Design and Development of Reliable Capacitated Facility Location for Distribution Network Design

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CERTIFICATE

I hereby certify that the matter which is being presented in the dissertation titled,"Design and Development ofReliable capacitated Facility Location for Distribution Network Design" in partial fulfillment of the requirements for the award of degree of Master of Technology in Computer Science and Applications submitted in Computer Science and Engineering Department of Thapar University, Patiala, is an authentic record of my own work carried out under the supervision of Dr. Vijay Kumar and refers other researcher's work which are duly listed in the reference section.

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The matter presented in the thesis has not been submitted for award of any other degree to this or any other University.

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ABSTRACT

The most of distribution centers allocation is often implicitly considered unaffected from any disruptions. However, the distribution centers may fail due to natural or economic factors. The reliability is an important factor to averse disruptions in distribution centers. Design of distribution centers with consideration of reliability is beneficial for long-term investment.

The aim of this dissertation is to propose a model for reliable distribution centers (DCs) in case of unexpected disruption. The site-dependent failure probabilities and random link disruptions have also utilized in the proposed model. The proposed model has considered three costs such as construction cost, transportation cost and improvising cost. The mixed-integer linear programming model (MILP) has been formulated to provide reliable DCs in case of random failures.

Three solution approach lagrangian relaxation, cross decomposition and firefly have been utilized to implement the proposed model. These approaches have been tested on randomly generated datasets for 200 customers and 30 DCs. The experimental results reveal that cross decomposition approach is applied to obtain better results in less time. However, cross decomposition approach performance is not always better than firefly algorithm due to the time required to obtain the cross decomposition bound.

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LIST OF ABBREVATIONS

CFLP	Capacitated Facility Location problem
CD	Cross Decomposition
DC	Distribution Center
FA	Firefly Algorithm
FLP	Facility Location problem
LR	Lagrangian Relaxation
RFLDP	Reliable Facility Location with Distribution Protection
UFLP	Un-Capacitated Facility Location problem

Chapter I

Introduction

This chapter provides a brief introduction about the topic of facility location problems. It subsequently presents the concepts about design of distribution network and need of reliability which are important for the research of this thesis.

1.1. Facility Location Problem

Location-specific fixed costs and shipping costs often play as major component in the overall product cost decisions. The Facility Location Problem (FLP), introduced by Balinski [1], consists of selecting locations for facilities and assignment of customers between facilities which satisfy the demand within the minimum possible cost. It is also known as location analysis problem. FLP is about selecting profitable, yet efficient location for opening of a facility and assigning of customers.

The basic facility location problem is to determine a facility location from a set of potential locations to fulfill customers demands in minimum cost.The inaugural of which facilities and by assigning customers to which open facilities for service of their demands, is a strategic decision [2]. By opening facilities, some fixed costs incurred to open/operate these facilities. Only after opening facilities, customers can be assigned to fulfill their demands. A second cost is incurred when we assign these customers/ retailers to facilities. This is the assignment costs; which may cover production and transportation costs of goods. It has to be a perfect balance between fixed cost of opening and assignment cost. If more facilities are opened, the distance between facility and assignment of customers can be decreased. That will result in decrease of assignment costs. Nevertheless, opening more facilities will surely escalate the fixed costs. This complexity emanate from various qualitative (i.e., responsiveness, proximity) and quantitative (i.e., stochastic, deterministic) factors that affect location decisions. The qualitative and quantitative of the FLP are dependent upon the organization preferences. The location of facilities so that the sum of the fixed costs of opening new facilities and variable costs of assigning customers to certain demands points is minimized [3], is the classical warehouse location problem.

Suppose that there are n possible places for facility location and m customer demand points, then following notions can be used:

$$y_j = \begin{cases} 1; & \text{if faciltiy } j \text{ is opened} \\ 0; & \text{otherwise} \end{cases}$$

$$x_{ij} = \begin{cases} 1; & \text{if customer } i \text{ is served by faciltiy } j \\ 0; & \text{otherwise} \end{cases}$$

c_{ij} - shipping cost of a unit product from facility i to customer area j

f_j - fixed cost of setting up a facility j.

d_i - demand generated at customer area i

It can be formulated as follows:

$$min\left(\sum_{i=1}^{m} f_i y_i + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} \right) \tag{1.1}$$

subject to

$$\sum_{i=1}^{m} x_{ij} = 1, \quad i = 1,\dots,m,\ j = 1,\dots,n \tag{1.2}$$

$$\sum_{j=1}^{n} d_i x_{ij} \le y_j, \quad i = 1,\dots,m,\ j = 1,\dots,n \tag{1.3}$$

$$x_{ij} \in \{0,1\}, y_i \in \{0,1\} \tag{1.4}$$

The objective function (1.1) represents the total fixed and assignment costs. The constraint (1.2) ensures that the demands must be satisfied. The constraint (1.3) guarantees that customer demand can be satisfied and shipped only from the opened facility sites. The constraints (1.4) are integrality constraints.

The FLP variants can be classified as uncapacitated and capacitated. The basic FLP mathematical formulation described above is known as Uncapacitated Facility Location Problem (UFLP) or Simple Plant Location Problem. As it is implied from the name itself, capacity of a facility is considered unlimited. It only considerers fixed costs and shipping costs in the formulation. Although, it is still an NP-hard problem. The second variant is Capacitated Facility Location problem (CFLP). This problem assumes each facility can produce/transport limited quantity of product. The capacity constraint can be added as upper limit on the quantity of demand can be served by a facility. Although mathematical formulation of CFLP is not very different, but solving it is much difficult. This is due to fact that the demand of a single customer can be divided across multiple facilities and this destroys the tightness of LP relaxations of the problem. The most efficient methods for CFLP are Lagrangian relaxation methods and the matrix column generation method. By combing above Equation (1.1) and capacity constraint (1.8), the mathematical formulation of CFLP is given below:

Suppose that there are *n* possible places for facility location and *m* customer demand points. The objective of CFLP is given below:

$$min\left(\sum_{i=1}^{m} f_i y_i + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}\right) \tag{1.5}$$

subject to

$$\sum_{i=1}^{m} x_{ij} = 1 \ , \quad i = 1,\ldots,m,\ j = 1,\ldots,n \tag{1.6}$$

$$x_{ij} \le y_j \,, \quad i = 1,\ldots,m,\ j = 1,\ldots,n \tag{1.7}$$

$$\sum_{j=1}^{n} d_i x_{ij} \le S_j y_j \,, \quad i = 1,\ldots,m,\ j = 1,\ldots,n \tag{1.8}$$

$$x_{ij} \in \{0,1\}, y_j \in \{0,1\} \tag{1.9}$$

Where S_j - maximum supply by facility i.

The objective function (1.6) represents the total fixed and assignment costs. The constraint (1.7) ensures that the demands must be satisfied. The Constraint (1.8) provides tighter Linear Programming relaxation. It guarantees demands must be less than supply. The constraint (1.8) implies total supply at facility I must be greater than total customer demands.

It is limited by S_j . The constraints (1.9) are integrality constraints.

The Uncapacitated and Capacitated FLP forms can be extended to various variations. These variations reliant on three factors. The first factor is the objective function such as minisum [4], minimax[5], maximum demand covering[6]. The second factor is time phasing scenarios or planning horizon (static[7], dynamic[8]). The third factor depends on the basis of types of parameter known are certain or uncertain (probabilistic [9], deterministic[10]). The main focus of this thesis is on minisum capacitated FLP under uncertainity.

In broad sense, facility in supply chain management could be physical manufacturing/service unit or a distribution/redistribution center that must be opened/operated to serve demands. Facility location decision is a strategic approach for organizations for selection of potential locations for plants, warehouse and distribution centers [11]. The next section introduces the integration FLP with distribution centers.

1.2. Distribution Network

Decisions on distribution network design involves the investment of substantial capital that has long-term impact on supply of products. It probably makes one of the decisive factor to determine success or failure of the related business in the long run. The key to success of an organization lies in making right location choices that will either maximize their profits or minimize their costs. The location decisions are dependent upon tangible or intangible factors of an organization. But, establishing a distribution center is a major cost component and organizations incur billions of dollars to establish new facilities each year, it is very crucial for companies to make good choice in this area.

Distribution centers (DC) plays pivotal role in the supply chain for delivery of products through cost-minimal solution by taking into consideration accessibility of customers. Finding the optimal DC location is a problem that occurs in all stages of the supply chain planning process. It is an important aspect in planning a new supply chain from scratch. Organizations entering new markets and need to establish a distribution structure with warehouses in order to ensure a smooth distribution. Design of Distribution system involves not only the shipping of goods from one destination to other, but also the decisions as follows:

i. Quantity to produce at each site,
ii. Quantity of products to hold in inventory for distribution,
iii. Location of distribution centers to minimize delivery time.

By incorporating above decisions factors in distribution design, demand anticipation for longer periods can be made with accuracy. Distribution networks (DN) design is a critical component in supply chain management. The major objectives of facility planning while designing distribution networks are:

i. Improve customer satisfaction conforming to customer promises,
ii. Increase return on assets,
iii. Maximize speed for efficient service level.

Efficiency is an important concept for doing business in the competitive markets of modern times, and its importance increases with the globalization of these markets. The efficiency of DN design is measured by its performance and robustness. The performance measures can be classified as qualitative and quantitative. Customer contentment, flexibility, and management under disruptions of facilitates are the measures to judge the quality of network design. And the measures that optimally and feasibly maximize or minimize objective functions defined by mathematical model are termed as quantitative.

Figure 1 demonstrates the basic facility location in distribution network design. In figure, $I = \{1,...,\text{m}\}$, a set of possible facility locations, each with a maximum capacity $a_i, i \in I$, and $J = \{1,...,\text{n}\}$, a set of customers, each with a demand $b_j, j \in J$, is assumed. None of the DCs are open, total cost is composed of fixed costs $f_i, i \in I$, , whenever DC i is open, and by the cost $c_{ij}, i \in I, j \in J$ of assigning each customer to one DC. The objective of this model is to minimize overall cost including facility installation cost and shipping costs.

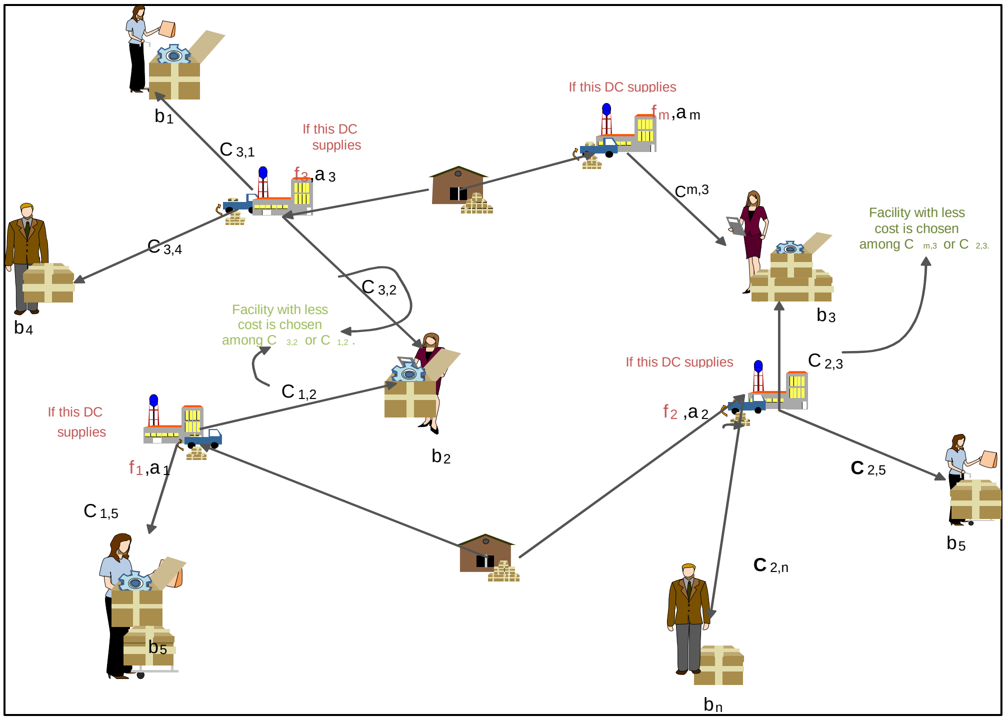

Figure 1.1 Distribution Network Design

Considering:

f_i fixed cost of setting up a DC at location i.

c_{ij} shipping cost of a unit product from location i to customer area j

b_j demand generated at customer area j

a_i Maximum capacity of DC i

$$y_i = \begin{cases} 1; \text{if DC } i \text{ is opened,} \\ 0; \text{otherwise.} \end{cases}$$

x_{ij} = the amount of products shipped from DC i to customer area j

It can be formulated as follows:

$$\min_{y_i, x_{ij}} \left(\sum_{i=1}^{m} f_i y_i + \sum_{i=1}^{m} \sum_{j=1}^{m} c_{ij} x_{ij} b_i \right) \tag{1.10}$$

subject to

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,\ldots,n \tag{1.11}$$

$$x_{ij} \le y_i, \quad i = 1,\ldots,m; \; j = 1,\ldots,n \tag{1.12}$$

$$\sum_{j=1}^{n} b_j x_{ij} \leq a_i y_i, \quad i = 1, \ldots, m \tag{1.13}$$

$$x_{ij} \geq 0, \quad i = 1, \ldots, m, \; j = 1, \ldots, n \tag{1.14}$$

$$y_i \in \{0, 1\}, i = 1, \ldots, m, \tag{1.15}$$

The objective function (1.10) minimizes the total cost of establishing a DC and shipment cost of products. Constraints (1.11) ensures that the total shipped products at site j must be able to satisfy customer demands at site j. The Constraint (1.12) provides tighter Linear Programming relaxation. It guarantees demands must be less than supply. The constraint (1.13) implies total supply at DC i must be greater than total customer demands. It is limited by capacity a_i. Constraint (1.14) and Constraint (1.15) are integrity constraint.

The characteristics, derived by supply chain actual requirements with regard to distribution centers, are a hierarchical setup, economies of scale in transportation cost, service time requirements and dynamic aspects like facility failure, additionally play an important role, as this allows companies to plan adaptations to their structures in the course of time. In the early studies on the facility location design problem, the facilities, once built, were assumed to remain operational during organization planning horizon. But in real scenario, the facilities may fail occasionally due to random disruptions. Some prominent happenings, such as September 11, 2001 terrorist attacks has disturbed many industries, 2002 west-coast port lockout for 10 days has made US to regain the work in 100 days, and Hurricane Katrina in 2005, has made facilities under disruptions topic for research [11].

Thus, decision makers can no longer ignore the influence of sensitive factors such as the population status of a candidate region, transportation conditions, market surroundings, location properties and cost factors related the alternative location. Moreover, the process could become highly judgmental if a wide variety of qualitative factors are present. In such cases, the selection process may lack consistency and flexibility. This makes overall decision for planning distribution network very complex.

1.2.1. Reliability in Distribution Network

Although, disruptions in a supply chain network are very rare. But, once a facility is disrupted, there is very little recourse regarding supply chain infrastructure because these strategic decisions cannot be altered quickly. Disruptions in a supply chain network have a numerous causes and may take numerous forms. These disruptions can be either man-made such as worker strikes, terrorist attacks, equipment breakdowns or natural disasters such as Floods, earthquakes, hurricanes. These disruptions substantially increase both the service costs and customer dissatisfaction. Due to disruption, customers demand may be delayed or even abandoned, which may lead to higher transportation costs, order delays, or a loss of market shares. The following are the case study that justify the need of reliability in distribution network.

In 2001, the performance of a semiconductor factory (belonging to Philips) in New Mexico, USA was halted by 8-minute fire. No work was able to perform for 9 months. It caused one of its customers, Ericsson, to lose USD 2.34 billion. Japan was struck by an earthquake with 8.9 magnitude in 2011 that resulted in tsunami. It put Japan into devastating situation. As Japanese supplier were unable to supply the materials, various production lines with international companies were discontinued for long periods. Also, plants of several industries were adversely affected [12].

Organizations entering new markets and need to establish a distribution structure which ensures a smooth distribution even in disruption conditions. Wal-Mart has an emergency operations center dedicated to preparing for and mitigating the effects of man-made and natural disasters. It is just as important for re-evaluating existing supply chains. The environment is not stable over a period of years, which requires adaptation of the existing network structure for distribution. Even a small disruption can have a devastating impact as it cascades through a supply chain. With the acquisition of reliability factor, while designing a network for the distribution system, can increase the efficiency and robustness of the design.

1.4. Lagrangian Relaxation

Lagrangian relaxation is a technique suitable for separating the constraints into two sets. First, good constraints, with which the problem can be solved easily. Second, complex constraints that makes the problem hard to solve. The main idea is to relax the problem by removing the complex constraints and putting them into the objective function, assigned with weights (the Lagrangian multiplier). The initial value of Lagrangian multiplier is zero which is calculated at each iteration. Each weight represents a penalty which is added to a solution that does not satisfy the particular constraint [13, 14].

1.5. Cross Decomposition

Cross decomposition was developed by Van Roy [15, 16]. It combines Benders decomposition and Lagrangian relaxation into single framework. Van Roy also claims that in all problems tested his method obtains in just a few decomposition iterations tight lower and upper bounds that differ by no more than 0.5%. Holmberg [17, 18] generalizes the concept of cross decomposition to pure integer problems and studies the lower bounds on the optimal objective function value of pure integer programming problems. The results shown that generalized Benders decomposition and generalized cross decomposition yield the best lower bound for pure integer problems. He also shows that while either ordinary Benders decomposition or Lagrangian relaxation may yield the best lower bound for a particular problem, cross decomposition can automatically yield the best of these bounds.

1.6. Firefly algorithm

Firefly algorithm (FA) is a metaheuristics approach based on luminescent flash, produced by bioluminescence, developed by Yang [19, 20]. The imitation of their rhythmic flash and synchronized behavior for communication via degree of flashing is done in algorithm. Female fireflies react to a male’s distinct rate of flashing in the same species, while in some species female fireflies can mimic the mating flashing pattern of other species so as to lure and eat the male fireflies who may mistake the flashes as a potential suitable mate. When a medium with light absorption coefficient γ, I varies with distance r

$$I = I_o e^{-\gamma r} \tag{1.16}$$

Where I_0 is associated with objective function value at $r = 0$.We know that the light intensity at a particular distance D from the light source obeys the inverse square law. That is to say, the light intensity I decreases as distance D increases in terms of I ∞ 1/D2. Distance between two fireflies can be defined by Cartesian product as

$$r_{ij} = \|x_i - x_j\| = \sqrt{(\sum_{d=1}^{D} x_{ik} - x_{jk})^2} \tag{1.17}$$

Here x_{ik} is the k_{th} dimension of the spatial coordinate x_i of the i_{th} firefly. Furthermore, the air absorbs light which becomes weaker and weaker as the distance increases. These two combined factors make most fireflies visible only to a limited distance, usually several hundred meters at night, which is good enough for fireflies to communicate.

1.6. Research Outline

This thesis deals with two aspects of reliability in distribution networks design. The first aspect is to consider the further advancement of existing models in the application of distribution network design. As less investigated area in FLP with DN design is Capacitated Facility Location Problems in distribution networks under disruption. This thesis studies a reliability issue in distribution network design. The focus is to provide the reliability for a facility by protecting some facilities, so that customers demands can be satisfied from protected ones in case of full disruptions. Hence, the model Reliable facility Location with Distribution Protection, RFLDP. Instead of making every facility protected and to counterbalance the investments, two types of facilities can be designed; the unreliable regular ones and the reliable protected ones. The second aspect is integrate the model with link disruptions. The disruptions are site-specific as disruption at one site is independent of others. The backup mechanism is provided in case of link failure so that each customer is served. The objective of RFLDP is to determine the facility location and protection decisions as well as customer assignments to minimize the fixed charges and expected service cost.

1.7. Dissertation layout

The remainder of the dissertation is outlined as follows:

- Chapter 2 summarize the literature by evaluating prior work done in the field of Facility Location in context of Distribution network. It also discuss the work referred to reliability context to analyze the gaps and propose the model.
- Chapter 3 discusses the research problem, formulates it as the mixed-integer linear programming model. The model formulation based on the gaps identified in the literature for the Reliability in Distribution center is proposed.
- Chapter 4 describes the detailed proposed approach for solving proposed model. First the Lagrangian Relaxation approach is discussed. Firefly algorithm is discussed followed by cross decomposition approach.
- Chapter 5 presents the test results performed on the proposed model. This chapter describes the tools and dataset used. The Performance evaluation on basis of test results is presented.
- Chapter 6 concludes the work done. This chapter describes the recommendations for future work in the field.

Chapter II

Literature Survey

The focus of this chapter is on prior work done in area of facility location problems and distribution network design. It subsequently surveys the earlier work done related to reliability of distribution network.

2.1. Facility Location Problems

Facility location problems (FLP) date back to the 17th century. Pierre de Fermat (1601-1665) is normally accredited to be the first one, who stated the problem in written form: "given three points in the plane, find a fourth point such that the sum of its distances to the three given points is a minimum" (Kuhn, 1967 as cited in [21]). The facility location problem is both a classical optimization and a fundamental problem. It is an extensive research problem studied by researchers over the past several decades due to its application in various areas [22-23]. The original problem have been modified and formulated for different industries and researched over the years. However, the most important reason is that there is no general location model, which can be applied to all potential or existing applications. The objective function, the constraints and variables look different, depending on the context and application they are applied in [24].

The models presented in section 1.1 form the basis for plenty of other facility location models that have been formulated. Extensions to those models include, but are not limited to, multi commodity settings, hierarchical setups, and capacitated versions, approaches dealing with uncertainty, multi-objective optimization and dynamic setups. The literature on facility location problems is extensive. Comprehensive reviews and surveys on models with those extensions can be found in [25-31].

2.2. Distribution Network Design

Location decisions are relevant to individuals and organizations in many aspects of daily life. They are of long-term, strategic nature and they can impose economic externalities, such as pollution or noise. It is furthermore very hard to solve location models optimally, especially when they grow in size.

As location decisions are being made by focusing only on organization budget constraints, not giving much attention to long-term profit maximizing objective [22]. Klose and Drexel [32] reviewed the contributions of the current distribution system design and focused on the continuous location models, network location models, mixed-integer programming models and their applications. The goal of this paper is to survey the models and approaches used in designing the distribution network for facility location. The key concepts and techniques arising from facility location are explained in detail. Fahimnia et al. [33] considered distribution models on basis of their complexity in their designs in the recent review. This degree of complexity is described by the number of products, plants, warehouses, end users, transport paths and time periods.

As the introduction pointed out, businesses impose various requirements towards their distribution structures. The basic mathematical formulation is a mixed integer linear program. It is formulated with assignment based decision variables, which means that each link of plant-warehouse-distribution center has its own decision variable x_{ijk}, where i denotes the plants, j the warehouse and k the distribution center. In contrast to that, also individual decision variables for each link can be established, e.g. plants-warehouse x_{ij} and warehouse-distribution center x_{jk} Additional constraints then ensure flow conservation, i.e. a warehouse is not able to send more than it received.

The Simple Plant Location Problem (SPLP) was outlined with 12 problem instances comprising combinations of three sets of factory locations and four levels of warehouse setup costs in Kuehn and Hamburger [34] through Linear programming. Hamer-Lavoie and Cordeau [35], the main contribution was stochastic demands and stock that were formulated with linear approximation and branch and bound. Holmberg *et al.* [36] developed a primal heuristic, by using repeated matching algorithm as a base and integrated into the Lagrangian heuristic. Khumawala [37] discussed branch and bound for simple warehouse problem. Sahinidis [38] reviewed a theory and methodology developed to cope with the complexity of optimization problems under uncertainty. The classical recourse-based stochastic programming, robust stochastic programming, probabilistic (chance-constraint) programming, fuzzy programming, and stochastic dynamic programming has been discussed. Melkote and Daskin [39] proposed branch and bound for capacitated FLND with up to 40 nodes and 160 candidate links in polynomial time. Jabalameli and Mortezaei [40] proposed a mixed integer programming for capacitated version. They presented a bi-objective formulation of the problem for cost minimization objective and developed a hybrid method. Alborzi *et al.* [41] proposed a mixed integer linear programming (MILP) model for stochastic environment. The formulation was bi-objective for cost minimization and demand satisfaction maximization. Ramezani *et al.* [42] analyzed the stochastic demands and formulated as MILP. The objective was profit maximization and maximum demand satisfaction. Nasiri e*t al.* [43] developed a mixed integer non-linear programming model for stochastic environment with cost minimization objective. Castillo-Villar *et al.* [44] developed MILP for deterministic environment. The objective was profit maximization. Abhijeet Ghadge *et al.* [45] developed Integer Linear Programming model with cost minimization objective.

Yao *et al.* [46] used Iterative heuristic on multi-source stocks on real and random datasets. Geoffrion and Graves [47] considered a single set of variables to represent the two-stage network. The increase in the number of decision variables provides a tighter formulation with benders' decomposition. Addis *et al.* [48] proposed a VLSN search, in which both dual ascent and branch-and-bound of CPLEX are applied to explore the search space. In a series of two papers, Pirkul and Jayaraman [49, 50] considered three stages and multi-product scenario in the location-allocation problem. Minimization of costs of both production and distribution is done by applying Lagrangian relaxation on the demand and the movement of the commodity from the warehouses.

In the case of distribution network with facility location, an amalgamation of exact and heuristics have proved better results. In [51], Lagrangian Relaxation (LR) with heuristic has showed better results, also in [52], a mixed-integer problem (MIP) is implemented BD and two Tabu search heuristics that made possible the convergence and solution quality. In [53], a multi-objective genetic algorithm is developed to solve a stochastic production-distribution network in which the objective function is optimizing the costs and service level. Drezner and Wesolowsky [54] developed a network design problem along with transportation routes, each having fixed cost of construction. Four basic problems subject to two objective functions were proposed. The problem was solved by using four solution approches such as a genetic algorithm, descent algorithm, tabu search, and a simulated annealing. Cocking [55, 56] proposed approaches by using heuristic and metaheuristics to solve the FLNDP under fixed budget. Classic greedy heuristics, a local search heuristic, metaheuristics including SA and variable neighborhood search (VNS), as well as a custom heuristic based on the problem-specific structure of FLNDP. The upper bound of branch-and-cut algorithm was improved by applying the heuristic solutions, and lower bounds were improved by applying cutting planes. This hybrid approach helped to reduce the number of nodes to approach optimality.

2.3. Reliability in Distribution Network

The majority of the literature addresses primarily demand and cost uncertainties [57], and relatively fewer studies consider the influence of facility disruptions. Most recently, both academics and practitioners have realized that facility disruptions may be triggered by various factors and may occur frequently, and an increasing number of studies are investigating ways to improve the reliability of facility networks by planning for facility disruptions [58-60]. A minor disruption can have a shattering impact as it affects a whole supply chain network. From the past few years, academics and researchers interest has been drawn towards random disruptions in supply chain networks, which can be caused by both natural disasters (floods, earthquakes, etc.) and man-made disruptions (accidents, terrorist attacks, labor strikes etc.). Drezner [61] was first to consider disruptions in a facility location model. Two models were introduced for facility under disruption. First, a reliability version of the classical p-median problem. Second model, called the "(p, q)-center problem," p facilities must be located to minimize the maximum cost that may occur when at most q facilities fail. Facilities under uncertainty gained more attention due to Snyder [60] implemented the reliable P-median problem and the reliable un-capacitated fixed-charge location problem. By analyzing the trade-off curves of both costs, the authors note that substantial improvements in reliability can always be obtained with only slight increases in the regular cost.

Zhan [62] studied un-capacitated fixed charge location problem (UFLP) in single-level and multi-levels by considering unstable (unreliable) facilities and developed a mixed-integer mathematical model. He also proposed useful algorithms based on a genetic algorithm (GA) to solve the problem. Besides, he studied the ways of enhancing the reliability of a designed and stable system from a mathematical view. Zhan *et al.* [63], introducing a GA, and Cui *et al.* [64], addressing the discrete UFLP, theorized that considering different failure probabilities for facilities in different places may have a considerable effect on the selection of facility location and site selection. The objective function is to minimize the initial setup and expected transportation costs. The Lagrangian relaxation and continuous approximation algorithm on Snyder's work. Also, by extending using site-specific failures. Berman *et al.* [65] study the reliable P-median problem on a network and propose several exact and heuristic algorithms, ultimately revealing that facilities become more centralized or even co-located as the failure probability increases. Shen *et al*. [66] propose a scenario-based stochastic program and a nonlinear integer program for the reliable facility location problem with heterogeneous failure probabilities. They prove that both models are equivalent. Li *et al.* [67] consider the correlated effect of disruptions and propose a continuous approximation approach for the reliable facility location problem. Tang *et al.* [68] proposed the uncapacitated reliability model with protection using Lagrangian relaxation approach with local search. The implemented model was simplified for one level of protection to facilities.

Jabbarzadeh *et al.* [69] formulated a supply chain design problem with the risk of disruptions to facilities to maximize the total profit for the entire system. Two solution methods based on Lagrangian Relaxation and Genetic Algorithm were developed to achieve near-optimal solutions for large-scale problem instances. Cui *et al.* [70] has proposed multi-level service assignments for suppliers by introducing the expedited shipments to assure delivery that may be expensive. The proposed integer nonlinear program model is solved using Lagrangian Relaxation. Peng *et al.* [71] developed a capacitated supply chain network design model with random disruptions, stochastic p-robustness criteria, and site-dependent disruption probabilities. A hybrid metaheuristic algorithm was proposed. Chen *et al.* [72] proposed a reliable integrated location and inventory problem using LR method. The probability is considered similar for all disrupted facilities. Ghezavati *et al.* [73] has proposed two-level hierarchical network in a supply chain of disaster relief under uncertainty in order to schedule the customers' services. The probability for roadways to become closed for relief operations when a disaster occurs. To solve the model, hybridization of three metaheuristics of genetic algorithm, simulated annealing and chance-constrained was proposed. Alcaraz *et al.* [74] proposed mathematical formulation based on set packing problem for reliable uncapacitated facility location. The investigated various aspects of their polyhedral properties. Yun *et al.* [75] proposed uncapacitated reliability model. This model assumes that the customer's visit sequence yields the minimum transportation cost, and developed a special LR algorithm. Zhang *et al.* [76] formulated a reliability joint-inventory model. The model used heterogeneous failure probability of facilities.

However, most of the current literature mainly deals with the receiver-side and/or in-between uncertainties. Also, exact algorithms provide exactness in the solution, yet are not much optimum for solving large instances. Several heuristics (or metaheuristics) have been used for the facility location. Hybridization of exact and heuristics algorithms is recently gaining attention of lot of researchers. Cross-decomposition which is hybridization of two exact algorithms is not much used. Algorithms such as genetic algorithm (GA), tabu search (TS), ant colony optimization (ACO), particle swarm optimization (PSO) have been widely used on facility location. The Firefly Algorithm (FA) being new has not been much applied on facility location as compared to GA and PSO. FA is a metaheuristic algorithm based on the flashing behavior of fireflies to attract other fireflies towards it and the phenomenon is bioluminescent communication.

Chapter III
Problem Statement

3.1. Rationale for Study

The majority of current literature on reliability in distribution networks mainly focus on homogeneous uncertainties and uncapacitated versions. The following are the research issues found in literature:

I. Limited use of site-specific failure probabilities

The disruptions at facilities site is dependent on the surroundings. As the random disruptions triggered by natural or accidental events entirely depends on the location of the site. So, consideration of site-specific failure probabilities while designing distribution network is more generic.

II. Limited use of Protection to facility assignment

The reliable facility work as a backup facility in case of facility failure. This mechanism ensures that customers are always served. Generally, it is assumed that under disruption conditions, facility may lost all of its capacity. But, in real scenario, we can protect the unreliable facilities by investing extra amount. In this way, we can save fraction of capacity and lost capacity can be served by other facility.

III. Ignorance towards Capacity issue

The literature has studied some mitigation approaches, but mostly on uncapacitated facility locations. Capacity depends on the demands to be met at that location. In real world scenarios, different locations may or may not be of different capacities. By considering limitless capacity of a facility, dynamic nature of real world system is being ignored.

3.2. Objectives

- To study and explore existing facility location and distribution network design problems
- To formulate a mixed-integer programming model, considering capacity issue and site-specific failure probabilities.
- To Implement and validate a proposed model using Lagrangian relaxation approach.
- To implement and validate the performance of the proposed model using Cross-Decomposition approach.

Chapter IV

Reliable Facility location in Distribution network

4.1. Basic Concept of Reliable Facility location in Distribution network

RFLDP (Reliable Facility Location with Distribution Protection) is an extension of the classical facility location problem (FLP). The assumption of facilities may fail independently and have site-specific failure probabilities is considered. The inspiration this model is from Tang *et al.* [68].The reliability of facilities can be protected through extra investments as they are more expensive than their unprotected counterparts. Possible protection measures include built-in redundancies, structural reinforcements, preventive monitoring and safety guarding, and outsourcing. So, two types of facilities are in the system: the unreliable regular ones and the reliable protected ones. Each customer should be assigned to a primary facility, which can be either a regular facility or a protected facility. If the primary facility of a customer is a regular one, it should be assigned to a protected facility as well, which works as the backup facility. In this way, the customer can obtain emergency services from the backup facility when the primary facility fails. This protection and backup mechanism ensures that each customer is served. RFLDP aims to determine the facility location and protection decisions as well as customer assignments to minimize the fixed charges and expected service cost.

4.2. Mathematical Formulation of proposed model

4.2.1. Facility Disruption

The mathematical formulation is regarded as a two-stage integer programming problem. For first stage, the decision about opening of DC locations are considered. In the second stage, customer are assigned to either reliable or unreliable facility, hedging against random disruptions. A set of I customers' demands and set of J distribution centers with M investment levels is considered. The demands of customer are static. The following definitions and ranges of the indices are used throughout this dissertation.

Notations

I Set of customer points, indexed by i

J Set of potential DCs sites, indexed by j

M Set of Investment level, indexed by m

Parameters

Cap_j Capacity of potential DC j

d_i Demand of customer i

$prob_j$ Failure probability of the unreliable DCs opened at site j

f_{jm}^U Fixed charge of opening and operating an unreliable DCs site j at level m

f_j^R Fixed charge of opening and operating a reliable DCs at site j

H_j Holding cost of product at DCs site j

S_{ji}^P Shipping cost of unit product from primary DCs j to customer i

S_{ji}^B Shipping cost of unit product from backup DCs j to customer i

Decision Variables

$$X_j^R = \begin{cases} 1; & \textit{if a reliable DC is opened at site } j \\ 0; & \textit{otherwise.} \end{cases}$$

$$X_{jm}^U = \begin{cases} 1; & \textit{if an unreliable DC is opened at site } j \textit{ with investment level } m \\ 0; & \textit{otherwise.} \end{cases}$$

$$\mathrm{Y}_{\mathrm{jki}} = \begin{cases} 1; & \textit{if DC at site } k \textit{ as the primary facility and reliable } \mathrm{DC}\ j \\ & \textit{as backup facility is assigned to customer } i \\ 0; & \textit{otherwise.} \end{cases}$$

$$Z_{ji} = \begin{cases} 1; & \textit{if } \text{reliable } \textit{DC at site } j \textit{ is assigned to customer } i \\ 0; & \textit{otherwise.} \end{cases}$$

The mathematical formulation for Reliable Facility Location with Distribution Protection is given below.

$$\sum_{\forall j \in J} f_{jl}^{U} X_{jm}^{U} + \sum_{\forall j \in J} f_{j}^{R} X_{j}^{R} + \sum_{\forall i \in I} \sum_{\forall j \in J} d_i S_{ji}^{P} Z_{ji} + \tag{4.1}$$

$$\sum_{\forall i \in I} \sum_{\forall k \in J, k != j} \sum_{\forall j \in J} (d_i S_{ji}^{P} (1 - prob_k) Y_{jki} + d_i S_{ji}^{P} prob_k Y_{jki}) + \sum_{\forall j \in J} \sum_{\forall i \in I} d_i H_j Y_{jki}$$

Subject to

$$X_{jl}^{U} + X_{jm}^{R} <= 1, \forall j \in J, \forall l \in L \tag{4.2}$$

$$\sum_{\forall i \in I} \sum_{\forall j \in J} \sum_{\forall k \in K} d_i Y_{ikj} \leq Cap_j \sum_{\forall l \in L} X_{jm}^{u} \tag{4.3}$$

$$\sum_{\forall i \in I} \sum_{\forall j \in J} \sum_{\forall k \in K} d_i Y_{ikj} \leq Cap_j X_{j}^{R} \tag{4.4}$$

$$\sum_{\forall i \in I} \sum_{\forall j \in J} Z_{ij} Y_{ikj} \leq Cap_j X_{j}^{R} \tag{4.5}$$

$$\sum_{\forall k \in J, k \neq j} Y_{ijk} + \sum_{\forall j \in Jj} Z_{ij} = 1, \forall i \in I \tag{4.6}$$

$$Z_{ij} <= X_{j}^{R}, \forall i \in I, \forall j \in J \tag{4.7}$$

$$\sum_{\forall k \in J, k \neq J} Y_{ijk} <= \sum_{\forall l \in L} X_{jm}^{U}, \forall i \in I, \forall j \in J \tag{4.8}$$

$$\sum_{\forall k \in J, k \neq j} Y_{ijk} <= X_{j}^{R}, \forall i \in I, \forall j \in J \tag{4.9}$$

$$\sum_{\forall j \in J} X_{j}^{R} \geq 1 \tag{4.10}$$

$$X_{jm}^{U}, X_{j}^{R} \in \{0,1\}, \forall j \in J, \forall m \in M \tag{4.11}$$

$$Z_{ij} \in \{0,1\}, \forall i \in I, \forall j \in J \tag{4.12}$$

$$Y_{ijk} \in \{0,1\}, \forall i \in I, \forall j, k \neq j \in J \tag{4.13}$$

The objective function mentioned in Equation (4.1) is to minimize the overall costs including the fixed cost of opened unreliable and reliable facilities, the deterministic service cost for customers who are served by a reliable facility, the expected service cost for customers who are served by a primary facility and a backup facility, and the holding cost. The constraint (4.2) designates that either a reliable facility or an unreliable facility can be opened at a single site, but not for both. The constraints (4.3), (4.4) and (4.5) are used for capacity bound. These constraints ensures that the production/ transportation goods does not exceed their maximum limit. Constraint (4.6) indicates that a customer is assigned either directly to a reliable primary facility or to an unreliable facility according to their situations. Constraint (4.7) state that if a customer is assigned to only one layer facility, then that facility should be reliable. Constraints (4.8) and (4.9) ensure that if a customer is assigned to two layer facilities, then the primary facility should be unreliable and the backup facility should be reliable. Constraint (4.10) guarantees that at least one reliable facility should be opened, which is a redundant constraint and can be derived by combining constraints (4.6), (4.7) and (4.9), but it tighten the bounds. Constraints (4.11)–(4.13) are the non-negative integrity constraints.

4.2.2. Link Disruption

Notations

I Set of customers zones, indexed by i

J Set of potential sites for distribution centers, indexed by j

M Set of available investment levels for opening unreliable distribution centers, indexed by m

L Set of available disrupted links between distribution centers and customers, indexed by l

Parameters

Cap_j Capacity of potential facility j

d_i Demand of customer i

H_j Holding cost per product unit in distribution center j

f_j^R Fixed cost of opening and operating reliable DC j

f_{jm}^U Fixed cost of opening and operating unreliable DC j with investment level m

S_{jil}^{U} Cost for transporting products from unreliable DC j to customer i through unreliable link l

S_{ji}^{P} Cost for transporting from primary facility j with reliable link to customer i

S_{ji}^{B} Cost for transporting from secondary facility j with reliable link to customer i

$prob_j$ Failure probability of the unreliable facility opened at site j

Δ_{jm} Percentage of capacity in disrupted unreliable DC j with investment level m

T_{jk} Transportation cost from unreliable DC j to reliable DC k $(j \neq k)$

θ_{jil} Failure probability between DC j and customer i due to unsafe transportation link l

Decision Variables

$$X_j^R = \begin{cases} 1; if\ a\ reliable\ DC\ is\ opened\ at\ site\ j; \\ 0; otherwise. \end{cases}$$

$$X_{jm}^U = \begin{cases} 1; if\ an\ unreliable\ DC\ is\ opened\ at\ site\ j\ with\ investment\ level\ m; \\ 0; otherwise. \end{cases}$$

$$Y_{ij} = \begin{cases} 1; if\ cutomer\ is\ assigned\ to\ an\ unreliable\ DC\ opened\ at\ site\ j; \\ 0; otherwise. \end{cases}$$

$$Z_{ij} = \begin{cases} 1; if\ customer\ i\ is\ assigned\ to\ a\ reliable\ DC\ j\ as\ the\ primary\ DC; \\ 0; otherwise. \end{cases}$$

$$YU_{ijl} = \begin{cases} 1; if\ customer\ is\ assigned\ to\ an\ unreliable\ DC\ j\ as\ the\ primary\ DC \\ through\ unsafe\ link\ l; \\ 0;\ otherwise. \end{cases}$$

$A_{jk} = Amount\ of\ products\ shipped\ from\ reliable\ DC\ j\ to\ unreliable\ DC\ k$

The model for the Reliable Facility Location with Link Disruption is as follows.

$$\begin{aligned} &\sum_{\forall j \in J}\sum_{\forall m \in M} f_{jm}^U X_{jm}^U + \sum_{\forall j \in J} f_j^R X_j^R + \sum_{\forall i \in I}\sum_{\forall j \in J}\left(d_i S_{ij}^P Z_{ij} + d_i S_{ij}^P Y_{ij}\right) \\ &+ \sum_{\forall i \in I}\sum_{\forall j \in J}\sum_{\forall l \in L}\left(d_i S_{jil}^U (1-\theta_{jil}) YU_{ijl} + d_i S_{jil}^B \theta_{jil} YU_{ijl}\right) \\ &+ \sum_{\forall i \in I}\sum_{\forall j \in J}\sum_{\forall m \in M} prob_j X_{jm}^U A_{jk} T_{jk} \end{aligned} \qquad (4.14)$$

Subject to

$$\sum_{\forall m \in M} X_{jm}^{U} + X_{j}^{R} <= 1, \forall j \in J \tag{4.15}$$

$$\sum_{\forall j \in J} (Y_{ij} + \sum_{\forall l \in L} YU_{ijl} + Z_{ij}) = 1, \forall i \in I \tag{4.16}$$

$$Z_{ij} <= X_{j}^{R}, \forall i \in I, \forall j \in J \tag{4.17}$$

$$A_{jk} <= Cap_{j} \sum_{\forall m \in M} X_{jm}^{U}, \forall j \in J, \forall k \in J (j \neq k) \tag{4.18}$$

$$A_{jk} <= \sum_{\forall i \in I} d_{i} X_{j}^{R}, \forall j \in J, \forall k \in J (j \neq k) \tag{4.19}$$

$$\sum_{\forall j \in J} A_{jk} + Cap_{j} \sum_{\forall m \in M} (1 - \theta_{jm}) X_{jm}^{U} \geq \sum_{\forall i \in I} d_{i} (YU_{ijl} + Y_{ij}), \quad \forall j \in J, \forall k \in J (j \neq k) \tag{4.20}$$

$$\sum_{\forall j \in J} X_{j}^{R} \geq 1 \tag{4.21}$$

$$X_{jm}^{U}, X_{j}^{R} \in \{0,1\}, \forall j \in J \tag{4.22}$$

$$Y_{ikj} \in \{0,1\}, \forall i \in I, \forall j, k \neq j \in J \tag{4.23}$$

$$Z_{ij} \in \{0,1\}, \forall i \in I, \forall j \in J \tag{4.24}$$

$$A_{jk} \geq 0, \forall j \in J, \forall k \in J (j \neq k) \tag{4.25}$$

The first term and second term in Equation (4.14) is to calculate the cost of opening unreliable and reliable facilities. The second term calculate the demands served by either reliable or unreliable. The third term calculates the cost incurred by failure of DC. The fourth term calculates the amount of product being sent to unreliable DC in case of failure. The fifth calculates the holding cost of product in DC assigned to as backup DCs. Constraint (4.15) states that only one DC can be opened at a single site either reliable or unreliable. Constraint (4.16) states that if unreliable DC is assigned, then secondary assigned DC will be protected. Constraint (4.17) states that if one DC is assigned it will be primary and protected. Constraint (4.18) states that in a disruption situation, products cannot be shipped from potential node j, unless a reliable DC is opened at it. Constraint (4.19) states that transportation of products can done only if unreliable is partial failed.Constraints (4.20) maintains the capacity of unreliable DCs equal to demand of assigned customers after disruption. Constraint (4.21) ensures that customer is assigned to one reliable DCs at least. Constraints (4.22)–(4.25) are the non-negative integrity constraints.

Chapter V

Proposed Methodology

5.1. Lagrangian Relaxation

Lagrangian Relaxation implements the problem by moving complicating constraints into the objective function and penalizing complicating constraint infeasibility with a Lagrangian multiplier. The same steps are followed in other models as well. First, the hard constraints of the given problem are relaxed which lead to a relaxation problem. Therefore, it can be solved easily and helps in obtaining a lower bound for the given problem. This approach can also construct a feasible solution to the original problem which provides an upper bound for the given problem. The Lagrange multipliers have been adjusted to reduce the amount of constraint violation.

5.1.1. Algorithm

The proposed algorithm consists of following steps.

Step 1. Initialize the parameters of Lagrangian relaxation-based approach. The lower bound (LB) and upper bound (UB) are set to $-\infty$ to $+\infty$ respectively. The Lagrange multiplier is set to zero. The maximum number of iterations and $L_{\max}$ are set to 300 and 10, respectively.

Step 2. (a) Compute

$$Z_{LR} = \min \left(\begin{array}{l} \sum_{\forall j \in J} f_{jl}^{U} X_{jl}^{U} + \sum_{\forall j \in J} f_{j}^{R} X_{j}^{R} + \sum_{\forall i \in I} \sum_{\forall j \in J} d_i S_{ji}^{P} Z_{ji} + \\ \sum_{\forall i \in I} \sum_{\forall k \in J, k != j} \sum_{\forall j \in J} (d_i S_{ji}^{P} (1 - prob_k) Y_{jki} + d_i S_{ji}^{P} prob_k Y_{jki}) + \sum_{\forall j \in J} \sum_{\forall i \in I} d_i H_j Y_{jki} + \\ \lambda \left(1 - \sum_{\forall i \in I} \sum_{\forall k \in J, k != j} \sum_{\forall j \in J} (Y_{jki} + Z_{ji}) \right) + \mu \left(\sum_{\forall i \in I} \sum_{\forall k \in J, k != j} \sum_{\forall j \in J} Y_{jki} - X_{jm}^{U} \right) \end{array} \right)$$

(b) If $Z_{LR} \geq LB$, then set $L_{count} = L_{count} + 1$

Else set $L_{count} = 0$.

(c) Set $LB = \min(Z_{LR}, LB)$

Step 3. (a) Update Lagrange multipliers using sub-gradient algorithm.

(b) If $L_{count} \geq L_{Max}$, then set the step size to half of the original value and $L_{count} = 0$

Endif

(c) Update iteration count

Step 4. (a) Compute

$$Z_{UB} = \min\left(\begin{array}{l} \sum_{\forall j \in J} f_{jl}^{U} X_{jl}^{U} + \sum_{\forall j \in J} f_{j}^{R} X_{j}^{R} + \sum_{\forall i \in I} \sum_{\forall j \in J} d_i S_{ji}^{P} Z_{ji} + \\ \sum_{\forall i \in I} \sum_{\forall k \in J, k != j} \sum_{\forall j \in J} (d_i S_{ji}^{P} (1 - prob_k) Y_{jki} + d_i S_{ji}^{P} prob_k Y_{jki}) + \sum_{\forall j \in J} \sum_{\forall i \in I} d_i H_j Y_{jki} \end{array} \right)$$

(b) Set $UB = \max(Z_{UB}, UB)$

Step 5. If termination criterion is satisfied, then terminate the algorithm; otherwise go to Step 2.

5.1.2. Solving the Lagrangian relaxation problem

To solve the Eq. (4.1), constraint (4.6) and (4.8) with the Lagrange multipliers λ and π, obtain the corresponding Lagrangian relaxation problem.

$$Z_{LR} = \begin{array}{l} \sum_{\forall j \in J} FC_{j}^{U} X_{j}^{U} + \sum_{\forall j \in J} FC_{j}^{R} X_{j}^{R} + \sum_{\forall i \in I} \sum_{\forall j \in J} D_i SC_{ji}^{P} Z_{ji} + \sum_{\forall i \in I} \sum_{\forall k \in J, k != j} \sum_{\forall j \in J} (D_i SC_{ji}^{P} (1 - FP_k) Y_{jki} + D_i SC_{ji}^{P} FP_k Y_{jki}) + \sum_{\forall j \in J} \sum_{\forall i \in I} D_i H_j Y_{jki} + ... \\ \lambda (1 - \sum_{\forall i \in I} \sum_{\forall k \in J, k != j} \sum_{\forall j \in J} (Y_{jki} + Z_{ji})) + \mu (\sum_{\forall i \in I} \sum_{\forall k \in J, k != j} \sum_{\forall j \in J} Y_{jki} - X_{j}^{U}) \end{array} \quad (5.1)$$

The constraints are same for equation (5.1), which are mentioned for equation (4.1). The above-mentioned equation calculates the lower bound for a given original objective function. The relaxed equation (5.1), which can be solved by using sub-gradient method. The algorithm for subgradient optimization is given in section 5.1.5. For the given Lagrange multipliers λ, μ the equation (5.1) can be decomposed into j independent facility sub-problems. Therefore, it can be easily solved as compared to the original problem. Figure 5.1 shows the systematic flow of lagrangian relaxation approach.

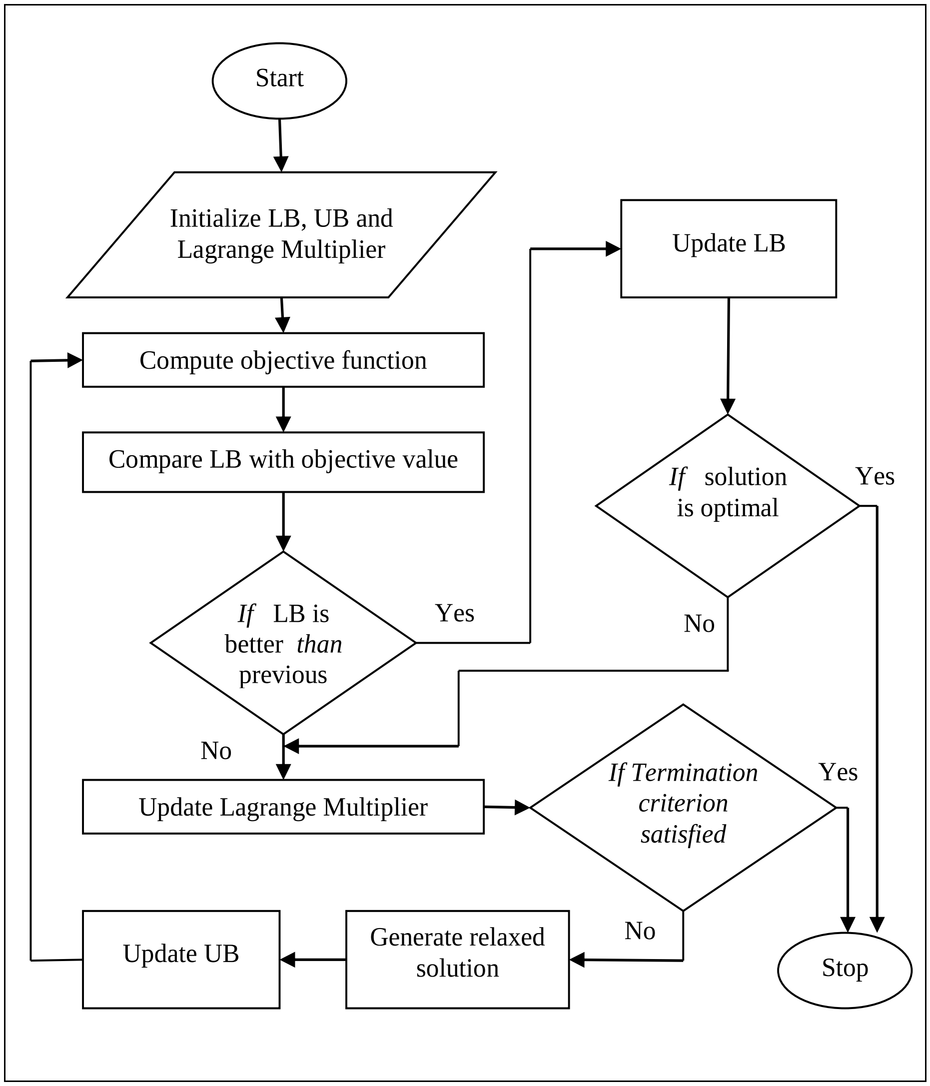

Figure 5.1 Systematic flow of lagrangian relaxation approach

5.1.3. Computation of the Lower Bound

The Lagrange multipliers helps to calculate the lower bound of the given objective function. The following steps are used iteratively to improve the bound to reach for an optimal value.

Step 1. Initialize decision variables such as $X_{jm}^{U}, X_{j}^{R}, Z_{ij}$ and Y_{ijk} to zero.

Step 2. Compute the objective function based on installation of facility and product assignment.

Step 3. Computed the lower bound using sub-gradient method

Step 4. If obtained lower bound is smaller than the previous value, then

Replace the lower bound

Endif

Step 5. Repeat Steps 2 to 4 until the termination criterion is satisfied.

5.1.4. Computation of the upper bound

A primal heuristic is used to obtain feasible solutions, which is generated through Lagrangian relaxation. Thereafter, configuration of feasible solution is obtained, which may not be the optimal configuration for the given problem. This provides an upper bound for the given problem. Therefore, the product assignment for feasible solution is the optimal assignment corresponding to the location configuration.

5.1.5. Lagrange Multiplier Initialization and Update

The Lagrangian dual problem can be solved using the sub-gradient algorithm, which further improves the lower bound by adjusting the Lagrange multipliers λ. The updating of Lagrange multipliers is done using standard subgradient optimization [13, 14]. The algorithm for subgradient optimization is as follows:

Step 1. Choose a starting point λ^0, μ^0 as iteration t=0.

Step 2. Choose a subgradient $s^t == \left(1 - \sum_{\forall k \in J, k \neq j} Y_{ijk} + \sum_{\forall j \in Jj} Z_{ij}\right)$ of the function Z at λ^t and

$s2^t = \left(\sum_{\forall k \in J, k \neq J} Y_{ijk} - \sum_{\forall l \in L} X_{jl}^U\right)$. If s^t = 0 or $s2^t$, then stop, because the optimal value

$L(\lambda^t, \mu^t)$ has been reached.

Step 3. Compute $\lambda^{0t+1} = \max\{0, \lambda^t + s^t \tau^t\}, \mu^{t+1} = \max\{0, \mu^t + s2^t \tau^t\}$, where θ^t denotes the step-size.

Step 4. Increment t and go to 2.

The formula for calculating the step-size proposed by Held and Karp is as follows [77]:

$$\tau^t = \frac{UB - L(\lambda^t, \mu^t)}{\| S^t + s2^t \|^2} \tag{5.2}$$

The updating process is repeated until termination conditions are met. The lower bound solution is computed and update the upper bound solution at the last iteration.

5.1.6. Termination Conditions

The algorithm terminates when the following conditions are met:

1. Optimality Gap: The algorithm terminates when the optimality gap is close to zero or is equal to the pre-specified tolerance value.
2. Maximum Iteration: The algorithm gets stop when it reaches a pre- specified number of iterations. The default value of maximum number of iteration is 1000.
3. Step size Constant: When the step size constant τ gets close to zero. The default value of step size constant is 2.

5.2. Cross Decomposition

Cross decomposition iterates between the primal benders subproblem and the dual Lagrangian subproblem. These problems sequentially provide bounds on the objective function of the RFLDP. It solves a primal or dual master problem for upper and lower bounds. When a convergence test fails, then subproblem phase is restarted. The algorithm terminates when the lower bound and upper bounds of selected problems converge to meet at an optimal value of objective function. These problems may provide tight bounds in the subproblem phase, but neither convergence, nor monotonic improvement is guaranteed. The convergence tests described below are used to determine when the subproblems fail to make progress toward an optimal solution. When a convergence test fails, cross decomposition solves a master problem that is formed using the cuts generated in the subproblem phase and then continues this phase with the next subproblem. Figure 5.2 describes the flowchart of cross-decomposition.

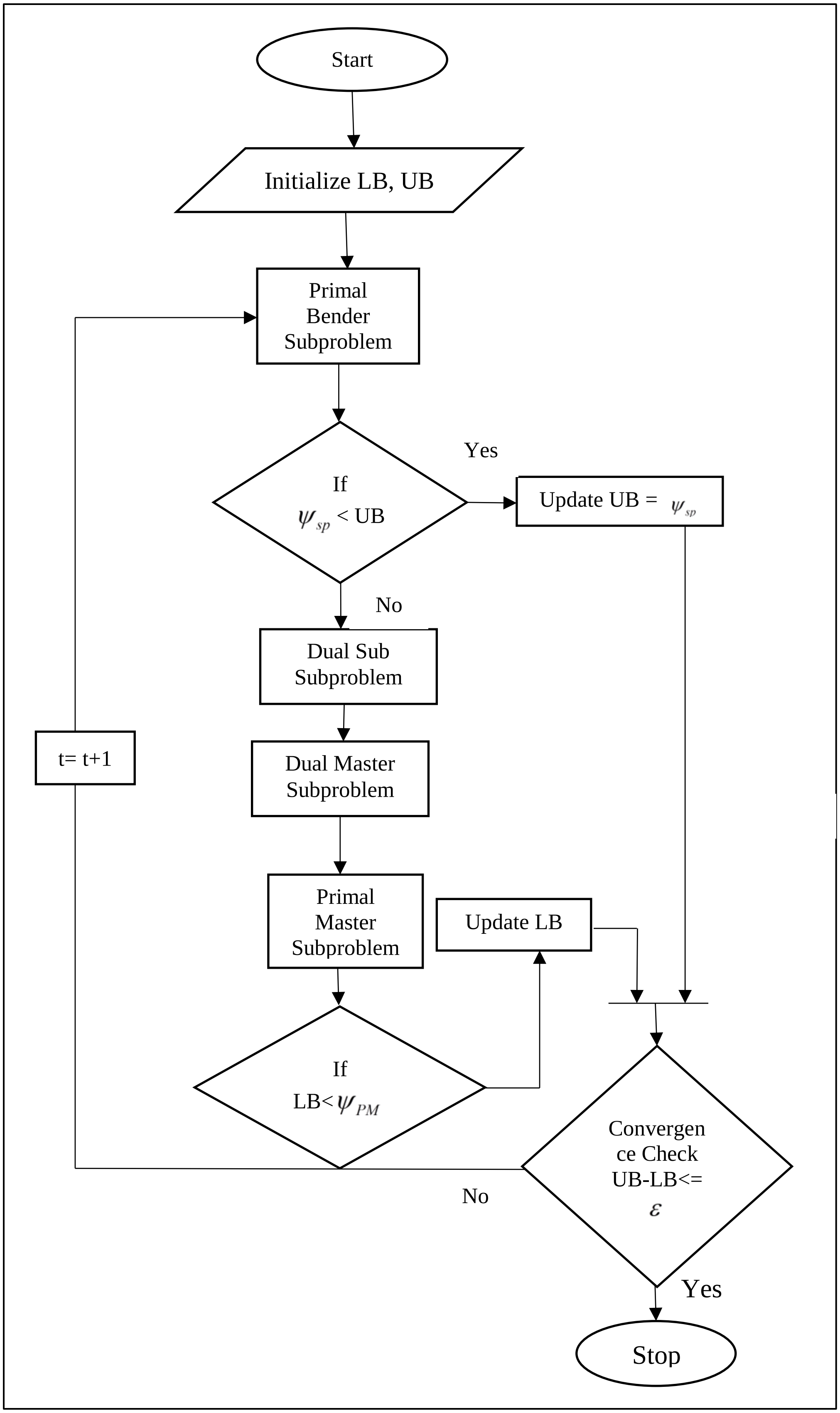

Figure 5.2 Systematic flow of cross decomposition approach

Benders decomposition [78] is a primal solution method. It separates the problem by fixing binary variables. Benders decomposition is an exact method that solves the RFLDP optimally by iterating between the primal subproblem and the primal master problem described below. Appendix A contains a derivation of the master problem and subproblem. The description below provides implementation details of primal subproblem and primal master problem.

5.2.1. The Primal Subproblem

The primal subproblem (PS) is the linear program obtained when the facility configuration is fixed at $\overline{X}, \overline{Y}, \overline{YU}, \overline{Z}$ in the RFLDP where $\overline{X}^{U}_{jm}, \overline{X}^{R}_{j}, \overline{YU}_{ijl}, \overline{Y}_{ij}, \overline{Z}_{ij}$ indicates that $X^{U}_{jm}, X^{R}_{j}, YU_{ijl}, Y_{ij}, Z_{ij}$ are fixed at either one or zero.

$$\begin{aligned}\psi_{SP} = \textbf{minimize} \; & \sum_{\forall j\in J}\sum_{\forall m\in M} f^{U}_{jm}\overline{X}^{U}_{jm} + \sum_{\forall j\in J} f^{R}_{j}\overline{X}^{R}_{j} + \sum_{\forall i\in I}\sum_{\forall j\in J}\left(d_i S^{P}_{ij}\overline{Z}_{ij} + d_i S^{P}_{ij}\overline{Y}_{ij}\right) \\ & + \sum_{\forall i\in I}\sum_{\forall j\in J}\sum_{\forall l\in L}\left(d_i S^{U}_{jil}(1-\theta_{jil})\overline{YU}_{ijl} + d_i S^{B}_{jil}\theta_{jil}\overline{YU}_{ijl}\right) \\ & + \sum_{\forall i\in I}\sum_{\forall j\in J}\sum_{\forall m\in M} P_j \overline{X}^{U}_{jm} A_{jk} T_{jk}\end{aligned} \tag{5.3}$$

subject to

$$A_{jk} \le C_j \sum_{\forall m\in M}\overline{X}^{U}_{jm}, \forall j\in J, \forall k\in J\,(j\ne k) \tag{5.4}$$

$$A_{jk} \le \sum_{\forall u\in I} d_i\, \overline{X}^{R}_{j}, \forall j\in J, \forall k\in J\,(j\ne k) \tag{5.5}$$

$$\sum_{\forall k\in J} A_{jk} + C_j \sum_{\forall m\in M}(1-\theta_{jm})\overline{X}^{U}_{jm} \ge \sum_{\forall i\in I} \mathrm{d_i}\left(\sum_{\forall l\in L}\overline{YU}_{ijl} + \overline{Y}_{ij}\right), \forall j\in J\,(j\ne k) \tag{5.6}$$

$$A_{jk} \ge 0, \forall j\in J, \forall k\in J\,(j\ne k) \tag{5.7}$$

Eq. (5.3) is a restriction of the RFLDP that consequently provides an upper bound on its optimal objective function value. The objective function value of the dual of eq. (5.3) is

$$\sum_{\forall k\in K} v_{\mathrm{k}}\left(\sum_{\forall \mathrm{i}\in \mathrm{I}} \mathrm{d_i}\left(\sum_{\forall l\in L}\overline{YU}_{ijl} + \overline{Y}_{ij}\right) - C_j\sum_{\forall m\in M}(1-\theta_{jm})\overline{X}^{U}_{jm}\right) - \sum_{\forall \mathrm{j}\in \mathrm{J}, \forall \mathrm{k}\in \mathrm{J}} \mu_{\mathrm{jk}} \sum_{\forall u\in I} d_i\, \overline{X}^{R}_{j} - \sum_{\forall \mathrm{j}\in \mathrm{J}, \forall \mathrm{k}\in \mathrm{J}} \lambda_{\mathrm{jk}}\left(C_j \sum_{\forall m\in M}\overline{X}^{U}_{jm}\right) \tag{5.8}$$

To maintain consistency with the objective function of the RFLDP, we add eq. (5.9), fixed facility installation cost and assignment cost to eq. (5.8)

$$\sum_{\forall j\in J}\sum_{\forall m\in M} f_{jm}^{U}\overline{X}_{jm}^{U} + \sum_{\forall j\in J} f_j^R \overline{X}_j^R$$
$$+\sum_{\forall i\in I}\sum_{\forall j\in J}\left(d_i S_{ij}^P \overline{Z}_{ij} + d_i S_{ij}^P \overline{Y}_{ij}\right) \tag{5.10}$$
$$+\sum_{\forall i\in I}\sum_{\forall j\in J}\sum_{\forall l\in L}\left(d_i S_{jil}^U (1-\theta_{jil})\overline{YU}_{ijl} + d_i S_{jil}^B \theta_{jil}\overline{YU}_{ijl}\right)$$

Following equation is obtained:

$$\sum_{\forall j\in J}\sum_{\forall m\in M} f_{jm}^{U}\overline{X}_{jm}^{U} + \sum_{\forall j\in J} f_j^R \overline{X}_j^R + \sum_{\forall i\in I}\sum_{\forall j\in J}\left(d_i S_{ij}^P \overline{Z}_{ij} + d_i S_{ij}^P * \overline{Y}_{ij}\right) + \ldots \tag{5.11}$$
$$\sum_{\forall i\in I}\sum_{\forall j\in J}\sum_{\forall l\in L}\left(d_i S_{jil}^U (1-\theta_{jil})\overline{YU}_{ijl} + d_i S_{jil}^B \theta_{jil}\overline{YU}_{ijl}\right) + \ldots$$
$$\sum_{\forall k\in K} v_{\mathrm{k}}\left(\sum_{\forall \mathrm{i}\in \mathrm{I}} \mathrm{d_i}\left(\sum_{\forall l\in L}\overline{YU}_{ijl} + \overline{Y}_{ij}\right) - Cap_j \sum_{\forall m\in M}\left(1-\theta_{jm}\right)\overline{X}_{jm}^U\right) - \sum_{\forall \mathrm{j}\in \mathrm{J},\forall \mathrm{k}\in \mathrm{J}} \mu_{\mathrm{jk}} \sum_{\forall u\in I} d_i\ \overline{X}_j^R - \sum_{\forall \mathrm{j}\in \mathrm{J},\forall \mathrm{k}\in \mathrm{J}} \lambda_{\mathrm{jk}}\left(Cap_j \sum_{\forall m\in M}\overline{X}_{jm}^U\right)$$

By duality, the value of the above expression for the optimal solution to eq. (5.5) is the maximum feasible value for the given facility configuration. This expression provides the basis for a primal cut. The optimal objective function value of the primal master problem must be less than or equal to for any set of feasible facility configurations (where the superscript *(t)* on the dual variables is the iteration number).

$$\omega_{t\in T} = \sum_{\forall j\in J}\sum_{\forall m\in M} f_{jm}^{U} X_{jm}^{U} + \sum_{\forall j\in J}\left(\left(f_j^R - \sum_{\forall \mathrm{k}\in \mathrm{J}}\mu_{jk}^t \sum_{\forall u\in I} d_i\right) X_j^R\right)$$
$$+\sum_{\forall i\in I}\sum_{\forall j\in J}\left(d_i S_{ij}^P Z_{ij} + d_i S_{ij}^P Y_{ij}\right)$$
$$+\sum_{\forall i\in I}\sum_{\forall j\in J}\sum_{\forall l\in L}\left(d_i S_{jil}^U (1-\theta_{jil}) YU_{ijl} + d_i S_{jil}^B \theta_{jil} YU_{ijl}\right) + \tag{5.12}$$
$$\sum_{\forall k\in K} v_k^t\left(\sum_{\forall \mathrm{i}\in \mathrm{I}} \mathrm{d_i}\left(\sum_{\forall l\in L} YU_{ijl} + Y_{ij}\right) - C_j \sum_{\forall m\in M}\left(1-\theta_{jm}\right) X_{jm}^U\right) - \sum_{\forall \mathrm{j}\in \mathrm{J},\forall \mathrm{k}\in \mathrm{K}} \lambda_{jk}^t\left(C_j \sum_{\forall m\in M} X_{jm}^U\right)$$

5.2.2. The Primal Master problem

The primal master problem is obtained by adding a primal cut eq. (5.14) each time the primal sub-problem is solved. Eq. (5.13) fixes the facility location variables for the primal sub-problem. PM provides a feasible facility configuration because constraint (5.19) ensures total supply meets total demand of disrupted facility. When using only a subset of all possible cuts (a relaxation), PM provides a lower bound on the optimal objective function ψ given in eq. (4.1) solution of the RFLDP.

$$\text{minimize } \psi_{PM} \tag{5.13}$$

subject to

$$\psi_{PM} \geq \omega \tag{5.14}$$

$$\sum_{\forall j \in J} X_j^R \geq 1 \tag{5.15}$$

$$\sum_{\forall j \in J} (Y_{ij} + \sum_{\forall l \in L} YU_{ijl} + Z_{ij}) = 1, \forall i \in I \tag{5.16}$$

$$Z_{ij} <= X_j^R, \forall i \in I, \forall j \in J \tag{5.17}$$

$$\sum_{\forall i \in I} d_i (\sum_{\forall j \in J} (Y_{ij} + \sum_{\forall l \in L} YU_{ijl})) <= Cap_j \sum_{\forall m \in M} X_{jm}^U \tag{5.18}$$

$$\sum_{\forall j \in J} d_i X_j^R >= \sum_{\forall j \in J, \forall k \in K} A_{jk}, (j \neq k) \tag{5.19}$$

$$X_j^U, X_j^R \in \{0,1\}, \forall j \in J \tag{5.20}$$

$$Y_{ikj} \in \{0,1\}, \forall i \in I, \forall j, k \neq j \in J \tag{5.21}$$

$$Z_{ij} \in \{0,1\}, \forall i \in I, \forall j \in J \tag{5.22}$$

Where ω is defined in eq. (5.23).

While the multi-cut version (5.14) and (5.23) provides faster convergence due to increased strength of the dual information, it increases the problem size compared to the single cut version [79]. Since PS determines the maximum value of for the facility configuration provided by PM, the optimal objective function value of the RFLDP is identified when the objective function value of PS equals the objective function value of PM.

$$\omega = \sum_{\forall j\in J}\sum_{\forall m\in M} f_{jm}^{U} X_{jm}^{U} + \sum_{\forall j\in J}\left(\left(f_{j}^{R} - \sum_{\forall k\in J}\mu_{jk}\sum_{\forall u\in I} d_{i}\right) X_{j}^{R}\right)$$

$$+ \sum_{\forall i\in I}\sum_{\forall j\in J}\left(d_{i} S_{ij}^{P} Z_{ij} + d_{i} S_{ij}^{P} Y_{ij}\right)$$

$$+ \sum_{\forall i\in I}\sum_{\forall j\in J}\sum_{\forall l\in L}\left(d_{i} S_{jil}^{U} * (1-\theta_{jil}) YU_{ijl} + d_{i} S_{jil}^{B}\theta_{jil} YU_{ijl}\right) + \tag{5.23}$$

$$\sum_{\forall k\in K} v_{k}\left(\sum_{\forall i\in I}\mathrm{d}_{i}\left(\sum_{\forall l\in L} YU_{ijl} + Y_{ij}\right) - C_{j}\sum_{\forall m\in M}\left(1-\theta_{jm}\right) X_{jm}^{U}\right) - \sum_{\forall j\in J,\forall k\in K}\lambda_{jk}\left(C_{j}\sum_{\forall m\in M} X_{jm}^{U}\right)$$

Lagrangian relaxation solves a relaxation of the RFLDP by iterating between the dual subproblem and the dual master problem described below.

5.2.3. The Dual Subproblem (DS)

DS is a relaxation of the RFLDP that consequently provides a lower bound on its optimal objective function value. DS provides for any set of dual values fixed facility locations X_{jm}^{U}, X_{j}^{R} and customer-assignments YU_{ijl}, Y_{ij}, Z_{ij} .

$$\phi = \sum_{\forall j\in J}\sum_{\forall m\in M} f_{jm}^{U} X_{jm}^{U} + \sum_{\forall j\in J} f_{j}^{R} X_{j}^{R} + \sum_{\forall i\in I}\sum_{\forall j\in J}\left(d_{i} S_{ij}^{P} Z_{ij} + d_{i} S_{ij}^{P} Y_{ij}\right)$$

$$+ \sum_{\forall i\in I}\sum_{\forall j\in J}\sum_{\forall l\in L}\left(d_{i} S_{jil}^{U}(1-\theta_{jil}) YU_{ijl} + d_{i} S_{jil}^{B}\theta_{jil} YU_{ijl}\right) \tag{5.24}$$

$$+ \sum_{\forall i\in I}\sum_{\forall j\in J}\sum_{\forall m\in M} P_{j} X_{jm}^{U} A_{jk} T_{jk} + \lambda\left(1 - \sum_{\forall i\in I}\sum_{\forall j\in J}\sum_{\forall l\in L}(Y_{ij} + YU_{ijl} + Z_{ij})\right) + \mu\left(\sum_{\forall i\in I}\sum_{\forall j\in J} Z_{ij} - X_{j}^{R}\right)$$

subject to

$$\sum_{\forall j\in J} X_{j}^{R} \geq 1 \tag{5.25}$$

$$\sum_{\forall j\in J}(Y_{ij} + \sum_{\forall l\in L} YU_{ijl} + \mathrm{Z}_{ij}) = 1, \forall i\in I \tag{5.26}$$

$$Z_{ij} <= X_{j}^{R}, \forall i\in I, \forall j\in J \tag{5.27}$$

$$\sum_{\forall i\in I} d_{i}\left(\sum_{\forall j\in J}(Y_{ij} + \sum_{\forall l\in L} YU_{ijl})\right) <= C_{j}\sum_{\forall m\in M} X_{jm}^{U} \tag{5.28}$$

$$\sum_{\forall j\in J} d_{i} X_{j}^{R} >= \sum_{\forall j\in J,\forall k\in K} A_{jk}, (j\neq k) \tag{5.29}$$

$$X_{j}^{U}, X_{j}^{R} \in \{0,1\}, \forall j\in J \tag{5.30}$$

$$Y_{ikj} \in \{0,1\}, \forall i \in I, \forall j, k \neq j \in J \tag{5.31}$$

$$Z_{ij} \in \{0,1\}, \forall i \in I, \forall j \in J \tag{5.32}$$

5.2.4. The Dual Master Problem (DM)

$$\text{maximize } \psi \tag{5.33}$$

subject to

$$\psi \leq \phi^t \; \forall t \tag{5.34}$$

The dual master problem is obtained by adding a dual cut eq. (5.34) each time the dual sub problem is solved. This is analogous to the multi-cut version of the Benders cut in 5.2.2, since Benders decomposition and Lagrangian relaxation can be seen as duals of each other [79]. With all possible cuts, eq. (5.33) is the Lagrangian dual. Eq. (5.33) fixes the dual variables for the dual sub problem. DM provides an upper bound on the optimal objective function value of the Lagrangian relaxation of the RFLDP because it is a relaxation of the Lagrangian dual.

5.2.5. The Convergence Test

The convergence test uses the following cuts:

$$UB - LB \leq \varepsilon \tag{5.35}$$

The lower bound (LB) and upper bound (UB) are set to $-\infty$ to $+\infty$ respectively. If these cuts are satisfied for all then cross decomposition continues by solving the primal and dual sub-problem. The convergence test uses the cuts in the primal master problem to determine if the upper bound or lower bound can be improved. If any cut *t* is not satisfied, a master problem is solved. The convergence test is sufficient to show that either: the sub problem can improve the upper bound or lower bound; or the primal sub problem can generate a new cut for the primal master problem.

5.3. Firefly Algorithm

Figure 5.3 explains the systematic approach of firefly algorithm.

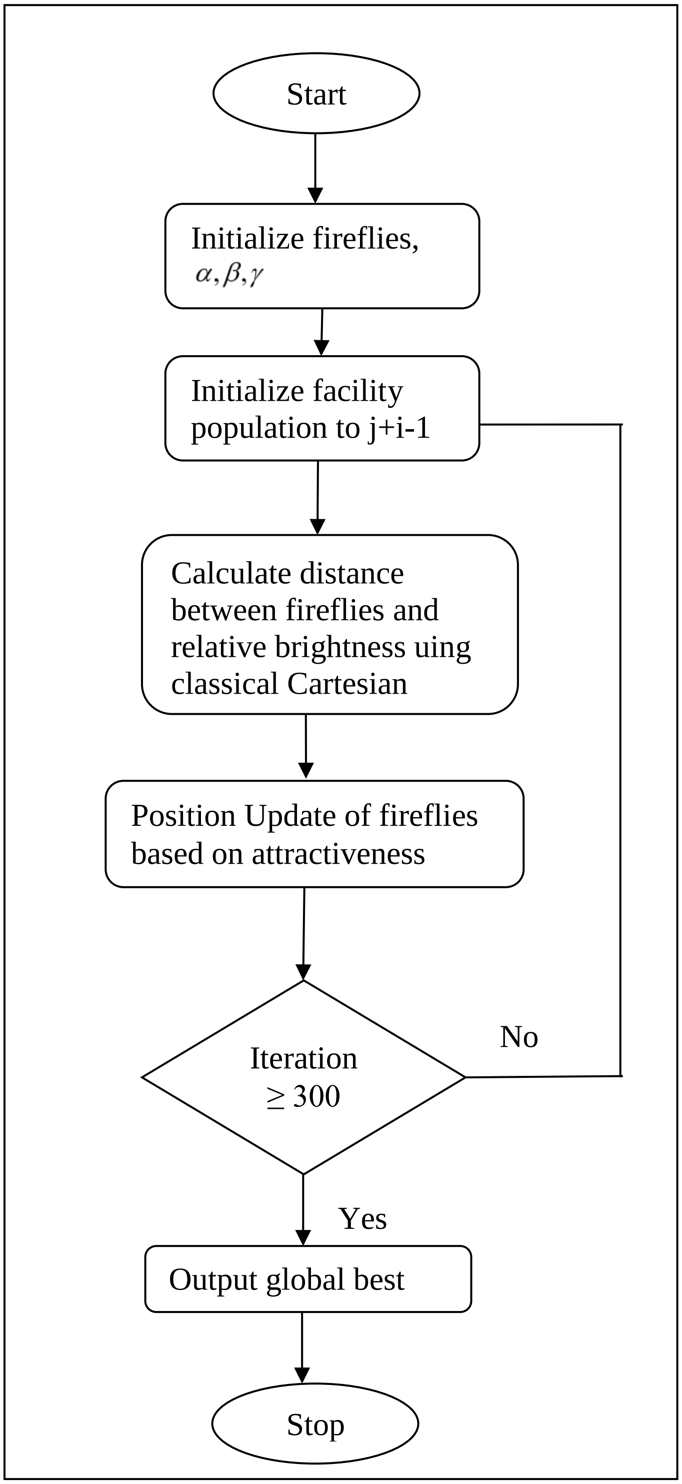

Figure 5.3 Systematic flow of Firefly Algorithm

The Firefly algorithm is decomposed in four steps:

5.3.1. Initialize firefly population

Initially, the number of fireflies are set to 25, it can be set between (15, 40) depending upon complexity of the problem. The degree of attraction β_o is set to 1, the degree of light attenuation γ is set at 1, the step factor α takes on values in range (0, 1), and the maximum number of iterations max_Iteration is set to 300**.**

5.3.2. Finding Best firefly

Evaluate the relative brightness I using Eq. (1.16) and compare the fireflies based on the brightness using Cartesian distance in Eq. (1.17). Calculate step factorαas

$$\alpha_t = a*(1-(1-\frac{10^{-4}}{9})^{1/\text{max\_Iterat ion}}) \qquad (5.36)$$

5.3.3. Movement of fireflies

Attractiveness of a firefly is proportional to the light intensity and can be expressed as

$$\beta = \beta_o e^{-\gamma r^2} \qquad (5.37)$$

Where β_o is the attractiveness at $r = 0$. r is the distance between two fireflies and γ is the light absorption coefficient.

- With distance, brightness decreases. This makes inverse relation between distance and brightness.
- As they move randomly, there is no chance to get attracted by firefly with little brightness than its own.

For minimization problem as stated in the proposed model, the brightness is inversely proportional to the value of the objective function. To get firefly with highest brightness, each firefly f change its position iteratively by cross-checking attractive among fellow fireflies. As each and every firefly possess unique attractiveness that is evaluated based on fireflies light intensity. The light intensity of each firefly f, I_f is directly associated objective function $\psi_{x,y,z}$. The position update

$$x_i = x_i + \beta_o e^{-\gamma r^2}(x_j - x_i) + \alpha(rand - 1/2) \quad (5.38)$$

Depends on light so, $I_f = \frac{1}{\psi_{(x,y,z)}}$. If $I_{i+1} > I_i$ then firefly i moves towards firefly $i+1$ with random step and if the new solution found is feasible, the movement is acceptable; otherwise, ignore.

5.3.4. Termination Condition

When the maximum iteration set is reached, the algorithm is stopped and global best output is produced to the model. Figure 5.3 demonstrates the overview of the firefly algorithm.

Chapter VI

Experimental Results and Analysis

6.1. IBM CPLEX 12.6.3

The term optimisation consists of maximizing and minimizing a function by selectively choosing input values from within defined set of values and then computing the value of the function. Optimization software provides better design and development of optimization solutions for real-life problems. The software generates different solutions under different constraints. Here IBM CPLEX is used for the purpose.

IBM CPLEX is one of the tools widely used to solve combinatorial optimization problems. It has a concert technology that provides interfaces to C++, C# and Java languages. It is accessible through independent modeling systems such as AMPL, and TOMLAB. It is also recognized as a constraint solving toolkit suitable for solving optimization models. It uses inbuilt procedures to solve the mixed integer programming in short time. It can be used to solve a variety of different optimization problems in a variety of computing environments.

6.2. Experimentation 1: Lagrangian Relaxation Approach

6.2.1. Datasets used and Parameter Setting

In order to demonstrate the superiority of proposed model, it is tested on twenty different test problem instances. These test problem instances are generated from parameters mentioned in Table 6.1. The customer range is set from 10 to 120 and number of DC is set from 2 to 20.

Table 6.1. Parameters for test instances

Parameters	Cap_j	d_i	$prob_j$	f_{jl}^U	S_{ji}^P	S_{ji}^B	H_j
Values	$U[250,300]$	$U[100,250]$	$U[0.025,0.15]$	$U[2000,3000]$	$U[30,35]$	$1.25 * S_{ji}^P$	$U[15,20]$

Besides these parameters, the fixed cost for opening reliable distribution center is set to 20000* $prob_j - f_{jl}^U$. The value of shipping cost from backup to customer is set to 1.25* SC_{ji}^P. It is also noticed that the shipping from back-up source is always costlier than the primary source.

Table 6.2 illustrates the initial values of Lagrangian parameters involved for solving proposed model. The four well-known parameters for Lagrangian Relaxation are step size, maximum number of iterations, Lagrange multiplier, and number of iterations before reduction in step size.

Table 6.2. Parameters for Lagrangian Relaxation

Parameter	Step Size	Maximum Number of Iterations	Initial Lagrange Multiplier	Number of Iterations before Reduce Step Size
Value	2	300	0	12

6.2.2. Results and Discussion

The performance of proposed model is evaluated using two well-known metrics. These are optimality gap and computational time. The optimality gap is a measure of how close the solution is to the optimal solution. It also measures how much the solution obtained can deviate from optimal solution for the given problem. The performance gap is computed as follows [59].

$$Performance\ gap\,\% = \frac{objective_x - objective_{CPLEX}}{objective_x} * 100 \qquad (6.1)$$

Where x is approach used to solve proposed model. Another performance metric is computation time. The computational time is the time taken by CPU to solve a particular problem. It is measured in terms of seconds. The proposed model run for 20 independent times for the fair comparison. The average value of the above-mentioned performance metrics is reported in the tables.

Table 6.3 describes the different parameters involved in the calculation of the problem such as constraints, non-zero coefficients and binary variables. It also describes the performance gap and computation time of different instances. The results obtained from table reveal that proposed approach provides gap than CPLEX. It is also noticed that there is a great decrease in CPU time as compared with CPLEX.

Table 6.3. Performance analysis on Test Instances

Instance	**I**	**J**	**Constant Variables**	**Non Zeros Variables**	**Binary Variables**	**Objective Value**	**% GAP**	**CPU Time (CPLEX)**	**CPU Time (LR)**
1	10	2	77	258	48	57724	0.00	0.23	0.21
2	13	3	140	690	129	76789	0.01	0.41	0.35
3	31	6	608	6006	1140	169675	0.02	0.68	0.50
4	27	5	448	3690	695	153854	0.03	0.63	0.60
5	15	3	160	792	147	86181	0.06	0.38	0.39
6	21	4	286	1884	352	119180	0.00	0.52	0.39
7	65	11	2244	40854	7909	351467	0.06	1.50	0.59
8	20	4	273	1796	336	116474	0.07	1.67	1.23
9	87	19	5684	178942	35093	528044	0.90	5.37	4.56
10	120	20	7381	244980	48080	658492	0.96	9.63	7.23
11	57	9	1624	24192	4653	309609	0.21	1.66	1.22
12	49	8	1250	16536	3168	272414	0.00	1.39	1.39
13	100	19	5858	184471	36176	542775	0.10	5.35	4.31
14	73	13	2960	63700	12389	399941	0.02	2.19	1.55
15	28	5	464	3825	720	157473	0.00	0.76	0.55
16	51	9	1456	21654	4167	278606	0.06	1.58	1.20
17	89	15	4140	102930	20085	489267	0.89	4.08	3.11
18	69	10	2170	35970	6940	372880	0.01	1.49	1.00
19	76	14	3311	76734	14952	417875	0.78	3.19	2.13
20	48	8	1225	16200	3104	267008	0.00	1.40	0.99

6.2.3. Convergence Analysis

Figures 6.1, 6.2, 6.3 and 6.4 show the convergence curves obtained from proposed model using test instances 2, 5, 10 and 14 respectively. These figures are drawn between objective function value and time.

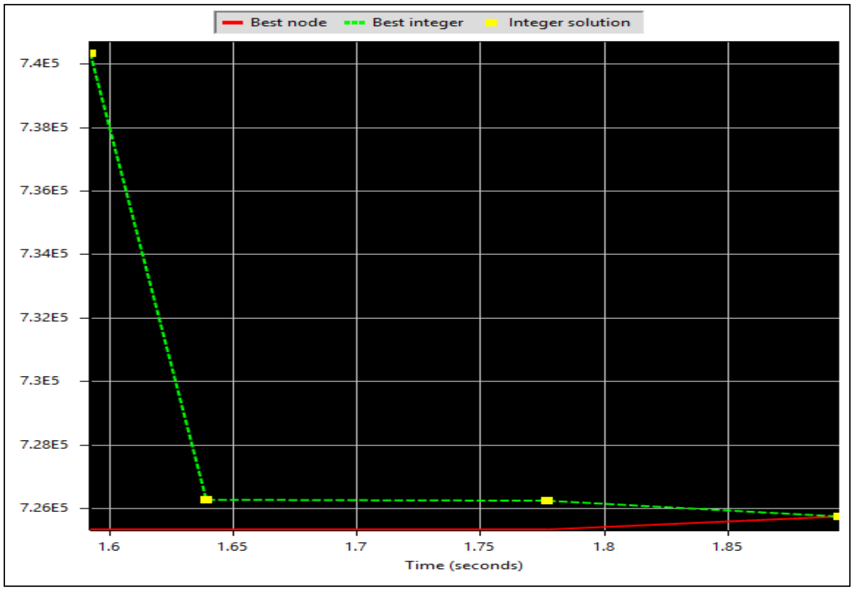

Figure 6.1. Convergence toward optimal value using Lagrangian Relaxation

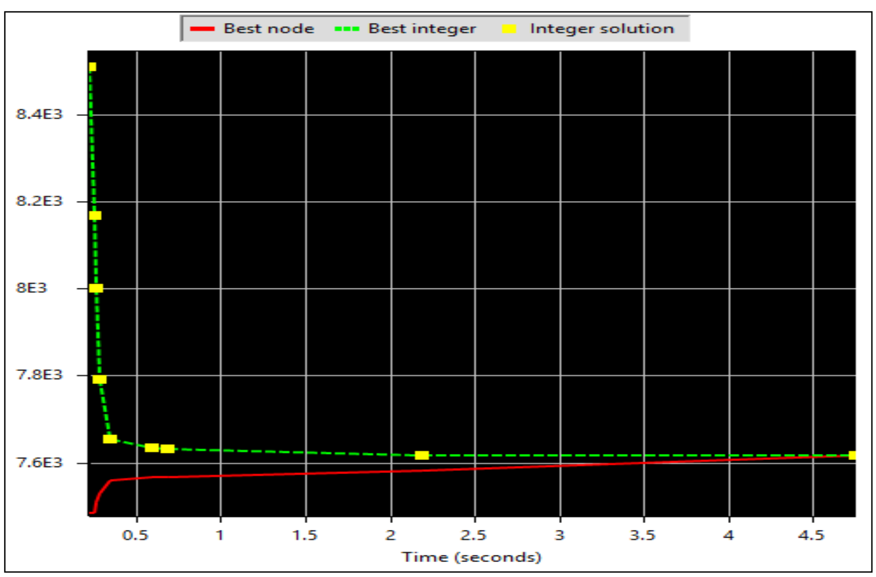

Figure 6.2. Convergence toward optimal value using Lagrangian Relaxation

In these figures, the yellow point indicates a node where an integer value has been found. The green line represents the evolution of best integer value computed. The red line gives a bound on the final solution. The convergence in the figures depends on duality gap, if gap of LR is small then the algorithm converges faster to lead optimal or near-optimal solutions.

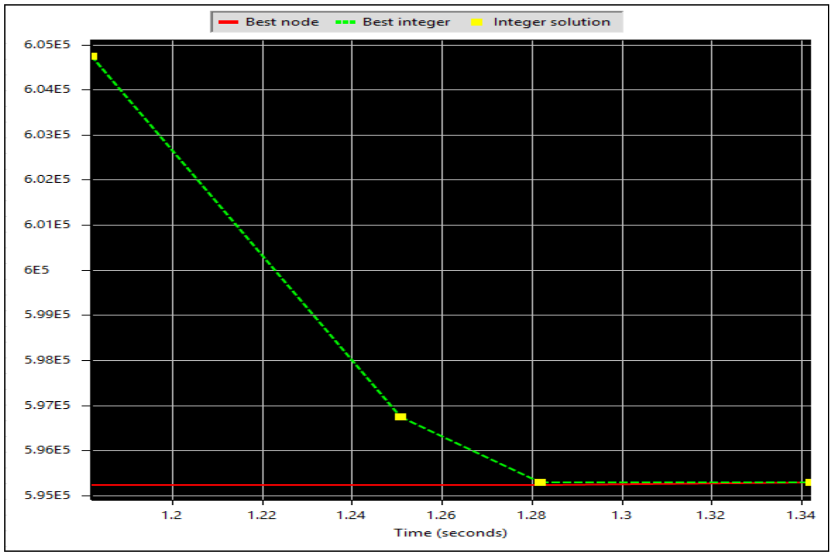

Figure 6.3. Convergence toward optimal value using Lagrangian Relaxation

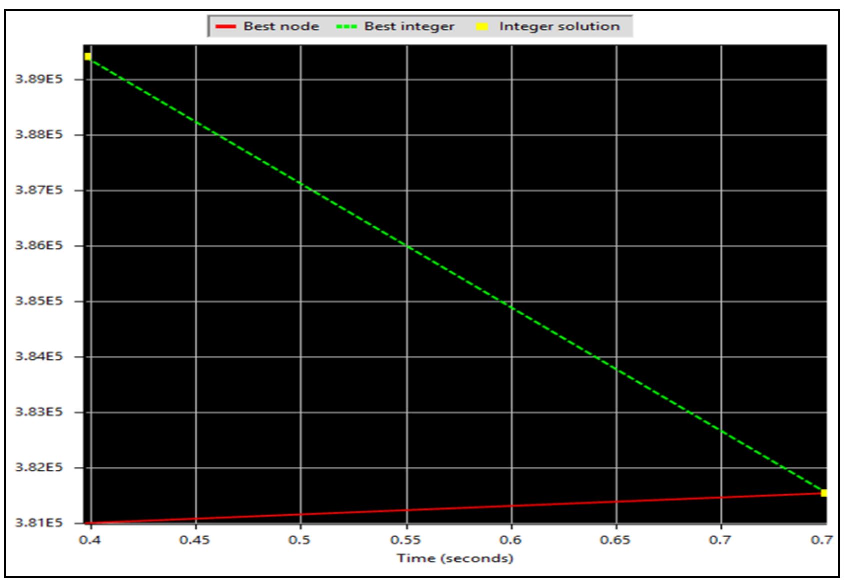

Figure 6.4. Convergence toward optimal value using Lagrangian Relaxation

6.2.4. Sensitivity Analysis

The influence of different parameters is shown in Table 6.7. Here, Ticks indicate the impact of parameters on the objective function and decision variables. Parameters which having lowest impact are opening reliable and unreliable facility cost. Parameters that have a medium impact are facility capacity and shipping cost from the primary. Parameter shipping cost from backup has also medium or high impact as it affects the scenario only when it fails. However, demand and probability contribute the highest impact.

Table 6.4. Sensitivity Analysis on different Parameters

	C_j	D_i	FP_j	FC_{jl}^U	FC_j^R	SC_{ji}^P	SC_{ji}^B	H_j
Objective	✓	✓	✓	✓	✓	✓	✓	✓
X_j^R	✓	✓	✓		✓	✓		
X_j^U	✓	✓	✓	✓			✓	
Y_{ikj}	✓	✓	✓			✓	✓	✓
Z_{ij}		✓	✓			✓		

The impact factor of parameters is computed using the data mentioned in Table 6.4. It is defined as the summation of check marks between the objective function and decision variables for each parameter involved is divided by its total check marks present in the table. It is observed from Fig. 6.5 that the impact factors are classified into four classes.

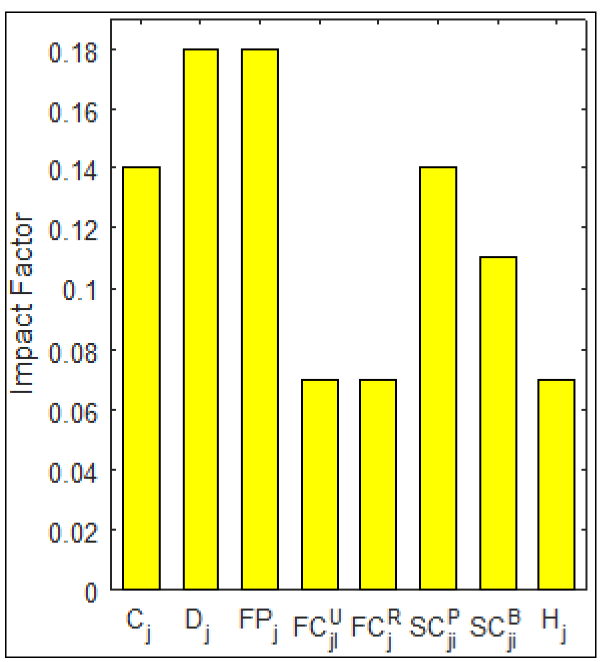

Figure 6.5 Effect of model parameters on the performance.

These are low, medium, high and very high. The value obtained from Fig. 2(b) for these impact factors are 0.07 (low), 0.11 (medium), 0.14 (high) and 0.18 (very high). It is also noticed that the demand and probability have great impact on the proposed model. By revealing, significant impact on the decision variables can help in the long-term planning of distribution design.

Fig. 6.6 shows the impact of the establishment of the reliable facility on the distribution network. It is also depicted that reliable values outweigh the additional shipping costs spent for the assignment of backup facility. There is increase in opening of reliable facility with increase in failure probability. Whereas, the establishment of unreliable facility is decreased with increase in failure probability.

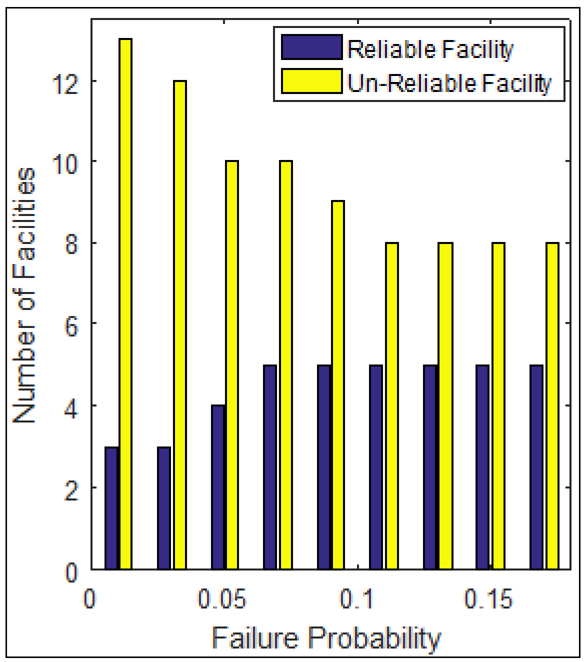

Figure 6.6 Effect of failure probability on the facility

6.3. Experimentation 2: Cross decomposition Approach

6.3.1. Datasets used and Parameter Setting

The proposed model with algorithm is tested on 30 randomly generated datasets of different problem sizes. This datasets has been divided into two categories small and medium. For customers size parameters are set between 15 to 200, the number of DCs is between 3 to 30, links are restricted with 3 to 6, and for investment levels are 3. The test problems are randomly generated by keeping realistic characteristics of small and medium sized organization. Table 6.5 presents the range of possible parameters for the costs, demands, supplies and capacities in each dataset. Table 6.9 and 6.10 shows the experimental analysis of the datasets. 10 runs has been done on each datasets value. Reported values correspond to the average values obtained in those runs.

Table 6.5 Range of parameters used

Parameter	**Value**
Demand (d_i)	U[80, 240]
Probability of site disruption $\left(prob_j\right)$	U[0.025,0.25]
f_{j1}^{u} f_{j2}^{u} f_{j3}^{u}	U[2000, 3000] 4000* P_j + f_{j1}^{u} 10000* P_j + f_{j1}^{u}
f_j^R	20000* $prob_j - f_{jl}^U$
Δ_{j1} Δ_{j2} Δ_{j3} Δ_{j4} Δ_{j5}	U[0.25, 0.75] 0.875 * Δ_{j1} 0.75 * Δ_{j1} 0.625* Δ_{j1} 0.5* Δ_{j1}
Primary DC cost with reliable link (S_{ji}^{P})	U[30, 35]
Secondary DC cost with reliable link (S_{ji}^{B}) $S_{ji}^{B} \geq S_{ji}^{P}$ because serving from demand from backup facility might be at longest route.	1.25* S_{ji}^{P}

Cost for transporting from unreliable DC j to customer i through unreliable link (S^{U}_{jil})	U[40, 45]
Shipping cost from unreliable DC j to reliable DC $k\ (j \neq k)\ (T_{jk})$	U[30, 40]
Failure probability of unsafe link between DC j and customer i due to unreliable transportation link l (θ_{jil})	U[0.025, 0.15]

6.3.2. Results and Discussion

Table 6.6 describes the different parameters involved in the calculation of the problem such as constraints, non-zero coefficients and binary variables.

Table 6.6 Size and characteristics of test instances

No.	I	J	M	L	*Int.	Cont.	Total	Const.
1	15	3	3	3	156	3	159	60
2			5	6	221	3	224	60
3	19	4	3	3	417	15	432	145
4			5	5	584	15	599	145
5	28	5	3	6	723	23	746	239
6			5	5	901	23	924	239
7	30	6	3	3	923	34	957	301
8			5	5	1295	34	1329	301
9	50	8	3	6	2029	63	1692	602
10			5	5	2848	63	2351	602
11	61	10	3	3	3043	106	3149	901
12			5	5	4269	106	4375	901
13	79	15	3	6	5663	199	5862	1625
14			5	5	7930	199	8129	1625
15	102	17	3	3	8571	293	8864	2441
16			5	5	12010	293	12303	2441
17	120	19	3	4	11474	359	11833	3179
18			5	5	16072	359	16431	3179

19	150	22	3	4	16583	481	17064	4481
20			5	6	23226	481	23707	4481
21	160	23	3	3	19072	563	18097	5001
22			5	5	22871	563	21082	5001
23	170	25	3	3	17098	592	19876	5982
24			5	6	23467	592	25672	5982
25	180	27	3	3	N/A	N/A	N/A	N/A
26			5	5	N/A	N/A	N/A	N/A
27	190	29	3	3	N/A	N/A	N/A	N/A
28			5	5	N/A	N/A	N/A	N/A
29	200	30	3	3	N/A	N/A	N/A	N/A
30			5	5	N/A	N/A	N/A	N/A

*Int. - Integer variable, cont. - Continuous variable, const. - Constraints

Table 6.7 demonstrates the objective value of the test instances. The computation time (in seconds) is recorded on these test instances. The proposed model run for 20 independent times for the fair comparison. The average value of performance measures are reported in tables.The algorithm was terminated after 20000s. It has performed well on 24 instances out of 30 instances. The last two columns provide the ratio between the computation times and solution costs, respectively. Below are the formulation used for the calculation of ratios, x b the algorithm used to solve the proposed model. Values less than 100 in the $\text{Ratio}_{\text{time}}$ (%) and $\text{Ratio}_{\text{objective}}$ (%) columns show that algorithm has outperformed CPLEX in response to speed and objective, respectively.

$$Ratio_{time} = \frac{Time_x}{Time_{CPLEX}} * 100 \quad (6.2)$$

$$Ratio_{objective} = \frac{objective_x}{objective_{CPLEX}} * 100 \quad (6.3)$$

Table 6.7 Performance evaluation on test instances

No.	**Objective**		**Time(in seconds)**		**Ratio(%)**	
	CPLEX	**CD**	**CPLEX**	**CD**	**$\text{Ratio}_{\text{objective}}$ (%)**	**$\text{Ratio}_{\text{time}}$(%)**
1	66088	66088	0.09	0.07	100	77.77
2	63160	63149	0.1	0.08	99.99	80
3	81129	81099	0.7	0.59	99.96	84.28

4	79832	78850	0.4	0.29	98.76	72.5
5	128154	128032	1.98	1.06	99.90	53.53
6	124220	129221	5.00	4.53	104.02	90.60
7	126850	126981	6.93	5.06	100.10	73.01
8	123645	124235	8.71	7.11	100.47	81.63
9	201143	197699	10.99	4.09	98.28	37.21
10	197646	207331	150.49	23.71	104.90	15.75
11	232298	226719	429.37	50.65	97.59	11.79
12	226836	203456	1590.81	67.87	89.96	4.26
13	308689	302456	5248.95	56.09	97.98	1.06
14	301915	296099	15341.90	130.04	98.07	0.84
15	379226	367605	17689.11	201.12	96.93	1.13
16	N/A	3621094	N/A	1009.67	N/A	N/A
17	N/A	3868721	N/A	3123.49	N/A	N/A
18	N/A	4610836	N/A	4125.03	N/A	N/A
19	N/A	4977614	N/A	7689.65	N/A	N/A
20	N/A	5014373	N/A	9875.99	N/A	N/A
21	N/A	5286735	N/A	19112.91	N/A	N/A
22	N/A	5678120	N/A	8279.13	N/A	N/A
23	N/A	5098109	N/A	8040.91	N/A	N/A
24	N/A	6109615	N/A	12896.00	N/A	N/A
25	N/A	N/A	N/A	1213.67	N/A	N/A
26	N/A	N/A	N/A	N/A	N/A	N/A
27	N/A	N/A	N/A	N/A	N/A	N/A
28	N/A	N/A	N/A	N/A	N/A	N/A
29	N/A	N/A	N/A	N/A	N/A	N/A
30	N/A	N/A	N/A	N/A	N/A	N/A

6.3.3. Sensitivity Analysis

In this subsection, impact of site-specific probability and link disruption probability are analyzed on opening of reliable and unreliable DCs. The results are discussed for two instances as 79 and 1500 are discussed below. Figure 6.7 demonstrate impact of failure probability of opening of reliable and unreliable DCs for 79 potential sites.

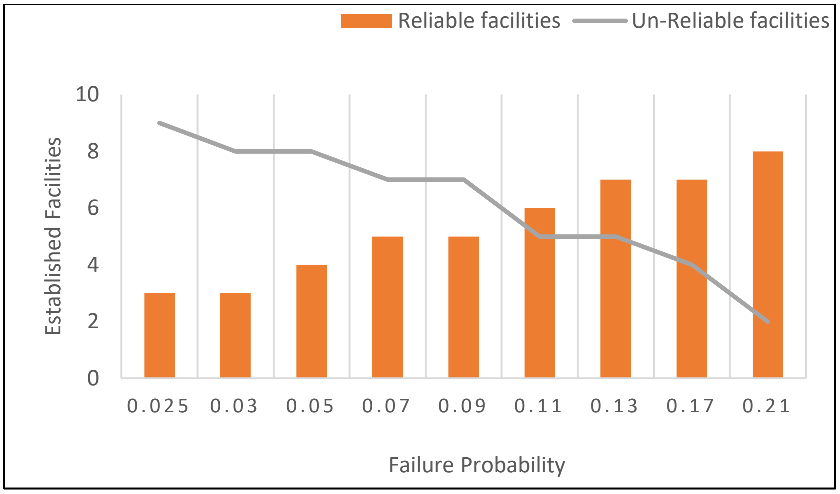

Figure 6.7 Effect of failure probability on 79 potential DCs

The figure reveals that with increase in failure probability ($prob$), opening of number of reliable DCs increases. With increase in establishment of reliable DCs, establishment of unreliable DCs fall down. Similar results are found for 150 potential sites for DCs. Figure 6.8 demonstrate opening of reliable and unreliable DCs for 150 potential sites.

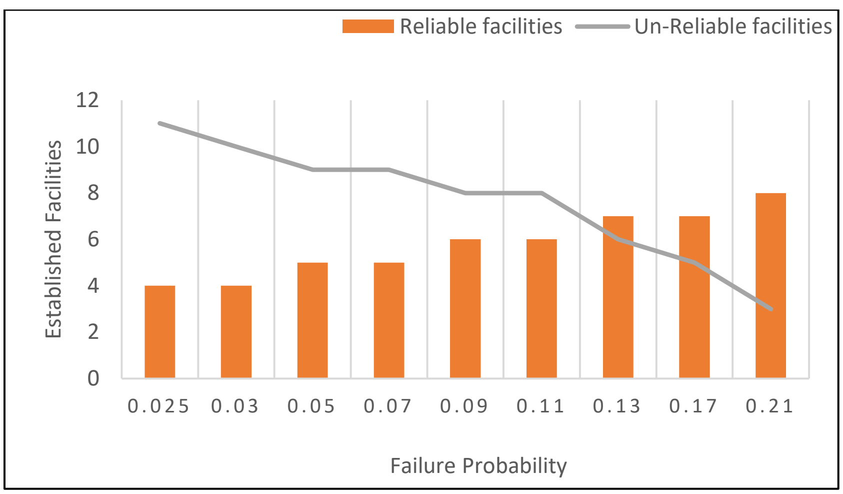

Figure 6.8 Effect of failure probability on 150 potential DCs

The impact of including link disruption (θ_{jil}) on opening of reliable and unreliable DCs is discussed in Figure 6.9. Figure 6.9 shows impact of link disruption probability of opening of reliable and unreliable DCs for 150 potential sites. As investing in reliable links can improve delivery and customer service level.

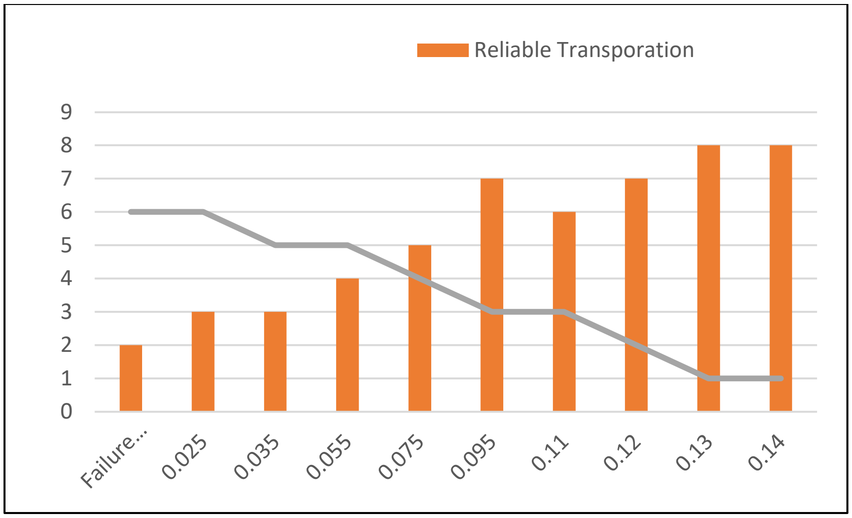

Figure 6.9 Effect of Transportation failure on 150 potential DCs

With increase in link disruption probability, establishment of unreliable DCs fall down. This figure reveals that when probability of transportation disruption increases, reliable links are more robust than unreliable links. Thereby, investing in reliable mode for shipping, the risk of failing to fulfill the demands can be averted.

6.4. Experimentation 3: Firefly Algorithm

6.4.1. Datasets used and Parameter Setting

The proposed model with algorithm is tested on 30 randomly generated datasets of different problem sizes. This datasets has been divided into two categories small and medium. For customers size parameters are set between 15 to 200, the number of DCs is between 3 to 30, links are restricted with 3 to 6, and for investment levels are 3. The test problems are randomly generated by keeping realistic characteristics of small and medium sized organization. Table 6.5 presents the range of possible parameters for the costs, demands, supplies and capacities in each dataset. Table 6.9 and 6.10 shows the experimental analysis of the datasets. 10 runs has been done on each datasets value. Reported values correspond to the average values obtained in those runs.

Table 6.8 Range of parameters used

Parameter	**Value**
Demand (d_i)	U[80, 240]

Probability of site disruption $\left(prob_j\right)$	U[0.025,0.25]
f_{j1}^{u} f_{j2}^{u} f_{j3}^{u}	U[2000, 3000] 4000* P_j + f_{j1}^{u} 10000* P_j + f_{j1}^{u}
f_j^R	20000* $prob_j - f_{jl}^U$
Δ_{j1} Δ_{j2} Δ_{j3} Δ_{j4} Δ_{j5}	U[0.25, 0.75] 0.875 * Δ_{j1} 0.75 * Δ_{j1} 0.625* Δ_{j1} 0.5* Δ_{j1}
Primary DC cost with reliable link (S_{ji}^{P})	U[30, 35]
Secondary DC cost with reliable link (S_{ji}^{B}) $S_{ji}^{B} \geq S_{ji}^{P}$ as serving from demand from backup facility might be at longest route.	1.25* S_{ji}^{P}
Cost for transporting from unreliable DC j to customer i through unreliable link (S_{jil}^{U})	U[40, 45]
Shipping cost from unreliable DC j to reliable DC k $(j \neq k)(T_{jk})$	U[30, 40]
Failure probability of unsafe link between DC j and customer i unreliable transportation link l (θ_{jil})	U[0.025, 0.15]

6.4.2. Results and Discussion

Table 6.9 describes the different parameters involved in the calculation of the problem such as constraints, non-zero coefficients and binary variables.

Table 6.9 Size and characteristics of test instances

No.	I	J	M	L	*Int.	Cont.	Total	Const.
1	15	3	3	3	156	3	159	60
2			5	5	221	3	224	60
3	19	4	3	3	417	15	432	145
4			5	5	584	15	599	145
5	28	5	3	3	723	23	746	239
6			5	5	901	23	924	239
7	30	6	3	3	923	34	957	301
8			5	5	1295	34	1329	301
9	50	8	3	3	2029	63	1692	602
10			5	5	2848	63	2351	602
11	61	10	3	3	3043	106	3149	901
12			5	5	4269	106	4375	901
13	79	15	3	3	5663	199	5862	1625
14			5	5	7930	199	8129	1625
15	102	17	3	3	8571	293	8864	2441
16			5	5	12010	293	12303	2441
17	120	19	3	3	11474	359	11833	3179
18			5	5	16072	359	16431	3179
19	150	22	3	3	16583	481	17064	4481
20			5	5	23226	481	23707	4481
21	160	23	3	3	19072	563	18097	5001
22			5	5	22871	563	21082	5001
23	170	25	3	3	17098	592	19876	5982
24			5	5	23467	592	25672	5982
25	180	27	3	3	19765	613	19265	6128
26			5	5	25671	613	29511	6128
27	190	29	3	3	23123	765	22181	6564
28			5	5	29875	765	29872	6564
29	200	30	3	3	27961	871	31721	6671

30			5	5	31267	871	35621	6671

*Int.- Integer variable, cont. - Continuous variable, const.- Constraints

Table 6.10 describes the objective value and computation time of different instances. The proposed model run for 20 independent times for the fair comparison. The average value of performance measures are reported in tables. The last two columns provide the ratio between the computation times and solution costs using eq. (6.2) and eq. (6.3), respectively. The algorithm was terminated after 20000s.

Table 6.10 Objective and Computation Time on Test Instances

No.	Objective		Time(in seconds)		Ratio(%)	
	CPLEX	FA	CPLEX	FA	$\text{Ratio}_{\text{objective}}$(%)	$\text{Ratio}_{\text{time}}$(%)
1	66088	66088	0.09	0.07	100	77.77
2	63149	63159	0.1	0.09	99.99	90
3	81099	81099	0.7	0.65	99.96	92.85
4	78850	79850	0.4	0.39	100	97.5
5	128032	134132	1.98	1.56	104.66	78.78
6	129221	134216	5.00	4.63	108.04	92.6
7	126981	127980	6.93	5.06	103.50	73.01
8	124235	134235	8.71	9.81	108.56	112.62
9	197699	197651	10.99	6.79	98.26	61.78
10	207331	207342	150.49	24.67	104.90	16.39
11	226719	226751	429.37	52.87	97.61	12.31
12	203456	213456	1590.81	89.87	94.10	5.64
13	302456	302456	5248.95	49.09	97.98	0.93
14	296099	298761	15341.90	149.04	98.95	0.97
15	367605	367801	17689.11	233.16	94.49	1.31
16	3621094	362390	N/A	1213.67	N/A	N/A
17	3868721	409871	N/A	3789.09	N/A	N/A
18	4610836	479087	N/A	4345.03	N/A	N/A
19	4977614	498076	N/A	7689.65	N/A	N/A
20	5014373	504327	N/A	9999.99	N/A	N/A
21	5286735	538373	N/A	10021.01	N/A	N/A
22	5678120	579389	N/A	12019.21	N/A	N/A
23	5098109	510917	N/A	9200.89	N/A	N/A
24	6109615	625361	N/A	15987.00	N/A	N/A

25	N/A	655399	N/A	13192.91	N/A	N/A
26	N/A	7693218	N/A	14652.78	N/A	N/A
27	N/A	7532061	N/A	15729.34	N/A	N/A
28	N/A	7984319	N/A	13874.33	N/A	N/A
29	N/A	8675215	N/A	12532.95	N/A	N/A
30	N/A	9875427	N/A	10921.73	N/A	N/A

6.4.3. Sensitivity Analysis

In this subsection, impact of site-specific probability and link disruption probability are analyzed on opening of reliable and unreliable DCs. The results are discussed for two instances as 150 and 180 are discussed below. Figure 6.7 demonstrate impact of failure probability of opening of reliable and unreliable DCs for 150 potential sites. The figure reveals that with increase in failure probability ($prob$), opening of number of reliable DCs increases. With increase in establishment of reliable DCs, establishment of unreliable DCs fall down. Similar results are found for 180 potential sites for DCs. Figure 6.8 demonstrate opening of reliable and unreliable DCs for 180 potential sites.

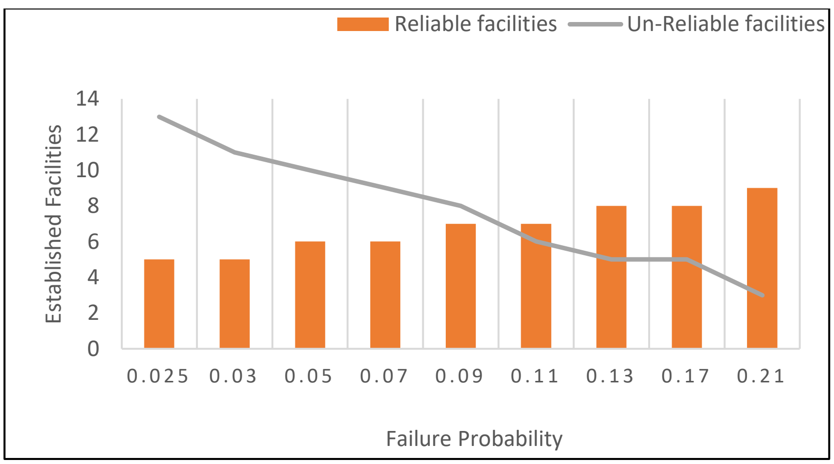

Figure 6.10 Effect of failure probability on 150 potential DCs

The impact of including link disruption (θ_{jil}) on opening of reliable and unreliable DCs is discussed in Figure 6.12. Figure 6.12 shows impact of link disruption probability of opening of reliable and unreliable DCs for 200 potential sites.

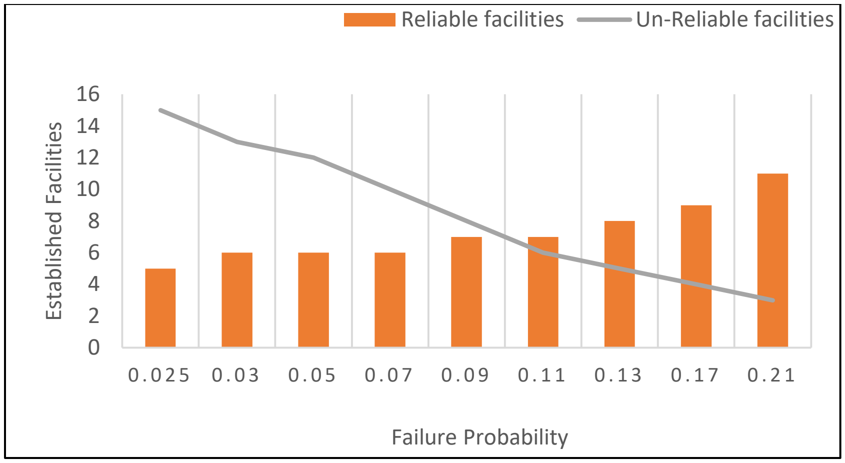

Figure 6.11 Effect of failure probability on 180 potential DCs

As investing in reliable links can improve delivery and customer service level. With increase in link disruption probability, establishment of unreliable DCs fall down. This figure reveals that when probability of transportation disruption increases, optimal number of reliable links are more robust than unreliable links. Thereby, investing in reliable mode for shipping helps to acquire minimum failure effect.

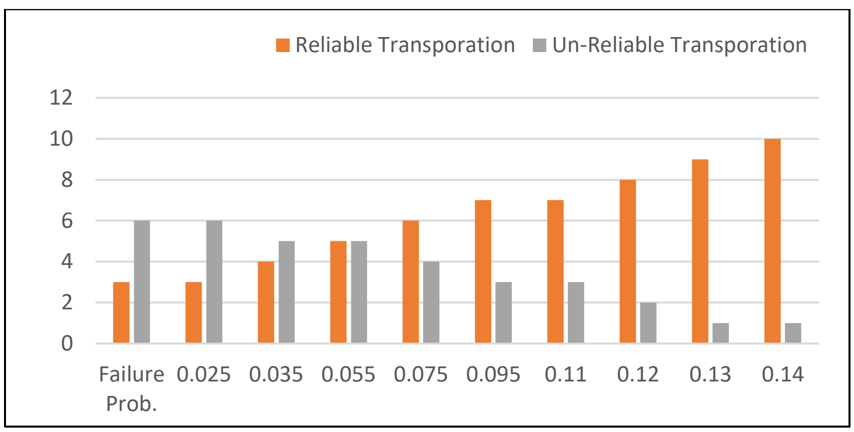

Figure 6.12 Effect of Transportation failure on 200 potential DCs

6.5. Comparison Between Cross Decomposition and Firefly Algorithm

Table 6.11 summarizes the performance of the Cross Decomposition (CD) with that of FA. Table uses the results from table 6.7 and table 6.10. Table 6.10 represents the run time in seconds and cost solution for the proposed model from both algorithms. The performance of CD when installing/operating new distribution centers is small, it has performed well in terms of computational speed. Whereas in case of large search space, FA has shown good results. The objective of both algorithms has been near about similar. As FA is approximate algorithm, objectives are approximately equal to that of CD.

Table 6.11 Comparison between CD and FA

No.	Objective		Time(in seconds)	
	CD	**FA**	**CD**	**FA**
1	66088	66088	0.07	0.07
2	63149	63159	0.08	0.09
3	81099	81099	0.59	0.65
4	78850	79850	0.29	0.39
5	128032	134132	1.06	1.56
6	129221	134216	4.53	4.63
7	126981	127980	5.06	5.06
8	124235	134235	7.11	9.81
9	197699	197651	4.09	6.79
10	207331	207342	23.71	24.67
11	226719	226751	50.65	52.87
12	203456	213456	67.87	89.87
13	302456	302456	56.09	49.09
14	296099	298761	130.04	149.04
15	367605	367801	201.12	233.16
16	3621094	3623901	1009.67	1213.67
17	3868721	4098712	3123.49	3789.09
18	4610836	4790879	4125.03	4345.03
19	4977614	4980767	7689.65	7689.65
20	5014373	5043273	9875.99	9999.99
21	5286735	5383739	19112.91	10021.01
22	5678120	5793894	8279.13	12019.21
23	5098109	5109173	8040.91	9200.89
24	6109615	6253612	12896.00	15987.00
25	N/A	6553991	1213.67	13192.91
26	N/A	7693218	N/A	14652.78
27	N/A	7532061	N/A	15729.34
28	N/A	7984319	N/A	13874.33
29	N/A	8675215	N/A	12532.95
30	N/A	9875427	N/A	10921.73

*N/A no feasible solution could be obtained for that instance in the allowed time (20000 s).

Above table show that FA has outperformed CD in some scenarios where search space is large. The solutions based on FA, on average, 1.1 % more expensive than those returned by CD (across all instances), and it was able to find the same or better solutions than CD for 25 of the 30 test problems (~84%).

Figure 6.13 illustrates the impact of failure probability and increase in number of distribution centers on the performance of both algorithms. The comparison showed that the cross decomposition algorithm performs much better than firefly on instances where the failure probability is low and search for potential distribution center is low. Indeed, the pruning phase of Firefly allows to quickly reduce the domain of the search, so that an exhaustive search on the remaining feasible solution is not computationally expensive. However, as the number of failure probability increases, the possibility for firefly to prune is reduced, and the computational time is raised.

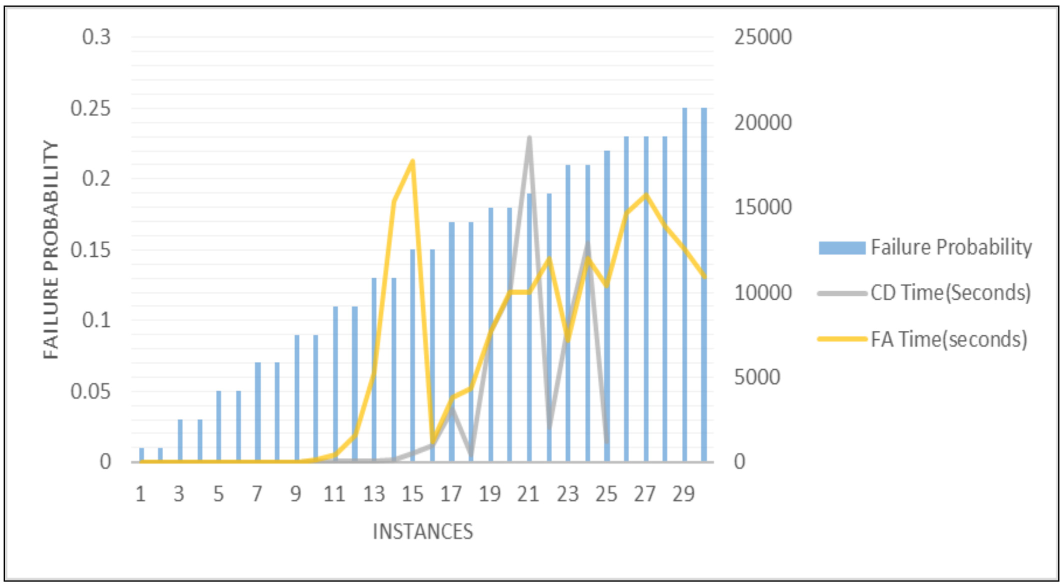

Figure 6.13 Comparison between CD and FA

Even though it is slower than cross decomposition on instances with few failure probability, it is much faster than cross decomposition when the number of failure probability is larger. Obviously, since Firefly is a meta-heuristic, there are no guarantees that the found solutions are the optimal ones, and usually more than one execution of the algorithm is needed.

Chapter VII

Conclusion and Future work

7.1. Conclusions

The reliability model offers a systematic approach to distribution facility location problem. It extends the previous research done in facility location by incorporating a consistent weighting of crucial factors (i.e., facility capacity, site-specific failures). In this work, a mixed-integer programming model has been proposed for reliable capacitated facility location problem. The proposed model uses both proactive and reactive measures when a customer's primary facility fails. It uses single-level backup mechanism that will increase facility availability. It can simultaneously determines both optimal number and location of capacitated distribution centers. The proposed model is implemented using Lagrangian Relaxation approach. The performance of proposed model has been tested on twenty test instances. The experimental results reveal that the computational time of proposed model is much lesser than CPLEX's model.

The proposed model has been further improved by incorporating the concept of link disruption. The proactive mitigation strategy was implemented through extra investment for reliable DCs during the design phase. It can simultaneously determines both optimal number and location of capacitated distribution centers. The optimal number of reliable transportation links are also determined. It is implemented using cross decomposition and firefly metaheuristic approaches. These have been tested on thirty different types of test instances. The experimental results demonstrate that firefly approach provides better results as compared to cross decomposition when the size of test instances and failure probability increases. Whereas, the cross decomposition approach performs better than firefly in case of small test instances and less failure probability.

7.2. Future Scope

Various mitigation strategies (i.e., diversification, expedition-shipment, fortification) can be compared to design the most resilient configuration for supply chain networks. Distribution networks under disruption with a multi-period setting can be a more practical research topic. The model can be extended to logistics management to design resilient supply chain networks under disruption. The proposed cross decomposition approach has shown promising results in less computational time. It can be applied on a real case study to assess the potential of approach. Multi-objective metaheuristics can be considered for solving large scale instances.

References

[1] M.L.Balinski, “Integer Programming: Methods, uses, computations.” *Management Science*, vol. 12, pp. 253-313, 1965.

[2] D. Shishebori and M. S. Jabalameli, “Improving the Efficiency of Medical Services Systems: A New Integrated Mathematical Modeling Approach,” *Mathematical Problems in Engineering*, vol. 2013, 2013.

[3] S. Tragantalerngsak, J. Holt, and M. Ronnqvist, “An exact method for the two-echelon, single-source, capacitated facility location problem”, *European Journal of Operational Research*, vol.123, pp.473–489, 2000.

[4] M. Kon, S. Kushimoto, “A Single Facility Minisum Location Problem under the A-Distance,” *Journal of the Operations Research, Society of Japan*, vol. 40(l), 1997.

[5] T. Matsutomi, and H. Ishii, “Minimax Location Problem with A-Distance,” *Journal of the Operations Research, Society of Japan*, vol. 41(2), 1998.

[6] R. Church, and C. ReVelle, “The Maximal Covering Location Problem,” The Johns Hopkins University, 1971.

[7] F.Plastria, “Static competitive facility location: An overview of optimisation approaches,” *European Journal of Operational Research*, vol.129 (3), pp. 461-470, 2001.

[8] G. O. Wesolowsky, “Dynamic Facility Location,” *Management Science*, vol. 19(11), pp. 1241-1248, 1973.

[9] G. O. Wesolowsky, “Probabilistic Weights in the One-Dimensional Facility Location Problem,” *Management Science*, vol. 24(2) pp.224 – 229, 1997.

[10] A. Anagnostopoulos, R. Bent, E. Upfal, and P. V. Hentenryck, “A simple and deterministic competitive algorithm for online facility location,” *Information and Computation*, vol. 194(2), pp.175–202, 2004.

[11] Y. Sheffi, The Resilient Enterprise: Overcoming Vulnerability for Competitive Advantage. Cambridge, MA: MIT Press, 2005.

[12] L. V. Snyder , Z. Atan, P. Peng, Y. Rong, A.J. Schmitt and B. Sinsoysal “OR/MS Models for Supply Chain Disruptions: A Review,” *IIE Transaction*, vol. 48, pp. 89-109, 2015.

[13] M. L. Fisher, “The Lagrangian Relaxation Method for Solving Integer Programming Problems,” *Management Science*, vol. 27(1), pp. 1-18, 1981.

[14] M. L. Fisher, “An Applications Oriented Guide to Lagrangian Relaxation,” *Znterfaces*, vol. 15(2), pp. 10-21, 1985.

[15] T.J. Van Roy, “Cross Decomposition for Mixed Integer Programming,” *Mathematical Programming*, vol. 25, pp.46-63, 1983.

[16] T.J. Van Roy, “A Cross Decomposition Algorithm for Capacitated Facility Location,” *Operations Research*, vol.34, pp. 145-163, 1986.

[17] K.Holmberg, “On the convergence of cross decomposition,” *Mathematical programming*. vol. 47, pp.269-296, 1990.

[18] K. Holmberg, “Cross decomposition applied to integer programming problems: duality gaps and convexification in parts,” *Operations research*, vol. 42, pp.657-668, 1994.

[19] X.Yang , “Firefly algorithms for multimodal optimization,” in Stochastic algorithms: foundations and applications, pp. 169-178, Springer Berlin Heidelberg, 2009.

[20] X.Yang, Nature-inspired metaheuristic algorithms, 2nd ed. Frome: Luniver press, 2010.

[21] Z. Drezner, K. Klamroth, A. Schöbel & G. O.Wesolowsky, “The Weber Problem,” in Z. Drezner & H. Hamacher (Eds.), *Facility Location: applications and theory* . Springer-Verlag Berlin, 2005, pp. 1–36.

[22] M. T. Melo, S. Nickel, & F. Saldanha-da-Gama, “Facility location and supply chain management – A review,” *European Journal of Operational Research*, vol. 196(2), pp. 401–412, 2009.

[23] Z. Drezner, and H. Hamacher, *Facility Location: Applications and Theory*, 1st ed., pp. 457. Springer Berlin Heidelberg, 2002.

[24] J.Current , M. S.Daskin, and D. Schilling,“Discrete Network Location Models,” in Z. Drezner & H. W. Hamacher (Eds.), *Facility Location: applications and theory*. Springer-Verlag Berlin, 2002, pp. 81–118.

[25] C. Revelle, and H. Eiselt, “Location analysis: A synthesis and survey,” *European Journal of Operational Research*, vol. 165(1), pp.1–19, 2005.

[26] S. H.Owen, and M.S. Daskin, "Strategic facility location: A review," *European Journal of Operational Research*, vol. 111(3), pp. 423–447, 1998.

[27] C. Revelle, H. Eiselt, and M. Daskin, "A bibliography for some fundamental problem categories in discrete location science," *European Journal of Operational Research*, vol. 184(3), pp. 817–848, 2008.

[28] G. Sahin, and H. Süral, "A review of hierarchical facility location models," *Computers & Operations Research*, vol. 34(8), pp. 2310–2331, 2007.

[29] M.E. O'Kelly, and D.L. Bryan, "Hub location with flow economies of scale,"*Transportation Research Part B: Methodological*, vol. 32(8), pp.605–616, 1998.

[30] R. Z. Farahani, N. Asgari, N. Heidari, M. Hosseininia, and M. Goh, "Covering problems in facility location: A review," *Computers & Industrial Engineering*, 62(1), 368–407, 2012.

[31] A. B.Arabani, and R. Z. Farahani, "Facility Location Dynamics: An Overview of Classifications and Applications," *Computers & Industrial Engineering*, vol.62 (1), pp.408– 420, 2011.

[32] K. Andreas and D. Andreas, "Facility location models for distribution system design," *European Journal of Operational Research*, 2004.

[33] B. Fahimnia, R. Farahani, R. Marian and L. Luong, "A review and critique on integrated production-distribution planning models and techniques", *Journal of Manufacturing Systems*, vol.32(1), pp.1–19, 2013.

[34] A.Kuehn and M. J.Hamburger, "A heuristic program for locating warehouses," *Management Science*, vol. 9, pp. 643–666, 1963.

[35] J.R.Hanley and R.L.Church, "Designing robust coverage networks to hedge against worst-case facility losses," *European Journal of Operational Research*, vol.209, pp.23–36, 2011.

[36] K. Holmberg, M. Ronnqvist, and D. Yuan, "An exact algorithm for the capacitated facility location problems with single sourcing," *European Journal of Operational Research*, vol. 113, pp. 544-559, 1999.

[37] B. M. Khumawala, "An efficient branch and bound algorithm for the warehouse location problem,"Management Science, vol.18, pp.B-718–B-731, 1972.

[38] NV Sahinidis, “Optimization under uncertainty: State of the art and opportunities,” *Comput. Chemical Eng.*, vol.28, pp.971-983, 2004.

[39] S. Melkote and M.S. Daskin, “Capacitated facility location-network design problems,” *European Journal of Operational Research*, vol.129 (3), pp.481-495, 2001.

[40] M. S.JabalAmeli and M.Mortezaei, “A hybrid model for multi-objective capacitated facility location network design problem,” *International Journal of Industrial Engineering Computations*, vol. 2, pp. 509–524, 2011.

[41] F. Alborzi, H.Vafaei, M.H. Gholami, and M.M.S. Esfahani, “A Multi-Objective Model for Supply Chain Network Design under Stochastic Demand,” *World Academy of Science, Engineering and Technology*, vol.5, pp.1651-1655, 2011.

[42] Majid Ramezani, Mahdi Bashiri, and Reza Tavakkoli-Moghaddam, “A new multi-objective stochastic model for a forward/reverse logistic network design with responsiveness and quality level,” *Applied Mathematical Modelling*, vol. 37, pp. 328–344, 2013.

[43] G. Reza Nasiri, Rohollah Zolfaghari, and Hamid Davoudpur, “An integrated supply chain production-distribution planning with stochastic demands,” *Computers and Industrial Engineeirng*, vol. 37, pp.35-45, 2014.

[44] K.K.Castillo-Villar, N.R.Smith and J.F.Herbert -Acero, “Design And Optimization of Capacitated Supply Chain Networks Including Quality Measures,” *Mathematical Problems in Engineering*,2014.

[45] Abhijeet Ghadge, Qifan Yang , Nigel Caldwell, Christian König and Manoj Kumar Tiwari, “Facility Location for a Closed-Loop Distribution Network: A Hybrid Approach,” *International Journal of Retail and Distribution Management*, vol. 44(9), pp. 884-902, 2016.

[46] Z.Yao, L. H.Lee, W.Jaruphongsa, V.Tan and C. F.Hui, “Multi-source facility location-allocation and inventory problem”, *European Journal of Operational Research*, vol. 207(2), pp.750-762, 2010

[47] A.M.Geoffrion and G.W.Graves, “Multicommodity distribution system design by Benders decomposition,” *Management Science*, vol. 20, pp. 822–844, 1974.

[48] B. Addis, G. Carello, and A. Ceselli, “Combining very large scale and ILP based neighborhoods for a two- level location problem”, *European Journal of Operational Research*, vol. 231, pp. 535–546, 2013.

[49] H. Pirkul, and V.Jayaraman, “Distribution planning in a multi-Commodity tri-echelon system*”, Transportation Science,* vol.30 (4), pp. 291-302, 1996.

[50] V. Jayaraman and H. Pirkul, “Planning and coordination of production and distribution facilities for multiple commodities”, *European Journal of Operational Research,* vol. 133(1), pp. 394-408, 2001.

[51] G. Easwaran and H. Uster, “Tabu search and benders decomposition approach for a capacitated closed-loop supply chain network design problem”, *Transportation Science,* vol.43(3), pp.301–320, 2009.

[52] J.Olhager, S.Pashaei, and H.Sternberg, “Design of global production and distribution networks: a literature review and research agenda,” *International Journal of Physical Distribution & Logistics Management,* vol.45, pp.138–158, 2015.

[53] Hongwei Ding, Lyes Benyoucef, and Xiaolan Xie, “Stochastic multi-objective production-distribution network design using simulation-based optimization”, *International Journal of Production Research*, vol. 47(2), pp.479–505, 2009.

[54] Z. Drezner, and G.O. Wesolowsky, “Network Design: Selection and Design of Links and Facility Location,” *Transportation Research Part A*, vol.37, pp.241–256, 2003.

[55] C. Cocking, Solutions to Facility Location–Network Design Problems. University of Heidelberg, Doctor of Philosophy Thesis, 2008.

[56] C. Cocking,and G. Reinelt, “Heuristics for budget facility location-network design problems with minisum objective,” in *Operations Research Proceedings* vol. 12, pp. 563-568, Springer Berlin Heidelberg,2008.

[57] L.V. Snyder, “Facility location under uncertainty: a review,” *IIE Transactions*, vol. 38(7), pp.547–64, 2004.

[58] C.S. Tang, “Robust strategies for mitigating supply chain disruptions,” *International Journal of Logistics Research & Applications***,** vol. 9(1), pp.33–45, 2006.

[59] L.V. Snyder, Z. Atan, P. Peng, Y. Rong, A.J. Schmitt, and B. Sinsoysal, "Models for supply chain disruptions: a review," *Ssrn Electronic Journal*, vol.48(2), 2012.

[60] L.V. Snyder and M.S.Daskin, "Reliability Models for Facility Location: The Expected Failure Cost Case," *Transportation Science*, vol. 39(3), pp.400– 16, 2005.

[61] Z. Drezner, "Heuristic solution methods for two location problems with unreliable facilities," *J. Oper. Res. Soc.*, vol.38, pp.509–514, 1987.

[62] R.L.Zhan, "Models and algorithms for reliable facility location problems and system reliability optimization", 2007,

[63] R. Zhan, Z. Shen, and M. Daskin, "System reliability with location-specific failure probabilities", Working paper, University of California, Berkeley,2007.

[64] T. Cui, Y. Ouyang, and Z.J.M.Shen, "Reliable Facility Location Design under the Risk of Disruptions," *Operations Research*, vol. 58(4-part-1), pp.998–1011, 2010.

[65] O. Berman, D. Krass, and M.B.C.Menezes, "Locating Facilities in the Presence of Disruptions and Incomplete," *Information Decision Sciences*, vol. 40(4), pp.845–68, 2009.

[66] Z.J.M.Shen, R.L.Zhan, and J. Zhang, "The Reliable Facility Location Problem: Formulations, Heuristics, and Approximation Algorithms," *Informs Journal on Computing*, vol. 23(3), pp.470–82, 2011.

[67] Q. Li, B.Zeng,and A. Savachkin, "Reliable facility location design under disruptions," *Computers & Operations Research*, vol. 40(4), pp.901–905, 2013.

[68] L. Tang, C. Zhu1, Z. Lin, J. Shi, and W. Zhang, "Reliable Facility Location Problem with Facility Protection," *PLOS ONE*, 2016.

[69] A. Jabbarzadeh, S. Gholamreza, J. Naini, H. Davoudpour, and N. Azad, "Designing a Supply Chain Network under the Risk of Disruptions," *Mathematical Problems in Engineering*, 2012. 2012: pp.1-23.

[70] J.Cui , M. Zhao, X. Li, and M. Parsafard, "An S. Reliable design of an integrated supply chain with expedited shipments under disruption risks," *Transportation Research Part E*, vol. 95, pp. 143-163, 2016.

[71] P.Peng et al., "Reliable logistics networks design with facility disruptions," *Transportation Research Part B: Methodological*, vol. 45(8), pp.1190-1211, 2011.

[72] Q. Chen, X. Li, and Y. Ouyang, "Joint inventory-location problem under the risk of probabilistic facility disruptions," *Transportation Research Part B: Methodological*, vol. 45(7), pp. 991-1003, 2011.

[73] V. Ghezavati, F. Soltanzadeh, and A. Hafezalkotob,"Optimization of reliability for a hierarchical facility location problem under disaster relief situations by a chance-constrained programming and robust optimization," *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, vol. 229(6), pp. 542-555, 2015.

[74] J. Alcaraz, M. Landete, J.F. Monge, and J.L. Sainz-Pardo, "Strengthening the reliability fixed-charge location model using clique constraints," *Computers & Operations Research*, vol. 60, pp. 14-26, 2015.

[75] L. Yun, et al., "A reliability model for facility location design under imperfect information," *Transportation Research Part B*, vol. 81(Part 2), pp. 596–615, 2015.

[76] Y. Zhang, L.V.Snyder, M. Qi, and L. Miao, "A heterogeneous reliable location model with risk pooling under supply disruptions," *Transportation Research Part B: Methodological*, vol.83, pp. 151-178, 2016.

[77] M.Held , R.M.Karp, "The Traveling-Salesman Problem and Minimum Spanning Trees," *Part ii. Mathematical Programming* , vol.1, pp. 6–25, 1971.

[78] T.L.Magnanti, and R.T. Wong, "Decomposition methods for facility location problems," in *Discrete Location Theory* edited by P.B. Mirchandani and R.L. Francis,John Wiley and Sons, Inc. New york, NY.

[79] S. Mitra, P. Garcia-Herreros, and I. E. Grossmann, "A novel cross-decomposition multi-cut scheme for two-stage stochastic programming," *Computer Aided Chemical Engineering*, vol. 33, pp. 241–246, 2014.

APPENDIX A. BENDERS DECOMPOSITION

In this section, Benders master problem for the reliable capacitated facility location problem is derived. In Benders decomposition, first the complicating binary variables that are associated with location decisions are fixed. By fixing the binary variables, the problem is decomposed into demand allocation problem. The master problem considers only a subset of often integer variables. A sub-problem (SP) tries to complete the assignment on binary variable. If it is possible, the problem is solved, but if not, a cut (rejecting at least the current assignment on integer variable) is produced and added to the master problem: it is called a Benders cut. The RFLDP can be formulated with an outer optimization over the complicating variables, and an inner optimization over the continuous variables.

$$\theta = \sum_{\forall i \in I} \sum_{\forall j \in J} \sum_{\forall m \in M} prob_j X_{jm}^U A_{jk} T_{jk} \tag{A.1}$$

Subject to

$$A_{jk} \le Cap_j \sum_{\forall m \in M} X_{jm}^U , \forall j \in J, \forall k \in J(j \ne k) \tag{A.2}$$

$$A_{jk} \le \sum_{\forall u \in I} d_i X_j^R , \forall j \in J, \forall k \in J(j \ne k) \tag{A.3}$$

$$\sum_{\forall k \in J} A_{jk} + Cap_j \sum_{\forall m \in M} (1-\theta_{jm}) X_{jm}^U \ge \sum_{\forall \mathrm{i} \in \mathrm{I}} \mathrm{d_i} \left(\sum_{\forall l \in L} YU_{ijl} + Y_{ij} \right), \forall j \in J(j \ne k) \tag{A.4}$$

$$A_{jk} \ge 0, \forall j \in J, \forall k \in J(j \ne k) \tag{A.5}$$

This cut has the form $\psi \ge \rho$, where ψ represents the objective function:

$$\psi = \sum_{\forall j \in J} \sum_{\forall m \in M} f_{jm}^U X_{jm}^U + \sum_{\forall j \in J} f_j^R X_j^R + \sum_{\forall i \in I} \sum_{\forall j \in J} \left(d_i S_{ij}^P Z_{ij} + d_i S_{ij}^P Y_{ij} \right)$$
$$+ \sum_{\forall i \in I} \sum_{\forall j \in J} \sum_{\forall l \in L} \left(d_i S_{jil}^U (1-\theta_{jil}) YU_{ijl} + d_i S_{jil}^B \theta_{jil} YU_{ijl} \right)$$
$$+ \sum_{\forall i \in I} \sum_{\forall j \in J} \sum_{\forall m \in M} prob_j X_{jm}^U A_{jk} T_{jk} \tag{A.6}$$

and constitutes the key point of the method, it is inferred by the dual of the sub-problem. Let us consider an assignment of products shipped from reliable DC to unreliable DC given by the master, the sub-problem (SP) and its dual (DSP) can be written as follows:

$$SP = \underset{}{\text{minimize}} \sum_{\forall j \in J} \sum_{\forall m \in M} f_{jm}^{U} X_{jm}^{U} + \sum_{\forall j \in J} f_{j}^{R} X_{j}^{R} + \sum_{\forall i \in I} \sum_{\forall j \in J} \left(d_i S_{ij}^{P} Z_{ij} + d_i S_{ij}^{P} Y_{ij} \right)$$

$$+ \sum_{\forall i \in I} \sum_{\forall j \in J} \sum_{\forall l \in L} \left(d_i S_{jil}^{U} (1-\theta_{jil}) YU_{ijl} + d_i S_{jil}^{B} \theta_{jil} YU_{ijl} \right) \quad (A.7)$$

subject to

$$\sum_{\forall j \in J} X_{j}^{R} \geq 1 \quad (A.8)$$

$$\sum_{\forall j \in J} (Y_{ij} + \sum_{\forall l \in L} YU_{ijl} + Z_{ij}) = 1, \forall i \in I \quad (A.9)$$

$$Z_{ij} <= X_{j}^{R}, \forall i \in I, \forall j \in J \quad (A.10)$$

$$\sum_{\forall i \in I} d_i (\sum_{\forall j \in J} (Y_{ij} + \sum_{\forall l \in L} YU_{ijl})) <= Cap_j \sum_{\forall m \in M} X_{jm}^{U} \quad (A.11)$$

$$\sum_{\forall j \in J} d_i \; X_{j}^{R} >= \sum_{\forall j \in J, \forall k \in K} A_{jk}, (j \neq k) \quad (A.12)$$

$$X_{j}^{U}, X_{j}^{R} \in \{0,1\}, \forall j \in J \quad (A.13)$$

$$Y_{ikj} \in \{0,1\}, \forall i \in I, \forall j, k \neq j \in J \quad (A.14)$$

$$Z_{i} \in \{0,1\}, \forall i \in I, \forall j \in J \quad (A.15)$$

The added constraint (A.12) in the outer optimization ensures that the facility configuration is feasible. By multiply eq. (A.2) by λ_{jk}, eq. (A.3) by μ_{jk} and eq.(A.4) by ν_k, the inner optimization with its linear programming dual:

$$DSP = \sum_{\forall k \in K} \nu_{\mathrm{k}} \left(\sum_{\forall \mathrm{i} \in \mathrm{I}} \mathrm{d}_{\mathrm{i}} \left(\sum_{\forall l \in L} \overline{YU}_{ijl} + \overline{Y}_{ij} \right) - C_j \sum_{\forall m \in M} (1-\theta_{jm}) \overline{X}_{jm}^{U} \right) - \sum_{\forall \mathrm{j} \in \mathrm{J}, \forall \mathrm{k} \in \mathrm{J}} \mu_{\mathrm{jk}} \sum_{\forall u \in I} d_i \; \overline{X}_{j}^{R} - \ldots$$

$$\sum_{\forall \mathrm{j} \in \mathrm{J}, \forall \mathrm{k} \in \mathrm{J}} \lambda_{\mathrm{jk}} \left(C_j \sum_{\forall m \in M} \overline{X}_{jm}^{U} \right) \quad (A.16)$$

subject to

$$\nu_k - \mu_{jk} - \lambda_{jk} \leq \sum_{\forall m \in M} prob_j X_{jm}^{U} T_{jk} \qquad \forall \mathrm{j} \in \mathrm{J}, \forall \mathrm{k} \in \mathrm{J} (\mathrm{j} \neq \mathrm{k}) \quad (A.17)$$

$$\nu_k \geq 0 \qquad \forall \mathrm{k} \in \mathrm{J} \quad (A.18)$$

$$\mu_{\mathrm{jk}}, \lambda_{jk} \geq 0 \qquad \forall \mathrm{j} \in \mathrm{J}, \forall \mathrm{k} \in \mathrm{J} (\mathrm{j} \neq \mathrm{k}) \quad (A.19)$$

$$\rho = \text{minimize} \sum_{\forall j\in J}\sum_{\forall m\in M} f_{jm}^{U} X_{jm}^{U} + \sum_{\forall j\in J} f_{j}^{R} X_{j}^{R} + \sum_{\forall i\in I}\sum_{\forall j\in J}\left(d_i S_{ij}^{P} Z_{ij} + d_i S_{ij}^{P} Y_{ij}\right)$$

$$+ \sum_{\forall i\in I}\sum_{\forall j\in J}\sum_{\forall l\in L}\left(d_i S_{jil}^{U}(1-\theta_{jil}) YU_{ijl} + d_i S_{jil}^{B}\theta_{jil} YU_{ijl}\right) + DSP \qquad \text{(A.19)}$$

subject to

$$\sum_{\forall j\in J} X_{j}^{R} \geq 1 \qquad \text{(A.20)}$$

$$\sum_{\forall j\in J}(Y_{ij} + \sum_{\forall l\in L} YU_{ijl} + Z_{ij}) = 1, \forall i \in I \qquad \text{(A.21)}$$

$$Z_{ij} <= X_{j}^{R}, \forall i \in I, \forall j \in J \qquad \text{(A.22)}$$

$$\sum_{\forall i\in I} d_i (\sum_{\forall j\in J}(Y_{ij} + \sum_{\forall l\in L} YU_{ijl})) <= C_j \sum_{\forall m\in M} X_{jm}^{U} \qquad \text{(A.23)}$$

$$\sum_{\forall j\in J} d_i\, X_{j}^{R} >= \sum_{\forall j\in J, \forall k\in K} A_{jk}, (j \neq k) \qquad \text{(A.24)}$$

$$X_{j}^{U}, X_{j}^{R} \in \{0,1\}, \forall j \in J \qquad \text{(A.25)}$$

$$Y_{ikj} \in \{0,1\}, \forall i \in I, \forall j, k \neq j \in J \qquad \text{(A.26)}$$

$$Z_{ij} \in \{0,1\}, \forall i \in I, \forall j \in J \qquad \text{(A.27)}$$

Let $T = \{ t \mid \nu^{t}, \mu^{t}, \lambda^{t}, \text{is a (dual) feasible extreme point solution} \}$ be the index set of all (dual) feasible extreme point solutions of the inner optimization problem above. Then the previous problem can be written as in Eq. A.28. The problem in Eq.A.28 is equivalent to the original problem since the dual of the inner optimization problem attains its optimal solution at one of a finite number of extreme points. (The dual problem is bounded if the primal problem is feasible.) Duality theory ensures that $SP \geq DSP$ is therefore a lower bound of SP. As feasibility of the dual is independent of integer variable A_{jk} , $SP \geq DSP$ and the inequality $\theta + SP \geq \theta + DSP$ is valid, leading to the Benders cut : $\psi \geq \rho$ Moreover, according to duality, the optimal value of $\nu^{t}, \mu^{t}, \lambda^{t}$ maximizing DSP corresponds to the same optimal value of SP. Even if the cut is derived from a particular $\nu^{t}, \mu^{t}, \lambda^{t}$, it is valid for all ν , μ , λ and excludes a large class of assignments which share common characteristics.

$$\left\{\begin{array}{l} \omega_{t\in T} = \sum_{\forall j\in J}\sum_{\forall m\in M} f_{jm}^{U} X_{jm}^{U} + \sum_{\forall j\in J}\left(\left(f_j^R - \sum_{\forall k\in J}\mu_{jk}^t \sum_{\forall u\in I} d_i\right)X_j^R\right) \\ + \sum_{\forall i\in I}\sum_{\forall j\in J}\left(d_i S_{ij}^P Z_{ij} + d_i S_{ij}^P Y_{ij}\right) \\ + \sum_{\forall i\in I}\sum_{\forall j\in J}\sum_{\forall l\in L}\left(d_i S_{jil}^U (1-\theta_{jil}) YU_{ijl} + d_i S_{jil}^B \theta_{jil} YU_{ijl}\right) + \\ \sum_{\forall k\in K} v_k^t \left(\sum_{\forall i\in I} \mathrm{d_i}\left(\sum_{\forall l\in L} YU_{ijl} + Y_{ij}\right) - C_j \sum_{\forall m\in M}\left(1-\theta_{jm}\right) X_{jm}^U\right) - \sum_{\forall j\in J, \forall k\in K}\lambda_{jk}^t\left(C_j \sum_{\forall m\in M} X_{jm}^U\right) \end{array}\right\} \tag{A.28}$$

subject to

$$\sum_{\forall j\in J} X_j^R \geq 1 \tag{A.29}$$

$$\sum_{\forall j\in J}(Y_{ij} + \sum_{\forall l\in L} YU_{ijl} + \mathrm{Z}_{ij}) = 1, \forall i \in I \tag{A.30}$$

$$Z_{ij} <= X_j^R, \forall i \in I, \forall j \in J \tag{A.31}$$

$$\sum_{\forall i\in I} d_i (\sum_{\forall j\in J}(Y_{ij} + \sum_{\forall l\in L} YU_{ijl})) <= C_j \sum_{\forall m\in M} X_{jm}^U \tag{A.32}$$

$$\sum_{\forall j\in J} d_i X_j^R >= \sum_{\forall j\in J, \forall k\in K} A_{jk}, (j \neq k) \tag{A.33}$$

$$X_j^U, X_j^R \in \{0,1\}, \forall j \in J \tag{A.34}$$

$$Y_{ikj} \in \{0,1\}, \forall i \in I, \forall j, k \neq j \in J \tag{A.35}$$

$$Z_{ij} \in \{0,1\}, \forall i \in I, \forall j \in J \tag{A.36}$$

The number of solutions to explore is reduced and the master problem (MP) can be written as:

$$MP : \text{minimize } \psi \tag{A.37}$$

$$\psi \geq \omega \tag{A.38}$$

The Benders master problem is a relaxation of this problem obtained when only a subset of the constraints associated with the index set *T* are known. The Benders master problem provides a feasible facility configuration and a lower bound on the optimal objective function value of the original problem. The strategy adopted by Benders decomposition is to solve a relaxed master problem that contains a subset of these constraints to obtain a feasible facility configuration and to then solve another problem, the Benders sub-problem, to determine if a lower cost facility configuration exists. If the facility configuration from the relaxed Benders master problem is not optimal, the sub-problem provides a new constraint that is violated by this facility configuration. This new constraint, called a Benders cut, is one of the constraints in the index set *T* that is not already in the master problem. The problem below is the Benders sub-problem.

$$\text{minimize} \sum_{\forall j \in J} \sum_{\forall m \in M} f_{jm}^{U} \overline{X}_{jm}^{U} + \sum_{\forall j \in J} f_{j}^{R} \overline{X}_{j}^{R} + \sum_{\forall i \in I} \sum_{\forall j \in J} \left(d_i S_{ij}^{P} \overline{Z}_{ij} + d_i S_{ij}^{P} \overline{Y}_{ij} \right)$$

$$+ \sum_{\forall i \in I} \sum_{\forall j \in J} \sum_{\forall l \in L} \left(d_i S_{jil}^{U} (1 - \theta_{jil}) \overline{YU}_{ijl} + d_i S_{jil}^{B} \theta_{jil} \overline{YU}_{ijl} \right) + \sum_{\forall i \in I} \sum_{\forall j \in J} \sum_{\forall m \in M} P_j \overline{X}_{jm}^{U} A_{jk} T_{jk} \qquad \text{(A.39)}$$

subject to

$$A_{jk} \le C_j \sum_{\forall m \in M} \overline{X}_{jm}^{U}, \forall j \in J, \forall k \in J (j \ne k) \qquad \text{(A.40)}$$

$$A_{jk} \le \sum_{\forall u \in I} d_i \overline{X}_{j}^{R}, \forall j \in J, \forall k \in J (j \ne k) \qquad \text{(A.41)}$$

$$\sum_{\forall k \in J} A_{jk} + C_j \sum_{\forall m \in M} (1 - \theta_{jm}) \overline{X}_{jm}^{U} \ge \sum_{\forall \mathrm{i} \in \mathrm{I}} \mathrm{d_i} \left(\sum_{\forall l \in L} \overline{YU}_{ijl} + \overline{Y}_{ij} \right), \forall j \in J (j \ne k) \qquad \text{(A.42)}$$

$$A_{jk} \ge 0, \forall j \in J, \forall k \in J (j \ne k) \qquad \text{(A.43)}$$

This problem is identical to the inner optimization problem in the reformulation of the RFLDP. The Benders sub problem is a restriction of the RFLDP obtained by fixing the facility locations. The optimal solution to the Benders sub problem provides a set of dual variable values that form a Benders cut and an upper bound on the optimal objective function value of the original problem. The objective function value of the Benders subproblem is equal to the optimal solution of the original problem when the facility configuration is optimal. Benders decomposition starts with a feasible facility configuration and iterates between the Benders subproblem and the Benders master problem. At each iteration, if the facility configuration is not optimal, the subproblem provides a new Benders cut. This cut is a violated constraint from the constraints associated with the index set *T*. In the extreme case, after a finite number of iterations the subproblem produces all of the constraints associated with the index set *T* and the master problem is equivalent to the original problem. Thus, after a finite number of iterations the master problem and the subproblem must converge in objective function value to the optimal objective function value of the original problem (RFLDP).

List of Publications

❖ Papers Published/ Accepted in International Journal

1. Rimmi Anand and Vijay Kumar, "Design of Reliable Capacitated Facility Protection in Distribution Centres", *Procedia Computer Science*, 2017 (Scopus Indexed).
2. Manpreet Singh Bhangu, Rimmi Anand and Vijay Kumar, "Lagrangian Relaxation for Distribution Networks with Cross-Docking Center", *International Journal of Intelligent Systems Technologies and Applications*, 2017 (Scopus Indexed).
3. Rimmi Anand, Divya Aggarwal and Vijay Kumar, "Comparative Analysis of Optimization Solvers", *Journal of Information and Optimization Sciences*, 2017 (ESCI).

❖ Papers Published/ Accepted in International Conference

1. Rimmi Anand and Vijay Kumar, "Firefly Algorithm for Reliable Protection in Distribution Networks", *IEEE International Conference on Intelligent Computing, Instrumentation & Control Technologies*, 2017.

Video Link

- https://youtu.be/WCqKUI8OHJQ