PHENOMENOLOGY OF A CLASS OF NEUTRINO MASS MATRICES

A THESIS SUBMITTED TO
THE FACULTY OF SCHOOL OF PHYSICS AND
MATERIAL SCIENCE
THAPAR UNIVERSITY

FOR THE AWARD OF
THE DEGREE OF
MASTER OF SCIENCE IN PHYSICS

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2009

Dedicated to

My Family

ACKNOWLEDGEMENT

Knowledge in itself is a continuous process. I would have never succeeded in completing my task without cooperation, encouragement and help provided to me by various personalities.

I would like to give my thanks to **Dr. O. P. Pandey**, Professor and Head, School of Physics and Material Science, for his full motivation and appreciation to my work. With deep sense of gratitude I express my sincere thanks to my worthy superviser, **Dr. Sanjeev Kumar**, for his valuable guidance in caring out work under his effective supervision, encouragement and cooperation.All my friends at the School of Physics and Material Sciences are acknowledged for providing me a friendly atmosphere and encouraging me throughout this work. Their assistance and partnership were of great pleasure. Their views were very insightful and helpful.

I am deeply thankful to **My Family**, their moral support and patience has bared fruit through completion of this Thesis which will result in award of the prestigious degree of M.Sc.

Above all I render my gratitude to the Almighty who bestowed self-confidence, ability, strength and path to me in accomplishing this work.

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CERTIFICATE

This is to certify that Mr. Kanwal jit Singh Channey, Roll No. 30704006 has worked on this thesis report as a partial fulfillment for award of the degree of **MASTERS OF SCIENCE** in physics. I certify that the matter embodied in this report is of candidate's own record and not submitted to any other university in any part or full form for the award of such a degree.

8.7.09

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Contents

Chapter 1

Introduction

Neutrinos (meaning: "Small neutral ones") are elementary particles that often travel close to the speed of light, lack an electric charge, are able to pass through ordinary matter almost undisturbed and are thus extremely difficult to detect. Neutrinos have a minuscule, but nonzero mass. They are usually denoted by the Greek letter ν (nu). Neutrinos are created as a result of certain types of radioactive decay or nuclear reactions such as those that take place in the Sun, in nuclear reactors, or when cosmic rays hit atoms. There are three types, or "flavors", of neutrinos: electron neutrinos, muon neutrinos and tau neutrinos; each type also has an antimatter partner, called an antineutrino. Electron neutrinos or antineutrinos are generated whenever neutrons change into protons or vice versa, the two forms of beta decay. Interactions involving neutrinos are generally mediated by the weak force. Most neutrinos passing through the Earth emanate from the Sun, and more than 50 trillion solar electron neutrinos pass through the human body every second

The neutrino was first postulated in 1930 by Wolfgang Pauli to preserve conservation of energy, conservation of momentum, and conservation of angular momentum in beta decay the decay of a neutron into a proton, an electron and an antineutrino. Pauli theorized that an undetected particle was carrying away the observed difference between the energy, momentum, and angular momentum of the initial and final particles. Pauli originally named his proposed light particle a neutron. When James Chadwick discovered a much more massive nuclear particle in 1932 and also named it a neutron, this left the two particles with the same name. The current name neutrino was coined by Enrico Fermi, who developed the theory of beta decay, as a clever way to resolve the confusion. It was a pun on neutron, the Italian equivalent of neutron: neutron seems to use the -one suffix (even though it is a complete word, not a compound), which in Italian indicates a large object, whereas -ino indicates a small one

We know that whole universe is made of electrone,neutron and protons but this statement is wrong these are only rarities of universe For every one of them, the universe contains a billion neutrinos.To understand the universe, we must understand the neutrinos.It has long been known that neutrinos are very light. However, it has not been known whether they are completely massless, or have small but nonzero masses. The answer to this question has major consequences, both for physics and for astrophysics. If the neutrino masses are very small but nonvanishing, then they are probably caused by new physics beyond the realm of the highly successful Standard Model of the weak and electromagnetic interactions. Measured values of the masses would be a clue to the nature of the new physics. In addition, because neutrinos are so abundant in the universe, even tiny nonzero neutrino masses can result in an important neutrino contribution to the mass density of the universe.

Three hints that neutrinos oscillate have been observed. The first behaviour of neutrinos produced in the earth's atmosphere by cosmic rays provides rather convincing evidence of oscillation. The second observed fluxes of neutrinos from the sunprovides a strong further hint of it. The third behavior of neutrinos in a beam at Los Alamos provides an additional but unconfirmed hint.

1.1 Neutrino Oscillation

In recent years much evidences have found that shows neutrinos have non-zero mass although very small.If we give neutrino enough time to travel through space they can change from one flavour to another flavour this is called neutrino oscillation or neutrino mixing.

The world of tiny particles is governed by Quantum mechanics which contain uncertainty in its core which means:: -

1. This particle may be here may be there.
2. It can be maybe this and maybe that.
3. It can be maybe a ν_μ and maybe a ν_τ.

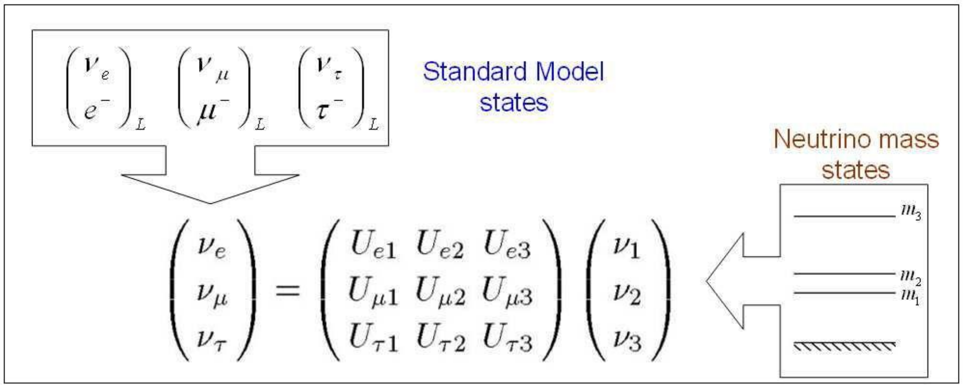
But now we know a proton is a proton is a proton. It does not oscillate into something else.Then how can we say that a neutrino change its flavour. This can be understood by this first of all we cannot take a netrino as a particle to begin with and secondly any flavour of neutrino is a mixture of different values of ν_1,ν_2 and ν_3. Now what are these ν_1,ν_2 and ν_3..??.These are different mass eigen states of neutrinos. In which ν_1 is the lightest and ν_3 is the heaviest. We can also understand this in an another way. Consider netrino flavour as a Soup having ingredients as ν_1,ν_2 and ν_3. By changing the amounts of these ingredients we can change the soup[Flavour]. Which is happening in neutrino mixing in which the amount of ν_1,ν_2 and ν_3 is changing.Reason for changing amount of ν_1,ν_2 and ν_3 is that because of there difference in mass they travel with different speeds [1]

1.2 Neutrino Oscillation and PMNS matrix

The first clues that neutrinos have mass came from an experiment deep underground, carried out by an American scientist Raymond Davis Jr., detecting solar neutrinos It revealed only about one-third of the number predicted by theories of how the sun works pioneered by John Bahcall. The result puzzled both solar and neutrino physicists. However, some Russian researchers, Mikheyev and Smirnov, developing ideas proposed previously by Wolfenstein in the U.S., suggested that the solar neutrinos might be changing into something else. Only electron neutrinos are emitted by the Sun and they could be converting into muon and tau neutrinos which were not being detected by the experiments. This effect called neutrino oscillations, as the types of neutrino inter convert over time from one kind to another, was first proposed some time earlier by Pontecorvo. The precise mechanism for solar neutrino oscillations proposed by Mikheyev, Smirnov and Wolfenstein involved the resonant enhancement of neutrino oscillations due to matter effects. Just as light passing through matter slows down, which is equivalent to the photon gaining a small effective mass, so neutrinos passing through matter also result in the neutrinos slowing down and gaining a small effective mass. The effective neutrino mass is largest when the matter density is highest, which in the case of solar neutrinos is in the core of the Sun. In particular electron neutrinos generated in the core of the Sun will be subject to such matter effects. It turns out that neutrino oscillations, which would be present in the vacuum due to neutrino mass and mixing, will exhibit strong resonant effects in the presence of matter as the effective mass of the neutrinos varies along the path length of the neutrinos

Neutrino oscillations are analagous to coupled pendulums, where oscillations in one pendulum induce oscillations in another pendulum. The coupling strength is defined in terms of something called the lepton mixing matrix U^2 which relates the basic Standard Model neutrino states, $\nu_e, \nu_\mu and \nu_\tau$ associated with the electron, muon and tau, to the neutrino mass states $\nu_1, \nu_2 and \nu_3$ with mass $m_1, m_2, and m_3$, as shown in Fig below

According to quantum mechanics it is not necessary that the Standard Model states



[3]

$\nu_e, \nu_\mu and \nu_\tau$ be identified in a one-one way with the mass eigenstates $\nu_1, \nu_2 and \nu_3$ and the matrix elements of U give the quantum amplitude that a particular Standard Model state contains an admixture of a particular mass eigenstate. As with all quantum amplitudes, the matrix elements of U are expected to be complex numbers in general.

The idea of neutrino oscillations gained support from the Japanese experiment SuperKamiokande which in 1998 showed that there was a deficit of muon neutrinos reaching Earth when cosmic rays strike the upper atmosphere, the so called atmospheric neutrinos. Since most neutrinos pass through the Earth unhindered, Super-Kamiokande was able to detect muon neutrinos coming from above and below, and found that while the correct number of muon neutrinos came from above, only about a half of the expected number came from below. The results were interpreted as half the muon neutrinos from below oscillating into tau neutrinos over an oscillation length L of the diameter of the Earth, with the muon neutrinos from above having a negligible oscillation length, and so not having time to oscillate, yielding the expected number of muon neutrinos from above. More recently, the Sudbury Neutrino Observatory (SNO) in Canada has spectacularly confirmed the solar neutrino oscillations. The experiment measured both the flux of the electron neutrinos and the total flux of all three types of neutrinos. The SNO data revealed that physicists theories of the Sun were correct after all, and the solar neutrinos were produced at the standard rate but were oscillating into $\nu_\tau and \nu_\mu$ with only about a third of the original Following these results several research groups showed that the electron neutrino has a mixing matrix element of $|U_{e_2}| \approx 1/\sqrt{3}$ which is the quantum amplitude for ν_e to contain an admixture of the mass eigenstate ν_2 corresponding to a massive neutrino of mass $m_2 \approx 0.007$ electronvolts or greater The muon and tau neutrinos were observed to contain approximately equal amplitudes of a heavier neutrino ν_3 of mass $m_3 \approx 0.05eV$ or greater $U_{\mu_3} \approx U_{\tau_3} \approx 1/\sqrt{2}$ corresponds to a 1/2 fraction of ν_3 in each of ν_μ and ν_τ, leading to a maximal mixing and oscillation of $\nu_\mu \longleftrightarrow \nu_\tau$ however, according to the results from the CHOOZ nuclear reactor experiment the electron neutrino must only mix very weakly (if at all) with this state, $|U_{e_3}| < 0.2$.

1.2.1 Neutrino mass and Standard Model

The most intuitive way to understand why neutrino mass is forbidden in the Standard Model, is to understand that the Standard Model predicts that neutrinos always have a left-handed spin - rather like rifle bullets which spin counter clockwise to the direction of travel More accurately, the handedness of a particle describes the direction of its spin vector along the direction of motion, and the neutrino being left-handed means that its spin vector always points in the opposite direction to its momentum vector. The fact that the neutrino is left-handed, written as ν_L implies that it must be massless. If the neutrino has mass then, according to special relativity, it can never travel at the speed of light. In principle, a fast moving observer could therefore overtake the spinning massive neutrino and would see it moving in the opposite direction. To the observer, the massive neutrino would therefore appear right-handed. Since the Standard Model predicts that neutrinos must be strictly left-handed, it follows that neutrinos are massless in the Standard Model. It also follows that the discovery of neutrino mass implies new physics Beyond the Standard Model, with profound implications for particle physics and cosmology

1.3 Survival Probability

1.3.1 Probability

The masslessness of neutrino was proposed by Pontecarvo and others many years ago. Neutrinos are produced and annihilated as flavour eigenstates and they propagate through space as superposition of mass eigenstates ν_e, ν_μ, ν_τ are flavour eigenstates which can be expressed as combination of mass eigen states ν_1, ν_2, ν_3 Now for mathematical treatment we will consider the case for two neutrino flavours say ν_e and ν_μ and they will be the combination of two eigen states ν_1, ν_2 using unitary transformation of matrices involving arbitrary mixing angle θ

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} Cos\theta & Sin\theta \\ -Sin\theta & Cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

On solving we get : -

$$\nu_\mu = \nu_1 Cos\theta + \nu_2 Sin\theta \tag{1.1}$$

$$\nu_e = -\nu_1 Sin\theta + \nu_2 Cos\theta \tag{1.2}$$

These two equations are showing wave functions in orthogonal states. Thus there propagation in space is given by

$$\nu_1(t) = \nu_1 e^{-\iota E_1 t} \tag{1.3}$$

$$\nu_2(t) = \nu_2 e^{-\iota E_2 t} \tag{1.4}$$

Here we are using natural system of units so c=1 and h=1. The states ν_1 and ν_2 will have the fixed mummentum p, so that if the masses are m_1 and m_2 then the equations become: -

$$\nu_1(t) = \nu_1(0) e^{-\iota \omega_1 t} \tag{1.5}$$

$$\nu_2(t) = \nu_2(0) e^{-\iota \omega_2 t} \tag{1.6}$$

Now let us consider two states at time t=0 and at time t=t then these can be written as

$$|i\rangle = |\nu_e(0)\rangle$$
$$\text{and}$$
$$|f\rangle = |\nu_e(t)\rangle$$

now

$$amp(\nu_e \to \nu_e) = \langle i|f\rangle = \langle \nu_e(0)|\nu_e(t)\rangle \tag{1.7}$$

$$amp(\nu_e \to \nu_\mu) = \langle i|F\rangle = \langle \nu_e(0)|\nu_\mu(t)\rangle \tag{1.8}$$

and

$$amp(\nu_e \to \nu e) = Cos^2\theta e^{-\iota\omega_1 t} + Sin^2\theta e^{-\iota\omega_2 t} \tag{1.9}$$

Probability of transition: -

$$(\nu_e \rightarrow \nu_e) = |amp|^2 = Re^2 + Im^2 \tag{1.10}$$

$$Re = Cos^2\theta Cos\omega_1 t + Sin^2\theta Cos\omega_2 t \tag{1.11}$$

$$Im = Cos^2\theta Sin\omega_1 t + Sin^2\theta Sin\omega_2 t \tag{1.12}$$

$$P(\nu_e \rightarrow \nu_e) = Cos^4\theta + Sin^4\theta + 2Sin^2\theta Cos^2\theta (Cos\omega_1 t Cos\omega_2 t + Sin\omega_1 t Sin\omega_2 t) \tag{1.13}$$

$$P(\nu_e \rightarrow \nu_e) = Cos^4\theta + Sin^4\theta + 2Sin^2\theta 2Cos^2\theta Cos(\omega_2 - \omega_1)t \tag{1.14}$$

Now as we know

$$Cos^2\theta + Sin^2\theta = 1 \tag{1.15}$$

$$Cos^4\theta + Sin^4\theta + 2Sin^2\theta Cos^2\theta = 1 \tag{1.16}$$

$$SoP = 1 - 2Sin^2\theta Cos^2\theta + 2Sin^2\theta Cos^2\theta Cos(\triangle\omega t) \tag{1.17}$$

$$P = 1 - 4Sin^2\theta Cos^2\theta Sin^2(\frac{\triangle\omega t}{2}) \tag{1.18}$$

$$P = 1 - Sin^2 2\theta Sin^2(\frac{\triangle\omega t}{2}) \tag{1.19}$$

Now here we get the probability relation for transision from ν_e to ν_e

So

$$P(\nu_e \rightarrow \nu_e) = 1 - Sin^2 2\theta Sin^2(\frac{\triangle\omega t}{2}) \tag{1.20}$$

Also we can write t = L/c

where L is the mixing length

So put it in above equation

we will get:-

$$P(\nu_e \rightarrow \nu_e) = 1 - Sin^2 2\theta Sin^2(\frac{\triangle \omega L}{2c}) \quad (1.21)$$

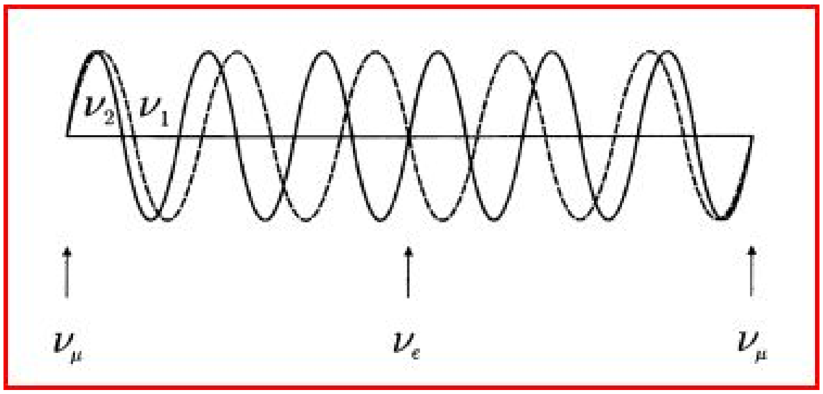


Figure 1.1: Two-neutrino oscillation scenario, showing the amplitudes of ν_1 and ν_2 mass eigen states for the states $\theta_1 = 45$ The two are in phase at the beginning and end of the plot, separated by one oscillatory wavelength, and thus correspond at these points to pure ν_μ flavour eigenstates. The two amplitudes are 180 out of phase in the centre of the plot, corresponding, to the ν_e weak eigenstate.

1.4 The MSW Effect

Wolfenstein (1978) and later Mikhaev and Smimov (1986) pointed out that matter could modify the oscillations by what is now called the MSW effect, after the initials of its proponents. They pointed out that, while all flavours of neu- trino undergo scattering from electrons via Z^0 exchange (neutral-current), in the MeV energy range only ν_e and $\bar{\nu_e}$ can scatter via $W^{\pm}$ exchange (charged-current), since ν_μ and ν_τ have insufficient energy to generate the corresponding charged leptons. Hence the ν_e suffers an extra potential affecting the forward scattering amplitude, which leads to a change in effective mass:

$$V_e = G\sqrt{2}N_e \tag{1.22}$$

$$m^2 = E^2 - p^2 \longrightarrow (E + V_e)^2 - p^2 = m^2 + 2EV_e \tag{1.23}$$

$$\triangle m_m^2 = 2\sqrt{2}GN_eE \tag{1.24}$$

Where N_e is the electron density, E the neutrino energy, G the fermi constant and $\triangle m_m^2$ the shift in mass squared. Suppose now that the vacuum mixing angle θ_v is very small. Then in the simple case of two flavours built from two mass eigenstates, the ν_e will consist predominantly of ν_1 with a little ν_2. The matter density in the Sun (relative to water) varies from $\rho \sim 150$ at the centre to $\rho \backsimeq 10^{-6}$ at the photosphere. If in some region, N_e and E are such that $\triangle m_m^2 \backsimeq \triangle m_v^2 = m_2^2 - m_1^2$ where m_m stands for matter and m_ν for vacuum, it was shown that a resonant-type transition can occur. The actual condition is that

$$\triangle m_m^2 = \triangle m_v^2 Cos(2\theta_v)[2] \tag{1.25}$$

Specifying from 1.23 a critical electron density for the transition. So basically what happens is that a ν_e starts out in the solar core, predominantly in what in a vacuum would be termed the ν_1 eigenstate of mass m_1, and the extra weak potential increases the effective mass of the ν_e to the mass value m_2, which is of course effectively the ν_μ flavour eigenstate in a vacuum. This mass eigenstate passes out of the sun without change provided that the interaction is adiabatic, i.e. the variation of N e per oscillation length is small (if not, only partial conversion will take place). So the end result is that the state of mass m_2, predominantly ν_μ' emerges from the Sun, a $\nu_e \longrightarrow \nu_\mu$ conversion having taken place. Because this transition depends, from (1.23), on the neutrino energy, the suppression of the ν_e flux is also energy dependent, and it is possible to obtain differentially more suppression in the region $E = 2 - 10 MeV then for E <$ 2MeV

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Figure 1.1 Introduction to High energy physics by Donald.H.Perkins

Chapter 2

The See Saw Mechanism

Assuming that neutrinos do have mass, we have to understand why they are nevertheless so much lighter than the charged leptons and quarks. The most popular explanation of this fact is the see-saw mechanism To understand how this mechanism works, let us recall that, unlike charged particles, neutrinos may be their own antiparticles. A neutrino which is its own antiparticle consists of just two states with a common mass: one with spin up and one with spin down. Such a neutrino is called a Majorana neutrino. By contrast, a neutrino which is distinct from its antiparticle consists of four states with a common mass: the spinup and spin-down neutrino, plus the spin-up and spin-down antineutrino. This collection of four states is called a Dirac neutrino. In the see-saw mechanism, a four-state Dirac neutrino N^d of mass M^d gets split by Majorana mass terms into a pair of two-state Majorana neutrinos. One of the latter neutrinos, ν_m,has a small mass M_ν and is identified as one of the observed light neutrinos. The other,N_M has a larger mass M_N reflecting the high mass scale of some new physics beyond the Standard Model, and has not been observed. The character of the breakup of N_d into ν_M andN_M is such that $M_\nu M_N \simeq (M_d)^2$ Now, it is reasonable to expect that the mass M_d of the Dirac particle N^d is of the same order as the typical mass,$M_{l\ or\ q}$ .of the charged leptons l and quarks q,since the latter are Dirac particles as well. Then,$M_\nu M_N \simeq (M_{l\ or\ q})^2$ with $M_{l\ or\ q} M_{l\ or\ q} and M_N$ [1] very large this see-saw relation explains why M_ν is very small. Very importantly, the see-saw mechanism predicts that neutrinos are Majorana particles.

In a typical model, the heavy Majorana neutral lepton N_M participates in some hypothetical feeble interaction beyond the familiar weak interaction. However, it does not participate in the weak interaction itself. For this reason, it (and any other neutrino which is free of normal weak interactions) is sometimes referred to as a sterile neutrino.

2.1 Seesaw and Standard model

Neutrino mass is zero in the Standard Model for three independent reasons:

1. There are no right-handed neutrinos ν_R
2. There are only Higgs doublets (H^+, H^0)
3. The theory is re-normalizable

In the SM these conditions all apply and so neutrinos are massless with $\nu_e, \nu_\mu and \nu_\tau$ distinguished by separate lepton numbers $L_e, L_\mu and L_\tau$ Neutrinos and antineutrinos are distinguished by total conserved lepton number $L = L_e + L_\mu + L_\tau$ to generate neutrino mass we must relax one or more of these conditions. For example, by adding right-handed neutrinos the Higgs mechanism of the Standard Model can give neutrinos the same type of mass as the electron mass or other charged lepton and quark masses. We begin by discussing the Higgs mechanism of the Standard Model. The Higgs mechanism, originally proposed by the British physicist Peter Higgs, is the mechanism that gives mass to all elementary particles in particle physics. It makes the W boson different from the photon

According to the Standard Model all of space is filled by a background Higgs field, which is somewhat analagous to the background electric and magnetic fields that are also present in deep space. In the Standard Model the background Higgs field is due to a single doublet consisting one charged and one neutral Higgs field (H^+, H^0) where only the neutral field (H^0) is switched on in the vacuum, breaking the symmetry of the doublet

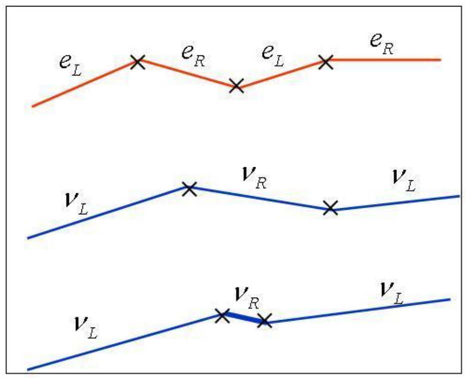

Figure 2.1: A diagramatic illustration of fermion masses in the presence of a background Higgs field H^0 which is uniformly switched on in the vacuum. In the upper diagram, a left-handed electron mass interacts with the background Higgs field to become a right-handed electron, then interacts again to become a lefthanded electron, and so on, resulting in a Dirac mass m_e for the electron. In the centre diagram a similar thing can happen to the neutrino provided right-handed neutrinos are introduced into the Standard Model, leading to a Dirac neutrino mass m_{LR} The lower diagram shows what happens when the right-handed neutrino acquires a large mass M_{RR} independently of the Higgs mechanism. In this case, the heavy righthanded neutrino cannot travel very far due to its large mass, and in the limit of extremely large M_{RR} when the length of its propagation goes to zero, the lower diagram looks effectively like a direct interaction between two left-handed neutrinos, resulting in an effective left-handed Majorana mass $m_{LR}^{eff} = m_{LR}^2/M_{RR}$. This is called the see-saw mechanism.

and hence breaking the symmetry between the weak and the electromagnetic interactions, resulting in W,Z masses. It also results in fermion masses due to their interaction with the background Higgs field. As an electron travels through space it is continually interacting with the background Higgs field as illustrated in the lower diagram in Fig

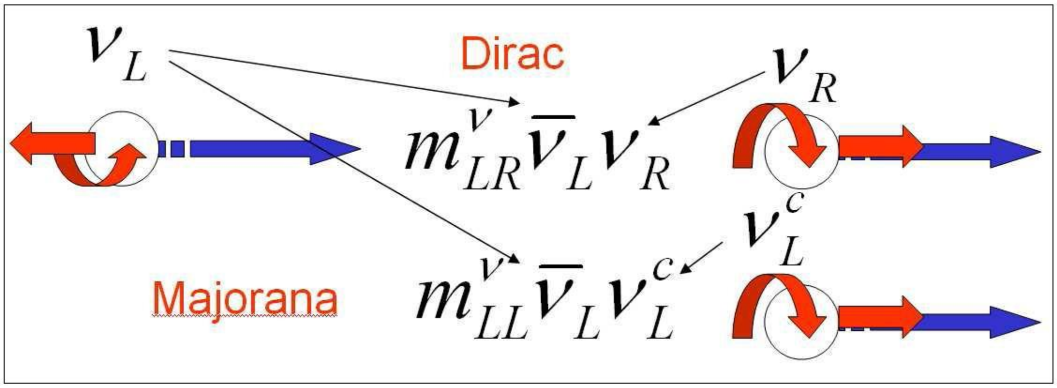

Figure 2.2: For neutrinos there are two types of mass that are possible. As in the case of the electron there is the Dirac mass m^{ν}_{LR} that couples a left-handed neutrino ν_L to a right-handed neutrino ν_R as shown in the upper part of the diagram. However, the role of a right-handed neutrino can be played by ν^c_L obtained by transforming the left-handed neutrino ν_L under the operations charge and parity conjugatation, where ν^c_L is a right-handed antineutrino. If ν_L interacts with ν^c_L then this results in a Majorana mass m^{ν}_{LL} Such mass terms appear in the Lagrangian density for the quantum field theory, where the bar over the ν_L has a conventional meaning that need not concern us here. From our point of view here such mass terms may simply be regarded as interactions.

2.1 resulting in its mass. However, with each interaction its handedness changes, so that its mass can be thought of as an interaction between a left-handed electron e^-_L and a right handed electron e^-_R such an interaction gives rise to what is known as a Dirac mass, named after Paul Dirac. Strictly speaking such mass terms appear in the Lagrangian density for the quantum field theory, but from our point of view here they may simply be regarded as interactions between a left-handed electron and a right-handed electron. It is possible to add right-handed neutrinos ν_R to the Standard Model, providing that the right-handed neutrinos do not take part in the weak interaction so as to not contradict with the result of Goldhaber et al that weakly interacting neutrinos are always left-handed. With right-handed neutrinos present a similar interaction can take place as for electrons, giving rise to a Dirac mass for the neutrino m_{LR}, as shown in the centre diagram in Fig.2.1 above and the upper part of the diagram in Fig.2.2 In principle it is also possible to give neutrinos a new kind of mass called a Majorana mass

m_{LL} if the left-handed neutrino ν_L interacts with its own charge and parity conjugated state, the right-handed antineutrino ν_l^c where the superscript c denotes the simultaneous operation of charge conjugation (C) (replacing the particle by the antiparticle) and parity (P) (replacing the particle by its mirror image, which has the effect of reversing the spin direction). Such a Majorana mass m_{LL} is shown in the lower part of Fig.2.2 In principle right-handed neutrinos ν_R can also independently acquire their own Majorana masses M_{RR} by interacting with their own CP conjugates ν_R^c as shown in Fig.2.2 Such Majorana masses m_{LL} or M_{RR} are only possible in principle for neutrinos since they are the only leptons which are electrically neutral. If such existed, however, they would violate total lepton number L.

Although left-handed Majorana masses m_{LL} are possible in principle, in the Standard Model they are zero since the background Higgs field H^0 is incapable of flipping a ν_L into a ν_L^c The Heisenberg Uncertainty Principle, which allows energy conservation to be violated on small time intervals, then allows a lefthanded neutrino to convert into a heavy right handed neutrino, via the Higgs interaction, for a brief moment before reverting back to being a left-handed neutrino, as shown in the lower diagram in Fig.2.1 for a very large M_{RR}, this effectively results in a very small effective Majorana mass for the left-handed neutrino $m_{LL}^{eff} = (m_{LR}^{\nu})^2/M_{RR}$ [2] The presence of large right-handed Majorana masses M_{RR} therefore leads to an attractive mechanism for explaining the smallness of neutrino masses compared to charged fermion masses. This is the so-called see-saw mechanism. The smallness of the neutrino mass m_{LL}^{eff} is associated with the heaviness of the right-handed neutrino mass M_{RR}. .

2.2 The see-saw mechanism

In this subsection we discuss the see-saw mechanism a little more quantitatively. Let us first summarize the different types of neutrino mass that are possible. There are

Majorana masses of the form:

$$m_{LL}\bar{\nu_L}\nu_L^c \tag{2.1}$$

Where ν_L is a left-handed neutrino field and ν_L^c is the CP conjugate of a left-handed neutrino field, in other words a right-handed anti-neutrino field. Such mass terms have been discussed in the previous section, and have been represented diagrammatically in Fig.2.2 Strictly speaking such mass terms appear in the Lagrangian density for the quantum field theory, but from our point of view here they may simply be regarded as interactions that enable left-handed neutrinos to interact with right-handed anti neutrinos, as depicted in Fig.2.2

Such Majorana masses are possible to since both the neutrino and the anti-neutrino are electrically neutral and so Majorana masses are not forbidden by electric charge conservation. For this reason a Majorana mass for the electron would be strictly forbidden. However such Majorana neutrino masses violate lepton number conservation, and in the standard model, assuming only the simplest Higgs bosons are present, are forbidden. The idea of the simplest version of the see-saw mechanism is to assume that such terms are zero to begin with, but are generated effectively, after right-handed neutrinos are introduced [3] If we introduce right-handed neutrino fields then there are two sorts of additional neutrino mass terms that are possible. There are additional Majorana masses of the form:

$$M_{RR}\bar{\nu_R}\nu_R^c \tag{2.2}$$

where ν_R is a right-handed neutrino field and ν^c is the CP conjugate of a right-handed neutrino field, in other words a left-handed anti-neutrino field. In addition there are Dirac masses of the form:

$$M_{LR}\bar{\nu_L}\nu_R \tag{2.3}$$

Such Dirac mass terms conserve lepton number, and are not forbidden by electric charge conservation even for the charged leptons and quarks.The Higgs mechanism, in its simplest form at least, forbids Majorana masses of the type M^{ν}_{LL} involving the left-handed neutrino ν_L and its CP conjugate ν^c_L but permits Majorana masses M_{RR} involving purely right-handed neutrinos ν_R and its CP conjugate ν^c_R In fact just as m^{ν}_{LL} must be zero in the Standard Model, so M_{RR} may be arbitrarily large. The reason is essentially that the left-handed neutrino ν_L takes part in weak interactions with the W,Z bosons, and if it were very heavy it would disturb the theory. The righthanded neutrino ν_R on the other hand does not take part in weak interactions with the W,Z bosons, and so its mass M_{RR} can be arbitarily large. With the types of neutrino mass discussed in equations 2.2,2.3 (but not Eq.2.3 since we assume no Higgs triplets) we have the see-saw mass matrix:

$$\begin{pmatrix} \bar{\nu_L} & \bar{\nu^c_R} \end{pmatrix} \begin{pmatrix} 0 & M_{LR} \\ M^T_{LR} & M_{RR} \end{pmatrix} \begin{pmatrix} \nu^c_L \\ \nu_R \end{pmatrix} \tag{2.4}$$

Since the right-handed neutrinos are electroweak singlets the Majorana masses of the righthanded neutrinos M_{RR} may be orders of magnitude larger than the electroweak scale. In the approximation that $M_{RR} >> m_{LR}$ the matrix in Eq.2.4 may be diagonalised to yield effective Majorana masses of the type in Eq 2.1,

$$m_{LL} = m_{LR} M^{-1}_{RR} m^T_{LR} \tag{2.5}$$

The effective left-handed Majorana masses m_{LL} are naturally suppressed by the heavy scale M_{RR} In one family example if we take $m_{LR} = M_W = 80GeV$ and $M_{RR} = M_{GUT} = 10^{16}GeV$ then we find $m_{LL} \approx 10^{-3}eV$ which looks good for solar neutrinos. Atmospheric neutrino masses would require a right-handed neutrino with a mass below the GUT scale.With three families of left-handed neutrinos and three right-handed neutrinos the Dirac masses m_{LR} are a 3×3 (complex) matrix and the heavy Majorana masses M_{RR} form a separate 3×3 (complex symmetric) matrix. The light effective Majorana masses

m_{LL} are also a 3×3 (complex symmetric) matrix and continue to be given from Eq.2.5 which is now interpreted as a matrix product. From a model building perspective the fundamental parameters which must be input into the see-saw mechanism are the Dirac mass matrix m_{LR} and the heavy right-handed neutrino Majorana mass matrix M_{RR} The light effective left-handed Majorana mass matrix m_{LL} arises as an output according to the see-saw formula in Eq.2.5

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[5] S. F. King, Rept. Prog. Phys. 67 (2004) 107 [arXiv:hep-ph/0310204]. *figure 2.1* Neutrino Mass by S.F King *figure 2.2* Neutrino Mass by S.F King

Chapter 3

Vanishing effective Majorana mass

In this paper The consequences of a texture zero at the ee entry of neutrino mass matrix in the flavor basis, which also implies a vanishing effective Majorana mass for neutrinoless double beta decay, have been studied for Majorana neutrinos. The neutrino parameter space under this condition has been constrained in the light of all available neutrino data including the CHOOZ bound on s_{13}^2

One of the fundamental goals of neutrino physics is to determine the neutrino mass matrix given by

$$(m_\nu)_{\alpha\beta} = (U m_\nu^{diag} U^T)_{\alpha\beta}; \qquad \alpha, \beta = e, \mu, \tau \tag{3.1}$$

for Majorana neutrinos where $m_\nu^{diag} = m_1, m_2, m_3$ Since m_ν is a symmetric matrix, it is specified by nine physical parameters viz. the three neutrino masses (m_1, m_2, m_3), three mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$, the Dirac-type CP-violating phase (δ), and two Majorana-type CP-violating phases (α, β). Out of these nine parameters, only the first six have been constrained to a reasonable degree of precision but the Dirac-type CP-violating phase and the two Majorana-type CP-violating phases remain unconstrained at present. While the Dirac-type CP-violating phase is expected to be constrained from the study of

CP-violation in long baseline neutrino oscillations, the two Majorana-type CP-violating phases can, hopefully, be constrained from neutrinoless double β decay ($0\nu\beta\beta$).

The rate for ($0\nu\beta\beta$) decay is proportional to the effective mass defined as

$$|m_{ee}| = |\Sigma_i m_i U_{ei}|^2 \tag{3.2}$$

Which, in fact, is the magnitude of the first element of the neutrino mass matrix in the charged lepton flavor basis. Thus, the $0\nu\beta\beta$ [1] decay provides us an unique opportunity to probe directly one of the elements of the neutrino mass matrix. The non-observation of $0\nu\beta\beta$ decay constrains the effective mass $|m_{ee}|$ to be close to zero. The possibility $m_{ee} = 0$ has been examined in the literature [2, 3, 4, 5, 6, 7] earlier under certain special conditions. However, a general study of the neutrino parameter space for $m_{ee} = 0$ with the full interplay of the Majorana phases is still lacking. The neutrino mixing matrix i.e. the PMNS matrix U can be parametrized in terms of three mixing angles ($\theta_{12}, \theta_{23}, \theta_{31}$), the Dirac CP-violating phase δ, and the two Majorana CP-violating phases (α, β) in the following manner [8]:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-\iota\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{\iota\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{\iota\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{\iota\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\iota\delta} & c_{23}c_{13} \end{pmatrix} \tag{3.3}$$

This parametrization has the advantage that the Dirac CP-violating phase δ is formally absent in the expression of the effective Majorana mass m_{ee} probed in $0\nu\beta\beta$ decays The neutrino mass matrix is a complex symmetric matrix described by nine parameters as discussed above. For the above parametrisation, the effective Majorana mass is given by

$$m_{ee} = c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e_{2\iota\alpha} + s_{13}^2 e_{2\iota\beta} \tag{3.4}$$

Which can be obtained easily by : -

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13}e_{\iota\alpha} & s_{13}e^{-\iota\delta}e^{\iota(\beta+\delta)} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{\iota\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{\iota\delta}e^{\iota\alpha} & s_{23}c_{13}e^{\iota(\beta+\delta)} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{\iota\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\iota\delta}e^{\iota\alpha} & c_{23}c_{13}e^{\iota(\beta+\delta)} \end{pmatrix} \tag{3.5}$$

$$U^T = \begin{pmatrix} c_{12}c_{13} & -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{\iota\delta} & s_{12}s_{23} - c_{12}c_{23}s_{13}e^{\iota\delta} \\ s_{12}c_{13}e_{\iota\alpha} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{\iota\delta}e^{\iota\alpha} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\iota\delta}e^{\iota\alpha} \\ s_{13}e^{-\iota\delta}e^{\iota(\beta+\delta)} & s_{23}c_{13}e^{\iota(\beta+\delta)} & c_{23}c_{13}e^{\iota(\beta+\delta)} \end{pmatrix} \tag{3.6}$$

$$m = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \tag{3.7}$$

$$m \times U^T = \begin{pmatrix} m_1c_{12}c_{13} & -m_1s_{12}c_{23} - m_1c_{12}c_{23}s_{13}e^{\iota\delta} & m_1s_{12}c_{23} - m_1c_{12}c_{23}s_{13}e^{\iota\delta} \\ s_{12}c_{13}m_2e^{\iota\alpha} & m_2c_{12}c_{23}e_{iota\alpha} - m_2s_{12}s_{23}s_{13}e_{\iota(\delta+\alpha)} & -m_2e^{\iota\alpha}c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\iota(\delta+\alpha)m_2} \\ m_3s_{13}e^{\iota\beta} & s_{23}c_{13}e^{\iota(\delta+\alpha)} & c_{23}c_{13}e^{\iota(\beta+\delta)} \end{pmatrix} \tag{3.8}$$

$$PMNS = U \times m \times U^T \tag{3.9}$$

$$PMNS = \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ m_{\mu e} & m_{\mu\mu} & m_{\mu\tau} \\ m_{\tau e} & m_{\tau\mu} & m_{\tau\tau} \end{pmatrix} \tag{3.10}$$

$$m_{ee} = c_{13}^2c_{12}^2m_1 + c_{13}^2s_{12}^2m_2e_{2\iota\alpha} + s_{13}^2e_{2\iota\beta} \tag{3.11}$$

The current experimental bound on the effective Majorana mass is [9, 10]

$$|m_{ee}\times = 0.35\zeta eV \tag{3.12}$$

The element m_{ee} of neutrino mass matrix depends on seven (out of a total of nine) parameters viz. $m_1, m_2, m_3, \theta_{12}, \theta_{13}, \alpha, and\beta$ The mixing angle θ is known from the solar and KamLAND neutrino data The masses m_1 and m_2 can be calculated from the two mass-squared differences $\triangle m_{12}^2$ and $\triangle m_{23}^2$ which has been measured experimentally in the solar and atmospheric neutrino experiments. For normal hierarchy (NH), m_1 is the lightest neutrino mass and the masses m_2 and m_3 can be expressed in terms of m_1 in the following way

$$m_2 = \sqrt{m_1^2 + \triangle m_{12}^2} \tag{3.13}$$

$$m_3 = \sqrt{m_2^2 + \triangle m_{23}^2} \tag{3.14}$$

For inverted hierarchy (IH), m3 is the lightest neutrino mass and the other two masses are given by: -

$$m_1 = \sqrt{m_3^2 - \triangle m_{13}^2} \tag{3.15}$$

$$m_2 = \sqrt{m_1^2 + \triangle m_{12}^2} \tag{3.16}$$

Thus, two neutrino masses are known from the solar and atmospheric mass-squared differences viz. $\triangle m_{12}^2$ and $\triangle m_{23}^2$ in terms of the lightest neutrino mass m_1 for NH and m_3 for IH. The best fit, 1 and 3 sigma values of the oscillation parameters are [11]

$$\triangle m_{12}^2 = 7.9_{-0.3,0.8}^{+0.3,1.0} \times 10^{-5} eV^2 \tag{3.17}$$

$$s_{12}^2 = 0.31_{0.03,0.07}^{0.02,0.09} \tag{3.18}$$

$$\triangle m_{23}^2 = \pm 2.2_{0.27,-0.8}^{0.37,1.1} \times 10^{-3} eV^2 \tag{3.19}$$

$$s_{23}^2 = 0.50_{-0.05,0.16}^{0.06,0.18} \tag{3.20}$$

$$s_{13}^2 lessthen 0.012(0.046) \tag{3.21}$$

The best fit value of s_{13}^2 is zero The positive and negative signs of $\triangle m^2$ correspond to the normal and inverted hierarchy, respectively. Thus, the element m_{ee} is now a function of four unknown parameters viz. the mixing angle θ_{13} and the two Majorana CP-violating phases (α, β) and the lightest neutrino mass The study of the element m_{ee} is, therefore, important in order $s_{13}^2 = 0$, the effective Majorana mass m_{ee} is given by

$$m_{ee} = c_{12}^2 m_1 + s_{12}^2 m_2 e_{2\iota\alpha} \tag{3.22}$$

now the condition for which it vanish can be obtained by : -

$$c_{12}^2 m_1 + s_{12}^2 m_2 Cos(2\alpha) + \iota s_{12}^2 m_2 Sin(2\alpha) \tag{3.23}$$

Compare real and imaginary parts on both sides

$$m_{ee} = c_{12}^2 m_1 + s_{12}^2 m_2 Cos(2\alpha) \tag{3.24}$$

$$s_{12}^2 m_2 Sin(2\alpha) = 0 \tag{3.25}$$

from Eq.3.25

$Sin(2\alpha) = 0$

$2\alpha = 0 or \pi$ But $\alpha \neq 0$ so $\alpha = \pi/2$ put it in eq.3.24

$$c_{12}^2 m_1 + s_{12}^2 m_2 cos(\pi) = 0 \tag{3.26}$$

$$\frac{m_1}{m_2} = \frac{s_{12}^2}{c_{12}^2} \tag{3.27}$$

hence both equations will hold simoultaneously for $\alpha = (2n+1)\pi/2$ i.e odd multiples of $\pi/2$

Now put values of m_2 from Eq.3.13 in equation 3.27

$$m_1^2 = \frac{s_{12}^2}{c_{12}^2}(\triangle m_{12}^2 + m_1^2) \tag{3.28}$$

$$m_1^2(1 - \frac{s_{12}^4}{c_{12}^4}) = \frac{s_{12}^4}{c_{12}^4}\triangle m_{12}^2 \tag{3.29}$$

$$m_1^2 = s_{12}^4 \frac{\triangle m_{12}^2}{(c_{12}^2 - s_{12}^2)(c_{12}^2 + s_{12}^2)} \tag{3.30}$$

$$m_1 = s_{12}^2 \sqrt{\frac{\triangle m_{12}^2}{Cos(2\theta_{12})}} \tag{3.31}$$

this value is both for normal and inverted hierarchies. Thus, there exists a point on the (αm_1) plane at which m_{ee} vanishes for s_{13}^2 for both the hierarchies which is a consequence of the fact that the effective Majorana mass m_{ee} is independent of m_3 and, hence, independent of the hierarchy for s_{13}^2. This case has been examined earlier [2]. Now we examine the consequences of a vanishing effective Majorana mass for non-zero s_{13}^2. The element m_{ee} depends on the four parameters viz. The lightest neutrino mass (m_1 for NH and m_3 for IH), the mixing angle θ_{13} and the two Majorana CP-violating phases α and β. Now, we demand that $m_{ee} = 0$ which requires that both the real and imaginary parts of m_{ee} vanish which yields two conditions on the neutrino parameter space viz.

$$Re(m_{ee}) = 0 = c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 Cos(2\alpha) + s_{13}^2 m_3 Cos(2\beta) \tag{3.32}$$

and

$$Im(m_{ee}) = 0 = c_{13}^2 s_{12}^2 m_2 Sin(2\alpha) + s_{13}^2 m_3 Sin(2\beta) \tag{3.33}$$

Let us begin by solving real and imaginary part one by one

Real:

$$(1 - s_{13}^2) c_{12}^2 m_1 + (1 - s_{13}^2) s_{12}^2 m_2 Cos(2\alpha) + s_{13}^2 m_3 Cos(2\beta) \tag{3.34}$$

$$s_{13}^2(-c_{12}^2 m_1 - s_{12}^2 m_2 Cos(2\alpha) + m_3 Cos(2\beta)) + c_{12}^2 m_1 + s_{12}^2 m_2 Cos(2\alpha) = 0 \tag{3.35}$$

$$s_{13}^2 = \frac{c_{12}^2 m_1 + s_{12}^2 m_2 Cos(2\alpha)}{c_{12}^2 m_1 + s_{12}^2 m_2 Cos(2\alpha) - m_3 Cos(2\beta)} \tag{3.36}$$

Imaginary:

$$(1 - s_{13}^2) s_{12}^2 m_2 Sin(2\alpha) + s_{13}^2 m_3 Sin(2\beta) = 0 \tag{3.37}$$

$$s_{12}^2 m_2 Sin(2\alpha) - s_{13}^2 s_{12}^2 m_2 Sin(2\alpha) + s_{13}^2 m_3 Sin(2\beta) = 0 \tag{3.38}$$

$$s_{13}^2 = \frac{s_{12}^2 m_2 Sin(2\alpha)}{m_2 s_{12}^2 Sin(2\alpha) - m_3 Sin(2\beta)} \tag{3.39}$$

Calculate Value of m_3 from equation 3.36 and 3.39 and equate them we will get:

$$\frac{m_1}{m_2} = \frac{s_{12}^2 sin2(\alpha - \beta)}{c_{12}^2 Sin(2\beta)} \tag{3.40}$$

Now find values of m_1 from equations 1.36 and 1.39 and equate them we will get :

$$\frac{m_2}{m_3} = \frac{s_{13}^2 Sin(2\beta)}{c_{13}^2 s_{12}^2 sin(2\alpha)} \tag{3.41}$$

Lets again equate eq. 3.36 and 3.39 to wind value of β

$$-m_1 m_3 c_{12}^2 Sin(2\beta) - m_2 m_3 s_{12}^2 Cos(2\alpha) Sin(2\beta) = -m_2 m_3 s_{12}^2 Sin(2\alpha) Cos(2\beta) \tag{3.42}$$

On squaring and solving we will get :

$$Sin(2\beta) = \frac{m_2 s_{12}^2 sin(2\alpha)}{M} \tag{3.43}$$

Where $M = \sqrt{m_1^2 c_{12}^4 + m_{12}^2 s_{12}^4 + 2 m_1 m_2 c_{12}^2 s_{12}^2 Cos(2\alpha)}$

We already have :

$$s_{13}^2 = \frac{m_2 s_{12}^2 Sin(2\alpha)}{m_2 s_{12}^2 Sin(2\alpha) - m_3 Sin(2\beta)} \tag{3.44}$$

Take square of both sides and put value of $Sin(2\beta)$ which we have obtained earlier we will get :

$$s_3^2 = \frac{M}{M \pm m_3} \tag{3.45}$$

The physical requirement s_{13}^2 Less then 1 implies that $m_{ee} = 0$ if

$$sin(2\beta) = -\frac{s_{12}^2 m_2 Sin(2\alpha)}{M} \tag{3.46}$$

And

$$s_{13}^2 = \frac{M}{M + m_3} \tag{3.47}$$

It is interesting to note that β becomes indeterminate when $\frac{m_1}{m_2} = \frac{s_{12}^2}{c_{12}^2}$ and $\alpha = (n + \frac{1}{2}\pi)$ for $s_{13} = 0$ since M = 0 at this point This is consistent with our earlier result that m_{ee} can vanish for arbitrary values of β when $s_{13}^2 = 0$ Substituting the value of β from Eq.3.46 in Eq.3.41 we obtain

$$m_3 = \frac{c_{13}^2 M}{s_{13}^2} \tag{3.48}$$

Note that R.H.S. of Eq.3.48 becomes indeterminate at the point $\frac{m_1}{m_2} = \frac{s_{12}^2}{c_{12}^2}$ for $s_{13} = 0$ since M $= 0$ at this point which is consistent with our earlier remark that m_{ee} can vanish for $s_{13} = 0$ irrespective of the values of m_3 One can solve Eq.3.48 for α to obtain

$$Cos(2\alpha) = \frac{m_3^2 s_{13}^4 - c_{13}^4(m_1^2)c_{12}^4 + m_2^2 s_{12}^4}{2m_1 m_2 s_{12}^2 c_{12}^2 c_{13}^4} \quad (3.49)$$

Which corresponds to :

$$Cos(2\beta) = -\frac{m_3^2 s_{13}^4 + c_{13}^4(m_2^2)s_{12}^4 - m_1^2 c_{12}^4}{2m_1 m_2 s_{12}^2 s_{13}^2 c_{13}^2} \quad (3.50)$$

Eqs.3.46 and 3.47 can be used to examine the allowed α, m_1 parameter space for $m_{ee} = 0$. Note that $sin(2\alpha)$ and $sin(2\beta)$ should have opposite signs. A natural choice is to take α centered around 90^o and β around 0^o With this particular choice, β should lie in the fourth quadrant , if α lies in the first quadrant and β should lie in the first quadrant if α lies in the second quadrant In other words, α should be ahead of β by one quadrant. This is apparent in Fig. 3.1 where we have plotted the contours of constant β on the α, m_1 plane The central vertical line corresponds to $\beta = 90^o$ and $\beta = 0^o$. The left half of the plot corresponds to α less then 90^o and β less then 0^o whereas the right half of the plot corresponds to α greater then 90^o and β greater then 0^o All the curves corresponding to different values of β intersect at a certain point on the central vertical line. This point has been obtained earlier while discussing the $s_{13}^2 = 0$ case. At this point, $m_{ee} = 0$ irrespective of the value of $beta$. For $\alpha = 90^o$, Eq. 3.47 gives

$$s_{13}^2 = \frac{|m_2 s_{12}^2 - m_1 c_{12}^2|}{|m_2 s_{12}^2 - m_1 c_{12}^2| + m_3} \quad (3.51)$$

for $m_1 = s_{12}^2 \sqrt{\frac{\triangle m_{12}^2}{cos(2\theta_{12})}}$ s_{13}^2becomes zero for which arbitrary values of β are allowed. This is the point through which all the curves corresponding to different values of β pass in Fig.3.1 Different points on the α, m_1 plane correspond to different values of s_{13}^2. However, the CHOOZ bound allows only a limited region on the α, m_1 plane, so we shall constrain the α, m_1 parameter space in the light of the CHOOZ bound. However,

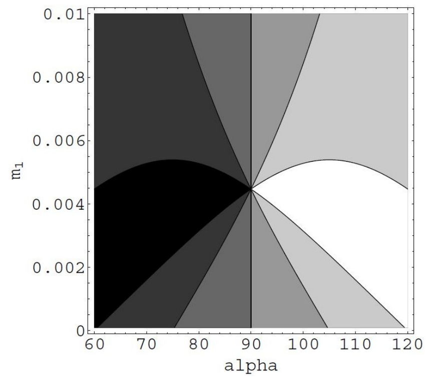

Figure 3.1: The contours of constant $Sin(2\beta)$ on α, m_1 plane as demanded by $m + ee = 0$. The central vertical line is for $sin(2\beta = 0)$. Immediately right (left) to it is the line $Sin(2\beta) = \frac{1}{2}, -\frac{1}{2}$ followed by the line for $sin(2\beta) = \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$

the Majorana CP-violating phase β can not be constrained. Since, M is bounded from above $\alpha = 0^o$ and from below $\alpha = 90^o$ s_{13}^2 is, also, bounded from above and below, i.e.

$$\frac{|m_2 s_{12}^2 - m_1 c_{12}^2|}{|m_2 s_{12}^2 - m_1 c_{12}^2| + m_3} \text{ less then } s_{13}^2 \text{ Less then } \frac{|m_1 c_{12}^2 + m_2 s_{12}^2|}{|m_1 c_{12}^2 + m_2 s_{12}^2| + m_3}$$

Thus, the region allowed by $m_{ee} = 0$ on the m_1, s_{13}^2 plane will be bounded by the two limiting values of s_{13}^2 corresponding to $\alpha = 0^o and 90^o$. As $_1 \longrightarrow 0$ 0, s_{13}^2 becomes independent of β and is given by

$$s_{13}^2 = \frac{\sqrt{\triangle m_{12}^2} s_{12}^2}{\sqrt{\triangle m_{12}^2} s_{12}^2 + \sqrt{\triangle m_{12}^2}} \tag{3.52}$$

This situation has been depicted in Fig. 2 where the upper and lower bounds on s_{13}^2 have been plotted as functions of m_1. The upper $\alpha = 0^o$ and lower $\alpha = 0^o$ curves come closer as $m_1 \longrightarrow 0$ and, eventually, merge for

$$s_{13}^2 = 0.055^{0.010,0.033}_{-0.009,0.024} \tag{3.53}$$

The central value of s_{13}^2 for $m_1 = 0$ given above is above the CHOOZ bound but is consistent with the CHOOZ bound at about 2.3 σ. Therefore, a vanishing m_{ee} element is disallowed at 2.3 σ C.L. for $m_1 = 0$ contrary to the results reported in Ref. [3] where the special case $m_1 = m_{ee} = 0$ has also been examined and it has been found to be consistent with the neutrino oscillation data. However, results are consistent with the recent results reported by Chauhan et al. [4] who conclude that the CHOOZ bound is violated when $m_1 = m_{ee} = 0$ in vanishing determinant neutrino mass matrix scenarios where $m_1 = 0$ is a natural choice. As m_1 increases,the lower curve (corresponding to $\alpha = 90^o$) dips to its minimum value $s_{13}^2 = 0$ for $m_1 = s_{12}^2\sqrt{\frac{\triangle m_{12}^2}{cos(2\theta_{12})}} = 0.045$ eV At this point, $m_{ee} = 0$ irrespective of the value of β with further increases in m_1, s_{13}^2 increases and, eventually, becomes larger than the CHOOZ bound. Thus, there exists a range of m_1 centered around 0.0045 eV and a corresponding range for α around 90^o, for which the condition $m_{ee} = 0$ and the CHOOZ bound hold simultaneously. Consequently, the condition $m_{ee} = 0$ combined with the CHOOZ bound yields both an upper and a lower bound on m_1. With further increase in m_1, we approach the quasi-degenerate (QD) region for normal neutrino mass ordering. For a certain value of m_1 the value of s_{13}^2 becomes larger than the CHOOZ bound. Thus, a vanishing mee element is not allowed in QD hierarchy with normal ordering of neutrino masses and, therefore, a texture zero at the ee entry in the neutrino mass matrix in the flavor basis can not accommodate a QD mass spectrum and small θ_{13} simultaneously

3.1 conclusions

The consequences of a vanishing effective Majorana mass have been examined in detail. It has been concluded that the effective Majorana mass can be zero only for normal hierarchy for certain ranges of values of m_1 and α. The Majorana phase β is left unconstrained but it should be one quadrant behind α. It is found that effective Majorana mass is not allowed to vanish at 2.3 σ C.L. when $m_1 = 0$ in normal hierarchy. Mass matrices with a texture zero at the ee entry naturally lead to a normal ordered neu-

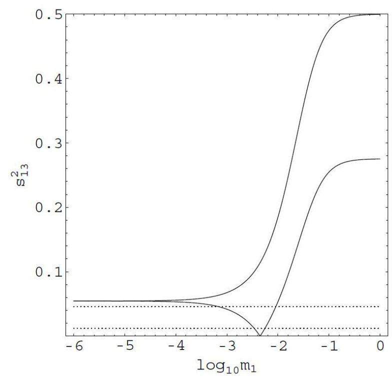

Figure 3.2: The s_{13}^2, m_1 parameter space allowed by $m_{ee} = 0$ for $0^o < \alpha < 90^o$. The upper solid line corresponds to $\alpha = 0^o$ and the lower solid line corresponds to $\alpha = 90^o$. We have also shown the 1 σ and 3 σ CHOOZ bounds as dotted lines.

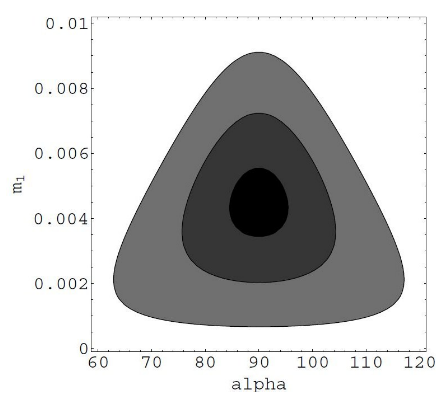

Figure 3.3: The parameter space on α, m_1 plane constrained by $m_{ee} = 0$ and the CHOOZ bound. The regions in the increasing order of lighting correspond to 1 σ, 2 σ and 3 σ bounds on s_{13}^2

trino mass spectrum and small θ_{13}. However, a neutrino mass spectrum with inverted or even quasi-degenerate hierarchies is not allowed by the condition of a vanishing effective Majorana mass in the light of the current neutrino data.

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[11] M. Maltony, T. Schwetz, M. A. Tortola and J. W. F. Valle, New J. Phys. 6, 122 (2004). *figure 3.1* Neutrino Parameter Space for a Vanishing ee Element in the

Neutrino Mass Matrix by S. DEV and SANJEEV KUMAR *figure 3.2* Neutrino Parameter Space for a Vanishing ee Element in the Neutrino Mass Matrix by S. DEV and SANJEEV KUMAR *figure 3.3* Neutrino Parameter Space for a Vanishing ee Element in the Neutrino Mass Matrix by S. DEV and SANJEEV KUMAR

Chapter 4

Texture zeros and vanishing determinant condition

4.1 Texture Zeros for Predictive Models

We will work in a basis in which the charged lepton Yukawa matrix is diagonal:

$$Y_e = Diag(\lambda_e, \lambda_\mu, \lambda_\tau) \tag{4.1}$$

As far as the RHN mass matrix M_N is concerned, we will assume that at high scale (identified with the GUT scale later on) it has the form

$$M_N = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M \tag{4.2}$$

This form of M_N is crucial for our studies.for building predictive neutrino scenarios Specific neutrino models consistent with resonant leptogenesis with a texture similar to (4.3) was investigated in [1] . Here we attempt to classify all possible scenarios with degenerate RHNs which lead to predictions consistent with experiments. Thus, with a basis (4.2) and the texture (4.3) we can discuss possible texture zeros in the matrix

Y_ν, which is of dimension 3×2. One can easily verify that two (and more) texture zeros in Y_ν do not lead to results compatible with the neutrino data. However, with only one texture zero, there are scenarios compatible with experiments and leading to interesting predictions. The matrix Y_ν contains two columns. Since due to the form of M_N (3) there is exchange invariance $N_1 \Longrightarrow N_2$, $N_2 \Longrightarrow N_1$ it does not matter in which column of Y_ν we set one element to zero. We choose here the second column of Y_ν having one texture zero. This leads to the three following possible forms for Y_ν [2]:

$$Texture\ A: \quad Y_\nu = \begin{pmatrix} a_1 & 0 \\ a_2 & b_2 \\ a_3 & b_3 \end{pmatrix} \tag{4.3}$$

$$Texture\ B_1: \quad Y_\nu = \begin{pmatrix} a_1 & b_1 \\ a_2 & 0 \\ a_3 & b_3 \end{pmatrix} \qquad Texture\ B_2: \quad Y_\nu = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & 0 \end{pmatrix} \tag{4.4}$$

A few words about the parametrization, used in (4.4) and (4.5), are in order. With the basis (4.2) and the form of MN given in (4.3), the one texture zero 3×2 matrix Y_ν has only one physical phase. Other phases can be rotated away by proper phase redefinitions of the fields. Moreover, in Y_ν there are five real parameters $|a_{1,2,3}|$ and two absolute values of the b-entries. The mass parameter M in (4.3) is in general complex, but its phase is not relevant for the physics of neutrino oscillations. These systems lead to predictive scenarios with texture A corresponding to normal mass hierarchy and textures B_1 and B_2 corresponding to inverted mass hierarchy. We will study these cases in turn.

4.1.1 Texture A: Normal Hierarchical Case

We will discuss this case in details. With (4.3), (4.4) and using the seesaw formula for the light neutrino mass matrix $M_\nu = \langle h_u^0 \rangle Y_\nu M_N^{-1} Y_\nu^T$, we arrive at

$$M_\nu \simeq \begin{pmatrix} 0 & a_1 b_2 & a_1 b_3 \\ a_1 b_2 & 2a_2 b_2 & a_2 b_3 + a_3 b_2 \\ a_1 b_3 & a_2 b_3 + a_3 b_2 & 2a_3 b_3 \end{pmatrix} \tag{4.5}$$

The matrix in (4.6) is rank two and leads to the one massless neutrino and two massive neutrinos labeled m_2 and m_3. This structure corresponds to the normal hierarchical case, i.e.

$$M_\nu = Diag(0, m_2, m_3) \tag{4.6}$$

with $m_3 \gg m_2$

Now, let us derive the prediction of this model. To achieve this and also get other useful relations we will use the equality

$$M_\nu = PU^* P^{`} M_\nu^{diag} U^\dagger P \tag{4.7}$$

where U is the lepton mixing matrix, given in a standard parameterization by:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-\iota\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{\iota\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{\iota\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{\iota\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\iota\delta} & c_{23}c_{13} \end{pmatrix} \tag{4.8}$$

The P and P' are diagonal phase matrices $P = Diag(e^{\iota\omega_1}, e^{\iota\omega_2}, e^{\iota\omega_3})$, $P' = Diag(1, e^{\iota\rho_1} e^{\iota\rho_2})$

4.1.2 Textures B_1 and B_2 : Inverted Hierarchical Cases

The textures B_1 and B_2 both lead to the inverted hierarchical neutrino mass pattern. Using these textures (4.5), the form of M_N given in (4.3) and the seesaw formula for the light neutrino mass matrices we obtain:

$$For\ Texture\ B_1: \qquad M_\nu \simeq \begin{pmatrix} 2a_1b_1 & a_2b_2 & a_1b_3+b_1a_3 \\ a_2b_2 & 0 & a_2b_3 \\ a_1b_3+b_1a_3 & a_2b_3 & 2a_3b_3 \end{pmatrix} \tag{4.9}$$

$$For\ Texture\ B_1: \qquad M_\nu \simeq \begin{pmatrix} 2a_1b_1 & a_1b_2+a_2b_1 & a_3b_1 \\ a_1b_2+a_2b_1 & 2a_2b_2 & a_2b_3 \\ a_3b_1 & a_2b_3 & 0 \end{pmatrix} \tag{4.10}$$

In order to derive predictions for both cases, we can still use the relation (4.8), which is general, but for M_ν use the forms corresponding to the cases $B_{1,2}$ and for M_ν^{Diag} an inverted hierarchical form:

$$M_\nu^{Diag} = Diag(m_1, m_2, 0) \tag{4.11}$$

We use the same form as before for the phase matrix P, while for the P' we use $P' = Diag(e^{\iota\rho_1}, e^{\iota\rho_2}, 1)$ Eq 4.7 can be writen as

$$M = U^* diag(m_1, m_2 e^{\iota\phi_1}, m_3 e^{\iota\phi_2}) U^\dagger \tag{4.12}$$

where ϕ_1, ϕ_2 are two extra CP violatingMajorana phases and $D = diag(m_1, m_2e^{\iota\phi_1}, m_3e^{\iota\phi_2})$. Applying determinants properties

$$detM = det(U^* D U^\dagger) = detD(Ureal) \neq detD(Ucomplex) \tag{4.13}$$

because if matrix U is real,$U^*U^\dagger = UU^T = 1$, which is satisfied provided $\delta = 0$ or $\theta_{13} = 0$ Thus the determinant is not in general basis independent. In order that $detD = detM$ it is necessary and sufficient that there is either no Dirac CP violation or that it is unobservable. The same arguments hold for the condition $TrD = TrM$ [3] From eq. (4.13) we get that $DetM = 0$ if and only if $DetD = 0$ because det$U^\dagger$ and det U^* are not zero The vanishing determinant condition is basis independent, corresponding to a zero eigenvalue of the mass matrix. So requiring det $M = 0$ is equivalent to assuming one of the neutrinos to be massless. This is realized for instance in the Affleck-Dine scenario for leptogenesis [4, 5, 6] which requires the lightest neutrino to be practically massless ($m \simeq 10^{-10} eV$) [7, 8]

4.2 Mass Matrices with vanishing determinant

We will consider sepratly the case for vanishing diagonal elements for normal and inverted hierarchy simultaneously with vanishing determinant

4.2.1 Normal Hierarchy (NH)

This is the case where the two mass eigenstates involved in the solar oscillations are assumed to be the lightest so that $\triangle m^2_{\otimes} = \triangle m^2_{32} > 0$

The case of vanishing m_{ee}

$$m_{ee} = \frac{1}{25}\sqrt{\frac{13}{10}} e^{2\iota\beta} s^2_{13} + \frac{e^{2\iota\alpha} s^2_{12}(1 - s^2_{13})}{50\sqrt{5}} = 0 \tag{4.14}$$

Now two complex numbers are same when there phases are same and there magnitudes are same so this equations takes up the form like

$$Ae^{2\iota\beta} + Be^{2\iota\alpha} = 0 \tag{4.15}$$

$$Ae^{2\iota\beta} = -Be^{2\iota\alpha} \tag{4.16}$$

$$Ae^{2\iota\beta} = Be^{2\iota\alpha} e^{\iota(2n+1)\pi} \tag{4.17}$$

Phase $2\beta = (2n + 1)\pi + 2\alpha$ Magnitude $A = B$

$$m_3 s^2_{13} = m_2 s^2_{12} c^2_{13} \tag{4.18}$$

$$s^2_{13} = 0.2205848 \tag{4.19}$$

and Phase

$$\beta = \frac{\pi}{2} - \alpha \tag{4.20}$$

Here the calculated value of of s_{13}^2 matches with the earler result given in ref [9] and phase shows that α is ahead of β by one quadrant as given in ref [9] also our value of s_{13}^2 is consistent with the values in range 0.02-0.06 [10]

4.2.2 The case of vanishing $m_{\mu\mu}$

$$m_{\mu\mu} = \frac{1}{25}\sqrt{\frac{13}{10}} e^{2\iota\beta} s_{23}^2 (1 - s_{13}^2) + \frac{e^{2\iota\alpha}(-s_{12}s_{13}s_{23} + \sqrt{1-s_{12}^2}\sqrt{1-s_{23}^2})^2}{50\sqrt{5}} \tag{4.21}$$

$$m_{\tau\tau} = \frac{1}{25}\sqrt{\frac{13}{10}} e^{2\iota\beta} s_{23}^2 (1 - s_{13}^2) + \frac{e^{2\iota\alpha}(-c_{12}s_{23} - s_{12}s_{13}c_{23})^2}{50\sqrt{5}} \tag{4.22}$$

We will solve these two equations in the same manner as for $m_{ee} = 0$.Then we will get the results of s_{13}. These results are not under the CHOOZ bound and also for $m_{\tau\tau}$
$s_{13} = 1.3473$ >CHOOZ bound for $m_{\mu\mu}$
$s_{13} = 0.5135$

4.2.3 Inverted Hierarchy (IH)

$m_{ee} = 0, m_{\mu\mu} = 0, m_{\tau\tau} = 0$

by puting $m_{ee} = 0$ term of s_{13} get cancel out.We will get the value of majorana phase $\alpha = 1.62349$ but for $m_{\tau\tau}$ and $m_{\mu\mu}$ we will get the variation of s_{13} with phase α as shown in figures 4.1 and 4.2 respectively.

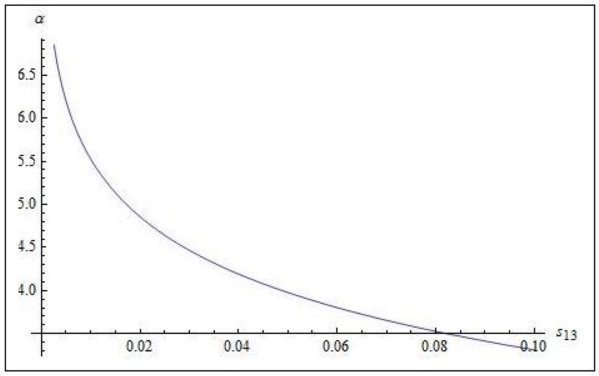

Figure 4.1: Variation of α with s_{13} in case of $m_{\mu\mu}$ for inverted hierarchy (IH)

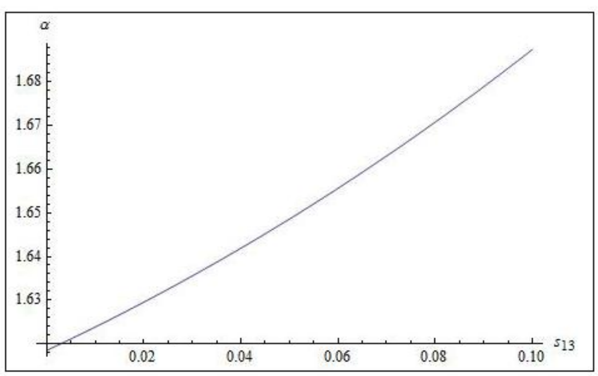

Figure 4.2: Variation of α with s_{13} in case of $m_{\tau\tau} = 0$ for inverted hierarchy (IH)

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Chapter 5

Summary

In the first chapter we have discussed general properties and behaviour of neutrinos in brief. Then a detailed description of neutrino oscillation from one flavour into another is explained following the description and relation of neutrino oscillation and PMNS matrix.After it analytic treatment of survival probability is discussed. After it description of MSW effect or mass effect is given in the last of chapter one. In second chapter whole chapter is used to discuss see saw mechanism. In which first need of see saw mechanism in standard model is discussed then idea about see saw mechanism is given followed by the analytical treatment of see saw mechanism. Third chapter a detailed review of paper ref [9] in chapter 3 is given which involves the effect of vanishing vanishing ee element in neutrino mass matrix. In second last chapter our new work in phenomenology of vanishing determinant with vanishing diagonal element in both inverted and normal hierarchy is given.In last chapter a brief summary of whole thesis is given assuming each paragraph corresponds to one chapter in the give order.

Neutrinos are elementary particles that often travel close to the speed of light, lack an electric charge, are able to pass through ordinary matter almost undisturbed and are thus extremely difficult to detect.Neutrinos are created as a result of certain types of radioactive decay or nuclear reactions such as those that take place in the Sun, in nuclear reactors, or when cosmic rays hit atoms. There are three types, of flavours", of neutrinos: electron neutrinos, muon neutrinos and tau neutrinos along with there antiparticles called anti-neutrinos Most neutrinos passing through the Earth emanate from the Sun, and more than 50 trillion solar electron neutrinos pass through the human body every second The neutrino was first postulated in 1930 by Wolfgang Pauli to preserve conservation of energy, conservation of momentum, and conservation of angular momentum in beta decay the decay of a neutron into a proton, an electron and an antineutrino. Pauli theorized that an undetected particle was carrying away the observed difference between the energy, momentum, and angular momentum of the initial and final particles.We know that whole universe is made of electron,neutron and protons but this statement is wrong these are only rarities of universe For every one of them, the universe contains a billion neutrinos.To understand the universe, we must understand the neutrinos.It has long been known that neutrinos are very light. However, it has not been known whether they are completely massless, or have small but nonzero masses. The answer to this question has major consequences, both for physics and for astrophysics. If the neutrino masses are very small but nonvanishing, then they are probably caused by new physics beyond the realm of the highly successful Standard Model of the weak and electromagnetic interactions. Measured values of the masses would be a clue to the nature of the new physics. In addition, because neutrinos are so abundant in the universe, even tiny nonzero neutrino masses can result in an important neutrino contribution to the mass density of the universe.Three hints that neutrinos oscillate have been observed. The first behaviour of neutrinos produced in the earth's atmosphere by cosmic rays provides rather convincing evidence of oscillation. The second of neutrinos from the sun provides a strong further hint of it. The third behaviour of neutrinos in a beam at Los Alamos provides an additional but unconfirmed hint.In recent years much evidences have found that shows neutrinos have non- zero mass although very small.If we give neutrino enough time to travel through space they can change from one flavour to another flavour this is called neutrino oscillation or neutrino mixing.This can be understood by this first of all we cannot take a neutrino as a particle to begin with and secondly any flavour of neutrino is a mixture of different values of ν_1, ν_2, ν_3 which are different mass eigenstates of neutrinos in which ν_1 is the lightest and ν_3 is the heaviest. Neutrino oscillations are analogous to coupled pendulums, where oscillations in one pendulum induce oscillations in another pendulum. The coupling strength is defined in terms of something called the lepton mixing matrix U^2 which relates the basic Standard Model neutrino states, $\nu_e, \nu_\mu and \nu_\tau$ associated with the electron, muon and tau, to the neutrino mass states $\nu_1, \nu_2 and \nu_3$ with mass $m_1, m_2, and m_3$,

The idea of neutrino oscillations gained support from the Japanese experiment SuperKamiokande which in 1998 showed that there was a deficit of muon neutrinos reaching Earth when cosmic rays strike the upper atmosphere, the so called atmospheric neutrinos. Since most neutrinos pass through the Earth unhindered, Super-Kamiokande was able to detect muon neutrinos coming from above and below, and found that while the correct number of muon neutrinos came from above, only about a half of the expected number came from below. The results were interpreted as half the muon neutrinos from below oscillating into tau neutrinos over an oscillation length L of the diameter of the Earth, with the muon neutrinos from above having a negligible oscillation length, and so not having time to oscillate, yielding the expected number of muon neutrinos from above. More recently, the Sudbury Neutrino Observatory (SNO) in Canada has spectacularly confirmed the solar neutrino oscillations The experiment measured both the flux of the electron neutrinos and the total flux of all three types of neutrinos. The SNO data revealed that physicist's theories of the Sun were correct after all, and the solar neutrinos were produced at the standard rate but were oscillating into $\nu_\tau and \nu_\mu$ with only about a third of the original Following these results several research groups showed that the electron neutrino has a mixing matrix element of $|U_{e_2}| \approx 1/\sqrt{3}$ which is the quantum amplitude for ν_e to contain an admixture of the mass eigenstate ν_2 corresponding to a massive neutrino of mass $m_2 \approx 0.007$ electronvolts or greater The muon and tau neutrinos were observed to contain approximately equal amplitudes of a heavier neutrino ν_3 of mass $m_3 \approx 0.05eV$ or greater $U_{\mu_3} \approx U_{\tau_3} \approx 1/\sqrt{2}$ corresponds to a 1/2 fraction of ν_3 in each of ν_μ and ν_τ, leading to a maximal mixing and oscillation of $\nu_\mu \longleftrightarrow \nu_\tau$ However, according to the results from the CHOOZ nuclear reactor experiment the electron neutrino must only mix very weakly (if at all) with this state, $|U_{e_3}| < 0.2$. The most intuitive way to understand why neutrino mass is forbidden in the Standard Model, is to understand that the Standard Model predicts that neutrinos always have a left-handed spin - rather like rifle bullets which spin counter clockwise to the direction of travel More accurately, the handedness of a particle describes the direction of its spin vector along the direction of motion, and the neutrino being left-handed means that its spin vector always points in the opposite direction to its momentum vector. The fact that the neutrino is left-handed, written as ν_L implies that it must be massless. If the neutrino has mass then, according to special relativity, it can never travel at the speed of light. In principle, a fast moving observer could therefore overtake the spinning massive neutrino and would see it moving in the opposite direction. To the observer, the massive neutrino would therefore appear right-handed. Since the Standard Model predicts that neutrinos must be strictly left-handed, it follows that neutrinos are massless in the Standard Model. It also follows that the discovery of neutrino mass implies new physics Beyond the Standard Model, with profound implications for particle physics and cosmology. Wolfenstein (1978) and later Mikhaev and Smimov (1986) pointed out that matter could modify the oscillations by what is now called the MSW effect, after the initials of its proponents. They pointed out that, while all flavours of neutrino undergo scattering from electrons via Z^0 exchange (neutral-current), in the MeV energy range only ν_e and $\bar{\nu}_e$ can scatter via $W^\pm$ exchange (charged-current), since ν_μ and ν_τ have insufficient energy to generate the corresponding charged leptons. Hence the ν_e suffers an extra potential affecting the forward scattering amplitude, which leads to a change in effective mass. Assuming that neutrinos do have mass, we have to understand why they are nevertheless so much lighter than the charged leptons and quarks.

The most popular explanation of this fact is the see-saw mechanism To understandhow this mechanism works, let us recall that, unlike charged particles, neutrinos may be their own antiparticles. A neutrino which is its own antiparticle consists of just two states with a common mass: one with spin up and one with spin down. Such a neutrino is called a Majorana neutrino. By contrast, a neutrino which is distinct from its antiparticle consists of four states with a common mass: the spinup and spin-down neutrino, plus the spin-up and spin-down antineutrino. This collection of four states is called a Dirac neutrino. In the see-saw mechanism, a four-state Dirac neutrino N^d of mass M^d gets split by Majorana mass terms into a pair of two-state Majorana neutrinos. One of the latter neutrinos, ν_m,has a small mass M_ν and is identified as one of the observed light neutrinos. The other,N_M has a larger mass M_N reflecting the high mass scale of some new physics beyond the Standard Model, and has not been observed. The character of the breakup of N_d into ν_M andN_M is such that $M_\nu M_N \simeq (M_d)^2$ Now, it is reasonable to expect that the mass M_d of the Dirac particle N^d is of the same order as the typical mass,$M_{l\ or\ q}$ .of the charged leptons l and quarks q,since the latter are Dirac particles as well. Then,$M_\nu M_N \simeq (M_{l\ or\ q})^2$ with $M_{l\ or\ q} M_{l\ or\ q} and M_N$ [1] very large this see-saw relation explains why M_ν is very small. Very importantly, the see-saw mechanism predicts that neutrinos are Majorana particles.

In a typical model, the heavy Majorana neutral lepton N_M participates in some hypothetical feeble interaction beyond the familiar weak interaction. However, it does not participate in the weak interaction itself. For this reason, it (and any other neutrino which is free of normal weak interactions) is sometimes referred to as a sterile neutrino.

The consequences of a texture zero at the ee entry of neutrino mass matrix in the flavour basis, which also implies a vanishing effective Majorana mass for neutrinoless double beta decay, have been studied for Majorana neutrinos. The neutrino parameter space under this condition has been constrained in the light of all available neutrino data including the CHOOZ bound on s_{13}^2 then the variation of majorana phase α with s_{13} has been plotted and the values of s_{13} in the assumed conditions of determinant M = 0 and vanishing diagonal elements.