A STUDY ON SOME METHODS FOR SOLVING FUZZY TRANSPORTATION PROBLEMS

A Thesis
Submitted in the Partial Fulfillment of the Requirements
For the Award of Degree of

Masters of Science
in
Mathematics and Computing

BY

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Patiala - 147004, INDIA
July 2018

Dedicated to my
Parents

CERTIFICATE

I hereby certify that the dissertation entitled, **"A study on some methods for solving fuzzy transportation problems"**, which is being submitted by **Akshay Sharma (301603001)**, in the partial fulfilment of the requirement for the award of the degree of Masters of Science in the School of Mathematics, Thapar Institute of Engineering and Technology (TIET), Patiala, comprises of candidate's own research work carried out under the supervision and guidance of **Dr. Amit Kumar** during the period from January 2018 to July 2018.

The part of work presented in this dissertation has not been submitted either in part or in full to this or any other university/ Institute for the award of any degree.

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This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

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ACKNOWLEDGEMENT

Gurur-Brahma Gurur-Vishnur-Gururdevo Maheshvarah |

Gurure va Param Brahma Tasmai Shrii-Gurave Namah ||

First and foremost, I express my deep and sincere gratitude to my supervisor Dr. Amit Kumar, Associate Professor School of Mathematics Thapar Institute of Engineering and Technology, Patiala for his expert guidance, valuable suggestions, support, advice and continuous encouragement throughout the period of my research work. The imperative comments, rendered by him during the discussions are deeply appericated.

I would like to acknowledge the innocent blessings of Mehar. My supervisor, Dr. Amit Kumar believe that Shri Mata Vashino Devi has appeared on the earth in the form of Mehar and without Mehar's blessings it would not have been possible to think the ideas presented in this thesis. Mehar is niece of Dr. Amit Kumar.

My heartfelt thanks goes to Satish Kumar Sharma (Head of School of Mathematics, TIET, Patiala), Prof. S.S. Bhatia (Dean of Academic Affairs, TIET, Patiala) and to all faculty members for their helpful and valuable suggestions.

I am forever grateful to my parents, who mean world to me. Thank you Mom and Dad for showing faith in me and giving me liberty to choose what I desired.

I am ever grateful to God, the Creator and the Guardian and to whom I owe my very existence. Thank you God for numerous blessings bestowed upon me in every aspect of my life.

Patiala **Akshay Sharma**

July 2018

ABSTRACT

Cost minimization problem is well known topic of Operations Research. The main aim of cost minimization transportation problem is to find the optimal quantity of the product that should be supplied from various sources to various destinations so, that the total transportation cost is minimum. The transportation problems, in which total availability of the product at all the sources and total demand at all the destination is equal, are said to be a balanced transportation problems otherwise the transportation problem is said to be an unbalanced transportation problems.

In general to find the optimal solution of a transportation problem, it is assumed that the value of each parameter is precisely known. However, in reality, there may exist uncertainty about some or all the parameters of a transportation problem e.g., the transportation problem between two fixed places varies according to the various situations like traffic jam, weather conditions, road condition etc. Therefore, it is inappropriate to assume that the parameters of transportation problem are precisely known. To handle the impreciseness, various ways have been adopted by the researchers in the literature. One way to handle the same is to represent some/all the parameters of the transportation problem as trapezoidal fuzzy number.

In the literature several methods have been proposed to solve fuzzy transportation problems. In this thesis out of these methods, three recently proposed methods [1-3], published in International Journals, are discussed.

Contents

CHAPTER 1

INTRODUCTION

Cost minimization problem is well known topic of Operations Research. The main aim of cost minimization transportation problem is to find the optimal quantity of the product that should be supplied from various sources to various destinations so, that the total transportation cost is minimum. The transportation problems, in which total availability of the product at all the sources and total demand at all the destination is equal, are said to be a balanced transportation problems otherwise the transportation problem is said to be an unbalanced transportation problems.

In general to find the optimal solution of a transportation problem, it is assumed that the value of each parameter is precisely known. However, in reality, there may exist uncertainty about some or all the parameters of a transportation problem e.g., the transportation problem between two fixed places varies according to the various situations like traffic jam, weather conditions, road condition etc. Therefore, it is inappropriate to assume that the parameters of transportation problem are precisely known. To handle the impreciseness, various ways have been adopted by the researchers in the literature. One way to handle the same is to represent some/all the parameters of the transportation problem as trapezoidal fuzzy number.

In the literature several methods have been proposed to solve fuzzy transportation problems. In this thesis out of these methods, three recently proposed methods [1-3], published in International Journals, are discussed.

1.1 SOME BASIC DEFINITIONS

In this section, some basic definitions are presented [1].

Definition 1.1: The fuzzy set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)), x \in X\}$ defined on the universal set X is said to be a fuzzy set, where, $\mu_{\tilde{A}}: X \to [0,1]$ is called membership function and $\mu_{\tilde{A}}(x)$ is called degree of membership of x in $\tilde{A}$

Definition 1.2: The fuzzy set $\tilde{A}$ is called a fuzzy number if

1) $\tilde{A}$ is normal i.e., $\exists x \in X$ such that $\mu_{\tilde{A}}(x) = 1$.

2) $\tilde{A}$ is convex i.e., $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}; \quad 0 \leq \lambda \leq 1$.

Definition 1.3: A fuzzy number $\tilde{A} = (a, b, c, d)$ is called a trapezoidal fuzzy number when the membership function is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x < b \\ 1, & b \leq x \leq c \\ \frac{x-d}{c-d}, & c < x \leq d \\ 0, & \text{otherwise} \end{cases}$$

1.2 ARITHMETIC OPERATIONS

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers. Then,

(i) $\tilde{A}_1 + \tilde{A}_2 = (a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2)$

(ii) $\tilde{A}_1 - \tilde{A}_2 = (a_1-d_2, b_1-c_2, c_1-b_2, d_1-a_2)$

(iii) $\lambda \times \tilde{A}_1 = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1), if\ \lambda \geq 0 \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1),\ if\ \lambda < 0 \end{cases}$

(iv) $\tilde{A}_1 \times \tilde{A}_2 = (a, b, c, d)$

where, $a = \min(a_1 a_2, a_1 d_2, a_2 d_1, d_1 d_2)$,

$b = \min(b_1 c_2, b_2 c_1, b_1 b_2, c_1 c_2)$,

$c = max(a_1 a_2, a_1 d_2, a_2 d_1, d_1 d_2)$, $d = max(b_1 c_2, b_2 c_1, b_1 b_2, c_1 c_2)$.

CHAPTER 2

MATHUR et al.'s METHOD FOR SOLVING FULLY FUZZY TRANSPORTATION PROBLEMS WITH TRAPEZOIDAL FUZZY NUMBERS

Mathur et al. [3] proposed a method for solving fully fuzzy transportation problems with trapezoidal fuzzy number (Transportation problems in which each parameter is represented as a trapezoidal fuzzy number). In this chapter Mathur et al.'s method [3] is discussed.

2.1 RANKING METHOD USED BY MATHUR et al.'s

To solve a fully fuzzy transportation problem with trapezoidal fuzzy number, there is need to compare/rank trapezoidal fuzzy numbers. Various methods have been proposed in literature for ranking of trapezoidal fuzzy numbers. In this section, the ranking method, used by Mathur et al. [3] in their proposed method, is discussed.

Mathur et al. [3] used the following ranking method for the ranking of trapezoidal fuzzy numbers.

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1) and\ \tilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers. Then,

(i) $\tilde{A}_1 > \tilde{A}_2\ if\ R(\tilde{A}_1) > R(\tilde{A}_2)$

(ii) $\tilde{A}_1 < \tilde{A}_2 if\ R(\tilde{A}_1) < R(\tilde{A}_2)$

(iii) $\tilde{A}_1 = \tilde{A}_2\ if\ R(\tilde{A}_1) = R(\tilde{A}_2)$

where, $R(\tilde{A}_1) = \frac{(a_1+b_1+c_1+d_1)}{4}\ and\ R(\tilde{A}_2) = \frac{(a_2+b_2+c_2+d_2)}{4}$.

2.2 MATHUR et al.'s METHOD

Mathur et al. [3] proposed the following method for solving fully fuzzy transportation problems with trapezoidal numbers.

Step 1 Represent the fully fuzzy transportation problem in tabular form as shown in Table 2.1

Table 2.1: Tabular representation of a fully fuzzy transportation problem

Sources ↓	Destinations ⟶					
	D_1	D_2	D_3	$\cdots$	D_n	Availability
S_1	$\tilde{C}_{11}$	$\tilde{C}_{12}$	$\tilde{C}_{13}$	$\cdots$	$\tilde{C}_{1n}$	$\tilde{A}_1$
. . .				. . .		. . .
S_m	$\tilde{C}_{n1}$	$\tilde{C}_{n2}$	$\tilde{C}_{n3}$	$\cdots$	$\tilde{C}_{nm}$	$\tilde{A}_m$
Demand	$\tilde{B}_1$	$\tilde{B}_2$	$\tilde{B}_3$	$\cdots$	$\tilde{B}_n$	

where, $\tilde{C}_{ij}$, $\tilde{A}_i$ and $\tilde{B}_j$ are trapezoidal fuzzy numbers $(C_{ij1}, C_{ij2}, C_{ij3}, C_{ij4})$, $(A_{i1}, A_{i2}, A_{i3}, A_{i4})$ and $(B_{j1}, B_{j2}, B_{j3}, B_{j4})$ respectively. Here $\tilde{C}_{ij}$, $\tilde{A}_i$, and $\tilde{B}_j$ represents the fuzzy transportation cost, fuzzy availability of the product and fuzzy demand of the product respectively.

Step 2 Transform the fully fuzzy transportation problem, represented by Table 2.1, into the crisp transportation problem, represented by Table 2.2, by replacing $\tilde{C}_{ij}$, $\tilde{A}_i$ and $\tilde{B}_j$ with $R(\tilde{C}_{ij}), R(\tilde{A}_i)$ and $R(\tilde{B}_j)$ respectively.

Table 2.2: Transformed crisp transportation problem

Sources ↓	Destinations ⟶					
	D_1	D_2	D_3	$\cdots$	D_n	Availability
S_1	$R(\tilde{C}_{11})$	$R(\tilde{C}_{12})$	$R(\tilde{C}_{13})$	$\cdots$	$R(\tilde{C}_{1n})$	$R(\tilde{A}_1)$
. . .				. . .		. . .
S_m	$R(\tilde{C}_{n1})$	$R(\tilde{C}_{n2})$	$R(\tilde{C}_{n3})$	$\cdots$	$R(\tilde{C}_{nm})$	$R(\tilde{A}_m)$
Demand	$R(\tilde{B}_1)$	$\mathrm{R}(\tilde{B}_2)$	$R(\tilde{B}_3)$	$\cdots$	$\mathrm{R}(\tilde{B}_n)$	

Step 3 Find the optimal solution $\{x_{ij}\}$ as well as optimal transportation cost of the crisp transportation problem represented by Table 2.2.

2.3 ILLUSTRATIVE EXAMPLES

Mathur et al. [3] solved two fuzzy transportation problems with trapezoidal fuzzy numbers to illustrate their proposed method. In this section, both the fully fuzzy transportation problems are solved by Mathur et al.'s method [3].

2.3.1 OPTIMAL SOLUTION OF FIRST FULLY FUZZY TRANSPORTATION PROBLEM

Mathur et al. [3] solved the fully fuzzy transportation problems with trapezoidal fuzzy numbers, represented by Table 2.3, by their proposed method. In this section, the solution of the same problem is discussed.

Table 2.3: First fully fuzzy transportation problem

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	(1,4,9,19)	(1,2,5,9)	(2,5,8,18)	(1,5,7,9)
S_2	(8,9,12,26)	(3,5,8,12)	(7,9,13,28)	(4,7,8,10)
S_3	(11,12,20,27)	(0,5,10,15)	(4,5,8,11)	(4,5,8,11)
Demand (B_j)	(3,5,8,12)	(4,8,9,10)	(2,4,6,8)	

Using Mathur et al.'s method [3], the optimal solution of the fully fuzzy transportation problem, represented by Table 2.3 can be obtained as follows.

Step 1 Using Step 2 of Mathur et al.'s method [3], discussed in Section 2.2, Table 2.3 can be transformed into Table 2.4.

Table 2.4: Transformed crisp transportation problem

Sources \ Destinations	D_1	D_2	D_3	Availability (A_i)
S_1	8.25	4.25	8.25	5.5
S_2	13.75	7	14.25	7.25
S_3	17.5	7.5	7	7
Demand (B_j)	7	7.75	5	

Step 2 According to Step 3 of Mathur et al.'s method [3], discussed in Section 2.2, there is need to find the optimal solution of the crisp transportation problem represented by Table 2.4.

Using the least cost method for finding the initial basic solution and using MODI method for finding the optimal solution with the help of the obtained initial basic feasible solution, the optimal solution of the crisp transportation problem, represented by Table 2.4, can be obtained as follows.

Step 2(a) The initial basic feasible solution of the crisp transportation problem, obtained by least cost method, is shown in Table 2.5.

Table 2.5: Initial basic feasible solution

Sources \ Destinations	D_1	D_2	D_3	Availability (A_i)
S_1	8.25	4.25 [5.5]	8.25	5.5
S_2	13.75 [5]	7 [2.25]	14.25	7.25
S_3	17.5 [2]	7.5	7 [5]	7
Demand (B_j)	7	7.75	5	

Step 2 (b) Since the basic variables are $x_{12}, x_{21}, x_{22}, x_{31}, x_{33}$ so, the values of dual variables $u_1, u_2, u_3, v_1, v_2, v_3$ will be obtained on solving the following system of equations by considering any one dual variable as "0"

(i) $u_1 + v_2 = 4.25,$

(ii) $u_2 + v_1 = 13.75,$

(iii) $u_2 + v_2 = 7,$

(iv) $u_3 + v_1 = 17.5,$

(v) $u_3 + v_3 = 7.$

Assuming $u_1 = 0,$ the values of remaining dual variables are $v_1 = 11, u_2 = 2.75, u_3 = 6.5, v_2 = 4.25, v_3 = 0.5$

The values of $(u_i + v_j - C_{ij})$ corresponding non-basic variables are

(i) $u_1 + v_1 - C_{11} = 2.75,$

(ii) $u_1 + v_3 - C_{13} = -7.75,$

(iii) $u_2 + v_3 - C_{23} = -11,$

(iv) $u_3 + v_2 - C_{32} = 3.25$

Since $u_1 + v_1 - C_{11}$ and $u_3 + v_2 - C_{32}$ are greater than zero so the obtained basic feasible solution is not optimal. Furthermore, as $u_3 + v_2 - C_{32}$ is most positive so, there is need to make a closed loop by considering $'\theta'$ corresponding to second column and third row. The possible loop is shown in Table 2.6.

Table 2.6: First iteration of MODI method

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	8.25	4.25 5.5	8.25	5.5
S_2	13.75 θ 5	$-\theta$ 7 2.25	14.25	7.25
S_3	17.5 2 $-\theta$	7.5 θ	7 5	7
Demand (B_j)	7	7.75	5	19.75

It is obvious from Table 2.6 that θ has been subtracted from 2.25 and 2 so $\theta = \min\{2, 2.25\}$ i.e. $\theta = 2$. Corresponding to θ the new initial basic feasible solution is shown in Table 2.7.

Table 2.7: The new basic feasible solution

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	8.25	4.25 5.5	8.25	5.5
S_2	13.75 7	7 0.25	14.25	7.25
S_3	17.5	7.5 2	7 5	7
Demand (B_j)	7	7.75	5	

Since the basic variables are $x_{12}, x_{21}, x_{22}, x_{32}, x_{33}$ so, the values of dual variables $u_1, u_2, u_3, v_1, v_2, v_3$ will be obtained on solving the following system of equations by considering any one dual variable as "0"

(i) $u_1 + v_2 = 4.25$,

(ii) $u_2 + v_1 = 13.75$,

(iii) $u_2 + v_2 = 7$,

(iv) $u_3 + v_2 = 7.5,$

(v) $u_3 + v_3 = 7.$

Assuming $u_1 = 0,$ the values of remaining dual variables are $v_1 = 11, u_2 = 2.75, u_3 = 3.25, v_2 = 4.25, v_3 = 3.75$

Now the values of $(u_i + v_j - C_{ij})$ corresponding non-basic variables are

(i) $u_1 + v_1 - C_{11} = 2.75,$

(ii) $u_1 + v_3 - C_{13} = -4.5,$

(iii) $u_2 + v_3 - C_{23} = -7.75,$

(iv) $u_3 + v_1 - C_{31} = -3.25$

Since $u_1 + v_1 - C_{11} = 2.75$ greater than zero so the obtained basic feasible solution is not optimal. Furthermore, as $u_1 + v_1 - C_{11}$ is most positive so, there is need to make a closed loop by considering 'θ' corresponding to first column and first row. The possible loop is shown in Table 2.8.

Table 2.8: Second iteration of MODI method

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	8.25 θ	4.25 $-\theta$ [5.5]	8.25	5.5
S_2	13.75 [7] $-\theta$	7 θ [0.25]	14.25	7.25
S_3	17.5	7.5 [2]	7 [5]	7
Demand (B_j)	7	7.75	5	

It is obvious from Table 2.8 that θ has been subtracted from 5.5 and 7 so $\theta = \min\{5.5, 7\}$ i.e., $\theta = 5.5$. Corresponding to θ the new initial basic feasible solution is shown in Table 2.9.

Table 2.9: The new basic feasible solution

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	8.25 [5.5]	4.25	8.25	5.5
S_2	13.75 [1.5]	7 [5.75]	14.25	7.25
S_3	17.5	7.5 [2]	7 [5]	7
Demand (B_j)	7	7.75	5	

Since the basic variables are $x_{11}, x_{21}, x_{22}, x_{32}, x_{33}$ so, the values of dual variables $u_1, u_2, u_3, v_1, v_2, v_3$ will be obtained on solving the following system of equations by considering any one dual variable as "0"

(i) $u_1 + v_1 = 8.25,$

(ii) $u_2 + v_1 = 13.75,$

(iii) $u_2 + v_2 = 7,$

(iv) $u_3 + v_2 = 7.5,$

(v) $u_3 + v_3 = 7.$

Assuming $u_1 = 0,$ the values of remaining dual variables are $v_1 = 8.25, u_2 = 5.5, u_3 = 6, v_2 = 1.5, v_3 = 1$

Now the values of $(u_i + v_j - C_{ij})$ corresponding non-basic variables are

(i) $u_1 + v_2 - C_{12} = -2.75,$

(ii) $u_1 + v_3 - C_{13} = -7.25,$

(iii) $u_2 + v_3 - C_{23} = -7.75,$

(iv) $u_3 + v_1 - C_{31} = -3.25$

Since all are less than zero so the obtained basic feasible solution is optimal. Therefore, total transportation cost = $5.5 \times 8.25 + 1.5 \times 13.25 + 5.75 \times 7 + 2 \times 7.5 + 5 \times 7 = 156.25$.

2.3.2 OPTIMAL SOLUTION OF SECOND FULLY FUZZY TRANSPORTATION PROBLEM

Mathur et al. [3] solved the fully fuzzy transportation problems with trapezoidal fuzzy numbers, represented by Table 2.10, by their proposed method. In this section, the solution of the same problem is discussed.

Table 2.10: Second fully fuzzy transportation problem

Destinations / Sources	D_1	D_2	D_3	D_4	Availability (A_i)
S_1	(1,2,3,4)	(1,3,4,6)	(9,11,12,4)	(5,7,8,11)	(1,6,7,12)
S_2	(0,1,2,4)	(1,0,1,2)	(5,6,7,8)	(0,1,2,3)	(0,1,2,3)
S_3	(3,5,6,8)	(5,8,9,12)	(12,15,16,19)	(7,9,10,12)	(5,10,12,17)
Demand (B_j)	(5,7,8,10	(1,5,6,10)	(1,3,4,6)	(1,2,3,4)	

Using Mathur et al.'s method [3], the optimal solution of the fully fuzzy transportation problem, represented by Table 2.10 can be obtained as follows.

Step 1 Using Step 2 of Mathur et al.'s method [3], discussed in Section 2.2, Table 2.10 can be transformed into Table 2.11

Table 2.11: Transformed crisp transportation problem

Destinations / Sources	D_1	D_2	D_3	D_4	Availability (A_i)
S_1	2.7	3.5	11.5	7.75	6.5
S_2	1.75	0.5	6.5	1.5	1.5
S_3	5.5	8.5	15.5	9.5	11
Demand (B_j)	7.5	5.5	3.5	2.5	19

Step 2: According to Step 3 of Mathur et al.'s method, discussed in Section 2.2, there is need to find the optimal solution of the crisp transportation problem represented by Table 2.11. Using the least cost method for finding the initial basic solution and MODI method for finding the optimal solution with the help of the obtained basic feasible solution, the optimal solution of the crisp transportation problem represented by Table 2.12 can be obtained as follows.

Step 2(a): The initial basic feasible solution of the crisp transportation problem represented by Table 2.11, obtained by least cost method, is shown in Table 2.12.

Table 2.12: Initial basic feasible solution

Destination / Sources	D_1	D_2	D_3	D_4	Availability (A_i)
S_1	2.7 [6.5]	3.5	11.5	7.75	6.5
S_2	1.75	0.5 [1.5]	6.5	1.5	1.5
S_3	5.5 [1]	8.5 [4]	15.5 [3.5]	9.5 [2.5]	11
Demand (B_j)	7.5	5.5	3.5	2.5	19

Step 2(b): Since the basic variables are $x_{11}, x_{22}, x_{31}, x_{32}, x_{33,}x_{34}$ so, the values of dual variables $u_1, u_2, u_3, v_1, v_2, v_3, v_4$ will be obtained on solving the following system of equations by considering any one dual variable as "0"

(i) $u_1 + v_1 = 2.7,$

(ii) $u_2 + v_2 = 0.5,$

(iii) $u_3 + v_1 = 5.5,$

(iv) $u_3 + v_2 = 8.5,$

(v) $u_3 + v_3 = 15.5,$

(vi) $u_3 + v_4 = 9.5.$

Assuming $u_1 = 0,$ the values of remaining dual variables are $v_1 = 2.7, u_2 = -5.2, u_3 = 2.8, v_2 = 5.7, v_3 = 12.7, v_4 = 6.7$

Now the values of $(u_i + v_j - C_{ij})$ corresponding non-basic variables are

(i) $u_1 + v_2 - C_{12} = 2.2,$

(ii) $u_1 + v_3 - C_{13} = 1.2,$

(iii) $u_2 + v_3 - C_{23} = 1,$

(iv) $u_2 + v_1 - C_{21} = -4.25,$

(v) $u_1 + v_4 - C_{14} = -1.05,$

(vi) $u_2 + v_4 - C_{24} = 0$

Since $u_2 + v_3 - C_{23}, u_1 + v_3 - C_{13}, u_1 + v_2 - C_{12}$ are greater than zero so the obtained basic feasible solution is not optimal. Furthermore, as $u_1 + v_2 - C_{12}$ is most positive so, there is need to make a closed loop by considering $'\theta'$ corresponding to second column and first row. The possible loop is shown in Table 2.13.

Table 2.13: First iteration of MODI method

Destinations / Sources	D_1	D_2	D_3	D_4	Availability (A_i)
S_1	2.7 $-\theta$ [6.5]	3.5 θ	11.5	7.75	6.5
S_2	1.75	0.5 [1.5]	6.5	1.5	1.5
S_3	5.5 [1] θ	8.5 $-\theta$ [4]	15.5 [3.5]	9.5 [2.5]	11
Demand (B_j)	7.5	5.5	3.5	2.5	19

It is obvious from Table 2.13 that θ has been subtracted from 6.5 and 4 so $\theta = \min\{6.5, 4\}$ i.e., $\theta = 4$. Corresponding to θ the new initial basic feasible solution is shown in Table 2.14.

Table 2.14: The new basic feasible solution

Destination / Sources	D_1	D_2	D_3	D_4	Availability (A_i)
S_1	2.7 [2.5]	3.5 [4]	11.5	7.75	6.5
S_2	1.75	0.5 [1.5]	6.5	1.5	1.5
S_3	5.5 [5]	8.5	15.5 [3.5]	9.5 [2.5]	11
Demand (B_j)	7.5	5.5	3.5	2.5	19

Since the basic variables are $x_{11}, x_{22}, x_{31}, x_{12}, x_{33,}x_{34}$ so, the values of dual variables $u_1, u_2, u_3, v_1, v_2, v_3, v_4$ will be obtained on solving the following system of equations by considering any one dual variable as "0"

(i) $u_1 + v_1 = 2.7$,

(ii) $u_2 + v_2 = 0.5$,

(iii) $u_3 + v_1 = 5.5$,

(iv) $u_1 + v_2 = 3.5$,

(v) $u_3 + v_3 = 15.5$,

(vi) $u_3 + v_4 = 9.5$.

Assuming $u_1 = 0$, the values of remaining dual variables are $v_1 = 2.7, u_2 = -3, u_3 = 2.8, v_2 = 3.5, v_3 = 12.7, v_4 = 6.7$

Now the values of $(u_i + v_j - C_{ij})$ corresponding non-basic variables are

(i) $u_3 + v_2 - C_{32} = -2.2$,

(ii) $u_1 + v_3 - C_{13} = 1.2$,

(iii) $u_2 + v_3 - C_{23} = 2.9,$

(iv) $u_2 + v_1 - C_{21} = -2.05,$

(v) $u_1 + v_4 - C_{14} = -1.05,$

(vi) $u_2 + v_4 - C_{24} = 2.2$

Since $u_2 + v_3 - C_{23}, u_1 + v_3 - C_{13}, u_2 + v_4 - C_{24}$ are greater than zero so the obtained basic feasible solution is not optimal. Furthermore, as $u_2 + v_3 - C_{23}$ is most positive so, there is need to make a closed loop by considering $'\theta'$ corresponding to third column and second row. The possible loop is shown in Table 2.15.

Table 2.15: Second iteration of MODI method

Destinations / Sources	D_1	D_2	D_3	D_4	Availability (A_i)
S_1	2.7 [2.5] $-\theta$	3.5 [4] θ	11.5	7.75	6.5
S_2	1.75	0.5 [1.5] $-\theta$	6.5 θ	1.5	1.5
S_3	5.5 [5] θ	8.5	15.5 [3.5] $-\theta$	9.5 [2.5]	11
Demand (B_j)	7.5	5.5	3.5	2.5	19

It is obvious from Table 2.15 that θ has been subtracted from 2.5, 1.5 and 3.5 so $\theta = \min\{2.5, 1.5, 3.5\}$ i.e., $\theta = 1.5$. Corresponding to θ the new initial basic feasible solution is shown in Table 2.16.

Table 2.16: The new basic feasible solution

Destination / Sources	D_1	D_2	D_3	D_4	Availability (A_i)
S_1	2.7 [1]	3.5 [5.5]	11.5	7.75	6.5
S_2	1.75	0.5	6.5 [1.5]	1.5	1.5
S_3	5.5 [6.5]	8.5	15.5 [2]	9.5 [2.5]	11
Demand (B_j)	7.5	5.5	3.5	2.5	19

Since the basic variables are $x_{11}, x_{23}, x_{31}, x_{12}, x_{33,} x_{34}$ so, the values of dual variables $u_1, u_2, u_3, v_1, v_2, v_3, v_4$ will be obtained on solving the following system of equations by considering any one dual variable as $''0''$

(i) $u_1 + v_1 = 2.7$,

(ii) $u_2 + v_3 = 6.5$,

(iii) $u_3 + v_1 = 5.5$,

(iv) $u_1 + v_2 = 3.5$,

(v) $u_3 + v_3 = 15.5$,

(vi) $u_3 + v_4 = 9.5$.

Assuming $u_1 = 0$, the values of remaining dual variables are $v_1 = 2.7, u_2 = -6.2, u_3 = 2.8, v_2 = 3.5, v_3 = 12.7, v_4 = 6.7$

Now the values of $(u_i + v_j - C_{ij})$ corresponding non-basic variables are

(i) $u_3 + v_2 - C_{32} = -2.2$,

(ii) $u_1 + v_3 - C_{13} = 1.2$,

(iii) $u_2 + v_3 - C_{23} = -1$,

(iv) $u_2 + v_1 - C_{21} = -5.25,$

(v) $u_1 + v_4 - C_{14} = -1.05,$

(vi) $u_2 + v_2 - C_{22} = -3.2$

Since $u_1 + v_3 - C_{13}$ greater than zero so the obtained basic feasible solution is not optimal. Furthermore, as $u_1 + v_3 - C_{13}$ is most positive so, there is need to make a closed loop by considering $'\theta'$ corresponding to third column and first row. The possible loop is shown in Table 2.17.

Table 2.17: Third iteration of MODI method

Destinations / Sources	D_1	D_2	D_3	D_4	Availability (A_i)
S_1	2.7 [1] $-\theta$	3.5 [5.5]	11.5 θ	7.75	6.5
S_2	1.75	0.5	6.5 [1.5]	1.5	1.5
S_3	5.5 [6.5] θ	8.5	15.5 [2] $-\theta$	9.5 [2.5]	11
Demand (B_j)	7.5	5.5	3.5	2.5	19

It is obvious from Table 2.17 that θ has been subtracted from 2 and 1 so $\theta = \min\{2,1\}$ i.e., $\theta = 1$. Corresponding to θ the new initial basic feasible solution is shown in Table 2.18.

Table 2.18: The new basic feasible solution

Destination / Sources	D_1	D_2	D_3	D_4	Availability (A_i)
S_1	2.7	3.5 [5.5]	11.5 [1]	7.75	6.5
S_2	1.75	0.5	6.5 [1.5]	1.5	1.5
S_3	5.5 [7.5]	8.5	15.5 [1]	9.5 [2.5]	11
Demand (B_j)	7.5	5.5	3.5	2.5	19

Since the basic variables are $x_{11}, x_{23}, x_{31}, x_{12}, x_{33,}x_{34}$ so, the values of dual variables $u_1, u_2, u_3, v_1, v_2, v_3, v_4$ will be obtained on solving the following system of equations by considering any one dual variable as $''0''$

(vii) $u_1 + v_3 = 11.5,$

(viii) $u_2 + v_3 = 6.5,$

(ix) $u_3 + v_1 = 5.5,$

(x) $u_1 + v_2 = 3.5,$

(xi) $u_3 + v_3 = 15.5,$

(xii) $u_3 + v_4 = 9.5.$

Assuming $u_1 = 0,$ the values of remaining dual variables are $v_1 = 1.5, u_2 = -5, u_3 = 4, v_2 = 3.5, v_3 = 11.5, v_4 = 5.5$

Now the values of $\left(u_i + v_j - C_{ij}\right)$ corresponding non-basic variables are

(i) $u_1 + v_1 - 2.7 = -1.2,$

(ii) $u_3 + v_2 - 7.75 = -1,$

(iii) $u_1 + v_4 - 1.75 = -2.25,$

(iv) $u_2 + v_1 - 0.5 = -5.25$,

(v) $u_2 + v_2 - 1.5 = -2$,

(vi) $u_2 + v_4 - 8.5 = -1$.

Since all are less than zero so the obtained basic feasible solution is optimal. Therefore,

the total transportation cost

$= 3.5 \times 5.5 + 11.5 \times 1 + 6.5 \times 1.5 + 5.5 \times 7.5 + 15.5 \times 1 + 9.5 \times 2.5 = 121$

2.4 CONCLUSION

The existing method [3] for solving fully fuzzy transportation problem with trapezoidal fuzzy number is discussed in a detailed manner.

CHAPTER 3

BISHT AND SRIVASTVA'S METHOD FOR SOLVING INTERVAL VALUED FULLY FUZZY TRANSPORTATION PROBLEMS WITH TRAPEZOIDAL FUZZY NUMBERS

Bisht and Srivastva [1] proposed a method for solving fully interval transportation problems (Transportation problems in which each parameters are represented by interval). Since in this approach each interval is transformed into a trapezoidal fuzzy number that is a fully interval transportation problem is transformed into a fully fuzzy transportation problem with trapezoidal fuzzy numbers. Therefore, the method proposed by Mathur et al. [3], can also be used for solving a fully interval transportation problem.

The only difference between the method, proposed by Mathur et al [3]. and the method proposed by Bisht and Srivastva [1], is that in Mathur et al.'s method [3], an existing ranking method is used for the ranking of trapezoidal fuzzy numbers, whereas in the method proposed by Bisht and Srivastva [1] the ranking method, proposed by Bisth and Srivastva [1] themselves, is used. In this chapter, the method, proposed by Bisth and Srivastva [1], is discussed.

3.1 METHOD FOR TRANSFORMING AN INTERVAL INTO A TRAPEZOIDAL FUZZY NUMBER

Bisth and Srivastva [1] proposed the following method to transform an interval $[A^L, A^U]$ into a trapezoidal fuzzy number.

Step 1 Find $d = \frac{(A^U - A^L)}{3}$.

Step 2 Find $A = A^L + d$ and $B = A^L + 2d$.

Step 3 The trapezoidal fuzzy number corresponding to the interval$[A^L, A^U]$ is (A^L, A, B, A^U).

Example:- The trapezoidal fuzzy number corresponding to interval $[A^L, A^U] = [1,19]$ can be obtained as follows.

Step 1 $d = \frac{19-1}{3} = 6,$

Step 2 $A = 1 + 6 = 7$ and $B = 1 + 12 = 13,$

Step 3 The trapezoidal fuzzy number corresponding to the interval $[1,19]$ is $(1,7,13,19)$.

3.2 RANKING METHOD USED BY BISTH AND SRIVASTVA'S

Bisth and Srivastva [1] proposed the following ranking method for the ranking of trapezoidal fuzzy numbers,

$\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$

Step 1: Find $x_i = \frac{a_i c_{i-b_i d_i}}{c_{i-d_i-b_i-a_i}}$ and $y_i = \frac{x_i - b_i}{b_i - a_i} + 1$;$i = 1,2$

Step 2: Find $p_i = d_i - a_i,$

$q_i = \sqrt{{y_i}^2 + (d_i - x_i)^2}$,

$r_i = \sqrt{{y_i}^2 + (x_i - a_i)^2}$; $i = 1,2$

Step 3: Find $R(\tilde{A}_i) = \frac{p_i x_i + q_i a_i + r_i d_i}{p_i + q_i + r_i}$;$i = 1,2.$

Step 4: Check that $R(\tilde{A}_1) > R(\tilde{A}_2)$ or $R(\tilde{A}_1) < R(\tilde{A}_2)$ or $R(\tilde{A}_1) = R(\tilde{A}_2)$ if

(i) $R(\tilde{A}_1) > R(\tilde{A}_2)$ then $\tilde{A}_1 > \tilde{A}_2$

(ii) $R(\tilde{A}_1) < R(\tilde{A}_2)$ then $\tilde{A}_1 < \tilde{A}_2$

(iii) $R(\tilde{A}_1) = R(\tilde{A}_2)$ then $\tilde{A}_1 = \tilde{A}_2$

3.3 ORIGIN OF RANKING METHOD

In Step 1 of Section 3.2, x_i, y_i represents the intersection of the lines joining the point $(a, 0)$, $(b, 1)$ and $(d, 0)$, $(c, 1)$ as shown in the Figure 3.1 given below

.

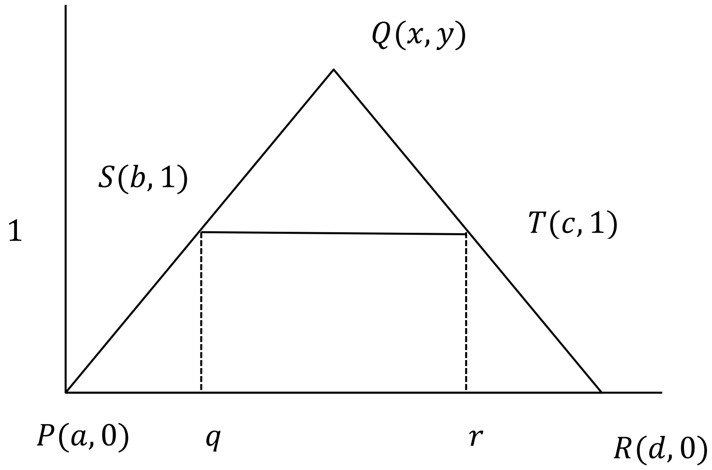

Figure 3.1 Normal trapezoidal fuzzy number.

In Step 2 of Section 3.2 p_i, q_i and r_i represents the length of the sides PR, RQ and PQ respectively of the triangle PQR as shown in the Figure 3.1. Finally in Step 3 $R(\tilde{A}_i)$ represent the center of incircle of triangle PQR as shown in the Figure 3.1. Moreover the values of x , y can be computed in the following manner.

Equation of line $P(a, 0)$ and $S(b, 1)$

$$y - 0 = \frac{1 - 0}{b - a}(x - a)$$

$$\Rightarrow \qquad y = \frac{x - a}{b - a} \qquad (1)$$

Equation of the line R$(d, 0)$ and T$(c, 0)$

$$y - 0 = \frac{1 - 0}{c - d}(x - d)$$

$$\Rightarrow \qquad y = \frac{x - d}{c - d} \qquad (2)$$

From (1) and (2) we get

$$\frac{x - d}{c - d} = \frac{x - a}{b - a}$$

$$\Longrightarrow \qquad (x - a)(c - d) = (x - d)\,(b - a)$$

$$\Longrightarrow \qquad xc - xd - ac + ad = xb - xa - db + da$$

$$\Longrightarrow \qquad x(c - d) - ac + ad = x(b - a)\ - db + da$$

$$x(c - d - b + a) = -db + da + ac - ad$$

$$x = \frac{-db+ac}{(c-d-b+a)}$$

On putting these value in equation (1) we have,

$$y = \frac{x-b}{b-a} + 1$$

3.4 NUMERICAL EXAMPLE

Bisht and Srivastva [1] solved the fully fuzzy transportation problems with trapezoidal fuzzy numbers, represented by Table 3.1, by their proposed method. In this section, the solution of the same problem is discussed.

Table 3.1: Interval transportation problem

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	[1,19]	[1,9]	[2,18]	[1,9]
S_2	[8,26]	[3,12]	[7,28]	[4,10]
S_3	[11,27]	[0,15]	[4,11]	[4,11]
Demand (B_j)	[3,12]	[4,10]	[2,8]	

In this problem the cost, demand and supply all are in interval form so according to Bisht and Srivastva's method [1] we first convert these interval into trapezoidal fuzzy number as shown in the Table 3.2.

Table 3.2: Fully fuzzy transportation problem

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	(1,7,13,19)	(1,3.67,6.34,9)	(2,7.33,12.66,18)	(1,3.67,6.33,9)
S_2	(8,1.67,13.33,16)	(3,6,9,12)	(7,14,21,28)	(4,6,8,10)
S_3	(11,16.3,21.6,27)	(0,5,10,15)	(4,6.33,8.67,11)	(4,6.33,8.67,11)
Demand (B_j)	(3,6,9,12)	(4,6,8,10)	(2,4,6,8)	

Using Step 3 of Bisht and Srivastva's method [1], discussed in Section 3.2, Table3.2 can be transformed into Table 3.3.

Table 3.3: Transformed crisp transportation problem

Sources \ Destinations	D_1	D_2	D_3	Availability (A_i)
S_1	9.75	4.77	9.48	4.77
S_2	14.72	6.76	15.48	6.51
S_3	17.61	7.50	6.87	6.87
Demand (B_j)	6.76	6.58	4.58	

Now applying least cost method for finding the initial solution and after that we have to used MODI method for finding the solution of the transportation problem.

Table 3.4: Initial basic feasible solution

Sources \ Destinations	D_1	D_2	D_3	D_4	Availability (A_i)
S_1	9.75	4.77 [4.54]	9.48	0 [0.23]	4.77
S_2	14.72 [4.47]	6.76 [2.04]	15.48	0	6.51
S_3	17.61 [2.29]	7.50	6.87 [4.58]	0	6.87
Demand (B_j)	6.76	6.58	4.58	0	

Since the basic variables are $x_{12}, x_{14}, x_{22}, x_{21}, x_{31}, x_{33}$ so, the values of dual variables $u_1, u_2, u_3, v_1, v_2, v_3, v_4$ will be obtained on solving the following system of equations by considering any one dual variable as $''0''$.

(i) $u_1 + v_2 = 4.77$,

(ii) $u_1 + v_4 = 0$,

(iii) $u_2 + v_2 = 6.76$,

(iv) $u_3 + v_1 = 17.61$,

(v) $u_3 + v_3 = 6.87$,

(vi) $u_2 + v_1 = 14.72$

Assuming $u_1 = 0$, the values of remaining dual variables are $v_1 = 12.73$, $u_2 = 1.99$, $u_3 = 4.88$, $v_2 = 4.77$, $v_3 = 1.99$, $v_4 = 0$

Now the values of $(u_i + v_j - C_{ij})$ corresponding non-basic variables are

(i) $u_1 + v_1 - C_{11} = 2.98$,

(ii) $u_1 + v_3 - C_{13} = -7.49$,

(iii) $u_2 + v_3 - C_{23} = -11.5$,

(iv) $u_3 + v_2 - C_{32} = 2.15$,

(v) $u_2 + v_4 - C_{24} = 1.99$,

(vi) $u_3 + v_4 - C_{34} = 4.88$

Since $u_1 + v_1 - C_{11}$, $u_3 + v_2 - C_{32}$, $u_2 + v_4 - C_{24}$, $u_3 + v_4 - C_{34}$ are greater than zero so the obtained basic feasible solution is not optimal. Furthermore, as $u_3 + v_4 - C_{34}$ is most positive so, there is need to make a closed loop by considering $'\theta'$ corresponding to fourth column and third row. The possible loop is shown in Table 3.5.

Table 3.5: First iteration of MODI method

Sources \ Destinations	D_1	D_2	D_3	D_4	Availability (A_i)
S_1	9.75	4.77 θ [4.54]	9.48	0 $-\theta$ [0.23]	4.77
S_2	14.72 θ [4.47]	$-\theta$ 6.76 [2.04]	15.48	0	6.51
S_3	17.61 [2.29] $-\theta$	7.50	6.87 [4.58]	0 θ	6.87
Demand (B_j)	6.76	6.58	4.58	0	

It is obvious from Table 3.5 that θ has been subtracted from 0.23, 2.29 and 2.04 so $\theta = \min\{0.23, 2.29, 2.04\}$ i.e. $\theta = 0.23$. Corresponding to θ the new initial basic feasible solution is shown in Table 3.6.

Table 3.6: The new basic solution

Sources \ Destinations	D_1	D_2	D_3	D_4	Availability (A_i)
S_1	9.75	4.77 [4.77]	9.48	0	4.77
S_2	14.72 [4.7]	6.76 [1.81]	15.48	0	6.51
S_3	17.61 [2.06]	7.50	6.87 [4.58]	0 [0.23]	6.87
Demand (B_j)	6.76	6.58	4.58	0	

Since the basic variables are $x_{12}, x_{14}, x_{22}, x_{21}, x_{31}, x_{33}$ so, the values of dual variables $u_1, u_2, u_3, v_1, v_2, v_3, v_4$ will be obtained on solving the following system of equations by considering any one dual variable as $''0''$.

(i) $u_1 + v_2 = 4.77,$

(ii) $u_3 + v_4 = 0,$

(iii) $u_2 + v_2 = 6.76,$

(iv) $u_3 + v_1 = 17.61,$

(v) $u_3 + v_3 = 6.87,$

(vi) $u_2 + v_1 = 14.72.$

Assuming $u_1 = 0,$ the values of remaining dual variables are $v_1 = 12.73,\ u_2 = 1.99,\ u_3 = 4.88,\ v_2 = 4.77,\ v_3 = 1.99,\ v_4 = -4.88$

Now the values of $(u_i + v_j - C_{ij})$ corresponding non-basic variables are

(i) $u_1 + v_1 - C_{11} = 2.98,$

(ii) $u_1 + v_3 - C_{13} = -7.49,$

(iii) $u_2 + v_3 - C_{23} = -11.5,$

(iv) $u_3 + v_2 - C_{32} = 2.15,$

(v) $u_2 + v_4 - C_{24} = -2.89,$

(vi) $u_1 + v_4 - C_{14} = -4.88$

Since $u_1 + v_1 - C_{11}, u_3 + v_2 - C_{32}$ are greater than zero so the obtained basic feasible solution is not optimal. Furthermore, as $u_1 + v_1 - C_{11}$ is most positive so, there is need to make a closed loop by considering $'\theta'$ corresponding to fourth column and third row. The possible loop is shown in Table 3.7.

Table 3.7: Second iteration of MODI method

Destinations / Sources	D_1	D_2	D_3	D_4	Availability (A_i)
S_1	9.75 θ	4.77 **4.77** $-\theta$	9.48	0	4.77
S_2	14.72 **4.7** $-\theta$	6.76 **1.81** θ	15.48	0	6.51
S_3	17.61 **2.06**	7.50	6.87 **4.58**	0 **0.23**	6.87
Demand (B_j)	6.76	6.58	4.58	0	

It is obvious from Table 3.7 that θ has been subtracted from 4.77 and 4.7 so $\theta = \min\{4.77,\ 4.7\}$ i.e., $\theta = 4.7$. Corresponding to θ the new initial basic feasible solution is shown in Table 3.8.

Table 3.8: The new basic solution

Destinations / Sources	D_1	D_2	D_3	D_4	Availability (A_i)
S_1	9.75 **4.7**	4.77 **0.07**	9.48	0	4.77
S_2	14.72	6.76 **6.51**	15.48	0	6.51
S_3	17.61 **2.06**	7.50	6.87 **4.58**	0 **0.23**	6.87
Demand (B_j)	6.76	6.58	4.58	0	

Since the basic variables are $x_{11}, x_{12}, x_{22}, x_{34}, x_{31}, x_{33}$ so, the values of dual variables $u_1, u_2, u_3, v_1, v_2, v_3, v_4$ will be obtained on solving the following system of equations by considering any one dual variable as $''0''$.

(i) $u_1 + v_2 = 4.77,$

(ii) $u_3 + v_4 = 0,$

(iii) $u_2 + v_2 = 6.76,$

(iv) $u_3 + v_1 = 17.61,$

(v) $u_3 + v_3 = 6.87,$

(vi) $u_1 + v_1 = 9.75$

Assuming $u_1 = 0,$ the values of remaining dual variables are $v_1 = 9.75,\ u_2 = 1.99,\ u_3 = 7.86,$ $v_2 = 4.77,\ v_3 = -0.99,\ v_4 = -7.86.$

Now the values of $(u_i + v_j - C_{ij})$ corresponding non-basic variables are

(i) $u_2 + v_1 - C_{11} = -2.98,$

(ii) $u_1 + v_3 - C_{13} = -10.47,$

(iii) $u_2 + v_3 - C_{23} = -14.48,$

(iv) $u_3 + v_2 - C_{32} = 5.13,$

(v) $u_2 + v_4 - C_{24} = -5.87,$

(vi) $u_1 + v_4 - C_{14} = -7.86$

Since $u_3 + v_2 - C_{32}$ is greater than zero so the obtained basic feasible solution is not optimal. Furthermore, as $u_3 + v_2 - C_{32}$ is most positive so, there is need to make a closed loop by considering $'\theta'$ corresponding to fourth column and third row. The possible loop is shown in Table 3.9.

Table 3.9: Third iteration of MODI method

Destinations / Sources	D_1	D_2	D_3	D_4	Availability (A_i)
S_1	9.75 θ [4.7]	4.77 $-\theta$ [0.07]	9.48	0	4.77
S_2	14.72	6.76 [6.51]	15.48	0	6.51
S_3	17.61 [2.06] $-\theta$	7.50 θ	6.87 [4.58]	0 [0.23]	6.87
Demand (B_j)	6.76	6.58	4.58	0	

It is obvious from Table 3.9 that θ has been subtracted from 2.06 and 0.07 so $\theta = \min\{2.06, 0.07\}$ i.e., $\theta = 0.07$. Corresponding to θ the new initial basic feasible solution is shown in Table 3.10.

Table 3.10: The new basic solution

Destinations / Sources	D_1	D_2	D_3	D_4	Availability (A_i)
S_1	9.75 [4.77]	4.77	9.48	0	4.77
S_2	14.72	6.76 [6.51]	15.48	0	6.51
S_3	17.61 [1.99]	7.50 [0.07]	6.87 [4.58]	0 [0.23]	6.87
Demand (B_j)	6.76	6.58	4.58	0	

Since the basic variables are $x_{11}, x_{32}, x_{22}, x_{34}, x_{31}, x_{33}$ so, the values of dual variables $u_1, u_2, u_3, v_1, v_2, v_3, v_4$ will be obtained on solving the following system of equations by considering any one dual variable as $''0''$.

(i) $u_3 + v_2 = 7.50$,

(ii) $u_3 + v_4 = 0$,

(iii) $u_2 + v_2 = 6.76$,

(iv) $u_3 + v_1 = 17.61$,

(v) $u_3 + v_3 = 6.87$,

(vi) $u_1 + v_1 = 9.75$

Assuming $u_1 = 0$, the values of remaining dual variables are $v_1 = 9.75$, $u_2 = 7.12$, $u_3 = 7.86$, $v_2 = -0.36$, $v_3 = -0.99$, $v_4 = -7.86$

Now the values of $(u_i + v_j - C_{ij})$ corresponding non-basic variables are

(i) $u_2 + v_1 - C_{11} = 2.15$,

(ii) $u_1 + v_3 - C_{13} = -10.47$,

(iii) $u_2 + v_3 - C_{23} = -9.35,$

(iv) $u_1 + v_2 - C_{12} = -5.13,$

(v) $u_2 + v_4 - C_{24} = -0.74,$

(vi) $u_1 + v_4 - C_{14} = -7.86$

Since $u_2 + v_1 - C_{11}$ is greater than zero so the obtained basic feasible solution is not optimal. Furthermore, as $u_2 + v_1 - C_{11}$ is most positive so, there is need to make a closed loop by considering $'\theta'$ corresponding to fourth column and third row. The possible loop is shown in Table 3.11.

Table 3.11: Fourth iteration of MODI method

Destinations / Sources	D_1	D_2	D_3	D_4	Availability (A_i)
S_1	9.75 [4.77]	4.77	9.48	0	4.77
S_2	14.72 θ	6.76 $-\theta$ [6.51]	15.48	0	6.51
S_3	17.61 [1.99] $-\theta$	7.50 θ [0.07]	6.87 [4.58]	0 [0.23]	6.87
Demand (B_j)	6.76	6.58	4.58	0	

It is obvious from Table 3.11 that θ has been subtracted from 1.99 and 6.51 so $\theta = \min\{1.99, 6.51\}$ i.e., $\theta = 1.99$. Corresponding to θ the new initial basic feasible solution is shown in Table 3.12.

Table 3.12: Optimal solution of the transportation problem

Sources \ Destinations	D_1	D_2	D_3	D_4	Availability (A_i)
S_1	9.75 [4.77]	4.77	9.48	0	4.77
S_2	14.72 [1.99]	6.76 [4.52]	15.48	0	6.51
S_3	17.61	7.50 [2.06]	6.87 [4.58]	0 [0.23]	6.87
Demand (B_j)	6.76	6.58	4.58	0	

Since the basic variables are $x_{11}, x_{32}, x_{22}, x_{34}, x_{21}, x_{33}$ so, the values of dual variables $u_1, u_2, u_3, v_1, v_2, v_3, v_4$ will be obtained on solving the following system of equations by considering any one dual variable as $''0''$.

(vii) $u_3 + v_2 = 7.50,$

(viii) $u_3 + v_4 = 0,$

(ix) $u_2 + v_2 = 6.76,$

(x) $u_2 + v_1 = 14.72,$

(xi) $u_3 + v_3 = 6.87,$

(xii) $u_1 + v_1 = 9.75$

Assuming $u_1 = 0,$ the values of remaining dual variables are $v_1 = 9.75,\ u_2 = 4.97,\ u_3 = 5.71, v_2 = 1.79,\ v_3 = 1.16,\ v_4 = -5.71$

Now the values of $(u_i + v_j - C_{ij})$ corresponding non-basic variables are

(vii) $u_3 + v_1 - C_{31} = -2.15,$

(viii) $u_1 + v_3 - C_{13} = -8.32,$

(ix) $u_2 + v_3 - C_{23} = -9.35,$

(x) $u_1 + v_2 - C_{12} = -2.98,$

(xi) $u_2 + v_4 - C_{24} = -0.74,$

(xii) $u_1 + v_4 - C_{14} = -5.71$

Since all are less than zero so the obtained basic feasible solution is optimal.'s Therefore,

the total transportation cost

$$= 9.75 \times 4.77 + 14.72 \times 1.99 + 6.76 \times 4.52 + 7.50 \times 2.06 + 6.87 \times 4.58 + 0 \times 0.23 = 153.27.$$

3.5 CONCLUSION

The existing method [1] for solving fully fuzzy transportation problem with trapezoidal fuzzy number is discussed in a detailed manner.

CHAPTER 4

PROPOSED METHOD FOR SOLVING FULLY FUZZY TRANSPORTATION PROBLEMS WITH PENTAGONAL FUZZY NUMBERS

Masheswari and Ganesan [2] defined the concept of pentagonal fuzzy number as well as proposed a method for solving fuzzy transportation problem with pentagonal fuzzy numbers. In this chapter, it is shown that more computational efforts are required to solve a fully fuzzy transportation problem with pentagonal fuzzy numbers by Maheswari and Ganesan's method [2]. Furthermore an alternative simplified method is proposed for solving a fuzzy transportation problems with pentagonal fuzzy numbers.

4.1 PRELIMINARIES

In this section definition of a pentagonal fuzzy number, arithmetic operations of two pentagonal fuzzy numbers and the method for comparing two pentagonal fuzzy numbers, proposed by Mashewari and Ganesan [2], are presented.

4.1.1 SOME BASIC DEFINITIONS

Definition 4.1: A fuzzy number $\tilde{A} = (a, b, c, d, e)$ is called a pentagonal fuzzy number when the membership function is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a \\ \left(\dfrac{x-a}{b-a}\right), & a \le x < b \\ \left(\dfrac{x-b}{c-b}\right), & b \le x < c \\ 1, & x = c \\ \left(\dfrac{d-x}{d-c}\right), & c < x < d \\ \left(\dfrac{e-x}{e-d}\right), & d \le x \le e \\ 0, & x > e \end{cases}$$

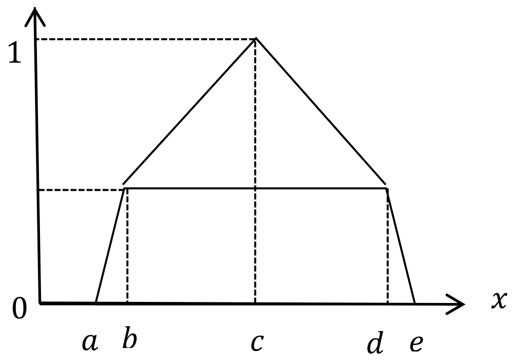

Figure 4.1 A pentagonal fuzzy number

4.1.2 ARITHMETIC OPERATIONS OF PENTAGONAL FUZZY NUMBER

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1, e_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2, e_2)$ be two pentagonal fuzzy numbers. Then,

(i) $\tilde{A}_1 + \tilde{A}_2 = (a_1{+}a_2, b_1{+}b_2, c_1{+}c_2, d_1{+}d_2, e_1 + e_2)$.

(ii) $\tilde{A}_1 - \tilde{A}_2 = (a_1{-}d_2, b_1{-}c_2, c_1{-}b_2, d_1{-}a_2, e_1 - a_2)$.

(iii) $\tilde{A}_1 \times \tilde{A}_2 = \left(\frac{a_1}{5}\beta_p, \frac{b_1}{5}\beta_p, \frac{c_1}{5}\beta_p, \frac{d_1}{5}\beta_p, \frac{e_1}{5}\beta_p\right), where\ \beta_p = (a_2 + b_2 + c_2 + d_2 + e_2)$.

(iv) $\tilde{A}_1 \div \tilde{A}_2 = \left(\frac{5a_1}{\beta_p}, \frac{5b_1}{\beta_p}, \frac{5c_1}{\beta_p}, \frac{5d_1}{\beta_p}, \frac{5e_1}{\beta_p}\right), if\ \beta_p \neq 0\ where\ \beta_p = (a_2 + b_2 + c_2 + d_2 + e_2)$.

(v) $k\tilde{A} = \begin{Bmatrix} (ka, kb, kc, kd, ke), & if\ k > 0 \\ (ke, kd, kc, kb, ka), & if\ k < 0 \end{Bmatrix}$

4.1.3 METHOD FOR COMPARING TWO PENTAGONAL FUZZY NUMBERS

Maheswari and Ganesan [2] proposed the following method for comparing two pentagonal fuzzy numbers.

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1, e_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2, e_2)$ be two pentagonal fuzzy numbers. Then,

(i) $\tilde{A}_1 > \tilde{A}_2\ if\ R(\tilde{A}_1) > R(\tilde{A}_2)$

(ii) $\tilde{A}_1 < \tilde{A}_2\ if\ R(\tilde{A}_1) < R(\tilde{A}_2)$

(iii) $\tilde{A}_1 = \tilde{A}_2\ if\ R(\tilde{A}_1) = R(\tilde{A}_2)$

where $R(\tilde{A}_1) = \left(\frac{a_1+b_1+c_1+d_1+e_1}{5}\right)$, $R(\tilde{A}_2) = \left(\frac{(a_2+b_2+c_2+d_2+e_2)}{5}\right)$.

4.2 MAHESWARI AND GANESAN'S METHOD

The method, proposed by Maheswari and Ganesan [2], is same as the classical methods for solving crisp transportation problems. The only difference is that Maheswari and Ganesan [2] have used the arithmetic operations of pentagonal fuzzy number instead of arithmetic operation of real numbers as well as Maheswari and Ganesan [2] have used the method, discussed in Section 4.1.2 for comparing pentagonal fuzzy numbers.

To illustrate the method, proposed by Maheswari and Ganesan [2], the fully fuzzy transportation problem, considered by Maheswari and Ganesan [2] to illustrate their proposed method, is solved in this section. Maheswari and Ganesan [2] solved the fully fuzzy transportation problem, represented by Table 4.1, in the following manner.

Table 4.1: Fully fuzzy transportation problem

	D_1	D_2	D_3	Availability (A_i)
S_1	(5,10,13,14,18)	(1,2,3,4,5)	(2,6,8,10,14)	(2,11,23,34,45)
S_2	(3,4,5,6,7)	(1,5,6,7,11)	(1,4,5,9,16)	(10,47,52,65,76)
S_3	(3,6,9,12,15)	(2,5,7,8,8)	(1,1,1,1,1)	(3,18,56,76,87)
Demand (B_j)	(11,16,51,67,75)	(20,40,60,80,100)	(15,30,45,75,110)	

Step 1 It is obvious from Table 4.1 that Total availability= $(2,11,23,34,45) + (10,47,52,65,76) + (3,18,56,76,87) = (15,76,131,175,208)$ and $(11,16,51,67,75) + (20,40,60,80,100) + (15,30,45,75,110) = (46,86,156,222,285) =$ Total demand

Since $R(15,76,131,175,208) \neq R(46,86,156,222,285)$ i.e., total availability is not equal to total demand so, the transportation problem, represented by Table 4.1, is an unbalanced transportation problem. Furthermore $R(15,76,131,175,208) < R(46,86,156,222,285)$ i.e., total availability is less than total demand. So, there is need to add a dummy source having dummy supply $(46,86,156,222,285) - (15,76,131,175,208) = (-162,-89,25,146,270)$. On adding the dummy source S_4 the transportation problem, represented by Table 4.1, will be transformed into Table 4.2

Table 4.2: Balanced fully fuzzy transportation problem

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	(5,10,13,14,18)	(1,2,3,4,5)	(2,6,8,10,14)	(2,11,23,34,45)
S_2	(3,4,5,6,7)	(1,5,6,7,11)	(1,4,5,9,16)	(10,47,52,65,76)
S_3	(3,6,9,12,15)	(2,5,7,8,8)	(1,1,1,1,1)	(3,18,56,76,87)
S_4	(0,0,0,0,0)	(0,0,0,0,0)	(0,0,0,0,0)	$(-162,-89,25,146,270)$
Demand (B_j)	(11,16,51,67,75)	(20,40,60,80,100)	(15,30,45,75,110)	

Step 2 In this step any classical method is used to find the initial basic feasible solution of the balanced fully fuzzy transportation problem. Using Vogel's approximation method, the initial basic feasible solution of the balanced fully fuzzy transportation problem, represented by Table 4.2, can be obtained as follows.

Step 2(a): The penalties corresponding to each row and column can be obtained as follows.

(i) Since $R(1,2,3,4,5) < \text{R}(2,6,8,10,14) < \text{R}(5,10,13,14,18)$ i.e. the smallest element of the first row is (1,2,3,4,5) and the next minimum element is (2,6,8,10,14) therefore the penalties corresponding to first row is (2,6,8,10,14)$-$ (1,2,3,4,5)= $(-3,2,5,8,13)$.

(ii) Since $R(3,4,5,6,7) < \text{R}(1,5,6,7,11) < \text{R}(2,6,8,10,14)$ i.e. the smallest element of the second row is (3,4,5,6,7) and the next minimum element is $(1,5,6,7,11)$ therefore the penalties corresponding to first row is $(1,5,6,7,11) - (3,4,5,6,7) = (-6,1,1,3.8)$.

(iii) Since $R(1,1,1,1,1) < \text{R}(2,5,7,8,8) < \text{R}(3,6,9,12,15)$ i.e. the smallest element of the third row is $(1,1,1,1,1)$ and the next minimum element is $(2,5,7,8,8)$ therefore the penalties corresponding to first row is $(2,5,7,8,8) - (1,1,1,1,1) = (1,4,6,7,7)$.

(iv) Since $R(0,0,0,0,0) \leq \text{R}(0,0,0,0,0) \leq \text{R}(0,0,0,0,0)$ i.e. the smallest element of the fourth row is (0,0,0,0,0) and the next minimum element is (0,0,0,0,0) therefore the penalties corresponding to first row is $(0,0,0,0,0) - (0,0,0,0,0) = (0,0,0,0,0)$.

(v) Since $R(0,0,0,0,0) < \mathrm{R}(3,4,5,6,7) < \mathrm{R}(3,6,9,12,15) < \mathrm{R}(5,10,13,14,18)$ i.e. the smallest element of the first column is $(0,0,0,0,0)$ and the next minimum element is $(3,4,5,6,7)$ therefore the penalties corresponding to first column is $(3,4,5,6,7) - (0,0,0,0,0) = (3,4,5,6,7)$.

(vi) Since $R(0,0,0,0,0) < \mathrm{R}(1,2,3,4,5) < \mathrm{R}(2,5,7,8,8) \leq \mathrm{R}(1,5,6,7,11)$ i.e. the smallest element of the second column is $(0,0,0,0,0)$ and the next minimum element is $\mathrm{R}(1,2,3,4,5)$ therefore the penalties corresponding to second column is $\mathrm{R}(1,2,3,4,5) - (0,0,0,0,0) = \mathrm{R}(1,2,3,4,5)$.

(vii) Since $R(0,0,0,0,0) < \mathrm{R}(1,1,1,1,1) \leq \mathrm{R}(1,4,5,9,16) < \mathrm{R}(2,6,8,10,14)$ i.e. the smallest element of the third column is $(0,0,0,0,0)$ and the next minimum element is $(1,1,1,1,1)$ therefore the penalties corresponding to third column is $(2,5,7,8,8) - (0,0,0,0,0) = (1,1,1,1,1)$.

Step2(b):

$\max\{R(-3,2,5,8,13), R(-6,1,1,3,8), R(1,4,6,7,7), R(0,0,0,0,0), R(3,4,5,6,7), R(1,2,3,4,5),$ $R(1,1,1,1,1)\} = R(3,4,5,6,7)$, i.e., the maximum penalty is corresponding to first column and $\min\{R(0,0,0,0,0), \mathrm{R}(3,6,9,12,15), \mathrm{R}(3,4,5,6,7), R(5,10,13,14,18)\}$ i.e., minimum cost corresponding to first column is $(0,0,0,0,0)$ so, $(4,1)$ the first basic cell.

Step 2(c): The availability and demand corresponding to the basic (4,1) are $(-162,-89,25,146,270)$ and (11,16,51,67,75) respectively. Furthermore $\min\{R(-162,-89,25,146,270), R(11,16,51,67,75)\} = R(-162,-89,25,146,270)$ therefore $x_{41} = (-162,-89,25,146,270)$ and the remaining demand is

$(11,16,51,67,75) - (-162,-89,25,146,270) = (-259,-130,26,156,237)$.

Step 2(d): Since the availability of source S_4 has been fulfilled therefore leaving source S_4, the transportation problem, represented by Table 4.3 is obtained.

Table 4.3: First iteration of VAM method

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	(5,10,13,14,18)	(1,2,3,4,5)	(2,6,8,10,14)	(2,11,23,34,45)
S_2	(3,4,5,6,7)	(1,5,6,7,11)	(1,4,5,9,16)	(10,47,52,65,76)
S_3	(3,6,9,12,15)	(2,5,7,8,8)	(1,1,1,1,1)	(3,18,56,76,87)
S_4	(0,0,0,0,0) $(-162,-89,25,146,270)$	(0,0,0,0,0)	(0,0,0,0,0)	$(-162,-89,25,146,270)$
Demand (B_j)	(11,16,51,67,75)	(20,40,60,80,100)	(15,30,45,75,110)	

Step 2(e): The penalties corresponding to each row and column can be obtained as follows.

(i) Since $R(1,2,3,4,5) < \text{R}(2,6,8,10,14) < \text{R}(5,10,13,14,18)$ i.e. the smallest element of the first row is (1,2,3,4,5) and the next minimum element is (2,6,8,10,14) therefore the penalties corresponding to first row is (2,6,8,10,14)$-$ (1,2,3,4,5)$=$ $(-3,2,5,8,13)$.

(ii) Since $R(3,4,5,6,7) < \text{R}(1,5,6,7,11) < \text{R}(2,6,8,10,14)$ i.e. the smallest element of the second row is $(3,4,5,6,7)$ and the next minimum element is $(1,5,6,7,11)$ therefore the penalties corresponding to first row is $(1,5,6,7,11) - (3,4,5,6,7) = (-6,1,1,3.8)$.

(iii) Since $R(1,1,1,1,1) < \text{R}(2,5,7,8,8) < \text{R}(3,6,9,12,15)$ i.e. the smallest element of the third row is $(1,1,1,1,1)$ and the next minimum element is $(2,5,7,8,8)$ therefore the penalties corresponding to first row is $(2,5,7,8,8) - (1,1,1,1,1) = (1,4,6,7,7)$.

(iv) Since $\text{R}(3,4,5,6,7) < \text{R}(3,6,9,12,15) < \text{R}(5,10,13,14,18)$ i.e. the smallest element of the first column is $(3,4,5,6,7)$ and the next minimum element is $(3,6,9,12,15)$ therefore the penalties corresponding to first column is $(3,6,9,12,15) - (3,4,5,6,7) = (-4,0,4,8,12)$.

(v) Since $\text{R}(2,5,7,8,8) \leq \text{R}(1,5,6,7,11) < \text{R}(2,6,8,10,14)$ i.e. the smallest element of the second column is $(2,5,7,8,8)$ and the next minimum element is $(1,5,6,7,11)$ therefore the penalties corresponding to second column is $(1,5,6,7,11) - (2,5,7,8,8) = (-7,-3,-1,2,9)$.

(vi) Since $R(1,1,1,1,1) \leq R(1,4,5,9,16) < R(2,6,8,10,14)$ i.e. the smallest element of the third column is $(1,1,1,1,1)$ and the next minimum element is $(1,4,5,9,16)$ therefore the penalties corresponding to third column is $(1,4,5,9,16) - (1,1,1,1,1) = (0,3,4,8,15)$.

Step2(f): $\max R(-3,2,5,8,13), R(-6,1,1,3.8), R(1,4,6,7,7), R(-4,0,4,8,12), R(-7,-3,-1,2,9)$ $R(0,3,4,8,15)\} = R(0,3,4,8,15)$ Since the maximum penalty is corresponding to third column and $\min\{R(1,1,1,1,1), R(1,4,5,9,16), R(2,6,8,10,14)\}$ i.e. minimum cost corresponding to second column is R(1,1,1,1,1) so, (3,3) is the second basic cell . Again from the Step 2(c) we have to allocate the demand in the basic cell. This can be shown in the Table 4.4 given below

Table 4.4: Second iteration of VAM method

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	(5,10,13,14,18)	(1,2,3,4,5)	(2,6,8,10,14)	(2,11,23,34,45)
S_2	(3,4,5,6,7)	(1,5,6,7,11)	(1,4,5,9,16)	(10,47,52,65,76)
S_3	(3,6,9,12,15)	(2,5,7,8,8)	(1,1,1,1,1) (3,18,56,76,87)	(3,18,56,76,87)
S_4	(0,0,0,0,0) $(-162,-89,25,146,270)$	(0,0,0,0,0)	(0,0,0,0,0)	$(-162,-89,25,146,270)$
Demand (B_j)	(11,16,51,67,75)	(20,40,60,80,100)	(15,30,45,75,110)	

Step 2(g): The penalties corresponding to each row and column can be obtained as follows.

(i) Since $R(1,2,3,4,5) < R(2,6,8,10,14) < R(5,10,13,14,18)$ i.e. the smallest element of the first row is (1,2,3,4,5) and the next minimum element is (2,6,8,10,14) therefore the penalties corresponding to first row is (2,6,8,10,14)$-$ (1,2,3,4,5)= $(-3,2,5,8,13)$.

(ii) Since $R(3,4,5,6,7) < R(1,5,6,7,11) < R(2,6,8,10,14)$ i.e. the smallest element of the second row is $(3,4,5,6,7)$ and the next minimum element is $(1,5,6,7,11)$ therefore the penalties corresponding to first row is $(1,5,6,7,11) - (3,4,5,6,7) = (-6,1,1,3.8)$.

(iii) Since $R(3,4,5,6,7) < R(5,10,13,14,18)$ i.e. the smallest element of the first column is $(3,4,5,6,7)$ and the next minimum element is $(5,10,13,14,18)$ therefore the penalties corresponding to first column is $(5,10,13,14,18) - (3,4,5,6,7) = (-2,4,8,10,15)$.

(iv) Since $R(1,5,6,7,11) < R(2,6,8,10,14)$ i.e. the smallest element of the second column is $(1,5,6,7,11)$ and the next minimum element is $(2,6,8,10,14)$ therefore the penalties corresponding to second column is $(2,6,8,10,14) - (1,5,6,7,11) = (-9,-1,2,5,13)$.

(v) Since $R(1,4,5,9,16) < R(2,6,8,10,14)$ i.e. the smallest element of the third column is $(1,4,5,9,16)$ and the next minimum element is $(2,6,8,10,14)$ therefore the penalties corresponding to third column is $(2,6,8,10,14) - (1,4,5,9,16) = (-14,-3,3,6,13)$.

Step2(h):

$\max\{R(-3,2,5,8,13), R(-6,1,1,3.8), R(-2,4,8,10,15), \mathrm{R}(-9,-1,2,5,13), \mathrm{R}(-14,-3,3,6,13)\} = R(-2,4,8,10,15)$. Since the maximum penalty is corresponding to first column and $\min\{R(5,10,13,14,18), \mathrm{R}(3,4,5,6,7)\}$ i.e. minimum cost corresponding to first column is (3,4,5,6,7) so, (2,1) is the third basic cell. Again from the Step 2(c) we have to allocate the demand in the basic cell. This can be shown in the Table 4.5 given below.

Table 4.5: Third iteration of VAM method

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	(5,10,13,14,18)	(1,2,3,4,5)	(2,6,8,10,14)	(2,11,23,34,45)
S_2	(3,4,5,6,7) (−259,−130,26,156,237)	(1,5,6,7,11)	(1,4,5,9,16)	(10,47,52,65,76)
S_3	(3,6,9,12,15)	(2,5,7,8,8)	(1,1,1,1,1) (3,18,56,76,87)	(3,18,56,76,87)
S_4	(0,0,0,0,0) (−162,−89,25,146,270)	(0,0,0,0,0)	(0,0,0,0,0)	(−162,−89,25,146,270)
Demand (B_j)	(11,16,51,67,75)	(20,40,60,80,100)	(15,30,45,75,110)	

Step 2(i): The penalties corresponding to each row and column can be obtained as follows.

(i) Since $R(1,2,3,4,5) < \mathrm{R}(2,6,8,10,14)$ i.e. the smallest element of the first row is (1,2,3,4,5) and the next minimum element is (2,6,8,10,14) therefore the penalties corresponding to first row is (2,6,8,10,14)− (1,2,3,4,5)= $(-3,2,5,8,13)$.

(ii) Since $\mathrm{R}(1,5,6,7,11) < \mathrm{R}(2,6,8,10,14)$ i.e. the smallest element of the second row is $(1,5,6,7,11)$ and the next minimum element is $(2,6,8,10,14)$ therefore the penalties corresponding to first row is $(2,6,8,10,14) - (1,5,6,7,11) = (-9,-1,2,5,13)$.

(iii) Since $\mathrm{R}(1,5,6,7,11) < \mathrm{R}(2,6,8,10,14)$ i.e. the smallest element of the second column is $(1,5,6,7,11)$ and the next minimum element is $(2,6,8,10,14)$ therefore the penalties corresponding to second column is $(2,6,8,10,14) - (1,5,6,7,11) = (-9,-1,2,5,13)$.

(iv) Since $\mathrm{R}(1,4,5,9,16) < \mathrm{R}(2,6,8,10,14)$ i.e. the smallest element of the third column is $(1,4,5,9,16)$ and the next minimum element is $(2,6,8,10,14)$ therefore the penalties corresponding to third column is $(2,6,8,10,14) - (1,4,5,9,16) = (-14,-3,3,6,13)$

Step 2(j):

$\max\{R(-3,2,5,8,13), R(-9,-1,2,5,13), \mathrm{R}(-9,-1,2,5,13), \mathrm{R}(-14,-3,3,6,13)\} =$ $R(-3,2,5,8,13)$. Since the maximum penalty is corresponding to first row and $\min\{R(1,2,3,4,5), \mathrm{R}(2,6,8,10,14)\}$ i.e., minimum cost corresponding to first column is $(1,2,3,4,5)$, so (1,2) is the fourth basic cell. Again from the Step 2(c) we have to allocate the demand in the basic cell. This can be shown in the Table 4.6 given below.

Table 4.6: Fourth iteration of VAM method

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	(5,10,13,14,18)	(1,2,3,4,5) (2,11,23,34,45)	(2,6,8,10,14)	(2,11,23,34,45)
S_2	(3,4,5,6,7) (−259,−130,26,156,237)	(1,5,6,7,11)	(1,4,5,9,16)	(10,47,52,65,76)
S_3	(3,6,9,12,15)	(2,5,7,8,8)	(1,1,1,1,1) (3,18,56,76,87)	(3,18,56,76,87)
S_4	(0,0,0,0,0) (−162,−89,25,146,270)	(0,0,0,0,0)	(0,0,0,0,0)	(−162,−89,25,146,270)
Demand (B_j)	(11,16,51,67,75)	(20,40,60,80,100)	(15,30,45,75,110)	

Step 3: Using VAM method we get the basic initial feasible solution for our fully fuzzy transportation problem. The final table is shown in the Table 4.7 below.

Table 4.7: Initial basic feasible solution

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	(5,10,13,14,18)	(1,2,3,4,5) (2,11,23,34,45)	(2,6,8,10,14)	(2,11,23,34,45)
S_2	(3,4,5,6,7) (−259,−130,26,156,237)	(1,5,6,7,11) (−334,166,37,241, 407)	(1,4,5,9,16) (−72,−46,−11,57,107)	(10,47,52,65,76)
S_3	(3,6,9,12,15)	(2,5,7,8,8)	(1,1,1,1,1) (3,18,56,76,87)	(3,18,56,76,87)
S_4	(0,0,0,0,0) (−162,−89,25,146,270)	(0,0,0,0,0)	(0,0,0,0,0)	(−162,−89,25,146,270)
Demand (B_j)	(11,16,51,67,75)	(20,40,60,80,100)	(15,30,45,75,110)	

The fuzzy basic initial basic feasible solution $=(1,2,3,4,5) \times (2,11,23,34,45) + (3,4,5,6,7) \times (-259,-130,26,156,237) + (1,5,6,7,11) \times (-334,-166,37,241,407) + (1,1,1,1,1) \times (3,18,56,76,87) + (1,4,5,9,16) \times (-72,-46,-11,57,107) + (0,0,0,0,0) \times (-162,-89,25,146,270) = (133,331,404,498,724)$

Step 4: Using the IBFS, presented in Table 4.7 and classical MODI method, the optimal solution of the fully fuzzy transportation problem, represented by Table 4.1 can be obtained as follows.

Step 4(a):

If $\tilde{u}_1 = (u_{11}, u_{12}, u_{13}, u_{14}, u_{15}), \tilde{u}_2 = (u_{21}, u_{22}, u_{23}, u_{24}, u_{25}), \tilde{u}_3 = (u_{31}, u_{32}, u_{33}, u_{34}, u_{35}),\quad \tilde{u}_4 = (u_{41}, u_{42}, u_{43}, u_{44}, u_{45}), \tilde{v}_1 = (v_{11}, v_{12}, v_{13}, v_{14}, v_{15}), \tilde{v}_1 = (v_{21}, v_{22}, v_{23}, v_{24}, v_{25}),\quad \tilde{v}_2 = (v_{21}, v_{22}, v_{23}, v_{24}, v_{25})$, $\widetilde{v}_3 = (v_{31}, v_{32}, v_{33}, v_{34}, v_{35})$ are dual variables corresponding to sources and destination respectively. Then,

$(u_{11}, u_{12}, u_{13}, u_{14}, u_{15}) + (v_{21}, v_{22}, v_{23}, v_{24}, v_{25}) = (1,2,3,4,5)$,

$(u_{21}, u_{22}, u_{23}, u_{24}, u_{25}) + (v_{11}, v_{12}, v_{13}, v_{14}, v_{15}) = (3,4,5,6,7)$,

$(u_{21}, u_{22}, u_{23}, u_{24}, u_{25}) + (v_{21}, v_{22}, v_{23}, v_{24}, v_{25}) = (1,5,6,7,11)$,,

$(u_{21}, u_{22}, u_{23}, u_{24}, u_{25}) + (v_{31}, v_{32}, v_{33}, v_{34}, v_{35}) = (1,4,5,9,16)$,

$(u_{31}, u_{32}, u_{33}, u_{34}, u_{35}) + (v_{31}, v_{32}, v_{33}, v_{34}, v_{35}) = (1,1,1,1,1)$,

$(u_{41}, u_{42}, u_{43}, u_{44}, u_{45}) + (v_{11}, v_{12}, v_{13}, v_{14}, v_{15}) = (0,0,0,0,0)$

assuming $(u_{11}, u_{12}, u_{13}, u_{14}, u_{15}) = (0,0,0,0,0)$ we have

$(v_{21}, v_{22}, v_{23}, v_{24}, v_{25}) = (1,2,3,4,5)$

$(u_{21}, u_{22}, u_{23}, u_{24}, u_{25}) = (-4,1,3,5,10)$

$(v_{11}, v_{12}, v_{13}, v_{14}, v_{15}) = (-7,-1,2,5,11)$

$(v_{31}, v_{32}, v_{33}, v_{34}, v_{35}) = (-9,-1,2,8,20)$

$(u_{31}, u_{32}, u_{33}, u_{34}, u_{35}) = (-19,-9,-1,2,10)$

$(u_{41}, u_{42}, u_{43}, u_{44}, u_{45}) = (-11,-5,-2,1,7)$

Now for non-basic cell we have

$(u_{11}, u_{12}, u_{13}, u_{14}, u_{15}) + (v_{11}, v_{12}, v_{13}, v_{14}, v_{15}) - (5,10,13,14,18) = (-25,-13,-11,-5,6)$

$(u_{11}, u_{12}, u_{13}, u_{14}, u_{15}) + (v_{31}, v_{32}, v_{33}, v_{34}, v_{35}) - (2,6,8,10,14) = (-23,-11,-6,2,18)$

$(u_{31}, u_{32}, u_{33}, u_{34}, u_{35}) + (v_{11}, v_{12}, v_{13}, v_{14}, v_{15}) - (3,6,9,12,15) = (-31,-22,-8,1,18)$

$(u_{31}, u_{32}, u_{33}, u_{34}, u_{35}) + (v_{21}, v_{22}, v_{23}, v_{24}, v_{25}) - (2,5,7,8,8) = (-16,-7,-2,7,23)$

$(u_{41}, u_{42}, u_{43}, u_{44}, u_{45}) + (v_{21}, v_{22}, v_{23}, v_{24}, v_{25}) - (0,0,0,0,0) = (-10,-3,1,5,12)$

$(u_{41}, u_{42}, u_{43}, u_{44}, u_{45}) + (v_{31}, v_{32}, v_{33}, v_{34}, v_{35}) - (0,0,0,0,0) = (-20, -6,0,9,27)$.

Now, $R(-25, -13, -11, -5,6) < 0$

$R(-23, -11, -6,2,18) < 0$,

$R(-31, -22, -8,1,18) < 0$,

$R(-16, -7, -2,7,23) > 0$,

$R(-10, -3,1,5,12) > 0$,

$R(-20, -6,0,9,27) > 0$.

Table 4.8: First iteration of MODI method

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	(5,10,13,14,18)	(1,2,3,4,5) (2,11,23,34,45)	(2,6,8,10,14)	(2,11,23,34,45)
S_2	(3,4,5,6,7) θ (−259,−130,26,156,237)	(1,5,6,7,11) (−334,−166,37,241, 407)	(1,4,5,9,16) $-\theta$ (−72,−46,−11,57,107)	(10,47,52,65,76)
S_3	(3,6,9,12,15)	(2,5,7,8,8)	(1,1,1,1,1) (3,18,56,76,87)	(3,18,56,76,87)
S_4	(0,0,0,0,0) $-\theta$ (−162,−89,25,146,270)	(0,0,0,0,0)	(0,0,0,0,0) θ	(−162,−89,25,146,270)
Demand (B_j)	(11,16,51,67,75)	(20,40,60,80,100)	(15,30,45,75,110)	

Now $min\{R(-72, -46,11,57,107), R((-162,89,25,146,270)\} =$ R$(-72, -46,11,57,107)$ hence $\theta = (-72, -46,11,57,107)$. Now we have to form a closed loop to rearrange the allocation in the basic cell. Clearly, here position of the basic cell also changed accordingly.

Step 4(b):

If $(u_{11}, u_{12}, u_{13}, u_{14}, u_{15}) + (v_{21}, v_{22}, v_{23}, v_{24}, v_{25}) = (1,2,3,4,5)$,

$(u_{21}, u_{22}, u_{23}, u_{24}, u_{25}) + (v_{11}, v_{12}, v_{13}, v_{14}, v_{15}) = (3,4,5,6,7)$,

$(u_{21}, u_{22}, u_{23}, u_{24}, u_{25}) + (v_{21}, v_{22}, v_{23}, v_{24}, v_{25}) = (1,5,6,7,11)$,,

$(u_{41}, u_{42}, u_{43}, u_{44}, u_{45}) + (v_{31}, v_{32}, v_{33}, v_{34}, v_{35}) = (0,0,0,0,0)$

$(u_{31}, u_{32}, u_{33}, u_{34}, u_{35}) + (v_{31}, v_{32}, v_{33}, v_{34}, v_{35}) = (1,1,1,1,1)$,

$(u_{41}, u_{42}, u_{43}, u_{44}, u_{45}) + (v_{11}, v_{12}, v_{13}, v_{14}, v_{15}) = (0,0,0,0,0)$

assuming $(u_{11}, u_{12}, u_{13}, u_{14}, u_{15}) = (0,0,0,0,0)$ we have

$(v_{21}, v_{22}, v_{23}, v_{24}, v_{25}) = (1,2,3,4,5)$

$(u_{21}, u_{22}, u_{23}, u_{24}, u_{25}) = (-4,1,3,5,10)$

$(v_{11}, v_{12}, v_{13}, v_{14}, v_{15}) = (-7,-1,2,5,11)$

$(v_{31}, v_{32}, v_{33}, v_{34}, v_{35}) = (-7,-1,2,5,11)$

$(u_{31}, u_{32}, u_{33}, u_{34}, u_{35}) = (-10,-4,-1,2,8)$

$(u_{41}, u_{42}, u_{43}, u_{44}, u_{45}) = (-11,-5,-2,1,7)$

Now for non-basic cell we have

$(u_{11}, u_{12}, u_{13}, u_{14}, u_{15}) + (v_{11}, v_{12}, v_{13}, v_{14}, v_{15}) - (5,10,13,14,18) = (-25,-13,-11,-5,6)$

$(u_{11}, u_{12}, u_{13}, u_{14}, u_{15}) + (v_{31}, v_{32}, v_{33}, v_{34}, v_{35}) - (2,6,8,10,14) = (-23,-11,-6,2,18)$

$(u_{31}, u_{32}, u_{33}, u_{34}, u_{35}) + (v_{11}, v_{12}, v_{13}, v_{14}, v_{15}) - (3,6,9,12,15) = (-32,-17,-8,1,16)$

$(u_{31}, u_{32}, u_{33}, u_{34}, u_{35}) + (v_{21}, v_{22}, v_{23}, v_{24}, v_{25}) - (2,5,7,8,8) = (-17,-10,-5,1,11)$

$(u_{21}, u_{22}, u_{23}, u_{24}, u_{25}) + (v_{31}, v_{32}, v_{33}, v_{34}, v_{35}) - (1,4,5,9,16) = (-27,-9,0,6,20)$

$(u_{41}, u_{42}, u_{43}, u_{44}, u_{45}) + (v_{21}, v_{22}, v_{23}, v_{24}, v_{25}) - (0,0,0,0,0) = (-10,-3,1,5,12)$.

now, $R(-25,-13,-11,-5,6) < 0$

$R(-23,-11,-6,2,18) < 0$,

$R(-32,-17,-8,1,16) < 0$,

$R(-17,-10,-5,1,11) < 0$,

$R(-10,-3,1,5,12) > 0$,

$R(-27,-9,0,6,20) < 0$.

Table 4.9: Second iteration of MODI method

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	(5,10,13,14,18)	(1,2,3,4,5) (2,11,23,34,45)	(2,6,8,10,14)	(2,11,23,34,45)
S_2	(3,4,5,6,7) (−331,−176,15,213,344)θ	(1,5,6,7,11) (−334,166,37,241,407) $-\theta$	(1,4,5,9,16)	(10,47,52,65,76)
S_3	(3,6,9,12,15)	(2,5,7,8,8)	(1,1,1,1,1) (3,18,56,76,87)	(3,18,56,76,87)
S_4	(0,0,0,0,0) $-\theta$ (−269,−146,36,192,342)	(0,0,0,0,0) θ	(0,0,0,0,0) (−72,−46,−11,57,107)	(−162,−89,25,146,270)
Demand (B_j)	(11,16,51,67,75)	(20,40,60,80,100)	(15,30,45,75,110)	

Now $min\{R(-334,166,37,241,407), R(-269,-146,36,192,342)\} = \text{R}(-72,-46,11,57,107)$ hence $\theta = (-269,-146,36,192,342)$. Now we have to form a closed loop to rearrange the allocation in the basic cell. Clearly, here position of the basic cell also changed accordingly.

$(u_{11},u_{12},u_{13},u_{14},u_{15}) + (v_{21},v_{22},v_{23},v_{24},v_{25}) = (1,2,3,4,5)$,

$(u_{21},u_{22},u_{23},u_{24},u_{25}) + (v_{11},v_{12},v_{13},v_{14},v_{15}) = (3,4,5,6,7)$,

$(u_{21},u_{22},u_{23},u_{24},u_{25}) + (v_{21},v_{22},v_{23},v_{24},v_{25}) = (1,5,6,7,11)$,

$(u_{41},u_{42},u_{43},u_{44},u_{45}) + (v_{31},v_{32},v_{33},v_{34},v_{35}) = (0,0,0,0,0)$,

$(u_{31},u_{32},u_{33},u_{34},u_{35}) + (v_{31},v_{32},v_{33},v_{34},v_{35}) = (1,1,1,1,1)$,

$(u_{41},u_{42},u_{43},u_{44},u_{45}) + (v_{21},v_{22},v_{23},v_{24},v_{25}) = (0,0,0,0,0)$

Assuming $(u_{11},u_{12},u_{13},u_{14},u_{15}) = (0,0,0,0,0)$ we have

$(v_{21},v_{22},v_{23},v_{24},v_{25}) = (1,2,3,4,5)$

$(u_{21},u_{22},u_{23},u_{24},u_{25}) = (-4,1,3,5,10)$

$(v_{11},v_{12},v_{13},v_{14},v_{15}) = (-7,-1,2,5,11)$

$(v_{31},v_{32},v_{33},v_{34},v_{35}) = (1,2,3,4,5)$

$(u_{31},u_{32},u_{33},u_{34},u_{35}) = (-4,-3,-2,-1,0)$

$(u_{41},u_{42},u_{43},u_{44},u_{45}) = (-5,-4,-3,-2,-1)$

Now for non-basic cell we have

$(u_{11}, u_{12}, u_{13}, u_{14}, u_{15}) + (v_{11}, v_{12}, v_{13}, v_{14}, v_{15}) - (5,10,13,14,18) = (-25, -13, -11, -5,6)$

$(u_{11}, u_{12}, u_{13}, u_{14}, u_{15}) + (v_{31}, v_{32}, v_{33}, v_{34}, v_{35}) - (2,6,8,10,14) = (-13, -8, -5, -4,3)$

$(u_{31}, u_{32}, u_{33}, u_{34}, u_{35}) + (v_{11}, v_{12}, v_{13}, v_{14}, v_{15}) - (3,6,9,12,15) = (-26, -16, -9, -2,8)$

$(u_{31}, u_{32}, u_{33}, u_{34}, u_{35}) + (v_{21}, v_{22}, v_{23}, v_{24}, v_{25}) - (2,5,7,8,8) = (-11, -9, -6, -2,3)$

$(u_{21}, u_{22}, u_{23}, u_{24}, u_{25}) + (v_{31}, v_{32}, v_{33}, v_{34}, v_{35}) - (1,4,5,9,16) = (-19, -6,1,5,14)$

$(u_{41}, u_{42}, u_{43}, u_{44}, u_{45}) + (v_{21}, v_{22}, v_{23}, v_{24}, v_{25}) - (0,0,0,0,0) = (-12, -5, -1,3,10)$.

now, $R(-25, -13, -11, -5,6) < 0$

$R(-13, -8, -5, -4,3) < 0$,

$R(-26, -16, -9, -2,8) < 0$,

$R(-11, -9, -6, -2,3) < 0$,

$R(-19, -6,1,5,14) < 0$,

$R(-12, -5, -1,3,10) < 0$.

Table 4.10: Final optimal solution

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	(5,10,13,14,18)	(1,2,3,4,5) (2,11,23,34,45)	(2,6,8,10,14)	(2,11,23,34,45)
S_2	(3,4,5,6,7) (−600,−322,51,405,686)	(1,5,6,7,11) (−676,−358,1,387,676)	(1,4,5,9,16)	(10,47,52,65,76)
S_3	(3,6,9,12,15)	(2,5,7,8,8)	(1,1,1,1,1) (3,18,56,76,87)	(3,18,56,76,87)
S_4	(0,0,0,0,0)	(0,0,0,0,0) (−269,−146,36,192,342)	(0,0,0,0,0) (−72,−46,−11,57,107)	(−162,−89,25,146,270)
Demand (B_j)	(11,16,51,67,75)	(20,40,60,80,100)	(15,30,45,75,110)	

The fuzzy optimal cost

$= (1,2,3,4,5) \times (2,11,23,34,45) + (3,4,5,6,7) \times (-600, -322,51,405,686) + (1,5,6,7,11) \times$

$(-676, -358,1,387,676) + (1,1,1,1,1) \times (3,18,56,76,87) + (0,0,0,0,0) \times$

$(-269,-146,36,192,342)+(0,0,0,0,0)\times(-72,-46,-11,57,107)=(6,33,69,102,135)+$
$(-3000,-1610,255,2025,3430)+(-4056,-2148,6,2322,4056)+(3,18,56,76,87)=$
$(-7047,-3707,386,4525,7708)$

4.3 PROPOSED METHOD

It is obvious from the Section 4.2 that more computational efforts are required for solving fully fuzzy transportation problem by Maheswari and Ganesan's method [2].

In this section, with the help of the methods, discussed in the previous chapters, a new method is proposed for solving fully fuzzy transportation problem with the help of pentagonal fuzzy numbers.

The steps of proposed method are discussed as follows.

Step 1 Represent the fully fuzzy transportation problem in tabular form as shown in Table 4.11

Table 4.11: Tabular representation of a fully fuzzy transportation problem

Sources ↓	Destinations ⟶					
	D_1	D_2	D_3	…	D_n	Availability
S_1	$\tilde{C}_{11}$	$\tilde{C}_{12}$	$\tilde{C}_{13}$	…	$\tilde{C}_{1n}$	$\tilde{A}_1$
. . .				. . .		. . .
S_m	$\tilde{C}_{n1}$	$\tilde{C}_{n2}$	$\tilde{C}_{n3}$	…	$\tilde{C}_{nm}$	$\tilde{A}_m$
Demand	$\tilde{B}_1$	$\tilde{B}_2$	$\tilde{B}_3$	…	$\tilde{B}_n$	

where, $\tilde{C}_{ij}$, $\tilde{A}_i$ and $\tilde{B}_j$ are pentagonal fuzzy numbers $(C_{ij_1},C_{ij_2},C_{ij_3},C_{ij_4},C_{ij_5})$, $(A_{i1},A_{i2},A_{i3},A_{i4},A_{i5})$ and $(B_{j1},B_{j2},B_{j3},B_{j4},B_{j5})$ respectively. Here $\tilde{C}_{ij}$, $\tilde{A}_i,\tilde{B}_j$ represents the fuzzy transportation cost, fuzzy availability of the product and fuzzy demand of the product respectively.

Step 2 Transform the fully fuzzy transportation problem represented by Table 4.11 into the crisp transportation problem represented by Table 4.12, by replacing $\tilde{C}_{ij}$, $\tilde{A}_i$ and $\tilde{B}_j$ with $R(\tilde{C}_{ij}),R(\tilde{A}_i)$ and $R(\tilde{B}_j)$ respectively.

Table 4.12: Transformed crisp transportation problem

Sources ↓	Destinations →					
	D_1	D_2	D_3	$\cdots$	D_n	Availability
S_1	$R(\tilde{C}_{11})$	$R(\tilde{C}_{12})$	$R(\tilde{C}_{13})$	$\cdots$	$R(\tilde{C}_{1n})$	$R(\tilde{A}_1)$
. . .				. . .		. . .
S_m	$R(\tilde{C}_{n1})$	$R(\tilde{C}_{n2})$	$R(\tilde{C}_{n3})$	$\cdots$	$R(\tilde{C}_{nm})$	$R(\tilde{A}_m)$
Demand	$R(\tilde{B}_1)$	R($\tilde{B}_2$)	$R(\tilde{B}_3)$	$\cdots$	R($\tilde{B}_n$)	

Step 3 Find the optimal solution $\{x_{ij}\}$ as well as optimal transportation cost of the crisp transportation problem, represented by Table 4.12.

4.4 ILLUSTRATIVE EXAMPLE

Using the proposed methods the fully fuzzy transportation problem, represented by Table 4.1 can be solved as follows.

Step 1: Using Step 1 of the proposed method, the fully fuzzy transportation problem represented by Table 4.1 can be converted into crisp transportation problem represented by Table 4.13 by using the ranking function which is used by Maheswari and Ganesan [2].

Table 4.13: Transformed crisp transportation problem

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	12	3	8	23
S_2	5	6	7	50
S_3	9	6	1	48
Demand (B_j)	44	60	55	

Clearly the above crisp transportation problem represented in Table 4.13 is an unbalanced transportation problem therefore a dummy source S_4 have to be added so that the above transportation problem can be converted into balanced transportation problem. After adding a dummy a source the balanced transportation problem is shown in the Table 4.14.

Table 4.14: Balanced crisp transportation problem

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	12	3	8	23
S_2	5	6	7	50
S_3	9	6	1	48
S_4	0	0	0	38
Demand (B_j)	44	60	55	

Using the least cost method for finding the initial basic solution and using MODI method for finding the optimal solution with the help of the obtained initial basic feasible solution, the optimal solution of the crisp transportation problem, represented by Table 4.14, can be obtained as follows.

Step 2(a) The initial basic feasible solution of the crisp transportation problem represented by Table 4.14 obtained by least cost method, is shown in Table 4.15.

Table 4.15: Initial basic feasible solution

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	12 [23]	3	8	23
S_2	5 [21]	6 [29]	7	50
S_3	9	6 [31]	1 [17]	48
S_4	0	0	0 [38]	38
Demand (B_j)	44	60	55	

Total initial transportation cost

$= 12 \times 23 + 5 \times 21 + 6 \times 29 + 6 \times 31 + 1 \times 17 + 0 \times 38 = 713.$

Step (2b) Since the basic variables are $x_{12}, x_{21}, x_{22}, x_{31}, x_{33}$ so, the values of dual variables $u_1, u_2, u_3, v_1, v_2, v_3$ will be obtained on solving the following system of equations by considering any one dual variable as $''0''$.

(i) $u_1 + v_1 = 2,$

(ii) $u_2 + v_1 = 5,$

(iii) $u_2 + v_2 = 6,$

(iv) $u_3 + v_2 = 6,$

(v) $u_3 + v_3 = 1,$

(vi) $u_4 + v_3 = 0.$

Assuming $u_1 = 0,$ the values of remaining dual variables are $v_1 = 12, u_2 = -7, u_3 = -7, v_2 = 13, v_3 = 8, u_4 = -8$ now, the values of $\left(u_i + v_j - C_{ij}\right)$ corresponding non-basic variables are

(i) $u_1 + v_2 - C_{12} = 10,$

(ii) $u_1 + v_3 - C_{13} = 0,$

(iii) $u_2 + v_3 - C_{23} = -6,$

(iv) $u_3 + v_1 - C_{31} = -4$

(v) $u_4 + v_1 - C_{41} = 4$

(vi) $u_4 + v_2 - C_{42} = 5$

Since $u_1 + v_2 - C_{12},\ u_4 + v_1 - C_{41}$ and $u_4 + v_2 - C_{42}$ are greater than zero so the obtained basic feasible solution is not optimal. Furthermore, as $u_1 + v_2 - C_{12} \geq 0$ is most positive so, there is need to make a closed loop by considering $'\theta'$ and corresponding to second column and first row the possible loop is shown in Table 4.16.

Table 4.16: First iteration of MODI method

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	12 [23] $-\theta$	3 θ	8	23
S_2	5 [21] θ	6 $-\theta$ [29]	7	50
S_3	9	6 [31] $-\theta$	1 θ [17]	48
S_4	0	0 $-\theta$	0 θ [38]	38
Demand (B_j)	44	60	55	

It is obvious from Table 4.17 that θ has been subtracted from 31 and 38 so $\theta = \min\{23, 29\}$ i.e., $\theta = 29$ corresponding to θ the new initial basic feasible solution is shown in Table 4.17.

Table 4.17: Second iteration of MODI method

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	12	3 [23]	8	23
S_2	5 [44]	6 [6]	7	50
S_3	9	6 [31] $-\theta$	1 θ [17]	48
S_4	0	0 θ	0 $-\theta$ [38]	38
Demand (B_j)	44	60	55	

(i) $u_1 + v_2 = 3,$

(ii) $u_2 + v_1 = 5,$

(iii) $u_2 + v_2 = 6,$

(iv) $u_3 + v_2 = 6,$

(v) $u_3 + v_3 = 1,$

(vi) $u_4 + v_3 = 0.$

Assuming $u_1 = 0,$ the values of remaining dual variables are $v_1 = 2, u_2 = 3, u_3 = 3, v_2 = 3,\ v_3 = -2\ , u_4 = 2$

Now the values of $(u_i + v_j - C_{ij})$ corresponding non-basic variables are

(i) $u_1 + v_1 - C_{11} = -10,$

(ii) $u_1 + v_3 - C_{13} = -10,$

(iii) $u_2 + v_3 - C_{23} = -6,$

(iv) $u_3 + v_1 - C_{31} = -4,$

(v) $u_4 + v_1 - C_{41} = 4,$

(vi) $u_4 + v_2 - C_{42} = 5.$

Since $, u_4 + v_1 - C_{41}$ and $u_4 + v_2 - C_{42}$ are greater than zero so the obtained basic feasible solution is not optimal. Furthermore, as $u_4 + v_2 - C_{42} \geq 0$ is most positive so, there is need to make a closed loop by considering $'\theta'$ and corresponding to second column and forth row the possible loop is shown in Table 4.18.

Table 4.18: Third iteration of MODI method

Destinations / Sources	D_1	D_2	D_3	Availability (A_i)
S_1	12	3 [23]	8	23
S_2	5 [44]	6 [6]	7	50
S_3	9	6	1 [48]	48
S_4	0	0 [31]	0 [7]	38
Demand (B_j)	44	60	55	

(i) $u_1 + v_2 = 3,$

(ii) $u_2 + v_1 = 5,$

(iii) $u_2 + v_2 = 6,$

(iv) $u_4 + v_2 = 0,$

(v) $u_3 + v_3 = 1,$

(vi) $u_4 + v_3 = 0.$

Assuming $u_1 = 0$, the values of remaining dual variables are $v_1 = 2, u_2 = 3, u_3 = -2, v_2 = 3, v_3 = 3, u_4 = -3$

Now the values of $(u_i + v_j - C_{ij})$ corresponding non-basic variables are

(i) $u_1 + v_1 - C_{11} = -10,$

(ii) $u_1 + v_3 - C_{13} = -5,$

(iii) $u_2 + v_3 - C_{23} = -1,$

(iv) $u_3 + v_1 - C_{31} = -9$

(v) $u_3 + v_2 - C_{32} = -4$

(vi) $u_4 + v_1 - C_{41} = -1$

Since all $(u_i + v_j - C_{ij}) < 0$

Total transportation cost

$$= 3 \times 23 + 5 \times 44 + 6 \times 6 + 48 \times 1 + 0 \times 31 + 0 \times 7 = 373$$

4.5 CONCLUSION

On the basis of present study, it can be concluded that less computational efforts are required to solve a fuzzy transportation problems with pentagonal fuzzy numbers by the proposed method as compared to Maheswari and Ganesan’s method [2].

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