“AN INTEGRATED INVENTORY MODEL WITH CONTROLLABLE LEAD TIME AND ADDITIONAL TRANSPORTATION COST”

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CERTIFICATE

This is to certify that the thesis entitled **"An integrated inventory model with controllable lead time and additional transportation cost"**, being presented in partial fulfilment of the requirements for the award of the degree of Master of Science in the School of Mathematics, Thapar Institute of Engineering & Technology, Patiala, is a bonafide work carried out under the supervision of **Dr. Mahesh Kumar Sharma**.

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

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This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

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There will be many hurdles, disbeliever, faults, confusions, misstep. Although by backbreaking one can achieve any goal, there is no end.

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ABSTRACT

In a manufacturing backlog structure, the craftsmen construct the products at a specific production scale, ships the placed orders to the purchaser in definite time periods and carries the extra inventory for the successive shipments. In actual world, an inventory supervisor has to carry a large number of primal matter, work in process items and polished goods in the backlog to survive in the aggressive merchandise. Numerous models have been proposed in literature for inventory management. The integrated models have been in light since they are concerned with the profit of both the parties- vendor and purchaser.

The present work has been split into three chapters.

In Chapter 1, the introduction of inventory models and the different approaches have been discussed. In Chapter 2, a research paper “A study of an integrated inventory with controllable lead time” Yang and Pan (2002) has been reviewed. In this paper, a joint inventory model for both vendor and purchaser is considered for reducing the joint total expected cost by reducing the lead time and thus obtaining the optimal solutions.

In Chapter 3, an integrated inventory model with controllable lead time given by Yang and Pan (2002) has been extended by introducing the additional transportation cost. In this model, the transportation cost is to be charged to the vendor for trading the primal matter from the purchaser. A numerical illustration is also mentioned in the support of the model.

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Chapter 1

Introduction

Inventory is attributed as stocks and it consists of the goods and primal matter that every employed company would carry and will be apt for marketing. For example- the minimarket in the locality has a considerable inventory of seasonally grown up fruits and vegetables which are available for trading.
Inventory model is a numerical standard which guides the trade in determining the ideal level of stockpile that should be sustained in a manufacturing course, controlling the density of ordering, decisive on abundance of goods or primal matter to be reserved, discovering the movement of supply of primal matter and goods to implement continual supply to clients without any lag in distribution.

1.1 General Approaches in an Inventory Structure

Inventory administration and supervision attributes to the outlining for optimal capacity of equipment during every phase of the manufacturing period and advancing the performance which will secure the availability of programmed backlogs. The advanced models keep on challenging the old hypothesis and the style of performing things. These contemporary approaches are entirely mobile which includes manufacturing mechanisms and the operations management. The motive of the supervisor of the manufacturing industries is to enhance the production capability. The supervisor must choose the desirable approach in order to meet the ongoing and future needs. These are the four important approaches

Material Requirement Planning (MRP), Just in Time Manufacturing (JIT), Optimized Production Technology (OPT), Flexible Manufacturing Systems (FMS).

In this chapter, the above specified approaches have been discussed briefly.

1.1.1Material Requirement Planning (MRP)

1.1.1.1 Economic Order Quantity Model (EOQ)

The most elementary and the finest acknowledged inventory decision model is the classical economic order quantity (EOQ) model. The upcoming challenges can be tackled easily by the use of EOQ model. Despite the fact, this model has been denounced due to its over-integrity for illustrating the most real-world scenario, but the positive side of this model is that it provided a skilful origin from which one can evolve more composite and rational inventory decision models. The critical EOQ model was matured by Harris (1915) and later Wilson (1934), an advisor who claimed it extensively, was given accommodation for its comprehensive investigation by Hax and Candea (1984).

The Economic Order Quantity (EOQ) represents the number of items that a firm will be going to add to the inventory with each order to diminish the overall amount of inventory which includes holding costs, order costs, and scarcity costs. Initially, the primitive assumptions include the no scarcity hypothesis for which the model calculates the ideal reorder point and the optimal reorder amount to assure the spontaneous decay of inventory. This type of models can be beneficial for modest business supervisor who have to make judgement regarding how much inventory to keep on hand in order to avoid shortages, how many units to order each time, and how often to backtrack to obtain the lowest possible costs.

The basic hypothesis of an EOQ model are as follows

1. The requirement is familiar, consistent and autonomous.
2. The lead time- which is the time interval among the placing of an order and the quittance of receiving it is also familiar.
3. The holding cost is familiar and consistent.
4. No discounts shall be provided.
5. No scarcity can occur.

The total expected cost of an inventory under this basic EOQ model associates an agreement between inventory holding costs (the cost of depot, along with the cost of confining up the capital in inventory rather than investing it) and order costs (any expense linked with placing orders, such as delivery charges). There can be two cases in such models- one can order a large quantity at a single time which will in turn increases the holding cost whereas in second case if one orders the units in instalments then the holding cost will be small as compared to the one in first case but the ordering cost will shoot up. The main role of this EOQ model is to obtain an optimal quantity which minimizes the sum of these costs. Kumar & Prajapati (2015) showed the 20% reduction in the total variable costs by using the EOQ model.

The basic EOQ model can further determine the production levels or order interval by making some changes in the model. By considering the holding costs and setup costs as the only variable costs, the basic above specified model can be used by the large firms across the world to make their stock management scheme much more economical, exclusively the ones with broad stockpile string and huge fluctuating costs per item of production and managing the desolation to the barest minimal, at the same time probing to accomplish the operational corporate target and objectives.

1.1.1.2 Economic Lot Size Model (ELS)

After this there is another economic lot size (ELS) model. Economic lot size (ELS) model was initially advanced in 1913. The costs of setup over an area of bunch of load is balanced counter to the costs of inventory. According to some defenders of Lean Manufacturing and Theory of Constraints, the Economic Lot Size (ELS) is dead. They argued that each and every agency should assemble the requirements of the customer in "batches" of one unit. In an optimal world, this approach may be correct whereas most of the industries are less than ideal where momentous setup costs occur but the batch amount is still a static controversy. Economic Lot Size model contributes to decisive observations for analytical opinion.

Lot Size Guidelines

- Predominantly, lot sizes should be flat at the edges of the ELS space. This cut down capital demand, flatten the production and causes the scheduling to be extra adjustable.

- If a peculiar tool is bottleneck and demands to engage at its maximal quantity, in such situations the lot sizes are appointed at the upper edge of the ELS space. This boosts the inventory buffers ahead and later the bottleneck operation, although it grants extra pace for production and lower pace for setups.
- When a chunk of machinery is preconditioned to be staffed and preserved even when unproductive, and if the machinery is not a bottleneck, the lot sizes are appointed at or below the ELS space. This boosts the setup cost whereas it declines the inventory.

The consequence of the Economic-lot-size equations tells us the order capacity which decreases the joint inventory and set-up (and ordering) costs. In assembling dilemma, economic-lot-size equations may result in absurd plan as in few production intervals, the market may not be able to accommodate demand, although at other times slack intervals that result in inactivity may occur.

1.1.1.3 Economic Production Size Model (EPS)

Economic production lot size model tells us the number of units of things generated or acquired at the minimum comprehensive inventory costs. It can be accomplished by stabilizing decreasing unit's setup cost and increasing unit's inventory cost. Taft (1918) advanced this model. This model is in contrast with the EPS model because Economic production lot size model presume that the assembler manufactures the units by themselves, consequently the goods are collected cumulatively during the time they are made. Differently, EOQ model presumes that the ordered amount turns up entirely after hiring, by cause as the items are made by different industry and are accessible for shipment whenever the order is acknowledged.

The model must satisfy the following 2 conditions in order to apply this economic production lot size model

1. The demand of the item remains stable all over the year.
2. The fresh order is produced incrementally when the inventory reaches zero.

The economic production lot size model takes into account three types of cost- setup costs, direct costs (precisely equivalent to the number of items generated, also incorporate the raw materials and employment costs) and inventory costs.

To formulate the model, a certain hypothesis should be considered-

1. The demand remains consistent all over the year;
2. The purchase rate is stable (no deduction applicable);
3. The lead time is stable;
4. Management runs to restore inventory are made at proper intermissions;
5. Additional replenishment;
6. Management setup cost is stable (does not depend on amount generated);
7. During the manufacture run, the production remains steady throughout.

   When buying larger portion:

   - The setup cost is flattened and the inventory costs is huge;
   - The larger assembling capabilities are advanced;
   - Capacity increases and utilization is reduced.

   When buying small portion:

   - The setup cost is huge and the inventory costs is flattened;
   - There is an adverse effect on assembling capabilities;
   - The accommodation decreases and utilization increases.

1.1.2 Just in Time Inventory

Quality enhancement and cost management are crucial to institutions for work performance improvement and satisfying purchasers needs. Inventories which consists of huge bundle of capacity may result in area exploitation and havoc.

To solve the above problem, the most powerful explanation is adopting Just in Time (JIT) approach as a way to diminish the respective costs, enhance the traits and meet the altering demands of the purchaser. In other words, JIT is a mainframe ideology which aims to eradicate the misuse and helps in yield expansion.

Just-in-time is an evolution and a concept which has achieved extensive approval in this fast-growing business era over the previous decagon. Due to the competing associations and the burden from Japan’s endless developed civilization, other companies are compelled to find ingenious actions to reduce the overall cost of an inventory. The approach trailing the JIT, or lean manufacturing, is to have the goods only at the time when they are ordered by the purchaser. This approach helps the firm to achieve its goal by minimizing the wastage and reinforcing its profit.

For example- an automobile producer works with very small inventory levels, depending upon its supplier to ship the components it requires to form an automobile. The components required to build an automobile does not buzz sooner or later they are required; rather, they appear only at the time of requirement.

The JIT incorporate the following aspects:

- The major obstacles are attacked to improve the whole model i.e. removing the traits that does not enumerate price to the respective item.
- The structure is crafted in such a manner to analyse the issues.
- The trait supervision is the liability of each labourer.
- The fool proof apparatus, approach, jigs etc. counters the blunder.
- Disposing off the wastage. There are seven types of waste- overproduction may generate the waste, waste due to delay in time, shipment waste, processing dissipation, backlog waste and waste due to defective items.
- The institute is kept clean which enforces good environment for workers.
- The setup time is diminished which in turn boosts resilience.
- Levelled construction – it allows the continuous generation of items in the industries.
- Kanban is an elementary mechanism to 'pull' items and segments through the development.
- Jidoka (Autonomation) is providing appliances with the self-governing effiency to use the knowledge, so that the laborer can do extra handy things instead of standing and observing them.
- Andon (trouble lights) provides signal the obstacles to trigger the remedial activity.

1.1.2.1 Just in Time (JIT) Inventory System Advantages and Disadvantages

Just in time inventory models have considerable dominance as compared to the classical models. The production rate remains short which gives the operator alliance to shift from one category of item to another very conveniently. This model diminishes the cost by eradicating stockroom storage requirements. The associations also contribute less capital on primal matter because they purchased only the sufficient amount of assets to make the ordered items and no more.

The disadvantages of just-in-time inventories holds interruption in the supply group. If a supplier of primal matter has a disruption due to some problem and cannot ship the items on time, then in such a case one supplier can close the integrated production process. An immediate order for items may outpace the outlook which in turn may postpone the shipment of finished products to the consumer.

1.1.2.2 JIT- Background and History

JIT is a Japanese administrative ideology which has been enforced in exercise since the initial 1970s in many Japanese construction institution. It was initially advanced within the Toyota assembling plants by Taiichi Ohno by virtue of fulfilling customer's requirements with minimal lag. The predecessor of JIT- Taiichi Ohno.
Toyota was apt to meet the upcoming challenges for continuity over an advent that fixate on nation. JIT would only be fruitful if each one of the sole within the management was sophisticated and devoted to it, i.e.- the plants and practice were organized for the best yield and expertise, and also the trait and manufacturing arrangements were anticipated to meet the definite requirements.

The blueprint in use at Toyota

JIT inventory controls were originated in 1970s by Toyota and it took additionally 15 years to become fool proof in its action. Toyota assigns the orders to acquirement production parts when it earns new orders from purchaser.

JIT model will be successful only if the association preserves a stable production standard, along with finest handiwork and no appliance disintegration. Also, it requires trustworthy provisional who can consistently ship the items instantaneously, and the capability to precisely summon parts that bring together its vehicles.

An example of interruption

A blaze occurred at a brake parts plant in 1997 purchased by the association Aisin which shattered its space to manufacture a P-valve segment for Toyota automobile. The association was sealed down for considerable weeks since Aisin was the exclusive provisional of this part for Toyota. As a result of Toyota's JIT inventory levels, it got exhausted of P-valve parts subsequently the day following the blaze.

Due to a blaze at company and there being only exclusive supplier could harshly ruin Toyota's supply chain. By good luck, one of Aisin's suppliers was capable of regenerating and started the construction of crucial P-valves subsequently after two days. Nonetheless, the blaze costed Toyota approximately $15 billion in forfeit revenue and 70,000 cars.

1.1.3 Optimized Production Technology (OPT)

Optimized Production Technology (OPT) is an automated formulation outlining and scheduling mechanism advanced by Creative Output of Milford, Connecticut, USA. Moreover, OPT pursue a set of convention known as the "Theory of Constraints" (TOC). This approach is concentrating the scrutiny on the accommodation constraints or blockage in chunk of the activity. One must analyse the area of constraints then try to discard them, and then again find next constraints an activity consistently focusing on the part that critically resolve the measure of the yield. A constraint is characterized as anything that inhibits an organization from accomplishing huge conduct analogous to its objective. The objective of this model is the expansion of throughput at the same time compressing both operational expenditure and backlog.

Aside, from MRP and many other approaches and systems have been advanced which admits the emphasis of planning to well-known space constraints, instead of overwhelming segment of the production system and thus failing to achieve the goal. The finest acknowledged approach adopted the Theory of Constraints (TOC). One must analyse the area of constraints then try to discard them, and then again find next constraints an activity consistently focusing on the part that critically resolve the measure of the yield. The pathway, which uses this ideology, is known optimised production technology (OPT). The authentic commerce of the software stock was originally given by Eliyahu Goldratt.

1.1.4 Flexible Manufacturing Systems

A flexible manufacturing system (FMS) is an assembling system in which there is some measure of resilience to counter in the cover of changes, whether anticipated or unanticipated. This affability can be split into two groups:

1. Apparatus affability,
2. Routing affability.

A definite number of instrument producers in Japan, the United States, and Europe are demanding to evolve flexible manufacturing systems. These structures are presumed to integrate planning and supervision of their machinery activities within their digital unified-supervised info systems. These info structure have inherent production planning practice; FMS parts-programming practice; and substantial-treatment practice for parts, gadgets, and embellishments; and stock regulation in the form of isolated section.

Although FMS is a digitalised approach, the director still performs a decisive part in interpreting associations target for the systems, which may consist of expansion of production yield, depreciation of unit production price, and expansion of periodical benefits.

The FMS model can freely reduce an employment strength in the midst of 10% and 15% of that which traditional adroitness required. Many of the upcoming academic works are being promoted by the ministry of numerous citizenry. Their eventual target is to formulate an FMS system that will need no trail force, will be exceptionally adjustable in charge of item and quantity mix, and will provide finest, economical yield with very short lead times.

1.1.5 Comparison between MRP, JIT and OPT

	MRP	JIT	OPT
Loading of activity	Analysed by quantity demands	Disciplined by Kanban system	Disciplined by bottleneck activity
Lot capacity	Outlining subsequently one week or more	Tiny as desirable	Fluctuating to exploit constraint
Concern of data efficiency	Analytical	Worthless	Analytical for bottleneck and feeder activity
Momentum of expected evolution	Slow	Very rapid	Rapid
Affability	Minimum	Maximum	Balanced
Price	Maximum	Minimum	Balanced
Target	Fulfil the demand	Fulfil the demand and dispose of the decay	Fulfil the demand and enlarge the benefits

1.2 Literature Review

The economic order quantity (EOQ) models are indeed one of the primeval models in the inventory scrutiny composition. Harris (1913) was the first one who worked on the issue of determining the economic lot size in production scheme. Subsequently, Wilson (1934) induced the model developed by Harris (1913) and he gave a formula to obtain economic order quantity. The classical Economic Production Quantity (EPQ) model was proposed by Taft (1918). EPQ designed that the replenishments are spontaneous and the corresponding amount exclusively contains the composition cost, stock-carrying cost of the polished stuff and the procuring cost of the resources. The stock-carrying amount for the primal matter was not added in the relevant cost by Taft (1918). Ghare and Schrader (1963) were the first one who studied the decaying inventory problems. A simple economic order quantity model was developed by them. In their study, they concluded that if there is consumption of decaying items then it is closely relative to the negative exponential function of time. Later, many authors developed the inventory models. Goyal (1973) studied a method for improving joint replenishment systems with a known frequency of replenishment orders and thought that the approach is generally relevant to the quantity refining cooperation where a particular volume is integrated and finally is gathered into different kind of packages- i.e. its main goal was to gather the products efficiently. Liberatore (1977) studied the EOQ model under stochastic lead time by assuming a regular deterministic-requirement, theoretical lead-time inventory model so that the specific unit needs are non-exchangeable and a lower bound, irrespective of the quantity space, was developed for the optimal charging pace.

Goyal (1977) proposed an integrated inventory model for a single supplier-single customer problem and determined that the joint model represented in his work can be continued and enforced in those situations where a vendor distributes a variety of items to the purchaser on an absolute ground. Dave and Patel (1981) too developed an inventory model for the deteriorating items with the time proportional demand. In this process, they assumed an EOQ model in which the demand rate was changing linearly with time and deterioration was assumed to be constant fraction of the on-hand inventory. Tersine (1982) studied the principles of Inventory and materials management and considered backlog string inventory models with tractable lead time, the initial one is planned beneath the centralized verdict approach whereas the later one studied under decentralized verdict approach. To obtain the optimal results, the explanatory plan was suggested. Additionally, shapely value measure and MCRS approach were used to coequal the welfare of both- supplier and the purchaser. Analytical illustration was given to interpret the conclusion of the developed models. Banerjee (1986), proposed a joint economic-lot-size model for purchaser and vendor and evolved a united economic-lot-size model for a certain situation where a supplier manufactures to charge for a customer on a lot-for-lot grounds under deterministic circumstances. The objective of this model was the collective total corresponding price and is concluded that a combined optimal charging approach, together with a relevant cost alteration, can be profitable efficiently for both the sides. The Banerjee (1986) model was extended by Goyal (1977) by leaving few hypotheses. A broad combined economical batch capacity model was advised and it resulted in lowering the cost as compared to the model of Banerjee (1986). Liao and Shyu (1991) studied an analytical determination of lead time with normal demand and developed an inventory model which can be used to resolve the portion of the lead time that diminishes the conventional absolute consistent cost. Daya and Raouf (1994) developed an inventory models involving lead time as decision variable where lead time is the decision inconstant. Richter (1996) proposed the extended EOQ repair and waste disposal model and he developed an additional EPQ model by accounting that all the resources are recyclable resources and the stock-carrying price of the resources was again taken into account. Also, it was hypothesized that all the commodity was screened for the affirmation of the items. Thomas and Griffin (1996) studied a coordinated supply chain management and analysed the composition inscribed the harmonized outlining amidst two or further phases of the supply segment, establishing appropriate attention on models which will grant themselves to a comprehensive supply chain model. Ouyang et al. (1996) studied the mixture inventory model with backorders and lost sales for variable lead time and pretended that the scarcity is granted and the chattels of framework. By enumerating the stockout cost, he continued the Ben-Daya and Raouf model (1994). Ouyang and Wu (1997) proposed the mixture inventory model involving variable lead time with a service level constraint where both lead time and the order batch were treated as decision inconstant for this mixture inventory model. Initially, it was hypothesised that the lead time requirement ensues a normal distribution, and subsequently reclining the supposition regarding the type of the distribution function of lead time requirement and finally practising the minimax distribution method to obtain the solution. Martinich (1997) studied the production and operations management and concluded that if the distributor accomplishes all the components of an order for the upcoming three to five years then in such a position the purchaser can demand the assurance of stable delivery, high quality, the declined expenses, a contribution in yield enhancements. Monczka (1998) proposed the purchasing and supply chain management and suggested that a good distributor would perform along with the purchaser firmly to diminish lead pace morally possible, and further be acceptable considering the distributor to control a balanced manufacturing and shipment record. Ouyang et al. (1999) developed a lead time and ordering cost reductions in continuous review inventory systems with partial backorders and inspected the encounter of ordering price cut back on the altered regular scrutiny inventory systems including inconstant lead time with a mixture of backorders and forfeit marketing. The goal is to spontaneously optimise the order space, charging price, reorder point and lead time. Initially, it was hypothesised that the lead time requirement ensues a normal distribution, and subsequently reclining the supposition to acknowledge the distribution free situation where only the mean and variance of lead time requirements are familiar. Yang and Pan (2002) developed a study of an integrated inventory with controllable lead time where they represented a joint inventory model for both supplier and customer for diminishing the total price by reducing the lead time. Yang and Pan (2004) studied just-in-time purchasing: an integrated inventory model involving deterministic variable lead time and quality improvement investment and investigated the integrated inventory model that reported for replenishment lead time reduction and quality enhancement expenditure considerations. The model minimized the total cost by simultaneously improving the quality and optimizing the order quantity, lead time and the count of deliveries. Salameh and Jaber (2000) created economic reduction quantity model for items with imperfect quality and in-sync with Shy-Der Lin (2013) developed an additional EPQ model by considering that all items were screened in the course of composing. The percentage of immature-quality resources were hooked and again the previous hypothesis- the stock-carrying price of resources was taken in debate. Su and Lin (2013) designed the optimal inventory policy of production management and Der & Lin (2013) was the first one to authorize an EPQ model by undertaking the stock-carrying price of resources into debate. Mandal and Giri (2015) studied a single-vendor multi-buyer integrated model with controllable lead time and quality improvement through reduction in defective items and relaxed the hypothesis of normal distribution where only the mean and standard deviation of lead-time demand are familiar and determined the optimum results of the decision variables in order to maximize the total profit. Wee and Wangsa (2017) developed an integrated vendor–buyer inventory model with transportation cost and stochastic demand and designed the heuristic model to minimise the combined total relevant cost by introducing a transportation cost which is a function of distance, shipping weight and transportation modes.

Chapter 2

A Study Of An Integrated Inventory With Controllable Lead Time (Yang and Pan (2002))

2.1 Introduction

Just-in-time (JIT) manufacturing or the Toyota production system (TPS), is a technique anticipated mainly at diminishing flow times within production system along with feedback from the purchaser to the distributor. Historically, a manufacturing company clashes on price, quality, multiplicity, service, etc. The faster an association responds to its purchaser, the more commercial it is. Currently, just-in-time (JIT) manufacturing performs an essential aspect in supply group ambiance. The associations nowadays adopt JIT manufacturing to grow and manage an ambitious preference. The main feature of this system is that it decreases the need for accumulating manufacturing materials, but increases the dependency on the distributor, quality control and a precise ordering task. The overall profit can be increased as a result of a JIT system, but it depends on distributor, supply and the respective industry. Altogether, a JIT system provides the manufacturing task and a quality-focused inventory. In this chapter, a research paper entitled *"A Study Of An Integrated Inventory With Controllable Lead Time"* (Yang and Pan (2002)) has been studied in detail.

This work represents a combined inventory model to diminish the cooperative total expected expenditure of distributor and purchaser by considering a new crashing cost which reduces the respective lead time although the probability distribution of the lead time requirement is normal. The structure presented here is appropriate especially for just in time backlog organizations where the distributor and its corresponding client design a critical collaboration for benefit allocations.

In today's era, companies are engaged in using a Just-in-Time manufacturing in order to improve their effectiveness, decrease the waste by cutting non-essential inventory and thereby reducing the total inventory cost. In this system, the distributor places an order only when the customer creates the demand. The system does not grant the company to produce or stock surplus inventory so that the holding cost can be shortened. The features of just in time organizations are uniform tremendous quality, limited batch sizes, persistent shipping, precise lead time and adjacent vendor ties. The lead time is the time that advances among the allocatement of an assortment and truly receiving the goods ordered. The controlling of lead time is identified to be feasible and is an active way to accomplish the objective of JIT. The lead time can be diminished by an extra crashing cost in order to enhance the purchaser's assistance level and diminish backlog in safety stocks. Safety stock inventory, also known as buffer stock, is an expression used to represent a level of extra stock that is preserved to mitigate risk of stockouts due to confusion in supply and demand. The equation for safety stock is given by

$$Safety\ stock = (maximal\ daily\ trade * maximal\ lead\ time) - (moderate\ daily\ trade * moderate\ lead\ time)$$

Historically, the lead time of a backlog miniature is mesmerized to be familiar or known with definite odds circulation. The lead time perhaps is diminished through a supplementary crashing cost, in order to enhance the client assistance level, and diminish the backlog in security stocks, which particularly means that the lead time is tractable. The following ingredients are contained in the crashing of lead time: order formation, order shipment, distributor lead time, and shipment pace. Many associations had established significant gain by joining hands with purchaser. If the distributor accomplishes all the components of an order for the upcoming three to five years then in such a position the purchaser can demand the assurance of stable delivery, high quality, a contribution in yield enhancements and the declined expenses. The two parties-vendor along with the purchaser should gain profit from the agreement and they should decide the criteria by what means a profit can be splitted between themselves. In 1991, Liao and Shyu represented a regular survey miniature where the placement capacity was fixed along with the lead time was a particular decisive variable. The Liao and Shyu model (1991) was extended by Ben-Daya and Raouf (1994) where both the order capacity as well as the lead time were considered as particular decisive variable. The total cost of an inventory which is the aggregate of the ordering cost, holding cost and the crashing cost was minimized through obtaining the optimal lead time and optimal order quantity. Ouyang et al. (1996) suggested a regular backlog miniature through the diminishing of the lead pace by granting shortfall and containing backorders and forfeited marketing.

Banerjee (1986) developed the approach of a combined economic lot size model (JELS) concealed by deterministic circumstances, concentrating on the combined total suitable expenditure for both the distributor as well as the customer. Banerjee (1986) model was postulated by Goyal (1988) by loosening up the hypothesis of the portion for portion approach of the distributor. There was compelling decline in inventory cost appropriately in the model advanced by Goyal (1988). Since it is an integrated model one's party gain is another's party loss. As a result, the net profit was mutual for vendor as well as purchaser in any stable sequence.

A good distributor would perform along with the purchaser firmly to diminish lead time morally possible, and further be acceptable by considering the distributor to control a balanced manufacturing and shipment record. The main query arises that who will be the one owing to pay the crashing amount is considered to be an open argument. There are many possibilities for this query (1) since the supply is exercised by the customer therefore he/she might pay the crashing cost, and (2) the distributor and customer collaborated in an acknowledged form. This miniature was evolved upon the basis of (1) and was approved to be appropriate. This work deals with tractable lead time for an integrated inventory model.

The suggested model resulted in lowering of combined total expected cost and the diminishing of lead time by using crashing cost components and is correlated with the two models of Banerjee (1986) and Goyal (1988). Goyal (1988) drawn out the Banerjee (1986) model by loosening up the portion for portion manufacturing hypothesis. This work in addition enhances the Goyal (1988) model by loosening up the manufacturing hypothesis. Since whenever the order is received from the respective industry, the lot is delivered to the purchaser thereby reducing the holding cost which is the motive of just in time inventory model. The classical obstacles have been proven to be collapsed by introducing such an integrated inventory model.

2.2 Notations and hypothesis

The following are the notations and hypothesis for this underlying paper:

D demand per annum,

P vendor's production rate per annum,

B purchaser's ordering cost per order,

T vendor's set-up per set-up,

h backlog holding cost per annum,

C_t unit production cost induced by vendor,

C_c unit production cost compensated by purchaser,

q placement or manufacture of order capacity,

u an integer corresponding to the figure of batches acquired in shipping the products from the vendor to purchaser,

R reorder point

SB safety stock

k security aspect

μ daily demand rate in units/day

σ predictable error of daily requirement in units/day

p_i minimal span of lead time factor i

q_i regular span of lead time factor i

cp_i crashing cost per unit time of lead time factor i

l range of lead time.

$$\sum_{i=1}^{n} p_i \leq l \leq \sum_{i=1}^{n} q_i$$

1. The order will be constructed alongside a fixed manufacturing estimate P, and $P > D$.
2. The requirement Y all along the lead time pursues the distribution normally with average μl and predictable error $\sigma\sqrt{l}$.
3. The inventory is analysed regularly. Whenever the backlog falls at the reorder point R, the order placement takes place.
4. In order to avoid overshooting at the reorder point, the items are demanded one at a time.
5. There are n components of the lead time and the respective components are crashed individually starting from the component having the minimal price/unit time.
6. Whenever the lead time will be diminished, an additional price will be induced in the cost of purchaser since the services are used by the purchaser.

2.3 Model formulation

Since R is the reorder point, SB is the safety stock and the daily requirement follows a distribution which is normal with average μl and predictable error $\sigma\sqrt{l}$.

Therefore, $R = conventional\ requirement\ during\ lead\ time + SB$

$$= \mu l + SB \qquad (1)$$

And $SB = k(predictable\ error\ of\ lead\ time\ requirement\ ) = \ k\sigma\sqrt{l}$ (2)

here k is called as security aspect.

Therefore equation (1) becomes

$$R = \mu l + k\sigma \qquad (3)$$

The total conventional cost per annum for the purchaser is -

$$TEC_p = ordering\ cost(O_p) + holding\ cost(H_p) + lead\ time\ crashing\ cost(L_p)$$

(4)

The ordering cost of the purchaser is calculated as follows-

Since B is the ordering cost per cost, therefore the total annual ordering cost is-

$$O_p = \left(\frac{D}{q}\right) B$$

If we assume that the replenishment occurs linearly over the period, then the moderate backlog for the purchaser is-

$$I_p \cong \frac{q}{2} + R - \mu l$$

$$I_p = \frac{q}{2} + \mu l + k\sigma\sqrt{l} - \mu l$$

$$I_p = \frac{q}{2} + k\sigma\sqrt{l}$$

Thus, the expected holding cost per annum is-

$$H_p = hC_c\left(\frac{q}{2} + k\sigma\sqrt{l}\right)$$

$$TEC_p = \left(\frac{D}{q}\right) B + hC_c\left(\frac{q}{2} + k\sigma\sqrt{l}\right) + \frac{D}{q} C(l) \quad (5)$$

Liao and Shyu (1991) proposed that the lead time conceivably be disintegrated into n mutually separate segments. Let p_i be the minimum duration of factor i, q_i be the normal duration of factor i, and cp_i be the crashing cost per unit time of lead time factor $i, i = (1,2,\ldots,n)$. By considering a linear tie, we can use the two endpoints p_i and q_i to obtain the linear equation describing the factor duration to its crashing cost. We are considering the following sequence of cp_i as $cp_1 \leq cp_2 \leq \ldots \leq cp_n$.

Now it is obvious to note that the factor 1 will be reduced first since it has the minimum crashing cost per unit time. After reducing the first factor the chain continues to reduce the factors according to their respective minimum crashing cost. Whenever the crashing cost on lead time exceeds the preserving on backlog holding cost in security stocks then at that particular point the further reduction of lead time is stopped.

We denote the maximum lead time as $l_0 = \sum_{j=1}^{n} q_j$

We define l_i as the range of lead time factor i crashed to its respective minimal span, $i = 1,2,\dots,n$, then l_i can be written as-

$$l_i = l_0 - \sum_{j=1}^{i} (q_j - p_j)$$

Also, we can write

$$l_i = \sum_{j=1}^{n} q_i - \sum_{j=1}^{i} (q_j - p_j)$$

$$l_i = q_1 + q_2 + \dots + q_n - [(q_1 - p_1) + (q_2 - p_2) + \dots + (q_n - p_n)]$$

$$l_i = p_1 + p_2 + \dots + p_n$$

Thus, we are left only with the minimum duration factors of lead time.

The lead time crashing cost

$$C(l) = cp_i(l_{i-1} - l) + \sum_{j=1}^{i-1} cp_j (q_j - p_j) \tag{6}$$

Thus, the expected lead time crashing cost is-

$$L_c = \frac{D}{q} C(l)$$

Total expected cost of the purchaser per annum is

$$TEC_p = \left(\frac{D}{q}\right) B + hC_c \left(\frac{q}{2} + k\sigma\sqrt{l}\right) + \frac{D}{q} C(l) \tag{7}$$

Since we have considered the case in which the extra cost will be incurred by the customer as he is the one using the services, so in vendor's inventory model there will be no crashing cost.

The total expected cost of vendor per annum is

$$TEC_v = ordering\ cost(O_v) + holding\ cost(H_v)$$

For ordering cost (O_v),since T is the ordering cost per set-up of vendor and in this work where we are considering the just-in-time inventory models, the production starts only when the purchaser places an order to the vendor. The placement of the order can be done in few number of lots which is given by an integer u.

Thus, $ordering\ cost(O_v) = \left(\frac{D}{uq}\right)T$

The cycle length for the purchaser as well as the vendor are given by $\frac{q}{D}$ and $\frac{q}{uD}$

conjointly.

The vendor manufactures the order in uq capacity and the purchaser will collect it in u parts, each having quantity q.

$$Average\ inventory\ time = \frac{weighted\ inventory\ for\ the\ vendor}{cycle\ length}$$

The average inventory for the vendor is given by-

$$I_v \cong \left\{ \frac{\left\{\left[uq\left(\frac{q}{P} + (u-1)\frac{q}{D}\right) - \frac{u^2q^2}{2P}\right] - \left[\frac{q}{D}(1 + 2 + \ldots + (u-1)q)\right]\right\}}{\left(\frac{uq}{D}\right)} \right\}$$

$$= \frac{\frac{uq^2}{P} + \frac{u(u-1)q^2}{D} - \frac{u^2q^2}{2P} - \frac{q^2u(u-1)}{D}}{\left(\frac{uq}{D}\right)}$$

$$= \frac{\frac{uq^2}{P}\left(1 - \frac{u}{2}\right) + \frac{u(u-1)q^2}{2D}}{\left(\frac{uq}{D}\right)}$$

$$= \frac{\frac{u(2-u)q^2}{2P} + \frac{u(u-1)q^2}{2D}}{\left(\frac{uq}{D}\right)}$$

$$= \frac{qD}{2}\left(\frac{2-u}{P} - \frac{u-1}{D}\right)$$

$$= \frac{q}{2}\left[u\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] \tag{8}$$

Thus, the expected holding cost of vendor per annum

$$Holding\ cost(H_v) = hC_t\frac{q}{2}\left[u\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right]$$

The total expected cost per annum of vendor is-

$$TEC_v(q,u) = \left[\frac{D}{uq}\right]T + hC_t\frac{q}{2}\left[u\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] \tag{9}$$

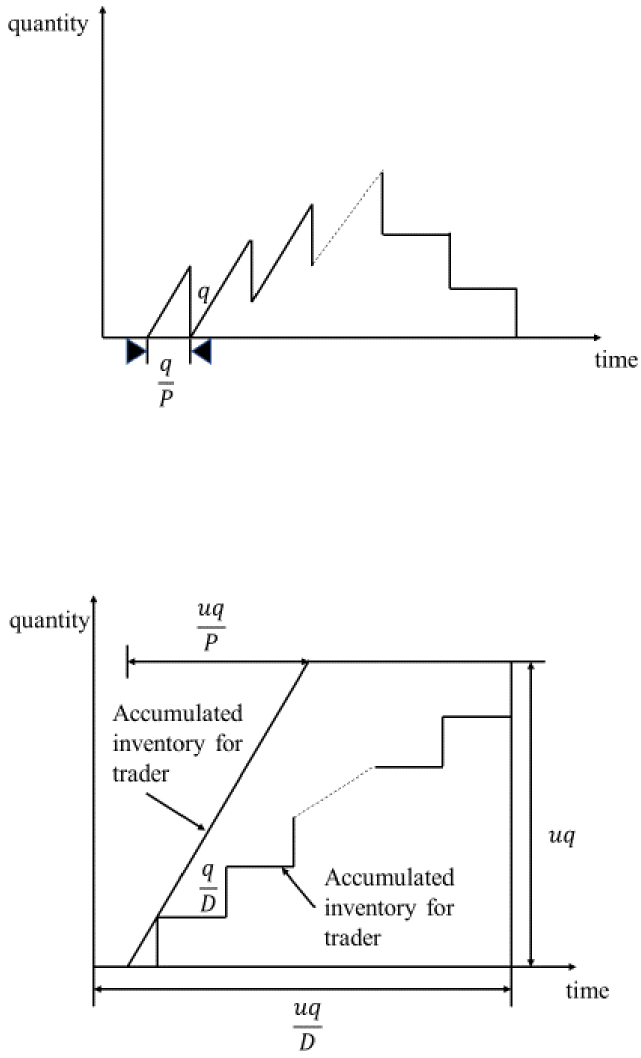


Figure 1. The inventory pattern for the vendor

The joint total expected cost per annum is-

$$JTEC(q,l,u) = \frac{D}{q}\left(B + \left(\frac{T}{u}\right) + C(l)\right) + \frac{qh}{2}\left[\left(u\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right)C_t + C_c\right] + hC_c k\sigma\sqrt{l} \quad (10.1)$$

In order to check the convexity and to calculate the value of order capacity q we have to proceed by taking partial derivative of $JTEC(q,l,u)$ with respect to q and l in every time spell (l_i, l_{i-1}).

$$\frac{\partial JTEC(q,l,u)}{\partial q} = -\frac{D}{q^2}\left(B+\frac{T}{u}+C(l)\right)+\frac{h}{2}\left[\left(u\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)C_t+C_c\right]$$
$$= 0 \tag{11}$$

$$\frac{D}{q^2}\left(B+\frac{T}{u}+C(l)\right)=\frac{h}{2}\left[\left(u\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)C_t+C_c\right]$$

$$q^2 = \frac{2D\left(B+\frac{T}{u}+C(l)\right)}{h\left(C_t\left(u\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)+C_c\right)}$$

$$q = \left(\frac{2D\left(B+\frac{T}{u}+C(l)\right)}{h\left(C_t\left(u\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)+C_c\right)}\right)^{\frac{1}{2}} \tag{12}$$

Also,

$$\frac{\partial^2 JTEC(q,l,u)}{\partial q^2} = \frac{2D}{q^3}\left(B+\frac{T}{u}+C(l)\right)>0$$

$JTEC(q,l,u)$ is convex in q for settled value of $l \in (l_i, l_{i-1})$

Now taking partial derivative w.r.t l we have,

$$\frac{\partial JTEC(q,l,u)}{\partial l} = -\frac{D}{q}cp_i+\frac{h}{2}C_c k\sigma l^{-1/2}=0 \tag{13}$$

Also, for fixed q, $JTEC(q,l,u)$ is concave in $l \in (l_i, l_{i-1})$ due to

$$\frac{\partial^2 JTEC(q,l,u)}{\partial l^2} = -\frac{h}{4} C_c k \sigma l^{-3/2} < 0$$

The joint total expected cost per annum for a particular value of u is given by

$$JTEC(u) = \left[2Dh\left(B + \tfrac{T}{u} + C(l)\right)\left(C_t\left(u\left(1 - \tfrac{D}{P}\right) - 1 + \tfrac{2D}{P}\right) + C_c\right)\right]^{\frac{1}{2}} + hC_c k\sigma\sqrt{l}$$

In order to minimise the joint total cost per annum, firstly take the square of above equation and then ignore the terms independent of u

$$\left(JTEC(u)\right)^2 = 2Dh\left(B + \tfrac{T}{u} + C(l)\right)\left(C_t\left(u\left(1 - \tfrac{D}{P}\right) - 1 + \tfrac{2D}{P}\right) + C_c\right)$$

In other form,

$$\left(JTEC(u)\right)^2 = 2Dh\left[\left(B + C(l)\right)\left(C_c - \left(1 - \tfrac{2D}{P}\right)C_t\right) + TC_t\left(1 - \tfrac{D}{P}\right) + uC_t\left(B + C(l)\right)\left(1 - \tfrac{D}{P}\right) + \tfrac{T}{u}\left(C_c - \left(1 - \tfrac{2D}{P}\right)C_t\right)\right]$$

Again, ignore the terms independent of u, we have

$$Z(u) = uC_t\left(B + C(l)\right)\left(1 - \frac{D}{P}\right) + \frac{T}{u}\left(C_c - \left(1 - \frac{2D}{P}\right)C_t\right) \tag{15}$$

Now we can minimise the above equation by using two constraints which will give an optimal value at point $u = u^*$

$$Z(u^*) \le Z(u^* - 1) \text{ and } Z(u^*) \le Z(u^* + 1) \tag{16}$$

After putting the values, we have the following expressions

$$uC_t\big(B+C(l)\big)\left(1-\frac{D}{P}\right)+\frac{T}{u}\left(C_c-\left(1-\frac{2D}{P}\right)C_t\right)$$
$$\leq (u-1)C_t\big(B+C(l)\big)\left(1-\frac{D}{P}\right)+\frac{T}{u-1}\left(C_c-\left(1-\frac{2D}{P}\right)C_t\right)$$

And

$$uC_t\big(B+C(l)\big)\left(1-\frac{D}{P}\right)+\frac{T}{u}\left(C_c-\left(1-\frac{2D}{P}\right)C_t\right)$$
$$\leq (u+1)C_t\big(B+C(l)\big)\left(1-\frac{D}{P}\right)+\frac{T}{u+1}\left(C_c-\left(1-\frac{2D}{P}\right)C_t\right)$$

Then we have,

$$\big((u+1)-u\big)C_t\big(B+C(l)\big)\left(1-\frac{D}{P}\right)\geq\left(\frac{1}{u}-\frac{1}{u+1}\right)T\left(C_c-\left(1-\frac{2D}{P}\right)C_t\right)$$

And

$$\big((u+1)-u\big)C_t\big(B+C(l)\big)\left(1-\frac{D}{P}\right)\leq\left(\frac{1}{u-1}-\frac{1}{u}\right)T\left(C_c-\left(1-\frac{2D}{P}\right)C_t\right)$$

Now we can write,

$$C_t\big(B+C(l)\big)\left(1-\frac{D}{P}\right)\leq\frac{1}{u(u-1)}T\left(C_c-\left(1-\frac{2D}{P}\right)C_t\right)$$

And

$$C_t\big(B+C(l)\big)\left(1-\frac{D}{P}\right)\geq\frac{1}{u(u+1)}T\left(C_c-\left(1-\frac{2D}{P}\right)C_t\right)$$

Identically we have,

$$u^*(u^*-1)\leq\frac{T\left(C_c-\left(1-\frac{2D}{P}\right)C_t\right)}{C_t\big(B+C(l)\big)\left(1-\frac{D}{P}\right)}$$

And

$$u^*(u^*+1) \geq \frac{T\left(C_c - \left(1 - \frac{2D}{P}\right) C_t\right)}{C_t\big(B + C(l)\big)\left(1 - \frac{D}{P}\right)}$$

Combining these two inequalities we get

$$u^*(u^*-1) \leq \frac{T\left(C_c - \left(1 - \frac{2D}{P}\right) C_t\right)}{C_t\big(B + C(l)\big)\left(1 - \frac{D}{P}\right)} \leq u^*(u^*+1) \qquad (17)$$

2.4 Solution Procedure

The aim be obliged to minimise the integrated total expected cost of an inventory model. To attain ideal values of $u, l\ and\ q$ we have to use the following steps.

STEP 1. Find the range of u using equation (17) which indicates the number of lots.

STEP 2. For all $l_i, i = 1,2, \dots, n$, find the analogous values of $C(l_i)$ using equation (6).

STEP 3. For all $l_i, i = 1,2, \dots, n$, find the analogous values of q_i using equation (12).

STEP 4. Find the value of $JTEC(q_i, l_i, u)$ for each factor $i, i = 1,2, \dots, n$.

STEP 5. Define $JTEC(q^*, l^*, u^*) = min_{i=0,1,..,n} JTEC(q_i, l_i, u)$. This provides the best result for the given problem.

2.5 Numerical Illustration

Examine the following problem of an inventory model. The parameters of the problem:

$D = 1000$ units/annum, $P = 3200$ units/annum, $B = \$25$/quantity, $T = \$400$/set-up, $h = 0.2$, $C_t = \$20$/unit, $C_c = \$25$/unit, $k = 2.33$, $\sigma = 7$ unit/week

The following table was presented in previous years paper by Banerjee (1986), Goyal (1988), Ouyang et al. (1999).

Lead time factor i	Regular span q_i(days)	Minimal span p_i(days)	Unit crashing cost cp_i(\$/day)
1	20	6	0.1
2	20	6	1.2
3	16	9	5.0

By following the above-mentioned procedure, we have $u = 3, 4\ and\ 5$.

		$u = 3$		$u = 4$		$u = 5$	
I	$C(l_i)$	q_i	$JTEC(q_i, l_i, u)$	q_i	$JTEC(q_i, l_i, u)$	q_i	$JTEC(q_i, l_i, u)$
0	0.0	164	2159.6 (2539.7)	131	2134.6 (2514.7)	110	2134.0 (2513.1)
1	1.4	165	2137.2 (2465.2)	**132***	**2114.3*** **(2443.3) ***	111	2115.7 (2444.7)
2	18.2	173	2200.0 (2647.1)	141	2200.9 (2648.9)	120	2224.8 (2672.8)
3	53.2	190	2370.8 (2603.8)	157	2414.5 (2646.5)	135	2477.5 (2701.5)

By following the above-mentioned solution procedure, we have optimal solution with **$u^* = 4$, optimal lead time, $l^* = 42\ days$, and optimal order capacity, $q^* = 132\ units$. The joint expected cost is \$2114.3/annum (\$2443.3/annum).**

Tersine (1982) advised in order to avoid stock-outs, due to arbitrary disturbance in the surroundings, an additional inventory i.e. safety stocks are kept. Keeping this in mind, for $u = 1$, the total cost is lowered as well as the lead time becomes shorter as compared to that of Banerjee (1986). For $u > 1$, the minimal price/annum is \$2505.6 for the Goyal (1988) model and is only \$2114.3 according to the work mentioned here. The comparison is done among the Banerjee (1986) model, Goyal (1988) model and this model and the results obtained are showed in below table.

	$u = 1$				$u > 1$	
	Purchaser decision	Vendor decision	Banerjee's model	This model	Goyal's model ($u = 2$)	This model ($u = 4$)
Purchaser's order size	100	800	369	369	198	132
Vendor's lot size	100	800	369	369	396	528
Purchaser's annual cost	730.7	2262.0	1221.0	1193.8	852.0	729.7
Vendor's annual cost	4062.5	1000.0	1314.6	1314.6	1653.6	1384.6
Joint total annual cost	4793.2	3262.0	2535.6	2508.4	2505.6	2114.3

Since purchaser and vendor resolve their respective inventory approach irrespective of one another. The purchaser can calculate the commercial order capacity using

equation (7).

Following the same steps as done before, the partial derivatives will be done of $TEC_c(q, l)$ w.r.t q and l for every time period (l_i, l_{i-1}), and equate it to zero to find the lot size with minimum cost.

$$\frac{\partial TEC_p(q, l)}{\partial q} = -\frac{D}{q^2}\big(B + C(l)\big) + \frac{h}{2}C_c = 0 \qquad (18)$$

$$q = \left[\frac{2D\left(B + C(l)\right)}{hC_c}\right]^{\frac{1}{2}} \tag{19}$$

Also,

$$\frac{\partial TEC_p(q,l)}{\partial l} = -\frac{D}{q}cp_i + \frac{h}{2}C_c k\sigma l^{\frac{-1}{2}} = 0 \tag{20}$$

Now,

$$\frac{\partial^2 TEC_p(q,l)}{\partial q^2} = \frac{2D}{q^3}\left(B + C(l)\right) > 0$$

$TEC_p\ (q,l)$ is convex in q for settled value of $l \in (l_i, l_{i-1})$

Again, for set q, $TEC_p\ (q,l)$ is concave in time period $l \in (l_i, l_{i-1})$ due to

$$\frac{\partial^2 TEC_p\ (q,l)}{\partial l} = -\frac{h}{4}C_c k\sigma l^{-3/2} < 0$$

From (19), the optimal approach is to place an amount of $q = 103\ units$ and the optimal lead time, $l = 42\ days$ at a total cost of \$713.6 per annum.

The commercial manufacturing capacity will be a numerical multiple of the purchaser's assets capacity. u is the exclusive foreign variable in (9). An arbitrary value of

$u = 1,2,3, \ldots$,was appointed and that value will be selected for which the total expected cost per annum of vendor will be minimal. The vendor gives $u = 5$ in consideration of $q = 103$ with interrelated total cost per annum of \$1407.5. Consequently, the combined cost per annum is \$2121.1. Goyal (1976) advised that the total annual cost should be designated to vendor and purchaser as

$$\beta = \frac{TEC_p(q^*,l^*)}{TEC_p(q^*,l^*) + TEC_v(q^*,u^*)}$$

Purchaser's cost $= \beta[JTEC(q^*,l^*,u^*)]$

Vendor's cost = $(1-\beta)[JTEC(q^*,l^*,u^*)]$

The results obtained are shown in below table.

Model type	Customer	Trader
Independent	Order capacity = 103	Production capacity= 515
	Total cost per annum = $713.6	Total cost per annum = $1407.5
	Order capacity = 132	Production capacity= 528
Integrated	Total cost per annum = $729.7	Total cost per annum = $1384.6
	Appropriate total cost per annum = $711.3	Appropriate total cost per annum = $1403.0

2.6 Conclusion

Lead time performs a crucial role in backlog manufacturing structure. By minimizing the lead time, the risk of scarcity can be reduced, safety stocks can be reduced, the services provided to the purchaser can be improved and the competition in this fast running era can be maintained is advised by Ouyang and Wu (1997). In this work, Yang and Pan (2002) concluded that the lead time is diminished by a supplementary crashing cost which implies that the lead time is tractable. Thus, the aggregate of ordering cost, holding cost and crashing cost which is the total cost of an integrated inventory is diminished. The basic assumptions of Banerjee (1986) and Goyal (1988) are taken in. This work concluded the decrease in joint total expected cost per annum and obtained the optimal order quantity, lead time and number of delivery lots.

Chapter 3

An Integrated Inventory Model With Controllable Lead Time And Additional Transportation Cost

3.1 Introduction

An integrated inventory model with controllable lead time by Yang and Pan (2002) has been reviewed in detail in chapter 2. In their work, they have reduced the aggregate of ordering cost, handling cost and crashing cost and provided the procedure to find the optimal order quantity, lead time and number of delivery lots. The additional crashing cost was credited to the purchaser if shortened lead time is requested.

In this chapter, we have considered the same model and methodology adopted by Yang and Pan (2002) with additional transportation cost to the vendor and a procedure has been developed to optimize this cost separately.

In this integrated inventory model, the supplier will transport the required raw materials to the vendor and the vendor in return will yield the required product to the purchaser. The total expected cost of the purchaser will remain same as given by Yang and Pan (2002) whereas there will be an additional cost- transportation cost which will be added to the cost of vendor i.e.

The total expected annual cost for the vendor is

$$TEC_{Nv} = ordering\ cost(O_v) + holding\ cost(H_v) + transportation\ cost(TPC_v)$$

The primitive no-setup model is considered for calculating the transportation cost from the supplier to the vendor. In this no-setup model, there are n intervals and each interval have finite production accommodation which further includes different production levels for example- regular time and supplementary time can be considered as two production levels. Since the total inventory cost has to be minimized, if there is some capacity for producing the raw material in the ongoing interval in that case advance raw materials are produced and stored for the future use. Also holding cost will be charged for storing raw material for future intervals.

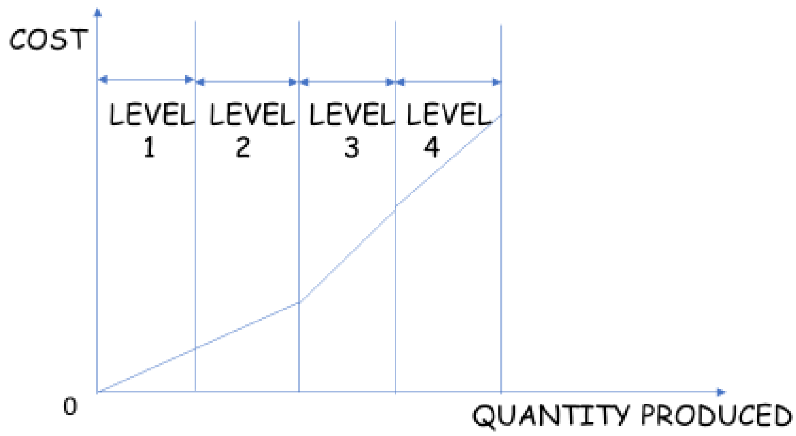
3.2 Hypothesis

The general assumptions of this model are as follows

1. There will be no setup cost in any interval.
2. There will be no scarcity.
3. The entity manufacturing price function in each interval either is stable or will have rising minimal cost (i.e. convex).
4. The unit holding cost will be constant in any interval.

3.3 Model formulation

Since there will be no scarcity means that the demand for an ongoing interval cannot be fulfilled from the production of upcoming intervals. This assumption stands in need for the condition that the aggregate production capacity for intervals $1,2,\dots,k$ be at least equal to aggregate requirement for the same inclusive intervals.



The entity manufacturing price function with increasing minimal cost is represented by the above graph. For instance, supplementary time and routine time represents two production zones in which the entity manufacturing price all along the supplementary time is larger as compared to that of routine time.

The k duration dilemma can be devised into transportation model with lk origins and k harbors, where l represents the figure of manufacturing zones per course. Since we have considered the routine time and supplementary time as two production levels implies $l = 2$.

The entity transportation cost from supplier to vendor is the sum of the production and holding cost per unit. The solution of the problem as a transportation model determines the minimum cost production amounts in each production zone.

The total expected cost of vendor per annum is-

$$TEC_{Nv}(q,u) = \left[\frac{D}{uq}\right]T + hC_t\frac{q}{2}\left[u\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] + TPC_v \tag{9.1}$$

where TPC_v is the transportation cost.

Since the additional transportation cost is charged to the vendor only, there will be no change in the cost of purchaser, the total expected annual cost of purchaser is same as given by Yang and Pan (2002)

$$TEC_p = \left(\frac{D}{q}\right)B + hC_c\left(\frac{q}{2} + k\sigma\sqrt{l}\right) + \frac{D}{q}C(l) \tag{7}$$

where $C(l) = cp_i(l_{i-1} - l) + \sum_{j=1}^{i-1} cp_j(q_j - p_j)$ (6)

Also, the integrated cost per annum as calculated in the previous chapter was convex and from our 3 hypothesis it is clear that the transportation cost (TPC_v) is also convex and thus the sum of two convex functions is also convex. Therefore, the new integrated annual inventory cost will be given by-

$$JTEC_N(q,l,u) = \frac{D}{q}\left(B + \left(\frac{T}{u}\right) + C(l)\right) + \frac{qh}{2}\left[\left(u\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right)C_t + C_c\right] + hC_c k\sigma\sqrt{l} + TPC_v \tag{10.1}$$

3.4 Solution Procedure

The transportation cost (TPC_v) is optimized using the following steps

STEP 1. To obtain the feasible solution calculate the cumulative production and cumulative demand as per the given problem.

STEP 2. In STEP 1, two cases can arise

i. aggregate production = aggregate requirement

ii. aggregate production ≠ aggregate requirement

In this case, if aggregate production $>$ aggregate requirement then an unused spare harbor is joined for the evenness of the model or if

aggregate production $<$ aggregate requirement then an unused spare origin is joined for the evenness of the model.

STEP 3. Since no scarcity is allowed, each of the transiting trail from the preceding to the ongoing interval are hindered.

STEP 4. The optimal solution will be obtained by starting from first column and choosing the cheapest route by assigning minimum of demand and production by Johnson (1957).

STEP 5. Repeat the STEP 4. until all the requirements is satisfied. This provides the optimal transportation cost for the vendor.

STEP 6. Find the range of u using equation (17) which indicates the number of lots.

STEP 7. For all $l_i, i = 1,2, \dots, n$, find the analogous values of $C(l_i)$ using equation (6).

STEP 8. For all $l_i, i = 1,2, \dots, n$, find the analogous values of q_i using equation (12).

STEP 9. Find the value of $JTEC(q_i, l_i, u)$ for each factor $i, i = 1,2, \dots, n$.

STEP 10. Define $JTEC(q^*, l^*, u^*) = min_{i=0,1,..,n} JTEC(q_i, l_i, u)$.

STEP 11. The transportation cost will be added to STEP 10 to obtain the optimal result for the given problem.

3.5 Numerical Illustration

An integrated inventory model with the following parameters remains same as considered by Yang and Pan (2002) in previous chapter 2.

$B = \$25/\text{order}$, $T = \$400/\text{composition}$, $h = 0.2$, $C_t = \$20/\text{unit}$, $C_c = \$25/\text{unit}$,

$k = 2.33$ and $\sigma = 7$ unit/week.

Lead pace factor i	Regular span q_i(days)	Minimal span p_i(days)	Unit crashing cost cp_i(\$/day)
1	20	6	0.1
2	20	6	1.2
3	16	9	5.0

In addition to this the data for additional transportation cost is given below. The following table represents the production capacity and the demand.

Interval	Routine(entities)	Supplementary(entities)	Demand(entities)
1	120	550	150
2	200	660	230
3	250	520	270
4	320	580	350

The unit manufacturing price in any interval is \$6 over routine time and \$9 over supplementary time. The carrying price per unit per interval is \$.10.

Solution-

Since no scarcity is granted, we are required to calculate aggregate production and aggregate requirement in order to obtain an appropriate result. Therefore, we have the following table

Interval	Aggregate production	Aggregate demand
1	120+550=670	150
2	670+200+660=1530	150+230=380
3	1530+250+520=2300	380+270=650
4	2300+320+580=3200	650+350=1000

Since aggregate production $>$ aggregate requirement, we need to add an unused spare harbor for the evenness of the given model. The unit cost to unused spare harbor is zero.

As per the solution procedure, each of the transiting trail from the preceding to the ongoing interval are hindered.

The symbols R_i and S_i represents the regular time and supplementary time for interval $i, i = 1,2,3,4$.

The unit transportation costs are calculated as the sum of the production and holding costs. Let us consider the unit cost from $R_1$to interval 1 equals the unit production cost \$6 only. The unit cost from 0_1 to interval 2 equals the unit production cost plus the unit holding cost for an interval 2- $\$9 + \$.1 = \$9.1$

Similarly, the unit cost 0_1 to interval 4 equals the unit production cost plus the unit holding cost for an interval 1 to 4- $\$9 + \$.1 + \$.1 + \$.1 = \$9.3$

Let us start from column 1, the journey $(R_1, 1)$ has the cheapest unit cost, thus we can assign $min(120,150) = 120$ to it by leaving 30 units unsatisfied in column 1. Now the next cheapest journey $(S_1, 1)$ in column 1 has the cheapest amount in column 1, thus we assign $min(550,30) = 30$. This now fulfils the demand for interval 1.

Similarly, in the same manner we can fulfil the demand for the remaining intervals 2,3 and 4. The following table is laid out to show the results obtained.

	1	2	3	4	Surplus	
R_1	6 (120)	6.1	6.2	6.3	0	120
S_1	9 (30)	9.1	9.2	9.3	0 (520)	550 → 520
R_2		6 (200)	6.1	6.2	0	200
S_2		9 (30)	9.1	9.2	0 (630)	660 → 630
R_3			6 (250)	6.1	0	250
S_3			9 (20)	9.1	0 (500)	520 → 500
R_4				6 (320)	0	320
S_4				9 (30)	0 (550)	580 → 550
	150 → 30	230 → 30	270 → 20	350 → 30	2200 → 1700 → 1180 → 630	

Interval 1 regular time- It produces 120 units for interval 1.

Interval 1 supplementary time- It produces 30 units for period 1 and leaves 520 units of unproductive capacity.

Interval 2 regular time- It produces 200 units for interval 2.

Interval 2 supplementary time- It produces 30 units for period 2 and leaves 630 units of unproductive capacity.

Interval 3 regular time- It produces 250 units for interval 3.

Interval 3 supplementary time- It produces 20 units for period 3 and leaves 500 units of unproductive capacity.

Interval 4 regular time- It produces 320 units for interval 4.

Interval 4 supplementary time- It produces 30 units for period 4 and leaves 550 units of unproductive capacity.

The joint transportation cost is $\$6 \times 120 + \$9 \times 30 + \$6 \times 200 + \$9 \times 30 + \$6 \times 250 + \$9 \times 20 + \$6 \times 320 + \$9 \times 30 = \$6330$.

$$TEC_{Nv}(q,u) = \left[\frac{D}{uq}\right]T + hC_t\frac{q}{2}\left[u\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] + \$6330$$

$$JTEC_N(q,l,u) = \frac{D}{q}\left(B + \left(\frac{T}{u}\right) + C(l)\right) + \frac{qh}{2}\left[\left(u\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right)C_t + C_c\right] + hC_c k\sigma\sqrt{l} + \$6330$$

By following the procedure mentioned by Yang and Pan (2002), we have

$u = 3, 4$ *and* 5.

		$u = 3$		$u = 4$		$u = 5$	
I	$C(l_i)$	q_i	$JTEC(q_i, l_i, u_i)$	q_i	$JTEC(q_i, l_i, u_i)$	q_i	$JTEC(q_i, l_i, u_i)$
0	0.0	164	8869.7	131	8844.7	110	8843.1
1	1.4	165	8795.2	**132***	**8773.3***	111	8774.7
2	18.2	173	8977.1	141	8978.9	120	9002.8
3	53.2	190	8933.8	157	8976.5	135	9031.5

By following the above-mentioned solution procedure, we have optimal solution with $u^* = 4$, optimum lead time, $l^* = 42\ days$, and ideal order capacity, $q^* = 132\ units$. The integrated conventional price is $8773.5/annum.

3.6 Conclusion

This study presents an integrated inventory with controllable lead time with an additional transportation cost of vendor. A three-stage integrated inventory model has been taken which includes the supplier, vendor and purchaser. Dynamic programming approach has been used to optimize the transportation cost. A numerical example has been considered and optimal solution is obtained.

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