WEIGHTED MULTILEVEL SPACE-TIME TRELLIS CODES FOR RAYLEIGH FADING CHANNELS

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DECLARATION

I, Jaspreet Singh Kaleka, hereby certify that the work which is being presented in this thesis entitled "WEIGHTED MULTILEVEL SPACE-TIME TRELLIS CODES FOR RAYLEIGH FADING CHANNELS" by me in partial fulfillment of the requirements for the award of degree of Master of Engineering in Electronics and Communication Engineering from Thapar University (Deemed University), Patiala, is an authentic record of my own work carried out under the supervision of Dr. Sanjay Sharma.

The matter presented in this thesis has not been submitted in any other University/Institute for the award of any other degree.

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It is certified that the above statement made by the student is correct to the best of my knowledge and belief.

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ABSTRACT

Wireless communications have been developed widely and rapidly in the modern world especially during the last decade. Recent advances in wireless communication systems have increased the throughput over wireless channels. The reliability of wireless communication has also been increased. But still the bandwidth and spectral availability demands are endless. The need to achieve reliable wireless systems with high spectral efficiency, low complexity and good error performance results in continued research in this field. The research in the field of space-time coding and multiple-input multiple-output systems has acquired a great interest in recent years.

In this thesis we present the concept of weighted multilevel space-time trellis codes. These codes are a combination of multilevel space-time codes and ideal beamforming. It has been shown that if perfect channel state information is available at the transmitter, the performance of a space-time coded system can be further improved by weighting the transmitted signals. In this thesis, we evaluate the performance of multilevel space-time trellis codes combined with ideal beamforming over slow fading channels. Simulation results show that the proposed scheme considerably outperforms the conventional multilevel space-time trellis without weighting.

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Chapter 1

Introduction

Marconi pioneered the wireless industry over 100 years ago. Today life does not seem possible without wireless in some form or the other. Wireless communication is one of the fastest growing industries [1, 2, 3]. It permeates every aspect of our lives. Recent advances in wireless communication systems have increased the throughput over wireless channels and also the reliability of wireless communication has been increased. The main driving force behind the rapid development of wireless communication is the promise of portability, mobility, and accessibility. Wired communication is more stable and highly reliable, but confines the users to a bounded environment. Logically, people choose freedom versus confinement. Therefore, there is a natural tendency towards getting rid of wires if possible. While, this freedom is the main driving force for users, the penalty for this freedom is often lower quality, privacy, security, or lower throughput compared to the equivalent wired solution. The demands on bandwidth and spectral availability are also endless. The need to achieve reliable wireless systems with high spectral efficiency, low complexity and good error performance results in continued research in this field.

Wireless designers face an uphill task of limited availability of radio frequency spectrum and complex time varying problems in the wireless channel, such as fading and multipath, as well as meeting the demand for high data rates. Simultaneously, there is an urgent need for better quality of service (QoS).

This chapter begins with a brief overview of the history of wireless communication, followed by some basics of wireless communication and highlights some of the recent works in the field. A brief background is also presented followed by main focus and the scope of this thesis. An overview of the thesis structure is also presented in this chapter. Key references are provided at the end for further reading and detailed exploration.

1.1 History of Wireless Communication

Since pre-modern man began yelling from hill-top to hill-top in order to transfer messages, there has been a desire to communicate in a convenient and efficient manner without wire. Other historical examples include Chinese fire signals along the Great Wall of China to warn the defenders of approaching invaders and smoke signals of Native Americans used in warfare. These may of course be considered as a form of wireless communications, but offer little modern interest. These methods predate the technological age of today but serve as the initial inspirations for the ideas of today.

The development of wireless communications began with the physicist Michael Faraday in the 19th century. He discovered the principle of electromagnetic induction in 1831, which demonstrated the concept of electric currents producing magnetism. Faraday's qualitative discovery paved the way for the mathematician James Maxwell to quantify the discoveries.

Maxwell's formulas for electricity and magnetism were published in the book "A Treatise on Electricity and Magnetism (1873)". These equations known as the Maxwell's equations implicitly showed that electromagnetic waves propagate through free space at the speed of light. This became a significant discovery because transferring signals via electromagnetic waves is hundreds of thousands of times faster than by sound and much more efficient than simple fire or smoke signals.

In 1873, Heinrich Hertz clarified and expanded the electromagnetic theory that had been developed by Maxwell and demonstrated the existence of radio waves. Through experimentation, he proved that transverse free space electromagnetic waves can travel over some distance. Hertz measured Maxwell's waves and demonstrated that the velocity of radio waves was equal to the velocity of light and that they possess many other properties of light. The electric field intensity and polarity was also measured by Hertz. In bulk, his work explained reflection, refraction, polarization, interference, and velocity of electric waves. The work of Maxwell and Hertz ignited the era of wireless communication.

With the discovery that electromagnetic waves are propagated at the speed of light, things got underway to find a method for transferring information across those waves. The first communication system based on these principles was built in 1894 by Oliver Lodge. The transmission distance of this system was only 150 meters.

It was not until the late 19th century when an Italian electrical engineer, Guglielmo Marconi [4], successfully transmitted the first wireless signal over a distance of one and a half miles. Marconi used electromagnetic waves at frequencies near those of radio frequencies to transmit and receive the signals. On December 12, 1901, Marconi successfully transmitted the first transatlantic wireless signal from Poldhu, Cornwall, to St. John's, Newfoundland, spanning a distance of 2100 miles. This event sparked a global interest in wireless communication and initiated an era of invention that would see the creation of radio, radar, and other innovations.

In 1906, Reginald Fessenden used a form of amplitude modulation, similar to what is used today, to translate signals to a higher frequency and thus circumvent the propagation limitations observed by Hertz at low frequencies. He managed to build the first system that could transmit voice and music [2].

In 1876, Alexander Graham Bell patented the telephone. The invention of the diode by Fleming in 1904 and the triode by Lee de Forest in 1906 made possible rapid development of long-distance (radio) telephony. John Bardeen, William Shockley, and Walter Brattain invented the transistor which later led to the development of the integrated circuits and paved the way for miniaturization of electronic systems. With the advent of such technologies the development of wireless phones began in the 1940s in America. However, it was not until the 1980s and 1990s that cell phones began to expand as a substantial force in the global market. As the number of cell phone users grew from around 50 thousand in the 1940s to over 1.4 million by the 1960s, companies began to have more interest in this market. With the advent of computers and digital signal processing in the 1960's and 1970's, studies in information theory proved incredibly fruitful. Utilizing Marconi's revolutionary discovery and the technological advancements of the past fifty years, cellular telephone networks flourished in the late Twentieth Century. Now, engineers seek to transmit not only voice, but also immense quantities of information over wireless networks. Along with mobile phones the development of wireless internet has also been prominent in the recent decade.

1.2 Wireless Applications

There are many systems in which wireless communication is applicable. Radio broadcasting is perhaps one of the earliest successful common applications. Television broadcasting and satellite communications are other important examples. However, the recent interest in wireless communication is perhaps inspired mostly by the establishment of the first generation cellular phones in the early 1980s. The first generation of mobile systems used analog transmission. The second generation of cellular communication systems, using digital transmission, was introduced in the 1990s. Both of these two systems were primarily designed to transmit speech.

Also, the industry has been actively involved in establishing new standards. Personal digital cellular (PDC), global system for mobile (GSM) communications, IS-54, IS-95, and IS-136 are some of the early examples of these standards. While they support data services up to 9.6 kbps, they are basically designed for speech. More advanced services for up to 100 kbps data transmission has been evolved from these standards and are called 2.5 generation. Recently, third generation mobile systems are being considered for high bit-rate services. With multimedia transmission in mind, the third generation systems are aiming towards the transmission of 144–384 kbps for fast moving users and up to 2.048 Mbps for slow moving users. It includes the enhanced data for global evolution (EDGE) standard, which is a time division multiple access (TDMA) system and an enhancement of GSM. It also includes two standards based on wideband code division multiple access (CDMA). One is a synchronous system called CDMA2000 and the other one is an asynchronous system named WCDMA. In addition to applications demanding higher bit rates, one can use multiple services in the third-generation standards simultaneously.

Another important application that drives the demand for high bit rates and spectral efficiency is wireless local area networks (WLANs). The most successful standard in this area is the IEEE 802.11 class of standards. IEEE 802.11a is based on orthogonal frequency division multiplexing (OFDM) to transmit up to 54 Mbps of data. IEEE 802.11b provides up to 11 Mbps over the 2.45 GHz unlicensed frequency band. IEEE 802.11g uses OFDM over the 2.45 GHz unlicensed frequency band to provide a data rate of up to 54 Mbps.

1.3 Wireless Channels

Wireless channels operate through electromagnetic radiation from the transmitter to the receiver. Electromagnetic waves propagate through environments where they are reflected, scattered, and diffracted by walls, terrain, buildings, and other objects. There are many different paths between the transmitter and the receiver which results in receiving different versions of the transmitted signal at the receiver. These separate versions experience different path loss and phases. At the receiver all received signals are accumulated together creating a non-additive white Gaussian noise (AWGN) model for the wireless channels. One could solve the electromagnetic field equations, in conjunction with the transmitted signal, to find the electromagnetic field impinging on the receiver antenna, taking into account the obstructions caused by ground, buildings, vehicles, etc. in the vicinity of this electromagnetic wave. But due to the amount and difficulty of these calculations, it is often difficult to obtain an accurate deterministic channel model. Thus, we resort to the statistical models to characterize the signal propagation.

A defining characteristic of the wireless channel is the variations of the channel strength over time and over frequency. The variations can be roughly divided into two types:

- Large-scale fading or *Attenuation*
- Small-scale fading or just *Fading*.

Attenuation, or large-scale fading, is caused by many factors including propagation losses, antenna losses, and filter losses. The average received signal decreases logarithmically with distance. The logarithm factor, or the path gain exponent, depends on the propagation medium and the environment between the transmitter and the receiver.

Fading, or equivalently small-scale fading, occurs due to the constructive and destructive interference of the multiple signal paths between the transmitter and receiver. The signals combine at the receiver antenna and provide an effective combined signal. This resulting signal can vary widely in amplitude and phase. The rapid fluctuation of the amplitude of the signal over a short period of time is such that the large-scale path loss effects may be ignored.

Out of *attenuation* and *fading*, the latter is more relevant to the design of reliable and efficient communication systems.

The *fading* channels can be classified based on their multipath time delay into flat and frequency selective and based on Doppler spread into slow and fast. These two phenomena are independent of each other and result in the following four types of fading channels:-

- Flat Slow Fading or Frequency Non-Selective Slow Fading: When the bandwidth of the signal is smaller than the coherence bandwidth (the range of frequencies in which the channel response appears to be “flat”, i.e., same gain and phase response) of the channel and the signal duration is smaller than the coherence time (the duration of time in which the channel appears to be static) of the channel.
- Flat Fast Fading or Frequency Non-Selective Fast Fading: When the bandwidth of the signal is smaller than the coherence bandwidth of the channel and the signal duration is larger than the coherence time of the channel.
- Frequency Selective Slow Fading: When the bandwidth of the signal is larger than the coherence bandwidth of the channel and the signal duration is smaller than the coherence time of the channel.
- Frequency Selective Fast Fading: When the bandwidth of the signal is larger than the coherence bandwidth of the channel and the signal duration is larger than the coherence time of the channel.

Here we review two of the most commonly used channel models, namely the additive white Gaussian noise (AWGN) and Rayleigh fading channel models. A more comprehensive study of different channel models can be found in the literature [5].

1.3.1 AWGN Channel Model

Additive white Gaussian noise (AWGN) is a channel model as shown in Fig. 1.1, in which the only impairment to communication is a linear addition of white noise with a constant spectral density (expressed as watts per hertz of bandwidth) and a Gaussian distribution of amplitude. The model may be described mathematically by considering signal transmission as

$$r(t) = x(t) + n(t) \tag{1.1}$$

where, at time t, $r(t)$ and $x(t)$ are the received and transmitted signals respectively and $n(t)$ is the noise, represented as a sample function from a Gaussian random process with zero mean and variance N_0. The noise $n(t)$ is assumed to be independent of the signal $r(t)$.

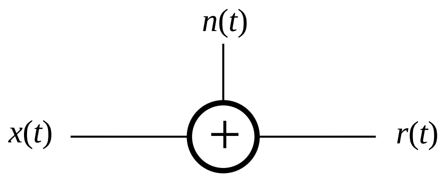

Figure 1.1 AWGN Channel Model.

1.3.2 Rayleigh Fading Channel Model

The Rayleigh fading [6] channel is a *fading* channel in which the received signal is corrupted by multipath fading as well as AWGN. It can be divided into two models based on the coherence bandwidth of the channel in comparison with the bandwidth of the transmitted signal.

In narrowband systems, the transmitted signals usually occupy a bandwidth smaller than the channel's coherence bandwidth. This type of fading is referred to as frequency *flat* or frequency *non-selective* as all frequency components of a signal are faded equally.

In wideband systems, the transmitted signals usually undergo frequency selective fading. This occurs when the transmitted signal bandwidth is greater than the channel coherence bandwidth, and the spectral components of the transmitted signal with a frequency separation larger than the coherence bandwidth are then faded independently.

When *flat* or *frequency non-selective* fading is present, the received signal in complex baseband form can be expressed as

$$r(t) = h(t)x(t) + n(t) \tag{1.2}$$

where, $x(t)$ and $r(t)$ are the transmitted and received complex baseband signals, respectively, and $h(t)$ is the complex baseband equivalent of the fading process, also known as channel state information (CSI). $h(t)$ introduces random phase rotations and random amplitude fluctuations to the transmitted signal. CSI may be estimated through pilot tones or pilot symbols.

1.4 Diversity

Fading results in the inefficient and unreliable transmission of data over many radio channels. The *fading* channel models suffer from sudden declines in the power due to the destructive addition of multipath signals in the propagation media. It can also be due to interference from other users. Therefore, the effective signal-to-noise ratio (SNR) at the receiver can be dropped dramatically. Some possible, but non-pragmatic, solutions to combat this degradation are to increase transmission power, antenna size, or antenna height. A practical alternative to these solutions would be is to provide different replicas of the transmitted signal to the receiver. If these different replicas fade independently, it is less probable to have all copies of the transmitted signal in deep fade simultaneously. Therefore, the receiver can reliably decode the transmitted signal using these received signals. This can be done, for example, by picking the signal with the highest SNR or by combining the multiple received signals. This scheme of transmission and reception is called *diversity* and is one of the most important techniques used to mitigate the effects of fading in wireless communications.

Several techniques of achieving diversity are available. Some of them are discussed below.

1.4.1 Temporal Diversity

In *temporal* or *time* diversity, replicas of the information signal are transmitted in different time slots. To achieve diversity, two adjacent time intervals must be separated for more than the coherence time of the channel so that the replicas of the signal experience independent fades. We get multiple, uncorrelated repetitions of the signal at the receiver. Time diversity can also be achieved through coding and interleaving.

Time diversity does not require increased transmit power, but it does decrease the data rate since data is repeated in the diversity time slots rather than sending new data in these time slots. Thus, it is not bandwidth efficient because of the underlying redundancy.

Time diversity can't be used for stationary applications, since the channel coherence time is infinite and thus fading is highly correlated over time.

1.4.2 Frequency Diversity

In *frequency* diversity, replicas of the information signal are transmitted from different carrier frequencies. To achieve diversity, the carrier frequencies must be separated by more than the coherence bandwidth of the channel so that the replicas of the signal experience independent fades.

Similar to *temporal* diversity, *frequency* diversity suffers from bandwidth deficiency. It also requires additional transmit power to send the signal over multiple frequency bands. Also the receiver needs to tune to different carrier frequencies.

1.4.3 Spatial Diversity

In *spatial* or *antenna* diversity, multiple transmit and/or receive antennas, also called an antenna array, are used, where the elements of the array are separated in distance. To achieve diversity, the distance between two adjacent antennas must be more than half of the signal wavelength so that the signals corresponding to different antennas fade independently.

Unlike *temporal* and *frequency* diversity, *spatial* diversity does not suffer from bandwidth deficiency. But the use of multiple antennas may not be possible in small handheld devices, due to the fact that a minimum physical separation is needed between different antennas to achieve spatial diversity.

Spatial diversity can be divided into two parts: *receive* diversity and *transmit* diversity. In case of receiver space diversity, independent fading paths are realized without an increase in transmit signal power or bandwidth. Coherent combining of the diversity signals leads to an increase in SNR at the receiver over the SNR that would be obtained with just a single receive antenna. In case of transmitter space diversity, the transmit power must be divided among multiple antennas. Thus, with coherent combining of the transmit signals the received SNR is the same as if there were just a single transmit antenna.

1.4.4 Angular Diversity

In *angular* or *directional* diversity, directional antennas are used at the receiver to achieve diversity. Directional antennas restrict the receive antenna beamwidth to a given angle. Different copies of the transmitted signal are collected from different angular directions at the receiver, thus the signals received via the multiple beams appear to be uncorrelated.

Unlike *spatial* diversity, it does not need separate physical locations. Therefore, it is also good for small devices. However, *angular* diversity requires a sufficient number of directional antennas to span all possible directions of arrival.

1.4.5 Polarization Diversity

In *polarization* diversity, vertically and horizontally polarized signals are used to achieve diversity. It uses either two transmit antennas or two receive antennas with different polarization. The two transmitted waves follow the same path. However, since the multiple random reflections distribute the power nearly equally relative to both polarizations, the average receive power corresponding to either polarized antenna is approximately the same. Since the scattering angle relative to each polarization is random, it is highly improbable that signals received on the two differently polarized antennas would be simultaneously in deep fades.

Unlike *spatial* diversity, *polarization* diversity does not require separate physical locations for the antennas. However, *polarization* diversity can only provide a diversity order of two and not more.

1.5 Capacity

Before the invention of Information theory by Claude Shannon in 1948, it was believed that the only way to achieve reliable communication over a noisy channel was to reduce the data rate. Shannon illustrated that by more intelligent coding of the information, one can communicate at a strictly positive rate but at the same time with as small an error probability as desired. However, there is a maximal rate, called the *capacity* of the channel, for which this can be done. If one attempts to communicate at rates above the channel capacity, then it is impossible to drive the error probability to zero.

The system capacity for a given channel is defined as the maximum achievable data rate for which an arbitrarily low probability of error can be achieved, provided the signal can be encoded over an arbitrarily long code word (to exploit the law of large numbers to average out the randomness of the noise). For digital transmission on a continuous AWGN channel, the capacity is given by the Shannon formula [7],

$$C = B \log_2 (1 + \rho) \tag{1.3}$$

where, C represents the Shannon capacity (measured in units of bits/sec), B represents the channel bandwidth and ρ is the signal-to-noise ratio (SNR). Shannon's coding theorem proves that codes exist, which achieve data rates arbitrarily close to capacity with arbitrarily small probability of error. The converse theorem shows that any code with rate $R > C$ has a probability of error bounded away from zero.

The proofs of the coding theorem and converse place no constraints on the complexity or delay of the communication system. Therefore, Shannon capacity is generally used as an upper bound on the data rates that can be achieved under real system constraints.

At the time that Shannon developed his theory of information, data rates over standard telephone lines were on the order of 100 bps. Thus, it was believed that Shannon capacity, which predicted speeds of roughly 30 Kbps over the same telephone lines, was not a very useful bound for real systems. However, breakthroughs in hardware, modulation, and coding techniques have brought commercial modems of today very close to the speeds predicted by Shannon in the 1950s. In fact, modems can exceed this 30 Kbps Shannon limit on some telephone channels, but that is because transmission lines today are of better quality than in Shannon's day and thus have a higher received power than that used in Shannon's initial calculation. On AWGN channels, Turbo codes [8] and low density parity check codes [9] have come within a fraction of a dB of the Shannon capacity limit.

The Shannon capacity of fading channels, with receiver CSI only can be shown to be less than that of an AWGN channel with the same average SNR. In other words, fading reduces the Shannon capacity when CSI is only available at the receiver. It has also been noted in the literature [2] that capacity-achieving codes for fading channels must be sufficiently long so that a received codeword is affected by all possible fading states. This can result in very long codewords and consequently long delays.

Recent research in information theory has shown that large gains in available channel capacity are possible for wireless channels by using multiple-input multiple-output (MIMO) systems [10]. MIMO channels result when multiple antennas are employed at both ends of the wireless link. In [10, 11], theoretical and experimental evidence demonstrates that the available channel capacity grows linearly when the number of transmit and receive antennas grow simultaneously. This provides added capacity with no increase in bandwidth.

1.6 MIMO wireless communication

Multiple-input and multiple-output, or MIMO, uses multiple antennas at both the transmitter and receiver to improve communication performance. MIMO technology has attracted attention in wireless communications, because it offers significant increases in data throughput and link range without additional bandwidth or increased transmit power. It achieves this goal by spreading the same total transmit power over the antennas to achieve an array gain that improves the spectral efficiency (more bits per second per hertz of bandwidth) or to achieve a diversity gain that improves the link reliability (reduced fading). MIMO schemes that assume the channel knowledge is only available at the receiver have in particular attracted a lot of research attention [11].

MIMO modulation schemes with receive-only channel knowledge are mainly of two types, diversity systems and spatial multiplexing systems. Diversity modulation, or space-time coding [12, 13, 14], uses codewords designed to maximize the diversity advantage of the transmitted information. Such codes tend to maximize diversity gain at the expense of some loss in available capacity. Spatial multiplexing or Bell Labs Layered Space Time (BLAST) type systems [11], on the other hand, transmit independent data streams from each transmitting antenna, allowing spectral efficiency to be achieved at the expense of a loss in diversity advantage for a fixed number of receive antennas.

The space-time coding work began with the 1994 paper by Wittenben [15], which proposes a system using transmit diversity and coding techniques, followed by the groundbreaking paper by Tarokh, Seshadri and Calderbank in 1998 [12] in which they stated the fundamental theory of space-time coding and introduce the first true space-time codes, namely space-time trellis codes (STTCs). This paper was followed by Alamouti's paper [13], which led to the development of what are now known as space-time block codes (STBCs) [16, 17].

The original BLAST structure was developed by Foschini [11] which uses a multi-element antenna array at both the transmitter and receiver, where every antenna transmits an independent sub stream of data. Advanced signal processing at the receiver is used to estimate and decode the received signal blocks. A BLAST system requires more receive than transmit antennas and a rich scattering environment, which often occurs indoors. Vertical-BLAST (V-BLAST) and Diagonal-BLAST (D-BLAST) [18] are the two major classes of BLAST transmission formats.

Initially MIMO was proposed for indoor WLANs and fixed wireless access networks. However, it has since found wider applications and some practical MIMO systems have been built and experimentally tested in industry. MIMO is an important part of modern wireless communication standards such as IEEE 802.11n (Wi-Fi), 4G, 3GPPLTE, WiMAX and HSPA+.

MIMO technology can be used in non-wireless communications systems. One example is the home networking standard ITU-T G.9963, which defines a power line communications system that uses MIMO techniques to transmit multiple signals over multiple AC wires (phase, neutral and ground).

1.7 Space-Time Coding

Space-time coding (STC) [12, 14] exploits the diversity provided by the MIMO channel in both space (antenna) and time domains, thus significantly increasing the system capacity as well as improving the reliability of the wireless link. The spatial-temporal correlation is then used to exploit the scattering environment and minimize transmission errors at the receiver. STC can achieve transmit diversity and coding gain compared to spatially uncoded systems without sacrificing bandwidth [14].

STBCs and STTCs can be considered to be the two main classes of space-time codes. In this thesis we focus on STTCs [12]. STTC is a class of signaling techniques that combine the design of the channel code with transmit and optionally receive antenna diversity. In addition to the diversity advantage, a certain amount of coding gain can be achieved by a well-designed STTC. Code design criteria for STTC are based upon upper bounds on the pair-wise error probability. The design criteria are the rank criterion, corresponding to maximization of diversity advantage and the determinant criterion, corresponding to maximization of coding gain. STTCs use a trellis encoder to introduce redundancy into the transmitted symbol stream, and to achieve coding gain. The coding gain is dependent on the construction criteria of the code, and on the length of the memory in the encoder. A number of different structures have been proposed for STTCs [19].

In this thesis, we focus on the STTCs originally proposed by Tarokh *et al.* [12] and later improved by others, most notably Baro *et al.* [20] and Vucetic *et al.* [21, 14]. These codes are discussed in more detail in Chapter 2.

1.8 Thesis Focus

Generally if CSI is not available at the transmitter the power is spread equally among all the transmit antennas. But if we have access to the channel information at the transmitter, we can use it to improve the error performance by setting the signal power on different transmit antennas differently, based on the wireless channel. This would make detection easier at the receiver. The space-time coded structures, in general, can benefit from having CSI at the transmitter.

If perfect CSI is available at the transmitter, the performance of a space-time coded system can be further improved by weighting the transmitted signals. In this thesis our main focus, is to evaluate the performance of multilevel space-time trellis codes (MLSTTCs) combined with ideal beamforming over slow fading channels. MLSTTCs use multilevel coding (MLC) [22, 23, 24, 25] techniques that have recently been used in MIMO systems. MLSTTCs and its building blocks are described in further chapters.

In this thesis, we will make use of MLSTTCs and combine them with an appropriate weighting scheme such that we are able to improve the error performance without sacrificing the benefits of MLSTTCs.

1.9 Structure of the Thesis

Brief information on STTCs and their corresponding design criteria is given in Chapter 2 along with an example space-time trellis coded system and some simulated results. Chapter 3 describes the basics of MLC techniques. The multi-level STTC scheme is illustrated in Chapter 4. The proposed weighted multi-level STTC scheme is discussed in Chapter 5. Simulation results for the proposed system are presented in Chapter 6. The thesis focus is concluded in the last chapter.

Chapter 2

Space-Time Trellis Codes

Space-Time Trellis Codes can simultaneously offer a substantial coding gain, spectral efficiency, and diversity improvement on flat fading channels. In this chapter, we consider, space-time trellis codes (STTC), which were first introduced by Tarokh, Seshadri and Calderbank [12] and those later proposed by Vucetic [21]. A brief outline of the derivations is included to provide an insight into these design criteria along with the encoder structure of STTC. A number of performance bounds involved in designing STTCs are also discussed in this chapter.

2.1 System Model

We consider a multiple-input multiple-output (MIMO) system with n_T transmit and n_R receive antennas as shown in Fig. 2.1. The transmitted data are encoded by a STTC encoder. At each time instant t, a block of m binary information symbols, denoted by

$$\mathbf{b}_t = (b_t^1, b_t^2, b_t^3, \dots, b_t^m) \tag{2.1}$$

is fed into the STTC encoder. The STTC encoder maps the block of m binary input data into n_T modulation symbols from a signal set of $M = 2^m$ points, represented by an $n_T \times 1$ column matrix $\mathbf{x}_t$,

$$\mathbf{x}_t = \left(x_t^1, x_t^2, x_t^3, \dots, x_t^{n_T}\right)^{\mathrm{T}} \tag{2.2}$$

where T means the transpose of a matrix. The n_T parallel outputs are simultaneously transmitted by n_T different antennas, whereby symbol x_t^i, $1 \le i \le n_T$, is transmitted by antenna i and all transmitted symbols have the same duration of T sec. The vector of coded modulation symbols from different antennas, as shown in (2.2), is called a *space-time symbol*.

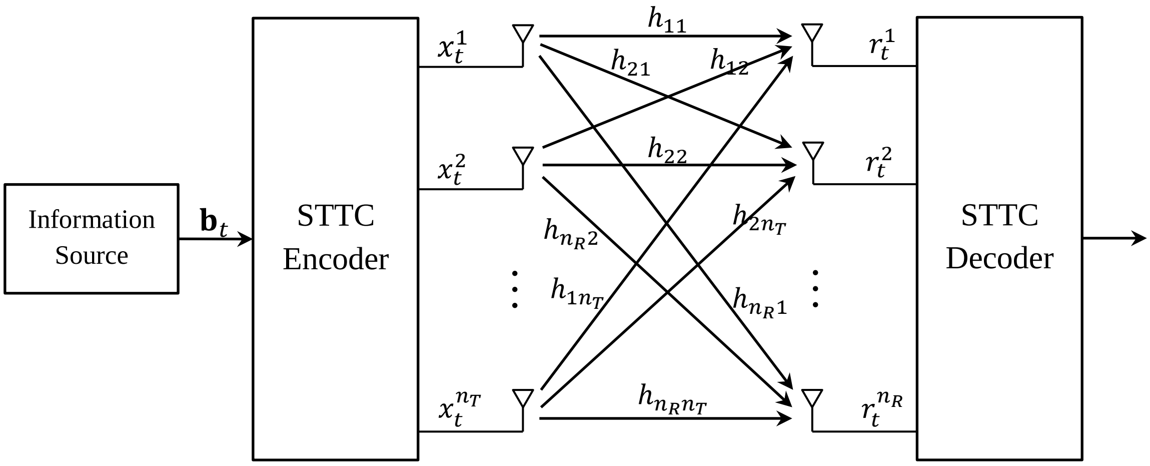

Figure 2.1 Block diagram of a MIMO system.

For wireless communications, we assume a quasi-static Rayleigh fading channel model, for which the fading coefficients are constant within one frame but vary independently from one frame to another. The channel matrix **H**, at time t is given by,

$$\mathbf{H}_t = \begin{bmatrix} h_{1,1}^t & h_{1,2}^t & \cdots & h_{1,n_T}^t \\ h_{2,1}^t & h_{2,2}^t & \cdots & h_{2,n_T}^t \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_R,1}^t & h_{n_R,2}^t & \cdots & h_{n_R,n_T}^t \end{bmatrix} \tag{2.3}$$

where the j,i-th element, denoted by $h_{j,i}^t$, is the fading attenuation coefficient for the path from transmit antenna i to receive antenna j.

At the receiver, the signal at each of the n_R receive antennas is a noisy superposition of the n_T transmitted signals degraded by channel fading. At time t, the received signal at antenna j, j = 1, 2, ... , n_R, denoted by r_t^j , is given by,

$$r_t^j = \sum_{i=1}^{n_T} h_{j,i}^t x_t^i + n_t^j \tag{2.4}$$

where n_t^j, is the noise component of receive antenna j at time t, which is an independent sample of complex Gaussian random variable with zero-mean and variance of $N_0/2$ per dimension.

Let us represent the received signals from n_R receive antennas at time t by an $n_R \times 1$ column matrix.

$$\mathbf{r}_t = \left(r_t^1, r_t^2, r_t^3, \dots, r_t^{n_R}\right)^{\mathrm{T}} \tag{2.5}$$

The noise at the receiver can be described by an $n_R \times 1$ column matrix, denoted by $\mathbf{n}_t$,

$$\mathbf{n}_t = \left(n_t^1, n_t^2, n_t^3, \dots, n_t^{n_R}\right)^{\mathrm{T}} \tag{2.6}$$

where each component refers to a sample of the noise at a receive antenna. Thus, the received signal vector can be represented as,

$$\mathbf{r}_t = \mathbf{H}_t \mathbf{x}_t + \mathbf{n}_t \tag{2.7}$$

We assume that the decoder at the receiver uses Viterbi algorithm to perform maximum likelihood decoding, to estimate the transmitted information sequence and that the receiver has ideal channel state information (CSI) on the MIMO channel. On the other hand, the transmitter has no information about the channel. At the receiver, the decision metric is computed based on the squared Euclidean distance between the hypothesized received sequence and the actual received sequence as,

$$\sum_t \sum_{j=1}^{n_R} \left| r_t^j - \sum_{i=1}^{n_T} h_{j,i}^t x_t^i \right|^2 \tag{2.8}$$

The Viterbi algorithm selects the path with the minimum path metric as the decoded sequence [26].

The spectral efficiency of the system is,

$$\eta = \frac{r_b}{B} = m \quad \text{bits/sec/Hz} \tag{2.9}$$

where r_b is the data rate and B is the channel bandwidth.

2.2 STTC Encoder

The encoder maps binary data to modulation symbols, where the mapping function is described by a trellis diagram.

Let us consider an encoder of space-time trellis coded M-PSK modulation with n_T transmit antennas as shown in Fig. 2.2. The input message stream, denoted by **B**, is given by

$$\mathbf{B} = (\mathbf{b_0}, \mathbf{b_1}, \mathbf{b_2}, \dots, \mathbf{b_t}, \dots) \tag{2.10}$$

where $\mathbf{b}_t$ is a group of $m = log_2 M$ information bits at time t and given by

$$\mathbf{b}_t = (b_t^1, b_t^2, b_t^3, \dots, b_t^m) \tag{2.11}$$

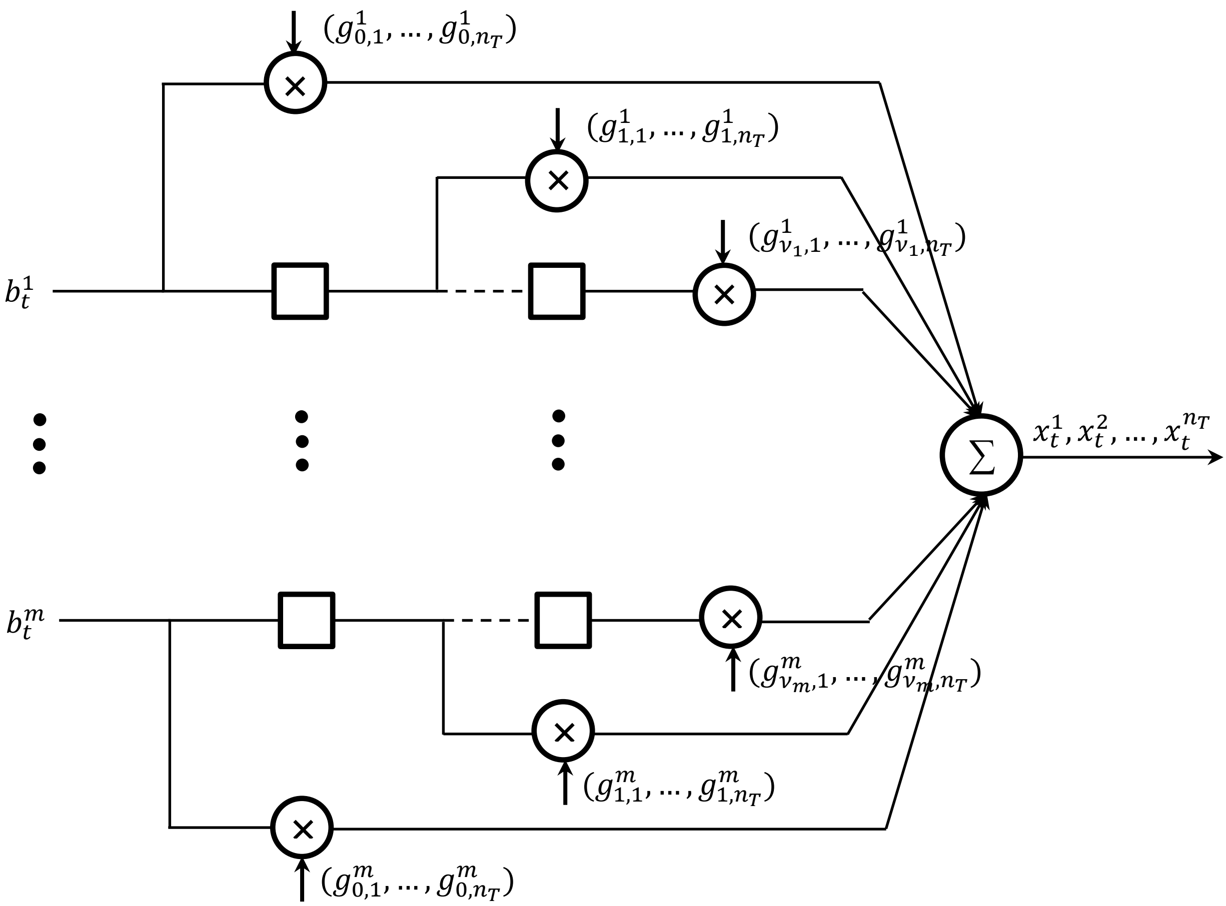

Figure 2.2 STTC Encoder.

The encoder maps the input sequence into an M-PSK modulated signal sequence, given by

$$\mathbf{X} = (\mathbf{x_0}, \mathbf{x_1}, \mathbf{x_2}, \dots, \mathbf{x}_t, \dots) \tag{2.12}$$

where $\mathbf{x}_t$ is a space-time symbol at time t and given by

$$\mathbf{x}_t = \left(x_t^1, x_t^2, x_t^3, \dots, x_t^{n_T}\right)^{\mathrm{T}} \tag{2.13}$$

The modulated signals, $x_t^1, x_t^2, x_t^3, \dots, x_t^{n_T}$, are transmitted simultaneously through n_T transmit antennas.

2.2.1 Generator Description

In the STTC encoder as shown in Fig. 2.1, m binary input sequences , $\mathbf{b}^1, \mathbf{b}^2, \dots, \mathbf{b}^m$ are fed into the encoder, which consists of m feed-forward shift registers. The k^{th} input sequence $\mathbf{b}^{k} = \left(b_0^k, b_1^k, b_2^k, \dots, b_t^k, \dots\right)$, k = 1, 2, … , m, is passed to the k^{th} shift register and multiplied by an encoder coefficient set. The multiplier outputs from all shift registers are added modulo M, giving the encoder output, $x_t^1, x_t^2, x_t^3, \dots, x_t^{n_T}$. The connections between the shift register elements and the modulo M adder can be described by the following m multiplication coefficient set sequences

$$\mathbf{g^1} = \left[\left(g_{0,1}^1, g_{0,2}^1, \dots, g_{0,n_T}^1\right), \left(g_{1,1}^1, g_{1,2}^1, \dots, g_{1,n_T}^1\right), \dots, \left(g_{\nu_1,1}^1, g_{\nu_1,2}^1, \dots, g_{\nu_1,n_T}^1\right)\right]$$
$$\mathbf{g^2} = \left[\left(g_{0,1}^2, g_{0,2}^2, \dots, g_{0,n_T}^2\right), \left(g_{1,1}^2, g_{1,2}^2, \dots, g_{1,n_T}^2\right), \dots, \left(g_{\nu_2,1}^2, g_{\nu_2,2}^2, \dots, g_{\nu_2,n_T}^2\right)\right]$$
$$\vdots$$
$$\mathbf{g^m} = \left[\left(g_{0,1}^m, g_{0,2}^m, \dots, g_{0,n_T}^m\right), \left(g_{1,1}^m, g_{1,2}^m, \dots, g_{1,n_T}^m\right), \dots, \left(g_{\nu_m,1}^m, g_{\nu_m,2}^m, \dots, g_{\nu_m,n_T}^m\right)\right]$$

where $g_{j,i}^k$, k = 1, 2, … , m, j = 0, 2, . . . , ν_k, i = 1, 2, . . . , n_T , is an element of M-PSK constellation set, and ν_k is the memory order of the k^{th} shift register.

The encoder output at time t for transmit antenna i, denoted by x_t^i , can be computed as

$$x_t^i = \sum_{k=1}^{m} \sum_{j=0}^{\nu_k} g_{j,i}^k b_{t-j}^k \qquad mod\ M \tag{2.14}$$

These outputs are elements of an M-PSK signal set. Modulated signals form the space-time symbol transmitted at time t,

$$\mathbf{x}_t = \left(x_t^1, x_t^2, x_t^3, \ldots, x_t^{n_T}\right)^{\mathrm{T}}$$

The space-time trellis coded M-PSK can achieve a bandwidth efficiency of m bits/s/Hz. The total memory order of the encoder, denoted by v, is given by

$$v = \sum_{k=1}^{m} v_k \qquad mod\ M \tag{2.15}$$

where v_k, k = 1, 2, … , m, is the memory order for the k^{th} encoder branch. The value of v_k for M-PSK constellations is determined by

$$v_k = \left\lfloor \frac{v + k - 1}{log_2 M} \right\rfloor \tag{2.16}$$

where $\lfloor Z \rfloor$ represents the maximum integer not larger than Z. The total number of states for the trellis encoder is 2^v. The m multiplication coefficient set sequences are also called the *generator sequences*, since they can fully describe the encoder structure. The *generator sequences* can be designed using the Rank & Determinant criteria [12] or the Trace Criterion [21] discussed later.

For example, in case of QPSK on slow Rayleigh fading channels, the generator sequence for 4-state STTC codes with two transmit antennas based on the rank & determinant criteria are given by,

$$\mathbf{g}^1 = [(0, 2), (2, 0)]$$

$$\mathbf{g}^2 = [(0, 1), (1, 0)]$$

and based on the trace criterion criteria are given by,

$$\mathbf{g}^1 = [(0, 2), (1, 2)]$$

$$\mathbf{g}^2 = [(2, 3), (2, 0)]$$

2.2.2 Example

Let us assume that the generator sequences of a 4-state space-time trellis coded QPSK scheme with 2 transmit antennas based on the rank & determinant criteria are,

$$\mathbf{g}^1 = [(0, 2), (2, 0)]$$

$$\mathbf{g}^2 = [(0, 1), (1, 0)]$$

The trellis structure for the code is shown in Fig. 2.3. The trellis consists of $2^\nu = 4$ states, represented by state nodes. The encoder takes $m = 2$ bits as its input at each time. There are $2^m = 4$ branches leaving from each state corresponding to four different input patterns. Each branch is labeled by $b_t^1, b_t^2 \mid x_t^1, x_t^2$, where b_t^1, b_t^2 is a pair of encoder input bits, and x_t^1, x_t^2 represents two coded QPSK symbols transmitted through antennas 1 and 2, respectively. The row listed next to a state node indicates the branch labels for transitions from that state corresponding to the encoder inputs 00, 01, 10, and 11, respectively.

Figure 2.3 Trellis structure for a 4-state STTC designed using Rank & Determinant criteria for two transmit antennas.

Assume that the input sequence is, **B** = (10, 01, 11, 00, 01, …).

The output sequence generated by the STTC encoder is, **X** = (02, 21, 13, 30, 01, …).

The transmitted signal sequences from the two transmit antennas are

$$\mathbf{X}^1 = (0, 2, 1, 3, 0, \ldots)$$

$$\mathbf{X}^2 = (2, 1, 3, 0, 1, \ldots)$$

2.3 STTC Decoder

STTC Decoder works on the same principle as explained in section 2.2. It uses Viterbi algorithm to perform maximum likelihood decoding. It is assumed that perfect CSI is available at the receiver and none at the transmitter. For a branch labeled by the symbol x_t, the branch metric is computed as the squared Euclidean distance between the hypothesized received symbols and the actual received signals as,

$$\sum_{j=1}^{n_R} \left| r_t^j - \sum_{i=1}^{n_T} h_{j,i}^t x_t^i \right|^2 \tag{2.17}$$

The Viterbi algorithm selects the path with the minimum path metric as the decoded sequence [26].

2.4 STTC Performance Analysis & Design Criteria

There are various design criteria proposed for STCs but the most widely used techniques are the rank and determinant criteria proposed by Tarokh [12] and the trace criterion proposed by Vucetic [21]. In this section we will briefly discuss about these methods considering the system model described in Section 2.1.

2.4.1 Pairwise Probability of Error

We assume that each transmitted data frame consists of L symbols for each antenna. We define an $n_T \times L$ space-time codeword matrix, obtained by arranging the transmitted sequence in an array, as

$$\mathbf{X} = [\mathbf{x_1}, \mathbf{x_2}, \dots, \mathbf{x_t}, \dots] = \begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_L^1 \\ x_1^2 & x_2^2 & \cdots & x_L^2 \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{n_T} & x_2^{n_T} & \cdots & x_L^{n_T} \end{bmatrix} \tag{2.18}$$

where each row represents the data sequence transmitted from each of the transmit antennas, and each column represents the space-time symbol at time t, respectively.

The pairwise error probability $P(\mathbf{X}, \widehat{\mathbf{X}})$ is defined as the probability that a decoder erroneously selects a sequence $\widehat{\mathbf{X}} = [\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2, \dots, \hat{\mathbf{x}}_t, \dots]$ when the transmitted sequence was in fact $\mathbf{X} = [\mathbf{x_1}, \mathbf{x_2}, \dots, \mathbf{x_t}, \dots]$.

This occurs in maximum likelihood decoding, if

$$\sum_{t=1}^{L}\sum_{j=1}^{n_R}\left|r_t^j - \sum_{i=1}^{n_T} h_{j,i}^t x_t^i\right|^2 \geq \sum_{t=1}^{L}\sum_{j=1}^{n_R}\left|r_t^j - \sum_{i=1}^{n_T} h_{j,i}^t \hat{x}_t^i\right|^2 \tag{2.19}$$

This inequality is equivalent to

$$\sum_{t=1}^{L}\sum_{j=1}^{n_R} 2Re\left\{\left(n_t^j\right)^* - \sum_{i=1}^{n_T} h_{j,i}^t\left(\hat{x}_t^i - x_t^i\right)\right\} \geq \sum_{t=1}^{L}\sum_{j=1}^{n_R}\left|\sum_{i=1}^{n_T} h_{j,i}^t\left(\hat{x}_t^i - x_t^i\right)\right|^2 \tag{2.20}$$

where $Re\{.\}$ means the real part of a complex number.

Assuming that ideal CSI is available at the receiver, for a given realization of the fading channel matrix sequence $\mathbf{H} = [\mathbf{H_1}, \mathbf{H_2}, \dots, \mathbf{H}_t, \dots]$, the term on the right hand side of (2.20) becomes a constant that represents a modified Euclidean distance between the two space-time code word matrices $\mathbf{X}$ and $\widehat{\mathbf{X}}$, denoted by $d^2\left(\mathbf{X}, \widehat{\mathbf{X}}\right)$ and the conditional pairwise error probability (PEP) is represented by

$$P\left(\mathbf{X}, \widehat{\mathbf{X}} | \mathbf{H}\right) = Q\left(\sqrt{\frac{\boldsymbol{E_s}}{\boldsymbol{2N_0}} d^2\left(\mathbf{X}, \widehat{\mathbf{X}}\right)}\right) \tag{2.21}$$

where E_s is the energy per symbol at each transmit antenna and $Q(x)$ is the complementary error function defined by

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2}\, dt \tag{2.22}$$

By using the inequality

$$Q(x) \leq \frac{1}{2} e^{-x^2/2}, \quad x \geq 0 \tag{2.23}$$

the conditional PEP shown in (2.21) can be upper bounded by

$$P\left(\mathbf{X}, \widehat{\mathbf{X}} | \mathbf{H}\right) \leq \frac{1}{2} exp\left(- d^2\left(\mathbf{X}, \widehat{\mathbf{X}}\right) \frac{E_s}{4N_0}\right) \tag{2.24}$$

2.4.2 Rank and Determinant Criterion

The rank & determinant criterion was the first criterion used for designing STCs, and was introduced by Tarokh [12] in 1998. The criterion is based on the rank and determinant of a matrix called as the *codeword difference matrix* defined by

$$\mathbf{B}(\mathbf{X},\widehat{\mathbf{X}}) = [\mathbf{X}-\widehat{\mathbf{X}}] = \begin{bmatrix} x_1^1 - \hat{x}_1^1 & x_2^1 - \hat{x}_2^1 & \cdots & x_L^1 - \hat{x}_L^1 \\ x_1^2 - \hat{x}_1^2 & x_2^2 - \hat{x}_2^2 & \cdots & x_L^2 - \hat{x}_L^2 \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{n_T} - \hat{x}_1^{n_T} & x_2^{n_T} - \hat{x}_2^{n_T} & \cdots & x_L^{n_T} - \hat{x}_L^{n_T} \end{bmatrix} \tag{2.25}$$

and an $n_T \times n_T$ *codeword distance matrix* $\mathbf{A}(\mathbf{X},\widehat{\mathbf{X}})$, defined as

$$\mathbf{A}(\mathbf{X},\widehat{\mathbf{X}}) = \mathbf{B}(\mathbf{X},\widehat{\mathbf{X}}) \cdot \mathbf{B}^{\boldsymbol{H}}(\mathbf{X},\widehat{\mathbf{X}}) \tag{2.26}$$

where $(.)^H$ denotes the Hermitian transpose of a matrix.

Assuming that the number of independent subchannels rn_R is small, then for high SNR, the upper bound on the PEP (2.20) for slow Rayleigh fading can be simplified to [12]

$$P(\mathbf{X},\widehat{\mathbf{X}}|\mathbf{H}) \leq \left(\prod_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{r}} \lambda_{\boldsymbol{i}}\right)^{-n_R} \left(\frac{E_s}{4N_0}\right)^{-rn_R} \tag{2.27}$$

where r denotes the rank of matrix $\mathbf{A}(\mathbf{X},\widehat{\mathbf{X}})$, and $\lambda_1, \lambda_2, \ldots, \lambda_r$ are the nonzero eigenvalues of matrix $\mathbf{A}(\mathbf{X},\widehat{\mathbf{X}})$.

Using a union bound technique [12], we can compute an upper bound of the code frame error probability, which sums the contributions of the pairwise error probabilities over all error events. The PEP in (2.27) decreases exponentially with the increasing SNR therefore, the frame error probability at high SNRs is dominated by the PEP with the minimum product rn_R (also called as the *diversity gain*). Thus, to achieve good performing codes, it is desirable to maximize the minimum rank r. As the matrix $\mathbf{A}(\mathbf{X},\widehat{\mathbf{X}})$ is an $n_T \times n_T$ matrix, thus, the maximum value of its rank r can be n_T, thereby, restricting the maximum possible value of rn_R to n_Tn_R, which is often not achievable due to restrictions on code structure.

Also from the PEP in (2.27) we can see that in order to minimize the error probability, the minimum product of nonzero eigenvalues, $\prod_{i=1}^{r}\lambda_i$, of the matrix $\mathbf{A}(\mathbf{X},\hat{\mathbf{X}})$ along the pairs of codewords with the minimum rank should be maximized. Therefore, for small values of rn_R (< 4), the space-time code design criteria for slow Rayleigh fading channels can be summarized as:

- Maximize the minimum rank r of matrix $\mathbf{A}(\mathbf{X},\hat{\mathbf{X}})$ over all pairs of distinct codewords.
- Maximize the minimum product, $\prod_{i=1}^{r}\lambda_i$, of the matrix $\mathbf{A}(\mathbf{X},\hat{\mathbf{X}})$ among the pairs of distinct codewords with the minimum rank.

This criterion is referred to as *rank and determinant* criteria [12].

2.4.3 Trace Criterion

The Trace Criterion was introduced by Vucetic [21] for cases where we have a *large* number of independent subchannels rn_R (≥ 4). Here, the PEP for high SNR's can be approximated by [21]

$$P(\mathbf{X},\hat{\mathbf{X}}|\mathbf{H}) \leq \frac{1}{4} exp\left(-n_R \frac{E_s}{N_0}\sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{r}} \lambda_{\boldsymbol{i}}\right) \tag{2.28}$$

Thus, to minimize the error probability, one should maximize the minimum sum of all eigenvalues, $\sum_{i=1}^{r}\lambda_i$, of matrix $\mathbf{A}(\mathbf{X},\hat{\mathbf{X}})$, among all pairs of distinct codewords. Since $\mathbf{A}(\mathbf{X},\hat{\mathbf{X}})$ is a square matrix, we can refer it as maximizing the minimum *trace* of the matrix among all pairs of distinct codewords. It can be expressed as

$$tr\left(\mathbf{A}(\mathbf{X},\hat{\mathbf{X}})\right) = \sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{r}} \lambda_{\boldsymbol{i}} = \sum_{\boldsymbol{i}=\mathbf{1}}^{n_T} \mathrm{A}^{\boldsymbol{i},\boldsymbol{i}} \tag{2.29}$$

where $\mathrm{A}^{\boldsymbol{i},\boldsymbol{i}}$ are the elements on the main diagonal of matrix $\mathbf{A}(\mathbf{X},\hat{\mathbf{X}})$, given by

$$\mathrm{A}^{\boldsymbol{i},\boldsymbol{i}} = \sum_{\boldsymbol{t}=\mathbf{1}}^{\boldsymbol{L}}\left(x_t^i - \hat{x}_t^i\right)\left(x_t^j - \hat{x}_t^j\right)^* \tag{2.30}$$

Substituting (2.30) into (2.29), we get

$$tr\left(\mathbf{A}(\mathbf{X},\hat{\mathbf{X}})\right) = \sum_{i=1}^{n_T}\sum_{t=1}^{L}\left|x_t^i - \hat{x}_t^i\right|^2 \tag{2.31}$$

Equation (2.31) shows that the trace of matrix $\mathbf{A}(\mathbf{X},\hat{\mathbf{X}})$ is equivalent to the squared minimum Euclidean distance between the codewords $\mathbf{X}$ and $\hat{\mathbf{X}}$. Thus, maximizing the minimum sum of all eigenvalues of the $\mathbf{A}(\mathbf{X},\hat{\mathbf{X}})$ matrix among the pairs of distinct codewords, or maximizing the minimum trace of the $\mathbf{A}(\mathbf{X},\hat{\mathbf{X}})$ matrix, is equivalent to maximizing the minimum Euclidean distance between all pairs of distinct codewords. This design criterion is referred to as the *trace* criterion.

2.5 Performance Evaluation on Slow Fading Channels

The code frame error rate (FER) performance is evaluated by simulations. In the simulations, each frame consisted of 130 symbols transmitted from each antenna. A maximum likelihood Viterbi decoder with perfect CSI is employed at the receiver. The performance curves are plotted against the signal-to-noise ratio (SNR) per receive antenna. See Appendix A, B, C, D, E & G for STTC source code developed in Matlab.

2.5.1 Performance Based on the Rank & Determinant Criteria

The performance of the 4-state space-time trellis coded QPSK scheme with 2 transmit antennas and various number of receive antennas, based on the rank & determinant criteria on slow Rayleigh fading channels is shown in Fig. 2.4. It can be seen that with increase in the number of receive antennaas the code performance improves.

2.5.2 Performance Based on the Trace Criterion

The performance of the 4-state space-time trellis coded QPSK scheme with 2 transmit antennas and various number of receive antennas, based on the trace criterion on slow Rayleigh fading channels is shown in Fig. 2.5. It can be seen that with increase in the number of receive antennas the code performance improves.

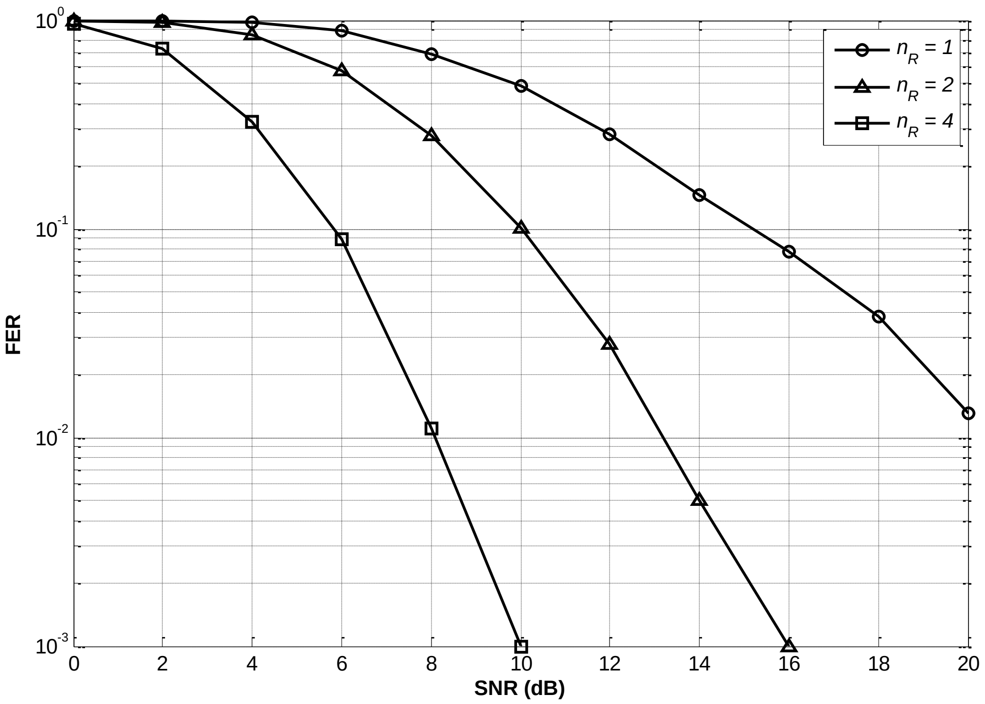

Figure 2.4 FER performance of 4-state QPSK STTCs designed using the Rank & Determinant criteria for two transmit and different number of receive antennas.

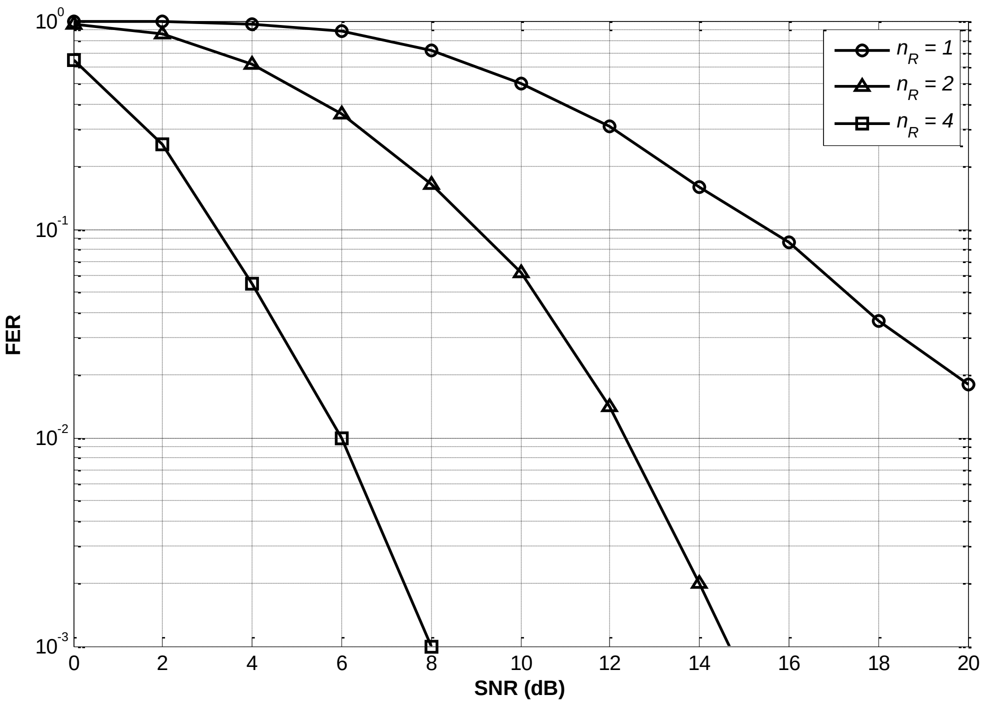

Figure 2.5 FER performance of 4-state QPSK STTCs designed using the Trace criteria for two transmit and different number of receive antennas.

2.6 Summary

In this chapter different design criteria for STCs were reviewed. As discussed, the design criteria for quasi-static Rayleigh fading channels depends on the rank of the codeword distance matrix r, and the number of receive antennas in the system, n_R. It was also briefly discussed that if $rn_R < 4$, then the *rank & determinant* criterion is suitable, else it's more appropriate to use the *trace* criterion. STTCs and their corresponding encoding and decoding algorithms were also discussed along with an example. Simulation results were provided for various STTC schemes.

In the next chapter we shall discuss multi-level codes (MLCs). These will be used in chapter 4 to design a multilevel space-time system.

Chapter 3

Multilevel Coded Modulation

Multilevel coding [22, 23, 24] is a *coded modulation* technique using which we can construct higher complexity code using simple component codes. It employs hierarchical partitioning of the signal constellation into various levels and defines a code over each level. These codes are generally decoded in a sequential manner using a multistage decoder (MSD). Multilevel codes were originally designed for the AWGN channel. Multilevel codes developed for multiple antenna systems have primarily used block component codes [25, 27].

In this chapter, we provide a brief introduction to multilevel codes and their encoding and decoding. In addition, some of the most commonly used multilevel code structures used for MIMO systems are outlined.

3.1 Introduction

The error-correction codes like block codes and convolutional codes when used in real time communication systems provide improvements in error performance at the cost of bandwidth expansion. For both block codes and convolutional codes, transforming each input data k-tuple into a larger output codeword n-tuple, requires additional transmission bandwidth. Therefore, in the past, coding generally was not popular for bandlimited channels such as telephone channels, where signal bandwidth expansion is not practical.

Coded modulation refers to a class of techniques in which coding and modulation is combined and jointly optimized in order to improve the performance of a given digital transmission scheme, usually without incurring bandwidth expansion. It is a bandwidth efficient signaling technique.

In the late 1970s, Ungerboeck [28] and Imai and Hirakawa [22] independently presented two of the most powerful applicable coded modulation techniques to date, namely trellis coded modulation (TCM) and multilevel coded modulation (MLC), respectively.

Ungerboeck's TCM is based on mapping by binary set partitioning, whereby the signal set, with an underlying signal constellation of $M = 2^m$ points, is successively binary partitioned in m or fewer steps to define a mapping of binary addresses to signal points. It maximizes the minimum intra-subset Euclidean distance. In the encoder, the binary addresses are usually divided into least significant binary symbols, which are convolutionally encoded, and most significant binary symbols, which if present are left uncoded. An exhaustive computer search is usually used to find the corresponding code parameters, in order to maximize the minimum distance between coded sequences in Euclidean space. A simple analogy might be helpful in understanding the overall goals in TCM. Imagine that there is an all-knowing wizard at the transmitter. As the message bits enter the system, the wizard recognizes that some of the bits are most vulnerable to the degradation effects of channel impairments; hence, they are assigned modulation waveforms associated with the best distance properties. Similarly, other bits are judged to be very robust, and hence, they are assigned waveforms with poorer distance properties. Modulation and coding take place together. The wizard is assigning waveforms to bits (modulation), but, the assignment is being performed according to the criterion of better or worse distance properties (channel coding).

MLC [22, 23, 29, 24] splits the transmission channel into several logical sub-channels. The number of sub-channels depends on the size of the signal constellation of the underlying modulation scheme. As the sub-channels are separated, we can employ a MSD which will decode the component codes sequentially starting with the most powerful component code first and use its output decisions (assumed to be correct) in the decoding of the subsequent and weaker code sequences. A MSD can potentially achieve the performance of a very large and complex code, but requires considerably lower decoding complexity [24]. The idea behind MLC, as originally described by Imai and Hirakawa [22], was to protect each bit in the label of a signal point with an independent binary code. This sort of protection implicitly assumes that some form of partitioning is being employed. Originally these codes were proposed for one-dimensional signaling combined with labeling by binary counting of the signal levels. The partitioning strategy was to maximize the minimum intra-subset Euclidean distance, in a similar manner to the TCM schemes developed by Ungerboeck. Unlike TCM, however, the MLC approach provides flexible transmission rates, through the use of multiple component codes that may have different rates. Furthermore, any code can be used as a component code.

3.2 Multilevel Encoder

Generally in a multilevel encoder design [23, 29], a signal constellation S_L is partitioned into a partition chain S_L / $S_{L\text{-}1}$ /… / S_0. Each set $S_{i\text{-}1}$ is a subset of the set above it, S_i, whereby it divides S_i exactly into $S_{i\text{-}1}$ and its co-subsets. The elements of the set formed by the partitioning of S_i into $S_{i\text{-}1}$ and its co-subsets are labeled by a set of labels x_i, whereby $S_{i\text{-}1}$ and its co-subsets map onto the elements of x_i and we can write $S_i / S_{i\text{-}1} \leftrightarrow x_i$. The labels x_i, are elements of a discrete alphabet over which a component code C_i can be defined. The combination consisting of the partition chain S_L / $S_{L\text{-}1}$ /… / S_0, the label sets x_1, x_2, …, x_L and the codes C_1, C_2, …, C_L form a multilevel code. The structure of a multilevel encoder is shown in Fig. 3.1.

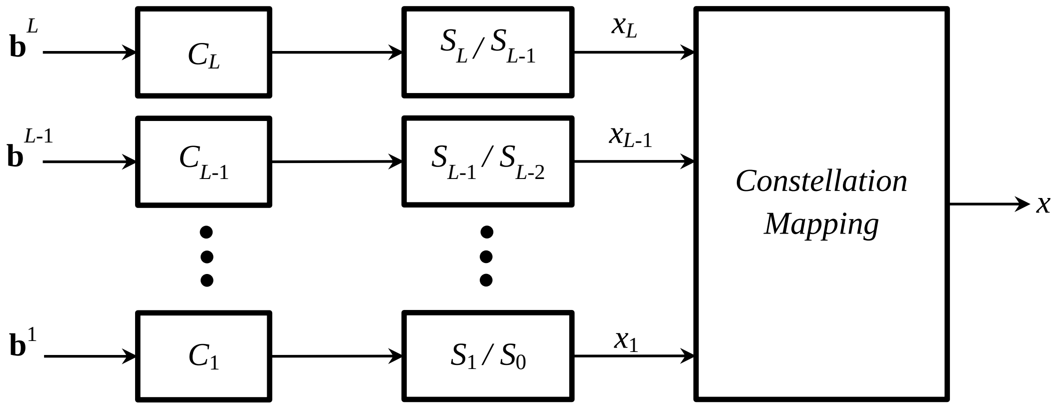

Figure 3.1 General encoder structure for a multilevel code.

Each code, C_i, accepts b_i input bits and outputs $|S_i| / |S_{i\text{-}1}|$ bits for each time slot. The output of the encoder for C_L selects a co-subset of S_L / $S_{L\text{-}1}$. The next encoder for $C_{L\text{-}1}$ selects a co-subset of $S_{L\text{-}1}$ / $S_{L\text{-}2}$, and so forth, until finally the code C_1 narrows down the selection to a single point on the underlying constellation, x, which will be transmitted. An overall code C, may be associated with the L-level multilevel code. This code is referred to as the multilevel code associated with the partition chain S_L / $S_{L\text{-}1}$ /… / S_0 and the L independent component codes C_1, C_2, …, C_L.

Several different criteria have been proposed for designing multilevel codes. These include distance based criteria [22, 23, 29], capacity based designs [24], those based on the cutoff rate [24] and the coding exponent [24].

Here, we use the balanced distance design rule, which was the original design rule proposed by Imai [22] based on the minimum Euclidean distance. It aims to maximize the minimum distance of the Euclidean space code by choosing component codes that result in equal minimum squared Euclidean distances on all levels. This then provides equal protection for all bits in each sequence of constellation points.

A variety of different partitioning strategies have been suggested [24, 30]. In this work, we consider a partitioning scheme based on multi-resolution modulation (MRM), originally introduced in the context of broadcast channels by Cover [31].

3.3 Multistage Decoder

Multilevel codes are usually decoded by a staged decoder as shown in Fig. 3.2. The decoder, on level i in Fig. 3.2 decodes the component code C_i. The staged decoder operates in a sequential manner. First the decoder at level L makes a decision on the code C_L and outputs the corresponding data bits, $\mathbf{b}_L$. This decision information is then passed on from stage L to stage $L-1$ and the decoder at level $L-1$ operates in a similar way, outputting $\mathbf{b}_L - 1$ and the corresponding co-subset information. The process continues down the partition chain until the received sequence is completely decoded.

The fact that the decision at each level assumes a correct decision from the previous level, means there can be error propagation through a MSD. Techniques such as interleaving and iterative multi-stage decoding have been used to combat these effects [32].

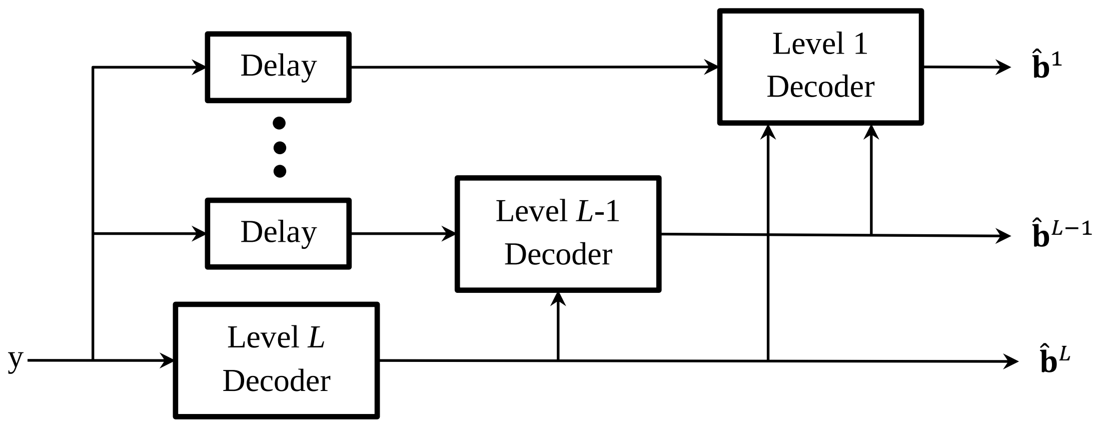

Figure 3.2 General multi-stage decoder for a multilevel code.

3.4 Summary

This chapter has presented background on MLC and highlighted the encoding and decoding structure of a multilevel coded system.

In the next chapter, the techniques highlighted in Chapter 2 and 3 are used to design a transmission scheme which makes use of multilevel coding concepts and STTCs in order to achieve good performance, high throughput and low complexity. This system offers a more integrated design, which spans both space and time.

Chapter 4

Multilevel Space-Time Trellis Codes

In Chapter 2 we discussed space-time trellis codes (STTCs) [21], which can simultaneously provide substantial coding gain, spectral efficiency and diversity improvement. But, the exponential increase in decoding complexity with the number of antennas and the size of the modulation set is a major drawback. So, practically we can’t use them for larger constellations.

In Chapter 3, we discussed multilevel coded (MLC) modulation [22, 23, 24], where a higher complexity coded signal constellation can be constructed using simple component codes.

In this chapter the techniques of Chapter 2 and 3 are combined to form multilevel space-time trellis codes (MLSTTCs). MLSTTC were developed by M. Baghaie [33] by combining MLC, STTC and MRM and are capable of simultaneously providing bandwidth efficiency, diversity improvement and coding gain with significantly reduced decoding complexity, especially for larger constellations and higher throughputs.

The general structure of MLSTTCs is flexible and can easily be tuned to achieve the required balance between spectral efficiency, error performance and decoding complexity. The overall structure and analytical model is described in this chapter.

4.1 Introduction

For high data rate transmissions we need high spectral efficiency, which requires us to construct STTCs using high order signal constellations. But, the STTC design criterion normally needs a vigorous computer search. The size of the search space increases exponentially with increase in the constellation size. It also increases if we increase the number of transmit antennas or the number of states in the code trellis. The decoding complexity of the STTCs also increases exponentially with the size of the underlying constellation.

MLSTTCs are a promising alternative to currently available STTCs, by simultaneously offering diversity improvement, coding gain and bandwidth efficiency at significantly lower decoding complexity than STTCs.

4.2 System Model

We consider a MIMO wireless system as shown in Fig. 4.1, with n_T transmit antennas and n_R receive antennas. The symbol transmitted at time t by the i^{th} transmit antenna is denoted by Q_t^i, $1 \le i \le n_T$. The channel exhibits quasi-static frequency flat Rayleigh fading over the frame duration. Thus, it is constant over one frame and varies independently between frames. We assume perfect CSI is available at the receiver only.

The received signal at time t, at the j^{th} receive antenna is a noisy superposition of independently Rayleigh faded versions of the n_T transmitted signals and is denoted by r_t^j, $1 \le j \le n_R$. The discrete complex baseband output of the j^{th} receive antenna at time t is given by,

$$r_t^j = \sum_{i=1}^{n_T} h_{j,i}^t Q_t^i + \eta_t^j \tag{4.1}$$

where, $h_{j,i}^t$is the path gain between the i^{th} transmit and j^{th} receive antennas and η_t^j is the noise associated with the j^{th} receive antenna at time t. The path gains, $h_{j,i}^t$, are modeled as samples of independent complex Gaussian random variables with zero mean and variance of ½ per dimension. The noise quantities are samples of independent complex Gaussian random variables with zero mean and variance of $N_0/2$ per dimension.

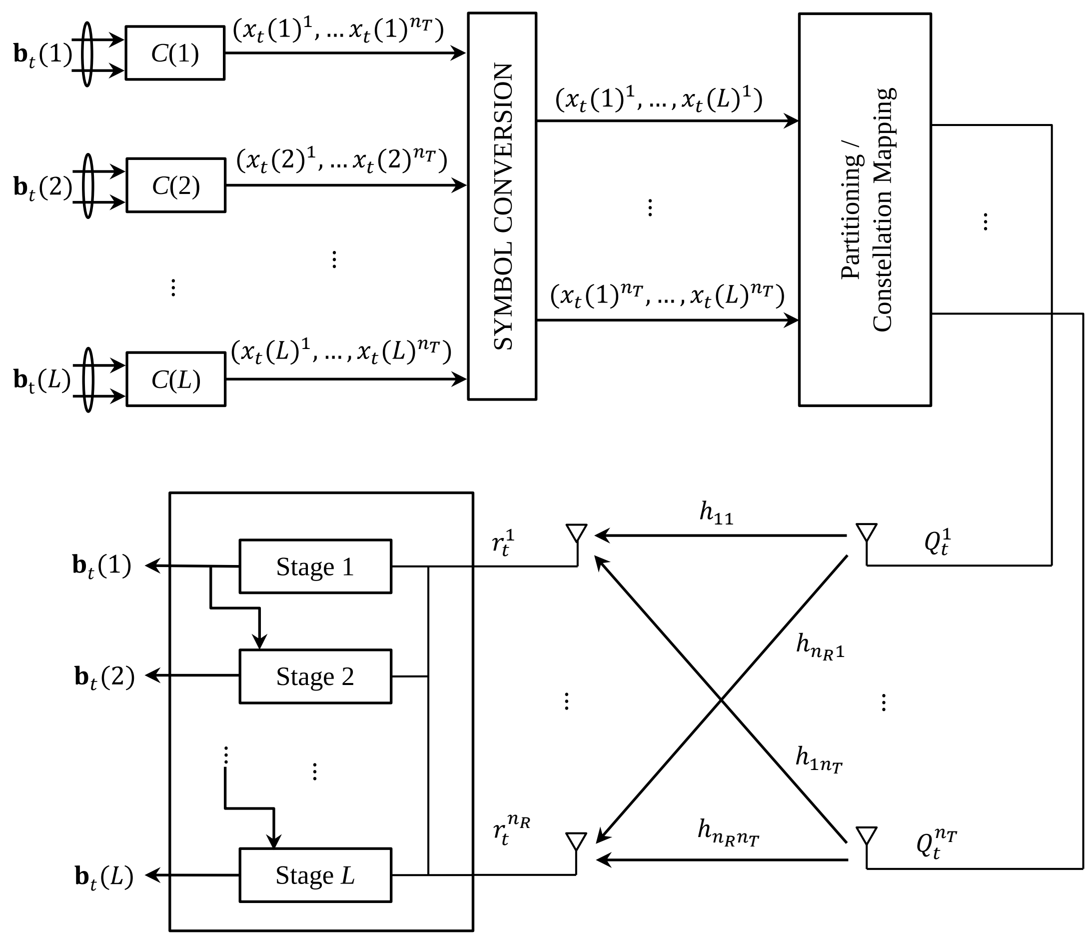

Figure 4.1 General structure of an MLSTTC system.

In matrix form, (4.1) can be represented as,

$$\begin{bmatrix} r_t^1 \\ r_t^2 \\ \vdots \\ r_t^{n_R} \end{bmatrix} = \begin{bmatrix} h_{1,1}^t & h_{1,2}^t & \cdots & h_{1,n_T}^t \\ h_{2,1}^t & h_{2,2}^t & \cdots & h_{2,n_T}^t \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_R,1}^t & h_{n_R,2}^t & \cdots & h_{n_R,n_T}^t \end{bmatrix} \begin{bmatrix} Q_t^1 \\ Q_t^2 \\ \vdots \\ Q_t^{n_T} \end{bmatrix} + \begin{bmatrix} \eta_t^1 \\ \eta_t^2 \\ \vdots \\ \eta_t^{n_R} \end{bmatrix} \tag{4.2}$$

or, in compact form as

$$\mathbf{r}_t = \mathbf{H}_t \mathbf{Q}_t + \boldsymbol{\eta}_t \tag{4.3}$$

where, $\mathbf{r_t} = \left(r_t^1, r_t^2, r_t^3, \dots, r_t^{n_R}\right)^{\mathrm{T}}$, $\mathbf{Q_t} = \left(Q_t^1, Q_t^2, \dots, Q_t^{n_T}\right)^{\mathrm{T}}$, $\boldsymbol{\eta}_\mathbf{t} = \left(\eta_t^1, \eta_t^2, \dots, \eta_t^{n_R}\right)^{\mathrm{T}}$ and $\mathbf{H_t}$ is the $n_R \times n_T$ channel matrix whose j,i-th entry is represented by $h_{j,i}^t$.

4.3 MLSTTC Encoder

MLSTTC encoder works by partitioning the underlying signal constellation into a hierarchy of subsets or clusters using the multi-resolution modulation (MRM) approach, originally introduced by Cover in 1972 [31]. Each cluster may itself have sub-clusters and so on. The incoming bits are encoded and mapped to the 2^m point MRM constellation, with the most significant coded bits being mapped to the clusters and the least significant bits to the sub-clusters and so forth. Ultimately, the last bits choose a signal point within the underlying constellation.

This clusterization provides up to L resolutions for an underlying M-QAM constellation, with $M = 4^L$, where each resolution can be considered as a 4-QAM constellation. Up to L component codes can be used to encode the incoming bits. Block diagram of a MLSTTC system is shown in Fig. 4.1. The component codes are denoted as $C(1)$, $C(2)$, … , $C(L)$ in this figure. Each of these component codes is designed for their corresponding cluster size. The output of each encoder is mapped to its corresponding cluster. Potentially, any code (including block codes) can be used as a component code. Here, STTCs [21] are used as component codes.

4.3.1 Partitioning and constellation mapping

The partitioning scheme used on each of the n_T transmit antennas is based on MRM, which was originally introduced in the context of broadcast channels [31]. The idea is to divide the underlying signal constellation into a hierarchy of clusters, where each cluster can have its own sub-clusters. The distance between clusters is greater than the distance between sub-clusters.

Fig. 4.2 shows the application of MRM to a 64-QAM constellation. The 64 points in the underlying constellation are divided into 4 clusters and each cluster into 4 sub-clusters and so forth. Thus, we use a 4-cluster as the basic unit of resolution. We partition [23] the multi-resolution constellation by treating each cluster as a 4-QAM constellation. This enables us to directly use STTCs designed for 4-QAM in our mapping process. The labeling of the signal constellation points based on the partitioning is also shown in the Fig. 4.2, where we have 3 clusters, each having 4 sub-clusters. The dashed rounded corner rectangles in the figure denote one sub-cluster of each cluster.

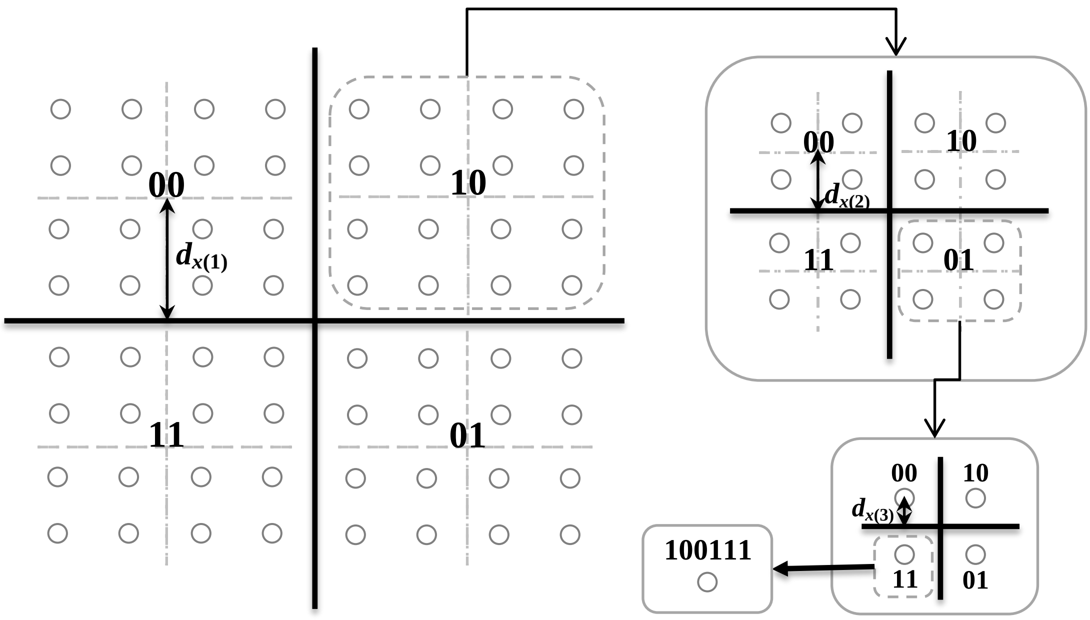

Figure 4.2 Partitioning and labeling of the underlying constellation for 64-QAM.

This clusterization allows L resolutions for a M-QAM constellation, with $M = 4^L$. We then map the output of the first component code, $C(1)$, to the clusters and the output of the next component code $C(2)$ to the sub-clusters and so forth with the output of $C(L)$ selecting the actual constellation points to be transmitted. For the 64-QAM constellation of Fig. 4.2, this results in $L = 3$ levels.

The output of $C(L)$, denoted as $x_t(L)$, gets mapped to the actual constellation points, while the outputs generated by $C(1)$ to $C(L-1)$, denoted $x_t(1), \dots, x_t(L-1)$ respectively, get mapped to the virtual cluster center points (centroids) as shown in Fig. 4.2, and thus label the corresponding clusters. These component codes are all designed for a 4-QAM constellation and their coded output symbols, $x_t(1)$ to $x_t(L)$, are all drawn from 4-QAM constellations and can be represented either in complex form as $x_t(1), = a + jb$, such that $a, b \in \{1,-1\}$ or mapped to a ring of integers $s_t(l) = \mu(x_t(l)) \mid s_t(l) \in \{0, 1, 2, 3\}$. Each point in the underlying M-QAM constellation can be represented in complex form as,

$$Q = d_{x(1)}x(1) + d_{x(2)}x(2) + \cdots + d_{x(L)}x(L) \tag{4.4}$$

where, $d_{x(1)}, d_{x(2)}, \dots d_{x(L)}$, are the cluster distances corresponding to $x_t(1), \dots, x_t(L)$, as shown in Fig. 4.2.

4.3.2 Mapping Symbols to Antennas

Considering the constellation shown Fig. 4.2. At time t, the symbol transmitted from the i^{th} transmit antenna can be represented by,

$$Q_t^i = d_{x(1)}x(1)^i + d_{x(2)}x(2)^i + \cdots + d_{x(L)}x(L)^i \tag{4.5}$$

where, $x_t(l)^i$ is the 4-QAM symbol generated by the l^{th} component code, $1 \leq l \leq L$.

A STTC designed for an M-ary constellation, where $M = 2^m$, has a throughput of m bits/sec/Hz [12]. If n_T transmit antennas are used, then the overall rate is $\frac{m}{m \times n_T}$. In MLSTTC, if we use L identical M-QAM STTCs designed for n_T transmit antennas as our component codes, then we will end up with an overall rate of $\frac{L\,m}{n_T\,(L\,m)}$.

4.4 Detection/Decoding

Multi-stage decoder with L stages is used to decode the received data, encoded by an L-level MLSTTC, shown in Fig. 4.1. The decoder starts by decoding the output of the first component code. The estimated values of $\mathbf{x}_t(1)$, $\hat{\mathbf{x}}_t(1)$ are then passed to the next decoding stage and are used to decode the values of $\mathbf{x}_t(L-1)$ and so forth. The final stage of the decoder uses the estimates obtained from levels 1 to L – 1, namely $\hat{\mathbf{x}}_t(1), \hat{\mathbf{x}}_t(2), \ldots, \hat{\mathbf{x}}_t(L-1)$ to obtain $\hat{\mathbf{x}}_t(L)$.

The received signal at the j^{th} receive antenna at time t is given by,

$$r_t^j = \sum_{l=1}^{L}\sum_{i=1}^{n_T} h_{j,i}^t d_{x(l)} x(l)^i + \eta_t^j \tag{4.6}$$

For an L-level MLSTTC system, if the decoding starts from stage L, the branch metric at stage k, where $1 < k \leq L$, for a transition labeled $x_t(k)^1$, $x_t(k)^2$, ..., $x_t(k)^i$ can be calculated as

$$\max_{\substack{\mathrm{x}_t(l),\\ l:\,1,\ldots,k-1}} \sum_{j=1}^{n_R} \left| r_t^j - \sum_{l=1}^{k}\sum_{i=1}^{n_T} h_{j,i}^t d_{x(l)} x(l)^i - \sum_{p=k+1}^{L}\sum_{i=1}^{n_T} h_{j,i}^t d_{x(p)} \hat{x}(p)^i \right|^2 \tag{4.7}$$

The Viterbi algorithm is then used to decode the path with the lowest accumulated metric.

4.5 Complexity Considerations

MLSTTC scheme makes use of a multilevel structure. The underlying constellation is partitioned and the encoding is done in stages using simpler component codes. A multi-stage decoder is then used to decode the succession of component codes. The overall structure results in reduced complexity, especially for larger signal constellations and larger numbers of states. In general, complexity of the MLSTTC system increases linearly with the size of the underlying constellation, while that of a STTC grows exponentially.

For a STTC transmitting at m bits/sec/Hz with diversity rn_R, it has been shown [12] the trellis complexity of the space–time code is at least $2^{m(r-1)}$, i.e., there are at least $2^{m(r-1)}$ states in the trellis and thus, complexity of the trellis grows exponentially with throughput. The number of branches leaving each state of the trellis diagram is 2^m. For the STTC with N_s states, and N_b branches per state, the STTC decoder complexity is approximately $N_s \times N_b \times (n_T + 1) \times n_R$. For same number of transmit and receive antennas we can use $N_s \times N_b$ as a measure of complexity the purpose of comparisons.

From the above complexity considerations, we can see that for a 4-QAM STTC with transmission rate of 2 bits/sec/Hz the minimum value of the complexity measure is 4×4. Similarly, for a 16-QAM STTC with transmission rate of 4 bits/sec/Hz the minimum value of the complexity is 16×16, and for a 64-QAM STTC with transmission rate of 6 bits/sec/Hz the minimum value of the complexity is 64×64. Thus, we see that incase of STTCs the complexity grows *exponentially* with throughput and the size of the underlying constellation.

A MLSTTC solves this by employing the MLC concept to break the complexity into parts. For a constellation size of 2^m, where $m \geq 4$, the MLSTTC can partition the underlying QAM constellation into L 4-QAM subsets, with $2^m = 4^L$. Multilevel encoding will then result in an overall minimum number of states equal to 4^L where 4 branches leave each state due to the 4-QAM subsets. Therefore, the complexity grows *linearly* with the size of constellation, giving a minimum complexity measure of $L \times 4 \times 4$ as opposed to $2^m \times 2^m$. Thus, the complexity of MLSTTC is manageable and can easily be extended to higher order constellations and higher throughputs.

As an example, for of a 16-QAM constellation partitioned into L = 2 levels, a MLSTTC can transmit 4 bits/sec/Hz with a minimum complexity of 2 × 4 × 4 (=32), as opposed to minimum complexity of 16 × 16 (=256) for an equivalent STTC with the same throughput, thereby offering the complexity saving of about 8 times. Similarly, for a 64-QAM constellation partitioned into L = 3 levels, MLSTTCs can transmit 6 bits/sec/Hz with a minimum complexity of 3 × 4 × 4 (=48), thereby resulting in complexity saving of over 85 times when compared with the minimum complexity of 64 × 64 (=4096) of the equivalent STTC with the same throughput.

It can be seen that the complexity reduction of MLSTTC compared to a STTC becomes more pronounced for higher throughputs and larger constellation sizes. Since we are transmitting at the same throughput using a smaller constellation, the MLSTTC is not only less complex in terms of receiver structure, but it has better distance properties than the comparable STTC. Therefore, we can achieve the same performance as that of STTC but using a smaller number of states.

4.6 Performance Evaluation

Here we evaluate the error performance of the MLSTTCs by simulations. We consider an example MLSTTC system with 2 transmit and different number of receive antennas, that uses L (= 2) level coding and set partitioning to partition a 16-QAM constellation 2 times using 4-way partitions into subset of constellation points [33]. The 16 points of the underlying constellation are divided into four clusters. Each cluster consists of 4 points and therefore can be treated as 4-QAM. The subset distances $d_{x(1)}$ and $d_{x(2)}$ are taken 2 and 1, respectively. Due to two cluster levels, we use two component code levels, $C(1)$ & $C(2)$, to encode the incoming bits. Each level, use 4-QAM STTCs designed using the trace criterion [21] for 2 transmit antennas, trellis diagram of which is shown in Fig. 4.3.

We assumed a quasi-static Rayleigh fading channel model which is constant over a frame and varies independently between frames. We consider a frame size of 130 symbols. Detection/Decoding of the received symbols is done via MSD. We assumed perfect CSI at the receiver only.

The error performance, namely, frame error rate (FER), symbol error rate (SER) and bit error rate (BER), curves are plotted against the signal-to-noise ratio (SNR) per receive antenna and are shown in Fig. 4.4, 4.5 and 4.6, respectively.

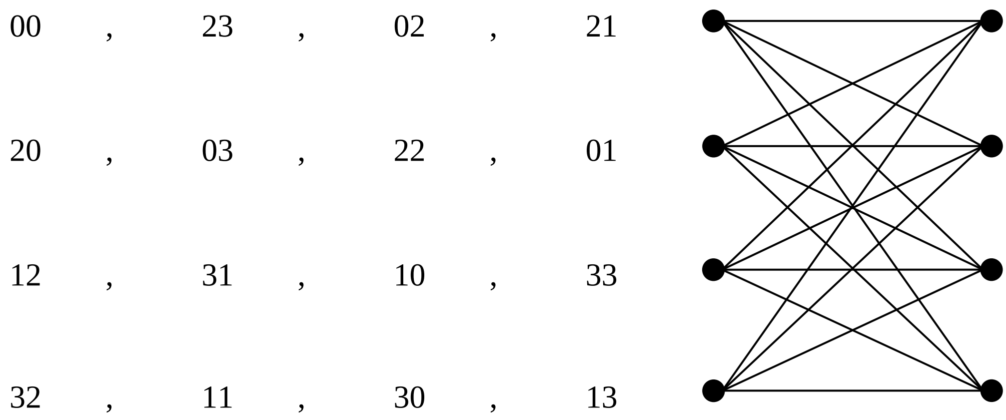

Figure 4.3 Trellis structure for a 4-state STTC designed using Trace Criterion for two transmit antennas.

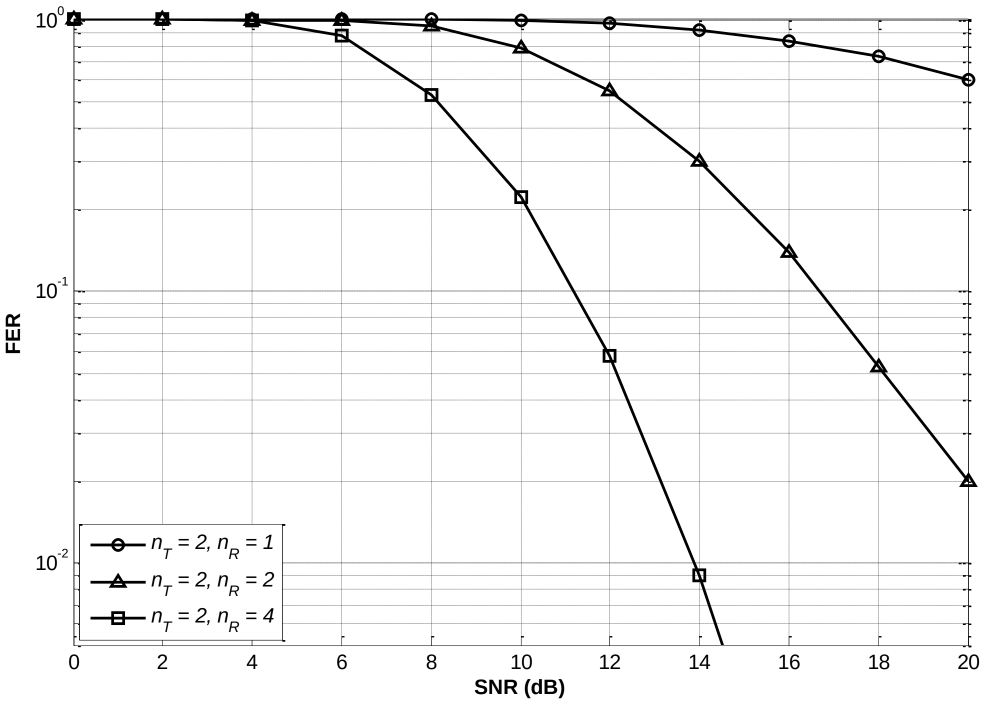

Figure 4.4 FER performance of a 2 level MLSTTC for two transmit and different number of receive antennas.

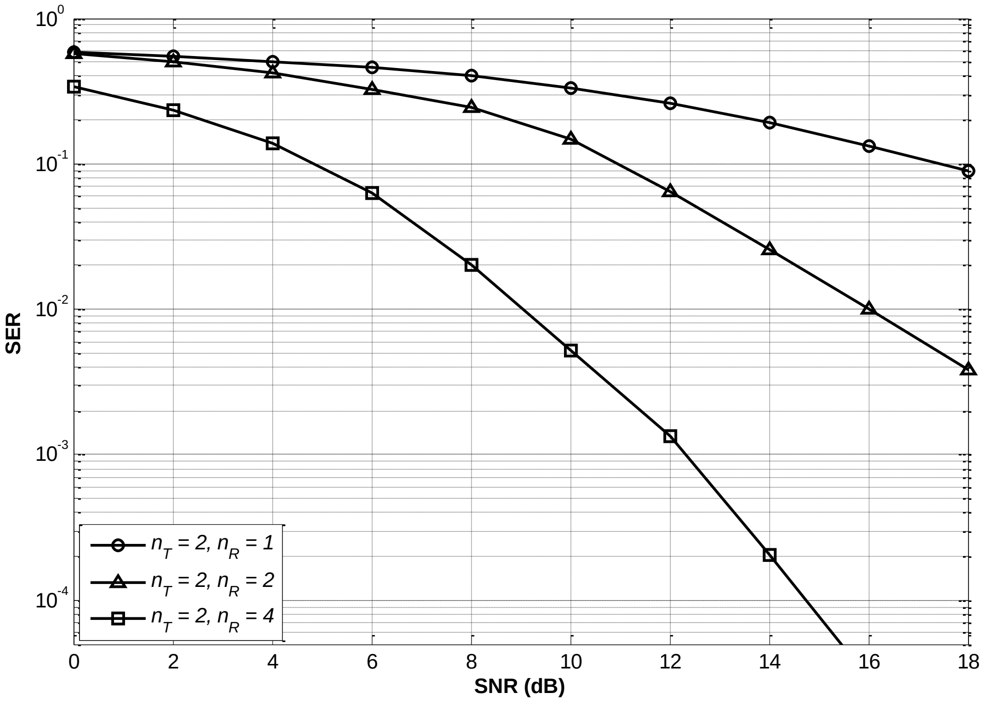

Figure 4.5 SER performance of a 2 level MLSTTC for two transmit and different number of receive antennas.

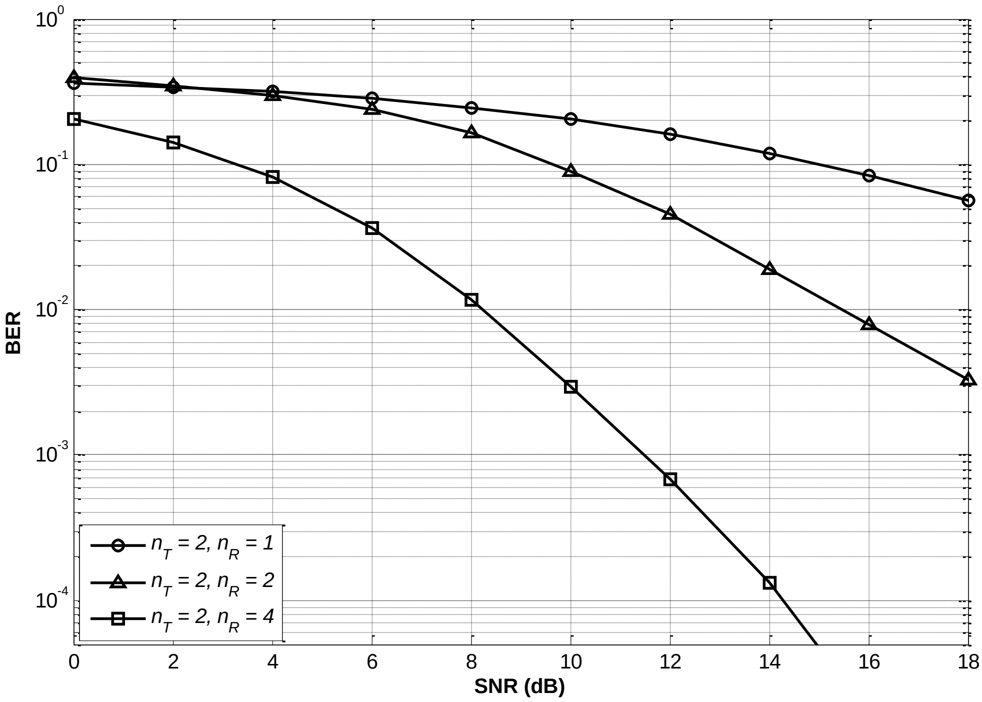

Figure 4.6 BER performance of a 2 level MLSTTC for two transmit and different number of receive antennas.

4.7 Summary

In this chapter, the techniques of STTC and MLC modulation were combined to form MLSTTC, capable of simultaneously providing bandwidth efficiency, diversity improvement and coding gain with reduced decoding complexity, especially for larger constellations and higher throughputs. MLSTTCs and their corresponding encoding and decoding structures were discussed along with an example. Simulation results were also provided for error performance of MLSTTC with 2 transmit and different receive antennas. Complexity considerations were also discussed.

In the next chapter, we make use of the MLSTTC technique and combine it with ideal beamforming to develop a new transmission scheme, weighted MLSTTC.

Chapter 5

Weighted Multilevel Space-Time Trellis Codes

Until now we have discussed about space-time trellis codes (STTCs) [21], multilevel coded (MLC) modulation [22, 23, 24] and multilevel space-time trellis codes (MLSTTCs) [33].

Space-time trellis codes (STTCs) [21] can provide coding gain as well as diversity gain on fading channels simultaneously, but typically transmit only one data symbol per time slot. Multilevel coding [22, 23, 24] can be used along with STTCs to transmit more than one data symbol per time slot. Combining STTC and MLC M. Baghaie [33] developed MLSTTCs. However it has been derived for a system with no CSI available at the transmitter. The system performance can be improved dramatically if perfect or partial CSI is made available at the transmitter.

In this chapter we extend the work of M. Baghaie [33] in which MLSTTC was designed by assuming perfect CSI available at receiver. A novel design criterion for MLSTTC with CSI at transmitter is proposed, that incorporates the statistical information concerning the channel estimates. New improved Weighed MLSTTC codes are obtained using a novel combination of beamforming and MLSTTC.

5.1 Introduction

In fading channels we have more than one source of randomness, viz., the fading process and the additive noise. It may happen that the receiver or the transmitter or both have perfect or at least partial knowledge of the realization of the fading process, which we call the channel-state information (CSI).

CSI at the receiver can be obtained by inserting suitable known pilot symbols in the transmitted signal periodically. These allow the demodulator to take advantage of its knowledge of the channel fading gain and hence to adjust its parameters to optimize operation.

On the other hand, CSI at the transmitter allows it to adjust its *transmitted power*, or its information rate, so as to adapt the transmission to channel conditions. Transmitter CSI can be relayed from the receiver through a *feedback* path, or, when transmissions in both directions are multiplexed in time, the signal from the opposite link can be used to measure the channel state. Transmit-power control can turn out to be the most effective technique to mitigate fading.

In [34] it has been shown that when perfect CSI is available at the transmitter, the performance of space-time coded system can be further improved by using the beamforming scheme which can enhance the performance of a STTC system by weighting the transmitted signals based on the available channel information

In this thesis work, we present the design of the combined MLSTTCs [33] and beamforming [34], henceforth referred to as weighted multilevel space-time trellis codes (WMLSTTCs), over slow Rayleigh fading channel, when *perfect* CSI is available at the transmitter. By assuming perfect CSI at the transmitter, we can use the information to improve the error performance by setting the signal power on different antennas differently (based on the channel), helping them be easier to detect in the receiver. We describe the overall structure and analytical model in this chapter. Performance evaluations are studied in Chapter 6.

5.2 System Model

We consider a WMLSTTC system as shown in Fig. 5.1, with n_T transmit antennas and n_R receive antennas. The symbol transmitted at time t by the i^{th} transmit antenna is denoted by c_t^i, $1 \le i \le n_T$.

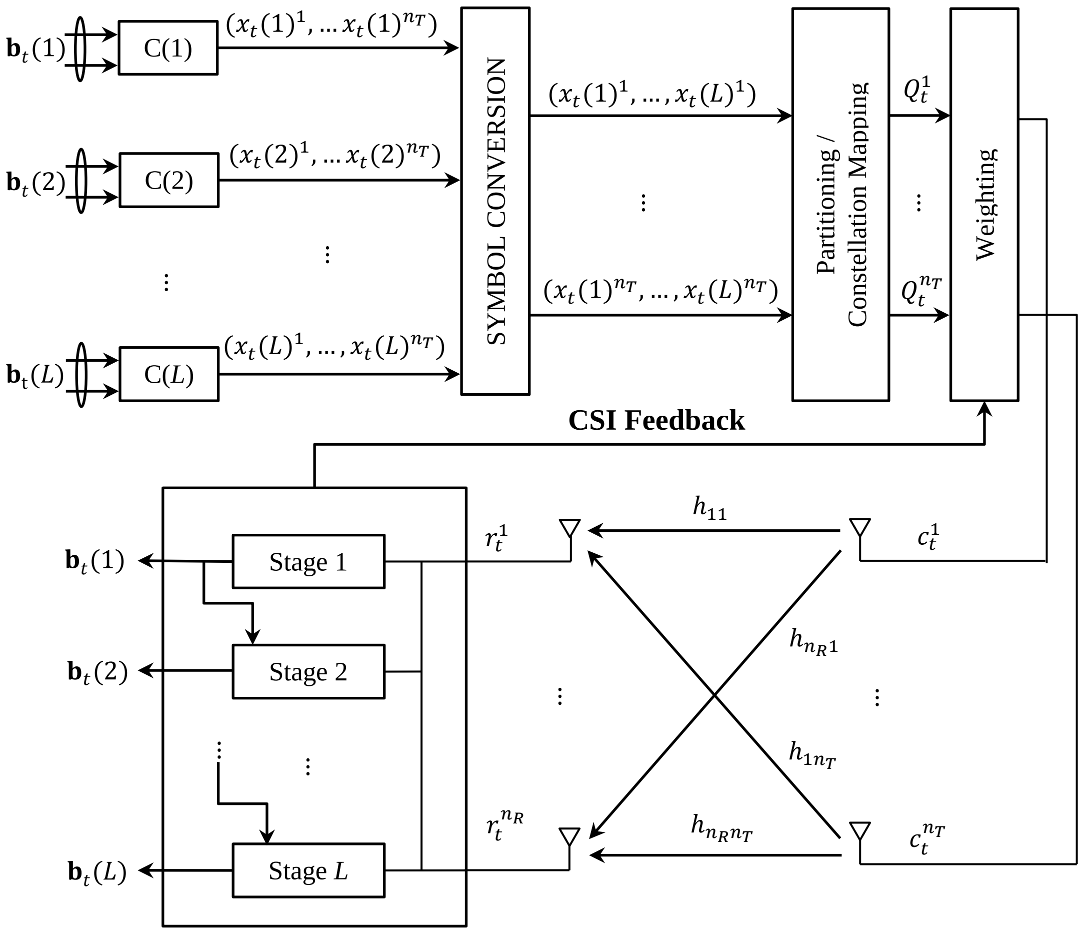

Figure 5.1 General structure of a WMLSTTC system.

We assume that the channel exhibits a quasi-static frequency flat Rayleigh fading over the frame duration. Thus, it is constant over one frame and varies independently between frames. We assume perfect CSI is available at the receiver and the transmitter both.

The received signal at time t, at the j^{th} receive antenna is a noisy superposition of independently Rayleigh faded versions of the n_T transmitted signals and is denoted by r_t^j, $1 \le j \le n_R$. The discrete complex baseband output of the j^{th} receive antenna at time t is given by,

$$r_t^j = \sum_{i=1}^{n_T} h_{j,i}^t c_t^i + \eta_t^j$$

(5.1)

where, $h_{j,i}^t$is the path gain between the i^{th} transmit and j^{th} receive antennas and η_t^j is the noise associated with the j^{th} receive antenna at time t. The path gains, $h_{j,i}^t$, are modeled as samples of independent complex Gaussian random variables with zero mean and variance of ½ per dimension. The noise quantities are samples of independent complex Gaussian random variables with zero mean and variance of $N_0/2$ per dimension.

In matrix form, (5.1) can be represented as,

$$\begin{bmatrix} r_t^1 \\ r_t^2 \\ \vdots \\ r_t^{n_R} \end{bmatrix} = \begin{bmatrix} h_{1,1}^t & h_{1,2}^t & \cdots & h_{1,n_T}^t \\ h_{2,1}^t & h_{2,2}^t & \cdots & h_{2,n_T}^t \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_R,1}^t & h_{n_R,2}^t & \cdots & h_{n_R,n_T}^t \end{bmatrix} \begin{bmatrix} c_t^1 \\ c_t^2 \\ \vdots \\ c_t^{n_T} \end{bmatrix} + \begin{bmatrix} \eta_t^1 \\ \eta_t^2 \\ \vdots \\ \eta_t^{n_R} \end{bmatrix}$$

(5.2)

or, in compact form as

$$\mathbf{r}_t = \mathbf{H}_t \mathbf{c}_t + \boldsymbol{\eta}_t \quad (5.3)$$

where, $\mathbf{r_t} = \left(r_t^1, r_t^2, r_t^3, \dots, r_t^{n_R}\right)^{\mathrm{T}}$, $\mathbf{c_t} = \left(c_t^1, c_t^2, \dots, c_t^{n_T}\right)^{\mathrm{T}}$, $\boldsymbol{\eta}_\mathbf{t} = \left(\eta_t^1, \eta_t^2, \dots, \eta_t^{n_R}\right)^{\mathrm{T}}$ and $\mathbf{H_t}$ is the $n_R \times n_T$ channel matrix whose j,i-th entry is represented by $h_{j,i}^t$.

5.3 WMLSTTC Encoder

WMLSTTC encoder employs the MLSTTC encoder described in chapter 4, followed by weighting as shown in Fig. 5.1. The information bits encoded by the MLSTTC encoder are weighted by the weighting matrix based on the CSI feedback from the receiver.

The weighted signal transmitted at time t, denoted by $\mathbf{c_t}$, can be written as

$$\mathbf{c_t} = \left(c_t^1, c_t^2, \dots, c_t^{n_T}\right)^{\mathrm{T}} = \left(w_t^1 Q_t^1, w_t^2 Q_t^2, \dots, w_t^{n_T} Q_t^{n_T}\right)^{\mathrm{T}} \quad (5.4)$$

where, T means the transpose of a matrix, Q_t^i, $1 \le i \le n_T$, is signal coded by the multilevel space-time trellis encoder at the stream i at time t, w_t^i is the weighting coefficient of signal c_t^i, $c_t^i = w_t^i Q_t^i$ denotes the weighted signal transmitted through antenna *i* at time *t*. Based on our assumption of perfect CSI available at the transmitter the beamforming coefficients [34] are given by

$$w_t^i = \sum_{j=1}^{n_R} \left(\frac{\left(h_{j,i}^t\right)^*}{\sqrt{\sum_{k=1}^{n_T} \left|h_{j,k}^t\right|^2}} \right) \tag{5.5}$$

where (.)* represents the complex conjugate operator.

5.4 Detection/Decoding

Multi-stage decoder with L stages is used to decode the received data, encoded by an L-level WMLSTTC, as shown in Fig. 5.1. The decoder starts by decoding the output of the first component code. The estimated values of $\mathbf{x}_t(1)$, $\hat{\mathbf{x}}_t(1)$ are then passed to the next decoding stage and are used to decode the values of $\mathbf{x}_t(L-1)$ and so forth. The final stage of the decoder uses the estimates obtained from levels 1 to L – 1, namely $\hat{\mathbf{x}}_t(1), \hat{\mathbf{x}}_t(2), \dots, \hat{\mathbf{x}}_t(L-1)$ to obtain $\hat{\mathbf{x}}_t(L)$.

The received signal at the j^{th} receive antenna at time *t* is given by,

$$r_t^j = \sum_{l=1}^{L} \sum_{i=1}^{n_T} h_{j,i}^t \, w_t^i \, d_{x(l)} x(l)^i + \eta_t^j \tag{5.6}$$

For an L-level WMLSTTC system, if the decoding starts from stage L, the branch metric at stage k, where $1 < k \leq L$, for a transition labeled $x_t(k)^1$, $x_t(k)^2$, ..., $x_t(k)^i$ can be calculated as

$$\max_{\substack{x_t(l), \\ l:\, 1,\dots,k-1}} \sum_{j=1}^{n_R} \left| r_t^j - \sum_{l=1}^{k} \sum_{i=1}^{n_T} h_{j,i}^t \, w_t^i \, d_{x(l)} x(l)^i - \sum_{p=k+1}^{L} \sum_{i=1}^{n_T} h_{j,i}^t \, w_t^i \, d_{x(p)} \hat{x}(p)^i \right|^2 \tag{5.7}$$

The Viterbi algorithm is then used to decode the path with the lowest accumulated metric.

5.5 Summary

In this chapter, we made use of the MLSTTC technique and combined it with ideal beamforming to develop a new transmission scheme, WMLSTTC, without sacrificing the ability of MLSTTC to simultaneously provide bandwidth efficiency, diversity improvement and coding gain with reduced decoding complexity, especially for larger constellations and higher throughputs.

The general structure of WMLSTTCs is very flexible and can easily be tuned to achieve the required balance between spectral efficiency, error performance and decoding complexity. We have described the overall structure and analytical model of WMLSTTC in this chapter.

Performance evaluations are studied in the next chapter.

Chapter 6

System Performance

In Chapter 5 we proposed a transmission scheme, WMLSTTC, as an extension for the MLSTTC [33] scheme, without sacrificing the capability of simultaneously providing bandwidth efficiency, diversity improvement and coding gain with reduced decoding complexity, especially for larger constellations and higher throughputs. The overall structure and analytical model of WMLSTTCs was described in Chapter 5.

In this chapter we evaluate the WMLSTTC system and its performance via examples based on an underlying 16-QAM constellation and using up to 4 transmit and 4 receive antennas. Concluding remarks and suggestions for future research are presented in Chapter 7.

6.1 Introduction

In this chapter, we consider WMLSTTC systems designed for an underlying 16-QAM constellation with up to 4 transmit and 4 receive antennas. We use STTCs [21] as component codes. We assume a quasi-static Rayleigh fading channel model which is constant over a frame and varies independently between frames. We consider a frame size of 130 symbols. Detection/Decoding of the received symbols is done via MSD described in chapter 5. We evaluate system performance and discuss the effects transmit and receive diversity.

For performance comparison, we consider MLSTTC system which has similar specifications as described above for the WMLSTTC system. We show that WMLSTTCs considerably outperforms the conventional MLSTTCs without weighting and without sacrificing the capability of simultaneously providing bandwidth efficiency, diversity improvement and coding gain with reduced decoding complexity.

6.2 An example WMLSTTC system

In this section we describe an example WMLSTTC system with n_T transmit and n_R receive antennas, designed for 16-QAM and using two component code levels as shown in Fig. 6.1. The setup is based on the WMLSTTC system model described in Section 5.2.

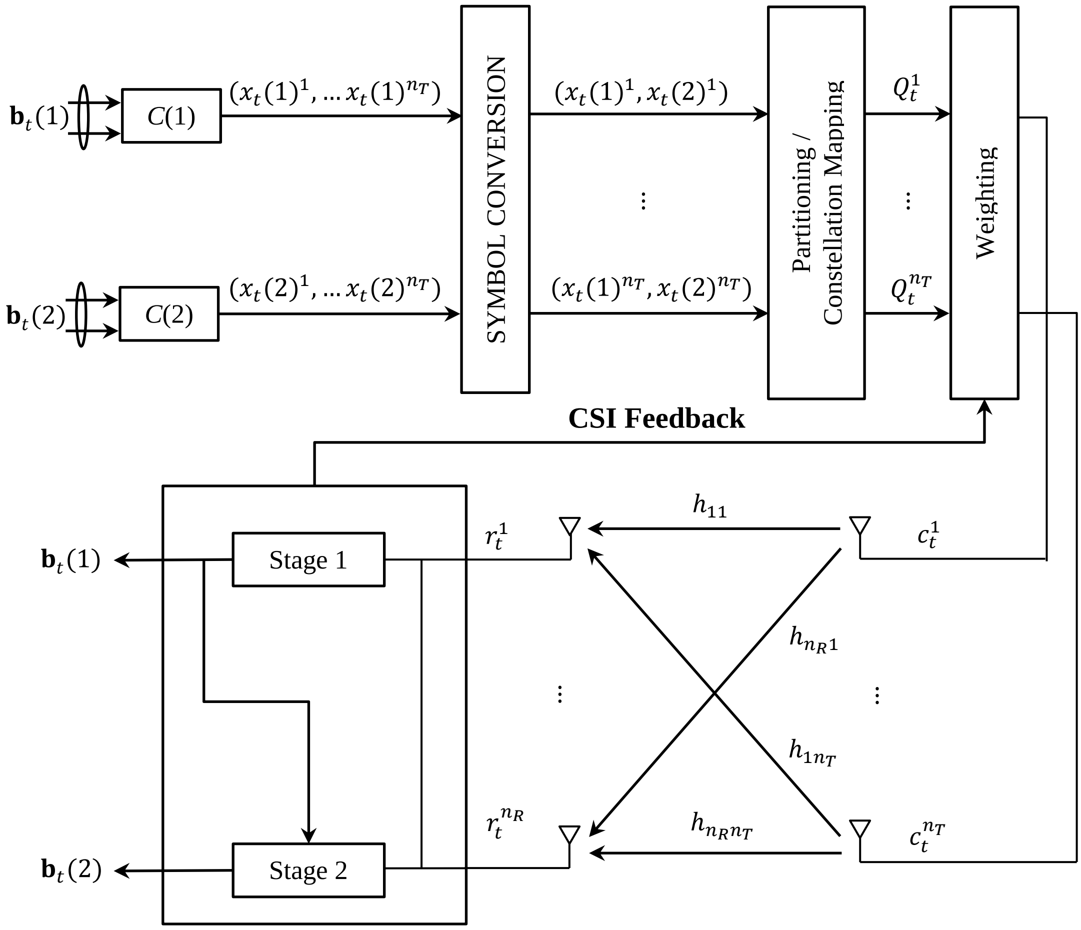

Figure 6.1 An example WMLSTTC system.

The example WMLSTTC system uses L (=2) level coding and set partitioning to partition a 16-QAM constellation 2 times using 4-way partitions into subset of constellation points [33] as shown in Fig. 6.2. The 16 points of the underlying constellation are divided into four clusters (division denoted with two bold lines crossing at the center in Fig. 6.2). Each cluster consists of 4 points and therefore can be treated as 4-QAM. This clusterization provides two resolutions for the 16-QAM constellation, where each resolution can be considered as a 4-QAM constellation.

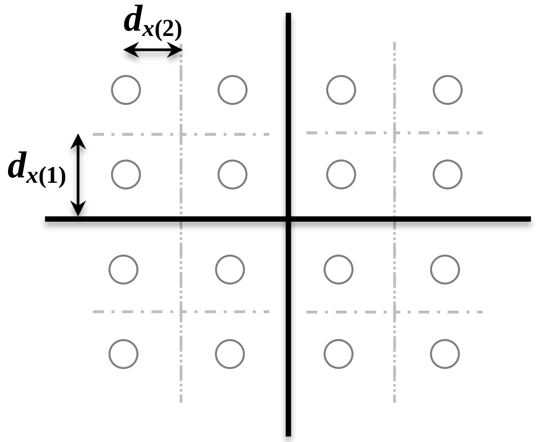

Figure 6.2 Partitioning of 16-QAM constellation.

The subset distances $d_{x(1)}$ and $d_{x(2)}$ are taken 2 and 1, respectively. Due to two cluster levels, we use two component code levels, $C(1)$ & $C(2)$, to encode the incoming bits. Each level, use 4-QAM STTCs designed using the trace criterion [21]. Fig. 6.3, shows the labeling of the signal constellation points.

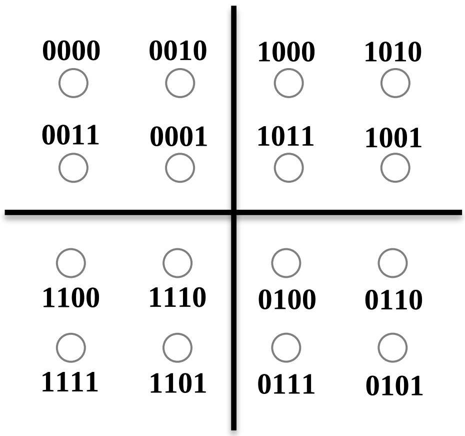

Figure 6.3 Labeling of 16-QAM MRM constellation.

A STTC designed for an M-ary constellation, where $M = 2^m$, has a throughput of m bits/sec/Hz and overall rate of $\frac{m}{m \times n_T}$. In WMLSTTC, if we use L identical 4-QAM STTCs designed for n_T transmit antennas as component codes, then the overall rate is $\frac{L\,m}{n_T\,(L\,m)}$. Thus, in the above described example WMLSTTC system with 16-QAM constellation, and two identical 4-QAM STTCs as component codes, in each time interval, one antenna transmits one 16-QAM symbol resulting in an overall rate of 4/8, for two antennas. This is equivalent to an overall throughput of 4 bits/sec/Hz.

6.3 Receive Diversity

Here we consider the effect of receive diversity on the error performance of the code. Performance is evaluated using 2 transmit antennas and different number of receive antennas. The Frame Error Rate (FER) performance of a WMLSTTC system described in Section 6.2 is shown in Fig. 6.4. The spectral efficiency is 4 bits/sec/Hz and the underlying constellation is 16 QAM. Two identical 4-state STTCs based on *trace* criterion are used as component codes. We use component codes designed for 2 transmit antennas, trellis diagram of which is shown in Fig. 4.3 (Chapter 4). The component codes are described in detail in Chapter 2. We assumed perfect CSI at the receiver and the transmitter both. The result shows that increasing the number of receive antennas yield a significant performance gain.

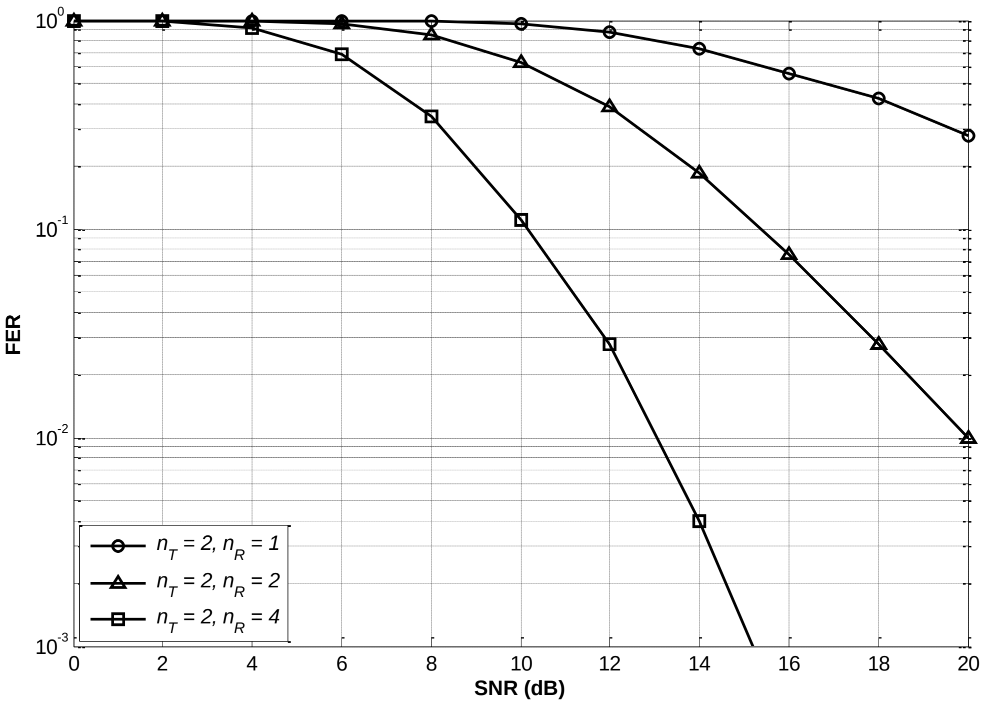

Figure 6.4 FER performance of a 2 level WMLSTTC for two transmit and different number of receive antennas.

The Symbol Error Rate (SER) and the Bit Error Rate (BER) performance of the WMLSTTC system are shown in Fig. 6.5 and 6.6, respectively. Performance is evaluated using 2 transmits antennas and different number of receive antennas. The result shows that increasing the number of receive antennas yield a significant performance gain as seen in case of FER performance.

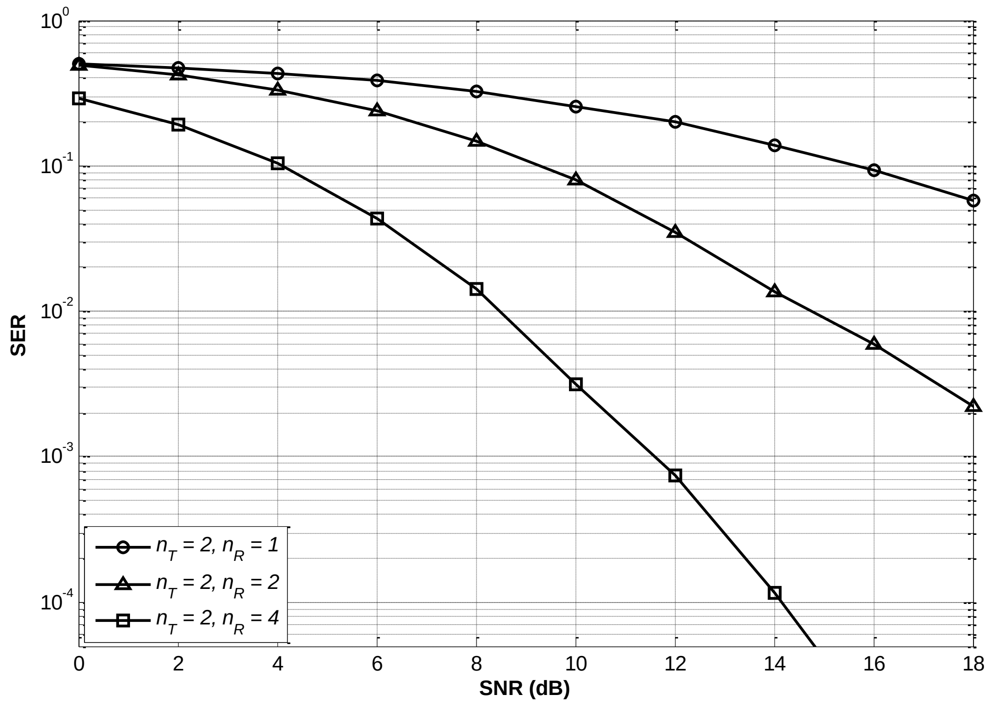

Figure 6.5 SER performance of a 2 level WMLSTTC for two transmit and different number of receive antennas.

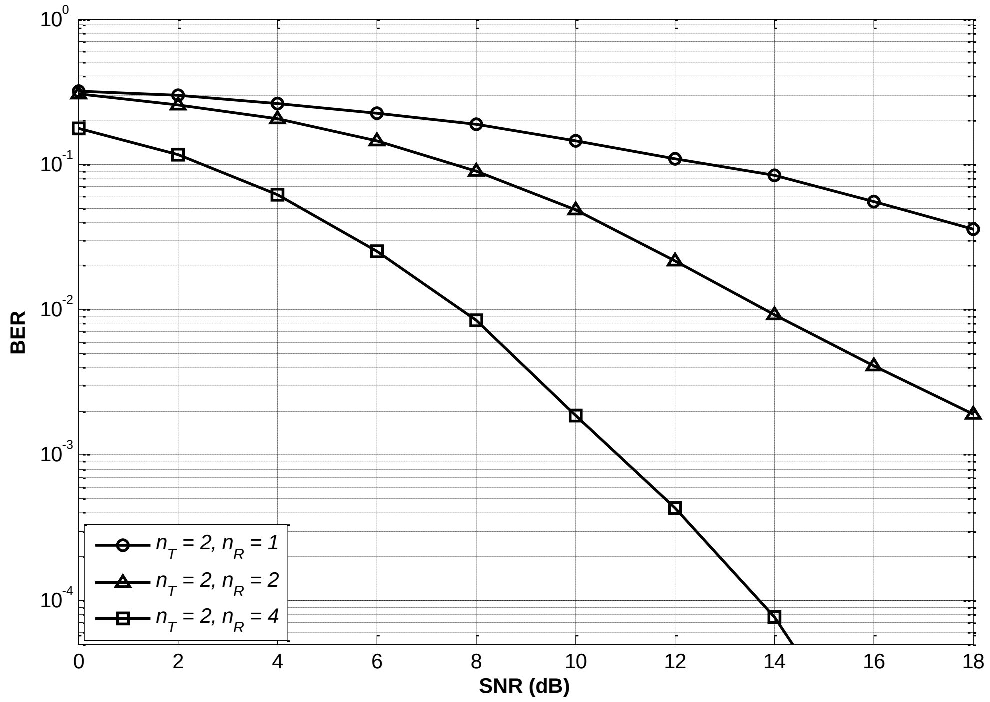

Figure 6.6 BER performance of a 2 level WMLSTTC for two transmit and different number of receive antennas.

6.4 Transmit Diversity

Here we consider the effect of transmit diversity on the error performance of the code. The WMLSTTC system is same as that described in section 6.2. The spectral efficiency is 4 bits/sec/Hz and the underlying constellation is 16-QAM. Two identical 4-state STTCs based on *trace* criterion are used as component codes. Performance is evaluated using 2 and 4 transmit antennas, so, we use component codes designed for 2 and 4 transmit antennas [21], trellis structures of which are shown in Fig. 4.3 (Chapter 4) and Fig. 6.8 respectively. We assumed perfect CSI at the receiver and the transmitter both.

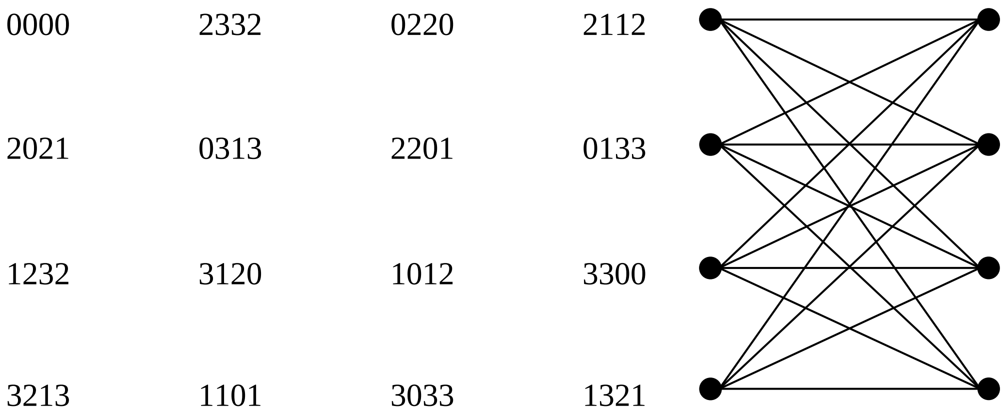

Figure 6.7 Trellis structure for a 4-state STTC designed using Trace Criterion for four transmit antennas.

Figure 6.8 illustrates the FER performance of the WMLSTTC system described above. Performance is evaluated using 2 and 4 transmit antennas. The number of receive antennas has been kept constant at 4 in both cases. As can be seen increasing the number of transmit antennas provides a slight improvement in FER performance.

Figure 6.9 illustrates the SER performance of the WMLSTTC system described above; using 2 and 4 transmit antennas and 4 receive antennas. As in case of FER performance here also we can be see that by increasing the number of transmit antennas we get a slight improvement in SER performance.

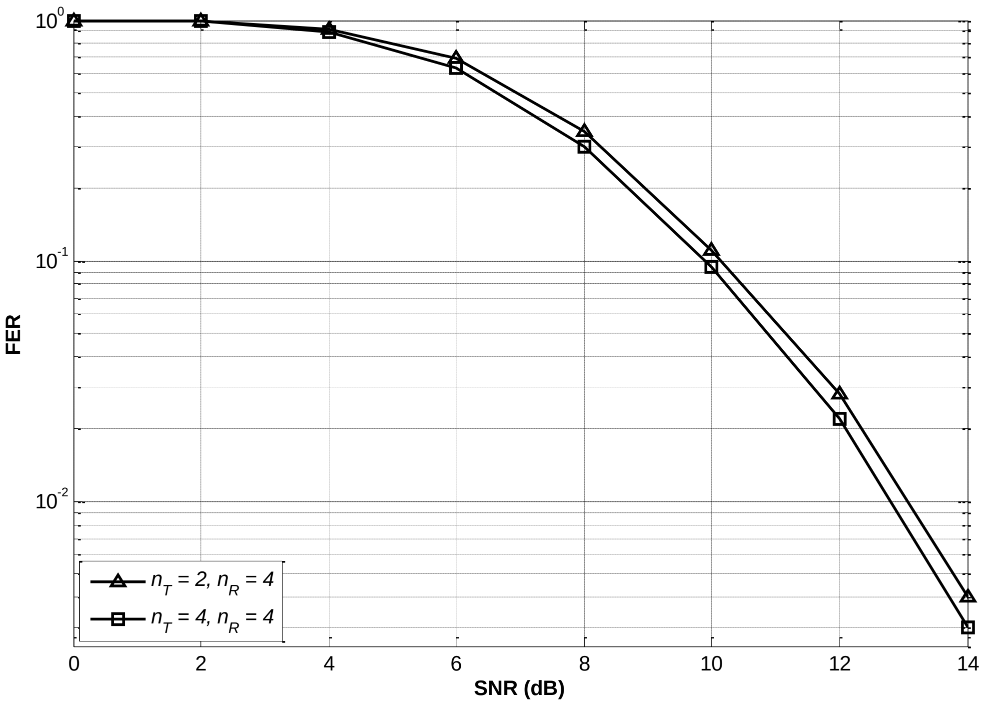

Figure 6.8 FER performance of a 2 level WMLSTTC for four receive and different number of transmit antennas.

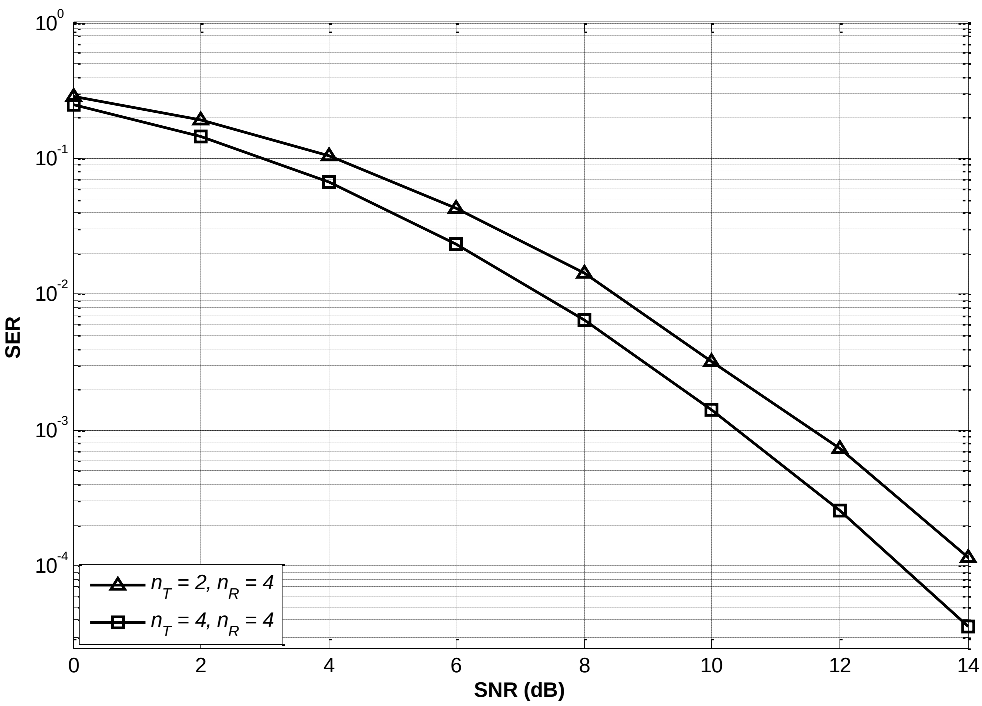

Figure 6.9 SER performance of a 2 level WMLSTTC for four receive and different number of transmit antennas.

6.5 Comparison with MLSTTCs

Here we compare the error performance of a WMLSTTC system, described in Section 6.2 with a MLSTTC system. For comparison purposes, we have used a MLSTTC system with same specifications as the above WMLSTTC system but without weighting. The spectral efficiency is 4 bits/sec/Hz and the underlying constellation is 16-QAM for both MLSTTC and WMLSTTC systems. Two identical 4-state STTCs based on *trace* criterion are used as component codes for both the systems. Performance is evaluated using 2 and 4 transmit antennas, so, we use component codes designed for 2 and 4 transmit antennas [21], trellis structures of which are shown in Fig. 4.3 (Chapter 4) and Fig. 6.8 respectively. We assumed perfect CSI at the receiver and the transmitter both.

Fig. 6.10, 6.11, 6.12 and 6.13 exhibits the FER and SER performance of WMLSTTC and MLSTTC system plotted against signal to noise ratio (SNR) for different transmit and receive antennas. It can be noted that the performance of the MLSTTC system is dramatically improved by the weighting.

Fig. 6.10 demonstrates the error performance comparison for two transmit and one receive antenna. It can be seen that WMLSTTC is superior to MLSTTC by about 1.7 dB at the SER of 10^{-1}.

Fig. 6.11 demonstrates the error performance comparison for two transmit and two receive antenna. It can be seen that WMLSTTC is superior to MLSTTC by about 1.3 dB at the FER of 10^{-1}.

Fig. 6.12 demonstrates the error performance comparison for two transmit and four receive antenna. It can be seen that WMLSTTC is superior to MLSTTC by about 0.9 dB at the FER of 10^{-2}.

Fig. 6.13 demonstrates the error performance comparison for four transmit and four receive antenna. It can be seen that WMLSTTC is superior to MLSTTC by about 0.5 dB at the FER of 10^{-2}.

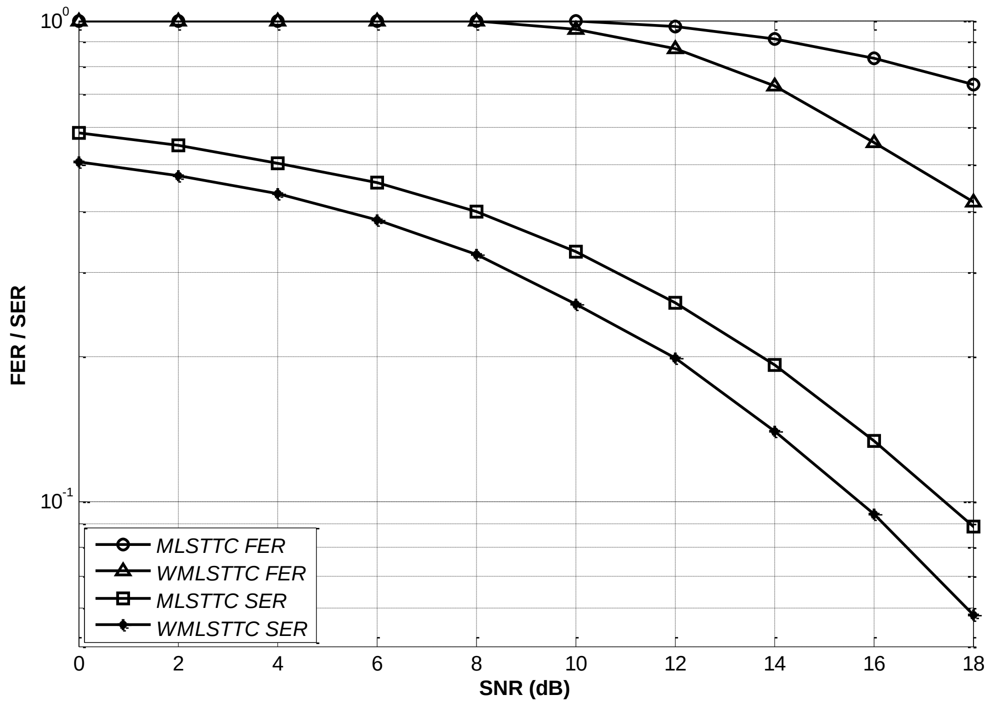

Figure 6.10 Error performance of WMLSTTC vs. MLSTTC for two transmit and one receive antenna.

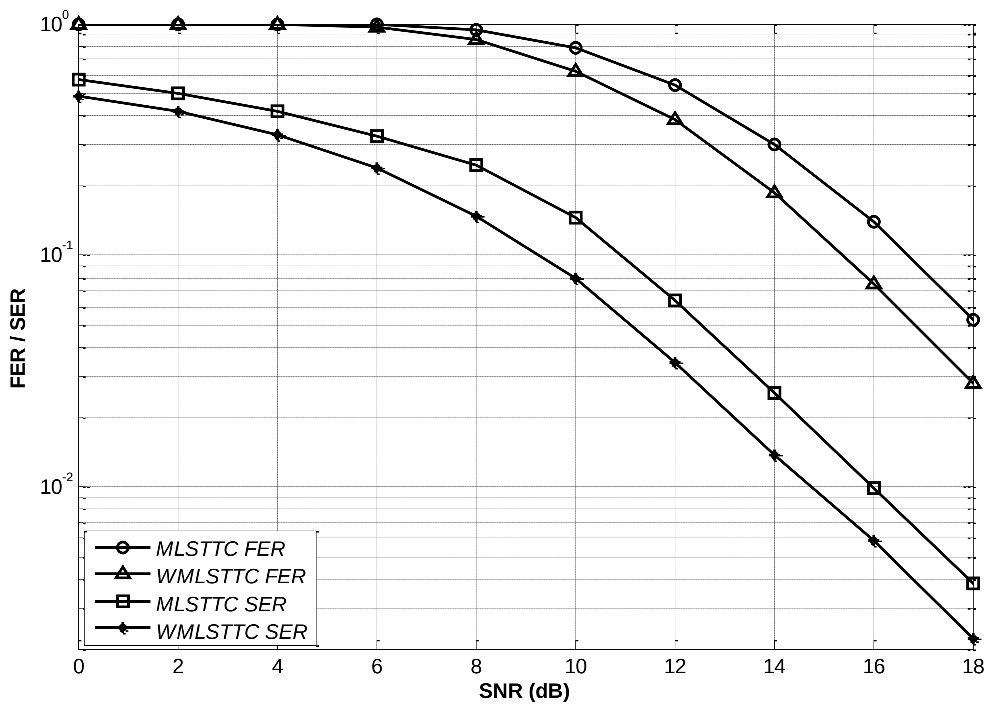

Figure 6.11 Error performance of WMLSTTC vs. MLSTTC for two transmit and two receive antennas.

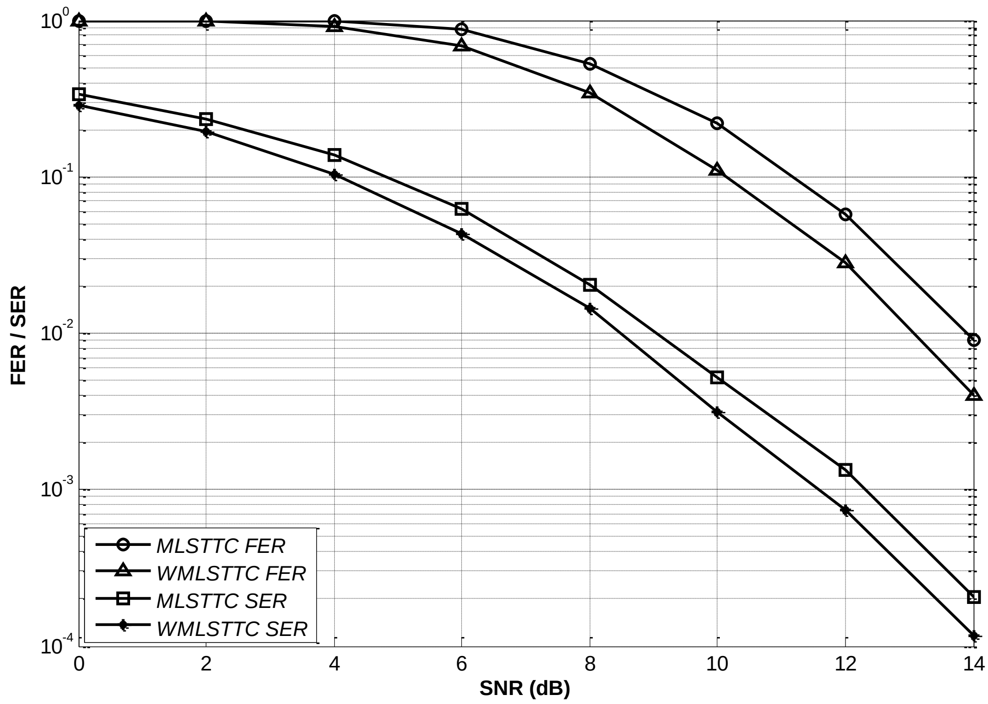

Figure 6.12 Error performance of WMLSTTC vs. MLSTTC for two transmit and four receive antennas.

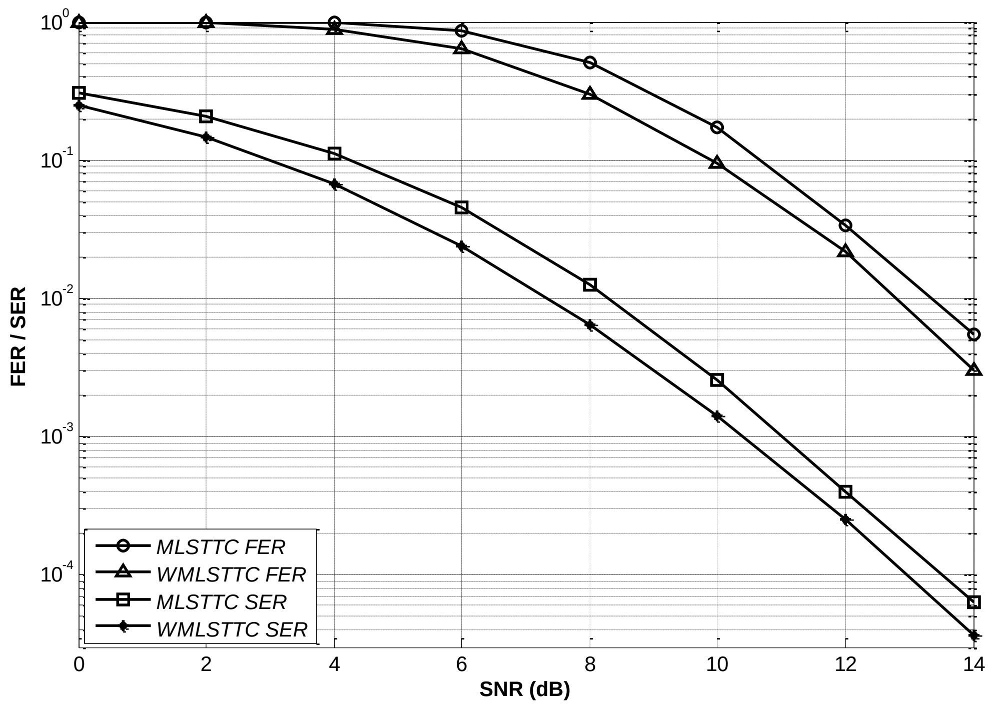

Figure 6.13 Error performance of WMLSTTC vs. MLSTTC for four transmit and four receive antennas.

6.6 Summary

In Chapter 5 we proposed a transmission scheme, WMLSTTC, as an extension for the MLSTTC [33] scheme, without sacrificing the capability of simultaneously providing bandwidth efficiency, diversity improvement and coding gain with reduced decoding complexity, especially for larger constellations and higher throughputs.

In this chapter we evaluated the WMLSTTC system and its performance via examples based on an underlying 16-QAM constellation and using up to 4 transmit and 4 receive antennas and throughput of up to 4 bits/sec/Hz. We observed the effect of transmit and receive diversity on the performance of the system. We also compared the WMLSTTC system with a similar MLSTTC system without weighting, for different number of transmit and receive antennas.

Simulation results show that the proposed WMLSTTC scheme considerably outperforms the conventional MLSTTCs without weighting and without sacrificing the capability of simultaneously providing bandwidth efficiency, diversity improvement and coding gain with reduced decoding complexity.

In the next chapter, summary of the thesis work and concluding remarks are presented.

Chapter 7

Conclusion

In this thesis we presented a new transmission scheme that we call weighted multilevel space-time trellis codes (WMLSTTCs), as an extension for the MLSTTC scheme, without sacrificing the capability of simultaneously providing bandwidth efficiency, diversity improvement and coding gain with reduced decoding complexity, especially for larger constellations and higher throughputs.

In this final chapter, we provide the summary of the work described in previous chapters followed by the conclusion.

7.1 Summary and conclusion

Recent advances in wireless communication systems have increased the throughput over wireless channels. The reliability of wireless communication has also been increased. But still the bandwidth and spectral availability demands are endless. The need to achieve reliable wireless systems with high spectral efficiency, low complexity and good error performance results in continued research in this field.

As the available radio spectrum is limited, higher data rates can only be achieved by designing more efficient signaling techniques. Thus, we need to construct STTCs with high order signal constellations but this normally involves the use of computer search, with the search space increasing exponentially with constellation size, the number of transmit antennas and the number of states in the code trellis. Decoding complexity of STTCs also increase in a similar manner. Therefore, despite their many benefits, STTCs are still faced with reluctance from system designers when it comes to implementation, especially when for systems which require the use of larger signal constellations or a larger number of antennas.

Recent works like that done in MLSTTCs [33], high rate CPFSK STTCs [35] and Grouped MLSTTC [36] have successfully achieved significant improvement over conventional STTCs. MLSTTCs solved the complexity problem faced by STTCs and that to without sacrificing the advantages of STTCs. MLSTTCs are a promising alternative to STTCs as they offer the flexibility of having a higher spectral efficiency and lower decoding complexity, especially for larger constellations and large number of states. MLSTTCs make use of MLC concepts, which are summarized in Chapter 3. The partitioning scheme used is based on MRM. The overall structure and analytical model of MLSTTCs is described in chapter 4.

In this work we extend the work of M. Baghaie [33] in which MLSTTCs were designed by assuming perfect CSI available at receiver. The system performance can be improved dramatically if perfect or partial CSI is made available at the transmitter. A novel design criterion for MLSTTC with CSI at transmitter is proposed, that incorporates the statistical information concerning the channel estimates. New improved Weighed MLSTTC codes are obtained using a novel combination of beamforming and MLSTTC. The overall structure and analytical model of WMLSTTCs is described in chapter 5.

Throughout the present work, we use STTCs as component codes, although it is possible to use other component codes (including block codes). To design STTCs, we use the trace criterion [21] which aims to maximize the minimum Euclidean distance between distinct codewords.

Performance of the WMLSTTC system is demonstrated in chapter 6, by simulation for up to 4 antennas and up to 4 bits/sec/Hz using an underlying 16 QAM signal constellation. Performance of the WMLSTTC is also compared with a similar MLSTTC system without weighting. It is shown that the proposed WMLSTTC scheme results in a significant performance improvement of the MLSTTC system. On average WMLSTTC is superior to MLSTTC by about 1 dB at same FER.

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Appendix A

STTC Generators Source Code (Matlab)

```
function [varargout] = STTC_Generators(M, numOfStates, Nt, criteria, code)

%  Returns  an  appropriate  GENERATOR  MATRIX  according  to  the  specified  input
%   arguments.
%
% Syntax:
%            [varargout] = STTC_Generators(M, Tr, Data_In)
%
% Description:
%
%            Inputs:
%                M           ->  Number of CONSTELLATION POINTS. (M-PSK, M-QAM).
%                numOfStates ->  Number of STATES.
%                Nt          ->  Number of TRANSMITTING (Tx) ANTENNAS.
%                criteria    ->  Criteria to use.
%                                1. RANK & DETERMINANT Criteria. ('R&Dc')
%                                2. TRACE Criteria.              ('Tc')
%                code        ->  Code to use.
%                                1. Tarokh/Seshadri/Calderbank (TSC) codes.
%                                2. Baro/Bauch/Hansmann (BBH) codes.
%                                3. Optimum (Opt) codes.
%
%            Outputs:
%                varargout(1), varargout(2), varargout(3) ...
%                The GENERATOR MATRICES.
%
% Examples:
%
%            For 4-PSK or QPSK (M = 4) STTC with 4 STATES and 2 Tx antennas,
%            the GENERATORs for R&D criteria using TSC codes will be:-
%
%                [g1 g2] = STTC_Generators(M, 4, 2, 'R&Dc', 'TSC')
%
%                 g1 =
%                      0     2
%                      2     0
%
%                 g2 =
%                      0     1
%                      1     0
```

```
%{

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Permission is hereby granted, free of charge, to any person obtaining a copy of
this software and associated documentation files (the "STTC_Generators.m"), to
deal in the Software without restriction, including without limitation the
rights to use, copy, modify, merge, publish, distribute, sublicense, and/or sell
copies of the Software, and to permit persons to whom the Software is furnished
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THE SOFTWARE IS PROVIDED "AS IS", WITHOUT WARRANTY OF ANY KIND, EXPRESS OR
IMPLIED, INCLUDING BUT NOT LIMITED TO THE WARRANTIES OF MERCHANTABILITY, FITNESS
FOR A PARTICULAR PURPOSE AND NONINFRINGEMENT. IN NO EVENT SHALL THE AUTHORS OR
COPYRIGHT HOLDERS BE LIABLE FOR ANY CLAIM, DAMAGES OR OTHER LIABILITY, WHETHER
IN AN ACTION OF CONTRACT, TORT OR OTHERWISE, ARISING FROM, OUT OF OR IN
CONNECTION WITH THE SOFTWARE OR THE USE OR OTHER DEALINGS IN THE SOFTWARE.

%}

%% Check Parameters

if strcmpi(criteria, 'Rank & Determinant Criteria')
    criteria = 'R&Dc';
elseif strcmpi(criteria, 'Trace Criteria')
    criteria = 'Tc';
end

if ~(strcmpi(criteria, 'R&Dc') || strcmpi(criteria, 'Tc'))
    error ('The CRITERIA must be R&Dc or Tc.');
end

if strcmpi(criteria, 'R&Dc')
    if strcmpi(code, 'Tarokh/Seshadri/Calderbank (TSC)')
        code = 'TSC';
    elseif strcmpi(code, 'Baro/Bauch/Hansmann (BBH)')
        code = 'BBH';
    elseif strcmpi(code, 'Optimal')
        code = 'Opt';
    end

    if ~(strcmpi(code, 'TSC') || strcmpi(code, 'BBH') || strcmpi(code, 'Opt'))
        error ('The CODE must be TSC, BBH or Opt.');
    end
end


%% Select the STTC GENERATOR MATRIX.

if M == 4
    if numOfStates == 4
        if Nt == 2
            if strcmpi(criteria, 'R&Dc')
                if strcmpi(code, 'TSC')
                    varargout(1) = { [0 2 ; 2 0] };
                    varargout(2) = { [0 1 ; 1 0] };
```

```
            elseif strcmpi(code, 'BBH')
                varargout(1) = { [2 2 ; 1 0] };
                varargout(2) = { [0 2 ; 3 1] };
            elseif strcmpi(code, 'Opt')
                varargout(1) = { [0 2 ; 1 0] };
                varargout(2) = { [2 2 ; 0 1] };
            end
        elseif strcmpi(criteria, 'Tc')
            varargout(1) = { [0 2 ; 1 2] };
            varargout(2) = { [2 3 ; 2 0] };
        end
    elseif Nt == 3
        if strcmpi(criteria, 'R&Dc')
            error ('GENERATORS not available for R&Dc criteria.');
        elseif strcmpi(criteria, 'Tc')
            varargout(1) = { [0 2 2 ; 1 2 3] };
            varargout(2) = { [2 3 3 ; 2 0 2] };
        end
    elseif Nt == 4
        if strcmpi(criteria, 'R&Dc')
            error ('GENERATORS not available for R&Dc criteria.');
        elseif strcmpi(criteria, 'Tc')
            varargout(1) = { [0 2 2 0 ; 1 2 3 2] };
            varargout(2) = { [2 3 3 2 ; 2 0 2 1] };
        end
    else
        error ('GENERATORS are available for Nt = 2, 3 and 4 only.');
    end
elseif numOfStates == 8
    if Nt == 2
        if strcmpi(criteria, 'R&Dc')
            if strcmpi(code, 'TSC')
                varargout(1) = { [0 2 ; 2 0] };
                varargout(2) = { [0 1 ; 1 0 ; 2 2] };
            elseif strcmpi(code, 'BBH')
                varargout(1) = { [2 2 ; 2 0] };
                varargout(2) = { [0 1 ; 1 0 ; 2 2] };
            elseif strcmpi(code, 'Opt')
                varargout(1) = { [0 2 ; 2 0] };
                varargout(2) = { [2 1 ; 1 2 ; 0 2] };
            end
        elseif strcmpi(criteria, 'Tc')
            varargout(1) = { [2 2 ; 2 1] };
            varargout(2) = { [2 0 ; 1 2 ; 0 2] };
        end
    elseif Nt == 3
        if strcmpi(criteria, 'R&Dc')
            error ('GENERATORS not available for R&Dc criteria.');
        elseif strcmpi(criteria, 'Tc')
            varargout(1) = { [2 2 2 ; 2 1 1] };
            varargout(2) = { [2 0 3 ; 1 2 0 ; 0 2 2] };
        end
    elseif Nt == 4
        if strcmpi(criteria, 'R&Dc')
            error ('GENERATORS not available for R&Dc criteria.');
        elseif strcmpi(criteria, 'Tc')
            varargout(1) = { [2 2 2 2 ; 2 1 1 2] };
            varargout(2) = { [2 0 3 1 ; 1 2 0 3 ; 0 2 2 1] };
        end
```

```
    else
        error ('GENERATORS are available for Nt = 2, 3 and 4 only.');
    end
elseif numOfStates == 16
    if Nt == 2
        if strcmpi(criteria, 'R&Dc')
            if strcmpi(code, 'TSC')
                varargout(1) = { [0 2 ; 2 0 ; 0 2] };
                varargout(2) = { [0 1 ; 1 2 ; 2 0] };
            elseif strcmpi(code, 'BBH')
                varargout(1) = { [0 2 ; 2 0 ; 0 2] };
                varargout(2) = { [2 1 ; 1 2 ; 2 0] };
            elseif strcmpi(code, 'Opt')
                varargout(1) = { [0 2 ; 1 2 ; 2 2] };
                varargout(2) = { [2 0 ; 1 1 ; 0 2] };
            end
        elseif strcmpi(criteria, 'Tc')
            varargout(1) = { [1 2 ; 1 3 ; 3 2] };
            varargout(2) = { [2 0 ; 2 2 ; 2 0] };
        end
    elseif Nt == 3
        if strcmpi(criteria, 'R&Dc')
            if strcmpi(code, 'TSC')
                varargout(1) = { [0 0 2 ; 0 1 2 ; 2 3 1] };
                varargout(2) = { [2 0 0 ; 1 2 0 ; 2 3 3] };
            else
                error ('GENERATORS not available for BBH and Opt codes.');
            end
        elseif strcmpi(criteria, 'Tc')
            varargout(1) = { [1 2 1 ; 1 3 2 ; 3 2 1] };
            varargout(2) = { [2 0 2 ; 2 2 0 ; 2 0 2] };
        end
    elseif Nt == 4
        if strcmpi(criteria, 'R&Dc')
            error ('GENERATORS not available for R&Dc criteria.');
        elseif strcmpi(criteria, 'Tc')
            varargout(1) = { [1 2 1 1 ; 1 3 2 2 ; 3 2 1 3] };
            varargout(2) = { [2 0 2 2 ; 2 2 0 0 ; 2 0 2 2] };
        end
    else
        error ('GENERATORS are available for Nt = 2, 3 and 4 only.');
    end
elseif numOfStates == 32
    if Nt == 2
        if strcmpi(criteria, 'R&Dc')
            if strcmpi(code, 'TSC')
                varargout(1) = { [0 2 ; 2 2 ; 3 3] };
                varargout(2) = { [0 1 ; 1 1 ; 2 0 ; 2 2] };
            elseif strcmpi(code, 'BBH')
                error ('GENERATORS not available for BBH codes.');
            elseif strcmpi(code, 'Opt')
                varargout(1) = { [2 0 ; 2 3 ; 0 2] };
                varargout(2) = { [2 2 ; 1 0 ; 1 2 ; 2 2] };
            end
        elseif strcmpi(criteria, 'Tc')
            varargout(1) = { [0 2 ; 2 3 ; 1 2] };
            varargout(2) = { [2 2 ; 1 2 ; 2 3 ; 2 0] };
        end
```

```
        elseif Nt == 3
            if strcmpi(criteria, 'R&Dc')
                if strcmpi(code, 'TSC')
                    varargout(1) = { [0 2 1 ; 2 0 0 ; 0 0 2] };
                    varargout(2) = { [3 1 0 ; 3 2 1 ; 3 2 2 ; 2 0 0] };
                else
                    error ('GENERATORS not available for BBH and Opt codes.');
                end
            elseif strcmpi(criteria, 'Tc')
                varargout(1) = { [0 2 2 ; 2 3 3 ; 1 2 2] };
                varargout(2) = { [2 2 0 ; 1 2 2 ; 2 3 1 ; 2 0 0] };
            end
        elseif Nt == 4
            if strcmpi(criteria, 'R&Dc')
                error ('GENERATORS not available for R&Dc criteria.');
            elseif strcmpi(criteria, 'Tc')
                varargout(1) = { [0 2 2 2 ; 2 3 3 2 ; 1 2 2 1] };
                varargout(2) = { [2 2 0 1 ; 1 2 2 0 ; 2 3 1 0 ; 2 0 0 2] };
            end
        else
            error ('GENERATORS are available for Nt = 2, 3 and 4 only.');
        end
    elseif numOfStates == 64
        if Nt == 2
            if strcmpi(criteria, 'R&Dc')
                if strcmpi(code, 'TSC')
                    error ('GENERATORS not available for TSC and BBH codes.');
                elseif strcmpi(code, 'BBH')
                    error ('GENERATORS not available for TSC and BBH codes.');
                elseif strcmpi(code, 'Opt')
                    varargout(1) = { [1 2 ; 2 2 ; 0 3 ; 2 0] };
                    varargout(2) = { [2 0 ; 2 0 ; 1 3 ; 0 2] };
                end
            elseif strcmpi(criteria, 'Tc')
                varargout(1) = { [0 2 ; 3 1 ; 3 3 ; 3 2] };
                varargout(2) = { [2 2 ; 2 2 ; 0 0 ; 2 0] };
            end
        elseif Nt == 3
            if strcmpi(criteria, 'R&Dc')
                if strcmpi(code, 'TSC')
                    varargout(1) = { [1 1 2 ; 2 1 2 ; 1 2 0 ; 2 0 0] };
                    varargout(2) = { [0 3 0 ; 0 3 2 ; 2 2 1 ; 0 2 2] };
                else
                    error ('GENERATORS not available for BBH and Opt codes.');
                end
            elseif strcmpi(criteria, 'Tc')
                varargout(1) = { [0 2 2 ; 3 1 0 ; 3 3 2 ; 3 2 1] };
                varargout(2) = { [2 2 0 ; 2 2 2 ; 0 0 3 ; 2 0 1] };
            end
        elseif Nt == 4
            if strcmpi(criteria, 'R&Dc')
                if strcmpi(code, 'TSC')
                    varargout(1) = { [0 3 0 2 ; 2 3 0 2 ; 2 1 1 1 ; 2 2 2 0] };
                    varargout(2) = { [3 0 2 0 ; 2 2 0 2 ; 0 0 3 2 ; 0 2 2 0] };
                else
                    error ('GENERATORS not available for BBH and Opt codes.');
                end
```

```
            elseif strcmpi(criteria, 'Tc')
                varargout(1) = { [0 2 2 1 ; 3 1 0 2 ; 3 3 2 2 ; 3 2 1 3] };
                varargout(2) = { [2 2 0 2 ; 2 2 2 0 ; 0 0 3 1 ; 2 0 1 2] };
            end
        else
            error ('GENERATORS are available for Nt = 2, 3 and 4 only.');
        end
    else
        error ('GENERATORS are available for STATES = 4, 8, 16, 32 and 64
only.');
    end
elseif M == 8
    if numOfStates == 8
        if Nt == 2
            if strcmpi(criteria, 'R&Dc')
                if strcmpi(code, 'TSC')
                    varargout(1) = { [0 4 ; 4 0] };
                    varargout(2) = { [0 2 ; 2 0] };
                    varargout(3) = { [0 1 ; 5 0] };
                elseif strcmpi(code, 'BBH')
                    error ('GENERATORS not available for BBH codes.');
                elseif strcmpi(code, 'Opt')
                    varargout(1) = { [0 2 ; 2 0] };
                    varargout(2) = { [0 4 ; 4 0] };
                    varargout(3) = { [4 5 ; 1 4] };
                end
            elseif strcmpi(criteria, 'Tc')
                varargout(1) = { [2 1 ; 3 4] };
                varargout(2) = { [4 6 ; 2 0] };
                varargout(3) = { [0 4 ; 4 0] };
            end
        elseif Nt == 3
            if strcmpi(criteria, 'R&Dc')

                error ('GENERATORS not available for R&Dc criteria.');
            elseif strcmpi(criteria, 'Tc')
                varargout(1) = { [2 1 3 ; 3 4 0] };
                varargout(2) = { [4 6 2 ; 2 0 4] };
                varargout(3) = { [0 4 4 ; 4 0 2] };
            end
        elseif Nt == 4
            if strcmpi(criteria, 'R&Dc')
                error ('GENERATORS not available for R&Dc criteria.');
            elseif strcmpi(criteria, 'Tc')
                varargout(1) = { [2 1 3 7 ; 3 4 0 5] };
                varargout(2) = { [4 6 2 2 ; 2 0 4 4] };
                varargout(3) = { [0 4 4 4 ; 4 0 2 0] };
            end
        else
            error ('GENERATORS are available for Nt = 2, 3 and 4 only.');
        end
    elseif numOfStates == 16
        if Nt == 2
            if strcmpi(criteria, 'R&Dc')
                if strcmpi(code, 'TSC')
                    varargout(1) = { [0 4 ; 4 4] };
                    varargout(2) = { [0 2 ; 2 2] };
                    varargout(3) = { [0 1 ; 5 1 ; 1 5] };
```

```
                elseif strcmpi(code, 'BBH')
                    error ('GENERATORS not available for BBH codes.');
                elseif strcmpi(code, 'Opt')
                    varargout(1) = { [0 4 ; 4 0] };
                    varargout(2) = { [0 2 ; 2 0] };
                    varargout(3) = { [2 0 ; 6 5 ; 1 4] };
                end
            elseif strcmpi(criteria, 'Tc')
                varargout(1) = { [2 4 ; 3 7] };
                varargout(2) = { [4 0 ; 6 6] };
                varargout(3) = { [7 2 ; 0 7 ; 4 4] };
            end
        elseif Nt == 3
            if strcmpi(criteria, 'R&Dc')
                error ('GENERATORS not available for R&Dc criteria.');
            elseif strcmpi(criteria, 'Tc')
                varargout(1) = { [2 4 2 ; 3 7 2 ] };
                varargout(2) = { [4 0 4 ; 6 6 4] };
                varargout(3) = { [7 2 2 ; 0 7 6 ; 4 4 0] };
            end
        elseif Nt == 4
            if strcmpi(criteria, 'R&Dc')
                error ('GENERATORS not available for R&Dc criteria.');
            elseif strcmpi(criteria, 'Tc')
                varargout(1) = { [2 4 2 2 ; 3 7 2 4] };
                varargout(2) = { [4 0 4 4 ; 6 6 4 0] };
                varargout(3) = { [7 2 2 0 ; 0 7 6 3 ; 4 4 0 2] };
            end
        else
            error ('GENERATORS are available for Nt = 2, 3 and 4 only.');
        end
    elseif numOfStates == 32
        if Nt == 2
            if strcmpi(criteria, 'R&Dc')
                if strcmpi(code, 'TSC')
                    varargout(1) = { [0 4 ; 4 4] };
                    varargout(2) = { [0 2 ; 2 2 ; 2 2] };
                    varargout(3) = { [0 1 ; 5 1 ; 3 7] };
                elseif strcmpi(code, 'BBH')
                    error ('GENERATORS not available for BBH codes.');
                elseif strcmpi(code, 'Opt')
                    varargout(1) = { [0 4 ; 4 4] };
                    varargout(2) = { [0 2 ; 2 2 ; 2 0] };
                    varargout(3) = { [3 5 ; 0 0 ; 4 0] };
                end
            elseif strcmpi(criteria, 'Tc')
                varargout(1) = { [0 4 ; 4 4] };
                varargout(2) = { [0 2 ; 2 3 ; 2 2] };
                varargout(3) = { [4 2 ; 4 2 ; 3 7] };
            end
        elseif Nt == 3
            if strcmpi(criteria, 'R&Dc')
                error ('GENERATORS not available for R&Dc criteria.');
            elseif strcmpi(criteria, 'Tc')
                varargout(1) = { [0 4 0 ; 4 4 4] };
                varargout(2) = { [0 2 4 ; 2 3 7 ; 2 2 7] };
                varargout(3) = { [4 2 6 ; 4 2 0 ; 3 7 2] };
            end
```

```
        elseif Nt == 4
            if strcmpi(criteria, 'R&Dc')
                error ('GENERATORS not available for R&Dc criteria.');
            elseif strcmpi(criteria, 'Tc')
                varargout(1) = { [0 4 0 3 ; 4 4 4 3] };
                varargout(2) = { [0 2 4 2 ; 2 3 7 1 ; 2 2 7 5] };
                varargout(3) = { [4 2 6 5 ; 4 2 0 7 ; 3 7 2 6] };
            end
        else
            error ('GENERATORS are available for Nt = 2, 3 and 4 only.');
        end
    else
        error ('GENERATORS are available for STATES = 8, 16, and 32 only.');
    end
else
    error ('GENERATORS are available for M = 4 and 8 only.');
end
```

Appendix B

STTC Generator to Trellis Source Code (Matlab)

```
function [Trellis varargout] = STTC_Generator_to_Trellis(varargin)

% Create the Space Time TRELLIS Structure from the GENERATOR MATRICES.
%
% Syntax:
%
%           [Trellis Tr1] = STTC_Generator_to_Trellis(g1)
%           [Trellis Tr1 Tr2] = STTC_Generator_to_Trellis(g1, g2)
%           [Trellis Tr1 Tr2 ...] = STTC_Generator_to_Trellis(g1, g2, ...)
%
% Description:
%
%           Inputs:
%               g1, g2, g3 ... are the GENERATOR MATRICES.
%
%           Outputs:
%               Trellis returns the whole Output Matrix.
%               Tr1, Tr2, Tr3 ... are the CURRENT STATE Trellis,
%                                        NEXT STATE Trellis,
%                                        NEXT STATE Trellis and so on.
%
% Examples:
%
%           For 4-PSK or QPSK STTC with 2 Tx antennas the generator
%           matrices are:-
%
%                       g1 = [0 2 ; 2, 0];
%                       g2 = [0 1 ; 1, 0];
%
%           There are 2 Tx antennas, thus, we have only 2 trellis:-
%
%                   Tr1 -> CURRENT STATE Trellis
%                   Tr2 -> NEXT STATE Trellis
%
%           [Tr Tr1 Tr2] = STTC_Generator_to_Trellis(g1, g2);
%
%           Tr =
%
%               0   0   0   0   1   1   1   1   2   2   2   2   3   3   3   3
%               0   1   2   3   0   1   2   3   0   1   2   3   0   1   2   3
%
```

```
%           Tr1 =
%
%               0     0     0     0
%               1     1     1     1
%               2     2     2     2
%               3     3     3     3
%
%           Tr2 =
%
%               0     1     2     3
%               0     1     2     3
%               0     1     2     3
%               0     1     2     3
%

%{

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%}

%% Extract the no. of STATES and no. of outputs (Tx Antennas).

m = size(varargin,2);      % No. of bits per symbol = No. of Generator Matrices.

M = 2^m;                   % No. of constellation points (M-PSK or M-QAM).

if m <= 0
    error ('Number of Generator Matrices must be >= 1.');
end

tempNt = 0;

v_stage = zeros(1, m);     % Memory order per stage. v1, v2 .... vm;

for i = 1 : m

    v_stage(i) = size(varargin{i},1) - 1;

    if i == 1
        tempNt = size(varargin{i},2);
```

```
    elseif tempNt ~= size(varargin{i},2)
        error ('Number of columns of each Generator Matrix must be same.');
    end

    if ndims(varargin{i}) ~= 2
        error ('Each Generator Matrix must be a 2D matrix.');
    end
end

v = sum(v_stage);          % Memory order, v = Total no. of registers used.

Nt = tempNt;               % No. of Tx Antennas.
numOfStates = 2^v;         % No. of STATES.


%% Create single GENERATOR Matrix, G.

numBr = M * numOfStates;   % Total no. of transition branches in trellis.

G = zeros(log2(numBr), Nt);

i = log2(numBr);
j = 1;

while i > 0
    for k = (m : -1 : 1)
        G(i, :) = varargin{k}(j , :);
        i = i - 1;

        if i <= 0
            break;
        end
    end
    j = j + 1;
end


%% Create Trellis.

outputMatrix = zeros(numBr, Nt);

for i = 1 : numBr
    b = dec2bin(i - 1, log2(numBr)) - 48;
    outputMatrix(i,:) = mod((b * G), M);
end

Trellis = outputMatrix';

for i = 1 : Nt
    varargout(i) = {(reshape((outputMatrix(:,i)), M, numOfStates))'};
end
```

Appendix C

STTC Encoder Source Code (Matlab)

```
function X = STTC_Encoder(M, Tr, Data_In)

% Encodes the data BITs using SPACE TIME TRELLIS CODES.
%
% Syntax:
%
%           X = STTC_Encoder(M, Tr, Data_In)
%
% Description:
%
%           Inputs:
%               M       ->  Number of CONSTELLATION POINTS (M-PSK or M-QAM).
%               Tr      ->  Output Matrix of the ENCODER TRELLIS.
%               Data_In ->  Data BITs to be encoded.
%
%           Outputs:
%               X       ->  Output CODEWORDS.
%
% Examples:
%
%           For 4-PSK or QPSK (M = 4) STTC with 2 Tx antennas the
%           generator matrices are:-
%                           g1 = [0 2 ; 2, 0];
%                           g2 = [0 1 ; 1, 0];
%           Trellis for the above generator matrices is:-
%
%           Tr = [
%
%               0   0   0   0   1   1   1   1   2   2   2   2   3   3   3   3
%               0   1   2   3   0   1   2   3   0   1   2   3   0   1   2   3 ];
%
%           Data_In = [ 1   0   0   1   1   1   0   0   0   1 ];
%
%           X = STTC_Encoder(M, Tr, Data_In)
%
%           X =
%
%               0     2     1     3     0
%               2     1     3     0     1
%
```

```
%{

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%}

%% Set Parameters.

if mod(length(Data_In) , log2(M)) ~= 0
    error ('Data_In length must be an integral multiple of log2(M).');
end

Nt = size(Tr,1);                        % Number of Tx Antennas.
numOfStates = length(Tr) / M;           % Number of STATES.
m = log2(M);                            % Number of BITs per SYMBOL.
num_of_Symbols = length(Data_In) / m;   % Number of symbols to encode.


%% Convert input data BITs into equivalent SYMBOLs.

C = zeros(1, (size(Data_In, 1) * size(Data_In, 2))/m);

bit_weights = (2*ones(m, 1)).^((m-1 : -1 : 0)');

for k = 1 : length(C)
    C(k) = sum(bit_weights .* (Data_In( m*(k-1) + 1 : m*(k-1) + m)'));
end


%% Encode.

X = zeros(Nt, num_of_Symbols);

Mapper = reshape(1 : numOfStates*M, M, numOfStates)';

currentState = 0;   % Initial STATE is always ZERO.

for k = 1 : num_of_Symbols
    X(:, k) = Tr(:, Mapper(currentState+1, C(k) + 1));
    currentState = mod(M*currentState, numOfStates) + C(k);
end
```

Appendix D

STTC Modulator Source Code (Matlab)

```
function modulated_Data = STTC_Modulator(Input_Data, M, Scheme)

% Modulates the input data BITs using M-PSK or M-QAM.
%
% Syntax:
%
%           modulated_Data = STTC_Modulator(Input_Data, M, Scheme)
%
% Description:
%
%           Inputs:
%               Input_Data  ->  Data BITs to be MODULATED.
%               M           ->  Number of CONSTELLATION POINTS.
%               Scheme      ->  'M-PSK' or 'M-QAM'.
%
%           Outputs:
%               modulated_Data  ->  MODULATED Output Data.
%
% Examples:
%
%           Consider 4-PSK or QPSK (M = 4) STTC with 2 Tx antennas and
%           ENCODED CODEWORDS X, given by
%
%           X =
%
%               0     2     1     3     0
%               2     1     3     0     1
%
%           modulated_X = STTC_Modulator(X, M, 'M-PSK')
%
%           modulated_X =
%
%               1    -1     j     -j    1
%              -1     j    -j      1    j
%
```

```
%{

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%}


if strcmpi(Scheme, 'M-PSK')
    modulated_Data = cos(2*pi*Input_Data/M) + 1i*sin(2*pi*Input_Data/M);
elseif strcmpi(Scheme, 'M-QAM')
    h = modem.qammod('M', M, 'SymbolOrder', 'Binary');
    modulated_Data = modulate(h, Input_Data);
else
    error ('Modulation scheme must be M-PSK or M-QAM.');
end
```

Appendix E

Viterbi Decoder Source Code (Matlab)

```
function [es_Symbols es_Bits] = Viterbi_Decoder(r, Tr, M, BM)

% VITERBI algorithm.
%
% Syntax:
%
%       [es_Symbols es_Bits] = Viterbi_Decoder(r, Tr, M, BM)
%
% Description:
%
%       Inputs:
%           r   ->  Recieved signal ( Fading + Noise ).
%           Tr  ->  Output Matrix of the ENCODER TRELLIS.
%           M   ->  Number of CONSTELLATION POINTS (M-PSK or M-QAM).
%           BM  ->  Branch metrics for whole trellis.
%
%       Outputs:
%           es_Symbols   ->  Estimation of the received SYMBOLs.
%           es_Bits      ->  Estimation of the received BITs.
%

%{

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%}
```

```
%% Set Parameters.

m = log2(M);                        % Number of bits per symbol.

nS = size(Tr, 2)/M;                 % Number of STATES.

num_of_Symbols = size(r, 2);        % Number of received SYMBOLs.

es_Symbols = zeros(1, num_of_Symbols);

es_Bits = zeros(1, length(es_Symbols) * m);


%% VITERBI ALGORITHM (MAXIMUM LIKELIHOOD DECODING).

PMs = zeros(nS, 1);
SH = zeros(nS, num_of_Symbols);

for k = 1 : num_of_Symbols
    BMs = reshape(BM(k, :), M, nS);
    BMs = BMs + repmat(PMs', M, 1);
    [PMs SH(:, k)] = min(reshape(reshape(BMs, 1, nS * M), nS, M), [], 2);
    SH(:, k) = (nS /M) * (SH(:, k) - 1) + floor((0: nS -1)/M)';
end

% Traceback.
for k = num_of_Symbols : -1 : 2
    es_Symbols(k-1) = SH(es_Symbols(k)+1 , k);
end

es_Symbols = mod(es_Symbols, M);


%% INITIAL & FINAL STATE is always assumed to be ZERO.

es_Symbols(1 : nS/M) = 0;
es_Symbols(end-nS/M + 1 : end) = 0;


%% Convert SYMBOLs to BITs.

for k = 1 : num_of_Symbols
    es_Bits(1, m*(k-1)+1 : m*(k-1) + m) = int8(dec2bin(es_Symbols(k), m)) - 48;
end
```

Appendix F

STTC Decoder Source Code (Matlab)

```
function [estimated_Symbols estimated_Bits] = STTC_Decoder(M, Tr, H, r, Scheme)

% Decodes the data BITs using MAXIMUM LIKELIHOOD DECODING employing the
% VITERBI algorithm.
%
% Syntax:
%
%     [estimated_Symbols estimated_Bits] = STTC_Decoder(M, Tr, H, r, Scheme)
%
% Description:
%
%     Inputs:
%           M       ->  Number of CONSTELLATION POINTS (M-PSK or M-QAM).
%           Tr      ->  Output Matrix of the ENCODER TRELLIS.
%           H       ->  Channel State Information (CSI) at receiver.
%           r       ->  Recieved signal ( Fading + Noise ).
%           Scheme  ->  'M-PSK' or 'M-QAM'.
%
%     Outputs:
%           estimated_Symbols   ->  Estimation of the received SYMBOLs.
%           estimated_Bits      ->  Estimation of the received BITs.
%
% Examples:
%
%     For 4-PSK or QPSK (M = 4) STTC with 2 Tx antennas (Nt) and
%     1 Rx antenna (Nr) the generator matrices are:-
%
%           g1 = [0 2 ; 2, 0];
%           g2 = [0 1 ; 1, 0];
%
%     Trellis for the above generator matrices is:-
%
%     Tr = [ 0   0   0   0   1   1   1   1   2   2   2   2   3   3   3   3
%            0   1   2   3   0   1   2   3   0   1   2   3   0   1   2   3 ];
%
%     Data_In = [ 0   0   1   1   1   1   0   1   0   0 ];
%
%     X = STTC_Encoder(M, Tr, Data_In);
%     modulated_X = STTC_Modulator(X, M, 'M-PSK');
%
%     % Rayleigh Fading.
%     H = rand(Nr, Nt) + 1i*rand(Nr, Nt);
%
%     % Faded Signal.
%     faded_r = H * modulated_X;
```

```
%      % Received Signal = Faded Signal + AWGN Noise.
%      r = awgn( faded_r , 15 );
%
%      [estimated_Symbols estimated_Bits] = STTC_Decoder(M, Tr, H, r, 'M-PSK')
%
%      estimated_Symbols =
%
%             2     1     3     0     1
%
%      estimated_Bits =
%
%             0     0     1     1     1     1     0     1     0     0

%{

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CONNECTION WITH THE SOFTWARE OR THE USE OR OTHER DEALINGS IN THE SOFTWARE.

%}


%% Set Parameters.

Nr = size(r,1);                    % Number of Rx Antennas.
numOfStates = size(Tr, 2)/M;       % Number of STATES.
num_of_Symbols = size(r, 2);       % Number of received SYMBOLs.


%% Decode using VITERBI ALGORITHM (MAXIMUM LIKELIHOOD DECODING).

X = STTC_Modulator(Tr, M, Scheme);

hX = (H * X);
BM = zeros(num_of_Symbols , numOfStates * M);

for k = 1 : num_of_Symbols
    for n = 1 : Nr
        BM(k, :) = BM(k, :) + ( abs( r(n, k) - ( hX(n, :) ) ) ).^2;
    end
end

[estimated_Symbols estimated_Bits] = Viterbi_Decoder(r, Tr, M, BM);
```

Appendix G

STTC Sample Code (Matlab)

```
% STTC Sample - Matlab Script.
%
% Evaluates FER, SER & BER performance of 4-state QPSK STTCs designed using
% the Rank & Determinant criteria for two transmit and 1 receive antenna.
%
% Value of the parameters like M, Scheme, numOfStates, Nt, Nr, Criterion etc.
% can be changed to evaluate the desired error performance.
%
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clc;
clear all;

startTime = tic;

%% Input Parameters.

M = 4;                          % Number of constellation points.
m = log2(M);                    % Number of bits per symbol.
Scheme = 'M-PSK';               % Modulation scheme to use. (M-PSK or M_QAM).

numOfStates = 4;                % Number of STATES (4, 8, 16, 32, 64).

Nt = 2;                         % Number of Tx Antennas.
Nr = 1;                         % Number of Rx Antennas.

Criterion = 'R&Dc';             % Criteria for GENERATORs. ('R&Dc' or 'Tc').
Code = 'TSC';                   % Code used for GENERATORs.('TSC', 'BBH' or 'Opt').

Tx_CSI = 0;                     % '1' -> Weighted STTC. '0' -> STTC.

Eb = 1;                         % Energy of each bit.
Es = log2(M) * Eb;              % Energy of each symbol.

SNRdB = 10:2:18;                % SNR in dB.
SNR = 10.^(SNRdB./10);          % SNR in linear scale.

N0 = Es ./ SNR;                 % Noise.

N_var = N0;                     % Noise Variance, SIGMA² = N0.
N_sd = sqrt(N_var);             % Noise Standard Deviation, SIGMA.

Fr = 1000;                      % Number of FRAMEs.
Sy = 130;                       % Number of symbols per FRAME.

disp('Started...');
```

```
%% Generate Trellis.

if M == 4
    [g1 g2] = STTC_Generators(M, numOfStates, Nt, Criterion, Code);
    [Tr Tr1 Tr2] = STTC_Generator_to_Trellis(g1, g2);
elseif M == 8
    [g1 g2 g3] = STTC_Generators(M, numOfStates, Nt, Criterion, Code);
    [Tr Tr1 Tr2] = STTC_Generator_to_Trellis(g1, g2, g3);
end


%% Generate random binary data BITs.

Data_In = round(rand(1, Sy*m));


%% Make appropriate number of BITs ZERO.

zerosNeeded = m * (numOfStates/M);

if zerosNeeded <= length(Data_In)
    Data_In(1 : zerosNeeded) = 0;
    Data_In(end-zerosNeeded + 1 : end) = 0;
else
    Data_In = zeros(1, zerosNeeded);
end


%% Convert Data BITs into equivalent SYMBOLs.

Data_In_Sy = zeros(1, (size(Data_In, 1) * size(Data_In, 2))/m);
bit_weights = (2*ones(m, 1)).^((m-1 : -1 : 0)');

for k = 1 : length(Data_In_Sy)
    Data_In_Sy(k) = sum(bit_weights .* (Data_In( m*(k-1) + 1 : m*(k-1) + m)'));
end


%% Encode.

X = STTC_Encoder(M, Tr, Data_In);


%% Modulate.

modulated_X = STTC_Modulator(X, M, Scheme);


%% Calculate BER, SER and FER.

FER = zeros(1, length(SNRdB));
SER = zeros(1, length(SNRdB));
BER = zeros(1, length(SNRdB));

for i = 1 : Fr
    % Complex White Gaussian Noise with mean ZERO and variance ONE.
    % Variance ONE means variance of 1/2 per dimension (Real & Imaginary).
    WGN = (randn(Nr, length(Data_In_Sy)) + 1i*randn(Nr, length(Data_In_Sy)));
    WGN = sqrt(1/2) .* WGN;
```

```
    % Rayleigh Fading.
    % Modelled as an i.i.d complex Gaussian random variable with zero mean
    % and variance 1/2 per dimension.
    % Variance 1/2 per dimension means that variance of real and imaginary
    % parts is 1/2 each thereby making the overall variance equal to 1, as
    % ( ( sqrt(1/2)^2 ) + ( sqrt(1/2)^2 ) ) = 1.
    H = sqrt(1/2) .* (randn(Nr, Nt) + 1i*randn(Nr, Nt));

    % Channel State Information is at the Transmitter.
    if Tx_CSI == 1
        Hstar = (H').';

        W = zeros(Nr, Nt);

        for j = 1 : Nr
            W(j, :) = (Hstar(j, :) ./ sqrt(sum(abs(H(j,:)).^2)));
        end

        H = H .* W;
    else
        H = sqrt(Es/Nt) .* H;
    end

    % Faded Signal.
    faded_r = (H * modulated_X);

    for j = 1 : length(SNRdB)
        % Additive White Gaussian Noise with mean ZERO and variance No/2.
        AWGN = N_sd(j) * WGN;

        % Received Signal = Faded Signal + AWGN Noise.
        r = faded_r + AWGN;

        % Demodulated & Decoded signal.
        [estimated_Symbols estimated_Bits] = STTC_Decoder(M, Tr, H, r, Scheme);

        % Count Error Bits.
        Error_Bits = sum(xor(Data_In, estimated_Bits));

        % Count Error Symbols.
        Error_Symbols = sum(bsxfun(@ne, Data_In_Sy, estimated_Symbols));

        % Calculate BER.
        BER(j) = BER(j) + Error_Bits;

        % Calculate SER.
        SER(j) = SER(j) + Error_Symbols;

        % Calculate FER.
        FER(j) = FER(j) + ( Error_Bits & 1 ) ;

        pause(.0000000001)
    end
end

BER = BER ./ (Fr * length(Data_In));
SER = SER ./ (Fr * length(Data_In_Sy));
FER = FER ./ Fr;
```

```
%% Plot FER, SER & BER.

figure;

semilogy(SNRdB, FER,'-kO'); hold on
semilogy(SNRdB, SER,'-r^'); hold on
semilogy(SNRdB, BER,'-bS'); hold on

xlabel('SNR (in dB)', 'FONTSIZE', 13);
ylabel('FER / SER / BER', 'FONTSIZE', 13);

legend('FER', 'SER', 'BER' );

hold off;
grid on;


%% Time Elapsed.

elapsedTime = toc(startTime);
disp(['Time Elapsed' ' - ' num2str(elapsedTime) ' seconds']);
disp('Stopped...');
```