

A Study of Anharmonic Pulsation of Rotationally and Tidally Distorted Stars

A Thesis

submitted in partial fulfillment of the requirements for the award of the degree of

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in

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by

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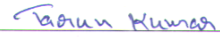
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September 2018

Certificate

I hereby certify that the work, which is being presented in the thesis, entitled **A Study of Anharmonic Pulsation of Rotationally and Tidally Distorted Stars**, in partial fulfillment of the requirements for the award of the degree of **Doctor of Philosophy** and submitted to the institution is an authentic record of my own work carried out during the period **January 2014** to **September 2018** under the supervision of **Dr. Arvind Kumar Lal**, Professor, School of Mathematics and **Dr. Ankush Pathania**, Assistant Professor, School of Mathematics, Thapar Institute of Engineering and Technology, Patiala.

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.....*Dedicated to My Divine Brothers, Parents and God*

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Abstract

The present thesis deals with the study of investigating the effects of rotation and tidal distortions on the shapes of the radial velocity curves of the pulsating variable stars. Such a study has practical relevance in astrophysics where it is expected to help in better understanding the problems of the shapes of the radial velocity curves of the rotating stars as well as stars in the binary and multiple systems.

A lot of work has been done on the problem of determining the equilibrium structures, periods of pulsation of pulsating variable stars and their light curves. However, the concept of anharmonic pulsations has not yet fully explored to understand the observed shape of the radial velocity curves. Most of the studies have included only the fundamental and first two higher modes of oscillations in the equation of anharmonic pulsation expecting that inclusion of higher modes will not substantially change the shape of the radial velocity curve. However, Mohan [42] has found that when higher modes are included in the case of a massive star in helium burning phase the shape of the radial velocity curve gets radically changed. But, after this study no systematic works have been done to check whether this was an isolated phenomenon, specific to that model or it is a general trend.

It is well known that in a pulsating star alongwith the radial modes, the nonradial modes of oscillations are also likely to be excited. However, no attempt has been made so far to see how the inclusion of nonradial modes of oscillations in the equation of anharmonic pulsation will affect the shape of the radial velocity curves.

Many of the pulsating variable stars are known to be rotating stars as well as stars in binary systems. Extensive study has been done on the problem of determining the effect of rotation and tidal distortion on the periods of oscillations of stellar models. However, the effect of rotation and tidal distortion on the radial velocity curve of such stars has not been investigated in detail. So it will be worthwhile to see how the rotation and tidal effects will affect the shapes of radial velocity curves of certain stellar models.

The following objectives have been accomplished in this study:

1. To investigate the effect of inclusion of higher modes of radial oscillations in the equation of anharmonic pulsation and hence to analyze its corresponding effect on the shapes of radial velocity curves of certain theoretical models of stars, representing different types of pulsating variable stars.
2. To study the effect of inclusion of nonradial modes of oscillation in the equation of anharmonic pulsations and hence to analyze the corresponding effect on the shape of radial velocity curve of certain nonradial pulsating stellar models.
3. To determine the effects of rotation and tidal distortions on the shapes of radial velocity

curves of certain radial and nonradial stellar models.

In the present thesis we have used the anharmonic theory of pulsating stars as proposed by Rosseland [61] and methodology of Prasad [58] to study the anharmonic pulsation of pulsating variable stars. The Mohan and saxena ([44], [45]) approach has been used to incorporate the effects of rotation and tidal distortions in our study. So, in the present work we have considered the possibility of further using Mohan and Saxena ([44], [45]) approach to determine the effects of rotation and tidal distortions on the shapes of radial velocity curves of the radial and non-radial oscillations of the polytropic models of pulsating variable stars using the technique of Rosseland [61] and Prasad [58].

The thesis consists of six chapters. Summary of the each chapter is presented as follows:

Chapter 1

Chapter 1 is introductory in nature. In this chapter we have briefly discussed the astrophysical significance of the problem of determining equilibrium structures, periods of small adiabatic oscillations and radial velocity curves of the rotationally and/or tidally distorted stellar models. We have discussed in brief fundamental system of differential equations that governs the equilibrium structure of stars, average technique of Kippenhahn and Thomas [25] and concepts of Roche equipotential. The equations that governs the equilibrium structures and small adiabatic modes of oscillations of rotationally and tidally distorted polytropic models of stars (as obtained earlier by Mohan and Saxena [45]) has been presented. The anharmonic theory of pulsating star as given by Rosseland [61] has been used in conjunction with Mohan and Saxena [45] approach to obtain the anharmonic pulsation equation of RTD stellar models of star. The methodology of Prasad [58] has been then used to solve the anharmonic pulsation equation of RTD stellar models. A brief survey of the literature available on the subject and summary of the work presented in the succeeding chapters of the thesis are also presented in this chapter.

Chapter 2

In Chapter 2, we formulated the anharmonic pulsation equation for rotationally and/or tidally distorted polytropic models of pulsating variable stars using the theory of Rosseland [61] along with Mohan and Saxena ([44], [45]) approach. The methodology of Prasad [58] has been used to solve these equations. Numerical computations have been performed to determine the solution of anharmonic equation for certain rotationally and/or tidally distorted polytropic models (with polytropic indices $N = 1.5$ and $N = 3.0$) of pulsating variable stars. The radial

velocity curves of certain rotationally and/or tidally distorted polytropic models of pulsating variable stars has been then obtained. The numerical results and figures has been then analysed to draw certain conclusions.

Chapter 3

In Chapter 3, we study the effects of the higher radial modes on the anharmonic pulsations and hence on the radial velocity curves of the polytropic models of rotationally and/or tidally distorted pulsating variable stars. For this purpose, we have considered the following cases of radial oscillations: (i) fundamental mode ; (ii) fundamental and the first mode ; (iii) fundamental and the next two modes and (iv) fundamental and the next three higher modes of pulsation. Numerical computations have been performed to determine the solution of anharmonic pulsation equation for certain rotationally distorted and rotationally and tidally distorted polytropic models (with polytropic indices $N = 1.5$, $N = 3.0$ and $N = 4.0$) of pulsating variable stars . Radial velocity curves have been then drawn for all the considered models. Finally, numerical results and figures have been analysed to obtain certain conclusions.

Chapter 4

The work done in Chapter 2 and Chapter 3 has further been extended and presented in Chapter 4. In this Chapter we have studied the effect of varying the relative amplitude of oscillations on the anharmonic radial modes of oscillations and hence on the shapes of the radial velocity curves of rotationally and tidally distorted polytropic models of pulsating variable stars. The interaction of various modes have also been taken into account (as discussed in Chapter 3) while studying the effect of relative amplitude. Numerical computations have been performed to determine the solution of anharmonic pulsation equation for certain rotationally and tidally distorted polytropic models of stars with indices 1.5, 3.0 and 4.0 and for certain values of relative amplitude of oscillation. The radial velocity curves of all the considered models have been then plotted and finally certain conclusions has been drawn.

Chapter 5

In Chapter 5, we have studied the effects of non-radial modes along with radial modes of oscillations, (that is mixing modes) on the shapes of radial velocity curves of rotationally and tidally distorted polytropic models of pulsating variable stars. For this purpose, following cases

of radial and nonradial modes of oscillations have been considered: (i) fundamental non-radial mode (ii) fundamental radial mode (iii) fundamental radial and fundamental non-radial modes (iv) fundamental radial, fundamental non-radial and first radial modes (v) fundamental radial, fundamental non-radial, first radial modes and first gravity modes. The anharmonic pulsation equation that governs the anharmonic pulsations (radial and nonradial) of certain rotationally and tidally distorted polytropic models of pulsating variable stars has been solved numerically for all such cases. Radial velocity curves has been drawn for all the considered models and certain conclusions drawn.

Chapter 6

The astrophysical significance of the present work along with the limitations and scope of the present work are discussed briefly in the concluding Chapter 6.

A research paper entitled **”Effects of rotation and tidal distortions on the shapes of radial velocity curves of polytropic models of pulsating variable stars ”** based on the contents of the work presented in chapter II has been published in the international Journal of Research in Astronomy and Astrophysics (RAA), Institute of Physics (IOP) publishing, Vol.18 No.6. 63(10pp), 2018. A research paper entitled **”Effect of interaction of the various modes on the radial velocity curves of the polytropic models of rotationally and tidally distorted pulsating variables stars”** based on the contents of the work presented in chapter III has been accepted for publication in the international Journal of Research in Astronomy and Astrophysics (RAA), Institute of Physics (IOP) publishing, 2018.

Based on the contents of chapter 4 a research paper entitled **”Effect of the relative amplitude of the radial pulsation on the radial velocity curves of the polytropic models of rotationally and tidally distorted pulsating variable stars”**, is to be communicated shortly. A research paper entitled **” Effect of the interaction of nonradial modes with radial modes on the radial velocity curves of the polytropic models of rotationally and distorted pulsating variable stars”**, based on contents of Chapter 5 is to be communicated shortly.

List of Publications

International Journal

1. Tarun Kumar, Arvind Kumar Lal and Ankush Pathania, ” Effects of rotation and tidal distortion on the shapes of radial velocity curves of polytropic models of pulsating variable stars”, Research in Astronomy and Astrophysics (RAA), Institute of Physics (IOP) publishing ,2018, Vol 18 No. 6, 63(10pp). **SCI, Impact Factor: 1.292**
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List of Notations

n	The rotation parameter that represents distortions due to rotation.
q	The tidal parameter that represents distortion due to tidal effects
N	Polytropic index
$q_b(t)$	The displacement at the surface of star at a time t
τ	A variable for time ($\tau = \sigma_1 t$)
σ_1	Eigenfrequency of fundamental mode
η	The relative amplitude of displacement of an element at a distance r from the center of star.
K	The Skewness coefficient
RTD	Rotationally and Tidally distorted
RD	Rotationally distorted
TD	Tidally distorted
RVC	Radial Velocity Curve(s)
f/f_r	fundamental radial mode
$f + 1$	fundamental and first radial mode
$f + 2$	fundamental and next two higher radial modes
$f + 3$	fundamental and next three higher radial modes
f_{nr}	fundamental nonradial mode
f_{rnr1}	fundamental radial, fundamental nonradial and first radial modes
f_{rnr1g}	fundamental radial, fundamental nonradial, first radial and fundamental gravity modes

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Chapter 1

Introduction

Chapter 1 is introductory in nature which is organised as follows: In section 1.1, we first explain in brief the astrophysical significance of the theoretical study to determine the equilibrium structures and periods of adiabatic oscillations of gaseous spheres in the presence of distortional parameters (due to rotation and / or tidal force). A literature review available on the topic is presented in Section 1.2. In section 1.3, we present the fundamental system of differential equations which governs the equilibrium structures of a gaseous sphere in hydrostatic and thermal stages. In subsection 1.3.1 we describe the application of averaging technique of Kippenhahn and Thomas [25] to obtain the equilibrium structures of rotationally and tidally distorted (RTD) gaseous spheres. The concepts of Roche equipotentials and Roche limits are then introduced in subsection 1.3.2. In section 1.4, equilibrium structures of RTD polytropic models of stars has been discussed. In section 1.5, we have presented an eigenvalued boundary value problem (as obtained by Mohan and Saxena [45]) that determines the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of RTD polytropic models of stars. In section 1.6, we present an eigenvalued boundary value problem that determine the eigenfrequencies of nonradial modes of oscillations of RTD polytropic models of stars. Section 1.7 deals with formulation of Anharmonic pulsation equation for a RTD stellar model using the theory of Rosseland [61] and Mohan and Saxena [45] . Method for solving the equation of anharmonic radial oscillations for RTD Polytropic model of star using the technique of Prasad [58] has been discussed in section 1.8. A brief summary of the work presented in the succeeding chapters of this thesis is finally presented in section 1.9.

1.1 Astrophysical significance of the problem determining the effects of rotational and tidal distortions on the equilibrium structure and the periods of oscillations of gaseous sphere

Like our sun, most of the stars observed in the sky are huge masses of gas in hydrostatic equilibrium radiating enormous amounts of energy in the form of heat and light. Theoretically a star is considered as a self gravitating gaseous sphere in hydrostatic and thermal equilibrium. So the problems of the equilibrium structure of gaseous spheres have often been studied to understand the inner structures, accountable for different observed feature of the stars. Observations

indicate that some of the stars are single others are in group of two or more . It is observed that some of the stars are rotating about their axes of rotation. This rotation may be either a uniform or a differential rotation. Observations indicate that many of the stars in binary and multiple systems are also known to be rotating about their axes as well as revolving around each other. Thus, if we assume the equilibrium model of a single non rotating star as a gaseous sphere, the equilibrium model of a rotating star will be rotationally distorted (RD) gaseous sphere. Similarly, the equilibrium model of a star appearing in a binary or a multiple system will be a tidally distorted (TD) gaseous sphere if it is not rotating and a rotationally and tidally distorted (RTD) gaseous sphere if the star is rotating as well.

Observation also indicates that the brightness of certain observed stars varies with time. These are called variable stars. In some of these variable stars, the variations in luminosity are periodic. It is reasonable to assume that such a regular variable stars are pulsating gaseous spheres in which the variation in luminosity is being due to the periodic contraction and expansion of the gaseous mass. The regular variable stars gained importance in astrophysics when it was discovered that there exists a definite relation between the periods of pulsation and the luminosities of such stars. This relationship has been utilized to determine the distance of stars. So, variable stars motivated theoretical astrophysicists to investigate the problems of small oscillations of the equilibrium models of the variable stars to understand the mechanism which could possibly be sustaining pulsations in these stars. Such investigations are also expected to help us knowing the nature of the internal structure of the stars. In most of these theoretical studies, the variable star is represented by a gaseous sphere undergoing radial and nonradial oscillations.

Besides periods of pulsations, another observable feature of variable star is the shape of their light curve which changes periodically with the period of pulsation. A star is expected to be brighter when it is most compressed and least brighter when it is fully expanded. The shape of the light curve is usually taken to represent the shape of radial velocity curve (RVC) of the star. On the basis of periodic pulsation, this curve should be Sine curve in nature. But, observations show that it is not.

Observations, also, show that some of the variable stars are also rotating stars. The theoretical models of such rotating stars can be regarded as RD gaseous spheres performing small oscillations about their equilibrium configuration. Similarly, some of the variable stars have also been observed in binary and multiple stellar systems. The theoretical model of such stars can be regarded as RTD gaseous spheres performing small oscillations about their equilibrium configuration. So, astrophysicist have turned their attention towards the analytical study of the problems of rotating stars and stars in binary systems have engaged the attention of astrophysicist since long with a view to analyze and understand the observational behavior of such stars.

Many of the pulsating stars are known to be rotating stars as well as stars in binary systems. Extensive study has been done on the problem of determining the effect of rotation and tidal

distortion on the periods of oscillations of stellar models. However, the effect of rotation and tidal distortion on the RVC of such stars has not been investigated in detail. So it will be worth to see how the rotation and tidal effects will affect the shapes of radial velocity curves of certain stellar models.

1.2 Literature Review

In a star there is a finite balancing act between the radiation force produced within itself that tends to expand the star and the gravitational force that tend to contract the star. In most of the theoretical studies, the equilibrium structures of the stars have been carried out in literature by assuming the star to be an undistorted gaseous sphere. A substantial literature is found on this subject Chandrasekhar [5], Schwarzschild [67], Eddington [16], Cox and Giuli [10], Kippenhahn and Weigert [26], Horedt [22]).

The prototype for one of the most prominent class of variables stars in astronomy was discovered by Goodricke [18]. The periodic and asymmetric light variation in δ -Cephei was first discovered by Goodricke [18] and in the same year he also discovered the eclipsing binary star β -Lyrae. After two years, he discovered the variability of Algol.

Mathematical study of the problems of small adiabatic oscillations of gaseous sphere gained importance in astrophysics when it was realized that many of the observed feature of certain types of variable stars could be explained on the basis of small periodic oscillations. Ritter was perhaps the first who suggested in the year 1879 that the periodic variation in the luminosity of variable stars may be due to radial oscillations. The two famous papers of Eddington [14, 15] on the pulsation problem, raised the pulsation hypothesis up to rank of a major astrophysical theory. Since then extensive studies have been made on the problems of small radial oscillation of gaseous spheres that are taken as representative models of the real star(for detailed see references Rosseland [61], Ledoux and Walraven [36] and Cox [9], Hilditch [21]).

In case of regular variables stars, the high symmetry of their observed properties favor the hypothesis of purely radial oscillations. However, purely radial oscillation may not be able to explain many other phenomena observed in case of certain variables stars. Ledoux and Walraven [36] pointed out that the dynamical instability leading to explosions in the stars might be easier to reach for some modes of nonradial oscillations. Chandrasekhar and Lebovitz [6] discussed that it might be possible to explain variability of β - Canis Majoris type star on the basis of resonance between the radial and nonradial modes of oscillations. Dalsgaard and Gough [11] suggested that certain observed phenomena in the outer layer of sun could be explained on the basis of certain nonradial modes of oscillations of the sun. Smith [73] studied zero-age main sequence **B** star and found that this star is pulsating nonradially.

Theoretical study of the problem of nonradial oscillations started with Kelvin's investiga-

tion of the oscillatory modes of an incompressible gaseous sphere. But the proper mathematical formulation of the problem is due to Pekeris [57] who derived the fourth order linear differential equation governing the adiabatic nonradial modes of oscillations of a compressible self-gravitating gaseous sphere. Since then, the theoretical studies of the problem of nonradial oscillations of spherical models have been carried out by many investigators. Several authors such as Cowling [8], Kopal [27], Ledoux [35], Owen [52], Hansen et al [20], Saio [63], Clement [7], Singh et al. [69], McDermott et al [40], Bhatia [2], Mohan et al. [49], Lee and Strohmayer [37], Savonije [64], Lovekin and Deupree [38] and Lal et al [33, 34] have made significant contribution to the studies of the problems of nonradial pulsation of stars.

Mohan and Saxena [44, 45] studied the effects of rotation and tidal distortions on the equilibrium structure and pulsations of the polytropic models of stars by using the averaging concept of Kippenhahn and Thomas [25] along with certain results of Roche equipotential. Saxena [65] extensively discussed this approach in his thesis. Later, Mohan and Agarwal [43] used it to study the structure and periods of small adiabatic oscillations of RTD composite models of stars. The technique was subsequently formalized by Mohan et al. ([48], [49]) to study the problems of equilibrium structures and oscillations of RTD main sequence stars. Singh [70] computed frequency of fundamental modes of oscillations for the White dwarf models with varying degeneracy parameters. Lal [32] discussed extensively the equilibrium structures and periods of oscillations of differentially rotating stellar models. The problems of oscillations of differentially rotating stars in binary system was also studied by Singh and Sharma [72]. Lal et al. [31] obtained the equilibrium structure of white dwarf stars distorted by differential rotation and tidal distortion. Further, Pathania et al. [53] used the concept of Roche equipotentials to analyse the problem of structure and oscillations of synchronous and nonsynchronous binary stars as well as single rotating stars. Pathania and Medupe [54] also computed the dimensions and equilibrium structure of the primary components of the nonsynchronous binaries stars. Saini et al [62] have computed the eigenfrequencies of pseudo-radial modes of oscillations of polytropic models of stars taking into account the mass variation inside the star that is rotating differentially. The equilibrium structures and various physical parameters of differentially rotating polytropic models of stars considering the effect of mass variation inside the star and on its equipotential surfaces has been studied by Kumar [30].

Sen [68] has studied the anharmonic pulsation of the Cepheid variable star. He has shown that only homogeneous stars are capable of uniform, radial, adiabatic pulsations and found a high degree of compressibility, in general, for the Cepheids. Rosseland [61] developed the theory of anharmonic pulsation to study the effect of higher modes and higher order of terms in the pulsation equation on the shape of radial velocity curve (RVC) of a pulsating stellar model. This theory has been subsequently used by different investigators such as Bhatnagar and Kothari [3], Schwarzschild and Savedoff [66], Prasad [58], Gurm [19], Van der Borgh and Murphy [75], Prasad and Mohan [60] and Mohan [42]. Prasad [58] applied the Fourier series

to solve the anharmonic equation in general and developed a method to evaluate the coefficient in the series. He applied the method to the standard model with two overtone and showed that the skewness in the radial velocity is in fair agreement with the observed one. The same study has been further applied to the homogeneous and the inverse square model by Prasad [59], the Roche model by Bhatnagar and Kuskwaha [4], the two phase model by Lal and Jain [23] and a composite model and main sequence star by Gurm [19].

In the equation of anharmonic pulsation, most of the authors have considered only the first three or four modes of pulsations. On the basis of their results, it was expected that the inclusion of additional higher modes in the equation will bring theoretical results closer to the observed RVC of the classical cepheids. However, the calculation of Van der Borgat and Murphy [75] have shown that the inclusion of fifth and sixth modes creates humps in the RVC in an early main-sequence star of ten solar masses. The RVC of the stellar model so obtained was different from the observed RVC of cepheids. Mohan [46] had considered the anharmonic pulsation of a 15.6 solar mass star in the helium burning phase taking into account the fifth and sixth modes in the anharmonic pulsation equation of the nonrotating stars. He has found that with the inclusion of the fifth mode the skewness coefficient again increases and create humps in the RVC of a nonrotating star with the inclusion of the sixth modes. However, with the inclusion of the modes higher than the fourth mode the RVC start deviating from the observed RVC of the cepheids. So, he concluded that modes higher than the fourth are not active in most of the cepheid type pulsating variable stars.

The pulsation properties of a sequence of massive stars have been discussed by Murphy [50]. Murphy [51] also found the skewness of the radial velocity at the surface of a pulsating massive star with higher modes of oscillations. Arentoft et al. [1], presented the light curves of V1162 Ori, including multiperiodicity and cyclic amplitude variability. Coupling coefficients for low-order radial modes have been calculated by Perdang & Blacher [56] for a polytropic model and by Takeuti et al. [74] for models of classical Cepheids. Van Hoolst [76] determined the coupling coefficients for low-order radial and second-degree nonradial modes of a polytropic model with index 3.0. Van Hoolst [77], have also calculated the self-coupling coefficients for radial and non-radial modes of oscillations of polytropic models in the isentropic approximations.

Goraya [17] has studied the spectrophotometric observations of four **Be** stars and one normal **B**- type star. Pandey et al. [55] have studied the anharmonic vibration in pulsating stars using the Hamiltonian formulation of Newtonian dynamics. Joshi et al. [24] discussed the light curve, frequency spectra and various astrophysical parameters of 108 chemically peculiar stars. In order to understand the pulsation mechanism of cepheids Dehnen et al. [13] have presented a model based on primary physical principles, namely energy conservations, mechanical adiabaticity and hydrodynamical equilibrium. They have found that the star pulsates anharmonically if the parameter a takes values close to 1 and harmonically when $a \approx 0$. Kjurkchieva

et al. [29] has conducted intensive photometric and spectral observations of the variable star V2551 Cyg. They have also studied its RVC and have found it to be a pulsating star that pulsate with the fundamental mode. Das et al. [12], analysed the light curve of RR Lyrae variable and also studied the multiband theoretical light curves of RR Lyrae stars.

1.3 Fundamental equations determining the equilibrium structure of a gaseous sphere

A system of equations which governs the equilibrium structures of a gaseous sphere in hydrostatics and thermal equilibrium, especially as they pertain to the problems of the equilibrium structure of stellar models, is well established in literature. Let P and ρ represent pressure and the density respectively at a point distant r from the center of the sphere and $M(r)$ the mass contained within the sphere of radius r . Also, let T be the temperature at a point distant r from the center of the sphere. $L(r)$ the net amount of energy crossing a spherical surface of radius r per second and ε the rate of energy generation from thermo-nuclear processes per gram per second,

The the equation of conservation of mass is given by

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho \quad (1.3.1)$$

The equation of hydrostatic equilibrium gives

$$\frac{dP}{dr} = -G \frac{M(r)}{r^2} \rho \quad (1.3.2)$$

where G is the gravitational constant.

The luminosity equation is

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho \varepsilon \quad (1.3.3)$$

and the energy transport equation states

$$\frac{dT}{dr} = -\frac{3\kappa}{4ac} \frac{\rho}{T^3} \frac{L(r)}{4\pi r^2} \quad (1.3.4)$$

in case of radiative equilibrium and for convective equilibrium,

$$\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right) \frac{T}{P} \frac{dP}{dr} \quad (1.3.5)$$

where κ is the mass absorption coefficient of the gas, c is the velocity of light, a is the Stefan-Boltzmann constant and γ is an adiabatic exponent which is equal to ratio of specific heats at constant pressure and volume (for a perfect non degenerate particle gas in the absence of radiation ($\gamma = 1$)). In addition to the above four differential equations we also require three more explicit relations which characterize more specifically the behavior of the interior gas. They are the equation of state, the equation for the mass absorption for energy generation by thermonuclear processes. The equations may be formally represented by

$$P = P(\rho, T, \text{chemical composition}) \quad (1.3.6)$$

$$\kappa = \kappa(\rho, T, \text{chemical composition}) \quad (1.3.7)$$

and

$$\varepsilon = \varepsilon(\rho, T, \text{chemical composition}) \quad (1.3.8)$$

Equations (1.3.1-1.3.5) are to be satisfied in every layer of gaseous sphere. In addition to these, certain boundary conditions are also to be satisfied. It is obvious from the definitions of $M(r)$ and $L(r)$ that at the center, we have

$$r = 0 : M(r) = 0, L(r) = 0 \quad (1.3.9)$$

$$r = R : M(r) = M, L(r) = L \quad (1.3.10)$$

where the symbol R , M and L represent total radius, total mass and total energy radiated by the gaseous sphere respectively. The pressure and the temperature at the outermost surface of a gaseous sphere, taken as the representative of star, are often approximated by what are generally known as zero boundary conditions. Under these conditions we take

$$r = R : P = 0 \text{ and } T = 0 \quad (1.3.11)$$

Conditions (1.3.11) are reasonably accurate for most of the theoretical stellar models. If moreover, more accurate computations are to be done then we may replace (1.3.11) by

$$r = R : P = P_s \text{ and } T = T_s \quad (1.3.12)$$

where P_s is determined from the pressure of the upper atmospheric layers and T_s is determined from the effective temperature of the star

The problem in which thermal properties are either not important or not to be investigated, the equilibrium structure of such gaseous sphere may be determined by solving equations (1.3.1-1.3.2) using some appropriate equation of state together with boundary conditions:

At the center

$$r = 0 : M(r) = 0 \quad (1.3.13)$$

At the surface

$$r = R : M(r) = M, P = 0 \text{ or } P_s, \rho = 0 \text{ or } \rho_s \quad (1.3.14)$$

A several theoretical as well as numerical studies for the equilibrium structures of gaseous spheres are available in literature (Chandrasekhar [5] , Schwarzschild [67], Eddington [16], Menzel et al. [41], Cox and Giuli [10], Kippenhahn and Weigert [26])

1.3.1 Averaging technique of Kippenhahn and Thomas

Kippenhahn and Thomas [25] defined topologically equivalent spherical surfaces to corresponding to the actual equipotential surfaces of a RTD model to study the effect of rotation and tidal distortion on the equilibrium structure of a gaseous sphere. On the equivalent spherical surfaces, they defined quantities such \bar{f} , \bar{g} etc., respectively certain averages of the quantities f , g respectively, on the actual equipotential surfaces. Thus, if ψ denotes the total potential arises due to gravitational, rotational and tidal forces of a RTD model at an arbitrary point (x, y, z) then $\psi(x, y, z) = \text{constant}$ represents an equipotential surface. Let V_ψ and S_ψ be the volume and the surface area of this equipotential surface $\psi = \text{constant}$. Then in analogy with a sphere, Kippenhahn and Thomas [25] defined a variable r_ψ by the relation.

$$V_\psi = \frac{4}{3}\Pi r_\psi^3, \quad (1.3.15)$$

where r_ψ is the radius of the topologically equivalent spherical surface. They also defined the mean value of any function $f(x, y, z)$ over the equipotential surface $\psi = \text{constant}$ by the relation

$$\bar{f} = \frac{1}{S_\psi} \int_{\psi=\text{const.}} f d\sigma, \quad (1.3.16)$$

where $d\sigma$ denotes the surface element of an equipotential surface $\psi = \text{constant}$. \bar{f} represents the value of f over the topologically equivalent spherical surface. Clearly \bar{f} is a function of ψ

only can be determined for each equipotential surface $\psi = \text{constant}$. Also by definition

$$S_\psi = \int_{\psi=\text{const.}} f d\sigma, \quad (1.3.17)$$

(clearly S_ψ is in general not equal to $4\pi r_\psi^2$) Corresponding to the usual definition of ' g ', the acceleration due to gravity, Kippenhahn and Thomas [25] defined a function $g(x, y, z)$ by the relation

$$g = \frac{d\psi}{dn}, \quad (1.3.18)$$

dn being the distance between two neighbouring equipotential surfaces $\psi = \text{constant}$ and $\psi + d\psi = \text{constant}$. The distance dn in general is not constant. Thus using the equation (1.3.16) the mean value \bar{g} and g^{-1} , respectively, may be defined by the relations

$$\bar{g} = \frac{1}{S_\psi} \int_{\psi=\text{const.}} \frac{d\psi}{dn} d\sigma \quad (1.3.19)$$

and

$$g^{-1} = \frac{1}{S_\psi} \int_{\psi=\text{const.}} \left(\frac{d\psi}{dn}\right)^{-1} d\sigma \quad (1.3.20)$$

The mean values \bar{g} and g^{-1} defined over the equipotential surface $\psi = \text{constant}$ are clearly function of ψ only and represent the value of g , g^{-1} , respectively, over the topologically equivalent spherical surface. The volume dV_ψ enclosed between the equipotential surfaces $\psi = \text{constant}$ and $\psi + d\psi = \text{constant}$ is given by

$$dV_\psi = \int_{\psi=\text{const.}} dn d\sigma = d\psi \int_{\psi=\text{const.}} \left(\frac{d\psi}{dn}\right)^{-1} d\sigma = S_\psi g^{-1} d\psi. \quad (1.3.21)$$

Kippenhahn and Thomas [25] also defined nondimensional parameters u, v, w as

$$u = \frac{S_\psi}{4\pi r_\psi^2}, \quad v = \bar{g} \frac{r_\psi^2}{GM_\psi}, \quad w = g^{-1} \frac{GM_\psi}{r_\psi^2} \quad (1.3.22)$$

where M_ψ is the mass enclosed by equipotential surface $\psi = \text{constant}$

Thus, the RTD equipotential surface $\psi = \text{constant}$ may be considered as topologically equivalent to a sphere of radius r_ψ for which the various quantities are defined by the above relations. (it is important to note that if ψ is the gravitational potential of a sphere then $\psi = \text{constant}$ are spherical surface with $r_\psi = r$ and therefore, $u = 1$. Also, on these spherical surfaces $g = \frac{GM_\psi}{r_\psi^2}$ is constant and hence $u = w = 1$.)

Equation (1.3.15) to (1.3.22) can be applied to gravitational fields of gaseous spheres dis-

torted by rotation and tidal forces. In hydrostatics equilibrium it is obvious that the equipotential surfaces are also surfaces of equipressure and equidensity. Therefore, pressure P_ψ and the density ρ_ψ are also constant on an equipotential surface .

Using equation (1.3.15), the mass dM_ψ within the equipotential surface $\psi = \text{constant}$ and $\psi + d\psi = \text{constant}$ is obtained by

$$dM_\psi = dV_\psi \rho_\psi = 4\pi r_\psi^2 \rho_\psi dr_\psi \quad (1.3.23)$$

Thus, we get

$$\frac{dM_\psi}{dr_\psi} = 4\pi r_\psi^2 \rho_\psi \quad (1.3.24)$$

From equation (1.3.21) and (1.3.36), we have

$$d\psi = \frac{d\psi}{dV_\psi} dV_\psi = \left(\frac{dV_\psi}{d\psi}\right)^{-1} \frac{dM_\psi}{\rho_\psi} = \frac{dM_\psi}{S_\psi \bar{g}^{-1} \rho_\psi} \quad (1.3.25)$$

Using relations (1.3.22), we get

$$d\psi = \frac{GM_\psi dM_\psi}{4\pi r_\psi^4 \rho_\psi \bar{g}} \quad (1.3.26)$$

The condition for hydrostatic equilibrium, $\frac{dP_\psi}{d\psi} = -\rho_\psi$ be written with equation in the form

$$\frac{dP_\psi}{dM_\psi} = -\frac{GM_\psi}{4\pi r_\psi^4} f_p \quad (1.3.27)$$

where

$$f_p = \frac{1}{\bar{g}} = \frac{4\pi r_\psi^4}{GM_\psi} \frac{1}{S_\psi \bar{g}^{-1}} \quad (1.3.28)$$

Clearly, the factor f_p is a function of ψ only. If ψ is known the equipotential surfaces can be determined and consequently with then values S_ψ , r_ψ , \bar{g} and \bar{g}^{-1} for each equipotential surface can also be obtained . The M_ψ which depends on the density distribution ρ_ψ can be determined by integrating the equation (1.3.36). Similarly, the other structure equations derived by Kippenhahn and Thomas [25] which include the effects of rotation and tidal distortions on the equilibrium structure of a gaseous sphere are as follows.

For chemically homogeneous gaseous spheres, the nuclear energy generation rate ϵ depends only on ρ_ψ and the temperature T_ψ and is, therefore, constant on equipotential surfaces. Thus if L_ψ is the energy which passes per second through the equipotential surface $\psi = \text{constant}$, then

$$\frac{dL_\psi}{dM_\psi} = \epsilon \quad (1.3.29)$$

using (1.3.36) it can be written as

$$\frac{dL_\psi}{dr_\psi} = 4\pi r_\psi^2 \rho_\psi \epsilon \quad (1.3.30)$$

if the energy is transported by radiation, then the energy transport equation is

$$F_\psi = -\frac{4acT_\psi^3}{3\kappa} \frac{d\psi}{dn} \frac{dT_\psi}{dM_\psi} \frac{uw4\pi r_\psi^4}{GM_\psi} \quad (1.3.31)$$

where (F_ψ) is the radiative flux on the equipotential surface $\psi = \text{constant}$. By integrating F_ψ over the equipotential surface $\psi = \text{constant}$. we get

$$L_\psi = \int_{\psi=\text{const.}} F_\psi d\sigma = -\frac{64\pi^2 acT_\psi^2 r_\psi^4 u^2 v w \frac{dT_\psi}{dM_\psi}}{3\kappa} \quad (1.3.32)$$

so that

$$\frac{dT_\psi}{dM_\psi} = -\frac{3\kappa L_\psi}{64\pi^2 acT_\psi^3 r_\psi^4 u^2 v w} \quad (1.3.33)$$

using (1.3.36) this equation can be expressed as

$$\frac{dT_\psi}{dr_\psi} = -\frac{3\kappa \rho_\psi L_\psi}{16\pi acT_\psi^3 r_\psi^2} f_T \quad (1.3.34)$$

with

$$f_T = \frac{1}{u^2 v w} \quad (1.3.35)$$

Thus these four Equations (1.3.36), (1.3.28), (1.3.29) and (1.3.33) governs the equilibrium structures of a gaseous sphere distorted by rotation and tidal forces may now be summarised as:

$$\frac{dM_\psi}{dr_\psi} = 4\pi r_\psi^2 \rho_\psi \quad (1.3.36)$$

$$\frac{dP_\psi}{dM_\psi} = -\frac{GM_\psi}{4\pi r_\psi^4} f_P \quad (1.3.37)$$

$$\frac{dL_\psi}{dM_\psi} = \epsilon \quad (1.3.38)$$

and

$$\frac{dT_\psi}{dr_\psi} = -\frac{3\kappa\rho_\psi L_\psi}{16\pi acT_\psi^3 r_\psi^2} f_T \quad (1.3.39)$$

where

$$f_p = \frac{1}{uw} \quad (1.3.40)$$

$$f_T = \frac{1}{u^2vw} \quad (1.3.41)$$

The boundary conditions to be satisfied are:

$$M_\psi = 0, L_\psi = 0 \quad (1.3.42)$$

at the center $r_\psi = 0$

$$\begin{aligned} M_\psi &= M_0, L_\psi = L_{\psi s} \\ P_\psi &= 0, T_\psi = 0 \text{ or } P_\psi = P_{\psi s}, T_\psi = T_{\psi s} \end{aligned} \quad (1.3.43)$$

at the free surface $r_\psi = R_\psi$

In the above conditions M_0 is the total mass of the model and $L_{\psi s}, P_{\psi s}, T_{\psi s}$ are the values of L_ψ, P_ψ, T_ψ respectively, on the outer most equipotential surface.

If there is no distortion, then $u = v = w = 1$ and also $f_p = f_T = 1$. Clearly, the above equations reduce to the usual equations governing the equilibrium structure of an undistorted gaseous sphere. Kippenhahn and Thomas [25] advocated the use of these equations to determine the inner structure of stars distorted by rotation and tidal forces.

1.3.2 Roche equipotential

A binary system of stars consists of a pair of stars rotating about their axes as well as revolving around their common center of mass. Of the two components of binary stars, one is generally larger in size and mass (called primary) than the other (called secondary). Because of rotation and tidal effects of the companion, the components of a binary system become RTD stars. In order to investigate the problems of structure and stability of such stars the concepts of Roche equipotential and Roche limits have often been discussed in literature. Following Kopal [28] and Mohan and Singh ([46], [47]), the results on Roche equipotentials of practical interest to the present study are summarized below.

Let M_0 and M_1 be the masses of the primary and secondary components of a binary system assumed to be gaseous spheres. Since primary is assumed to be much larger than the secondary ($M_0 > M_1$). Let D be the distance between the centers of these two masses. Further, suppose that the position of two components of this binary system is referred to a rectangular system of cartesian coordinates having the origin at the centers of the two components, and X – axis along the line joining the center of two components, and Z – axis perpendicular to the plane of the orbit of the two components. Then ψ the total potential of the gravitational, rotational and other disturbing forces acting at an arbitrary point $P(x, y, z)$, which is not inside any of these two gaseous spheres, can be expressed as

$$\psi = G \frac{M_0}{r_1} + G \frac{M_1}{r_2} + \frac{1}{2} \Omega^2 \left[\left(x - \frac{M_1 D}{M_0 + M_1} \right)^2 + y^2 \right] \quad (1.3.44)$$

where $r_1^2 = x^2 + y^2 + z^2$, $r_2^2 = (D - x)^2 + y^2 + z^2$, G is the gravitational constant, r_1 and r_2 , respectively, the distance of the arbitrary point P from the centers of these two masses M_0 and M_1 , and Ω the angular velocity of rotation of the system about an axis perpendicular to the XY – plane and passing through the center of gravity of the system. The three terms on the right hand side of (1.3.44) are, respectively, the potential arising from the mass primary M_0 , the disturbing potential of its companion of mass M_1 and the potential arising from the centrifugal forces. Equation (1.3.44) strictly holds at points which are outside the components of binary system.

If we assume Roche model for the primary and a point mass for the secondary component, equation (1.3.44) holds everywhere. Also, if the angular Velocity Ω is assumed to be identical with Keplerian angular Velocity Ω_k , that is

$$\Omega^2 = \Omega_k^2 = G \frac{M_0 + M_1}{D^3} \quad (1.3.45)$$

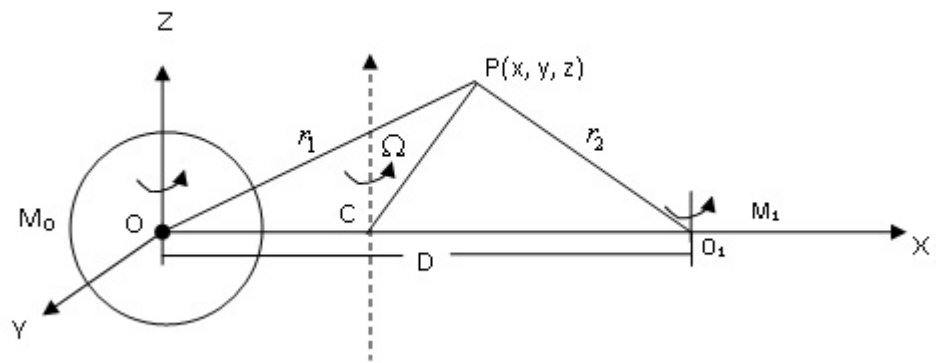


Figure 1.1. Axis of Reference for a binary system

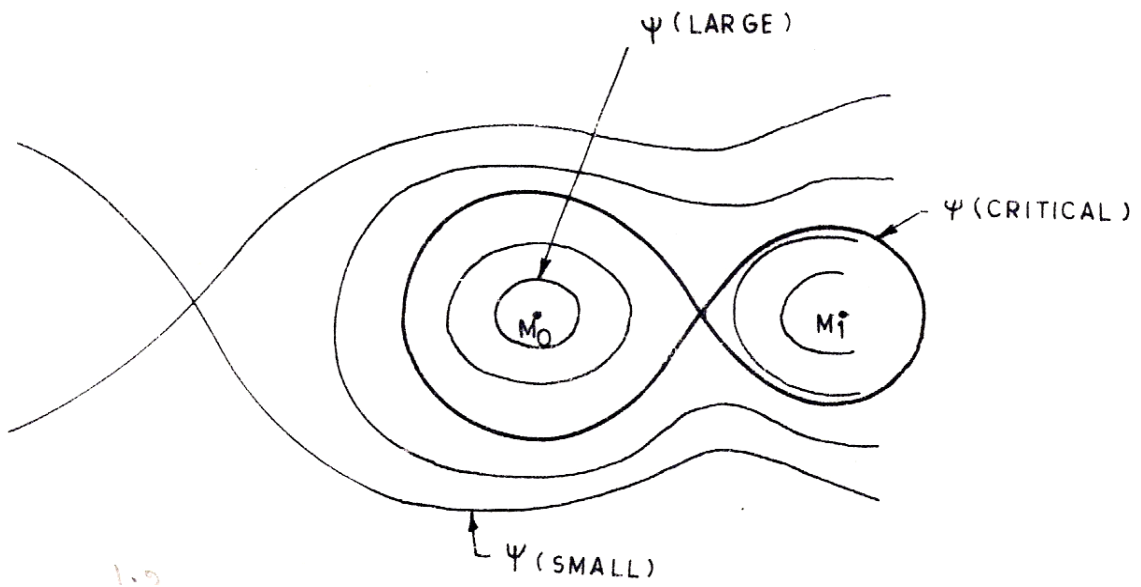


Figure 1.2. Roche equipotential surfaces (two dimensional)

Then

$$n = \frac{q + 1}{2} \quad (1.3.46)$$

So, in nondimensional form equation (1.3.44) can be expressed as

$$\psi^* = \frac{1}{r^*} + q \left[\frac{1}{\sqrt{1 - 2\lambda r^* + r^{*2}}} - \lambda r^* \right] + n r^{*2} (1 - \nu^2) \quad (1.3.47)$$

where

$$\psi^* = \frac{D\psi}{GM_0} - \frac{M_1^2}{2M_0(M_0 + M_1)}$$

is the nondimensional form of total potential ψ and $r^* = \frac{r}{D}$ is nondimensional form of r . Also $\lambda = \sin \theta \cos \phi$, $\mu = \sin \theta \sin \phi$ and $\nu = \cos \theta$ (r, θ, ϕ being the polar spherical coordinate of the point P). Moreover,

$$q = \frac{M_1}{M_0} \quad (1.3.48)$$

ψ^* is a nondimensional parameter representing the ratio of mass of the secondary over primary and $2n$ represents the square of the normalized angular velocity Ω . The equation (1.3.44) reduces to the potential of a purely rotating spherical model if $q = 0$. For $n = 0$, it reduces to the potential of a non-rotating spherical model distorted by the tidal effects of the companion alone. The surfaces generated by setting $\psi = \text{constant}$ on the left hand side of the equation (1.3.44) are referred to the Roche equipotentials. Roche equipotentials in nondimensional form may be represented by $\psi^* = \text{constant}$ where ψ^* is same as defined in equation (1.3.47). The form of Roche-equipotential depends entirely upon the values of ψ . If ψ is large the corresponding equipotentials consist of two separate ovals, around each of the two mass point. For specified values of M_0, M_1, Ω and D the right hand side of equation (1.3.44) can be large only if r_1 and r_2 becomes small. Therefore, large values of ψ correspond to equipotentials which differ little from spheres surrounding each of the two mass centers. With decreasing values of ψ , these spherical equipotential surface become oval shaped and get elongated in the direction of the center of gravity of the system until for a certain critical value of ψ , which is characteristics of each mass ratio, both oval shaped surface unite at a single point on the x-axis to form a dumbbell like configuration. These limiting values of ψ are called Roche limits. For certain mass, ratios Kopal [28] computed the numerical values of Roche limits in the case of synchronous binary stars for values of q ranging from zero to one.

Defining a non-dimensional variable r_0 by the relation

$$r_0 = \frac{1}{\psi^* - q} \quad (1.3.49)$$

Kopal [28] has also shown that on the surface of Roche equipotential (r, θ, ϕ) are connected through the relation

$$r^* = r_0[1 + C_3r_0^3 + C_4r_0^4 + C_5r_0^5 + C_6r_0^6 + C_7r_0^7 + C_8r_0^8 + C_9r_0^9 + C_{10}r_0^{10} \dots] \quad (1.3.50)$$

where

$$\begin{aligned} C_3 &= qP_2 + n(1 - \nu^2), C_4 = qP_3, C_5 = qP_4, \\ C_6 &= qP_5 + 3C_3^2, C_7 = qP_6 + 7qC_3^2P_3, \\ C_8 &= qP_7 + 8qC_3P_4 + 4q^2P_3^2 \\ C_9 &= qP_8 + 9qC_3P_5 + 9q^2P_3P_4 \\ C_{10} &= qP_9 + 10qC_3P_6 + 5q^2(P_4^2 + 2P_3P_5) \end{aligned} \quad (1.3.51)$$

Here, $P_j = p_j(\lambda)$ are the Legendre polynomials and terms upto second order of smallness in n and q have been in equation (1.3.50). This relation helps to obtain the shape of a Roche equipotentials $\psi = \text{constant}$. The volume enclosed by the equipotential surface $\psi^* = \text{constant}$ is given by

$$V_\psi = \frac{2}{3} \int_{-1}^1 \int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}} \frac{r^3}{\mu} d\lambda d\nu \quad (1.3.52)$$

Kopal [28] has shown that the explicit expression of V_ψ in term of r_0 defined by equation (1.3.49), can be explicitly obtained as

$$V_\psi = \frac{4}{3}\pi D^3 r_0^3 \left[1 + 2nr_0^3 + \left(\frac{12}{15}q^2 + \frac{8}{5}nq + \frac{32}{5}n^2 \right) r_0^6 + \frac{15}{7}q^2r_0^8 + 2q^2r_0^{10} + \dots \right] \quad (1.3.53)$$

Also, averaging parameter r_ψ given by

$$r_\psi = \left(\frac{3V_\psi}{4\pi} \right)^{\frac{1}{3}} = Dr_0 \left[1 + \frac{2n}{3}r_0^3 + \left(\frac{4}{5}q^2 + \frac{8}{15}nq + \frac{76}{45}n^2 \right) r_0^6 + \frac{5}{7}q^2r_0^8 + \frac{2}{3}q^2r_0^{10} + \dots \right] \quad (1.3.54)$$

where terms upto second order of smallness in n and q have been retained.

1.4 Equilibrium structures of RTD polytropic models of stars

If we assumed a polytropic model of star subjected to rotational and tidal forces then its structure will be a RTD polytropic model. Using the approach discussed in section 1.3.2, we may further approximate the equipotential surfaces of this distorted model by Roche equipotentials and use Kippenhahn and Thomas [25] averaging technique to determine its structure.

Let P_ψ and ρ_ψ denotes pressure and density respectively the equipotential surface $\psi = \text{constant}$ of the distorted model. Then the value of the density and the pressure on the equivalent equipotential surface of the corresponding spherical shell will also be P_ψ and ρ_ψ respectively. We shall assume that the topologically equivalent spherical model of the RTD polytropic model also behave like a polytropic model of the same polytropic index so that P_ψ and ρ_ψ of this distorted model are connected through the polytropic relations of the type

$$P_\psi = P_{c\psi} \theta_\psi^{N+1}, \quad \rho_\psi = \rho_{c\psi} \theta_\psi^N \quad (1.4.1)$$

where $P_{c\psi}$ and $\rho_{c\psi}$ are values of P_ψ and ρ_ψ at the center, N is Polytropic index, θ ($0 \leq \theta \leq 1$) is a parameter depending upon the distance of the chosen point from the center and θ_ψ is the value of θ on the equipotential surface $\psi = \text{constant}$. Using the pressure and mass equation, Mohan and Saxena [44] developed structure equation of RTD polytropic model given as

$$\frac{d}{dr_0} \left[H(n, q, r_0) \frac{d\theta_\psi}{dr_0} \right] = -\frac{\xi_u}{K^2} \theta_\psi^N r_0^2 f_1(n, q, r_0) \quad (1.4.2)$$

where

$$H(n, q, r_0) = \frac{r_0^2}{f_2} = r_0^2 \left[1 + \left(\frac{2}{5}q^2 + \frac{4}{15}nq + \frac{16}{15}n^2 \right) r_0^6 + \frac{9}{14}q^2 r_0^8 + \frac{8}{9}q^2 r_0^{10} \right] \quad (1.4.3)$$

and

$$f_1(n, q, r_0) = 1 + 4nr_0^3 + \left[\left(\frac{36}{5}q^2 + \frac{24}{5}nq + \frac{96}{5}n^2 \right) r_0^6 + \frac{55}{7}q^2 r_0^8 + \frac{26}{3}q^2 r_0^{10} \right] \quad (1.4.4)$$

$$f_2(n, q, r_0) = 1 - \left[\left(\frac{2}{5}q^2 + \frac{4}{15}nq + \frac{48}{45}n^2 \right) r_0^6 - \frac{9}{14}q^2 r_0^8 + \frac{8}{9}q^2 r_0^{10} \right] \quad (1.4.5)$$

In above expressions $r_0 = \frac{1}{\psi - q}$, (ψ is nondimensional form of Roche equipotential), $K = \frac{R_u}{D}$ (Where R_u is the undistorted radius of primary and D is the separation between the centers of the primary and secondary components of the binary system) and ξ_u is the value of ξ (where ξ is Lane-Emden variable: specifically we have value of $\xi_u = 3.65375, 6.89685$ and 14.97155

corresponding to polytropic index $N = 1.5, 3.0$ and 4.0 respectively) at the outermost surface of Polytropic models. The symbols n and q are the same as define in section 1.3 As regards the boundary conditions, P_ψ and ρ_ψ must be maximum at the center and zero at the free surface. These require θ_ψ to be maximum at the center and zero at the free surface. This leads to the condition: θ_ψ and $\frac{d\theta_\psi}{dr_0}$ at the center and $\theta_\psi = 0$ at the free surface. Thus the boundary conditions which equation (1.4.2) has to satisfy are

$$\theta_\psi = 1, \frac{d\theta_\psi}{dr_0} \quad (1.4.6)$$

at the center $r_0 = 0$

$$\theta_\psi = 0 \quad (1.4.7)$$

at the surface $r_0 = r_{0s}$

where r_{0s} is the value of r_0 at surface For no distortions ($f_1=f_2=1$), equation (1.4.2) reduces to the usual Lane-Emden equation governing the equilibrium structure of an undistorted polytropic model of index N in nondimensional form. To find the numerical solution of the second-order nonlinear differential equation (1.4.2) subject to the boundary conditions (1.4.6-1.4.7), we integrate equation (1.4.2) for certain choices of the values of N, ξ, k, n and q with boundary conditions (1.4.6-1.4.7). The integration may be continued till θ_ψ first becomes zero. The value of r_0 (i.e r_{0s}) when θ_ψ first becomes zero determines the outermost free surface of the topologically equivalent spherical model. Once the solution of the equation (1.4.2) are obtained, we know the values of θ_ψ for various values of the nondimensional independent variable r_0 varying from zero to r_{0s} . The pressure P_ψ and the density ρ_ψ on various equipotentials of the distorted model may now be obtained through the relation (1.4.1) in the same manner as is done for undistorted polytropic model. Also, radius r_ψ of the topologically equivalent spherical surface corresponding to the equipotential surface $\psi^{**} = \text{constant}$ can be written as

$$r_\psi = \left(\frac{\alpha \xi_u}{K} \right) r_0 \left[1 + \frac{2}{3} n r_0^3 + \left(\frac{4}{5} q^2 + \frac{76}{45} n^2 + \frac{8}{15} n q \right) r_0^6 + \frac{5}{7} q^2 r_0^8 + \frac{2}{3} q^2 r_0^{10} + \dots \right] \quad (1.4.8)$$

1.5 Eigenvalued boundary value problem determining the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of RTD polytropic models of stars

In order to determine the eigenfrequencies of pseudo radial modes of oscillations of RTD polytropic models, we have used the equation developed by Mohan and Saxena [45]. This equation

in nondimensional form is given by

$$H_1 \frac{d^2 \eta}{dr_0^2} + H_2 \frac{d\eta}{dr_0^2} + (H_3 \omega^2 - H_4) \eta = 0 \quad (1.5.1)$$

where

$$\begin{aligned} H_1 &= 1 - \frac{16}{3} n r_0^3 - \left(\frac{56}{5} q^2 + \frac{112}{15} n q + \frac{104}{45} n^2 \right) r_0^6 - \frac{90}{7} q^2 r_0^8 - \frac{44}{3} q^2 r_0^{10} + \dots \\ H_2 &= \frac{1}{r_0} \left[\left(4 - \frac{64}{3} n r_0^3 - \left(\frac{296}{5} q^2 + \frac{592}{15} n q + \frac{1064}{45} n^2 \right) r_0^6 - \frac{560}{7} q^2 r_0^8 \right. \right. \\ &\quad \left. \left. - \frac{316}{3} q^2 r_0^{10} + \dots \right) + (N+1) \left(\frac{1}{\theta_\psi} \frac{d\theta_\psi}{dx} \right) r_0 H_1 \right] \\ H_3 &= \frac{(N+1)}{3\gamma r_{0s}^2} \xi_u^2 K \frac{\bar{\rho}}{\rho_c} \frac{1}{\theta_\psi} \\ H_4 &= - \left(3 - \frac{4}{\gamma_1} \right) (N+1) \left(\frac{1}{\theta_\psi} \frac{d\theta_\psi}{dr_0} \right) \frac{1}{r_0} \left[1 - \frac{10n}{3} n r_0^3 - \left(\frac{32}{5} q^2 + \frac{64}{15} n q + \frac{188}{45} n^2 \right) r_0^6 \right. \\ &\quad \left. - \frac{50}{7} q^2 r_0^8 - 8q^2 r_0^{10} + \dots \right], \end{aligned}$$

where

$$\omega^2 = \frac{D^3 r_{0s}^2 \sigma^2}{GM_0} \text{ and } r_0 = x r_{0s},$$

n is a rotation parameter ($2n = \Omega^2$, Ω is the normalized angular velocity of rotation), q is the ratio of the mass of secondary to primary star, D is the separation between the two components of a binary stars, G is the universal gravitational constant, M_0 is the total mass of the star, ω^2 is the non-dimensional form of the actual eigenfrequencies of oscillation σ , r_{0s} being the value of r_0 on the outermost surface, N is the polytropic index, H_1 , H_2 , H_3 and H_4 are nonlinear functions of distortions parameters n and q , ρ_c represents density at center, $\bar{\rho}$ is the average density of the undistorted polytropic model of star and ξ_u is the value of ξ (where ξ is Lane-Emden variable: specifically we have value of $\xi_u = 3.65375, 6.89685$ corresponding to polytropic index $N = 1.5, 3.0$ respectively) at the outermost surface of Polytropic models.

Equation (1.5.1) is the general equation in nondimensional form which determine the eigenfrequencies of small pseudo-radial modes of oscillations of RTD polytropic model when terms upto second order of smallness in n , q and upto r_0^{10} in r_0 are retained. For numerically evaluation of the eigenfrequencies, the second order differential equation (1.5.1) is to be solved numerically subject which require η to be finite at points corresponding to the center ($r_0 = 0$) and the free surface ($r_0 = r_{0s}$) of the model.

On setting $n = q = 0$, equation (1.5.1) reduces to the usual equation which determines the eigenfrequencies of small adiabatic radial oscillations of an undistorted polytropic model (cf. Singh [71]). We can study the effects of rotational distortions alone by setting $q = 0$ in (1.5.1) and by setting $n = 0$ in (1.5.1) the effects of tidal distortions alone on the eigenfrequencies of small radial oscillations of polytropic models can be studied. Also by setting $n = \frac{(q+1)}{2}$, we can discuss the problem of small adiabatic pseudo-radial oscillations of the primary component that the primary component has a polytropic structure.

1.6 Eigenvalued boundary value problem determining the eigenfrequencies of small adiabatic nonradial modes of oscillations of RTD polytropic models of stars

Mohan and Saxena ([45]) formulated the eigenvalued boundary value problem for determining the eigenfrequencies of small adiabatic nonradial modes of oscillations of RTD polytropic models of stars, which are given by

$$\begin{aligned} \frac{d\zeta}{dx} + B_1\zeta + \left(B_2 + \frac{1}{\omega^2} B_3 \right) \eta + \frac{1}{\omega^2} B_3 \phi &= 0, \\ \frac{d\eta}{dx} + (E_1\omega^2 + E_2) \zeta + E_3\eta + E_4\phi + \frac{d\phi}{dx} &= 0, \end{aligned} \quad (1.6.1)$$

and

$$\frac{d^2\phi}{dx^2} + F_1\frac{d\phi}{dx} + F_2\zeta + F_3\eta + F_4\phi = 0$$

$$B_1 = \frac{l+1}{x} + \frac{N+1}{\gamma} \left(\frac{1}{\theta_\psi} \frac{d\theta_\psi}{dx} \right),$$

$$\begin{aligned} B_2 = \frac{(N+1)\xi_u^2 r_{0s}^3 x}{2\gamma k^2 \theta_\psi} \left[1 + 4n(xr_{0s})^3 + \left(\frac{36}{5}q^2 + \frac{72}{15}nq + \frac{864}{45}n^2 \right) (xr_{0s})^6 \right. \\ \left. + \frac{55}{7}q^2(xr_{0s})^8 + \frac{26}{3}q^2(xr_{0s})^{10} + \dots \right], \end{aligned}$$

$$\begin{aligned} B_3 = -\frac{3l(l+1)r_{0s}^4}{2k^3x} \left(\frac{\rho_c}{\bar{\rho}} \right) \left[1 + \frac{8n}{3}(xr_{0s})^3 + \left(\frac{28}{5}q^2 + \frac{56}{15}nq + \frac{532}{45}n^2 \right) (xr_{0s})^6 \right. \\ \left. + \frac{45}{7}q^2(xr_{0s})^8 + \frac{22}{3}q^2(xr_{0s})^{10} + \dots \right], \end{aligned}$$

$$E_1 = -\frac{2k^3}{3r_{0s}^4x} \left(\frac{\rho_c}{\bar{\rho}} \right) \left[1 + \frac{4n}{3}(xr_{0s})^3 + \left(4q^2 + \frac{40}{15}nq + \frac{280}{45}n^2 \right) (xr_{0s})^6 \right]$$

$$\begin{aligned}
& + 5q^2(xr_{0s})^8 + 6q^2(xr_{0s})^{10} + \dots \Big], \\
E_2 &= 2 \frac{k^2}{\xi_u^2} \left(N - \frac{N+1}{\gamma} \right) \frac{1}{\theta_\psi} \left(\frac{d\theta_\psi}{dx} \right)^2 \left(\frac{1}{xr_{0s}^3} \right) \left[1 - 4n(xr_{0s})^3 \right. \\
& \quad \left. - \left(\frac{36}{5}q^2 + \frac{144}{45}nq + \frac{72}{15}n^2 \right) (xr_{0s})^6 - \frac{55}{7}q^2(xr_{0s})^8 - \frac{26}{3}6q^2(xr_{0s})^{10} + \dots \right], \\
E_3 &= \frac{l}{x} + \left(N - \frac{N+1}{\gamma} \right) \frac{1}{\theta_\psi} \frac{d\theta_\psi}{dx}, \\
E_4 &= \frac{l}{x}, \\
F_1 &= \frac{1}{x} \left[2(l+1) - 4n(xr_{0s})^3 - \left(24q^2 + 16nq + 32n^2 \right) (xr_{0s})^6 \right. \\
& \quad \left. - 40q^2(xr_{0s})^8 - 60q^2(xr_{0s})^{10} + \dots \right], \\
F_2 &= \frac{2}{r_{0s}x} \left(N - \frac{N+1}{\gamma} \right) \theta_\psi^{N-1} \frac{d\theta_\psi}{dx} \left[1 + \frac{4n}{3}(xr_{0s})^3 + \left(4q^2 + \frac{8}{3}nq + \frac{56}{9}n^2 \right) (xr_{0s})^6 \right. \\
& \quad \left. + 5q^2(xr_{0s})^8 + 6q^2(xr_{0s})^{10} + \dots \right], \\
F_3 &= - \left(\frac{N+1}{\gamma} \right) \frac{\xi_u^2}{k^2} \theta_\psi^{N-1} r_{0s}^2 \left[1 + \frac{16n}{3}(xr_{0s})^3 + \left(\frac{56}{5}q^2 + \frac{112}{15}nq + \frac{1384}{45}n^2 \right) (xr_{0s})^6 \right. \\
& \quad \left. + \frac{90}{7}q^2(xr_{0s})^8 + \frac{44}{3}q^2(xr_{0s})^{10} + \dots \right], \\
F_4 &= - \frac{l}{x^2} \left[l \left\{ 4n(xr_{0s})^3 + \left(\frac{48}{5}q^2 + \frac{96}{15}nq + \frac{972}{45}n^2 \right) (xr_{0s})^6 \right. \right. \\
& \quad \left. \left. + \frac{80}{7}q^2(xr_{0s})^8 + \frac{40}{3}q^2(xr_{0s})^{10} + \dots \right\} + 8n(xr_{0s})^3 \right. \\
& \quad \left. + \left(\frac{168}{5}q^2 + \frac{336}{15}nq + \frac{804}{5}n^2 \right) (xr_{0s})^6 + \frac{360}{7}q^2(xr_{0s})^8 + \frac{220}{3}q^2(xr_{0s})^{10} + \dots \right]
\end{aligned}$$

and

$$\omega^2 = \frac{D^3 r_{0s}^3 \sigma^2}{GM_0}$$

ω being the nondimensional form of the eigenfrequency σ . $\xi_u, \rho_c, \bar{\rho}$

The eigenvalue problem (1.6.1) determining the eigenfrequencies of nonradial modes of oscillations of RTD polytropic model of star has to be solved subject to the boundary conditions

at the center and the free space.

The boundary condition at the center ($x = 0$) for the case distorted polytropic model is give by

$$\eta + \phi = \frac{2K^3\omega^3}{3lr_{0s}^4} \left(\frac{\bar{\rho}}{\rho_c} \right) \zeta, \quad \frac{d\phi}{dx} = 0 \quad (1.6.2)$$

The boundary condition at the free space ($x=1$) for the case of polytropic models is given by

$$\eta r_{0s}^3 \left[1 + 4nr_{0s}^3 + \left(\frac{36}{5}q^2 + \frac{72}{15}nq + \frac{864}{45}n^2 \right) r_{0s}^6 + \frac{55}{7}q^2r_{0s}^8 + \frac{26}{3}q^2r_{0s}^{10} + 2\frac{K^2}{\zeta_u^2} \frac{d\theta_\psi}{dx} \zeta = 0 \right] \quad (1.6.3)$$

and

$$\frac{d\phi}{dx} + \phi \left[(l+1) \left\{ 1 + 2nr_{0s}^3 + \left(\frac{24}{5}q^2 + \frac{48}{15}nq + \frac{396}{45}n^2 \right) r_{0s}^6 + \frac{407}{7}q^2r_{0s}^8 + \frac{20}{3}q^2r_{0s}^{10} \right\} + l \right] = 0 \quad (1.6.4)$$

The system of differential equations (1.6.1) with the boundary conditions (1.6.2 - 1.6.4) constitute a eigenvalued boundary value problem determining the effects of rotation and tidal distortions on the eigenfrequencies of nonradial oscillations of polytropic models.

For distortion i.e. $n = q = 0$, the system of differential equations (1.6.1) along with the boundary conditions (1.6.2-1.6.4) reduce to the usual eigenvalued boundary value problem determining the eigenfrequencies of nonradial oscillations of undistorted polytropic models.

On setting $q = 0$ or $n = 0$ separately in the system of the equation (1.6.1) along with the boundary conditions (1.6.2-1.6.4) we can study the effect of rotational distortion alone or the effect of tidal distortion alone on the eigenfrequencies of the various modes of nonradial oscillations of the polytropic models. By setting $n = \frac{(q+1)}{2}$, we may also discuss the problem of non-radial oscillations of a RTD primary component of a synchronously rotating binary system assuming that the primary component has polytropic structure.

1.7 Anharmonic radial pulsation equation for a RTD Stellar model

Following, the approach used by Rosseland [61], the equation of anharmonic pulsation of RTD stellar model can be written by applying it to the topological equivalent spherical model. Using the averaging concept of Kippenhahn and Thomas [25] as used by Mohan and Saxena [45], we assume $r_{1\psi}$ to be average of the displacement on the equipotential surface ($\psi = constant$)

and write it as

$$r_{1\psi} = \eta_1(r_{0\psi})q_1(t) + \eta_2(r_{0\psi})q_2(t) + \eta_3(r_{0\psi})q_3(t) + \dots, \quad (1.7.1)$$

where $\eta_1, \eta_2, \eta_3, \dots$ are the solutions for the various modes of pulsation equation as discussed by Mohan and Saxena [45] and $q_1(t), q_2(t), q_3(t), \dots$ are the functions of time to be determined by substituting in the exact equation of motion.

Let r_ψ denote the radius of the topologically equivalent spherical model which corresponds to an equipotential surface $\psi=\text{constant}$ of RTD model. Also, let R_ψ be the value of r_ψ on the outermost equipotential surface of the model. So, the equation of motion of a RTD gaseous sphere can be written as

$$\ddot{r}_\psi = -g - \frac{1}{\rho_\psi} \frac{dP_\psi}{dr_\psi} \quad (1.7.2)$$

On substituting $r_\psi = r_{0\psi}(1 + r_{1\psi})$ in equation (1.7.2) and using the adiabatic condition $P_\psi = \text{constant } \rho_\psi^\gamma$ and the equation of continuity $\rho_\psi r_\psi^2 dr_\psi = \rho_{0\psi} r_{0\psi}^2 dr_{0\psi}$ (subscript zero refers to the equilibrium value) we get after some simplification

$$r_{0\psi} \ddot{r}_{1\psi} = - (1 + r_{1\psi})^{-2} g_0 - \frac{(1 + r_{1\psi})^2}{\rho_{0\psi}} \frac{\partial}{\partial r_{0\psi}} \left[P_{0\psi} (1 + r_{1\psi})^{-2\gamma} \left(1 + r_{1\psi} + r_{0\psi} \frac{\partial r_{1\psi}}{\partial r_{0\psi}} \right)^{-\gamma} \right] \quad (1.7.3)$$

where $P_{0\psi}, \rho_{0\psi}$ and $r_{0\psi}$ are the equilibrium values of pressure P_ψ , density ρ_ψ , and distance from the center r_ψ , γ is the ratio of specific heat, and $r_{1\psi}$ is the relative displacement $\frac{r_\psi - r_{0\psi}}{r_{0\psi}}$. The dots denote differentiation with respect to time t .

The terms q_1, q_2, q_3, \dots appearing in Equations (1.7.1) can be separated by using the orthogonality property

$$\int_0^{R_\psi} \rho_{0\psi} r_{0\psi}^4 \eta_j \eta_k dr_{0\psi} = 0, \quad j \neq k \quad (1.7.4)$$

On expanding the right hand side and retaining terms up to second order, Equation (1.7.3) can be written as

$$\begin{aligned} \rho_{0\psi} r_{0\psi} \ddot{r}_\psi &= P'_{0\psi} [(3\gamma - 4)r_{1\psi} + \gamma r_{0\psi} r'_{1\psi}] + \gamma P_{0\psi} (4r'_{1\psi} + r_{0\psi} r''_{1\psi}) \\ &- P'_{0\psi} \left[\frac{1}{2} (3\gamma - 4)(3\gamma + 1) r_{1\psi}^2 + \gamma (3\gamma - 1) r_{0\psi} r_{1\psi} r'_{1\psi} + \frac{1}{2} \gamma (\gamma + 1) r_{0\psi}^2 r_{1\psi}'^2 \right] \\ &- \gamma P_{0\psi} \left[4(3\gamma - 1) r_{1\psi} r'_{1\psi} + 2(2\gamma + 1) r_{0\psi}^2 r_{1\psi}'^2 + (3\gamma - 1) r_{0\psi} r_{1\psi} r''_{1\psi} + (\gamma + 1) r_{0\psi}^2 r_{1\psi}' r''_{1\psi} \right] \end{aligned} \quad (1.7.5)$$

where the prime symbols denote differentiation with respect to $r_{0\psi}$. On multiplying Equation (1.7.5) by $r_{0\psi}^3 \eta_k dr_{0\psi}$, integrating over the entire star and then using Equation (1.7.1), the left hand side of Equation (1.7.5) gives

$$\int_0^{R\psi} \rho_{0\psi} r_{0\psi}^4 \eta_k \left(\sum_j (\eta_j \ddot{q}_j) \right) dr_{0\psi} = I_k \ddot{q}_k, \quad (1.7.6)$$

where

$$I_k = \int_0^{R\psi} \rho_{0\psi} r_{0\psi}^4 \eta_k^2 dr_{0\psi}. \quad (1.7.7)$$

The other terms vanish due to orthogonality in Equation (1.7.4). On the right hand side of Equation (1.7.5), the linear terms yield

$$\int_0^{R\psi} \gamma P_{0\psi} r_{0\psi}^4 \eta_k \left[r_{1\psi}'' + \frac{4 - \mu}{r_{0\psi}} r_{1\psi}' - \left(3 - \frac{4}{\gamma} \right) \frac{\mu}{r_{0\psi}^2} r_{1\psi} \right] dr_{0\psi} \quad (1.7.8)$$

where

$$\mu = -\frac{r_{0\psi}}{P_{0\psi}} \frac{dP_{0\psi}}{dr_{0\psi}}$$

Substituting for $r_{1\psi}$ from Equation (1.7.1) and using the fact that $\eta_1, \eta_2, \eta_3, \dots$ are the eigenfunctions of pulsation equation discussed by Mohan and Saxena [45] with corresponding periods $(\frac{2\pi}{\sigma_1}, \frac{2\pi}{\sigma_2}, \frac{2\pi}{\sigma_3}, \dots)$, Equation (1.7.8) can further be written as

$$-\int_0^{R\psi} \gamma P_{0\psi} r_{0\psi}^4 \eta_k \sum_j \frac{\sigma_j^2 \rho_{0\psi}}{\gamma P_{0\psi}} (\eta_j q_j) dr_{0\psi} = -I_k \sigma_k^2 q_k. \quad (1.7.9)$$

Now, the second order terms on the right hand side of Equation (1.7.5), after integrating the last two terms by parts to get rid of $r_{1\psi}''$, gives

$$\begin{aligned} & -\frac{1}{2}(3\gamma - 4)(3\gamma + 1) \int_0^{R\psi} P_{0\psi}' r_{0\psi}^3 \eta_k r_{1\psi}^2 dr_{0\psi} + \frac{1}{2}\gamma(3\gamma - 1) \int_0^{R\psi} P_{0\psi} r_{0\psi}^4 \eta_k r_{1\psi}'^2 dr_{0\psi} \\ & + \gamma(3\gamma - 1) \int_0^{R\psi} P_{0\psi} r_{0\psi}^4 \eta_k r_{1\psi} r_{1\psi}' dr_{0\psi} + \frac{1}{2}\gamma(\gamma + 1) \int_0^{R\psi} P_{0\psi} r_{0\psi}^5 \eta_k' r_{1\psi}'^2 dr_{0\psi} \end{aligned} \quad (1.7.10)$$

Substituting this expression (1.7.1) for $r_{1\psi}$, it may further be expressed as

$$\gamma \left[\sum_i D_{ii,k} q_i^2 + 2 \sum_{\substack{i,j \\ i \neq j}} D_{ij,k} q_i q_j \right], \quad (j \geq i), \quad (1.7.11)$$

where

$$\begin{aligned}
D_{ij,k} = & -\frac{1}{2} \left(3 - \frac{4}{\gamma} \right) (3\gamma + 1) \int_0^{R_\psi} P'_{0\psi} r_{0\psi}^3 \eta_i \eta_j \eta_k dr_{0\psi} \\
& + \frac{1}{2} (3\gamma - 1) \int_0^{R_\psi} P_{0\psi} r_{0\psi}^4 (\eta_i \eta'_j \eta'_k + \eta'_i \eta_j \eta'_k + \eta'_i \eta'_j \eta_k) dr_{0\psi} \\
& + \frac{1}{2} (\gamma + 1) \int_0^{R_\psi} P_{0\psi} r_{0\psi}^5 \eta'_i \eta'_j \eta'_k dr_{0\psi}
\end{aligned} \tag{1.7.12}$$

Thus, the pulsation equation breaks up into the equations

$$\ddot{q}_k + \sigma_k^2 q_k = \frac{\gamma}{I_k} \left[\sum_i D_{ii,k} q_i^2 + 2 \sum_{\substack{i,j \\ i \neq j}} D_{ij,k} q_i q_j \right], \quad (j \geq i) \tag{1.7.13}$$

where $k=1,2,3,\dots$. It is more convenient to take the time variable τ instead of t such that

$$\tau = \sigma_1 t$$

where σ_1 is the eigenfrequency of fundamental mode for stellar model. This makes the period of oscillation in the fundamental mode as 2π in the variable τ . So using $\tau = \sigma_1 t$, the Equation (1.7.13) becomes

$$\frac{d^2 q_k}{d\tau^2} + \beta_k q_k = \frac{\gamma}{I_k \sigma_1^2} \left[\sum_i D_{ii,k} q_i^2 + 2 \sum_{\substack{i,j \\ i \neq j}} D_{ij,k} q_i q_j \right], \quad (j \geq i) \text{ for } k = 1, 2, 3, \dots \tag{1.7.14}$$

where

$$\beta_k = \frac{\sigma_k^2}{\sigma_1^2} \tag{1.7.15}$$

Using r_ψ , ρ_ψ and P_ψ (that denote the equilibrium values on the equipotential surface) in place of $r_{0\psi}$, $\rho_{0\psi}$ and $P_{0\psi}$ respectively, the expression for I_k and $D_{ij,k}$ finally becomes.

$$I_k = \int_0^{R_\psi} \rho_\psi r_\psi^4 \eta_k^2 dr_\psi. \tag{1.7.16}$$

and

$$D_{ij,k} = -\frac{1}{2} \left(3 - \frac{4}{\gamma} \right) (3\gamma + 1) \int_0^{R_\psi} P'_\psi r_\psi^3 \eta_i \eta_j \eta_k dr_\psi$$

$$\begin{aligned}
& + \frac{1}{2}(3\gamma - 1) \int_0^{R_\psi} P_\psi r_\psi^4 (\eta_i \eta_j' \eta_k' + \eta_i' \eta_j \eta_k' + \eta_i' \eta_j' \eta_k) dr_\psi \\
& + \frac{1}{2}(\gamma + 1) \int_0^{R_\psi} P_\psi r_\psi^5 \eta_i' \eta_j' \eta_k' dr_\psi
\end{aligned} \tag{1.7.17}$$

So, Equations (1.7.14-1.7.17) constitute the equations of anharmonic radial pulsations for a RTD stellar model correct up to second order. The coefficients $D_{ij,k}$ and I_k are constants which can be computed for a given stellar model. In calculating $D_{ij,k}$ and I_k we take the amplitude of pulsation to be normalized to unity at the star's surface, so that the displacement q_b at the surface is given by

$$q_b = q_1 + q_2 + q_3 + \dots \tag{1.7.18}$$

1.8 Method for solving the equation of Anharmonic pulsation for a RTD Stellar model

In order to solve the equation (1.7.14), we consider the equations for q_1 , q_2 and q_3 . These may be written as

$$\begin{aligned}
\frac{d^2 q_1}{d\tau^2} + q_1 &= A_{11,1} q_1^2 + 2A_{12,1} q_1 q_2 + 2A_{13,1} q_1 q_3 \\
\frac{d^2 q_2}{d\tau^2} + \beta_2 q_2 &= A_{11,2} q_1^2 + 2A_{12,2} q_1 q_2 + 2A_{13,2} q_1 q_3 \\
\frac{d^2 q_3}{d\tau^2} + \beta_3 q_3 &= A_{11,3} q_1^2 + 2A_{12,3} q_1 q_2 + 2A_{13,3} q_1 q_3
\end{aligned} \tag{1.8.1}$$

where $A_{11,1}$, $A_{12,1}$, $A_{13,1}$...are constants that are to be determined. Following Prasad [58], we assume the solution for these equations of the form

$$\begin{aligned}
q_1 &= a_{0,1} + a_{1,1} \cos n_1 \tau + a_{2,1} \cos 2n_1 \tau + a_{3,1} \cos 3n_1 \tau + \dots \\
q_2 &= a_{0,2} + a_{1,2} \cos n_1 \tau + a_{2,2} \cos 2n_1 \tau + a_{3,2} \cos 3n_1 \tau + \dots \\
q_3 &= a_{0,3} + a_{1,3} \cos n_1 \tau + a_{2,3} \cos 2n_1 \tau + a_{3,3} \cos 3n_1 \tau + \dots
\end{aligned} \tag{1.8.2}$$

where $a_{0,1}$, $a_{1,1}$, $a_{2,1}$, \dots , $a_{0,2}$, $a_{1,2}$, $a_{2,2}$, \dots , $a_{0,3}$, $a_{1,3}$, $a_{2,3}$, \dots and n_1 are constants that are to be determined. These values of q_1 , q_2 and q_3 are substituted in Equation (1.8.1). On equating the constant terms and the coefficients of $\cos kn_1 \tau$ (for different k) to zero we get

$$\begin{aligned}
a_{0,1} &= A_{11,1} \left[a_{0,1}^2 + \frac{1}{2} a_{1,1}^2 + \frac{1}{2} a_{2,1}^2 + \frac{1}{2} a_{3,1}^2 + \dots \right] \\
&+ 2A_{12,1} \left[a_{0,1} a_{0,2} + \frac{1}{2} a_{1,1} a_{1,2} + \frac{1}{2} a_{2,1} a_{2,2} + \frac{1}{2} a_{3,1} a_{3,2} + \dots \right]
\end{aligned}$$

$$\begin{aligned}
& + 2A_{13,1} \left[a_{0,1}a_{0,3} + \frac{1}{2}a_{1,1}a_{1,3} + \frac{1}{2}a_{2,1}a_{2,3} + \frac{1}{2}a_{3,1}a_{3,3} + \dots \right] \quad (1.8.3a) \\
(1 - n_1^2) a_{1,1} = & A_{11,1} \left[2a_{0,1}a_{1,1} + a_{1,1}a_{2,1} + a_{2,1}a_{3,1} + \dots \right] \\
& + 2A_{12,1} \left[a_{1,1}a_{0,2} + a_{0,1}a_{1,2} + a_{2,1}a_{0,2} + \frac{1}{2} \left(a_{1,1}a_{2,2} + a_{2,1}a_{1,2} \right) \right. \\
& \left. + \frac{1}{2} (a_{2,1}a_{3,2} + a_{3,1}a_{3,2}) + \dots \right] \\
& + 2A_{13,1} \left[2a_{0,1}a_{1,3} + a_{1,1}a_{0,3} + \frac{1}{2}a_{1,1}a_{2,3} + \frac{1}{2}a_{2,1}a_{1,3} + \frac{1}{2}a_{3,1}a_{2,3} + \dots \right] \quad (1.8.3b)
\end{aligned}$$

$$\begin{aligned}
(1 - n_1^2 k^2) a_{k,1} = & A_{11,1} \left[\frac{1}{2} \sum_{i=0}^k a_{i,1} a_{k-i,1} + \sum_{i=0}^{\infty} a_{i,1} a_{k+i,1} \right] \\
& + A_{12,1} \left[\frac{1}{2} \sum_{i=0}^k a_{i,1} a_{k-i,2} + \frac{1}{2} \sum_{i=0}^{\infty} \left(a_{i,1} a_{k+i,2} + a_{k+i,1} a_{i,2} \right) \right] \\
& + A_{13,1} \left[\frac{1}{2} \sum_{i=0}^k a_{i,1} a_{k-i,3} + \frac{1}{2} \sum_{i=0}^{\infty} \left(a_{i,1} a_{k+i,3} + a_{k+i,1} a_{i,3} \right) \right] \quad (1.8.3c)
\end{aligned}$$

We derive similar equations from the second equation in Equation (1.8.1) which are same as above but with $(\beta_2 - k^2 n_1^2) a_{k,2}$ in place of $(1 - k^2 n_1^2) a_{k,1}$ and $A_{11,2}$, $A_{13,2}$ and $A_{22,2}$ in place of $A_{11,1}$, $A_{12,1}$ and $A_{13,1}$ respectively.

In order to solve these algebraic equations, defined in Equation (1.8.3a-1.8.3c), we suppose that $a_{1,1}$ is known small quantity and we determine the other a's in terms of $a_{1,1}$. Considering Equation (1.8.3a) we see that $a_{0,1}$ is a small quantity of the second order, all other terms are square or products and they contain the term $a_{1,1}^2$.

Similarly we find that $a_{2,1}$ is a quantity of the second order, $a_{3,1}$ is of the third and in general $a_{k,1} (k > 1)$ is of the k^{th} order. Considering the equation for $a_{1,2}$ is of third order and $a_{2,2}$ is of the second order and in general $a_{k,2} (k > 1)$ is of the k^{th} order. Therefore, a first approximation to the solution of the equation (1.8.3a-1.8.3c) can be given as:

$$\begin{aligned}
a_{0,1} &= \frac{1}{2} A_{11,1} a_{1,1}^2, \\
a_{2,1} &= -\frac{1}{6} A_{11,1} a_{1,1}^2, \\
a_{3,1} &= \frac{1}{16} \left[\frac{1}{3} A_{11,1}^2 + \frac{A_{12,1}}{(4 - \beta_2)} A_{11,2} \right] a_{1,1}^3, \\
&\dots\dots\dots \\
&\dots\dots\dots
\end{aligned}$$

$$\begin{aligned}
a_{0,2} &= \frac{1}{2} A_{11,2} a_{1,1}^2 / \beta_2, \\
a_{1,2} &= \left[\frac{5}{6} A_{11,1} + \frac{(8 - 3\beta_2)}{2\beta_2(4 - \beta_2)} A_{12,2} \right] \frac{A_{11,2} a_{1,1}^3}{(\beta_2 - 1)} \\
a_{2,2} &= -\frac{1}{2} \frac{A_{11,2} a_{1,1}^2}{(4 - \beta_2)} \\
a_{3,2} &= \frac{1}{2} \left[\frac{1}{3} A_{11,1} + \frac{A_{12,2}}{(4 - \beta_2)} \right] \frac{A_{11,2} a_{1,1}^3}{(9 - \beta_2)}
\end{aligned}$$

.....
.....

$$\begin{aligned}
a_{0,3} &= \frac{1}{2} A_{11,3} a_{1,1}^2 / \beta_3, \\
a_{1,3} &= \left[\frac{5}{6} A_{11,1} + \frac{(8 - 3\beta_3)}{2\beta_3(4 - \beta_3)} A_{13,3} \right] \frac{A_{11,3} a_{1,1}^3}{(\beta_3 - 1)} \\
a_{2,3} &= -\frac{1}{2} \frac{A_{11,3} a_{1,1}^2}{(4 - \beta_3)} \\
a_{3,3} &= \frac{1}{2} \left[\frac{1}{3} A_{11,1} + \frac{A_{13,3}}{(4 - \beta_3)} \right] \frac{A_{11,3} a_{1,1}^3}{(9 - \beta_3)}
\end{aligned}$$

.....
.....

and

$$n_1^2 = 1 - \left[\frac{5}{6} A_{11,1}^2 + (8 - 3\beta_2) A_{12,1} A_{11,2} / (2\beta_2 (4 - \beta_2)) \right] a_{1,1}^2$$

These value of $a_{i,1}$, $a_{i,2}$ and n_1 are substituted in Equations (1.8.3a-1.8.3c) and a better approximation is obtained. The new values are resubstituted in (1.8.3a-1.8.3c) and the process is repeated a number of times till we attain the desired accuracy.

1.9 The Present Work

The present study is primarily a possibility of using the the methodology of Mohan and Saxena [45] that utilizes the averaging technique of Kippenhahn and Thomas [25] and results of Roche equipotentials as given by Kopal [28] to incorporate the effects of rotations and/or tidal distortions in our present study. The various assumptions that have been made in this methodology are (i) Star is highly centrally condensed so that the actual equipotential surfaces of rotationally and tidally distorted star can be approximated by Roche equipotential surfaces. (ii) Binary systems are considered to be circular, synchronous and aligned so that the axis of rotation of the star under investigation (primary component) is at right angle to the line joining the center of

the primary and secondary component of the binary system. (iii) The rotational velocity as well as ratio of the mass of the companion causing tidal distortions to the mass of the primary star are not too large so that models do not deviate too much from spherical symmetry and, hence the terms beyond second order of smallness in rotational distortion parameter and tidal distortion parameter can be neglected in various mathematical expressions used. (iv) The distorted model of the star is well within its Roche lobe. (v) The equipotential surfaces of the distorted star are also the surfaces of equipressure and equidensity. (vi) Oscillations are barotropic so that fluid elements on the equipotential surface of the star in its equilibrium position always remain on it during the entire period of small adiabatic oscillations. So in the view of these assumptions our present analysis is applicable to highly centrally condensed pulsating variable stars with small oscillations that are (i) Single and rotating slowly. (ii) Slow rotating primary components of synchronous, circular and aligned binary systems in which the mass of the secondary is very less than the mass of the primary star.

In the present work we have formulated the anharmonic pulsation equation for RTD polytropic models of pulsating variable stars using the theory of Rosseland [61] along with Mohan and Saxena ([44], [45]) approach. The methodology of Prasad [58] has been then used to solve these equations. We have considered the problem of determining the effects of rotation and/or tidal distortions on the shapes of RVC of polytropic models of pulsating variable stars. This approach has also been extended to study the effect of amplitude on the anharmonic radial oscillations and hence on the shapes of the RVC of polytropic models of rotationally distorted (RD) and RTD polytropic models of pulsating variable stars. We have also studied the effects of non-radial modes along with radial modes, that is mixing modes on the shapes of RVC of RD and RTD polytropic models of pulsating variable stars.

Chapterwise summary of the work presented in the subsequent chapters of this thesis is as follows:

Chapter 1

Chapter 1 is introductory in nature. In this chapter we have briefly discussed the astrophysical significance of the problem of determining equilibrium structures, periods of small adiabatic oscillations and radial velocity curves of the rotationally and/or tidally distorted stellar models. We have discussed in brief fundamental system of differential equations that governs the equilibrium structure of stars, average technique of Kippenhahn and Thomas [25] and concepts of Roche equipotential. The equations that governs the equilibrium structures and small adiabatic modes of oscillations of rotationally and tidally distorted polytropic models of stars (as obtained earlier by Mohan and Saxena [45]) has been presented. The anharmonic theory of pulsating star as given by Rosseland [61] has been used in conjunction with Mohan and Saxena

[45] approach to obtain the anharmonic pulsation equation of RTD stellar models of star. The methodology of Prasad [58] has been then used to solve the anharmonic pulsation equation of RTD stellar models. A brief survey of the literature available on the subject and summary of the work presented in the succeeding chapters of the thesis are also presented in this chapter.

Chapter 2

In Chapter 2, we formulated the anharmonic pulsation equation for rotationally and/or tidally distorted polytropic models of pulsating variable stars using the theory of Rosseland [61] along with Mohan and Saxena ([44], [45]) approach. The methodology of Prasad [58] has been used to solve these equations. Numerical computations have been performed to determine the solution of anharmonic equation for certain rotationally and/or tidally distorted polytropic models (with polytropic indices $N = 1.5$ and $N = 3.0$) of pulsating variable stars. The radial velocity curves of certain rotationally and/or tidally distorted polytropic models of pulsating variable stars has been then obtained. The numerical results and figures has been then analysed to draw certain conclusions.

Chapter 3

In Chapter 3, we study the effects of the higher radial modes on the anharmonic oscillations and hence on the radial velocity curves of the polytropic models of rotationally and/or tidally distorted pulsating variable stars. For this purpose, we have considered the following cases of radial oscillations: (i) fundamental mode ; (ii) fundamental and the first mode ; (iii) fundamental and the next two modes and (iv) fundamental and the next three higher modes of pulsation. Numerical computations have been performed to determine the solution of anharmonic pulsation equation for certain rotationally distorted and rotationally and tidally distorted polytropic models (with polytropic indices $N = 1.5$, $N = 3.0$ and $N = 4.0$) of pulsating variable stars . Radial velocity curves have been then drawn for all the considered models. Finally, numerical results and figures have been analysed to obtain certain conclusions.

Chapter 4

The work done in Chapter 2 and Chapter 3 has further been extended and presented in Chapter 4. In this Chapter we have studied the effect of varying the relative amplitude on the anharmonic radial modes of oscillations and hence on the shapes of the radial velocity curves of rotationally

and tidally distorted polytropic models of pulsating variable stars. The interaction of various modes have also been taken into account (as discussed in Chapter 3) while studying the effect of relative amplitude. Numerical computations have been performed to determine the solution of anharmonic pulsation equation for certain rotationally and tidally distorted polytropic models of stars with indices 1.5, 3.0 and 4.0 and for certain values of relative amplitude of oscillation. The radial velocity curves of all the considered models have been then plotted and finally certain conclusions has been drawn.

Chapter 5

In Chapter 5, we have studied the effects of non-radial modes along with radial modes of oscillations,(that is mixing modes) on the shapes of radial velocity curves of rotationally and tidally distorted polytropic models of pulsating variable stars. For this purpose, following cases of radial and nonradial modes of oscillations have been considered: (i) fundamental non-radial mode (ii) fundamental radial mode (iii) fundamental radial and fundamental non-radial modes (iv) fundamental radial, fundamental non-radial and first radial modes (v) fundamental radial, fundamental non-radial, first radial modes and first gravity modes. The anharmonic pulsation equation that governs the anharmonic pulsations (radial and nonradial) of certain rotationally and tidally distorted polytropic models of pulsating variable stars has been solved numerically for all such cases. Radial velocity curves has been drawn for all the considered models and certain conclusions drawn.

Chapter 6

The astrophysical significance of the present work along with the limitations and scope of the present work are discussed briefly in the concluding Chapter 6.

Chapter 2

Effects of rotation and tidal distortion on the shapes of RVC of polytropic models of pulsating variable stars ¹

The linear theory of the radial pulsations predicts a symmetrical sine curve as the shapes of radial velocity curve for a pulsating stars. However, observations show that Cepheid variables have their velocity curves, and also their light curves which are far from being represented by simple sine curves. The discrepancies in the observed and predicted data are attributed to various effects such as coupling of the various modes of oscillations. Theory of anharmonic pulsations has frequently used in literature to discuss the effects of coupling of various modes on the radial velocity of a variable star (Rosseland ([61]))

In this chapter, we have considered the problem of determining the effects of rotation and/or tidal distortions on the radial velocity curve of a pulsating variable star based on the theory of anharmonic pulsation as given by Rosseland [61]. Anharmonic radial pulsation equation of a RTD polytropic model of a star have been formulated in section 2.1. The equations of anharmonic pulsation for the first three modes of RTD polytropic models of stars have been solved numerically in section 2.2 to obtain their effects on the RVC . Certain observations based on the results obtained in the present chapter are finally made in section 2.3. Numerical solution of anharmonic equation has been discussed in section 2.4.

2.1 Anharmonic radial pulsation equation of a RTD polytropic model of a star

Suppose a polytropic model of a star is subjected to rotation and tidal distortions. Then structure of such a star will be RTD polytropic model. Following the approach adopted by Mohan and Saxena [45], we may approximate the equipotential surfaces of this distorted model by Roche equipotential and use Kippenhahn and Thomas [25] averaging technique determine its structure.

On Substituting the values of P_ψ and ρ_ψ given in equation (1.4.1) of chapter 1, in the

¹Kumar,Tarun, et al. “*Effects of rotation and tidal distortion on the shapes of radial velocity curves of polytropic models of pulsating variable stars*”, Research in Astronomy and Astrophysics, Iop, Vol 18, pages 63, 2018.

anharmonic pulsation equation for RTD stellar model discussed in Equation (1.7.14) of Chapter 1, we obtain the anharmonic pulsation equation for RTD polytropic model of stars as:

$$\frac{d^2 q_k}{d\tau^2} + \beta_k q_k = \frac{\gamma}{I_k^* \omega_1^2} \left[\sum_i A_{ii,k}^* q_i^2 + 2 \sum_{\substack{i,j \\ i \neq j}} A_{ij,k}^* q_i q_j \right], \quad (j \geq i) \text{ for } k = 1, 2, 3 \dots \quad (2.1.1)$$

where

$$\beta_k = \frac{\omega_k^2}{\omega_1^2}; A_{ii,k}^* = \frac{D_{ii,k}^*}{\omega_1^2 I_k^*}; A_{ij,k}^* = \frac{D_{ij,k}^*}{\omega_1^2 I_k^*}$$

$$I_k^* = \int_0^1 \theta^N x^4 \eta_k^2 dx$$

$$\begin{aligned} D_{ij,k}^* = & -\frac{(N+1)}{2} \left(3 - \frac{4}{\gamma} \right) (3\gamma + 1) \int_0^1 \theta^N \theta' x^3 \eta_i \eta_j \eta_k dx \\ & + \frac{1}{2} (3\gamma - 1) \int_0^1 x^4 \theta^{(N+1)} (\eta_i \eta_j' \eta_k' + \eta_i' \eta_j \eta_k' + \eta_i' \eta_j' \eta_k) dx \\ & + \frac{1}{2} (\gamma + 1) \int_0^1 x^5 \theta^{(N+1)} \eta_i' \eta_j' \eta_k' dx \end{aligned}$$

where γ is the ratio of specific heat, ω_1^2 is eigenfrequency of fundamental radial mode, x is non-dimensional variable of displacement varying from center to surface for a given polytropic model of a star and $\eta_1, \eta_2, \eta_3, \dots$ are the eigenfunctions of various modes of radial oscillation of RTD polytropic models of stars (obtained by solving pulsation Equation (1.5.1)). So, the Equation (2.1.1) governs the anharmonic radial pulsations of rotationally and /or tidally distorted polytropic models of stars. The coefficients $D_{ij,k}^*$ and I_k^* are constants which can be computed for a given rotationally and/or tidally distorted polytropic model of a star. Also the displacement q_b at the surface of the star can be obtained by using Equation (1.7.18)

2.2 Numerical Computations

Following the approach as discussed in Chapter 1, Section (1.8) to solve the anharmonic equation, we have solved the equation of anharmonic oscillation for the first three modes of pseudo radial oscillations of the RTD polytropic models of stars for polytropic indices 1.5, 3.0 and for

various values of distortion parameters n and q with $\gamma = \frac{5}{3}$. Simpson's one third rule has been used to numerically evaluate the coefficients I_k^* and $D_{ij,k}^*$ appearing in the anharmonic pulsation Equation (2.1.1). The values of $A_{ij,k}^*$ are presented in the Tables (2.4-2.9) for the possible models.

While solving the anharmonic equation we require various eigenfrequencies of pseudo radial modes of oscillations of RTD polytropic models. These are obtained by solving Equation (1.5.1) and presented in Table (2.1-2.3) for certain polytropic models of stars.

The skewness coefficient K (the ratio of the time of rise of the radial velocity curve from minimum to the maximum to the total pulsation period) has also been computed in each case and given in Table (2.10-2.12). Finally, radial velocity curves for each model (graph of $\frac{dq_b}{d\tau}$ against $n_1\tau$) of polytropic indices 1.5 and 3.0 has been obtained and are shown in Figs. (2.1-2.3).

In the present work, to study the effects of rotation and/or tidal distortion on the anharmonic radial oscillations and hence on the radial velocity curves of pulsating variable stars, we have considered the following polytropic models of pulsating variable stars: (i) undistorted star (single non-rotating star, no rotational or tidal effects ($n = 0, q = 0$)), (ii) single rotating star or RD star ($q = 0, n$ has some value), (iii) tidally distorted star ($n = 0, q$ has some value, although stars with only tidal distortion and no rotation does not exist observationally, however, from theoretical point of view and for the completeness of our study we have also considered the case of tidally distorted (TD) stars), (iv) a rotating star which is primary component in a synchronous, circular and aligned binary system or RTD star (n and q has some value, $2n = q + 1$ is the same as what was derived by Kopal [28] for synchronous binary stars assuming Keplerian angular velocity).

2.3 Concluding Observations

The results presented in Tables (2.1-2.3) represent the eigenfrequencies of the fundamental, first and second pseudo radial modes of oscillations, for RTD polytropic models for $\gamma = \frac{5}{3}$ and with polytropic index $N = 1.5, 3.0$. The eigenfrequencies of radial modes of RD, TD and, RTD polytropic models are smaller as compared to the values of undistorted polytropic model. These results are in accordance with previous results of Mohan and Saxena [45].

From Fig.(2.1), we observed that with increase in the value of n (or angular velocity of rotation of star or rotational distortion) the RVC for rotating polytropic models of pulsating variable stars deviates more from the RVC of undistorted polytropic model and this effect is more appreciable near points of maxima and minima. Also in the case of polytropic index $N = 1.5$, more deviation is observed near points of maxima and minima with increasing value of n as compared to polytropic model with index $N = 3.0$.

From Fig.(2.2), we can conclude that with increase in value of q (tidal distortions due to companion or secondary component) there is no appreciable effect on the shapes of radial velocity curves for TD polytropic models as compared to radial velocity curve for undistorted polytropic model.

It is clear from Fig.(2.3), that when combined effects of rotation (n) and tidal distortions (q) are considered then again radial velocity curves of these RTD polytropic models of pulsating variable stars deviate from radial velocity curve of undistorted model and these deviations are more near points of maxima and minima.

From Table (2.10-2.12), it can be observed that, as compared to undistorted model, there is decrease in the value of skewness coefficient K under the effect of rotational distortions , tidal distortions and rotational and tidal distortions. Also from Table (2.10), it can be concluded that the value of K decreases with increase in rotational distortions. However, from Table (2.11-2.12) we can observe that with increase in tidal distortions and rotation and tidal distortions there is no change in the value of skewness coefficient K .

So from the present study we can conclude that rotational effects causes more deviations in the shapes of RVC for pulsating variables stars as compared to tidal effects. Also these deviations in the shapes of RVC increases with increase in the effects of rotational distortions.

Table 2.1: Eigenvalues for Various Modes of RD Polytropic Models of Stars

	$N = 1.5$			$N = 3.0$		
	ω_1^2	ω_2^2	ω_3^2	ω_1^2	ω_2^2	ω_3^2
(n=0,q=0)	2.7058	12.5337	26.533	9.2545	16.9839	28.5540
(n=0.01,q=0.0)	2.6442	12.0984	25.6116	9.1138	16.6188	27.8497
(n=0.03,q=0.0)	2.5747	11.6478	24.5983	8.8233	15.8699	26.4272
(n=0.05,q=0.0)	2.4865	11.0428	23.2970	8.5190	15.0990	24.5980
(n=0.07,q=0.0)	2.3974	10.3877	21.9816	8.3061	14.6425	23.9485
(n=0.09,q=0.0)	2.3072	9.7802	20.6470	7.8535	13.4209	21.1310

Table 2.2: Eigenvalues for Various Modes of TD Polytropic Models of Stars

	$N = 1.5$			$N = 3.0$		
	ω_1^2	ω_2^2	ω_3^2	ω_1^2	ω_2^2	ω_3^2
(n=0,q=0)	2.7058	12.5337	26.5330	9.2545	16.9839	28.5540
(n=0.0,q=0.1)	2.7050	12.5260	26.5160	9.2522	16.9754	28.5377
(n=0.0,q=0.2)	2.7024	12.5026	26.4640	9.2450	16.9497	28.4760
(n=0.0,q=0.3)	2.6982	12.4636	26.3770	9.2333	16.9105	28.3776
(n=0.0,q=0.4)	2.6931	12.3947	26.2333	9.2162	16.8496	28.2289
(n=0.0,q=0.5)	2.6855	12.3378	26.1021	9.1941	16.7664	28.0370

Table 2.3: Eigenvalues for Various modes of RTD Polytropic Models of Stars

	$N = 1.5$			$N = 3.0$		
	ω_1^2	ω_2^2	ω_3^2	ω_1^2	ω_2^2	ω_3^2
(n=0,q=0)	2.7058	12.5337	26.5330	9.2545	16.9839	28.5540
(n=0.505,q=0.01)	2.4279	10.6362	22.4308	8.3099	14.5690	23.7700
(n=0.515,q=0.03)	2.4218	10.5909	22.3374	8.2874	14.5112	23.6428
(n=0.525,q=0.05)	2.4182	10.5123	22.3225	8.2654	14.4514	23.5060
(n=0.535,q=0.07)	2.4094	10.4994	22.1448	8.2418	14.3914	23.3520
(n=0.55,q=0.1)	2.3999	10.4282	21.9953	8.2064	14.3000	23.1500

Table 2.4: Coefficients $A_{ij,k}^*$ of the anharmonic pulation equation of the RD Polytropic model of index $N = 1.5$

i	j	k	$A_{ij,k}^*$					
			(n=0.0, q=0.0)	(n=0.01, q=0.0)	(n=0.03, q=0.0)	(n=0.05, q=0.0)	(n=0.07, q=0.0)	(n=0.09, q=0.0)
1	1	1	3.31231	3.40219	3.51283	3.66106	3.82290	4.00063
1	2	1	1.35696	1.38073	1.41236	1.45613	1.50643	1.56479
1	3	1	0.02743	-0.00573	0.015847	0.00873	0.00264	-0.00138
1	1	2	14.4451	14.95689	15.97793	17.15933	18.51124	20.08999
1	2	2	16.46809	17.24269	18.37087	19.88555	21.6597	23.77456
1	3	2	6.65729	6.80944	7.38326	8.00170	8.76204	9.70755
1	1	3	1.22131	-0.25376	0.75385	0.43140	0.13484	-0.073196
1	2	3	27.83933	27.80873	31.04755	33.53074	36.41042	39.87174
1	3	3	36.12107	37.64862	40.67610	44.24114	48.33155	53.06956

Table 2.5: Coefficients $A_{ij,k}^*$ of the anharmonic pulation equation of the RD Polytropic model of index $N = 3.0$

i	j	k	$A_{ij,k}^*$					
			(n=0.0, q=0.0)	(n=0.01, q=0.0)	(n=0.03, q=0.0)	(n=0.05, q=0.0)	(n=0.07, q=0.0)	(n=0.09, q=0.0)
1	1	1	0.97465	0.99671	1.04606	1.10458	1.09927	1.28304
1	2	1	0.44855	0.46174	0.49076	0.52392	0.51298	0.63582
1	3	1	0.10505	0.10985	0.11926	0.12982	0.12424	0.18971
1	1	2	5.29616	5.46698	5.85807	6.34924	6.75130	8.22679
1	2	2	3.83660	3.96707	4.25913	4.61944	4.90935	6.25329
1	3	2	1.64354	1.71116	1.84751	2.01451	2.14087	3.25355
1	1	3	8.01184	8.38203	9.26522	10.44854	11.47172	16.26754
1	2	3	10.6158	11.02734	12.0238	13.37862	15.01983	21.56199
1	3	3	8.54528	8.86136	9.60193	10.64789	12.34753	20.22074

Table 2.6: Coefficients $A_{ij,k}^*$ of the anharmonic pulation equation of the TD Polytropic model of index $N = 1.5$

i	j	k	$A_{ij,k}^*$					
			(n=0.0, q=0.0)	(n=0.0, q=0.1)	(n=0.0, q=0.2)	(n=0.0, q=0.3)	(n=0.0, q=0.4)	(n=0.0, q=0.5)
1	1	1	3.31231	3.31336	3.31684	3.32233	3.32922	3.33901
1	2	1	1.35696	1.35715	1.35779	1.35872	1.36019	1.36121
1	3	1	0.02743	0.02737	0.02714	0.02675	0.02685	0.02553
1	1	2	14.44512	14.45846	14.49666	14.55945	14.65903	14.76138
1	2	2	16.46809	16.48454	16.53318	16.61336	16.73216	16.87151
1	3	2	6.65729	6.66589	6.69143	6.73375	6.80064	6.87126
1	1	3	1.22131	1.21925	1.21047	1.19568	1.20474	1.14900
1	2	3	27.83933	27.86864	27.94965	28.08307	28.30783	28.51527
1	3	3	36.12107	36.16305	36.28221	36.47876	36.77976	37.11134

Table 2.7: Coefficients $A_{ij,k}^*$ of the anharmonic pulation equation of the TD Polytropic model of index $N = 3.0$

i	j	k	$A_{ij,k}^*$					
			(n=0.0, q=0.0)	(n=0.0, q=0.1)	(n=0.0, q=0.2)	(n=0.0, q=0.3)	(n=0.0, q=0.4)	(n=0.0, q=0.5)
1	1	1	0.97465	0.97469	0.97499	0.97542	0.97618	0.97721
1	2	1	0.44855	0.44858	0.44859	0.44857	0.44852	0.44860
1	3	1	0.10505	0.10502	0.10487	0.10455	0.10429	0.10397
1	1	2	5.29616	5.29548	5.31008	5.33448	5.37192	5.42118
1	2	2	3.83660	3.83610	3.84392	3.85703	3.87788	3.90596
1	3	2	1.64354	1.64329	1.64378	1.64395	1.64756	1.65292
1	1	3	8.01184	7.98046	8.02535	8.10116	8.21549	8.37344
1	2	3	10.61583	10.5780	10.62612	10.71098	10.83637	11.01595
1	3	3	8.54528	8.52565	8.55073	8.59833	8.67292	8.79246

Table 2.8: Coefficients $A_{ij,k}^*$ of the anharmonic pulsation equation of the RTD Polytopic model of index $N = 1.5$

i	j	k	$A_{ij,k}^*$					
			(n=0.0, q=0.0)	(n=0.505, q=0.01)	(n=0.515, q=0.03)	(n=0.525, q=0.03)	(n=0.535, q=0.07)	(n=0.55, q=0.1)
1	1	1	3.31231	3.76598	3.76720	3.75879	3.79993	3.81758
1	2	1	1.35696	1.48840	1.48893	1.48186	1.49876	1.50413
1	3	1	0.02743	0.00456	0.00442	0.00334	0.00340	0.00285
1	1	2	14.4451	18.02925	18.03482	18.1668	18.33277	18.49502
1	2	2	16.4680	21.02290	21.03273	21.2376	21.4241	21.63938
1	3	2	6.65729	8.48588	8.48923	8.59366	8.66572	8.76365
1	1	3	1.22131	0.23034	0.22330	0.17031	0.17320	0.14563
1	2	3	27.8393	35.37746	35.39046	35.7175	36.0303	36.38150
1	3	3	36.1210	46.87448	46.89429	47.3876	47.7916	48.28095

Table 2.9: Coefficients $A_{ij,k}^*$ of the anharmonic pulsation equation of the RTD Polytopic model of index $N = 3.0$

i	j	k	$A_{ij,k}^*$					
			(n=0.0, q=0.0)	(n=0.505, q=0.01)	(n=0.515, q=0.03)	(n=0.525, q=0.03)	(n=0.535, q=0.07)	(n=0.55, q=0.1)
1	1	1	0.97465	1.15018	1.15519	1.16060	1.16586	1.17459
1	2	1	0.44855	0.54983	0.55267	0.55590	0.55889	0.56389
1	3	1	0.10505	0.13892	0.14009	0.14151	0.14321	0.14532
1	1	2	5.29616	6.75115	6.80429	6.87199	6.92002	7.03432
1	2	2	3.83660	4.92309	4.96424	5.01643	5.05595	5.14737
1	3	2	1.64354	2.17605	2.20088	2.23217	2.26317	2.31989
1	1	3	8.01184	11.42663	11.57716	11.8660	11.9216	12.35315
1	2	3	10.6158	14.57736	14.77283	15.1410	15.2152	15.80812
1	3	3	8.54528	11.76512	11.96513	12.3094	12.4369	13.04652

2.4 Solution of Anharmonic equation for RD polytropic models with index $N = 1.5$

Case 1: (n=0.0, q=0.0)

$$\begin{aligned}
 q_1(\tau) &= 0.00538 + 0.05700 \cos n_1\tau - 0.00179 \cos 2n_1\tau - 0.00031 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots \\
 q_2(\tau) &= 0.00506 + 0.01424 \cos n_1\tau + 0.03713 \cos 2n_1\tau + 0.00071 \cos 3n_1\tau + 0.00004 \cos 4n_1\tau \dots \\
 q_3(\tau) &= 0.00253 + 0.00903 \cos n_1\tau + 0.00603 \cos 2n_1\tau + 0.03528 \cos 3n_1\tau + 0.00009 \cos 4n_1\tau \dots \\
 q_b(\tau) &= 0.01297 + 0.08027 \cos n_1\tau + 0.04137 \cos 2n_1\tau + 0.03568 \cos 3n_1\tau + 0.00013 \cos 4n_1\tau \dots
 \end{aligned}$$

Case 2: (n=0.01, q=0.0)

$$\begin{aligned}
 q_1(\tau) &= 0.00552 + 0.05700 \cos n_1\tau - 0.00184 \cos 2n_1\tau - 0.00037 \cos 3n_1\tau + 0.00001 \cos 4n_1\tau \dots \\
 q_2(\tau) &= 0.00531 + 0.01672 \cos n_1\tau + 0.04223 \cos 2n_1\tau - 0.00018 \cos 3n_1\tau + 0.00011 \cos 4n_1\tau \dots \\
 q_3(\tau) &= 0.00262 + 0.00991 \cos n_1\tau + 0.00619 \cos 2n_1\tau + 0.04009 \cos 3n_1\tau + 0.00027 \cos 4n_1\tau \dots \\
 q_b(\tau) &= 0.01345 + 0.08363 \cos n_1\tau + 0.04658 \cos 2n_1\tau + 0.03954 \cos 3n_1\tau + 0.00039 \cos 4n_1\tau \dots
 \end{aligned}$$

Case 3: (n=0.03, q=0.0)

$$\begin{aligned}
 q_1(\tau) &= 0.00570 + 0.05700 \cos n_1\tau - 0.00190 \cos 2n_1\tau - 0.00045 \cos 3n_1\tau + 0.00001 \cos 4n_1\tau \dots \\
 q_2(\tau) &= 0.00573 + 0.02059 \cos n_1\tau + 0.04954 \cos 2n_1\tau + 0.00077 \cos 3n_1\tau + 0.00009 \cos 4n_1\tau \dots \\
 q_3(\tau) &= 0.00389 + 0.01324 \cos n_1\tau + 0.00936 \cos 2n_1\tau + 0.05270 \cos 3n_1\tau + 0.00020 \cos 4n_1\tau \dots \\
 q_b(\tau) &= 0.01532 + 0.09083 \cos n_1\tau + 0.05700 \cos 2n_1\tau + 0.05302 \cos 3n_1\tau + 0.00030 \cos 4n_1\tau \dots
 \end{aligned}$$

Case 4: (n=0.05, q=0.0)

$$\begin{aligned}
 q_1(\tau) &= 0.00594 + 0.05700 \cos n_1\tau - 0.00198 \cos 2n_1\tau - 0.00060 \cos 3n_1\tau + 0.00001 \cos 4n_1\tau \dots \\
 q_2(\tau) &= 0.00627 + 0.02776 \cos n_1\tau + 0.06319 \cos 2n_1\tau + 0.00075 \cos 3n_1\tau + 0.00016 \cos 4n_1\tau \dots \\
 q_3(\tau) &= 0.00559 + 0.01804 \cos n_1\tau + 0.01337 \cos 2n_1\tau + 0.06971 \cos 3n_1\tau + 0.00038 \cos 4n_1\tau \dots \\
 q_b(\tau) &= 0.01780 + 0.10280 \cos n_1\tau + 0.07458 \cos 2n_1\tau + 0.06986 \cos 3n_1\tau + 0.00055 \cos 4n_1\tau \dots
 \end{aligned}$$

Case 5: (n=0.07, q=0.0)

$$\begin{aligned}
 q_1(\tau) &= 0.00621 + 0.05700 \cos n_1\tau - 0.00207 \cos 2n_1\tau - 0.00091 \cos 3n_1\tau + 0.00003 \cos 4n_1\tau \dots \\
 q_2(\tau) &= 0.00694 + 0.04189 \cos n_1\tau + 0.09037 \cos 2n_1\tau + 0.00021 \cos 3n_1\tau + 0.00038 \cos 4n_1\tau \dots \\
 q_3(\tau) &= 0.00911 + 0.02741 \cos n_1\tau + 0.02118 \cos 2n_1\tau + 0.09376 \cos 3n_1\tau + 0.00103 \cos 4n_1\tau \dots \\
 q_b(\tau) &= 0.02226 + 0.12630 \cos n_1\tau + 0.10948 \cos 2n_1\tau + 0.09306 \cos 3n_1\tau + 0.00144 \cos 4n_1\tau \dots
 \end{aligned}$$

Case 6: (n=0.09, q=0.0)

$$\begin{aligned}q_1(\tau) &= 0.00662 + 0.05750 \cos n_1\tau - 0.00220 \cos 2n_1\tau - 0.00150 \cos 3n_1\tau + 0.00005 \cos 4n_1\tau \dots \\q_2(\tau) &= 0.00783 + 0.06925 \cos n_1\tau + 0.13907 \cos 2n_1\tau + 0.00126 \cos 3n_1\tau + 0.00079 \cos 4n_1\tau \dots \\q_3(\tau) &= 0.01680 + 0.04572 \cos n_1\tau + 0.03767 \cos 2n_1\tau + 0.12206 \cos 3n_1\tau + 0.00263 \cos 4n_1\tau \dots \\q_b(\tau) &= 0.03125 + 0.17247 \cos n_1\tau + 0.17454 \cos 2n_1\tau + 0.12182 \cos 3n_1\tau + 0.00347 \cos 4n_1\tau \dots\end{aligned}$$

2.4.1 Solution of Anharmonic equation for TD polytropic models with index $N = 1.5$

Case 1: (n=0.0, q=0.1)

$$\begin{aligned}q_1(\tau) &= 0.00538 + 0.05700 \cos n_1\tau - 0.00179 \cos 2n_1\tau - 0.00031 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots \\q_2(\tau) &= 0.00507 + 0.01430 \cos n_1\tau + 0.03724 \cos 2n_1\tau + 0.00072 \cos 3n_1\tau + 0.00005 \cos 4n_1\tau \dots \\q_3(\tau) &= 0.00254 + 0.00907 \cos n_1\tau + 0.00606 \cos 2n_1\tau + 0.03547 \cos 3n_1\tau + 0.00009 \cos 4n_1\tau \dots \\q_b(\tau) &= 0.01292 + 0.08037 \cos n_1\tau + 0.04151 \cos 2n_1\tau + 0.03588 \cos 3n_1\tau + 0.00014 \cos 4n_1\tau \dots\end{aligned}$$

Case 2: (n=0.0, q=0.2)

$$\begin{aligned}q_1(\tau) &= 0.00538 + 0.05700 \cos n_1\tau - 0.00179 \cos 2n_1\tau - 0.00032 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots \\q_2(\tau) &= 0.00509 + 0.01446 \cos n_1\tau + 0.03759 \cos 2n_1\tau + 0.00072 \cos 3n_1\tau + 0.00005 \cos 4n_1\tau \dots \\q_3(\tau) &= 0.00257 + 0.00918 \cos n_1\tau + 0.00614 \cos 2n_1\tau + 0.03599 \cos 3n_1\tau + 0.00009 \cos 4n_1\tau \dots \\q_b(\tau) &= 0.01304 + 0.08064 \cos n_1\tau + 0.04194 \cos 2n_1\tau + 0.03639 \cos 3n_1\tau + 0.00014 \cos 4n_1\tau \dots\end{aligned}$$

Case 3: (n=0.0, q=0.3)

$$\begin{aligned}q_1(\tau) &= 0.00539 + 0.05700 \cos n_1\tau - 0.00179 \cos 2n_1\tau - 0.00032 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots \\q_2(\tau) &= 0.00512 + 0.01473 \cos n_1\tau + 0.03819 \cos 2n_1\tau + 0.00074 \cos 3n_1\tau + 0.00005 \cos 4n_1\tau \dots \\q_3(\tau) &= 0.00263 + 0.00936 \cos n_1\tau + 0.00628 \cos 2n_1\tau + 0.03691 \cos 3n_1\tau + 0.00009 \cos 4n_1\tau \dots \\q_b(\tau) &= 0.01314 + 0.08109 \cos n_1\tau + 0.04268 \cos 2n_1\tau + 0.03733 \cos 3n_1\tau + 0.00014 \cos 4n_1\tau \dots\end{aligned}$$

Case 4: (n=0.0, q=0.4)

$$\begin{aligned}q_1(\tau) &= 0.00540 + 0.05700 \cos n_1\tau - 0.00180 \cos 2n_1\tau - 0.00034 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots \\q_2(\tau) &= 0.00517 + 0.01530 \cos n_1\tau + 0.03954 \cos 2n_1\tau + 0.00079 \cos 3n_1\tau + 0.00005 \cos 4n_1\tau \dots \\q_3(\tau) &= 0.00275 + 0.00975 \cos n_1\tau + 0.00660 \cos 2n_1\tau + 0.03887 \cos 3n_1\tau + 0.00009 \cos 4n_1\tau \dots \\q_b(\tau) &= 0.01332 + 0.08205 \cos n_1\tau + 0.04434 \cos 2n_1\tau + 0.03932 \cos 3n_1\tau + 0.00014 \cos 4n_1\tau \dots\end{aligned}$$

Case 5: (n=0.0, q=0.5)

$$\begin{aligned}q_1(\tau) &= 0.00542 + 0.05700 \cos n_1\tau - 0.00180 \cos 2n_1\tau - 0.00034 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots \\q_2(\tau) &= 0.00522 + 0.01570 \cos n_1\tau + 0.04033 \cos 2n_1\tau + 0.00079 \cos 3n_1\tau + 0.00005 \cos 4n_1\tau \dots \\q_3(\tau) &= 0.00283 + 0.01002 \cos n_1\tau + 0.00679 \cos 2n_1\tau + 0.04020 \cos 3n_1\tau + 0.00010 \cos 4n_1\tau \dots \\q_b(\tau) &= 0.01347 + 0.08272 \cos n_1\tau + 0.04532 \cos 2n_1\tau + 0.04065 \cos 3n_1\tau + 0.00015 \cos 4n_1\tau \dots\end{aligned}$$

2.4.2 Solution of Anharmonic equation for RTD polytropic models with index $N = 1.5$

Case 1: (n=0.505, q=0.01)

$$\begin{aligned}q_1(\tau) &= 0.00611 + 0.05700 \cos n_1\tau - 0.00203 \cos 2n_1\tau - 0.00076 \cos 3n_1\tau + 0.00002 \cos 4n_1\tau \dots \\q_2(\tau) &= 0.00670 + 0.03520 \cos n_1\tau + 0.07714 \cos 2n_1\tau + 0.00068 \cos 3n_1\tau + 0.00025 \cos 4n_1\tau \dots \\q_3(\tau) &= 0.00745 + 0.02303 \cos n_1\tau + 0.01764 \cos 2n_1\tau + 0.08411 \cos 3n_1\tau + 0.00063 \cos 4n_1\tau \dots \\q_b(\tau) &= 0.02026 + 0.11523 \cos n_1\tau + 0.09275 \cos 2n_1\tau + 0.08403 \cos 3n_1\tau + 0.00090 \cos 4n_1\tau \dots\end{aligned}$$

Case 2: (n=0.515, q=0.03)

$$\begin{aligned}q_1(\tau) &= 0.00612 + 0.05700 \cos n_1\tau - 0.00204 \cos 2n_1\tau - 0.00075 \cos 3n_1\tau + 0.00002 \cos 4n_1\tau \dots \\q_2(\tau) &= 0.00668 + 0.03491 \cos n_1\tau + 0.07641 \cos 2n_1\tau + 0.00064 \cos 3n_1\tau + 0.00025 \cos 4n_1\tau \dots \\q_3(\tau) &= 0.00739 + 0.02283 \cos n_1\tau + 0.01747 \cos 2n_1\tau + 0.08343 \cos 3n_1\tau + 0.00063 \cos 4n_1\tau \dots \\q_b(\tau) &= 0.02019 + 0.11474 \cos n_1\tau + 0.09185 \cos 2n_1\tau + 0.08332 \cos 3n_1\tau + 0.00090 \cos 4n_1\tau \dots\end{aligned}$$

Case 3: (n=0.525, q=0.05)

$$\begin{aligned}q_1(\tau) &= 0.00589 + 0.05600 \cos n_1\tau - 0.00196 \cos 2n_1\tau - 0.00080 \cos 3n_1\tau + 0.00002 \cos 4n_1\tau \dots \\q_2(\tau) &= 0.00655 + 0.03689 \cos n_1\tau + 0.08229 \cos 2n_1\tau + 0.00071 \cos 3n_1\tau + 0.000271 \cos 4n_1\tau \dots \\q_3(\tau) &= 0.00776 + 0.02417 \cos n_1\tau + 0.01844 \cos 2n_1\tau + 0.09035 \cos 3n_1\tau + 0.00066 \cos 4n_1\tau \dots \\q_b(\tau) &= 0.02020 + 0.11706 \cos n_1\tau + 0.09877 \cos 2n_1\tau + 0.09026 \cos 3n_1\tau + 0.00095 \cos 4n_1\tau \dots\end{aligned}$$

Case 4: (n=0.535, q=0.07)

$$\begin{aligned}q_1(\tau) &= 0.00617 + 0.05700 \cos n_1\tau - 0.00205 \cos 2n_1\tau - 0.00083 \cos 3n_1\tau + 0.00002 \cos 4n_1\tau \dots \\q_2(\tau) &= 0.00683 + 0.03847 \cos n_1\tau + 0.08330 \cos 2n_1\tau + 0.00066 \cos 3n_1\tau + 0.00030 \cos 4n_1\tau \dots \\q_3(\tau) &= 0.00807 + 0.02453 \cos n_1\tau + 0.01900 \cos 2n_1\tau + 0.08722 \cos 3n_1\tau + 0.00073 \cos 4n_1\tau \dots \\q_b(\tau) &= 0.02107 + 0.12000 \cos n_1\tau + 0.10025 \cos 2n_1\tau + 0.08705 \cos 3n_1\tau + 0.00105 \cos 4n_1\tau \dots\end{aligned}$$

Case 5: (n=0.55, q=0.1)

$$\begin{aligned}q_1(\tau) &= 0.00620 + 0.05700 \cos n_1\tau - 0.00206 \cos 2n_1\tau - 0.00087 \cos 3n_1\tau + 0.00002 \cos 4n_1\tau \dots \\q_2(\tau) &= 0.00691 + 0.04044 \cos n_1\tau + 0.08702 \cos 2n_1\tau + 0.00066 \cos 3n_1\tau + 0.00032 \cos 4n_1\tau \dots \\q_3(\tau) &= 0.00881 + 0.02655 \cos n_1\tau + 0.02069 \cos 2n_1\tau + 0.09284 \cos 3n_1\tau + 0.00083 \cos 4n_1\tau \dots \\q_b(\tau) &= 0.02192 + 0.12399 \cos n_1\tau + 0.10565 \cos 2n_1\tau + 0.09263 \cos 3n_1\tau + 0.00117 \cos 4n_1\tau \dots\end{aligned}$$

2.4.3 Solution of Anharmonic equation for RD polytropic models with index $N = 3.0$

Case 1: (n=0.0, q=0.0)

$$\begin{aligned}q_1(\tau) &= 0.00168 + 0.05880 \cos n_1\tau - 0.00056 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots \\q_2(\tau) &= 0.00498 + 0.00260 \cos n_1\tau - 0.00422 \cos 2n_1\tau + 0.00133 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots \\q_3(\tau) &= 0.00501 + 0.00089 \cos n_1\tau - 0.01650 \cos 2n_1\tau + 0.00178 \cos 3n_1\tau - 0.00003 \cos 4n_1\tau \dots \\q_b(\tau) &= 0.01167 + 0.06229 \cos n_1\tau - 0.02128 \cos 2n_1\tau + 0.00313 \cos 3n_1\tau - 0.00004 \cos 4n_1\tau \dots\end{aligned}$$

Case 2: (n=0.01, q=0.0)

$$\begin{aligned}q_1(\tau) &= 0.00173 + 0.05900 \cos n_1\tau - 0.00057 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots \\q_2(\tau) &= 0.00521 + 0.00285 \cos n_1\tau - 0.00437 \cos 2n_1\tau + 0.00140 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots \\q_3(\tau) &= 0.00540 + 0.00113 \cos n_1\tau - 0.01709 \cos 2n_1\tau + 0.00189 \cos 3n_1\tau - 0.00003 \cos 4n_1\tau \dots \\q_b(\tau) &= 0.01234 + 0.06298 \cos n_1\tau - 0.02203 \cos 2n_1\tau + 0.00330 \cos 3n_1\tau - 0.00004 \cos 4n_1\tau \dots\end{aligned}$$

Case 3: (n=0.03, q=0.0)

$$\begin{aligned}q_1(\tau) &= 0.00182 + 0.05900 \cos n_1\tau - 0.00060 \cos 2n_1\tau + 0.00002 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots \\q_2(\tau) &= 0.00566 + 0.00342 \cos n_1\tau - 0.00463 \cos 2n_1\tau + 0.00156 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots \\q_3(\tau) &= 0.00626 + 0.00175 \cos n_1\tau - 0.01837 \cos 2n_1\tau + 0.00213 \cos 3n_1\tau - 0.00004 \cos 4n_1\tau \dots \\q_b(\tau) &= 0.01374 + 0.06417 \cos n_1\tau - 0.02360 \cos 2n_1\tau + 0.00371 \cos 3n_1\tau - 0.00005 \cos 4n_1\tau \dots\end{aligned}$$

Case 4: (n=0.05, q=0.0)

$$\begin{aligned}q_1(\tau) &= 0.00197 + 0.05900 \cos n_1\tau - 0.00065 \cos 2n_1\tau + 0.00002 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots \\q_2(\tau) &= 0.00633 + 0.00436 \cos n_1\tau - 0.00503 \cos 2n_1\tau + 0.00185 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots \\q_3(\tau) &= 0.00780 + 0.00293 \cos n_1\tau - 0.02072 \cos 2n_1\tau + 0.00254 \cos 3n_1\tau - 0.00006 \cos 4n_1\tau \dots \\q_b(\tau) &= 0.01610 + 0.06629 \cos n_1\tau - 0.02640 \cos 2n_1\tau + 0.00441 \cos 3n_1\tau - 0.00008 \cos 4n_1\tau \dots\end{aligned}$$

Case 5: (n=0.07, q=0.0)

$$q_1(\tau) = 0.00191 + 0.05900 \cos n_1\tau - 0.00063 \cos 2n_1\tau + 0.00002 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$$

$$q_2(\tau) = 0.00666 + 0.00473 \cos n_1\tau - 0.00525 \cos 2n_1\tau + 0.00220 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots$$

$$q_3(\tau) = 0.00860 + 0.00347 \cos n_1\tau - 0.02220 \cos 2n_1\tau + 0.00299 \cos 3n_1\tau - 0.00007 \cos 4n_1\tau \dots$$

$$q_b(\tau) = 0.01718 + 0.06720 \cos n_1\tau - 0.02808 \cos 2n_1\tau + 0.00521 \cos 3n_1\tau - 0.00009 \cos 4n_1\tau \dots$$

Case 6: (n=0.09, q=0.0)

$$q_1(\tau) = 0.00223 + 0.05900 \cos n_1\tau - 0.00074 \cos 2n_1\tau + 0.00003 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$$

$$q_2(\tau) = 0.00837 + 0.00801 \cos n_1\tau - 0.00625 \cos 2n_1\tau + 0.00420 \cos 3n_1\tau - 0.00005 \cos 4n_1\tau \dots$$

$$q_3(\tau) = 0.01591 + 0.00994 \cos n_1\tau - 0.03544 \cos 2n_1\tau + 0.00554 \cos 3n_1\tau - 0.00021 \cos 4n_1\tau \dots$$

$$q_b(\tau) = 0.02651 + 0.07695 \cos n_1\tau - 0.04243 \cos 2n_1\tau + 0.00977 \cos 3n_1\tau - 0.00026 \cos 4n_1\tau \dots$$

2.4.4 Solution of Anharmonic equation for TD polytropic models with index $N = 3.0$

Case 1: (n=0.0, q=0.1)

$$q_1(\tau) = 0.00169 + 0.05900 \cos n_1\tau - 0.00056 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$$

$$q_2(\tau) = 0.00502 + 0.00262 \cos n_1\tau - 0.00425 \cos 2n_1\tau + 0.00133 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$$

$$q_3(\tau) = 0.00503 + 0.00090 \cos n_1\tau - 0.01656 \cos 2n_1\tau + 0.00179 \cos 3n_1\tau - 0.00003 \cos 4n_1\tau \dots$$

$$q_b(\tau) = 0.01174 + 0.06252 \cos n_1\tau - 0.02137 \cos 2n_1\tau + 0.00313 \cos 3n_1\tau - 0.00004 \cos 4n_1\tau \dots$$

Case 2: (n=0.0, q=0.2)

$$q_1(\tau) = 0.00169 + 0.05900 \cos n_1\tau - 0.00056 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$$

$$q_2(\tau) = 0.00504 + 0.00264 \cos n_1\tau - 0.00426 \cos 2n_1\tau + 0.00133 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$$

$$q_3(\tau) = 0.00507 + 0.00093 \cos n_1\tau - 0.01660 \cos 2n_1\tau + 0.00180 \cos 3n_1\tau - 0.00003 \cos 4n_1\tau \dots$$

$$q_b(\tau) = 0.01180 + 0.06257 \cos n_1\tau - 0.02142 \cos 2n_1\tau + 0.00314 \cos 3n_1\tau - 0.00004 \cos 4n_1\tau \dots$$

Case 3: (n=0.0, q=0.3)

$$q_1(\tau) = 0.00169 + 0.05900 \cos n_1\tau - 0.00056 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$$

$$q_2(\tau) = 0.00507 + 0.00267 \cos n_1\tau - 0.00428 \cos 2n_1\tau + 0.00134 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$$

$$q_3(\tau) = 0.00514 + 0.00098 \cos n_1\tau - 0.01667 \cos 2n_1\tau + 0.00181 \cos 3n_1\tau - 0.00003 \cos 4n_1\tau \dots$$

$$q_b(\tau) = 0.01190 + 0.06265 \cos n_1\tau - 0.02151 \cos 2n_1\tau + 0.00316 \cos 3n_1\tau - 0.00004 \cos 4n_1\tau \dots$$

Case 4: (n=0.0, q=0.4)

$$\begin{aligned}q_1(\tau) &= 0.00169 + 0.05900 \cos n_1\tau - 0.00056 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots \\q_2(\tau) &= 0.00511 + 0.00272 \cos n_1\tau - 0.00430 \cos 2n_1\tau + 0.00136 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots \\q_3(\tau) &= 0.00525 + 0.00105 \cos n_1\tau - 0.01678 \cos 2n_1\tau + 0.00183 \cos 3n_1\tau - 0.00003 \cos 4n_1\tau \dots \\q_b(\tau) &= 0.01205 + 0.06277 \cos n_1\tau - 0.02164 \cos 2n_1\tau + 0.00320 \cos 3n_1\tau - 0.00004 \cos 4n_1\tau \dots\end{aligned}$$

Case 5: (n=0.0, q=0.5)

$$\begin{aligned}q_1(\tau) &= 0.00170 + 0.05900 \cos n_1\tau - 0.00056 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots \\q_2(\tau) &= 0.00517 + 0.00278 \cos n_1\tau - 0.00433 \cos 2n_1\tau + 0.00138 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots \\q_3(\tau) &= 0.00539 + 0.00115 \cos n_1\tau - 0.01694 \cos 2n_1\tau + 0.00186 \cos 3n_1\tau - 0.00003 \cos 4n_1\tau \dots \\q_b(\tau) &= 0.01226 + 0.06293 \cos n_1\tau - 0.02183 \cos 2n_1\tau + 0.00325 \cos 3n_1\tau - 0.00004 \cos 4n_1\tau \dots\end{aligned}$$

2.4.5 Solution of Anharmonic equation for RTD polytropic models with index $N = 3.0$

Case 1: (n=0.505, q=0.01)

$$\begin{aligned}q_1(\tau) &= 0.00200 + 0.05900 \cos n_1\tau - 0.00066 \cos 2n_1\tau + 0.00002 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots \\q_2(\tau) &= 0.00670 + 0.00491 \cos n_1\tau - 0.00523 \cos 2n_1\tau + 0.00204 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots \\q_3(\tau) &= 0.00873 + 0.00375 \cos n_1\tau - 0.02200 \cos 2n_1\tau + 0.00281 \cos 3n_1\tau - 0.00007 \cos 4n_1\tau \dots \\q_b(\tau) &= 0.01743 + 0.06766 \cos n_1\tau - 0.02789 \cos 2n_1\tau + 0.00487 \cos 3n_1\tau - 0.00009 \cos 4n_1\tau \dots\end{aligned}$$

Case 2: (n=0.515, q=0.03)

$$\begin{aligned}q_1(\tau) &= 0.00201 + 0.05900 \cos n_1\tau - 0.00067 \cos 2n_1\tau + 0.00002 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots \\q_2(\tau) &= 0.00676 + 0.00501 \cos n_1\tau - 0.00526 \cos 2n_1\tau + 0.00209 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots \\q_3(\tau) &= 0.00892 + 0.00391 \cos n_1\tau - 0.02229 \cos 2n_1\tau + 0.00287 \cos 3n_1\tau - 0.00007 \cos 4n_1\tau \dots \\q_b(\tau) &= 0.01769 + 0.06792 \cos n_1\tau - 0.02822 \cos 2n_1\tau + 0.00498 \cos 3n_1\tau - 0.00009 \cos 4n_1\tau \dots\end{aligned}$$

Case 3: (n=0.525, q=0.05)

$$\begin{aligned}q_1(\tau) &= 0.00202 + 0.05900 \cos n_1\tau - 0.00067 \cos 2n_1\tau + 0.00002 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots \\q_2(\tau) &= 0.00684 + 0.00513 \cos n_1\tau - 0.00531 \cos 2n_1\tau + 0.00218 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots \\q_3(\tau) &= 0.00923 + 0.00413 \cos n_1\tau - 0.02286 \cos 2n_1\tau + 0.00298 \cos 3n_1\tau - 0.00007 \cos 4n_1\tau \dots \\q_b(\tau) &= 0.01809 + 0.06826 \cos n_1\tau - 0.02884 \cos 2n_1\tau + 0.00518 \cos 3n_1\tau - 0.00009 \cos 4n_1\tau \dots\end{aligned}$$

Case 4: (n=0.535, q=0.07)

$$q_1(\tau) = 0.00202 + 0.05900 \cos n_1\tau - 0.00067 \cos 2n_1\tau + 0.00002 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$$

$$q_2(\tau) = 0.00689 + 0.00522 \cos n_1\tau - 0.00534 \cos 2n_1\tau + 0.00219 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots$$

$$q_3(\tau) = 0.00936 + 0.00429 \cos n_1\tau - 0.02294 \cos 2n_1\tau + 0.00300 \cos 3n_1\tau - 0.00007 \cos 4n_1\tau \dots$$

$$q_b(\tau) = 0.01827 + 0.06851 \cos n_1\tau - 0.02895 \cos 2n_1\tau + 0.00521 \cos 3n_1\tau - 0.00009 \cos 4n_1\tau \dots$$

Case 5: (n=0.55, q=0.1)

$$q_1(\tau) = 0.00204 + 0.05900 \cos n_1\tau - 0.00068 \cos 2n_1\tau + 0.00002 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$$

$$q_2(\tau) = 0.00702 + 0.00542 \cos n_1\tau - 0.00542 \cos 2n_1\tau + 0.00235 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots$$

$$q_3(\tau) = 0.00984 + 0.00463 \cos n_1\tau - 0.02386 \cos 2n_1\tau + 0.00320 \cos 3n_1\tau - 0.00008 \cos 4n_1\tau \dots$$

$$q_b(\tau) = 0.01890 + 0.06905 \cos n_1\tau - 0.02996 \cos 2n_1\tau + 0.00557 \cos 3n_1\tau - 0.00010 \cos 4n_1\tau \dots$$

Table 2.10: Values of n_1^2 and Skewness Coefficient K for RD Polytropic Models of Stars

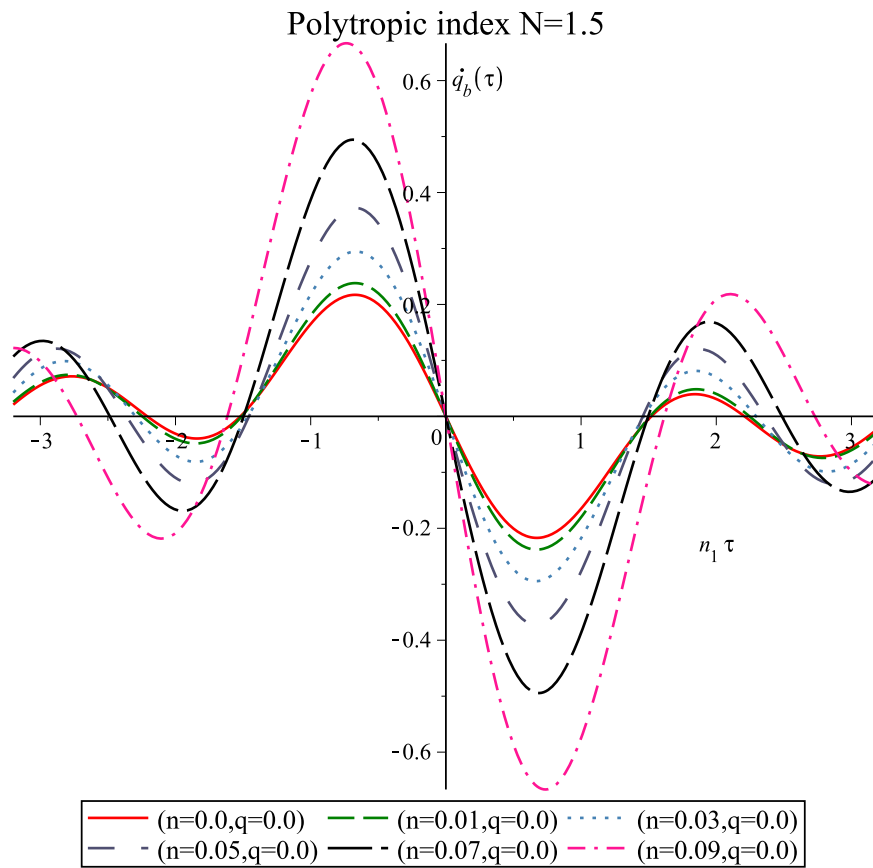
	$N = 1.5$		$N = 3.0$	
	n_1^2	K	n_1^2	K
(n=0,q=0)	0.90616	0.79	0.99468	0.29
(n=0.01,q=0.0)	0.89568	0.79	0.99431	0.29
(n=0.03,q=0.0)	0.88040	0.79	0.99353	0.28
(n=0.05,q=0.0)	0.85341	0.78	0.99213	0.28
(n=0.07,q=0.0)	0.80337	0.78	0.99235	0.27
(n=0.1,q=0.0)	0.71363	0.77	0.98854	0.26

Table 2.11: Values of n_1^2 and Skewness Coefficient K for TD Polytropic Models of Stars

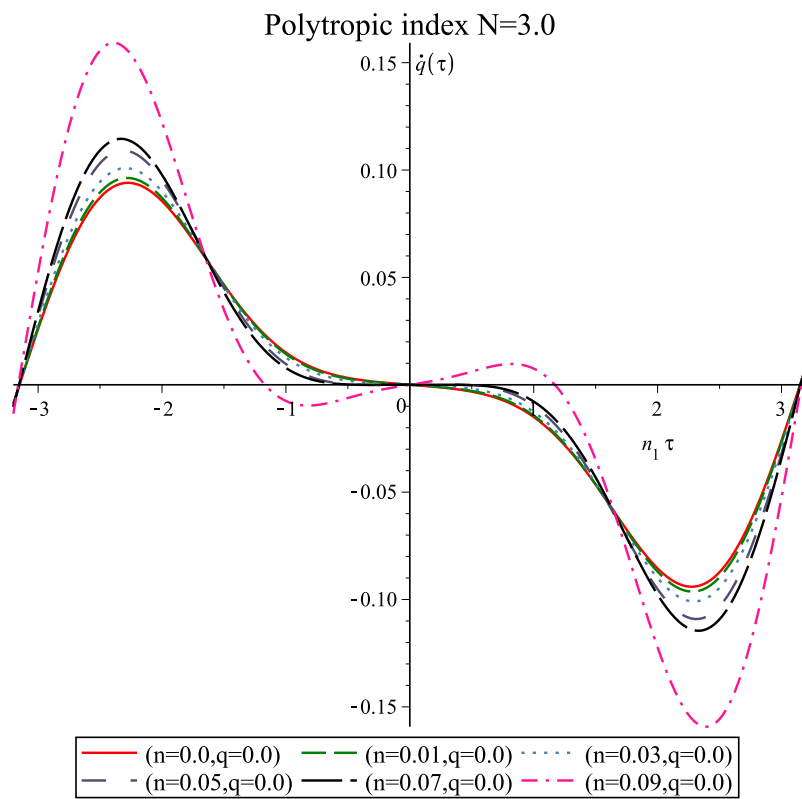
	$N = 1.5$		$N = 3.0$	
	n_1^2	K	n_1^2	K
(n=0,q=0)	0.90616	0.79	0.99468	0.29
(n=0.0,q=0.1)	0.90595	0.79	0.99464	0.28
(n=0.0,q=0.2)	0.90534	0.79	0.99633	0.28
(n=0.0,q=0.3)	0.90430	0.79	0.99461	0.28
(n=0.0,q=0.4)	0.90212	0.79	0.99457	0.28
(n=0.0,q=0.5)	0.90067	0.79	0.99453	0.28

Table 2.12: Values of n_1^2 and Skewness Coefficient K for RTD Polytropic Models of Stars

	$N = 1.5$		$N = 3.0$	
	n_1^2	K	n_1^2	K
(n=0,q=0)	0.90616	0.79	0.99468	0.29
(n=0.505,q=0.01)	0.82681	0.78	0.99166	0.27
(n=0.515,q=0.03)	0.82789	0.78	0.99156	0.27
(n=0.525,q=0.05)	0.82170	0.78	0.99143	0.27
(n=0.535,q=0.07)	0.81556	0.78	0.99133	0.27
(n=0.55,q=0.1)	0.80884	0.78	0.99113	0.27



(a)



(b)

Figure 2.1. Radial velocity curves of RD Polytropic models (n varies and $q = 0$)

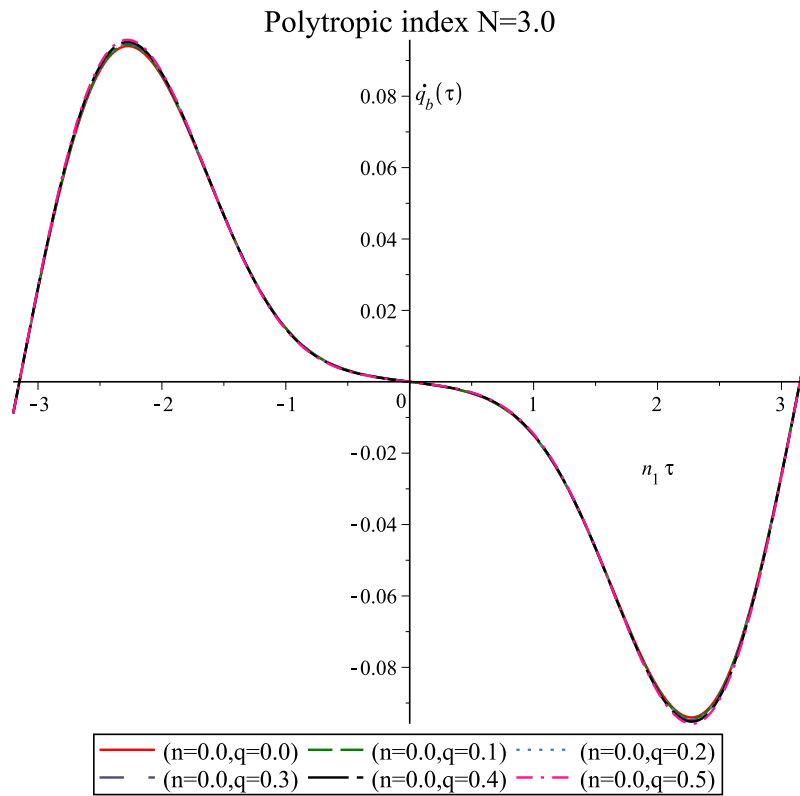
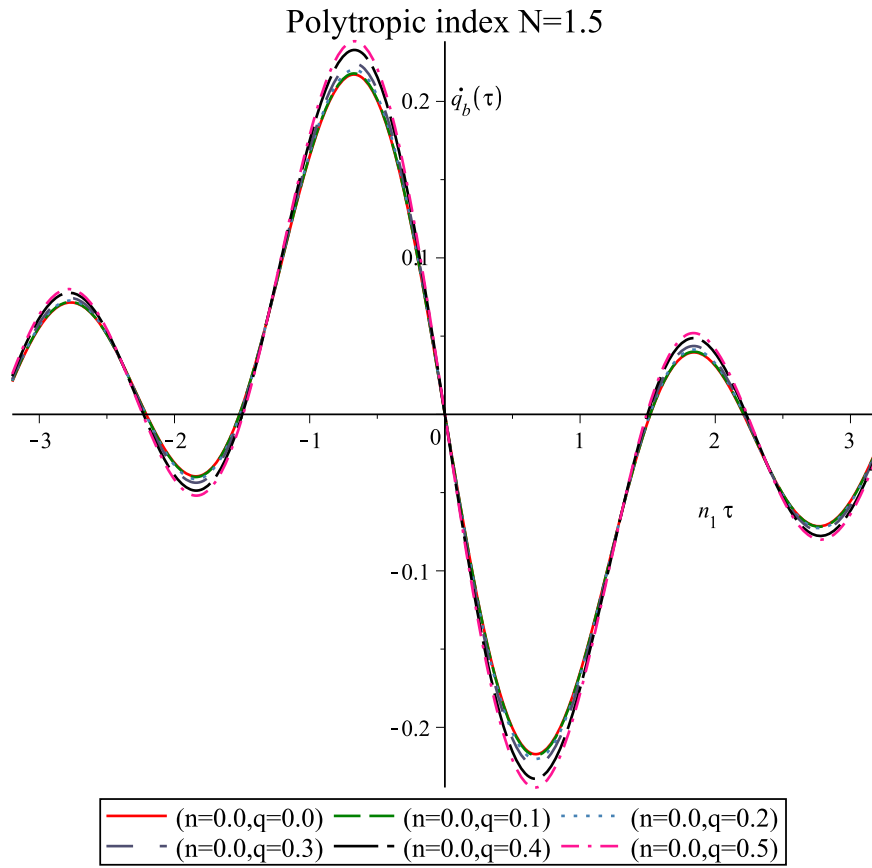
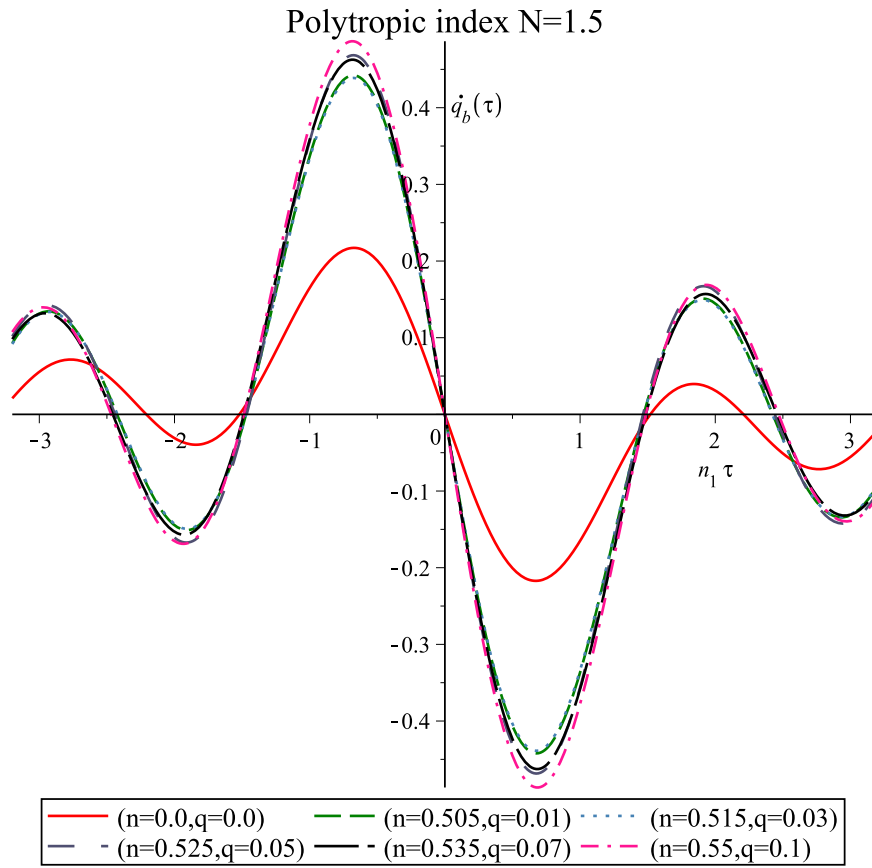
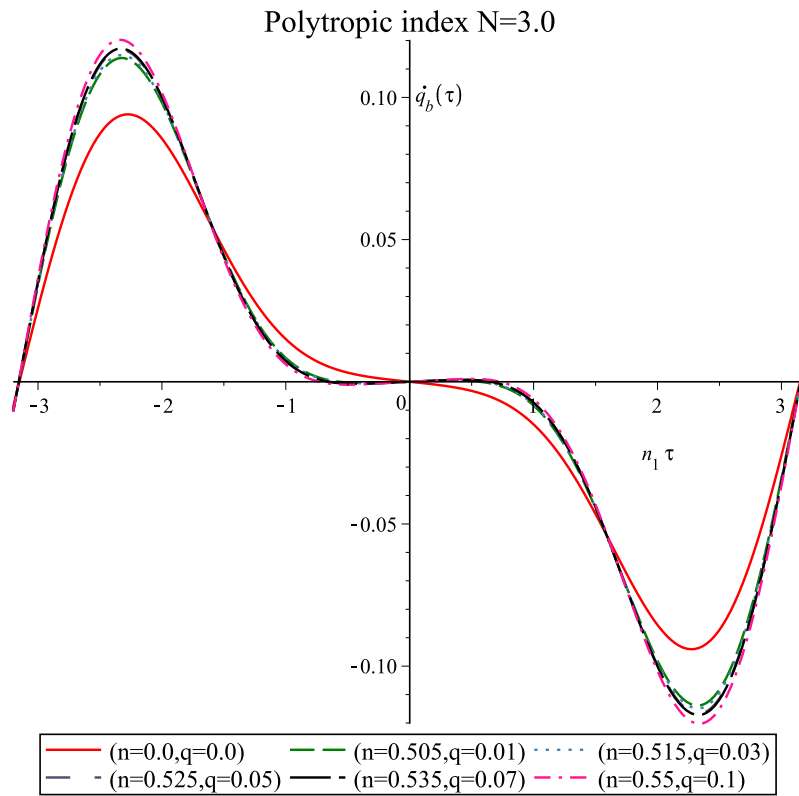


Figure 2.2. Radial velocity curves of TD Polytropic models (q varies and $n = 0$)



(a)



(b)

Figure 2.3. Radial velocity curves of RTD Polytropic models(both n and q varies)

Chapter 3

Effect of interaction of the various modes on the RVC of the polytropic models of RTD pulsating variables stars ²

In this chapter , we considered the problems of determining the effects of including the higher modes on the anharmonic radial pulsation and hence on the RVC of the polytropic models of rotationally and/or tidally distorted pulsating variable stars . We use the same approach as discussed in Chapter-2 to study the effect of interaction of the various modes on the radial velocity curves of the polytropic model of RTD pulsating variable star.

Using the approach of section 2.1 of Chapter-2, Anharmonic radial pulsation equation of a RTD polytropic model of a star have been formulated in section 3.1. In section 3.2 we describe in brief the technique which can be used to solve the equations of anharmonic pulsation of a RTD polytropic models of pulsating variable stars. The equations of anharmonic pulsation obtained in section 3.1 have been solved numerically in section 3.3 for the first four radial modes of oscillations of RTD polytropic models with polytropic indices 1.5, 3.0 and 4.0. The effects of including the various modes of oscillations on the RVC have also been discussed in this section. Certain observations based on the results obtained in the present chapter are finally made in section 3.4.

3.1 Anharmonic radial pulsation equation of a RTD polytropic model of a star

Following the methodology as developed in section (2.1) the equation governing the anharmonic radial pulsations of the RTD polytropic model of a star for first four modes can be expressed as

$$\frac{d^2 q_k}{d\tau^2} + \beta_k q_k = \frac{\gamma}{I_k^* \omega_1^2} \left[\sum_i A_{ii,k}^* q_i^2 + 2 \sum_{\substack{i,j \\ i \neq j}} A_{ij,k}^* q_i q_j \right], \quad (j \geq i) \text{ for } k = 1, 2, 3, 4 \dots \quad (3.1.1)$$

²Tarun Kumar, Ankush Pathania and Arvind Kumar Lal, "Effect of interaction of the various modes on the radial velocity curves of the polytropic models of rotationally and tidally distorted pulsating variables stars", Research in Astronomy and Astrophysics, IOP, (2018). [Accepted for publication]

where

$$\tau = \omega_1 t; \beta_k = \frac{\omega_k^2}{\omega_1^2}; A_{ii,k}^* = \frac{D_{ii,k}^*}{\omega_1^2 I_k^*}; A_{ij,k}^* = \frac{D_{ij,k}^*}{\omega_1^2 I_k^*}$$

$$I_k^* = \int_0^1 \theta^N x^4 \eta_k^2 dx$$

$$\begin{aligned} D_{ij,k}^* = & -\frac{(N+1)}{2} \left(3 - \frac{4}{\gamma}\right) (3\gamma + 1) \int_0^1 \theta^N \theta' x^3 \eta_i \eta_j \eta_k dx \\ & + \frac{1}{2} (3\gamma - 1) \int_0^1 x^4 \theta^{(N+1)} (\eta_i \eta_j' \eta_k' + \eta_i' \eta_j \eta_k' + \eta_i' \eta_j' \eta_k) dx \\ & + \frac{1}{2} (\gamma + 1) \int_0^1 x^5 \theta^{(N+1)} \eta_i' \eta_j' \eta_k' dx \end{aligned}$$

Where the symbols γ , τ , θ , ω_1^2 , x , $D_{ij,k}^*$ and I_k^* have the same meaning as explained in section (2.1) and $\eta_1, \eta_2, \eta_3, \eta_4, \dots$, are the eigenfuctions of the various modes of pulsation Equation (1.5.1) of the radial oscillation of the RTD polytropic models of the stars. So, the displacement q_b at the surface in this case becomes

$$q_b = q_1 + q_2 + q_3 + q_4 + \dots \quad (3.1.2)$$

3.2 Method for solving the equation of the anharmonic radial oscillation of RTD polytropic model of a star

In order to solve the equation (3.1.1) we have followed the same approach as discussed in section (1.8) in Chapter 1. In this case, the equation for q_1, q_2, q_3 and q_4 becomes

$$\begin{aligned} \frac{d^2 q_1}{d\tau^2} + q_1 &= A_{11,1} q_1^2 + 2A_{12,1} q_1 q_2 + 2A_{13,1} q_1 q_3 + 2A_{14,1} q_1 q_4 \\ \frac{d^2 q_2}{d\tau^2} + \beta_2 q_2 &= A_{11,2} q_1^2 + 2A_{12,2} q_1 q_2 + 2A_{13,2} q_1 q_3 + 2A_{14,2} q_1 q_4 \\ \frac{d^2 q_3}{d\tau^2} + \beta_3 q_3 &= A_{11,3} q_1^2 + 2A_{12,3} q_1 q_2 + 2A_{13,3} q_1 q_3 + 2A_{14,3} q_1 q_4 \\ \frac{d^2 q_4}{d\tau^2} + \beta_4 q_4 &= A_{11,4} q_1^2 + 2A_{12,4} q_1 q_2 + 2A_{13,4} q_1 q_3 + 2A_{14,4} q_1 q_4 \end{aligned} \quad (3.2.1)$$

where $A_{11,1}, A_{12,1}, A_{13,1}, A_{14,1}, \dots$ are the constants that are to be determined. Following Prasad [59], we assume the solution of these equations in the form

$$\begin{aligned}
q_1 &= a_{0,1} + a_{1,1} \cos n_1 \tau + a_{2,1} \cos 2n_1 \tau + a_{3,1} \cos 3n_1 \tau + a_{4,1} \cos 4n_1 \tau + \dots \\
q_2 &= a_{0,2} + a_{1,2} \cos n_1 \tau + a_{2,2} \cos 2n_1 \tau + a_{3,2} \cos 3n_1 \tau + a_{4,2} \cos 4n_1 \tau + \dots \\
q_3 &= a_{0,3} + a_{1,3} \cos n_1 \tau + a_{2,3} \cos 2n_1 \tau + a_{3,3} \cos 3n_1 \tau + a_{4,3} \cos 4n_1 \tau + \dots \\
q_4 &= a_{0,4} + a_{1,4} \cos n_1 \tau + a_{2,4} \cos 2n_1 \tau + a_{3,4} \cos 3n_1 \tau + a_{4,4} \cos 4n_1 \tau + \dots
\end{aligned} \tag{3.2.2}$$

where $a_{0,1}, a_{1,1}, a_{2,1}, \dots, a_{0,2}, a_{1,2}, a_{2,2}, \dots, a_{0,3}, a_{1,3}, a_{2,3}, \dots, a_{0,4}, a_{1,4}, a_{2,4}, \dots$ and n_1 are constants that are to be determined. The values of q_1, q_2, q_3 and q_4 are substituted in (3.2.1) and on equating the constant terms and the coefficients of $\cos kn_1 \tau$ (for different k) to zero we get

$$\begin{aligned}
a_{0,1} &= A_{11,1} \left[a_{0,1}^2 + \frac{1}{2} a_{1,1}^2 + \frac{1}{2} a_{2,1}^2 + \frac{1}{2} a_{3,1}^2 + \dots \right] \\
&+ 2A_{12,1} \left[a_{0,1} a_{0,2} + \frac{1}{2} a_{1,1} a_{1,2} + \frac{1}{2} a_{2,1} a_{2,2} + \frac{1}{2} a_{3,1} a_{3,2} + \dots \right] \\
&+ 2A_{13,1} \left[a_{0,1} a_{0,3} + \frac{1}{2} a_{1,1} a_{1,3} + \frac{1}{2} a_{2,1} a_{2,3} + \frac{1}{2} a_{3,1} a_{3,3} + \dots \right] \\
&+ 2A_{14,1} \left[a_{0,1} a_{0,4} + \frac{1}{2} a_{1,1} a_{1,4} + \frac{1}{2} a_{2,1} a_{2,4} + \frac{1}{2} a_{3,1} a_{3,4} + \dots \right] \tag{3.2.3a}
\end{aligned}$$

$$\begin{aligned}
\left(1 - n_1^2\right) a_{1,1} &= A_{11,1} \left[2a_{0,1} a_{1,1} + a_{1,1} a_{2,1} + a_{2,1} a_{3,1} + \dots \right] \\
&+ 2A_{12,1} \left[a_{1,1} a_{0,2} + a_{0,1} a_{1,2} + a_{2,1} a_{0,2} + \frac{1}{2} \left(a_{1,1} a_{2,2} + a_{2,1} a_{1,2} \right) \right. \\
&\left. + \frac{1}{2} (a_{2,1} a_{3,2} + a_{3,1} a_{3,2}) + \dots \right] \\
&+ 2A_{13,1} \left[2a_{0,1} a_{1,3} + a_{1,1} a_{0,3} + \frac{1}{2} a_{1,1} a_{2,3} + \frac{1}{2} a_{2,1} a_{1,3} + \frac{1}{2} a_{3,1} a_{2,3} + \dots \right] \\
&+ 2A_{14,1} \left[2a_{0,1} a_{1,4} + a_{1,1} a_{0,4} + \frac{1}{2} a_{1,1} a_{2,4} + \frac{1}{2} a_{2,1} a_{1,4} + \frac{1}{2} a_{3,1} a_{2,4} + \dots \right] \tag{3.2.3b}
\end{aligned}$$

$$\begin{aligned}
\left(1 - n_1^2 k^2\right) a_{k,1} &= A_{11,1} \left[\frac{1}{2} \sum_{i=0}^k a_{i,1} a_{k-i,1} + \sum_{i=0}^{\infty} a_{i,1} a_{k+i,1} \right] \\
&+ A_{12,1} \left[\frac{1}{2} \sum_{i=0}^k a_{i,1} a_{k-i,2} + \frac{1}{2} \sum_{i=0}^{\infty} \left(a_{i,1} a_{k+i,2} + a_{k+i,1} a_{i,2} \right) \right]
\end{aligned}$$

$$\begin{aligned}
& + A_{13,1} \left[\frac{1}{2} \sum_{i=0}^k a_{i,1} a_{k-i,3} + \frac{1}{2} \sum_{i=0}^{\infty} \left(a_{i,1} a_{k+i,3} + a_{k+i,1} a_{i,3} \right) \right] \\
& + A_{14,1} \left[\frac{1}{2} \sum_{i=0}^k a_{i,1} a_{k-i,4} + \frac{1}{2} \sum_{i=0}^{\infty} \left(a_{i,1} a_{k+i,4} + a_{k+i,1} a_{i,4} \right) \right] \quad (3.2.3c)
\end{aligned}$$

We get similar equation from the second equation in (3.2.1) which are same as above but with $(\beta_2 - k^2 n_1^2) a_{k,2}$ in place of $(1 - k^2 n_1^2) a_{k,1}$ and $A_{11,2}$, $A_{12,2}$, $A_{13,2}$ and $A_{14,2}$ in place of $A_{11,1}$, $A_{12,1}$, $A_{13,1}$ and $A_{14,1}$ respectively.

In order to solve these algebraic (3.2.3a-3.2.3c) equations, we suppose that $a_{1,1}$ is a known small quantity and we determine the other a's in terms of $a_{1,1}$. Considering (3.2.3a) we see that $a_{0,1}$ is a small quantity of the second order, all other terms are square or products and they contain the term $a_{1,1}^2$.

Similarly we find that $a_{2,1}$ is a quantity of the second order, $a_{3,1}$ is of the third and in general $a_{k,1}$ ($k > 1$) is of the k^{th} order. Considering equation for $a_{1,2}$ of third order and $a_{2,2}$ of the second order and in general $a_{k,2}$ ($k > 1$) of the k^{th} order. Therefore, a first approximation to the solution of the equation (3.2.3a-3.2.3c) can be given as:

$$\begin{aligned}
a_{0,1} &= \frac{1}{2} A_{11,1} a_{1,1}^2, \\
a_{2,1} &= -\frac{1}{6} A_{11,1} a_{1,1}^2, \\
a_{3,1} &= \frac{1}{16} \left[\frac{1}{3} A_{11,1}^2 + \frac{A_{12,1}}{(4 - \beta_2)} A_{11,2} \right] a_{1,1}^3, \\
&\dots\dots\dots \\
&\dots\dots\dots \\
a_{0,2} &= \frac{1}{2} A_{11,2} a_{1,1}^2 / \beta_2, \\
a_{1,2} &= \left[\frac{5}{6} A_{11,1} + \frac{(8 - 3\beta_2)}{2\beta_2(4 - \beta_2)} A_{12,2} \right] \frac{A_{11,2} a_{1,1}^3}{(\beta_2 - 1)} \\
a_{2,2} &= -\frac{1}{2} \frac{A_{11,2} a_{1,1}^2}{(4 - \beta_2)} \\
a_{3,2} &= \frac{1}{2} \left[\frac{1}{3} A_{11,1} + \frac{A_{12,2}}{(4 - \beta_2)} \right] \frac{A_{11,2} a_{1,1}^3}{(9 - \beta_2)} \\
&\dots\dots\dots \\
&\dots\dots\dots \\
a_{0,3} &= \frac{1}{2} A_{11,3} a_{1,1}^2 / \beta_3,
\end{aligned}$$

$$\begin{aligned}
a_{1,3} &= \left[\frac{5}{6} A_{11,1} + \frac{(8-3\beta_3)}{2\beta_3(4-\beta_3)} A_{13,3} \right] \frac{A_{11,3} a_{1,1}^3}{(\beta_3-1)} \\
a_{2,3} &= -\frac{1}{2} \frac{A_{11,3} a_{1,1}^2}{(4-\beta_3)} \\
a_{3,3} &= \frac{1}{2} \left[\frac{1}{3} A_{11,1} + \frac{A_{13,3}}{(4-\beta_3)} \right] \frac{A_{11,3} a_{1,1}^3}{(9-\beta_3)} \\
&\dots\dots\dots \\
&\dots\dots\dots \\
a_{0,4} &= \frac{1}{2} A_{11,4} a_{1,1}^2 / \beta_4, \\
a_{1,4} &= \left[\frac{5}{6} A_{11,1} + \frac{(8-3\beta_4)}{2\beta_4(4-\beta_4)} A_{14,4} \right] \frac{A_{11,4} a_{1,1}^3}{(\beta_4-1)} \\
a_{2,4} &= -\frac{1}{2} \frac{A_{11,4} a_{1,1}^2}{(4-\beta_4)} \\
a_{3,4} &= \frac{1}{2} \left[\frac{1}{3} A_{11,1} + \frac{A_{14,4}}{(4-\beta_4)} \right] \frac{A_{11,4} a_{1,1}^3}{(9-\beta_4)} \\
&\dots\dots\dots \\
&\dots\dots\dots
\end{aligned}$$

and

$$n_1^2 = 1 - \left[\frac{5}{6} A_{11,1}^2 + (8-3\beta_2) A_{12,1} A_{11,2} / (2\beta_2(4-\beta_2)) \right] a_{1,1}^2$$

These value of $a_{i,1}$, $a_{i,2}$, $a_{i,3}$, $a_{i,4}$ and n_1 are substituted in (3.2.3a-3.2.3c) and a better approximation is obtained. The new values are resubstituted in (3.2.3a-3.2.3c) and the process is repeated a number of times till we attain the desired accuracy.

3.3 Numerical Computations

We have solved the equation of the anharmonic radial oscillation upto first four modes for certain RTD polytropic models of the stars. We have considered the polytropic models with indices 1.5, 3.0 and 4.0 for the different values of the rotational distortion parameter (n) and the tidal distortion parameter (q) with $\gamma = \frac{5}{3}$. Simpson's rule has been used to numerically evaluate the coefficients I_k^* and $D_{ij,k}^*$ in the anharmonic pulsation equation (3.1.1). Numerical technique discussed in section (3.2) has been used to solve the anharmonic pulsation equation (3.1.1). The values of $A_{ij,k}^*$ are presented in tables (3.2-3.7). The solution of anharmonic equation has been given in the tables (3.8-3.10).

The eigenfrequencies of the pseudo radial modes of the oscillations of the RTD polytropic models have been obtained using the approach of Mohan and Saxena [45] and are presented in

Table (3.1) for certain polytropic models of the stars. The RVC for each model of polytropic index $N = 1.5, 3.0$ and 4.0 have been obtained and are shown in Fig. (3.1-3.13). The value of the skewness coefficient K has also been computed in each case and are given in the Table (3.11).

In the present work, to study the effect of the interaction of the various modes on the radial velocity curves of the RTD polytropic models of the stars we have considered the following cases: (i) fundamental mode (from hereafter f mode) (ii) fundamental and the first mode (from hereafter $f + 1$ modes) (iii) fundamental and the next two modes (from hereafter $f + 2$ modes) and finally (iv) fundamental and the next three higher modes (from hereafter $f + 3$ modes) of the pulsation. For computational and numerical purpose following polytropic models of the pulsating variable stars have been considered (i) undistorted star (no rotational or tidal distortion) (ii) single rotating star (only rotational distortion) (iii) rotating star which is a primary component of the synchronous, circular and aligned binary system (or RTD star) in which the mass of the secondary is assumed to be very less than the mass of the primary star.

3.4 Concluding Observations

The results shown in the Table (3.1) represents the eigenfrequencies of the fundamental, first, second and third pseudo radial modes of the oscillations of the RTD polytropic models with index $N = 1.5, 3.0$ and 4.0 . From this table it is clear that the eigenfrequencies of the radial modes of the RD and the RTD polytropic models are small as compared to the corresponding values of the undistorted polytropic model. Also, from this table it can be observed that with increase in the rotational distortions (the value of n or angular velocity of the rotation of the star) and the rotational and tidal distortions (the value of both n and q where q represent tidal distortions due to secondary component) the value of the eigenfrequencies decreases. These results are in accordance with the previous results of Mohan and Saxena [45].

From Figs. (3.1-3.7), it can be observed that as we consider the interaction of the fundamental mode with the higher modes there is appreciable deviation in the shape of the radial velocity curve of the RD and the RTD polytropic models of the pulsating variable stars. These deviations in the shape of the RVC (as compared to the shapes of the RVC obtained for the f mode) are more in the case of the polytropes with index 1.5 then for the polytropes with index 3.0 and 4.0 . However, for the polytropes with index 1.5 the $f + 3$ modes do not seems to change the radial velocity curve further, infact the radial velocity curve coincides with the radial velocity curve of the $f + 1$ modes in all the considered cases.

From Figs. (3.8-3.13), it can be observed that with increase in the value of n (rotation distortions) and both n and q (rotational and tidal distortions) there is no change in the shape of the RVC (as compared to the shape of the radial velocity curve of the undistorted model)

when only f mode is considered. However, on considering the interaction of the f mode with the higher modes there is appreciable change in the shape of the RVC of the various polytropic models with increasing value of n and both n and q . Also the radial velocity curve show more deviation in case of the rotational distortion (n) than the rotational and tidal distortion (both n and q). These results are in accordance with the results of our paper 1 except for the case when we have considered only f mode.

From Table (3.11), it is clear that when only f mode is considered and when the interaction of f mode with the higher modes is taken into account then, in general, there is slight decrease in the value of K with increasing value of n and both n and q . The value of K decreases more in case of RD than RTD. Again these results are in accordance with the results of our Chapter 2.

However, in Table (3.11), we have also got some results that are contrary to the usual trend and also contrary to the trend we obtained in Chapter 1. Firstly for polytropes of index $N = 3.0$ when $f + 3$ modes are considered then the value of K shows opposite behavior. It tends to increase with increase in the value of n and both n and q . Secondly there is appreciable decrease in the value of K with increase in the value of n and both n and q for polytropes of index $N = 4.0$ when higher modes $f + 2$ and $f + 3$ are considered. Whereas, in general, vary slight decrease is observed in the values of K with increasing values of n and both n and q in all other cases.

Again from Table (3.11), it can be observed that for polytropes of index $N = 1.5$ with increase in the number of modes (upto $f + 2$ modes) the value of K increases and then it decreases for $f + 3$ modes. However, for polytropes of index $N = 3.0$ and $N = 4.0$ with increase in the number of modes the value of K decreases, except for polytropes with index $N = 3.0$ where after $f + 2$ modes the value start increasing.

So from the present study we can conclude that interaction of the fundamental mode with the higher modes appreciably changes the shape of the RVC of the RD and RTD polytropic models of the pulsating variable stars.

Table 3.1: Eigenfrequencies of the various modes of the Polytropic models of the stars

	$N = 1.5$				$N = 3.0$				$N = 4.0$			
	ω_1^2	ω_2^2	ω_3^2	ω_4^2	ω_1^2	ω_2^2	ω_3^2	ω_4^2	ω_1^2	ω_2^2	ω_3^2	ω_4^2
	Undistorted											
(n=0,q=0)	2.705	12.533	26.533	44.509	9.254	16.983	28.554	44.749	15.149	24.834	35.477	48.859
	Rotationally distorted											
(n=0.03,q=0.0)	2.574	11.647	24.598	41.247	8.823	15.869	26.427	41.452	14.157	23.029	33.112	45.684
(n=0.05,q=0.0)	2.486	11.042	23.297	39.138	8.519	15.099	24.598	38.495	13.463	21.827	31.618	43.568
(n=0.07,q=0.0)	2.397	10.387	21.981	37.074	8.306	14.642	23.948	35.977	12.996	20.707	30.933	41.514
	Rotationally and Tidally distorted											
(n=0.525,q=0.05)	2.418	10.512	22.322	37.645	8.265	14.451	23.506	35.449	12.887	20.938	31.120	42.012
(n=0.535,q=0.07)	2.409	10.499	22.144	37.337	8.241	14.391	23.352	35.110	12.835	20.850	31.058	41.701
(n=0.55,q=0.1)	2.399	10.428	21.995	37.113	8.206	14.300	23.150	34.599	12.697	20.726	31.011	40.456

Table 3.2: Coefficients $A_{ij,k}^*$ of the anharmonic pulation equation of the RD Polytropic model of index $N = 1.5$

i	j	k	$A_{ij,k}^*$			
			(n=0.0, q=0.0)	(n=0.03, q=0.0)	(n=0.05, q=0.0)	(n=0.07, q=0.0)
1	1	1	3.31231	3.51283	3.66106	3.82290
1	2	1	1.35696	1.41236	1.45613	1.50643
1	3	1	0.02743	0.015847	0.00873	0.00264
1	4	1	-0.00860	-0.00667	-0.00426	-0.00046
1	1	2	14.4451	15.97793	17.15933	18.51124
1	2	2	16.46809	18.37087	19.88555	21.6597
1	3	2	6.65729	7.38326	8.00170	8.76204
1	4	2	0.43958	0.458271	0.50262	0.58983
1	1	3	1.22131	0.75385	0.43140	0.13484
1	2	3	27.83933	31.04755	33.53074	36.41042
1	3	3	36.12107	40.67610	44.24114	48.33155
1	4	3	16.20271	18.21582	19.76019	21.41668
1	1	4	-1.05032	-0.87048	-0.58046	-0.06629
1	2	4	5.04100	5.28091	5.81220	6.92110
1	3	4	44.4322	49.91790	54.5296	60.47534
1	4	4	62.41050	70.78386	77.0786	84.06400

Table 3.3: Coefficients $A_{ij,k}^*$ of the anharmonic pulation equation of the RD Polytropic model of index $N = 3.0$

i	j	k	$A_{ij,k}^*$			
			(n=0.0, q=0.0)	(n=0.03, q=0.0)	(n=0.05, q=0.0)	(n=0.07, q=0.0)
1	1	1	0.97465	1.04606	1.10458	1.09927
1	2	1	0.44855	0.49076	0.52392	0.51298
1	3	1	0.10505	0.11926	0.12982	0.12424
1	4	1	0.01105	0.01359	0.01427	0.12656
1	1	2	5.29616	5.85807	6.34924	6.75130
1	2	2	3.83660	4.25913	4.61944	4.90935
1	3	2	1.64354	1.84751	2.01451	2.14087
1	4	2	0.37458	0.42577	0.45963	0.51551
1	1	3	8.01184	9.26522	10.44854	11.47172
1	2	3	10.6158	12.0238	13.37862	15.01983
1	3	3	8.54528	9.60193	10.64789	12.34753
1	4	3	3.81192	4.17756	4.62528	6.03474
1	1	4	3.82727	5.13360	5.66245	4.80950
1	2	4	10.97957	13.4689	15.04909	14.88510
1	3	4	17.29856	20.30580	22.80309	24.83664
1	4	4	14.09451	15.68404	17.73249	22.61527

Table 3.4: Coefficients $A_{ij,k}^*$ of the anharmonic pulation equation of the RTD Polytrropic model of index $N = 1.5$

i	j	k	$A_{ij,k}^*$		
			(n=0.525, q=0.03)	(n=0.535, q=0.07)	(n=0.55, q=0.1)
1	1	1	3.75879	3.79993	3.81758
1	2	1	1.48186	1.49876	1.50413
1	3	1	0.00334	0.00340	0.00285
1	4	1	-0.00058	-0.00090	-0.00030
1	1	2	18.1668	18.33277	18.49502
1	2	2	21.2376	21.4241	21.63938
1	3	2	8.59366	8.66572	8.763654
1	4	2	0.57970	0.58191	0.598324
1	1	3	0.17031	0.17320	0.145634
1	2	3	35.7175	36.0303	36.38150
1	3	3	47.3876	47.7916	48.28095
1	4	3	21.0128	21.20292	21.38807
1	1	4	-0.08463	-0.12901	-0.04430
1	2	4	6.79706	6.81449	7.02989
1	3	4	59.27892	59.71772	60.53256
1	4	4	82.42178	83.11356	83.91493

Table 3.5: Coefficients $A_{ij,k}^*$ of the anharmonic pulation equation of the RTD Polytropic model of index $N = 3.0$

i	j	k	$A_{ij,k}^*$		
			(n=0.525, q=0.03)	(n=0.535, q=0.07)	(n=0.55, q=0.1)
1	1	1	1.16060	1.16586	1.17459
1	2	1	0.55590	0.55889	0.56389
1	3	1	0.14151	0.14321	0.14532
1	4	1	0.01549	0.01592	0.01651
1	1	2	6.87199	6.92002	7.03432
1	2	2	5.01643	5.05595	5.14737
1	3	2	2.23217	2.26317	2.31989
1	4	2	0.54441	0.56315	0.59605
1	1	3	11.8660	11.9216	12.35315
1	2	3	15.1410	15.2152	15.80812
1	3	3	12.3094	12.4369	13.04652
1	4	3	5.88115	6.07011	6.58884
1	1	4	5.59654	5.42658	5.58154
1	2	4	15.91093	15.50161	16.14662
1	3	4	25.33959	24.85333	26.19331
1	4	4	22.11620	22.16785	24.05023

Table 3.6: Coefficients $A_{ij,k}^*$ of the anharmonic pulation equation of the RD Polytropic model of index $N = 4.0$

i	j	k	$A_{ij,k}^*$			
			(n=0.0, q=0.0)	(n=0.03, q=0.0)	(n=0.05, q=0.0)	(n=0.07, q=0.0)
1	1	1	0.44815	0.48779	0.51922	0.64149
1	2	1	0.19850	0.21256	0.21516	0.31405
1	3	1	0.07512	0.07487	0.05450	0.11754
1	4	1	0.03389	0.02759	-0.00095	0.03744
1	1	2	2.84184	3.07914	3.14942	4.70009
1	2	2	2.14585	2.25631	2.01911	3.41354
1	3	2	1.49776	1.50833	0.93520	1.75900
1	4	2	1.14211	1.11172	0.37423	0.65936
1	1	3	5.33696	5.09683	4.46411	18.2299
1	2	3	7.43179	7.08760	5.23286	18.22865
1	3	3	8.57790	8.18978	4.76193	11.84319
1	4	3	9.20889	9.00431	4.62496	4.68669
1	1	4	2.65455	1.77130	-0.08414	18.02847
1	2	4	6.24807	4.92599	2.26529	21.21085
1	3	4	10.15299	8.49075	5.00333	14.54821
1	4	4	13.62364	12.10364	8.33263	5.02924

Table 3.7: Coefficients $A_{ij,k}^*$ of the anharmonic pulation equation of the RTD Polytropic model of index $N = 4.0$

i	j	k	$A_{ij,k}^*$		
			(n=0.525, q=0.03)	(n=0.535, q=0.07)	(n=0.55, q=0.1)
1	1	1	0.60241	0.62250	0.65150
1	2	1	0.27884	0.29798	0.32637
1	3	1	0.08944	0.10448	0.12930
1	4	1	0.01652	0.02809	0.05042
1	1	2	4.15692	4.43460	4.88539
1	2	2	2.87423	3.14874	3.65073
1	3	2	1.32043	1.54184	1.97868
1	4	2	0.34987	0.51439	0.87330
1	1	3	12.1415	15.3274	21.26884
1	2	3	12.0237	15.19819	21.74330
1	3	3	7.22367	9.51668	14.73403
1	4	3	2.33096	3.46873	6.76375
1	1	4	4.82860	10.86599	32.61130
1	2	4	6.85970	13.36766	37.72879
1	3	4	5.01882	9.14478	26.59176
1	4	4	2.78649	3.43955	10.25171

Table 3.8: Solution of the Anharmonic equation for RD and RTD for Polytropes with index $N = 1.5$

Models	Value of displacement on the surface of Star ($q_b(\tau)$)
<i>f</i> -mode	
(n=0.0,q=0.0)	$q_b(\tau) = 0.00538 + 0.05700 \cos n_1\tau - 0.00179 \cos 2n_1\tau + 0.00004 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.00570 + 0.05700 \cos n_1\tau - 0.00190 \cos 2n_1\tau + 0.00004 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.00584 + 0.05650 \cos n_1\tau - 0.00194 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.07,q=0.0)	$q_b(\tau) = 0.00610 + 0.05650 \cos n_1\tau - 0.00203 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.00599 + 0.05650 \cos n_1\tau - 0.00199 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.535,q=0.07)	$q_b(\tau) = 0.00606 + 0.05650 \cos n_1\tau - 0.00202 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.00609 + 0.05650 \cos n_1\tau - 0.00203 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
<i>f</i> + 1-modes	
(n=0.0,q=0.0)	$q_b(\tau) = 0.01044 + 0.07124 \cos n_1\tau + 0.03533 \cos 2n_1\tau - 0.00795 \cos 3n_1\tau + 0.00092 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.01144 + 0.07758 \cos n_1\tau + 0.04763 \cos 2n_1\tau - 0.01165 \cos 3n_1\tau + 0.00155 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.01222 + 0.08476 \cos n_1\tau + 0.06121 \cos 2n_1\tau - 0.01589 \cos 3n_1\tau + 0.00023 \cos 4n_1\tau \dots$
(n=0.07,q=0.0)	$q_b(\tau) = 0.01315 + 0.09890 \cos n_1\tau + 0.08831 \cos 2n_1\tau - 0.02435 \cos 3n_1\tau + 0.00429 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.01289 + 0.09590 \cos n_1\tau + 0.08323 \cos 2n_1\tau - 0.02257 \cos 3n_1\tau + 0.00381 \cos 4n_1\tau \dots$
(n=0.535,q=0.07)	$q_b(\tau) = 0.01300 + 0.09547 \cos n_1\tau + 0.08124 \cos 2n_1\tau - 0.02228 \cos 3n_1\tau + 0.00380 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.01311 + 0.09744 \cos n_1\tau + 0.08497 \cos 2n_1\tau - 0.02347 \cos 3n_1\tau + 0.00409 \cos 4n_1\tau \dots$
<i>f</i> + 2-modes	
(n=0.0,q=0.0)	$q_b(\tau) = 0.01298 + 0.08028 \cos n_1\tau + 0.04137 \cos 2n_1\tau + 0.03569 \cos 3n_1\tau + 0.00014 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.01534 + 0.09083 \cos n_1\tau + 0.05700 \cos 2n_1\tau + 0.05303 \cos 3n_1\tau + 0.00031 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.01781 + 0.01028 \cos n_1\tau + 0.07458 \cos 2n_1\tau + 0.06987 \cos 3n_1\tau + 0.00056 \cos 4n_1\tau \dots$
(n=0.07,q=0.0)	$q_b(\tau) = 0.02222 + 0.12631 \cos n_1\tau + 0.10948 \cos 2n_1\tau + 0.09306 \cos 3n_1\tau + 0.00143 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.02021 + 0.11707 \cos n_1\tau + 0.09877 \cos 2n_1\tau + 0.09026 \cos 3n_1\tau + 0.00096 \cos 4n_1\tau \dots$
(n=0.535,q=0.07)	$q_b(\tau) = 0.02108 + 0.12000 \cos n_1\tau + 0.10025 \cos 2n_1\tau + 0.08706 \cos 3n_1\tau + 0.00106 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.02193 + 0.12399 \cos n_1\tau + 0.10565 \cos 2n_1\tau + 0.09263 \cos 3n_1\tau + 0.00119 \cos 4n_1\tau \dots$
<i>f</i> + 3-modes	
(n=0.0,q=0.0)	$q_b(\tau) = 0.01054 + 0.07137 \cos n_1\tau + 0.03554 \cos 2n_1\tau - 0.00729 \cos 3n_1\tau + 0.00089 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.01148 + 0.07765 \cos n_1\tau + 0.04774 \cos 2n_1\tau - 0.01093 \cos 3n_1\tau + 0.00151 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.01223 + 0.08480 \cos n_1\tau + 0.06126 \cos 2n_1\tau - 0.01517 \cos 3n_1\tau + .002336 \cos 4n_1\tau \dots$
(n=0.07,q=0.0)	$q_b(\tau) = 0.01316 + 0.09893 \cos n_1\tau + 0.08834 \cos 2n_1\tau - 0.02376 \cos 3n_1\tau + 0.00425 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.01291 + 0.09594 \cos n_1\tau + 0.08327 \cos 2n_1\tau - 0.02183 \cos 3n_1\tau + 0.00377 \cos 4n_1\tau \dots$
(n=0.535,q=0.07)	$q_b(\tau) = 0.01302 + 0.09550 \cos n_1\tau + 0.08128 \cos 2n_1\tau - 0.02162 \cos 3n_1\tau + 0.00376 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.01313 + 0.09748 \cos n_1\tau + 0.08500 \cos 2n_1\tau - 0.02281 \cos 3n_1\tau + 0.00405 \cos 4n_1\tau \dots$

Table 3.9: Solution of the Anharmonic equation for RD and RTD Polytropes with index $N = 3.0$

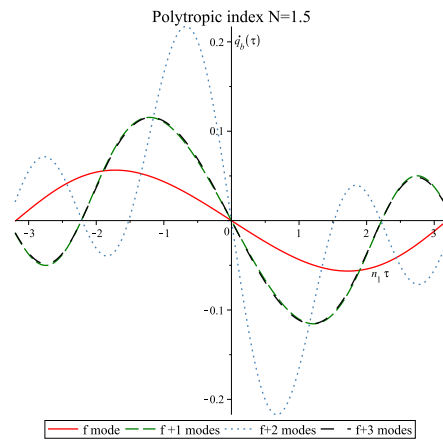
Models	Value of displacement on the surface of Star ($q_b(\tau)$)
<i>f</i> -mode	
(n=0.0,q=0.0)	$q_b(\tau) = 0.00169 + 0.05900 \cos n_1\tau - 0.00056 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.00182 + 0.05900 \cos n_1\tau - 0.00060 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.00192 + 0.05900 \cos n_1\tau - 0.00064 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.07,q=0.0)	$q_b(\tau) = 0.00191 + 0.05900 \cos n_1\tau - 0.00063 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.00202 + 0.05900 \cos n_1\tau - 0.00067 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.535,q=0.07)	$q_b(\tau) = 0.00202 + 0.05900 \cos n_1\tau - 0.00067 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.00204 + 0.05900 \cos n_1\tau - 0.00068 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
<i>f</i> + 1-modes	
(n=0.0,q=0.0)	$q_b(\tau) = 0.00672 + 0.06162 \cos n_1\tau - 0.04823 \cos 2n_1\tau + 0.00017 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.00751 + 0.06249 \cos n_1\tau - 0.05219 \cos 2n_1\tau + 0.00021 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.00815 + 0.06320 \cos n_1\tau - 0.00560 \cos 2n_1\tau + 0.00024 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.07,q=0.0)	$q_b(\tau) = 0.00857 + 0.06373 \cos n_1\tau - 0.00589 \cos 2n_1\tau + 0.00027 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.00886 + 0.06413 \cos n_1\tau - 0.00598 \cos 2n_1\tau + 0.00028 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.535,q=0.07)	$q_b(\tau) = 0.00892 + 0.06423 \cos n_1\tau - 0.00602 \cos 2n_1\tau + 0.00028 \cos 3n_1\tau - 0.00010 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.00906 + 0.06442 \cos n_1\tau - 0.00610 \cos 2n_1\tau + 0.00029 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
<i>f</i> + 2-modes	
(n=0.0,q=0.0)	$q_b(\tau) = 0.01169 + 0.06229 \cos n_1\tau - 0.02130 \cos 2n_1\tau + 0.00313 \cos 3n_1\tau - 0.00004 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.01375 + 0.06418 \cos n_1\tau - 0.02361 \cos 2n_1\tau + 0.00372 \cos 3n_1\tau - 0.00006 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.01610 + 0.06629 \cos n_1\tau - 0.02641 \cos 2n_1\tau + 0.00442 \cos 3n_1\tau - 0.00008 \cos 4n_1\tau \dots$
(n=0.07,q=0.0)	$q_b(\tau) = 0.01718 + 0.06720 \cos n_1\tau - 0.02809 \cos 2n_1\tau + 0.00521 \cos 3n_1\tau - 0.00010 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.01809 + 0.06827 \cos n_1\tau - 0.02884 \cos 2n_1\tau + 0.00519 \cos 3n_1\tau - 0.00010 \cos 4n_1\tau \dots$
(n=0.535,q=0.07)	$q_b(\tau) = 0.01828 + 0.06851 \cos n_1\tau - 0.02896 \cos 2n_1\tau + 0.00522 \cos 3n_1\tau - 0.00010 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.01891 + 0.06906 \cos n_1\tau - 0.02996 \cos 2n_1\tau + 0.00559 \cos 3n_1\tau - 0.00011 \cos 4n_1\tau \dots$
<i>f</i> + 3-modes	
(n=0.0,q=0.0)	$q_b(\tau) = 0.01261 + 0.06326 \cos n_1\tau - 0.01208 \cos 2n_1\tau - 0.00041 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.01480 + 0.06622 \cos n_1\tau - 0.00846 \cos 2n_1\tau - 0.00093 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.01656 + 0.07003 \cos n_1\tau - 0.00346 \cos 2n_1\tau - 0.00233 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
(n=0.07,q=0.0)	$q_b(\tau) = 0.01743 + 0.07526 \cos n_1\tau + 0.00149 \cos 2n_1\tau - 0.00471 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.01839 + 0.07962 \cos n_1\tau + 0.00987 \cos 2n_1\tau - 0.00682 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.535,q=0.07)	$q_b(\tau) = 0.01846 + 0.08096 \cos n_1\tau + 0.01252 \cos 2n_1\tau - 0.00750 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.01899 + 0.08675 \cos n_1\tau + 0.02063 \cos 2n_1\tau - 0.01064 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$

Table 3.10: Solution of the Anharmonic equation for RD and RTD Polytropes with index $N = 4.0$

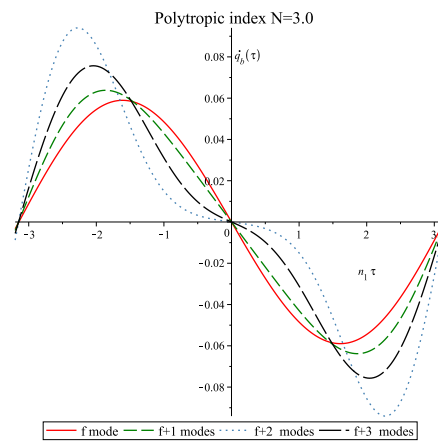
Models	Value of displacement on the surface of Star ($q_b(\tau)$)
<i>f</i> -mode	
(n=0.0,q=0.0)	$q_b(\tau) = 0.00080 + 0.06000 \cos n_1\tau - 0.00026 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.00087 + 0.06000 \cos n_1\tau - 0.00029 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.00093 + 0.06000 \cos n_1\tau - 0.00031 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.07,q=0.0)	$q_b(\tau) = 0.00115 + 0.06000 \cos n_1\tau - 0.00038 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.00108 + 0.06000 \cos n_1\tau - 0.00036 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.535,q=0.07)	$q_b(\tau) = 0.00112 + 0.06000 \cos n_1\tau - 0.00037 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.00117 + 0.06000 \cos n_1\tau - 0.00039 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
<i>f</i> + 1-modes	
(n=0.0,q=0.0)	$q_b(\tau) = 0.00398 + 0.06127 \cos n_1\tau - 0.00240 \cos 2n_1\tau + 0.00004 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.00428 + 0.06140 \cos n_1\tau - 0.00262 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.00443 + 0.06137 \cos n_1\tau - 0.00269 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.07,q=0.0)	$q_b(\tau) = 0.00635 + 0.06311 \cos n_1\tau - 0.00394 \cos 2n_1\tau + 0.00012 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.00569 + 0.06239 \cos n_1\tau - 0.00351 \cos 2n_1\tau + 0.00009 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.535,q=0.07)	$q_b(\tau) = 0.00603 + 0.06275 \cos n_1\tau - 0.00372 \cos 2n_1\tau + 0.00010 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.00656 + 0.06335 \cos n_1\tau - 0.00410 \cos 2n_1\tau + 0.00013 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
<i>f</i> + 2-modes	
(n=0.0,q=0.0)	$q_b(\tau) = 0.00859 + 0.06384 \cos n_1\tau - 0.00942 \cos 2n_1\tau + 0.00108 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.00879 + 0.06408 \cos n_1\tau - 0.00929 \cos 2n_1\tau + 0.00101 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.00443 + 0.06137 \cos n_1\tau - 0.00269 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.07,q=0.0)	$q_b(\tau) = 0.02290 + 0.07299 \cos n_1\tau - 0.03236 \cos 2n_1\tau + 0.00525 \cos 3n_1\tau - 0.00011 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.01599 + 0.06778 \cos n_1\tau - 0.01992 \cos 2n_1\tau + 0.00225 \cos 3n_1\tau - 0.00003 \cos 4n_1\tau \dots$
(n=0.535,q=0.07)	$q_b(\tau) = 0.01944 + 0.07027 \cos n_1\tau - 0.02592 \cos 2n_1\tau + 0.00363 \cos 3n_1\tau - 0.00006 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.02633 + 0.07538 \cos n_1\tau - 0.04051 \cos 2n_1\tau + 0.00762 \cos 3n_1\tau - 0.00019 \cos 4n_1\tau \dots$
<i>f</i> + 3-modes	
(n=0.0,q=0.0)	$q_b(\tau) = 0.00953 + 0.06135 \cos n_1\tau - 0.01445 \cos 2n_1\tau + 0.00139 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.00920 + 0.06197 \cos n_1\tau - 0.01224 \cos 2n_1\tau + 0.00102 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.0780 + 0.06212 \cos n_1\tau - 0.00736 \cos 2n_1\tau + 0.00025 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.07,q=0.0)	$q_b(\tau) = 0.02983 + 0.06537 \cos n_1\tau - 0.06861 \cos 2n_1\tau + 0.00481 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.01740 + 0.06440 \cos n_1\tau - 0.02902 \cos 2n_1\tau + 0.00140 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.535,q=0.07)	$q_b(\tau) = 0.02343 + 0.06532 \cos n_1\tau - 0.04724 \cos 2n_1\tau + 0.00265 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.04056 + 0.06074 \cos n_1\tau - 0.10102 \cos 2n_1\tau + 0.01132 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$

Table 3.11: Values of n_1^2 and the skewness coefficient K for RD and RTD Polytropic models

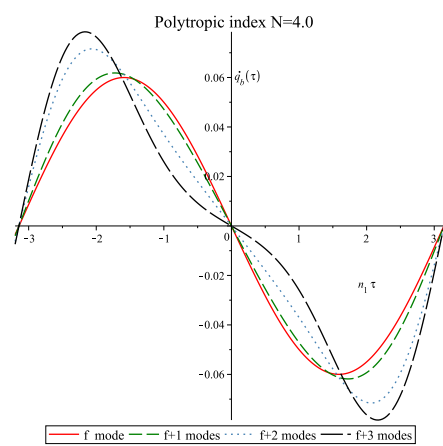
Models	f mode		$f + 1$ modes		$f + 2$ modes		$f + 3$ modes	
	n_1^2	K	n_1^2	K	n_1^2	K	n_1^2	K
Polytropic index ($N = 1.5$)								
(n=0.0,q=0.0)	0.97029	0.4632	0.90616	0.6207	0.90616	0.7926	0.90616	0.6219
(n=0.03,q=0.0)	0.96658	0.4588	0.88041	0.6152	0.88040	0.7920	0.88040	0.6213
(n=0.05,q=0.0)	0.96434	0.4569	0.85341	0.6082	0.85341	0.7830	0.85341	0.6131
(n=0.07,q=0.0)	0.96114	0.4541	0.80336	0.6082	0.80337	0.7821	0.80335	0.5983
(n=0.525,q=0.05)	0.96243	0.4551	0.82527	0.5944	0.82170	0.7822	0.82526	0.6035
(n=0.535,q=0.07)	0.96160	0.4544	0.81557	0.5975	0.81556	0.7813	0.81556	0.6013
(n=0.55,q=0.1)	0.96123	0.4541	0.80885	0.5956	0.80884	0.7812	0.80882	0.5993
Polytropic index ($N = 3.0$)								
(n=0.0,q=0.0)	0.99724	0.4965	0.99467	0.4181	0.99468	0.2891	0.99464	0.3615
(n=0.03,q=0.0)	0.99682	0.4955	0.99349	0.4126	0.99353	0.2830	0.99349	0.4038
(n=0.05,q=0.0)	0.99646	0.4947	0.99252	0.4074	0.99252	0.2770	0.99252	0.4754
(n=0.07,q=0.0)	0.99649	0.4948	0.99235	0.4038	0.99235	0.2716	0.99235	0.5106
(n=0.525,q=0.05)	0.99609	0.4939	0.99143	0.4026	0.99143	0.2718	0.99143	0.5541
(n=0.535,q=0.07)	0.99605	0.4938	0.99133	0.4022	0.99133	0.2717	0.99133	0.5620
(n=0.55,q=0.1)	0.99599	0.4937	0.99113	0.4021	0.99111	0.2698	0.99113	0.5735
Polytropic index ($N = 4.0$)								
(n=0.0,q=0.0)	0.99939	0.5033	0.99855	0.4608	0.99855	0.3764	0.99855	0.3433
(n=0.03,q=0.0)	0.99928	0.5028	0.99845	0.4567	0.99833	0.3764	0.99833	0.3388
(n=0.05,q=0.0)	0.99919	0.5024	0.99819	0.4555	0.99819	0.3763	0.99819	0.3385
(n=0.07,q=0.0)	0.99876	0.5003	0.99661	0.4443	0.99661	0.2760	0.99661	0.2701
(n=0.525,q=0.05)	0.99891	0.5013	0.99723	0.4414	0.99723	0.3021	0.99722	0.3008
(n=0.535,q=0.07)	0.99883	0.5010	0.99691	0.4377	0.99691	0.2882	0.99691	0.2818
(n=0.55,q=0.1)	0.99872	0.5006	0.99642	0.4318	0.99642	0.2643	0.99642	0.2501



(a)



(b)



(c)

Figure 3.1. Radial velocity curves of the undistorted Polytropic models ($n = 0.0, q = 0.0$)

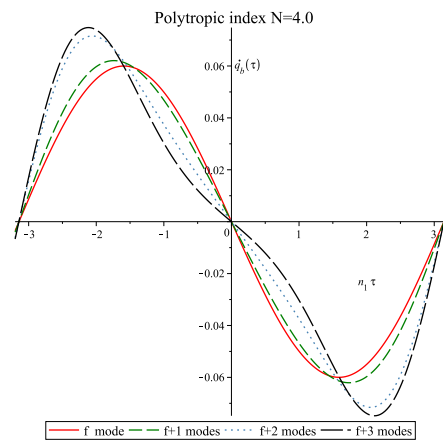
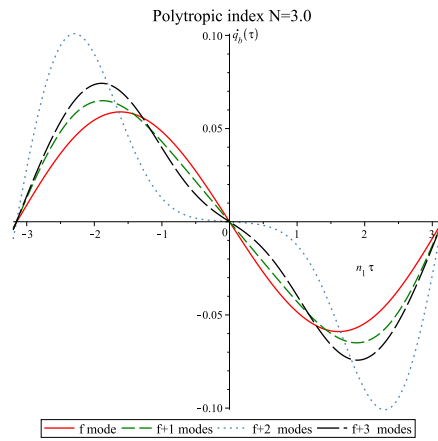
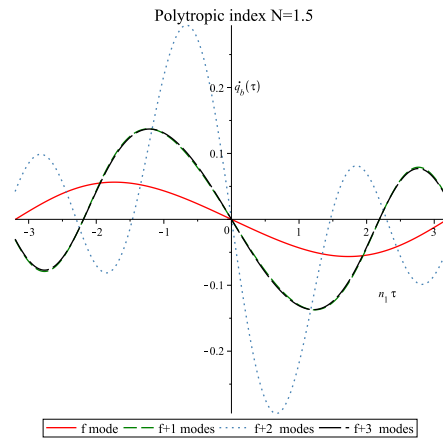


Figure 3.2. Radial velocity curves of the rotationally distorted Polytropic models ($n = 0.03$, $q = 0.0$)

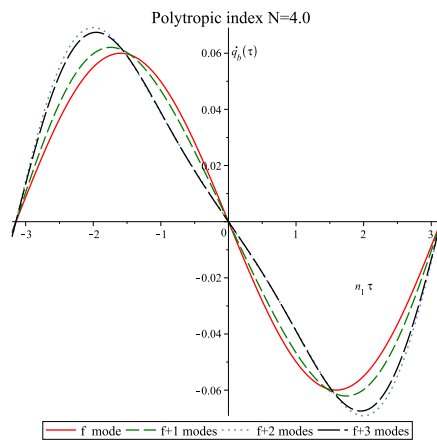
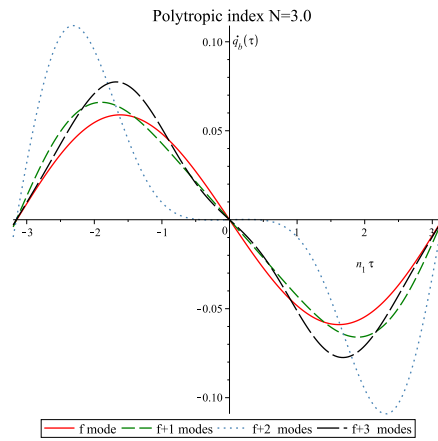
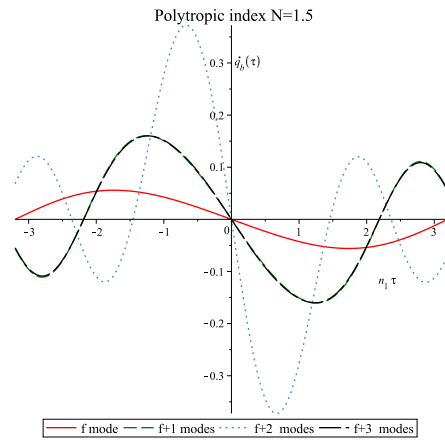


Figure 3.3. Radial velocity curves of the rotationally distorted Polytropic models ($n = 0.05$, $q = 0.0$)

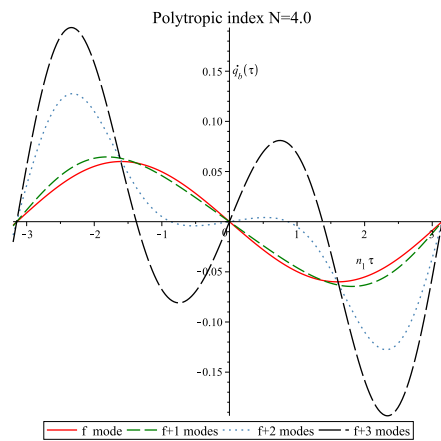
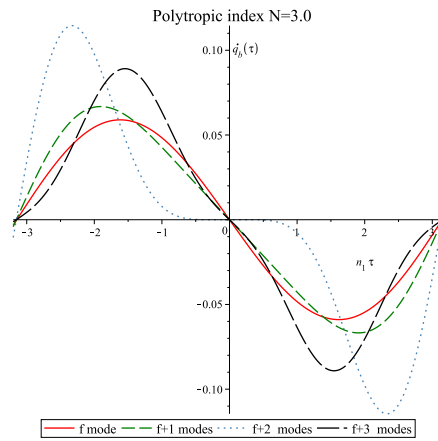
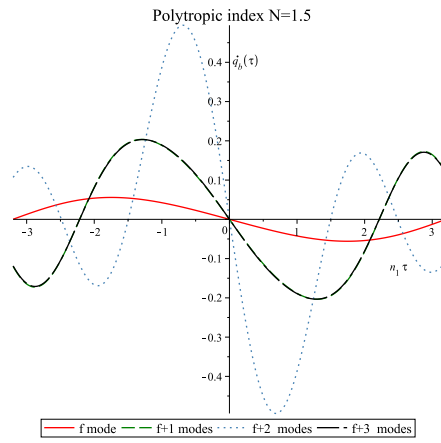
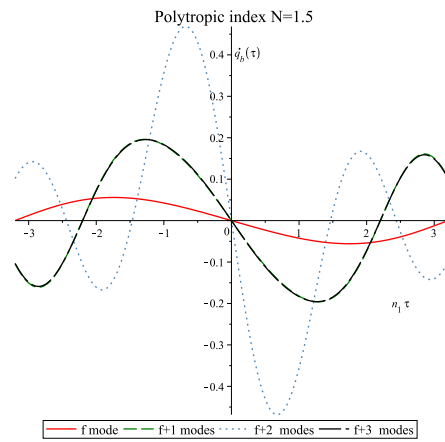
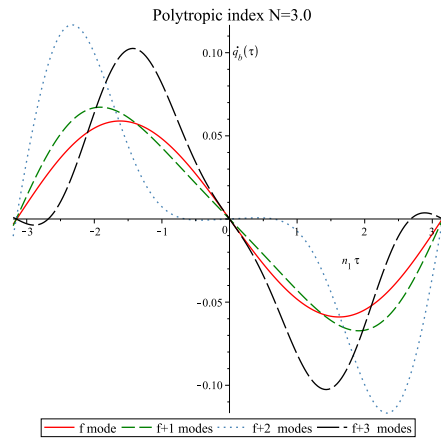


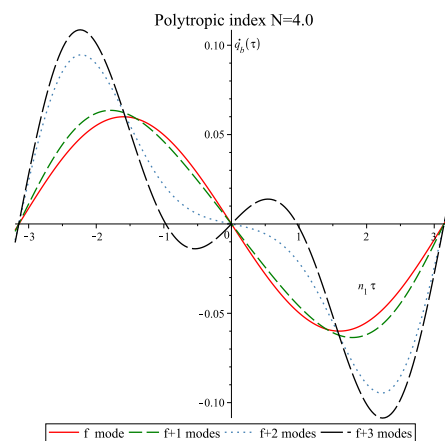
Figure 3.4. Radial velocity curves of the rotationally distorted Polytropic models ($n = 0.07$, $q = 0.0$)



(a)

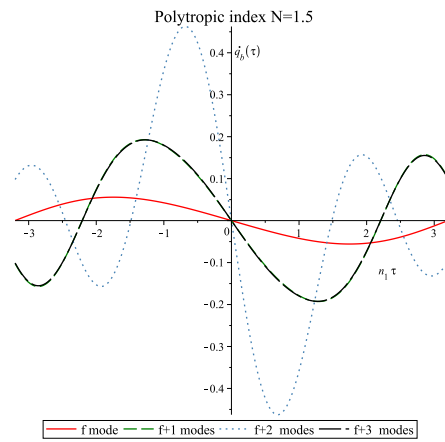


(b)

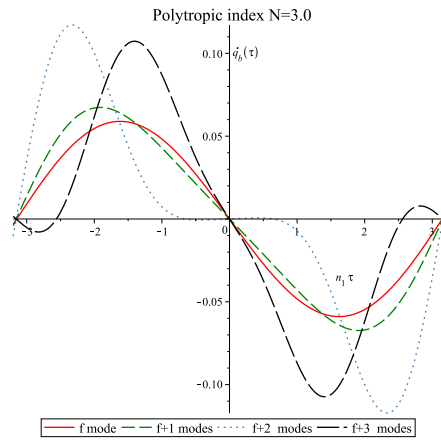


(c)

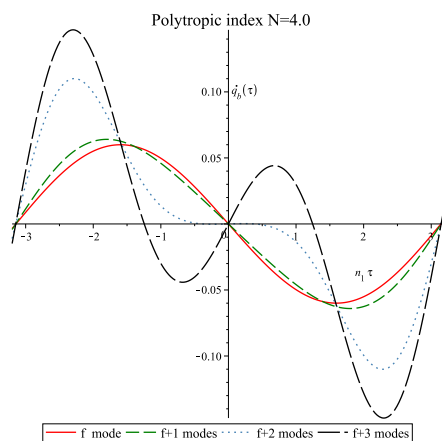
Figure 3.5. Radial velocity curves of the RTD Polytropic models ($n = 0.525$, $q = 0.05$)



(a)

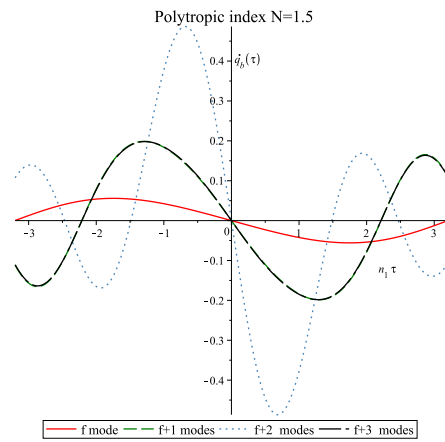


(b)

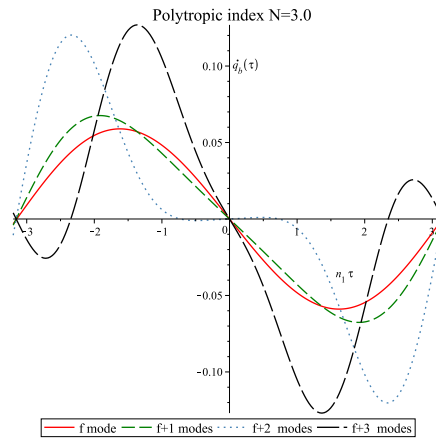


(c)

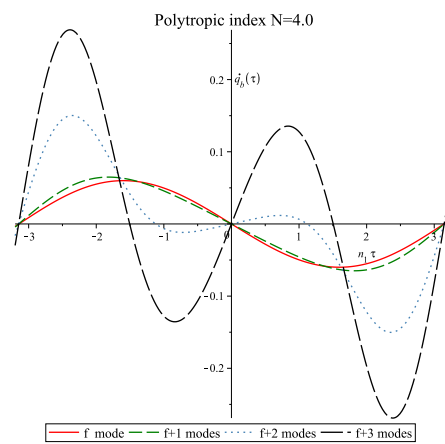
Figure 3.6. Radial velocity curves of the RTD Polytropic models ($n = 0.535, q = 0.07$)



(a)

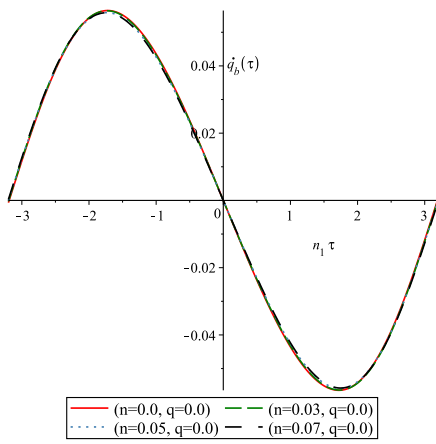


(b)

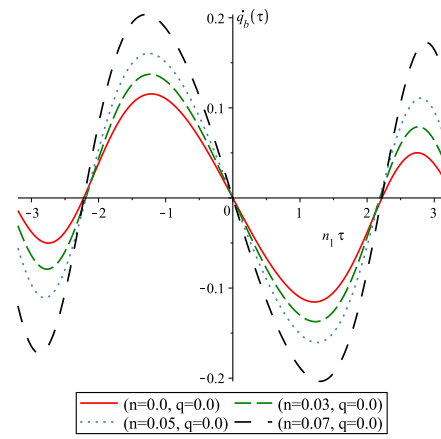


(c)

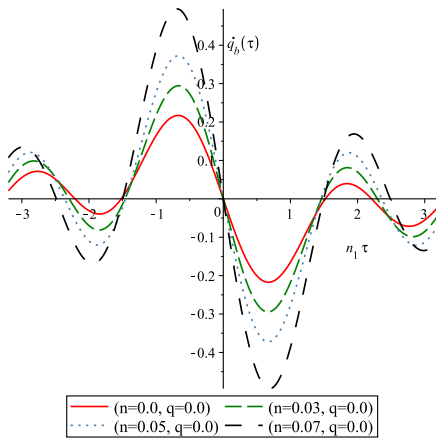
Figure 3.7. Radial velocity curves of the RTD Polytropic models ($n = 0.55, q = 0.1$)



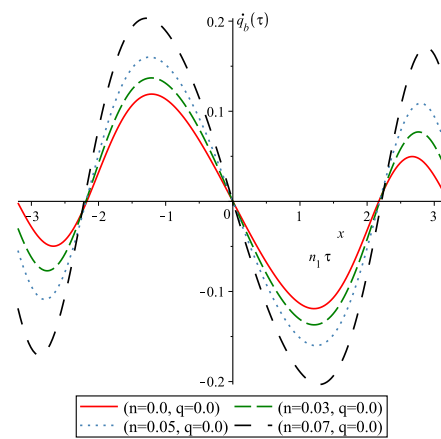
(a) f mode



(b) $f + 1$ modes

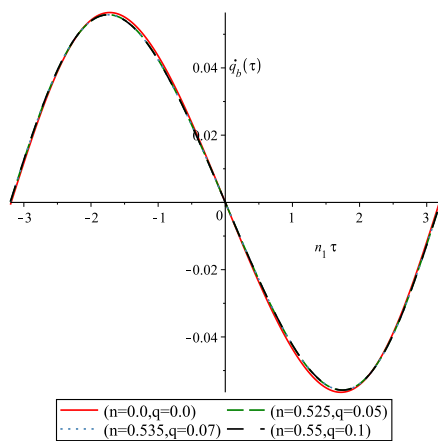


(c) $f + 2$ modes

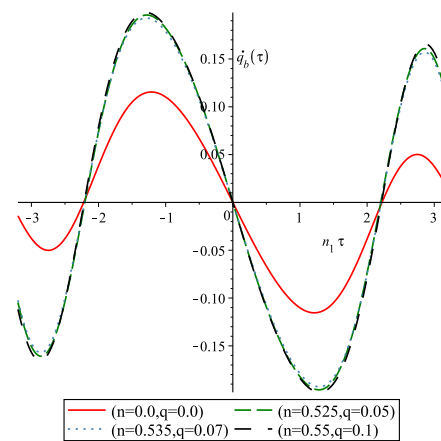


(d) $f + 3$ modes

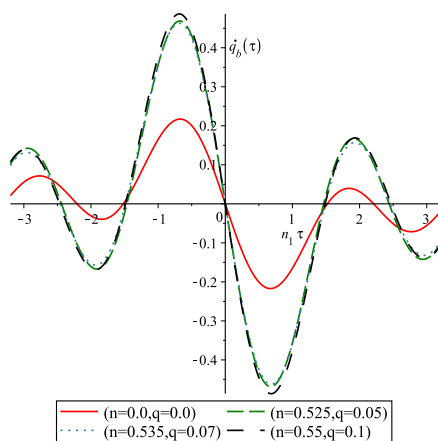
Figure 3.8. Radial velocity curves of the rotationally distorted Polytropic models $N = 1.5$ (n varies and $q = 0$)



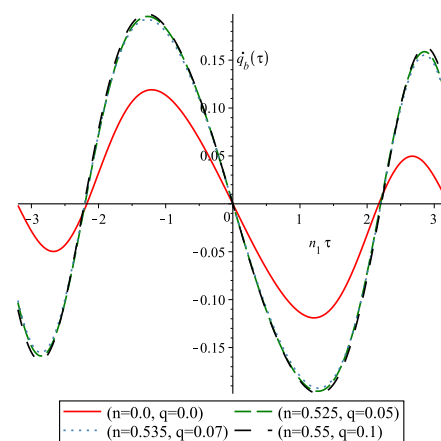
(a) f mode



(b) $f + 1$ modes



(c) $f + 2$ modes



(d) $f + 3$ modes

Figure 3.9. Radial velocity curves of the RTD Polytropic models $N = 1.5$ (both n and q varies)

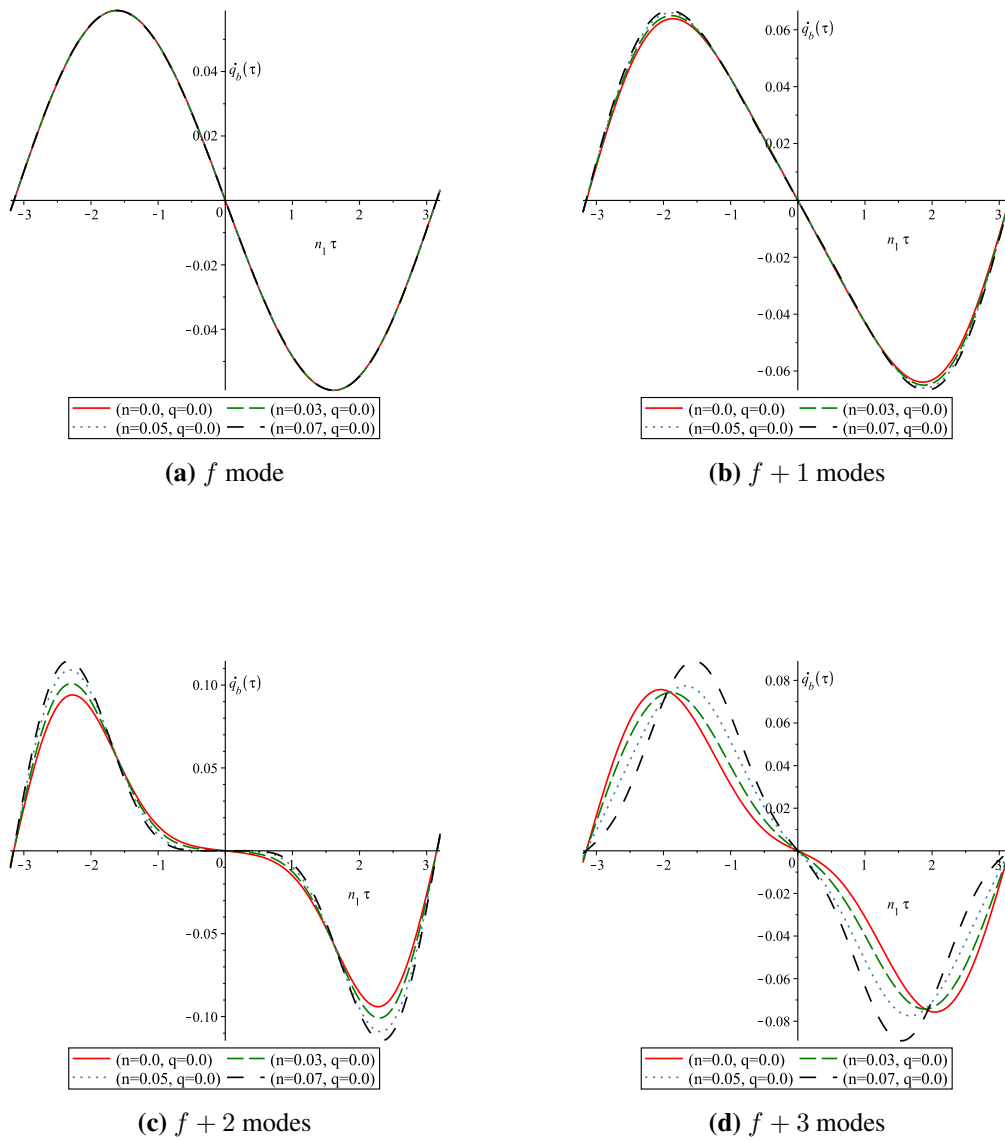
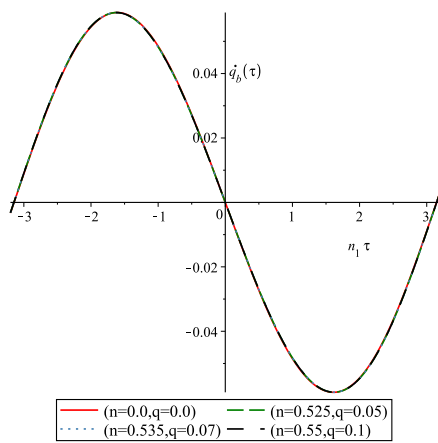
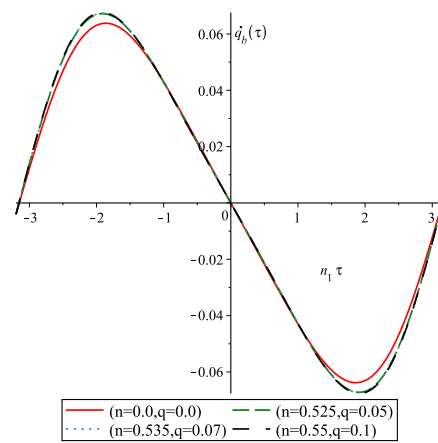


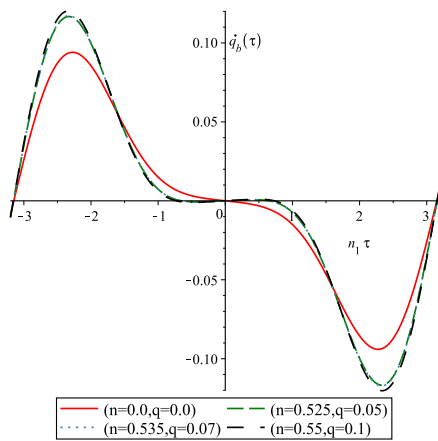
Figure 3.10. Radial velocity curves of the rotationally distorted Polytropic models $N = 3.0$ (n varies and $q = 0$)



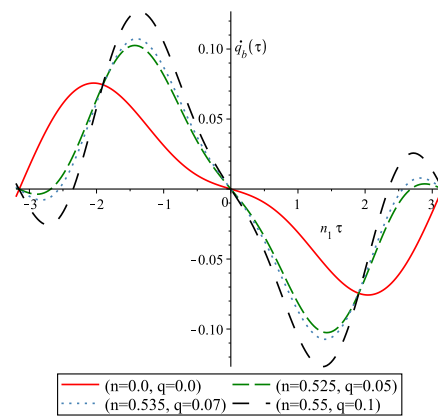
(a) f mode



(b) $f + 1$ modes



(c) $f + 2$ modes



(d) $f + 3$ modes

Figure 3.11. Radial velocity curves of the RTD Polytropic models $N = 3.0$ (both n and q varies)

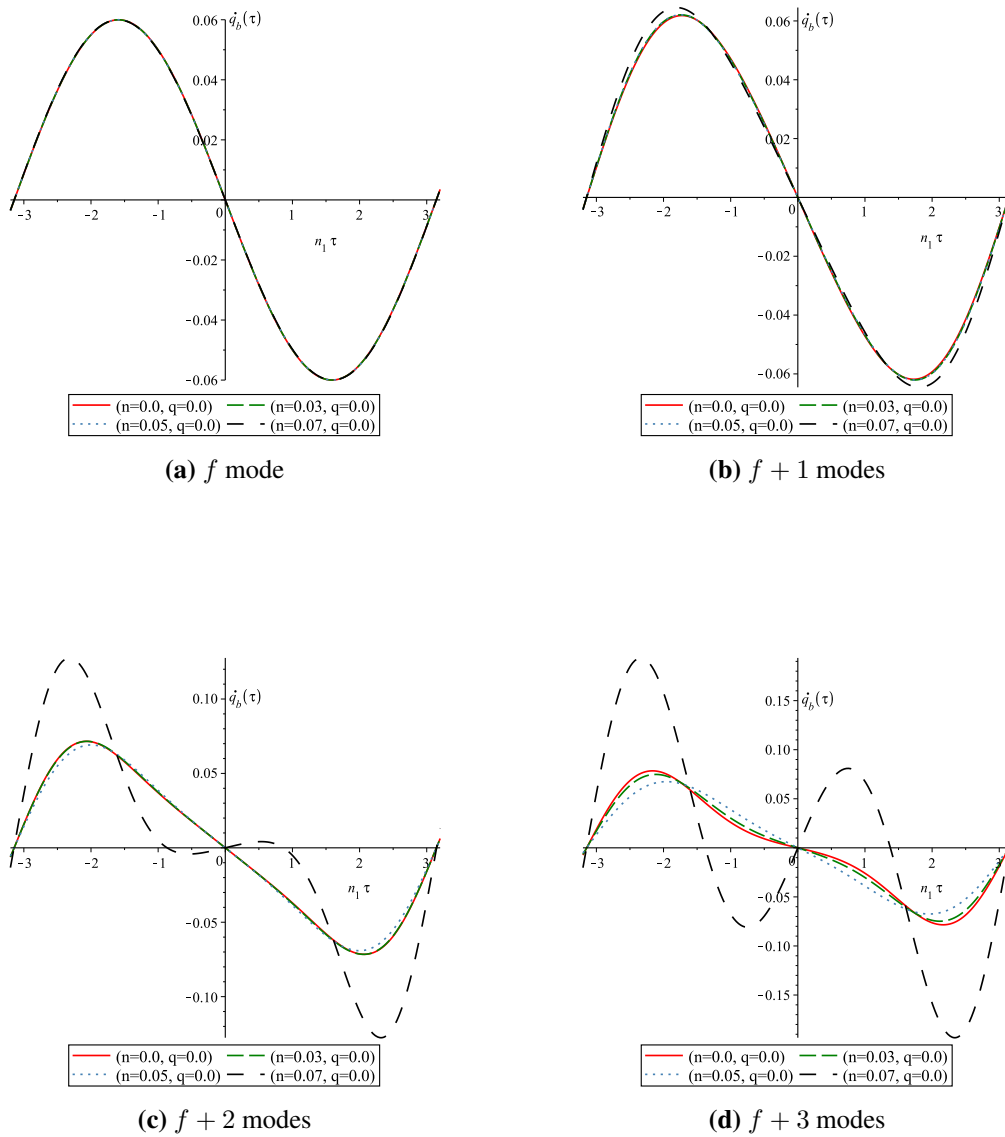
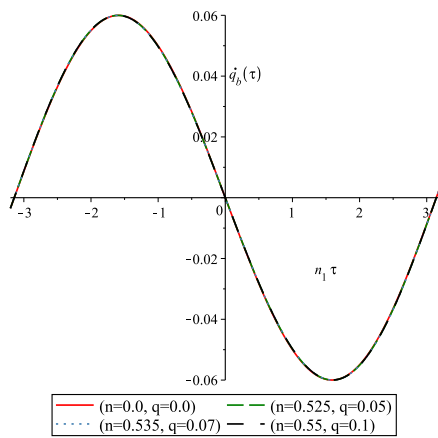
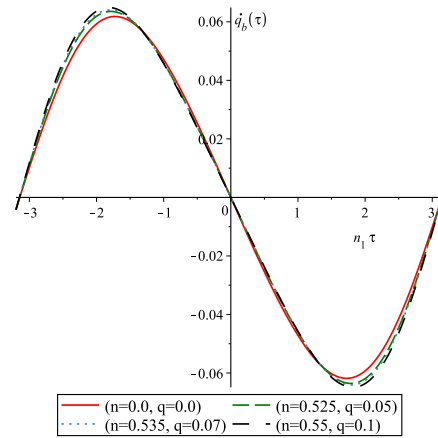


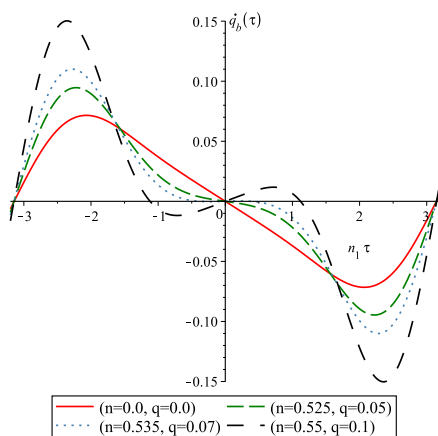
Figure 3.12. Radial velocity curves of the rotationally distorted Polytropic models $N = 4.0$ (n varies and $q = 0$)



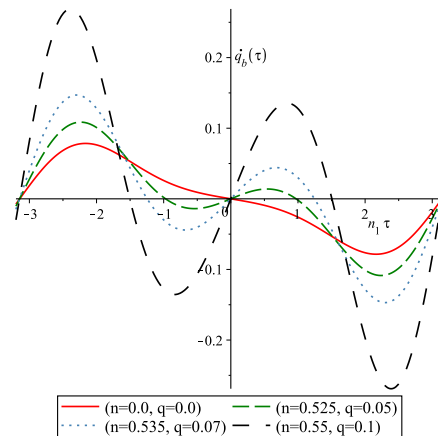
(a) f mode



(b) $f + 1$ modes



(c) $f + 2$ modes



(d) $f + 3$ modes

Figure 3.13. Radial velocity curves of the RTD Polytropic models $N = 4.0$ (both n and q varies)

Chapter 4

Effects of the amplitude of the radial pulsation on the shapes of the RVC of the RTD polytropic models of the pulsating variable stars

In chapter two we have studied the effect of rotation/or tidal distortions on the RVC of the polytropic models of the pulsating variable stars. In chapter three we studied the effect of interaction of the various modes on the radial velocity curves of the RD and RTD polytropic models of the pulsating variable stars. In these two chapters while solving the anharmonic pulsation equation we have considered the initial amplitude of the fundamental mode equal to 0.06 ($q_1(0) = 0.06$) in each case. That is, the relative amplitude of pulsation in the fundamental mode has been assumed to be 0.06 at the surface of the star as this value corresponds to the mean observed relative amplitude (Eddington [16]). Now in the present chapter we have further extended our work to check how change in the value of relative amplitude affects the RVC of the RD and RTD polytropic models of the pulsating variable stars.

In section 4.1, the equations of anharmonic pulsation as discussed in Chapter 3 has been solved numerically for first four modes of RD and RTD polytropic models of the pulsating variable stars considering certain values of relative amplitude . The results so obtained has been used to study how amplitude of pulsation affects the shape of the radial velocity curves of RD and RTD Polytropic models of pulsating variable stars. Certain conclusion based on the results obtained in the present chapter are finally discussed in section 4.2.

4.1 Numerical Computations

We have solved the equation of the anharmonic radial oscillation upto first four modes for certain RD and RTD polytropic models of the stars with polytropic indices 1.5, 3.0 and 4.0. We have considered the polytropic models (same as considered in chapter 3) with different values of the rotational distortion parameter (n) and the tidal distortion parameter (q) with $\gamma = \frac{5}{3}$. Simpson's one third rule has been further used to numerically evaluate the coefficients I_k^* and $D_{ij,k}^*$ in the anharmonic pulsation equation (3.1.1). Numerical technique as discussed in section (3.2) has been used to solve the anharmonic pulsation equation (3.1.1).

In Chapter 2 and Chapter 3 the equation of anharmonic pulsation have been solved with

$q_1(0) = 0.06$ in each case. This corresponds to the value of the relative amplitude of pulsation in the fundamental mode being assumed to be $q_1(0) = 0.06$ at the surface of the star. Now, in the present chapter we have started with $q_1(0) = 0.02$ and the solution have been obtained for the higher values upto $q_1(0) = 0.08$ with an intervals of 0.02. That is, we have considered $q_1(0) = 0.02, 0.04, 0.06$ and 0.08 . We have solved the equation of anharmonic equation (3.1.1) for fundamental (f), fundamental and first mode (f+1), fundamental and first two modes (f+2), fundamental and next three higher modes (f+3) modes of pulsation for certain values of amplitude of pulsation.

The eigenfrequencies of the pseudo radial modes of the oscillations of the RD and RTD polytropic models have been obtained earlier in chapter 3 using the approach of Mohan and Saxena [45] and are presented in Table (3.1) for certain polytropic models of the stars. The solution of the anharmonic equation has been presented in tables (4.1-4.12). The RVC for each model of polytropic indices $N = 1.5, 3.0$ and 4.0 have been obtained and are shown in Figs. (4.1-4.24). The value of the skewness coefficient K has also been computed in each case for certain values of amplitude and are given in the Table (4.13.4.15).

4.2 Concluding Observations

From Figs. (4.1-4.24), it can be observed that with increase in the value of relative amplitude of pulsation , in general, there is deviation in the shape of the radial velocity curves of the RD and RTD models of the pulsating variable stars. This effect is more appreciable near the point of maxima and minima. Also these changes in the shapes of RVC, in general, are more predominant in polytropes with index 1.5 then polytropes with index 3.0 and 4.0.

From Table (4.13-4.15), it can be observed that with increase in the value of relative amplitude there is decrease in the value of skewness coefficient (K) for RD and RTD polytropic models of pulsating variable stars. This result hold true for all polytropes with index 1.5, 3.0 and 4.0.

Hence, from the present study we can conclude that with increase in the value of relative amplitude there is decrease in the value of K and also there is appreciable deviation in the shape of radial velocity curves of RD and RTD polytropic models of the pulsating variable stars.

Table 4.1: Solution of the Anharmonic equation for RD and RTD Polytropes with index $N = 1.5$ for f mode

$q_1(0)$	Value of displacement on the surface of Star $q_b(\tau)$
$(n = 0.0, q = 0.0)$	
0.02	$q_b(\tau) = 0.00066 + 0.02000 \cos n_1\tau - 0.00022 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00251 + 0.03900 \cos n_1\tau - 0.00084 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00538 + 0.05700 \cos n_1\tau - 0.00179 \cos 2n_1\tau + 0.00004 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.00906 + 0.07400 \cos n_1\tau - 0.00302 \cos 2n_1\tau + 0.00009 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.03, q = 0.0)$	
0.02	$q_b(\tau) = 0.00070 + 0.02000 \cos n_1\tau - 0.00023 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00267 + 0.03900 \cos n_1\tau - 0.00089 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00570 + 0.05700 \cos n_1\tau - 0.00190 \cos 2n_1\tau + 0.00004 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.00961 + 0.07400 \cos n_1\tau - 0.00320 \cos 2n_1\tau + 0.00010 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.05, q = 0.0)$	
0.02	$q_b(\tau) = 0.00073 + 0.02000 \cos n_1\tau - 0.00024 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00278 + 0.03900 \cos n_1\tau - 0.00092 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00584 + 0.05650 \cos n_1\tau - 0.00194 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01002 + 0.07400 \cos n_1\tau - 0.00334 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.07, q = 0.0)$	
0.02	$q_b(\tau) = 0.00076 + 0.02000 \cos n_1\tau - 0.00025 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00290 + 0.03900 \cos n_1\tau - 0.00096 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00610 + 0.05650 \cos n_1\tau - 0.00203 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01032 + 0.07350 \cos n_1\tau - 0.00344 \cos 2n_1\tau + 0.00012 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.525, q = 0.05)$	
0.02	$q_b(\tau) = 0.00075 + 0.02000 \cos n_1\tau - 0.00025 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00285 + 0.03900 \cos n_1\tau - 0.00095 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00600 + 0.05650 \cos n_1\tau - 0.00200 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01015 + 0.07350 \cos n_1\tau - 0.00338 \cos 2n_1\tau + 0.00011 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.535, q = 0.07)$	
0.02	$q_b(\tau) = 0.00076 + 0.02000 \cos n_1\tau - 0.00025 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00289 + 0.03900 \cos n_1\tau - 0.00096 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00606 + 0.05650 \cos n_1\tau - 0.00200 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01026 + 0.07350 \cos n_1\tau - 0.00342 \cos 2n_1\tau + 0.00011 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.55, q = 0.1)$	
0.02	$q_b(\tau) = 0.00076 + 0.02000 \cos n_1\tau - 0.00025 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00290 + 0.03900 \cos n_1\tau - 0.00096 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00609 + 0.05650 \cos n_1\tau - 0.00203 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01031 + 0.07350 \cos n_1\tau - 0.00343 \cos 2n_1\tau + 0.00012 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$

Table 4.2: Solution of the Anharmonic equation for RD and RTD Polytropes with index $N = 1.5$ for $f + 1$ mode

$q_1(0)$	Value of displacement on the surface of Star $q_b(\tau)$
$(n = 0.0, q = 0.0)$	
0.02	$q_b(\tau) = 0.00128 + 0.02061 \cos n_1\tau + 0.00435 \cos 2n_1\tau - 0.00034 \cos 3n_1\tau + 0.00001 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00489 + 0.04356 \cos n_1\tau + 0.01654 \cos 2n_1\tau - 0.00254 \cos 3n_1\tau + 0.00018 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01044 + 0.07124 \cos n_1\tau + 0.03533 \cos 2n_1\tau - 0.00795 \cos 3n_1\tau + 0.00092 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01808 + 0.10745 \cos n_1\tau + 0.06117 \cos 2n_1\tau - 0.01812 \cos 3n_1\tau + 0.00313 \cos 4n_1\tau \dots$
$(n = 0.03, q = 0.0)$	
0.02	$q_b(\tau) = 0.00140 + 0.02088 \cos n_1\tau + 0.00586 \cos 2n_1\tau - 0.00050 \cos 3n_1\tau + 0.00002 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00535 + 0.04559 \cos n_1\tau + 0.02230 \cos 2n_1\tau - 0.00373 \cos 3n_1\tau + 0.00030 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01144 + 0.07759 \cos n_1\tau + 0.04764 \cos 2n_1\tau - 0.01165 \cos 3n_1\tau + 0.00155 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01981 + 0.12190 \cos n_1\tau + 0.08248 \cos 2n_1\tau - 0.02654 \cos 3n_1\tau + 0.00548 \cos 4n_1\tau \dots$
$(n = 0.05, q = 0.0)$	
0.02	$q_b(\tau) = 0.00150 + 0.02119 \cos n_1\tau + 0.00748 \cos 2n_1\tau - 0.00068 \cos 3n_1\tau + 0.00002 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00572 + 0.04784 \cos n_1\tau + 0.02845 \cos 2n_1\tau - 0.00505 \cos 3n_1\tau + 0.00045 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01222 + 0.08460 \cos n_1\tau + 0.06078 \cos 2n_1\tau - 0.01579 \cos 3n_1\tau + 0.00235 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.02115 + 0.13787 \cos n_1\tau + 0.10523 \cos 2n_1\tau - 0.03597 \cos 3n_1\tau + 0.00874 \cos 4n_1\tau \dots$
$(n = 0.07, q = 0.0)$	
0.02	$q_b(\tau) = 0.00161 + 0.02181 \cos n_1\tau + 0.01087 \cos 2n_1\tau - 0.00105 \cos 3n_1\tau + 0.00004 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00615 + 0.05242 \cos n_1\tau + 0.04134 \cos 2n_1\tau - 0.00780 \cos 3n_1\tau + 0.00077 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01315 + 0.09890 \cos n_1\tau + 0.08831 \cos 2n_1\tau - 0.02435 \cos 3n_1\tau + 0.00429 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.02276 + 0.17045 \cos n_1\tau + 0.15290 \cos 2n_1\tau - 0.05548 \cos 3n_1\tau + 0.01784 \cos 4n_1\tau \dots$
$(n = 0.525, q = 0.05)$	
0.02	$q_b(\tau) = 0.00158 + 0.02168 \cos n_1\tau + 0.01024 \cos 2n_1\tau - 0.00092 \cos 3n_1\tau + 0.00004 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00603 + 0.05146 \cos n_1\tau + 0.03895 \cos 2n_1\tau - 0.00722 \cos 3n_1\tau + 0.00070 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01289 + 0.09590 \cos n_1\tau + 0.08322 \cos 2n_1\tau - 0.02256 \cos 3n_1\tau + 0.00381 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.02232 + 0.16362 \cos n_1\tau + 0.14407 \cos 2n_1\tau - 0.05141 \cos 3n_1\tau + 0.01539 \cos 4n_1\tau \dots$
$(n = 0.535, q = 0.07)$	
0.02	$q_b(\tau) = 0.00160 + 0.02166 \cos n_1\tau + 0.01000 \cos 2n_1\tau - 0.00096 \cos 3n_1\tau + 0.00004 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00608 + 0.05132 \cos n_1\tau + 0.03803 \cos 2n_1\tau - 0.00713 \cos 3n_1\tau + 0.00069 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01300 + 0.09547 \cos n_1\tau + 0.08124 \cos 2n_1\tau - 0.02228 \cos 3n_1\tau + 0.00380 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.02252 + 0.16264 \cos n_1\tau + 0.14066 \cos 2n_1\tau - 0.05076 \cos 3n_1\tau + 0.01534 \cos 4n_1\tau \dots$
$(n = 0.55, q = 0.1)$	
0.02	$q_b(\tau) = 0.00161 + 0.02174 \cos n_1\tau + 0.01045 \cos 2n_1\tau - 0.00101 \cos 3n_1\tau + 0.00004 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00614 + 0.05195 \cos n_1\tau + 0.03977 \cos 2n_1\tau - 0.00751 \cos 3n_1\tau + 0.00074 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01311 + 0.09744 \cos n_1\tau + 0.08495 \cos 2n_1\tau - 0.02346 \cos 3n_1\tau + 0.00409 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.02270 + 0.16713 \cos n_1\tau + 0.14708 \cos 2n_1\tau - 0.05346 \cos 3n_1\tau + 0.01677 \cos 4n_1\tau \dots$

Table 4.3: Solution of the Anharmonic equation for RD and RTD Polytropes with index $N = 1.5$ for $f + 2$ mode

$q_1(0)$	Value of displacement on the surface of Star $q_b(\tau)$
$(n = 0.0, q = 0.0)$	
0.02	$q_b(\tau) = 0.00134 + 0.02099 \cos n_1\tau + 0.00448 \cos 2n_1\tau + 0.00283 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00549 + 0.04640 \cos n_1\tau + 0.01799 \cos 2n_1\tau + 0.01583 \cos 3n_1\tau + 0.00002 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01298 + 0.08028 \cos n_1\tau + 0.04137 \cos 2n_1\tau + 0.03571 \cos 3n_1\tau + 0.00014 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.02542 + 0.12849 \cos n_1\tau + 0.07806 \cos 2n_1\tau + 0.05854 \cos 3n_1\tau + 0.00058 \cos 4n_1\tau \dots$
$(n = 0.03, q = 0.0)$	
0.02	$q_b(\tau) = 0.00148 + 0.02144 \cos n_1\tau + 0.00605 \cos 2n_1\tau + 0.00555 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00624 + 0.04977 \cos n_1\tau + 0.02454 \cos 2n_1\tau + 0.02652 \cos 3n_1\tau + 0.00005 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01534 + 0.09088 \cos n_1\tau + 0.05710 \cos 2n_1\tau + 0.05312 \cos 3n_1\tau + 0.00031 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.03135 + 0.15292 \cos n_1\tau + 0.10886 \cos 2n_1\tau + 0.08040 \cos 3n_1\tau + 0.00136 \cos 4n_1\tau \dots$
$(n = 0.05, q = 0.0)$	
0.02	$q_b(\tau) = 0.00159 + 0.02195 \cos n_1\tau + 0.00770 \cos 2n_1\tau + 0.00981 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00696 + 0.05357 \cos n_1\tau + 0.03179 \cos 2n_1\tau + 0.03884 \cos 3n_1\tau + 0.00009 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01781 + 0.10282 \cos n_1\tau + 0.07460 \cos 2n_1\tau + 0.06989 \cos 3n_1\tau + 0.00056 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.03783 + 0.18040 \cos n_1\tau + 0.14308 \cos 2n_1\tau + 0.09953 \cos 3n_1\tau + 0.00281 \cos 4n_1\tau \dots$
$(n = 0.07, q = 0.0)$	
0.02	$q_b(\tau) = 0.00176 + 0.02296 \cos n_1\tau + 0.01124 \cos 2n_1\tau + 0.02088 \cos 3n_1\tau + 0.00001 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00816 + 0.06105 \cos n_1\tau + 0.04641 \cos 2n_1\tau + 0.05929 \cos 3n_1\tau + 0.00019 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.02227 + 0.26334 \cos n_1\tau + 0.10966 \cos 2n_1\tau + 0.09354 \cos 3n_1\tau + 0.00130 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.05007 + 0.23437 \cos n_1\tau + 0.21079 \cos 2n_1\tau + 0.12505 \cos 3n_1\tau + 0.00988 \cos 4n_1\tau \dots$
$(n = 0.525, q = 0.05)$	
0.02	$q_b(\tau) = 0.00171 + 0.02275 \cos n_1\tau + 0.01059 \cos 2n_1\tau + 0.02016 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00787 + 0.05950 \cos n_1\tau + 0.04364 \cos 2n_1\tau + 0.05806 \cos 3n_1\tau + 0.00016 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.02123 + 0.12145 \cos n_1\tau + 0.10297 \cos 2n_1\tau + 0.09212 \cos 3n_1\tau + 0.00105 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.04727 + 0.22316 \cos n_1\tau + 0.19786 \cos 2n_1\tau + 0.12357 \cos 3n_1\tau + 0.00696 \cos 4n_1\tau \dots$
$(n = 0.535, q = 0.07)$	
0.02	$q_b(\tau) = 0.00173 + 0.02271 \cos n_1\tau + 0.01034 \cos 2n_1\tau + 0.01864 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00791 + 0.05925 \cos n_1\tau + 0.04265 \cos 2n_1\tau + 0.05578 \cos 3n_1\tau + 0.00016 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.02130 + 0.12070 \cos n_1\tau + 0.10079 \cos 2n_1\tau + 0.08955 \cos 3n_1\tau + 0.00108 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.04736 + 0.22149 \cos n_1\tau + 0.19395 \cos 2n_1\tau + 0.12064 \cos 3n_1\tau + 0.00710 \cos 4n_1\tau \dots$
$(n = 0.55, q = 0.1)$	
0.02	$q_b(\tau) = 0.00175 + 0.02286 \cos n_1\tau + 0.01082 \cos 2n_1\tau + 0.02061 \cos 3n_1\tau + 0.00001 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00807 + 0.06031 \cos n_1\tau + 0.04468 \cos 2n_1\tau + 0.05865 \cos 3n_1\tau + 0.00017 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.02193 + 0.12400 \cos n_1\tau + 0.10566 \cos 2n_1\tau + 0.09261 \cos 3n_1\tau + 0.00119 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.04790 + 0.22906 \cos n_1\tau + 0.20333 \cos 2n_1\tau + 0.12387 \cos 3n_1\tau + 0.00843 \cos 4n_1\tau \dots$

Table 4.4: Solution of the Anharmonic equation for RD and RTD Polytropes with index $N = 1.5$ for $f + 3$ mode

$q_1(0)$	Value of displacement on the surface of Star $q_b(\tau)$
$(n = 0.0, q = 0.0)$	
0.02	$q_b(\tau) = 0.00129 + 0.02062 \cos n_1\tau + 0.00437 \cos 2n_1\tau - 0.00315 \cos 3n_1\tau + 0.00001 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00493 + 0.04360 \cos n_1\tau + 0.01663 \cos 2n_1\tau - 0.00233 \cos 3n_1\tau + 0.00018 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01054 + 0.07137 \cos n_1\tau + 0.03554 \cos 2n_1\tau - 0.00729 \cos 3n_1\tau + 0.00089 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01825 + 0.10775 \cos n_1\tau + 0.06153 \cos 2n_1\tau - 0.01660 \cos 3n_1\tau + 0.00304 \cos 4n_1\tau \dots$
$(n = 0.03, q = 0.0)$	
0.02	$q_b(\tau) = 0.00141 + 0.02089 \cos n_1\tau + 0.00588 \cos 2n_1\tau - 0.00047 \cos 3n_1\tau + 0.00001 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00537 + 0.04562 \cos n_1\tau + 0.02239 \cos 2n_1\tau - 0.00350 \cos 3n_1\tau + 0.00300 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01148 + 0.07768 \cos n_1\tau + 0.04783 \cos 2n_1\tau - 0.01095 \cos 3n_1\tau + 0.00152 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01988 + 0.12212 \cos n_1\tau + 0.08280 \cos 2n_1\tau - 0.02492 \cos 3n_1\tau + 0.00537 \cos 4n_1\tau \dots$
$(n = 0.05, q = 0.0)$	
0.02	$q_b(\tau) = 0.00150 + 0.02120 \cos n_1\tau + 0.00754 \cos 2n_1\tau - 0.00065 \cos 3n_1\tau + 0.00002 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00573 + 0.04790 \cos n_1\tau + 0.02868 \cos 2n_1\tau - 0.00486 \cos 3n_1\tau + 0.00044 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01223 + 0.08480 \cos n_1\tau + 0.06127 \cos 2n_1\tau - 0.01517 \cos 3n_1\tau + 0.00233 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.02119 + 0.13834 \cos n_1\tau + 0.10608 \cos 2n_1\tau - 0.03457 \cos 3n_1\tau + 0.00868 \cos 4n_1\tau \dots$
$(n = 0.07, q = 0.0)$	
0.02	$q_b(\tau) = 0.00162 + 0.02181 \cos n_1\tau + 0.01087 \cos 2n_1\tau - 0.00102 \cos 3n_1\tau + 0.00004 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00616 + 0.05243 \cos n_1\tau + 0.04136 \cos 2n_1\tau - 0.00761 \cos 3n_1\tau + 0.00077 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01316 + 0.09893 \cos n_1\tau + 0.08834 \cos 2n_1\tau - 0.02376 \cos 3n_1\tau + 0.00425 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.02279 + 0.17052 \cos n_1\tau + 0.15295 \cos 2n_1\tau - 0.05413 \cos 3n_1\tau + 0.01770 \cos 4n_1\tau \dots$
$(n = 0.525, q = 0.05)$	
0.02	$q_b(\tau) = 0.00159 + 0.02168 \cos n_1\tau + 0.01025 \cos 2n_1\tau - 0.00094 \cos 3n_1\tau + 0.00004 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00604 + 0.05147 \cos n_1\tau + 0.03897 \cos 2n_1\tau - 0.00699 \cos 3n_1\tau + 0.00069 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01291 + 0.09594 \cos n_1\tau + 0.08326 \cos 2n_1\tau - 0.02182 \cos 3n_1\tau + 0.00377 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.02236 + 0.16371 \cos n_1\tau + 0.14414 \cos 2n_1\tau - 0.04972 \cos 3n_1\tau + 0.01523 \cos 4n_1\tau \dots$
$(n = 0.535, q = 0.07)$	
0.02	$q_b(\tau) = 0.00160 + 0.02166 \cos n_1\tau + 0.01000 \cos 2n_1\tau - 0.00093 \cos 3n_1\tau + 0.00004 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00609 + 0.05133 \cos n_1\tau + 0.03805 \cos 2n_1\tau - 0.00692 \cos 3n_1\tau + 0.00069 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01302 + 0.09550 \cos n_1\tau + 0.08128 \cos 2n_1\tau - 0.02162 \cos 3n_1\tau + 0.00376 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.02255 + 0.16271 \cos n_1\tau + 0.14072 \cos 2n_1\tau - 0.04926 \cos 3n_1\tau + 0.01519 \cos 4n_1\tau \dots$
$(n = 0.55, q = 0.1)$	
0.02	$q_b(\tau) = 0.00161 + 0.02174 \cos n_1\tau + 0.01046 \cos 2n_1\tau - 0.00098 \cos 3n_1\tau + 0.00004 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00615 + 0.05196 \cos n_1\tau + 0.03979 \cos 2n_1\tau - 0.00730 \cos 3n_1\tau + 0.00074 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01313 + 0.09748 \cos n_1\tau + 0.08500 \cos 2n_1\tau - 0.02281 \cos 3n_1\tau + 0.00405 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.02274 + 0.16722 \cos n_1\tau + 0.14717 \cos 2n_1\tau - 0.05197 \cos 3n_1\tau + 0.01662 \cos 4n_1\tau \dots$

Table 4.5: Solution of the Anharmonic equation for RD and RTD Polytropes with index $N = 3.0$ for f mode

$q_1(0)$	Value of displacement on the surface of Star $q_b(\tau)$
$(n = 0.0, q = 0.0)$	
0.02	$q_b(\tau) = 0.00019 + 0.02000 \cos n_1\tau - 0.00006 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00780 + 0.04000 \cos n_1\tau - 0.00026 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00169 + 0.05900 \cos n_1\tau - 0.00056 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.00300 + 0.07850 \cos n_1\tau - 0.00100 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.03, q = 0.0)$	
0.02	$q_b(\tau) = 0.00020 + 0.02000 \cos n_1\tau - 0.00007 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00083 + 0.04000 \cos n_1\tau - 0.00027 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00182 + 0.05900 \cos n_1\tau - 0.00060 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.00322 + 0.07850 \cos n_1\tau - 0.00107 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.05, q = 0.0)$	
0.02	$q_b(\tau) = 0.00022 + 0.02000 \cos n_1\tau - 0.00007 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00088 + 0.04000 \cos n_1\tau - 0.00029 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00192 + 0.05900 \cos n_1\tau - 0.00064 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.00340 + 0.07850 \cos n_1\tau - 0.00113 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.07, q = 0.0)$	
0.02	$q_b(\tau) = 0.00022 + 0.02000 \cos n_1\tau - 0.00007 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00087 + 0.04000 \cos n_1\tau - 0.00029 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00191 + 0.05900 \cos n_1\tau - 0.00063 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.00338 + 0.07850 \cos n_1\tau - 0.00112 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.525, q = 0.05)$	
0.02	$q_b(\tau) = 0.00023 + 0.02000 \cos n_1\tau - 0.00007 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00092 + 0.04000 \cos n_1\tau - 0.00030 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00202 + 0.05900 \cos n_1\tau - 0.00067 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.00357 + 0.07850 \cos n_1\tau - 0.00119 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.535, q = 0.07)$	
0.02	$q_b(\tau) = 0.00023 + 0.02000 \cos n_1\tau - 0.00007 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00093 + 0.04000 \cos n_1\tau - 0.00031 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00202 + 0.05900 \cos n_1\tau - 0.00067 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.00359 + 0.07850 \cos n_1\tau - 0.00119 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.55, q = 0.1)$	
0.02	$q_b(\tau) = 0.00023 + 0.02000 \cos n_1\tau - 0.00007 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00094 + 0.04000 \cos n_1\tau - 0.00031 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00204 + 0.05900 \cos n_1\tau - 0.00068 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.00361 + 0.07850 \cos n_1\tau - 0.00120 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$

Table 4.6: Solution of the Anharmonic equation for RD and RTD Polytropes with index $N = 3.0$ for $f + 1$ mode

$q_1(0)$	Value of displacement on the surface of Star $q_b(\tau)$
$(n = 0.0, q = 0.0)$	
0.02	$q_b(\tau) = 0.00077 + 0.02010 \cos n_1\tau - 0.00055 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00308 + 0.04081 \cos n_1\tau - 0.00221 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00672 + 0.06162 \cos n_1\tau - 0.00482 \cos 2n_1\tau + 0.00017 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01189 + 0.08468 \cos n_1\tau - 0.00853 \cos 2n_1\tau + 0.00041 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
$(n = 0.03, q = 0.0)$	
0.02	$q_b(\tau) = 0.00086 + 0.02013 \cos n_1\tau - 0.00060 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00344 + 0.04106 \cos n_1\tau - 0.00240 \cos 2n_1\tau + 0.00006 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00748 + 0.06242 \cos n_1\tau - 0.00523 \cos 2n_1\tau + 0.00021 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01325 + 0.08656 \cos n_1\tau - 0.00927 \cos 2n_1\tau + 0.00050 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
$(n = 0.05, q = 0.0)$	
0.02	$q_b(\tau) = 0.00093 + 0.02016 \cos n_1\tau - 0.00064 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00375 + 0.04131 \cos n_1\tau - 0.00257 \cos 2n_1\tau + 0.00007 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00815 + 0.06320 \cos n_1\tau - 0.00560 \cos 2n_1\tau + 0.00024 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01444 + 0.08840 \cos n_1\tau - 0.00991 \cos 2n_1\tau + 0.00057 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots$
$(n = 0.07, q = 0.0)$	
0.02	$q_b(\tau) = 0.00098 + 0.02018 \cos n_1\tau - 0.00067 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00394 + 0.04147 \cos n_1\tau - 0.00270 \cos 2n_1\tau + 0.00008 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00857 + 0.06373 \cos n_1\tau - 0.00589 \cos 2n_1\tau + 0.00027 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01518 + 0.08964 \cos n_1\tau - 0.01042 \cos 2n_1\tau + 0.00063 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots$
$(n = 0.525, q = 0.05)$	
0.02	$q_b(\tau) = 0.00101 + 0.02020 \cos n_1\tau - 0.00068 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00407 + 0.04160 \cos n_1\tau - 0.00275 \cos 2n_1\tau + 0.00008 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00886 + 0.06413 \cos n_1\tau - 0.00598 \cos 2n_1\tau + 0.00028 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01568 + 0.09060 \cos n_1\tau - 0.01059 \cos 2n_1\tau + 0.00066 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots$
$(n = 0.535, q = 0.07)$	
0.02	$q_b(\tau) = 0.00102 + 0.02020 \cos n_1\tau - 0.00069 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00410 + 0.04163 \cos n_1\tau - 0.00276 \cos 2n_1\tau + 0.00008 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00892 + 0.06423 \cos n_1\tau - 0.00602 \cos 2n_1\tau + 0.00028 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01580 + 0.09081 \cos n_1\tau - 0.01065 \cos 2n_1\tau + 0.00067 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots$
$(n = 0.55, q = 0.1)$	
0.02	$q_b(\tau) = 0.00104 + 0.02021 \cos n_1\tau - 0.00070 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00416 + 0.04169 \cos n_1\tau - 0.00280 \cos 2n_1\tau + 0.00009 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00906 + 0.06442 \cos n_1\tau - 0.00610 \cos 2n_1\tau + 0.00029 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01605 + 0.09128 \cos n_1\tau - 0.01080 \cos 2n_1\tau + 0.00069 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots$

Table 4.7: Solution of the Anharmonic equation for RD and RTD Polytropes with index $N = 3.0$ for $f + 2$ mode

$q_1(0)$	Value of displacement on the surface of Star $q_b(\tau)$
$(n = 0.0, q = 0.0)$	
0.02	$q_b(\tau) = 0.00127 + 0.02013 \cos n_1\tau - 0.00232 \cos 2n_1\tau + 0.00012 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00527 + 0.04109 \cos n_1\tau - 0.00951 \cos 2n_1\tau + 0.00098 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01177 + 0.06252 \cos n_1\tau - 0.02145 \cos 2n_1\tau + 0.00316 \cos 3n_1\tau - 0.00004 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.02157 + 0.08684 \cos n_1\tau - 0.03996 \cos 2n_1\tau + 0.00748 \cos 3n_1\tau - 0.00014 \cos 4n_1\tau \dots$
$(n = 0.03, q = 0.0)$	
0.02	$q_b(\tau) = 0.00149 + 0.02020 \cos n_1\tau - 0.00247 \cos 2n_1\tau + 0.00014 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00610 + 0.04160 \cos n_1\tau - 0.01026 \cos 2n_1\tau + 0.00115 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01375 + 0.06418 \cos n_1\tau - 0.02361 \cos 2n_1\tau + 0.00372 \cos 3n_1\tau - 0.00006 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.02556 + 0.09077 \cos n_1\tau - 0.04514 \cos 2n_1\tau + 0.00880 \cos 3n_1\tau - 0.00019 \cos 4n_1\tau \dots$
$(n = 0.05, q = 0.0)$	
0.02	$q_b(\tau) = 0.00167 + 0.02026 \cos n_1\tau - 0.00262 \cos 2n_1\tau + 0.00016 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00688 + 0.04215 \cos n_1\tau - 0.01103 \cos 2n_1\tau + 0.00133 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01569 + 0.06593 \cos n_1\tau - 0.02588 \cos 2n_1\tau + 0.00430 \cos 3n_1\tau - 0.00007 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.02957 + 0.09495 \cos n_1\tau - 0.05072 \cos 2n_1\tau + 0.01019 \cos 3n_1\tau - 0.00024 \cos 4n_1\tau \dots$
$(n = 0.07, q = 0.0)$	
0.02	$q_b(\tau) = 0.00180 + 0.02031 \cos n_1\tau - 0.00278 \cos 2n_1\tau + 0.00020 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00748 + 0.04254 \cos n_1\tau - 0.01182 \cos 2n_1\tau + 0.00162 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01718 + 0.06720 \cos n_1\tau - 0.02809 \cos 2n_1\tau + 0.00521 \cos 3n_1\tau - 0.00010 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.03270 + 0.09797 \cos n_1\tau - 0.05593 \cos 2n_1\tau + 0.01233 \cos 3n_1\tau - 0.00033 \cos 4n_1\tau \dots$
$(n = 0.525, q = 0.05)$	
0.02	$q_b(\tau) = 0.00187 + 0.02035 \cos n_1\tau - 0.00280 \cos 2n_1\tau + 0.00020 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00782 + 0.04287 \cos n_1\tau - 0.01199 \cos 2n_1\tau + 0.00161 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01809 + 0.06827 \cos n_1\tau - 0.02884 \cos 2n_1\tau + 0.00519 \cos 3n_1\tau - 0.00010 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.03472 + 0.10053 \cos n_1\tau - 0.05827 \cos 2n_1\tau + 0.01230 \cos 3n_1\tau - 0.00033 \cos 4n_1\tau \dots$
$(n = 0.535, q = 0.07)$	
0.02	$q_b(\tau) = 0.00189 + 0.02036 \cos n_1\tau - 0.00280 \cos 2n_1\tau + 0.00020 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00790 + 0.04295 \cos n_1\tau - 0.01201 \cos 2n_1\tau + 0.00162 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01829 + 0.06852 \cos n_1\tau - 0.02896 \cos 2n_1\tau + 0.00522 \cos 3n_1\tau - 0.00010 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.03516 + 0.10113 \cos n_1\tau - 0.05870 \cos 2n_1\tau + 0.01237 \cos 3n_1\tau - 0.00033 \cos 4n_1\tau \dots$
$(n = 0.55, q = 0.1)$	
0.02	$q_b(\tau) = 0.00194 + 0.02038 \cos n_1\tau - 0.00286 \cos 2n_1\tau + 0.00021 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00814 + 0.04311 \cos n_1\tau - 0.01235 \cos 2n_1\tau + 0.00173 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01891 + 0.06906 \cos n_1\tau - 0.02996 \cos 2n_1\tau + 0.00559 \cos 3n_1\tau - 0.00011 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.03651 + 0.10242 \cos n_1\tau - 0.06116 \cos 2n_1\tau + 0.01323 \cos 3n_1\tau - 0.00037 \cos 4n_1\tau \dots$

Table 4.8: Solution of the Anharmonic equation for RD and RTD Polytropes with index $N = 3.0$ for $f + 3$ mode

$q_1(0)$	Value of displacement on the surface of Star $q_b(\tau)$
$(n = 0.0, q = 0.0)$	
0.02	$q_b(\tau) = 0.00145 + 0.02016 \cos n_1\tau - 0.00138 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00580 + 0.04132 \cos n_1\tau - 0.00555 \cos 2n_1\tau - 0.00001 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01261 + 0.06326 \cos n_1\tau - 0.01208 \cos 2n_1\tau - 0.00004 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.02205 + 0.08785 \cos n_1\tau - 0.02112 \cos 2n_1\tau - 0.00009 \cos 3n_1\tau - 0.00003 \cos 4n_1\tau \dots$
$(n = 0.03, q = 0.0)$	
0.02	$q_b(\tau) = 0.00169 + 0.02027 \cos n_1\tau - 0.00097 \cos 2n_1\tau - 0.00003 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00679 + 0.04223 \cos n_1\tau - 0.00389 \cos 2n_1\tau - 0.00029 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01477 + 0.06616 \cos n_1\tau - 0.00848 \cos 2n_1\tau - 0.00093 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.02615 + 0.09536 \cos n_1\tau - 0.01501 \cos 2n_1\tau - 0.00218 \cos 3n_1\tau - 0.00004 \cos 4n_1\tau \dots$
$(n = 0.05, q = 0.0)$	
0.02	$q_b(\tau) = 0.00190 + 0.02043 \cos n_1\tau - 0.00039 \cos 2n_1\tau - 0.00009 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00761 + 0.04344 \cos n_1\tau - 0.00159 \cos 2n_1\tau - 0.00072 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01656 + 0.07003 \cos n_1\tau - 0.00346 \cos 2n_1\tau - 0.00233 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.02932 + 0.10450 \cos n_1\tau - 0.00613 \cos 2n_1\tau - 0.00550 \cos 3n_1\tau - 0.00004 \cos 4n_1\tau \dots$
$(n = 0.07, q = 0.0)$	
0.02	$q_b(\tau) = 0.00200 + 0.02063 \cos n_1\tau + 0.00017 \cos 2n_1\tau - 0.00018 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00801 + 0.04506 \cos n_1\tau + 0.00068 \cos 2n_1\tau - 0.00146 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01743 + 0.07526 \cos n_1\tau + 0.00149 \cos 2n_1\tau - 0.00471 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.03086 + 0.11679 \cos n_1\tau + 0.00265 \cos 2n_1\tau - 0.01109 \cos 3n_1\tau - 0.00004 \cos 4n_1\tau \dots$
$(n = 0.525, q = 0.05)$	
0.02	$q_b(\tau) = 0.00211 + 0.02080 \cos n_1\tau + 0.00113 \cos 2n_1\tau - 0.00026 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00845 + 0.04642 \cos n_1\tau + 0.00453 \cos 2n_1\tau - 0.00212 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01839 + 0.07962 \cos n_1\tau + 0.00987 \cos 2n_1\tau - 0.00682 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.03256 + 0.12706 \cos n_1\tau + 0.01747 \cos 2n_1\tau - 0.01607 \cos 3n_1\tau - 0.00030 \cos 4n_1\tau \dots$
$(n = 0.535, q = 0.07)$	
0.02	$q_b(\tau) = 0.00212 + 0.02079 \cos n_1\tau + 0.00102 \cos 2n_1\tau - 0.00025 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00848 + 0.04634 \cos n_1\tau + 0.00408 \cos 2n_1\tau - 0.00204 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01845 + 0.07935 \cos n_1\tau + 0.00888 \cos 2n_1\tau - 0.00654 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.03266 + 0.12643 \cos n_1\tau + 0.01572 \cos 2n_1\tau - 0.01542 \cos 3n_1\tau - 0.00003 \cos 4n_1\tau \dots$
$(n = 0.55, q = 0.1)$	
0.02	$q_b(\tau) = 0.00218 + 0.02108 \cos n_1\tau + 0.00237 \cos 2n_1\tau - 0.00041 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00873 + 0.04864 \cos n_1\tau + 0.00948 \cos 2n_1\tau - 0.00331 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01899 + 0.08675 \cos n_1\tau + 0.00206 \cos 2n_1\tau - 0.01064 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.03362 + 0.14386 \cos n_1\tau + 0.03652 \cos 2n_1\tau - 0.02507 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$

Table 4.9: Solution of the Anharmonic equation for RD and RTD Polytropes with index $N = 4.0$ for f mode

$q_1(0)$	Value of displacement on the surface of Star $q_b(\tau)$
$(n = 0.0, q = 0.0)$	
0.02	$q_b(\tau) = 0.00009 + 0.02000 \cos n_1\tau - 0.00003 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00035 + 0.04000 \cos n_1\tau - 0.00012 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00080 + 0.06000 \cos n_1\tau - 0.00026 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.00143 + 0.08000 \cos n_1\tau - 0.00047 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.03, q = 0.0)$	
0.02	$q_b(\tau) = 0.00009 + 0.02000 \cos n_1\tau - 0.00003 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00039 + 0.04000 \cos n_1\tau - 0.00013 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00087 + 0.06000 \cos n_1\tau - 0.00029 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.00152 + 0.07900 \cos n_1\tau - 0.00050 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.05, q = 0.0)$	
0.02	$q_b(\tau) = 0.00010 + 0.02000 \cos n_1\tau - 0.00003 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00041 + 0.04000 \cos n_1\tau - 0.00013 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00093 + 0.06000 \cos n_1\tau - 0.00031 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.00166 + 0.08000 \cos n_1\tau - 0.00055 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.07, q = 0.0)$	
0.02	$q_b(\tau) = 0.00012 + 0.02000 \cos n_1\tau - 0.00004 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00051 + 0.04000 \cos n_1\tau - 0.00017 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00115 + 0.06000 \cos n_1\tau - 0.00038 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.00200 + 0.07900 \cos n_1\tau - 0.00066 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.525, q = 0.05)$	
0.02	$q_b(\tau) = 0.00012 + 0.02000 \cos n_1\tau - 0.00004 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00048 + 0.04000 \cos n_1\tau - 0.00016 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00108 + 0.06000 \cos n_1\tau - 0.00036 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.00188 + 0.07900 \cos n_1\tau - 0.00062 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.535, q = 0.07)$	
0.02	$q_b(\tau) = 0.00012 + 0.02000 \cos n_1\tau - 0.00004 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00498 + 0.04000 \cos n_1\tau - 0.00016 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01121 + 0.06000 \cos n_1\tau - 0.00037 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00001 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01943 + 0.07900 \cos n_1\tau - 0.00064 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.55, q = 0.1)$	
0.02	$q_b(\tau) = 0.00013 + 0.02000 \cos n_1\tau - 0.00004 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00521 + 0.04000 \cos n_1\tau - 0.00017 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01173 + 0.06000 \cos n_1\tau - 0.00039 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.00203 + 0.07900 \cos n_1\tau - 0.00067 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$

Table 4.10: Solution of the Anharmonic equation for RD and RTD Polytropes with index $N = 4.0$ for $f + 1$ mode

$q_1(0)$	Value of displacement on the surface of Star $q_b(\tau)$
$(n = 0.0, q = 0.0)$	
0.02	$q_b(\tau) = 0.00044 + 0.02004 \cos n_1\tau - 0.00026 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00173 + 0.04037 \cos n_1\tau - 0.00107 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00398 + 0.06127 \cos n_1\tau - 0.00240 \cos 2n_1\tau + 0.00004 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.00709 + 0.08301 \cos n_1\tau - 0.00427 \cos 2n_1\tau + 0.00011 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.03, q = 0.0)$	
0.02	$q_b(\tau) = 0.00047 + 0.02005 \cos n_1\tau - 0.00029 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00190 + 0.04041 \cos n_1\tau - 0.00116 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00428 + 0.06140 \cos n_1\tau - 0.00262 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.00743 + 0.08220 \cos n_1\tau - 0.00455 \cos 2n_1\tau + 0.00012 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.05, q = 0.0)$	
0.02	$q_b(\tau) = 0.00049 + 0.02005 \cos n_1\tau - 0.00029 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00196 + 0.04040 \cos n_1\tau - 0.00119 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00443 + 0.06137 \cos n_1\tau - 0.00269 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.00768 + 0.08213 \cos n_1\tau - 0.00467 \cos 2n_1\tau + 0.00011 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.07, q = 0.0)$	
0.02	$q_b(\tau) = 0.00070 + 0.02011 \cos n_1\tau - 0.00043 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00282 + 0.04092 \cos n_1\tau - 0.00175 \cos 2n_1\tau + 0.00003 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00635 + 0.06310 \cos n_1\tau - 0.00394 \cos 2n_1\tau + 0.00012 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01102 + 0.08608 \cos n_1\tau - 0.00684 \cos 2n_1\tau + 0.00028 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.525, q = 0.05)$	
0.02	$q_b(\tau) = 0.00063 + 0.02008 \cos n_1\tau - 0.00039 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00252 + 0.04071 \cos n_1\tau - 0.00156 \cos 2n_1\tau + 0.00002 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00569 + 0.06239 \cos n_1\tau - 0.00351 \cos 2n_1\tau + 0.00009 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.00986 + 0.08446 \cos n_1\tau - 0.00608 \cos 2n_1\tau + 0.00021 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.535, q = 0.07)$	
0.02	$q_b(\tau) = 0.00067 + 0.02010 \cos n_1\tau - 0.00041 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00268 + 0.04081 \cos n_1\tau - 0.00165 \cos 2n_1\tau + 0.00003 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00603 + 0.06275 \cos n_1\tau - 0.00373 \cos 2n_1\tau + 0.00010 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01046 + 0.08528 \cos n_1\tau - 0.00647 \cos 2n_1\tau + 0.00024 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.55, q = 0.1)$	
0.02	$q_b(\tau) = 0.00072 + 0.02012 \cos n_1\tau - 0.00045 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00291 + 0.04099 \cos n_1\tau - 0.00182 \cos 2n_1\tau + 0.00004 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00656 + 0.06335 \cos n_1\tau - 0.00410 \cos 2n_1\tau + 0.00013 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01137 + 0.08665 \cos n_1\tau - 0.00711 \cos 2n_1\tau + 0.00031 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$

Table 4.11: Solution of the Anharmonic equation for RD and RTD Polytropes with index $N = 4.0$ for $f + 2$ mode

$q_1(0)$	Value of displacement on the surface of Star $q_b(\tau)$
$(n = 0.0, q = 0.0)$	
0.02	$q_b(\tau) = 0.00090 + 0.02014 \cos n_1\tau - 0.00093 \cos 2n_1\tau + 0.00004 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00368 + 0.04113 \cos n_1\tau - 0.00390 \cos 2n_1\tau + 0.00032 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00859 + 0.06384 \cos n_1\tau - 0.00942 \cos 2n_1\tau + 0.00108 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01603 + 0.08913 \cos n_1\tau - 0.01835 \cos 2n_1\tau + 0.00257 \cos 3n_1\tau - 0.00003 \cos 4n_1\tau \dots$
$(n = 0.03, q = 0.0)$	
0.02	$q_b(\tau) = 0.00092 + 0.02015 \cos n_1\tau - 0.00091 \cos 2n_1\tau + 0.00003 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00376 + 0.04120 \cos n_1\tau - 0.00384 \cos 2n_1\tau + 0.00030 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00879 + 0.06408 \cos n_1\tau - 0.00929 \cos 2n_1\tau + 0.00101 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01597 + 0.08834 \cos n_1\tau - 0.01757 \cos 2n_1\tau + 0.00231 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots$
$(n = 0.05, q = 0.0)$	
0.02	$q_b(\tau) = 0.00087 + 0.02011 \cos n_1\tau - 0.00084 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00355 + 0.04049 \cos n_1\tau - 0.00346 \cos 2n_1\tau + 0.00016 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00815 + 0.06318 \cos n_1\tau - 0.00808 \cos 2n_1\tau + 0.00056 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01453 + 0.08628 \cos n_1\tau - 0.01468 \cos 2n_1\tau + 0.00128 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
$(n = 0.07, q = 0.0)$	
0.02	$q_b(\tau) = 0.00225 + 0.02047 \cos n_1\tau - 0.00287 \cos 2n_1\tau + 0.00019 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00946 + 0.04382 \cos n_1\tau - 0.01263 \cos 2n_1\tau + 0.00158 \cos 3n_1\tau - 0.00020 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.02297 + 0.07298 \cos n_1\tau - 0.03268 \cos 2n_1\tau + 0.00535 \cos 3n_1\tau - 0.00011 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.04369 + 0.10881 \cos n_1\tau - 0.06648 \cos 2n_1\tau + 0.01223 \cos 3n_1\tau - 0.00035 \cos 4n_1\tau \dots$
$(n = 0.525, q = 0.05)$	
0.02	$q_b(\tau) = 0.00165 + 0.02028 \cos n_1\tau - 0.00195 \cos 2n_1\tau + 0.00008 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00679 + 0.04229 \cos n_1\tau - 0.00820 \cos 2n_1\tau + 0.00066 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01599 + 0.06778 \cos n_1\tau - 0.01992 \cos 2n_1\tau + 0.00225 \cos 3n_1\tau - 0.00030 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.02932 + 0.09683 \cos n_1\tau - 0.03791 \cos 2n_1\tau + 0.00515 \cos 3n_1\tau - 0.00009 \cos 4n_1\tau \dots$
$(n = 0.535, q = 0.07)$	
0.02	$q_b(\tau) = 0.00196 + 0.02037 \cos n_1\tau - 0.00241 \cos 2n_1\tau + 0.00013 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00814 + 0.04303 \cos n_1\tau - 0.01036 \cos 2n_1\tau + 0.00107 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01944 + 0.07027 \cos n_1\tau - 0.02592 \cos 2n_1\tau + 0.00363 \cos 3n_1\tau - 0.00006 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.03627 + 0.10257 \cos n_1\tau - 0.05097 \cos 2n_1\tau + 0.00830 \cos 3n_1\tau - 0.00019 \cos 4n_1\tau \dots$
$(n = 0.55, q = 0.1)$	
0.02	$q_b(\tau) = 0.00251 + 0.02056 \cos n_1\tau - 0.00335 \cos 2n_1\tau + 0.00028 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.01066 + 0.04453 \cos n_1\tau - 0.01513 \cos 2n_1\tau + 0.00225 \cos 3n_1\tau - 0.00003 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.02633 + 0.07538 \cos n_1\tau - 0.04050 \cos 2n_1\tau + 0.00762 \cos 3n_1\tau - 0.00019 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.05098 + 0.11436 \cos n_1\tau - 0.08520 \cos 2n_1\tau + 0.01744 \cos 3n_1\tau - 0.00059 \cos 4n_1\tau \dots$

Table 4.12: Solution of the Anharmonic equation for RD and RTD Polytropes with index $N = 4.0$ for $f + 3$ mode

$q_1(0)$	Value of displacement on the surface of Star $q_b(\tau)$
$(n = 0.0, q = 0.0)$	
0.02	$q_b(\tau) = 0.00105 + 0.02005 \cos n_1\tau - 0.00160 \cos 2n_1\tau + 0.000005 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00423 + 0.04040 \cos n_1\tau - 0.00642 \cos 2n_1\tau + 0.00041 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00953 + 0.06135 \cos n_1\tau - 0.01445 \cos 2n_1\tau + 0.00139 \cos 3n_1\tau - 0.00008 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01695 + 0.08321 \cos n_1\tau - 0.02569 \cos 2n_1\tau + 0.00331 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots$
$(n = 0.03, q = 0.0)$	
0.02	$q_b(\tau) = 0.00102 + 0.02007 \cos n_1\tau - 0.00136 \cos 2n_1\tau + 0.00003 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00409 + 0.04058 \cos n_1\tau - 0.00543 \cos 2n_1\tau + 0.00030 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00920 + 0.06197 \cos n_1\tau - 0.01223 \cos 2n_1\tau + 0.00102 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01595 + 0.08349 \cos n_1\tau - 0.02121 \cos 2n_1\tau + 0.00233 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
$(n = 0.05, q = 0.0)$	
0.02	$q_b(\tau) = 0.00086 + 0.02007 \cos n_1\tau - 0.00081 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00346 + 0.04062 \cos n_1\tau - 0.00327 \cos 2n_1\tau + 0.00007 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.00780 + 0.06212 \cos n_1\tau - 0.00736 \cos 2n_1\tau + 0.00025 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.01353 + 0.08384 \cos n_1\tau - 0.01276 \cos 2n_1\tau + 0.00058 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
$(n = 0.07, q = 0.0)$	
0.02	$q_b(\tau) = 0.00334 + 0.02019 \cos n_1\tau - 0.00768 \cos 2n_1\tau + 0.00018 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.01338 + 0.04155 \cos n_1\tau - 0.03075 \cos 2n_1\tau + 0.00147 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.03012 + 0.06524 \cos n_1\tau - 0.06920 \cos 2n_1\tau + 0.00496 \cos 3n_1\tau - 0.00003 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.05222 + 0.09096 \cos n_1\tau - 0.11996 \cos 2n_1\tau + 0.01132 \cos 3n_1\tau - 0.00009 \cos 4n_1\tau \dots$
$(n = 0.525, q = 0.05)$	
0.02	$q_b(\tau) = 0.00193 + 0.02016 \cos n_1\tau - 0.00322 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.00773 + 0.04130 \cos n_1\tau - 0.01290 \cos 2n_1\tau + 0.00041 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.01740 + 0.06440 \cos n_1\tau - 0.02904 \cos 2n_1\tau + 0.00140 \cos 3n_1\tau - 0.00008 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.03017 + 0.08905 \cos n_1\tau - 0.05034 \cos 2n_1\tau + 0.00320 \cos 3n_1\tau - 0.00002 \cos 4n_1\tau \dots$
$(n = 0.535, q = 0.07)$	
0.02	$q_b(\tau) = 0.00260 + 0.02019 \cos n_1\tau - 0.00525 \cos 2n_1\tau + 0.00009 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.01041 + 0.04157 \cos n_1\tau - 0.02101 \cos 2n_1\tau + 0.00078 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.02343 + 0.06531 \cos n_1\tau - 0.04728 \cos 2n_1\tau + 0.00265 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.04062 + 0.09114 \cos n_1\tau - 0.08197 \cos 2n_1\tau + 0.00606 \cos 3n_1\tau - 0.00004 \cos 4n_1\tau \dots$
$(n = 0.55, q = 0.1)$	
0.02	$q_b(\tau) = 0.00450 + 0.02002 \cos n_1\tau - 0.01128 \cos 2n_1\tau + 0.00042 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
0.04	$q_b(\tau) = 0.01802 + 0.04021 \cos n_1\tau - 0.04491 \cos 2n_1\tau + 0.00335 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
0.06	$q_b(\tau) = 0.04056 + 0.06072 \cos n_1\tau - 0.10105 \cos 2n_1\tau + 0.01133 \cos 3n_1\tau - 0.00006 \cos 4n_1\tau \dots$
0.08	$q_b(\tau) = 0.07031 + 0.08066 \cos n_1\tau - 0.17519 \cos 2n_1\tau + 0.02586 \cos 3n_1\tau - 0.00020 \cos 4n_1\tau \dots$

Table 4.13: Values of n_1^2 and the skewness coefficient K for RD and RTD Polytropes with index $N = 1.5$

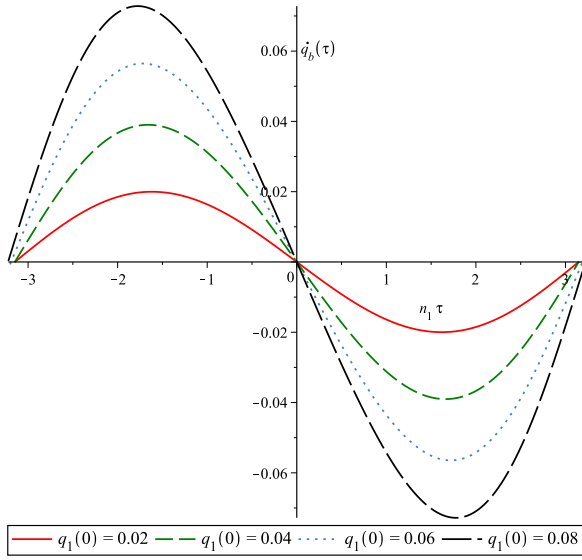
$q_1(0)$	f mode		$f + 1$ modes		$f + 2$ modes		$f + 3$ modes	
	n_1^2	K	n_1^2	K	n_1^2	K	n_1^2	K
$(n = 0.0, q = 0.0)$								
0.02	0.99634	0.4944	0.98844	0.6431	0.98844	0.7926	0.98844	0.6477
0.04	0.98609	0.4812	0.95607	0.6428	0.95607	0.7905	0.95607	0.6473
0.06	0.97029	0.4621	0.90616	0.6226	0.90616	0.7899	0.90616	0.6219
0.08	0.94993	0.4449	0.83753	0.5833	0.83753	0.7807	0.83753	0.5935
$(n = 0.03, q = 0.0)$								
0.02	0.99588	0.4934	0.98527	0.6534	0.98526	0.7999	0.98526	0.6551
0.04	0.98435	0.4682	0.94401	0.6464	0.94395	0.7992	0.94395	0.6502
0.06	0.96658	0.4587	0.88040	0.6152	0.88028	0.7920	0.88028	0.6213
0.08	0.94368	0.4400	0.79294	0.5643	0.79273	0.7775	0.79273	0.5700
$(n = 0.05, q = 0.0)$								
0.02	0.99553	0.4927	0.98203	0.6627	0.98195	0.8048	0.98195	0.6643
0.04	0.98301	0.4750	0.93167	0.6476	0.93136	0.8032	0.99833	0.6501
0.06	0.96434	0.4568	0.85404	0.6081	0.85338	0.7910	0.99819	0.6131
0.08	0.93883	0.4364	0.74730	0.5418	0.74617	0.7717	0.99661	0.6131
$(n = 0.07, q = 0.0)$								
0.02	0.99512	0.4919	0.97579	0.6549	0.97579	0.8138	0.97579	0.6756
0.04	0.98147	0.4733	0.90794	0.6480	0.90794	0.8042	0.90794	0.6500
0.06	0.96112	0.4540	0.80335	0.5951	0.80335	0.7849	0.80335	0.5983
0.08	0.93420	0.4329	0.65955	0.4917	0.65955	0.7600	0.65955	0.4961
$(n = 0.525, q = 0.05)$								
0.02	0.99529	0.4922	0.97725	0.6734	0.97725	0.8139	0.97725	0.6744
0.04	0.98209	0.4740	0.91352	0.6490	0.91349	0.8052	0.91352	0.6515
0.06	0.96241	0.4551	0.81528	0.5993	0.81522	0.7872	0.81528	0.6035
0.08	0.93639	0.4345	0.68020	0.5068	0.68010	0.7619	0.68020	0.5127
$(n = 0.535, q = 0.07)$								
0.02	0.99518	0.4921	0.97729	0.6721	0.97729	0.8127	0.97729	0.6731
0.04	0.98169	0.4735	0.91365	0.6477	0.91365	0.8047	0.91365	0.6499
0.06	0.96158	0.4544	0.81556	0.5975	0.81556	0.7870	0.81556	0.6013
0.08	0.93495	0.4355	0.68068	0.5039	0.68068	0.7626	0.68068	0.5090
$(n = 0.55, q = 0.1)$								
0.02	0.99514	0.4919	0.97646	0.6734	0.97646	0.8140	0.97646	0.6743
0.04	0.98152	0.4733	0.91051	0.6476	0.91050	0.8047	0.91050	0.6498
0.06	0.96123	0.4540	0.80884	0.5956	0.80882	0.7860	0.80882	0.5993
0.08	0.93438	0.4330	0.66905	0.4962	0.66901	0.7611	0.66901	0.5011

Table 4.14: Values of n_1^2 and the skewness coefficient K for RD and RTD Polytropes with index $N = 3.0$

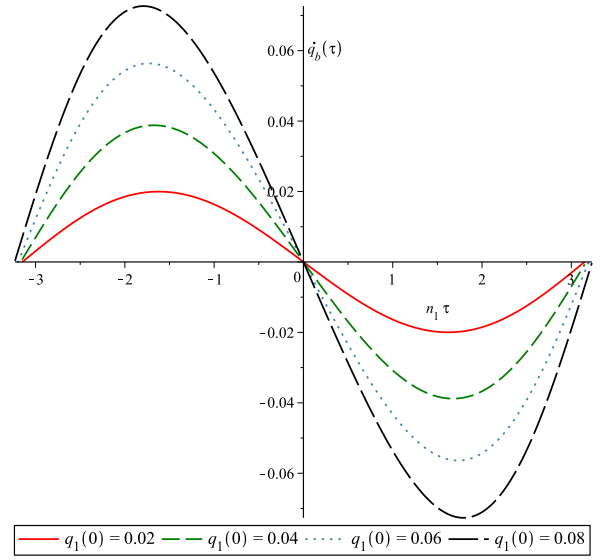
$q_1(0)$	f mode		$f + 1$ modes		$f + 2$ modes		$f + 3$ modes	
	n_1^2	K	n_1^2	K	n_1^2	K	n_1^2	K
$(n = 0.0, q = 0.0)$								
0.02	0.99968	0.5049	0.99938	0.4749	0.99938	0.3873	0.99938	0.4321
0.04	0.99873	0.5006	0.99745	0.4437	0.99753	0.3226	0.99753	0.3866
0.06	0.99724	0.4964	0.99464	0.4181	0.99464	0.2889	0.99464	0.3615
0.08	0.99512	0.4919	0.99052	0.3960	0.99052	0.2670	0.99064	0.3455
$(n = 0.03, q = 0.0)$								
0.02	0.99963	0.5046	0.99925	0.4721	0.99925	0.3814	0.99925	0.4546
0.04	0.99854	0.5000	0.99702	0.4388	0.99702	0.3161	0.99702	0.4228
0.06	0.51253	0.4954	0.99353	0.4122	0.99353	0.2830	0.99353	0.4085
0.08	0.50937	0.4906	0.98855	0.3897	0.98855	0.2621	0.98855	0.4027
$(n = 0.05, q = 0.0)$								
0.02	0.99959	0.5044	0.99914	0.4696	0.99914	0.3759	0.99914	0.4873
0.04	0.99837	0.4995	0.99656	0.4347	0.99656	0.3102	0.99656	0.4773
0.06	0.99646	0.4946	0.99252	0.4074	0.99252	0.2780	0.99252	0.4755
0.08	0.99373	0.4895	0.98676	0.3847	0.98676	0.2579	0.98676	0.4682
$(n = 0.07, q = 0.0)$								
0.02	0.99959	0.5044	0.99912	0.4677	0.99912	0.3694	0.99912	0.5195
0.04	0.99838	0.4995	0.99648	0.4316	0.99648	0.3031	0.99648	0.5186
0.06	0.99649	0.4947	0.99235	0.4039	0.99235	0.2716	0.99235	0.5168
0.08	0.99379	0.4895	0.98645	0.3810	0.98645	0.2526	0.98645	0.5143
$(n = 0.525, q = 0.05)$								
0.02	0.99955	0.5042	0.99901	0.4670	0.99901	0.3692	0.99901	0.5618
0.04	0.99820	0.4990	0.99606	0.4305	0.99606	0.3031	0.99606	0.5611
0.06	0.99609	0.4938	0.99143	0.4026	0.99143	0.2718	0.99143	0.5531
0.08	0.99308	0.4884	0.98484	0.3798	0.98484	0.2529	0.98484	0.5443
$(n = 0.535, q = 0.07)$								
0.02	0.99954	0.5041	0.99900	0.4682	0.99900	0.3692	0.99900	0.5577
0.04	0.99818	0.4989	0.99601	0.4302	0.99601	0.3030	0.99601	0.5572
0.06	0.99605	0.4938	0.99133	0.4022	0.99133	0.2717	0.99133	0.5620
0.08	0.99302	0.4883	0.98465	0.3793	0.98465	0.2528	0.98465	0.5416
$(n = 0.55, q = 0.1)$								
0.02	0.99954	0.5041	0.99898	0.4662	0.99898	0.3366	0.99898	0.5916
0.04	0.99816	0.4988	0.99592	0.4292	0.99592	0.3003	0.99592	0.5883
0.06	0.99599	0.4937	0.99113	0.4011	0.99113	0.2695	0.99113	0.5735
0.08	0.99291	0.4881	0.98430	0.3783	0.98430	0.2510	0.98430	0.5600

Table 4.15: Values of n_1^2 and the skewness coefficient K for RD and RTD Polytropes with index $N = 4.0$

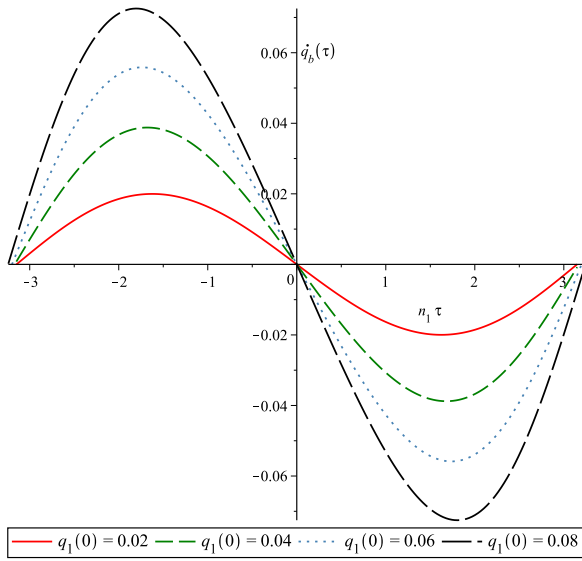
$q_1(0)$	f mode		$f + 1$ modes		$f + 2$ modes		$f + 3$ modes	
	n_1^2	K	n_1^2	K	n_1^2	K	n_1^2	K
$(n = 0.0, q = 0.0)$								
0.02	0.99993	0.5072	0.99983	0.4924	0.99983	0.4517	0.99983	0.4184
0.04	0.99973	0.5053	0.99935	0.4761	0.99935	0.3975	0.99935	0.3598
0.06	0.99939	0.5033	0.99855	0.4608	0.99855	0.3764	0.99855	0.3403
0.08	0.99892	0.5013	0.99743	0.4465	0.99743	0.3217	0.99743	0.2996
$(n = 0.03, q = 0.0)$								
0.02	0.99992	0.5070	0.99981	0.4909	0.99981	0.4527	0.99914	0.4302
0.04	0.99968	0.5049	0.99925	0.4732	0.99925	0.3996	0.99925	0.3750
0.06	0.99928	0.5028	0.99833	0.4567	0.99833	0.3764	0.99833	0.3388
0.08	0.99876	0.5007	0.99711	0.4421	0.99711	0.3261	0.99711	0.3151
$(n = 0.05, q = 0.0)$								
0.04	0.99991	0.5069	0.99979	0.4905	0.99979	0.4577	0.99979	0.4598
0.06	0.99964	0.5047	0.99919	0.4724	0.99919	0.4113	0.99919	0.4191
0.06	0.99919	0.5024	0.99819	0.4555	0.99819	0.3763	0.99819	0.3885
0.08	0.99856	0.5001	0.99687	0.4407	0.99687	0.3482	0.99687	0.3666
$(n = 0.07, q = 0.0)$								
0.02	0.99986	0.5064	0.99962	0.4819	0.99962	0.3684	0.99962	0.3158
0.04	0.99945	0.5036	0.99849	0.4566	0.99849	0.3043	0.99849	0.2838
0.06	0.99876	0.5008	0.99661	0.4343	0.99661	0.2744	0.99661	0.2688
0.08	0.99786	0.4980	0.99414	0.4162	0.99414	0.2590	0.99413	0.2592
$(n = 0.525, q = 0.05)$								
0.02	0.99987	0.5065	0.99969	0.4849	0.99969	0.4033	0.99969	0.3712
0.04	0.99951	0.5039	0.99876	0.4619	0.99876	0.3407	0.99876	0.3245
0.06	0.99891	0.5013	0.99722	0.4414	0.99722	0.3055	0.99722	0.3008
0.08	0.99811	0.4987	0.99518	0.4242	0.99518	0.2851	0.99518	0.2897
$(n = 0.535, q = 0.07)$								
0.02	0.99987	0.5064	0.99965	0.4834	0.99965	0.3847	0.99965	0.3379
0.04	0.99948	0.5038	0.99862	0.4592	0.99862	0.3204	0.99862	0.3379
0.06	0.99883	0.5010	0.99691	0.4377	0.99691	0.2876	0.99691	0.2818
0.08	0.99798	0.4983	0.99484	0.4201	0.99464	0.2698	0.99464	0.2711
$(n = 0.55, q = 0.1)$								
0.02	0.99985	0.5064	0.99960	0.4809	0.99960	0.3532	0.99960	0.2959
0.04	0.99943	0.5035	0.99840	0.4547	0.99840	0.2902	0.99840	0.2678
0.06	0.99872	0.5006	0.99642	0.4318	0.99642	0.2633	0.99642	0.2535
0.08	0.99779	0.4978	0.99379	0.4134	0.99379	0.2502	0.99379	0.2439



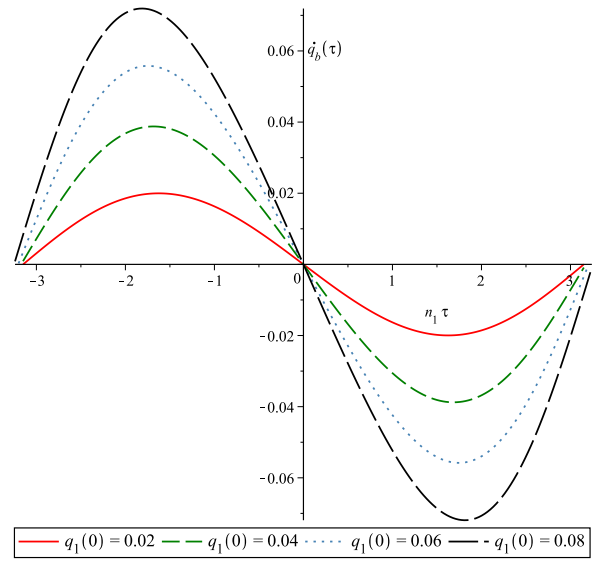
(a) ($n=0.0, q=0.0$)



(b) ($n=0.03, q=0.0$)

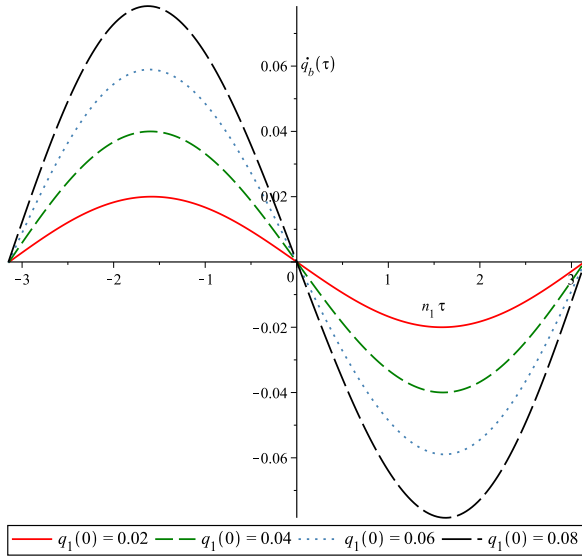


(c) ($n=0.05, q=0.0$)

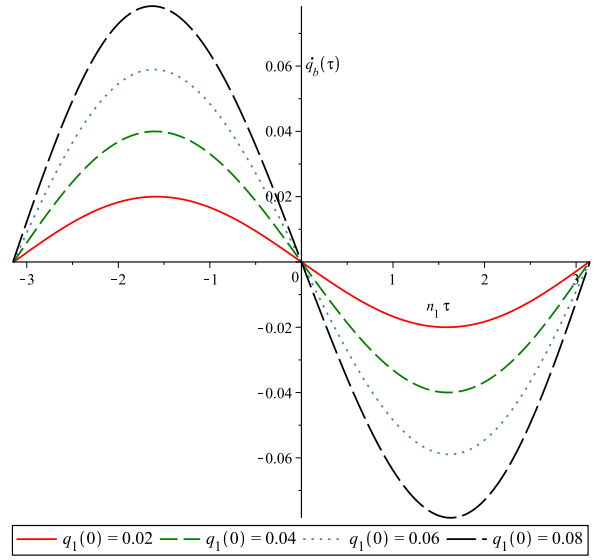


(d) ($n=0.07, q=0.0$)

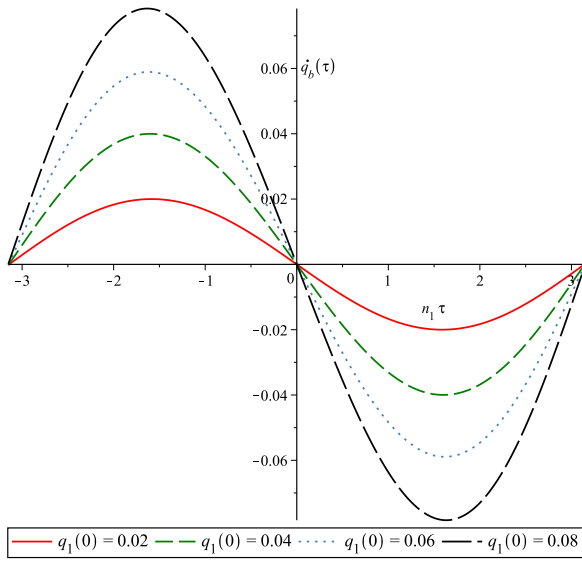
Figure 4.1. Radial Velocity Curves of RD Polytropic models with index $N = 1.5$ for fundamental (f) mode



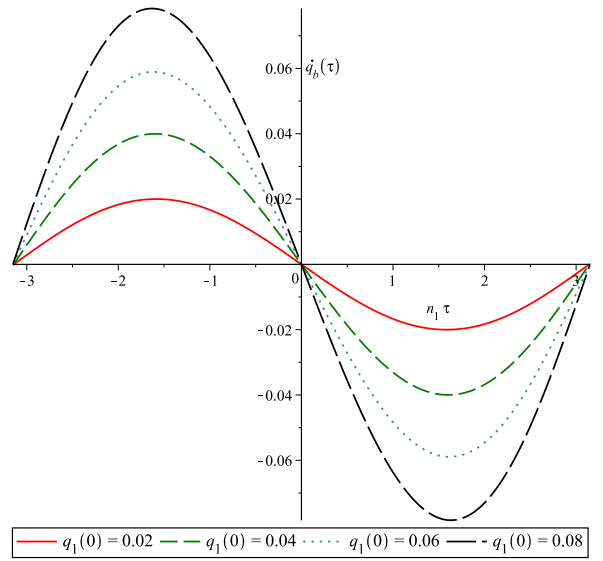
(a) ($n=0.0, q=0.0$)



(b) ($n=0.03, q=0.0$)

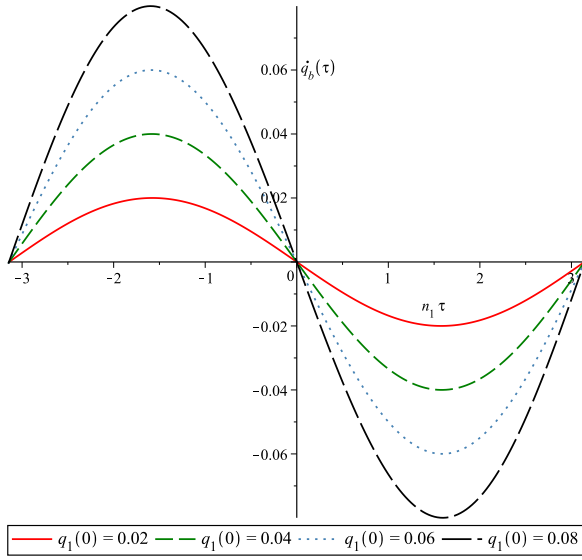


(c) ($n=0.05, q=0.0$)

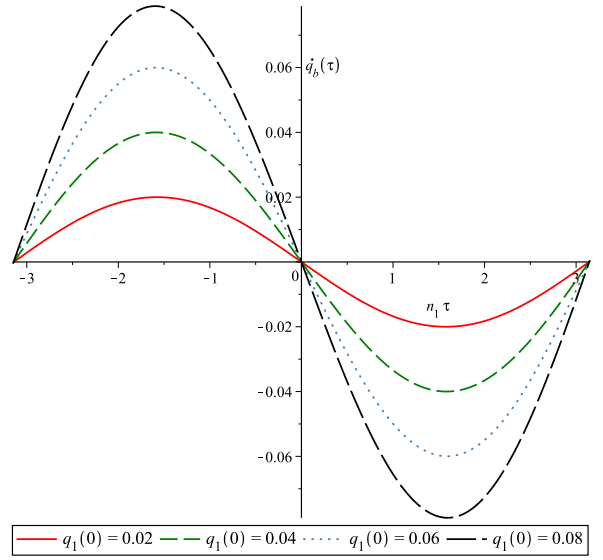


(d) ($n=0.07, q=0.0$)

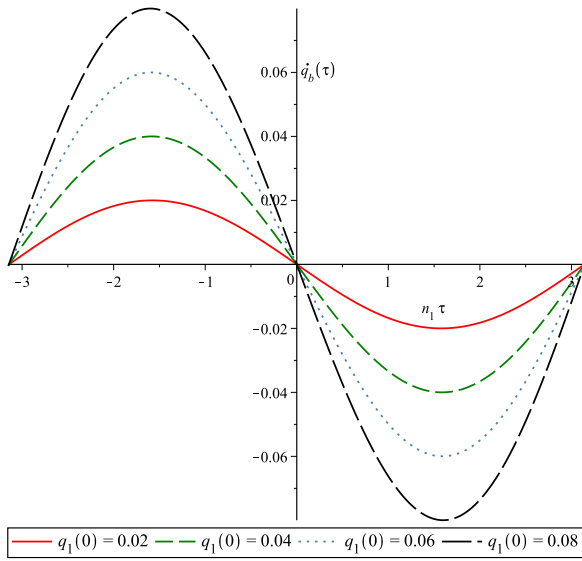
Figure 4.2. Radial Velocity Curves of RD Polytropic models with index $N = 3.0$ for fundamental (f) mode



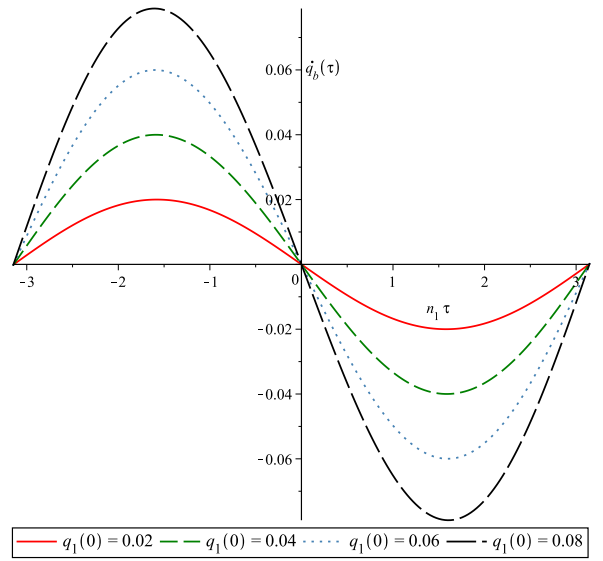
(a) ($n=0.0, q=0.0$)



(b) ($n=0.03, q=0.0$)

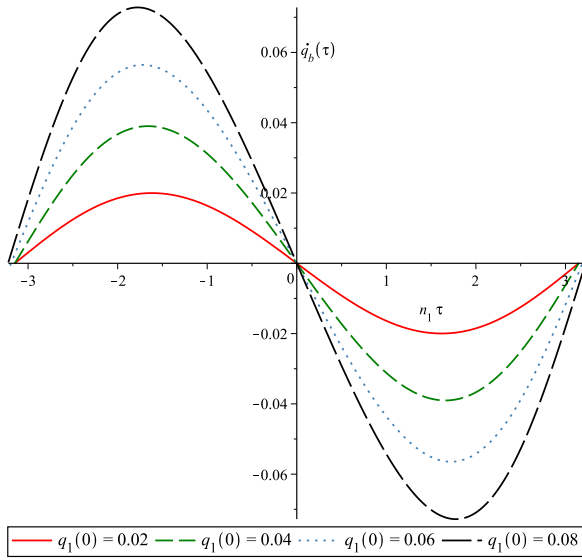


(c) ($n=0.05, q=0.0$)

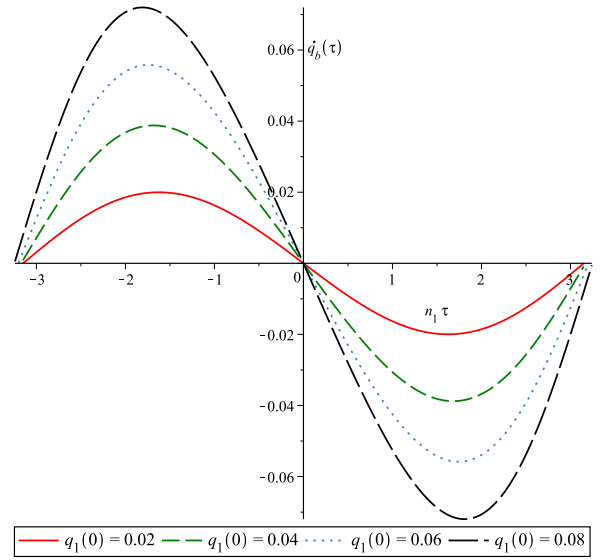


(d) ($n=0.07, q=0.0$)

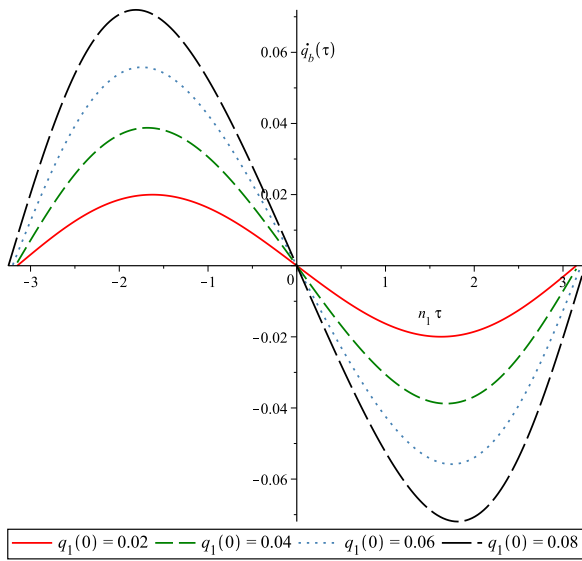
Figure 4.3. Radial Velocity Curves of RD Polytropic models with index $N = 4.0$ for fundamental (f) mode



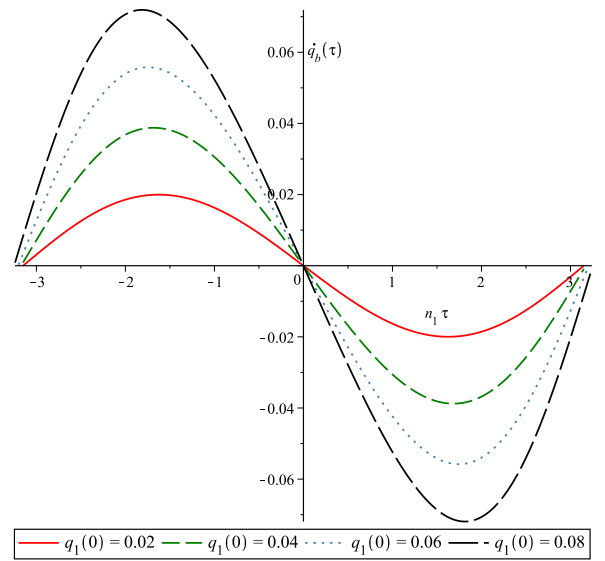
(a) $(n=0.0, q=0.0)$



(b) $(n=0.525, q=0.05)$

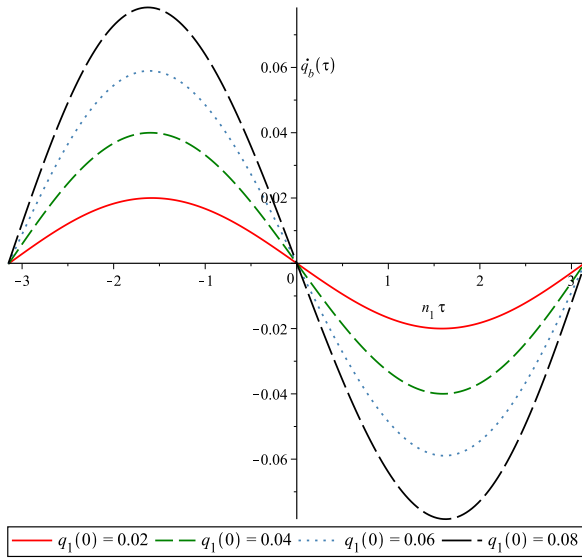


(c) $(n=0.535, q=0.07)$

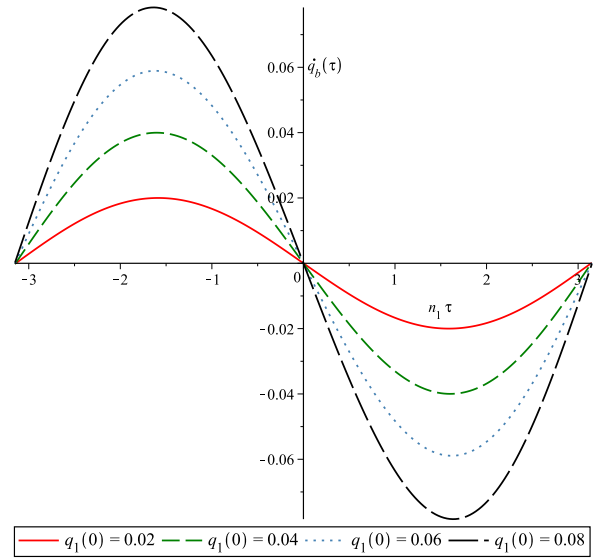


(d) $(n=0.55, q=0.1)$

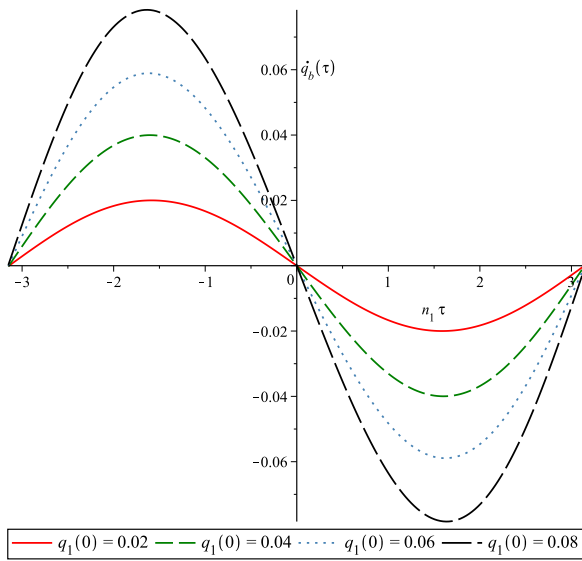
Figure 4.4. Radial Velocity Curves of RTD Polytropic models with index $N = 1.5$ for fundamental (f) mode



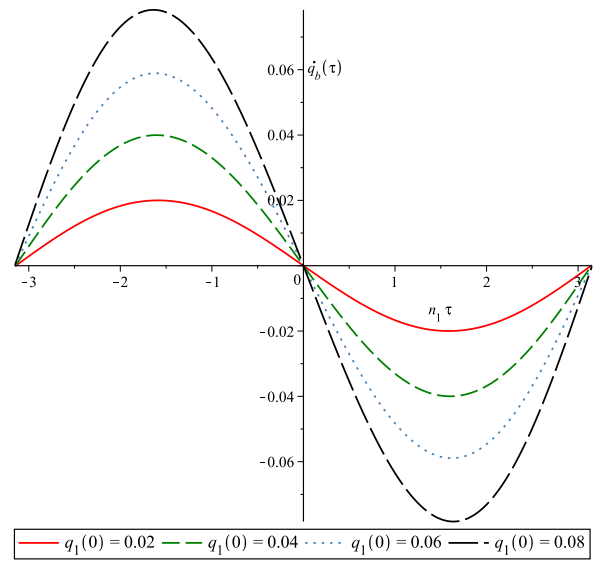
(a) $(n=0.0, q=0.0)$



(b) $(n=0.525, q=0.05)$

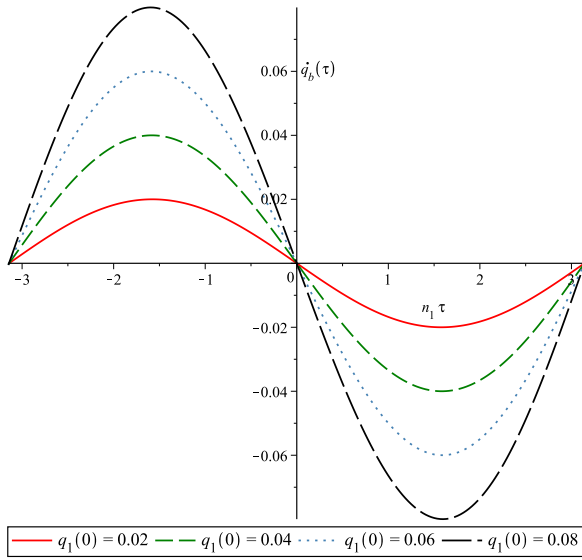


(c) $(n=0.535, q=0.07)$

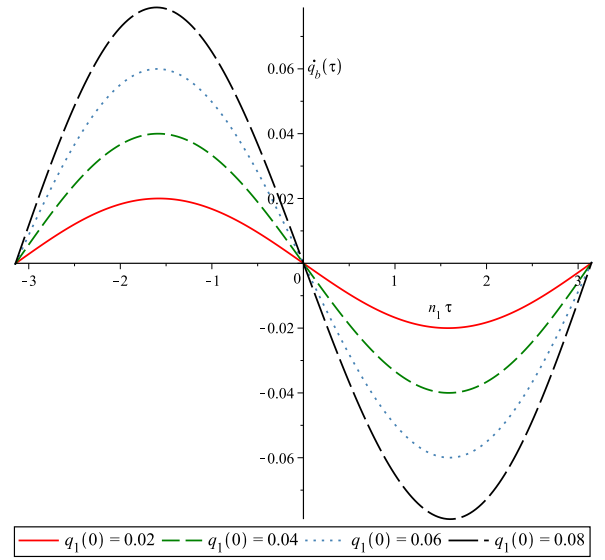


(d) $(n=0.55, q=0.1)$

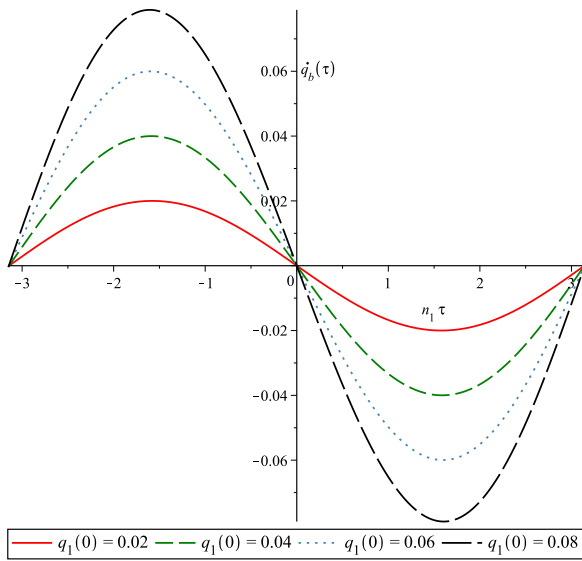
Figure 4.5. Radial Velocity Curves of RTD Polytropic models with index $N = 3.0$ for fundamental (f) mode



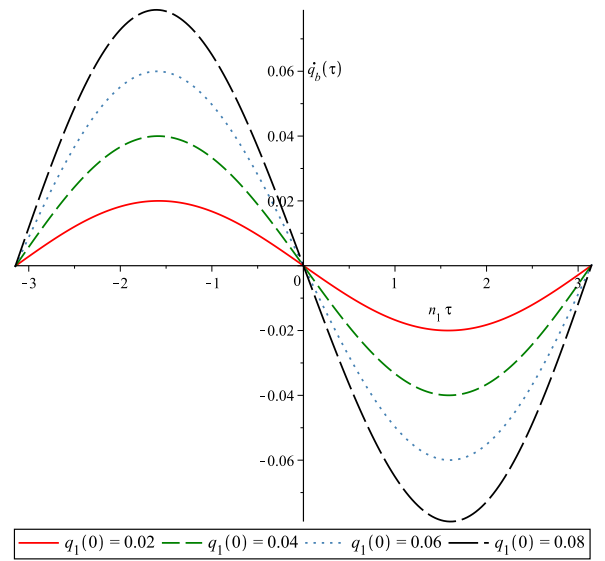
(a) $(n=0.0, q=0.0)$



(b) $(n=0.525, q=0.05)$

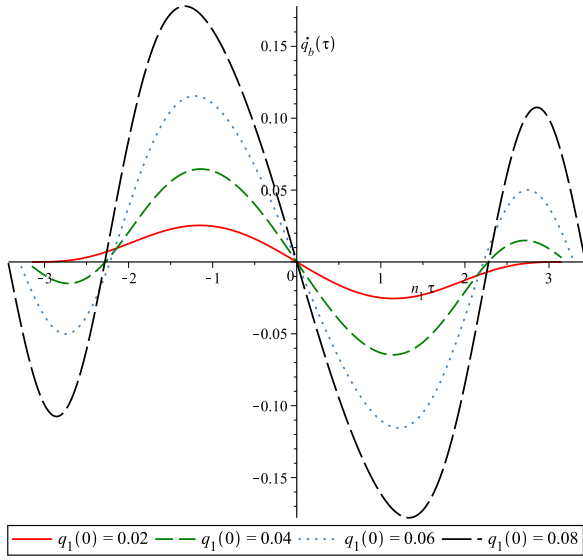


(c) $(n=0.535, q=0.07)$

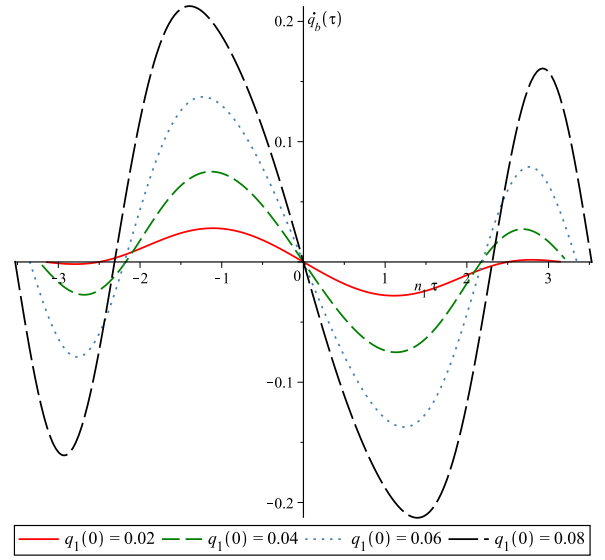


(d) $(n=0.55, q=0.1)$

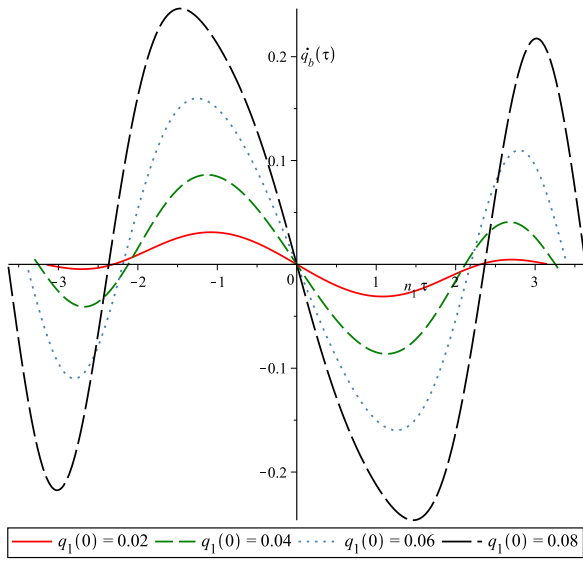
Figure 4.6. Radial Velocity Curves of RTD Polytropic models with index $N = 4.0$ for fundamental (f) mode



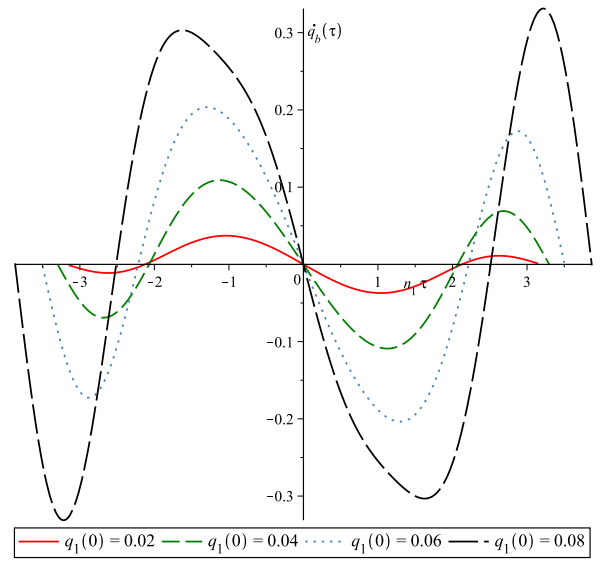
(a) ($n=0.0, q=0.0$)



(b) ($n=0.03, q=0.0$)

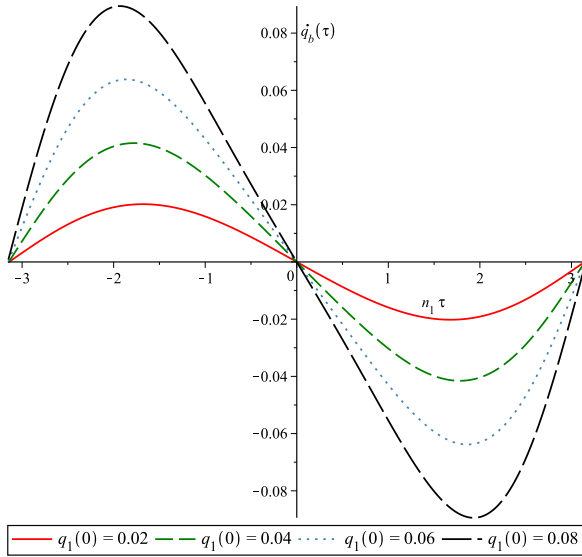


(c) ($n=0.05, q=0.0$)

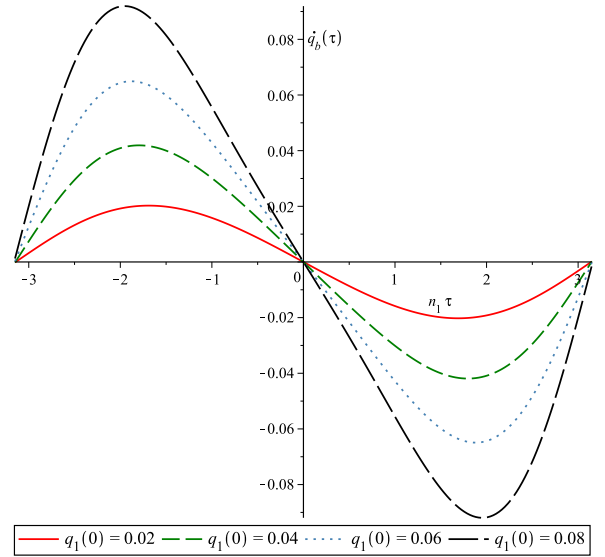


(d) ($n=0.07, q=0.0$)

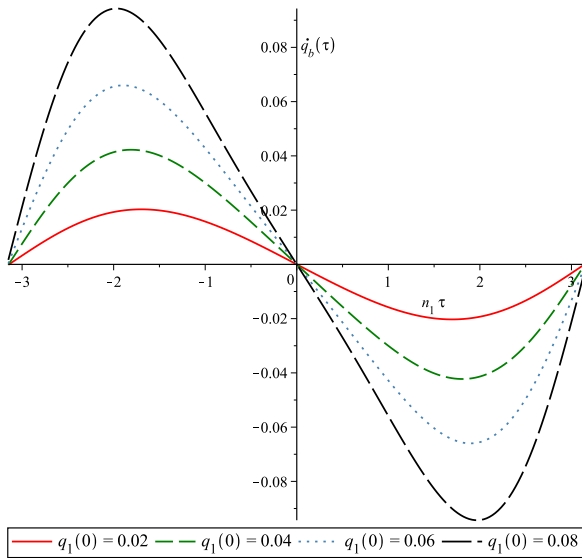
Figure 4.7. Radial Velocity Curves of RD Polytropic models with index $N = 1.5$ for fundamental and first $(f + 1)$ modes



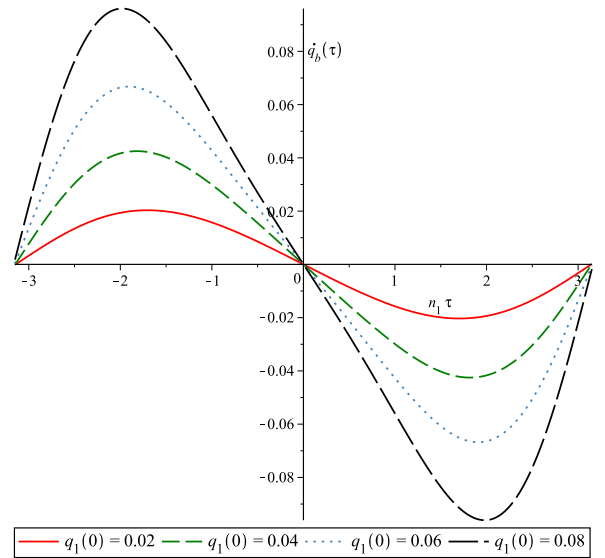
(a) ($n=0.0, q=0.0$)



(b) ($n=0.03, q=0.0$)

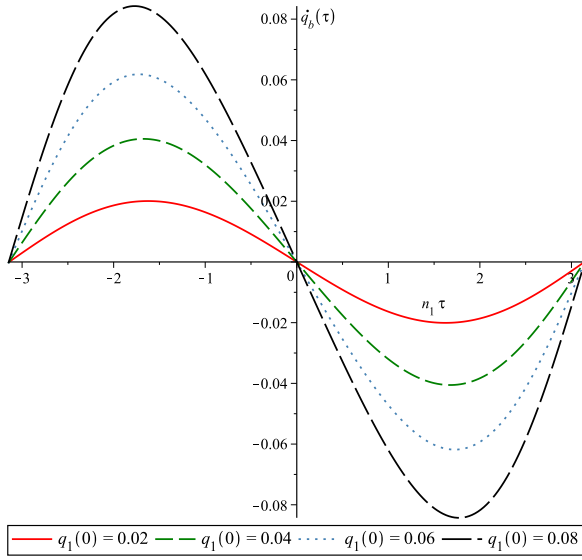


(c) ($n=0.05, q=0.0$)

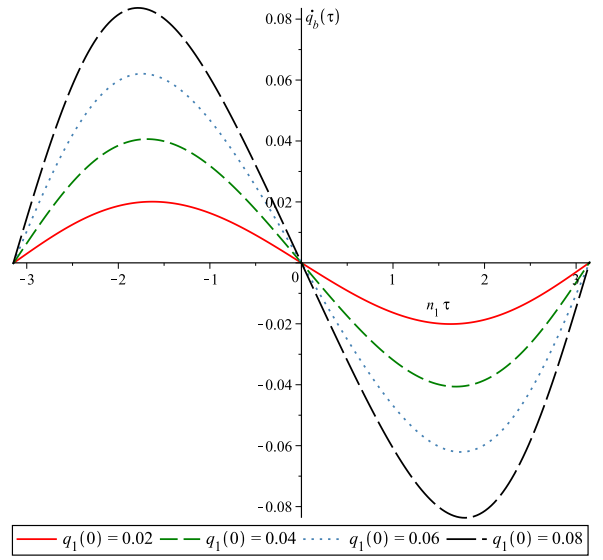


(d) ($n=0.07, q=0.0$)

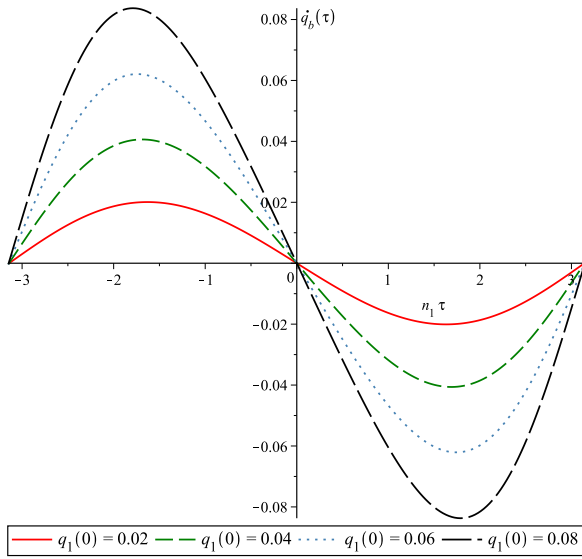
Figure 4.8. Radial Velocity Curves of RD Polytropic models with index $N = 3.0$ for fundamental and first $(f + 1)$ modes



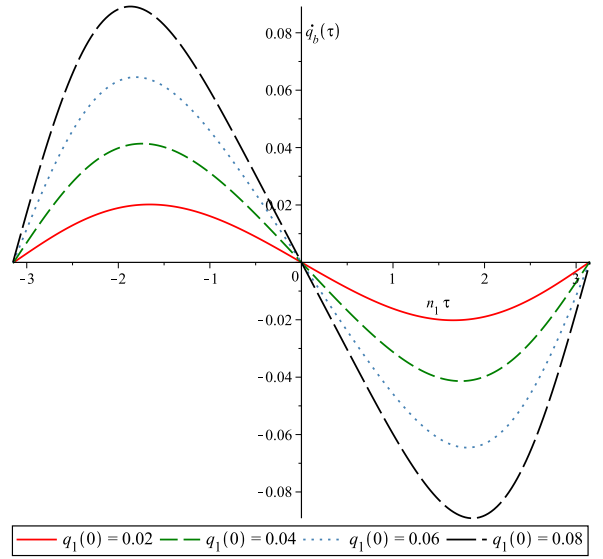
(a) ($n=0.0, q=0.0$)



(b) ($n=0.03, q=0.0$)

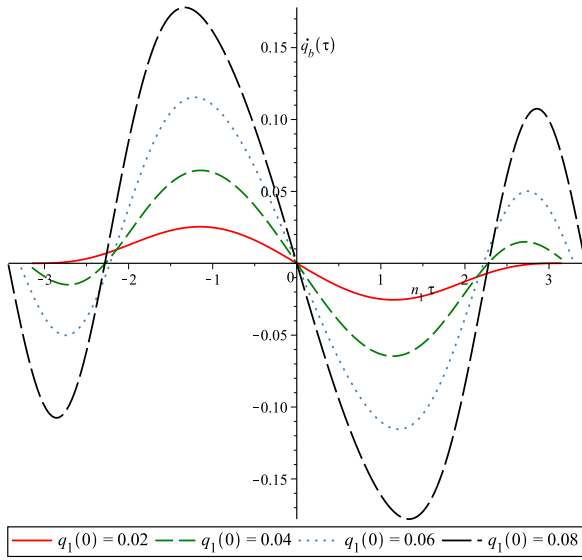


(c) ($n=0.05, q=0.0$)

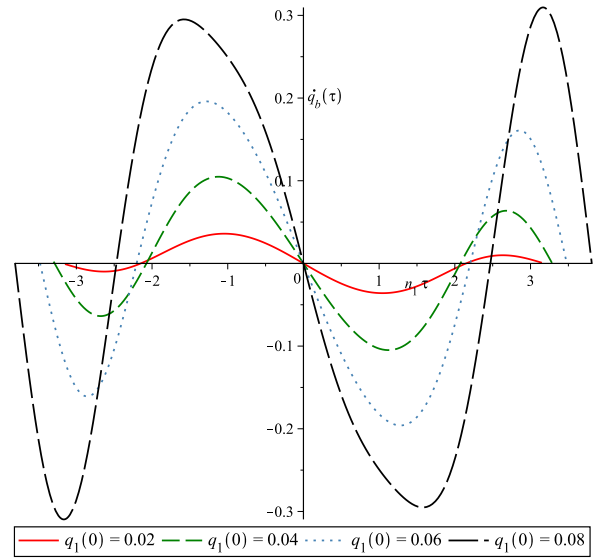


(d) ($n=0.07, q=0.0$)

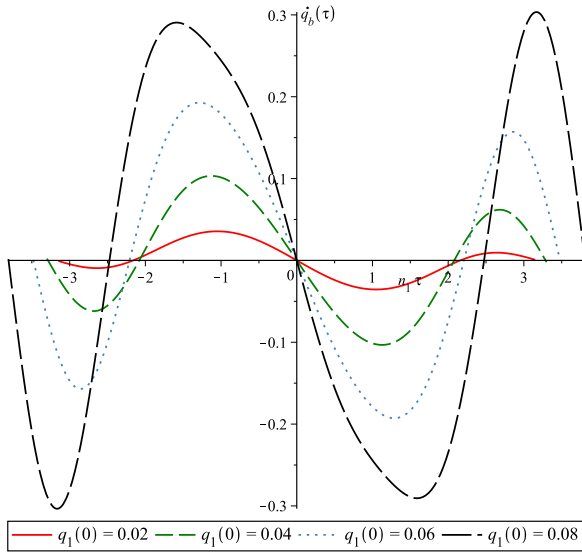
Figure 4.9. Radial Velocity Curves of RD Polytropic models with index $N = 4.0$ for fundamental and first ($f + 1$) modes



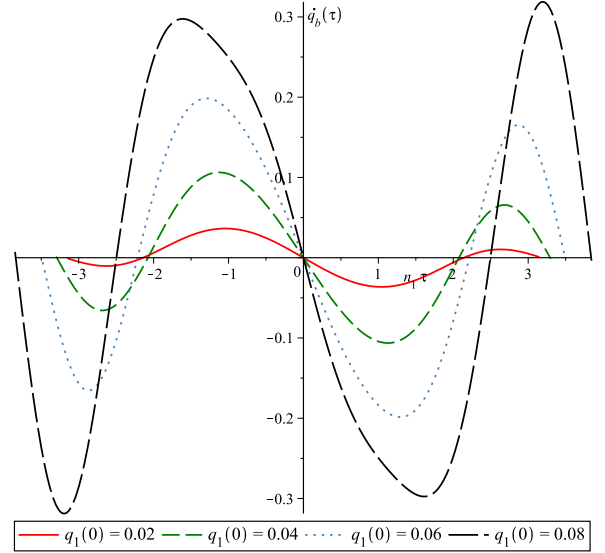
(a) ($n=0.0, q=0.0$)



(b) ($n=0.525, q=0.05$)

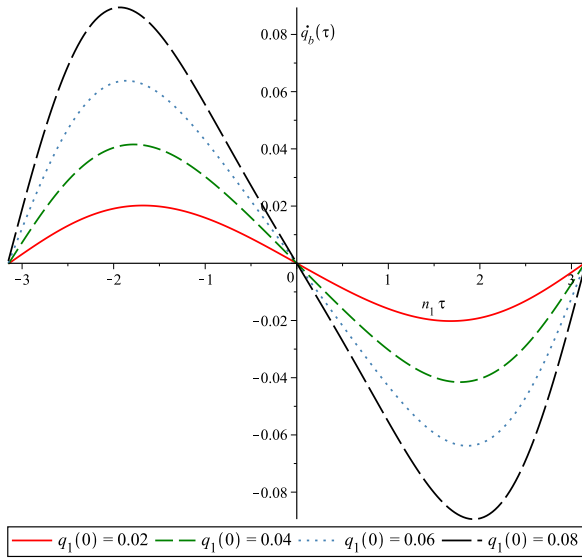


(c) ($n=0.535, q=0.07$)

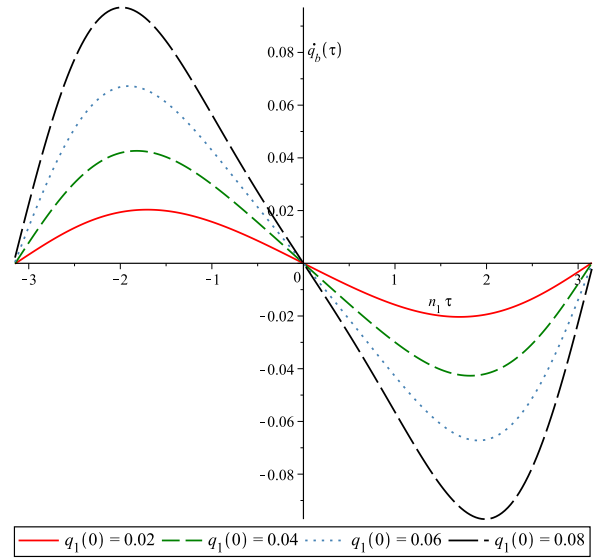


(d) ($n=0.55, q=0.1$)

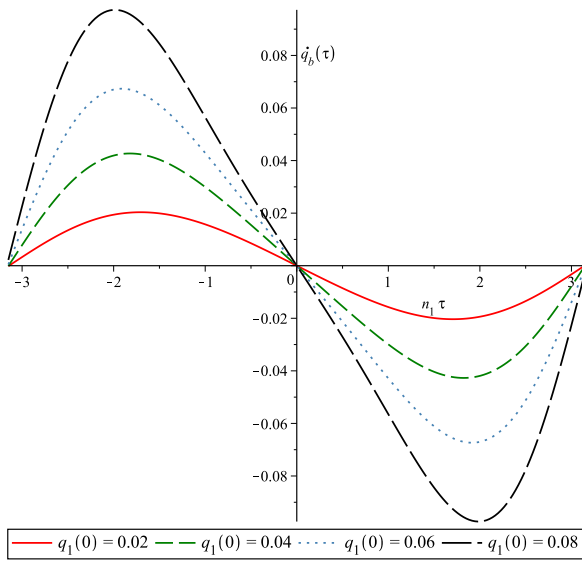
Figure 4.10. Radial Velocity Curves of RTD Polytropic models with index $N = 1.5$ for fundamental and first $(f + 1)$ modes



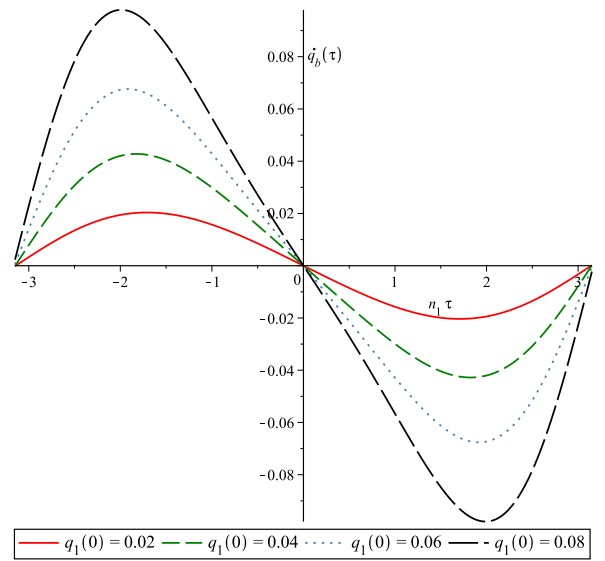
(a) $(n=0.0, q=0.0)$



(b) $(n=0.525, q=0.05)$

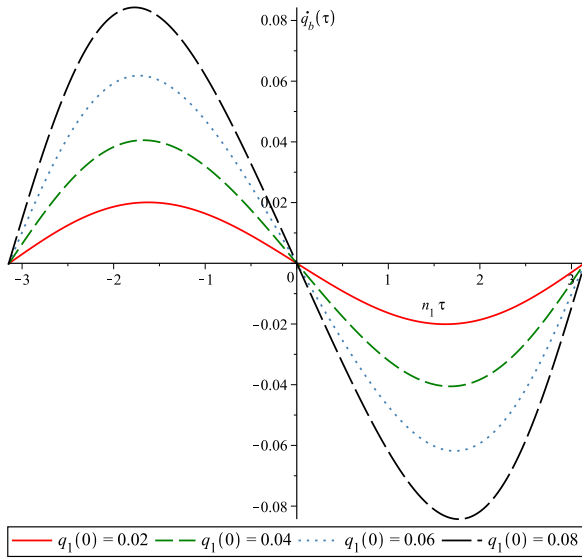


(c) $(n=0.535, q=0.07)$

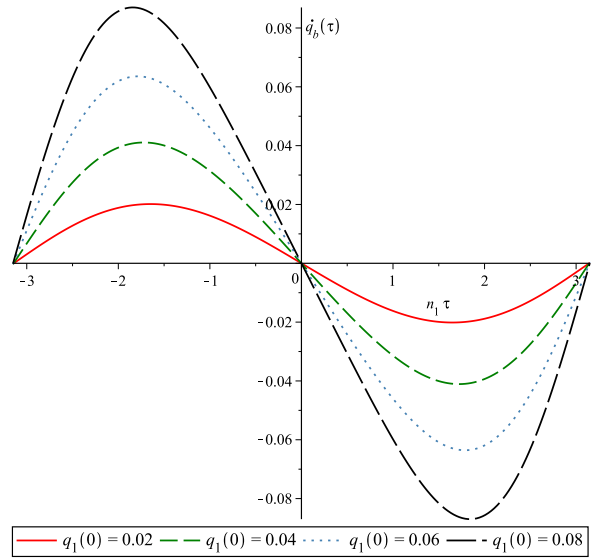


(d) $(n=0.55, q=0.1)$

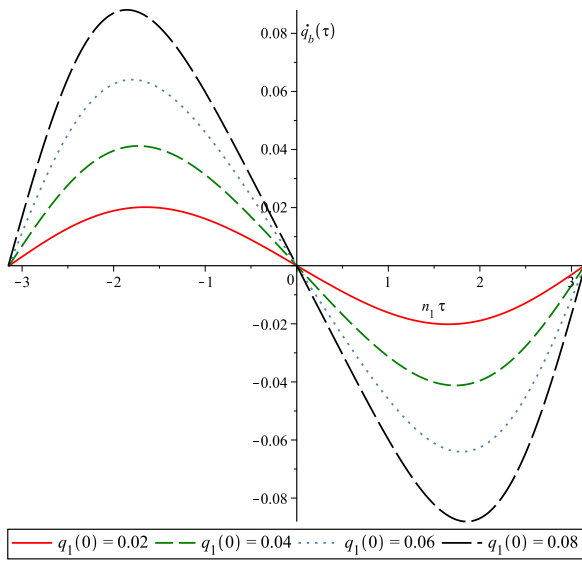
Figure 4.11. Radial Velocity Curves of RTD Polytropic models with index $N = 3.0$ for fundamental and first $(f + 1)$ modes



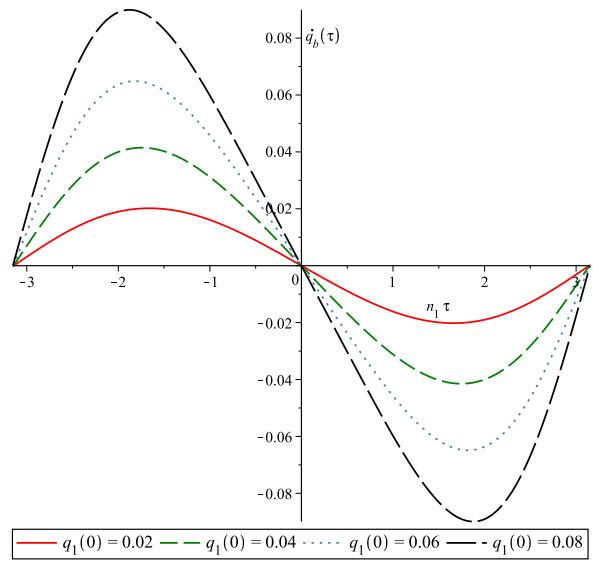
(a) $(n=0.0, q=0.0)$



(b) $(n=0.525, q=0.05)$

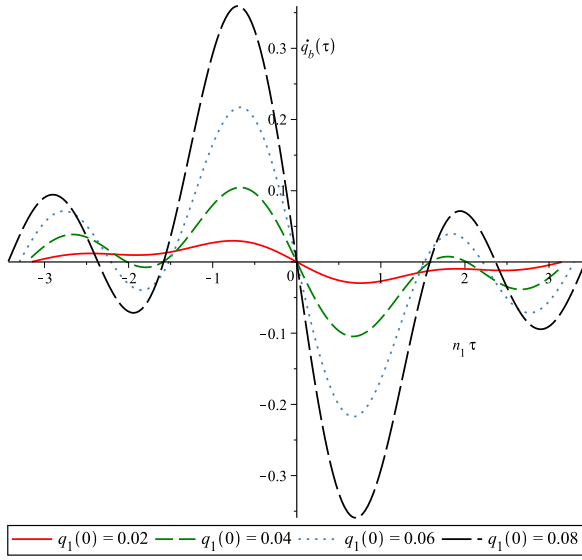


(c) $(n=0.535, q=0.07)$

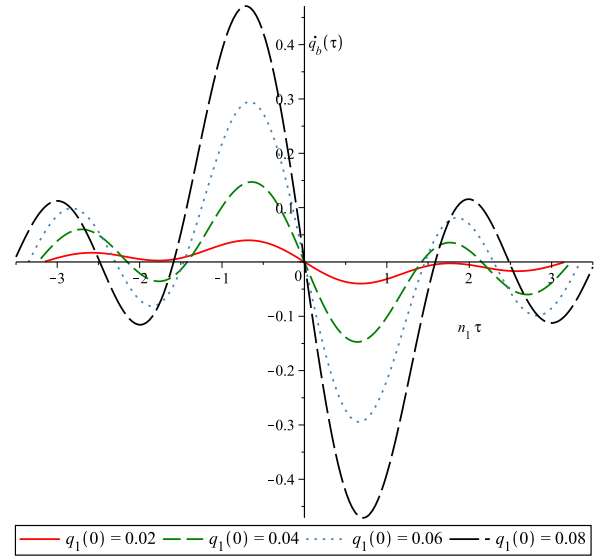


(d) $(n=0.55, q=0.1)$

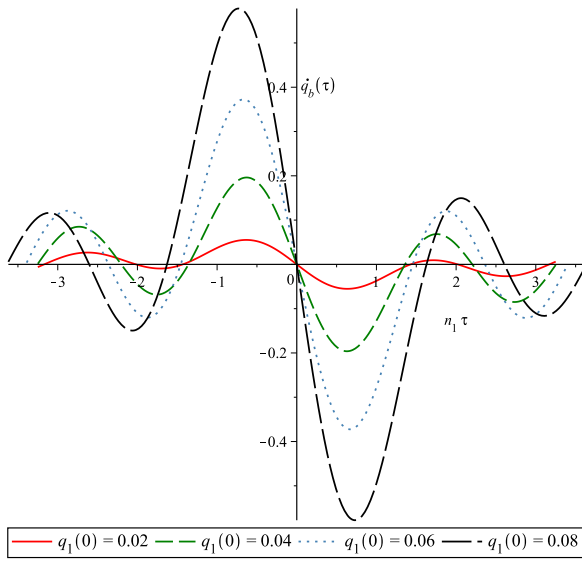
Figure 4.12. Radial Velocity Curves of RTD Polytropic models with index $N = 4.0$ for fundamental and first $(f + 1)$ modes



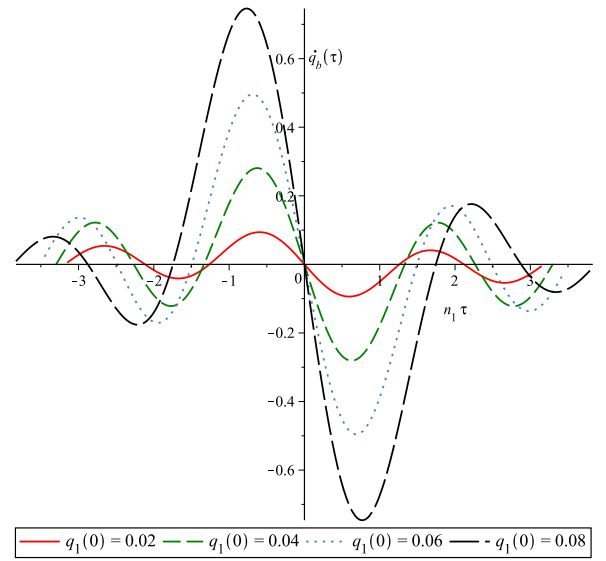
(a) ($n=0.0, q=0.0$)



(b) ($n=0.03, q=0.0$)

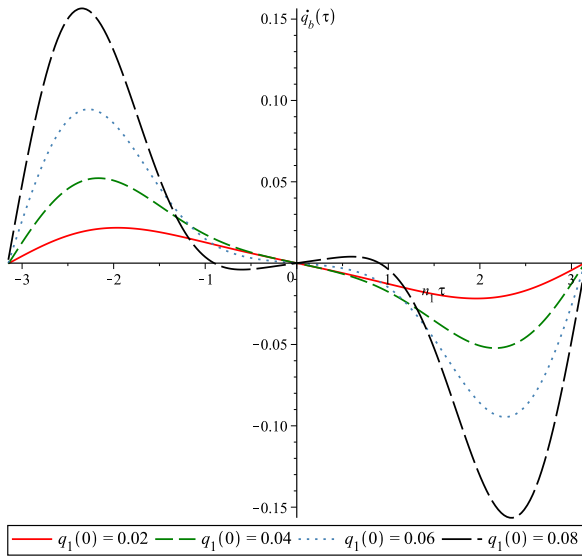


(c) ($n=0.05, q=0.0$)

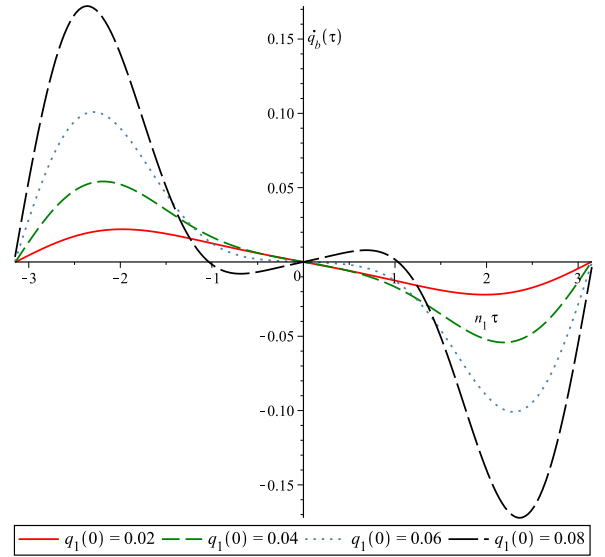


(d) ($n=0.07, q=0.0$)

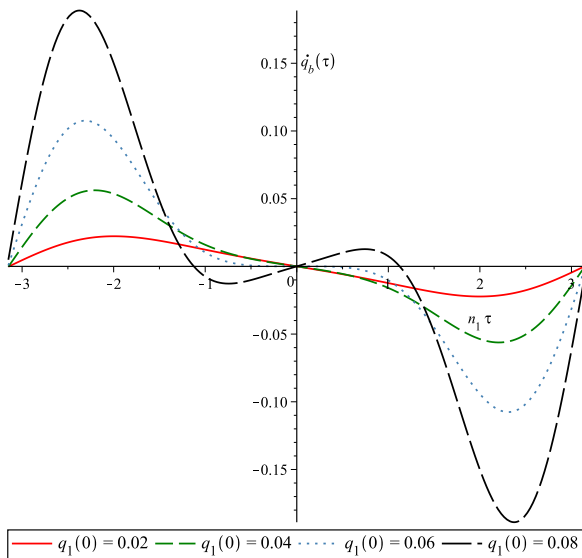
Figure 4.13. Radial Velocity Curves of RD Polytropic models with index $N = 1.5$ for fundamental and next two higher ($f + 2$) modes



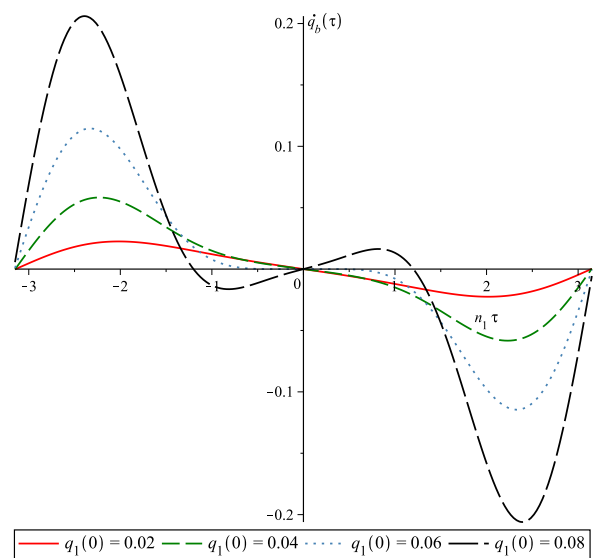
(a) ($n=0.0, q=0.0$)



(b) ($n=0.03, q=0.0$)

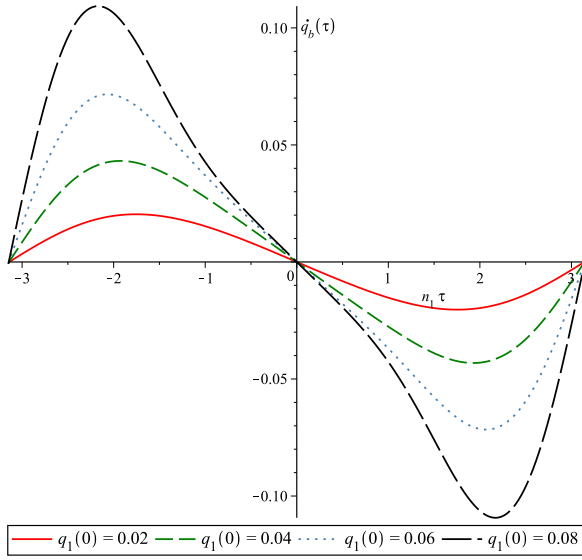


(c) ($n=0.05, q=0.0$)

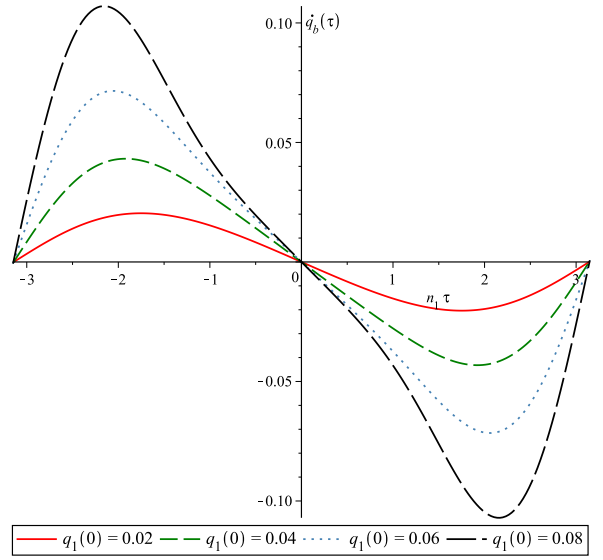


(d) ($n=0.07, q=0.0$)

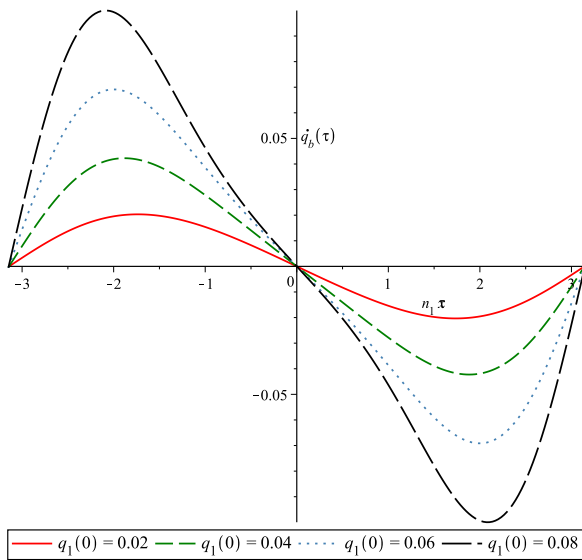
Figure 4.14. Radial Velocity Curves of RD Polytropic models with index $N = 3.0$ for fundamental and next two higher ($f + 2$) modes



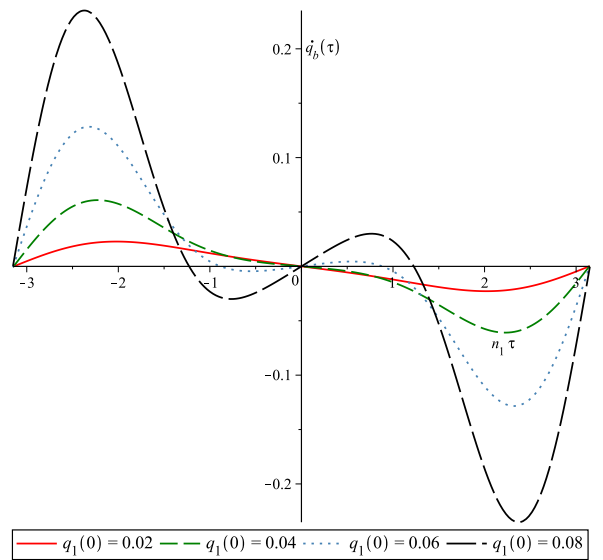
(a) ($n=0.0, q=0.0$)



(b) ($n=0.03, q=0.0$)

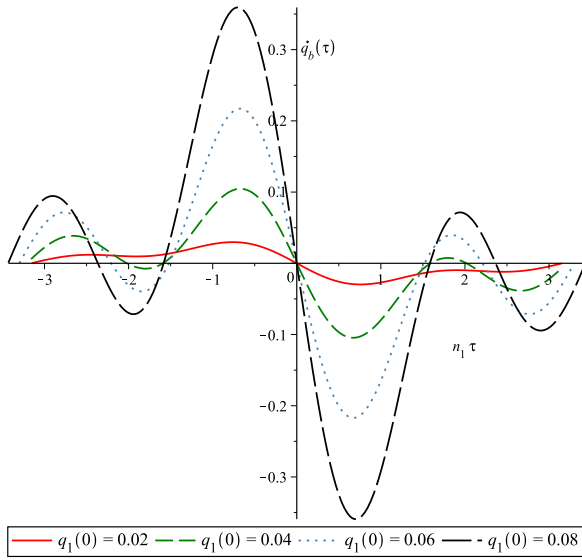


(c) ($n=0.05, q=0.0$)

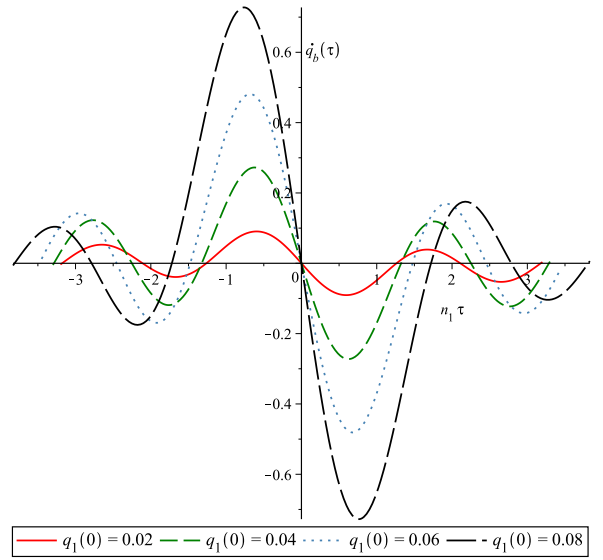


(d) ($n=0.07, q=0.0$)

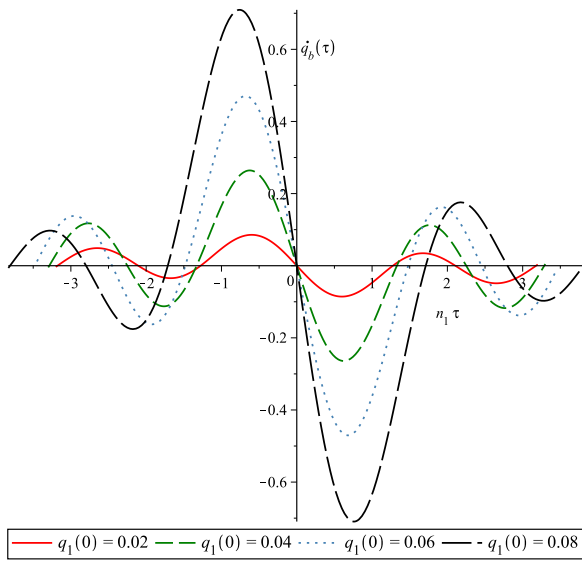
Figure 4.15. Radial Velocity Curves of RD Polytropic models with index $N = 4.0$ for fundamental and next two higher ($f + 2$) modes



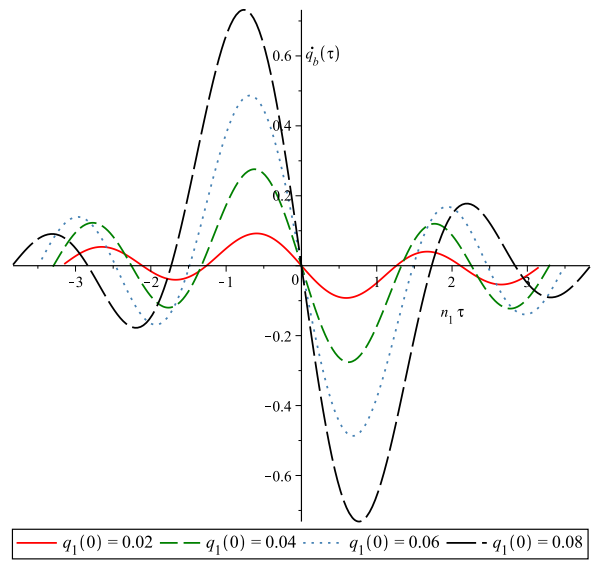
(a) ($n=0.0, q=0.0$)



(b) ($n=0.525, q=0.05$)

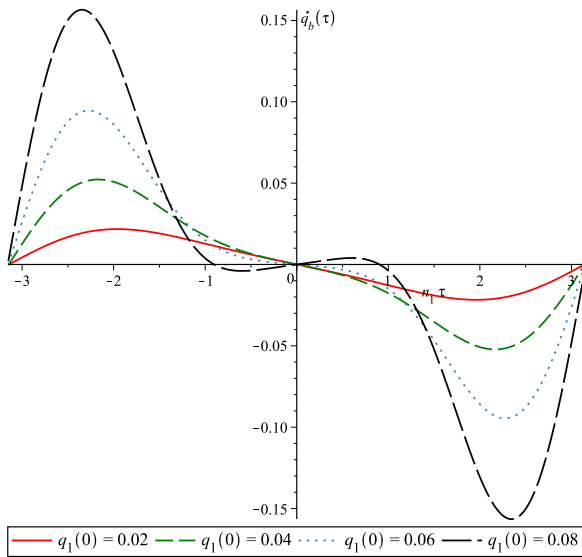


(c) ($n=0.535, q=0.07$)

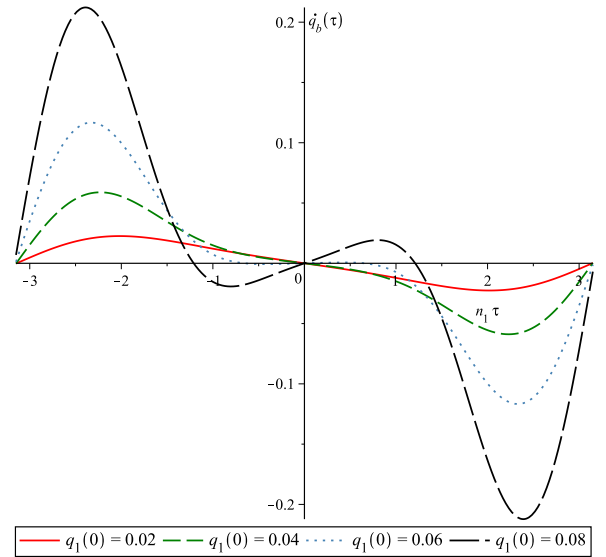


(d) ($n=0.55, q=0.1$)

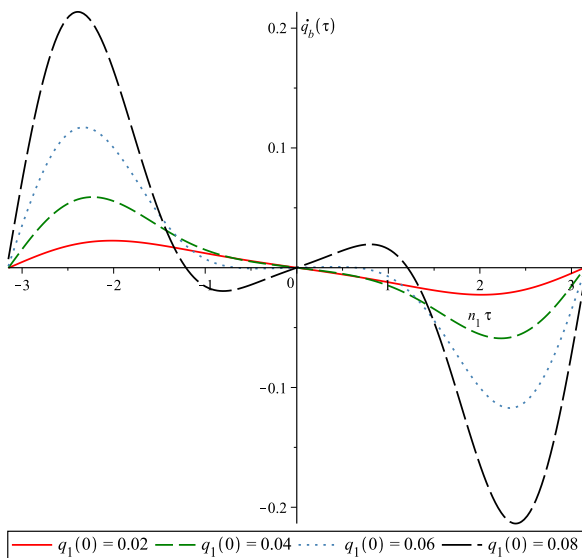
Figure 4.16. Radial Velocity Curves of RTD Polytropic models with index $N = 1.5$ for fundamental and next two higher ($f + 2$) modes



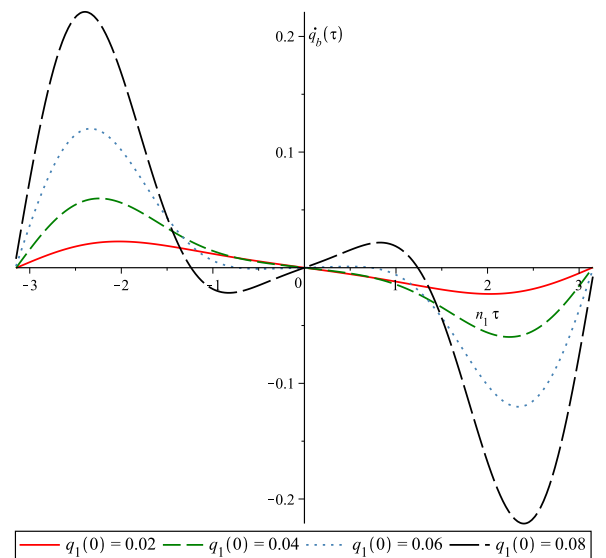
(a) $(n=0.0, q=0.0)$



(b) $(n=0.525, q=0.05)$

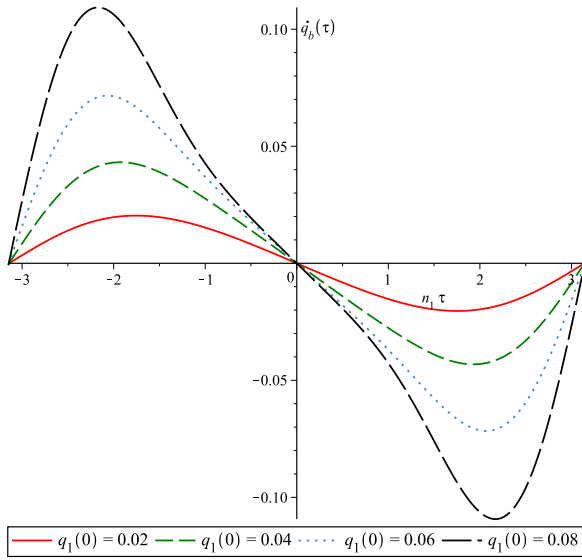


(c) $(n=0.535, q=0.07)$

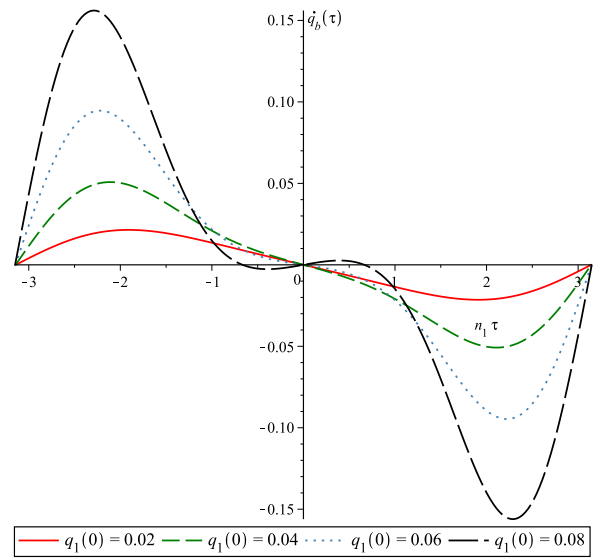


(d) $(n=0.55, q=0.1)$

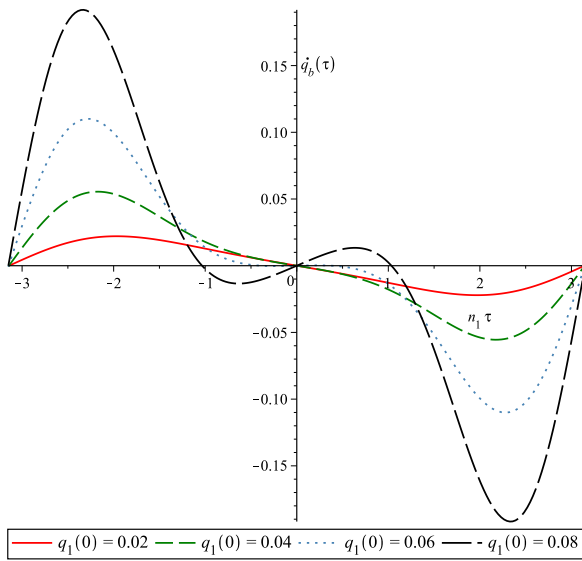
Figure 4.17. Radial Velocity Curves of RTD Polytropic models with index $N = 3.0$ for fundamental and next two higher ($f + 2$) modes



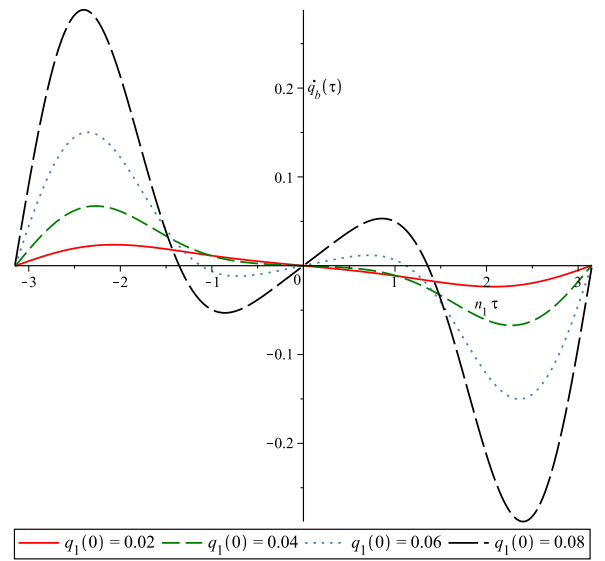
(a) $(n=0.0, q=0.0)$



(b) $(n=0.525, q=0.05)$

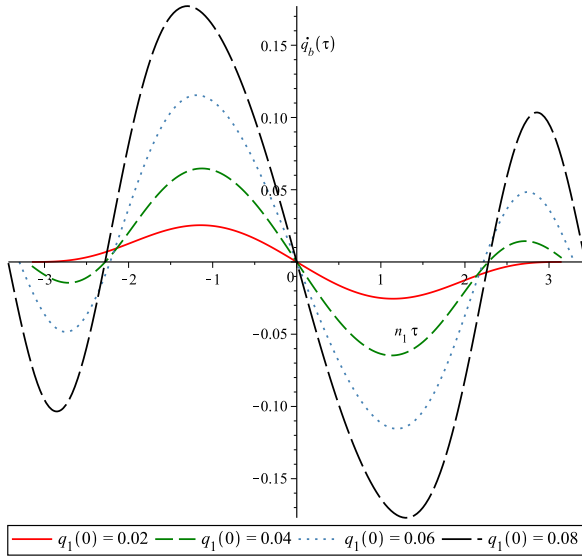


(c) $(n=0.535, q=0.07)$

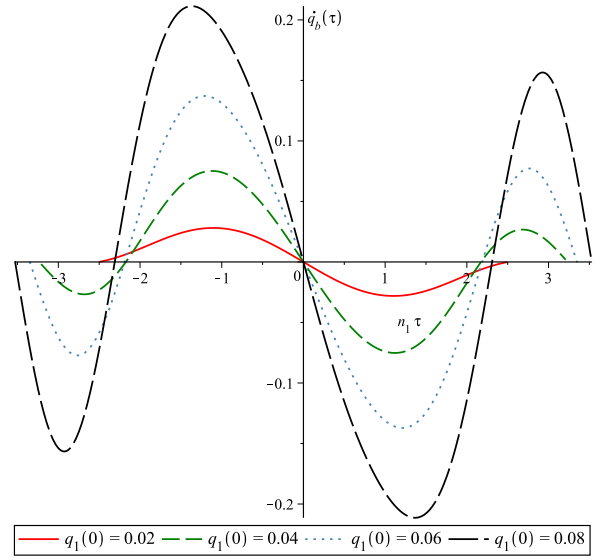


(d) $(n=0.55, q=0.1)$

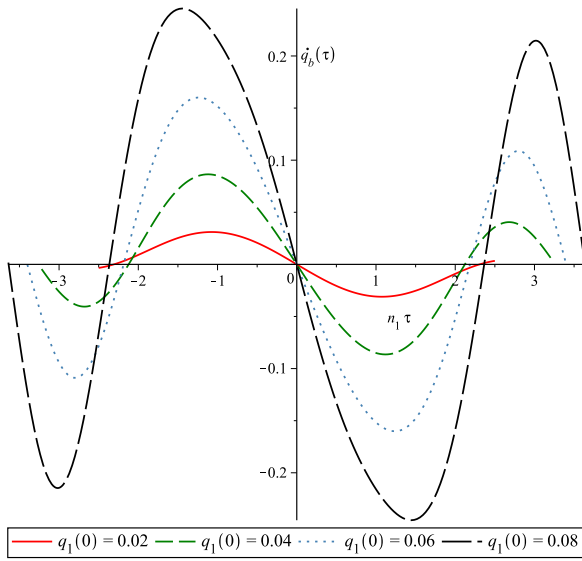
Figure 4.18. Radial Velocity Curves of RTD Polytropic models with index $N = 4.0$ for fundamental and next two higher ($f + 2$) modes



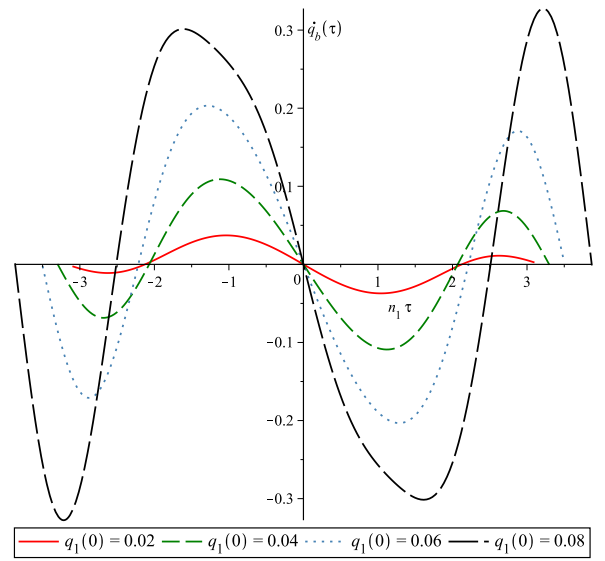
(a) ($n=0.0, q=0.0$)



(b) ($n=0.03, q=0.0$)

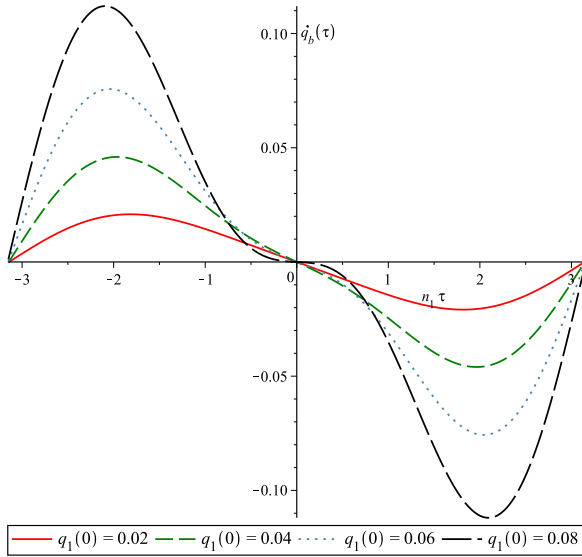


(c) ($n=0.05, q=0.0$)

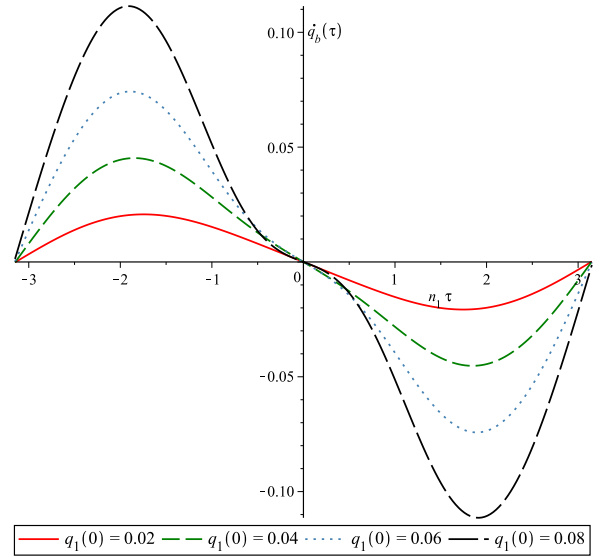


(d) ($n=0.07, q=0.0$)

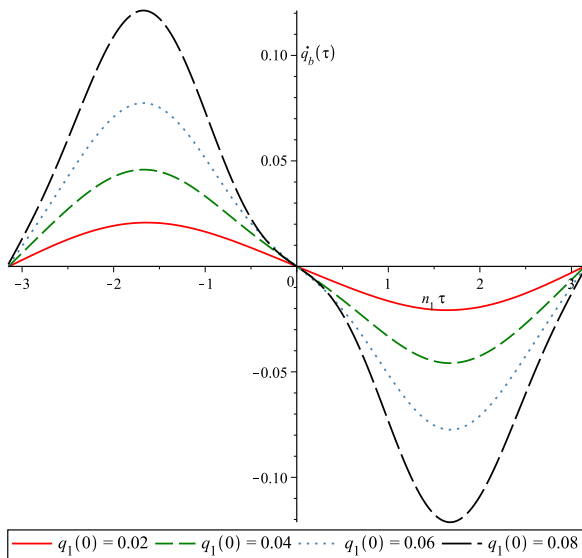
Figure 4.19. Radial Velocity Curves of RD Polytropic models with index $N = 1.5$ for fundamental and next three higher ($f + 3$) modes



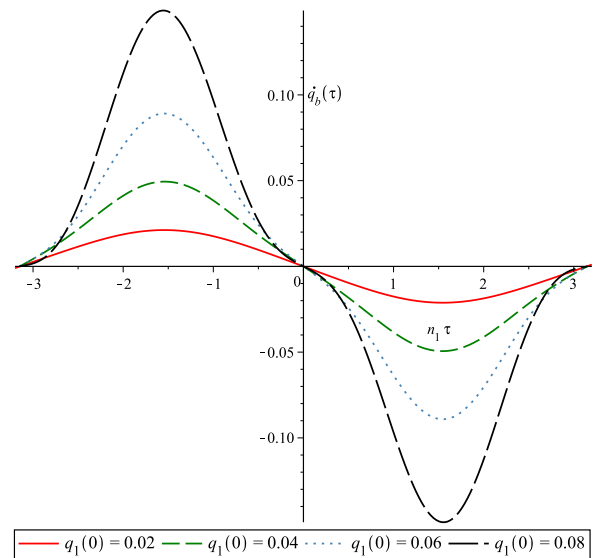
(a) ($n=0.0, q=0.0$)



(b) ($n=0.03, q=0.0$)

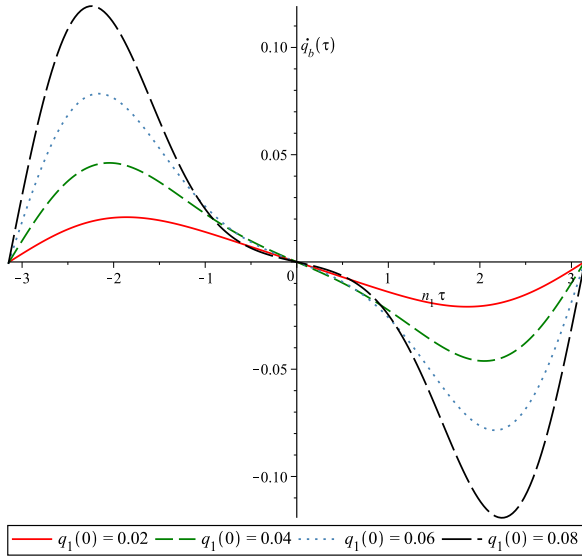


(c) ($n=0.05, q=0.0$)

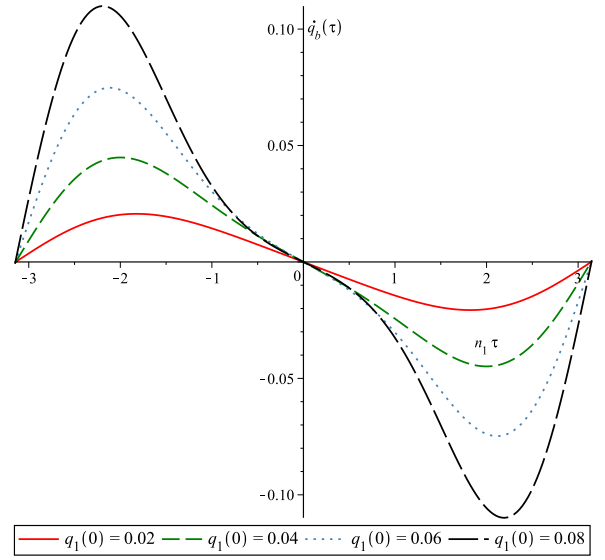


(d) ($n=0.07, q=0.0$)

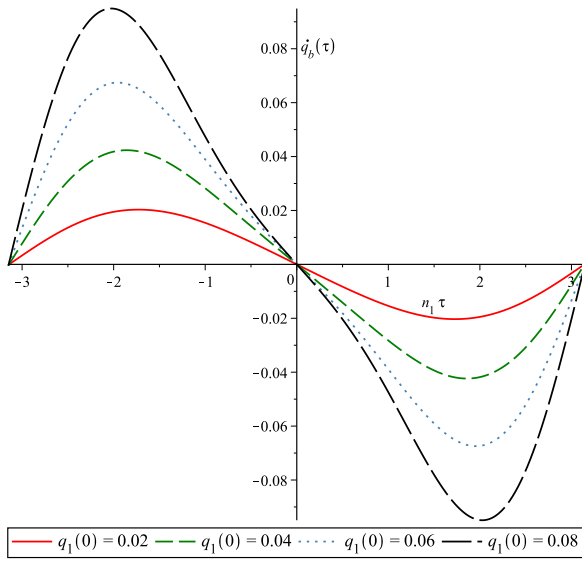
Figure 4.20. Radial Velocity Curves of RD Polytropic models with index $N = 3.0$ for fundamental and next three higher ($f + 3$) modes



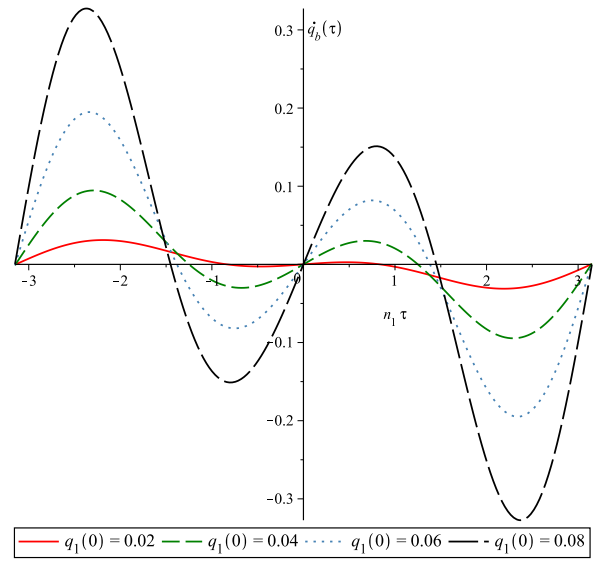
(a) ($n=0.0, q=0.0$)



(b) ($n=0.03, q=0.0$)

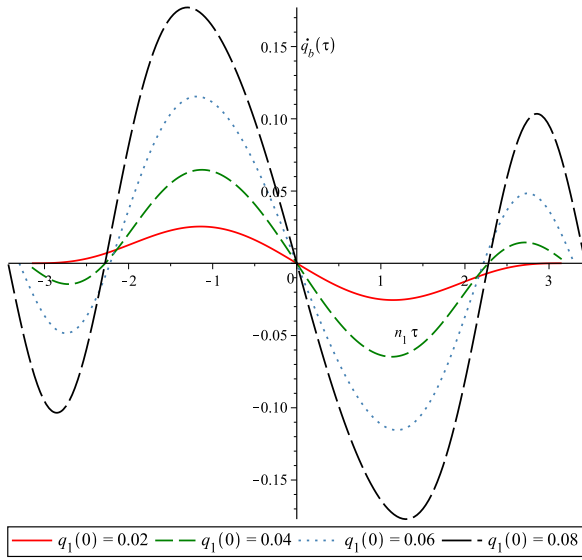


(c) ($n=0.05, q=0.0$)

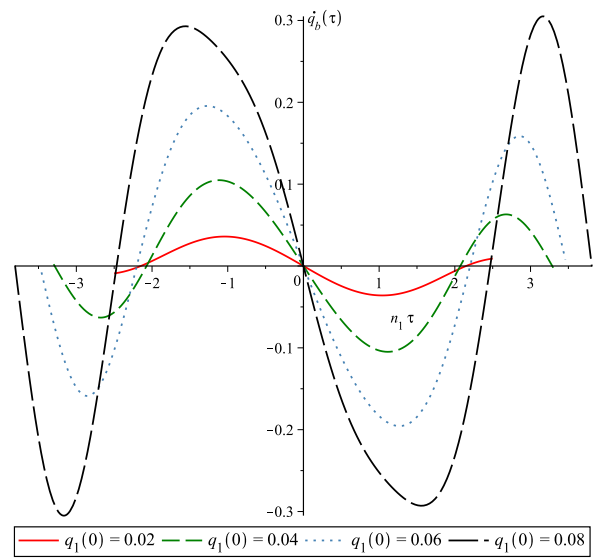


(d) ($n=0.07, q=0.0$)

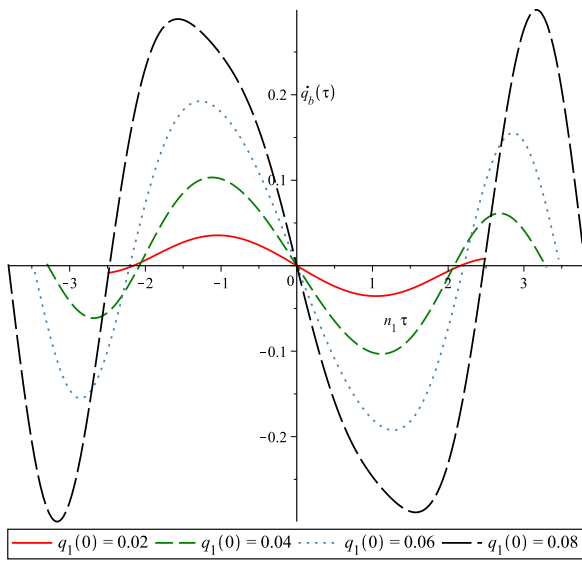
Figure 4.21. Radial Velocity Curves of RD Polytropic models with index $N = 4.0$ for fundamental and next three higher ($f + 3$) modes



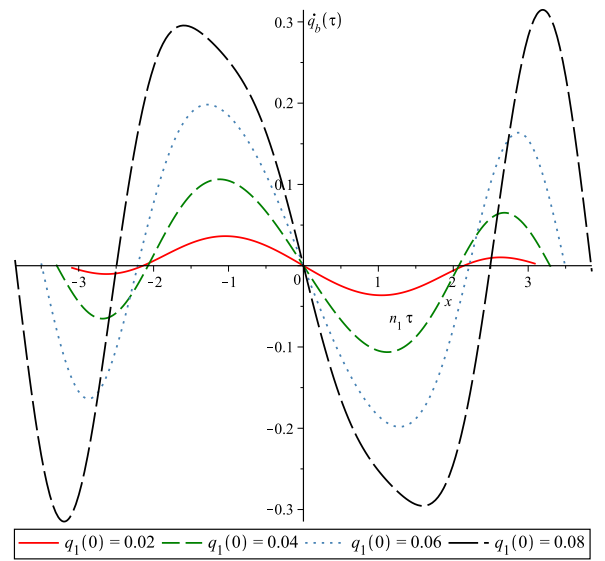
(a) $(n=0.0, q=0.0)$



(b) $(n=0.525, q=0.05)$

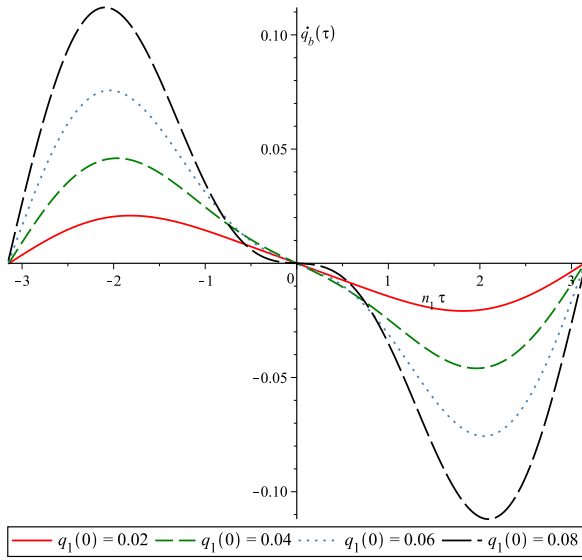


(c) $(n=0.535, q=0.07)$

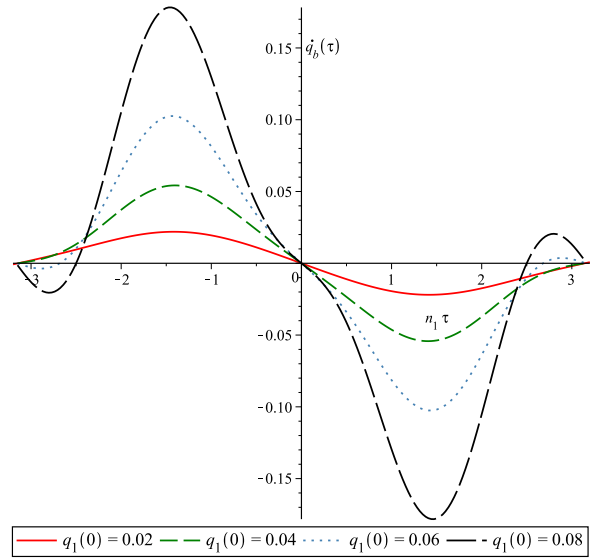


(d) $(n=0.55, q=0.1)$

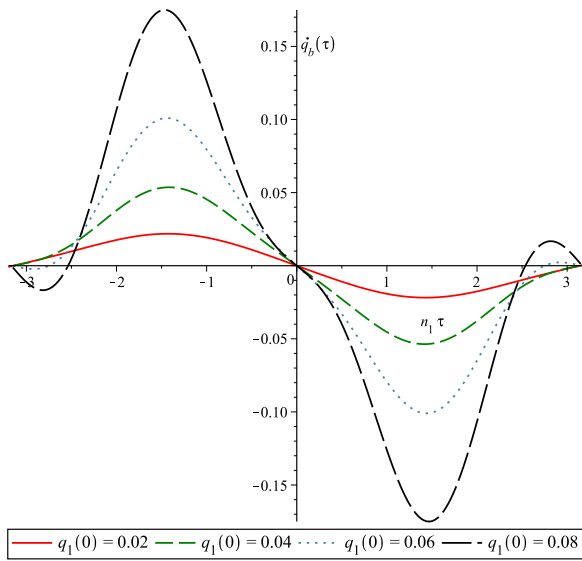
Figure 4.22. Radial Velocity Curves of RTD Polytropic models with index $N = 1.5$ for fundamental and next three higher ($f + 3$) modes



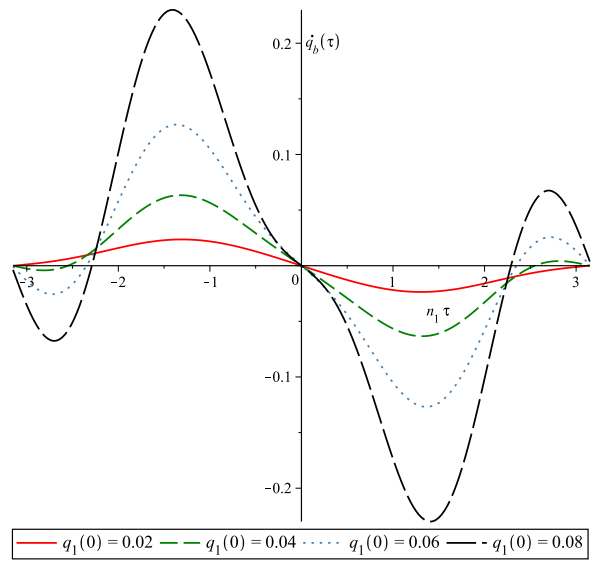
(a) $(n=0.0, q=0.0)$



(b) $(n=0.525, q=0.05)$

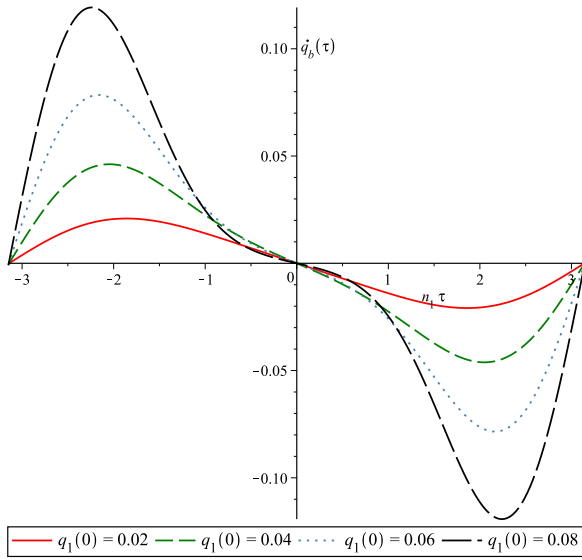


(c) $(n=0.535, q=0.07)$

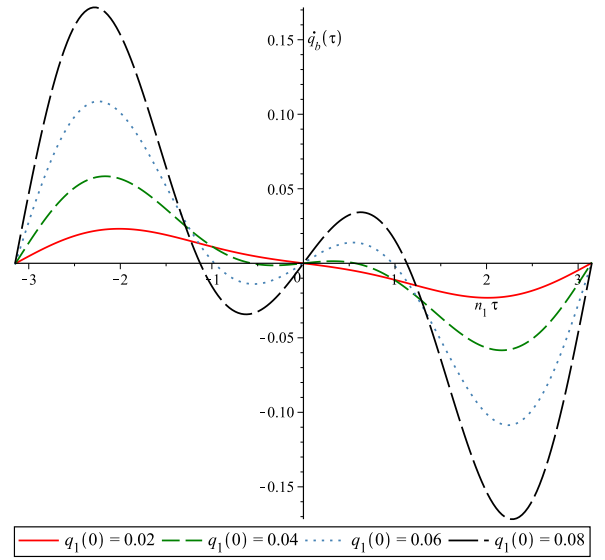


(d) $(n=0.55, q=0.1)$

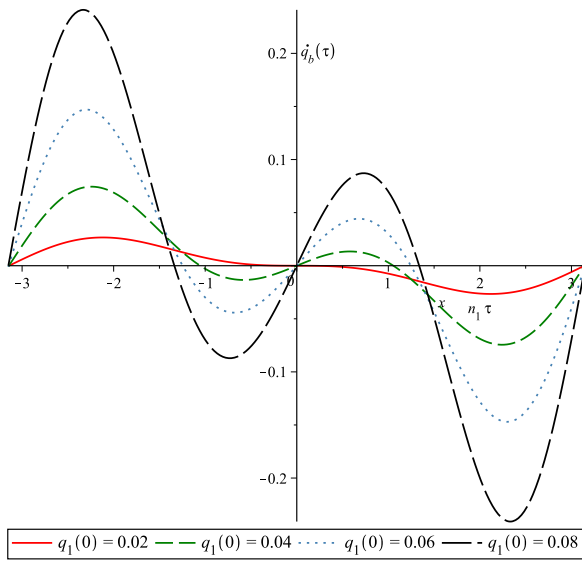
Figure 4.23. Radial Velocity Curves of RTD Polytropic models with index $N = 3.0$ for fundamental and next three higher ($f + 3$) modes



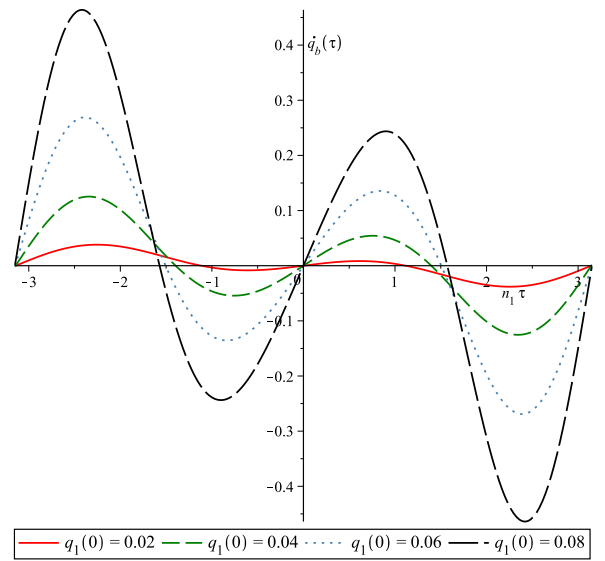
(a) $(n=0.0, q=0.0)$



(b) $(n=0.525, q=0.05)$



(c) $(n=0.535, q=0.07)$



(d) $(n=0.55, q=0.1)$

Figure 4.24. Radial Velocity Curves of Polytropic models with index $N = 4.0$ for fundamental and next three higher ($f + 3$) modes

Chapter 5

Effects of interaction of the radial modes with non radial modes on the shapes of the RVC of the polytropic models of the pulsating variable stars

In all our previous work upto chapter four we have studied the anharmonic radial pulsation of the RD and RTD polytropic models of the pulsating variable stars. Observations show that along with radial modes certain nonradial modes of oscillations can also get excited. So, in this chapter, we have studied the effects of the interaction of the radial modes with non radial modes (that is , mixing modes) on the shapes of the RVC of RD and RTD polytropic models of the pulsating variable stars. For this purpose, following cases have been considered: (i) fundamental non-radial mode (ii) fundamental radial mode (iii) fundamental radial and fundamental non-radial modes (iv) fundamental radial, fundamental non-radial and first radial modes (v) fundamental radial, fundamental non-radial, first radial and first gravity modes.

Anharmonic pulsation equation of RD and RTD polytropic model of a star have been formulated for mixing modes (radial and non-radial oscillations) in section 5.1. The equations obtained for the mixing modes of oscillations have been solved numerically in section 5.2. The results so obtained has been then used to study the effects of nonradial modes of oscillation on the shapes of the RVC of certain polytropic models of the pulsating variable stars. Certain conclusions based on the results obtained in the present chapter are finally discussed in section 5.3.

5.1 Anharmonic pulsation equation of RD and RTD polytropic model of a star for mixing mode of oscillations

As discussed in Chapter 2 and Chapter 3, the equation that governs the anharmonic pulsations of the RD and RTD polytropic model of a star can be written as

$$\frac{d^2 q_k}{d\tau^2} + \beta_k q_k = \frac{\gamma}{I_k^* \omega_1^2} \left[\sum_i A_{ii,k}^* q_i^2 + 2 \sum_{\substack{i,j \\ i \neq j}} A_{ij,k}^* q_i q_j \right], \quad (j \geq i) \text{ for } k = 1, 2, 3, 4 \quad (5.1.1)$$

where

$$\tau = \omega_1 t; \beta_k = \frac{\omega_k^2}{\omega_1^2}; A_{ii,k}^* = \frac{D_{ii,k}^*}{\omega_1^2 I_k^*}; A_{ij,k}^* = \frac{D_{ij,k}^*}{\omega_1^2 I_k^*}$$

$$I_k^* = \int_0^1 \theta^N x^4 \eta_k^2 dx$$

$$\begin{aligned} D_{ij,k}^* = & -\frac{(N+1)}{2} \left(3 - \frac{4}{\gamma}\right) (3\gamma + 1) \int_0^1 \theta^N \theta' x^3 \eta_i \eta_j \eta_k dx \\ & + \frac{1}{2} (3\gamma - 1) \int_0^1 x^4 \theta^{(N+1)} (\eta_i \eta_j' \eta_k' + \eta_i' \eta_j \eta_k' + \eta_i' \eta_j' \eta_k) dx \\ & + \frac{1}{2} (\gamma + 1) \int_0^1 x^5 \theta^{(N+1)} \eta_i' \eta_j' \eta_k' dx \end{aligned}$$

where the symbols γ , τ , θ ($0 \leq \theta \leq 1$), N , ω_1^2 , x , $D_{ij,k}^*$ and I_k^* are same as discussed in Chapter 2. Now, here $\eta_1, \eta_2, \eta_3, \eta_4, \dots$, are the eigenfuctions of the various modes of the pulsation equation as obtained in chapter 1 ((1.5.1) for radial modes and equation (1.6.1) for nonradial modes of pulsation of RD and RTD polytropic models of stars as derived by Mohan and Saxena [45]). As done earlier we will obtain the radial modes of oscillations using equation (1.5.1), however, to obtain the nonradial modes of oscillations we will use the equation (1.6.1).

Also the displacement q_b at the surface is given by

$$q_b = q_1 + q_2 + q_3 + q_4 + \dots \quad (5.1.2)$$

5.2 Numerical Computations

Following the approach as discussed in Chapter 2 and Chapter 3 the anharmonic pulsation equation (5.1.1) has been solved for the mixing modes of RD and RTD polytropic models of the pulsating variable stars. In order to study how the interaction of nonradial modes with the radial modes of oscillations affects the shapes of RVC of RD and RTD polytropic models of pulsating variable stars, we have considered the following cases: (i) fundamental nonradial (f_{nr}) mode (ii) fundamental radial (f_r) mode (iii) fundamental radial and fundamental non-radial modes ($f_r + f_{nr}$ from hereafter f_{rnr} mode) (iv) fundamental radial, fundamental nonradial and first radial ($f_r + f_{nr} + f_{1r}$ from hereafter f_{rnr1} mode) modes of oscillations (v) fundamental radial, fundamental nonradial, first radial and first gravity ($f_r + f_{nr} + f_{1r} + g_1$, from hereafter f_{rnr1g} mode) modes of oscillations.

The value of coefficients $A_{ij,k}^*$ has been given in table (5.3-5.6) for all the considered cases

where nonradial modes has been taken into account. The solution of anharmonic equation has been presented in tables (5.7-5.9). The RVC obtained in each case are shown in Figs. (5.1-5.5). The value of skewness ratio K obtained for different models are presented in Table (5.10-5.11).

5.3 Concluding Observations

Table (5.1) represents the eigenfrequencies of the fundamental non-radial modes of the oscillations of the RD and RTD polytropic models with indices 1.5, 3.0 and 4.0, whereas Table (5.2) represents the eigenfrequencies of the first gravity modes of the non-radial oscillations of RD and RTD polytropic models with indices 3.0 and 4.0. From these tables it is clear that the eigenfrequencies of the non-radial modes of the RD and the RTD polytropic models are small as compared to the corresponding values of the undistorted polytropic model. Also, from this table it can be observed that with increase in the rotational distortions (the value of n or angular velocity of the rotation of the star) and the rotational and tidal distortions (the value of both n and q where q represent tidal distortions due to secondary component) the value of the eigenfrequencies decreases. These results are in accordance with the previous results of Mohan and Saxena [45].

From figs. (5.1-5.5), it can be observed that with the inclusion of non radial modes there is deviation in the shapes of radial velocity curves (as compared to shapes of RVC when only radial modes are considered) of the RD and RTD polytropic modes of the pulsating variable stars. This effect is more dominant near the points of maxima and minima. It can also be observed from the these figures that due to the interaction of non radial modes with the radial modes more deviation in the shapes of RVC is observed in case of polytropes with index 1.5 then with index 3.0 and 4.0. Also, there is more deviation in the shapes of the RVC of the RD and RTD models of pulsating variable stars as we consider more nonradial modes.

From table (5.10) and (5.11) we can conclude that when the interaction of fundamental nonradial modes is considered with fundamental radial modes then there is decrease in the value of skewness coefficient. However, when fundamental g_1 mode is considered along with radial modes (fundamental and first radial modes) then there is increase in the value of K .

So, from this study we can conclude that due to interaction of nonradial modes with radial modes there is appreciable deviation in the RVC of RD and RTD polytropic models of pulsating variable stars. And these deviations in the RVC increases when more nonradial modes are considered.

Table 5.1: Eigenfrequencies of the fundamental non-radial modes of the Polytropic models of the stars

	$N = 1.5$	$N = 3.0$	$N = 4.0$
	ω_1^2	ω_1^2	ω_1^2
Undistorted (n=0,q=0)	2.705	9.254	15.149
Rotationally distorted			
(n=0.03,q=0.0)	2.574	8.823	14.157
(n=0.05,q=0.0)	2.486	8.519	13.463
(n=0.07,q=0.0)	2.397	8.306	12.996
Rotationally and Tidally distorted			
(n=0.525,q=0.05)	2.418	8.265	12.887
(n=0.535,q=0.07)	2.409	8.241	12.835
(n=0.55,q=0.1)	2.399	8.206	12.697

Table 5.2: Eigenfrequencies of the fundamental gravity non-radial modes (g_1 modes) of the Polytropic models of the stars

	$N = 3.0$	$N = 4.0$
	ω_1^2	ω_1^2
Undistorted (n=0,q=0)	4.9152	27.590
Rotationally distorted		
(n=0.03,q=0.0)	4.8208	26.500
(n=0.05,q=0.0)	4.7471	25.678
Rotationally and Tidally distorted		
(n=0.525,q=0.05)	4.6920	24.998
(n=0.55,q=0.1)	4.6790	24.9760

Table 5.3: Coefficients $A_{ij,k}^*$ of the anharmonic pulsation equation of the RD and RTD
Polytropic model of index $N = 1.5$ for f_{nr}

i	j	k	$A_{ij,k}^*$				
			(n=0.0, q=0.0)	(n=0.03, q=0.0)	(n=0.05, q=0.0)	(n=0.525, q=0.05)	(n=0.55, q=0.1)
Polytropic Index $N = 1.5$							
1	1	1	3.30942	3.52437	3.66129	4.0034	3.89636
Polytropic Index $N = 3.0$							
1	1	1	0.63479	0.63541	0.63693	0.61446	0.63380
Polytropic Index $N = 4.0$							
1	1	1	1.61708	1.73672	2.09065	2.01045	2.31602

Table 5.4: Coefficients $A_{ij,k}^*$ of the anharmonic pulsation equation of the RD and RTD
Polytropic model of index $N = 1.5$ for f_{rnr1}

i	j	k	$A_{ij,k}^*$				
			(n=0.0, q=0.0)	(n=0.03, q=0.0)	(n=0.05, q=0.0)	(n=0.525, q=0.05)	(n=0.55, q=0.1)
1	1	1	3.31241	3.51283	3.61862	3.78842	3.81760
1	2	1	2.99412	3.07492	3.12022	3.30507	3.23378
1	3	1	1.35702	1.41236	1.43739	1.49525	1.50414
1	1	2	3.66300	4.09744	4.34227	4.55228	4.73581
1	2	2	3.48504	3.81091	4.00458	4.31841	4.31771
1	3	2	1.60760	1.81763	1.93984	2.08631	2.13932
1	1	3	14.44663	15.97793	17.0278	18.22858	18.49512
1	2	3	13.98916	15.43127	16.51277	18.46579	17.96226
1	3	3	16.46942	18.37087	19.75046	21.28627	21.63951

Table 5.5: Coefficients $A_{ij,k}^*$ of the anharmonic pulation equation of the RD and RTD
Polytropic model of index $N = 3.0$ for f_{rnr1g}

i	j	k	$A_{ij,k}^*$				
			(n=0.0, q=0.0)	(n=0.03, q=0.0)	(n=0.05, q=0.0)	(n=0.525, q=0.05)	(n=0.55, q=0.1)
1	1	1	0.97465	1.04599	1.10442	1.16063	1.17446
1	2	1	0.83638	0.84188	1.06426	0.88885	0.90705
1	3	1	0.44846	0.49080	0.52390	0.55567	0.56378
1	4	1	1.13898	1.15105	1.42564	1.32292	1.33607
1	1	2	0.59844	0.70342	0.49666	0.85299	0.85459
1	2	2	0.85348	0.67852	0.56527	0.71231	0.80723
1	3	2	0.27092	0.33370	0.22124	0.42512	0.42597
1	4	2	0.50966	0.66839	0.67551	1.12008	1.22812
1	1	3	5.29235	5.85030	6.33856	6.86254	7.02087
1	2	3	4.46838	4.76057	5.73598	5.47104	5.63035
1	3	3	3.83474	4.25459	4.61245	5.00994	5.13783
1	4	3	4.78411	5.13713	6.05124	6.33086	6.46950
1	1	4	0.29146	0.32328	0.21972	0.33530	0.33688
1	2	4	0.18228	0.22467	0.22316	0.29582	0.32866
1	3	4	0.10374	0.12104	0.07711	0.62615	1.30990
1	4	4	0.71709	0.89246	0.61332	0.62615	0.63738

Table 5.6: Coefficients $A_{ij,k}^*$ of the anharmonic pulation equation of the RD and RTD
Polytropic model of index $N = 4.0$ for f_{rnr1g}

i	j	k	$A_{ij,k}^*$				
			(n=0.0, q=0.0)	(n=0.03, q=0.0)	(n=0.05, q=0.0)	(n=0.525, q=0.05)	(n=0.55, q=0.1)
1	1	1	0.44815	0.48762	0.51921	0.60233	0.64847
1	2	1	0.02288	0.01381	0.00667	0.02635	0.04074
1	3	1	0.19850	0.21244	0.21516	0.27952	0.32486
1	4	1	0.07680	0.06664	0.05567	0.07145	0.08554
1	1	2	2.64658	1.69727	0.96761	3.41953	6.22891
1	2	2	2.73791	3.07198	2.79829	2.30117	1.98971
1	3	2	4.97430	4.40245	3.40399	5.32510	8.82607
1	4	2	3.37558	3.23890	3.42267	2.27053	2.36366
1	1	3	2.84184	3.07556	3.14938	4.13752	4.86288
1	2	3	0.61564	0.51858	0.34359	0.60756	0.86410
1	3	3	2.14585	2.25122	2.01908	2.86214	3.63393
1	4	3	1.09615	1.04032	0.85771	1.08164	1.35419
1	1	4	2.77099	3.44893	3.74835	5.63748	7.62496
1	2	4	1.05286	1.36390	1.58919	1.38070	1.37805
1	3	4	2.76245	3.71900	3.94541	5.76492	8.06422
1	4	4	1.65589	2.21645	2.57196	2.51416	2.43827

Table 5.7: Solution of the Anharmonic equation of the RD and RTD polytropic models of
Index $N = 1.5$

Models	Value of displacement on the surface of Star $q_b(\tau)$
f_{nr} -mode	
(n=0.0,q=0.0)	$q_b(\tau) = 0.00537 + 0.05700 \cos n_1\tau - 0.00179 \cos 2n_1\tau + 0.00004 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.00572 + 0.05700 \cos n_1\tau - 0.00190 \cos 2n_1\tau + 0.00004 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.00584 + 0.05650 \cos n_1\tau - 0.00194 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.00639 + 0.05650 \cos n_1\tau - 0.00213 \cos 2n_1\tau + 0.00006 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.00621 + 0.05650 \cos n_1\tau - 0.00207 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
f_r -modes	
(n=0.0,q=0.0)	$q_b(\tau) = 0.00538 + 0.05700 \cos n_1\tau - 0.00179 \cos 2n_1\tau + 0.00004 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.00570 + 0.05700 \cos n_1\tau - 0.00190 \cos 2n_1\tau + 0.00004 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.00587 + 0.05700 \cos n_1\tau - 0.00195 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.00604 + 0.05650 \cos n_1\tau - 0.00201 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.00609 + 0.05650 \cos n_1\tau - 0.00203 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
f_{rnr} -modes	
(n=0.0,q=0.0)	$q_b(\tau) = 0.01202 + 0.01756 \cos n_1\tau - 0.00371 \cos 2n_1\tau + 0.00017 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.01333 + 0.01719 \cos n_1\tau - 0.00403 \cos 2n_1\tau + 0.00020 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.01395 + 0.02000 \cos n_1\tau - 0.00412 \cos 2n_1\tau + 0.00021 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.01463 + 0.01601 \cos n_1\tau - 0.00432 \cos 2n_1\tau + 0.00024 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.01504 + 0.01479 \cos n_1\tau - 0.00442 \cos 2n_1\tau + 0.00024 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
f_{rnr1} -modes	
(n=0.0,q=0.0)	$q_b(\tau) = 0.01347 + 0.03331 \cos n_1\tau + 0.01672 \cos 2n_1\tau - 0.01598 \cos 3n_1\tau + 0.00037 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.01641 + 0.04007 \cos n_1\tau + 0.03645 \cos 2n_1\tau - 0.02354 \cos 3n_1\tau + 0.00061 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.01962 + 0.05074 \cos n_1\tau + 0.06817 \cos 2n_1\tau - 0.03199 \cos 3n_1\tau + 0.00088 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.02184 + 0.05621 \cos n_1\tau + 0.09606 \cos 2n_1\tau - 0.04276 \cos 3n_1\tau + 0.00131 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.02336 + 0.05833 \cos n_1\tau + 0.11238 \cos 2n_1\tau - 0.04663 \cos 3n_1\tau + 0.00139 \cos 4n_1\tau \dots$

Table 5.8: Solution of the Anharmonic equation of the RD and RTD polytropic models of Index $N = 3.0$

Models	Value of displacement on the surface of Star $q_b(\tau)$
f_{nr} -mode	
(n=0.0,q=0.0)	$q_b(\tau) = 0.00114 + 0.06000 \cos n_1\tau - 0.00381 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.00114 + 0.06000 \cos n_1\tau - 0.00038 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.00114 + 0.06000 \cos n_1\tau - 0.00038 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.00110 + 0.06000 \cos n_1\tau - 0.00036 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.00114 + 0.06000 \cos n_1\tau - 0.00038 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
f_r -modes	
(n=0.0,q=0.0)	$q_b(\tau) = 0.00169 + 0.05900 \cos n_1\tau - 0.00056 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.00182 + 0.05900 \cos n_1\tau - 0.00060 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.00192 + 0.05900 \cos n_1\tau - 0.00064 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.00202 + 0.05900 \cos n_1\tau - 0.00067 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.00204 + 0.05900 \cos n_1\tau - 0.00068 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
f_{rnr} -modes	
(n=0.0,q=0.0)	$q_b(\tau) = 0.00287 + 0.05727 \cos n_1\tau - 0.00090 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.00319 + 0.05692 \cos n_1\tau - 0.00100 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.00287 + 0.05746 \cos n_1\tau - 0.00092 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.00366 + 0.05611 \cos n_1\tau - 0.00115 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.03694 + 0.05589 \cos n_1\tau - 0.00116 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00001 \cos 4n_1\tau \dots$
f_{rnr1} -modes	
(n=0.0,q=0.0)	$q_b(\tau) = 0.08029 + 0.06052 \cos n_1\tau - 0.00550 \cos 2n_1\tau + 0.00033 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.00908 + 0.06115 \cos n_1\tau - 0.00615 \cos 2n_1\tau + 0.00040 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.00955 + 0.06238 \cos n_1\tau - 0.00668 \cos 2n_1\tau + 0.00046 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.01095 + 0.06243 \cos n_1\tau - 0.00741 \cos 2n_1\tau + 0.00054 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.01119 + 0.06256 \cos n_1\tau - 0.00759 \cos 2n_1\tau + 0.00056 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
f_{rnr1g} -modes	
(n=0.0,q=0.0)	$q_b(\tau) = 0.00885 + 0.05963 \cos n_1\tau - 0.00530 \cos 2n_1\tau + 0.00017 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.00988 + 0.05999 \cos n_1\tau - 0.00578 \cos 2n_1\tau + 0.00020 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.00979 + 0.06146 \cos n_1\tau - 0.00598 \cos 2n_1\tau + 0.00023 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.01152 + 0.06091 \cos n_1\tau - 0.00662 \cos 2n_1\tau + 0.00027 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.01173 + 0.06098 \cos n_1\tau - 0.00674 \cos 2n_1\tau + 0.00028 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$

Table 5.9: Solution of the Anharmonic equation of the RD and RTD polytropic models of Index $N = 4.0$

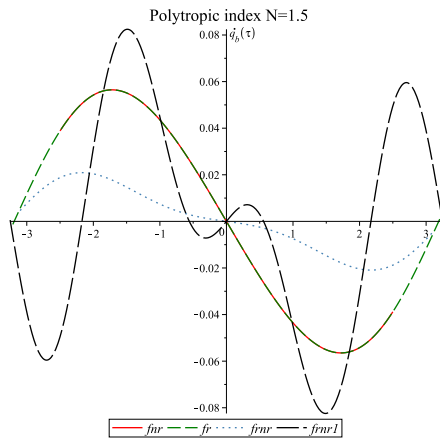
Models	Value of displacement on the surface of Star $q_b(\tau)$
f_{nr} -mode	
(n=0.0,q=0.0)	$q_b(\tau) = 0.00281 + 0.05900 \cos n_1\tau - 0.00938 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.00297 + 0.05850 \cos n_1\tau - 0.00099 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.00363 + 0.05900 \cos n_1\tau - 0.00121 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.00389 + 0.05800 \cos n_1\tau - 0.00112 \cos 2n_1\tau + 0.00001 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.00344 + 0.05850 \cos n_1\tau - 0.00114 \cos 2n_1\tau + 0.00002 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
f_r -modes	
(n=0.0,q=0.0)	$q_b(\tau) = 0.00080 + 0.06000 \cos n_1\tau - 0.00026 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.00087 + 0.06000 \cos n_1\tau - 0.00029 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.00093 + 0.06000 \cos n_1\tau - 0.00031 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.00116 + 0.06000 \cos n_1\tau - 0.00038 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.00108 + 0.06000 \cos n_1\tau - 0.00036 \cos 2n_1\tau + 0.00000 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
f_{rnr} -modes	
(n=0.0,q=0.0)	$q_b(\tau) = 0.00290 + 0.06035 \cos n_1\tau - 0.00301 \cos 2n_1\tau + 0.00007 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.00218 + 0.06021 \cos n_1\tau - 0.00213 \cos 2n_1\tau + 0.00005 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.00167 + 0.06012 \cos n_1\tau - 0.00136 \cos 2n_1\tau + 0.00003 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.00585 + 0.06072 \cos n_1\tau - 0.00735 \cos 2n_1\tau + 0.00015 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.03718 + 0.06043 \cos n_1\tau - 0.00406 \cos 2n_1\tau + 0.00009 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
f_{rnr1} -modes	
(n=0.0,q=0.0)	$q_b(\tau) = 0.00613 + 0.06162 \cos n_1\tau - 0.00531 \cos 2n_1\tau + 0.00010 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.00562 + 0.06152 \cos n_1\tau - 0.00465 \cos 2n_1\tau + 0.00011 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.00528 + 0.06151 \cos n_1\tau - 0.00388 \cos 2n_1\tau + 0.00009 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.01174 + 0.06430 \cos n_1\tau - 0.01166 \cos 2n_1\tau + 0.00030 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.00858 + 0.06292 \cos n_1\tau - 0.00754 \cos 2n_1\tau + 0.00019 \cos 3n_1\tau + 0.00000 \cos 4n_1\tau \dots$
f_{rnr1g} -modes	
(n=0.0,q=0.0)	$q_b(\tau) = 0.00876 + 0.06219 \cos n_1\tau - 0.00747 \cos 2n_1\tau + 0.00015 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.03,q=0.0)	$q_b(\tau) = 0.00848 + 0.06221 \cos n_1\tau - 0.00783 \cos 2n_1\tau + 0.00017 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.05,q=0.0)	$q_b(\tau) = 0.00870 + 0.06253 \cos n_1\tau - 0.00697 \cos 2n_1\tau + 0.00015 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$
(n=0.525,q=0.05)	$q_b(\tau) = 0.01823 + 0.06610 \cos n_1\tau - 0.01777 \cos 2n_1\tau + 0.00044 \cos 3n_1\tau - 0.00001 \cos 4n_1\tau \dots$
(n=0.55,q=0.1)	$q_b(\tau) = 0.01355 + 0.06437 \cos n_1\tau - 0.01210 \cos 2n_1\tau + 0.00029 \cos 3n_1\tau - 0.00000 \cos 4n_1\tau \dots$

Table 5.10: Values of n_1^2 and the skewness coefficient K for the radial and nonradial Modes

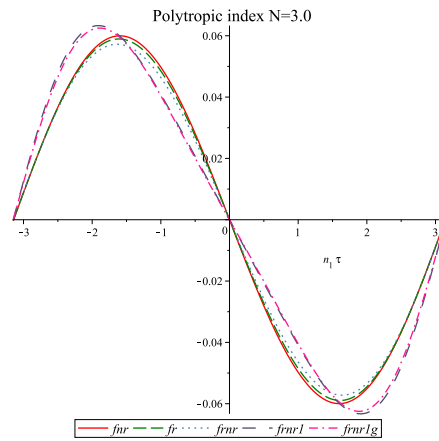
Models	f_{nr} mode		f_r modes		f_{rnr} modes		f_{rnr1} modes	
	n_1^2	K	n_1^2	K	n_1^2	K	n_1^2	K
Polytropic index ($N = 1.5$)								
(n=0.0,q=0.0)	0.97034	0.4605	0.97029	0.4632	0.93622	0.3318	0.93622	0.1683
(n=0.03,q=0.0)	0.96636	0.4585	0.96658	0.4588	0.96576	0.3238	0.92621	0.1772
(n=0.05,q=0.0)	0.96433	0.4567	0.96434	0.4569	0.92099	0.3161	0.92099	0.1856
(n=0.525,q=0.05)	0.95736	0.4509	0.96243	0.4551	0.91268	0.3132	0.91268	0.1896
(n=0.55,q=0.1)	0.95961	0.4527	0.96123	0.4541	0.91105	0.3063	0.91105	0.1910

Table 5.11 : Values of n_1^2 and the skewness coefficient K for the radial and nonradial Modes

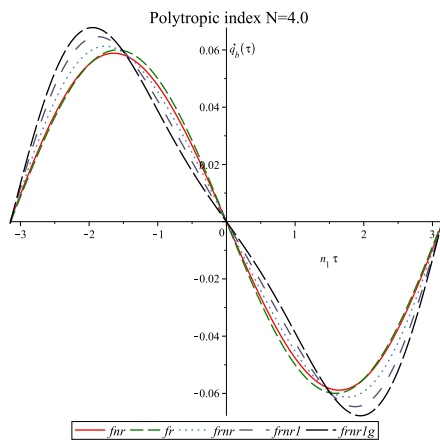
Models	f_{nr} mode		f_r modes		f_{nr} modes		f_{nr1} modes		f_{nr1g} modes	
	n_1^2	K	n_1^2	K	n_1^2	K	n_1^2	K	n_1^2	K
Polytropic index ($N = 3.0$)										
(n=0.0,q=0.0)	0.99879	0.4929	0.99724	0.4965	0.99555	0.4802	0.99555	0.3939	0.99555	0.4003
(n=0.03,q=0.0)	0.99894	0.4929	0.99682	0.4955	0.99485	0.4777	0.99485	0.3847	0.99485	0.3935
(n=0.05,q=0.0)	0.99878	0.4929	0.99646	0.4947	0.99472	0.4796	0.99472	0.3790	0.99472	0.3922
(n=0.525,q=0.05)	0.99886	0.4932	0.99605	0.4938	0.99358	0.4736	0.99358	0.3693	0.99358	0.3828
(n=0.55,q=0.1)	0.99879	0.4929	0.99599	0.4937	0.99344	0.4733	0.99344	0.3671	0.99344	0.3812
Polytropic index ($N = 4.0$)										
(n=0.0,q=0.0)	0.99241	0.4792	0.99939	0.5033	0.99936	0.4400	0.99906	0.4054	0.99936	0.3808
(n=0.03,q=0.0)	0.99139	0.4776	0.99928	0.5028	0.99927	0.4567	0.99927	0.4143	0.99926	0.3809
(n=0.05,q=0.0)	0.98753	0.4722	0.99919	0.5024	0.99918	0.4954	0.99918	0.4722	0.99918	0.3861
(n=0.525,q=0.05)	0.98886	0.4737	0.99891	0.5013	0.99887	0.4222	0.99887	0.3800	0.99887	0.3468
(n=0.55,q=0.1)	0.98496	0.4690	0.99872	0.5006	0.99864	0.3801	0.99864	0.3490	0.99864	0.2713



(a)

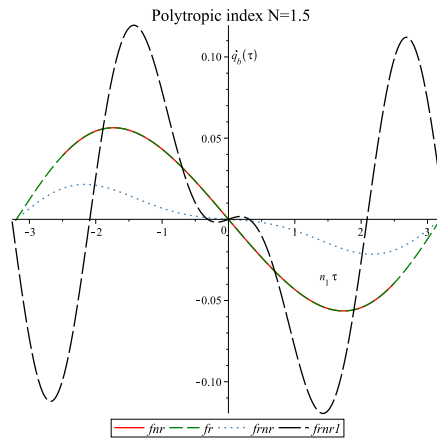


(b)

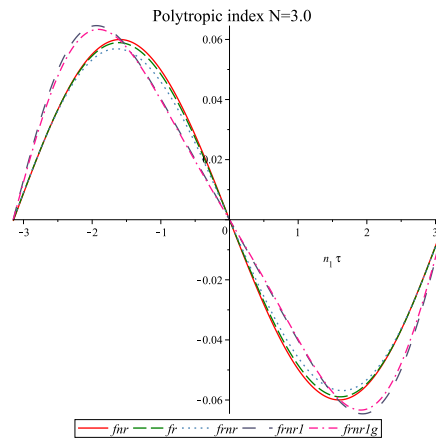


(c)

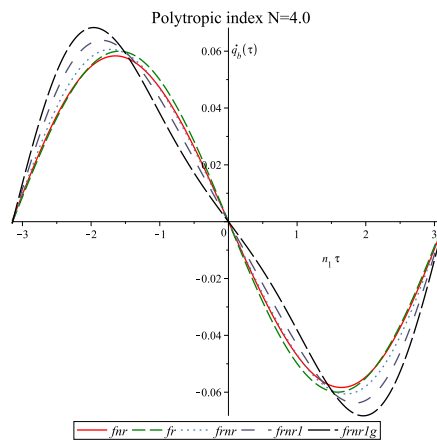
Figure 5.1. Radial velocity curves of the undistorted Polytropic models ($n = 0.0, q = 0.0$) for mixing modes



(a)

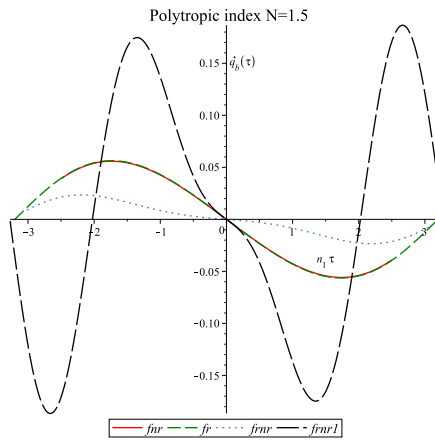


(b)

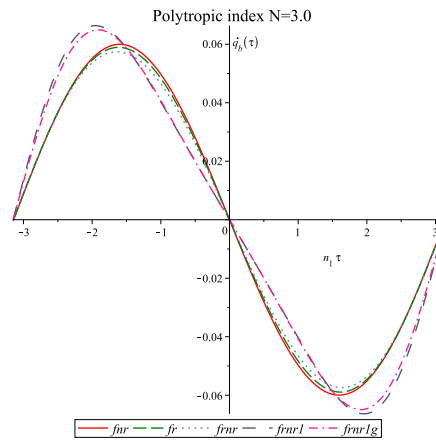


(c)

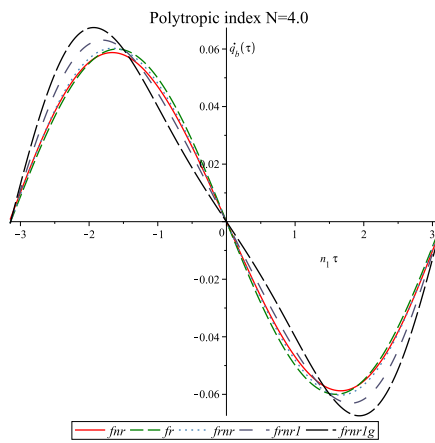
Figure 5.2. Radial velocity curves of the RD Polytropic models ($n = 0.03, q = 0.0$) for mixing modes



(a)

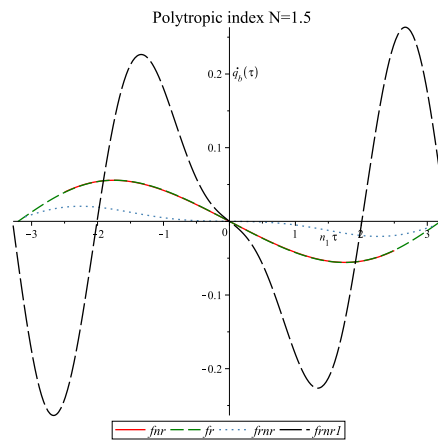


(b)

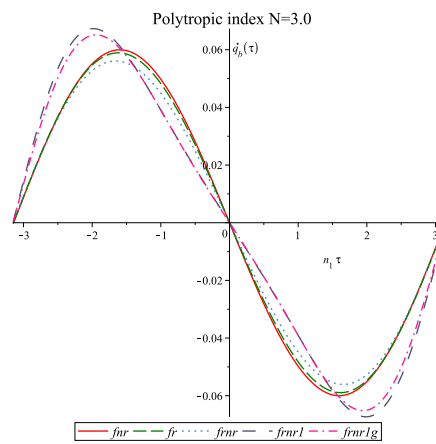


(c)

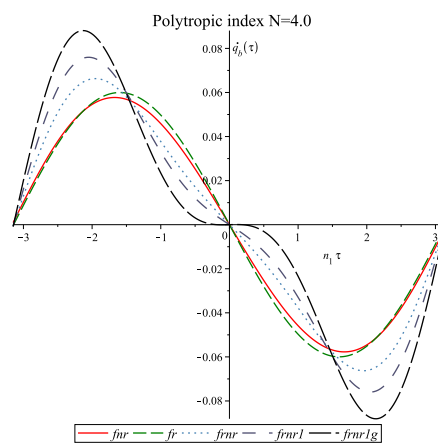
Figure 5.3. Radial velocity curves of the RD Polytropic models ($n = 0.05, q = 0.0$) for mixing modes



(a)

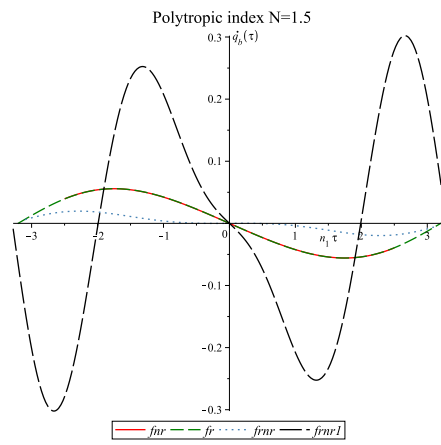


(b)

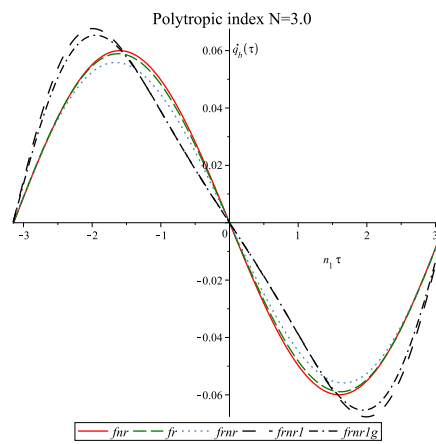


(c)

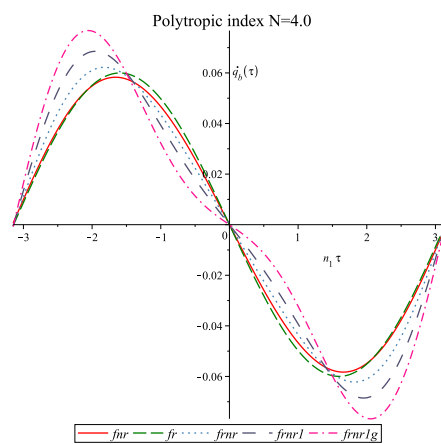
Figure 5.4. Radial velocity curves of the RTD Polytropic models ($n = 0.525$, $q = 0.05$) for mixing modes



(a)



(b)



(c)

Figure 5.5. Radial velocity curves of the RTD Polytropic models ($n = 0.55$, $q = 0.1$) for mixing modes

Chapter 6

Concluding Observations

In this chapter we have critically overviewed in brief the work accomplished in the earlier chapters of this thesis. Besides discussing the limitations and scope of the method developed in preceding chapters for determining the effects of rotation and tidal distortions, interactions of higher and mixing modes of oscillations on the shapes of RVC of RTD polytropic modes of stars, we have also analysed the astrophysical significance of results obtained in these chapters. This chapter ends with some suggestions for further work in this field.

In section 6.1 we critically review the method discussed in this thesis for determining the effect of rotation and tidal distortion on the RVC of RTD polytropic models of pulsating variable stars. The limitations and scope of the method discussed in the thesis analysing the effects of interaction of higher modes and mixing modes on the RVC and RTD polytropic models of stars are next presented in section 6.2. Section 6.3 deals with astrophysical significance of the results obtained in earlier chapters. The scope of further research work on this topic is finally discussed in section 6.4.

6.1 Limitations and scope of the method proposed in the thesis for determining the effects of rotation and tidal distortion on the RVC of polytropic models of stars

In the present thesis we have primarily considered the feasibility of using Mohan and Saxena [45] approach to determine the effect of rotation and tidal distortions on the RVC of polytropic models of variable stars by using the technique of Rosseland [61] and Prasad [58]. The methodology of Mohan and Saxena [45] utilizes the average concepts of Kippenhahn and Thomas [25] in conjunction with the results of Roche equipotentials obtained by Kopal [28] to incorporate the effects of rotation and tidal distortion on the stars. The eigenvalued boundary value problem determining pseudo radial and nonradial modes of oscillations of RTD stars, presented in sections 1.5 and 1.6 are the same as discussed by Mohan and Saxena [45]. However, it is important to mention that while developing the explicit formulations of these eigenvalued boundary value problems, it is assumed that:

1. Actual equipotential surfaces of the distorted star can be approximated by the appropriate Roche equipotential surfaces.

2. Effect of rotation and tidal distortion are not too large so that terms beyond second order of smallness in rotational and tidal distortion parameters can be neglected.
3. The distorted model is well within its appropriate Roche lobes.
4. During the oscillations, the points on each equipotential surface oscillate in unison about their equilibrium positions (i.e. oscillations are assumed to be barotropic).
5. Effects of Coriolis force can be neglected.

In order to determine the effects of rotation and tidal distortion on the RVC predicted on the basis of the theory of anharmonic oscillation, we first formulate in Chapter 1, the general equation of anharmonic oscillations of a RTD model using the theory of Rosseland [61] and Mohan and Saxena [45] and then use it in Chapter 2 to determine the RVC of certain RTD polytropic models of variable stars with index $N = 1.5, 3.0$ and 4.0 . The results show that with the introduction of rotational effects, the RVC for polytropic models of pulsating variable stars deviates more as compared to the tidal effect. The shapes of the radial velocity curves depicted in the graphs of this chapter also show that distortional effect are more dominant near the point of maxima and minima. This flattening effect is considerably enlarged in the case of synchronous binary system.

While studying the effect of rotation and tidal distortion on the RVC of polytropic models of stars we have considered only first four radial modes of oscillations. A difficulty arises to find higher (fifth onward) modes of radial oscillations of RTD polytropic models of stars.

Our investigations on the shapes of RVC of rotating variable stars and variable stars in binary system can also be of some help in better understanding the nature of the stars. Present results show that rotation and tidal pull of the companion star can be some of the possible causes for exciting resonance phenomenon between different modes of oscillation of a variable stars.

6.2 Limitations and scope of the method determining the effects of higher modes, mixing modes and amplitude of pulsation on the shapes of RVC of RTD polytropic models of stars

The equation of anharmonic pulsation developed in Chapter 2 to analyse the effects of rotation and tidal distortion on the RVC of RTD polytropic models has been next used in Chapter 3 to determine the effect of higher modes of radial oscillations on the shapes of RVC of RTD polytropic pulsating models of stars. In this Chapter, we have considered the interaction of various

modes such as : (i) fundamental mode (ii) fundamental and first mode (iii) fundamental and the next two modes and finally (iv) fundamental and next three higher modes. The methodology developed in Chapter 2 and Chapter 3 has been further used in Chapter 4 to determine the effect of amplitude of radial oscillations on the shapes of RVC of above mentioned distorted stars. The same methodology is used in Chapter 5 along with the results of nonradial oscillations discussed in Chapter 1 (subsection 1.6) to determine the combined effect of radial and nonradial modes, that is , mixing modes on the shape of RVC of polytropic models of variable stars. The methodology developed in this thesis can be conveniently used to exhibit the behaviour of RVC of RTD polytropic models of variable star under the assumptions specified earlier.

In the case of combined effects of radial and nonradial modes of oscillations on the RVC of RTD polytropic models, we have only considered the fundamental and gravity g_1 modes of nonradial oscillations. It is assumed that fundamental mode is more dominant over the other modes.

Our results in Chapter 4 show that physically possible solution of the anharmonic equations are only possible upto a certain value of amplitude of pulsation. The upper limit of the amplitude of pulsation giving a physically possible solutions of the RVC changes from model to model and decrease with the inclusion of higher modes in the anharmonic equations. On the basis of these results, we may, therefore, say that if the stars are pulsating with small radial oscillations and shapes of the observed RVC is due to the anharmonic effects, then there is an upper limit to the amplitude with which a star pulsates.

The present study is expected to give an answer to the question whether theory of anharmonic pulsation can properly explain the shapes of RVC of observed variable stars or the observed shape of the light curve is due to some other physical cause working in the star. In case we are able to find a mathematical model of the star whose RVC resembles the shape of RVC of an observed variable star then it would not only validate the theory of anharmonic pulsation but will also give an insight into the nature of the interior structure of such stars.

6.3 Astrophysical Significance of the present work

The computational methods developed in this thesis are primarily designed to study the effects of rotation and tidal distortion on the RVC of polytropic models of pulsating variable stars . Since polytropic models are often used in astrophysical literature to depict the inner structure of a variety of realistic stars, the results obtained in chapter (2-5) have practical significance.

In order to compare the the radial velocity curves for the present undistorted polytropic model ($n = q = 0$) with that of standard model of star obtained by Prasad [58], we have show that both the curves in Figs. (6.1-6.2). In Fig. (6.1) the solid red line represents the radial velocity curve for fundamental and first mode for standard model obtained by Prasad

[58] and black dotted line represent the radial velocity curve for fundamental and first mode for undistorted polytropic model of index 3.0 obtained in the present study. We observed that both the curves are in good agreement

In Fig.(6.2) the solid red line represents the radial velocity curve for fundamental and next two higher modes for standard model obtained by Prasad [58] and black dotted line represent the radial velocity curve for next two higher modes for undistorted polytropic model of index 3.0. The value of skewness coefficient for the radial velocity curve for fundamental and next two higher modes for standard model obtained by Prasad [58] was 0.307 and the value of skewness coefficient for fundamental and next two higher modes for undistorted polytropic model of index 3.0 is 0.29 which is again in good agreement. The values of skewness ratio K for different undistorted models are collected and presented in Table (6.1).

Table 6.1: Comparison of Skewness ratio K of present result with some realistic stars

S.No	Star	K
1	Homogenous model	0.46*
2	Standard model ($N = 3.0$)	0.307**
3	Standard model (Commensurable period)	0.287 ⁺
4	$K(10)$ model	0.268 [†]
5	The present model $N = 1.5$	0.79
6	The present model $N = 3.0$	0.29
7	The present model $N = 4.0$	0.376
8	η Aquilae	0.24 [‡]
9	δ Cephei	0.20 [‡]

* Prasad[59], ** Prasad [58], † Gurm [19], ‡ Schwarzschild and Savedoff [66]

No specific conclusion of astrophysical significance can be straightway drawn on the basis of the work reported in this thesis. However, the developed method and numerical results reported in this thesis can be of some help in arriving at conclusion which can have astrophysical significance. The technique discussed in the present thesis can be used to determine the shapes of RVC of rotating stars and stars in binary systems. The methods reported in this thesis can also help in deciding whether an observed rotating star has a uniform rotation or a differential type of rotation depending upon which law best predicts its observed features. Investigations on the effects of rotation and tidal distortions on the shapes of RVC of rotating variable stars and variable stars in binary system can also be of some help in better understanding the true nature of these stars. Present results also that rotation and tidal pull of the companion star can

be some of the possible causes for exciting resonance phenomena between different modes of oscillations of a variable stars.

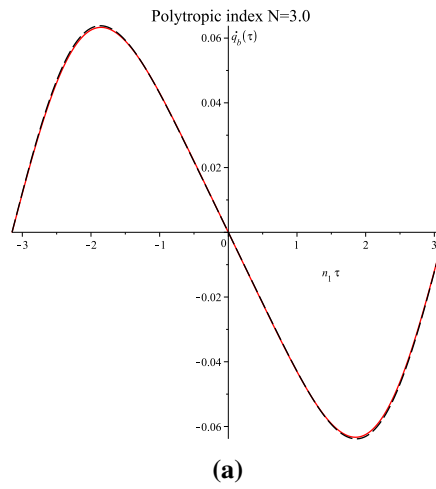


Figure 6.1. Radial velocity curves of the undistorted Polytropic models ($n = 0.0, q = 0.0$)

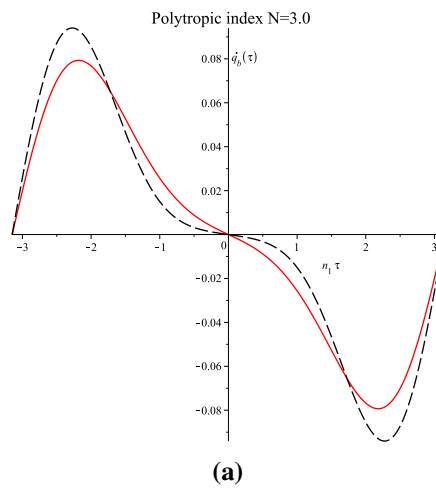


Figure 6.2. Radial velocity curves of the undistorted Polytropic models ($n = 0.0, q = 0.0$)

6.4 Future Scope

Determining the effect of rotation and tidal distortion on the RTD gaseous sphere are very complex mathematical problems. Therefore, it is not easy to provide general results applicable to all type of rotating variable star which exists as single or binary star. Due to its practical utility, analysis of such problem is of practical interest to the theory of stellar structure. During the progress of the present study on the anharmonic pulsation of variable star, several interesting topics have come up to our attention. However, it is not possible to pursue all these within the research framework of this thesis. Hence, in the concluding chapter, some of them will be proposed as a focus for further research.

In the present work we have primarily considered the effectiveness of Mohan and Saxena [45] approach for studying the anharmonic pulsation of RTD stars. We have formulated the equation of anharmonic for RTD polytropic model of stars correct upto second order terms. It will be worthwhile to extend further the analysis of the present methods to include higher order terms, so that method can be used to study the more realistic problems of stars.

While formulating the equipotential surfaces of the distorted models of stars, Mohan and Saxena [45] considered the rotational and tidal forces only. This approximation is much better if we include Coriolis force, magnetic field and radiation pressure other than rotation and tidal force. It will be of our interest to see how the combined forces affects the RVC of the pulsating variable stars.

In the present investigation it has been assumed that in a RTD model only those eigenfrequencies excited are radial and nonradial modes of oscillations of undistorted spherically symmetric star (barotropic). There is every likelihood of modes other than these (baroclinic modes of oscillations). The present study can be extended to investigate the eigenfrequencies of RTD stars without barotropic assumptions. Also, the present approach can be modified by including the effects of higher modes of radial oscillations and pressure and gravity modes in the case of nonradial oscillations.

In the present approach we have considered the effect of rotation and tidal distortion on the RVC only. This can be further used to analyse the effects of rotation and tidal distortion on the rate of energy generation inside the core and damping effects of the outer ionization zone. It will help us in deciding the pulsations stability of rotating variable stars and variable stars in binary systems.

The present approach to study the behaviour of RVC of RTD polytropic models of variable stars in the presence of distortional force only . It will be of our interest to see how the shape of RVC of other realistic variable stars (such as White dwarf, Composite model, Roche model etc).

From the view point of astrophysical importance, it will also be worthwhile to apply the technique developed in thesis to determine the radial velocity curves of some realistic models

of rotating stars and stars in binary systems by using the approach of section 2.1 in conjunction with some available computer code for determining the radial velocity curves of RTD theoretical models of realistic stars.

From the astrophysical view point, it will be worthwhile to incorporate the present methodology into certain computer codes for anharmonic pulsation and apply it to determine the various shapes of RVC and skewness of distorted models of star and also trace the evolutionary traces of certain realistic models of rotating variable stars and stars in binary systems. Use of the method developed in this thesis for determining the radial velocity curves of rotating variable stars and variable stars in binary systems is also likely to yield results of some practical interest to astronomy.

We shall also develop necessary computational algorithms to determine RVC of certain radial and nonradial pulsating stellar models. These algorithms will be helpful for those who are working in these fields either in observations or in theoretical work.

It is to be noted that the theory of anharmonic pulsation is not only of interest to the problem of variable stars in astronomy and astrophysics but is of vital interest in studies of vibrations of mechanical structures in engineering as well as analysis of earthquakes in geophysics.

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