generalized intuitionistic fully fuzzy transportation problem, considered by Chakraborty et al. [18], is solved by proposed method.

Chapter 7 Future scope

It is noticed that ranking of generalized exponential trapezoidal fuzzy numbers, obtained by using the existing method [121], is independent from height of generalized exponential trapezoidal fuzzy numbers. While, the ranking of generalized exponential trapezoidal fuzzy numbers should be dependent on its height. Hence, it is not genuine to use the existing method [121] for comparing the generalized exponential trapezoidal fuzzy numbers. In this chapter, the flaws of the existing method [121] are pointed out and a modified method for ranking of generalized exponential trapezoidal fuzzy numbers is proposed. In future, the proposed ranking method may be extended for generalized exponential trapezoidal intuitionistic fuzzy numbers and a new method may be proposed to find the solution of generalized exponential trapezoidal intuitionistic fully fuzzy transportation problems (transportation problems in which cost, availability and demand are represented by generalized exponential trapezoidal intuitionistic fuzzy numbers).

List of Publications

1. Gupta G., Kumar A. and Sharma M. K., A note on "A new method for solving fuzzy linear programming problems based on the fuzzy linear complementary problem (FLCP)", International Journal of Fuzzy Systems, Vol. 18, pp. 333-337, 2016. **(Impact factor – 1.095) (SCI).**
2. Gupta G., Kumar A. and Appadoo S. S., A note on ''Ranking generalized exponential trapezoidal fuzzy numbers based on variance'', Journal of Intelligent and Fuzzy Systems, Vol. 31, pp. 213-215, 2016. **(Impact factor – 1.812) (SCI)**.
3. Gupta G., Kumar A. and Sharma M. K., Additive and multiplicative inverse of generalized fuzzy numbers with different left heights and right heights, "Proceedings of the Third International Conference on Soft Computing for Problem Solving: SocProS 2013, Published as the book, Vol. 258, pp. 535-538, 2014 in the bool series "Advances in Intelligent System and Computing".
4. Gupta G., Kumar A. and Sharma M. K., A new method for solving transportation problem with generalized fuzzy sets having different left heights and right heights **(Communicated in International Journal of Fuzzy Systems) (SCI)**.
5. Gupta G., Kumar A. and Sharma M. K., Intuitionistic fuzzy optimal solution of transportation problem under impreciseness **(Communicated in Hacettepe Journal of Mathematics and Statistics) (SCI)**.
6. Gupta G., Kumar A. and Sharma M. K., A new method for finding exact fuzzy optimal solution of fully fuzzy linear programming problems **(Presented in an International Conference on Recent Advances in Optimization Theory and Application (RAOTA-2016), organized in memory of Prof. M. C. Puri (1939-2005) and organized by Mathematical Programming Group, Delhi Department**

of Operational Research, University of Delhi, Department of Mathematics, University of Delhi at University of Delhi on January 30 – 31, 2016).

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Chapter 1

Introduction

1.1 Introduction

In several real life situations, there is need to transport the product from different sources to different destinations. In the literature, different methods have been proposed to find the best way of transporting the product i.e., to find the quantity of the product to be supplied from different sources to different destinations so that the total transportation cost is minimum. However, the classical methods (Modified Distribution method or linear programming approaches etc.) are proposed by assuming that the values of all the parameters (cost, availability and demand) are precise and hence can be represented as a real numbers. However, none of the classical methods can be used if there exist impreciseness about some or all the parameters. In the literature [7,34,36,51,52,56,63,78,87-91,96,99,110,113-115,125], fuzzy numbers [144] are used to represent these imprecise data (cost, availability and demand). The transportation problems in which some or all the parameters are represented by fuzzy numbers may be classified as:

(i) Transportation problems in which availability and demand are represented as fuzzy numbers whereas cost for transporting unit quantity of the product from a particular source to a particular destination is represented by a real numbers, is named as fuzzy transportation problems of type – I.

(ii) Transportation problems in which cost for transporting unit quantity of the product from a particular source to a particular destination is represented by a fuzzy numbers whereas availability and demand are represented as real numbers, is named as fuzzy transportation problems of type – II.

(iii) Transportation problems in which all the parameters i.e., cost for transporting unit quantity of the product from a particular source to a particular destination, availability and demand, are represented as fuzzy numbers, is named as fully fuzzy transportation problems.

In the literature [1,3,9,18,29,45,46,48,49,64-66,84-86,112,118,119,123,131-134,138], it is pointed out that if there exist hesitation with impreciseness about the data then such type of data can not be represented as fuzzy numbers [4] and hence intuitionistic fuzzy numbers is used to represent such data. The transportation problems in which some or all data are represented as intuitionistic fuzzy numbers may be classified as:

(i) Transportation problems in which availability and demand are represented as intuitionistic fuzzy numbers whereas cost for transporting unit quantity of the product from a particular source to a particular destination is represented by real numbers, is named as intuitionistic fuzzy transportation problems of type – I [9,46,48,64-66,118, 132].

(ii) Transportation problems in which cost for transporting unit quantity of the product from a particular source to a particular destination is represented by intuitionistic fuzzy numbers whereas availability and demand are represented as real numbers, is named as intuitionistic fuzzy transportation problems of type – II [1,3,119,131].

(iii) Transportation problems in which all the parameters i.e., cost for transporting unit quantity of the product from a particular source to a particular destination, availability and demand, are represented as intuitionistic fuzzy numbers, is named as intuitionistic fully fuzzy transportation problems [18,29,45,49,84-86,112,123,133,134,138].

It is pertinent to mention that in the literature, the classical methods (North West Corner method, Least Cost method, Vogel's Approximation method etc.) are extended to intuitionistic fuzzy methods (Intuitionistic fuzzy North West Corner method, Intuitionistic fuzzy Least Cost method, Intuitionistic fuzzy Vogel's Approximation method etc.) for solving different types of intuitionistic fuzzy transportation problems. The only difference between the classical methods and the intuitionistic fuzzy methods is that in the intuitionistic fuzzy methods, the arithmetic operations of intuitionistic fuzzy numbers are used instead of arithmetic operations of real numbers. Also, it is pertinent to mention that there is no unique way to compare intuitionistic fuzzy numbers. Therefore, different methods have been proposed for same type of intuitionistic fuzzy transportation problems by changing the method for comparing intuitionistic fuzzy numbers.

Keeping the same in mind, instead of explaining the work of all the published papers, only the work done in these recently published papers [131,132] are discussed in this chapter in a detailed manner.

1.1.1 Singh and Yadav method for solving intuitionistic fuzzy transportation problems of type – I

In this section, intuitionistic fuzzy methods (Intuitionistic fuzzy North – West Corner method, Intuitionistic fuzzy Least Cost method, Intuitionistic fuzzy Vogel's Approximation method) to find initial basic feasible solutions as well as the Intuitionistic fuzzy Modified Distribution method to find optimal solution of intuitionistic fuzzy transportation problems of type – I, proposed by Singh and Yadav [132], are presented in a brief manner.

1.1.1.1 Intuitionistic fuzzy methods to find initial basic feasible solutions

In this section, the intuitionistic fuzzy methods (Intuitionistic fuzzy North – West Corner method, Intuitionistic fuzzy Least Cost method, Intuitionistic fuzzy Vogel's Approximation method) to find the initial basic feasible solutions of intuitionistic fuzzy transportation problems of type – I, proposed by Singh and Yadav [132], is presented in a brief manner.

1.1.1.1.1 Intuitionistic fuzzy North – West Corner method for intuitionistic fuzzy transportation problems of type – I

Singh and Yadav [132] proposed an intuitionistic fuzzy North – West Corner method to find the initial basic feasible solution of intuitionistic fuzzy transportation problem of type – I.

The steps of this method are as follows:

Step 1: Start from the north – west corner cell, say (i, j), of intuitionistic fuzzy transportation problem. To assign this (i, j) cell, there are three cases which may arise: either $\min\left\{R\left(\tilde{a}_i^I\right), R\left(\tilde{b}_j^I\right)\right\} = R\left(\tilde{a}_i^I\right)$ or $\min\left\{R\left(\tilde{a}_i^I\right), R\left(\tilde{b}_j^I\right)\right\} = R\left(\tilde{b}_j^I\right)$ or $\min\left\{R\left(\tilde{a}_i^I\right), R\left(\tilde{b}_j^I\right)\right\} = R\left(\tilde{a}_i^I\right) = R\left(\tilde{b}_j^I\right)$.

(i) If $\min\left\{R\left(\tilde{a}_i^I\right), R\left(\tilde{b}_j^I\right)\right\} = R\left(\tilde{a}_i^I\right)$, then assign $\tilde{a}_i^I$ to (i, j) cell. Block the i^{th} row of given intuitionistic fuzzy transportation problem. Replace $\tilde{b}_j^I$ with $\tilde{b}_j^I \ominus \tilde{a}_i^I$ and then go to Step 2.

(ii) If $\min\left\{R\left(\tilde{a}_i^I\right), R\left(\tilde{b}_j^I\right)\right\} = R\left(\tilde{b}_j^I\right)$, then assign $\tilde{b}_j^I$ to (i, j) cell. Block the j^{th} column of given intuitionistic fuzzy transportation problem. Replace $\tilde{a}_i^I$ with $\tilde{a}_i^I \ominus \tilde{b}_j^I$ and then go to Step 2.

(iii) If $R\left(\tilde{a}_i^I\right) = R\left(\tilde{b}_j^I\right)$, this implies that $R\left(\tilde{a}_i^I\right) \le R\left(\tilde{b}_j^I\right)$ and $R\left(\tilde{a}_i^I\right) \ge R\left(\tilde{b}_j^I\right)$. Therefore, follow any one of the cases stated above, but not both together.

Step 2: Repeat Step 1, until all the availability and demands are satisfied.

Step 3: The initial intuitionistic fuzzy basic feasible solution is $\{\tilde{x}_{ij}^I\}$ and the initial intuitionistic fuzzy transportation cost is $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \times \tilde{x}_{ij}^I$.

1.1.1.1.2 Intuitionistic fuzzy Least Cost method for intuitionistic fuzzy transportation problems of type – I

Singh and Yadav [132] proposed an intuitionistic fuzzy Least Cost method to find the initial basic feasible solution of intuitionistic fuzzy transportation problem of type – I.

The steps of this method are as follows:

Step 1: Find $\min\{c_{ij}; i=1,2,...,m, j=1,2,...,n\}$. Let it be at (i, j) cell. To assign this (i, j) cell, there are three cases which may arise: either $\min\{R(\tilde{a}_i^I), R(\tilde{b}_j^I)\} = R(\tilde{a}_i^I)$ or $\min\{R(\tilde{a}_i^I), R(\tilde{b}_j^I)\} = R(\tilde{b}_j^I)$ or $\min\{R(\tilde{a}_i^I), R(\tilde{b}_j^I)\} = R(\tilde{a}_i^I) = R(\tilde{b}_j^I)$.

(i) If $\min\{R(\tilde{a}_i^I), R(\tilde{b}_j^I)\} = R(\tilde{a}_i^I)$, then assign $\tilde{a}_i^I$ to (i, j) cell. Block the i^{th} row of given intuitionistic fuzzy transportation problem. Replace $\tilde{b}_j^I$ with $\tilde{b}_j^I \ \Theta\ \tilde{a}_i^I$ and then go to Step 2.

(ii) If $\min\{R(\tilde{a}_i^I), R(\tilde{b}_j^I)\} = R(\tilde{b}_j^I)$, then assign $\tilde{b}_j^I$ to (i, j) cell. Block the j^{th} column of given intuitionistic fuzzy transportation problem. Replace $\tilde{a}_i^I$ with $\tilde{a}_i^I \,\Theta\, \tilde{b}_j^I$ and then go to Step 2.

(iii) If $R(\tilde{a}_i^I) = R(\tilde{b}_j^I)$, this implies that $R(\tilde{a}_i^I) \le R(\tilde{b}_j^I)$ and $R(\tilde{a}_i^I) \ge R(\tilde{b}_j^I)$. Therefore, follow any one of the cases stated above, but not both together.

Step 2: Repeat Step 1, until all the availability and demands are satisfied.

Step 3: The initial intuitionistic fuzzy basic feasible solution is $\left\{\tilde{x}_{ij}^I\right\}$ and the initial intuitionistic fuzzy transportation cost is $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \times \tilde{x}_{ij}^I$.

1.1.1.1.3 Intuitionistic fuzzy Vogel's Approximation method for intuitionistic fuzzy transportation problems of type – I

Singh and Yadav [132] proposed an intuitionistic fuzzy Vogel's Approximation method to find the initial basic feasible solution of intuitionistic fuzzy transportation problem of type – I.

The steps of this method are as follows:

Step 1: Find the penalty of each row by taking the difference of smallest entry and next smallest entry of corresponding row and similarly find the penalty of each column by taking the difference of smallest entry and next smallest entry of corresponding column.

Step 2: Choose a row or column having largest penalty ranking value among all rows and columns. Let it be p^{th} row. Select a cell (p, j) which has $\min\left\{c_{pj}; j = 1, 2, ..., n\right\}$. To assign this (p, j) cell, there are three cases which may arise: either $\min\left\{R\left(\tilde{a}_i^I\right), R\left(\tilde{b}_j^I\right)\right\} = R\left(\tilde{a}_i^I\right)$ or $\min\left\{R\left(\tilde{a}_i^I\right), R\left(\tilde{b}_j^I\right)\right\} = R\left(\tilde{b}_j^I\right)$ or $\min\left\{R\left(\tilde{a}_i^I\right), R\left(\tilde{b}_j^I\right)\right\} = R\left(\tilde{a}_i^I\right) = R\left(\tilde{b}_j^I\right)$.

(i) If $\min\left\{R\left(\tilde{a}_i^I\right), R\left(\tilde{b}_j^I\right)\right\} = R\left(\tilde{a}_i^I\right)$, then assign $\tilde{a}_i^I$ to (i, j) cell. Block the i^{th} row of given intuitionistic fuzzy transportation problem. Replace $\tilde{b}_j^I$ with $\tilde{b}_j^I \ominus \tilde{a}_i^I$ and then go to Step 3.

(ii) If $\min\left\{R\left(\tilde{a}_i^I\right), R\left(\tilde{b}_j^I\right)\right\} = R\left(\tilde{b}_j^I\right)$, then assign $\tilde{b}_j^I$ to (i, j) cell. Block the j^{th} column of given intuitionistic fuzzy transportation problem. Replace $\tilde{a}_i^I$ with $\tilde{a}_i^I \ominus \tilde{b}_j^I$ and then go to Step 3.

(iii) If $R\left(\tilde{a}_i^I\right) = R\left(\tilde{b}_j^I\right)$, this implies that $R\left(\tilde{a}_i^I\right) \le R\left(\tilde{b}_j^I\right)$ and $R\left(\tilde{a}_i^I\right) \ge R\left(\tilde{b}_j^I\right)$. Therefore, follow any one of the cases stated above, but not both together.

Step 3: Repeat Step 1 and Step 2, until all the intuitionistic fuzzy availability and intuitionistic fuzzy demands are satisfied.

Step 4: The initial intuitionistic fuzzy basic feasible solution is $\left\{\tilde{x}_{ij}^I\right\}$ and the initial intuitionistic fuzzy transportation cost is $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \times \tilde{x}_{ij}^I$.

1.1.1.2 Intuitionistic fuzzy Modified Distribution method to find optimal solution for intuitionistic fuzzy transportations problem of type – I

Singh and Yadav [132] proposed an intuitionistic fuzzy Modified Distribution method to find optimal solution of intuitionistic fuzzy transportation problem of type – I.

The steps of this method are as follows:

Step 1: Find the initial intuitionistic fuzzy basic feasible solution by using any of these three methods: intuitionistic fuzzy North – West Corner method, intuitionistic fuzzy Least Cost method, intuitionistic fuzzy Vogel's Approximation method.

Step 2: Take dual variables u_i and v_j corresponding to i^{th} row and j^{th} column respectively such that $u_i + v_j = c_{ij}$ for the basic cell (i, j). Find the value of u_i and v_j by using the relations $u_i = c_{ij} - v_j$ or $v_j = c_{ij} - u_i$.

Step 3: Calculate $d_{ij} = u_i + v_j - c_{ij}$ for all non-basic cells.

(i) If $d_{ij} \le 0$ for all non-basic cells, then the initial intuitionistic fuzzy basic feasible solution is the intuitionistic fuzzy optimal solution.

(ii) If there is at least one d_{ij} for which $d_{ij} > 0$, then go to Step 4.

Step 4: Select a cell $\{(i, j); \max\{d_{ij}; \text{for all non-basic cells}\}\}$ and assign a quantity $\tilde{\theta}$ to this cell.

Now making a loop, move horizontally and vertically from θ - cell to the nearest basic cell and turn to loop to another nearest basic cell. In this way, the loop will return to θ - cell. The value of $\tilde{\theta}$ is the minimum quantity at the corner basic cell from which $\tilde{\theta}$ is subtracted.

Step 5: Add and subtract $\tilde{\theta}$ successively to/from the assigned intuitionistic fuzzy quantity in the basic cell from where the loop takes turn. The value of $\tilde{\theta}$ will be the minimum of those assigned intuitionistic fuzzy quantity from which $\tilde{\theta}$ is subtracted. While inserting this value of $\tilde{\theta}$, a basic cell assume zero value and this cell becomes non-basic cell. Resultant problem gives the improved intuitionistic fuzzy basic feasible solution.

Step 6: Repeat Step 1 to Step 5 until $d_{ij} \le 0; \forall i, j$.

Step 7: Using the intuitionistic fuzzy optimal solution $\{\tilde{x}_{ij}^I\}$, obtained in Step 6, the optimal intuitionistic fuzzy transportation cost is $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \times \tilde{x}_{ij}^I$.

1.1.2 Solution of intuitionistic fuzzy transportation problem of type – I obtained by Singh and Yadav

The aim of Chapter 2 is to propose an alternative method to find initial basic feasible solution and optimal solution of intuitionistic fuzzy transportation problem of type – I which is computationally simple as compared to the existing methods [132]. To show that proposed alternative method is computationally simple as compared to the existing method [132], there is need to solve the same problem by the existing method [132] as well as by the proposed method. In this section, the existing intuitionistic fuzzy transportation problem of type – I, presented by Table 1.1, is solved by the existing method [132]. The same problem will be solved by proposed method in Chapter 2.

Table 1.1: Intuitionistic fuzzy transportation problem [132]

Sources	Destinations				Int. fuzzy availability $\left(\tilde{a}_i^I\right)$
	D_1	D_2	D_3	D_4	
S_1	16	1	8	13	(2,4,5;1,4,6)
S_2	11	4	7	10	(4,6,8;3,6,9)
S_3	8	15	9	2	(3,7,12;2,7,13)
S_4	6	12	5	14	(8,10,13;5,10,16)
Int. fuzzy demand $\left(\tilde{b}_j^I\right)$	(3,4,6;1,4,8)	(2,5,7;1,5,8)	(10,15,20;8,15,22)	(2,3,5;1,3,6)	(17,27,38; 11,27,44)

1.1.2.1 Initial basic feasible solution

Using the existing method [132], the initial basic feasible solution of the intuitionistic fuzzy transportation problem of type – I, presented by Table 1.1, can be obtained as follows:

1.1.2.1.1 Intuitionistic fuzzy North – West Corner method

Using the existing intuitionistic fuzzy North – West Corner method [132], the initial basic feasible solution of the intuitionistic fuzzy transportation problem, presented in Table 1.1, can be obtained as follows:

Step 1: The availability and demand, corresponding to North – West Corner cell i.e., (1, 1), is (2,4,5;1,4,6) and (3, 4, 6; 1, 4, 8) respectively. Since, $\left[R(2,4,5;1,4,6)=3.75\right]<\left[R(3,4,6;1,4,8)=4.25\right]$. So, using Step 1 of existing method [132], discussed in Section 1.1.1.1.1, $\tilde{a}_1^I=(2,4,5;1,4,6)$ will be assigned in the North – West Corner cell of the Table 1.1, the first row will be blocked and the remaining demand of destination D_1 will be $(3,4,6;1,4,8)\,\Theta\,(2,4,5;1,4,6)=(-2,0,4;-5,0,7)$, as shown in Table 1.2.

Table 1.2: First allocation by Intuitionistic fuzzy North – West Corner method

Sources	Destinations				Int. fuzzy availability $\left(\tilde{a}_i^I\right)$
	D_1	D_2	D_3	D_4	
S_1	16 **(2,4,5;1,4,6)**	1	8	13	~~(2,4,5;1,4,6)~~
S_2	11	4	7	10	(4,6,8;3,6,9)
S_3	8	15	9	2	(3,7,12;2,7,13)
S_4	6	12	5	14	(8,10,13;5,10,16)
Int. fuzzy demand $\left(\tilde{b}_j^I\right)$	~~(3,4,6;1,4,8)~~ $(-2,0,4;-5,0,7)$	(2,5,7;1,5,8)	(10,15,20; 8,15,22)	(2,3,5;1,3,6)	

Step 2: By repeating the Step 1, the next allocations are shown in Table 1.3 to Table 1.7.

Table 1.3: Second allocation by Intuitionistic fuzzy North – West Corner method

Sources	Destinations				Int. fuzzy availability $\left(\tilde{a}_i^I\right)$
	D_1	D_2	D_3	D_4	
S_2	11 **(–2,0,4;–5,0,7)**	4	7	10	~~(4,6,8;3,6,9)~~ (0,6,10;–4,6,14)
S_3	8	15	9	2	(3,7,12;2,7,13)

S_4	6	12	5	14	(8,10,13;5,10,16)
Int. fuzzy demand $\left(\tilde{b}_j^I\right)$	~~(−2,0,4; −5,0,7)~~	(2,5,7;1,5,8)	(10,15,20; 8,15,22)	(2,3,5;1,3,6)	

Table 1.4: Third allocation by Intuitionistic fuzzy North – West Corner method

Sources	Destinations			Int. fuzzy availability $\left(\tilde{a}_i^I\right)$
	D_2	D_3	D_4	
S_2	4 **(2,5,7;1,5,8)**	7	10	~~(0,6,10; −4,6,14)~~ (−7,1,8; −12,1,13)
S_3	15	9	2	(3,7,12;2,7,13)
S_4	12	5	14	(8,10,13;5,10,16)
Int. fuzzy demand $\left(\tilde{b}_j^I\right)$	~~(2,5,7;1,5,8)~~	(10,15,20;8,15,22)	(2,3,5;1,3,6)	

Table 1.5: Forth allocation by Intuitionistic fuzzy North – West Corner method

Sources	Destinations		Int. fuzzy availability $\left(\tilde{a}_i^I\right)$
	D_3	D_4	
S_2	7 **(−7,1,8; −12,1,13)**	10	~~(−7,1,8; −12,1,13)~~
S_3	9	2	(3,7,12;2,7,13)
S_4	5	14	(8,10,13;5,10,16)
Int. fuzzy demand $\left(\tilde{b}_j^I\right)$	~~(10,15,20;8,15,22)~~ (2,14,27; −5,14,34)	(2,3,5;1,3,6)	

Table 1.6: Fifth allocation by Intuitionistic fuzzy North – West Corner method

Sources	Destinations		Int. fuzzy availability $\left(\tilde{a}_i^I\right)$
	D_3	D_4	
S_3	9 **(3,7,12;2,7,13)**	2	~~(3,7,12;2,7,13)~~
S_4	5	14	(8,10,13;5,10,16)
Int. fuzzy demand $\left(\tilde{b}_j^I\right)$	~~(2,14,27; −5,14,34)~~ (−10,7,24; −8,7,32)	(2,3,5;1,3,6)	

Table 1.7: Sixth and Seventh allocation by Intuitionistic fuzzy North – West Corner method

Sources	Destinations		Int. fuzzy availability $\left(\tilde{a}_i^I\right)$
	D_3	D_4	
S_4	5 **(−10,7,24; −8,7,32)**	14 **(2,3,5;1,3,6)**	~~(8,10,13;5,10,16)~~
Int. fuzzy demand $\left(\tilde{b}_j^I\right)$	~~(−10,7,24; −8,7,32)~~	~~(2,3,5;1,3,6)~~	

Step 3: The initial basic feasible solution, obtained by using Table 1.2 to Table 1.7, is shown in Table 1.8. Using the initial basic feasible solution, shown in Table 1.8, the obtained initial intuitionistic fuzzy transportation cost is $16\times(2,4,5;1,4,6)\oplus 11\times(-2,0,4;-5,0,7)\oplus 4\times$ $(2,5,7;1,5,8)\oplus 7\times(-7,1,8;-12,1,13)\oplus 9\times(3,7,12;2,7,13)\oplus 5\times(-10,7,24;-18,7,32)\oplus 14\times$ $(2,3,5;1,3,6)=(-26,231,506;-177,231,657)$.

Table 1.8: Stating basic feasible solution by Intuitionistic fuzzy North – West Corner method

Sources	Destinations			
	D_1	D_2	D_3	D_4
S_1	16 **(2,4,5;1,4,6)**	1	8	13
S_2	11 **(−2,0,4; −5,0,7)**	4 **(2,5,7;1,5,8)**	7 **(−7,1,8; −12,1,13)**	10
S_3	8	15	9 **(3,7,12;2,7,13)**	2
S_4	6	12	5 **(−10,7,24; −18,7,32)**	14 **(2,3,5;1,3,6)**

1.1.2.1.2 Intuitionistic fuzzy Least Cost method

Using the existing intuitionistic fuzzy Least Cost method [132], the initial basic feasible solution of existing intuitionistic fuzzy transportation problem can be obtained as follows:

Step 1: The availability and demand, corresponding to a cell having least cost i.e., (1, 2), is (2,4,5;1,4,6) and (2,5,7;1,5,8) respectively. Since, $[R(2,4,5;1,4,6)=3.75]<[R(2,5,7;1,5,8)=4.75]$. So, using Step 1 of existing method [132], discussed in Section 1.1.1.1.2, $\tilde{a}_1^I=(2,4,5;1,4,6)$ will be assigned in the least cost cell of the Table 1.1, the first row will be blocked and the remaining demand of destination D_2 will be $(2,5,7;1,5,8)\,\Theta\,(2,4,5;1,4,6)=(-3,1,5;-5,1,7)$, as shown in Table 1.9.

Table 1.9: First allocation by Intuitionistic fuzzy Least Cost method

Sources	Destinations				Int. fuzzy availability $\left(\tilde{a}_i^I\right)$
	D_1	D_2	D_3	D_4	
S_1	16	1 **(2,4,5;1,4,6)**	8	13	~~(2,4,5;1,4,6)~~
S_2	11	4	7	10	(4,6,8;3,6,9)
S_3	8	15	9	2	(3,7,12;2,7,13)
S_4	6	12	5	14	(8,10,13;5,10,16)
Int. fuzzy demand $\left(\tilde{b}_j^I\right)$	(3,4,6;1,4,8)	~~(2,5,7;1,5,8)~~ (−3,1,5; −5,1,7)	(10,15,20; 8,15,22)	(2,3,5;1,3,6)	

Step 2: Repeating the Step 1, the next allocations are shown in Table 1.10 to Table 1.14.

Table 1.10: Second allocation by Intuitionistic fuzzy Least Cost method

Sources	Destinations				Int. fuzzy availability $\left(\tilde{a}_i^I\right)$
	D_1	D_2	D_3	D_4	
S_2	11	4	7	10	(4,6,8;3,6,9)
S_3	8	15	9	2 **(2,3,5;1,3,6)**	~~(3,7,12;2,7,13)~~ (−2,4,10; −4,4,7)
S_4	6	12	5	14	(8,10,13;5,10,16)
Int. fuzzy demand $\left(\tilde{b}_j^I\right)$	(3,4,6;1,4,8)	(−3,1,5; −5,1,7)	(10,15,20; 8,15,22)	~~(2,3,5;1,3,6)~~	

Table 1.11: Third allocation by Intuitionistic fuzzy Least Cost method

Sources	Destinations			Int. fuzzy availability $\left(\tilde{a}_i^I\right)$
	D_1	D_2	D_3	
S_2	11	4 **(−3,1,5; −5,1,7)**	7	~~(4,6,8;3,6,9)~~ (-1,5,11;-4,5,14)
S_3	8	15	9	(−2,4,10; −4,4,7)
S_4	6	12	5	(8,10,13;5,10,16)
Int. fuzzy demand $\left(\tilde{b}_j^I\right)$	(3,4,6;1,4,8)	~~(−3,1,5; −5,1,7)~~	(10,15,20; 8,15,22)	

Table 1.12: Forth allocation by Intuitionistic fuzzy Least Cost method

Sources	Destinations		Int. fuzzy availability $\left(\tilde{a}_i^I\right)$
	D_1	D_3	
S_2	11	7	(−1,5,11; −4,5,14)
S_3	8	9	(−2,4,10; −4,4,7)
S_4	6	5 **(8,10,13;5,10,16)**	~~(8,10,13;5,10,16)~~
Int. fuzzy demand $\left(\tilde{b}_j^I\right)$	(3,4,6;1,4,8)	~~(10,15,20; 8,15,22)~~ (−3,5,12; −8,5,17)	

Table 1.13: Fifth allocation by Intuitionistic fuzzy Least Cost method

Sources	Destinations		Int. fuzzy availability $\left(\tilde{a}_i^I\right)$
	D_1	D_3	
S_2	11	7 **(−3,5,12; −8,5,17)**	~~(−1,5,11; −4,5,14)~~ (−13,0,14; −21,0,22)
S_3	8	9	(−2,4,10; −4,4,7)
Int. fuzzy demand $\left(\tilde{b}_j^I\right)$	(3,4,6;1,4,8)	~~(−3,5,12; −8,5,17)~~	

Table 1.14: Sixth and seventh allocation by Intuitionistic fuzzy Least Cost method

Sources	Destinations	Int. fuzzy availability $\left(\tilde{a}_i^I\right)$
	D_1	
S_2	11 **(−13,0,14; −21,0,22)**	~~(−13,0,14; −21,0,22)~~
S_3	8	~~(−2,4,10; −4,4,7)~~

	(−2,4,10; −4,4,7)	
Int. fuzzy demand $\left(\tilde{b}_j^I\right)$	~~(3,4,6;1,4,8)~~	

Step 3: The initial basic feasible solution, obtained by using Table 1.9 to Table 1.14, is shown in Table 1.15. Using the initial basic feasible solution, shown in Table 1.15, the obtained initial intuitionistic fuzzy transportation cost is $1\times(2,4,5;1,4,6)\oplus 11\times(-13,0,14;-21,0,22)\oplus 4\times(-3,1,5;-5,1,7)\oplus 7\times(-3,5,12;-8,5,17)\oplus 8\times(-2,4,10;-4,4,7)\oplus 2\times(2,3,5;1,3,6)\oplus 5\times(8,10,13;5,10,16)=(-146,131,418;-314,131,543)$.

Table 1.15: Stating basic feasible solution by Intuitionistic fuzzy Least Cost method

Sources	Destinations			
	D_1	D_2	D_3	D_4
S_1	16	1 **(2,4,5;1,4,6)**	8	13
S_2	11 **(−13,0,14; −21,0,22)**	4 **(−3,1,5; −5,1,7)**	7 **(−3,5,12; −8,5,17)**	10
S_3	8 **(−2,4,10; −4,4,7)**	15	9	2 **(2,3,5;1,3,6)**
S_4	6	12	5 **(8,10,13;5,10,16)**	14

1.1.2.1.3 Intuitionistic Vogel's Approximation method

Using the existing intuitionistic fuzzy Vogel's Approximation method [132], the initial basic feasible solution of existing intuitionistic fuzzy transportation problem can be obtained as follows:

Step 1: The availability and demand, corresponding to a cell having minimum cost among a row or column which has largest penalty i.e., (3, 4), is (3,7,12;2,7,13) and (2,3,5;1,3,6) respectively. Since, $\left[R(3,7,12;2,7,13)=7.125\right]>\left[R(2,3,5;1,3,6)=3.25\right]$. So, using Step 2 of existing method [132], discussed in Section 1.1.1.1.3, $\tilde{b}_4^I=(2,3,5;1,3,6)$ will be assigned in that cell of the Table

1.1, fourth destination will be blocked and the remaining availability of source S_3 will be $(3,7,12;2,7,13)\,\Theta\,(2,3,5;1,3,6)=(-2,4,10;-4,4,12)$, as shown in Table 1.16.

Table 1.16: First allocation by Intuitionistic Vogel's Approximation method

Sources	Destinations				Intuitionistic fuzzy availability $\left(\tilde{a}_i^I\right)$	Penalty
	D_1	D_2	D_3	D_4		
S_1	16	1	8	13	(2,4,5;1,4,6)	7
S_2	11	4	7	10	(4,6,8;3,6,9)	3
S_3	8	15	9	2 **(2,3,5;1,3,6)**	~~(3,7,12;2,7,13)~~ (−2,4,10; −4,4,12)	6
S_4	6	12	5	14	(8,10,13;5,10,16)	1
Intuitionistic fuzzy demand $\left(\tilde{b}_j^I\right)$	(3,4,6; 1,4,8)	(2,5,7; 1,5,8)	(10,15,20; 8,15,22)	~~(2,3,5;1,3,6)~~		
Penalty	2	3	2	8		

Step 2: Repeating the Step 1, the next allocations are shown in Table 1.16 to Table 1.21.

Table 1.17: Second allocation by Intuitionistic Vogel's Approximation method

Sources	Destinations			Int. fuzzy availability $\left(\tilde{a}_i^I\right)$	Penalty
	D_1	D_2	D_3		
S_1	16	1 **(2,4,5;1,4,6)**	8	~~(2,4,5;1,4,6)~~	7
S_2	11	4	7	(4,6,8;3,6,9)	3
S_3	8	15	9	(−2,4,10; −4,4,12)	1
S_4	6	12	5	(8,10,13;5,10,16)	1
Int. fuzzy demand $\left(\tilde{b}_j^I\right)$	(3,4,6; 1,4,8)	~~(2,5,7; 1,5,8)~~ (−3,1,5; −5,1,7)	(10,15,20; 8,15,22)		
Penalty	2	3	2		

Table 1.18: Third allocation by Intuitionistic Vogel's Approximation method

Sources	Destinations			Int. fuzzy availability $\left(\tilde{a}_i^I\right)$	Penalty
	D_1	D_2	D_3		
S_2	11	4 **(−3,1,5; −5,1,7)**	7	~~(4,6,8;3,6,9)~~ (−1,5,11; −4,5,14)	3
S_3	8	15	9	(−2,4,10; −4,4,12)	1
S_4	6	12	5	(8,10,13;5,10,16)	1
Int. fuzzy demand $\left(\tilde{b}_j^I\right)$	(3,4,6; 1,4,8)	~~(−3,1,5; −5,1,7)~~	(10,15,20; 8,15,22)		
Penalty	2	8	2		

Table 1.19: Forth allocation by Intuitionistic Vogel's Approximation method

Sources	Destinations		Int. fuzzy availability $\left(\tilde{a}_i^I\right)$	Penalty
	D_1	D_3		
S_2	11	7 **(−1,5,11; −4,5,14)**	~~(−1,5,11; −4,5,14)~~	4
S_3	8	9	(−2,4,10; −4,4,12)	1
S_4	6	5	(8,10,13;5,10,16)	1
Int. fuzzy demand $\left(\tilde{b}_j^I\right)$	(3,4,6; 1,4,8)	~~(10,15,20; 8,15,22)~~ (−1,10,21; −6,10,26)		
Penalty	2	2		

Table 1.20: Fifth allocation by Intuitionistic Vogel's Approximation method

Sources	Destinations		Int. fuzzy availability $\left(\tilde{a}_i^I\right)$	Penalty
	D_1	D_3		
S_3	8	9	(-2,4,10;-4,4,12)	1
S_4	6	5 **(−1,10,21; −6,10,26)**	~~(8,10,13;5,10,16)~~ (−13,0,14; −21,0,22)	1
Int. fuzzy demand $\left(\tilde{b}_j^I\right)$	(3,4,6; 1,4,8)	~~(−1,10,21; −6,10,26)~~		

Penalty	2	4

Table 1.21: Sixth and seventh allocation by Intuitionistic Vogel's Approximation method

Sources	Destinations	Int. fuzzy availability $\left(\tilde{a}_i^I\right)$	Penalty
	D_1		
S_3	8 **(−2,4,10; −4,4,12)**	~~(−2,4,10; −4,4,12)~~	8
S_4	6 **(−13,0,14; −21,0,22)**	~~(−13,0,14; −21,0,22)~~	6
Int. fuzzy demand $\left(\tilde{b}_j^I\right)$	~~(3,4,6; 1,4,8)~~		
Penalty	2		

Step 3: The initial basic feasible solution, obtained by using Table 1.16 to Table 1.21, is shown in Table 1.22. Using the initial basic feasible solution, shown in Table 1.22, the obtained initial intuitionistic fuzzy transportation cost is $1\times(2,4,5;1,4,6)\oplus 4\times(-3,1,5;-5,1,7)\oplus 7\times(-1,5,11;-4,5,14)\oplus 8\times(-2,4,10;-4,4,12)\oplus 2\times(2,3,5;1,3,6)\oplus 6\times(-13,0,14;-21,0,22)\oplus 5\times(-1,10,21;-6,10,26)=(-112,131,381;-233,131,502)$.

Table 1.22: Stating basic feasible solution by Intuitionistic Vogel's Approximation method

Sources	Destinations			
	D_1	D_2	D_3	D_4
S_1	16	1 **(2,4,5;1,4,6)**	8	13
S_2	11	4 **(−3,1,5; −5,1,7)**	7 **(−1,5,11; −4,5,14)**	10
S_3	8 **(−2,4,10; −4,4,12)**	15	9	2 **(2,3,5;1,3,6)**
S_4	6 **(−13,0,14; −21,0,22)**	12	5 **(−1,10,21; −6,10,26)**	14

1.1.2.2 Optimal solution

Using the existing intuitionistic fuzzy Modified Distribution method [132], the optimal solution of existing intuitionistic fuzzy transportation problem of type – I by considering the initial basic feasible solution, obtained in Table 1.8, can be obtained as follows:

Step 1: Using Step 2 and Step 3 of the existing method [132], discussed in Section 1.1.1.2, the following values of dual variables and the values of $d_{ij} = u_i + v_j - c_{ij}$ for all non-basic cells are obtained,

$u_1 = 5, u_2 = 0, u_3 = 2, u_4 = -2, v_1 = 11, v_2 = 4, v_3 = 7, v_4 = 16$,

$d_{12} = 8, d_{13} = 4, d_{14} = 8, d_{24} = 6, d_{31} = 5, d_{32} = -9, d_{34} = 16, d_{41} = 3, d_{42} = -10$.

Step 2: Using Step 4 and Step 5 of the existing method [132], discussed in Section 1.1.1.2, among all the value of d_{ij}, obtained in Step 1, d_{34} has the largest value i.e., 16 therefore the cell (3, 4) will enter in the basic cell. The loop is shown in Table 1.23 and the quantity allocated to the cell (3, 4) is $\tilde{\theta} = \min\{R(2,3,5;1,3,6), R(3,7,12;2,7,13)\} = (2,3,5;1,3,6)$.

Table 1.23: First loop from (3, 4) cell [132]

Sources	Destinations			
	D_1	D_2	D_3	D_4
S_1	16 **(2,4,5;1,4,6)**	1	8	13
S_2	11 **(−2,0,4; −5,0,7)**	4 **(2,5,7;1,5,8)**	7 **(−7,1,8; −12,1,13)**	10
S_3	8	15	9 **(3,7,12;2,7,13)**	2 $(\tilde{\theta})$
S_4	6	12	5 **(−10,7,24; −18,7,32)**	14 **(2,3,5;1,3,6)**

After adding and subtracting the value of $\tilde{\theta}$, next (second) intuitionistic fuzzy basic feasible solution, shown in Table 1.24, is obtained and again using the Step 2 and Step 3 of existing method [132], discussed in Section 1.1.1.2, the following values of dual variables and the values of $d_{ij} = u_i + v_j - c_{ij}$ for all non-basic cells are obtained,

$u_1 = 5, u_2 = 0, u_3 = 2, u_4 = -2, v_1 = 11, v_2 = 4, v_3 = 7, v_4 = 0$,

$d_{12} = 8, d_{13} = 4, d_{14} = -8, d_{24} = -10, d_{31} = 5, d_{32} = -9, d_{41} = 3, d_{42} = -10, d_{44} = -16$.

Using Step 4 and Step 5 of existing method [132], discussed in Section 1.1.1.2, it can be concluded that the non-basic cell (1, 2) will enter in the basic cell and make a loop from this cell as given in Table 1.24 and the value of $\tilde{\theta}$ is (2,4,5;1,4,6).

Table 1.24: Second intuitionistic fuzzy basic feasible solution [132]

Sources	Destinations			
	D_1	D_2	D_3	D_4
S_1	16 **(2,4,5;1,4,6)**	1 $(\tilde{\theta})$	8	13
S_2	11 **(−2,0,4; −5,0,7)**	4 **(2,5,7;1,5,8)**	7 **(−7,1,8; −12,1,13)**	10
S_3	8	15	9 **(−2,4,10; −3,4,11)**	2 **(2,3,5;1,3,6)**
S_4	6	12	5 **(−8,10,29; −17,10,38)**	14

After adding and subtracting the value of $\tilde{\theta}$, next (third) intuitionistic fuzzy basic feasible solution is obtained, given in Table 1.25, and again using the Step 2 and Step 3 of existing method [132], discussed in Section 1.1.1.2, the following values of dual variables and the values of $d_{ij} = u_i + v_j - c_{ij}$ for all non-basic cells are obtained,

$u_1 = -3, u_2 = 0, u_3 = 2, u_4 = -2, v_1 = 11, v_2 = 4, v_3 = 7, v_4 = 0$,

$d_{11} = -8, d_{13} = -4, d_{14} = -16, d_{24} = -10, d_{31} = 5, d_{32} = -9, d_{41} = 3, d_{42} = -10, d_{44} = -16$.

By using Step 4 and Step 5 of existing method [132], discussed in Section 1.1.1.2, it can be concluded that the non-basic cell (3, 1) will enter in the basic cell and make a loop from this cell as given in Table 1.25 and the value of $\tilde{\theta}$ is (–2,4,10; –3,4,11).

After adding and subtracting the value of $\tilde{\theta}$, next (fourth) intuitionistic fuzzy basic feasible solution is obtained, given in Table 1.26, and again using the Step 2 and Step 3 of existing method [132], discussed in Section 1.1.1.2, the following values of dual variables and the values of $d_{ij} = u_i + v_j - c_{ij}$ for all non-basic cells are obtained,

Table 1.25: Third intuitionistic fuzzy basic feasible solution [132]

Sources	Destinations			
	D_1	D_2	D_3	D_4
S_1	16	1 **(2,4,5;1,4,6)**	8	13
S_2	11 **(0,4,9; −4,4,13)**	4 **(−3,1,5; −5,1,7)**	7 **(−7,1,8; −12,1,13)**	10
S_3	8 $(\tilde{\theta})$	15	9 **(-2,4,10; −3,4,11)**	2 **(2,3,5;1,3,6)**
S_4	6	12	5 **(−8,10,29; −17,10,38)**	14

$u_1 = -3, u_2 = 0, u_3 = -3, u_4 = -2, v_1 = 11, v_2 = 4, v_3 = 7, v_4 = 5$,

$d_{11} = -8, d_{13} = -4, d_{14} = -11, d_{24} = -5, d_{32} = -14, d_{33} = -5, d_{41} = 3, d_{42} = -10, d_{44} = -11$.

Using Step 4 and Step 5 of existing method [132], discussed in Section 1.1.1.2, it can be concluded that the non-basic cell (3, 1) will enter in the basic cell and make a loop from this cell as given in Table 1.26 and the value of $\tilde{\theta}$ is (–2,4,10; –3,4,11).

Table 1.26: Fourth intuitionistic fuzzy basic feasible solution [132]

Sources	Destinations			
	D_1	D_2	D_3	D_4

S_1	16	1 **(2,4,5;1,4,6)**	8	13
S_2	11 **(–10,0,11; –15,0,16)**	4 **(–3,1,5; –5,1,7)**	7 **(–9,5,18; –15,5,24)**	10
S_3	8 **(–2,4,10; –3,4,11)**	15	9	2 **(2,3,5;1,3,6)**
S_4	6 $\left(\tilde{\theta}\right)$	12	5 **(–8,10,29; –17,10,38)**	14

After getting the fifth intuitionistic fuzzy basic feasible solution, given in Table 1.27, again applying the Step 2 and Step 3 of existing method [132], discussed in Section 1.1.1.2, and the following values of dual variables and the values of $d_{ij} = u_i + v_j - c_{ij}$ for all non-basic cells are obtained,

$u_1 = -3, u_2 = 0, u_3 = 0, u_4 = -2, v_1 = 8, v_2 = 4, v_3 = 7, v_4 = 2$,

$d_{11} = -9, d_{13} = -4, d_{14} = -14, d_{21} = -3, d_{24} = -8, d_{32} = -11, d_{33} = -2, d_{42} = -10, d_{44} = -14$.

Here, $d_{ij} \le 0;$ for all non-basic cells. Therefore, the fifth intuitionistic fuzzy basic feasible solution, shown in Table 1.27, is the intuitionistic fuzzy optimal solution.

Table 1.27: Fifth intuitionistic fuzzy basic feasible solution [132]

Sources	Destinations			
	D_1	D_2	D_3	D_4
S_1	16	1 **(2,4,5;1,4,6)**	8	13
S_2	11	4 **(–3,1,5; –5,1,7)**	7 **(–19,5,29; –30,5,40)**	10
S_3	8 **(–2,4,10; –3,4,11)**	15	9	2 **(2,3,5;1,3,6)**
S_4	6 **(–10,0,11; –15,0,16)**	12	5 **(–19,10,39; –33,10,53)**	14

Step 3: The intuitionistic fuzzy optimal solution of intuitionistic fuzzy transportation problem of type – I, shown in Table 1.1, is $\tilde{x}_{12}^I = (2,4,5;1,4,6)$, $\tilde{x}_{22}^I = (-3,1,5;-5,1,7)$, $\tilde{x}_{23}^I = (-19,5,29;$

$-30,5,40)$, $\tilde{x}_{31}^I=(-2,4,10;-3,4,11)$, $\tilde{x}_{34}^I=(2,3,5;1,3,6)$, $\tilde{x}_{41}^I=(-10,0,11;-15,0,16)$, $\tilde{x}_{43}^I=(-19,$ $10,39;-33,10,53)$ and the optimal intuitionistic fuzzy transportation cost is

$1\times(2,4,5;1,4,6)+4\times(-3,1,5;-5,1,7)+7\times(-19,5,29;-30,5,40)+8\times(-2,4,10;-3,4,11)+2\times$

$(2,3,5;1,3,6)+6\times(-10,0,11;-15,0,16)+5\times(-19,10,39;-33,10,53)$

$=(-310,131,579;-506,131,775)$.

1.1.3 Singh and Yadav method for solving intuitionistic fuzzy transportation problems of type – II

In this section, the intuitionistic fuzzy methods (Intuitionistic fuzzy North West Corner method, Intuitionistic fuzzy Least Cost method, Intuitionistic fuzzy Vogel's Approximation method) to find initial basic feasible solutions as well as the intuitionistic fuzzy Modified Distribution method to find optimal solution of intuitionistic fuzzy balanced transportation problem of type – II, proposed by Singh and Yadav [131], are presented in a brief manner.

1.1.3.1 Intuitionistic fuzzy methods to find initial basic feasible solutions

In this section, the intuitionistic fuzzy methods (Intuitionistic fuzzy North – West Corner method, Intuitionistic fuzzy Least Cost method, Intuitionistic fuzzy Vogel's Approximation method) to find the initial basic feasible solutions of intuitionistic fuzzy balanced transportation problems of type – II, proposed by Singh and Yadav [131], is presented in a brief manner.

1.1.3.1.1 Intuitionistic fuzzy North - West Corner method for intuitionistic fuzzy balanced transportation problem of type – II

Singh and Yadav [131] proposed an intuitionistic fuzzy North – West Corner method to find the initial basic feasible solution of intuitionistic fuzzy balanced transportation problems of type – II.

The steps of this method are as follows:

Step 1: Start from the North-West corner cell, say (i, j), of intuitionistic fuzzy balanced transportation problems. To assign this (i, j) cell, there are three cases which may arise: either $\min\{a_i, b_j\} = a_i$ or $\min\{a_i, b_j\} = b_j$ or $a_i = b_j$.

(i) If $\min\{a_i, b_j\} = a_i$, then assign a_i to (i, j) cell. Block the i^{th} row of given intuitionistic fuzzy balanced transportation problems. Replace b_j with $b_j - a_i$ and then go to Step 2.

(ii) If $\min\{a_i, b_j\} = b_j$, then assign b_j to (i, j) cell. Block the j^{th} column of given intuitionistic fuzzy balanced transportation problems of type – II. Replace a_i with $a_i - b_j$ and then go to Step 2.

(iii) If $a_i = b_j$, this implies that $a_i \le b_j$ and $a_i \ge b_j$. Therefore, follow any one of the cases stated above, but not both together.

Step 2: Repeat Step 1, until all the availability and demands are satisfied.

Step 3: The initial basic feasible solution is $\{x_{ij}\}$ and the initial intuitionistic fuzzy transportation cost is $\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}^{I} \times x_{ij}$.

1.1.3.1.2 Intuitionistic fuzzy Least Cost method for intuitionistic fuzzy balanced transportation problems of type – II

Singh and Yadav [131] proposed an intuitionistic fuzzy Least Cost method to find the initial basic feasible solution of intuitionistic fuzzy balanced transportation problems of type – II.

The steps of this method are as follows:

Step 1: Find $\min\left\{R\left(\tilde{c}_{ij}^{I}\right); i=1,2,...,m,\ j=1,2,...,n\right\}$. Let it be at (i, j) cell. To assign this (i, j) cell, there are three cases which may arise: either $\min\left\{a_i, b_j\right\}=a_i$ or $\min\left\{a_i, b_j\right\}=b_j$ or $a_i=b_j$.

(i) If $\min\left\{a_i, b_j\right\}=a_i$, then assign a_i to (i, j) cell. Block the i^{th} row of given intuitionistic fuzzy balanced transportation problems of type – II. Replace b_j with b_j-a_i and then go to Step 2.

(ii) If $\min\left\{a_i, b_j\right\}=b_j$, then assign b_j to (i, j) cell. Block the j^{th} column of given intuitionistic fuzzy balanced transportation problems of type – II. Replace a_i with a_i-b_j and then go to Step 2.

(iii) If $a_i=b_j$, this implies that $a_i \le b_j$ and $a_i \ge b_j$. Therefore, follow any one of the cases stated above, but not both together.

Step 2: Repeat Step 1, until all the availability and demands are satisfied.

Step 3: The initial basic feasible solution is $\left\{x_{ij}\right\}$ and the initial intuitionistic fuzzy transportation cost is $\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}^{I}\times x_{ij}$.

1.1.3.1.3 Intuitionistic fuzzy Vogel's Approximation method for intuitionistic fuzzy balanced transportation problems of type – II

Singh and Yadav [131] proposed an intuitionistic fuzzy Vogel's Approximation method to find the initial basic feasible solution of intuitionistic fuzzy balanced transportation problems of type – II.

The steps of this method are as follows:

Step 1: Find the penalty of each row by taking the difference of smallest entry and next smallest entry of corresponding row and find the penalty of each column by taking the difference of smallest entry and next smallest entry of corresponding column.

Step 2: Choose a row or column whose penalty is largest among all rows and columns. Let it be p^{th} row. Select a cell which has $\min\left\{R\left(\tilde{c}_{pj}^{I}\right); j=1,2,...,n\right\}$. To assign this (p, j) cell, there are three cases which may arise: either $\min\left\{a_p, b_j\right\}=a_p$ or $\min\left\{a_p, b_j\right\}=b_j$ or $a_p=b_j$.

(i) If $\min\left\{a_p, b_j\right\}=a_p$, then assign a_i to (i, j) cell. Block the p^{th} row of given intuitionistic fuzzy balanced transportation problems of type – II. Replace b_j with b_j-a_p and then go to Step 3.

(ii) If $\min\left\{a_p, b_j\right\}=b_j$, then assign b_j to (i, j) cell. Block the j^{th} column of given intuitionistic fuzzy balanced transportation problems of type – II. Replace a_p with a_p-b_j and then go to Step 3.

(iii) If $a_p=b_j$, this implies that $a_p \leq b_j$ and $a_p \geq b_j$. Therefore, follow any one of the cases stated above, but not both together.

Step 3: Repeat Step 1 and Step 2, until all the availability and demands are satisfied.

Step 4: The initial basic feasible solution is $\{x_{ij}\}$ and the initial intuitionistic fuzzy transportation cost is $\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}^{I} \times x_{ij}$.

1.1.3.2 Intuitionistic fuzzy Modified Distribution method to find optimal solution of intuitionistic fuzzy balanced transportation problems of type – II

Singh and Yadav [131] proposed an intuitionistic fuzzy Modified Distribution method to find the optimal solution of intuitionistic fuzzy balanced transportation problems of type – II.

The steps of this method are as follows:

Step 1: Find the initial basic feasible solution by using any of these three methods: intuitionistic fuzzy North West Corner method, intuitionistic fuzzy Least Cost method, intuitionistic fuzzy Vogel's Approximation method.

Step 2: Take intuitionistic fuzzy dual variables $\tilde{u}_i^I = (u_1, u_2, u_3; u_1', u_2, u_3')$ and $\tilde{v}_j^I = (v_1, v_2, v_3; v_1', v_2, v_3')$ corresponding to i^{th} row and j^{th} column respectively such that $R(\tilde{u}_i^I \oplus \tilde{v}_i^I) = R(\tilde{c}_{ij}^I)$ for the basic cell (i, j). Find the value of $\tilde{u}_i^I$ and $\tilde{v}_j^I$ by using the relations $\tilde{u}_i^I \approx \tilde{c}_{ij}^I \ominus \tilde{v}_j^I$ or $\tilde{v}_j^I \approx \tilde{c}_{ij}^I \ominus \tilde{u}_i^I$.

Step 3: Calculate $\tilde{d}_{ij}^I \approx \tilde{v}_j^I \oplus \tilde{u}_i^I \ominus \tilde{c}_{ij}^I$ for all non-basic cells.

(i) If $R(\tilde{d}_{ij}^I) \leq 0$ for all non-basic cells, then the initial basic feasible solution is the optimal solution.

(ii) If there is at least one $\tilde{d}_{ij}^I$ for which $R\left(\tilde{d}_{ij}^I\right) > 0$, then go to Step 4.

Step 4: Select a cell $\left\{(i, j); \max\left\{R\left(\tilde{d}_{ij}^I\right) : R\left(\tilde{d}_{ij}^I\right) > 0 \text{ for all non-basic cells}\right\}\right\}$ and assign a quantity θ to this cell.

Now making a loop, move horizontally and vertically from θ - cell to the nearest basic cell and turn to loop to another nearest basic cell. In this way, the loop will return to θ - cell. The value of θ is the minimum quantity of the corner basic cell from which θ is subtracted.

Step 5: Add and subtract θ successively to/from the assigned quantity in the basic cell from where the loop takes turn. The value of θ will be the minimum of those assigned quantity from which θ is subtracted. While, inserting this value of θ, a basic cell assume zero value and this cell becomes non-basic cell. Resultant problem gives the improved basic feasible solution.

Step 6: Repeat Step 1 to Step 5 until $R\left(\tilde{d}_{ij}^I\right) \leq 0; \forall i, j$.

Step 7: Using the optimal solution $\left\{x_{ij}\right\}$, obtained in Step 6, the optimal intuitionistic fuzzy transportation cost is $\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}^I \times x_{ij}$.

1.1.4 Initial basic feasible solution of intuitionistic fuzzy balanced transportation problems of type – II obtained by Singh and Yadav method

The aim of Chapter 3 is to propose an alternative method to find initial basic feasible solution and optimal solution of intuitionistic fuzzy balanced transportation problem of type – II which is computationally simple as compared to the existing method [131]. To show that proposed alternative method is computationally simple as compared to the existing method [131], there is need to solve the same problem by the existing method [131] as well as by the proposed method. In this section, the existing intuitionistic fuzzy balanced transportation problem of type – II, presented in Table 1.28, is solved by the existing method [131]. The same problem will be solved by proposed method in Chapter 3.

Table 1.28: Intuitionistic fuzzy balanced transportation problem of type – II

Sources	Destinations				Avail-ability (a_i)
	D_1	D_2	D_3	D_4	
S_1	(2,4,5;1,4,6)	(2,5,7;1,5,8)	(4,6,8;3,6,9)	(4,7,8;3,7,9)	11
S_2	(4,6,8;3,6,9)	(3,7,12;2,7,13)	(10,15,20;8,15,22)	(11,12,13;10,12,14)	11
S_3	(3,4,6;1,4,8)	(8,10,13;5,10,16)	(2,3,5;1,3,6)	(6,10,14;5,10,15)	11
S_4	(2,4,6;1,4,7)	(3,9,10;2,9,12)	(3,6,10;2,6,12)	(3,4,5;2,4,8)	12
Demand (b_j)	16	10	8	11	15

1.1.4.1 Initial basic feasible solution

Using the existing method [131], the initial basic feasible solution of existing intuitionistic fuzzy balanced transportation problem of type – II can be obtained as follows:

1.1.4.1.1 Intuitionistic fuzzy North – West Corner method

Using the existing intuitionistic fuzzy north - west corner method [131], the initial basic feasible solution of existing intuitionistic fuzzy balanced transportation problem of type – II can be obtained as follows:

Step 1: The availability and demand, corresponding to North-West Corner cell i.e., (1, 1), is 11 and 16 respectively. Since, 11 < 16. So, using Step 1 of existing method, discussed in Section 1.1.3.1.1, $a_1 = 11$ will be assigned in the North – West Corner cell of the Table 1.28, the first row will be blocked and the remaining demand of destination D_1 will be 16 – 11 = 5, as shown in Table 1.29.

Table 1.29: First allocation by Intuitionistic fuzzy North – West Corner method

Sources	Destinations				Availability (a_i)
	D_1	D_2	D_3	D_4	
S_1	(2,4,5;1,4,6) **(11)**	(2,5,7;1,5,8)	(4,6,8;3,6,9)	(4,7,8;3,7,9)	~~11~~
S_2	(4,6,8;3,6,9)	(3,7,12;2,7,13)	(10,15,20;8,15,22)	(11,12,13;10,12,14)	11
S_3	(3,4,6;1,4,8)	(8,10,13;5,10,16)	(2,3,5;1,3,6)	(6,10,14;5,10,15)	11
S_4	(2,4,6;1,4,7)	(3,9,10;2,9,12)	(3,6,10;2,6,12)	(3,4,5;2,4,8)	12
Demand (b_j)	~~16~~ 5	10	8	11	

Step 2: By repeating the Step 1, the next allocations are shown from Table 1.30 to Table 1.34.

Table 1.30: Second allocation by Intuitionistic fuzzy North – West Corner method

Sources	Destinations				Availability (a_i)
	D_1	D_2	D_3	D_4	
S_2	(4,6,8;3,6,9) **(5)**	(3,7,12;2,7,13)	(10,15,20;8,15,22)	(11,12,13;10,12,14)	~~11~~ 6
S_3	(3,4,6;1,4,8)	(8,10,13;5,10,16)	(2,3,5;1,3,6)	(6,10,14;5,10,15)	11
S_4	(2,4,6;1,4,7)	(3,9,10;2,9,12)	(3,6,10;2,6,12)	(3,4,5;2,4,8)	12
Demand (b_j)	~~5~~	10	8	11	

Table 1.31: Third allocation by Intuitionistic fuzzy North – West Corner method

Sources	Destinations			Availability (a_i)
	D_2	D_3	D_4	
S_2	(3,7,12;2,7,13) **(6)**	(10,15,20;8,15,22)	(11,12,13;10,12,14)	~~6~~
S_3	(8,10,13;5,10,16)	(2,3,5;1,3,6)	(6,10,14;5,10,15)	11
S_4	(3,9,10;2,9,12)	(3,6,10;2,6,12)	(3,4,5;2,4,8)	12
Demand (b_j)	~~10~~ 4	8	11	

Table 1.32: Fourth allocation by Intuitionistic fuzzy North – West Corner method

Sources	Destinations			Availability (a_i)
	D_2	D_3	D_4	
S_3	(8,10,13;5,10,16) **(4)**	(2,3,5;1,3,6)	(6,10,14;5,10,15)	~~11~~ 7
S_4	(3,9,10;2,9,12)	(3,6,10;2,6,12)	(3,4,5;2,4,8)	12
Demand (b_j)	~~4~~	8	11	

Table 1.33: Fifth allocation by Intuitionistic fuzzy North – West Corner method

Sources	Destinations		Availability (a_i)
	D_3	D_4	
S_3	(2,3,5;1,3,6) **(7)**	(6,10,14;5,10,15)	~~7~~
S_4	(3,6,10;2,6,12)	(3,4,5;2,4,8)	12
Demand (b_j)	~~8~~ 1	11	

Table 1.34: Sixth and seventh allocation by Intuitionistic fuzzy North – West Corner method

Sources	Destinations		Availability (a_i)
	D_3	D_4	
S_4	(3,6,10;2,6,12) **(1)**	(3,4,5;2,4,8) **(11)**	~~12~~
Demand (b_j)	~~1~~	~~11~~	

Step 3: The initial basic feasible solution obtained by using Table 1.29 to Table 1.34, is shown in Table 1.35. Using the initial basic feasible solution, shown in Table 1.35, the obtained initial intuitionistic fuzzy transportation cost is $11\times(2,4,5;1,4,6)\oplus 5\times(4,6,8;3,6,9)\oplus 6\times$ $(3,7,12;2,7,13)\oplus 4\times(8,10,13;5,10,16)\oplus 7\times(2,3,5;1,3,6)\oplus 1\times(3,6,10;2,6,12)\oplus 11\times(3,4,5;2,4,8)$

$=(142,227,319;89,227,395)$.

Table 1.35: Initial basic feasible solution by Intuitionistic fuzzy North – West Corner method

Sources	Destinations			
	D_1	D_2	D_3	D_4
S_1	(2,4,5;1,4,6) **(11)**	(2,5,7;1,5,8)	(4,6,8;3,6,9)	(4,7,8;3,7,9)
S_2	(4,6,8;3,6,9) **(5)**	(3,7,12;2,7,13) **(6)**	(10,15,20;8,15,22)	(11,12,13;10,12,14)
S_3	(3,4,6;1,4,8)	(8,10,13;5,10,16) **(4)**	(2,3,5;1,3,6) **(7)**	(6,10,14;5,10,15)
S_4	(2,4,6;1,4,7)	(3,9,10;2,9,12)	(3,6,10;2,6,12) **(1)**	(3,4,5;2,4,8) **(11)**

1.1.4.1.2 Intuitionistic fuzzy Least Cost method

Using the existing intuitionistic fuzzy Least Cost method [131], the initial basic feasible solution of existing intuitionistic fuzzy balanced transportation problem of type – II can be obtained as follows:

Step 1: The rank of cost in each cell is $R\left(\tilde{c}_{11}^{I}\right)=3.75, R\left(\tilde{c}_{12}^{I}\right)=4.75, R\left(\tilde{c}_{13}^{I}\right)=6, R\left(\tilde{c}_{14}^{I}\right)=6.5,$ $R\left(\tilde{c}_{21}^{I}\right)=6,$ $R\left(\tilde{c}_{22}^{I}\right)=7.25,$ $R\left(\tilde{c}_{23}^{I}\right)=15, R\left(\tilde{c}_{24}^{I}\right)=12,$ $R\left(\tilde{c}_{31}^{I}\right)=4.25,$ $R\left(\tilde{c}_{32}^{I}\right)=10.25, R\left(\tilde{c}_{33}^{I}\right)=3.25,$ $R\left(\tilde{c}_{34}^{I}\right)=10, R\left(\tilde{c}_{41}^{I}\right)=4, R\left(\tilde{c}_{42}^{I}\right)=7.875, R\left(\tilde{c}_{43}^{I}\right)=6.375, R\left(\tilde{c}_{11}^{I}\right)=4.25.$ The availability and demand, corresponding to cell having least rank of the cost i.e., (3, 3), is 11 and 8 respectively. Since, 8 < 11. So, using Step 1 of existing method, discussed in Section 1.1.3.1.2, $a_3=3$ will be assigned in that cell of the Table 1.28, the third column will be blocked and the remaining availability of source S_3 will be 11 – 8 = 3, as shown in Table 1.36.

Table 1.36: First allocation by Intuitionistic fuzzy Least Cost method

Sources	Destinations				Avail-ability (a_i)
	D_1	D_2	D_3	D_4	
S_1	(2,4,5;1,4,6)	(2,5,7;1,5,8)	(4,6,8;3,6,9)	(4,7,8;3,7,9)	11
S_2	(4,6,8;3,6,9)	(3,7,12;2,7,13)	(10,15,20;8,15,22)	(11,12,13;10,12,14)	11
S_3	(3,4,6;1,4,8)	(8,10,13;5,10,16)	(2,3,5;1,3,6) **(8)**	(6,10,14;5,10,15)	~~11~~ 3
S_4	(2,4,6;1,4,7)	(3,9,10;2,9,12)	(3,6,10;2,6,12)	(3,4,5;2,4,8)	12
Demand (b_j)	16	10	~~8~~	11	

Step 2: By repeating the Step 1, the next allocations are shown from Table 1.37 to Table 1.41.

Table 1.37: Second allocation by Intuitionistic fuzzy Least Cost method

Sources	Destinations			Availability
	D_1	D_2	D_4	(a_i)
S_1	(2,4,5;1,4,6)	(2,5,7;1,5,8)	(4,7,8;3,7,9)	11
S_2	(4,6,8;3,6,9)	(3,7,12;2,7,13)	(11,12,13;10,12,14)	11
S_3	(3,4,6;1,4,8)	(8,10,13;5,10,16)	(6,10,14;5,10,15)	3
S_4	(2,4,6;1,4,7) **(12)**	(3,9,10;2,9,12)	(3,4,5;2,4,8)	~~12~~
Demand (b_j)	~~16~~ 4	10	11	

Table 1.38: Third allocation by Intuitionistic fuzzy Least Cost method

Sources	Destinations			Availability
	D_1	D_2	D_4	(a_i)
S_1	(2,4,5;1,4,6)	(2,5,7;1,5,8)	(4,7,8;3,7,9)	11
S_2	(4,6,8;3,6,9)	(3,7,12;2,7,13)	(11,12,13;10,12,14)	11
S_3	(3,4,6;1,4,8) **(3)**	(8,10,13;5,10,16)	(6,10,14;5,10,15)	~~3~~
Demand (b_j)	~~4~~ 1	10	11	

Table 1.39: Fourth allocation by Intuitionistic fuzzy Least Cost method

Sources	Destinations			Availability
	D_1	D_2	D_4	(a_i)
S_1	(2,4,5;1,4,6)	(2,5,7;1,5,8) **(10)**	(4,7,8;3,7,9)	~~11~~ 1
S_2	(4,6,8;3,6,9)	(3,7,12;2,7,13)	(11,12,13;10,12,14)	11
Demand (b_j)	1	~~10~~	11	

Table 1.40: Fifth allocation by Intuitionistic fuzzy Least Cost method

Sources	Destinations		Availability (a_i)
	D_1	D_4	
S_1	(2,4,5;1,4,6)	(4,7,8;3,7,9)	1
S_2	(4,6,8;3,6,9) **(1)**	(11,12,13;10,12,14)	~~11~~ 10
Demand (b_j)	~~1~~	11	

Table 1.41: Sixth and seventh allocation by Intuitionistic fuzzy Least Cost method

Sources	Destinations	Availability (a_i)
	D_4	
S_1	(4,7,8;3,7,9) **(1)**	~~1~~
S_2	(11,12,13;10,12,14) **(10)**	~~10~~
Demand (b_j)	~~11~~	

Step 3: The initial basic feasible solution obtained, by using Table 1.36 to Table 1.41, is shown in Table 1.42. Using the initial basic feasible solution, shown in Table 1.42, the obtained initial intuitionistic fuzzy transportation cost is $10\times(2,5,7;1,5,8)\oplus 1\times(4,7,8;3,7,9)\oplus 1\times (4,6,8;3,6,9)\oplus 10\times(11,12,13;10,12,14)\oplus 3\times(3,4,6;1,4,8)\oplus 8\times(2,3,5;1,3,6)\oplus 12\times(2,4,6;1,4,7)$

$=(187,267,346;139,267,394)$.

Table 1.42: Initial basic feasible solution by Intuitionistic fuzzy Least Cost method

Sources	Destinations			
	D_1	D_2	D_3	D_4
S_1	(2,4,5;1,4,6)	(2,5,7;1,5,8) **(10)**	(4,6,8;3,6,9)	(4,7,8;3,7,9) **(1)**

S_2	(4,6,8;3,6,9) **(1)**	(3,7,12;2,7,13)	(10,15,20;8,15,22)	(11,12,13;10,12,14) **(10)**
S_3	(3,4,6;1,4,8) **(3)**	(8,10,13;5,10,16)	(2,3,5;1,3,6) **(8)**	(6,10,14;5,10,15)
S_4	(2,4,6;1,4,7) **(12)**	(3,9,10;2,9,12)	(3,6,10;2,6,12)	(3,4,5;2,4,8)

1.1.4.1.3 Intuitionistic Vogel's Approximation method

Using the existing intuitionistic fuzzy Vogel's Approximation method [131], the initial basic feasible solution of existing intuitionistic fuzzy balanced transportation problem of type – II can be obtained as follows:

Step 1: With the help of ranking function, it can be easily seen that destination D_3 has the largest penalty and the cell (3, 3) has the smallest cost in third column, as shown in Table 1.43. The availability and demand, corresponding to a cell (3, 3), is 11 and 8 respectively. Since, 8 < 11. So, using Step 2 of existing method, discussed in Section 1.1.3.1.3, $b_3 = 3$ will be assigned in (3, 3) cell of the Table 1.28, third destination will be blocked and the remaining availability of source S_3 will be 11 – 8 = 3, as shown in Table 1.43.

Table 1.43: First allocation by Intuitionistic Vogel's Approximation method

Sources	Destinations				Availability	Inst. Fuzzy
	D_1	D_2	D_3	D_4	$\left(\tilde{a}_i^I\right)$	Penalty
S_1	(2,4,5; 1,4,6)	(2,5,7; 1,5,8)	(4,6,8; 3,6,9)	(4,7,8; 3,7,9)	11	(–3,1,5; –5,1,7)
S_2	(4,6,8; 3,6,9)	(3,7,12; 2,7,13)	(10,15,20; 8,15,22)	(11,12,13; 10,12,14)	11	(–5,1,8; –7,1,10)

S_3	(3,4,6; 1,4,8)	(8,10,13; 5,10,16)	(2,3,5; 1,3,6) **(8)**	(6,10,14; 5,10,15)	~~11~~ 3	(–2,1,4; –5,1,7)
S_4	(2,4,6; 1,4,7)	(3,9,10; 2,9,12)	(3,6,10; 2,6,12)	(3,4,5; 2,4,8)	12	(–3,0,3; –5,0,7)
Demand $\left(\tilde{b}_j^I\right)$	16	10	~~8~~	11		
Penalty	(–9,0,4; –5,0,6)	(–4,2,10; –6,5,12)	**(–1,3,6; -3,3,8)**	(–1,3,5; –5,3,7)		

Step 2: Repeating the Step 1, the next allocations are shown from Table 1.44 to Table 1.48.

Table 1.44: Second allocation by Intuitionistic Vogel's Approximation method

Sources	Destinations			Availability $\left(\tilde{a}_i^I\right)$	Inst. Fuzzy Penalty
	D_1	D_2	D_4		
S_1	(2,4,5; 1,4,6)	(2,5,7; 1,5,8)	(4,7,8; 3,7,9)	11	(–3,1,5; –5,1,7)
S_2	(4,6,8; 3,6,9)	(3,7,12; 2,7,13)	(11,12,13;10,12,14)	11	(–5,1,8; –7,1,10)
S_3	(3,4,6; 1,4,8) **(3)**	(8,10,13;5,10,16)	(6,10,14;5,10,15)	~~3~~	**(0,6,11; –3,6,14)**
S_4	(2,4,6; 1,4,7)	(3,9,10; 2,9,12)	(3,4,5; 2,4,8)	12	(–3,0,3; –5,0,7)
Demand $\left(\tilde{b}_j^I\right)$	~~16~~ 13	10	11		
Penalty	(–9,0,4;–5,0,6)	(–4,2,10;–6,5,12)	(–1,3,5;–5,3,7)		

Table 1.45: Third allocation by Intuitionistic Vogel's Approximation method

Sources	Destinations			Availability $\left(\tilde{a}_i^I\right)$	Inst. Fuzzy Penalty
	D_1	D_2	D_4		
S_1	(2,4,5; 1,4,6)	(2,5,7; 1,5,8) **(10)**	(4,7,8; 3,7,9)	~~11~~ 1	(–3,1,5; –5,1,7)
S_2	(4,6,8; 3,6,9)	(3,7,12; 2,7,13)	(11,12,13;10,12,14)	11	(–5,1,8; –7,1,10)
S_4	(2,4,6; 1,4,7)	(3,9,10; 2,9,12)	(3,4,5; 2,4,8)	12	(–3,0,3;

					–5,0,7)
Demand $\left(\tilde{b}_j^I\right)$	13	~~10~~	11		
Penalty	(–9,0,4;–5,0,6)	**(–4,2,10;–6,5,12)**	(–1,3,5;–5,3,7)		

Table 1.46: Fourth allocation by Intuitionistic Vogel's Approximation method

Sources	Destinations		Availability $\left(\tilde{a}_i^I\right)$	Inst. Fuzzy Penalty
	D_1	D_4		
S_1	(2,4,5; 1,4,6)	(4,7,8; 3,7,9)	1	(–3,1,5;–5,1,7)
S_2	(4,6,8; 3,6,9) **(11)**	(11,12,13;10,12,14)	~~11~~	**(–5,1,8;–7,1,10)**
S_4	(2,4,6; 1,4,7)	(3,4,5; 2,4,8)	12	(–3,0,3;–5,0,7)
Demand $\left(\tilde{b}_j^I\right)$	~~13~~ 2	11		
Penalty	(–9,0,4;–5,0,6)	(–1,3,5;–5,3,7)		

Table 1.47: Fifth allocation by Intuitionistic Vogel's Approximation method

Sources	Destinations		Availability $\left(\tilde{a}_i^I\right)$	Inst. Fuzzy Penalty
	D_1	D_4		
S_1	(2,4,5; 1,4,6) **(1)**	(4,7,8; 3,7,9)	~~1~~	**(–3,1,5;–5,1,7)**
S_4	(2,4,6; 1,4,7)	(3,4,5; 2,4,8)	12	(–3,0,3;–5,0,7)
Demand $\left(\tilde{b}_j^I\right)$	~~2~~ 1	11		
Penalty	(–9,0,4;–5,0,6)	(–1,3,5;–5,3,7)		

Table 1.48: Sixth and seventh allocation by Intuitionistic Vogel's Approximation method

Sources	Destinations		Availability $\left(\tilde{a}_i^I\right)$
	D_1	D_4	
S_4	(2,4,6; 1,4,7) **(1)**	(3,4,5; 2,4,8) **(11)**	~~12~~

Demand $\left(\tilde{b}_j^I\right)$	~~1~~	~~11~~

Step 3: The initial basic feasible solution obtained, by using Table 1.43 to Table 1.48, is shown in Table 1.49. Using the initial basic feasible solution, shown in Table 1.49, the obtained initial intuitionistic fuzzy transportation cost is $1\times(2,4,5;1,4,6)\oplus10\times(2,5,7;1,5,8)\oplus11\times(4,6,8;3,6,9)$

$\oplus3\times(3,4,6;1,4,8)\oplus8\times(2,3,5;1,3,6)\oplus1\times(2,4,6;1,4,7)\oplus11\times(3,4,5;2,4,8)$

$=(126,204,282;78,204,352)$.

Table 1.49: Initial basic feasible solution by Intuitionistic Vogel's Approximation method

Sources	Destinations			
	D_1	D_2	D_3	D_4
S_1	(2,4,5;1,4,6) **(1)**	(2,5,7;1,5,8) **(10)**	(4,6,8;3,6,9)	(4,7,8;3,7,9)
S_2	(4,6,8;3,6,9) **(11)**	(3,7,12;2,7,13)	(10,15,20;8,15,22)	(11,12,13;10,12,14)
S_3	(3,4,6;1,4,8) **(3)**	(8,10,13;5,10,16)	(2,3,5;1,3,6) **(8)**	(6,10,14;5,10,15)
S_4	(2,4,6;1,4,7) **(1)**	(3,9,10;2,9,12)	(3,6,10;2,6,12)	(3,4,5;2,4,8) **(11)**

1.1.4.2 Optimal solution

Using the existing intuitionistic fuzzy Modified Distribution method [131] discussed in Section 1.1.3.2, the optimal solution of existing intuitionistic fuzzy balanced transportation problem of type – II by considering the initial basic feasible solution, obtained in Table 1.35, can be obtained as follows:

Step 1: The intuitionistic fuzzy dual variables corresponding to i^{th} row and j^{th} column are $\tilde{u}_i^I = (u_1, u_2, u_3; u_1', u_2, u_3')$ and $\tilde{v}_j^I = (v_1, v_2, v_3; v_1', v_2, v_3'); i = 1,2,3,4; j = 1,2,3,4$ respectively such that for the basic cells i.e., (1, 1), (2, 1), (2, 2), (3, 2), (3, 3), (4, 3) and (4, 4),

$\tilde{u}_1^I = (2,4,5;1,4,6)\,\Theta\,\tilde{v}_1^I$, $\tilde{u}_2^I = (4,6,8;3,6,9)\,\Theta\,\tilde{v}_1^I$, $\tilde{u}_2^I = (3,7,12;2,7,13)\,\Theta\,\tilde{v}_2^I$, $\tilde{u}_3^I = (8,10,13;5,10,16)\,\Theta\,\tilde{v}_2^I$, $\tilde{u}_3^I = (2,3,5;1,3,6)\,\Theta\,\tilde{v}_3^I$, $\tilde{u}_4^I = (3,6,10;2,6,12)\,\Theta\,\tilde{v}_3^I$, $\tilde{u}_4^I = (3,4,5;2,4,8)\,\Theta\,\tilde{v}_4^I$.

By putting the value of any one dual variable as zero intuitionistic fuzzy number, i.e., $\tilde{u}_4^I = (0,0,0;0,0,0)$, the value of remaining dual variables are under as,

$\tilde{u}_1^I = (-24,-8,7;-33,-8,15), \tilde{u}_2^I = (-18,-6,6;-25,-6,12), \tilde{u}_3^I = (-8,-3,2;-11,-3,4), \tilde{u}_4^I = (0,0,0;0,0,0), \tilde{v}_1^I = (-2,12,26;-9,12,34), \tilde{v}_2^I = (6,13,21;1,13,27), \tilde{v}_3^I = (3,6,10;2,6,12), \tilde{v}_4^I = (3,4,5;2,4,8).$

Step 2: Using Step 3 of the existing method [131], discussed in Section 1.1.3.2, the rank of $\tilde{d}_{ij}^I$ for all non basic cells are

$$R\left(\tilde{d}_{12}^I\right) = \frac{2}{8}, R\left(\tilde{d}_{13}^I\right) = -8, R\left(\tilde{d}_{14}^I\right) = -\frac{85}{8}, R\left(\tilde{d}_{23}^I\right) = -\frac{118}{8}, R\left(\tilde{d}_{24}^I\right) = -\frac{111}{8}, R\left(\tilde{d}_{31}^I\right) = \frac{38}{8}, R\left(\tilde{d}_{34}^I\right) = -\frac{71}{8}, R\left(\tilde{d}_{41}^I\right) = \frac{65}{8}, R\left(\tilde{d}_{42}^I\right) = \frac{44}{8}.$$

Step 3: Using Step 4 and Step 5 of existing method [131], discussed in Section 1.1.3.2, the non – basic cell (4, 1) will enter in the basic cell. The loop is given in Table 1.50.

Table 1.50: First loop from cell (4, 1)

Sources	Destinations				Availability (a_i)
	D_1	D_2	D_3	D_4	
S_1	(2,4,5;1,4,6) **11**	(2,5,7;1,5,8) **(2/8)**	(4,6,8;3,6,9) **(–8)**	(4,7,8;3,7,9) **(–85/8)**	11

S_2	(4,6,8;3,6,9) **5**	(3,7,12;2,7,13) **6**	(10,15,20;8,15,22) **(–118/8)**	(11,12,13;10,12,14) **(–111/8)**	11
S_3	(3,4,6;1,4,8) **(38/8)**	(8,10,13;5,10,16) **4**	(2,3,5;1,3,6) **7**	(6,10,14;5,10,15) **(–71/8)**	11
S_4	(2,4,6;1,4,7) θ **(65/8)**	(3,9,10;2,9,12) **(44/8)**	(3,6,10;2,6,12) **1**	(3,4,5;2,4,8) **11**	12
Demand (b_j)	16	10	8	11	15

After adding and subtracting the value of θ, next (second) basic feasible solution, shown in Table 1.51, is obtained and using the Step 1 and Step 2, discussed above, the following values of intuitionistic fuzzy dual variables and the ranking values of $\tilde{d}_{ij}^I \approx \tilde{v}_j^I \oplus \tilde{u}_i^I \Theta \tilde{c}_{ij}^I$ for all non-basic cells are obtained,

$$\tilde{u}_1^I = (-6,-2,1;-8,-2,3), \tilde{u}_2^I = (0,0,0;0,0,0), \tilde{u}_3^I = (-4,3,10;-8,3,14), \tilde{u}_4^I = (-6,-2,2;-8,-2,4)$$

$$\tilde{v}_1^I = (4,6,8;3,6,9), \tilde{v}_2^I = (3,7,12;2,7,13), \tilde{v}_3^I = (-8,0,9;-13,0,14), \tilde{v}_4^I = (1,6,11;-2,6,16)$$

$$R\left(\tilde{d}_{12}^I\right) = \frac{2}{8}, R\left(\tilde{d}_{13}^I\right) = -8, R\left(\tilde{d}_{14}^I\right) = -\frac{20}{8}, R\left(\tilde{d}_{23}^I\right) = -\frac{118}{8}, R\left(\tilde{d}_{24}^I\right) = -\frac{46}{8}, R\left(\tilde{d}_{31}^I\right) = \frac{38}{8}, R\left(\tilde{d}_{34}^I\right)$$

$$= -\frac{6}{8}, R\left(\tilde{d}_{42}^I\right) = -\frac{15}{8}, R\left(\tilde{d}_{43}^I\right) = -\frac{65}{8}.$$

By using Step 4 and Step 5 of existing method [131], discussed in Section 1.1.3.2, the non-basic cell (3, 1) will enter in the basic cell and make a loop from this cell as given in Table 1.51.

Table 1.51: Second basic feasible solution

Sources	Destinations				Avail-ability (a_i)
	D_1	D_2	D_3	D_4	
S_1	(2,4,5;1,4,6) **11**	(2,5,7;1,5,8) **(2/8)**	(4,6,8;3,6,9) **(–8)**	(4,7,8;3,7,9) **(–20/8)**	11
S_2	(4,6,8;3,6,9) **4**	(3,7,12;2,7,13) **7**	(10,15,20;8,15,22) **(–118/8)**	(11,12,13;10,12,14) **(–46/8)**	11
S_3	(3,4,6;1,4,8) θ **(38/8)**	(8,10,13;5,10,16) **3**	(2,3,5;1,3,6) **8**	(6,10,14;5,10,15) **(–6/8)**	11

S_4	(2,4,6;1,4,7) **1**	(3,9,10;2,9,12) **(–15/8)**	(3,6,10;2,6,12) **(–65/8)**	(3,4,5;2,4,8) **11**	12
Demand (b_j)	16	10	8	11	15

After adding and subtracting the value of θ i.e., $\theta = \min\{3,4\} = 3$, third basic feasible solution, shown in Table 1.52, is obtained and again using the Step 1 and Step 2, discussed above, the following values of intuitionistic fuzzy dual variables and the ranking values of $\tilde{d}_{ij}^I \approx \tilde{v}_j^I \oplus \tilde{u}_i^I \Theta \tilde{c}_{ij}^I$ for all non-basic cells are obtained,

$\tilde{u}_1^I = (2,4,5;1,4,6), \tilde{u}_2^I = (4,6,8;3,6,9), \tilde{u}_3^I = (3,4,6;1,4,8), \tilde{u}_4^I = (2,4,6;1,4,7)$

$\tilde{v}_1^I = (0,0,0;0,0,0), \tilde{v}_2^I = (-5,1,8;-7,1,10), \tilde{v}_3^I = (-4,-1,2;-7,-1,5), \tilde{v}_4^I = (-3,0,3;-5,0,7)$

$$R(\tilde{d}_{12}^I) = \frac{2}{8}, R(\tilde{d}_{13}^I) = -\frac{26}{8}, R(\tilde{d}_{14}^I) = -\frac{20}{8}, R(\tilde{d}_{23}^I) = -\frac{80}{8}, R(\tilde{d}_{24}^I) = -\frac{46}{8}, R(\tilde{d}_{32}^I) = -\frac{38}{8}, R(\tilde{d}_{34}^I) = -\frac{44}{8}, R(\tilde{d}_{42}^I) = -\frac{21}{8}, R(\tilde{d}_{43}^I) = -\frac{27}{8}.$$

Using Step 4 and Step 5 of existing method [131], discussed in Section 1.1.3.2, the non-basic cell (1, 2) will enter in the basic cell and make a loop from this cell as shown in Table 1.52.

Table 1.52: Third basic feasible solution

Sources	Destinations				Avail-ability (a_i)
	D_1	D_2	D_3	D_4	
S_1	(2,4,5;1,4,6) **11**	(2,5,7;1,5,8) θ **(2/8)**	(4,6,8;3,6,9) **(–26/8)**	(4,7,8;3,7,9) **(–20/8)**	11
S_2	(4,6,8;3,6,9) **1**	(3,7,12;2,7,13) **10**	(10,15,20;8,15,22) **(–80/8)**	(11,12,13;10,12,14) **(–46/8)**	11
S_3	(3,4,6;1,4,8) **3**	(8,10,13;5,10,16) **(–38/8)**	(2,3,5;1,3,6) **8**	(6,10,14;5,10,15) **(–44/8)**	11
S_4	(2,4,6;1,4,7) **1**	(3,9,10;2,9,12) **(–21/8)**	(3,6,10;2,6,12) **(–27/8)**	(3,4,5;2,4,8) **11**	12
Demand (b_j)	16	10	8	11	15

After getting the fourth basic feasible solution, shown in Table 1.52, again using the Step 1 and Step 2, discussed above, the following values of intuitionistic fuzzy dual variables and the ranking values of $\tilde{d}_{ij}^I \approx \tilde{v}_j^I \oplus \tilde{u}_i^I \,\Theta\, \tilde{c}_{ij}^I$ for all non-basic cells are obtained,

$$\tilde{u}_1^I = (2,4,5;1,4,6), \tilde{u}_2^I = (4,6,8;3,6,9), \tilde{u}_3^I = (3,4,6;1,4,8), \tilde{u}_4^I = (2,4,6;1,4,7)$$

$$\tilde{v}_1^I = (0,0,0;0,0,0), \tilde{v}_2^I = (-3,1,5;-5,1,7), \tilde{v}_3^I = (-4,-1,2;-7,-1,5), \tilde{v}_4^I = (-3,0,3;-5,0,7)$$

$$R\left(\tilde{d}_{13}^I\right) = -\frac{26}{8}, R\left(\tilde{d}_{14}^I\right) = -\frac{20}{8}, R\left(\tilde{d}_{22}^I\right) = -\frac{1}{4}, R\left(\tilde{d}_{23}^I\right) = -\frac{80}{8}, R\left(\tilde{d}_{24}^I\right) = -\frac{46}{8}, R\left(\tilde{d}_{32}^I\right) = -\frac{15}{4},$$

$$R\left(\tilde{d}_{34}^I\right) = -\frac{44}{8}, R\left(\tilde{d}_{42}^I\right) = -\frac{15}{8}, R\left(\tilde{d}_{43}^I\right) = -\frac{27}{8}.$$

Here, $R\left(\tilde{d}_{ij}^I\right) \leq 0$ for all non-basic cells. Therefore, the fourth basic feasible solution, shown in Table 1.53, is the optimal solution.

Table 1.53: Optimal solution

Sources	Destinations				Availability (a_i)
	D_1	D_2	D_3	D_4	
S_1	(2,4,5;1,4,6) **1**	(2,5,7;1,5,8) **10**	(4,6,8;3,6,9) **(–26/8)**	(4,7,8;3,7,9) **(–20/8)**	11
S_2	(4,6,8;3,6,9) **11**	(3,7,12;2,7,13) **(–1/4)**	(10,15,20;8,15,22) **(–80/8)**	(11,12,13;10,12,14) **(–46/8)**	11
S_3	(3,4,6;1,4,8) **3**	(8,10,13;5,10,16) **(–15/4)**	(2,3,5;1,3,6) **8**	(6,10,14;5,10,15) **(–44/8)**	11
S_4	(2,4,6;1,4,7) **1**	(3,9,10;2,9,12) **(–15/8)**	(3,6,10;2,6,12) **(–27/8)**	(3,4,5;2,4,8) **11**	12
Demand (b_j)	16	10	8	11	15

Step 4: The optimal solution of intuitionistic fuzzy balanced transportation problem, presented by Table 1.28, is $x_{11} = 1, x_{12} = 10, x_{21} = 11, x_{31} = 3, x_{32} = 8, x_{41} = 1, x_{44} = 11$ and the optimal intuitionistic fuzzy transportation cost is $1 \times (2,4,5;1,4,6) + 10 \times (2,5,7;1,5,8) + 11 \times (4,6,8;3,6,9)$

$+3\times(3,4,6;1,4,8)+8\times(2,3,5;1,3,6)+1\times(2,4,6;1,4,7)+11\times(3,4,5;2,4,8)$

$=(126,204,282;78,204,352)$.

1.2 Brief review about the work

After a deep study of all the existing methods [1,3,9,18,29,45,46,48,49,64-66,84-86,112,118,119,123,131-134,138], it is noticed that there are flaws in the existing methods [18,29,45,49,84-86,112,123,133,134,138] for solving intuitionistic fully fuzzy transportation problems and alternative simple methods can be proposed for solving intuitionistic fuzzy transportation problems of type – I and intuitionistic fuzzy transportation problems of type – II.

Keeping the same in mind, in this thesis, alternative simple methods are proposed for solving intuitionistic fuzzy transportation problems of type – I and intuitionistic fuzzy transportation problems of type – II [1,3,9,46,48,64-66,118,119,131,132] and new methods are proposed for solving intuitionistic fully fuzzy transportation problems [18,29,45,49,84-86,112,123,133,134, 138].

The chapter wise summary of thesis is as follows,

Chapter 2 A simplified method for solving intuitionistic fuzzy transportation problems of type – I

In this chapter, an alternative method for solving intuitionistic fuzzy transportation problems of type – I is proposed. Also, the advantages of proposed method over the existing methods [9,46,48,64-66,118,132] are discussed. Furthermore, to illustrate the proposed method, the numerical examples, considered by Singh and Yadav [132], are solved by the proposed method.

Chapter 3 A simplified method for solving intuitionistic fuzzy transportation problems of type – II

In this chapter, an alternative method for solving intuitionistic fuzzy transportation problems of type – II is proposed. Also, the advantages of proposed method over the existing methods [1,3,119,131], are discussed. Furthermore, to illustrate the proposed method, the numerical examples, considered by Singh and Yadav [131], are solved by the proposed method.

Chapter 4 Modified approach for solving intuitionistic fully fuzzy transportation problems

Kumar and Hussain [84-86] proposed methods for solving intuitionistic fully fuzzy transportation problems. In this chapter, it is pointed out that for the ranking function, used by Kumar and Hussain [84-86], the linearity property is not satisfying. However, in the existing method [84-86], this property is used. Therefore, the existing methods [84-86] are not valid. Hence, the result of numerical problems, obtained by Kumar and Hussain [84-86] by their proposed method, is also not correct. Furthermore, it is shown that for the ranking function, used by Singh and Yadav [132], linearity property is satisfying. Hence, the existing methods [84-86], will be valid if the ranking function, used by Kumar and Hussain [84-86], is replaced with the ranking function, used by Singh and Yadav [132]. Also, the exact results of numerical problems, considered by Kumar and Hussain [86], are obtained.

Chapter 5 A new method for solving intuitionistic fully fuzzy transportation problems

In this chapter, flaws of the existing methods [18,29,45,49,112,118,123,133,134,138] for solving intuitionistic fully fuzzy transportation problems are pointed out. Also, a new method is proposed for solving intuitionistic fully fuzzy transportation problems. To illustrate the proposed method, the intuitionistic fully fuzzy transportation problem, considered by Roseline and Amirtharaj [123], is solved by proposed method.

Chapter 6 A new method for solving generalized intuitionistic fully fuzzy transportation problems

Chakraborty et al. [18] proposed the arithmetic operations on generalized trapezoidal intuitionistic fuzzy numbers and used these arithmetic operations to find the solution of such intuitionistic fully fuzzy transportation problems in which cost, availability and demand all are represented by generalized trapezoidal intuitionistic fuzzy numbers. In this chapter, it is shown that Chakraborty et al. [18] have used the property $R\left(\tilde{A}^I \otimes \tilde{B}^I\right) = R\left(\tilde{A}^I\right) \times R\left(\tilde{B}^I\right)$ in their proposed method. While, for the ranking function *R*, considered by Chakraborty et al. [18], this property is not satisfying. Hence, it is not genuine to used the method, proposed by Chakraborty et al. [18], to find the solution of generalized intuitionistic fully fuzzy transportation problem. Furthermore, a new method is proposed to resolve the flaws of the existing method [18]. To illustrate the proposed method, the generalized intuitionistic fully fuzzy transportation problem, considered by Chakraborty et al. [18], is solved by proposed method.

Chapter 7 Future scope

It is noticed that ranking of generalized exponential trapezoidal fuzzy numbers, obtained by using the existing method [121], is independent from height of generalized exponential trapezoidal fuzzy numbers. While, the ranking of generalized exponential trapezoidal fuzzy numbers should be dependent on its height. Hence, it is not genuine to use the existing method [121] for comparing the generalized exponential trapezoidal fuzzy numbers. In this chapter, the flaws of the existing method [121] are pointed out and a modified method for ranking of generalized exponential trapezoidal fuzzy numbers is proposed. In future, the proposed ranking method may be extended for generalized exponential trapezoidal intuitionistic fuzzy numbers and a new method may be proposed to find the solution of generalized exponential trapezoidal intuitionistic fully fuzzy transportation problems (transportation problems in which cost, availability and demand are represented by generalized exponential trapezoidal intuitionistic fuzzy numbers).

1.3 Intuitionistic fuzzy sets

In this section, some basic definitions (intuitionistic fuzzy set, intuitionistic fuzzy number, triangular intuitionistic fuzzy number etc.), arithmetic operations of triangular / trapezoidal intuitionistic fuzzy numbers and existing method for ordering of triangular / trapezoidal intuitionistic fuzzy numbers are presented.

1.3.1 Some basic definitions

In this section, some basic definitions are presented [132].

Definition 1.1: Let X be a universal set. Then, a fuzzy set $\tilde{A}$ in X is defined by $\tilde{A}=\left\{\left(x,\mu_{\tilde{A}}(x)\right):\right.$ $\left.x\in X\right\}$, where $\mu_{\tilde{A}}:X\rightarrow[0,1]$ is called membership function.

Definition 1.2: Let X be a universe of discourse. Then, an intuitionistic fuzzy set $\tilde{A}^I$ in X is defined by a set of ordered triples $\tilde{A}^I=\left\{\left\langle x,\mu_{\tilde{A}^I}(x),\nu_{\tilde{A}^I}(x)\right\rangle:x\in X\right\}$ where $\mu_{\tilde{A}^I}(x),\nu_{\tilde{A}^I}(x):X\rightarrow\left[0,1\right]$ are functions such that $0\leq\mu_{\tilde{A}^I}(x),\nu_{\tilde{A}^I}(x)\leq 1,\ \forall x\in X$. $\mu_{\tilde{A}^I}(x)$ and $\nu_{\tilde{A}^I}(x)$ represent the degree of membership and degree of non-membership of the element $x\in X$

being in $\tilde{A}^I$, respectively. The degree of hesitation for the element $x \in X$ being in $\tilde{A}^I$ is given by $h(x) = 1 - \mu_{\tilde{A}^I}(x) - \nu_{\tilde{A}^I}(x) \le 1, \forall x \in X$.

Definition 1.3: An intuitionistic fuzzy subset, $\tilde{A}^I = \left\{ \left\langle x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x) \right\rangle : x \in X \right\}$, of the real line $\mathbb{R}$ is called an intuitionistic fuzzy number if the following holds:

(i) There exist $m \in \mathbb{R}$, $\mu_{\tilde{A}^I}(m) = 1$ and $\nu_{\tilde{A}^I}(m) = 0$, (m is called the mean value of $\tilde{A}^I$).

(ii) $\mu_{\tilde{A}^I}(x)$ and $\nu_{\tilde{A}^I}(x)$ are piecewise continuous mapping from $\mathbb{R}$ to the closed interval [0, 1] and the relation $0 \le \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x) \le 1, \forall x \in \mathbb{R}$ holds.

The membership and non-membership function of $\tilde{A}^I$ is of the following form:

$$\mu_{\tilde{A}^I}(x) = \begin{cases} 0 & ; \ -\infty < x \le m - \alpha \\ f_1(x) & ; \ x \in (m - \alpha, m] \\ 1 & ; \ x = m \\ h_1(x) & ; \ x \in [m, m + \beta) \\ 0 & ; \ m + \beta \le x < \infty \end{cases}$$

where $f_1(x)$ and $h_1(x)$ are strictly increasing and decreasing function in $(m - \alpha, m]$ and $[m, m + \beta)$ respectively.

$$\nu_{\tilde{A}^I}(x) = \begin{cases} 1 & ; \ -\infty < x \le m - \alpha' \\ f_2(x) & ; \ x \in (m - \alpha', m]; 0 \le f_1(x) + f_2(x) \le 1 \\ 0 & ; \ x = m \\ h_2(x) & ; \ x \in [m, m + \beta'); 0 \le h_1(x) + h_2(x) \le 1 \\ 0 & ; \ m + \beta' \le x < \infty \end{cases}$$

where $f_2(x)$ and $h_2(x)$ are strictly increasing and decreasing function in $(m - \alpha', m]$ and $[m, m + \beta')$ respectively. Here α and β are called left and right spreads of membership function $\mu_{\tilde{A}^I}(x)$ respectively. α' and β' are called left and right spreads of non-membership function

$\nu_{\tilde{A}^I}(x)$ respectively. The intuitionistic fuzzy number $\tilde{A}^I$ is represented by $\tilde{A}^I = (m;\alpha,\beta;\alpha',\beta')$.

Definition 1.4: An intuitionistic fuzzy number $\tilde{A}^I = \left(a_1, a_2, a_3; a_1', a_2, a_3'\right)$ is said to be triangular intuitionistic fuzzy number if its membership function $\mu_{\tilde{A}^I}(x)$ and non-membership function $\nu_{\tilde{A}^I}(x)$ is given by

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & ;\ a_1 < x \le a_2 \\ \dfrac{a_1-x}{a_3-a_2} & ;\ a_2 < x \le a_3 \\ 0 & ;\ \text{otherwise} \end{cases} \quad \text{and} \quad \nu_{\tilde{A}^I}(x) = \begin{cases} \dfrac{a_1'-x}{a_2-a_1} & ;\ a_1' < x \le a_2 \\ \dfrac{x-a_2}{a_3'-a_2} & ;\ a_2 < x \le a_3' \\ 1 & ;\ \text{otherwise} \end{cases}$$

Definition 1.5: An intuitionistic fuzzy number $\tilde{A}^I = \left(a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4'\right)$ is said to be trapezoidal intuitionistic fuzzy number if its membership function $\mu_{\tilde{A}^I}(x)$ and non-membership function $\nu_{\tilde{A}^I}(x)$ is given by

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & ;\ a_1 \le x < a_2 \\ 1 & ;\ a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3} & ;\ a_3 < x \le a_4 \\ 0 & ;\ \text{otherwise} \end{cases} \quad \text{and} \quad \nu_{\tilde{A}^I}(x) = \begin{cases} \dfrac{a_2-x}{a_2-a_1} & ;\ a_1' \le x < a_2 \\ 0 & ;\ a_2 \le x \le a_3 \\ \dfrac{a_3-x}{a_3-a_4} & ;\ a_3 < x \le a_4' \\ 1 & ;\ \text{otherwise} \end{cases}$$

1.3.2 Arithmetic operations on intuitionistic fuzzy numbers

In this section, arithmetic operations on triangular and trapezoidal intuitionistic fuzzy numbers are presented [132].

1.3.2.1 Arithmetic operations on triangular intuitionistic fuzzy numbers

In this section, arithmetic operations on triangular intuitionistic fuzzy numbers are presented [132].

Let $\tilde{A}^I = (a_1, a_2, a_3; a_1', a_2, a_3')$ and $\tilde{B}^I = (b_1, b_2, b_3; b_1', b_2, b_3')$ be two triangular intuitionistic fuzzy numbers. Then,

(i) $\tilde{A}^I \oplus \tilde{B}^I = (a_1 + b_1, a_2 + b_2, a_3 + b_3; a_1' + b_1', a_2 + b_2, a_3' + b_3')$

(ii) $\tilde{A}^I \ominus \tilde{B}^I = (a_1 - b_3, a_2 - b_2, a_3 - b_1; a_1' - b_3', a_2 - b_2, a_3' - b_1')$

(iii) $\tilde{A}^I \otimes \tilde{B}^I = (m_1, m_2, m_3; m_1', m_2, m_3')$,

where, $m_1 = \min\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$, $m_2 = a_2b_2$, $m_3 = \max\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$, $m_1' = \min\{a_1'b_1',$ $a_1'b_3', a_3'b_1', a_3'b_3'\}$, $m_3' = \max\{a_1'b_1', a_1'b_3', a_3'b_1', a_3'b_3'\}$.

(iv) $$\lambda\tilde{A}^I = \begin{cases} (\lambda a_1, \lambda a_2, \lambda a_3; \lambda a_1', \lambda a_2, \lambda a_3'); & \lambda \geq 0, \\ (\lambda a_3, \lambda a_2, \lambda a_1; \lambda a_3', \lambda a_2, \lambda a_1'); & \lambda < 0. \end{cases}$$

1.3.2.2 Arithmetic operations on trapezoidal intuitionistic fuzzy numbers

In this section, arithmetic operations on trapezoidal intuitionistic fuzzy numbers are presented [133].

Let $\tilde{A}^I = (a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4')$ and $\tilde{B}^I = (b_1, b_2, b_3, b_4; b_1', b_2, b_3, b_4')$ be two trapezoidal intuitionistic fuzzy numbers. Then,

(i) $\tilde{A}^I \oplus \tilde{B}^I = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; a_1' + b_1', a_2 + b_2, a_3 + b_3, a_4' + b_4')$

(ii) $\tilde{A}^I \ominus \tilde{B}^I = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1; a_1' - b_4', a_2 - b_2, a_3 - b_2, a_4' - b_1')$

(iii) $\tilde{A}^I \otimes \tilde{B}^I = (m_1, m_2, m_3, m_4; m_1', m_2, m_3, m_4')$, where, $m_1 = \min\{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$, $m_2 = \min\{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$, $m_3 = \max\{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$, $m_4 = \max\{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$, $m_1' = \min\{a_1'b_1', a_1'b_4', a_4'b_1', a_4'b_4'\}$, $m_4' = \max\{a_1'b_1', a_1'b_4', a_4'b_1', a_4'b_4'\}$.

(iv) $$\lambda \tilde{A}^I = \begin{cases} (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4; \lambda a_1', \lambda a_2, \lambda a_3, \lambda a_4'); & \lambda \geq 0, \\ (\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1; \lambda a_4', \lambda a_3, \lambda a_2, \lambda a_1'); & \lambda < 0. \end{cases}$$

1.3.3 Ordering of intuitionistic fuzzy numbers

In this section, ordering of triangular / trapezoidal intuitionistic fuzzy numbers are presented [132].

1.3.3.1 Ordering of triangular intuitionistic fuzzy numbers

Singh and Yadav [132] pointed out the shortcomings of existing methods [46,64] for ordering of triangular intuitionistic fuzzy numbers and proposed the following method for ordering of triangular intuitionistic fuzzy numbers.

Let $\tilde{A}^I = (a_1, a_2, a_3; a_1', a_2, a_3')$ and $\tilde{B}^I = (b_1, b_2, b_3; b_1', b_2, b_3')$ be two triangular intuitionistic fuzzy numbers. Then,

(i) $\tilde{A}^I \succeq \tilde{B}^I$ if $R(\tilde{A}^I) \geq R(\tilde{B}^I)$

(ii) $\tilde{A}^I \approx \tilde{B}^I$ if $R(\tilde{A}^I) = R(\tilde{B}^I)$

where,

$$R(\tilde{A}^I) = \left(\frac{a_1 + 2a_2 + a_3 + a_1' + 2a_2 + a_3'}{8}\right) \text{ and } R(\tilde{B}^I) = \left(\frac{b_1 + 2b_2 + b_3 + b_1' + 2b_2 + b_3'}{8}\right).$$

1.3.3.2 Ordering of trapezoidal intuitionistic fuzzy numbers

In this section, ordering of trapezoidal intuitionistic fuzzy numbers are presented [133].

Let $\tilde{A}^I = (a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4')$ and $\tilde{B}^I = (b_1, b_2, b_3, b_4; b_1', b_2, b_3, b_4')$ be two trapezoidal intuitionistic fuzzy numbers. Then,

(i) $\tilde{A}^I \succeq \tilde{B}^I$ if $R(\tilde{A}^I) \geq R(\tilde{B}^I)$

(ii) $\tilde{A}^I \approx \tilde{B}^I$ if $R(\tilde{A}^I) = R(\tilde{B}^I)$

where, $R(\tilde{A}^I) = \left(\dfrac{a_1 + a_2 + a_3 + a_4 + a_1' + a_2 + a_3 + a_4'}{8} \right)$

and $R(\tilde{B}^I) = \left(\dfrac{b_1 + b_2 + b_3 + b_4 + b_1' + b_2 + b_3 + b_4'}{8} \right)$.

Chapter 2

A simplified method for solving intuitionistic fuzzy transportation problems of type – I*

In this chapter, an alternative method for solving intuitionistic fuzzy transportation problems of type – I is proposed. Also, the advantages of proposed method over the existing methods [9,46,48,64-66,118,132] are discussed. Furthermore, to illustrate the proposed method, the numerical examples, considered by Singh and Yadav [132], are solved by the proposed method.

2.1 Intuitionistic fuzzy linear programming problem of intuitionistic fuzzy transportation problems of type – I

It is well known that problem (P2.1) represents the linear programming problem of such balanced transportation problems (total availability = total demand) having m sources and n destinations for which the precise information about the unit transportation cost (c_{ij}) for i^{th} source to j^{th} destination, availability (a_i) at i^{th} source and demand (b_j) at j^{th} destination is available.

$$\text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i;\, i = 1,2,\ldots,m, \qquad \text{(P2.1)}$$

$$\sum_{i=1}^{m} x_{ij} = b_j;\, j = 1,2,\ldots,n,$$

$$x_{ij} \geq 0;\, i = 1,2,\ldots,m;\; j = 1,2,\ldots,n.$$

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where, x_{ij} is the quantity of the product need to be supplied from i^{th} source to j^{th} destination.

In the literature [9,46,48,64-66,118,132], the intuitionistic fuzzy transportation problem of type – I (P2.2) is obtained by replacing the availability (a_i) and demand (b_j) and the quantity (x_{ij}) with triangular intuitionistic fuzzy numbers $\left(a_{i1}, a_{i2}, a_{i3}; a'_{i1}, a_{i2}, a'_{i3}\right)$, $\left(b_{j1}, b_{j2}, b_{j3}; b'_{j1}, b_{j2}, b'_{j3}\right)$ and $(x_{ij1}, x_{ij2}, x_{ij3};\ x'_{ij1}, x_{ij2}, x'_{ij3})$ respectively.

$$\text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right)$$

Subject to

$$\begin{aligned}
&\sum_{j=1}^{n}\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right) \approx \left(a_{i1}, a_{i2}, a_{i3}; a'_{i1}, a_{i2}, a'_{i3}\right); i = 1, 2, \ldots, m, \\
&\sum_{i=1}^{m}\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right) \approx \left(b_{j1}, b_{j2}, b_{j3}; b'_{j1}, b_{j2}, b'_{j3}\right);\ j = 1, 2, \ldots, n, \\
&\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right) \succeq (0, 0, 0; 0, 0, 0); i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n.
\end{aligned} \qquad \text{(P2.2)}$$

where, $\sum_{i=1}^{m}\left(a_{i1}, a_{i2}, a_{i3}; a'_{i1}, a_{i2}, a'_{i3}\right) \approx \sum_{j=1}^{n}\left(b_{j1}, b_{j2}, b_{j3}; b'_{j1}, b_{j2}, b'_{j3}\right)$.

2.2 Proposed method

In this section, an alternative method to find the initial basic feasible solution and optimal solution of intuitionistic fuzzy transportation problem of type – I is proposed.

The steps of proposed method are as follows:

Step 1: Using Section 1.3.3.1, the problem (P2.2) can be transformed into the problem (P2.3).

$$\text{Minimize } R\left(\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right)\right)$$

Subject to

$$R\left(\sum_{j=1}^{n}\left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)\right)=R\left(a_{i1},a_{i2},a_{i3};a'_{i1},a_{i2},a'_{i3}\right);i=1,2,...,m, \qquad \text{(P2.3)}$$

$$R\left(\sum_{i=1}^{m}\left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)\right)=R\left(b_{j1},b_{j2},b_{j3};b'_{j1},b_{j2},b'_{j3}\right);\ j=1,2,...,n,$$

$$R\left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)\geq R(0,0,0;0,0,0);i=1,2,...,m;\ j=1,2,...,n.$$

Step 2: Using the relation, $R\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(a_{ij},b_{ij},c_{ij};a'_{ij},b_{ij},c'_{ij}\right)\right)=\sum_{i=1}^{m}\sum_{j=1}^{n}R\left(a_{ij},b_{ij},c_{ij};a'_{ij},b_{ij},c'_{ij}\right)$, the problem (P2.3) can be transformed into the problem (P2.4).

Minimize $\sum_{i=1}^{m}\sum_{j=1}^{n}R\left(c_{ij}\left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)\right)$

Subject to

$$\sum_{j=1}^{n}R\left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)=R\left(a_{i1},a_{i2},a_{i3};a'_{i1},a_{i2},a'_{i3}\right);i=1,2,...,m, \qquad \text{(P2.4)}$$

$$\sum_{i=1}^{m}R\left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)=R\left(b_{j1},b_{j2},b_{j3};b'_{j1},b_{j2},b'_{j3}\right);\ j=1,2,...,n,$$

$$R\left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)\geq R(0,0,0;0,0,0);i=1,2,...,m;\ j=1,2,...,n.$$

Step 3: Using the relation, $R\left(\lambda\times\left(a,b,c;a',b,c'\right)\right)=\lambda\times R\left(a,b,c;a',b,c'\right)$, the problem (P2.4) can be transformed into the problem (P2.5).

Minimize $\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}\times R\left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)$

Subject to

Constraints of problem (P2.4). (P2.5)

Step 4: Since, $R\left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)$ will be a real number. So, assuming $R\left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)=x_{ij}$, the problem (P2.5) can be transformed into the problem (P2.6).

Minimize $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}$

Subject to

$$\sum_{j=1}^{n} x_{ij} = R\left(a_{i1}, a_{i2}, a_{i3}; a'_{i1}, a_{i2}, a'_{i3}\right); i = 1, 2, ..., m, \quad \text{(P2.6)}$$

$$\sum_{i=1}^{m} x_{ij} = R\left(b_{j1}, b_{j2}, b_{j3}; b'_{j1}, b_{j2}, b'_{j3}\right); j = 1, 2, ..., n,$$

$$x_{ij} \geq 0; i = 1, 2, ..., m; j = 1, 2, ..., n.$$

Step 5: Using the expression, $R\left(a, b, c; a', b, c'\right) = \dfrac{a + 2b + c + a' + 2b + c'}{8}$, the problem (P2.6) can be transformed into the problem (P2.7).

Minimize $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}$

Subject to

$$\sum_{j=1}^{n} x_{ij} = \frac{a_{i1} + 2a_{i2} + a_{i3} + a'_{i1} + 2a_{i2} + a'_{i3}}{8}; i = 1, 2, ..., m, \quad \text{(P2.7)}$$

$$\sum_{i=1}^{m} x_{ij} = \frac{b_{j1} + 2b_{j2} + b_{j3} + b'_{j1} + 2b_{j2} + b'_{j3}}{8}; j = 1, 2, ..., n,$$

$$x_{ij} \geq 0; i = 1, 2, ..., m; j = 1, 2, ..., n.$$

Step 6: Find the optimal solution of crisp linear programming problem (P2.7).

Step 7: If $\left\{x_{ij} = p_{ij}\right\}$ is the set of optimal solution of problem (P2.7) then $\left\{\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right) : \dfrac{x_{ij1} + 2x_{ij2} + x_{ij3} + x'_{ij1} + 2x_{ij2} + x'_{ij3}}{8} = p_{ij}\right\}$ represents the set of optimal solution of problem (P2.2) and the minimum intuitionistic fuzzy transportation cost of intuitionistic fuzzy transportation problem is $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right)$.

Remark 2.1: The existing intuitionistic fuzzy transportation problem [138], in which availability and demand are represented by hexagonal intuitionistic fuzzy numbers can also be easily solved by using the proposed method by replacing the triangular intuitionistic fuzzy numbers with hexagonal intuitionistic fuzzy numbers.

2.3 Illustrative example

To illustrate the proposed method, the existing intuitionistic fuzzy transportation problem of type – I [132], presented by Table 1.1, is solved. Using the proposed method, the initial basic feasible solution and hence optimal solution of the existing intuitionistic fuzzy transportation problem of type – I can be obtained as follows:

Step 1: Using Section 2.1, the intuitionistic fuzzy transportation problem of type – I, presented in Table 1.1, can be transformed into the problem (P2.8).

$$\begin{aligned}\text{Minimize } \Big[&16\left(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113}\right)\oplus\left(x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123}\right)\oplus 8(x_{131},x_{132},x_{133};x'_{131},x_{132},\\&x'_{133})\oplus 13\left(x_{141},x_{142},x_{143};x'_{141},x_{142},x'_{143}\right)\oplus 11\left(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213}\right)\oplus 4(x_{221},x_{222},x_{223};\\&x'_{221},x_{222},x'_{223})\oplus 7\left(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233}\right)\oplus 10\left(x_{241},x_{242},x_{243};x'_{241},x_{242},x'_{243}\right)\oplus 8(x_{311},\\&x_{312},x_{313};x'_{311},x_{312},x'_{313})\oplus 15\left(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323}\right)\oplus 9\left(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333}\right)\\&\oplus 2\left(x_{341},x_{342},x_{343};x'_{341},x_{342},x'_{343}\right)\oplus 6\left(x_{411},x_{412},x_{413};x'_{411},x_{412},x'_{413}\right)\oplus 12(x_{421},x_{422},x_{423};x'_{421},\\&x_{422},x'_{423})\oplus 5\left(x_{431},x_{432},x_{433};x'_{431},x_{432},x'_{433}\right)\oplus 14\left(x_{441},x_{442},x_{443};x'_{441},x_{442},x'_{443}\right)\Big] \qquad \text{(P2.8)}\end{aligned}$$

Subject to

$$\begin{aligned}&\left(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113}\right)\oplus\left(x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123}\right)\oplus\left(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133}\right)\\&\qquad\oplus\left(x_{141},x_{142},x_{143};x'_{141},x_{142},x'_{143}\right)\approx\left(2,4,5;1,4,6\right),\\&\left(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213}\right)\oplus\left(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223}\right)\oplus\left(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233}\right)\\&\qquad\oplus\left(x_{241},x_{242},x_{243};x'_{241},x_{242},x'_{243}\right)\approx\left(4,6,8;3,6,9\right),\\&\left(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313}\right)\oplus\left(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323}\right)\oplus\left(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333}\right)\\&\qquad\oplus\left(x_{341},x_{342},x_{343};x'_{341},x_{342},x'_{343}\right)\approx\left(3,7,12;2,7,13\right),\\&\left(x_{411},x_{412},x_{413};x'_{411},x_{412},x'_{413}\right)\oplus\left(x_{421},x_{422},x_{423};x'_{421},x_{422},x'_{423}\right)\oplus\left(x_{431},x_{432},x_{433};x'_{431},x_{432},x'_{433}\right)\\&\qquad\oplus\left(x_{441},x_{442},x_{443};x'_{441},x_{442},x'_{443}\right)\approx\left(8,10,13;5,10,16\right),\\&\left(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113}\right)\oplus\left(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213}\right)\oplus\left(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313}\right)\\&\qquad\oplus\left(x_{411},x_{412},x_{413};x'_{411},x_{412},x'_{413}\right)\approx\left(3,4,6;1,4,8\right),\end{aligned}$$

$$\left(x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123}\right)\oplus\left(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223}\right)\oplus\left(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323}\right)$$
$$\oplus\left(x_{421},x_{422},x_{423};x'_{421},x_{422},x'_{423}\right)\approx\left(2,5,7;1,5,8\right),$$
$$\left(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133}\right)\oplus\left(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233}\right)\oplus\left(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333}\right)$$
$$\oplus\left(x_{431},x_{432},x_{433};x'_{431},x_{432},x'_{433}\right)\approx\left(10,15,20;8,15,22\right),$$
$$\left(x_{141},x_{142},x_{143};x'_{141},x_{142},x'_{143}\right)\oplus\left(x_{241},x_{242},x_{243};x'_{241},x_{242},x'_{243}\right)\oplus\left(x_{341},x_{342},x_{343};x'_{341},x_{342},x'_{343}\right)$$
$$\oplus\left(x_{441},x_{442},x_{443};x'_{441},x_{442},x'_{443}\right)\approx\left(2,3,5;1,3,6\right),$$
$$\left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)\succeq\left(0,0,0;0,0,0\right);i=1,2,3,4;\ j=1,2,3,4.$$

Step 2: Using Step 1 of the proposed method, the problem (P2.8) can be transformed into the problem (P2.9).

$$\text{Minimize}\Big[R\big(16\left(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113}\right)\oplus\left(x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123}\right)\oplus 8(x_{131},x_{132},x_{133};x'_{131},x_{132},$$
$$x'_{133})\oplus 13\left(x_{141},x_{142},x_{143};x'_{141},x_{142},x'_{143}\right)\oplus 11\left(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213}\right)\oplus 4(x_{221},x_{222},x_{223};$$
$$x'_{221},x_{222},x'_{223})\oplus 7\left(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233}\right)\oplus 10\left(x_{241},x_{242},x_{243};x'_{241},x_{242},x'_{243}\right)\oplus 8(x_{311},$$
$$x_{312},x_{313};x'_{311},x_{312},x'_{313})\oplus 15\left(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323}\right)\oplus 9\left(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333}\right)$$
$$\oplus 2\left(x_{341},x_{342},x_{343};x'_{341},x_{342},x'_{343}\right)\oplus 6\left(x_{411},x_{412},x_{413};x'_{411},x_{412},x'_{413}\right)\oplus 12(x_{421},x_{422},x_{423};x'_{421},$$
$$x_{422},x'_{423})\oplus 5\left(x_{431},x_{432},x_{433};x'_{431},x_{432},x'_{433}\right)\oplus 14\left(x_{441},x_{442},x_{443};x'_{441},x_{442},x'_{443}\right)\big)\Big] \qquad \text{(P2.9)}$$

Subject to

$$R\big(\left(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113}\right)\oplus\left(x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123}\right)\oplus\left(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133}\right)$$
$$\oplus\left(x_{141},x_{142},x_{143};x'_{141},x_{142},x'_{143}\right)\big)=R\left(2,4,5;1,4,6\right),$$
$$R\big(\left(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213}\right)\oplus\left(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223}\right)\oplus\left(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233}\right)$$
$$\oplus\left(x_{241},x_{242},x_{243};x'_{241},x_{242},x'_{243}\right)\big)=R\left(4,6,8;3,6,9\right),$$
$$R\big(\left(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313}\right)\oplus\left(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323}\right)\oplus\left(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333}\right)$$
$$\oplus\left(x_{341},x_{342},x_{343};x'_{341},x_{342},x'_{343}\right)\big)=R\left(3,7,12;2,7,13\right),$$
$$R\big(\left(x_{411},x_{412},x_{413};x'_{411},x_{412},x'_{413}\right)\oplus\left(x_{421},x_{422},x_{423};x'_{421},x_{422},x'_{423}\right)\oplus\left(x_{431},x_{432},x_{433};x'_{431},x_{432},x'_{433}\right)$$
$$\oplus\left(x_{441},x_{442},x_{443};x'_{441},x_{442},x'_{443}\right)\big)=R\left(8,10,13;5,10,16\right),$$
$$R\big(\left(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113}\right)\oplus\left(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213}\right)\oplus\left(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313}\right)$$
$$\oplus\left(x_{411},x_{412},x_{413};x'_{411},x_{412},x'_{413}\right)\big)=R\left(3,4,6;1,4,8\right),$$
$$R\big(\left(x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123}\right)\oplus\left(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223}\right)\oplus\left(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323}\right)$$
$$\oplus\left(x_{421},x_{422},x_{423};x'_{421},x_{422},x'_{423}\right)\big)=R\left(2,5,7;1,5,8\right),$$
$$R\big(\left(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133}\right)\oplus\left(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233}\right)\oplus\left(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333}\right)$$
$$\oplus\left(x_{431},x_{432},x_{433};x'_{431},x_{432},x'_{433}\right)\big)=R\left(10,15,20;8,15,22\right),$$

$$R\left(\left(x_{141},x_{142},x_{143};x'_{141},x_{142},x'_{143}\right)\oplus\left(x_{241},x_{242},x_{243};x'_{241},x_{242},x'_{243}\right)\oplus\left(x_{341},x_{342},x_{343};x'_{341},x_{342},x'_{343}\right)\right.$$
$$\left.\oplus\left(x_{441},x_{442},x_{443};x'_{441},x_{442},x'_{443}\right)\right)=R\left(2,3,5;1,3,6\right),$$
$$R\left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)\geq R\left(0,0,0;0,0,0\right);i=1,2,3,4;\ j=1,2,3,4.$$

Step 3: Using the relation, $R\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(a_{ij},b_{ij},c_{ij};a'_{ij},b_{ij},c'_{ij}\right)\right)=\sum_{i=1}^{m}\sum_{j=1}^{n}R\left(a_{ij},b_{ij},c_{ij};a'_{ij},b_{ij},c'_{ij}\right)$, the problem (P2.9) can be transformed into the problem (P2.10).

$$\text{Minimize}\ \Big[R\left(16(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113})\right)+R\left(x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123}\right)+R(8(x_{131},x_{132},x_{133};$$
$$x'_{131},x_{132},x'_{133}))+R\left(13(x_{141},x_{142},x_{143};x'_{141},x_{142},x'_{143})\right)+R\left(11(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213})\right)+$$
$$R(4(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223}))+R\left(7(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233})\right)+R(10(x_{241},x_{242},x_{243};$$
$$x'_{241},x_{242},x'_{243}))+R(8(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313}))+R\left(15(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323})\right)$$
$$+R\left(9(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333})\right)+R\left(2(x_{341},x_{342},x_{343};x'_{341},x_{342},x'_{343})\right)+R(6(x_{411},x_{412},$$
$$x_{413};x'_{411},x_{412},x'_{413}))+R(12(x_{421},x_{422},x_{423};x'_{421},x_{422},x'_{423}))+R\left(5(x_{431},x_{432},x_{433};x'_{431},x_{432},x'_{433})\right)$$
$$+R\left(14(x_{441},x_{442},x_{443};x'_{441},x_{442},x'_{443})\right)\Big]$$

Subject to (P2.10)

$$R\left(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113}\right)+R\left(x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123}\right)+R\left(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133}\right)$$
$$+R\left(x_{141},x_{142},x_{143};x'_{141},x_{142},x'_{143}\right)=R\left(2,4,5;1,4,6\right),$$
$$R\left(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213}\right)+R\left(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223}\right)+R\left(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233}\right)$$
$$+R\left(x_{241},x_{242},x_{243};x'_{241},x_{242},x'_{243}\right)=R\left(4,6,8;3,6,9\right),$$
$$R\left(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313}\right)+R\left(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323}\right)+R\left(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333}\right)$$
$$+R\left(x_{341},x_{342},x_{343};x'_{341},x_{342},x'_{343}\right)=R\left(3,7,12;2,7,13\right),$$
$$R\left(x_{411},x_{412},x_{413};x'_{411},x_{412},x'_{413}\right)+R\left(x_{421},x_{422},x_{423};x'_{421},x_{422},x'_{423}\right)\oplus R\left(x_{431},x_{432},x_{433};x'_{431},x_{432},x'_{433}\right)$$
$$+R\left(x_{441},x_{442},x_{443};x'_{441},x_{442},x'_{443}\right)=R\left(8,10,13;5,10,16\right),$$
$$R\left(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113}\right)+R\left(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213}\right)+R\left(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313}\right)$$
$$+R\left(x_{411},x_{412},x_{413};x'_{411},x_{412},x'_{413}\right)=R\left(3,4,6;1,4,8\right),$$
$$R\left(x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123}\right)+R\left(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223}\right)+R\left(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323}\right)$$
$$+R\left(x_{421},x_{422},x_{423};x'_{421},x_{422},x'_{423}\right)=R\left(2,5,7;1,5,8\right),$$
$$R\left(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133}\right)+R\left(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233}\right)+R\left(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333}\right)$$
$$+R\left(x_{431},x_{432},x_{433};x'_{431},x_{432},x'_{433}\right)=R\left(10,15,20;8,15,22\right),$$
$$R\left(\left(x_{141},x_{142},x_{143};x'_{141},x_{142},x'_{143}\right)+\left(x_{241},x_{242},x_{243};x'_{241},x_{242},x'_{243}\right)+\left(x_{341},x_{342},x_{343};x'_{341},x_{342},x'_{343}\right)\right.$$
$$\left.+\left(x_{441},x_{442},x_{443};x'_{441},x_{442},x'_{443}\right)\right)=R\left(2,3,5;1,3,6\right),$$

$$R\left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)\geq R\left(0,0,0;0,0,0\right);i=1,2,3,4;\ j=1,2,3,4.$$

Step 4: Using the relation, $R\left(\lambda\times\left(a,b,c;a',b,c'\right)\right)=\lambda\times R\left(a,b,c;a',b,c'_{ij}\right)$, the problem (P2.10) can be transformed into the problem (P2.11).

$$\begin{aligned}
\text{Minimize } \Big[&16\times R\left(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113}\right)+R\left(x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123}\right)+8\times R(x_{131},x_{132},x_{133};\\
&x'_{131},x_{132},x'_{133})+13\times R\left(x_{141},x_{142},x_{143};x'_{141},x_{142},x'_{143}\right)+11\times R\left(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213}\right)+\\
&4\times R(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223})+7\times R\left(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233}\right)+10\times R(x_{241},x_{242},x_{243};\\
&x'_{241},x_{242},x'_{243})+8\times R(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313})+15\times R\left(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323}\right)\\
&+9\times R\left(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333}\right)+2\times R\left(x_{341},x_{342},x_{343};x'_{341},x_{342},x'_{343}\right)+6\times R(x_{411},x_{412},\\
&x_{413};x'_{411},x_{412},x'_{413})+12\times R(x_{421},x_{422},x_{423};x'_{421},x_{422},x'_{423})+5\times R\left(x_{431},x_{432},x_{433};x'_{431},x_{432},x'_{433}\right)\\
&+14\times R\left(x_{441},x_{442},x_{443};x'_{441},x_{442},x'_{443}\right)\Big]
\end{aligned}$$

Subject to (P2.11)

Constraints of problem (P2.10).

Step 5: Assuming $R\left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)=x_{ij}$, the problem (P2.11) can be transformed into problem (P2.12).

$$\begin{aligned}
\text{Minimize } [&16x_{11}+x_{12}+8x_{13}+13x_{14}+11x_{21}+4x_{22}+7x_{23}+10x_{24}+8x_{31}+15x_{32}+9x_{33}+2x_{34}+6x_{41}\\
&+12x_{42}+5x_{43}+14x_{44}]
\end{aligned}$$

Subject to

$$\begin{aligned}
&x_{11}+x_{12}+x_{13}+x_{14}=R\left(2,4,5;1,4,6\right),\\
&x_{21}+x_{22}+x_{23}+x_{24}=R\left(4,6,8;3,6,9\right), \qquad \text{(P2.12)}\\
&x_{31}+x_{32}+x_{33}+x_{34}=R\left(3,7,12;2,7,13\right),\\
&x_{41}+x_{42}+x_{43}+x_{44}=R\left(8,10,13;5,10,16\right),\\
&x_{11}+x_{21}+x_{31}+x_{41}=R\left(3,4,6;1,4,8\right),\\
&x_{12}+x_{22}+x_{32}+x_{42}=R\left(2,5,7;1,5,8\right),\\
&x_{13}+x_{23}+x_{33}+x_{43}=R\left(10,15,20;8,15,22\right),\\
&x_{14}+x_{24}+x_{34}+x_{44}=R\left(2,3,5;1,3,6\right),\\
&x_{ij}\geq 0;i=1,2,3,4;\ j=1,2,3,4.
\end{aligned}$$

Step 6: Using the expression, $R(a,b,c;a',b,c') = \frac{a+2b+c+a'+2b+c'}{8}$, the problem (P2.12) can be transformed into the problem (P2.13).

$$\begin{aligned}
&\text{Minimize } [16x_{11} + x_{12} + 8x_{13} + 13x_{14} + 11x_{21} + 4x_{22} + 7x_{23} + 10x_{24} + 8x_{31} + 15x_{32} + 9x_{33} + 2x_{34} + 6x_{41} \\
&\qquad + 12x_{42} + 5x_{43} + 14x_{44}] \\
&\text{Subject to} \\
&\qquad x_{11} + x_{12} + x_{13} + x_{14} = 3.75, \\
&\qquad x_{21} + x_{22} + x_{23} + x_{24} = 6, \\
&\qquad x_{31} + x_{32} + x_{33} + x_{34} = 7.25, \qquad\qquad \text{(P2.13)} \\
&\qquad x_{41} + x_{42} + x_{43} + x_{44} = 10.25, \\
&\qquad x_{11} + x_{21} + x_{31} + x_{41} = 4.25, \\
&\qquad x_{12} + x_{22} + x_{32} + x_{42} = 4.75, \\
&\qquad x_{13} + x_{23} + x_{33} + x_{43} = 15, \\
&\qquad x_{14} + x_{24} + x_{34} + x_{44} = 3.25, \\
&\qquad x_{ij} \geq 0; i = 1,2,3,4;\ j = 1,2,3,4.
\end{aligned}$$

Step 7: On solving the crisp linear programming problem (P2.13), the obtained optimal solution is $x_{11} = 0,\ x_{12} = 3.75,\ x_{13} = 0,\ x_{14} = 0,\ x_{21} = 0,\ x_{22} = 1,\ x_{23} = 5,\ x_{24} = 0,$

$x_{31} = 4, x_{32} = 0, x_{33} = 0, x_{34} = 3.25, x_{41} = 0.25, x_{42} = 0, x_{43} = 10, x_{44} = 0$ and the obtained optimal value is 132.75.

Step 8: Using the optimal solution, obtained in Step 7, the optimal solution of the problem (P2.8) is $\tilde{x}^I_{12} = \left\{ (x_{121}, x_{122}, x_{123}; x'_{121}, x_{122}, x'_{123}) : \frac{x_{121} + 2x_{122} + x_{123} + x'_{121} + 2x_{122} + x'_{123}}{8} = 3.75 \right\}$,

$\tilde{x}^I_{22} = \left\{ (x_{221}, x_{222}, x_{223}; x'_{221}, x_{222}, x'_{223}) : \frac{x_{221} + 2x_{222} + x_{223} + x'_{221} + 2x_{222} + x'_{223}}{8} = 1 \right\}$,

$\tilde{x}^I_{23} = \left\{ (x_{231}, x_{232}, x_{233}; x'_{231}, x_{232}, x'_{233}) : \frac{x_{231} + 2x_{232} + x_{233} + x'_{231} + 2x_{232} + x'_{233}}{8} = 5 \right\}$,

$$\tilde{x}_{31}^{I}=\left\{\left(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313}\right):\frac{x_{311}+2x_{312}+x_{313}+x'_{311}+2x_{312}+x'_{313}}{8}=4\right\},$$

$$\tilde{x}_{34}^{I}=\left\{\left(x_{341},x_{342},x_{343};x'_{341},x_{342},x'_{343}\right):\frac{x_{341}+2x_{342}+x_{343}+x'_{341}+2x_{342}+x'_{343}}{8}=3.25\right\},$$

$$\tilde{x}_{41}^{I}=\left\{\left(x_{411},x_{412},x_{413};x'_{411},x_{412},x'_{413}\right):\frac{x_{411}+2x_{412}+x_{413}+x'_{411}+2x_{412}+x'_{413}}{8}=0.25\right\},$$

$$\tilde{x}_{43}^{I}=\left\{\left(x_{431},x_{432},x_{433};x'_{431},x_{432},x'_{433}\right):\frac{x_{431}+2x_{432}+x_{433}+x'_{431}+2x_{432}+x'_{433}}{8}=10\right\},$$

and the optimal value of the problem (P2.8) is

$$\left\{\left(c_1,c_2,c_3;c'_1,c_2,c'_3\right):\frac{c_1+2c_2+c_3+c'_1+2c_2+c'_3}{8}=132.75\right\}.$$

2.4 Advantages of proposed method over existing methods

In this section, advantages of proposed method over existing methods are discussed.

1. To apply the existing methods [9,46,48,64-66,118,132], there is need to used arithmetic operations of intuitionistic fuzzy numbers. While, to apply the proposed method there is need to use arithmetic operation of real numbers. Since, it is much easy to apply arithmetic operations of real numbers as compared to intuitionistic fuzzy numbers. So, it is much easy to apply the proposed method as compared to the existing methods [9,46,48,64-66,118,132]. The same is also obvious from Section 1.1.2 and Section 2.3 in which same problem is solved by the existing methods [132] and proposed method respectively.
2. To solve the problem of large size by the existing methods [9,46,48,64-66,118,132], there is need to implement the existing methods [9,46,48,64-66,118,132] into a new programming language as no software is available in the market to deal with intuitionistic fuzzy

transportation problem of type – I. However, as in the proposed method, there is need to find the initial basic feasible solution or optimal solution of a crisp transportation problem. So, the existing software like TORA can be used for the same purpose and there is no need to develop new software.

3. There will always exist infinite number of basic feasible solution/optimal solution for intuitionistic fuzzy transportation problem of type – I. However, using the the existing methods [9,46,48,64-66,118,132], only one of these infinite initial basic feasible solution/optimal solution is obtained. While, using the proposed method, all infinite number of initial basic feasible solution or optimal solution are obtained.

2.5 Conclusion

On the basis of the present study, it can be concluded that it is much easy to apply the proposed method as compared to the existing methods [9,46,48,64-66,118,132], for finding the optimal solution of intuitionistic fuzzy transportation problem of type – I. Hence, it is better to use the proposed method instead of the existing methods [9,46,48,64-66,118,132] for solving intuitionistic fuzzy transportation problems of type – I.

Chapter 3

A simplified method for solving intuitionistic fuzzy transportation problems of type – II*

In this chapter, an alternative method for solving intuitionistic fuzzy transportation problems of type – II is proposed. Also, the advantages of proposed method over the existing methods [1,3,119,131], are discussed. Furthermore, to illustrate the proposed method, the numerical examples, considered by Singh and Yadav [131], are solved by the proposed method.

3.1 Intuitionistic fuzzy linear programming problem of intuitionistic fuzzy transportation problems of type – II

In the literature [1,3,119,131], the intuitionistic fuzzy linear programming problem (P3.1) of the intuitionistic fuzzy transportation problem of type – II is obtained by replacing the transportation cost $\left(c_{ij}\right)$ with triangular intuitionistic fuzzy numbers $\left(c_{ij1}, c_{ij2}, c_{ij3}; c'_{ij1}, c_{ij2}, c'_{ij3}\right)$

$$\text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij1}, c_{ij2}, c_{ij3}; c'_{ij1}, c_{ij2}, c'_{ij3}\right)x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i; i = 1, 2, ..., m, \qquad \text{(P3.1)}$$

$$\sum_{i=1}^{m} x_{ij} = b_j;\ j = 1, 2, ..., n,$$

$$x_{ij} \geq 0; i = 1, 2, ..., m;\ j = 1, 2, ..., n.$$

where, $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.

* The contents of this chapter are to be communicated in Iranian Journal of Fuzzy Systems, Springer.

3.2 Proposed method

In this section, an alternative method to find the optimal solution of intuitionistic fuzzy transportation problem of type – II is proposed.

The steps of proposed method are as follows:

Step 1: Using Section 1.3.3.2, the problem (P3.1) can be transformed into the problem (P3.2).

$$\begin{aligned}&\text{Minimize } R\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij1},c_{ij2},c_{ij3};c'_{ij1},c_{ij2},c'_{ij3}\right)x_{ij}\right)\\&\text{Subject to}\\&\qquad\sum_{j=1}^{n}x_{ij}=a_i;i=1,2,...,m, \qquad\qquad (P3.2)\\&\qquad\sum_{i=1}^{m}x_{ij}=b_j;\ j=1,2,...,n,\\&\qquad x_{ij}\geq 0;i=1,2,...,m;\ j=1,2,...,n.\end{aligned}$$

Step 2: Using the relation, $R\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(a_{ij},b_{ij},c_{ij};a'_{ij},b_{ij},c'_{ij}\right)\right)=\sum_{i=1}^{m}\sum_{j=1}^{n}R\left(a_{ij},b_{ij},c_{ij};a'_{ij},b_{ij},c'_{ij}\right)$, the problem (P3.2) can be transformed into problem (P3.3).

$$\begin{aligned}&\text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n}R\left(\left(c_{ij1},c_{ij2},c_{ij3};c'_{ij1},c_{ij2},c'_{ij3}\right)x_{ij}\right)\\&\text{Subject to}\\&\qquad\sum_{j=1}^{n}x_{ij}=a_i;i=1,2,...,m, \qquad\qquad (P3.3)\\&\qquad\sum_{i=1}^{m}x_{ij}=b_j;\ j=1,2,...,n,\\&\qquad x_{ij}\geq 0;i=1,2,...,m;\ j=1,2,...,n.\end{aligned}$$

Step 3: Using the relation, $R\left(\lambda\times\left(a,b,c;a',b,c'\right)\right)=\lambda\times R\left(a,b,c;a',b,c'\right)$, the problem (P3.3) can be transformed into problem (P3.4).

$$\text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n} R\left(c_{ij1}, c_{ij2}, c_{ij3}; c'_{ij1}, c_{ij2}, c'_{ij3}\right) \times x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i; i = 1, 2, ..., m, \qquad \text{(P3.4)}$$

$$\sum_{i=1}^{m} x_{ij} = b_j; j = 1, 2, ..., n,$$

$$x_{ij} \geq 0; i = 1, 2, ..., m; j = 1, 2, ..., n.$$

Step 4: Using the expression, $R\left(a, b, c; a', b, c'\right) = \dfrac{a + 2b + c + a' + 2b + c'}{8}$, the problem (P3.4) can be transformed into the problem (P3.5).

$$\text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n} \left(\frac{c_{ij1} + 2c_{ij2} + c_{ij3} + c'_{ij1} + 2c_{ij2} + c'_{ij3}}{8}\right) \times x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i; i = 1, 2, ..., m, \qquad \text{(P3.5)}$$

$$\sum_{i=1}^{m} x_{ij} = b_j; j = 1, 2, ..., n,$$

$$x_{ij} \geq 0; i = 1, 2, ..., m; j = 1, 2, ..., n.$$

Step 5: Find the optimal solution of crisp linear programming problem (P3.5).

Step 6: The optimal solution $\{x_{ij}\}$, obtained in Step 5, represents the optimal solution of intuitionistic fuzzy transportation problem of type – II and the minimum intuitionistic fuzzy transportation cost is $\sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij1}, c_{ij2}, c_{ij3}; c'_{ij1}, c_{ij2}, c'_{ij3}\right) \times x_{ij}$.

3.3 Illustrative example

To illustrate the proposed method, existing intuitionistic fuzzy balanced transportation problem of type – II, presented in Table 1.28, is solved.

Using the proposed method, the initial basic feasible solution and hence optimal solution of existing intuitionistic fuzzy balanced transportation problem of type – II, presented in Table 1.28, can be obtained as follows:

Step 1: Using Section 3.1, the intuitionistic fuzzy balanced transportation problem of type – II, presented by Table 1.28, can be transformed into the intuitionistic fuzzy linear programming problem (P3.6).

$$\text{Minimize } \big[(2,4,5;1,4,6)x_{11} \oplus (2,5,7;1,5,8)x_{12} \oplus (4,6,8;3,6,9)x_{13} \oplus (4,7,8;3,7,9)x_{14} \oplus (4,6,8;3,6,9) x_{21} \oplus (3,7,12;2,7,13)x_{22} \oplus (10,15,20;8,15,22)x_{23} \oplus (11,12,13;10,12,14)x_{24} \oplus (3,4,6;1,4,8) x_{31} \oplus (8,10,13;5,10,16)x_{32} \oplus (2,3,5;1,3,6)x_{33} \oplus (6,10,14;5,10,15)x_{34} \oplus (2,4,6;1,4,7)x_{41} \oplus (3,9,10;2,9,12)x_{42} \oplus (3,6,10;2,6,12)x_{43} \oplus (3,4,5;2,4,8)x_{44}\big]$$

Subject to

$$\begin{aligned} &x_{11}+x_{12}+x_{13}+x_{14}=11, \qquad \text{(P3.6)}\\ &x_{21}+x_{22}+x_{23}+x_{24}=11,\\ &x_{31}+x_{32}+x_{33}+x_{34}=11,\\ &x_{41}+x_{42}+x_{43}+x_{44}=12,\\ &x_{11}+x_{21}+x_{31}+x_{41}=16\\ &x_{12}+x_{22}+x_{32}+x_{42}=10\\ &x_{13}+x_{23}+x_{33}+x_{43}=8\\ &x_{14}+x_{24}+x_{34}+x_{44}=11\\ &x_{ij}\ge 0; i=1,2,3,4;\ j=1,2,3,4. \end{aligned}$$

Step 2: Using Step 1 of the proposed method, the problem (P3.6) can be transformed into the problem (P3.7).

$$\text{Minimize } R\big((2,4,5;1,4,6)x_{11} \oplus (2,5,7;1,5,8)x_{12} \oplus (4,6,8;3,6,9)x_{13} \oplus (4,7,8;3,7,9)x_{14} \oplus (4,6,8;3, 6,9)x_{21} \oplus (3,7,12;2,7,13)x_{22} \oplus (10,15,20;8,15,22)x_{23} \oplus (11,12,13;10,12,14)x_{24} \oplus (3,4,6; 1,4,8)x_{31} \oplus (8,10,13;5,10,16)x_{32} \oplus (2,3,5;1,3,6)x_{33} \oplus (6,10,14;5,10,15)x_{34} \oplus (2,4,6;1,4,7) x_{41} \oplus (3,9,10;2,9,12)x_{42} \oplus (3,6,10;2,6,12)x_{43} \oplus (3,4,5;2,4,8)x_{44}\big)$$

Subject to

Constraint of problem (P3.6) (P3.7)

Step 3: Using the relation, $R\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(a_{ij},b_{ij},c_{ij};a'_{ij},b_{ij},c'_{ij}\right)\right)=\sum_{i=1}^{m}\sum_{j=1}^{n}R\left(a_{ij},b_{ij},c_{ij};a'_{ij},b_{ij},c'_{ij}\right)$, the problem (P3.7) can be transformed into problem (P3.8).

$$\begin{aligned}\text{Minimize } &\left[R\left((2,4,5;1,4,6)x_{11}\right)+R\left((2,5,7;1,5,8)x_{12}\right)+R\left((4,6,8;3,6,9)x_{13}\right)+R\left((4,7,8;3,7,9)x_{14}\right)\right.\\ &+R\left((4,6,8;3,6,9)x_{21}\right)+R\left((3,7,12;2,7,13)x_{22}\right)+R\left((10,15,20;8,15,22)x_{23}\right)+R((11,12,13;\\ &10,12,14)x_{24})+R\left((3,4,6;1,4,8)x_{31}\right)+R\left((8,10,13;5,10,16)x_{32}\right)+R\left((2,3,5;1,3,6)x_{33}\right)\\ &+R\left((6,10,14;5,10,15)x_{34}\right)+R\left((2,4,6;1,4,7)x_{41}\right)+R\left((3,9,10;2,9,12)x_{42}\right)+R((3,6,10;2,6,\\ &\left.12)x_{43})+R\left((3,4,5;2,4,8)x_{44}\right)\right]\end{aligned}$$

Subject to

Constraint of problem (P3.6) (P3.8)

Step 4: Using the relation $R\left(\lambda\times\left(a,b,c;a',b,c'\right)\right)=\lambda\times R\left(a,b,c;a',b,c'_{ij}\right)$, the problem (P3.8) can be transformed into problem (P3.9).

$$\begin{aligned}\text{Minimize } &\left[R(2,4,5;1,4,6)x_{11}+R(2,5,7;1,5,8)x_{12}+R(4,6,8;3,6,9)x_{13}+R(4,7,8;3,7,9)x_{14}\right.\\ &+R(4,6,8;3,6,9)x_{21}+R(3,7,12;2,7,13)x_{22}+R(10,15,20;8,15,22)x_{23}+R(11,12,13;\\ &10,12,14)x_{24}+R(3,4,6;1,4,8)x_{31}+R(8,10,13;5,10,16)x_{32}+R(2,3,5;1,3,6)x_{33}+R(6,10,14;\\ &5,10,15)x_{34}+R(2,4,6;1,4,7)x_{41}+R(3,9,10;2,9,12)x_{42}+R(3,6,10;2,6,12)x_{43}\\ &\left.+R(3,4,5;2,4,8)x_{44}\right]\end{aligned}$$

Subject to

Constraint of problem (P3.6) (P3.9)

Step 5: Using the expression, $R\left(a,b,c;a',b,c'\right)=\dfrac{a+2b+c+a'+2b+c'}{8}$, the problem (P3.9) can be transformed into problem (P3.10).

$$\begin{aligned}\text{Minimize } &\left(3.75x_{11}+4.75x_{12}+6x_{13}+6.5x_{14}+6x_{21}+7.25x_{22}+15x_{23}+12x_{24}+4.25x_{31}+10.25x_{32}\right.\\ &\left.+3.25x_{33}+10x_{34}+4x_{41}+7.875x_{42}+6.375x_{43}+4.25x_{44}\right)\end{aligned}$$

Subject to

Constraint of problem (P3.6) (P3.10)

Step 6: On solving the crisp linear programming problem (P3.10), the obtained optimal solution is $x_{11}=1, x_{12}=10, x_{13}=0, x_{14}=0, x_{21}=11, x_{22}=0,\ x_{23}=0, x_{24}=0,\ x_{31}=3,\ x_{32}=0, x_{33}=8,\ x_{34}=0,$ $x_{41}=1, x_{42}=0, x_{43}=0, x_{44}=11$.

Step 7: Using the optimal solution, obtained in Step 6, the minimum intuitionistic fuzzy transportation cost is $(2,4,5;1,4,6)\times 1 \oplus (2,5,7;1,5,8)\times 10 \oplus (4,6,8;3,6,9)\times 11 \oplus (3,4,6;1,4,8)\times 3$ $\oplus (2,3,5;1,3,6)\times 8 \oplus (2,4,6;1,4,7)\times 1 \oplus (3,4,5;2,4,8)\times 11 = (126,204,282;78,204,352)$.

3.4 Intuitionistic fuzzy optimal solution of a real life problem

Singh and Yadav [131] solved the real life intuitionistic fuzzy transportation problem, presented by Table 3.1, to illustrate their proposed method. In this section, the same problem is solved by the proposed method.

Table 3.1: Real life intuitionistic fuzzy transportation problem

Sources	Destinations				Supply (s_i)
	D_1 Ludhiana	D_2 Delhi	D_3 Kullu	D_4 Leh Ladakh	
S_1 Mandi Govindgarh (Fortune Metals)	(210,250,270; 200,250,280)	(600,700,750; 600,700,800)	(950,1000,1100; 900,1050,1150)	(3500,3700,3900; 3400,3700,4100)	4500
S_2 Bhwarhi (Kamdhenu Saria)	(650,750,800; 600,750,850)	(350,400,450; 340,400,480)	(1000,1050,1100; 950,1050,1150)	(3600,3900,4600; 3500,3900,4600)	3500
S_3 Raipur (Goel Group)	(3600,2800,3000; 2100,2200,2350)	(2100,2200,2300; 2011,2200,2350)	(2900,3100,3300; 2800,3100,3400)	(5400,5600,5800; 5300,5600,6000)	2000

Demand (d_j)	3500	3000	2000	1500	

Using the proposed method, the intuitionistic fuzzy optimal solution of the intuitionistic fully fuzzy transportation problem, presented in Table 3.1, can be obtained as follows:

Step 1: Using Section 3.1, the intuitionistic fuzzy balanced transportation problem of type – II, presented by Table 3.1, can be transformed into the intuitionistic fuzzy linear programming problem (P3.11).

$$\begin{aligned}
\text{Minimize } & [(210,250,270;200,250,280)x_{11} \oplus (600,700,750;600,700,800)x_{12} \oplus (950,1000,1050;\\
& 900,1000,1100)x_{13} \oplus (3500,3700,3900;3400,3700,4100)x_{14} \oplus (650,750,800;600,750,\\
& 850)x_{21} \oplus (350,400,450;340,400,480)x_{22} \oplus (1000,1050,1100;950,1050,1150)x_{23} \oplus\\
& (3600,3900,4600;3500,3900,4600)x_{24} \oplus (2600,2800,3000;2500,2800,3100)x_{31} \oplus (2100,\\
& 2200,2300;2100,2200,2350)x_{32} \oplus (2900,3100,3300;2800,3100,3400)x_{33} \oplus (5400,5600,\\
& 5800;5300,5600,6000)x_{34}]
\end{aligned}$$

Subject to

$$\begin{aligned}
& x_{11}+x_{12}+x_{13}+x_{14}=4500, && \text{(P3.11)}\\
& x_{21}+x_{22}+x_{23}+x_{24}=3500,\\
& x_{31}+x_{32}+x_{33}+x_{34}=2000,\\
& x_{11}+x_{21}+x_{31}=3500,\\
& x_{12}+x_{22}+x_{32}=3000,\\
& x_{13}+x_{23}+x_{33}=2000,\\
& x_{14}+x_{24}+x_{34}=1500,\\
& x_{ij}\geq 0; i=1,2,3;\ j=1,2,3,4.
\end{aligned}$$

Step 2: Using Step 1 of the proposed method, problem (P3.11) can be transformed into problem (P3.12).

$$\begin{aligned}
\text{Minimize } R\big(& (210,250,270;200,250,280)x_{11} \oplus (600,700,750;600,700,800)x_{12} \oplus (950,1000,1050;\\
& 900,1000,1100)x_{13} \oplus (3500,3700,3900;3400,3700,4100)x_{14} \oplus (650,750,800;600,750,\\
& 850)x_{21} \oplus (350,400,450;340,400,480)x_{22} \oplus (1000,1050,1100;950,1050,1150)x_{23} \oplus
\end{aligned}$$

$(3600,3900,4600;3500,3900,4600)x_{24} \oplus (2600,2800,3000;2500,2800,3100)x_{31} \oplus (2100,$
$2200,2300;2100,2200,2350)x_{32} \oplus (2900,3100,3300;2800,3100,3400)x_{33} \oplus (5400,5600,$
$5800;5300,5600,6000)x_{34})$

Subject to

Constraint of problem (P3.11) (P3.12)

Step 3: Using the relation $R\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(a_{ij},b_{ij},c_{ij};a'_{ij},b_{ij},c'_{ij}\right)\right)=\sum_{i=1}^{m}\sum_{j=1}^{n}R\left(a_{ij},b_{ij},c_{ij};a'_{ij},b_{ij},c'_{ij}\right)$, the problem (P3.12) can be transformed into the problem (P3.13).

Minimize $\left[R\left((210,250,270;200,250,280)x_{11}\right)+R\left((600,700,750;600,700,800)x_{12}\right)+R\left((950,\right.\right.$
$1000,1050;900,1000,1100)x_{13})+R\left((3500,3700,3900;3400,3700,4100)x_{14}\right)+R\left((650,\right.$

$750,800;600,750,850)x_{21})+R\left((350,400,450;340,400,480)x_{22}\right)+R\left((1000,1050,1100;\right.$
$950,1050,1150)x_{23}+R\left((3600,3900,4600;3500,3900,4600)x_{24}\right)+R\left((2600,2800,3000;\right.$
$2500,2800,3100)x_{31}+R\left((2100,2200,2300;2100,2200,2350)x_{32}\right)+R\left((2900,3100,3300;\right.$
$\left.2800,3100,3400)x_{33})+R\left((5400,5600,5800;5300,5600,6000)x_{34}\right)\right]$

Subject to

Constraint of problem (P3.11) (P3.13)

Step 4: Using the relation, $R\left(\lambda\times\left(a,b,c;a',b,c'\right)\right)=\lambda\times R\left(a,b,c;a',b,c'_{ij}\right)$, the problem (P3.13) can be transformed into the problem (P3.14).

Minimize $[R(210,250,270;200,250,280)x_{11}+R(600,700,750;600,700,800)x_{12}+R(950,1000,$
$1050;900,1000,1100)x_{13}+R(3500,3700,3900;3400,3700,4100)x_{14}+R(650,750,800;$
$600,750,850)x_{21}+R(350,400,450;340,400,480)x_{22}+R(1000,1050,1100;950,1050,$

$1150)x_{23}+R(3600,3900,4600;3500,3900,4600)x_{24}+R(2600,2800,3000;2500,2800,$
$3100)x_{31}+R(2100,2200,2300;2100,2200,2350)x_{32}+R(2900,3100,3300;2800,3100,$
$3400)x_{33}+R(5400,5600,5800;5300,5600,6000)x_{34}]$

Subject to

Constraint of problem (P3.11) (P3.14)

Step 5: Using the expression, $R(a,b,c;a',b,c') = \frac{a+2b+c+a'+2b+c'}{8}$, the problem (P3.14) can be transformed into the problem (P3.15).

$$\begin{aligned}
&\text{Minimize } \big(245x_{11} + 693.75x_{12} + 1000x_{13} + 3712.5x_{14} + 737.5x_{21} + 402.5x_{22} + 1050x_{23} + 3987.5x_{24} \\
&\qquad + 2800x_{31} + 2206.25x_{32} + 3100x_{33} + 5612.5x_{34}\big) \\
&\text{Subject to} \\
&\qquad\qquad \text{Constraint of problem (P3.11)} \qquad\qquad\qquad\qquad \text{(P3.15)}
\end{aligned}$$

Step 6: On solving the crisp linear programming problem (P3.15), obtained optimal solution is $x_{11} = 3500, x_{12} = 0, x_{13} = 0, x_{14} = 1000, x_{21} = 11, x_{22} = 1500, x_{23} = 2000, x_{24} = 0, x_{31} = 0, x_{32} = 1500,$ $x_{33} = 0, x_{34} = 500$.

Step 7: Using the optimal solution, obtained in Step 6, the minimum intuitionistic fuzzy transportation cost is

$$\begin{aligned}
&(210,250,270;200,250,280)\times 3500 \oplus (3500,3700,3900;3400,3700,4100)\times 1000 \oplus (350,400,450; \\
&340,400,480)\times 1500 \oplus (1000,1050,1100;950,1050,1150)\times 2000 \oplus (2100,2200,2300;2100,2200, \\
&2350)\times 1500 \oplus (5400,5600,5800;5300,5600,6000)\times 500 \\
&= (12710000,13425000,14070000;12400000,13425000,14605000).
\end{aligned}$$

Remark 3.2: The advantages of the proposed method over the existing methods [1,3,119,131] are same as discussed in Section 2.4 of Chapter 2.

3.5 Conclusion

On the basis of the present study, it can be concluded that it is much easy to apply the proposed method as compared to the existing methods [1,3,119,131], for finding the optimal solution of intuitionistic fuzzy transportation problems of type – II.

Chapter 4

Modified method for solving intuitionistic fully fuzzy transportation problems*

Kumar and Hussain [84-86] proposed methods for solving intuitionistic fully fuzzy transportation problems. In this chapter, it is pointed out that for the ranking function, used by Kumar and Hussain [84-86], the linearity property is not satisfying. However, in the existing method [84-86], this property is used. Therefore, the existing methods [84-86] are not valid. Hence, the result of numerical problems, obtained by Kumar and Hussain [84-86] by their proposed method, is also not correct. Furthermore, it is shown that for the ranking function, used by Singh and Yadav [132], linearity property is satisfying. Hence, the existing methods [84-86], will be valid if the ranking function, used by Kumar and Hussain [84-86], is replaced with the ranking function, used by Singh and Yadav [132]. Also, the exact results of numerical problems, considered by Kumar and Hussain [86], are obtained.

4.1 Ordering of triangular intuitionistic fuzzy numbers

Kumar and Hussain [84-86] used the following method for comparing two triangular intuitionistic fuzzy numbers.

Let $\tilde{A}^I = (a_1, a_2, a_3; a_1', a_2, a_3')$ and $\tilde{B}^I = (b_1, b_2, b_3; b_1', b_2, b_3')$ be two triangular intuitionistic fuzzy numbers. Then,

* The contents of this chapter are to be communicated in International Journal of System Assurance Engineering and Management, Springer.

(i) $\tilde{A}^I \succeq \tilde{B}^I$ if $R(\tilde{A}^I) \geq R(\tilde{B}^I)$

(ii) $\tilde{A}^I \approx \tilde{B}^I$ if $R(\tilde{A}^I) = R(\tilde{B}^I)$

where,

$$R(\tilde{A}^I) = \frac{1}{3}\left[\frac{(a_3' - a_1')(a_2 - 2a_3' - 2a_1') + (a_3 - a_1)(a_1 + a_2 + a_3) + 3(a_3'^2 - a_1'^2)}{a_3' - a_1' + a_3 - a_1}\right] \text{ and}$$

$$R(\tilde{B}^I) = \frac{1}{3}\left[\frac{(b_3' - b_1')(b_2 - 2b_3' - 2b_1') + (b_3 - b_1)(b_1 + b_2 + b_3) + 3(b_3'^2 - b_1'^2)}{b_3' - b_1' + b_3 - b_1}\right].$$

4.2 Multiplication of triangular intuitionistic fuzzy numbers

If $\tilde{A}^I = (a_1, a_2, a_3; a_1', a_2, a_3')$ and $\tilde{B}^I = (b_1, b_2, b_3; b_1', b_2, b_3')$ are two non-negative triangular intuitionistic fuzzy numbers then the multiplication of $\tilde{A}^I$ and $\tilde{B}^I$ i.e., $\tilde{A}^I \otimes \tilde{B}^I$ is defined as,

$\tilde{A}^I \otimes \tilde{B}^I = (a_1b_1, a_2b_2, a_3b_3; a_1'b_1', a_2b_2, a_3'b_3')$.

Since, $R\left(\tilde{A}^I \otimes \tilde{B}^I\right) \neq R\left(\tilde{A}^I\right) \otimes R\left(\tilde{B}^I\right)$. So, Kumar and Hussain [84-86] defined the multiplication of triangular intuitionistic fuzzy numbers $\tilde{A}^I$ and $\tilde{B}^I$ i.e., $\tilde{A}^I \otimes \tilde{B}^I$ in such a manner so that the property $R\left(\tilde{A}^I \otimes \tilde{B}^I\right) = R\left(\tilde{A}^I\right) \otimes R\left(\tilde{B}^I\right)$ will always be satisfied.

The multiplication of triangular intuitionistic fuzzy numbers $\tilde{A}^I$ and $\tilde{B}^I$, defined by Kumar and Hussain [84-86], is as follows:

$$\tilde{A}^I \otimes \tilde{B}^I = \left(a_1 R\left(\tilde{B}^I\right), a_2 R\left(\tilde{B}^I\right), a_3 R\left(\tilde{B}^I\right); a_1' R\left(\tilde{B}^I\right)_1, a_2 R\left(\tilde{B}^I\right), a_3' R\left(\tilde{B}^I\right)\right) \text{ if } R\left(\tilde{A}^I\right), R\left(\tilde{B}^I\right) \geq 0.$$

4.3 Intuitionistic fuzzy linear programming problem of intuitionistic fully fuzzy transportation problem

Kumar and Hussain [84-86] proposed the following intuitionistic fully fuzzy linear programming problem of intuitionistic fully fuzzy transportation problem which is obtained by replacing the crisp parameters c_{ij}, x_{ij}, a_i and b_j of crisp linear programming problem (P2.1) of transportation problem by the triangular intuitionistic fuzzy numbers $\left(c_{ij1}, c_{ij2}, c_{ij3}; c'_{ij1}, c_{ij2}, c'_{ij3}\right)$, $\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right)$, $\left(a_{i1}, a_{i2}, a_{i3}; a'_{i1}, a_{i2}, a'_{i3}\right)$ and $\left(b_{j1}, b_{j2}, b_{j3}; b'_{j1}, b_{j2}, b'_{j3}\right)$ respectively.

$$\text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij1}, c_{ij2}, c_{ij3}; c'_{ij1}, c_{ij2}, c'_{ij3}\right)\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right)$$

Subject to

$$\begin{aligned}
&\sum_{j=1}^{n}\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right) \approx \left(a_{i1}, a_{i2}, a_{i3}; a'_{i1}, a_{i2}, a'_{i3}\right); i = 1, 2, \ldots, m, \qquad \text{(P4.1)}\\
&\sum_{i=1}^{m}\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right) \approx \left(b_{j1}, b_{j2}, b_{j3}; b'_{j1}, b_{j2}, b'_{j3}\right);\ j = 1, 2, \ldots, n,\\
&\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right) \succeq (0,0,0;0,0,0); i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n.
\end{aligned}$$

4.4 Kumar and Hussain method

Kumar and Hussain [84-86] proposed methods for solving intuitionistic fully fuzzy transportation problem.

The steps of these methods are follows:

Step 1: Using Section 4.1, the intuitionistic fully fuzzy transportation problem (P4.1) can be transformed into the problem (P4.2).

$$\text{Minimize } R\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij1}, c_{ij2}, c_{ij3}; c'_{ij1}, c_{ij2}, c'_{ij3}\right)\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right)\right)$$

Subject to

$$R\left(\sum_{j=1}^{n}\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right)\right) = R\left(a_{i1}, a_{i2}, a_{i3}; a'_{i1}, a_{i2}, a'_{i3}\right); i = 1, 2, ..., m, \qquad \text{(P4.2)}$$

$$R\left(\sum_{i=1}^{m}\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right)\right) = R\left(b_{j1}, b_{j2}, b_{j3}; b'_{j1}, b_{j2}, b'_{j3}\right);\ j = 1, 2, ..., n,$$

$$R\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right) \geq R(0,0,0;0,0,0); i = 1, 2, ..., m;\ j = 1, 2, ..., n.$$

Step 2: Using the relation, $R\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(a_{ij}, b_{ij}, c_{ij}; a'_{ij}, b_{ij}, c'_{ij}\right)\right) = \sum_{i=1}^{m}\sum_{j=1}^{n} R\left(a_{ij}, b_{ij}, c_{ij}; a'_{ij}, b_{ij}, c'_{ij}\right)$, the problem (P4.2) can be transformed into problem (P4.3).

Minimize $\sum_{i=1}^{m}\sum_{j=1}^{n} R\left(\left(c_{ij1}, c_{ij2}, c_{ij3}; c'_{ij1}, c_{ij2}, c'_{ij3}\right)\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right)\right)$

Subject to

$$\sum_{j=1}^{n} R\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right) = R\left(a_{i1}, a_{i2}, a_{i3}; a'_{i1}, a_{i2}, a'_{i3}\right); i = 1, 2, ..., m, \qquad \text{(P4.3)}$$

$$\sum_{i=1}^{m} R\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right) = R\left(b_{j1}, b_{j2}, b_{j3}; b'_{j1}, b_{j2}, b'_{j3}\right);\ j = 1, 2, ..., n,$$

$$R\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right) \geq R(0,0,0;0,0,0); i = 1, 2, ..., m;\ j = 1, 2, ..., n.$$

Step 3: Using Section 4.2, the problem (P4.3) can be transformed into the problem (P4.4).

Minimize $\sum_{i=1}^{m}\sum_{j=1}^{n} R\left(c_{ij1}, c_{ij2}, c_{ij3}; c'_{ij1}, c_{ij2}, c'_{ij3}\right) R\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right)$

Subject to

$$\sum_{j=1}^{n} R\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right) = R\left(a_{i1}, a_{i2}, a_{i3}; a'_{i1}, a_{i2}, a'_{i3}\right); i = 1, 2, ..., m, \qquad \text{(P4.4)}$$

$$\sum_{i=1}^{m} R\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right) = R\left(b_{j1}, b_{j2}, b_{j3}; b'_{j1}, b_{j2}, b'_{j3}\right);\ j = 1, 2, ..., n,$$

$$R\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right) \geq R(0,0,0;0,0,0); i = 1, 2, ..., m;\ j = 1, 2, ..., n.$$

Step 4: Since, $R\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right)$ will always be a real number. So, assuming $R\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right) = x_{ij}$, problem (P4.4) can be transformed into problem (P4.5).

Minimize $\sum_{i=1}^{m}\sum_{j=1}^{n} R\left(c_{ij1}, c_{ij2}, c_{ij3}; c'_{ij1}, c_{ij2}, c'_{ij3}\right) x_{ij}$

Subject to

$$\sum_{j=1}^{n} x_{ij} = R\left(a_{i1}, a_{i2}, a_{i3}; a'_{i1}, a_{i2}, a'_{i3}\right); i = 1, 2, ..., m, \qquad \text{(P4.5)}$$

$$\sum_{i=1}^{m} x_{ij} = R\left(b_{j1}, b_{j2}, b_{j3}; b'_{j1}, b_{j2}, b'_{j3}\right);\ j = 1, 2, ..., n,$$

$$x_{ij} \geq 0; i = 1, 2, ..., m;\ j = 1, 2, ..., n.$$

Step 5: Find the optimal solution of the crisp linear programming problem (P4.5).

Step 6: If $\{x_{ij} = p_{ij}\}$ is the set of optimal solution of problem (P4.5) then $\tilde{x}_{ij}^{I} = \left\{\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right) : \frac{x_{ij1} + 2x_{ij2} + x_{ij3} + x'_{ij1} + 2x_{ij2} + x'_{ij3}}{8} = p_{ij}\right\}$ represents the set of optimal solution of problem (P4.1) and the minimum intuitionistic fuzzy transportation cost is $\sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij1}, c_{ij2}, c_{ij3}; c'_{ij1}, c_{ij2}, c'_{ij3}\right) x_{ij}$.

4.5 Flaw in Kumar and Hussain method

It is obvious from Step 2 of the existing methods [84-86], presented in Section 4.4, that Kumar and Hussain [84-86] have used the property $R\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(a_{ij}, b_{ij}, c_{ij}; a'_{ij}, b_{ij}, c'_{ij}\right)\right)$ $= \sum_{i=1}^{m}\sum_{j=1}^{n} R\left(a_{ij}, b_{ij}, c_{ij}; a'_{ij}, b_{ij}, c'_{ij}\right)$ for transforming problem (P4.2) to problem (P4.3). However, it can be easily verified that for the ranking function, used by Kumar and Hussain [84-86], this property is not satisfying. Hence, problem (P4.2) can not be transformed into problem (P4.3).

4.6 Suggested modifications in Kumar and Hussain method

It can be easily verified that for the ranking function, used by Singh and Yadav [132], the linearity property, $R\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(a_{ij},b_{ij},c_{ij};a'_{ij},b_{ij},c'_{ij}\right)\right)=\sum_{i=1}^{m}\sum_{j=1}^{n}R\left(a_{ij},b_{ij},c_{ij};a'_{ij},b_{ij},c'_{ij}\right)$, will always be satisfied. Hence, the flaws of the existing methods [84-86], can be resolved by replacing the ranking function of Kumar and Hussain [84-86] with the ranking function of Singh and Yadav [132].

4.7 Exact results of numerical examples

Kumar and Hussain [84-86] solved the intuitionistic fully fuzzy transportation problems, presented by Table 4.1 and Table 4.2, to illustrate their proposed method. However, as discussed in Section 4.5 that the existing methods [84-86] is not valid. Therefore, the results of numerical problems, obtained by Kumar and Hussain [84-86], are also not exact results. In this section, the intuitionistic fully fuzzy transportation problems, presented by Table 4.1 and Table 4.2, are solved with the modification suggested in Section 4.6.

Table 4.1: Real life intuitionistic fully fuzzy transportation problem

Sources	Destinations			Int. fuzzy availability $\left(\tilde{a}_i^I\right)$
	Tirunelveli D_1	Trichy D_2	Chennai D_3	
Sivakasi S_1	(1,4,9;0,4,12)	(3,13,14;2,13,15)	(4,6,16;1,6,33)	(6,7,10;2,7,11)
Kollam S_2	(4,5,7;1,5,9)	(5,10,15;0,10,39)	(7,16,24;0,16,41)	(6,15,23;1,15,29)
Nagercoil S_3	(1,3,6;0,3,10)	(5,13,21;5,13,35)	(8,18,27;6,18,48)	(2,10,16;0,10,21)
Int. fuzzy demand $\left(\tilde{b}_j^I\right)$	(3,8,16;0,8,19)	(1,6,7;0,6,14)	(10,18,26;3,18,28)	(14,32,49;3,32,61)

Table 4.2: Real life intuitionistic fully fuzzy transportation problem

Sources	Destinations				Intuitionistic fuzzy availability $\left(\tilde{a}_i^I\right)$
	Coimbatore D_1	Karur D_2	Tirupur D_3	Namakkal D_4	

Trichy S_1	(14,16,18; 12,16,20)	(0,1,2; –1,1,3)	(7,8,9; 6,8,10)	(11,13,15; 10,13,16)	(2,4,6; 1,4,7)
Madurai S_2	(8,11,14; 7,11,15)	(3,4,5; 2,4,6)	(5,7,9; 4,7,10)	(8,10,12; 6,10,14)	(5,6,7; 4,6,8)
Dindugal S_3	(6,8,10; 5,8,11)	(13,15,17; 12,15,18)	(7,9,11; 6,9,12)	(1,2,3; 0,2,4)	(7,8,9; 5,8,11)
Palani S_4	(5,6,7; 4,6,8)	(11,12,13; 10,12,14)	(3,5,7; 1,5,9)	(12,14,16; 11,14,17)	(8,10,12; 6,10,14)
Intuitionistic fuzzy demand $\left(\tilde{b}_j^I\right)$	(3,4,5;2,4,6)	(3,5,7; 1,5,9)	(10,12,14; 8,12,16)	(6,7,8; 5,7,9)	(22,28,34; 16,28,40)

4.7.1 Intuitionistic fuzzy optimal solution of fully fuzzy transportation problem presented by Table 4.1

Using the proposed method, the intuitionistic fuzzy optimal solution of intuitionistic fully fuzzy transportation problem, presented by Table 4.1, can be obtained as follows:

Step 1: Using Section 4.3, the intuitionistic fully fuzzy transportation problem, presented by Table 4.1, can be transformed into intuitionistic fully fuzzy linear programming problem (P4.6).

$$\begin{aligned}
\text{Minimize}\Big(&(1,4,9;0,4,12)\otimes(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113})\oplus(3,13,14;2,13,15)\otimes(x_{121},x_{122},x_{123};x'_{121},\\
&x_{122},x'_{123})\oplus(4,6,16;1,6,33)\otimes(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133})\oplus(4,5,7;1,5,9)\otimes(x_{211},x_{212},x_{213};\\
&x'_{211},x_{212},x'_{213})\oplus(5,10,15;0,10,39)\otimes(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223})\oplus(7,16,24;0,16,41)\otimes\\
&(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233})\oplus(1,3,6;0,3,10)\otimes(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313})\oplus(5,13,21;5,\\
&13,35)\otimes(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323})\oplus(8,18,27;6,18,48)\otimes(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333})\Big)
\end{aligned}$$

Subject to (P4.6)

$$\begin{aligned}
&(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113})\oplus(x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123})\oplus(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133})\\
&\qquad\approx(6,7,10;2,7,11),\\
&(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213})\oplus(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223})\oplus(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233})\\
&\qquad\approx(6,15,23;1,15,29),
\end{aligned}$$

$$\left(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313}\right)\oplus\left(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323}\right)\oplus\left(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333}\right) \approx \left(2,10,16;0,10,21\right),$$

$$\left(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113}\right)\oplus\left(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213}\right)\oplus\left(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313}\right) \approx \left(3,8,16;0,8,19\right),$$

$$\left(x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123}\right)\oplus\left(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223}\right)\oplus\left(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323}\right) \approx \left(1,6,7;0,6,14\right),$$

$$\left(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133}\right)\oplus\left(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233}\right)\oplus\left(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333}\right) \approx \left(10,18,26;3,18,28\right),$$

$$\left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)\succeq(0,0,0;0,0,0); i=1,2,3;\ j=1,2,3.$$

Step 2: Using Section 4.1, the intuitionistic fully fuzzy transportation problem (P4.6) can be transformed into the problem (P4.7).

$$\text{Minimize } R\Big(\left(1,4,9;0,4,12\right)\otimes\left(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113}\right)\oplus\left(3,13,14;2,13,15\right)\otimes\left(x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123}\right)\oplus\left(4,6,16;1,6,33\right)\otimes\left(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133}\right)\oplus\left(4,5,7;1,5,9\right)\otimes\left(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213}\right)\oplus\left(5,10,15;0,10,39\right)\otimes\left(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223}\right)\oplus\left(7,16,24;0,16,41\right)\otimes\left(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233}\right)\oplus\left(1,3,6;0,3,10\right)\otimes\left(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313}\right)\oplus\left(5,13,21;5,13,35\right)\otimes\left(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323}\right)\oplus\left(8,18,27;6,18,48\right)\otimes\left(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333}\right)\Big)$$

Subject to (P4.7)

$$R\Big(\left(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113}\right)\oplus\left(x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123}\right)\oplus\left(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133}\right)\Big) = R\left(6,7,10;2,7,11\right),$$

$$R\Big(\left(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213}\right)\oplus\left(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223}\right)\oplus\left(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233}\right)\Big) = R\left(6,15,23;1,15,29\right),$$

$$R\Big(\left(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313}\right)\oplus\left(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323}\right)\oplus\left(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333}\right)\Big) = R\left(2,10,16;0,10,21\right),$$

$$R\Big(\left(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113}\right)\oplus\left(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213}\right)\oplus\left(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313}\right)\Big) = R\left(3,8,16;0,8,19\right),$$

$$R\Big(\left(x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123}\right)\oplus\left(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223}\right)\oplus\left(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323}\right)\Big) = R\left(1,6,7;0,6,14\right),$$

$$R\left(\left(x_{131}, x_{132}, x_{133}; x'_{131}, x_{132}, x'_{133}\right) \oplus \left(x_{231}, x_{232}, x_{233}; x'_{231}, x_{232}, x'_{233}\right) \oplus \left(x_{331}, x_{332}, x_{333}; x'_{331}, x_{332}, x'_{333}\right)\right) = R(10,18,26;3,18,28),$$

$$R\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right) \geq (0,0,0;0,0,0); i=1,2,3;\ j=1,2,3.$$

Step 3: Using the relation, $R\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(a_{ij}, b_{ij}, c_{ij}; a'_{ij}, b_{ij}, c'_{ij}\right)\right) = \sum_{i=1}^{m}\sum_{j=1}^{n} R\left(a_{ij}, b_{ij}, c_{ij}; a'_{ij}, b_{ij}, c'_{ij}\right)$, the problem (P4.7) can be transformed into the problem (P4.8).

$$\begin{aligned}
\text{Minimize } \Big(& R\left((1,4,9;0,4,12) \otimes \left(x_{111}, x_{112}, x_{113}; x'_{111}, x_{112}, x'_{113}\right)\right) + R\left((3,13,14;2,13,15) \otimes (x_{121}, x_{122},\right.\\
& \left. x_{123}; x'_{121}, x_{122}, x'_{123})\right) + R\left((4,6,16;1,6,33) \otimes \left(x_{131}, x_{132}, x_{133}; x'_{131}, x_{132}, x'_{133}\right)\right) + R\left((4,5,7;1,5,9)\right.\\
& \left.\otimes (x_{211}, x_{212}, x_{213}; x'_{211}, x_{212}, x'_{213})\right) + R\left((5,10,15;0,10,39) \otimes \left(x_{221}, x_{222}, x_{223}; x'_{221}, x_{222}, x'_{223}\right)\right)\\
& + R\left((7,16,24;0,16,41) \otimes \left(x_{231}, x_{232}, x_{233}; x'_{231}, x_{232}, x'_{233}\right)\right) + R\left((1,3,6;0,3,10) \otimes (x_{311}, x_{312}, x_{313};\right.\\
& \left. x'_{311}, x_{312}, x'_{313})\right) + R\left((5,13,21;5,13,35) \otimes \left(x_{321}, x_{322}, x_{323}; x'_{321}, x_{322}, x'_{323}\right)\right) + R\left((8,18,27;6,18,48)\right.\\
& \left.\otimes \left(x_{331}, x_{332}, x_{333}; x'_{331}, x_{332}, x'_{333}\right)\right)\Big)
\end{aligned}$$

Subject to (P4.8)

$$R\left(x_{111}, x_{112}, x_{113}; x'_{111}, x_{112}, x'_{113}\right) + R\left(x_{121}, x_{122}, x_{123}; x'_{121}, x_{122}, x'_{123}\right) + R\left(x_{131}, x_{132}, x_{133}; x'_{131}, x_{132}, x'_{133}\right) = R(6,7,10;2,7,11),$$

$$R\left(x_{211}, x_{212}, x_{213}; x'_{211}, x_{212}, x'_{213}\right) + R\left(x_{221}, x_{222}, x_{223}; x'_{221}, x_{222}, x'_{223}\right) + R\left(x_{231}, x_{232}, x_{233}; x'_{231}, x_{232}, x'_{233}\right) = R(6,15,23;1,15,29),$$

$$R\left(x_{311}, x_{312}, x_{313}; x'_{311}, x_{312}, x'_{313}\right) + R\left(x_{321}, x_{322}, x_{323}; x'_{321}, x_{322}, x'_{323}\right) + R\left(x_{331}, x_{332}, x_{333}; x'_{331}, x_{332}, x'_{333}\right) = R(2,10,16;0,10,21),$$

$$R\left(x_{111}, x_{112}, x_{113}; x'_{111}, x_{112}, x'_{113}\right) + R\left(x_{211}, x_{212}, x_{213}; x'_{211}, x_{212}, x'_{213}\right) + R\left(x_{311}, x_{312}, x_{313}; x'_{311}, x_{312}, x'_{313}\right) = R(3,8,16;0,8,19),$$

$$R\left(x_{121}, x_{122}, x_{123}; x'_{121}, x_{122}, x'_{123}\right) + R\left(x_{221}, x_{222}, x_{223}; x'_{221}, x_{222}, x'_{223}\right) + R\left(x_{321}, x_{322}, x_{323}; x'_{321}, x_{322}, x'_{323}\right) = R(1,6,7;0,6,14),$$

$$R\left(\left(x_{131}, x_{132}, x_{133}; x'_{131}, x_{132}, x'_{133}\right) + R\left(x_{231}, x_{232}, x_{233}; x'_{231}, x_{232}, x'_{233}\right) + R\left(x_{331}, x_{332}, x_{333}; x'_{331}, x_{332}, x'_{333}\right)\right) = R(10,18,26;3,18,28),$$

$$R\left(x_{ij1}, x_{ij2}, x_{ij3}; x'_{ij1}, x_{ij2}, x'_{ij3}\right) \geq (0,0,0;0,0,0); i=1,2,3;\ j=1,2,3.$$

Step 4: Using Section 4.2, the problem (P4.8) can be transformed into the problem (P4.9).

Minimize $\left(R(1,4,9;0,4,12)\times R\left(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113}\right)+R(3,13,14;2,13,15)\times R(x_{121},x_{122},\right.$
$x_{123};x'_{121},x_{122},x'_{123})+R(4,6,16;1,6,33)\times R\left(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133}\right)+R(4,5,7;1,5,9)$
$\times R\left(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213}\right)+R(5,10,15;0,10,39)\times R\left(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223}\right)$
$+R(7,16,24;0,16,41)\times R\left(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233}\right)+R(1,3,6;0,3,10)\times R(x_{311},x_{312},x_{313};$
$x'_{311},x_{312},x'_{313})+R(5,13,21;5,13,35)\times R\left(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323}\right)+R(8,18,27;6,18,48)$
$\left.\times R\left(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333}\right)\right)$

Subject to (P4.9)

Constraints of problem (P4.8).

Step 5: Since, $R\left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)$ will always be a real number. So, assuming $R\left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)=x_{ij}$, the problem (P4.9) can be transformed into the problem (P4.10).

Minimize $\left(R(1,4,9;0,4,12)\times x_{11}+R(3,13,14;2,13,15)\times x_{12}+R(4,6,16;1,6,33)\times x_{13}\right.$
$+R(4,5,7;1,5,9)\times x_{21}+R(5,10,15;0,10,39)\times x_{22}+R(7,16,24;0,16,41)\times x_{23}$
$\left.+R(1,3,6;0,3,10)\times x_{31}+R(5,13,21;5,13,35)\times x_{32}+R(8,18,27;6,18,48)\times x_{33}\right)$

Subject to

$x_{11}+x_{12}+x_{13}=R(6,7,10;2,7,11),$
$x_{21}+x_{22}+x_{23}=R(6,15,23;1,15,29),$ (P4.10)
$x_{31}+x_{32}+x_{33}=R(2,10,16;0,10,21),$
$x_{11}+x_{21}+x_{31}=R(3,8,16;0,8,19),$
$x_{12}+x_{22}+x_{32}=R(1,6,7;0,6,14),$
$x_{13}+x_{23}+x_{33}=R(10,18,26;3,18,28),$
$x_{ij}\geq 0; i=1,2,3;\ j=1,2,3.$

Step 6: Using the relation, $R(a,b,c;a',b,c')=\dfrac{a+2b+c+a'+2b+c'}{8}$, the problem (P4.10) can be transformed into the problem (P4.11).

Minimize $\left(4.75x_{11}+10.75x_{12}+9.75x_{13}+5.125x_{21}+12.375x_{22}+17x_{23}+3.625x_{31}\right.$
$\left.+14.75x_{32}+20.125x_{33}\right)$

Subject to

$x_{11}+x_{12}+x_{13}=7.125,$
$x_{21}+x_{22}+x_{23}=14.875,$ (P4.11)

$$x_{31}+x_{32}+x_{33}=9.875,$$
$$x_{11}+x_{21}+x_{31}=8.75,$$
$$x_{12}+x_{22}+x_{32}=5.75,$$
$$x_{13}+x_{23}+x_{33}=17.375,$$
$$x_{ij}\geq 0; i=1,2,3;\ j=1,2,3.$$

Step 7: The optimal solution of problem (P4.11) is $x_{13}=7.125, x_{22}=4.625, x_{23}=10.25,$ $x_{31}=8.75,\ x_{32}=1.125$.

Step 8: Using the optimal solution, obtained in Step 6, the optimal solution of problem (P4.6) is

$$\tilde{x}_{13}^{I}=\left\{\left(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133}\right):\frac{x_{131}+2x_{132}+x_{133}+x'_{131}+2x_{132}+x'_{133}}{8}=7.125\right\},$$

$$\tilde{x}_{22}^{I}=\left\{\left(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223}\right):\frac{x_{221}+2x_{222}+x_{223}+x'_{221}+2x_{222}+x'_{223}}{8}=4.625\right\},$$

$$\tilde{x}_{23}^{I}=\left\{\left(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233}\right):\frac{x_{231}+2x_{232}+x_{233}+x'_{231}+2x_{232}+x'_{233}}{8}=10.25\right\},$$

$$\tilde{x}_{31}^{I}=\left\{\left(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313}\right):\frac{x_{311}+x_{312}+2x_{313}+x'_{311}+2x_{312}+x'_{313}}{8}=8.75\right\},$$

$$\tilde{x}_{32}^{I}=\left\{\left(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323}\right):\frac{x_{321}+x_{322}+2x_{323}+x'_{321}+2x_{322}+x'_{323}}{8}=1.125\right\},$$

and the optimal value is $\left\{\left(c_1,c_2,c_3;c'_1,c_2,c'_3\right):\frac{c_1+2c_2+c_3+c'_1+2c_2+c'_3}{8}=349.267\right\}$.

4.7.2 Intuitionistic fuzzy optimal solution of fully fuzzy transportation problem presented by Table 4.2

Using the proposed method, the intuitionistic fuzzy optimal solution of intuitionistic fully fuzzy transportation problem, presented by Table 4.2, can be obtained as follows:

Step 1: Using Section 4.3, the intuitionistic fully fuzzy transportation problem, presented in

Table 4.2, can be transformed into the intuitionistic fully fuzzy linear programming problem (P4.12).

$$
\begin{aligned}
\text{Minimize } & (14,16,18;12,16,20)\otimes\left(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113}\right)\oplus(0,1,2;-1,1,3)\otimes(x_{121},x_{122},x_{123};\\
& x'_{121},x_{122},x'_{123})\oplus(7,8,9;6,8,10)\otimes\left(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133}\right)\oplus(11,13,15;10,13,16)\otimes\\
& \left(x_{141},x_{142},x_{143};x'_{141},x_{142},x'_{143}\right)\oplus(8,11,14;7,11,15)\otimes(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213})\oplus(3,4,5;\\
& 2,4,6)\otimes\left(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223}\right)\oplus(5,7,9;4,7,10)\otimes\left(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233}\right)\oplus\\
& (8,10,12;6,10,14)\otimes\left(x_{241},x_{242},x_{243};x'_{241},x_{242},x'_{243}\right)\oplus(6,8,10;5,8,11)\otimes(x_{311},x_{312},x_{313};x'_{311},\\
& x_{312},x'_{313})\oplus(13,15,17;12,15,18)\otimes\left(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323}\right)\oplus(7,9,11;\ 6,9,12)\otimes\\
& \left(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333}\right)\oplus(1,2,3;0,2,4)\otimes\left(x_{341},x_{342},x_{343};x'_{341},x_{342},x'_{343}\right)\oplus(5,6,7;4,\\
& 6,8)\otimes\left(x_{411},x_{412},x_{413};x'_{411},x_{412},x'_{413}\right)\oplus(11,12,13;10,12,14)\otimes\left(x_{421},x_{422},x_{423};x'_{421},x_{422},x'_{423}\right)\\
& \oplus(3,5,7;1,5,9)\otimes\left(x_{431},x_{432},x_{433};x'_{431},x_{432},x'_{433}\right)\oplus(12,14,16;11,14,17)\otimes(x_{441},x_{442},x_{443};\\
& x'_{441},x_{442},x'_{443})
\end{aligned}
$$

Subject to (P4.12)

$$
\begin{aligned}
& \left(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113}\right)\oplus\left(x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123}\right)\oplus\left(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133}\right)\\
& \qquad\oplus\left(x_{141},x_{142},x_{143};x'_{141},x_{142},x'_{143}\right)\approx\left(2,4,6;1,4,7\right),\\
& \left(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213}\right)\oplus\left(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223}\right)\oplus\left(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233}\right)\\
& \qquad\oplus\left(x_{241},x_{242},x_{243};x'_{241},x_{242},x'_{243}\right)\approx\left(5,6,7;4,6,8\right),\\
& \left(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313}\right)\oplus\left(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323}\right)\oplus\left(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333}\right)\\
& \qquad\oplus\left(x_{341},x_{342},x_{343};x'_{341},x_{342},x'_{343}\right)\approx\left(7,8,9;5,8,11\right),\\
& \left(x_{411},x_{412},x_{413};x'_{411},x_{412},x'_{413}\right)\oplus\left(x_{421},x_{422},x_{423};x'_{421},x_{422},x'_{423}\right)\oplus\left(x_{431},x_{432},x_{433};x'_{431},x_{432},x'_{433}\right)\\
& \qquad\oplus\left(x_{441},x_{442},x_{443};x'_{441},x_{442},x'_{443}\right)\approx\left(8,10,12;6,10,14\right),\\
& \left(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113}\right)\oplus\left(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213}\right)\oplus\left(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313}\right)\\
& \qquad\oplus\left(x_{411},x_{412},x_{413};x'_{411},x_{412},x'_{413}\right)\approx\left(3,4,5;2,4,6\right),\\
& \left(x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123}\right)\oplus\left(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223}\right)\oplus\left(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323}\right)\\
& \qquad\oplus\left(x_{421},x_{422},x_{423};x'_{421},x_{422},x'_{423}\right)\approx\left(3,5,7;1,5,9\right),\\
& \left(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133}\right)\oplus\left(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233}\right)\oplus\left(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333}\right)\\
& \qquad\oplus\left(x_{431},x_{432},x_{433};x'_{431},x_{432},x'_{433}\right)\approx\left(10,12,14;8,12,16\right),\\
& \left(x_{141},x_{142},x_{143};x'_{141},x_{142},x'_{143}\right)\oplus\left(x_{241},x_{242},x_{243};x'_{241},x_{242},x'_{243}\right)\oplus\left(x_{341},x_{342},x_{343};x'_{341},x_{342},x'_{343}\right)\\
& \qquad\oplus\left(x_{441},x_{442},x_{443};x'_{441},x_{442},x'_{443}\right)\approx\left(6,7,8;5,7,9\right),\\
& \left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)\succeq(0,0,0;0,0,0);i=1,2,3,4;\ j=1,2,3,4.
\end{aligned}
$$

Step 2: Using Section 4.1, the intuitionistic fully fuzzy transportation problem (P4.12) can be transformed into the problem (P4.13).

$$\begin{aligned}
\text{Minimize } R\big(&(14,16,18;12,16,20)\otimes(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113})\oplus(0,1,2;-1,1,3)\otimes(x_{121},x_{122},x_{123};\\
&x'_{121},x_{122},x'_{123})\oplus(7,8,9;6,8,10)\otimes(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133})\oplus(11,13,15;10,13,16)\otimes\\
&(x_{141},x_{142},x_{143};x'_{141},x_{142},x'_{143})\oplus(8,11,14;7,11,15)\otimes(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213})\oplus(3,4,5;\\
&2,4,6)\otimes(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223})\oplus(5,7,9;4,7,10)\otimes(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233})\oplus\\
&(8,10,12;6,10,14)\otimes(x_{241},x_{242},x_{243};x'_{241},x_{242},x'_{243})\oplus(6,8,10;5,8,11)\otimes(x_{311},x_{312},x_{313};x'_{311},\\
&x_{312},x'_{313})\oplus(13,15,17;12,15,18)\otimes(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323})\oplus(7,9,11;\ 6,9,12)\otimes\\
&(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333})\oplus(1,2,3;0,2,4)\otimes(x_{341},x_{342},x_{343};x'_{341},x_{342},x'_{343})\oplus(5,6,7;4,\\
&6,8)\otimes(x_{411},x_{412},x_{413};x'_{411},x_{412},x'_{413})\oplus(11,12,13;10,12,14)\otimes(x_{421},x_{422},x_{423};x'_{421},x_{422},x'_{423})\\
&\oplus(3,5,7;1,5,9)\otimes(x_{431},x_{432},x_{433};x'_{431},x_{432},x'_{433})\oplus(12,14,16;11,14,17)\otimes(x_{441},x_{442},x_{443};\\
&x'_{441},x_{442},x'_{443})\big)
\end{aligned}$$

Subject to (P4.13)

$$\begin{aligned}
&R\big((x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113})\oplus(x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123})\oplus(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133})\\
&\qquad\oplus(x_{141},x_{142},x_{143};x'_{141},x_{142},x'_{143})\big)=R(2,4,6;1,4,7),\\
&R\big((x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213})\oplus(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223})\oplus(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233})\\
&\qquad\oplus(x_{241},x_{242},x_{243};x'_{241},x_{242},x'_{243})\big)=R(5,6,7;4,6,8),\\
&R\big((x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313})\oplus(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323})\oplus(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333})\\
&\qquad\oplus(x_{341},x_{342},x_{343};x'_{341},x_{342},x'_{343})\big)=R(7,8,9;5,8,11),\\
&R\big((x_{411},x_{412},x_{413};x'_{411},x_{412},x'_{413})\oplus(x_{421},x_{422},x_{423};x'_{421},x_{422},x'_{423})\oplus(x_{431},x_{432},x_{433};x'_{431},x_{432},x'_{433})\\
&\qquad\oplus(x_{441},x_{442},x_{443};x'_{441},x_{442},x'_{443})\big)=R(8,10,12;6,10,14),\\
&R\big((x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113})\oplus(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213})\oplus(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313})\\
&\qquad\oplus(x_{411},x_{412},x_{413};x'_{411},x_{412},x'_{413})\big)=R(3,4,5;2,4,6),\\
&R\big((x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123})\oplus(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223})\oplus(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323})\\
&\qquad\oplus(x_{421},x_{422},x_{423};x'_{421},x_{422},x'_{423})\big)=R(3,5,7;1,5,9),\\
&R\big((x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133})\oplus(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233})\oplus(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333})\\
&\qquad\oplus(x_{431},x_{432},x_{433};x'_{431},x_{432},x'_{433})\big)=R(10,12,14;8,12,16),\\
&R\big((x_{141},x_{142},x_{143};x'_{141},x_{142},x'_{143})\oplus(x_{241},x_{242},x_{243};x'_{241},x_{242},x'_{243})\oplus(x_{341},x_{342},x_{343};x'_{341},x_{342},x'_{343})\\
&\qquad\oplus(x_{441},x_{442},x_{443};x'_{441},x_{442},x'_{443})\big)=R(6,7,8;5,7,9),
\end{aligned}$$

$$R\left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)\geq R(0,0,0;0,0,0);i=1,2,3,4;\ j=1,2,3,4.$$

Step 3: Using the relation, $R\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(a_{ij},b_{ij},c_{ij};a'_{ij},b_{ij},c'_{ij}\right)\right)=\sum_{i=1}^{m}\sum_{j=1}^{n}R\left(a_{ij},b_{ij},c_{ij};a'_{ij},b_{ij},c'_{ij}\right)$, the problem (P4.13) can be transformed into the problem (P4.14).

$$\begin{aligned}\text{Minimize}\Big[&R\left((14,16,18;12,16,20)\otimes\left(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113}\right)\right)+R\left((0,1,2;-1,1,3)\otimes(x_{121},x_{122}\right.\\&\left.,x_{123};x'_{121},x_{122},x'_{123})\right)+R\left((7,8,9;6,8,10)\otimes\left(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133}\right)\right)+R\left((11,13,15;\right.\\&\left.10,13,16)\otimes\left(x_{141},x_{142},x_{143};x'_{141},x_{142},x'_{143}\right)\right)+R\left((8,11,14;7,11,15)\otimes(x_{211},x_{212},x_{213};x'_{211},x_{212}\right.\\&\left.,x'_{213})\right)+R\left((3,4,5;2,4,6)\otimes\left(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223}\right)\right)+R\left((5,7,9;4,7,10)\otimes(x_{231},x_{232},\right.\\&\left.x_{233};x'_{231},x_{232},x'_{233})\right)+R\left((8,10,12;6,10,14)\otimes\left(x_{241},x_{242},x_{243};x'_{241},x_{242},x'_{243}\right)\right)+R\left((6,8,10;5,\right.\\&\left.8,11)\otimes(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313})\right)+R\left((13,15,17;12,15,18)\otimes(x_{321},x_{322},x_{323};x'_{321},x_{322},\right.\\&\left.x'_{323})\right)+R\left((7,9,11;\ 6,9,12)\otimes\left(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333}\right)\right)+R\left((1,2,3;0,2,4)\otimes(x_{341},\right.\\&\left.x_{342},x_{343};x'_{341},x_{342},x'_{343})\right)+R\left((5,6,7;4,6,8)\otimes\left(x_{411},x_{412},x_{413};x'_{411},x_{412},x'_{413}\right)\right)+R\left((11,12,13;\right.\\&\left.10,12,14)\otimes\left(x_{421},x_{422},x_{423};x'_{421},x_{422},x'_{423}\right)\right)+R\left((3,5,7;1,5,9)\otimes\left(x_{431},x_{432},x_{433};x'_{431},x_{432},x'_{433}\right)\right)\\&+R\left((12,14,16;11,14,17)\otimes(x_{441},x_{442},x_{443};x'_{441},x_{442},x'_{443})\right)\Big]\end{aligned}$$

Subject to (P4.14)

$$\begin{aligned}&R\left(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113}\right)+R\left(x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123}\right)\\&\qquad+R\left(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133}\right)+R\left(x_{141},x_{142},x_{143};x'_{141},x_{142},x'_{143}\right)=R\left(2,4,6;1,4,7\right),\\&R\left(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213}\right)+R\left(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223}\right)\\&\qquad+R\left(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233}\right)+R\left(x_{241},x_{242},x_{243};x'_{241},x_{242},x'_{243}\right)=R\left(5,6,7;4,6,8\right),\\&R\left(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313}\right)+R\left(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323}\right)\\&\qquad+R\left(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333}\right)+R\left(x_{341},x_{342},x_{343};x'_{341},x_{342},x'_{343}\right)=R\left(7,8,9;5,8,11\right),\\&R\left(x_{411},x_{412},x_{413};x'_{411},x_{412},x'_{413}\right)+R\left(x_{421},x_{422},x_{423};x'_{421},x_{422},x'_{423}\right)\\&\qquad+R\left(x_{431},x_{432},x_{433};x'_{431},x_{432},x'_{433}\right)+R\left(x_{441},x_{442},x_{443};x'_{441},x_{442},x'_{443}\right)=R\left(8,10,12;6,10,14\right),\\&R\left(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113}\right)+R\left(x_{211},x_{212},x_{213};x'_{211},x_{212},x'_{213}\right)\\&\qquad+R\left(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313}\right)+R\left(x_{411},x_{412},x_{413};x'_{411},x_{412},x'_{413}\right)=R\left(3,4,5;2,4,6\right),\\&R\left(x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123}\right)+R\left(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223}\right)\\&\qquad+R\left(x_{321},x_{322},x_{323};x'_{321},x_{322},x'_{323}\right)+R\left(x_{421},x_{422},x_{423};x'_{421},x_{422},x'_{423}\right)=R\left(3,5,7;1,5,9\right),\\&R\left(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133}\right)+R\left(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233}\right)\\&\qquad+R\left(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333}\right)+\left(x_{431},x_{432},x_{433};x'_{431},x_{432},x'_{433}\right)=R\left(10,12,14;8,12,16\right),\\&R\left(x_{141},x_{142},x_{143};x'_{141},x_{142},x'_{143}\right)+R\left(x_{241},x_{242},x_{243};x'_{241},x_{242},x'_{243}\right)\end{aligned}$$

$$+R\left(x_{341},x_{342},x_{343};x'_{341},x_{342},x'_{343}\right)+R\left(x_{441},x_{442},x_{443};x'_{441},x_{442},x'_{443}\right)=R\left(6,7,8;5,7,9\right),$$
$$R\left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)\geq R(0,0,0;0,0,0); i=1,2,3,4;\ j=1,2,3,4.$$

Step 4: Using Section 4.2, the problem (P4.14) can be transformed into the problem (P4.15).

$$\begin{aligned}\text{Minimize } & \left[R(14,16,18;12,16,20)\times R\left(x_{111},x_{112},x_{113};x'_{111},x_{112},x'_{113}\right)+R(0,1,2;-1,1,3)\times R(x_{121},x_{122}\right.\\ & ,x_{123};x'_{121},x_{122},x'_{123})+R(7,8,9;6,8,10)\times R\left(x_{131},x_{132},x_{133};x'_{131},x_{132},x'_{133}\right)+R(11,13,15;\\ & 10,13,16)\times R\left(x_{141},x_{142},x_{143};x'_{141},x_{142},x'_{143}\right)+R(8,11,14;7,11,15)\times R(x_{211},x_{212},x_{213};x'_{211},\\ & x_{212},x'_{213})+R(3,4,5;2,4,6)\times R\left(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223}\right)+R(5,7,9;4,7,10)\times R(x_{231},\\ & x_{232},x_{233};x'_{231},x_{232},x'_{233})+R(8,10,12;6,10,14)\times R\left(x_{241},x_{242},x_{243};x'_{241},x_{242},x'_{243}\right)+R(6,8,\\ & 10;5,8,11)\times R(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313})+R(13,15,17;12,15,18)\times R(x_{321},x_{322},x_{323};\\ & x'_{321},x_{322},x'_{323})+R(7,9,11;\ 6,9,12)\times R\left(x_{331},x_{332},x_{333};x'_{331},x_{332},x'_{333}\right)+R(1,2,3;0,2,4)\times\\ & R(x_{341},x_{342},x_{343};x'_{341},x_{342},x'_{343})+R(5,6,7;4,6,8)\times R\left(x_{411},x_{412},x_{413};x'_{411},x_{412},x'_{413}\right)+R(11,\\ & 12,13;10,12,14)\times R\left(x_{421},x_{422},x_{423};x'_{421},x_{422},x'_{423}\right)+R(3,5,7;1,5,9)\times R(x_{431},x_{432},x_{433};\\ & \left.x'_{431},x_{432},x'_{433})+R(12,14,16;11,14,17)\times R(x_{441},x_{442},x_{443};x'_{441},x_{442},x'_{443})\right]\end{aligned}$$

Subject to (P4.15)

Constraints of problem (P4.14).

Step 5: Since, $R\left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)$ will always be a real number. So, assuming $R\left(x_{ij1},x_{ij2},x_{ij3};x'_{ij1},x_{ij2},x'_{ij3}\right)=x_{ij}$, problem (P4.15) can be transformed into problem (P4.16).

$$\begin{aligned}\text{Minimize } & \left[R(14,16,18;12,16,20)\times x_{11}+R(0,1,2;-1,1,3)\times x_{12}+R(7,8,9;6,8,10)\times x_{13}\right.\\ & +R(11,13,15;10,13,16)\times x_{14}+R(8,11,14;7,11,15)\times x_{21}+R(3,4,5;2,4,6)\times x_{22}\\ & +R(5,7,9;4,7,10)\times x_{23}+R(8,10,12;6,10,14)\times x_{24}+R(6,8,10;5,8,11)\times x_{31}\\ & +R(13,15,17;12,15,18)\times x_{32}+R(7,9,11;\ 6,9,12)\times x_{33}+R(1,2,3;0,2,4)\times x_{34}\\ & +R(5,6,7;4,6,8)\times x_{41}+R(11,12,13;10,12,14)\times x_{42}+R(3,5,7;1,5,9)\times x_{43}\\ & \left.+R(12,14,16;11,14,17)\times x_{44}\right]\end{aligned}$$

Subject to (P4.16)

$$x_{11}+x_{12}+x_{13}+x_{14}=R\left(2,4,6;1,4,7\right),$$
$$x_{21}+x_{22}+x_{23}+x_{24}=R\left(5,6,7;4,6,8\right),$$
$$x_{31}+x_{32}+x_{33}+x_{34}=R\left(7,8,9;5,8,11\right),$$

$$x_{41}+x_{42}+x_{43}+x_{44}=R(8,10,12;6,10,14),$$
$$x_{11}+x_{21}+x_{31}+x_{41}=R(3,4,5;2,4,6),$$
$$x_{12}+x_{22}+x_{32}+x_{42}=R(3,5,7;1,5,9),$$
$$x_{13}+x_{23}+x_{33}+x_{43}=R(10,12,14;8,12,16),$$
$$x_{14}+x_{24}+x_{34}+x_{44}=R(6,7,8;5,7,9),$$
$$x_{ij}\geq 0; i=1,2,3,4;\ j=1,2,3,4.$$

Step 6: Using the relation, $R(a,b,c;a',b,c')=\dfrac{a+2b+c+a'+2b+c'}{8}$, the problem (P4.16) can be transformed into the problem (P4.17).

$$\text{Minimize } [16x_{11}+x_{12}+8x_{13}+13x_{14}+11x_{21}+4x_{22}+7x_{23}+10x_{24}+8x_{31}+15x_{32}+9x_{33}+2x_{34}+6x_{41}$$
$$+12x_{42}+5x_{43}+14x_{44}]$$

Subject to (P4.17)

$$x_{11}+x_{12}+x_{13}+x_{14}=4,$$
$$x_{21}+x_{22}+x_{23}+x_{24}=6,$$
$$x_{31}+x_{32}+x_{33}+x_{34}=8,$$
$$x_{41}+x_{42}+x_{43}+x_{44}=10,$$
$$x_{11}+x_{21}+x_{31}+x_{41}=4,$$
$$x_{12}+x_{22}+x_{32}+x_{42}=5,$$
$$x_{13}+x_{23}+x_{33}+x_{43}=12,$$
$$x_{14}+x_{24}+x_{34}+x_{44}=7,$$
$$x_{ij}\geq 0; i=1,2,3,4;\ j=1,2,3,4.$$

Step 7: The optimal solution of problem (P4.17) is $x_{12}=4, x_{22}=1, x_{23}=5, x_{31}=1, x_{34}=7, x_{41}=3,$ $x_{43}=7$.

Step 8: Using the optimal solution, obtained in Step 7, the optimal solution of problem, presented by Table 4.2, is

$$\tilde{x}_{12}^{I}=\left\{(x_{121},x_{122},x_{123};x'_{121},x_{122},x'_{123}):\frac{x_{121}+2x_{122}+x_{123}+x'_{121}+2x_{122}+x'_{123}}{8}=4\right\},$$

$$\tilde{x}_{22}^{I}=\left\{\left(x_{221},x_{222},x_{223};x'_{221},x_{222},x'_{223}\right):\frac{x_{221}+2x_{222}+x_{223}+x'_{221}+2x_{222}+x'_{223}}{8}=1\right\},$$

$$\tilde{x}_{23}^{I}=\left\{\left(x_{231},x_{232},x_{233};x'_{231},x_{232},x'_{233}\right):\frac{x_{231}+2x_{232}+x_{233}+x'_{231}+2x_{232}+x'_{233}}{8}=5\right\},$$

$$\tilde{x}_{31}^{I}=\left\{\left(x_{311},x_{312},x_{313};x'_{311},x_{312},x'_{313}\right):\frac{x_{311}+x_{312}+2x_{313}+x'_{311}+2x_{312}+x'_{313}}{8}=1\right\},$$

$$\tilde{x}_{34}^{I}=\left\{\left(x_{341},x_{341},x_{343};x'_{341},x_{342},x'_{343}\right):\frac{x_{341}+2x_{341}+x_{343}+x'_{341}+2x_{342}+x'_{343}}{8}=7\right\},$$

$$\tilde{x}_{41}^{I}=\left\{\left(x_{411},x_{412},x_{413};x'_{411},x_{412},x'_{413}\right):\frac{x_{411}+2x_{412}+x_{413}+x'_{411}+2x_{412}+x'_{413}}{8}=3\right\},$$

$$\tilde{x}_{43}^{I}=\left\{\left(x_{431},x_{432},x_{433};x'_{431},x_{432},x'_{433}\right):\frac{x_{431}+2x_{432}+x_{433}+x'_{431}+2x_{432}+x'_{433}}{8}=7\right\},$$

and the optimal value is $\left\{\left(c_1,c_2,c_3;c'_1,c_2,c'_3\right):\frac{c_1+2c_2+c_3+c'_1+2c_2+c'_3}{8}=118\right\}$.

4.8 Conclusion

On the basis of present study, it can be concluded that the existing methods [84-86] is valid only if the ranking function, used in the existing methods [84-86], is replaced with either the ranking function, used in the existing method [132], or with any other appropriate ranking function for which the linear property $R\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(a_{ij},b_{ij},c_{ij};a'_{ij},b_{ij},c'_{ij}\right)\right)$ $=\sum_{i=1}^{m}\sum_{j=1}^{n}R\left(a_{ij},b_{ij},c_{ij};a'_{ij},b_{ij},c'_{ij}\right)$ will always be satisfied.

Chapter 5

A new method for solving intuitionistic fully fuzzy transportation problems*

In this chapter, flaws of the existing methods [18,29,45,49,112,118,123,133,134,138] for solving intuitionistic fully fuzzy transportation problems are pointed out. Also, a new method is proposed for solving intuitionistic fully fuzzy transportation problems. To illustrate the proposed method, the intuitionistic fully fuzzy transportation problem, considered by Roseline and Amirtharaj [123], is solved by proposed method.

5.1 Flaws of the existing methods

In Chapter 4, the ranking function, proposed by Singh and Yadav [132] and a special multiplication, proposed by Kumar and Hussain [84-86], is used. It can be easily verified that if $\tilde{A}^I = (a_1, a_2, a_3, a_4; b_1, b_2, b_3, b_4)$ and $\tilde{B}^I = (a'_1, a'_2, a'_3, a'_4; b'_1, b'_2, b'_3, b'_4)$ are two trapezoidal intuitionistic fuzzy numbers and $\tilde{A}^I \otimes \tilde{B}^I$ is a trapezoidal intuitionistic fuzzy number obtained by using special multiplication, proposed by Kumar and Hussain [84-86], then for the ranking function *R*, used by Singh and Yadav [132], the property $R(\tilde{A}^I \otimes \tilde{B}^I) = R(\tilde{A}^I) \times R(\tilde{B}^I)$ will always be satisfied and hence in Step 3 of the method, modified in Chapter 4, the problem (P4.3) can be transformed into problem (P4.4).

However, if instead of special multiplication, the generally used multiplication of intuitionistic fuzzy numbers is used then the property $R(\tilde{A}^I \otimes \tilde{B}^I) = R(\tilde{A}^I) \times R(\tilde{B}^I)$ will not be necessarily

* The contents of this chapter are to be communicated in Applied Mathematical Modelling.

satisfied. For example, if $\tilde{A}^I=(3,7,10,14;2,5,12,15)$ and $\tilde{B}^I=(3,6,9,11;1,5,10,13)$ are two trapezoidal intuitionistic fuzzy numbers then $\tilde{A}^I\otimes\tilde{B}^I=(9,42,90,154;2,25,120,195)$ and hence $R\left(\tilde{A}^I\otimes\tilde{B}^I\right)=79.625$. While, $R\left(\tilde{A}^I\right)\otimes R\left(\tilde{B}^I\right)=61.625$. It is obvious that $R\left(\tilde{A}^I\otimes\tilde{B}^I\right)\neq R\left(\tilde{A}^I\right)\times R\left(\tilde{B}^I\right)$.

Therefore, the problem (P4.3) can not be transformed into problem (P4.4) and hence the method, modified in Chapter 4, can not be used for solving such intuitionistic fully fuzzy transportation problems in which general multiplication instead of special multiplication is used. While, it can be easily verified that in the existing methods [18,29,45,49,112,118,123,133,134, 138], the mathematical incorrect property $R\left(\tilde{A}^I\otimes\tilde{B}^I\right)=R\left(\tilde{A}^I\right)\times R\left(\tilde{B}^I\right)$ is used to transform the problem (P4.3) into the problem (P4.4). Hence, it is not genuine to use these existing methods.

5.2 Proposed method

In this section, to resolve the flaws of the existing methods [18,29,45,49,112,118,123,133,134, 138], modified in Chapter 4, a new method is proposed for solving intuitionistic fully fuzzy transportation problem without considering the special multiplication of intuitionistic fuzzy numbers.

The steps of the proposed method are as follows:

Step 1: Replacing c_{ij},x_{ij},a_i and b_j of the crisp linear programming problem (P2.1) of transportation problem with the trapezoidal intuitionistic fuzzy numbers $\left(c_{ij1},c_{ij2},c_{ij3},c_{ij4};c'_{ij1},c'_{ij2},c'_{ij3},c'_{ij4}\right)$, $\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x'_{ij2},x'_{ij3},x'_{ij4}\right)$, $(a_{i1},a_{i2},a_{i3},a_{i4};a'_{i1},a'_{i2},a'_{i3},$

a'_{i4}) and $\left(b_{j1}, b_{j2}, b_{j3}, b_{j4}; b'_{j1}, b'_{j2}, b'_{j3}, b'_{j4}\right)$ respectively, the problem (P2.1) is transformed into problem (P5.1).

$$\text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij1}, c_{ij2}, c_{ij3}, c_{ij4}; c'_{ij1}, c'_{ij2}, c'_{ij3}, c'_{ij4}\right) \otimes \left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x'_{ij2}, x'_{ij3}, x'_{ij4}\right)$$

Subject to (P5.1)

$$\sum_{j=1}^{n}\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x'_{ij2}, x'_{ij3}, x'_{ij4}\right) \approx \left(a_{i1}, a_{i2}, a_{i3}, a_{i4}; a'_{i1}, a'_{i2}, a'_{i3}, a'_{i4}\right); i = 1, 2, ..., m,$$

$$\sum_{i=1}^{m}\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x'_{ij2}, x'_{ij3}, x'_{ij4}\right) \approx \left(b_{j1}, b_{j2}, b_{j3}, b_{j4}; b'_{j1}, b'_{j2}, b'_{j3}, b'_{j4}\right); j = 1, 2, ..., n,$$

$$\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x'_{ij2}, x'_{ij3}, x'_{ij4}\right) \succeq (0, 0, 0, 0, 0, 0, 0, 0); i = 1, 2, ..., m; j = 1, 2, ..., n,$$

$\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x'_{ij2}, x'_{ij3}, x'_{ij4}\right)$ is a non-negative trapezoidal intuitionistic fuzzy number.

Step 2: Using the arithmetic operations of trapezoidal intuitionistic fuzzy numbers, the problem (P5.1) can be transformed into the problem (P5.2).

$$\text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij1}x_{ij1}, c_{ij2}x_{ij2}, c_{ij3}x_{ij3}, c_{ij4}x_{ij4}; c'_{ij1}x'_{ij1}, c'_{ij2}x'_{ij2}, c'_{ij3}x'_{ij3}, c'_{ij4}x'_{ij4}\right) a$$

Subject to (P5.2)

$$\sum_{j=1}^{n}\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x'_{ij2}, x'_{ij3}, x'_{ij4}\right) \approx \left(a_{i1}, a_{i2}, a_{i3}, a_{i4}; a'_{i1}, a'_{i2}, a'_{i3}, a'_{i4}\right); i = 1, 2, ..., m,$$

$$\sum_{i=1}^{m}\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x'_{ij2}, x'_{ij3}, x'_{ij4}\right) \approx \left(b_{j1}, b_{j2}, b_{j3}, b_{j4}; b'_{j1}, b'_{j2}, b'_{j3}, b'_{j4}\right); j = 1, 2, ..., n,$$

$$\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x'_{ij2}, x'_{ij3}, x'_{ij4}\right) \succeq (0, 0, 0, 0; 0, 0, 0, 0); i = 1, 2, ..., m; j = 1, 2, ..., n,$$

$\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x'_{ij2}, x'_{ij3}, x'_{ij4}\right)$ is a non-negative trapezoidal intuitionistic fuzzy number.

Step 3: Using the Section 1.3.3.2, the problem (P5.2) can be transformed into the problem (P5.3).

$$\text{Minimize } R\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij1}x_{ij1}, c_{ij2}x_{ij2}, c_{ij3}x_{ij3}, c_{ij4}x_{ij4}; c'_{ij1}x'_{ij1}, c'_{ij2}x'_{ij2}, c'_{ij3}x'_{ij3}, c'_{ij4}x'_{ij4}\right)\right)$$

Subject to (P5.3)

$$R\left(\sum_{j=1}^{n}\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x'_{ij2},x'_{ij3},x'_{ij4}\right)\right)=R\left(a_{i1},a_{i2},a_{i3},a_{i4};a'_{i1},a'_{i2},a'_{i3},a'_{i4}\right);i=1,2,...,m,$$

$$R\left(\sum_{i=1}^{m}\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x'_{ij2},x'_{ij3},x'_{ij4}\right)\right)=R\left(b_{j1},b_{j2},b_{j3},b_{j4};b'_{j1},b'_{j2},b'_{j3},b'_{j4}\right);\ j=1,2,...,n,$$

$$R\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x'_{ij2},x'_{ij3},x'_{ij4}\right)\geq R(0,0,0,0;0,0,0,0);x'_{ij1}\geq 0;i=1,2,...,m;\ j=1,2,...,n,$$

$\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x'_{ij2},x'_{ij3},x'_{ij4}\right)$ is a non-negative trapezoidal intuitionistic fuzzy number.

Step 4: Using the relation, $R\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(a_{ij},b_{ij},c_{ij},d_{ij};a'_{ij},b'_{ij},c'_{ij},d'_{ij}\right)\right)=\sum_{i=1}^{m}\sum_{j=1}^{n}R(a_{ij},b_{ij},c_{ij},d_{ij};a'_{ij},b'_{ij},c'_{ij},d'_{ij})$, the problem (P5.3) can be transformed into the problem (P5.4).

Minimize $\sum_{i=1}^{m}\sum_{j=1}^{n}R\left(c_{ij1}x_{ij1},c_{ij2}x_{ij2},c_{ij3}x_{ij3},c_{ij4}x_{ij4};c'_{ij1}x'_{ij1},c'_{ij2}x'_{ij2},c'_{ij3}x'_{ij3},c'_{ij4}x'_{ij4}\right)$

Subject to (P5.4)

$$\sum_{j=1}^{n}R\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x'_{ij2},x'_{ij3},x'_{ij4}\right)=R\left(a_{i1},a_{i2},a_{i3},a_{i4};a'_{i1},a'_{i2},a'_{i3},a'_{i4}\right);i=1,2,...,m,$$

$$\sum_{i=1}^{m}R\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x'_{ij2},x'_{ij3},x'_{ij4}\right)=R\left(b_{j1},b_{j2},b_{j3},b_{j4};b'_{j1},b'_{j2},b'_{j3},b'_{j4}\right);\ j=1,2,...,n,$$

$$R\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x'_{ij2},x'_{ij3},x'_{ij4}\right)\geq R(0,0,0,0;0,0,0,0);x'_{ij1}\geq 0;i=1,2,...,m;\ j=1,2,...,n,$$

$\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x'_{ij2},x'_{ij3},x'_{ij4}\right)$ is a non-negative trapezoidal intuitionistic fuzzy number.

Step 5: Using the expression, $R\left(a,b,c,d;a',b',c',d'\right)=\dfrac{a+b+c+d+a'+b'+c'+d'}{8}$, the problem (P5.4) can be transformed into the problem (P5.5).

Minimize $\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\dfrac{c_{ij1}x_{ij1}+c_{ij2}x_{ij2}+c_{ij3}x_{ij3}+c_{ij4}x_{ij4}+c'_{ij1}x'_{ij1}+c'_{ij2}x'_{ij2}+c'_{ij3}x'_{ij3}+c'_{ij4}x'_{ij4}}{8}\right)$

Subject to (P5.5)

$$\sum_{j=1}^{n}\left(\frac{x_{ij1}+x_{ij2}+x_{ij3}+x_{ij4}+x'_{ij1}+x'_{ij2}+x'_{ij3}+x'_{ij4}}{8}\right)=\frac{a_{i1}+a_{i2}+a_{i3}+a_{i4}+a'_{i1}+a'_{i2}+a'_{i3}+a'_{i4}}{8};i=1,2,...,m,$$

$$\sum_{i=1}^{m}\left(\frac{x_{ij1}+x_{ij2}+x_{ij3}+x_{ij4}+x'_{ij1}+x'_{ij2}+x'_{ij3}+x'_{ij4}}{8}\right)=\frac{b_{j1}+b_{j2}+b_{j3}+b_{j4}+b'_{j1}+b'_{j2}+b'_{j3}+b'_{j4}}{8};\ j=1,2,...,n,$$

$$x_{ijk}, x'_{ijk} \geq 0; i=1,2,...,m;\ j=1,2,...,n; k=1,2,3,4.$$

Step 6: Find the optimal solution of crisp linear programming problem (P5.5).

Step 7: Using the optimal solution, obtained in Step 6, the intuitionistic fuzzy optimal solution of problem (P5.1) is $\left\{\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x'_{ij2}, x'_{ij3}, x'_{ij4}\right)\right\}$.

Step 8: Using the optimal solution, obtained in Step 7, the minimum intuitionistic fuzzy transportation cost of intuitionistic fully fuzzy transportation problem (P5.1) is $\sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij1}, c_{ij2}, c_{ij3}, c_{ij4}; c'_{ij1}, c'_{ij2}, c'_{ij3}, c'_{ij4}\right) \otimes \left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x'_{ij2}, x'_{ij3}, x'_{ij4}\right)$.

5.3 Illustrative example

Roseline and Amirtharaj [123] solved the intuitionistic fully fuzzy transportation problem, presented by Table 5.1, to illustrate their proposed method. In this section, the intuitionistic fuzzy intuitionistic fuzzy optimal solution of the same problem is obtained by the proposed method.

Table 5.1: Intuitionistic fully fuzzy transportation problem

Sources	Destinations				Inst. Fuzzy Availability ($\tilde{a}_i^I$)
	D_1	D_2	D_3	D_4	
S_1	(2,5,7,10; 1,4,8,12)	(3,6,9,12; 2,5,10,14)	(3,6,9,11; 1,5,10,13)	(4,6,8,11; 3,5,9,12)	(4,6,10,14; 2,5,11,15)
S_2	(4,7,9,13; 3,6,11,15)	(2,6,10,14; 1,4,12,16)	(3,7,10,14; 2,5,12,15)	(4,7,10,13; 1,5,12,15)	(5,9,12,16; 4,8,14,18)
S_3	(4,8,10,12; 2,6,11,14)	(3,5,8,10; 2,4,9,11)	(4,7,11,13; 3,5,12,14)	(3,6,9,11; 1,4,10,14)	(8,15,20,27; 6,11,23,33)

Inst. Fuzzy Demand $(\tilde{b}_j^I)$	(5,8,11,15; 4,7,13,16)	(4,8,12,14; 3,6,13,18)	(3,6,9,14; 2,5,10,16)	(5,8,10,14; 3,6,12,16)	

Using the proposed method, the intuitionistic fuzzy optimal solution of intuitionistic fully fuzzy transportation problem, presented by Table 5.1, can be obtained as follows:

Step 1: The intuitionistic linear programming problem of intuitionistic fully fuzzy transportation problem, presented by Table 5.1, can be transformed into the problem (P5.6).

$$
\begin{aligned}
&\text{Minimize}\Big((2,5,7,10;1,4,8,12)\otimes(x_{111},x_{112},x_{113},x_{114};x'_{111},x'_{112},x'_{113},x'_{114})\oplus(3,6,9,12;2,5,10,14)\\
&\quad\otimes(x_{121},x_{122},x_{123},x_{124};x'_{121},x'_{122},x'_{123},x'_{124})\oplus(3,6,9,11;1,5,10,13)\otimes(x_{131},x_{132},x_{133},x_{134};x'_{131},\\
&\quad x'_{132},x'_{133},x'_{134})\oplus(4,6,8,11;3,5,9,12)\otimes(x_{141},x_{142},x_{143},x_{144};x'_{141},x'_{142},x'_{143},x'_{144})\oplus(4,7,9,13;\\
&\quad 3,6,11,15)\otimes(x_{211},x_{212},x_{213},x_{214};x'_{211},x'_{212},x'_{213},x'_{214})\oplus(2,6,10,14;1,4,12,16)\otimes(x_{221},x_{222},\\
&\quad x_{223},x_{224};x'_{221},x'_{222},x'_{223},x'_{224})\oplus(3,7,10,14;2,5,12,15)\otimes(x_{231},x_{232},x_{233},x_{234};x'_{231},x'_{232},x'_{233},\\
&\quad x'_{234})\oplus(4,7,10,13;1,5,12,15)\otimes(x_{241},x_{242},x_{243},x_{244};x'_{241},x'_{242},x'_{243},x'_{244})\oplus(4,8,10,12;2,6,\\
&\quad 11,14)\otimes(x_{311},x_{312},x_{313},x_{314};x'_{311},x'_{312},x'_{313},x'_{314})\oplus(3,5,8,10;2,4,9,11)\otimes(x_{321},x_{322},x_{323},x_{324};\\
&\quad x'_{321},x'_{322},x'_{323},x'_{324})\oplus(4,7,11,13;3,5,12,14)\otimes(x_{331},x_{332},x_{333},x_{334};x'_{331},x'_{332},x'_{333},x'_{334})\oplus\\
&\quad (3,6,9,11;1,4,10,14)\otimes(x_{341},x_{342},x_{343},x_{344};x'_{341},x'_{342},x'_{343},x'_{344})\Big)\\
&\text{Subject to} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\text{(P5.6)}\\
&\quad (x_{111},x_{112},x_{113},x_{114};x'_{111},x'_{112},x'_{113},x'_{114})\oplus(x_{121},x_{122},x_{123},x_{124};x'_{121},x'_{122},x'_{123},x'_{124})\oplus(x_{131},x_{132},x_{133},\\
&\quad x_{134};x'_{131},x'_{132},x'_{133},x'_{134})\oplus(x_{141},x_{142},x_{143},x_{144};x'_{141},x'_{142},x'_{143},x'_{144})\approx(4,6,10,14;2,5,11,15),\\
&\quad (x_{211},x_{212},x_{213},x_{214};x'_{211},x'_{212},x'_{213},x'_{214})\oplus(x_{221},x_{222},x_{223},x_{224},x'_{221};x'_{222},x'_{223},x'_{224})\oplus(x_{231},x_{232},x_{233},\\
&\quad x_{234};x'_{231},x'_{232},x'_{233},x'_{234})\oplus(x_{241},x_{242},x_{243},x_{244};x'_{241},x'_{242},x'_{243},x'_{244})\approx(5,9,12,16;4,8,14,18),\\
&\quad (x_{311},x_{312},x_{313},x_{314};x'_{311},x'_{312},x'_{313},x'_{314})\oplus(x_{321},x_{322},x_{323},x_{324};x'_{321},x'_{322},x'_{323},x'_{324})\oplus(x_{331},x_{332},x_{333},\\
&\quad x_{334};x'_{331},x'_{332},x'_{333},x'_{334})\oplus(x_{341},x_{342},x_{343},x_{344};x'_{341},x'_{342},x'_{343},x'_{344})\approx(8,15,20,27;6,11,23,33),\\
&\quad (x_{111},x_{112},x_{113},x_{114};x'_{111},x'_{112},x'_{113},x'_{114})\oplus(x_{211},x_{212},x_{213},x_{214};x'_{211},x'_{212},x'_{213},x'_{214})\oplus(x_{311},x_{312},x_{313},\\
&\qquad\qquad\qquad\qquad\qquad\qquad x_{314};x'_{311},x'_{312},x'_{313},x'_{314})\approx(5,8,11,15;4,7,13,16),\\
&\quad (x_{121},x_{122},x_{123},x_{124},x'_{121},x'_{122},x'_{123},x'_{124})\oplus(x_{221},x_{222},x_{223},x_{224},x'_{221},x'_{222},x'_{223},x'_{224})\oplus(x_{321},x_{322},x_{323},\\
&\qquad\qquad\qquad\qquad\qquad\qquad x_{324},x'_{321},x'_{322},x'_{323},x'_{324})\approx(4,8,12,14;\ 3,6,13,18),
\end{aligned}
$$

$$\left(x_{131},x_{132},x_{133},x_{134};x'_{131},x'_{132},x'_{133},x'_{134}\right)\oplus\left(x_{231},x_{232},x_{233},x_{234};x'_{231},x'_{232},x'_{233},x'_{234}\right)\oplus(x_{331},x_{332},x_{333},$$
$$x_{334};x'_{331},x'_{332},x'_{333},x'_{334})\approx\left(3,6,9,14;2,5,10,16\right),$$
$$\left(x_{141},x_{142},x_{143},x_{144};x'_{141},x'_{142},x'_{143},x'_{144}\right)\oplus\left(x_{241},x_{242},x_{243},x_{244};x'_{241},x'_{242},x'_{243},x'_{244}\right)\oplus(x_{341},x_{342},x_{343},$$
$$x_{344};x'_{341},x'_{342},x'_{343},x'_{344})\approx\left(5,8,10,14;3,6,12,16\right)$$
$$\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x'_{ij2},x'_{ij3},x'_{ij4}\right)\succeq(0,0,0,0;0,0,0,0);i=1,2,3;\ j=1,2,3,4.$$
$\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x'_{ij2},x'_{ij3},x'_{ij4}\right)$ is a non-negative trapezoidal intuitionistic fuzzy number.

Step 2: Using the arithmetic operations of trapezoidal intuitionistic fuzzy numbers, the problem (P5.6) can be transformed into the problem (P5.7).

$$\text{Minimize}\big(2x_{111}+3x_{121}+3x_{131}+4x_{141}+4x_{211}+2x_{221}+3x_{231}+4x_{241}+4x_{311}+3x_{321}+4x_{331}+3x_{341},$$
$$5x_{112}+6x_{122}+6x_{132}+6x_{142}+7x_{212}+6x_{222}+7x_{232}+7x_{242}+8x_{312}+5x_{322}+7x_{332}+6x_{342},$$
$$7x_{113}+9x_{123}+9x_{133}+8x_{143}+9x_{213}+10x_{223}+10x_{233}+10x_{243}+10x_{313}+8x_{323}+11x_{333}+9x_{343},$$
$$10x_{114}+12x_{124}+11x_{134}+11x_{144}+13x_{214}+14x_{224}+14x_{234}+13x_{244}+12x_{314}+10x_{324}+13x_{334}$$
$$+11x_{344};x'_{111}+2x'_{121}+x'_{131}+3x'_{141}+3x'_{211}+x'_{221}+2x'_{231}+x'_{241}+2x'_{311}+2x'_{321}+3x'_{331}+x'_{341},$$
$$4x'_{112}+5x'_{122}+5x_{132}+5x'_{142}+6x'_{212}+4x'_{222}+5x'_{232}+5x'_{242}+6x'_{312}+4x'_{322}+5x'_{332}+4x'_{342},$$
$$8x'_{113}+10x'_{123}+10x_{133}+9x'_{143}+11x'_{213}+12x'_{223}+12x'_{233}+12x'_{243}+11x'_{313}+9x'_{323}+12x'_{333}$$
$$+10x'_{343},12x'_{114}+14x'_{124}+13x'_{134}+12x'_{144}+15x'_{214}+16x'_{224}+15x'_{234}+15x'_{244}+14x'_{314}+11x'_{324}$$
$$+14x'_{334}+14x'_{344}\big)$$

Subject to (P5.7)

$$\big(x_{111}+x_{121}+x_{131}+x_{141},x_{112}+x_{122}+x_{132}+x_{142},x_{113}+x_{123}+x_{133}+x_{143},x_{114}+x_{124}+x_{134}+x_{144};$$
$$x'_{111}+x'_{121}+x'_{131}+x'_{141},x'_{112}+x'_{122}+x'_{132}++x'_{142},\ x'_{113}+x'_{123}+x'_{133}+x'_{143},x'_{114}+x'_{124}+x'_{134}+x'_{144}\big)$$
$$\approx\left(4,6,10,14;2,5,11,15\right),$$
$$\big(x_{211}+x_{221}+x_{231}+x_{241},x_{212}+x_{222}+x_{232}+x_{242},x_{213}+x_{223}+x_{233}+x_{243},x_{214}+x_{224}+x_{234}+x_{244},$$
$$x'_{211}+x'_{221}+x'_{231}+x'_{241},x'_{212}+x'_{222}+x'_{232}+x'_{242},x'_{213}+x'_{223}+x'_{233}+x'_{243},x'_{214}+x'_{224}+x'_{234}+x'_{244}\big)$$
$$\approx\left(5,9,12,16;4,8,14,18\right),$$
$$\big(x_{311}+x_{321}+x_{331}+x_{341},x_{312}+x_{322}+x_{332}+x_{342},x_{313}+x_{323}+x_{333}+x_{343},x_{314}+x_{324}+x_{334}+x_{344};$$
$$x'_{311}+x'_{321}+x'_{331}+x'_{341},x'_{312}+x'_{322}+x'_{332}+x'_{342},x'_{313}+x'_{323}+x'_{333}+x'_{343},x'_{314}+x'_{324}+x'_{334}+x'_{344}\big)$$
$$\approx\left(8,15,20,27;6,11,23,33\right),$$
$$\big(x_{111}+x_{211}+x_{311},x_{112}+x_{212}+x_{312},x_{113}+x_{213}+x_{313},x_{114}+x_{214}+x_{314};x'_{111}+x'_{211}+x'_{311},x'_{112}$$
$$+x'_{212}+x'_{312},x'_{113}+x'_{213}+x'_{313},x'_{114}+x'_{214}+x'_{314}\big)\approx\left(5,8,11,15;4,7,13,16\right),$$
$$\big(x_{121}+x_{221}+x_{321},x_{122}+x_{222}+x_{322},x_{123}+x_{223}+x_{323},x_{124}+x_{224}+x_{324};x'_{121}+x'_{221}+x'_{321},x'_{122}$$
$$+x'_{222}+x'_{322},x'_{123}+x'_{223}+x'_{323},x'_{124}+x'_{224}+x'_{324}\big)\approx\left(4,8,12,14;3,6,13,18\right),$$

$$\left(x_{131}+x_{231}+x_{331}, x_{132}+x_{232}+x_{332}, x_{133}+x_{233}+x_{333}, x_{134}+x_{234}+x_{334}; x'_{131}+x'_{231}+x'_{331}, x'_{132}\right.$$
$$\left.+x'_{232}+x'_{332}, x'_{133}+x'_{233}+x'_{333}, x'_{134}+x'_{234}+x'_{334}\right) \approx (3,6,9,14;2,5,10,16),$$
$$\left(x_{141}+x_{241}+x_{341}, x_{142}+x_{242}+x_{342}, x_{143}+x_{243}+x_{343}, x_{144}+x_{244}+x_{344}; x'_{141}+x'_{241}+x'_{341}, x'_{142}\right.$$
$$\left.+x'_{242}+x'_{342}, x'_{143}+x'_{243}+x'_{343}, x'_{144}+x'_{244}+x'_{344}\right) \approx (5,8,10,14;3,6,12,16)$$
$$\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x'_{ij2}, x'_{ij3}, x'_{ij4}\right) \succeq (0,0,0,0;0,0,0,0); i=1,2,3;\ j=1,2,3,4,$$
$\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x'_{ij2}, x'_{ij3}, x'_{ij4}\right)$ is a non-negative trapezoidal intuitionistic fuzzy number.

Step 3: Using Section 1.3.3.2, the problem (P5.7) can be transformed into the problem (P5.8).

Minimize $R\left(2x_{111}+3x_{121}+3x_{131}+4x_{141}+4x_{211}+2x_{221}+3x_{231}+4x_{241}+4x_{311}+3x_{321}+4x_{331}+3x_{341},\right.$
$$5x_{112}+6x_{122}+6x_{132}+6x_{142}+7x_{212}+6x_{222}+7x_{232}+7x_{242}+8x_{312}+5x_{322}+7x_{332}+6x_{342},$$
$$7x_{113}+9x_{123}+9x_{133}+8x_{143}+9x_{213}+10x_{223}+10x_{233}+10x_{243}+10x_{313}+8x_{323}+11x_{333}+9x_{343},$$
$$10x_{114}+12x_{124}+11x_{134}+11x_{144}+13x_{214}+14x_{224}+14x_{234}+13x_{244}+12x_{314}+10x_{324}+13x_{334}$$
$$+11x_{344}; x'_{111}+2x'_{121}+x'_{131}+3x'_{141}+3x'_{211}+x'_{221}+2x'_{231}+x'_{241}+2x'_{311}+2x'_{321}+3x'_{331}+x'_{341},$$
$$4x'_{112}+5x'_{122}+5x_{132}+5x'_{142}+6x'_{212}+4x'_{222}+5x'_{232}+5x'_{242}+6x'_{312}+4x'_{322}+5x'_{332}+4x'_{342},$$
$$8x'_{113}+10x'_{123}+10x_{133}+9x'_{143}+11x'_{213}+12x'_{223}+12x'_{233}+12x'_{243}+11x'_{313}+9x'_{323}+12x'_{333}$$
$$+10x'_{343}, 12x'_{114}+14x'_{124}+13x'_{134}+12x'_{144}+15x'_{214}+16x'_{224}+15x'_{234}+15x'_{244}+14x'_{314}+11x'_{324}$$
$$\left.+14x'_{334}+14x'_{344}\right)$$

Subject to (P5.8)

$$R\left(x_{111}+x_{121}+x_{131}+x_{141}, x_{112}+x_{122}+x_{132}+x_{142}, x_{113}+x_{123}+x_{133}+x_{143}, x_{114}+x_{124}+x_{134}+x_{144};\right.$$
$$\left.x'_{111}+x'_{121}+x'_{131}+x'_{141}, x'_{112}+x'_{122}+x'_{132}++x'_{142},\ x'_{113}+x'_{123}+x'_{133}+x'_{143}, x'_{114}+x'_{124}+x'_{134}+x'_{144}\right)$$
$$=R(4,6,10,14;2,5,11,15)$$
$$\left(x_{211}+x_{221}+x_{231}+x_{241}, x_{212}+x_{222}+x_{232}+x_{242}, x_{213}+x_{223}+x_{233}+x_{243}, x_{214}+x_{224}+x_{234}+x_{244},\right.$$
$$\left.x'_{211}+x'_{221}+x'_{231}+x'_{241}, x'_{212}+x'_{222}+x'_{232}+x'_{242}, x'_{213}+x'_{223}+x'_{233}+x'_{243}, x'_{214}+x'_{224}+x'_{234}+x'_{244}\right)$$
$$=R(5,9,12,16;4,8,14,18)$$
$$R\left(x_{311}+x_{321}+x_{331}+x_{341}, x_{312}+x_{322}+x_{332}+x_{342}, x_{313}+x_{323}+x_{333}+x_{343}, x_{314}+x_{324}+x_{334}+x_{344},\right.$$
$$\left.x'_{311}+x'_{321}+x'_{331}+x'_{341}, x'_{312}+x'_{322}+x'_{332}+x'_{342}, x'_{313}+x'_{323}+x'_{333}+x'_{343}, x'_{314}+x'_{324}+x'_{334}+x'_{344}\right)$$
$$=R(8,15,20,27;6,11,23,33)$$
$$R\left(x_{111}+x_{211}+x_{311}, x_{112}+x_{212}+x_{312}, x_{113}+x_{213}+x_{313}, x_{114}+x_{214}+x_{314}; x'_{111}+x'_{211}+x'_{311}, x'_{112}\right.$$
$$\left.+x'_{212}+x'_{312}, x'_{113}+x'_{213}+x'_{313}, x'_{114}+x'_{214}+x'_{314}\right)=R(5,8,11,15;4,7,13,16)$$
$$R\left(x_{121}+x_{221}+x_{321}, x_{122}+x_{222}+x_{322}, x_{123}+x_{223}+x_{323}, x_{124}+x_{224}+x_{324}; x'_{121}+x'_{221}+x'_{321}, x'_{122}\right.$$
$$\left.+x'_{222}+x'_{322}, x'_{123}+x'_{223}+x'_{323}, x'_{124}+x'_{224}+x'_{324}\right)=R(4,8,12,14;3,6,13,18)$$
$$R\left(x_{131}+x_{231}+x_{331}, x_{132}+x_{232}+x_{332}, x_{133}+x_{233}+x_{333}, x_{134}+x_{234}+x_{334}; x'_{131}+x'_{231}+x'_{331}, x'_{132}\right.$$
$$\left.+x'_{232}+x'_{332}, x'_{133}+x'_{233}+x'_{333}, x'_{134}+x'_{234}+x'_{334}\right)=R(3,6,9,14;2,5,10,16)$$

$$R\left(x_{141}+x_{241}+x_{341},x_{142}+x_{242}+x_{342},x_{143}+x_{243}+x_{343},x_{144}+x_{244}+x_{344};x'_{141}+x'_{241}+x'_{341},x'_{142}\right.$$
$$\left.+x'_{242}+x'_{342},x'_{143}+x'_{243}+x'_{343},x'_{144}+x'_{244}+x'_{344}\right)=R\left(5,8,10,14;3,6,12,16\right)$$
$$R\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x'_{ij2},x'_{ij3},x'_{ij4}\right)\geq R(0,0,0,0;0,0,0,0);i=1,2,3;\ j=1,2,3,4.$$

Step 4: Using the expression, $R\left(a,b,c,d;a',b',c',d'\right)=\dfrac{a+b+c+d+a'+b'+c'+d'}{8}$, the problem (P5.8) can be transformed into the problem (P5.9).

$$\text{Minimize}\left[\frac{1}{8}\left(2x_{111}+3x_{121}+3x_{131}+4x_{141}+4x_{211}+2x_{221}+3x_{231}+4x_{241}+4x_{311}+3x_{321}+4x_{331}+3x_{341}+\right.\right.$$
$$5x_{112}+6x_{122}+6x_{132}+6x_{142}+7x_{212}+6x_{222}+7x_{232}+7x_{242}+8x_{312}+5x_{322}+7x_{332}+6x_{342}+$$
$$7x_{113}+9x_{123}+9x_{133}+8x_{143}+9x_{213}+10x_{223}+10x_{233}+10x_{243}+10x_{313}+8x_{323}+11x_{333}+9x_{343}+$$
$$10x_{114}+12x_{124}+11x_{134}+11x_{144}+13x_{214}+14x_{224}+14x_{234}+13x_{244}+12x_{314}+10x_{324}+13x_{334}$$
$$+11x_{344}+x'_{111}+2x'_{121}+x'_{131}+3x'_{141}+3x'_{211}+x'_{221}+2x'_{231}+x'_{241}+2x'_{311}+2x'_{321}+3x'_{331}+x'_{341}+$$
$$4x'_{112}+5x'_{122}+5x_{132}+5x'_{142}+6x'_{212}+4x'_{222}+5x'_{232}+5x'_{242}+6x'_{312}+4x'_{322}+5x'_{332}+4x'_{342}+$$
$$8x'_{113}+10x'_{123}+10x_{133}+9x'_{143}+11x'_{213}+12x'_{223}+12x'_{233}+12x'_{243}+11x'_{313}+9x'_{323}+12x'_{333}$$
$$+10x'_{343}+12x'_{114}+14x'_{124}+13x'_{134}+12x'_{144}+15x'_{214}+16x'_{224}+15x'_{234}+15x'_{244}+14x'_{314}+11x'_{324}$$
$$\left.\left.+14x'_{334}+14x'_{344}\right)\right]$$

Subject to (P5.9)

$$x_{111}+x_{121}+x_{131}+x_{141}+x_{112}+x_{122}+x_{132}+x_{142}+x_{113}+x_{123}+x_{133}+x_{143}+x_{114}+x_{124}+x_{134}+x_{144}+x'_{111}$$
$$+x'_{121}+x'_{131}+x'_{141}+x'_{112}+x'_{122}+x'_{132}+x'_{142}+x'_{113}+x'_{123}+x'_{133}+x'_{143}+x'_{114}+x'_{124}+x'_{134}+x'_{144}=67,$$
$$x_{211}+x_{221}+x_{231}+x_{241}+x_{212}+x_{222}+x_{232}+x_{242}+x_{213}+x_{223}+x_{233}+x_{243}+x_{214}+x_{224}+x_{234}+x_{244}+x'_{211}$$
$$+x'_{221}+x'_{231}+x'_{241}+x'_{212}+x'_{222}+x'_{232}+x'_{242}+x'_{213}+x'_{223}+x'_{233}+x'_{243}+x'_{214}+x'_{224}+x'_{234}+x'_{244}=86,$$
$$x_{311}+x_{321}+x_{331}+x_{341}+x_{312}+x_{322}+x_{332}+x_{342}+x_{313}+x_{323}+x_{333}+x_{343}+x_{314}+x_{324}+x_{334}+x_{344}+x'_{311}$$
$$+x'_{321}+x'_{331}+x'_{341}+x'_{312}+x'_{322}+x'_{332}+x'_{342}+x'_{313}+x'_{323}+x'_{333}+x'_{343}+x'_{314}+x'_{324}+x'_{334}+x'_{344}=143,$$
$$x_{111}+x_{211}+x_{311}+x_{112}+x_{212}+x_{312}+x_{113}+x_{213}+x_{313}+x_{114}+x_{214}+x_{314}+x'_{111}+x'_{211}+x'_{311}+x'_{112}+x'_{212}$$
$$+x'_{312}+x'_{113}+x'_{213}+x'_{313}+x'_{114}+x'_{214}+x'_{314}=79,$$
$$x_{121}+x_{221}+x_{321}+x_{122}+x_{222}+x_{322}+x_{123}+x_{223}+x_{323}+x_{124}+x_{224}+x_{324}+x'_{121}+x'_{221}+x'_{321}+x'_{122}+x'_{222}$$
$$+x'_{322}+x'_{123}+x'_{223}+x'_{323}+x'_{124}+x'_{224}+x'_{324}=78,$$
$$x_{131}+x_{231}+x_{331}+x_{132}+x_{232}+x_{332}+x_{133}+x_{233}+x_{333}+x_{134}+x_{234}+x_{334}+x'_{131}+x'_{231}+x'_{331}+x'_{132}+x'_{232}$$
$$+x'_{332}+x'_{133}+x'_{233}+x'_{333}+x'_{134}+x'_{234}+x'_{334}=65,$$
$$x_{141}+x_{241}+x_{341}+x_{142}+x_{242}+x_{342}+x_{143}+x_{243}+x_{343}+x_{144}+x_{244}+x_{344}+x'_{141}+x'_{241}+x'_{341}+x'_{142}$$
$$+x'_{242}+x'_{342}+x'_{143}+x'_{243}+x'_{343}+x'_{144}+x'_{244}+x'_{344}=74,$$
$$x_{ijk},x'_{ijk}\geq 0;i=1,2,3;\ j=1,2,3,4;k=1,2,3,4.$$

Step 5: The optimal solution of the linear programming problem (P5.9), is

$$x_{111}=\frac{67}{8},\ x_{112}=\frac{67}{8},\ x_{113}=\frac{67}{8},\ x_{114}=\frac{67}{8},\ x'_{111}=\frac{67}{8},\ x'_{112}=\frac{67}{8},\ x'_{113}=\frac{67}{8},\ x'_{114}=\frac{67}{8}, x_{211}=\frac{3}{2},$$

$$x_{212}=\frac{3}{2}, x_{213}=\frac{3}{2}, x_{214}=\frac{3}{2}, x'_{211}=\frac{3}{2}, x'_{212}=\frac{3}{2}, x'_{213}=\frac{3}{2}, x'_{214}=\frac{3}{2}, x_{231}=\frac{65}{8}, x_{232}=\frac{65}{8}, x_{233}=\frac{65}{8},$$

$$x_{234}=\frac{65}{8}, x'_{231}=\frac{65}{8}, x'_{232}=\frac{65}{8}, x'_{233}=\frac{65}{8}, x'_{234}=\frac{65}{8}, x'_{241}=\frac{9}{4}, x'_{242}=\frac{9}{4}, x'_{243}=\frac{9}{4}, x'_{244}=\frac{9}{4}, x_{321}=\frac{39}{4},$$

$$x_{322}=\frac{39}{4},\ x_{323}=\frac{39}{4},\ x_{324}=\frac{39}{4}, x'_{321}=\frac{39}{4},\ x'_{322}=\frac{39}{4},\ x'_{323}=\frac{39}{4},\ x'_{324}=\frac{39}{4}, x_{341}=\frac{65}{8}, x_{342}=\frac{65}{8},$$

$$x_{343}=\frac{65}{8}, x_{344}=\frac{65}{8}, x'_{341}=\frac{65}{8}, x'_{342}=\frac{65}{8}, x'_{343}=\frac{65}{8}, x'_{344}=\frac{65}{8}.$$

Step 5: Using the optimal solution, obtained in Step 4, the obtained intuitionistic fuzzy optimal solution is

$$\tilde{x}^I_{11}=\left(\frac{67}{8},\frac{67}{8},\frac{67}{8},\frac{67}{8};\frac{67}{8},\frac{67}{8},\frac{67}{8},\frac{67}{8}\right),\quad \tilde{x}^I_{21}=\left(\frac{3}{2},\frac{3}{2},\frac{3}{2},\frac{3}{2};\frac{3}{2},\frac{3}{2},\frac{3}{2},\frac{3}{2}\right),\quad \tilde{x}^I_{23}=\left(\frac{65}{8},\frac{65}{8},\frac{65}{8},\frac{65}{8};\right.$$

$$\left.\frac{65}{8},\frac{65}{8},\frac{65}{8},\frac{65}{8}\right),\quad \tilde{x}^I_{24}=\left(0,0,0,0;\frac{9}{4},\frac{9}{4},\frac{9}{4},\frac{9}{4}\right),\quad \tilde{x}^I_{32}=\left(\frac{39}{4},\frac{39}{4},\frac{39}{4},\frac{39}{4};\frac{39}{4},\frac{39}{4},\frac{39}{4},\frac{39}{4}\right)\quad \text{and}$$

$$\tilde{x}^I_{34}=\left(\frac{65}{8},\frac{65}{8},\frac{65}{8},\frac{65}{8};\frac{65}{8},\frac{65}{8},\frac{65}{8},\frac{65}{8}\right).$$

Step 6: Using the intuitionistic fuzzy optimal solution, obtained in Step 5, the optimal intuitionistic fuzzy transportation cost of the problem, presented by Table 5.1, is

$$(2,5,7,10;1,4,8,12)\otimes\left(\frac{67}{8},\frac{67}{8},\frac{67}{8},\frac{67}{8};\frac{67}{8},\frac{67}{8},\frac{67}{8},\frac{67}{8}\right)\oplus(4,7,9,13;3,6,11,15)\otimes\left(\frac{3}{2},\frac{3}{2},\frac{3}{2},\frac{3}{2};\right.$$

$$\left.\frac{3}{2},\frac{3}{2},\frac{3}{2},\frac{3}{2}\right)\oplus(3,7,10,14;2,5,12,15)\otimes\left(\frac{65}{8},\frac{65}{8},\frac{65}{8},\frac{65}{8};\frac{65}{8},\frac{65}{8},\frac{65}{8},\frac{65}{8};\right)\oplus(4,7,10,13;1,5,12,15)$$

$$\otimes\left(0,0,0,0;\frac{9}{4},\frac{9}{4},\frac{9}{4},\frac{9}{4}\right)\oplus(3,5,8,10;2,4,9,11)\otimes\left(\frac{39}{4},\frac{39}{4},\frac{39}{4},\frac{39}{4};\frac{39}{4},\frac{39}{4},\frac{39}{4},\frac{39}{4}\right)\oplus(3,6,9,11;1,$$

$$4,10,14)\otimes\left(\frac{65}{8},\frac{65}{8},\frac{65}{8},\frac{65}{8};\frac{65}{8},\frac{65}{8},\frac{65}{8},\frac{65}{8}\right)=\left(\frac{403}{4},\frac{1855}{8},\frac{609}{2},\frac{3231}{8};59,\frac{1327}{8},377,\frac{3997}{8}\right).$$

5.4 Conclusion

It is shown that in the existing methods [18,29,45,49,112,123,133,134,138], some mathematical incorrect assumptions are used. Hence, it is not genuine to use the existing methods [18,29,45,49, 112,118,123,133,134,138]. Also, a new method is proposed for solving intuitionistic fully fuzzy transportation problems.

Chapter 6

A new method for solving generalized intuitionistic fully fuzzy transportation problems*

Chakraborty et al. [18] proposed the arithmetic operations on generalized trapezoidal intuitionistic fuzzy numbers and used these arithmetic operations to find the solution of such intuitionistic fully fuzzy transportation problems in which cost, availability and demand all are represented by generalized trapezoidal intuitionistic fuzzy numbers. In this chapter, it is shown that Chakraborty et al. [18] have used the property $R\left(\tilde{A}^I \otimes \tilde{B}^I\right) = R\left(\tilde{A}^I\right) \times R\left(\tilde{B}^I\right)$ in their proposed method. While, for the ranking function *R*, considered by Chakraborty et al. [18], this property is not satisfying. Hence, it is not genuine to use the method, proposed by Chakraborty et al. [18], to find the solution of generalized intuitionistic fully fuzzy transportation problem. Furthermore, a new method is proposed to resolve the flaws of the existing method [18]. To illustrate the proposed method, the generalized intuitionistic fully fuzzy transportation problem, considered by Chakraborty et al. [18], is solved by proposed method.

6.1 Generalized intuitionistic fuzzy numbers

In this section, some basic definitions (generalized intuitionistic fuzzy numbers, generalized trapezoidal intuitionistic fuzzy numbers etc.), arithmetic operations of generalized trapezoidal intuitionistic fuzzy numbers and a method for ordering of generalized trapezoidal intuitionistic fuzzy numbers [18], are presented [18].

* The contents of this chapter are to be communicated in Applied Soft Computing.

6.1.1 Some basic definitions

In this section, some basic definitions are presented [18].

Definition 6.1 An intuitionistic fuzzy number $\tilde{A}^I = \{\langle x, \mu_{\tilde{A}^I}, \nu_{\tilde{A}^I} \rangle\}$ of the real line $\mathbb{R}$ is called generalized intuitionistic fuzzy number, if the following holds,

(i) there exist $m \in \mathbb{R}$, $\mu_{\tilde{A}^I}(m) = w, \nu_{\tilde{A}^I}(m) = 0, 0 < w \le 1$.

(ii) $\mu_{\tilde{A}^I}$ is continuous mapping from $\mathbb{R}$ to the interval (0, w] and $x \in \mathbb{R}$, the relation $0 \le \mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \le 1$ holds.

The membership function and non – membership function of $\tilde{A}^I$ is of the following form,

$$\mu_{\tilde{A}^I}(x) = \begin{cases} wf_1(x) & ; \quad m - \alpha \le x < m, \\ w & ; \quad x = m, \\ wh_1(x) & ; \quad m < x \le m + \beta \\ 0 & ; \quad \text{otherwise} \end{cases}$$

$$\nu_{\tilde{A}^I}(x) = \begin{cases} wf_2(x) & ; \quad m - \alpha' \le x < m, \ 0 < w(f_1(x) + f_2(x)) \le w; \\ 0 & ; \quad x = m, \\ wh_2(x) & ; \quad m < x \le m + \beta', \ 0 < w(h_1(x) + h_2(x)) \le w; \\ w & ; \quad \text{otherwise.} \end{cases}$$

where $f_1(x)$ and $h_1(x)$ are strictly increasing and decreasing function in $[m - \alpha, m]$ and $[m, m + \beta]$ respectively and where $f_2(x)$ and $h_2(x)$ are strictly decreasing and increasing function in $[m - \alpha', m]$ and $[m, m + \beta']$ respectively. α and β are called left and right spreads of membership function $\mu_{\tilde{A}^I}(x)$ respectively. α' and β' are called left and right spreads of non-membership function $\nu_{\tilde{A}^I}(x)$ respectively.

Definition 6.2 A generalized intuitionistic fuzzy number $\tilde{A}^I = \left(a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4'; w\right)$ is said to be generalized intuitionistic trapezoidal fuzzy number if its membership function $\mu_{\tilde{A}^I}(x)$ and non-membership function $\nu_{\tilde{A}^I}(x)$ is given by

$$\mu_{\tilde{A}^I}(x) = \begin{cases} w\dfrac{x-a_1}{a_2-a_1} & ;\ a_1 \le x < a_2, \\ w & ;\ a_2 \le x \le a_3, \\ w\dfrac{a_4-x}{a_4-a_3} & ;\ a_3 < x \le a_4, \\ 0 & ;\ \text{otherwise.} \end{cases} \quad \text{and} \quad \nu_{\tilde{A}^I}(x) = \begin{cases} w\dfrac{a_2-x}{a_2-a_1'} & ;\ a_1' \le x < a_2, \\ 0 & ;\ a_2 \le x \le a_3 \\ w\dfrac{x-a_3}{a_4'-a_3} & ;\ a_3 < x \le a_4' \\ w & ;\ \text{otherwise} \end{cases}$$

Definition 6.3 A generalized trapezoidal intuitionistic fuzzy number $\tilde{A}^I = (a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4'; w)$ is said to be non-negative if and only if $a_1' \ge 0$.

Definition 6.4 A generalized trapezoidal intuitionistic fuzzy number $\tilde{A}^I = (a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4'; w_1)$ and $\tilde{B}^I = \left(b_1, b_2, b_3, b_4; b_1', b_2, b_3, b_4'; w_2\right)$ are said to be equal if and only if $a_1 = b_1, a_2 = b_2,$ $a_3 = b_3, a_4 = b_4, a_1' = b_1', a_4' = b_4'$ and $w_1 = w_2$.

6.1.2 Arithmetic operations of generalized trapezoidal intuitionistic fuzzy numbers

Chakraborty et al. [18] proposed the following arithmetic operations for generalized trapezoidal intuitionistic fuzzy numbers.

Let $\tilde{A}^I = \left(a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4'; w_1\right)$ and $\tilde{B}^I = \left(b_1, b_2, b_3, b_4; b_1', b_2, b_3, b_4'; w_2\right)$ be two generalized trapezoidal intuitionistic fuzzy numbers. Then,

(i) $\tilde{A}^I \oplus \tilde{B}^I = \left(a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4; a_1'+b', a_2+b_2, a_3+b_3, a_4'+b_4'; w\right);\ w = \min\{w_1, w_2\}$,

(ii) $\tilde{A}^I \ominus \tilde{B}^I = \left(a_1-b_4, a_2-b_3, a_3-b_2, a_4-b_1; a_1'-b_4', a_2-b_3, a_3-b_2, a_4'-b_1'; w\right);\ w = \min\{w_1, w_2\}$,

(iii) $\lambda\tilde{A}^I = \begin{cases} (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4; \lambda a_1', \lambda a_2, \lambda a_3, \lambda a_4'; w_1); & \lambda \ge 0, \\ (\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1; \lambda a_4', \lambda a_3, \lambda a_2, \lambda a_1'; w_1); & \lambda < 0, \end{cases}$

(iv) $\tilde{A}^I \otimes \tilde{B}^I = (a_1b_1, a_2b_2, a_3b_3, a_4b_4; a_1'b_1', a_2b_2, a_3b_3, a_4'b_4'; w)$, if $a_1' \ge 0$ and $b_1' \ge 0$; $w =$

$\min\{w_1, w_2\}$.

6.1.3 Ordering of generalized trapezoidal intuitionistic fuzzy numbers

Chakraborty et al. [18] proposed the following method for comparing two generalized trapezoidal intuitionistic fuzzy numbers,

Let $\tilde{A}^I = (a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4'; w_1)$ and $\tilde{B}^I = (b_1, b_2, b_3, b_4; b_1', b_2, b_3, b_4'; w_2)$ be two generalized trapezoidal intuitionistic fuzzy numbers. Then, the ordering of $\tilde{A}^I$ and $\tilde{B}^I$ are as follows:

(i) $\tilde{A}^I \succeq \tilde{B}^I$ if $R(\tilde{A}^I) \ge R(\tilde{B}^I)$

(ii) $\tilde{A}^I \approx \tilde{B}^I$ if $R(\tilde{A}^I) = R(\tilde{B}^I)$

where,

$$R(\tilde{A}^I) = \frac{w_1(a_1 + a_1' + 2a_2 + 2a_3 + a_4 + a_4')}{8} \text{ and } R(\tilde{B}^I) = \frac{w_2(b_1 + b_1' + 2b_2 + 2b_3 + b_4 + b_4')}{8}.$$

6.2 Existing mathematical formulation of generalized intuitionistic fully fuzzy transportation problem

Chakraborty et al. [18] claimed that problem (P6.1) represents the mathematical formulation of generalized intuitionistic fully fuzzy transportation problems.

$$\text{Minimize} \sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij1},c_{ij2},c_{ij3},c_{ij4};c'_{ij1},c_{ij2},c_{ij3},c'_{ij4};w_{ij}\right)x_{ij}$$

Subject to (P6.1)

$$\sum_{j=1}^{n} x_{ij} \approx \left(a_{i1},a_{i2},a_{i3},a_{i4};a'_{i1},a_{i2},a_{i3},a'_{i4};w_i\right); i=1,2,...,m,$$

$$\sum_{i=1}^{m} x_{ij} \approx \left(b_{j1},b_{j2},b_{j3},b_{j4};b'_{j1},b_{j2},b_{j3},b'_{j4};w_j\right); j=1,2,...,n,$$

$$x_{ij} \geq 0; i=1,2,...,m;\ j=1,2,...,n.$$

This mathematical formulation is obtained as follows:

Step 1: On replacing the crisp parameters c_{ij}, x_{ij}, a_i and b_j of crisp linear programming problem (P2.1) of transportation problem with the generalized trapezoidal intuitionistic fuzzy numbers $\left(c_{ij1},c_{ij2},c_{ij3},c_{ij4};c'_{ij1},c_{ij2},c_{ij3},c'_{ij4};w_{ij}\right)$, $(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x_{ij2},x_{ij3},x'_{ij4};w_{ij})$, $(a_{i1},a_{i2},a_{i3},a_{i4};a'_{i1},a_{i2},a_{i3},a'_{i4};w_i)$ and $\left(b_{j1},b_{j2},b_{j3},b_{j4};b'_{j1},b_{j2},b_{j3},b'_{j4};w_j\right)$ respectively, it is transformed into the problem (P6.2).

$$\text{Minimize} \sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij1},c_{ij2},c_{ij3},c_{ij4};c'_{ij1},c_{ij2},c_{ij3},c'_{ij4};w_{ij}\right)\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x_{ij2},x_{ij3},x'_{ij4};w_{ij}\right)$$

Subject to (P6.2)

$$\sum_{j=1}^{n}\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x_{ij2},x_{ij3},x'_{ij4};w_{ij}\right) \approx \left(a_{i1},a_{i2},a_{i3},a_{i4};a'_{i1},a_{i2},a_{i3},a'_{i4};w_i\right); i=1,2,...,m,$$

$$\sum_{i=1}^{m}\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x_{ij2},x_{ij3},x'_{ij4};w_{ij}\right) \approx \left(b_{j1},b_{j2},b_{j3},b_{j4};b'_{j1},b_{j2},b_{j3},b'_{j4};w_j\right); j=1,2,...,n,$$

$$\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x_{ij2},x_{ij3},x'_{ij4};w_{ij}\right) \succeq (0,0,0,0;0,0,0,0;w_{ij}); i=1,2,...,m;\ j=1,2,...,n,$$

$\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x_{ij2},x_{ij3},x'_{ij4};w_{ij}\right)$ is a non-negative generalized trapezoidal intuitioni – stic fuzzy number.

Step 2: Replacing the generalized trapezoidal intuitionistic fuzzy numbers $\left(c_{ij1},c_{ij2},c_{ij3},c_{ij4};c'_{ij1},c_{ij2},c_{ij3},c'_{ij4};w_{ij}\right)$, $(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x_{ij2},x_{ij3},x'_{ij4};w_{ij})$, $(a_{i1},a_{i2},a_{i3},a_{i4};a'_{i1}, a_{i2},a_{i3},a'_{i4};w_i)$ and

$\left(b_{j1}, b_{j2}, b_{j3}, b_{j4}; b'_{j1}, b_{j2}, b_{j3}, b'_{j4}; w_j\right)$ with $\left(c_{ij1}, c_{ij2}, c_{ij3}, c_{ij4}; c'_{ij1}, c_{ij2}, c_{ij3}, c'_{ij4}; w\right)$, $(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w)$, $(a_{i1}, a_{i2}, a_{i3}, a_{i4}; a'_{i1}, a_{i2}, a_{i3}, a'_{i4}; w)$ and $(b_{j1}, b_{j2}, b_{j3}, b_{j4}; b'_{j1}, b_{j2}, b_{j3}, b'_{j4}; w)$ respectively, where $w = \min\left\{w_{ij}, w_i, w_j; i = 1, 2, ..., m; j = 1, 2, ..., n\right\}$, the problem (P6.2) is transformed into problem (P6.3).

$$\text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij1}, c_{ij2}, c_{ij3}, c_{ij4}; c'_{ij1}, c_{ij2}, c_{ij3}, c'_{ij4}; w\right)\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right)$$

Subject to (P6.3)

$$\sum_{j=1}^{n}\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right) \approx \left(a_{i1}, a_{i2}, a_{i3}, a_{i4}; a'_{i1}, a_{i2}, a_{i3}, a'_{i4}; w\right); i = 1, 2, ..., m,$$

$$\sum_{i=1}^{m}\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right) \approx \left(b_{j1}, b_{j2}, b_{j3}, b_{j4}; b'_{j1}, b_{j2}, b_{j3}, b'_{j4}; w\right); j = 1, 2, ..., n,$$

$$\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right) \succeq (0, 0, 0, 0; 0, 0, 0, 0; w); i = 1, 2, ..., m; j = 1, 2, ..., n,$$

$\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right)$ is a non-negative generalized trapezoidal intuit – ionistic fuzzy number.

Step 3: Using Section 6.1.3, the problem (P6.3) can be transformed into the problem (P6.4).

$$\text{Minimize } R\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij1}, c_{ij2}, c_{ij3}, c_{ij4}; c'_{ij1}, c_{ij2}, c_{ij3}, c'_{ij4}; w\right)\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right)\right)$$

Subject to (P6.4)

$$R\left(\sum_{j=1}^{n}\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right)\right) = R\left(a_{i1}, a_{i2}, a_{i3}, a_{i4}; a'_{i1}, a_{i2}, a_{i3}, a'_{i4}; w\right); i = 1, 2, ..., m,$$

$$R\left(\sum_{i=1}^{m}\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right)\right) = R\left(b_{j1}, b_{j2}, b_{j3}, b_{j4}; b'_{j1}, b_{j2}, b_{j3}, b'_{j4}; w\right); j = 1, 2, ..., n,$$

$$R\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right) \geq R(0, 0, 0, 0; 0, 0, 0, 0); i = 1, 2, ..., m; j = 1, 2, ..., n,$$

$\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right)$ is a non-negative generalized trapezoidal intuit-ionistic fuzzy number.

Step 4: Using the relation, $R\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(a_{ij}, b_{ij}, c_{ij}, d_{ij}; a'_{ij}, b_{ij}, c_{ij}, d'_{ij}; w\right)\right) = \sum_{i=1}^{m}\sum_{j=1}^{n} R(a_{ij}, b_{ij}, c_{ij}, d_{ij}; a'_{ij}, b_{ij},$

$c_{ij}, d'_{ij}; w)$, the problem (P6.4) can be transformed into the problem (P6.5).

$$\text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n} R\left(\left(c_{ij1}, c_{ij2}, c_{ij3}, c_{ij4}; c'_{ij1}, c_{ij2}, c_{ij3}, c'_{ij4}; w\right)\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right)\right)$$

Subject to (P6.5)

$$\sum_{j=1}^{n} R\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right) = R\left(a_{i1}, a_{i2}, a_{i3}, a_{i4}; a'_{i1}, a_{i2}, a_{i3}, a'_{i4}; w\right); i = 1, 2, ..., m,$$

$$\sum_{i=1}^{m} R\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right) = R\left(b_{j1}, b_{j2}, b_{j3}, b_{j4}; b'_{j1}, b_{j2}, b_{j3}, b'_{j4}; w\right); j = 1, 2, ..., n,$$

$$R\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right) \geq R(0,0,0,0;0,0,0,0); i = 1, 2, ..., m; j = 1, 2, ..., n,$$

$\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right)$ is a non-negative generalized trapezoidal intuitionistic fuzzy number.

Step 5: The problem (P6.5) can be transformed into the problem (P6.6).

$$\text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n} R\left(c_{ij1}, c_{ij2}, c_{ij3}, c_{ij4}; c'_{ij1}, c_{ij2}, c_{ij3}, c'_{ij4}; w\right) R\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right)$$

Subject to (P6.6)

Constraints of problem (P6.5).

Step 6: Since, $R\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right)$ will always be a real number. So, assuming $R\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right) = x_{ij}$, the problem (P6.6) can be transformed into the problem (P6.7).

$$\text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n} R\left(c_{ij1}, c_{ij2}, c_{ij3}, c_{ij4}; c'_{ij1}, c_{ij2}, c_{ij3}, c'_{ij4}; w\right) x_{ij}$$

Subject to (P6.7)

$$\sum_{j=1}^{n} x_{ij} = R\left(a_{i1}, a_{i2}, a_{i3}, a_{i4}; a'_{i1}, a_{i2}, a_{i3}, a'_{i4}; w\right); i = 1, 2, ..., m,$$

$$\sum_{i=1}^{m} x_{ij} = R\left(b_{j1}, b_{j2}, b_{j3}, b_{j4}; b'_{j1}, b_{j2}, b_{j3}, b'_{j4}; w\right); j = 1, 2, ..., n,$$

$$x_{ij} \geq 0; i = 1, 2, ..., m; j = 1, 2, ..., n.$$

6.3 Flaws in existing mathematical formulation

It is obvious from Step 4 of Section 6.2 that Chakraborty et al. [18] have assumed the property $R\left(\tilde{A}^I \otimes \tilde{B}^I\right) = R\left(\tilde{A}^I\right) \otimes R\left(\tilde{B}^I\right)$ for transforming problem (P6.5) into problem (P6.6). However, for the ranking function R, considered by Chakraborty et al. [18], this property is not satisfying. For example, if $\tilde{A}^I = (2,4,5,6;1,4,5,7;0.2)$ and $\tilde{B}^I = (4,6,7,8;3,6,7,9;0.2)$ are two generalized trapezoidal intuitionistic fuzzy numbers then $\tilde{A}^I \otimes \tilde{B}^I = (8,24,35,48;3,24,35,63;0.2)$ and hence $R\left(\tilde{A}^I \otimes \tilde{B}^I\right) = 6$ while, $R\left(\tilde{A}^I\right) \otimes R\left(\tilde{B}^I\right) = 1$. It is obvious that $R\left(\tilde{A}^I \otimes \tilde{B}^I\right) \neq R\left(\tilde{A}^I\right) \times R\left(\tilde{B}^I\right)$. Therefore, it is not genuine to use existing method [18].

6.4 Proposed method

In this section, a new method to find the optimal solution of generalized intuitionistic fully fuzzy transportation problem is proposed.

The steps of proposed method are as follows:

Step 1: Using the arithmetic operations of generalized trapezoidal intuitionistic fuzzy numbers, discussed in Section 6.1.2, the problem (P6.3) can be transformed into the problem (P6.8).

$$\text{Minimize} \sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij1}x_{ij1}, c_{ij2}x_{ij2}, c_{ij3}x_{ij3}, c_{ij4}x_{ij4}; c'_{ij1}x'_{ij1}, c_{ij2}x_{ij2}, c_{ij3}x_{ij3}, c'_{ij4}x'_{ij4}; w\right)$$

Subject to (P6.8)

$$\sum_{j=1}^{n}\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right) \approx \left(a_{i1}, a_{i2}, a_{i3}, a_{i4}; a'_{i1}, a_{i2}, a_{i3}, a'_{i4}; w\right); i = 1,2,...,m,$$

$$\sum_{i=1}^{m}\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right) \approx \left(b_{j1}, b_{j2}, b_{j3}, b_{j4}; b'_{j1}, b_{j2}, b_{j3}, b'_{j4}; w\right); j = 1,2,...,n,$$

$$\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right) \succeq (0,0,0,0;0,0,0,0); i = 1,2,...,m; j = 1,2,...,n,$$

$\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right)$ is non-negative trapezoidal intuitionistic fuzzy number.

Step 2: Using the ranking function of generalized trapezoidal intuitionistic fuzzy numbers, discussed in Section 6.1.3, the problem (P6.8) can be transformed into the problem (P6.9).

$$\text{Minimize } R\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij1}x_{ij1},c_{ij2}x_{ij2},c_{ij3}x_{ij3},c_{ij4}x_{ij4};c'_{ij1}x'_{ij1},c_{ij2}x_{ij2},c_{ij3}x_{ij3},c'_{ij4}x'_{ij4};w\right)\right)$$

Subject to (P6.9)

$$R\left(\sum_{j=1}^{n}\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x_{ij2},x_{ij3},x'_{ij4};w\right)\right)=R\left(a_{i1},a_{i2},a_{i3},a_{i4};a'_{i1},a_{i2},a_{i3},a'_{i4};w\right);i=1,2,...,m,$$

$$R\left(\sum_{i=1}^{m}\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x_{ij2},x_{ij3},x'_{ij4};w\right)\right)=R\left(b_{j1},b_{j2},b_{j3},b_{j4};b'_{j1},b_{j2},b_{j3},b'_{j4};w\right);\ j=1,2,...,n,$$

$$R\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x_{ij2},x_{ij3},x'_{ij4};w\right)\geq R(0,0,0,0;0,0,0,0),$$

$$x'_{ij1}\geq 0, i=1,2,...,m;\ j=1,2,...,n.$$

Step 3: Using the relation, $R\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(a_{ij},b_{ij},c_{ij},d_{ij};a'_{ij},b'_{ij},c'_{ij},d'_{ij};w\right)\right)=\sum_{i=1}^{m}\sum_{j=1}^{n}R(a_{ij},b_{ij},c_{ij},d_{ij};a'_{ij},b'_{ij},$ $c'_{ij},d'_{ij};w)$, the problem (P6.9) can be transformed into the problem (P6.10).

$$\text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n}R\left(c_{ij1}x_{ij1},c_{ij2}x_{ij2},c_{ij3}x_{ij3},c_{ij4}x_{ij4};c'_{ij1}x'_{ij1},c_{ij2}x_{ij2},c_{ij3}x_{ij3},c'_{ij4}x'_{ij4};w\right)$$

Subject to (P6.10)

$$\sum_{j=1}^{n}R\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x_{ij2},x_{ij3},x'_{ij4};w\right)=R\left(a_{i1},a_{i2},a_{i3},a_{i4};a'_{i1},a_{i2},a_{i3},a'_{i4};w\right);i=1,2,...,m,$$

$$\sum_{i=1}^{m}R\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x_{ij2},x_{ij3},x'_{ij4};w\right)=R\left(b_{j1},b_{j2},b_{j3},b_{j4};b'_{j1},b_{j2},b_{j3},b'_{j4};w\right);\ j=1,2,...,n,$$

$$R\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x_{ij2},x_{ij3},x'_{ij4};w\right)\geq R(0,0,0,0;0,0,0,0),$$

$$x'_{ij1}\geq 0; i=1,2,...,m;\ j=1,2,...,n.$$

Step 4: Using the expression, $R\left(a,b,c,d;a',b,c,d';w\right)=\dfrac{w\left(a+a'+2b+2c+d+d'\right)}{8}$, the problem (P6.10) can be transformed into the problem (P6.11).

$$\text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{w\left(c_{ij1}x_{ij1}+c'_{ij1}x'_{ij1}+2c_{ij2}x_{ij2}+2c_{ij3}x_{ij3}+c_{ij4}x_{ij4}+c'_{ij4}x'_{ij4}\right)}{8}$$

Subject to (P6.11)

$$\sum_{j=1}^{n}\frac{w\left(x_{ij1}+x'_{ij1}+2x_{ij2}+2x_{ij3}+x_{ij4}+x'_{ij4}\right)}{8}=\frac{w\left(a_{i1}+a'_{i1}+2a_{i2}+2a_{i3}+a_{i4}+a'_{i4}\right)}{8}; i=1,2,...,m,$$

$$\sum_{i=1}^{m}\frac{w\left(x_{ij1}+x'_{ij1}+2x_{ij2}+2x_{ij3}+x_{ij4}+x'_{ij4}\right)}{8}=\frac{w\left(b_{j1}+b'_{j1}+2b_{j2}+2b_{j3}+b_{j4}+b'_{j4}\right)}{8}; j=1,2,...,n,$$

$$x_{ijk}, x'_{ijp} \geq 0; i=1,2,...,m; j=1,2,...,n; k=1,2,3,4; p=1,4.$$

Step 5: Find the solution of linear programming problem (P6.11).

Step 6: Using the optimal solution, obtained in Step 5, the generalized intuitionistic fuzzy optimal solution of problem (P6.2) is $\left\{\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right)\right\}$.

Step 7: The minimum generalized intuitionistic fuzzy transportation cost of generalized intuitionistic fully fuzzy transportation problem (P6.2) is

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij1}, c_{ij2}, c_{ij3}, c_{ij4}; c'_{ij1}, c_{ij2}, c_{ij3}, c'_{ij4}; w\right)\left(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; x'_{ij1}, x_{ij2}, x_{ij3}, x'_{ij4}; w\right).$$

6.5 Illustrative example

Chakraborty et al. [18] solved the generalized intuitionistic fully fuzzy transportation problem, presented by Table 6.1, to illustrate their proposed method. In this section, the generalized intuitionistic fuzzy optimal solution of the same problem is obtained by the proposed method.

Table 6.1: Intuitionistic fully fuzzy transportation problem

Sources	Destinations			Instuitionistic Fuzzy Availability ($\tilde{a}_i^I$)
	D_1	D_2	D_3	

S_1	(2,4,5,6;1,4,5,6;0.5)	(4,6,7,8;3,6,7,9;0.2)	(3,7,8,12;2,7,8,13;0.3)	(4,6,8,9; 2,6,8,10;0.6)
S_2	(1,3,4,5;0.5,3,4,5;0.6)	(3,5,6,7;2,5,6,8;0.6)	(2,6,7,11;1,6,7,12;0.4)	(0,0.5,1,2; 0,0.5,1,5;0.7)
S_3	(3,4,5,8;2,4,5,9;0.7)	(1,2,3,4;0.5,2,3,5;0.8)	(2,4,5,10;1,4,5,11;0.2)	(8,9.5,10,11; 6.5,9.5,10,11 ;0.8)
Instuitionistic Fuzzy Demand $(\tilde{b}_j^I)$	(6,7,8,9;5,7,8,11;1)	(4,5,6,7;3,5,6,8;0.8)	(2,4,5,6;0.5,4,5,7;0.6)	

Using the Step 2 of existing formulation, discussed in Section 6.2, the problem, given in Table 6.1 can be transformed into problem, given in Table 6.2.

Table 6.2: Transformed intuitionistic fully fuzzy transportation problem

Sources	Destinations			Inst. Fuzzy Availability $(\tilde{a}_i^I)$
	D_1	D_2	D_3	
S_1	(2,4,5,6;1,4,5,6;0.2)	(4,6,7,8;3,6,7,9;0.2)	(3,7,8,12;2,7,8,13;0.2)	(4,6,8,9; 2,6,8,10;0.2)
S_2	(1,3,4,5;0.5,3,4,5;0.2)	(3,5,6,7;2,5,6,8;0.2)	(2,6,7,11;1,6,7,12;0.2)	(0,0.5,1,2; 0,0.5,1,5;0.2)
S_3	(3,4,5,8;2,4,5,9;0.2)	(1,2,3,4;0.5,2,3,5;0.2)	(2,4,5,10;1,4,5,11;0.2)	(8,9.5,10,11; 6.5,9.5,10,11 ;0.2)
Inst. Fuzzy Demand $(\tilde{b}_j^I)$	(6,7,8,9;5,7,8,11;0.2)	(4,5,6,7;3,5,6,8;0.2)	(2,4,5,6;0.5,4,5,7;0.2)	

Using the proposed method, the generalized intuitionistic fuzzy optimal solution of generalized intuitionistic fully fuzzy transportation problem, presented by Table 6.1, can be obtained as follows:

Step 1: The generalized intuitionistic fully fuzzy transportation problem, presented by Table 6.1, can be transformed into the problem (P6.12).

$$\begin{aligned}
\text{Minimize } & (2,4,5,6;1,4,5,6;0.2)\otimes(x_{111},x_{112},x_{113},x_{114};x'_{111},x_{112},x_{113},x'_{114};0.2)\oplus(4,6,7,8;3,6,7,9;\\
& 0.2)\otimes(x_{121},x_{122},x_{123},x_{124};x'_{121},x_{122},x_{123},x'_{124};0.2)\oplus(3,7,8,12;2,7,8,13;0.2)\otimes(x_{131},x_{132},\\
& x_{133},x_{134};x'_{131},x_{132},x_{133},x'_{134};0.2)\oplus(1,3,4,5;0.5,3,4,5;0.2)\otimes(x_{211},x_{212},x_{213},x_{214};x'_{211},x_{212},\\
& x_{213},x'_{214};0.2)\oplus(3,5,6,7;2,5,6,8;0.2)\otimes(x_{221},x_{222},x_{223},x_{224};x'_{221},x_{222},x_{223},x'_{224};0.2)\oplus\\
& (2,6,7,11;1,6,7,12;0.2)\otimes(x_{231},x_{232},x_{233},x_{234};x'_{231},x_{232},x_{233},x'_{234};0.2)\oplus(3,4,5,8;2,4,5,9;\\
& 0.2)\otimes(x_{311},x_{312},x_{313},x_{314};x'_{311},x_{312},x_{313},x'_{314};0.2)\oplus(1,2,3,4;0.5,2,3,5;0.2)\otimes(x_{321},x_{322},\\
& x_{323},x_{324};x'_{321},x_{322},x_{323},x'_{324};0.2)\oplus(2,4,5,10;1,4,5,11;0.2)\otimes(x_{331},x_{332},x_{333},x_{334};x'_{331},x_{332},\\
& x_{333},x'_{334};0.2)\\
\text{Subject to } & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(P6.12)}\\
& (x_{111},x_{112},x_{113},x_{114};x'_{111},x_{112},x_{113},x'_{114};0.2)\oplus(x_{121},x_{122},x_{123},x_{124};x'_{121},x_{122},x_{123},x'_{124};0.2)\\
& \qquad\oplus(x_{131},x_{132},x_{133},x_{134};x'_{131},x_{132},x_{133},x'_{134};0.2)\approx(4,6,8,9;2,6,8,10;0.2),\\
& (x_{211},x_{212},x_{213},x_{214};x'_{211},x_{212},x_{213},x'_{214};0.2)\oplus(x_{221},x_{222},x_{223},x_{224};x'_{221},x_{222},x_{223},x'_{224};0.2)\\
& \qquad\oplus(x_{231},x_{232},x_{233},x_{234};x'_{231},x_{232},x_{233},x'_{234};0.2)\approx(0,0.5,1,2;0,0.5,1,5;0.2),\\
& (x_{311},x_{312},x_{313},x_{314};x'_{311},x_{312},x_{313},x'_{314};0.2)\oplus(x_{321},x_{322},x_{323},x_{324};x'_{321},x_{322},x_{323},x'_{324};0.2)\\
& \qquad\oplus(x_{331},x_{332},x_{333},x_{334};x'_{331},x_{332},x_{333},x'_{334};0.2)\approx(8,9.5,10,11;6.5,9.5,10,11;0.2),\\
& (x_{111},x_{112},x_{113},x_{114};x'_{111},x_{112},x_{113},x'_{114};0.2)\oplus(x_{211},x_{212},x_{213},x_{214};x'_{211},x_{212},x_{213},x'_{214};0.2)\\
& \qquad\oplus(x_{311},x_{312},x_{313},x_{314};x'_{311},x_{312},x_{313},x'_{314};0.2)\approx(6,7,8,9;5,7,8,11;0.2),\\
& (x_{121},x_{122},x_{123},x_{124};x'_{121},x_{122},x_{123},x'_{124};0.2)\oplus(x_{221},x_{222},x_{223},x_{224};x'_{221},x_{222},x_{223},x'_{224};0.2)\\
& \qquad\oplus(x_{321},x_{322},x_{323},x_{324};x'_{321},x_{322},x_{323},x'_{324};0.2)\approx(4,5,6,7;3,5,6,8;0.2),\\
& (x_{131},x_{132},x_{133},x_{134};x'_{131},x_{132},x_{133},x'_{134};0.2)\oplus(x_{231},x_{232},x_{233},x_{234};x'_{231},x_{232},x_{233},x'_{234};0.2)\\
& \qquad\oplus(x_{331},x_{332},x_{333},x_{334};x'_{331},x_{332},x_{333},x'_{334};0.2)\approx(2,4,5,6;0.5,4,5,7;0.2),\\
& \left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x_{ij2},x_{ij3},x'_{ij4};0.2\right)\succeq(0,0,0,0;0,0,0,0;0.2);\, i=1,2,3;\; j=1,2,3,\\
& \left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x_{ij2},x_{ij3},x'_{ij4};0.2\right)\text{is a non-negative trapezoidal intuitionistic fuzzy}\\
& \text{number.}
\end{aligned}$$

Step 2: Using the arithmetic operations of generalized trapezoidal intuitionistic fuzzy numbers, discussed in Section 6.1.2, the problem (P6.12) can be transformed into the problem (P6.13).

$$\begin{aligned}
\text{Minimize } & (2x_{111}+4x_{121}+3x_{131}+x_{211}+3x_{221}+2x_{231}+3x_{311}+x_{321}+2x_{331},4x_{112}+6x_{122}+7x_{132}+3x_{212}\\
& +5x_{222}+6x_{232}+4x_{312}+2x_{322}+4x_{332},5x_{113}+7x_{123}+8x_{133}+4x_{213}+6x_{223}+7x_{233}+5x_{313}+3x_{323}
\end{aligned}$$

$$+5x_{333},6x_{114}+8x_{124}+12x_{134}+5x_{214}+7x_{224}+11x_{234}+8x_{314}+4x_{324}+10x_{334};x'_{111}+3x'_{121}+2x'_{131}$$
$$+0.5x'_{211}+2x'_{221}+x'_{231}+2x'_{311}+0.5x'_{321}+x'_{331},4x_{112}+6x_{122}+7x_{132}+3x_{212}+5x_{222}+6x_{232}+4x_{312}$$
$$+2x_{322}+4x_{332},5x_{113}+7x_{123}+8x_{133}+4x_{213}+6x_{223}+7x_{233}+5x_{313}+3x_{323}+5x_{333},6x'_{114}+8x'_{124}$$
$$+13x'_{134}+5x'_{214}+8x'_{224}+12x'_{234}+9x'_{314}+5x'_{324}+11x'_{334};0.2)$$

Subject to (P6.13)

$$(x_{111}+x_{121}+x_{131},x_{112}+x_{122}+x_{132},x_{113}+x_{123}+x_{133},x_{114}+x_{124}+x_{134};x'_{111}+x'_{121}+x'_{131},x_{112}$$
$$+x_{122}+x_{132},x_{113}+x_{123}+x_{133},x'_{114}+x'_{124}+x'_{134};0.2)\approx(4,6,8,9;2,6,8,10;0.2),$$
$$(x_{211}+x_{221}+x_{231},x_{212}+x_{222}+x_{232},x_{213}+x_{223}+x_{233},x_{214}+x_{224}+x_{234};x'_{211}+x'_{221}+x'_{231},x_{212}$$
$$+x_{222}+x_{232},x_{213}+x_{223}+x_{233},x'_{214}+x'_{224}+x'_{234};0.2)\approx(0,0.5,1,2;0,0.5,1,5;0.2),$$
$$(x_{311}+x_{321}+x_{331},x_{312}+x_{322}+x_{332},x_{313}+x_{323}+x_{333},x_{314}+x_{324}+x_{334};x'_{311}+x'_{321}+x'_{331},x_{312}$$
$$+x_{322}+x_{332},x_{313}+x_{323}+x_{333},x'_{314}+x'_{324}+x'_{334};0.2)\approx(8,9.5,10,11;6.5,9.5,10,11;0.2),$$
$$(x_{111}+x_{211}+x_{311},x_{112}+x_{212}+x_{312},x_{113}+x_{213}+x_{313},x_{114}+x_{214}+x_{314};x'_{111}+x'_{211}+x'_{311},x_{112}$$
$$+x_{212}+x_{312},x_{113}+x_{213}+x_{313},x'_{114}+x'_{214}+x'_{314};0.2)\approx(6,7,8,9;5,7,8,11;0.2),$$
$$(x_{121}+x_{221}+x_{321},x_{122}+x_{222}+x_{322},x_{123}+x_{223}+x_{323},x_{124}+x_{224}+x_{324};x'_{121}+x'_{221}+x'_{321},x_{122}$$
$$+x_{222}+x_{322},x_{123}+x_{223}+x_{323},x'_{124}+x'_{224}+x'_{324};0.2)\approx(4,5,6,7;3,5,6,8;0.2),$$
$$(x_{131}+x_{231}+x_{331},x_{132}+x_{232}+x_{332},x_{133}+x_{233}+x_{333},x_{134}+x_{234}+x_{334};x'_{131}+x'_{231}+x'_{331},x_{132}$$
$$+x_{232}+x_{332},x_{133}+x_{233}+x_{333},x'_{134}+x'_{234}+x'_{334};0.2)\approx(2,4,5,6;0.5,4,5,7;0.2),$$
$$\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x_{ij2},x_{ij3},x'_{ij4};0.2\right)\succeq(0,0,0,0;0,0,0,0;0.2);i=1,2,3;\ j=1,2,3,$$

$\left(x_{ij1},x_{ij2},x_{ij3},x_{ij4};x'_{ij1},x_{ij2},x_{ij3},x'_{ij4};0.2\right)$ is a non-negative trapezoidal intuitionistic fuzzy number.

Step 3: Using the expression, $R\left(a,b,c,d;a',b,c,d';w\right)=\dfrac{w\left(a+a'+2b+2c+d+d'\right)}{8}$, the problem (P6.13) can be transformed into the problem (P6.14).

$$\text{Minimize } \frac{1}{40}\left(2x_{111}+4x_{121}+3x_{131}+x_{211}+3x_{221}+2x_{231}+3x_{311}+x_{321}+2x_{331}+8x_{112}+12x_{122}+14x_{132}\right.$$
$$+6x_{212}+10x_{222}+12x_{232}+8x_{312}+4x_{322}+8x_{332}+10x_{113}+14x_{123}+16x_{133}+8x_{213}+12x_{223}$$
$$+14x_{233}+10x_{313}+6x_{323}+10x_{333}+6x_{114}+8x_{124}+12x_{134}+5x_{214}+7x_{224}+11x_{234}+8x_{314}+4x_{324}$$
$$+10x_{334}+x'_{111}+3x'_{121}+2x'_{131}+0.5x'_{211}+2x'_{221}+x'_{231}+2x'_{311}+0.5x'_{321}+x'_{331}+6x'_{114}+8x'_{124}$$
$$\left.+13x'_{134}+5x'_{214}+8x'_{224}+12x'_{234}+9x'_{314}+5x'_{324}+11x'_{334}\right)$$

Subject to (P6.14)

$$x_{111}+x_{121}+x_{131}+2x_{112}+2x_{122}+2x_{132}+2x_{113}+2x_{123}+2x_{133}+x_{114}+x_{124}+x_{134}+x'_{111}+x'_{121}$$
$$+x'_{131}+x'_{114}+x'_{124}+x'_{134}=1.325,$$

$$x_{211}+x_{221}+x_{231}+2x_{212}+2x_{222}+2x_{232}+2x_{213}+2x_{223}+2x_{233}+x_{214}+x_{224}+x_{234}+x'_{211}+x'_{221}$$
$$+x'_{231}+x'_{214}+x'_{224}+x'_{234}=0.25,$$
$$x_{311}+x_{321}+x_{331}+2x_{312}+2x_{322}+2x_{332}+2x_{313}+2x_{323}+2x_{333}+x_{314}+x_{324}+x_{334}+x'_{311}+x'_{321}$$
$$+x'_{331}+x'_{314}+x'_{324}+x'_{334}=1.8875,$$
$$x_{111}+x_{211}+x_{311}+2x_{112}+2x_{212}+2x_{312}+2x_{113}+2x_{213}+2x_{313}+x_{114}+x_{214}+x_{314}+x'_{111}+x'_{211}$$
$$+x'_{311}+x'_{114}+x'_{214}+x'_{314}=1.525,$$
$$x_{121}+x_{221}+x_{321}+2x_{122}+2x_{222}+2x_{322}+2x_{123}+2x_{223}+2x_{323}+x_{124}+x_{224}+x_{324}+x'_{121}+x'_{221}$$
$$+x'_{321}+x'_{124}+x'_{224}+x'_{324}=1.1,$$
$$x_{131}+x_{231}+x_{331}+2x_{132}+2x_{232}+2x_{332}+2x_{133}+2x_{233}+2x_{333}+x_{134}+x_{234}+x_{334}+x'_{131}+x'_{231}$$
$$+x'_{331}+x'_{134}+x'_{234}+x'_{334}=0.8375,$$
$$x_{ijk}\geq 0; i=1,2,3;\ j=1,2,3.$$

Step 4: On solving the linear programming problem (P6.14), the obtained optimal solution is,

$x_{111}=\frac{53}{320}$, $x_{112}=\frac{53}{320}$, $x_{113}=\frac{53}{320}$, $x_{114}=\frac{53}{320}$, $x'_{111}=\frac{53}{320}$, $x'_{114}=\frac{53}{320}$, $x_{211}=\frac{1}{40}$, $x_{212}=\frac{1}{40}$, $x_{213}=\frac{1}{40}$, $x_{214}=\frac{1}{40}$, $x'_{211}=\frac{1}{40}$, $x'_{214}=\frac{1}{40}$, $x_{231}=\frac{1}{160}$, $x_{232}=\frac{1}{160}$, $x_{233}=\frac{1}{160}$, $x_{234}=\frac{1}{160}$, $x'_{231}=\frac{1}{160}$, $x'_{234}=\frac{1}{160}$, $x_{321}=\frac{11}{80}$, $x_{322}=\frac{11}{80}$, $x_{323}=\frac{11}{80}$, $x_{324}=\frac{11}{80}$, $x'_{321}=\frac{11}{80}$, $x'_{324}=\frac{11}{80}$, $x_{331}=\frac{63}{640}$, $x_{332}=\frac{63}{640}$,

$x_{333}=\frac{63}{640}$, $x_{334}=\frac{63}{640}$, $x'_{331}=\frac{63}{640}$, $x'_{332}=\frac{63}{640}$ $x'_{333}=\frac{63}{640}$ $x'_{334}=\frac{63}{640}$.

Step 5: Using the optimal solution, obtained in Step 4, the generalized intuitionistic fuzzy optimal solution of problem (P6.12) is $\tilde{x}^I_{11}=\left(\frac{53}{320},\frac{53}{320},\frac{53}{320},\frac{53}{320};\frac{53}{320},\frac{53}{320},\frac{53}{320},\frac{53}{320};0.2\right)$,

$\tilde{x}^I_{21}=\left(\frac{1}{40},\frac{1}{40},\frac{1}{40},\frac{1}{40};\frac{1}{40},\frac{1}{40},\frac{1}{40},\frac{1}{40};0.2\right)$, $\tilde{x}^I_{23}=\left(\frac{1}{160},\frac{1}{160},\frac{1}{160},\frac{1}{160};\frac{1}{160},\frac{1}{160},\frac{1}{160},\frac{1}{160};0.2\right)$, $\tilde{x}^I_{32}=\left(\frac{11}{80},\frac{11}{80},\frac{11}{80},\frac{11}{80};\frac{11}{80},\frac{11}{80},\frac{11}{80},\frac{11}{80};0.2\right)$ and $\tilde{x}^I_{33}=\left(\frac{63}{640},\frac{63}{640},\frac{63}{640},\frac{63}{640};\frac{63}{640},\frac{63}{640},\frac{63}{640},\frac{63}{640};0.2\right)$.

Step 6: Using the generalized intuitionistic fuzzy optimal solution, obtained in Step 5, the optimal generalized intuitionistic fuzzy transportation cost of problem, presented by Table 6.1, is

$$(2,4,5,6;1,4,5,6;0.2) \otimes \left(\frac{53}{320},\frac{53}{320},\frac{53}{320},\frac{53}{320};\frac{53}{320},\frac{53}{320},\frac{53}{320},\frac{53}{320};0.2\right) \oplus (1,3,4,5;0.5,3,4,5;$$

$$0.6) \otimes \left(\frac{1}{40},\frac{1}{40},\frac{1}{40},\frac{1}{40};\frac{1}{40},\frac{1}{40},\frac{1}{40},\frac{1}{40};0.2\right) \oplus (2,6,7,11;1,6,7,12;0.4) \otimes \left(\frac{1}{160},\frac{1}{160},\frac{1}{160},\frac{1}{160};\right.$$

$$\left.\frac{1}{160},\frac{1}{160},\frac{1}{160},\frac{1}{160};0.2\right) \oplus (1,2,3,4;0.5,2,3,5;0.8) \otimes \left(\frac{11}{80},\frac{11}{80},\frac{11}{80},\frac{11}{80};\frac{11}{80},\frac{11}{80},\frac{11}{80},\frac{11}{80};0.2\right) \oplus (2,$$

$$4,5,10;1,4,5,11;0.2) \otimes \left(\frac{63}{640},\frac{63}{640},\frac{63}{640},\frac{63}{640};\frac{63}{640},\frac{63}{640},\frac{63}{640},\frac{63}{640};0.2\right)$$

$$= \left(\frac{45}{64},\frac{249}{160},\frac{1201}{640},\frac{871}{320};\frac{45}{128},\frac{231}{160},\frac{1201}{640},\frac{1897}{640};0.2\right).$$

6.6 Conclusion

The flaws of the existing method [18] for solving generalized intuitionistic fully fuzzy transportation problems are pointed out. Also, to resolve these flaws, a new method is proposed. Furthermore, the exact generalized intuitionistic fuzzy optimal solution of the problem, solved by Chakraborty et al. [18], is obtained by the proposed method.

Chapter 7

Future scope*

It is noticed that ranking of generalized exponential trapezoidal fuzzy numbers, obtained by using the existing method [121], is independent from height of generalized exponential trapezoidal fuzzy numbers. While, the ranking of generalized exponential trapezoidal fuzzy numbers should be dependent on its height. Hence, it is not genuine to use the existing method [121] for comparing the generalized exponential trapezoidal fuzzy numbers. In this chapter, the flaws of the existing method [121] are pointed out and a modified method for ranking of generalized exponential trapezoidal fuzzy numbers is proposed. In future, the proposed ranking method may be extended for generalized exponential trapezoidal intuitionistic fuzzy numbers and a new method may be proposed to find the solution of generalized exponential trapezoidal intuitionistic fully fuzzy transportation problems (transportation problems in which cost, availability and demand are represented by generalized exponential trapezoidal intuitionistic fuzzy numbers).

7.1 Existing results

Rezvani [122; Theorem 1, pp. 192] proved that the probability density function $f_{\tilde{A}}(x)$, corresponding to exponential trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)_E$, is given as:

$$f_{\tilde{A}}(x) = \begin{cases} \dfrac{we^{\left[\frac{(x-a)}{(b-a)}\right]}}{a(1-e)-b+c+(e-1)d} & a \le x < b, \\ \dfrac{we}{a(1-e)-b+c+(e-1)d} & b \le x \le c, \\ \dfrac{we^{\left[\frac{(d-x)}{(d-c)}\right]}}{a(1-e)-b+c+(e-1)d} & c < x \le d \end{cases} \tag{7.1}$$

* The contents of this chapter are published in Journal of Intelligent and Fuzzy Systems.

Also, Rezvani [122] used this $f_{\tilde{A}}(x)$ to prove the remaining results [122;Theorem 2, pp. 193; Theorem 3, pp. 193] of the published paper Rezvani [122].

Rezvani [122; Theorem 1, pp. 192] has used the following procedure to prove that $f_{\tilde{A}}(x)$ is the probability density function corresponding to exponential trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)_E$.

Step 1: Let $f_{\tilde{A}}(x) = C\mu_{\tilde{A}}(x)$, where C is a non-negative real number and $\mu_{\tilde{A}}(x) = \begin{cases} we^{-\left[\frac{(b-x)}{(b-a)}\right]} & a \leq x < b, \\ w & b \leq x \leq c, \\ we^{-\left[\frac{(x-c)}{(d-c)}\right]} & c < x \leq d \end{cases}$ is the membership function of exponential trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)_E$.

Step 2: Since, $f_{\tilde{A}}(x)$ is assumed as the probability density function. Therefore, $\int_{-\infty}^{\infty} f_{\tilde{A}}(x)dx$ should be 1. Also,

$$\int_{-\infty}^{\infty} f_{\tilde{A}}(x)dx = 1 \Rightarrow \int_{-\infty}^{\infty} C\mu_{\tilde{A}}(x)dx = 1$$

$$\Rightarrow \int_a^b Ce^{-\left[\frac{(b-x)}{(b-a)}\right]}dx + \int_b^c Cdx + \int_c^d Ce^{-\left[\frac{(x-c)}{(d-c)}\right]}dx = 1$$

$$\Rightarrow C = \frac{e}{a(1-e)-b+c+(e-1)d} \tag{7.2}$$

Step 3: Putting the value of C, obtained from Step 2, in $f_{\tilde{A}}(x) = C\mu_{\tilde{A}}(x)$, the probability density function $f_{\tilde{A}}(x)$, defined in (7.1) , is obtained.

7.2 Mathematical error in existing results

It is obvious from Step 2 that Rezvani [122] has used the wrong expression

$$\mu_{\tilde{A}}(x) = \begin{cases} e^{-\left[\frac{(b-x)}{(b-a)}\right]} & a \le x < b, \\ 1 & b \le x \le c, \\ e^{-\left[\frac{(x-c)}{(d-c)}\right]} & c < x \le d \end{cases}$$ instead of the exact expression

$$\mu_{\tilde{A}}(x) = \begin{cases} we^{-\left[\frac{(b-x)}{(b-a)}\right]} & a \le x < b, \\ w & b \le x \le c, \\ we^{-\left[\frac{(x-c)}{(d-c)}\right]} & c < x \le d \end{cases}$$ for calculating the value of C and hence to obtain the probability density function $f_{\tilde{A}}(x)$ defined in (7.1), to prove the existing results [122; Theorem 2, pp. 193; Theorem 3, pp. 193] and to solve the numerical examples [122; Example 1, pp. 194; Example 2, pp. 195; Example 3, pp. 195; Example 4, pp. 195; Example 5, pp. 196; Example 6, pp. 196; Example 7, pp. 197].

7.3 Exact form of existing results

It can be easily verified that on using the exact expression $\mu_{\tilde{A}}(x) = \begin{cases} we^{-\left[\frac{(b-x)}{(b-a)}\right]} & a \le x < b, \\ w & b \le x \le c, \\ we^{-\left[\frac{(x-c)}{(d-c)}\right]} & c < x \le d \end{cases}$ instead of using the wrong expression $\mu_{\tilde{A}}(x) = \begin{cases} e^{-\left[\frac{(b-x)}{(b-a)}\right]} & a \le x < b, \\ 1 & b \le x \le c, \\ e^{-\left[\frac{(x-c)}{(d-c)}\right]} & c < x \le d \end{cases}$, for calculating the value of C, the obtained exact value of C is $\frac{e}{w[a(1-e)-b+c+(e-1)d]}$. Also, putting this value of C in $f_{\tilde{A}}(x) = C\mu_{\tilde{A}}(x)$, the obtained exact probability density function $f_{\tilde{A}}(x)$ corresponding to exponential trapezoidal fuzzy number

$\tilde{A} = (a, b, c, d; w)_E$ is $$f_{\tilde{A}}(x) = \begin{cases} \frac{e^{\left[\frac{(x-a)}{(b-a)}\right]}}{a(1-e)-b+c+(e-1)d} & a \le x < b, \\ \frac{e}{a(1-e)-b+c+(e-1)d} & b \le x \le c, \\ \frac{e^{\left[\frac{(d-x)}{(d-c)}\right]}}{a(1-e)-b+c+(e-1)d} & c < x \le d \end{cases}$$

Hence, the error occurring in the existing results [122; Theorem 1, pp. 192, Theorem 2, pp. 193; Theorem 3, pp. 193] and in the solution of numerical examples [122; Example 1, pp. 194; Example 2, pp. 195; Example 3, pp. 195; Example 4, pp.195; Example 5, pp.196; Example 6, pp.196; Example 7, pp.197] can be resolved by using the exact probability density function $f_{\tilde{A}}(x)$ i.e., $f_{\tilde{A}}(x) = \begin{cases} \frac{e^{\left[\frac{(x-a)}{(b-a)}\right]}}{a(1-e)-b+c+(e-1)d} & a \le x < b, \\ \frac{e}{a(1-e)-b+c+(e-1)d} & b \le x \le c, \\ \frac{e^{\left[\frac{(d-x)}{(d-c)}\right]}}{a(1-e)-b+c+(e-1)d} & c < x \le d, \end{cases}$ instead of using the wrong probability density function $f_{\tilde{A}}(x)$ i.e., $f_{\tilde{A}}(x) = \begin{cases} \frac{we^{\left[\frac{(x-a)}{(b-a)}\right]}}{a(1-e)-b+c+(e-1)d} & a \le x < b, \\ \frac{we}{a(1-e)-b+c+(e-1)d} & b \le x \le c, \\ \frac{we^{\left[\frac{(d-x)}{(d-c)}\right]}}{a(1-e)-b+c+(e-1)d} & c < x \le d, \end{cases}$.

Furthermore, it can be concluded that the existing results [122; Theorem 1, pp. 192, Theorem 2, pp. 193; Theorem 3, pp. 193] and existing solution the numerical examples [122; Example 1, pp. 194; Example 2, pp. 195; Example 3, pp. 195; Example 4, pp.195; Example 5, pp.196; Example 6, pp.196; Example 7, pp.197] are not valid. However, if w , present in existing results [122; Theorem 2, pp. 193; Theorem 3, pp. 193], is replaced by 1 then the existing results [122; Theorem 1, pp. 192, Theorem 2, pp. 193; Theorem 3, pp. 193] will be valid. Also, the exact solution of numerical examples [122; Example 1, pp. 194; Example 2, pp. 195; Example 3, pp. 195; Example 4, pp. 195; Example 5, pp. 196; Example 6, pp. 196; Example 7, pp. 197] can be obtained by using the exact expressions

$C = \frac{e}{w[a(1-e)-b+c+(e-1)d]}$, $\mu = \frac{1}{a(1-e)-b+c+(e-1)d}\left[\frac{1.72b^2-1.72c^2-a^2+d^2}{2}\right]$ and

$$\sigma^2 = \frac{1}{a(1-e)-b+c+(e-1)d}\left[\frac{1.72b^3-1.72c^3-a^3+d^3}{3}\right] - \frac{1}{[a(1-e)-b+c+(e-1)d]^2}\left[\frac{(1.72b^2-1.72c^2-a^2+d^2)^2}{4}\right]$$

instead of using the wrong expressions $C = \frac{e}{a(1-e)-b+c+(e-1)d}$, $\mu = \frac{w}{a(1-e)-b+c+(e-1)d}\left[\frac{1.72b^2-1.72c^2-a^2+d^2}{2}\right]$ and $\sigma^2 = \frac{w}{a(1-e)-b+c+(e-1)d}\left[\frac{1.72b^3-1.72c^3-a^3+d^3}{3}\right] - \frac{w^2}{[a(1-e)-b+c+(e-1)d]^2}\left[\frac{\left(1.72b^2-1.72c^2-a^2+d^2\right)^2}{4}\right]$ respectively for calculating the constants $C_{\tilde{A}}$, $C_{\tilde{B}}$, $C_{\tilde{C}}$; means $\mu_{\tilde{A}}$, $\mu_{\tilde{B}}$, $\mu_{\tilde{C}}$ and variance $\sigma^2_{\tilde{A}}$, $\sigma^2_{\tilde{B}}$, $\sigma^2_{\tilde{C}}$ of exponential trapezoidal fuzzy numbers $\tilde{A}$; $\tilde{B}$ and $\tilde{C}$ respectively.

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