

# **ROLE OF MOMENTUM DEPENDENT INTERACTION IN NUCLEAR STOPPING**

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*Dedicated*

*to*

*My parents*

## CERTIFICATE

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## **ABSTRACT**

The relative role of momentum dependent interactions in nuclear stopping at intermediate energy heavy ion collisions is studied using isospin-dependent quantum dynamics model. The calculations have been done for colliding systems in the energy range between 50 and 1000 MeV/nucleon in the presence of symmetry energy. It is observed that nuclear stopping is sensitive to impact parameter, incident energies and the mass of the colliding system; however it is insensitive to N/Z ratio of the colliding system. The degree of stopping is suppressed by the inclusion of momentum dependent interactions, whereas the particle production is not affected by the MDI.

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# CHAPTER 1

## INTRODUCTION

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### 1.1 Heavy ion physics

Nuclear physics (at low/intermediate/high energies) is one of the most extensively studied fields. After many decades, nuclear physics has been differentiated into different branches. One of its branches that separate itself from the traditional nuclear physics to leap into the unexplored space of *deconfinement and quark gluon plasma* is known as high energy physics. Its second branch led to study of *low energy  $\gamma$ -ray spectroscopy*. Main interest to study the low energy nuclear physics ( $\leq 10 \text{ MeV/nucleon}$ ) or heavy ion collisions is to look for the low density phenomena. The low energy heavy-ion reactions give unique possibility to look for the nuclear interactions, fusion-fission, cluster radioactivity, formation of super heavy nuclei, and possibilities of synthesis of super heavy elements, halo nuclei [1,2] etc. Thus low energy nuclear physics focuses mainly on the structure of nuclei.

The reaction cross section at low energies consists of three types i.e., the fusion, quasi-elastic and deep inelastic scattering [3]. All these processes depend on the projectile-target combinations, on the bombarding energy of projectile and on the angular momentum. At low energies the available free phase space is very small, therefore 98% of attempted collisions are blocked. The whole dynamics at low energies is governed by the mean field. The shortcoming of low energy physics is that it explains the properties of nucleus at or below the normal nuclear matter density. Due to lots of efforts made by people, the understanding of different phenomena is quite rich in low energies heavy ion collisions.

The third branch deals with the study of *nuclear collectivity through giant resonances* including supernova explosion, birth of neutron stars etc. These phenomena are rare and limited in space. Lastly the branch that grew out of the compound nucleus

and matured through the low energy deep inelastic scattering is now exploring the field of *intermediate energy heavy-ion reaction*.

On the basis of energy of colliding nuclei, heavy ion physics can be classified in the following branches.

Low energy heavy ion reactions :  $E \leq 10 \text{ MeV/nucl.}$

Intermediate energy heavy-ion reactions :  $10 \text{ MeV/nucl.} < E < 1 \text{ GeV/nucl.}$

High energy heavy ion reactions :  $E \geq 1 \text{ GeV/nucl.}$

Heavy ion physics at intermediate energies has created a great interest among the nuclear physics community. Primary goal of this field of research is to create a state of matter at extreme conditions of temperature and pressure. Heavy ion refers to an ionized atom which is usually heavier than helium. Heavy-ion physics is devoted to the study of extremely hot and dense nuclear matter and the collective effects appearing in such systems, differing from particle physics, which studies the interactions between elementary particles.

The present scenario displays a twofold picture of heavy ion collision. One of the interests in understanding the time evolution of the reaction starting from a highly disturbed situation towards a possible thermalized system by means of dissipation. Secondly, the reaction products are in extreme states in the sense that they are supposed to be highly exotic or hot. Hotter nuclei have excitation energies close or even higher than their total binding energies.

The main interest to study the heavy-ion collisions is the investigation of the properties of nuclear matter at extreme densities and excitation energies. The study of the behavior of the hadrons in hot and dense nuclear matter is one of the central topics in present day research. Naturally like all other materials, the properties of nuclear matter are also influenced by the pressure, density and temperature. The hadronic matter may

have rich structure in this unexplored domain of high excitation energies and compressions.

At relativistic energies ( $\geq 2$  GeV/nucleon) there is a presence of large free phase space, so only 4% of attempted collisions are blocked and the dynamics is termed as cascade approach. Whereas at intermediate energies, both the cascade picture as well the mean field are taken into consideration.

The major phenomena in these intermediate energy heavy ion reactions are the multifragmentation, collective flow as well as nuclear stopping. The multifragmentation is the breakup of colliding nuclei into different kind of fragments like free particles, light charge particles (LCP's), as well as intermediate mass fragments (IMF's) . On the other hand, the collective flow is the movement of particles along different directions. The last but not the least nuclear stopping is the phenomena of stopping of the projectile nucleons by the target nucleons and vice-versa. All these phenomena are studied in the literature time independently. Recently nuclear stopping is studied by Kumar et.al. by showing the reaction between LCP's and nuclear stopping by using IQMD model [4].

No one has tried to correlate the LCP's with Nuclear stopping in the presence of momentum dependent interactions. Before going through a deep analysis we must have a detailed knowledge of nuclear stopping.

## **1.2 Nuclear stopping**

The global stopping is defined as the complete destruction of initial correlations. This makes the matter homogeneous and one can have global stopping. There is considerable interest in the stopping power of nuclei at relativistic energies. The nuclear stopping power can be viewed as a measure of the degree to which the energy of the relative motion of the two colliding nuclei is transformed into other degrees of freedom [5]. The nuclear stopping can be measured as full stopping, partial as well as transparency. Full stopping, means the whole of the projectile nucleons are stopped by

target nucleons and vice-versa. The partial stopping means some of the nucleons are stopped, while the rest of the nucleons are allowed to cross. Lastly transparency of the nucleons denotes the crossing of whole of the projectile nucleons towards the target and vice-versa.

The amount of nuclear stopping determines parameters, such as the energy and volume of the interaction region (and therefore energy density), which govern the reaction dynamics and the extent to which conditions might be favorable for the formation of a highly dense, deconfined phase of matter.

At intermediate energies, the mixture of attractive mean field and repulsive nucleon-nucleon scattering exists [6]

This leads the whole matter from a fused state to total disassembly. The degree of stopping varies drastically with:

1. Incident energies.
2. Mass of colliding nuclei
3. Geometry of the system.

This has also been linked with the thermalization in heavy ion collisions. This can happen in the participant zone only. More the initial memory of nucleons is erased, better it is stopped and better one has average mixing of projectile and target momentum. We shall here consider few different quantities capable of estimating the degree of global stopping.

Moreover LCP's are expected to originate from the participant zone. It is expected that one can correlate the LCP's with nuclear stopping in intermediate energy heavy ion collisions, one have tried in the literature to correlate it in QMD as well as IQMD models. In IQMD model detailed analysis is performed by Kumar et.al. [4], but keep in mind that study is performed with static equation of state (EOS). For more classification the literature related to nuclear stopping and fragments is discussed in the proceeding sections.

### 1.3 Experimental review

Some of the important experiments performed in recent years are as follow. The first experiments were done at Berkley. The motive behind these experiments was to get the theoreticians and experimentalists aware of the problems and pitfall of the way from medium energy heavy ion collision to equation of state. Later on, several accelerators were built at Michigan state university (USA), GANIL (France), and at GSI (Germany). The SIS (heavy ion synchrotron) accelerator at GSI is specially designed to study the heavy-ion collisions at intermediate energies. The MSU group at Michigan state university is very active in studying the fragment's spectra at lower side of the bombarding energies. Similarly efforts are also made by INDRA group at GANIL. The ALADIN group at GSI has provided complete spectra of the fragments.

The main interest of INDRA collaboration at GANIL is to study the collisions where large multiplicities of the nucleons are observed in the exit channel and they have studied the influence of different parameters on multifragmentation including role of size of system in entrance channel, Coulomb instabilities etc. In particular, the size effects are studied in symmetric collisions of  $^{36}\text{Ar}+^{136}\text{Xe}$  [7],  $^{58}\text{Ni}+^{58}\text{Ni}$ ,  $^{129}\text{Xe}+^{118}\text{Sn}$  [8],  $^{181}\text{Ta}+^{197}\text{Au}$  and  $^{238}\text{U}+^{238}\text{U}$ . On the other hand the entrance channel effects are studied by keeping the total mass equal to 250 units. Naturally, for studying the gentle compression and Coulomb instabilities, they have to go to heavy fragments. A detailed theoretical study of INDRA experimental findings was carried out by Aichelin and coworkers [9].

The Berkley group has concentrated mainly on the asymmetric reactions like  $^{197}\text{Au}+^{27}\text{Al}$ ,  $^{51}\text{V}$ , and  $^{64}\text{Cu}$  at 60 MeV/nucleon [10].  $^{56}\text{Fe}+^{197}\text{Au}$  at 50 and 100 MeV/nucl. [11] etc. Their main motive was to look for probabilities which include the excitation energy, angular distribution and velocity distribution etc. another symmetric reactions studied were carried out using AMPHORA detector at SARA (France) [12]. The MSU group studied reactions like  $^{36}\text{Ar}+^{197}\text{Au}$ ,  $^{129}\text{Xe}+^{197}\text{Au}$  at 50-110 MeV/nucl.,  $^{40}\text{Ar}+^{45}\text{Sc}$  etc.

The FOPI and ALADIN groups at GSI are studying the variety of reactions giving nearly all kind of possibilities. It ranges from  $^{12}\text{C}$  to  $^{208}\text{Pb}$ , and with incident energy between 100 to 1000 MeV/nucl.[13]. A lot of physical conclusions are also drawn from these studies.

Therefore, in the past years a very active program has developed at many places. SIS at GSI has replaced Bevalac as the main accelerator in what is now called intermediate energy (100 MeV/nucl. up to 2 GeV/nucleon). This facility has made a big impact with three new second generation experiments, FOPI, ALADIN and KAOS.

At present, several experimental groups are engaged in extracting information of nuclear EOS from multifragmentation and nuclear flow/fragment flow. Many body nuclear dynamics group at INDIANA University Bloomington, USA studied the role of nuclear equation of state (EOS) in particular how the density dependence of the symmetry energy affects the properties of nuclear matter [14]. TAPS collaboration studied the role of nuclear incompressibility in the production of hard photons in heavy ion collisions [15]. Recently E802 collaboration studied mid rapidity energy distribution [16].

Moreover NA49 collaboration has recently performed a series of measurements of  $^{208}\text{Pb} + ^{208}\text{Pb}$  collisions at 20, 30, 40, 80 and 158 MeV/nucl. [17].

#### **1.4 Theoretical review of nuclear stopping**

Theoretically, several models have been developed to study heavy ion collisions (HIC). HIC's involve very complicated non-equilibrium physics. Therefore, its numerical modeling is not straight forward. At low incident energies the available free phase is very less, due to this reason 98% of the attempted collisions are blocked and the whole dynamics of the reaction is governed by the mean field or by mutual two and three body interactions. In contrary, at relativistic energy ( $\geq 2 \text{ GeV}/\text{nucl.}$ ) the available free phase space is very large. Thus only 4% collisions are blocked and hence the dynamics of the

reaction is governed by the cascade picture. On the other hand, both cascade and mean field emerges at intermediate energies.

The best suitable approaches at intermediate energy heavy ion collision are the conventional theories like Time dependent Hartree-Fock (TDHF) [18], or its semi-classical version that so called Vlasov equation (in phase space). At intermediate energy heavy ion physics, one should keep in mind to treat the nucleon-nucleon collisions and the mean field on equal footing. Some attempts are made in the literature to extend the TDHF to take care of the residual nucleon-nucleon (NN) interactions which are responsible for the two body collisions (ETDHF) [19]

A new realization named as BUU equation is developed by coupling the semi-classical version of TDHF with nucleon-nucleon collisions. This approach is being used till date to study the large deviation problems of low, intermediate and relativistic heavy-ion collisions. The same realization is also named as Landau-Vlasov (LV) equation or Vlasov-Uehling-Uhlenbeck (VUU) or Boltzmann-Nordheim equation. The solution of BUU equation provides the time evolution of one body distribution function in six dimensional phase space. Some attempts were also made to extend the BUU equation by including stochastic two-body correlation so that the N-body phenomena like multifragmentation can also be studied [20].

Naturally, one would like to have the method where correlations and fluctuations among the nucleons are preserved. The classical molecular dynamics [21] is capable of treating both the compression and fragment formation. This model predicts the collective flow in qualitative agreement with data. It incorporates the complete classical N-body dynamics which is necessary to describe the formation of fragments. Naturally the model needs a major refinement which should also include the quantum features. This approach was later extended to incorporate with the quantum features by Aichelin and Stöcker [22]. This new approach was dubbed as Quantum Molecular Dynamics (QMD) [23] Model. In past decade, several refinements and improvements were made over the original QMD. The QMD model takes into account the Pauli principle, but through two-body collisions, which is somewhat contrary to MD picture. In view of the difficulties

encountered with taking the Pauli principle in classical MD methods properly into account, a new class of approaches was developed in the beginning of the 1990s, the so called Fermionic Molecular Dynamics (FMD) [24], and AMD for Antisymmetrized Molecular Dynamics [25]. The serious numerical problems have restricted the use of FMD and AMD approaches to light nuclei.

IQMD model is used for studying the isospin effects on nuclear transverse collective flow, on nuclear radial flow and nuclear fragmentation in heavy ion collisions at intermediate energies [4]. The dynamics in the formation of the transient state is mainly governed by three components, namely, the mean field, two body collisions, and Pauli blocking. For an isospin dependent reaction dynamics algorithm it is essential that all the three components should reasonably include isospin degrees of freedom. In addition, it is also important that, in initialization of projectile and target nuclei, the samples of neutrons and protons in phase space should be treated separately since there exists a large difference between neutron and proton density distributions for nuclei. IQMD model is developed just on above basis. It has been shown that the IQMD can be used with large success for studying the effects of isospin in heavy ion collisions at intermediate energies.

In all the dynamical models (QMD)[22,23], (MD)[21], (BUU)[26], (RQMD)[27] one start with the two well defined nuclei and then follow the time evolution of the reaction. In other words the correlations and fluctuations among the nucleons are preserved. All the dynamical models follow the dynamics of single nucleon only and no model deals with the fragments directly. One mostly chooses construction time of the fragments  $t = 200\text{fm}/c$  i.e. the saturation time of the reaction [28]

It is important to investigate whether an equilibrium is reached or not for a colliding system in order to obtain correctly the information of equation of state (EOS) and the reaction mechanism. This problem has been studied extensively both theoretically and experimentally [29,30]. Following the establishment of radioactive beam facilities in many laboratories, it has become possible to study neutron-rich (or proton-rich) nuclear collisions at intermediate energies. Therefore it is necessary to study the effect of isospin

asymmetry on the equilibrium process in a colliding system. The sensitivity of nuclear stopping to the isospin dependence of the cross section was studied but the isospin dependence of the medium correction of two-body cross sections and the isospin asymmetry of the initial system were not tested [31]. As expected, the degree of equilibrium is influenced by the mean field, the medium correction of two-body cross section as well as the size of the colliding system.

In nucleus-nucleus collisions, stopping can be seen as a shift of the rapidity distribution of the incident nucleons towards mid-rapidity, i.e. the centre-of-mass of the collision. Thus, the shape of rapidity distributions provides key information in terms of the nuclear stopping power. Bauer *et al.* pointed out that in intermediate energy HIC, nuclear stopping power is determined by both the mean field and the in-medium nucleon-nucleon ( $N-N$ ) cross section [32], but the mean field he used did not include symmetry potential. Bass, Yennello, Johnston, Li, and co-workers suggested that the degree of approaching isospin equilibration provides a means to probe the mechanism and the power of nuclear stopping in heavy ion collisions (HIC) [31].

**The thesis is organized as follow:**

The chapter 2 includes the detailed analysis of the theoretical model, which is of our interest in the present study. Chapter 3 includes the results and discussion. The detailed analysis of the light charge particles (LCP's) and nuclear stopping is performed in the presence of MDI's for different impact parameters, system mass, energy, etc. the results are concluded in chapter 4.

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### 2.1 Isospin-dependent Quantum Molecular Dynamics model (IQMD)

The dynamics in HIC at intermediate energies can be described by the three important components namely, the mean field, two-body collisions, and Pauli blocking. So it is essential that these three components should reasonably include the ‘isospin degree of freedom’. The QMD model is a classical aspect because the time evolution of the system is determined by the classical canonical equation of motion. However QMD [1] model include many important quantum features like Pauli blocking, stochastic collisions, sub-threshold particle production, etc. In IQMD, it is also important that, in initialization of target and projectile nuclei, the samples of neutrons and protons in the phase space should be treated separately since there exist a large difference in density distribution of neutrons and protons for the nuclei far from the  $\beta$  stability line. The IQMD model was used successfully for the analysis of the observables from low to relativistic energies [2,3]. The Isospin degree of freedom enters the calculations via both cross sections and mean field [4,5]

The simulation models require three steps. First, one has to generate the nuclei. This procedure is called as *initialization*. These nucleons then propagate under the influence of mean field. This process is termed as *propagation*. Finally, nucleons are bound to collide if they come too close to each other. This part is dubbed as *collision*. Each of these is discussed as follows.

#### 2.1.1 Initialization:

In the IQMD model, the neutrons and protons are distinguished from each other in the initialization of projectile and target nuclei. The neutron and proton density for the initial projectile and target nuclei are determined by Skyrme-Hartree-Fock method [6]. The

IQMD model treats different charge states of nucleons, deltas, and pions explicitly [7]. In this model the baryons are represented by Gaussian –shaped density distributions

$$f_i(\vec{r}, \vec{p}, t) = \frac{1}{\pi^2 \hbar^2} e^{-[\vec{r} - \vec{r}_i(t)]^2 \frac{1}{2L}} e^{-[\vec{p} - \vec{p}_i(t)]^2 \frac{2L}{\hbar^2}} \quad (2.1)$$

In IQMD model the centroid of the Gaussians in a nucleus are randomly distributed in a phase space sphere ( $r \leq R$  and  $p \leq p_F$ ) with  $R = 1.12A^{1/3}$  fm corresponding to a ground state density of

$$\rho_o = 0.17 \text{ fm}^{-3}.$$

The Fermi momentum  $p_F$  depends upon the ground state density. For  $\rho_o = 0.17 \text{ fm}^{-3}$ , the value of  $p_F \approx 268$  MeV/c. The momenta are uniformly distributed within a momentum sphere without any local constraints. Therefore the nucleons near the surface, because of low potential energy, are unbound initially. This possibility however gives a reduced binding energy per nucleon as compared to Weizsacker mass formula. So the initialized nuclei are less stable against spurious particle evaporation compared to oriented QMD model.

### 2.1.1.1 Interaction range:

Recently studies have been done to see the effect of width of the Gaussian wave packet on fragmentation using QMD model [8]. It was observed that the variation of the width ( $L$ ) has a sizable effect on the clusterization. A broader Gaussian binds more nucleons into a fragment. As a result the fragment turns much heavier. The interaction range parameter  $L$  influences the interaction density for finite systems. For (homogeneous) infinite nuclear matter the density (and thus the potential energy) does not depend anymore on the extension of the Gaussian wave packets. Thus, the equation of state of infinite nuclear matter is independent of  $L$ . In finite matter  $E=A$  also depends on  $L$ . Thus even two parameterizations which yield the same EOS may produce different results for the reaction of two heavy ions. Therefore we have to adjust  $L$  to have reasonable surface properties. In order to allow a physical interpretation  $L$  should be in the order of the size

one expects for the range of the nuclear interaction. There exists a range of values for  $L$ , which allows fixing these properties. Larger values of  $L$  increase the effective range of the interaction and thus lead to some smearing of fluctuations, which are stronger for more located wave packets (small values of  $L$ ). The Gaussian width can be regarded as a description of the interaction range of a particle. Its influence disappears for infinite nuclear matter whereas for finite systems it may play a non negligible role. In IQMD the Gaussian width can be used as an optional input parameter. The system dependence of  $L$  in IQMD has been introduced in order to obtain maximum stability of the nucleonic density profiles. As an example for Au + Au a value of  $L = 8.66 \text{ fm}^2$  is chosen, for Ca + Ca and lighter nuclei  $L = 4.33 \text{ fm}^2$ .

### 2.1.1.2 Two body collisions

In IQMD model, two different parameterizations of  $N$ - $N$  cross sections may be used optionally. One of the parameterization of Cugnon ( $\sigma_{Cug}$ ) cross section [9], which is the isospin independent, and the other is the experimental parameterization  $\sigma_{exp}$  which is isospin dependent. It is shown that for experimental parameterization at energies lower than 300 MeV/nucleon, the neutron-proton cross section is about three times larger than the neutron-neutron or proton-proton cross section [10].

### 2.1.1.3 Pauli blocking

Whenever a collision occurred, in the phase space we assume that each nucleon occupies a six dimensional sphere with a volume of  $h^3/2$  and then calculate the phase volume  $V$ , of the scattered nucleons being occupied by the rest nucleons with the same isospin as that of the scattered ones. Then we compare  $2V/h^3$  with a random number and decide whether the collision is blocked or not. So the Pauli blocking is isospin dependent. Pauli blocking of neutron and proton is treated separately.

## 2.1.2 Propagation

The successfully initialized nuclei are then boosted towards each other with a proper center of mass velocity using relativistic kinematics. The nucleons of target and projectile

interact via two and three body Skyrme forces and the Yukawa potential. The isospin degree of freedom is treated explicitly by employing a symmetry potential and explicit Coulomb forces between the protons of colliding target and projectile. All this provide us a correct distribution of protons and neutrons within the nucleus.

The Hamilton equations of motion for the propagation of hadrons are

$$\frac{dr_i}{dt} = \frac{d\langle H \rangle}{d p_i}, \quad (2.2)$$

$$\frac{dp_i}{dt} = - \frac{d\langle H \rangle}{d r_i} \quad (2.3)$$

### 2.1.2.1 Potentials used in IQMD

A total Hamiltonian function with a kinetic energy T and a potential energy V is given by

$$\begin{aligned} \langle H \rangle &= \langle T \rangle + \langle V \rangle \\ &= \sum_i \frac{p_i^2}{2m_i} + \sum_i \sum_{j>i} \int f_i(\vec{r}, \vec{p}, t) V^{ij}(\vec{r}', \vec{r}) \\ &\quad \times f_j(\vec{r}', \vec{p}', t) d\vec{r} d\vec{r}' d\vec{p} d\vec{p}' \end{aligned} \quad (2.4)$$

The potential in equation (4) is the sum of the following specific elementary potentials,

$$V = V_{Sky} + V_{Yuk} + V_{Coul} + V_{mdi} + V_{loc} \quad (2.5)$$

With a local hard core repulsion,

$$\begin{aligned} V_{loc} &= \sum_i \sum_{j>i} t_1 \delta(\vec{r}_i - \vec{r}_j) \\ &\quad + \sum_i \sum_{j>i} \sum_{k>j} t_2 \delta(\vec{r}_i - \vec{r}_j) \delta(\vec{r}_i - \vec{r}_k), \end{aligned} \quad (2.6)$$

The Yukawa short-range term,

$$V_{Yuk} = \sum_i \sum_{j>i} t_3 \frac{\exp(-|\vec{r}_i - \vec{r}_j|/\mu)}{(|\vec{r}_i - \vec{r}_j|/\mu)} \quad (2.7)$$

The Yukawa potential  $V_{Yuk}$  in IQMD is very short ranged and weak ( $\mu = 0.4$  fm). Additional attractive Yukawa forces hence modify the EOS. Yukawa forces stabilize the nuclei because of the increase of the interaction range as compared to a  $\delta$ -like Skyrme potential. Thus nucleons notice earlier that they will arrive at the surface and are more effectively decelerated without this potential. In addition the fluctuations are reduced.

The Coulomb term with only a mean value for the nucleon charge because in QMD the isospin of the particles is not included,

$$V_{Coul} = \sum_i \sum_{j>i} \frac{Z_i Z_j e^2}{|\vec{r}'_i - \vec{r}'_j|} \quad (2.8)$$

And the momentum dependent interaction

$$V_{mdi} = \sum_i \sum_{j>i} t_4 \ln(1 + t_5 (\vec{p}_i - \vec{p}_j)^2) \delta(\vec{r}'_i - \vec{r}'_j) \quad (2.9)$$

The IQMD-model offers rather stable density distributions and good energy conservation, however for the price of nucleon evaporation and improper binding energies ( $E_{bind} \approx 4-5$  MeV/nucleon for heavy nuclei instead of 8 MeV/nucleon). In addition to the potential used in QMD, a symmetry potential between protons and neutrons corresponding to the Bethe-Weizsacker mass formula has been included

$$V_{Sym}^{ij} = t_6 \frac{1}{\rho_0} T_3^i T_3^j \delta(\vec{r}'_i - \vec{r}'_j) \quad (2.10)$$

Where  $T_3^i$  and  $T_3^j$  denote the isospin projections of particles  $i$  and  $j$ . Other baryonic potentials like  $V_{Sym}^{ij}$  and  $V_{mdi}$  are defined isospin-independent like in all other flavors. The parameters are propagated under the total interaction calculated by the Hamiltonian equations of motion (2.2) and (2.3).

Parameters used in equation (2.6) to (2.10) are shown in table below:

IQMD parameters	
$t_3$	15 MeV
$t_4$	1.57 MeV
$t_5$	$5.10^{-4}$ MeV <sup>-2</sup>
$t_6$	25 MeV
$\mu$	0.4 fm

Table 2.2: IQMD parameters used in equation (2.6) to (2.10)

### 2.1.3 Collision

During the propagation, two nucleons are supposed to suffer a binary collision if the distance between their centroids is

$$|r_i - r_j| \leq \sqrt{\frac{\sigma_{tot}}{\pi}}, \quad \sigma_{tot} = \sigma(\sqrt{s}, type), \quad (2.11)$$

‘type’ denotes the ingoing collision parameter ( $N-N$ ,  $N-\Delta$ ,  $N-\pi$ , etc.). And  $\sqrt{s}$  is the c.m. energy in GeV. Pions are formed model via the decay of delta resonances. The colliding particles can scatter elastically as well as inelastically. The main processes include:

$$\text{Elastic} \begin{cases} N + N \rightarrow N + N & (a) \\ N + \Delta \rightarrow N + \Delta & (b) \\ \Delta + \Delta \rightarrow \Delta + \Delta & (c) \end{cases}$$

$$\text{Inelastic} \begin{cases} N + N \rightarrow N + \Delta & (d) \\ N + \Delta \rightarrow N + N & (e) \end{cases}$$

Experimental cross sections are used for the processes (a) and (d). The cross section for (e) is obtained by the detailed balance method. The cross section for processes (b) and (c) are taken to be same as that of process (a).

In addition to this the Pauli blocking of baryons is also taken into account by checking the phase space densities in the final state. The final phase space fractions  $P_i$  and  $P_j$ , which are already occupied by other nucleons, are determined for each of the scattering baryons. For each collision the phase space densities in the final state are checked in order to assure that the final distribution in phase space is in agreement with the Pauli principle ( $P \leq 1$ ). The collision is then blocked with a possibility

$$P_{block} = 1 - (1 - P_i)(1 - P_j) \quad (2.12)$$

Where  $P_i$  and  $P_j$  are the already occupied phase space fractions by other nucleons.

## 2.2 Method of clusterization:

### Minimum spanning tree method (MST)

One of the most extensively used methods to clusterize the nucleons is MST (Minimum spanning tree) [11,12]. According to this method two nucleons share the same fragment if their centroids are closer than a distance  $d_{min}$ ,

$$|\vec{r}_i - \vec{r}_j| \leq d_{min}$$

Where  $\vec{r}_i$  and  $\vec{r}_j$  are the spatial positions of both nucleons. The value of  $d_{min}$  can vary between 2-4 fm. It has small effect on multifragmentation [13,14]. However, this method can not address the question of time scale as it will give a big fragment at the time of

highest density when the interactions between the nucleons are still active. This method can only be used to analyze asymptotic configuration in which the fragmenting system can be viewed as a very dilute mixture of free particles and almost equilibrated fragments.

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### 3.1 Introduction

Momentum dependent interactions (MDI) play a very important role in determining the nuclear dynamics. So it is necessary to include the momentum dependence of the mean field generated in the heavy ion collision [1] to acquire a reasonable theoretical prediction and it is an important feature for the fundamental understanding of nuclear matter properties over a wide range of densities and temperature.

The initial attempts with MDI have showed a drastic effect on the collective flow as well as particle production. The pion yields were found to be suppressed by almost 30% in the presence of momentum dependence [2].

During the initial phase of collision the effect of momentum dependent interaction is very strong. The particles propagating with momentum dependent interaction are accelerated in the transverse direction during the early phase of the reaction. As a result fewer collisions take place and the transverse flow increases considerably. To study the role of equation of state (EOS) and momentum dependent interaction (MDI) in nuclear stopping one start with simple form of nuclear EOS which has couple of parameters that are specified by global properties of nuclear matter (i.e. by fitting the binding energy, saturation densities, etc.). The different compressibilities lead to different EOS. By simulating the heavy ion collisions with different EOS's one can study their effect and role of momentum dependent interactions. Free nucleon-nucleon cross section has been used in the analysis. We simulate the heavy ion collision with variety of equation of state and study the formation of fragments. This gives us the possibility to check the role of different equations of state and momentum dependent interactions in nuclear stopping.

From the parameters given in Table 3.1 one can see two kinds of equations of state. One is so-called hard EOS (H, HMD) with the value of compressibility ( $K$ ) = 380 MeV, and the other is the soft EOS (S, SMD) with  $K= 200$  MeV[1].

$$U = \alpha \left( \frac{\rho}{\rho_0} \right) + \beta \left( \frac{\rho}{\rho_0} \right)^\gamma + \delta \ln^2 [\epsilon (\Delta \vec{p})^2 + 1] \left( \frac{\rho}{\rho_0} \right)$$

M refers to the inclusion of momentum dependent interaction. The parameters used in the above equation are summarized in the table given below

K(MeV)	$\alpha$ (MeV)	$\beta$ (MeV)	$\gamma$	$\delta$ (MeV)	$\epsilon$	EOS
200	-356	303	1.7	—	—	S
380	-124	70.5	2	—	—	H
200	-390	320	1.14	1.57	21.54	SMD
380	-130	59	2.09	1.57	21.54	HMD

Table 3.1: The parameters for the nuclear equations of state used in IQMD model.

Nuclear stopping in HIC has been studied by means of rapidity distribution [3] and asymmetry of momentum distribution [4-6]. It is important quantity in determining the outcome of the reaction.

The degree of stopping can be described by the parameters described in the next section.

### 3.2 Stopping parameters:

#### 3.2.1 Anisotropy Ratio R (Stopping ratio)

Anisotropic ratio is a possibility to probe the degree of stopping. It is also called as the transverse-parallel ratio of momentum ( $R$ ) and is given by

$$R = \frac{2 [\sum_i^A |p_{\perp}(i)|]}{\pi [\sum_i^A |p_{\parallel}(i)|]}$$

The values of the transverse and parallel (longitudinal) components of the momentum of the  $i^{th}$  nucleon are

$$p_{\perp}(i) = \sqrt{p_x^2(i) + p_y^2(i)}$$

$$\text{and } p_{\parallel}(i) = p_z(i)$$

respectively. Naturally for a complete stopping, R ratio should be close to unity. More close is the value of R to one, more is the stopping in colliding nuclei. Super-stopping leads to  $R > 1$  while large transparency leads to  $R < 1$ .

### 3.2.2 Quadrupole Moment

The momentum quadrupole  $Q_{zz}$  is defined as

$$Q_{zz} = \sum_i^A [2p_z^2(i) - p_x^2(i) - p_y^2(i)]$$

Here the total mass  $A$  is the sum of the projectile mass  $A_p$  and the target mass  $A_T$ . From the expression, it is clear that for spherical distribution of particles, the quadrupole moment  $Q_{zz}$  vanishes. For a complete stopping,  $Q_{zz}$  should be close to 0. It implies  $1/Q_{zz}$  must have larger value for complete stopping. It has been observed that with the passage of time, when the beam energy is less, the asymptotic value of  $Q_{zz}$  decreases towards zero, indicating an isotropic nucleon momentum distribution of the whole composite system and consequently a full stopping at beam energy below Fermi energy. As beam energy increases above Fermi energy, because of the pre-equilibrium particle emission and longitudinal motion of the target like nuclei,  $Q_{zz}$  will get certain non-vanishing value, indicating partial transparency. Also, the relaxation time decreases with increasing beam energy, which shows the fact that high energy leads to more violent N-N collisions and faster dissipation. This is consistent with the isospin equilibrium process as shown by Li *et al.* [7].

### 3.2.3 Rapidity distribution

An important possibility for describing the nuclear stopping is rapidity distribution, which is defined as

$$Y(i) = \frac{1}{2} \ln \frac{E(i) + P_z(i)}{E(i) - P_z(i)}$$

Where  $E(i)$  and  $P_z(i)$  are respectively the total energy and longitudinal momentum of  $i^{th}$  particle. For complete stopping a single Gaussian shape of rapidity is expected. Rapidity distribution can also be used to define the nature of emitting source.

## 3.3 Results and discussion

In the present analysis , thousand of events were simulated for the neutron rich reaction of  ${}_{54}\text{Xe}^{131} + {}_{54}\text{Xe}^{131}$  , at incidents energies between 50 and 1000 MeV/nucleon using hard momentum dependent (HMD) and  $E_{\text{sym}} = 32$  MeV. To see the effect of compressibilities on nuclear stopping and fragmentation soft equation of state (EOS) is also used in Fig. 9. We shall also correlate the degree of stopping with the emission of light charge particles. The fragments are constructed within the minimum spanning tree (MST) method, which binds the nucleons if they are within a distance of 4 fm.

### 3.3.1 Phase space analysis

In figure 1, we display the final phase space of a single event of  ${}_{54}\text{Xe}^{131} + {}_{54}\text{Xe}^{131}$  at incident energy of 400 MeV/nucleon with symmetry energy and for hard momentum dependent interactions (HMD). The top, middle and bottom panels are at  $\hat{b} = 0, 0.3$  and  $0.6$  respectively. Here the phase space of free nucleons [ $A=1$ ], light charged particles (LCP's) [ $2 \leq A \leq 4$ ], and intermediate mass fragments (IMF's) [ $5 \leq A \leq 44$ ] is displayed. It is clear from the figure that in the central collisions, there is complete

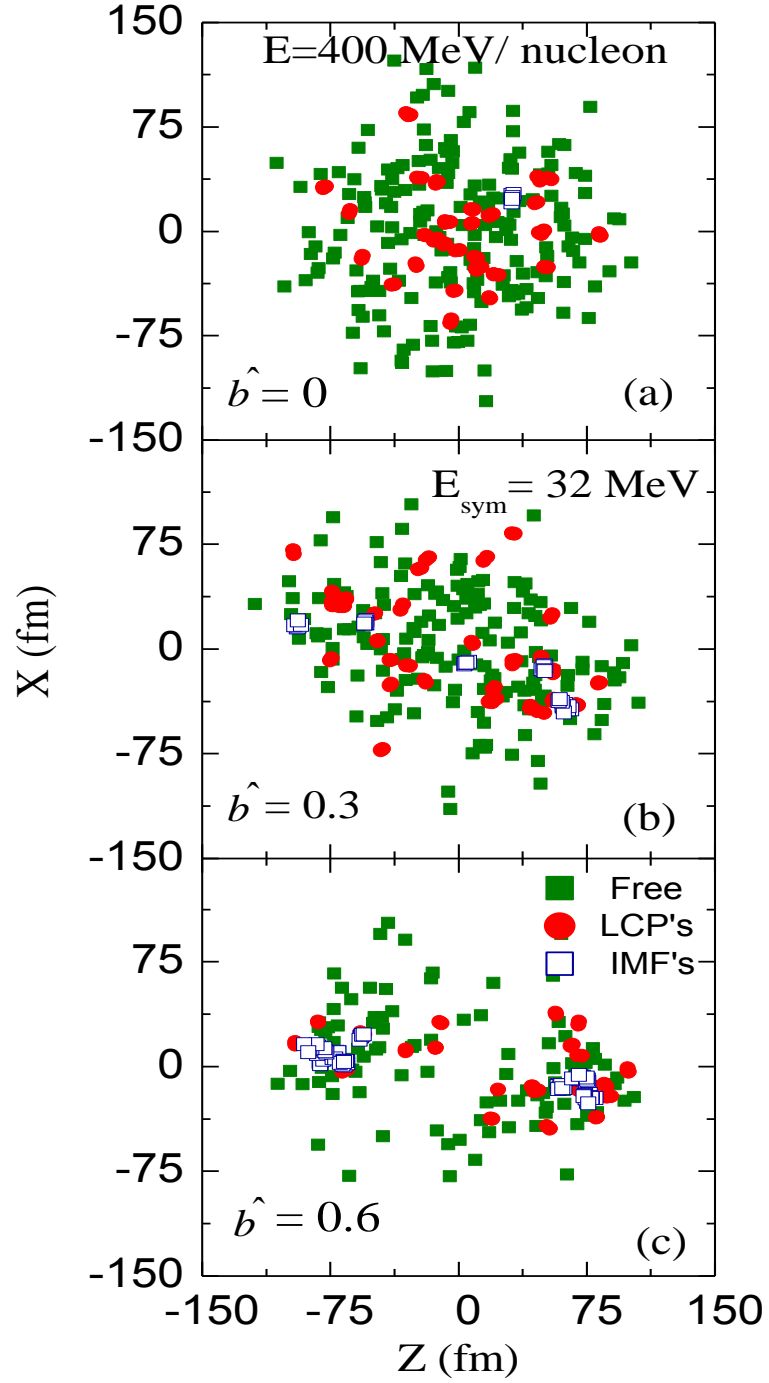


Fig. 1. The final phase space of a single event for the reaction of  ${}_{54}\text{Xe}^{131} + {}_{54}\text{Xe}^{131}$  with symmetry energy= 32 MeV. The top (a), middle (b), and bottom (c) panels are respectively , for scaled impact parameter  $\hat{b} = 0, 0.3$  and  $0.6$  different symbols are used for free nucleons, LCP's and IMF's.

spherical distribution of particles, indicating the spreading of nucleons in all directions, which implies that the breaking of initial correlation is maximal in this region. As a result of this, randomization increases and stopping in hot and compressed nuclear matter occurs. However the MDI's have a sizable effect on the charged particle multiplicity.

It is found that in comparison to the simple static equation of state, the MDI's lead to large number of fragments at peripheral collisions, whereas it yields fewer fragments in central collision [8]. The stopping power decreases with impact parameter. Since the free particles as well as the LCP's originate from the mid rapidity region, they are better suited for studying the degree of stopping reached in HIC. Whereas the IMF's originated from the target or projectile region, are treated as the residue of the spectator matter [9-11].

### 3.3.2 Rapidity distribution

Fig. 2 displays the rapidity distribution  $dN/dY$  for the emission of free nucleons as well as LCP's and IMF's. Rapidity distribution gives a large value with MDI's. The most striking point is that the rapidity is less affected in the central collisions, whereas it leads to entirely different results at peripheral collisions. It is clear from the figure that free particles and LCP's emitted in the central collisions form a single narrow Gaussian shape, whereas IMF's have broader Gaussian indicating less thermalization. With the increase in impact parameter, the single Gaussian distribution splits into two Gaussian (at target and projectile rapidities), indicating correlated matter.

From the shape of the Gaussian, one sees that free particles and LCP's are a better indicator of the thermalization [12]. At peripheral collisions, the spectator zone increases and participant zone becomes very small, so most of the matter goes without any collision and hence the rapidity of IMF's increases considerably. From the figure, one sees a one to one relation between the degree of stopping and the emission of LCP's [10].

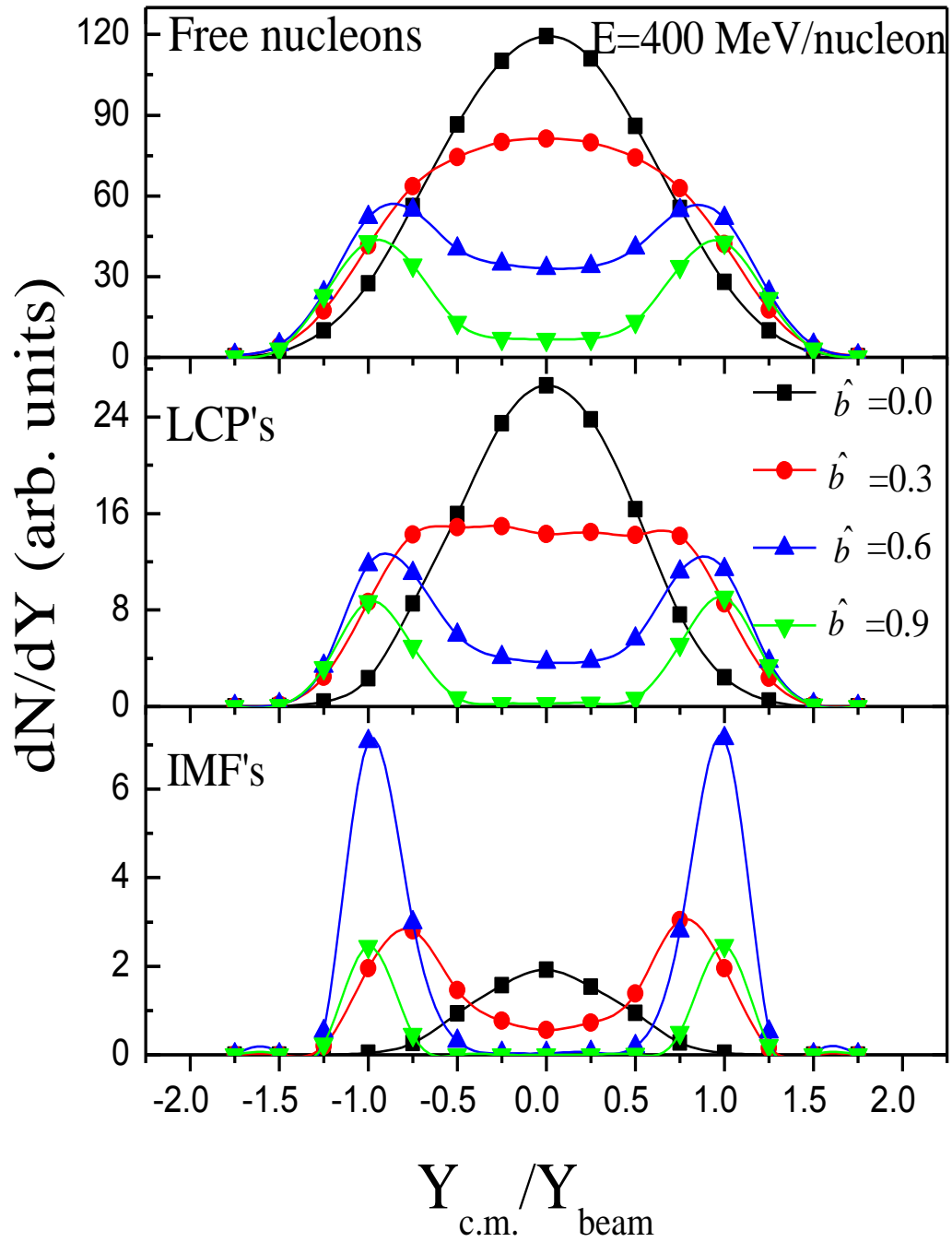


Fig. 2. The rapidity distribution  $dN/dY$  as a function of reduced rapidity for free nucleons (a), LCP's (b), and IMF's (c) at different impact parameters, with symmetry energy. The reaction under study is  ${}_{54}\text{Xe}^{131} + {}_{54}\text{Xe}^{131}$  at incident energy  $E = 400\text{MeV/nucleon}$ .

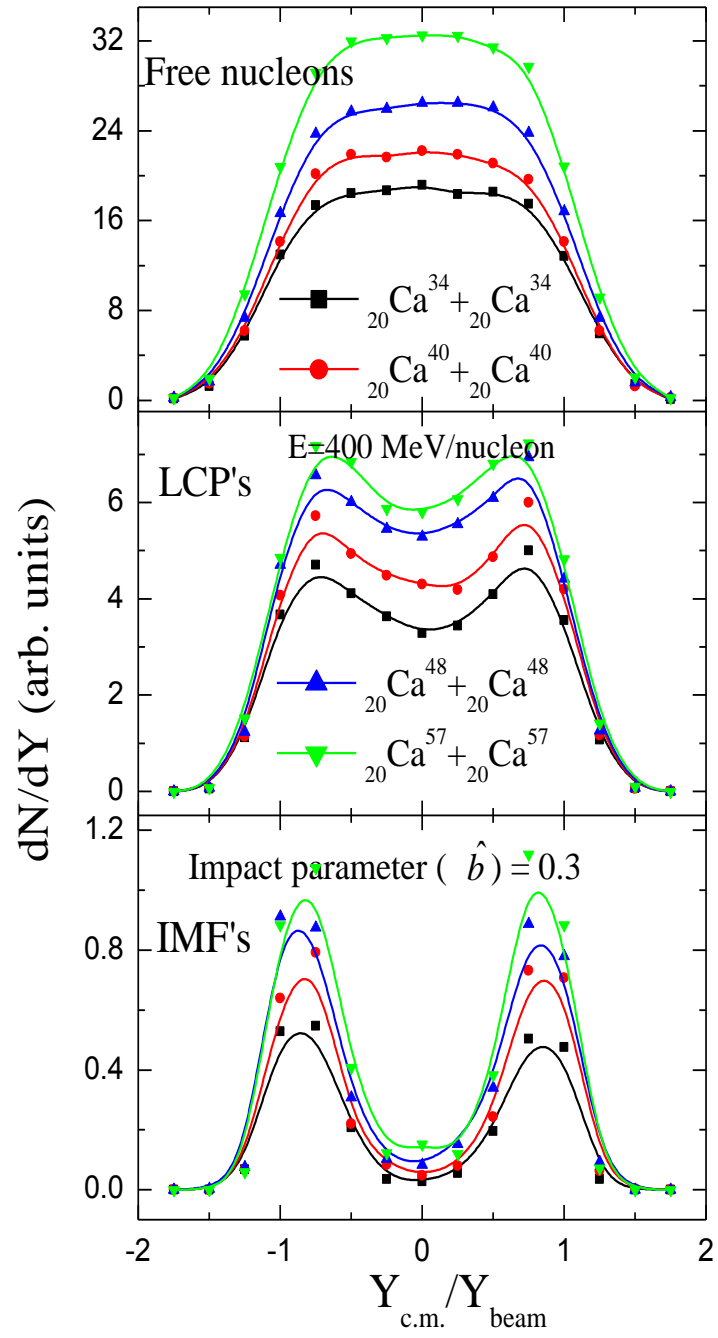


Fig. 3. The rapidity distribution  $dN/dY$  as a function of  $Y_{\text{c.m.}}/Y_{\text{beam}}$  for free nucleons(a), LCP's (b) and IMF's(c) at energy= 400 MeV/nucleon and 0.3 impact parameter .The results are obtained with HMD.

In fig.3, we display the rapidity distribution of free particles, LCP's and IMF's for different isotopes of Ca. It is clear that with the increase in total mass number of the system, the rapidity of all the fragments also increases. For free particles a single narrow Gaussian is observed, whereas for LCP's and IMF's the single Gaussian split into two Gaussians. The first Gaussian is indicating the projectile and the other is indicating the target rapidities. One can also conclude from here that with the increase in neutron number the collision suppressed and stopping increases.

### 3.3.3 Time evolution

Fig. 4, represents the time evolution of the nuclear stopping parameters  $R$  and  $\frac{1}{Q_{zz}/\text{nucl.}}$  at scaled impact parameter  $(\hat{b}) = 0.3$ , at different energies with MDI. It is noted that with increase in energy the nuclear stopping decreases with the time evolution the relaxation time decreases [13]. When  $R$  starts decreasing from 1 towards the lower values, it shows a partial transparency.

In fig. 5, the time evolution of the reaction  $_{54}\text{Xe}^{131} + _{54}\text{Xe}^{131}$  is displayed at impact parameter = 0.3 and at different energies with HMD. It is noted that with increase in energy the particle production increases in both, free particles as well as LCP's. One can clearly observe that the maximum change comes at energy = 400 MeV /nucleon[14,15]. After 200 fm/c saturation in the production of fragments takes place. Hence we take all the observations at 200 fm/c.

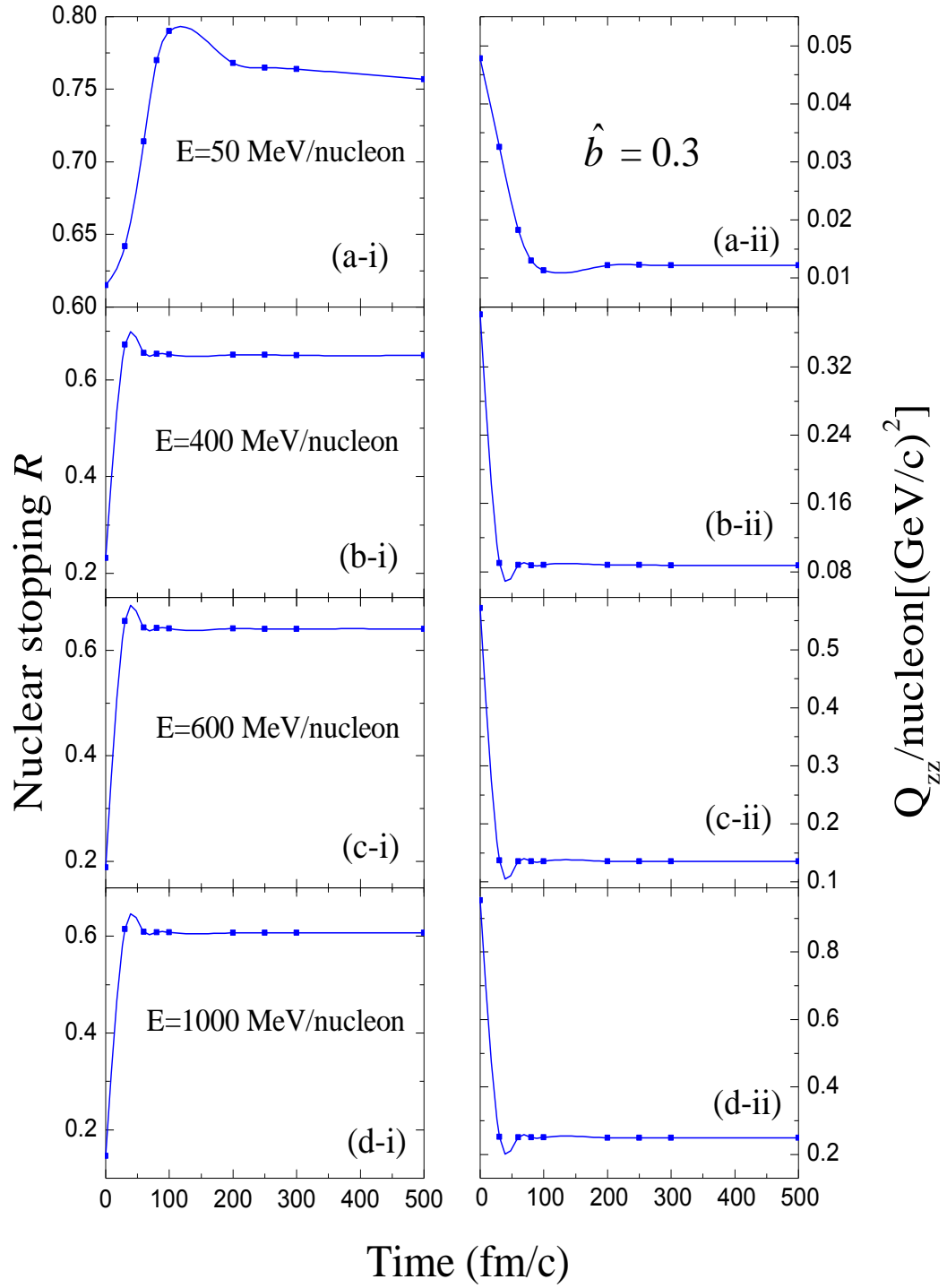


Fig. 4. Time evolution of anisotropic ratio  $R$  (i) and quadrupole moment ( $Q_{zz}/\text{nucleon}$ ) (ii) at energies equal to 50(a), 400 (b), 600 (c) and 1000 MeV/nucleon(d), respectively.

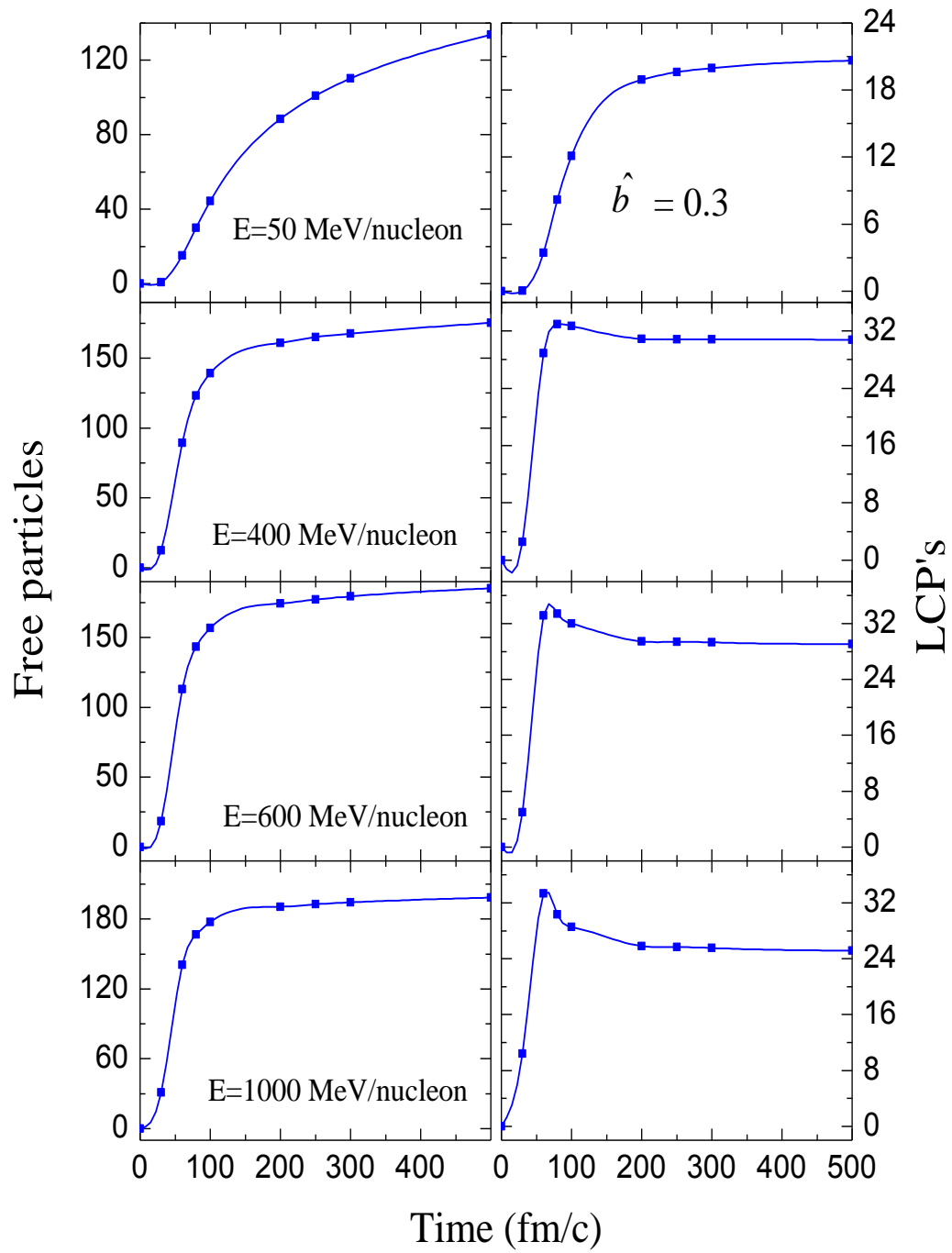


Fig.5. Time evolution of free nucleons (a) and LCP's (b) at different energies at impact parameter  $\hat{b} = 0.3$ .

### 3.3.4 Energy dependence

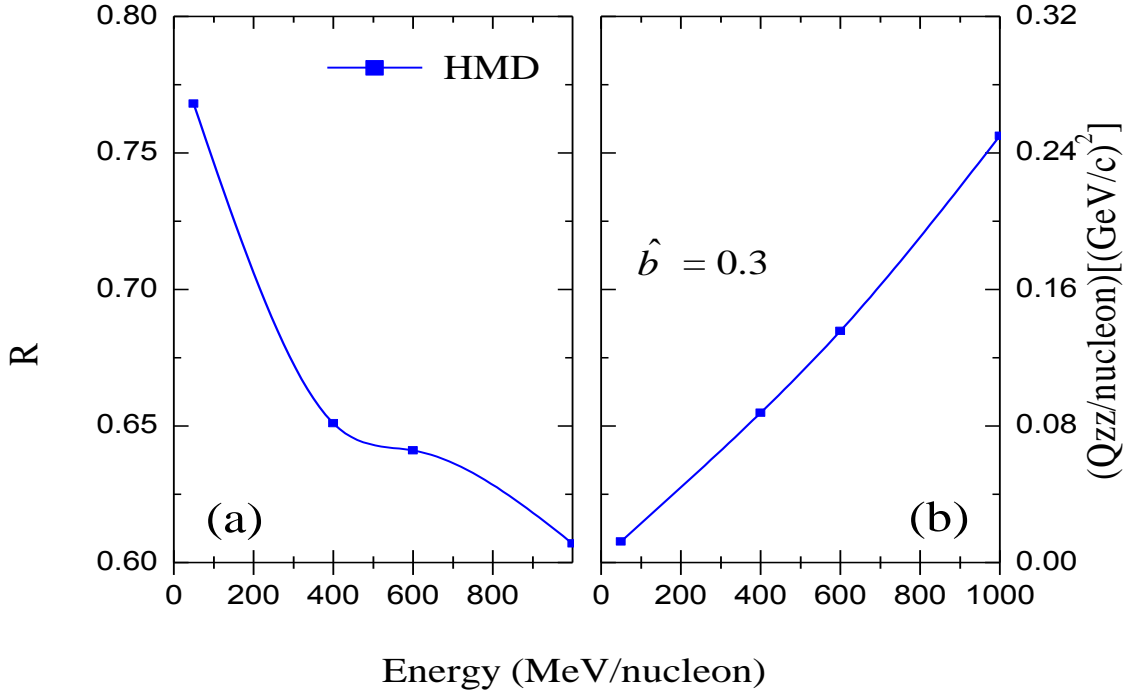


Fig. 6. The anisotropic ratio  $R$  (a), quadrupole moment (b) as a function of beam energies using HMD equation of state.

In fig. 6, we display the variation of stopping parameters and multiplicity of fragments with respect to energy with MDI's. One can easily observe that stopping decreases with increase in energy. This is due to increase in transparency of the colliding nuclei. At higher energies number of collisions decreases. Thus  $R$  approaches to 0 and  $Q_{zz}$  increases. At higher incident energies the  $z$  component of momentum becomes stronger compared to  $x$  and  $y$  component, and hence the value of  $Q_{zz}$  also increases.

### 3.3.5 Impact parameter dependence

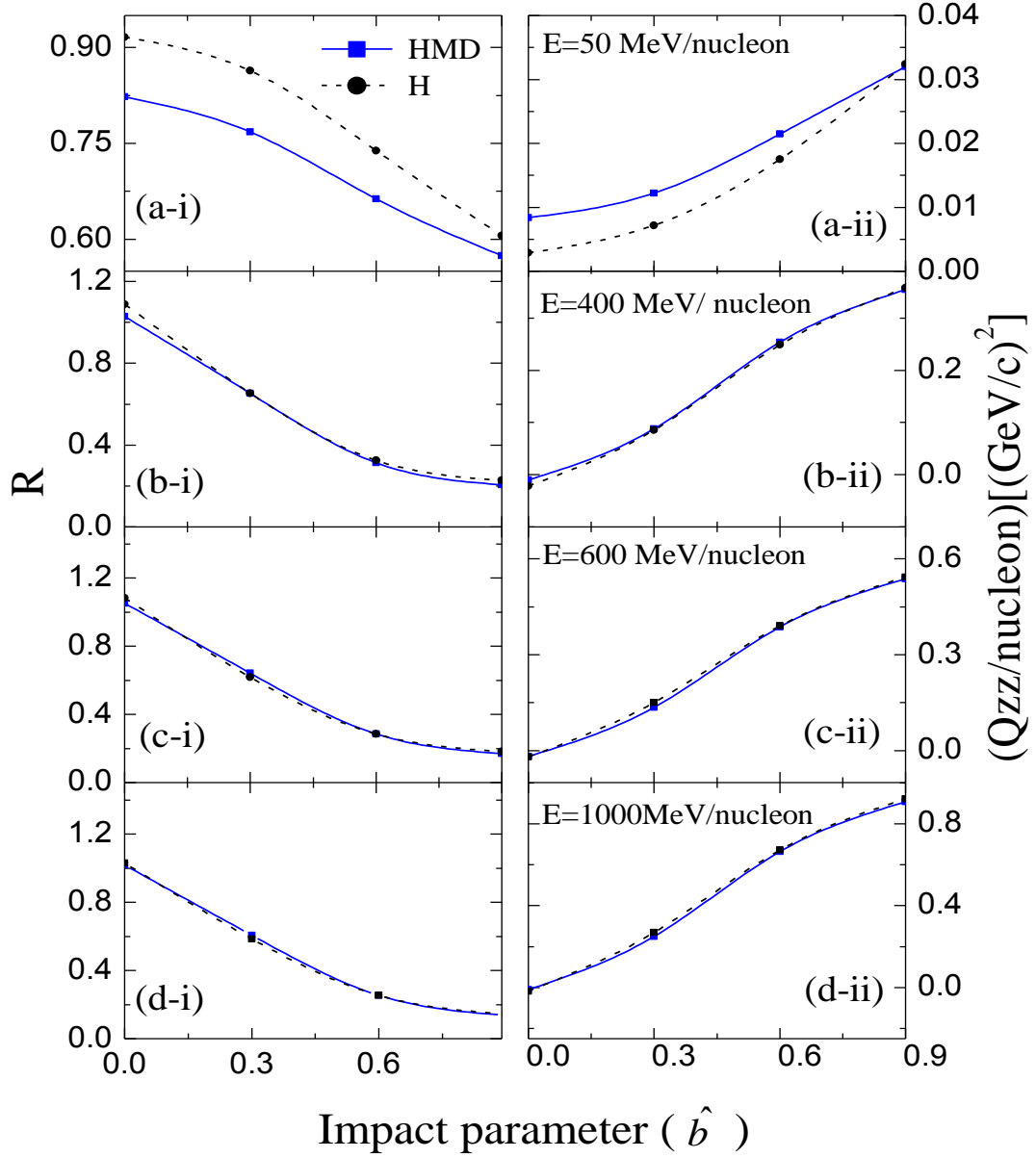


Fig.7. Anisotropic ratio  $R$  (i) and  $Q_{zz}/\text{nucleon}$  (ii) as a function of impact parameter with symmetry energy and MDI's. The results obtained with hard and HMD, respectively, are represented by solid and dashed lines at energies = 50 (a), 400 (b), 600 (c) and 1000 MeV/nucleon (d).

Impact parameter dependence of global variables ( $R$  and  $Q_{zz}/\text{nucleon}$ ) has been displayed in fig 7 at  $E_{\text{Sym}} = 32$  MeV. We observe that  $R$  and  $Q_{zz}/\text{nucleon}$  behave in opposite fashion. For  $R > 1$  and  $Q_{zz}/\text{nucleon} < 0$ , can be explained by the preponderance of momentum flow perpendicular to the beam direction [13]. As the energy increases,  $P_z$  becomes stronger than  $P_x$  and  $P_y$ . So the net value of  $R$  decreases, indicating less stopping. Whereas, with the increase in energy, the violence of N-N collisions also increases. The maximum stopping is observed around 400 MeV/nucleon, which is in supportive nature with the findings of Reisdorf *et al.*[16]. In their work, they measured the nuclear stopping for the energy range from 0.09 to 1.93 GeV/nucleon and maximal stopping was observed around 400 MeV/nucleon. At certain energy, when the reaction reaches a maximal stopping, the composite matter reach to a stage of least transparency and thus most of the particles are preferentially out of plane.

The effect of momentum dependent interactions (MDI's) is also visible in nuclear stopping at lower energies. This effect decreases with the increasing incident energy. At high incident energy (1000 MeV/nucleon) anisotropic ratio  $R$  is independent of equation of state (EOS). At low values of incident energy (50 MeV/nucleon) the value of  $R$  with HMD is less compared to simple static EOS. This indicates that MDI suppress the nuclear stopping [17].

In Fig.8, we display the impact parameter dependence of multiplicity of free nucleons and LCP's at 200 fm/c. The behavior of all the curves is in analogy with nuclear stopping parameters  $R$  and  $\frac{1}{Q_{zz}/\text{nucl.}}$ , as discussed in fig 7. It is noted that the multiplicity of free particles is increasing monotonically. Whereas the multiplicity of LCP's is maximum around 400 MeV/nucleon. We see that the multiplicity of free nucleons decreases with the increase in impact parameter due to decrease in the participant region. As a result LCP's also decreases with the increase in impact parameter. The effect of MDI's decreases with the increase in incident energy. Looking at the behavior of LCP's, one can say that production of LCP's can act as an indicator of nuclear stopping.

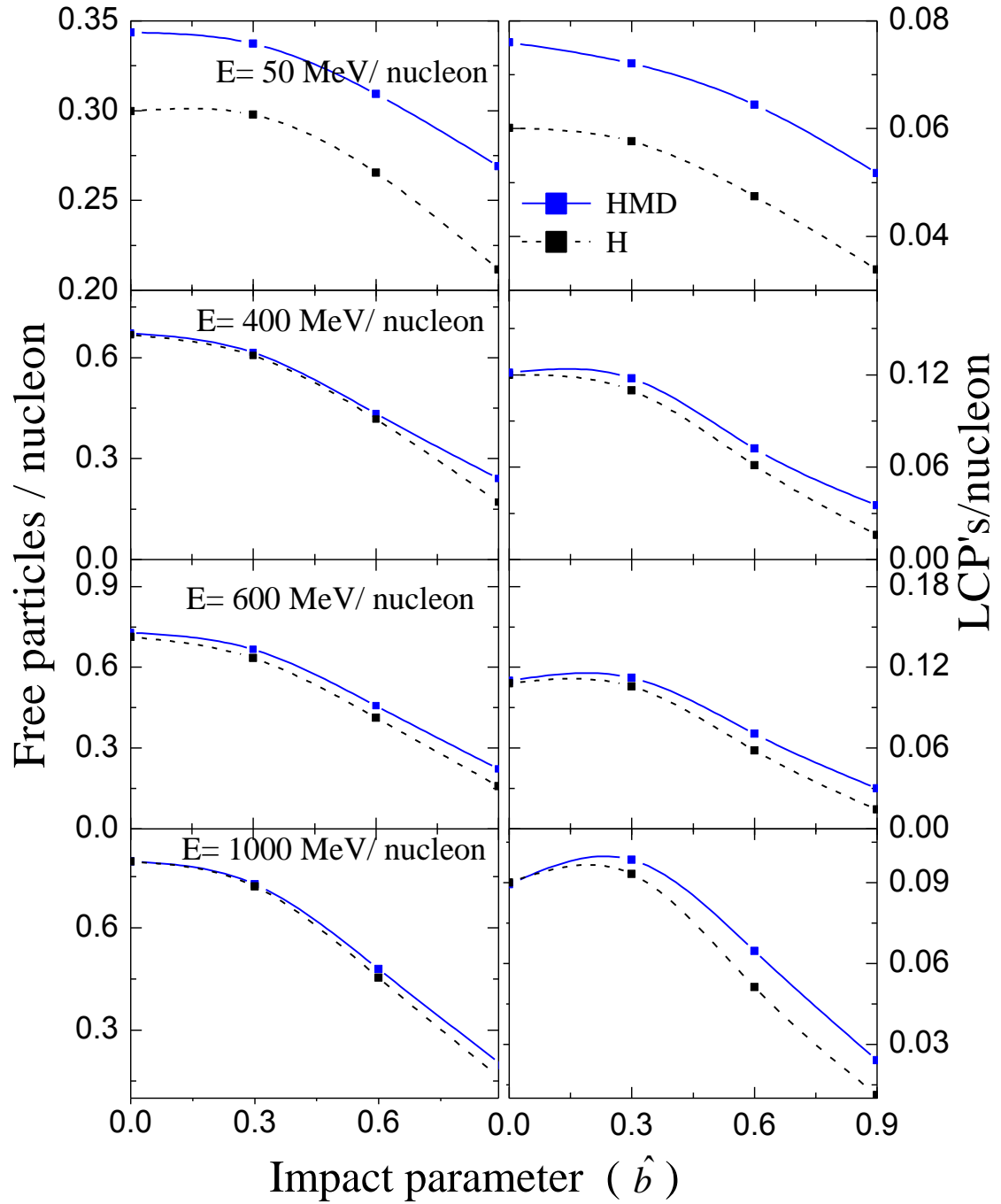


Fig.8. Impact parameter dependence of multiplicity of free particles and LCP's ) at energies equal to 50(a), 400 (b), 600 (c) and 1000 MeV/nucleon(d), respectively, with(solid line) and without momentum dependence(dotted line).

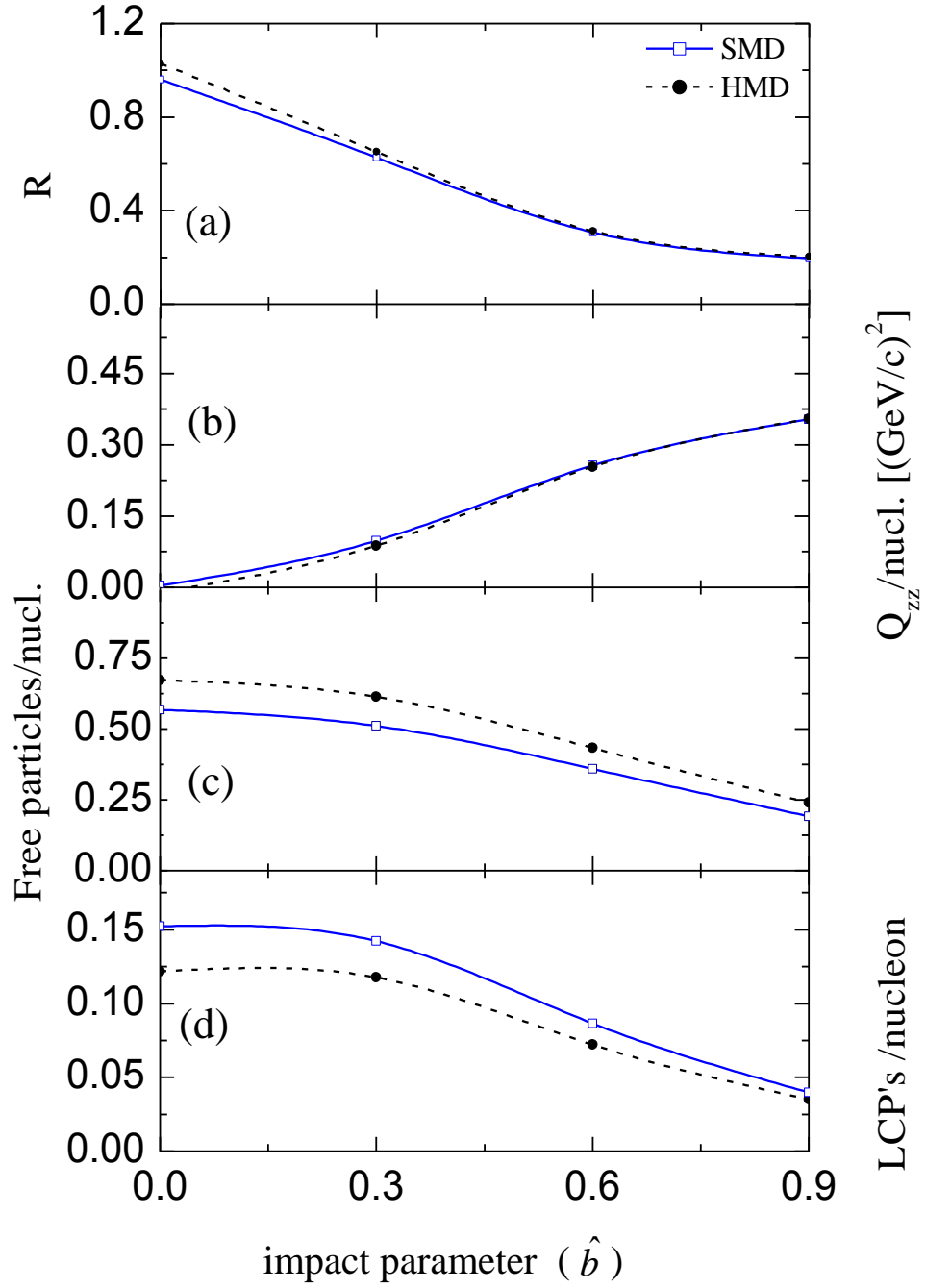


Fig. 9 impact parameter dependence of  $R$  (a),  $\frac{1}{Q_{zz}/\text{nucl.}}$  (b), free nucleons(c) and LCP's (d) with hard (HMD) and soft (SMD) equation of state with symmetry energy=32 MeV

In fig. 9. We check the sensitivity of nuclear stopping as well as the multiplicity of free particles and LCP's for nuclear equation of states with MDI's. For this purpose, a hard and soft equation of state with compressibility  $\kappa = 380$  and  $200$  MeV are employed, respectively. Repulsive interactions are stronger with HMD compared to SMD. Since at lower impact parameter the participant zone is more. Hence one can clearly see more stopping in the presence of HMD. This difference keeps on decreasing as we move from central to peripheral collisions. Production of LCP's and Free particles follow the same trend as stopping parameter R. This simply implies that the fragment production can act as a global indicator for the nuclear stopping.

To have a clear picture of nuclear stopping in HIC with MDI's we further elaborate the above findings in fig. 10.

Fig.10 display fragment multiplicity/nucleon as well as R and  $\frac{1}{Q_{zz}/\text{nucl.}}$  with MDI's. We normalize the free nucleon, LCP's with R at the starting point of impact parameter. We see that the behavior of the fragments with respect to the impact parameter comes out to be in great analogy with the anisotropic ratio R, whereas a visible difference occurs with reference to the quadrupole moment. Hence the similarity in all three quantities in the presence of momentum dependent interaction makes LCP's a good indicator of global stopping in HIC.

### 3.3.6 Mass dependence

In fig.11, the system size dependence of R,  $1/Q_{zz}/\text{nucleon}$ , free particles and LCP's with MDI has been displayed. We display the results for the reactions of  ${}_{20}\text{Ca}^{40}+{}_{20}\text{Ca}^{40}$ ,  ${}_{28}\text{Ni}^{58}+{}_{28}\text{Ni}^{58}$ ,  ${}_{41}\text{Nb}^{93}+{}_{41}\text{Nb}^{93}$ ,  ${}_{54}\text{Xe}^{131}+{}_{54}\text{Xe}^{131}$ , and  ${}_{79}\text{Au}^{197}+{}_{79}\text{Au}^{197}$ .

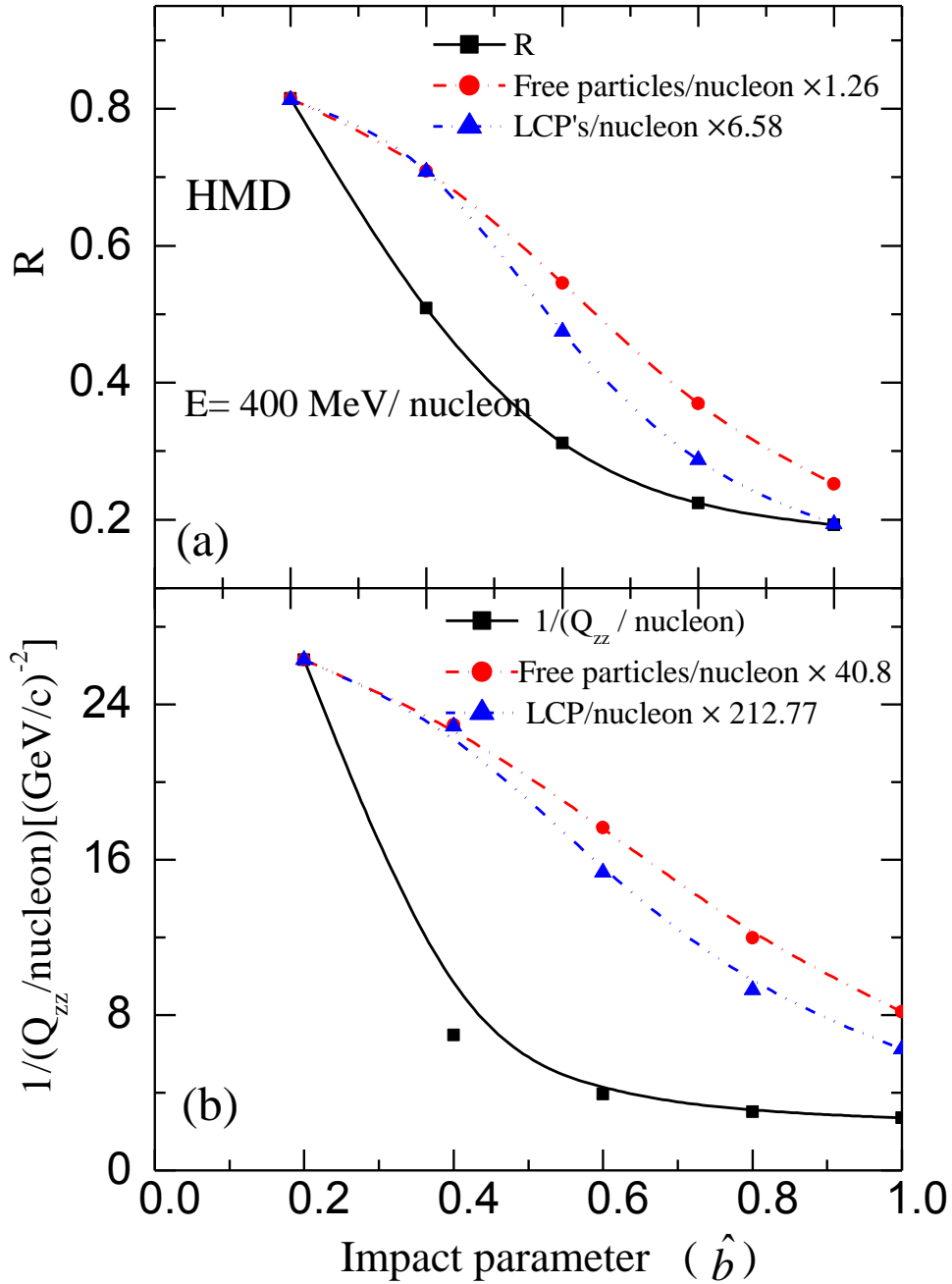


Fig.10. The scaled free particles/nucleon and LCP's/nucleon along with anisotropic ratio  $R$  (a) and  $\frac{1}{Q_{zz}/\text{nucl.}}$  (b) as a function of normalized impact parameter. In the presence of symmetry energy and incident energy = 400 MeV/nucleon with HMD.

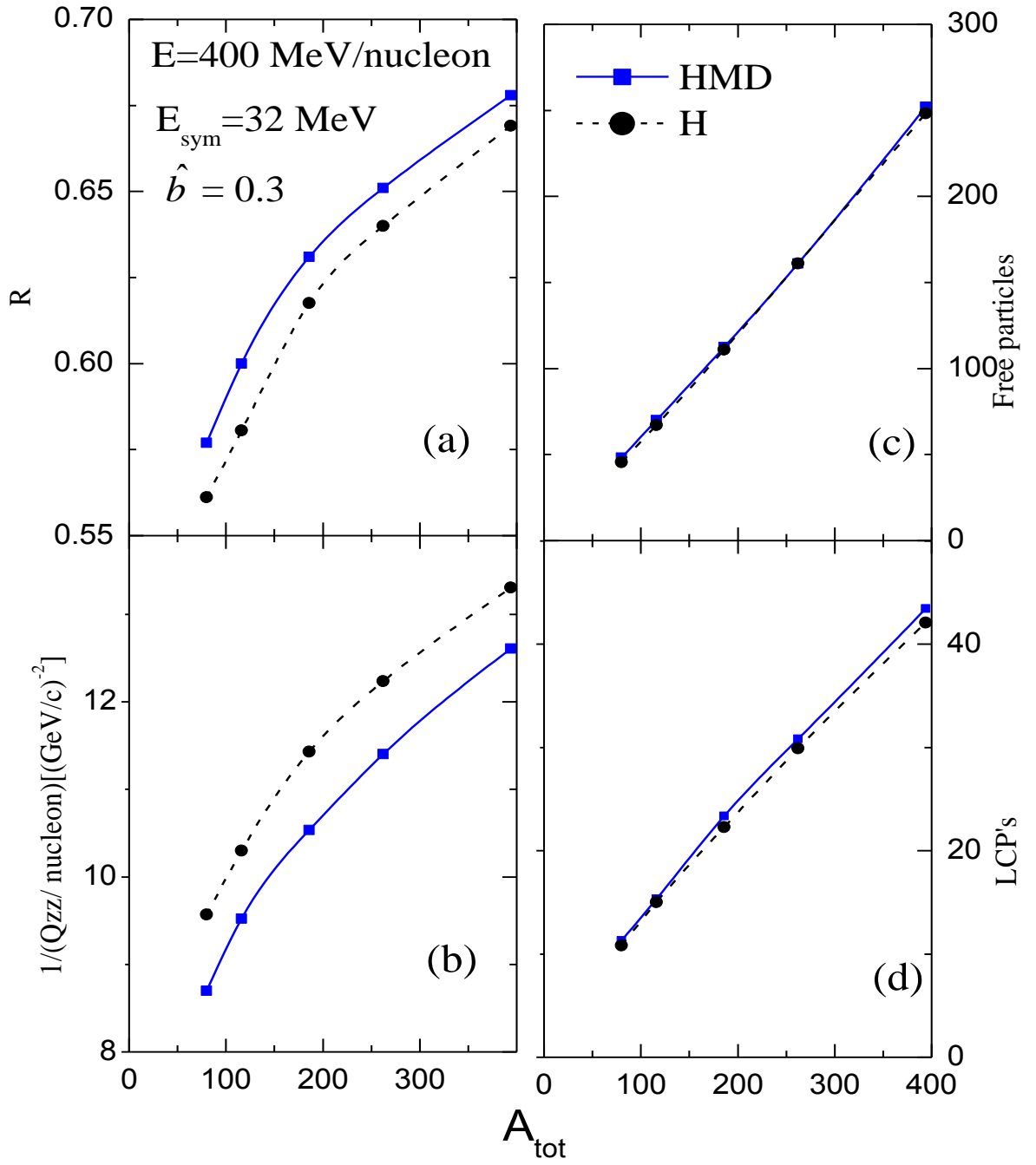


Fig. 11. System size dependence of  $R$  (a),  $\frac{1}{Q_{zz}/\text{nucl.}}$  (b), free particles (c) and LCP's (d) in the presence of symmetry energy and HMD.

From fig. 11 it is observed that the parameter  $R, \frac{1}{Q_{zz}/\text{nucl.}}$ , free particles and LCP's are varying in a similar fashion with the composite mass of the system. For a fixed geometry as the composite mass of the system increases, the compressed zone becomes hotter which in turn results in more global stopping. Similarly, as we know that the LCP's and free particles are generated from the participant zone only. With increase in composite mass of the system the number of participants increases and the production of free particles as well as LCP's also goes on increasing [17].

It is also clear from the fig.11 that the extent of nuclear stopping decreases when the MDI's are taken into account.

### 3.3.7 Isospin dependence

Results are displayed in fig. 12 for the reactions of  ${}_{20}\text{Ca}^{34} + {}_{20}\text{Ca}^{34}$  ( $N/Z=0.7$ ),  ${}_{20}\text{Ca}^{40} + {}_{20}\text{Ca}^{40}$  ( $N/Z=1$ ),  ${}_{20}\text{Ca}^{48} + {}_{20}\text{Ca}^{48}$  ( $N/Z=1.4$ ),  ${}_{20}\text{Ca}^{57} + {}_{20}\text{Ca}^{57}$  ( $N/Z=1.85$ ) having same  $Z$  and different  $A$  in the presence of symmetry energy  $=32$  MeV and isospin dependent cross-section. The results displayed are in supportive evidence with the findings of fig. 11. With the increase in  $N/Z$  ratio the number of neutrons decreases and hence the number of collisions increases. This in turn increase the stopping parameters ( $R$  and  $\frac{1}{Q_{zz}/\text{nucl.}}$ ). From the above displayed results one may conclude that the nuclear stopping and LCP's can also be used as a tool to investigate the stopping nature of nucleons in the presence of MDI's.

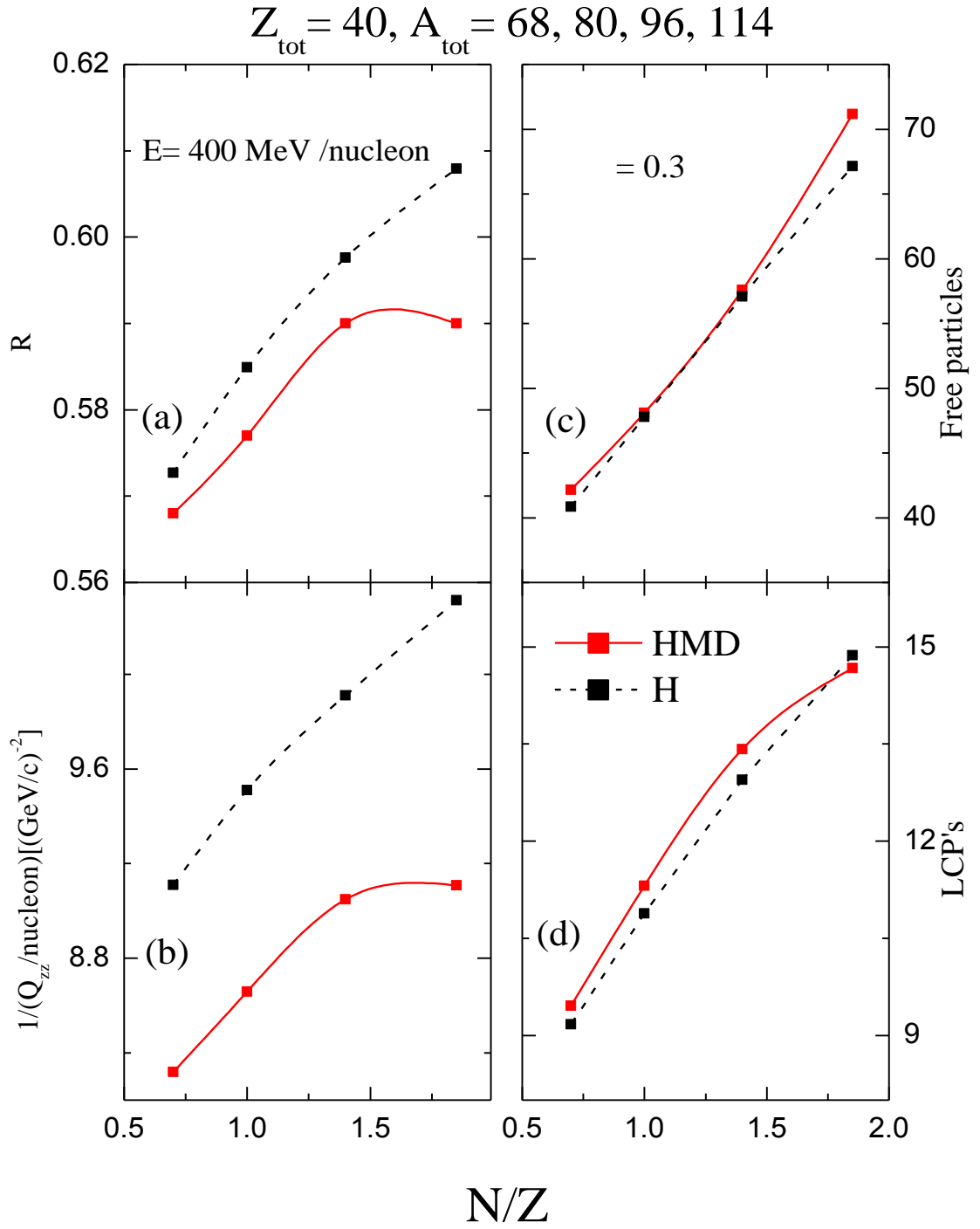


FIG.12. N/Z dependence of R (a),  $\frac{1}{Q_{zz}/\text{nucl.}}$  (b), free particles (c) and LCP's (d) in the presence of symmetry energy and HMD.

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## CHAPTER 4

### SUMMARY

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The thesis contains a brief review of one of the important branch of nuclear physics i.e. heavy ion physics. It also contains the theoretical study of nuclear stopping. The isospin dependent quantum molecular dynamics model (IQMD) is used to describe the effect of momentum dependence on nuclear stopping.

In chapter 2 isospin quantum molecular dynamics model is explained in detail. Then method of clusterization i.e. minimum spanning tree method is also discussed.

In chapter 3 there is complete theory of nuclear stopping with momentum dependent interaction in heavy ion collision. Some important parameters used to describe nuclear stopping like quadrupole moment ( $Q_{zz}$ ), stopping ( $R$ ) are discussed. The analysis of the result obtained for describing the role of momentum dependant interaction in HIC is also shown in details. Phase space analysis, rapidity distribution, dependence of energy, impact parameter, mass and N/Z are shown in figures for different mass fragments for example LCP's, IMF's and Free particles.

Chapter 3 gives the following conclusions.

- MDI lead to large number of fragments at peripheral collisions.
- Nuclear stopping increases with increase in number of neutrons and it decreases with increase in incident energy and impact parameter.
- Momentum dependent interactions suppress the nuclear stopping.