# LOAD FLOW SOLUTION OF RADIAL DISTRIBUTION SYSTEMS USING TWO PORT PARAMETERS

Thesis submitted in partial fulfillment of the requirements for the award of degree of

**Master of Engineering**
in
**Power Systems & Electric Drives**

**Thapar University, Patiala**


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## CERTIFICATE

I hereby certify that the work which is being presented in the thesis entitled, **"Load Flow Solution of Radial Distribution Systems Using Two Port Parameters"**, in partial fulfillment of the requirements for the award of degree of Master of Engineering in Power Systems and Electric Drives submitted in Electrical and Instrumentation Engineering Department of Thapar University, Patiala, is an authentic record of my own work carried out under the supervision of Mr. N.Patnaik and Dr. S. Ghosh.

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

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This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

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# CONTENTS


| | |
|---|---|
| CERTIFICATE | i |
| ACKNOWLEDGEMENTS | ii |
| CONTENTS | iii |
| LIST OF FIGURES | v |
| LIST OF TABLES | vi |
| KEYWORDS | vii |
| ABSTRACT | viii |
| **CHAPTER 1: INTRODUCTION** | **1-5** |
| 1.1 Introduction | 1 |
| 1.2 Literature Review | 1 |
| 1.3 Scope of Thesis | 4 |
| 1.4 Organisation of Thesis | 5 |
| **CHAPTER 2: NETWORK MODELLING** | **6-38** |
| 2.1 Introduction | 6 |
| 2.2 Generalized two port parameter modeling | 6 |
| 2.3 Derivation of two port parameter | 8 |
| 2.3.1 Shunt Capacitors | 8 |
| 2.3.2 Shunt Reactors | 14 |
| 2.3.3 Branch | 20 |
| 2.3.6 Series Reactor | 21 |
| 2.3.5 Distribution line | 21 |
| 2.3.6 Single phase transformer | 25 |
| 2.3.7 Three phase transformer | 29 |
| 2.3.8 Three phase regulator | 35 |

| | |
|---|---|
| **CHAPTER 3: DISTRIBUTION SYSTEM LOAD FLOW** | **38-41** |
| 3.1 Introduction | 38 |
| 3.2 Load flow algorithm | 39 |
| 3.3 Enerlyser Software | 40 |
| **CHAPTER 4: RESUTS** | **42-55** |
| 4.1 IEEE 123 node Distribution System | 42 |
| 4.2 Comparison of results | 43 |
| **CHAPTER 5: CONCLUSION AND FUTURE SCOPE** | **56** |
| **APPENDIX - A** | **57-63** |
| **REFERENCES** | **64-66** |

# LIST OF FIGURES


| Caption | PP. No. |
|---|---|
| Fig. 2.1 Shunt Capacitors (phase to neutral connection) | 8 |
| Fig. 2.2 Shunt Capacitors (phase to phase connection) | 10 |
| Fig. 2.3 Shunt Capacitors (ungrounded wye connection) | 12 |
| Fig. 2.4 Shunt Reactors (phase to neutral connection) | 14 |
| Fig. 2.5 Shunt Reactors (phase to phase connection) | 16 |
| Fig. 2.6 Shunt Capacitors (ungrounded wye connection) | 18 |
| Fig. 2.7 Distribution line model | 21 |
| Fig. 2.8 Three phase transformer (delta-star grounded) | 29 |
| Fig. 2.9 Three phase transformer (star ungrounded-star grounded) | 31 |
| Fig. 2.10 Three phase transformer (star grounded-star grounded) | 33 |
| Fig. 2.11 Three phase voltage regulator (star connected) | 35 |
| Fig. 4.1 IEEE 123 node Test Feeder | 42 |

## LIST OF TABLES


| Title | PP. No. |
|---|---|
| Table 4.1 Comparison of Current | 43 |
| Table 4.2 Comparison of Line losses | 47 |
| Table 4.3 Comparison of Voltages | 51 |
| Table 4.4 Summary of Load Flow results | 55 |
| Table A.1 Regulator data | 57 |
| Table A.2 Line segment data | 58 |
| Table A.3 Overhead line Configurations | 59 |
| Table A.4 Transformer data | 59 |
| Table A.5 Shunt Capacitors | 59 |
| Table A.6 Three phase switches | 59 |
| Table A.7 Spot loads data | 60 |

# KEY WORDS

| | |
|---|---|
| E | Earth |
| f | System frequency |
| $I_n$ | Load side neutral current |
| $I_N$ | Source side neutral current |
| $L_x$ | Per phase shunt inductance |
| $l$ | Length of branch |
| N | Neutral |
| $n_{ta}$ | Transformer winding ratio |
| 0 | Zero matrix |
| $R_x$ | Per phase shunt resistance |
| U | Unity matrix |
| $V_{NE}$ | Source side neutral to earth voltage |
| $V_{ne}$ | Load side neutral to earth voltage |
| Y | Shunt branch admittance |
| $Z_{ta}$ | Series impedance of transformer referred to secondary |
| $Z_n$ | Transformer neutral impedance |
| $a_{Ra,}$ $a_{Rb,}$ $a_{Rc}$ | Effective turns ratio for single phase regulator |
| $I_a$, $I_b$, $I_c$, | Load side line currents |
| $I_A$, $I_B$, $I_C$, | Source side line currents |
| $V_{ae}$,$V_{be}$,$V_{ce}$, | Load side line to earth voltages |
| $V_{AE}$,$V_{BE}$,$V_{CE}$, | Source side line to earth voltages |

# ABSTRACT

A load flow is an essential tool for the steady state analysis of a power system. Load flow involves calculation of bus voltages, line currents, line flows and losses given full network parameters, complex loads and one bus voltage as reference at steady state condition. Distribution load flow is a very important tool for the analysis of distribution systems and is used in operational as well as it's planning. Generally, for distribution system load flow solution, Forward- backward sweep algorithm is used in which voltages are updated in forward sweep and currents in backward sweep.

The aim of the present work is to develop the various distribution system network elements models in terms of two port parameter. With these network elements modeling, a load flow algorithm based on Forward- backward sweep technique has been developed .This algorithm is tested on standard IEEE 123 node distribution system with the help of Enerlyser software. The results from Enerlyser software and IEEE results are compared. This proposed algorithm converges fast as there in no matrix factorization and inversion.

# CHAPTER −1

# INTRODUCTION

## 1.1 INTRODUCTION

A load-flow is an essential tool for the steady state analysis of a power system. The main objective of the load-flow analysis is to find out the real and reactive powers flowing in each line along with the magnitude and phase angle of the voltage at each bus of the system for the specific loading conditions. Distribution load-flow is a very important tool for the analysis of distribution systems and is used in operational as well as planning environments. Many real-time applications in the distribution automation system (DAS) and distribution management system (DMS), such as network optimization, VAr planning, switching, state estimation and so forth, need the support of a robust and efficient power-flow method. Such a power-flow solution must be able to model the special features of distribution systems in sufficient detail. Some of the inherent features of electric distribution systems are:

(1) a radial network structure,

(2) an unbalanced distributed load and unbalanced operation,

(3) an extremely large number of branches/nodes, and

(4) a wide range of resistance and reactance values. The distribution networks are radial in nature having high R/X ratio where as transmission networks are loop in nature having high X/R ratio.

These features cause the traditional power-flow methods, the Gauss-Seidel and Newton-Raphson techniques, which arise from the transmission area, to lack either computer economy or robustness in distribution applications. The Gauss-Seidel, Newton-Raphson and Fast decoupled load-flow fail to converge to distribution systems because these networks have high R/X ratio.

## 1.2 LITERATURE OVERVIEW

Many load-flow techniques are available in literature for transmission systems. A few load-flow techniques had been proposed for distribution systems. The literature survey for distribution systems is discussed below.

Shirmohammadi *et al.* [1], need Kirchhoff's voltage and current laws for solving radial distribution networks. In their method, a branch-numbering scheme was employed to enhance the computations. The method was then extended to apply to weakly meshed networks. In their method, they first break the interconnected grid at a number of points (breakpoints) in order to convert it into one simple radial network. Each breakpoint would open one simple loop. The radial network was solved by direct application of Kirchhoff's laws. They accounted for the flows at the breakpoints by injecting currents at their two end nodes. The numerical efficiency of their method, however, diminished as the number of breakpoints required to convert the meshed network to a radial configuration increases. This restricts the practical application of the method to weakly meshed networks.

Goswami and Basu [2] presented a direct method for solving radial and meshed distribution networks. Their method had the advantages of a no convergence problem, a guaranteed accurate solution for any realistic distribution system, and the ease with which composite loads could be represented. The disadvantages were difficulty numbering the nodes and branches, and that no node in the network is the junction of more than three branches.

Das *et al.* [3] presented a load-flow method for radial distribution networks based on evaluating the total real and reactive power fed through any node. They created a unique node, branch, and lateral numbering scheme to enhance the evaluations of real and reactive loads fed through any node and receiving-end voltages. Their method has the advantage that all data can be stored in vector forms, saving an enormous amount of computer memory.

Baran and Wu [4, 5] performed load-flow analysis in a distribution system using an iterative solution of three fundamental equations representing real and reactive power, and voltage magnitude. The three equations were useful because they could be used in real systems rather than in other classically known forms. Here, they computed the system Jacobian matrix using a chain rule. The mismatches and the Jacobian matrix involved only evaluating simple algebraic expressions and no trigonometric functions. The formulation and evaluation of Jacobians were time consuming and require large amounts of computer memory storage.

Ghosh and Das [6] presented a method for solving radial distribution networks by evaluating an algebraic expression of receiving-end voltages. In this method, the authors assumed an initial flat voltage for all nodes. They proposed an "IDENT" software to computer nodes beyond each branch. The loads and charging currents were calculated at first and then the branch currents. The modified nodal voltages were recalculated, as were the losses. Evaluating the difference between new and previous voltage values and then comparing it with an accepted tolerance verified the convergence for this method. Their method was simple and had good and fast convergence, and could be used efficiently for composite load modeling, provided the composition of the loads is known.

Ciric *et al*. [7] presented a general power-flow algorithm for three-phase four-wire radial distribution networks considering neutral grounding, based on which backward-forward technique. In their method, both the neutral wire and ground were explicitly represented. A problem of three-phase distribution system with earth return, as a special case of a four-wire network, was also elucidated.

Chang *et al.* [8] presented a simplified forward and backward approach for load-flow analysis in radial distribution system. This method involved two phases. In Phase I (forward sweep), the KCL and KVL were used to compare voltage for each bus located at upstream of each line segment or transformer. In Phase II (backward sweep), the linear proportion concept for real and imaginary decomposition is  adopted to find the ratios of real and imaginary parts of specified voltage to the calculated voltage at each upstream bus. Then, the voltage at each downstream bus was updated by real and imaginary parts of initial or calculated voltage multiplying with the corresponding ratios, respectively. The solution procedure was terminated after the mismatch of calculated voltage and specified voltage of substation was less than the tolerance value.

Golkar [9] presented a novel method for load-flow analysis in radially operated 3-phase distribution networks without solving the well-known conventional load-flow equations. The method could be applied for distribution systems in which the loads were unbalanced. The size of matrix used was very small compared to those in conventional methods, the amount of memory used was very small. Therefore, the speed is very high, and the relative speed of calculation increases with the size of the system.

Teng [10] developed a network-topology-based three-phase distribution power-flow algorithm. The special topology of a distribution network had been fully exploited to make obtaining a direct solution possible. Two developed matrices were enough to obtain the power-flow solution: they are the bus-injection to branch-current matrix and the branch-current to bus-voltage matrix. The traditional Newton Raphson and Gauss implicit Z matrix algorithms, which needed LU decomposition and forward/backward substitution of the Jacobian matrix or the Y admittance matrix, were not needed for this new development. The features of this method were robustness and computer economy.

Kersting [11] presented the complete data for the three four-wire wye and one three-wire delta radial distribution feeders The data is for 123-bus, 34-bus and 13 bus four-wire wye radial distribution feeders and 37-bus three-wire delta radial distribution feeders..

Stot et al. [12] presented a fast decoupled load-flow method to solve the unbalanced three-phase distribution systems. They used the Newton Raphson and Fast decoupled method and developed the new fast decoupled method which would help to calculate the voltage corrections and angle.

Brit *et al* [13] developed the 3-phase load-flow program which would use for steady state initialization of real time system. The program used the standard 3-phase models and converts these models into its equivalent 1-phase models. In order to had a better convergence for large system, they used the Newton-Raphson algorithm instead of Gauss-Seidal algorithm.

Chen *et al.* [14] presented the detailed three-phase cogenerator and transformer models for analyzing a large-scale distribution systems. The cogenerator model presented had inherent generator phase imbalance due to distribution systems imbalance. The transformer model considers the copper and core losses, the winding connection, three-phase shifting between primary and secondary windings and off-nominal tapping

Chen *et al.* [15] introduced a rigid approach to three phase distribution power-flow analysis for large scale distribution systems. This approach is oriented towards applications in distribution systems operational analysis rather then planning analysis. The system that could be analyzed was balanced or unbalanced, radial or mixed distribution system.

Teng *et al.* [16] presented a new solution for the unbalanced three-phase power system using Newton-Raphson method. They written the three-phase current injection equations in rectangular coordinates and used the forward-backward sweep method to test the proposed method.

## 1.3 SCOPE OF THESIS

This thesis will present models and a method to solve the distribution system load-flow
problem using Enerlyser software. Specifically, in this dissertation works following objectives are considered: −

1. Modified approach to model components for analysis of unbalanced distribution system.
2. Development of distributed power-flow solution algorithm.
3. Investigation of the convergence property of algorithm.
4. Verification of proposed work.

## 1.4 ORGANISATION OF THESIS.

The thesis will progress in the following manner.

1) Chapter 1 covers the Introduction and Literature survey of the distribution systems.

2) Chapter 2 covers the modeling of unbalanced distribution system network elements using 2−port parameter.

3) Chapter 3 covers the distribution system load-flow and a simple efficient load-flow algorithm for radial distribution system using forward-backward sweep solution scheme.

4) Chapter 4 covers the results which are obtained using Enerlyser software for IEEE 123 node system and the results are compared with standard IEEE 123 node system.

5) Chapter 5 provides the conclusion of the work done along with the scope for further work.

# CHAPTER −2

# NETWORK MODELLING

## 2.1INTRODUCTION

It has become very important and necessary to operate a distribution system at its maximum capacity. Keeping in view the operating limits of distribution system so as to operate distribution system efficiently. For this, the distribution system can be modeled accurately. In this chapter the accurate models for all the major components of a distribution system are developed in terms of two port parameters.

Three-phase four-wire distribution networks are widely adopted in modern power distribution systems. A multi grounded three-phase four-wire service has higher sensitivity for fault protection than a three-phase three-wire service. The return current is due to both the unbalanced load and the nonlinear characteristics of electrical equipment through the distribution feeder. The return current may be larger than the phase currents if three-phase loads are seriously unbalanced in some segments. The neutral play an important role in power quality and safety problems.

In thesis, three-phase four-wire distribution system with single earth wire as a return is considered. The distribution system is unbalanced and neutral is solidly grounded to earth.

## 2.2. GENERALISED TWO PORT PARAMETER MODELLING

Here, the distribution system network elements are modeled using generalized two port parameters defined as follows:

$$\begin{pmatrix} [I_{ABCN}] \\ [V_{abcn}] \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} [V_{ABCN}] \\ [I_{abcn}] \end{pmatrix} \qquad 2.1$$

$[I_{ABCN}]$ =A vector of current on source side

$= [I_A \quad I_B \quad I_C \quad I_N]^T$

$[V_{ABCN}]$= A vector of line to earth voltages on source side

$= [V_{AE} \quad V_{BE} \quad V_{CE} \quad V_{NE}]^T$

$[I_{abcn}]$ = A vector of current on load side

$= [I_a \quad I_b \quad I_c \quad I_n]^T$

$[V_{abcn}]$ = A vector of line to earth voltages on load side

$= [V_{ae} \quad V_{be} \quad V_{ce} \quad V_{ne}]^T$

Where,

$I_A$, $I_B$, $I_C$, is source side line currents.

$I_N$ source side neutral current

$V_{AE}$, $V_{BE}$, $V_{CE}$, are source side line to earth voltages.

$V_{NE}$ source side neutral to earth voltage.

$I_a$, $I_b$, $I_c$, are load side line currents.

$I_n$ is load side neutral current.

$V_{ae}$, $V_{be}$, $V_{ce}$, are load side line to earth voltages.

$V_{ne}$ is load side neutral to earth voltage.

Here, all the voltages, currents , admittances and impedances are phasor quantities.

Hence, every device is characterized by 4 matrices (A, B, C, D) of dimension 4X4. Hence, the steps of the load-flow algorithm can be fairly standardized for all devices without having to actually check the type of the device.

Formulation of A, B, C, D matrices for various network elements is described as:

## 2.3 DERIVATION OF TWO PORT PARAMETER

### 2.3.1. Shunt Capacitors

(A) Phase to neutral connection：

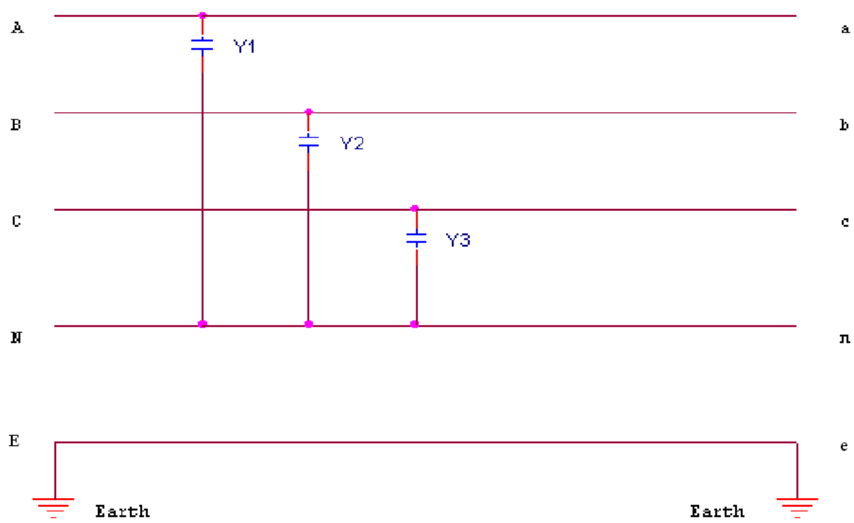


Fig 2.1 Shunt capacitors ( phase to neutral )

(i) Sending-end Currents:

$I_A = (V_{AE} - V_{NE})\, Y_1 + I_a$

$\quad = Y_1 V_{AE} - Y_1 V_{NE} + I_a$

$I_B = (V_{BE} - V_{NE})\, Y_2 + I_b$

$\quad = Y_2 V_{BE} - Y_2 V_{NE} + I_b$

$I_C = (V_{CE} - V_{NE})\, Y_3 + I_c$

$\quad = Y_3 V_{CE} - Y_3 V_{NE} + I_c$

$I_N = -Y_1 (V_{AE} - V_{NE}) - Y_2 (V_{BE} - V_{NE}) - Y_3 (V_{CE} - V_{NE}) + I_n$

$\quad = -Y_1 V_{AE} - Y_2 V_{BE} - Y_3 V_{CE} + (Y_1 + Y_2 + Y_3) V_{NE} + I_n$

Hence,

$$\begin{bmatrix} I_A \\ I_B \\ I_C \\ I_N \end{bmatrix} = \begin{bmatrix} Y_1 & 0 & 0 & -Y_1 \\ 0 & Y_2 & 0 & -Y_2 \\ 0 & 0 & Y_3 & -Y_3 \\ -Y_1 & -Y_2 & -Y_3 & (Y_1+Y_2+Y_3) \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \qquad 2.2$$

(ii) Receiving-end Voltages:

$V_{ae} = V_{AE}$

$V_{be} = V_{BE}$

$V_{ce} = V_{CE}$

$V_{ne} = V_{NE}$

Hence,

$$\begin{bmatrix} V_{ae} \\ V_{be} \\ V_{ce} \\ V_{ne} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \qquad 2.3$$

Comparing equations (2.2) and (2.3) with equation (2.1)

$$\mathbf{A} = \begin{pmatrix} Y_1 & 0 & 0 & -Y_1 \\ 0 & Y_2 & 0 & -Y_2 \\ 0 & 0 & Y_3 & -Y_3 \\ -Y_1 & -Y_2 & -Y_3 & Y_1 + Y_2 + Y_3 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = U$$

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = U$$

$$\mathbf{D} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = 0$$

(B). Phase to phase connection:

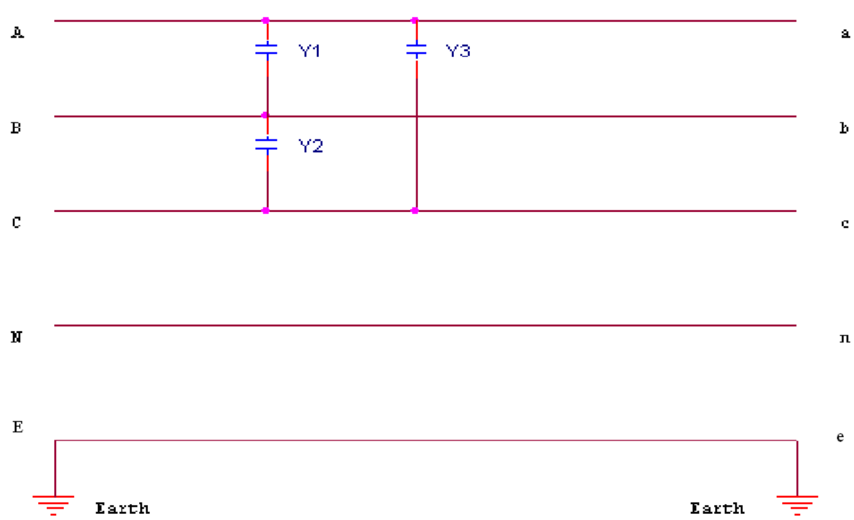


Fig 2.2 Shunt capacitors ( phase to phase)

(i) Sending-end Currents:

$$I_A = Y_1 (V_{AE} - V_{BE}) + Y_3 (V_{AE} - V_{CE}) + I_a$$

$$= (Y_1 + Y_3) V_{AE} - Y_1 V_{BE} - Y_3 V_{CE} + I_a$$

$I_B = Y_2 (V_{BE} - V_{CE}) + Y_1(V_{BE} - V_{AE}) + I_b$

$= -Y_1 V_{AE} + (Y_1 + Y_2) V_{BE} - Y_2 V_{CE} + I_b$

$I_C = Y_3 (V_{CE} - V_{AE}) + Y_2 (V_{CE} - V_{BE}) + I_c$

$= -Y_3 V_{AE} - Y_2 V_{BE} + (Y_2 + Y_3) V_{CE} + I_c$

Hence,

$$\begin{bmatrix} I_A \\ I_B \\ I_C \\ I_N \end{bmatrix} = \begin{bmatrix} Y_3 + Y_1 & -Y_1 & -Y_3 & 0 \\ -Y_1 & Y_1 + Y_2 & -Y_2 & 0 \\ -Y_3 & -Y_2 & Y_2 + Y_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \qquad 2.4$$

(ii) Receiving-end Voltages:

$V_{ae} = V_{AE}$

$V_{be} = V_{BE}$

$V_{ce} = V_{CE}$

$V_{ne} = V_{NE}$

Hence,

$$\begin{bmatrix} V_{ae} \\ V_{be} \\ V_{ce} \\ V_{ne} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \qquad 2.5$$

Comparing equations (2.4) and (2.5) with equation (2.1)

$$\mathbf{A} = \begin{pmatrix} Y_1 + Y_3 & -Y_1 & -Y_3 & 0 \\ -Y_1 & Y_1 + Y_2 & -Y_2 & 0 \\ -Y_3 & -Y_2 & Y_2 + Y_3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

**B = U**

**C = U**

**D = 0**

(C) Ungrounded Wye Connection:

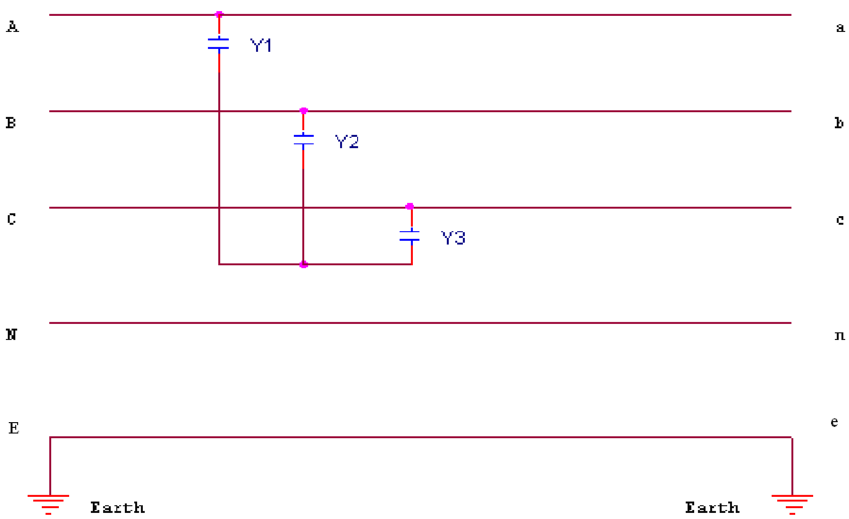


Fig 2.3 Shunt capacitors (ungrounded wye)

(i) Sending-end Currents:

Let the ungrounded star point be x.

$I_A = Y_1 V_{Ax} + I_a$

$= Y_1 V_{AE} - Y_1 V_{xE} + I_a$

$I_B = Y_2 V_{BE} - Y_2 V_{xE} + I_b$

$I_C = Y_3 V_{CE} - Y_3 V_{xE} + I_c$

$I_N = I_n$

Also, $Y_1 V_{Ax} + Y_2 V_{Bx} + Y_3 V_{Cx} = 0$

$Y_1 (V_{AE} - V_{xE}) + Y_2 (V_{BE} - V_{xE}) + Y3 (V_{CE} - V_{xE}) = 0$

$$V_{xE} = \frac{Y_1 V_{AE} + Y_2 V_{BE} + Y_3 V_{CE}}{Y_1 + Y_2 + Y_3}$$

$$I_A = \left(Y_1 - \frac{Y_1^2}{Y_T}\right) V_{AE} - \frac{Y_1 Y_2}{Y_T} V_{BE} - \frac{Y_1 Y_3}{Y_T} V_{CE} + I_a$$

$$I_B = -\frac{Y_1 Y_2}{Y_T} V_{AE} + \left(Y_2 - \frac{Y_2^2}{Y_T}\right) V_{BE} - \frac{Y_2 Y_3}{Y_T} V_{CE} + I_b$$

$$I_C = -\frac{Y_1 Y_3}{Y_T} V_{AE} - \frac{Y_2 Y_3}{Y_T} V_{BE} + \left( Y_3 - \frac{Y_3^2}{Y_T} \right) V_{CE} + I_c$$

where $Y_T = Y_1 + Y_2 + Y_3$

Hence,

$$\begin{bmatrix} I_A \\ I_B \\ I_C \\ I_N \end{bmatrix} = \begin{bmatrix} Y_1 - Y_1^2/Y_T & -Y_1 Y_2/Y_T & -Y_1 Y_3/Y_T & 0 \\ -Y_1 Y_2/Y_T & Y_2 - Y_2^2/Y_T & -Y_2 Y_3/Y_T & 0 \\ -Y_1 Y_3/Y_T & -Y_2 Y_3/Y_T & Y_3 - Y_3^2/Y_T & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \quad 2.6$$

(ii) Receiving-end Voltages:

$V_{ae} = V_{AE}$

$V_{be} = V_{BE}$

$V_{ce} = V_{CE}$

$V_{ne} = V_{NE}$

Hence,

$$\begin{bmatrix} V_{ae} \\ V_{be} \\ V_{ce} \\ V_{ne} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \quad 2.7$$

Comparing equations (2.6) and (2.7) with equation (2.1)

$$\mathbf{A} = \begin{pmatrix} Y_1 - (Y_1 * Y_1)/Y_T & -Y_1 * Y_2/Y_T & -Y_1 * Y_3/Y_T & 0 \\ -Y_1 * Y_2/Y_T & Y_2 - (Y_2 * Y_2)/Y_T & -Y_2 * Y_3/Y_T & 0 \\ -Y_1 * Y_3/Y_T & -Y_2 * Y_3/Y_T & Y_3 - (Y_3 * Y_3)/Y_T & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

**B = U**

**C = U**

**D = 0**

### 2.3.2 Shunt Reactors

$Z_1 = R_1 + j\,\omega\,L_1$ ; $Y_1 = 1/Z_1$

$Z_2 = R_2 + j\,\omega\,L_2$ ; $Y_2 = 1/Z_2$

$Z_3 = R_3 + j\,\omega\,L_3$ ; $Y_3 = 1/Z_3$

Where,

$\omega = 2\Pi\, f$

$R_x$ and $L_x$ are per phase shunt resistance and inductance and f is system frequency.

x= 1, 2, 3.

(A) Phase to neutral connection:

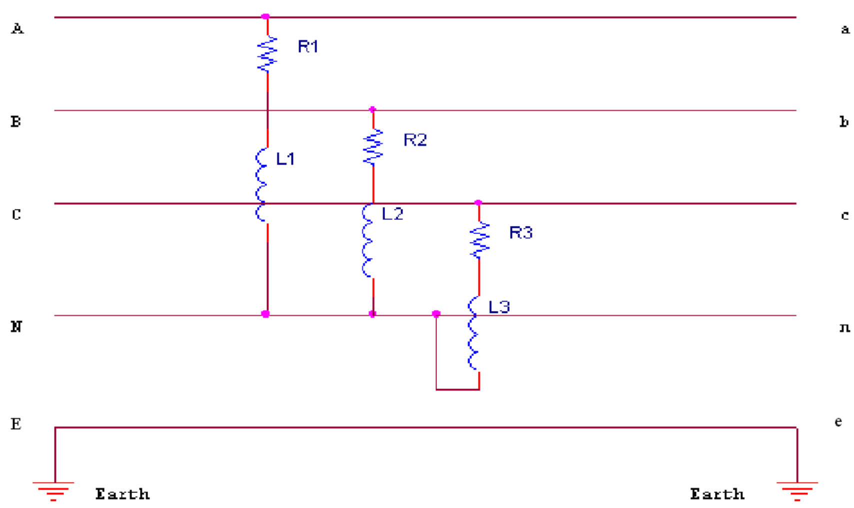


Fig 2.4 Shunt reactors (phase to neutral)

(i) Sending-end Currents:

$I_A = Y_1\,V_{AE} - Y_1\,V_{NE} + I_a$

$I_B = Y_2\,V_{BE} - Y_2\,V_{NE} + I_b$

$I_C = Y_3\,V_{CE} - Y_3\,V_{NE} + I_c$

$I_N = -\,Y_1\,(V_{AE} - V_{NE}) - Y_2\,(V_{BE} - V_{NE}) - Y_3\,(V_{CE} - V_{NE}) + I_n$

$\quad = -\,Y_1\,V_{AE} - Y_2\,V_{BE} - Y_3\,V_{CE} + (Y_1 + Y_2 + Y_3)\,V_{NE} + I_n$

Hence,

$$\begin{bmatrix} I_A \\ I_B \\ I_C \\ I_N \end{bmatrix} = \begin{bmatrix} Y_1 & 0 & 0 & -Y_1 \\ 0 & Y_2 & 0 & -Y_2 \\ 0 & 0 & Y_3 & -Y_3 \\ -Y_1 & -Y_2 & -Y_3 & (Y_1+Y_2+Y_3) \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \quad 2.8$$

(ii) Receiving-end Voltages:

$V_{ae} = V_{AE}$

$V_{be} = V_{BE}$

$V_{ce} = V_{CE}$

$V_{ne} = V_{NE}$

Hence,

$$\begin{bmatrix} V_{ae} \\ V_{be} \\ V_{ce} \\ V_{ne} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \quad 2.9$$

Comparing equations (2.8) and (2.9) with equation (2.1)

$$\mathbf{A} = \begin{pmatrix} Y_1 & 0 & 0 & -Y_1 \\ 0 & Y_2 & 0 & -Y_2 \\ 0 & 0 & Y_3 & -Y_3 \\ -Y_1 & -Y_2 & -Y_3 & Y_1+Y_2+Y_3 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = U$$

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = U$$

$$\mathbf{D} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = 0$$

(B). Phase to phase connection:

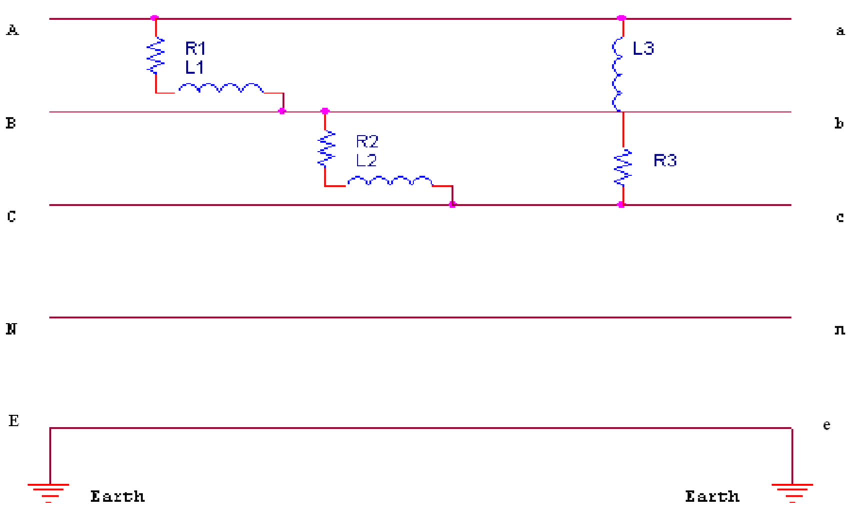


Fig 2.5 Shunt reactors (phase to phase)

(i) Sending-end Currents:

$I_A = Y_1 (V_{AE} - V_{BE}) + Y_3 (V_{AE} - V_{CE}) + I_a$

$\quad = (Y_1 + Y_3) V_{AE} - Y_1 V_{BE} - Y_3 V_{CE} + I_a$

$I_B = Y_2 (V_{BE} - V_{CE}) + Y_1(V_{BE} - V_{AE}) + I_b$

$\quad = -Y_1 V_{AE} + (Y_1 + Y_2) V_{BE} - Y_2 V_{CE} + I_b$

$I_C = Y_3 (V_{CE} - V_{AE}) + Y_2 (V_{CE} - V_{BE}) + I_c$

$= -Y_3 V_{AE} - Y_2 V_{BE} + (Y_2 + Y_3) V_{CE} + I_c$

Hence,

$$\begin{bmatrix} I_A \\ I_B \\ I_C \\ I_N \end{bmatrix} = \begin{bmatrix} Y_3 + Y_1 & -Y_1 & -Y_3 & 0 \\ -Y_1 & Y_1 + Y_2 & -Y_2 & 0 \\ -Y_3 & -Y_2 & Y_2 + Y_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \qquad 2.10$$

(ii) Receiving-end Voltages:

$V_{ae} = V_{AE}$

$V_{be} = V_{BE}$

$V_{ce} = V_{CE}$

$V_{ne} = V_{NE}$

Hence,

$$\begin{bmatrix} V_{ae} \\ V_{be} \\ V_{ce} \\ V_{ne} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \qquad 2.11$$

Comparing equations (2.10) and (2.11) with equation (2.1)

$$\mathbf{A} = \begin{pmatrix} Y_1 + Y_3 & -Y_1 & -Y_3 & 0 \\ -Y_1 & Y_1 + Y_2 & -Y_2 & 0 \\ -Y_3 & -Y_2 & Y_2 + Y_3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

**B = U**

**C = U**

**D = 0**

(C) Ungrounded Wye Connection:

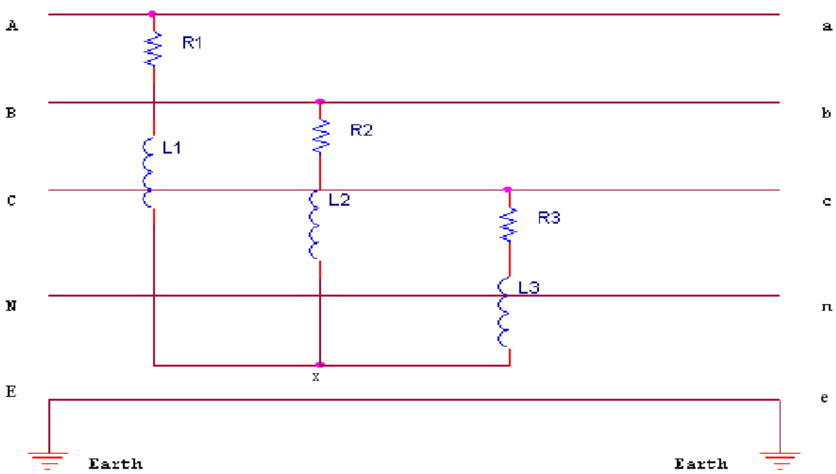


Fig 2.6 Shunt reactors (ungrounded wye)

(i) Sending-end Currents:

Let the ungrounded star point be x.

$I_A = Y_1 V_{Ax} + I_a$

$= Y_1 V_{AE} - Y_1 V_{xE} + I_a$

$I_B = Y_2 V_{BE} - Y_2 V_{xE} + I_b$

$I_C = Y_3 V_{CE} - Y_3 V_{xE} + I_c$

$I_N = I_n$

Also, $Y_1 V_{Ax} + Y_2 V_{Bx} + Y_3 V_{Cx} = 0$

$Y_1 (V_{AE} - V_{xE}) + Y_2 (V_{BE} - V_{xE}) + Y3 (V_{CE} - V_{xE}) = 0$

$$V_{xE} = \frac{Y_1 V_{AE} + Y_2 V_{BE} + Y_3 V_{CE}}{Y_1 + Y_2 + Y_3}$$

$$I_A = \left( Y_1 - \frac{Y_1^2}{Y_T} \right) V_{AE} - \frac{Y_1 Y_2}{Y_T} V_{BE} - \frac{Y_1 Y_3}{Y_T} V_{CE} + I_a$$

$$I_B = -\frac{Y_1Y_2}{Y_T}V_{AE} + \left(Y_2 - \frac{Y_2^2}{Y_T}\right)V_{BE} - \frac{Y_2Y_3}{Y_T}V_{CE} + I_b$$

$$I_C = -\frac{Y_1Y_3}{Y_T}V_{AE} - \frac{Y_2Y_3}{Y_T}V_{BE} + \left(Y_3 - \frac{Y_3^2}{Y_T}\right)V_{CE} + I_c$$

where $Y_T = Y_1 + Y_2 + Y_3$

Hence,

$$\begin{bmatrix} I_A \\ I_B \\ I_C \\ I_N \end{bmatrix} = \begin{bmatrix} Y_1 - Y_1^2/Y_T & -Y_1Y_2/Y_T & -Y_1Y_3/Y_T & 0 \\ -Y_1Y_2/Y_T & Y_2 - Y_2^2/Y_T & -Y_2Y_3/Y_T & 0 \\ -Y_1Y_3/Y_T & -Y_2Y_3/Y_T & Y_3 - Y_3^2/Y_T & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \quad 2.12$$

(ii) Receiving-end Voltages:

$V_{ae} = V_{AE}$

$V_{be} = V_{BE}$

$V_{ce} = V_{CE}$

$V_{ne} = V_{NE}$

Hence,

$$\begin{bmatrix} V_{ae} \\ V_{be} \\ V_{ce} \\ V_{ne} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \quad 2.13$$

Comparing equations (2.12) and (2.13) with equation (2.1)

$$\mathbf{A} = \begin{pmatrix} -Y_3\,(Y_1 * Y_1)/Y_T & -Y_1 * Y_2/Y_T & -Y_1 * Y_3/Y_T & 0 \\ -Y_1 * Y_2/Y_T & Y_2 - Y_3\,(Y_2 * Y_2)/Y_T & -Y_2 * Y_3/Y_T & 0 \\ -Y_1 * Y_3/Y_T & -Y_2 * Y_3/Y_T & Y_3 - Y_3\,(Y_3 * Y_3)/Y_T & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

**B = U**

**C = U**

**D = 0**

**2.3.3. Branch**

(i) Sending-end Currents:

$[I_S] = [Y]\, l\, [V_S] + [I_r]$

Hence, $A = l\, [Y]$

$B = U$

(ii) Receiving-end Voltages:

$[V_r] = [V_s] - [Z]\, l\, [I_r]$

Hence, $C = U$

$D = -l\, [Z]$

Where,

[Z] and [Y] are 4×4 impedance and admittance matrices of the line per unit length

$l$ is length of the branch.

**A =** $l$ [Y]

**B =** U

**C =** U

**D =** $-l$ [Z]

**Note:** For implementation purposes, in case of 3 wire systems, if on the load side of the branch there is a 3 phase transformer with its primary winding wye connected and neutral point grounded, the neutral voltage on the load side of the branch (Vne) is calculated separately. For this, a separate function is included that would calculate the correct value of Vne during voltage update (i.e., backward sweep) and overwrite the value of Vne calculated using A, B, C, D matrices with this correct value.

$Vne = Z_N (I_a + I_b + I_c)$

Where, $Z_N$ is the transformer primary neutral point grounding impedance and $I_a$, $I_b$, $I_c$ are the line currents on the load side of the branch which are known during the backward sweep. If the neutral is not grounded then the neutral to earth voltage is simply equal to the vector sum of the three line to earth voltages.

### 2.3.4. Series reactor:

$\bar{Z}_{ii} = R_i + \mathbf{j}\omega L_i$

$\bar{Z}_{ij} = 0$

Hence, $[\bar{Z}]$ is formulated similar to that of a line.
Using Kron's reduction, [Z] is evaluated.
[Y] = 0
Finally, A, B, C, D matrices for series reactor are evaluated similar to that of a branch.

**Implementation detail**: For ease of implementation, $\bar{Z}_{ii}$, $\bar{Z}_{ij}$, [Z], [Y] are all assumed to be per unit length and while calculating the A,B,C,D matrices *l* is taken to be 1 unit.

### 2.3.5. Distribution line

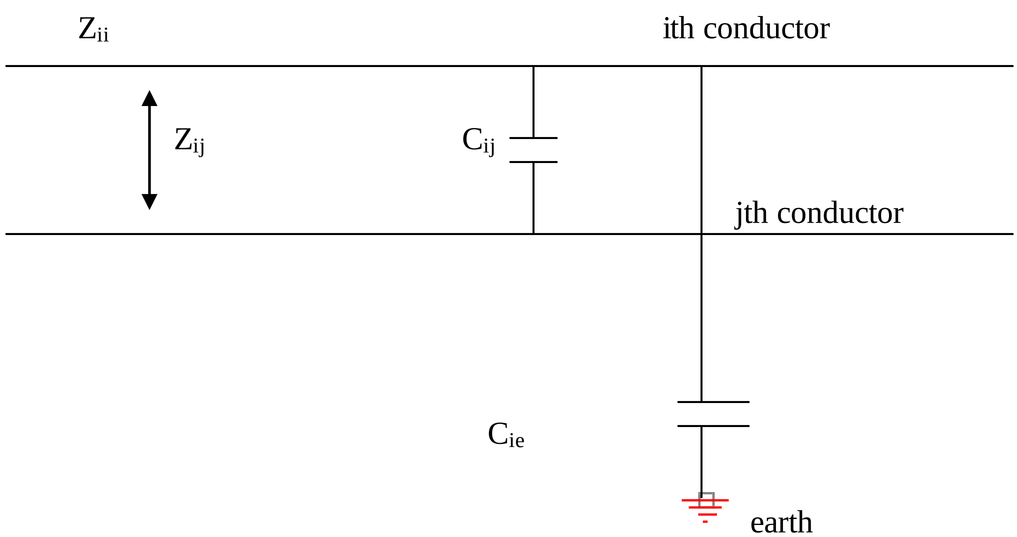


Fig 2.7 Distribution line model

Series impedance and shunt admittance matrices of distribution lines

Expressions for self and mutual impedance (for conductor lines and single core cable lines) and potential coefficients (in case of conductor lines only) :

$\bar{Z}_{ii}$ = $R_i$ + 0.00098696f + **j**0.00125664f [−ln (GM $R_i$) + 6.4905 + 0.5ln (ρ/f)] Ω/km

$\bar{Z}_{ij}$ = 0.00098696f + **j**0.00125664f [−ln ($D_{ij}$) + 6.4905 + 0.5ln (ρ/f)] Ω/km

$\bar{P}_{ii}$ = 17.98746 ln ($S_{ii}$ /$RD_i$ ) km/μF

$\bar{P}_{ij}$ = 17.98746 ln ($S_{ij}$ / $D_{ij}$) km/μF

Where

$\bar{Z}_{ii}$ : Self-impedance of ith conductor per unit length (Ω/km)

$\bar{Z}_{ij}$ : Mutual impedance between conductors i and j per unit length (Ω/km)

$\bar{P}_{ii}$ : Self-potential coefficient of ith conductor per unit length (km/μF)

$\bar{P}_{ij}$ : Mutual potential coefficient between ith and jth conductor (km/μF)

$S_{ii}$: distance (m) from conductor i to its image = 2 x height of ith conductor above ground (m) = 2 x y coordinate (m) of the ith conductor, origin being on the surface of the earth
$S_{ij}$: distance from ith conductor to the image of jth conductor = $\sqrt{(x_i - x_j)^2 + (y_i + y_j)^2}$ where $x_i$ and $x_j$ are the x coordinates (m) of the ith and jth conductors and $y_i$ and $y_j$ are the y coordinates (m) of the ith and jth conductor, origin being on the surface of the earth
f : frequency (Hz)
$R_i$: resistance of the ith conductor (Ω/km)
ρ : resistivity of earth(Ω-m)
$GMR_i$ : Geometric mean radius of ith conductor (m)
$RD_i$: radius of ith conductor (m)
$D_{ij}$: distance between conductors i and j (m) = $\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$

GMR calculations:-

GMR = 0.5×D×Kg
D = overall conductor diameter (m)
Kg = layer factor

Table containing values of Kg for ACSR conductors for 4 common stranding arrangements:

| **ACSR** | **Kg** |
|---|---|
| 6/1 | 0.7160 |
| 30/7 | 0.8250 |
| 54/7 | 0.8099 |

Table containing values of Kg for AAC conductors for 4 common stranding arrangements:

| **AAC** | **Kg** |
|---|---|
| 7 | 0.7256 |
| 19 | 0.7577 |
| 37 | 0.7678 |
| 61 | 0.7722 |

The distances ($GMR_i$ and $D_{ij}$) necessary for calculating self and mutual impedance and admittance of lines are summarized as follows:

A. Conductor lines :
   1. GMR of each conductor (phase, neutral and ground)
   2. $D_{ij}$ for each pair of conductors where $D_{ij}$ = center to center distance between the two conductors.

B. Concentric neutral Cable lines :
   1. GMR of each phase conductor and external neutral conductor (if any).
   2. GMR of each concentric neutral = $[GMR_s \times k \times R^{k-1}]^{1/k}$
      Where
      GMRs = GMR of one neutral strand
      R = 0.5× (total diameter of the cable – diameter of one neutral strand)
      k = no. of neutral strands per cable
   3. $D_{ij}$ (i, j: phase conductors or external neutral) = distance between centers of the corresponding conductors
   4. $D_{ij}$ (i, j: concentric neutral) = distance between centers of the corresponding conductors
   5. $D_{ij}$ (i, j : phase conductor and corresponding concentric neutral) = R
   6. $D_{ij}$ (i, j: phase conductor of one cable and concentric neutral of another cable) = $[Dnm^k - R^k]^{1/k}$ where Dnm = distance between centers of the phase conductors of the respective cables

C. Tape shielded cables :
   1. GMR of each phase conductor and neutral conductor (if any).
   2. GMR (tape shield) = 0.5× (outside diameter of tape shield – thickness of the tape shield)
   3. $D_{ij}$ (i, j: phase conductors or neutral) = distance between centers of the corresponding conductors

4. $D_{ij}$ (i, j: tape shield) = distance between centers of the corresponding conductors
5. $D_{ij}$ (i, j : phase conductor and corresponding tape shield) = GMR of the shield
6. $D_{ij}$ (i, j: phase conductor of one cable and tape shield of another cable) = distance between centers of the phase conductors of the respective cables

Formulation of impedance matrix [Z] (for conductor lines and single core cable lines) from primitive impedance matrix [Z]

$$[\bar{Z}] = \begin{pmatrix} [\bar{Z}_{ij}] & [\bar{Z}_{in}] \\ [\bar{Z}_{nj}] & [\bar{Z}_{nn}] \end{pmatrix}$$

Kron's reduction: $[Z] = [\bar{Z}_{ij}] - [\bar{Z}_{in}][\bar{Z}_{nn}]^{-1}[\bar{Z}_{nj}]$

Where
n: set of multi-grounded conductors
i, j: set of phase conductors and conductor(s) that is (are) not multi-grounded

Formulation of admittance matrix [Y] (for conductor lines only) from primitive potential coefficient matrix [P]

$$[\bar{P}] = \begin{pmatrix} [\bar{P}_{ij}] & [\bar{P}_{in}] \\ [\bar{P}_{nj}] & [\bar{P}_{nn}] \end{pmatrix}$$

Kron's reduction: $[P] = [\bar{P}_{ij}] - [\bar{P}_{in}][\bar{P}_{nn}]^{-1}[\bar{P}_{nj}]$

$[C] = [P]^{-1}$

$[Y] = j\omega\,[C]$

where
n : set of multi-grounded conductors
i,j : set of phase conductors and conductor(s) that is (are) not multi-grounded

Calculation of shunt admittance of single core cable lines [Ref 14]:

1. Concentric**:** $Y_{pg} = \mathbf{j}\ 40.0587 / [\ln (R_b / RD_c) - (1/k) \ln (k \times RD_s / R_b)]$
   where $Y_{pg}$ = shunt admittance between one phase and earth (µS/km)

$R_b$ = 0.5 × (overall diameter – diameter of one neutral strand)
k = no. of neutral strands per cable
$RD_c$ = radius of phase conductor
$RD_s$ = radius of one neutral strand

2. Tape shielded cable**:** $Y_{pg}$ = **j** 40.0587 / [ln ($R_b$ / $RD_c$)] μS/km

   In both cases, ($Y_{ii}$ = 0 for single core cable)

   **Note**: Calculation of impedance and admittance of 3 phase 3 core, 3[1]/2 core and 4 core cables is done using the sequence impedance and admittance values as follows:

   $Z_s = 1/3\ (Z_0 + 2Z_+)$
   $Z_m = 1/3\ (Z_0 - Z_+)$
   where Zs = self impedance and Zm = mutual impedance

   $Y_s = 1/3\ (Y_0 + 2Y_+)$
   $Y_m = 1/3\ (Y_0 - Y_+)$

Where $Y_s$ = self admittance and $Y_m$ = mutual admittance

### 2.3.6. Single phase transformer

## (A) Phase to phase (between phases A and B):

(i) Sending-end Currents:

$I_A = Y\,(V_{AE} - V_{BE}) + I_a/n_t$

$I_B = -I_A = -Y\,V_{AE} + YV_{BE} - I_a/n_t$

$I_C = I_N = 0$

Hence,

$$\begin{bmatrix} I_A \\ I_B \\ I_C \\ I_N \end{bmatrix} = \begin{bmatrix} Y & -Y & 0 & 0 \\ -Y & Y & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + \begin{bmatrix} 1/n_t & 0 & 0 & 0 \\ -1/n_t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \qquad 2.14$$

(ii) Receiving-end Voltages:

$V_{ae} = V_{an} + V_{ne}$

$= V_{an}$ (assuming solidly grounded secondary neutral)

$= \dfrac{1}{n_t} V_{AB} - Zt_a\, I_a$

$$= \frac{1}{n_t}V_{AE} - \frac{1}{n_t}V_{BE} - Zt_a I_a$$

$V_{ne} = 0$

$V_{be} = 0$

$V_{ce} = 0$

Where Y = Shunt branch admittance referred to primary side.

$Z_{ta}$ = Series impedance of transformer referred to secondary side.

$n_t$ = turns ratio

Hence,

$$\begin{bmatrix} V_{ae} \\ V_{be} \\ V_{ce} \\ V_{ne} \end{bmatrix} = \begin{bmatrix} \frac{1}{n_t} & \frac{-1}{n_t} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + \begin{bmatrix} -Z_{ta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \qquad 2.15$$

Comparing equations (2.14) and (2.15) with equation (2.1)

$$\mathbf{A} = \begin{pmatrix} Y & -Y & 0 & 0 \\ -Y & Y & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 1/n_t & 0 & 0 & 0 \\ -1/n_t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 1/n_t & -1/n_t & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{D} = \begin{pmatrix} -Z_{ta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(B) Phase to neutral (between phase A and neural N):

(i) Sending-end Currents:

$I_A = Y\,(V_{AE} - V_{NE}) + I_a/n_t$

$I_B = 0$

$I_C = 0$

$I_N = -I_A = -Y\,V_{AE} + Y\,V_{NE} - I_a/n_t$

Hence,

$$\begin{bmatrix} I_A \\ I_B \\ I_C \\ I_N \end{bmatrix} = \begin{bmatrix} Y & 0 & 0 & -Y \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -Y & 0 & 0 & Y \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + \begin{bmatrix} 1/n_t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1/n_t & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \qquad 2.16$$

(ii) Receiving-end Voltages:

$V_{ne} = 0$ (assuming the secondary neutral to be solidly grounded)

$V_{ae} = V_{an}$

$$= \frac{V_{AN}}{n_t} - Zt_a I_a$$

$$= \frac{1}{n_t} V_{AE} - \frac{1}{nt} V_{NE} - Zt_a I_a$$

$V_{be} = V_{ce} = 0$

where Y = Shunt branch admittance referred to primary side.

Zta = Series impedance of transformer referred to

secondary side.

$n_t$ = winding ratio.

Hence,

$$\begin{bmatrix} V_{ae} \\ V_{be} \\ V_{ce} \\ V_{ne} \end{bmatrix} = \begin{bmatrix} \frac{1}{n_t} & 0 & 0 & -\frac{1}{n_t} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + \begin{bmatrix} -Z_{ta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \qquad 2.17$$

Comparing equations (2.16) and (2.17) with equation (2.1)

$$\mathbf{A} = \begin{pmatrix} Y & 0 & 0 & -Y \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -Y & 0 & 0 & Y \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 1/n_t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1/n_t & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 1/n_t & 0 & 0 & -1/n_t \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{D} = \begin{pmatrix} -Z_{ta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

### 2.3.7. Three phase transformer:

(A) Delta−Star (gnd) :

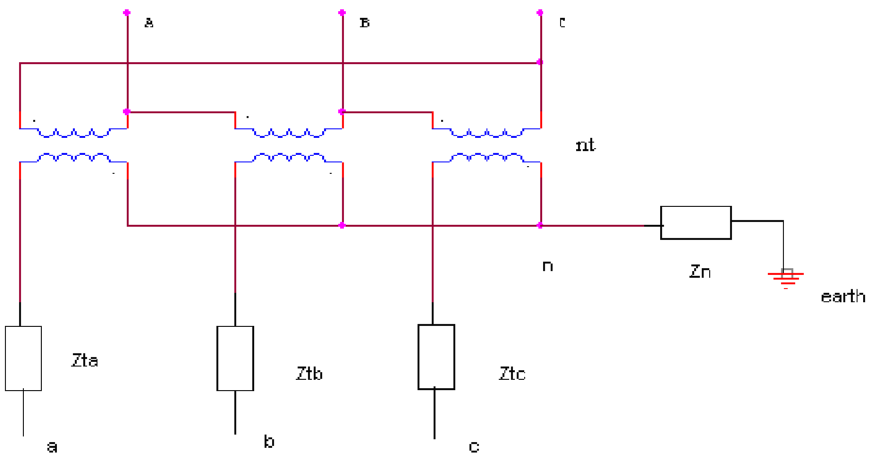


Fig 2.8 delta- star grounded connection

(i) Sending-endcurrents:

$I_A = Y_3 (V_{AE} - V_{CE}) + Y_1 (V_{BE} - V_{AE}) + (I_a - I_b)/ n_t$

$\quad = (Y_1 + Y_3) V_{AE} - Y_1 V_{BE} - Y_3 V_{CE} + I_a / n_t - I_b / n_t$

$I_B = Y_1 (V_{BE} - V_{AE}) + Y_2(V_{CE} - V_{BE}) + (I_b - I_c)/ n_t$

$\quad = -Y_1 V_{AE} + (Y_1 + Y_2) V_{BE} - Y_2 V_{CE} + I_b / n_t - I_c / n_t$

$I_C = Y_2 (V_{CE} - V_{BE}) + Y_3 (V_{AE} - V_{CE}) + (I_c - I_a)/ n_t$

$\quad = -Y_3 V_{AE} - Y_2 V_{BE} + (Y_2 + Y_3) V_{CE} + I_c / n_t - I_a / n_t$

$I_N=0$

Hence,

$$\begin{bmatrix} I_A \\ I_B \\ I_C \\ I_N \end{bmatrix} = \begin{bmatrix} Y_3+Y_1 & -Y_1 & -Y_3 & 0 \\ -Y_1 & Y_1+Y_2 & -Y_2 & 0 \\ -Y_3 & -Y_2 & Y_2+Y_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + 1/n_t \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \qquad 2.18$$

(iii) Receiving−end voltages:

$V_{ae} = -Z_{ta} I_a + V_{ne} + (V_{AE} - V_{CE})/ n_t$

$V_{be} = -Z_{tb} I_b + V_{ne} + (V_{BE} - V_{AE})/ n_t$

$V_{ce} = -Z_{tc} I_c + V_{ne} + (V_{CE} - V_{BE})/ n_t$

$V_{ne} = -(I_a + I_b + I_c + I_n) Z_n$

Hence,

$$\begin{bmatrix} V_{ae} \\ V_{be} \\ V_{ce} \\ V_{ne} \end{bmatrix} = 1/n_t \begin{bmatrix} 1 & 0 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + \begin{bmatrix} -(Z_{ta}+Z_n) & -Z_n & -Z_n & -Z_n \\ -Z_n & -(Z_{tb}+Z_n) & -Z_n & -Z_n \\ -Z_n & -Z_n & -(Z_{tc}+Z_n) & -Z_n \\ -Z_n & -Z_n & -Z_n & -Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix}$$

__________________2.19

Comparing equations (2.18) and (2.19) with equation (2.1)

$$\mathbf{A} = \begin{pmatrix} Y_1+Y_3 & -Y_1 & -Y_3 & 0 \\ -Y_1 & Y1+Y_2 & -Y_2 & 0 \\ -Y_1 & -Y_2 & Y_2+Y_3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{B} = 1/n_t \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{C} = 1/n_t \begin{pmatrix} 1 & 0 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{D} = \begin{bmatrix} -(Z_{ta}+Z_n) & -Z_n & -Z_n & -Z_n \\ -Z_n & -(Z_{tb}+Z_n) & -Z_n & -Z_n \\ -Z_n & -Z_n & -(Z_{tc}+Z_n) & -Z_n \\ -Z_n & -Z_n & -Z_n & -Z_n \end{bmatrix}$$

(B) Star (ungnd)–Star (gnd) :

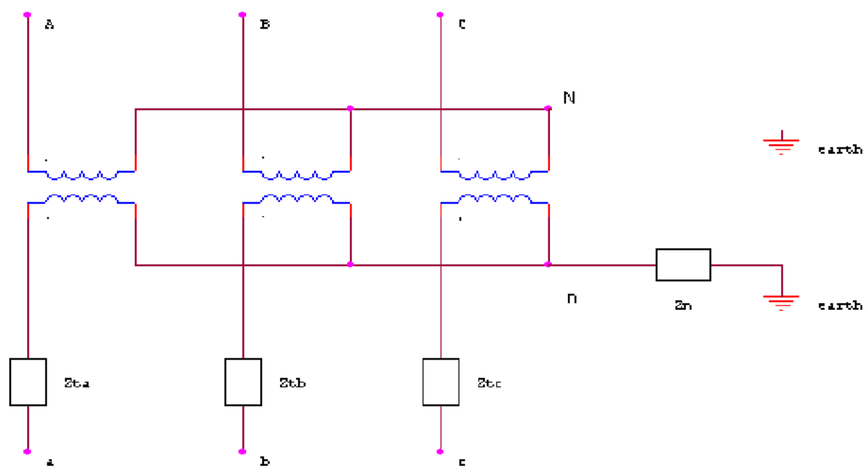


Fig 2.9 Srar ungrounded - Star grounded connection

(i) Sending–endcurrents :

$I_A = Y_1 V_{AE} - Y_1 V_{NE} + I_a / n_t$

$I_B = Y_2 V_{BE} - Y_2 V_{NE} + I_b / n_t$

$I_C = Y_2 V_{CE} - Y_3 V_{NE} + I_c / n_t$

$I_N = -(I_A + I_B + I_C)$

$$= -\left(Y_1 V_{AN} + \frac{I_a}{n_t}\right) - \left(Y_2 V_{BN} + \frac{I_b}{n_t}\right) - \left(Y_3 V_{CN} + \frac{I_c}{n_t}\right)$$

$$= -Y_1 V_{AE} - Y_2 V_{BE} - Y_3 V_{CE} + (Y_1 + Y_2 + Y_3) V_{NE} - \frac{I_a}{n_t} - \frac{I_b}{n_t} - \frac{I_c}{n_t}$$

Hence,

$$\begin{bmatrix} I_A \\ I_B \\ I_C \\ I_N \end{bmatrix} = \begin{bmatrix} Y_1 & 0 & 0 & -Y_1 \\ 0 & Y_2 & 0 & -Y_2 \\ 0 & 0 & Y_3 & -Y_3 \\ -Y_1 & -Y_2 & -Y_3 & Y_1+Y_2+Y_3 \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + 1/n_t \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \qquad 2.20$$

(ii) Receiving−endvoltages :

$V_{ae} = -Z_{ta} I_a + V_{ne} + (V_{AE} - V_{NE})/ n_t$

$V_{be} = -Z_{tb} I_b + V_{ne} + (V_{BE} - V_{NE})/ n_t$

$V_{ce} = -Z_{tc} I_c + V_{ne} + (V_{CE} - V_{NE})/ n_t$

$V_{ne} = -(I_a + I_b + I_c + I_n) Z_n$

Hence,

$$\begin{bmatrix} V_{ae} \\ V_{be} \\ V_{ce} \\ V_{ne} \end{bmatrix} = 1/n_t \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + \begin{bmatrix} -(Z_{ta}+Z_n) & -Z_n & -Z_n & -Z_n \\ -Z_n & -(Z_{tb}+Z_n) & -Z_n & -Z_n \\ -Z_n & -Z_n & -(Z_{tc}+Z_n) & -Z_n \\ -Z_n & -Z_n & -Z_n & -Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \qquad 2.21$$

Comparing equations (2.20) and (2.21) with equation (2.1)

$$\mathbf{A} = \begin{pmatrix} Y_1 & 0 & 0 & -Y_1 \\ 0 & Y_2 & 0 & -Y_2 \\ 0 & 0 & Y_3 & -Y_3 \\ -Y_1 & -Y_2 & -Y_3 & Y_1+Y_2+Y_3 \end{pmatrix}$$

$$\mathbf{B} = 1/n_t \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & 0 \end{pmatrix}$$

$$\mathbf{C} = 1/n_t \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{D} = \begin{pmatrix} -(Z_{ta} + Z_n) & -Z_n & -Z_n & -Z_n \\ -Z_n & -(Z_{tb} + Z_n) & -Z_n & -Z_n \\ -Z_n & -Z_n & -(Z_{tc} + Z_n) & -Z_n \\ -Z_n & -Z_n & -Z_n & -Z_n \end{pmatrix}$$

(C) Star (gnd) – Star (gnd) :

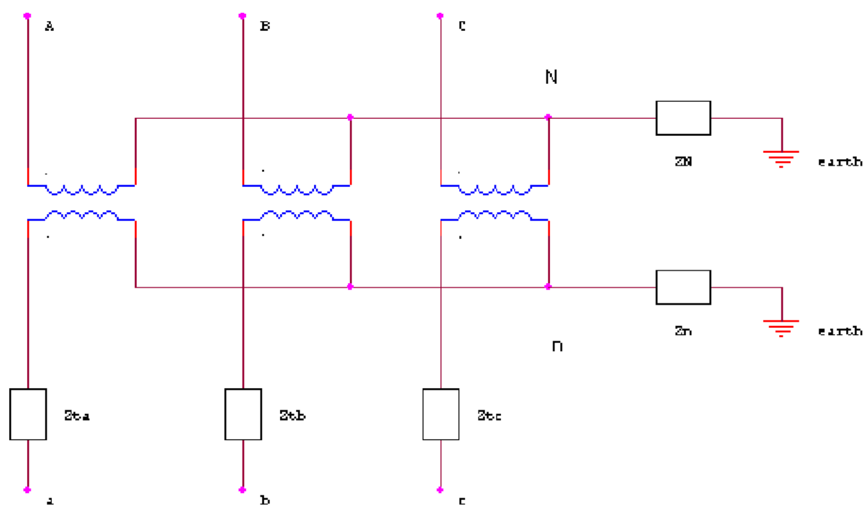


Fig 2.10 Star grounded - Star grounded connection

(i) Sending−endcurrents :

$I_A = Y_1 V_{AE} - Y_1 V_{NE} + I_a / n_t$

$I_B = Y_2 V_{BE} - Y_2 V_{NE} + I_b / n_t$

$I_C = Y_2 V_{CE} - Y_3 V_{NE} + I_c / n_t$

$I_N = V_{NE} / Z_N - (I_A + I_B + I_C)$

$$= V_{NE}/Z_N - \left(Y_1 V_{AN} + \frac{I_a}{n_t}\right) - \left(Y_2 V_{BN} + \frac{I_b}{n_t}\right) - \left(Y_3 V_{CN} + \frac{Ic}{n_t}\right)$$

$$= -Y_1 V_{AE} - Y_2 V_{BE} - Y_3 V_{CE} + (Y_1 + Y_2 + Y_3 + 1/Z_N) V_{NE} - \frac{I_a}{n_t} - \frac{I_b}{n_t} - \frac{I_c}{n_t}$$

Hence,

$$\begin{bmatrix} I_A \\ I_B \\ I_C \\ I_N \end{bmatrix} = \begin{bmatrix} Y_1 & 0 & 0 & -Y_1 \\ 0 & Y_2 & 0 & -Y_2 \\ 0 & 0 & Y_3 & -Y_3 \\ -Y_1 & -Y_2 & -Y_3 & Y_1 + Y_2 + Y_3 + 1/Z_N \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + 1/n_t \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix}$$

______________________________ 2.22

(ii) Receiving−endvoltages :

$V_{ae} = -Z_{ta} I_a + V_{ne} + (V_{AE} - V_{NE})/n_t$

$V_{be} = -Z_{tb} I_b + V_{ne} + (V_{BE} - V_{NE})/n_t$

$V_{ce} = -Z_{tc} I_c + V_{ne} + (V_{CE} - V_{NE})/n_t$

$V_{ne} = -(I_a + I_b + I_c + I_n) Z_n$

Hence,

$$\begin{bmatrix} V_{ae} \\ V_{be} \\ V_{ce} \\ V_{ne} \end{bmatrix} = 1/n_t \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + \begin{bmatrix} -(Z_{ta} + Z_n) & -Z_n & -Z_n & -Z_n \\ -Z_n & -(Z_{tb} + Z_n) & -Z_n & -Z_n \\ -Z_n & -Z_n & -(Z_{tc} + Z_n) & -Z_n \\ -Z_n & -Z_n & -Z_n & -Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix}$$

______________________________ 2.23

Comparing equations (2.22) and (2.23) with equation (2.1)

$$\mathbf{A} = \begin{pmatrix} Y_1 & 0 & 0 & -Y_1 \\ 0 & Y_2 & 0 & -Y_2 \\ 0 & 0 & Y_3 & -Y_3 \\ -Y_1 & -Y_2 & -Y_3 & Y_1 + Y_2 + Y_3 + 1/Z_N \end{pmatrix}$$

$$\mathbf{B} = 1/n_t \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & 0 \end{pmatrix}$$

$$\mathbf{C} = 1/n_t \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{D} = \begin{pmatrix} -(Z_{ta} + Z_n) & -Z_n & -Z_n & -Z_n \\ -Z_n & -(Z_{tb} + Z_n) & -Z_n & -Z_n \\ -Z_n & -Z_n & -(Z_{tc} + Z_n) & -Z_n \\ -Z_n & -Z_n & -Z_n & -Z_n \end{pmatrix}$$

**2.3.8. Three phase regulator:**

(A) Star connected regulator:

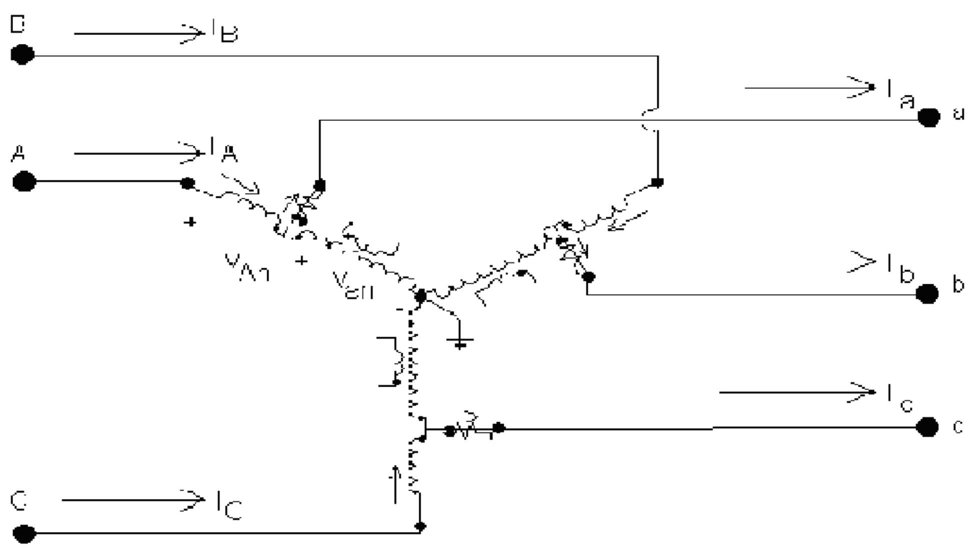


Fig 2.11 Star connected three phase voltage regulator

(i) Sending−endcurrent :

$I_A = Y_1 V_{AE} - Y_1 V_{NE} + I_a / a_{Ra}$

$I_B = Y_2 V_{BE} - Y_2 V_{NE} + I_b / a_{Rb}$

$I_C = Y_2 V_{CE} - Y_3 V_{NE} + I_c / a_{Rc}$

$$I_N = \frac{V_{NE}}{Z_N} - (I_A + I_B + I_C) + I_a + I_b + I_c$$

$$= \frac{V_{NE}}{Z_N} - \left(Y_1 V_{AN} + \frac{I_a}{aR_a}\right) - \left(Y_2 V_{BN} + \frac{I_b}{aR_b}\right) - \left(Y_3 V_{CN} + \frac{I_c}{a_{Rc}}\right) + I_a + I_b + I_c$$

$$= -Y_1 V_{AE} - Y_2 V_{BE} - Y_3 V_{CE} + \left(Y_1 + Y_2 + Y_3 + \frac{1}{Z_N}\right) V_{NE} + \left(1 - \frac{1}{a_{Ra}}\right) I_a + \left(1 - \frac{1}{a_{Rb}}\right) I_b + \left(1 - \frac{1}{a_{Rc}}\right) I_c$$

Hence,

$$\begin{bmatrix} I_A \\ I_B \\ I_C \\ I_N \end{bmatrix} = \begin{bmatrix} Y_1 & 0 & 0 & -Y_1 \\ 0 & Y_2 & 0 & -Y_2 \\ 0 & 0 & Y_3 & -Y_3 \\ -Y_1 & -Y_2 & -Y_3 & Y_1 + Y_2 + Y_3 + 1/Z_N \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + \begin{bmatrix} 1/a_{Ra} & 0 & 0 & 0 \\ 0 & 1/a_{Rb} & 0 & 0 \\ 0 & 0 & 1/a_{Rc} & 0 \\ 1-1/a_{Ra} & 1-1/a_{Rb} & 1-1/a_{Rc} & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix}$$

______________________________2.24

(ii) Receiving−endvoltages :

$V_{ae} = V_{ne} + (V_{AE} - V_{NE})/ a_{Ra}$

$V_{be} = V_{ne} + (V_{BE} - V_{NE})/ a_{Rb}$

$V_{ce} = V_{ne} + (V_{CE} - V_{NE})/ a_{Rc}$

$V_{ne} = V_{NE}$

Hence,

$$\begin{bmatrix} V_{ae} \\ V_{be} \\ V_{ce} \\ V_{ne} \end{bmatrix} = \begin{bmatrix} 1/a_{Ra} & 0 & 0 & 1-1/a_{Ra} \\ 0 & 1/a_{Rb} & 0 & 1-1/a_{Rb} \\ 0 & 0 & 1/a_{Rc} & 1-1/a_{Rc} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{AE} \\ V_{BE} \\ V_{CE} \\ V_{NE} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \qquad 2.25$$

Where,

$a_{Ra}$, $a_{Rb}$, $a_{Rc}$ are effective turns ratio for single phase regulator.

Comparing equations (2.24) and (2.25) with equation (2.1)

$$\mathbf{A} = \begin{pmatrix} Y1 & 0 & 0 & -Y1 \\ 0 & Y2 & 0 & -Y2 \\ 0 & 0 & Y3 & -Y3 \\ -Y1 & -Y2 & -Y3 & Y1+Y2+Y3+1/Z_N \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 1/aRa & 0 & 0 & 0 \\ 0 & 1/aRb & 0 & 0 \\ 0 & 0 & 1/aRc & 0 \\ 1-1/aRa & 1-1/aRb & 1-1/aRc & 1 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 1/aRa & 0 & 0 & 1-1/aRa \\ 0 & 1/aRb & 0 & 1-1/aRb \\ 0 & 0 & 1/aRc & 1-1/aRc \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{D} = \mathbf{0}$$

# CHAPTER −3

## DISTRIBUTION SYSTEM LOAD-FLOW

### 3.1 INTRODUCTION

Load-flow solution involves calculation of bus voltages, line currents, line flows and losses provided full network parameters, complex loads and one bus voltage as reference at steady state condition is known. Classical load-flow algorithms like Gauss Seidel, Newton Raphson and fast decoupled load-flow are widely used especially for transmission systems. They can also be used, in general, for distribution systems. However, a distribution system is simpler in terms of configuration than a transmission system in the sense that the former is mostly a radial system as opposite to the latter which is in general highly meshed. Hence, simpler load-flow algorithms are employed in solving distribution system networks. However, these algorithms must calculate the node voltages for all the phases separately since, unlike a more or less balanced transmission system, a distribution system is significantly unbalanced.

Different techniques have been developed to solve load-flow for such distribution networks. Literature survey shows that the two most commonly used techniques for radial distribution load-flow are one based on a Newton like method involving formation of Jacobians and computation of power mismatches at the end of the feeder and laterals**,** and the other based on the backward and forward sweeps involving computation of branch current flows **.** It involves of backward update of power flows and losses and forward updates of voltages. In this method it is assumed that voltage at the sending end of the feeder and loads at the receiving end of the feeder branch section are known in advance.

Distribution system load-flow algorithm primarily involves calculation of node voltages and line currents given the substation transformer secondary side voltages, full network parameters and complex loads. In this study the network elements are modeled using generalized 2−port parameters.

The algorithm used here is applicable for radial distribution systems. It does not involve any matrix factorization or inversion for that matter and is hence computationally less costly. Since, the distribution systems commonly encountered are simple in terms of configuration, this algorithm converges very fast. Hence, it is efficient and quick and can be comfortably used in solving radial distribution systems. This load-flow algorithm is tested using PlanwoRx software.

## 3.2 LOAD-FLOWALGORITHM

The load-flow algorithm consists of the following steps:

Step 0: Initialize:

The A, B, C, D parameters are evaluated for each element of network. The branch currents are initialized to zero. Feeder primary voltage is initialized to the substation voltage.

Step 1a: Voltage Update: (Forward sweep)

Starting from the feeder, the network is traversed recursively to the load ends and the receiving end voltages of each device are evaluated using the C and D matrices of the device and the sending end voltage (out of most recent iteration voltage) and receiving end currents (currents of the previous iteration) of the device.

Step 1b: Load assignment:

Using the latest voltages and currents of previous iteration and depending upon the load type (constant impedance, constant current, constant power or a combination of two or more of these), the complex load power demand are evaluated and assigned to each load.

Step 2: Current Update: (Backward sweep)

The network is traversed from the load end up to the source end by updating the sending end current and summing them up at the nodes having more than one lateral. The sending end currents are evaluated using the A and B matrices of the device, the sending end voltages (voltages of the previous iteration) and the receiving end currents (latest currents of the current iteration) of the device. The load currents are evaluated using the complex load powers assigned to them in the previous iteration and the load voltages of the previous iteration. A check is also made on the device currents so that if the magnitude of the device current of any phase is greater than a certain value (i.e., abnormally high), the program terminates printing a message that the load-flow has diverged.

Step 3: Convergence check:

The feeder current values of the previous iteration and the current iteration are compared. If the magnitude of the difference between them is less than a tolerance value, the load-flow is said to have converged. If the load-flow converges the next step (step 4) is carried out else the program loops to step 1. If the number of iterations exceeds a certain value, the program terminates.

Step 4: Line flow and loss calculation:

Starting from the feeder end, the network is traversed recursively to the load ends calculating the three phase sending end complex power flows and complex losses for each network element and the final assigned loads.

Step 5: Load-flow Results output:

Starting from the feeder end, the network is traversed recursively to the load ends, printing the complex device voltages, complex device currents, final loads assigned, three phase complex power flows and device losses.

## 3.3 ENERLTSER SOFTWARE

Enerlyser is an integrated software package for analyzing electrical distribution systems. It is an engineering analysis program, which allows the operator to perform a range of different analyses on models of electrical distribution systems.

**Salient-Feature of Enerlyser**

1 Integrated with an graphic electrical documentation and GIS software.

2. Data entry is easy and flexible

3. Results can be produced in tabular form, or exported for displaying graphically in excel for better understanding.

4. The GIS software provides for single line diagram display of the network preferably including schematic diagram of substation and access to any or all equipment and network information with zoom facility.

5. The software provides the list of all elements drawn in the drawing, which can be found out for the purpose of consolidating assets in the network. This list contains the details such as circuit km length of lines, number of transformers, etc. for different voltage levels, and for any part of the drawing. It also provides the HT, LT line length ratio of distribution feeders and other lines.

The load-flow analysis module is the principal analysis module in Enerlyser. The load-flow module helps to pinpoint problems in the existing system and determine the effects of additions and upgrades before implementing them in actual fieldwork. It gives the data needed to confidently carry out load-flow for identify improvements in the system. Given the system model, a load-flow analysis solves for the flows and voltages at all points in the system. On a typical system, those values are measured only at a handful no. of points (typically substations). Load-flow allows the operator to determine conditions throughout the system.

In Enerlyser, load-flow analysis is fast, versatile, and powerful. Traditional load-flow algorithms are not always suitable for the configuration, size or loading conditions found in distribution systems.

# CHAPTER −4

# RESULTS

## 4.1 IEEE 123 NODE DISTRIBUTION SYSTEM

The load-flow algorithm was tested on a IEEE−123 node distribution system drawn in Enerlyser. The layout of the network and load-flow results obtained from Enerlyser is also shown.

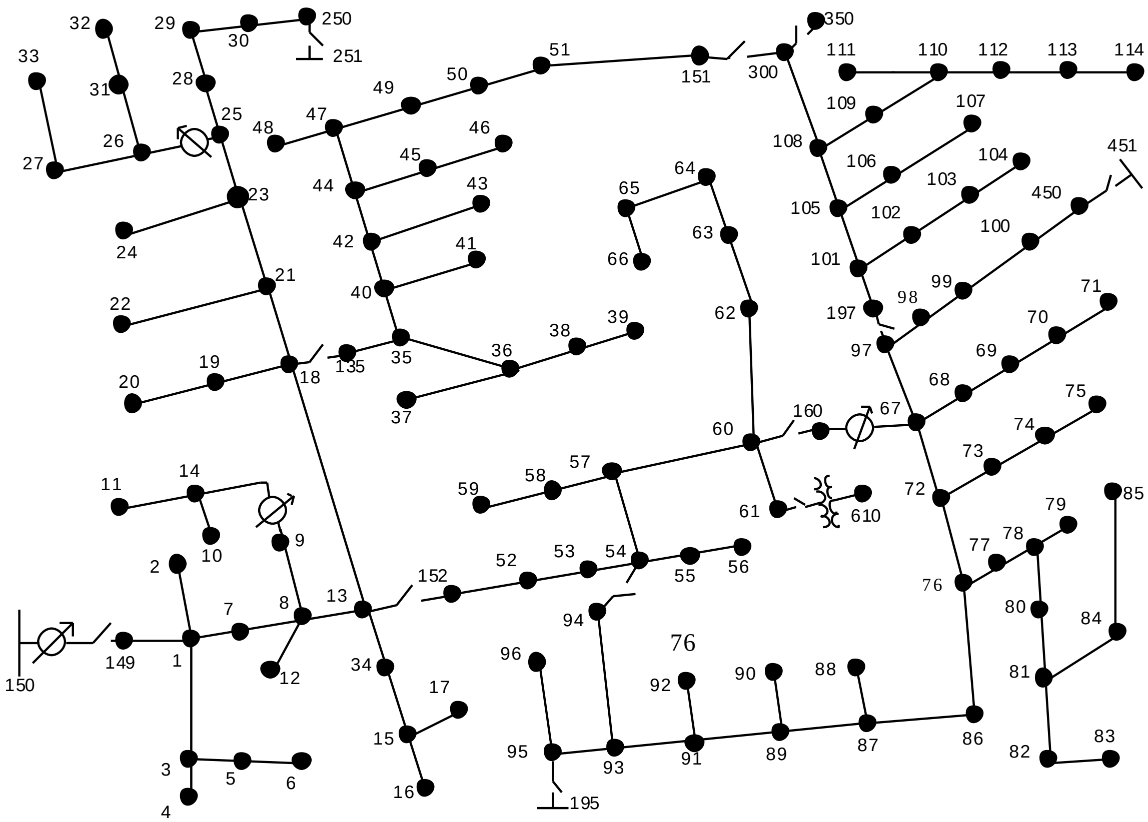


Fig. 4.1 IEEE 123 node Test Feeder

## 4.2 COMPARISON OF RESULTS

The following tables shows the comparison difference between Enerlyser (E'er) software results and those published in IEEE 123 bus feeder system [13].

**Table 1: Comparison of Current (Amp)**

| From node | To node | Phase R | | | Phase Y | | | Phase B | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | E'er | IEEE | Diff | E'er | IEEE | Diff | E'er | IEEE | Diff |
| 150 | RG1 | 655.1 | 655.91 | 0.81 | 425.3 | 425.91 | 0.61 | 524.2 | 523.82 | −0.38 |
| RG1 | 149 | 628.2 | 628.42 | 0.22 | 408.0 | 408.06 | 0.06 | 502.3 | 501.86 | −0.44 |
| 149 | 1 | 628.2 | 628.42 | 0.22 | 408.0 | 408.06 | 0.06 | 502.3 | 501.86 | −0.44 |
| 1 | 2 | | | | 8.93 | 8.94 | 0.01 | | | |
| 1 | 3 | | | | | | | 46.55 | 46.54 | −0.01 |
| 1 | 7 | 610.2 | 610.45 | 0.25 | 400.3 | 399.99 | −0.31 | 455.4 | 455.89 | 0.49 |
| 3 | 4 | | | | | | | 17.92 | 18.01 | 0.09 |
| 3 | 5 | | | | | | | 28.62 | 28.51 | −0.11 |
| 5 | 6 | | | | | | | 19.31 | 19.20 | −0.11 |
| 7 | 8 | 601.1 | 601.39 | 0.29 | 400.3 | 399.99 | −0.31 | 455.4 | 455.89 | 0.49 |
| 8 | 12 | | | | 8.99 | 8.97 | −0.02 | | | |
| 8 | 13 | 555.1 | 555.51 | 0.41 | 390.4 | 390.31 | −0.09 | 455.4 | 455.89 | 0.49 |
| 8 | 9 | 46.12 | 46.22 | 0.1 | | | | | | |
| 13 | 152 | 332.0 | 332.01 | 0.01 | 243.9 | 244.34 | 0.44 | 265.1 | 265.02 | −0.08 |
| 13 | 18 | 228.2 | 228.85 | 0.65 | 155.2 | 155.56 | 0.36 | 153.8 | 153.35 | −0.45 |
| 13 | 34 | | | | | | | 48.12 | 48.42 | 0.3 |
| 152 | 52 | 332.0 | 332.01 | 0.01 | 243.9 | 244.34 | 0.44 | 265.1 | 265.02 | −0.08 |
| 52 | 53 | 313.9 | 314.05 | 0.15 | 244.0 | 244.34 | 0.34 | 265.1 | 265.02 | −0.08 |
| 53 | 54 | 296.2 | 296.14 | −0.06 | 244.1 | 244.34 | 0.24 | 265.1 | 265.02 | −0.08 |
| 54 | 55 | 9.12 | 9.28 | 0.16 | 9.09 | 9.01 | −0.08 | 0.00 | 0.00 | 0 |
| 54 | 57 | 287.1 | 287.25 | 0.15 | 234.8 | 235.82 | 1.02 | 265.1 | 265.02 | −0.08 |
| 55 | 56 | 0.00 | 0.00 | 0 | 9.10 | 9.01 | −0.09 | 0.00 | 0.00 | 0 |
| 57 | 58 | | | | 18.46 | 18.35 | −0.11 | | | |
| 57 | 60 | 286.7 | 287.25 | 0.55 | 218.2 | 218.62 | 0.42 | 266.1 | 265.02 | −1.08 |
| 58 | 59 | | | | 9.15 | 9.04 | −0.11 | | | |
| 60 | 160 | 241.1 | 240.09 | −1.01 | 171.8 | 172.00 | 0.2 | 191.0 | 191.06 | 0.06 |
| 60 | 61 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0 |
| 60 | 62 | 45.32 | 45.37 | 0.05 | 52.24 | 52.24 | 0 | 80.61 | 80.73 | 0.12 |
| 160 | RG4 | 241.1 | 240.09 | −1.01 | 171.8 | 172.00 | 0.2 | 191.0 | 191.06 | 0.06 |
| RG4 | 67 | 228.3 | 228.66 | 0.36 | 170.8 | 170.93 | 0.13 | 185.6 | 185.25 | −0.35 |

| From node | To node | Phase R | | | Phase Y | | | Phase B | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | E’er | IEEE | Diff | E’er | IEEE | Diff | E’er | IEEE | Diff |
| 67 | 68 | 55.02 | 54.15 | −0.87 | | | | | | |
| 67 | 72 | 118.6 | 118.75 | 0.15 | 126.4 | 126.10 | −0.3 | 132.2 | 132.91 | 0.71 |
| 67 | 97 | 82.69 | 82.68 | −0.01 | 54.03 | 54.30 | 0.27 | 64.19 | 64.43 | 0.24 |
| 68 | 69 | 45.11 | 45.14 | 0.03 | | | | | | |
| 69 | 70 | 27.09 | 27.10 | 0.01 | | | | | | |
| 70 | 71 | 17.99 | 18.07 | 0.08 | | | | | | |
| 72 | 73 | | | | | | | 55.29 | 55.31 | 0.02 |
| 72 | 76 | 118.2 | 118.75 | 0.55 | 126.5 | 126.10 | −0.4 | 100.2 | 100.03 | −0.17 |
| 73 | 74 | | | | | | | 37.28 | 37.27 | −0.01 |
| 74 | 75 | | | | | | | 18.06 | 18.09 | 0.03 |
| 76 | 77 | 77.29 | 77.82 | 0.53 | 77.34 | 77.40 | 0.06 | 77.56 | 77.45 | −0.11 |
| 76 | 86 | 32.12 | 32.69 | 0.57 | 57.01 | 57.67 | 0.66 | 22.01 | 21.03 | −0.98 |
| 77 | 78 | 77.28 | 77.82 | 0.54 | 79.98 | 80.01 | 0.03 | 77.54 | 77.45 | −0.09 |
| 78 | 79 | 19.38 | 19.31 | −0.07 | | | | | | |
| 78 | 80 | 80.12 | 80.52 | 0.4 | 80.21 | 80.01 | −0.2 | 77.21 | 77.45 | 0.24 |
| 80 | 81 | 80.13 | 80.52 | 0.39 | 86.34 | 86.40 | 0.06 | 77.23 | 77.45 | 0.22 |
| 81 | 82 | 80.13 | 80.52 | 0.39 | 86.34 | 86.40 | 0.06 | 82.85 | 82.90 | 0.05 |
| 81 | 84 | | | | | | | 27.08 | 27.01 | −0.07 |
| 82 | 83 | 86.76 | 86.90 | 0.14 | 86.35 | 86.40 | 0.05 | 82.84 | 82.90 | 0.06 |
| 84 | 85 | | | | | | | 18.07 | 18.01 | −0.06 |
| 86 | 87 | 31.76 | 32.69 | 0.93 | 49.17 | 49.21 | 0.04 | 20.98 | 21.02 | 0.04 |
| 87 | 88 | 20.89 | 21.00 | 0.11 | | | | | | |
| 87 | 89 | 17.89 | 18.03 | 0.14 | 33.45 | 33.25 | −0.2 | 20.98 | 21.02 | 0.04 |
| 89 | 90 | | | | 20.99 | 21.16 | 0.17 | | | |
| 89 | 91 | 17.94 | 18.03 | 0.09 | 17.99 | 18.15 | 0.16 | 21.21 | 21.02 | −0.19 |
| 91 | 92 | | | | | | | 21.11 | 21.02 | −0.09 |
| 91 | 93 | 17.98 | 18.03 | 0.05 | 18.01 | 18.15 | 0.14 | 0.02 | 0.00 | −0.02 |
| 93 | 94 | 17.91 | 18.03 | 0.12 | | | | | | |
| 93 | 95 | 0.00 | 0.00 | 0 | 18.00 | 18.15 | 0.15 | 0.00 | 0.00 | 0 |
| 95 | 96 | | | | 9.01 | 9.08 | 0.07 | | | |
| 97 | 197 | 64.65 | 64.68 | 0.03 | 36.12 | 36.21 | 0.09 | 45.34 | 45.20 | −0.14 |
| 97 | 98 | 17.97 | 18.00 | 0.03 | 18.01 | 18.08 | 0.07 | 19.21 | 19.23 | 0.02 |
| 197 | 101 | 65.13 | 64.68 | −0.45 | 35.97 | 36.21 | 0.24 | 44.98 | 45.20 | 0.22 |
| 101 | 102 | | | | | | | 45.05 | 45.20 | 0.15 |
| 101 | 105 | 65.23 | 64.68 | −0.55 | 35.97 | 36.21 | 0.24 | 0.00 | 0.00 | 0 |
| 102 | 103 | | | | | | | 36.06 | 36.18 | 0.12 |
| 103 | 104 | | | | | | | 18.05 | 18.11 | 0.06 |
| 105 | 106 | | | | 35.87 | 36.22 | 0.35 | | | |

| From node | To node | Phase R | | | Phase Y | | | Phase B | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | E'er | IEEE | Diff | E'er | IEEE | Diff | E'er | IEEE | Diff |
| 105 | 108 | 64.11 | 64.68 | 0.57 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0 |
| 106 | 107 | | | 0 | 17.94 | 18.12 | 0.18 | | | 0 |
| 108 | 109 | 64.11 | 64.68 | 0.57 | | | 0 | | | 0 |
| 109 | 110 | 46.14 | 46.54 | 0.4 | | | 0 | | | 0 |
| 110 | 111 | 8.97 | 9.09 | 0.12 | | | 0 | | | 0 |
| 110 | 112 | 37.21 | 37.45 | 0.24 | | | 0 | | | 0 |
| 112 | 113 | 28.21 | 28.14 | −0.07 | | | 0 | | | 0 |
| 113 | 114 | 8.97 | 9.11 | 0.14 | | | 0 | | | 0 |
| 98 | 99 | 0.01 | 0.00 | −0.01 | 17.97 | 18.08 | 0.11 | 19.25 | 19.23 | −0.02 |
| 99 | 100 | 0.06 | 0.00 | −0.06 | 0.00 | 0.00 | 0 | 19.27 | 19.23 | −0.04 |
| 100 | 450 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0 |
| 61 | XF1 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0 |
| XF1 | 610 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0 |
| 62 | 63 | 45.13 | 45.37 | 0.24 | 52.24 | 52.24 | 0 | 61.96 | 62.06 | 0.1 |
| 63 | 64 | 27.11 | 27.13 | 0.02 | 52.23 | 52.24 | 0.01 | 61.96 | 62.06 | 0.1 |
| 64 | 65 | 27.10 | 27.13 | 0.03 | 17.97 | 17.99 | 0.02 | 61.96 | 62.06 | 0.1 |
| 65 | 66 | 0.00 | 0.00 | 0 | 0.00 | 0.01 | 0.01 | 34.68 | 34.61 | −0.07 |
| 18 | 135 | 135.12 | 135.82 | 0.7 | 136.2 | 136.56 | 0.36 | 97.97 | 98.12 | 0.15 |
| 18 | 19 | 37.15 | 37.29 | 0.14 | | | 0 | | | 0 |
| 18 | 21 | 55.45 | 55.79 | 0.34 | 19.10 | 19.19 | 0.09 | 55.40 | 55.41 | 0.01 |
| 135 | 35 | 135.25 | 135.82 | 0.57 | 136.2 | 136.56 | 0.36 | 99.21 | 98.12 | −1.09 |
| 35 | 36 | 18.53 | 18.51 | −0.02 | 18.43 | 18.37 | −0.06 | | | 0 |
| 35 | 40 | 107.96 | 108.64 | 0.68 | 108.2 | 108.47 | 0.27 | 99.14 | 98.12 | −1.02 |
| 36 | 37 | 18.98 | 18.51 | −0.47 | | | 0 | | | 0 |
| 36 | 38 | | | 0 | 18.43 | 18.37 | −0.06 | | | 0 |
| 38 | 39 | | | 0 | 9.12 | 9.06 | −0.06 | | | 0 |
| 40 | 41 | | | 0 | | | 0 | 9.16 | 9.22 | 0.06 |
| 40 | 42 | 109.69 | 108.64 | −1.05 | 108.2 | 108.47 | 0.27 | 90.04 | 88.95 | −1.09 |
| 42 | 43 | | | 0 | 18.96 | 19.10 | 0.14 | | | 0 |
| 42 | 44 | 99.02 | 99.32 | 0.3 | 89.50 | 89.57 | 0.07 | 89.89 | 88.95 | −0.94 |
| 44 | 45 | 18.62 | 18.71 | 0.09 | | | 0 | | | 0 |
| 44 | 47 | 80.24 | 80.76 | 0.52 | 89.51 | 89.57 | 0.06 | 89.89 | 88.95 | −0.94 |
| 45 | 46 | 9.16 | 9.40 | 0.24 | | | 0 | | | 0 |
| 47 | 48 | 36.12 | 35.48 | −0.64 | 36.47 | 36.71 | 0.24 | 36.39 | 36.01 | −0.38 |
| 47 | 49 | 27.39 | 27.40 | 0.01 | 35.92 | 34.95 | −0.97 | 35.19 | 35.15 | −0.04 |
| 49 | 50 | 9.45 | 9.40 | −0.05 | 0.00 | 0.00 | 0 | 18.36 | 18.49 | 0.13 |

| From node | To node | Phase R | | | Phase Y | | | Phase B | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | E’er | IEEE | Diff | E’er | IEEE | Diff | E’er | IEEE | Diff |
| 50 | 51 | 9.16 | 9.40 | 0.24 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0 |
| 51 | 151 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0 |
| 19 | 20 | 18.53 | 18.62 | 0.09 | | | 0 | | | 0 |
| 21 | 22 | | | 0 | 15.04 | 14.19 | −0.85 | | | 0 |
| 21 | 23 | 55.17 | 55.80 | 0.63 | 0.00 | 0.00 | 0 | 55.11 | 55.41 | 0.3 |
| 23 | 24 | | | 0 | | | 0 | 18.34 | 18.46 | 0.12 |
| 23 | 25 | 55.16 | 55.80 | 0.64 | 0.00 | 0.00 | 0 | 36.74 | 36.95 | 0.21 |
| 23 | 28 | 36.79 | 37.18 | 0.39 | 0.00 | 0.00 | 0 | 18.12 | 18.47 | 0.35 |
| 25 | RG3 | 18.33 | 18.62 | 0.29 | | | 0 | 18.13 | 18.47 | 0.34 |
| 28 | 29 | 18.25 | 18.56 | 0.31 | 0.00 | 0.00 | 0 | 18.37 | 18.47 | 0.1 |
| 29 | 30 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0 | 18.34 | 18.47 | 0.13 |
| 30 | 250 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0 |
| RG3 | 26 | 18.33 | 18.62 | 0.29 | | | | 18.13 | 18.59 | 0.46 |
| 26 | 27 | 17.98 | 18.62 | 0.64 | | | | 0.00 | 0.00 | 0 |
| 26 | 31 | | | 0 | | | | 18.33 | 18.59 | 0.26 |
| 27 | 33 | 18.62 | 18.62 | 0 | | | | | | 0 |
| 31 | 32 | | | 0 | | | | 9.17 | 9.30 | 0.13 |
| 34 | 15 | | | 0 | | | | 27.32 | 27.45 | 0.13 |
| 15 | 16 | | | 0 | | | | 18.22 | 18.30 | 0.08 |
| 15 | 17 | | | 0 | | | | 9.10 | 9.15 | 0.05 |
| 9 | RG2 | 27.23 | 27.86 | 0.63 | | | | | | |
| RG2 | 14 | 27.98 | 28.03 | 0.05 | | | | | | |
| 14 | 10 | 9.31 | 9.31 | 0 | | | | | | |
| 14 | 11 | 19.11 | 18.73 | −0.38 | | | | | | |

## Table 2: Comparison of Line Losses (kW)

| From node | To node | Phase R | | | Phase Y | | | Phase B | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | E'er | IEEE | Diff | E'er | IEEE | Diff | E'er | IEEE | Diff |
| 150 | RG1 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 |
| RG1 | 149 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 |
| 149 | 1 | 11.15 | 11.176 | 0.026 | 0.420 | 0.429 | 0.009 | 7.550 | 7.554 | 0.004 |
| 1 | 2 | | | 0 | 0.004 | 0.004 | 0 | | | 0 |
| 1 | 3 | | | 0 | | | 0 | 0.140 | 0.136 | −0.004 |
| 1 | 7 | 8.400 | 8.408 | 0.008 | 0.160 | 0.167 | 0.007 | 4.500 | 4.515 | 0.015 |
| 3 | 4 | | | 0 | | | 0 | 0.015 | 0.016 | 0.001 |
| 3 | 5 | | | 0 | | | 0 | 0.060 | 0.066 | 0.006 |
| 5 | 6 | | | 0 | | | 0 | 0.020 | 0.023 | 0.003 |
| 7 | 8 | 5.420 | 5.420 | 0 | 0.190 | 0.184 | −0.006 | 3.000 | 2.979 | −0.021 |
| 8 | 12 | | | 0 | 0.001 | 0.005 | 0.004 | | | 0 |
| 8 | 13 | 6.700 | 6.704 | 0.004 | 0.710 | 0.700 | −0.01 | 4.340 | 4.341 | 0.001 |
| 8 | 9 | 0.120 | 0.122 | 0.002 | | | 0 | | | 0 |
| 13 | 152 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 |
| 13 | 18 | 2.440 | 2.436 | −0.004 | 0.340 | 0.341 | 0.001 | 2.200 | 2.131 | −0.069 |
| 13 | 34 | | | 0 | | | 0 | 0.080 | 0.081 | 0.001 |
| 152 | 52 | 3.550 | 3.538 | −0.012 | 0.360 | 0.363 | 0.003 | 1.800 | 1.719 | −0.081 |
| 52 | 53 | 1.550 | 1.555 | 0.005 | 0.270 | 0.271 | 0.001 | 0.840 | 0.837 | −0.003 |
| 53 | 54 | 0.851 | 0.846 | −0.005 | 0.220 | 0.226 | 0.006 | 0.500 | 0.508 | 0.008 |
| 54 | 55 | 0.001 | 0.003 | 0.002 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 |
| 54 | 57 | 1.540 | 1.543 | 0.003 | 1.320 | 1.328 | 0.008 | 1.400 | 1.369 | −0.031 |
| 55 | 56 | 0.000 | 0.000 | 0 | 0.020 | 0.020 | 0 | 0.000 | 0.000 | 0 |
| 57 | 58 | | | 0 | 0.020 | 0.021 | 0.001 | | | 0 |
| 57 | 60 | 3.200 | 3.190 | −0.01 | 2.300 | 2.296 | −0.004 | 3.200 | 3.294 | 0.094 |
| 58 | 59 | | | 0 | 0.004 | 0.005 | 0.001 | | | 0 |
| 60 | 160 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 |
| 60 | 61 | 0.000 | 0.000 | 0 | 0.001 | 0.001 | 0 | 0.000 | 0.000 | 0 |
| 60 | 62 | 0.070 | 0.072 | 0.002 | 0.150 | 0.151 | 0.001 | 0.340 | 0.341 | 0.001 |
| 160 | RG4 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 |
| RG4 | 67 | 0.920 | 0.940 | 0.02 | 0.430 | 0.429 | −0.001 | 1.000 | 1.001 | 0.001 |

| From node | To node | Phase R | | | Phase Y | | | Phase B | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | E'er | IEEE | Diff | E'er | IEEE | Diff | E'er | IEEE | Diff |
| 67 | 68 | 0.150 | 0.148 | −0.002 | | | 0 | | | |
| 67 | 72 | 0.110 | 0.117 | 0.007 | 0.40 | 0.396 | −0.004 | 0.250 | 0.256 | 0.006 |
| 67 | 97 | 0.100 | 0.094 | −0.006 | 0.030 | 0.035 | 0.005 | 0.070 | 0.077 | 0.007 |
| 68 | 69 | 0.122 | 0.141 | 0.019 | | | 0 | | | 0 |
| 69 | 70 | 0.050 | 0.060 | 0.01 | | | 0 | | | 0 |
| 70 | 71 | 0.023 | 0.023 | 0 | | | 0 | | | 0 |
| 72 | 73 | | | 0 | | | 0 | 0.200 | 0.212 | 0.012 |
| 72 | 76 | 0.153 | 0.168 | 0.015 | 0.240 | 0.248 | 0.008 | 0.020 | 0.020 | 0 |
| 73 | 74 | | | 0 | | | 0 | 0.100 | 0.122 | 0.022 |
| 74 | 75 | | | 0 | | | 0 | 0.030 | 0.033 | 0.003 |
| 76 | 77 | 0.101 | 0.101 | 0 | 0.150 | 0.151 | 0.001 | 0.160 | 0.166 | 0.006 |
| 76 | 86 | 0.072 | 0.075 | 0.003 | 0.110 | 0.191 | 0.081 | −0.03 | −0.036 | −0.006 |
| 77 | 78 | 0.031 | 0.034 | 0.003 | 0.030 | 0.031 | 0.001 | 0.040 | 0.043 | 0.003 |
| 78 | 79 | 0.007 | 0.008 | 0.001 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 |
| 78 | 80 | 0.110 | 0.126 | 0.016 | 0.140 | 0.139 | −0.001 | 0.250 | 0.259 | 0.009 |
| 80 | 81 | 0.165 | 0.175 | 0.01 | 0.120 | 0.120 | 0 | 0.260 | 0.267 | 0.007 |
| 81 | 82 | 0.093 | 0.091 | −0.002 | 0.110 | 0.112 | 0.002 | 0.500 | 0.500 | 0 |
| 81 | 84 | | | 0 | | | 0 | 0.120 | 0.124 | 0.004 |
| 82 | 83 | 0.080 | 0.081 | 0.001 | 0.100 | 0.102 | 0.002 | 0.136 | 0.135 | −0.001 |
| 84 | 85 | | | 0 | | | 0 | 0.040 | 0.039 | −0.001 |
| 86 | 87 | 0.060 | 0.059 | −0.001 | 0.010 | 0.066 | 0.056 | −0.01 | −0.009 | 0.001 |
| 87 | 88 | 0.020 | 0.019 | −0.001 | | | 0 | | | 0 |
| 87 | 89 | 0.020 | 0.019 | −0.001 | 0.030 | 0.031 | 0.001 | −0.01 | −0.009 | 0.001 |
| 89 | 90 | | | 0 | 0.020 | 0.025 | 0.005 | | | 0 |
| 89 | 91 | 0.010 | 0.009 | −0.001 | 0.007 | 0.008 | 0.001 | 0.000 | 0.001 | 0.001 |
| 91 | 92 | | | 0 | | | 0 | 0.021 | 0.033 | 0.012 |
| 91 | 93 | 0.010 | 0.010 | 0 | 0.000 | 0.001 | 0.001 | 0.000 | 0.000 | 0 |
| 93 | 94 | 0.022 | 0.023 | 0.001 | | | 0 | | | 0 |
| 93 | 95 | 0.000 | 0.000 | 0 | 0.007 | 0.008 | 0.001 | 0.000 | 0.000 | 0 |
| 95 | 96 | | | 0 | 0.001 | 0.004 | 0.003 | | | 0 |
| 97 | 197 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 |
| 97 | 98 | 0.004 | 0.004 | 0 | 0.001 | 0.007 | 0.006 | 0.001 | 0.005 | 0.004 |
| 197 | 101 | 0.050 | 0.060 | 0.01 | 0.100 | 0.101 | 0.001 | 0.040 | 0.044 | 0.004 |
| 101 | 102 | | | 0 | | | 0 | 0.120 | 0.116 | −0.004 |
| 101 | 105 | 0.122 | 0.133 | 0.011 | −0.01 | −0.019 | −0.009 | 0.000 | 0.000 | 0 |
| 102 | 103 | | | 0 | | | | 0.100 | 0.107 | 0.007 |
| 103 | 104 | | | | | | | 0.050 | 0.058 | 0.008 |
| 105 | 106 | | | | 0.050 | 0.074 | 0.024 | | | |

| From node | To node | Phase R | | | Phase Y | | | Phase B | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | E'er | IEEE | Diff | E'er | IEEE | Diff | E'er | IEEE | Diff |
| 105 | 108 | 0.010 | 0.119 | 0.109 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 |
| 106 | 107 | | | 0 | 0.040 | 0.048 | 0.008 | | | 0 |
| 108 | 109 | | | 0 | | | 0 | | | 0 |
| 109 | 110 | 0.010 | 0.164 | 0.154 | | | 0 | | | 0 |
| 110 | 111 | 0.001 | 0.012 | 0.011 | | | 0 | | | 0 |
| 110 | 112 | 0.021 | 0.044 | 0.023 | | | 0 | | | 0 |
| 112 | 113 | 0.040 | 0.105 | 0.065 | | | 0 | | | 0 |
| 113 | 114 | 0.010 | 0.007 | −0.003 | | | 0 | | | 0 |
| 98 | 99 | 0.000 | 0.000 | 0 | 0.020 | 0.028 | 0.008 | 0.000 | 0.000 | 0 |
| 99 | 100 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 | 0.010 | 0.010 | 0 |
| 100 | 450 | 0.000 | 0.000 | 0 | −0.010 | −0.001 | 0.009 | 0.000 | 0.001 | 0.001 |
| 61 | XF1 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 |
| XF1 | 610 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 |
| 62 | 63 | 0.061 | 0.065 | 0.004 | 0.101 | 0.106 | 0.005 | 0.120 | 0.125 | 0.005 |
| 63 | 64 | 0.030 | 0.032 | 0.002 | 0.240 | 0.245 | 0.005 | 0.250 | 0.256 | 0.006 |
| 64 | 65 | 0.024 | 0.024 | 0 | 0.030 | 0.031 | 0.001 | 0.370 | 0.398 | 0.028 |
| 65 | 66 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 | 0.100 | 0.112 | 0.012 |
| 18 | 135 | 0.007 | 0.009 | 0.002 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0 |
| 18 | 19 | 0.080 | 0.087 | 0.007 | | | 0 | | | 0 |
| 18 | 21 | 0.010 | 0.011 | 0.001 | −0.010 | −0.002 | 0.008 | 0.110 | 0.116 | 0.006 |
| 135 | 35 | 0.460 | 0.555 | 0.095 | 0.032 | 0.397 | 0.365 | 0.070 | 0.074 | 0.004 |
| 35 | 36 | 0.030 | 0.030 | 0 | 0.010 | 0.003 | −0.007 | | | 0 |
| 35 | 40 | 0.250 | 0.246 | −0.004 | 0.010 | 0.126 | 0.116 | 0.110 | 0.110 | 0 |
| 36 | 37 | 0.022 | 0.026 | 0.004 | | | 0 | | | 0 |
| 36 | 38 | | | 0 | 0.020 | 0.021 | 0.001 | | | 0 |
| 38 | 39 | | | 0 | 0.010 | 0.007 | −0.003 | | | 0 |
| 40 | 41 | | | 0 | | | 0 | 0.004 | 0.007 | 0.003 |
| 40 | 42 | 0.250 | 0.265 | 0.015 | 0.010 | 0.110 | 0.1 | 0.050 | 0.083 | 0.033 |
| 42 | 43 | | | 0 | 0.040 | 0.046 | 0.006 | | | 0 |
| 42 | 44 | 0.101 | 0.151 | 0.05 | 0.620 | 0.663 | 0.043 | 0.067 | 0.086 | 0.019 |
| 44 | 45 | 0.010 | 0.018 | 0.008 | | | 0 | | | 0 |
| 44 | 47 | 0.120 | 0.125 | 0.005 | 0.101 | 0.117 | 0.016 | 0.066 | 0.085 | 0.019 |
| 45 | 46 | 0.005 | 0.007 | 0.002 | | | 0 | | | 0 |
| 47 | 48 | 0.010 | 0.012 | 0.002 | 0.010 | 0.015 | 0.005 | 0.010 | 0.007 | −0.003 |
| 47 | 49 | 0.010 | 0.010 | 0 | 0.020 | 0.030 | 0.01 | 0.010 | 0.007 | −0.003 |
| 49 | 50 | −0.004 | −0.002 | 0.002 | 0.000 | 0.000 | 0 | 0.010 | 0.010 | 0 |

| From node | To node | Phase R | | | Phase Y | | | Phase B | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | E'er | IEEE | Diff | E'er | IEEE | Diff | E'er | IEEE | Diff |
| 50 | 51 | 0.001 | 0.002 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 51 | 151 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.001 |
| 19 | 20 | 0.020 | 0.028 | 0.008 | | | | | | |
| 21 | 22 | | | | 0.040 | 0.049 | 0.009 | | | |
| 21 | 23 | −0.002 | −0.006 | −0.004 | 0.000 | 0.000 | 0.000 | 0.090 | 0.117 | 0.027 |
| 23 | 24 | | | | | | | 0.040 | 0.047 | 0.007 |
| 23 | 25 | 0.011 | 0.021 | 0.010 | 0.000 | 0.000 | 0.000 | 0.070 | 0.070 | 0.000 |
| 23 | 28 | 0.010 | 0.011 | 0.001 | 0.000 | 0.000 | 0.000 | 0.010 | 0.015 | 0.005 |
| 25 | RG3 | 0.000 | 0.000 | 0.000 | | | | 0.000 | 0.000 | 0.000 |
| 28 | 29 | −0.001 | −0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.010 | 0.015 | 0.005 |
| 29 | 30 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.010 | 0.010 | 0.000 |
| 30 | 250 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| RG3 | 26 | 0.001 | 0.001 | 0.000 | | | | 0.010 | 0.016 | 0.006 |
| 26 | 27 | 0.001 | 0.009 | 0.008 | | | | 0.000 | 0.000 | 0.000 |
| 26 | 31 | | | | | | | 0.020 | 0.020 | 0.000 |
| 27 | 33 | 0.023 | 0.044 | 0.021 | | | | | | |
| 31 | 32 | | | | | | | 0.010 | 0.007 | −0.003 |
| 34 | 15 | | | | | | | 0.018 | 0.019 | 0.001 |
| 15 | 16 | | | | | | | 0.034 | 0.032 | −0.002 |
| 15 | 17 | | | | | | | 0.010 | 0.007 | −0.003 |
| 9 | RG2 | 0.000 | 0.000 | 0.000 | | | | | | |
| RG2 | 14 | 0.075 | 0.084 | 0.009 | | | | | | |
| 14 | 10 | 0.001 | 0.005 | 0.004 | | | | | | |
| 14 | 11 | 0.020 | 0.022 | 0.002 | | | | | | |

**Table 3: Comparison of Voltages (p.u.)**

| Node | Phase R | | | Phase Y | | | Phase B | | |
|---|---|---|---|---|---|---|---|---|---|
| | E'er | IEEE | Diff | E'er | IEEE | Diff | E'er | IEEE | Diff |
| 150 | 1.0000 | 1.0000 | 0.0000 | 1.0000 | 1.0000 | 0.0000 | 1.0000 | 1.0000 | 0.0000 |
| RG1 | 1.0436 | 1.0437 | 0.0001 | 1.0437 | 1.0438 | 0.0001 | 1.0437 | 1.0438 | 0.0001 |
| 149 | 1.0436 | 1.0436 | 0.0000 | 1.0436 | 1.0437 | 0.0001 | 1.0436 | 1.0436 | 0.0000 |
| 1 | 1.0311 | 1.0311 | 0.0000 | 1.0411 | 1.0412 | 0.0001 | 1.0347 | 1.0348 | 0.0001 |
| 2 | | | | 1.0410 | 1.0410 | 0.0000 | | | |
| 3 | | | | | | | 1.0330 | 1.0331 | 0.0001 |
| 4 | | | | | | | 1.0325 | 1.0326 | 0.0001 |
| 5 | | | | | | | 1.0315 | 1.0318 | 0.0003 |
| 6 | | | | | | | 1.0310 | 1.0311 | 0.0001 |
| 7 | 1.0217 | 1.0218 | 0.0001 | 1.0393 | 1.0395 | 0.0002 | 1.0290 | 1.0291 | 0.0001 |
| 8 | 1.0157 | 1.0158 | 0.0001 | 1.0380 | 1.0382 | 0.0002 | 1.0250 | 1.0253 | 0.0003 |
| 12 | | | | 1.0375 | 1.0379 | 0.0003 | | | |
| 13 | 1.0080 | 1.0079 | 0.0001 | 1.0361 | 1.0360 | 0.0001 | 1.0194 | 1.0196 | 0.0002 |
| 152 | 1.0078 | 1.0078 | 0.0000 | 1.0360 | 1.0360 | 0.0000 | 1.0194 | 1.0196 | 0.0002 |
| 52 | 1.0018 | 1.0018 | 0.0000 | 1.0346 | 1.0348 | 0.0002 | 1.0165 | 1.0164 | 0.0001 |
| 53 | 0.9990 | 0.9991 | 0.0001 | 1.0341 | 1.0340 | 0.0001 | 1.0150 | 1.0148 | 0.0002 |
| 54 | 0.9975 | 0.9976 | 0.0001 | 1.0332 | 1.0334 | 0.0002 | 1.0137 | 1.0138 | 0.0001 |
| 55 | 0.9975 | 0.9974 | 0.0001 | 1.0333 | 1.0334 | 0.0001 | 1.0140 | 1.0139 | 0.0001 |
| 56 | 0.9975 | 0.9974 | 0.0001 | 1.0330 | 1.0332 | 0.0002 | 1.0140 | 1.0140 | 0.0000 |
| 57 | 0.9946 | 0.9945 | 0.0001 | 1.0303 | 1.0306 | 0.0003 | 1.0110 | 1.0113 | 0.0003 |
| 58 | | | | 1.0300 | 1.0300 | 0.0000 | | | |
| 59 | | | | 1.0297 | 1.0296 | 0.0001 | | | |
| 60 | 0.9981 | 0.9880 | 0.0001 | 1.0256 | 1.0256 | 0.0000 | 1.0050 | 1.0052 | 0.0002 |
| 160 | 0.9980 | 0.9880 | 0.0000 | 1.0255 | 1.0256 | 0.0001 | 1.0050 | 1.0052 | 0.0002 |
| RG4 | 1.0373 | 1.0374 | 0.0001 | 1.0321 | 1.0320 | 0.0001 | 1.0364 | 1.0366 | 0.0002 |
| 67 | 1.0355 | 1.0355 | 0.0000 | 1.0310 | 1.0311 | 0.0001 | 1.0347 | 1.0345 | 0.0002 |
| 68 | 1.0341 | 1.0340 | 0.0001 | | | | | | |
| 69 | 1.0321 | 1.0322 | 0.0001 | | | | | | |
| 70 | 1.0310 | 1.0310 | 0.0000 | | | | | | |
| 71 | 1.0302 | 1.0303 | 0.0001 | | | | 1.0342 | 1.0343 | 0.0001 |
| 72 | 1.0358 | 1.0359 | 0.0001 | 1.0300 | 1.0302 | 0.0002 | 1.0320 | 1.0321 | 0.0001 |
| 73 | | | | | | | 1.0300 | 1.0303 | 0.0003 |
| 74 | | | | | | | 1.0292 | 1.0293 | 0.0001 |
| 75 | | | | | | | 1.0349 | 1.0349 | 0.0000 |
| 76 | 1.0358 | 1.0358 | 0.0000 | 1.0298 | 1.0297 | 0.0001 | 1.0358 | 1.0358 | 0.0000 |
| 77 | 1.0371 | 1.0370 | 0.0001 | 1.0308 | 1.0308 | 0.0000 | 1.0358 | 1.0360 | 0.0002 |

| Node | Phase R | | | Phase Y | | | Phase B | | |
|---|---|---|---|---|---|---|---|---|---|
| | E'er | IEEE | Diff | E'er | IEEE | Diff | E'er | IEEE | Diff |
| 78 | 1.0371 | 1.0373 | 0.0003 | 1.0310 | 1.0312 | 0.0002 | 1.0358 | 1.0360 | 0.0002 |
| 79 | 1.0371 | 1.0370 | 0.0001 | 1.0315 | 1.0313 | 0.0002 | 1.0358 | 1.0359 | 0.0002 |
| 80 | 1.0395 | 1.0394 | 0.0001 | 1.0330 | 1.0329 | 0.0001 | 1.0366 | 1.0368 | 0.0002 |
| 81 | 1.0413 | 1.0415 | 0.0002 | 1.0350 | 1.0352 | 0.0002 | 1.0375 | 1.0374 | 0.0001 |
| 82 | 1.0422 | 1.0424 | 0.0002 | 1.0362 | 1.0364 | 0.0002 | 1.0380 | 1.0382 | 0.0002 |
| 83 | 1.0435 | 1.0436 | 0.0001 | 1.0374 | 1.0375 | 0.0001 | 1.0390 | 1.0390 | 0.0000 |
| 84 | | | | | | | 1.0346 | 1.0348 | 0.0002 |
| 85 | | | | | | | 1.0335 | 1.0336 | 0.0001 |
| 86 | 1.0350 | 1.0349 | 0.0001 | 1.0277 | 1.0279 | 0.0002 | 1.0365 | 1.0364 | 0.0001 |
| 87 | 1.0340 | 1.0342 | 0.0002 | 1.0271 | 1.0272 | 0.0001 | 1.0370 | 1.0369 | 0.0001 |
| 88 | 1.0340 | 1.0342 | 0.0002 | | | | | | |
| 89 | 1.0336 | 1.0338 | 0.0002 | 1.0270 | 1.0270 | 0.0000 | 1.0370 | 1.0373 | 0.0003 |
| 90 | | | | 1.0270 | 1.0269 | 0.0001 | | | |
| 91 | 1.0336 | 1.0336 | 0.0000 | 1.0266 | 1.0266 | 0.0000 | 1.0375 | 1.0376 | 0.0001 |
| 92 | | | | | | | 1.0375 | 1.0375 | 0.0000 |
| 93 | 1.0332 | 1.0333 | 0.0001 | 1.0266 | 1.0265 | 0.0001 | 1.0375 | 1.0377 | 0.0002 |
| 94 | 1.0325 | 1.0326 | 0.0001 | | | | | | |
| 95 | 1.0332 | 1.0332 | 0.0000 | 1.0260 | 1.0261 | 0.0001 | 1.0376 | 1.0378 | 0.0002 |
| 96 | | | | 1.0255 | 1.0258 | 0.0003 | | | |
| 97 | 1.0343 | 1.0345 | 0.0002 | 1.0305 | 1.0306 | 0.0001 | 1.0337 | 1.0338 | 0.0001 |
| 197 | 1.0343 | 1.0345 | 0.0002 | 1.0305 | 1.0306 | 0.0001 | 1.0337 | 1.0338 | 0.0001 |
| 101 | 1.0335 | 1.0337 | 0.0002 | 1.0302 | 1.0303 | 0.0001 | 1.0330 | 1.0332 | 0.0002 |
| 102 | | | | | | | 1.0318 | 1.0318 | 0.0000 |
| 103 | | | | | | | 1.0300 | 1.0301 | 0.0001 |
| 104 | | | | | | | 1.0281 | 1.0283 | 0.0002 |
| 105 | 1.0324 | 1.0323 | 0.0001 | 1.0300 | 1.0301 | 0.0001 | 1.0333 | 1.0335 | 0.0002 |
| 106 | | | | 1.0291 | 1.0290 | 0.0001 | | | |
| 107 | | | | 1.0275 | 1.0275 | 0.0000 | | | |
| 108 | 1.0310 | 1.0309 | 0.0001 | 1.0310 | 1.0308 | 0.0002 | 1.0333 | 1.0334 | 0.0002 |
| 109 | 1.0265 | 1.0267 | 0.0002 | | | | | | |
| 110 | 1.0250 | 1.0248 | 0.0002 | | | | | | |
| 111 | 1.0240 | 1.0240 | 0.0000 | | | | | | |
| 112 | 1.0240 | 1.0241 | 0.0001 | | | | | | |
| 113 | 1.0220 | 1.0220 | 0.0000 | | | | | | |
| 114 | 1.0215 | 1.0216 | 0.0001 | | | | | | |
| 300 | 1.0310 | 1.0309 | 0.0001 | 1.0310 | 1.0308 | 0.0002 | 1.0333 | 1.0334 | 0.0001 |

| Node | Phase R | | | Phase Y | | | Phase B | | |
|---|---|---|---|---|---|---|---|---|---|
| | E'er | IEEE | Diff | E'er | IEEE | Diff | E'er | IEEE | Diff |
| 98 | 1.0340 | 1.0343 | 0.0003 | 1.0310 | 1.0308 | 0.0002 | 1.0335 | 1.0336 | 0.0001 |
| 99 | 1.0345 | 1.0346 | 0.0001 | 1.0300 | 1.0303 | 0.0003 | 1.0330 | 1.0332 | 0.0002 |
| 100 | 1.0348 | 1.0348 | 0.0000 | 1.0295 | 1.0295 | 0.0000 | 1.0330 | 1.0328 | 0.0002 |
| 450 | 1.0348 | 1.0348 | 0.0000 | 1.0295 | 1.0294 | 0.0001 | 1.0330 | 1.0328 | 0.0002 |
| 61 | 0.9881 | 0.9880 | 0.0001 | 1.0295 | 1.0294 | 0.0001 | 1.0050 | 1.0052 | 0.0002 |
| XF1 | 0.9881 | 0.9880 | 0.0001 | 1.0255 | 1.0256 | 0.0001 | 1.0050 | 1.0052 | 0.0002 |
| 610 | 0.9881 | 0.9880 | 0.0001 | 1.0255 | 1.0256 | 0.0001 | 1.0050 | 1.0052 | 0.0002 |
| 62 | 0.9870 | 0.9872 | 0.0001 | 1.0255 | 1.0256 | 0.0001 | 1.0030 | 1.0032 | 0.0002 |
| 63 | 0.9865 | 0.9866 | 0.0001 | 1.0242 | 1.0245 | 0.0003 | 1.0023 | 1.0022 | 0.0001 |
| 64 | 0.9864 | 0.9863 | 0.0001 | 1.0235 | 1.0236 | 0.0001 | 1.0000 | 1.0000 | 0.0000 |
| 65 | 0.9855 | 0.9856 | 0.0001 | 1.0216 | 1.0217 | 0.0001 | 0.9970 | 0.9970 | 0.0000 |
| 66 | 0.9856 | 0.9858 | 0.0002 | 1.0215 | 1.0214 | 0.0001 | 0.9955 | 0.9955 | 0.0000 |
| 18 | 0.9986 | 0.9988 | 0.0002 | 1.0215 | 1.0216 | 0.0001 | 1.0120 | 1.0122 | 0.0002 |
| 135 | 0.9986 | 0.9988 | 0.0002 | 1.0318 | 1.0319 | 0.0001 | 1.0120 | 1.0122 | 0.0002 |
| 35 | 0.9962 | 0.9960 | 0.0002 | 1.0317 | 1.0318 | 0.0001 | 1.0111 | 1.0112 | 0.0001 |
| 36 | 0.9950 | 0.9951 | 0.0001 | 1.0290 | 1.0293 | 0.0003 | | | |
| 37 | 0.9941 | 0.9943 | 0.0002 | | | | | | |
| 38 | | | | 1.0280 | 1.0282 | 0.0002 | | | |
| 39 | | | | 1.0277 | 1.0278 | 0.0001 | | | |
| 40 | 0.9944 | 0.9945 | 0.0001 | 1.0280 | 1.0282 | 0.0002 | 1.0100 | 1.0101 | 0.0001 |
| 41 | | | | | | | 1.0096 | 1.0097 | 0.0001 |
| 42 | 0.9930 | 0.9929 | 0.0001 | 1.0270 | 1.0270 | 0.0000 | 1.0090 | 1.0092 | 0.0002 |
| 43 | | | | 1.0255 | 1.0257 | 0.0002 | | | |
| 44 | 0.9916 | 0.9918 | 0.0002 | 1.0261 | 1.0263 | 0.0002 | 1.0082 | 1.0084 | 0.0002 |
| 45 | 0.9910 | 0.9913 | 0.0003 | | | | | | |
| 46 | 0.9910 | 0.9909 | 0.0001 | | | | | | |
| 47 | 0.9910 | 0.9908 | 0.0002 | 1.0251 | 1.0253 | 0.0002 | 1.0072 | 1.0074 | 0.0002 |
| 48 | 0.9905 | 0.9905 | 0.0000 | 1.0250 | 1.0250 | 0.0000 | 1.0072 | 1.0072 | 0.0000 |
| 49 | 0.9905 | 0.9905 | 0.0000 | 1.0245 | 1.0247 | 0.0002 | 1.0070 | 1.0071 | 0.0001 |
| 50 | 0.9905 | 0.9905 | 0.0000 | 1.0245 | 1.0247 | 0.0002 | 1.0065 | 1.0067 | 0.0002 |
| 51 | 0.9904 | 0.9903 | 0.0001 | 1.0246 | 1.0248 | 0.0002 | 1.0065 | 1.0067 | 0.0002 |
| 151 | 0.9905 | 0.9903 | 0.0002 | 1.0246 | 1.0248 | 0.0002 | 1.0065 | 1.0067 | 0.0002 |
| 19 | 0.9976 | 0.9975 | 0.0001 | | | | | | |
| 20 | 0.9965 | 0.9967 | 0.0002 | | | | | | |
| 21 | 0.9980 | 0.9983 | 0.0003 | 1.0320 | 1.0320 | 0.0000 | 1.0110 | 1.0111 | 0.0001 |
| 22 | | | | 1.0304 | 1.0305 | 0.0001 | | | |

| Node | Phase R | | | Phase Y | | | Phase B | | |
|---|---|---|---|---|---|---|---|---|---|
| | E'er | IEEE | Diff | E'er | IEEE | Diff | E'er | IEEE | Diff |
| 23 | 0.9978 | 0.9979 | 0.0001 | 1.0320 | 1.0323 | 0.0003 | 1.0101 | 1.0100 | 0.0001 |
| 24 | | | | | | | 1.0085 | 1.0085 | 0.0000 |
| 25 | 0.9971 | 0.9972 | 0.0001 | 1.0330 | 1.0328 | 0.0002 | 1.0090 | 1.0091 | 0.0001 |
| 26 | 0.9968 | 0.9968 | 0.0000 | 1.0330 | 1.0330 | 0.0000 | 1.0085 | 1.0087 | 0.0002 |
| 27 | 0.9968 | 0.9967 | 0.0001 | 1.0331 | 1.0332 | 0.0001 | 1.0082 | 1.0083 | 0.0001 |
| 28 | 0.9968 | 0.9969 | 0.0001 | 1.0331 | 1.0331 | 0.0000 | 1.0076 | 1.0078 | 0.0002 |
| 29 | 0.9968 | 0.9969 | 0.0001 | 1.0331 | 1.0331 | 0.0000 | 1.0076 | 1.0078 | 0.0002 |
| 30 | 0.9971 | 0.9972 | 0.0001 | | | | 1.0025 | 1.0028 | 0.0003 |
| 250 | 0.9974 | 0.9976 | 0.0002 | | | | 1.0020 | 1.0023 | 0.0003 |
| RG3 | 0.9965 | 0.9967 | 0.0002 | | | | 1.0020 | 1.0022 | 0.0002 |
| 266 | 0.9950 | 0.9953 | 0.0003 | | | | | | |
| 27 | | | | | | | 1.0015 | 1.0017 | 0.0002 |
| 33 | | | | | | | 1.0015 | 1.0013 | 0.0002 |
| 31 | | | | | | | 1.0190 | 1.0187 | 0.0003 |
| 32 | | | | | | | 1.0180 | 1.0183 | 0.0003 |
| 34 | | | | | | | 1.0170 | 1.0173 | 0.0003 |
| 15 | | | | | | | 1.0177 | 1.0178 | 0.0001 |
| 16 | | | | | | | | | |
| 17 | | | | | | | | | |
| 9 | 1.0145 | 1.0144 | 0.0001 | | | | | | |
| RG2 | 1.0080 | 1.0080 | 0.0000 | | | | | | |
| 14 | 1.0060 | 1.0063 | 0.0003 | | | | | | |
| 10 | 1.0060 | 1.0060 | 0.0000 | | | | | | |
| 11 | 1.0055 | 1.0057 | 0.0002 | | | | | | |

**Table 4: Summary of Load-flow Results**

| Quantities | Enerlyser | | | IEEE | | |
|---|---|---|---|---|---|---|
| | Phase R | Phase Y | Phase B | Phase R | Phase Y | Phase B |
| Feeder power flow(kW) | 1460.231 | 963.142 | 1193.264 | 1463.861 | 963.484 | 1193.153 |
| Feeder power flow(kVAr) | 581.901 | 343.267 | 399.102 | 582.101 | 343.687 | 398.976 |
| Total system load (kW) | 1425.041 | 931.021 | 1166.712 | 1425.022 | 930.965 | 1168.900 |
| Total system load(kVAr) | 777.777 | 524.571 | 642.651 | 777.767 | 524.544 | 637.773 |
| Total system loss (kW) | 84.742 | | | 95.611 | | |
| Total system loss(kVAr) | 139.944 | | | 193.727 | | |

# CHAPTER- 5

# CONCLUSION AND FUTURE SCOPE

## CONCLUSION

In this thesis work, generalized two-port parameters (A, B, C, D matrices) have been developed for the network elements. Also, a simple yet efficient load-flow algorithm has been implemented for radial distribution systems that converge very fast. The algorithm was tested on a standard IEEE123-bus distribution system. The algorithm converged and results were obtained. All KCL and KVL equations are satisfied on current and voltages so obtained. In this thesis, only three phase regulators with the same tap setting in all the three phases were modeled (equivalent to transformer) i.e., single phase and two phase regulators and three phase regulators with different tap settings in the three phases were not modeled . Hence, the branch currents of some phases and total system losses both kWh and kVAr showed some deviations from the results of the standard test system. However, the voltage magnitudes and phase angles matched with the results of the test system. The algorithm developed does not involve any matrix factorization or inversion for that matter and is hence computationally less costly.

## FUTURE SCOPE

Due to time constraints certain features of distribution networks could not be included in the model. Some of these listed below can be explored in future:

1. Lines with multiple earth wires
2. Street light modeling
3. Three phase regulators with different taps in different phases
4. Inclusion of Single phase and two phase regulators

# APPENDIX - A

## IEEE 123 node Test Feeder data

### Table 5: Regulator Data

**Regulator Data**

| | | | |
|---|---|---|---|
| Regulator ID: | 1 | | |
| Line Segment: | 150 - 149 | | |
| Location: | 150 | | |
| Phases: | A-B-C | | |
| Connection: | 3-Ph, Wye | | |
| Monitoring Phase: | A | | |
| Bandwidth: | 2.0 volts | | |
| PT Ratio: | 20 | | |
| Primary CT Rating: | 700 | | |
| Compensator: | Ph-A | | |
| R - Setting: | 3 | | |
| X - Setting: | 7.5 | | |
| Voltage Level: | 120 | | |
| | | | |
| Regulator ID: | 2 | | |
| Line Segment: | 9 - 14 | | |
| Location: | 9 | | |
| Phases: | A | | |
| Connection: | 1-Ph, L-G | | |
| Monitoring Phase: | A | | |
| Bandwidth: | 2.0 volts | | |
| PT Ratio: | 20 | | |
| Primary CT Rating: | 50 | | |
| Compensator: | Ph-A | | |
| R - Setting: | 0.4 | | |
| X - Setting: | 0.4 | | |
| Voltage Level: | 120 | | |

| | | | |
|---|---|---|---|
| Regulator ID: | 3 | | |
| Line Segment: | 25 - 26 | | |
| Location: | 25 | | |
| Phases: | A-C | | |
| Connection: | 2-Ph,L-G | | |
| Monitoring Phase: | A & C | | |
| Bandwidth: | 1 | | |
| PT Ratio: | 20 | | |
| Primary CT Rating: | 50 | | |
| Compenator: | Ph-A | Ph-C | |
| R - Setting: | 0.4 | 0.4 | |
| X - Setting: | 0.4 | 0.4 | |
| Voltage Level: | 120 | 120 | |
| | | | |
| Regulator ID: | 4 | | |
| Line Segment: | 160 - 67 | | |
| Location: | 160 | | |
| Phases: | A-B-C | | |
| Connection: | 3-Ph, LG | | |
| Monitoring Phase: | A-B-C | | |
| Bandwidth: | 2 | | |
| PT Ratio: | 20 | | |
| Primary CT Rating: | 300 | | |
| Compensator: | Ph-A | Ph-B | Ph-C |
| R - Setting: | 0.6 | 1.4 | 0.2 |
| X - Setting: | 1.3 | 2.6 | 1.4 |
| Voltage Level: | 124 | 124 | 124 |

## Table 6: Line Segment Data

### Line Segment Data

| Node A | Node B | Length (ft.) | Config. |
|---|---|---|---|
| 1 | 2 | 175 | 10 |
| 1 | 3 | 250 | 11 |
| 1 | 7 | 300 | 1 |
| 3 | 4 | 200 | 11 |
| 3 | 5 | 325 | 11 |
| 5 | 6 | 250 | 11 |
| 7 | 8 | 200 | 1 |
| 8 | 12 | 225 | 10 |
| 8 | 9 | 225 | 9 |
| 8 | 13 | 300 | 1 |
| 9 | 14 | 425 | 9 |
| 13 | 34 | 150 | 11 |
| 13 | 18 | 825 | 2 |
| 14 | 11 | 250 | 9 |
| 14 | 10 | 250 | 9 |
| 15 | 16 | 375 | 11 |
| 15 | 17 | 350 | 11 |
| 18 | 19 | 250 | 9 |
| 18 | 21 | 300 | 2 |
| 19 | 20 | 325 | 9 |
| 21 | 22 | 525 | 10 |
| 21 | 23 | 250 | 2 |
| 23 | 24 | 550 | 11 |
| 23 | 25 | 275 | 2 |
| 25 | 26 | 350 | 7 |
| 25 | 28 | 200 | 2 |
| 26 | 27 | 275 | 7 |
| 26 | 31 | 225 | 11 |
| 27 | 33 | 500 | 9 |
| 28 | 29 | 300 | 2 |
| 29 | 30 | 350 | 2 |
| 30 | 250 | 200 | 2 |
| 31 | 32 | 300 | 11 |
| 34 | 15 | 100 | 11 |
| 35 | 36 | 650 | 8 |
| 35 | 40 | 250 | 1 |
| 36 | 37 | 300 | 9 |
| 36 | 38 | 250 | 10 |
| 38 | 39 | 325 | 10 |
| 40 | 41 | 325 | 11 |
| 40 | 42 | 250 | 1 |
| 42 | 43 | 500 | 10 |
| 42 | 44 | 200 | 1 |
| 44 | 45 | 200 | 9 |
| 44 | 47 | 250 | 1 |
| 45 | 46 | 300 | 9 |
| 47 | 48 | 150 | 4 |
| 47 | 49 | 250 | 4 |
| 49 | 50 | 250 | 4 |
| 50 | 51 | 250 | 4 |
| 52 | 53 | 200 | 1 |
| 53 | 54 | 125 | 1 |
| 54 | 55 | 275 | 1 |
| 54 | 57 | 350 | 3 |
| 55 | 56 | 275 | 1 |
| 57 | 58 | 250 | 10 |
| 57 | 60 | 750 | 3 |
| 58 | 59 | 250 | 10 |
| 60 | 61 | 550 | 5 |
| 60 | 62 | 250 | 12 |
| 62 | 63 | 175 | 12 |
| 63 | 64 | 350 | 12 |
| 64 | 65 | 425 | 12 |
| 65 | 66 | 325 | 12 |
| 67 | 68 | 200 | 9 |
| 67 | 72 | 275 | 3 |
| 67 | 97 | 250 | 3 |
| 68 | 69 | 275 | 9 |
| 69 | 70 | 325 | 9 |
| 70 | 71 | 275 | 9 |
| 72 | 73 | 275 | 11 |
| 72 | 76 | 200 | 3 |
| 73 | 74 | 350 | 11 |
| 74 | 75 | 400 | 11 |
| 76 | 77 | 400 | 6 |
| 76 | 86 | 700 | 3 |
| 77 | 78 | 100 | 6 |
| 78 | 79 | 225 | 6 |
| 78 | 80 | 475 | 6 |
| 80 | 81 | 475 | 6 |
| 81 | 82 | 250 | 6 |
| 81 | 84 | 675 | 11 |
| 82 | 83 | 250 | 6 |
| 84 | 85 | 475 | 11 |
| 86 | 87 | 450 | 6 |
| 87 | 88 | 175 | 9 |
| 87 | 89 | 275 | 6 |

**Line Segment Data (Contd.)**

| | | | |
|---|---|---|---|
| 89 | 90 | 225 | 10 |
| 89 | 91 | 225 | 6 |
| 91 | 92 | 300 | 11 |
| 91 | 93 | 225 | 6 |
| 93 | 94 | 275 | 9 |
| 93 | 95 | 300 | 6 |
| 95 | 96 | 200 | 10 |
| 97 | 98 | 275 | 3 |
| 98 | 99 | 550 | 3 |
| 99 | 100 | 300 | 3 |
| 100 | 450 | 800 | 3 |
| 101 | 102 | 225 | 11 |
| 101 | 105 | 275 | 3 |
| 102 | 103 | 325 | 11 |
| 103 | 104 | 700 | 11 |
| 105 | 106 | 225 | 10 |
| 105 | 108 | 325 | 3 |
| 106 | 107 | 575 | 10 |
| 108 | 109 | 450 | 9 |
| 108 | 300 | 1000 | 3 |
| 109 | 110 | 300 | 9 |
| 110 | 111 | 575 | 9 |
| 110 | 112 | 125 | 9 |
| 112 | 113 | 525 | 9 |
| 113 | 114 | 325 | 9 |
| 135 | 35 | 375 | 4 |
| 149 | 1 | 400 | 1 |
| 152 | 52 | 400 | 1 |
| 160 | 67 | 350 | 6 |
| 197 | 101 | 250 | 3 |

**Three Phase Switches**

| Node A | Node B | Normal |
|---|---|---|
| 13 | 152 | closed |
| 18 | 135 | closed |
| 60 | 160 | closed |
| 61 | 610 | closed |
| 97 | 197 | closed |
| 150 | 149 | closed |
| 250 | 251 | open |
| 450 | 451 | open |
| 54 | 94 | open |
| 151 | 300 | open |
| 300 | 350 | open |

**Table 10: Three Phase Switches Data**

**Table 7 :**

**Overhead Line Configurations (Config.)**

| Config. | Phasing | Phase Cond. ACSR | Neutral Cond. ACSR | Spacing ID |
|---|---|---|---|---|
| 1 | A B C N | 336,400 26/7 | 4/0 6/1 | 500 |
| 2 | C A B N | 336,400 26/7 | 4/0 6/1 | 500 |
| 3 | B C A N | 336,400 26/7 | 4/0 6/1 | 500 |
| 4 | C B A N | 336,400 26/7 | 4/0 6/1 | 500 |
| 5 | B A C N | 336,400 26/7 | 4/0 6/1 | 500 |
| 6 | A C B N | 336,400 26/7 | 4/0 6/1 | 500 |
| 7 | A C N | 336,400 26/7 | 4/0 6/1 | 505 |
| 8 | A B N | 336,400 26/7 | 4/0 6/1 | 505 |
| 9 | A N | 1/0 | 1/0 | 510 |
| 10 | B N | 1/0 | 1/0 | 510 |
| 11 | C N | 1/0 | 1/0 | 510 |

**Underground Line Configuration (Config.)**

| Config. | Phasing | Cable | Spacing ID |
|---|---|---|---|
| 12 | A B C | 1/0 AA, CN | 515 |

**Transformer Data**

| | kVA | kV-high | kV-low | R - % | X - % |
|---|---|---|---|---|---|
| Substation | 5,000 | 115 - D | 4.16 Gr-W | 1 | 8 |
| XFM - 1 | 150 | 4.16 - D | .480 - D | 1.27 | 2.72 |

**Table 8: Transformer Data**

**Shunt Capacitors**

| Node | Ph-A kVAr | Ph-B kVAr | Ph-C kVAr |
|---|---|---|---|
| 83 | 200 | 200 | 200 |
| 88 | 50 | | |
| 90 | | 50 | |
| 92 | | | 50 |
| Total | 250 | 250 | 250 |

**Table 9: Shunt Capacitor Data**

## Table 11: Spot Loads Data

| **Spot Loads** | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| Node | Load Model | Ph-1 kW | Ph-1 kVAr | Ph-2 kW | Ph-2 kVAr | Ph-3 kW | Ph-4 kVAr |
| 1 | Y-PQ | 40 | 20 | 0 | 0 | 0 | 0 |
| 2 | Y-PQ | 0 | 0 | 20 | 10 | 0 | 0 |
| 4 | Y-PQ | 0 | 0 | 0 | 0 | 40 | 20 |
| 5 | Y-I | 0 | 0 | 0 | 0 | 20 | 10 |
| 6 | Y-Z | 0 | 0 | 0 | 0 | 40 | 20 |
| 7 | Y-PQ | 20 | 10 | 0 | 0 | 0 | 0 |
| 9 | Y-PQ | 40 | 20 | 0 | 0 | 0 | 0 |
| 10 | Y-I | 20 | 10 | 0 | 0 | 0 | 0 |
| 11 | Y-Z | 40 | 20 | 0 | 0 | 0 | 0 |
| 12 | Y-PQ | 0 | 0 | 20 | 10 | 0 | 0 |
| 16 | Y-PQ | 0 | 0 | 0 | 0 | 40 | 20 |
| 17 | Y-PQ | 0 | 0 | 0 | 0 | 20 | 10 |
| 19 | Y-PQ | 40 | 20 | 0 | 0 | 0 | 0 |
| 20 | Y-I | 40 | 20 | 0 | 0 | 0 | 0 |
| 22 | Y-Z | 0 | 0 | 40 | 20 | 0 | 0 |
| 24 | Y-PQ | 0 | 0 | 0 | 0 | 40 | 20 |
| 28 | Y-I | 40 | 20 | 0 | 0 | 0 | 0 |
| 29 | Y-Z | 40 | 20 | 0 | 0 | 0 | 0 |
| 30 | Y-PQ | 0 | 0 | 0 | 0 | 40 | 20 |
| 31 | Y-PQ | 0 | 0 | 0 | 0 | 20 | 10 |
| 32 | Y-PQ | 0 | 0 | 0 | 0 | 20 | 10 |
| 33 | Y-I | 40 | 20 | 0 | 0 | 0 | 0 |
| 34 | Y-Z | 0 | 0 | 0 | 0 | 40 | 20 |
| 35 | D-PQ | 40 | 20 | 0 | 0 | 0 | 0 |
| 37 | Y-Z | 40 | 20 | 0 | 0 | 0 | 0 |
| 38 | Y-I | 0 | 0 | 20 | 10 | 0 | 0 |
| 39 | Y-PQ | 0 | 0 | 20 | 10 | 0 | 0 |
| 41 | Y-PQ | 0 | 0 | 0 | 0 | 20 | 10 |
| 42 | Y-PQ | 20 | 10 | 0 | 0 | 0 | 0 |
| 43 | Y-Z | 0 | 0 | 40 | 20 | 0 | 0 |
| 45 | Y-I | 20 | 10 | 0 | 0 | 0 | 0 |
| 46 | Y-PQ | 20 | 10 | 0 | 0 | 0 | 0 |
| 47 | Y-I | 35 | 25 | 35 | 25 | 35 | 25 |
| 48 | Y-Z | 70 | 50 | 70 | 50 | 70 | 50 |
| 49 | Y-PQ | 35 | 25 | 70 | 50 | 35 | 25 |
| 50 | Y-PQ | 0 | 0 | 0 | 0 | 40 | 20 |
| 51 | Y-PQ | 20 | 10 | 0 | 0 | 0 | 0 |
| 52 | Y-PQ | 40 | 20 | 0 | 0 | 0 | 0 |
| 53 | Y-PQ | 40 | 20 | 0 | 0 | 0 | 0 |
| 55 | Y-Z | 20 | 10 | 0 | 0 | 0 | 0 |
| 56 | Y-PQ | 0 | 0 | 20 | 10 | 0 | 0 |
| 58 | Y-I | 0 | 0 | 20 | 10 | 0 | 0 |
| 59 | Y-PQ | 0 | 0 | 20 | 10 | 0 | 0 |
| 60 | Y-PQ | 20 | 10 | 0 | 0 | 0 | 0 |
| 62 | Y-Z | 0 | 0 | 0 | 0 | 40 | 20 |
| 63 | Y-PQ | 40 | 20 | 0 | 0 | 0 | 0 |
| 64 | Y-I | 0 | 0 | 75 | 35 | 0 | 0 |
| 65 | D-Z | 35 | 25 | 35 | 25 | 70 | 50 |
| 66 | Y-PQ | 0 | 0 | 0 | 0 | 75 | 35 |
| 68 | Y-PQ | 20 | 10 | 0 | 0 | 0 | 0 |
| 69 | Y-PQ | 40 | 20 | 0 | 0 | 0 | 0 |
| 70 | Y-PQ | 20 | 10 | 0 | 0 | 0 | 0 |
| 71 | Y-PQ | 40 | 20 | 0 | 0 | 0 | 0 |
| 73 | Y-PQ | 0 | 0 | 0 | 0 | 40 | 20 |
| 74 | Y-Z | 0 | 0 | 0 | 0 | 40 | 20 |
| 75 | Y-PQ | 0 | 0 | 0 | 0 | 40 | 20 |
| 76 | D-I | 105 | 80 | 70 | 50 | 70 | 50 |
| 77 | Y-PQ | 0 | 0 | 40 | 20 | 0 | 0 |
| 79 | Y-Z | 40 | 20 | 0 | 0 | 0 | 0 |
| 80 | Y-PQ | 0 | 0 | 40 | 20 | 0 | 0 |
| 82 | Y-PQ | 40 | 20 | 0 | 0 | 0 | 0 |
| 83 | Y-PQ | 0 | 0 | 0 | 0 | 20 | 10 |
| 84 | Y-PQ | 0 | 0 | 0 | 0 | 20 | 10 |
| 85 | Y-PQ | 0 | 0 | 0 | 0 | 40 | 20 |
| 86 | Y-PQ | 0 | 0 | 20 | 10 | 0 | 0 |
| 87 | Y-PQ | 0 | 0 | 40 | 20 | 0 | 0 |
| 88 | Y-PQ | 40 | 20 | 0 | 0 | 0 | 0 |
| 90 | Y-I | 0 | 0 | 40 | 20 | 0 | 0 |
| 92 | Y-PQ | 0 | 0 | 0 | 0 | 40 | 20 |
| 94 | Y-PQ | 40 | 20 | 0 | 0 | 0 | 0 |
| 95 | Y-PQ | 0 | 0 | 20 | 10 | 0 | 0 |
| 96 | Y-PQ | 0 | 0 | 20 | 10 | 0 | 0 |
| 98 | Y-PQ | 40 | 20 | 0 | 0 | 0 | 0 |
| 99 | Y-PQ | 0 | 0 | 40 | 20 | 0 | 0 |
| 100 | Y-Z | 0 | 0 | 0 | 0 | 40 | 20 |
| 102 | Y-PQ | 0 | 0 | 0 | 0 | 20 | 10 |
| 103 | Y-PQ | 0 | 0 | 0 | 0 | 40 | 20 |
| 104 | Y-PQ | 0 | 0 | 0 | 0 | 40 | 20 |
| 106 | Y-PQ | 0 | 0 | 40 | 20 | 0 | 0 |
| 107 | Y-PQ | 0 | 0 | 40 | 20 | 0 | 0 |
| 109 | Y-PQ | 40 | 20 | 0 | 0 | 0 | 0 |
| 111 | Y-PQ | 20 | 10 | 0 | 0 | 0 | 0 |
| 112 | Y-I | 20 | 10 | 0 | 0 | 0 | 0 |
| 113 | Y-Z | 40 | 20 | 0 | 0 | 0 | 0 |
| 114 | Y-PQ | 20 | 10 | 0 | 0 | 0 | 0 |
| Total | | 1420 | 775 | 915 | 515 | 1155 | 635 |

## Impedance Data:

**Configuration 1:**

Z (R +jX) in ohms per mile

| | | | | | |
|---|---|---|---|---|---|
| 0.4576 | 1.0780 | 0.1560 | 0.5017 | 0.1535 | 0.3849 |
| | | 0.4666 | 1.0482 | 0.1580 | 0.4236 |
| | | | | 0.4615 | 1.0651 |

B in micro Siemens per mile

| | | |
|---|---|---|
| 5.6765 | -1.8319 | -0.6982 |
| | 5.9809 | -1.1645 |
| | | 5.3971 |

**Configuration 2:**

Z (R +jX) in ohms per mile

| | | | | | |
|---|---|---|---|---|---|
| 0.4666 | 1.0482 | 0.1580 | 0.4236 | 0.1560 | 0.5017 |
| | | 0.4615 | 1.0651 | 0.1535 | 0.3845 |
| | | | | 0.4576 | 1.0780 |

B in micro Siemens per mile

| | | |
|---|---|---|
| 5.9809 | -1.1645 | -1.8319 |
| | 5.3971 | -0.6982 |
| | | 5.6765 |

**Configuration 3:**

Z (R +jX) in ohms per mile

| | | | | | |
|---|---|---|---|---|---|
| 0.4615 | 1.0651 | 0.1535 | 0.3849 | 0.1580 | 0.4236 |
| | | 0.4576 | 1.0780 | 0.1560 | 0.5017 |
| | | | | 0.4666 | 1.0482 |

B in micro Siemens per mile

| | | |
|---|---|---|
| 5.3971 | -0.6982 | -1.1645 |
| | 5.6765 | -1.8319 |
| | | 5.9809 |

**Configuration 4:**

Z (R +jX) in ohms per mile

| | | | | | |
|---|---|---|---|---|---|
| 0.4615 | 1.0651 | 0.1535 | 0.3849 | 0.1580 | 0.4236 |
| | | 0.4666 | 1.0482 | 0.1560 | 0.5017 |
| | | | | 0.4576 | 1.0780 |

B in micro Siemens per mile

| | | |
|---|---|---|
| 5.3971 | -1.1645 | -0.6982 |
| | 5.9809 | -1.8319 |
| | | 5.6765 |

**Configuration 5:**

Z (R +jX) in ohms per mile

| | | | | | |
|---|---|---|---|---|---|
| 0.4666 | 1.0482 | 0.1560 | 0.5017 | 0.1580 | 0.4236 |
| | | 0.4576 | 1.0780 | 0.1535 | 0.3849 |
| | | | | 0.4615 | 1.0651 |

B in micro Siemens per mile

| | | |
|---|---|---|
| 5.9809 | -1.8319 | -1.1645 |
| | 5.6765 | -0.6982 |
| | | 5.3971 |

**Configuration 6**:

Z (R +jX) in ohms per mile

| | | | | | |
|---|---|---|---|---|---|
| 0.4576 | 1.0780 | 0.1535 | 0.3849 | 0.1560 | 0.5017 |
| | | 0.4615 | 1.0651 | 0.1580 | 0.4236 |
| | | | | 0.4666 | 1.0482 |

B in micro Siemens per mile

| | | |
|---|---|---|
| 5.6765 | -0.6982 | -1.8319 |
| | 5.3971 | -1.1645 |
| | | 5.9809 |

**Configuration 7:**

Z (R +jX) in ohms per mile

| | | | | | |
|---|---|---|---|---|---|
| 0.4576 | 1.0780 | 0.0000 | 0.0000 | 0.1535 | 0.3849 |
| | | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | | | | 0.4615 | 1.0651 |

B in micro Siemens per mile

| | | |
|---|---|---|
| 5.1154 | 0.0000 | -1.0549 |
| | 0.0000 | 0.0000 |
| | | 5.1704 |

**Configuration 8:**

Z (R +jX) in ohms per mile

| | | | | | |
|---|---|---|---|---|---|
| 0.4576 | 1.0780 | 0.1535 | 0.3849 | 0.0000 | 0.0000 |
| | | 0.4615 | 1.0651 | 0.0000 | 0.0000 |
| | | | | 0.0000 | 0.0000 |

B in micro Siemens per mile

| | | |
|---|---|---|
| 5.1154 | -1.0549 | 0.0000 |
| | 5.1704 | 0.0000 |
| | | 0.0000 |

**Configuration 9:**

Z (R +jX) in ohms per mile

| | | | | | |
|---|---|---|---|---|---|
| 1.3292 | 1.3475 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | | | | 0.0000 | 0.0000 |

B in micro Siemens per mile

| | | |
|---|---|---|
| 4.5193 | 0.0000 | 0.0000 |
| | 0.0000 | 0.0000 |
| | | 0.0000 |

**Configuration 10:**

Z (R +jX) in ohms per mile

| | | | | | |
|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | | 1.3292 | 1.3475 | 0.0000 | 0.0000 |
| | | | | 0.0000 | 0.0000 |

B in micro Siemens per mile

| | | |
|---|---|---|
| 0.0000 | 0.0000 | 0.0000 |
| | 4.5193 | 0.0000 |
| | | 0.0000 |

**Configuration 11:**

Z (R +jX) in ohms per mile

| | | | | | |
|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | | | | 1.3292 | 1.3475 |

B in micro Siemens per mile

| | | |
|---|---|---|
| 0.0000 | 0.0000 | 0.0000 |
| | 0.0000 | 0.0000 |
| | | 4.5193 |

**Configuration 12:**

Z (R +jX) in ohms per mile

| | | | | | |
|---|---|---|---|---|---|
| 1.5209 | 0.7521 | 0.5198 | 0.2775 | 0.4924 | 0.2157 |
| | | 1.5329 | 0.7162 | 0.5198 | 0.2775 |
| | | | | 1.5209 | 0.7521 |

B in micro Siemens per mile

| | | |
|---|---|---|
| 67.2242 | 0.0000 | 0.0000 |
| | 67.2242 | 0.0000 |
| | | 67.2242 |

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