

**MULTI-OBJECTIVE
OPTIMIZATION PROBLEMS
WITH A BEARING ON REAL-LIFE**

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CANDIDATE'S DECLARATION

I hereby certify that the work being presented in the thesis entitled “**Multi-Objective Optimization Problems with a Bearing on Real-Life**” in fulfillment of the requirements for the award of degree of Doctor of Philosophy, submitted in the School of Mathematics & Computer Applications of Thapar University, Patiala is an authentic record of my own work carried out under the supervision of Dr. Satya Prakash, Professor Emeritus and HOD, Applied Sciences, Amity School of Engineering & Technology, New Delhi and Dr. Mahesh K. Sharma, Assistant Professor, School of Mathematics & Computer Applications, Thapar University, Patiala.

The matter embodied in this thesis has not been submitted by me to any other university or institute for the award of any other degree.

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This is to certify that the above statement made by the candidate is correct to the best of our knowledge.

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ABSTRACT

The work consists of 7 Chapters. Chapter 1 is introductory. Chapters 2-6 give brief details of the new work which consists of 5 problems. Each one is briefly described in a separate chapter. Chapter 7 concludes the work.

In Chapter 1, Scope and Background of the work, New Work, Methodology, Purpose of the Work and the Road Ahead are discussed.

In Chapter 2, the problem of selecting upto a fixed number of sites, from among a given number of potential warehouse sites for clustering a given number of ration shops to them subject to several constraints with two objectives, is considered. The objectives are to minimize the total cost and the duration of meeting the requirements of all the ration shops from their assigned warehouses at the selected sites. A heuristic iterative algorithm incorporating tabu search is developed to find the set of efficient solutions of the problem. The incorporation of tabu search allows search for global optimal solution in a wider region thereby increasing the possibility of arriving at the global optimal solution or a solution close to it and thus leading to better results. This is brought out here.

In Chapter 3, a bulk transportation problem with two objectives to minimize the total cost and the duration of transportation without prioritizing them is considered. A heuristic algorithm to obtain the set of efficient solutions of this problem is developed. This algorithm is simple to apply and provides an alternative method of solving the bulk transportation problem with two objectives solved earlier by an analytical approach.

In Chapter 4, Chinese postman problem with two objectives to minimize the distance and time to cover all the streets is considered. The two objectives are not accorded priorities. An algorithm is developed for finding the set of efficient solutions of this problem. The algorithm has three steps. In the first step, the Chinese postman problem is represented through a graph. In the second step, the graph is made Eulerian if it is not. In the third step, we apply Fleury's algorithm after modifying it to obtain efficient solutions of the Chinese postman problem with two objectives.

In Chapter 5, a generalized assignment problem with two objectives has been considered, with the constraints that a job is assigned to one and only one worker though he/she may be assigned more than one job depending upon the time available with him/her. An algorithm is proposed to find the set of Pareto optimal solutions of the problem. The algorithm is an extended and modified version of the adaptive search heuristic algorithm used for solving single objective generalized assignment problem.

In Chapter 6, the problem of selecting upto a fixed number of sites from among a given number of potential fire station sites for clustering a given number of areas to them subject to several constraints with two objectives is considered. A heuristic algorithm is developed to find the set of efficient solutions of the problem.

In Chapter 7, utility and future possible extensions of the work are indicated.

Contents

CANDIDATE’S DECLARATION	i
ACKNOWLEDGEMENTS	ii
ABSTRACT	iii
1 INTRODUCTION	1
1.1 Scope	1
1.2 Background	3
1.3 New Work	3
1.4 Methodology	6
1.5 Purpose of the Work	7
1.6 The Road Ahead	7
REFERENCES	9
2 SELECTION OF WAREHOUSE SITES FOR CLUSTERING RATION SHOPS TO THEM WITH TWO OBJECTIVES THROUGH A HEURISTIC ALGORITHM INCORPORATING TABU SEARCH	16
2.1 Preview	17
2.2 Formulation of the Problem	19
2.3 Solution Procedure	21
2.3.1 Procedure to Obtain 1 st Efficient Solution	22

2.3.2	Procedure to Obtain 2 nd and Subsequent Efficient Solutions	25
2.4	Numerical Problem	26
2.5	Closure	29
	REFERENCES	30
3	A HEURISTIC FOR MULTI-OBJECTIVE BULK TRANSPORTATION PROBLEM WITH COST-TIME TRADE-OFFS	32
3.1	Preview	33
3.2	Formulation of the Problem	35
3.3	Solution Procedure	37
3.3.1	Procedure to Obtain 1 st Efficient Solution	37
3.3.2	Procedure to Obtain 2 nd and Subsequent Efficient Solutions	40
3.4	Numerical Problem	41
3.5	Closure	46
	REFERENCES	47
4	MULTI-OBJECTIVE CHINESE POSTMAN PROBLEM WITH DISTANCE- TIME TRADE-OFFS	50
4.1	Preview	51
4.2	Definition of the Problem	54
4.3	Solution Procedure	55
4.3.1	Procedure to Determine the Set E_1 of Edges to be “doubled” to Con- vert the Graph into an Eulerian Graph	56
4.3.2	Procedure to Determine the Minimum Weighted Set E_1	57
4.3.3	Procedure to Determine the Eulerian Circuit	57
4.4	Numerical Problem	58
4.5	Closure	61
	REFERENCES	62

5	EFFICIENT SOLUTIONS FOR MULTI-OBJECTIVE GENERALIZED ASSIGNMENT PROBLEM	65
5.1	Preview	66
5.2	Formulation of the Problem	68
5.3	Solution Procedure	70
5.3.1	Procedure to Obtain 1 st Efficient Solution	70
5.3.2	Procedure to Obtain 2 nd and Subsequent Efficient Solutions	72
5.3.3	Some Additional Assignment Procedures	73
5.4	Numerical Problem	75
5.5	Closure	81
	REFERENCES	83
6	MULTI-OBJECTIVE FIRE STATION LOCATION PROBLEM	86
6.1	Preview	87
6.2	Formulation of the Problem	90
6.3	Solution Procedure	92
6.3.1	Procedure to Obtain 1 st Efficient Solution	93
6.3.2	Procedure to Obtain 2 nd and Subsequent Efficient Solutions	94
6.4	Numerical Problem	95
6.5	Closure	102
	REFERENCES	103
7	CONCLUSIONS	106
A	PUBLICATIONS OUT OF THE PRESENT WORK	108

Chapter 1

INTRODUCTION

In this Chapter, scope of the work is first explained. Then the background and the new work are described. After this, the methodology and the purpose of the work are discussed. A brief description of all the Chapters is also given.

1.1 Scope

The present work deals with multi-objective optimization problems which have a bearing on real-life. We come across these problems in everyday life. Multi-objective problems differ from single-objective problems in the sense that the former problems have more than one objective whereas the latter problems have only one objective. There are many approaches for solving multi-objective optimization problems whereas there is a universally accepted single approach seeking to optimize the single objective function. The various approaches for solving multi-objective optimization problems are weighting approach, goal programming,

lexicographic/ranking/prioritized and Pareto optimal/efficient/non-dominated solution approach. A discussion about them can be found in the works of Zeleny (1974), Prakash (1981), Ignizio (1982), Sharma and Prakash (1986), Steuer (1986), Prakash, Agarwal and Shah (1988), Prakash and Pradeep (1991), Shah and Prakash (1992), Sutardi, Bector, Goulter and Cheng (1994), Schniederjans (1995), Prakash, Balaji and Tuteja (1999), Prakash and Gupta (2006), Zhang and Ong (2007), Prakash, Kumar, Prasad and Gupta (2008), Taha (2008). The first two approaches reduce the multi-objective optimization problems to single-objective optimization problems while the last two approaches do not alter the nature of the problems. There are two main approaches for solving the multi-objective optimization problems - (a) analytic and (b) heuristic. The analytical approach yields an exact solution of the problem whereas the heuristic approach yields a satisfying solution which can also be at times an exact solution. There are many real-life problems which either cannot be solved or even if can be solved through analytical approach, the amount of time and labour spent on solving them through analytic approach renders it unsuitable. In such cases, heuristic approach comes to our rescue. The heuristic approach does not require specialized knowledge of the subject. It is based on intuition, experience and judgment, thereby making it easy to apply and has wider applicability. A discussion about heuristic approach used for solving problems can be found in the works of Ignizio (1982), Glover (1989, 1990), Reeves (1993), Ignizio and Cavalier (1994), Osman and Kelly (1996), Aarts and Lenstra (1997), Rayward-Smith (1996), Rardin (1998), Prakash, Natarajan and Roy (1999), Praveena, Prasad and Prakash (1999), Zitzler and Thiele (1999), Deb (2001), Prakash and Gupta (2006), Prakash, Madhusudan and Kunal (2007), Prakash, Tuli and Sharma (2009), Prakash and Tuli (2010).

1.2 Background

In the remote past, there were no real-life optimization problems to warrant attention and even if they cropped up, they were solved by rudimentary techniques based on common sense and the rule of thumb. But with the passage of time, real-life optimization problems grew in size and complexity, with the result rudimentary techniques were unable to give proper solutions of these problems. This led to the need to solve them through analytical or heuristic approach. Many real-life optimization problems have been solved through analytical and heuristic approaches by many workers, such as, Ignizio (1968, 1971), Mukherjee and Mehta (1978), Hwang and Masud (1979), Prakash (1981, 1982a, 1982b, 1983), Osyczka, (1985), Prakash and Gupta (1985), Steuer (1986), Agarwal and Dhingra (1989), Shah and Prakash (1992), Prakash and Prasad (1997), Badri, Mortagy and Alsayed (1998), Taboada, Baheranwala, Coit and Wattanapongsakorn (2007), Taha (2008). The above list of researchers and authors of books is not exhaustive. It is given here merely to have a glimpse at enormous work done related to real-life optimization problems. Still many problems remain unsolved and many more even unformulated and unknown.

1.3 New Work

New work consists of five problems related to multi-objective programming with a bearing on real-life. The details of new work are given in Chapters 2-6. Each problem is described in a separate Chapter. Below we give brief details of the new work.

In Chapter 2, the problem of selecting upto a fixed number of sites from among a given number of potential warehouse sites for clustering a given number of ration shops to them subject to several constraints with two objectives, is considered. The two objectives are to minimize the total cost and the duration of meeting the requirements of all the ration shops from their assigned warehouses at the selected sites. A heuristic iterative algorithm incorporating tabu search is developed to find the set of efficient solutions of the problem. The incorporation of tabu search allows search for global optimal solution in a wider region thereby increasing the possibility of arriving at the global optimal solution or a solution close to it and thus, leading to better results. This work extends the work of Ignizio and Cavalier (1994), Praveena, Prasad and Prakash (1999). The model considered in this work is versatile. It can be adapted to new situations. It can deal with large size problems after computerization and is capable of incorporating different types of constraints and objective functions other than those considered in the model. This can be achieved if the terms warehouses and ration shops are stretched to include any supply and demand points.

In Chapter 3, a bulk transportation problem with objectives to minimize the total cost and the duration of transportation without prioritizing them is considered. A heuristic algorithm to obtain the set of efficient solutions of this problem is developed using extremum difference of costs. This algorithm is simple to apply and provides an alternative method of solving the bulk transportation problem with two objectives considered by Prakash, Kumar, Prasad and Gupta (2008). Maio and Roveda (1971) have considered a single objective bulk transportation problem to minimize the total cost of bulk transportation prior to solving the bulk transportation problem with two objectives mentioned above. Thus, this work extends

their works.

In Chapter 4, a Chinese Postman Problem with two objectives to minimize the distance and time to cover all the streets is considered. The two objectives are not accorded priorities. An algorithm is developed for finding the set of efficient solutions of this problem. This work extends the works of Chinese mathematician Mei-Ko Kwan (1962).

In Chapter 5, a generalized assignment problem with two objectives wherein each job is assigned to one worker but a worker may be assigned to no job or one or more than one job subject to the availability of time at his/her disposal. The two objectives are to minimize the total cost of assignment and the duration of completion of work without priorities being accorded to them. A heuristic algorithm is developed to obtain the set of efficient solutions of this problem. The algorithm is applied to solve several numerical problems confirming its validity. The generalized assignment problem differs from the usual assignment problem in that herein a worker may be assigned to no job or one or more than one job subject to the availability of time at his/ her disposal compared to the usual assignment problem wherein each worker has to be assigned to no job or one job only. Chu and Beasley (1997) have considered a single objective generalized assignment problem and have solved it using a genetic algorithm. Lourenço and Serra (2002) have considered a single objective generalized assignment problem and have solved it using adaptive approach heuristic. Thereafter, Kasana and Kumar (2004) have solved the generalized assignment problem using Extremum Difference Method. Thus, this work extends and modifies the earlier work done on assignment problems.

In Chapter 6, the problem of selecting upto a fixed number of sites from among a given

number of potential fire station sites for clustering a given number of areas to them subject to several constraints with two objectives is considered. The two objectives are to minimize the total operational cost to cover the areas and the maximum individual time among all times to reach the areas from the fire stations. A heuristic algorithm is developed to find the set of efficient solutions of this problem. This work is an attempt to formulate a problem of selecting fire station sites and then an attempt to solve it through a heuristic method. Badri, Mortagy and Alsayed (1998) have considered the problem related to the selection of sites for fire stations and have solved using the goal programming. Liu, Huang and Chandramouli (2006) have solved the problem using Geographic Information Systems approach. This way our work extends the earlier work done related to the selection of sites for fire stations.

The first two problems have been considered in the past but new algorithms have been developed to solve them. These methods not only substantially differ from all the existing ones but are also superior to them in some respect or the other. The third and fourth problems extend the single-objective problem to problems with two objectives. The fifth problem provides a heuristic approach for selecting sites for fire stations with two objectives which has so far been tackled through rudimentary techniques which fail many time.

1.4 Methodology

New work has been done in the following way. We have formulated new problems and then have developed algorithms to solve them with or without modification and extension of the existing algorithms. We have extended single-objective problems to multi-objective problems

and have developed algorithms for solving them while in some cases we have improved the existing algorithms for solving the existing problems. Each Chapter is independent in itself to a large extent. Every attempt has been made to make the reading easy keeping this in view the references appearing in a particular Chapter are kept at the end of that Chapter itself.

1.5 Purpose of the Work

Purpose of this work is to help persons in the industry and other walks of life in optimizing the use of their resources through minimizing expenditure and maximizing benefit. This is accomplished by formulating multi-objective problems having a bearing on real-life and providing algorithms for solving them.

1.6 The Road Ahead

Chapter 1 is an introduction to the Thesis. Here scope of the work, background new work, methodology, purpose of the work and the road ahead are briefly discussed. Chapters 2-6 contain new work which consists of 5 problems. Each problem is described in a separate Chapter. Chapter 7 concludes the work. Herein utility and possible future extensions of the work are indicated.

Part of the new work has been published in research journals and in the Proceedings of

International Conferences. Some of the work is being communicated to research journals. This is explained under the heading “PUBLICATIONS OUT OF THE PRESENT WORK” at the end of the Thesis.

REFERENCES

- [1] Aarts, E.H.L., Lenstra, J.K., 1997. "Local search in Combinatorial Optimization", John-Wiley & Sons, New York.
- [2] Agarwal, A.K., Dhingra, S.L., 1989. "An optimal programme for augmentation of capacities of depots and shipment of buses from depots to starting points of routes", Indian Journal of Pure and Applied Mathematics, Vol. 20, No. 2, pp. 111-120.
- [3] Badri, M.A., Mortagy, A.K., Alsayed, A., 1998. "A multi-objective model for locating fire stations", European Journal of Operational Research, Vol. 110, pp. 243-260.
- [4] Chu, P.C., Beasley, J.E., 1997. "A genetic algorithm for the generalized assignment problem", Computers & Operations Research, Vol. 24, pp. 17-23.
- [5] Deb, K., 2001. "Multiobjective Optimization Using Evolutionary Algorithms", John-Wiley & Sons, New York.
- [6] Glover, F., 1989. "Tabu Search - part i", ORSA Journal on Computing, Vol. 1, No. 3, pp. 190-206.
- [7] Glover, F., 1990. "Tabu Search - part ii", ORSA Journal on Computing, Vol. 2, No. 1, pp. 4-32.
- [8] Hwang, C.L., Masud, A.S.M., 1979. "Multiple Objective Decision Making - Methods and Applications", Springer-Verlag, New York.

-
- [9] Ignizio, J.P., 1968. "A Method to Achieve Optimum Air Defense Sensor Allocation", MS Thesis, University of Alabama, Alabama.
- [10] Ignizio, J.P., 1971. "A Heuristic Solution to Generalized Covering Problems", Ph.D. Dissertation, Virginia Polytechnic Institute, Blacksburg.
- [11] Ignizio, J.P., 1982. "Linear Programming in Single- & Multi- Objective Systems", Prentice-Hall, Englewood Cliffs, New Jersey, pp. 374-376, 475-483.
- [12] Ignizio, J.P., Cavalier, T.M., 1994. "Linear Programming", Prentice-Hall, Englewood Cliffs, New Jersey, pp. 457-505.
- [13] Kasana, H.S., Kumar, K.D., 2004. "Introductory Operations Research. Theory and Applications.", Springer-Verlag, Berlin.
- [14] Kwan, M., 1962. "Graphics programming using odd and even points", Chinese Mathematics, Vol. 1, pp. 83-97.
- [15] Liu, N., Huang, B., Chandramouli, M., 2006. "Optimal Siting of Fire Stations Using GIS and ANT Algorithm", Journal of Computing in Civil Engineering, Vol. 20. No. 5, pp. 361-369.
- [16] Lourenço, H.R., Serra, D., 2002. "Adaptive approach heuristics for the generalized assignment problem", Mathware & Soft Computing, Vol. 9, pp. 209-234.
- [17] Maio, A.D., Roveda, C., 1971. "An all zero-one algorithm for a certain class of transportation problems", Operations Research, Vol. 19, pp. 1406-1418.

-
- [18] Mukherjee, S.K., Mehta, A.C., 1978. "A linear programming model for optimal water transmission system: A case study for Ahmedabad", *OPSEARCH*, Vol. 15, No. 4, pp. 169.
- [19] Osman, I.H., Kelly, J.P., 1996. "Meta-Heuristics: Theory and Applications", Kluwer Academic Publishers, Norwell, Massachusetts.
- [20] Osyczka, A., 1985. "Multicriteria optimization for engineering design", *Design Optimization*, Academic Press, Cambridge, pp. 193-227.
- [21] Prakash, S., 1981. "A transportation problem with objectives to minimize total cost and duration of transportation", *OPSEARCH*, Vol. 18, pp. 235-238.
- [22] Prakash, S., 1982a. "A two-objective capacity expansion and water allocation programme for reservoirs with restrictions on augmentation of their capacities to meet drinking water requirements", *Indian Journal of Pure and Applied Mathematics*, Vol. 13, No. 8, pp. 861-870.
- [23] Prakash, S., 1982b. "On minimizing the duration of transportation", *Proceedings of Indian Academy of Sciences (Mathematical Sciences)*, Vol. 91, No. 1, pp. 53-57.
- [24] Prakash, S., 1983. "A two-objective capacity expansion and water allocation programme for reservoirs to meet the water requirements of regions", *Proceedings of the National Academy of Sciences, India*, Vol. 53, Sec. A, No. 1, pp. 155-163.
- [25] Prakash, S., Agarwal, A.K., Shah, S., 1988. "Nondominated solutions of cost-time

-
- trade-off transportation and assignment problems”, *OPSEARCH*, Vol. 25, pp. 126-131.
- [26] Prakash, S., Balaji, B.V., Tuteja, D., 1999. “Optimizing dead mileage in urban bus routes through a non-dominated solution approach”, *European Journal of Operational Research*, Vol. 114, pp. 465-473.
- [27] Prakash, S., Gupta, A., 2006. “Efficient solutions for the problem selecting sites for polling stations and assigning voter areas to them with two objectives”, *Proceedings of the National Conference on Management Science and Practice*, held at IIM, Ahmedabad, India, Allied Publishers Pvt. Ltd., New Delhi, pp. 141-151.
- [28] Prakash, S., Gupta, S.S., 1985. “On the problem of optimal programme for augmentation of capacity of reservoirs and allocation of water from them to regions”, *Proceedings of the National Symposium on Formulation and Appraisal of Irrigation Projects*, held at Bihar Engineering College, Patna during March 28-30, 1985, Sec. 2, pp. 9-18.
- [29] Prakash, S., Kumar, P., Prasad, B.V.N.S., Gupta, G., 2008. “Pareto optimal solutions of a cost-time trade-off bulk transportation problem”, *European Journal of Operational Research*, Vol. 188, pp. 85-100.
- [30] Prakash, S., Madhusudan, A., Kunal, 2007. “Clustering offices of a company to internet service providers”, *Proceedings of the National Academy of Sciences, India*, Vol. 77(A), No. 1, pp. 31-35.

-
- [31] Prakash, S., Natarajan, B.R., Roy, Y.Y.C., 1999. "Site location with lexicographic minimization of three objectives", (i) Proceedings of the National Academy of Sciences, India, Vol. 69(A), No. 4, pp. 459-464. (ii) Mathematics and Statistics in Engineering and Technology, Edited by Chattopadhyaya, A., Narosa Publishing House, New Delhi, pp. 186-190.
- [32] Prakash, S., Pradeep, J.S.K.C., 1991. "Nondominated solutions of deviation-time trade-off assignment problem", Bulletin of Calcutta Mathematical Society, Vol. 83, pp. 364-370.
- [33] Prakash, S., Prasad, T.D., 1997. "Determination of optimal pumping head and diameters of pipes in lines of a water distribution system of given geometrical configuration", News Bulletin of Calcutta Mathematical Society, Vol. 20, No. 7, pp. 5-11.
- [34] Prakash, S., Tuli, R., 2010. "Efficient solutions of a bicriterion routing network problem through a modified and extended version of Dijkstra's algorithm", Proceedings of Third National Conference on Mathematical Techniques: Emerging Paradigms for Electronics and IT Industries, held at Deen Dayal Upadhyaya College, New Delhi, pp. TS 3.2.1-3.2.6.
- [35] Prakash, S., Tuli, R., Sharma, S., 2009. "Efficient solution for a bicriterion problem selecting sites for polling stations and assigning voter-areas to them through Hungarian method incorporating concept of lexicographic minimization of ordered pairs", News Bulletin of Calcutta Mathematical Society, Vol. 32, No. 4-6, pp. 11-19.

-
- [36] Praveena, V., Prasad, B.V.N.S., Prakash, S., 1999. "Selection of warehouse sites for clustering ration shops to them with two objectives", Proceedings of the Seminar on Recent Trends and Advances of Mathematics and Statistics in Engineering & Technology, pp. 174-179.
- [37] Rardin, R.L., 1998. "Optimization in Operations Research", Pearson Education (Singapore) Pte Ltd, Indian Branch, New Delhi, pp. 15-16, 670-705.
- [38] Rayward-Smith, V.J., 1996. "Modern Heuristic Search Methods", John-Wiley & Sons, New York.
- [39] Reeves, C.R., 1993. "Modern Heuristic Techniques for Combinatorial Problems", Halsted Press.
- [40] Schneiderjans, M.J., 1995. "Goal Programming: Methodology and Applications", Kluwer Academic Publishers, Norwell, Massachusetts, USA.
- [41] Shah, S., Prakash, S., 1992. "Nondominated Solution of an Assignment Problem with Two Objectives", The Mathematics Student, Vol. 61, No. 1-4, pp. 223-228.
- [42] Sharma, V., Prakash, S., 1986. "Optimizing dead mileage in urban bus routes", Journal of Transportation Engineering, Vol. 112, pp. 121-129.
- [43] Steuer, R.E, 1986. "Multiple Criteria Optimization: Theory, Computation and Application", John-Wiley & Sons, New York.
- [44] Sutardi, Bector, C.R., Goulter, I., Cheng, T.C.E., 1994. "Multiobjective water

- resources investment planning under budgetary and social-economic uncertainties”, IEEE Transactions on Emerging Management, Vol. 41, No. 1, pp. 50-68.
- [45] Taboada, H.A, Baheranwala, F., Coit, D.W., Wattanapongsakorn, N., 2007. “Practical solutions for multiobjective optimization : An application to system reliability design problems”, Reliability Engineering and System Safety, Vol. 92, pp. 314-322.
- [46] Taha, H.A., 2008. “Operations Research: An Introduction”, Eighth Edition, Prentice-Hall Learning Private Limited, New Delhi, pp. 333-348.
- [47] Zhang, C.W., Ong, H.L., 2007. “An efficient solution to biobjective generalized assignment problem”, Advances in Software Engineering, Vol. 38, No. 1, pp. 50-58.
- [48] Zitzler, E., Thiele, L., 1999. “Multiobjective Evolutionary algorithms : A comparative case study and the strength pareto approach”, IEEE Transactions on Evolutionary Computation, Vol. 3, No. 4, pp. 257-271.
- [49] Zeleny, M., 1974. “Linear Multi-objective Programming”, Springer-Verlag, Berlin.

Chapter 2

SELECTION OF WAREHOUSE SITES FOR CLUSTERING RATION SHOPS TO THEM WITH TWO OBJECTIVES THROUGH A HEURISTIC ALGORITHM INCORPORATING TABU SEARCH

The problem of selecting upto a fixed number of sites, from among a given number of potential warehouse sites for clustering a given number of ration shops to them subject to several constraints with two objectives, is considered. One of the constraint is that each ration shop should be clustered to a unique warehouse site, which is selected for locating a warehouse at it; however there is no restriction on the number of ration shops to be clustered to a selected warehouse site. Another constraint is that the total cost of warehouses to be

set up should not exceed a budgetary amount. The two objectives are to minimize the total cost and the duration of meeting the requirements of all the ration shops from their assigned warehouses at the selected sites. A heuristic iterative algorithm incorporating tabu search is developed to find the set of efficient solutions of this problem.

2.1 Preview

Ignizio and Cavalier (1994) have formulated and solved the problem of selecting upto a fixed number of sites from among a given number of potential sites for locating warehouses at them and clustering customers to the selected sites in such a way that each customer is assigned to a unique selected site. The problem has a single objective to minimize the sum of distances from each customer to his/her assigned site. Praveena, Prasad and Prakash (1999) have modified and extended this problem to a new situation. The problem selects up to a fixed number of sites from among a given number of potential sites for clustering ration shops to them subject to several constraints with two objectives. One of the constraints is that each ration shop should be clustered to a unique site which is selected for locating a warehouse at it; however there is no restriction on the number of ration shops to be clustered to a warehouse at the selected site. Another constraint is that the setup cost of the warehouses should not exceed a certain budgetary amount. The two objectives are to minimize the total cost and the duration of meeting requirements of all the ration shops from their assigned warehouses to the selected sites. The two objectives are not accorded priorities. The set of efficient solutions of this problem has been found through solving a sequence of

prioritized bicriterion problems. Each of the prioritized bicriterion problem is solved using a heuristic algorithm consisting of a combination of add and drop rules.

The present work solves this very problem through solving a sequence of prioritized bicriterion problems incorporating tabu search in the heuristic algorithm used for solving the prioritized bicriterion problems. The incorporation of tabu search leads to better results than those obtained earlier without its incorporation. The incorporation of tabu search allows search for global optimal solution in a wider region thereby increasing the possibility of arriving at the global optimal solution or a solution close to it and thus, leading to better results. Tabu search is associated with a heuristic algorithm seeking global optimal solution. This is because most of the heuristic algorithms select moves in the neighbourhood of the current solution which improve the value of the objective function; this results, at times, into getting stuck at the local optimum and terminating the procedure at the arrival of the local optimum instead of the global optimum.

The important characteristics of tabu search are the following: (i) It allows moves, which even worsen the value of the objective function in order to get out of the local optimum, (ii) It temporarily prevents cycling, i.e. revisiting the solutions already examined recently, by using a tabu list. The tabu list contains forbidden moves which prohibit revisiting the solutions having been examined recently. The tabu list changes as we move to the next solution whether it is better or worse. (iii) It uses the concept of an incumbent solution which is the best solution obtained so far. The incumbent solution changes to a better current solution as and when we arrive at it. (iv) The procedure terminates when either we revisit an already examined solution or we are not able to find a new current solution. A detailed discussion

about heuristic approach and the methods based on this approach incorporating tabu search can be found in the works of Ignizio and Cavalier (1994) and Rardin (1998).

A solution of a multi-objective problem is efficient if no solution of the problem exists that is superior to it with respect to at least one objective but is not inferior to it with respect to any one of the objectives. Terms “non-dominated solution” and “Pareto optimal solution” are used in the literature for “efficient solution”. Specifically, for the above bicriterion problem in question with C and T denoting the total cost and the duration of meeting the requirements of all the ration shops from their assigned warehouses at selected sites as the two minimizing objective functions without being accorded priorities to them, a solution (\bar{X}, \bar{Y}) is called an efficient solution if there exists no solution (\bar{x}, \bar{y}) satisfying the conditions (i) $C(\bar{X}) \geq C(\bar{x})$ and (ii) $T(\bar{X}) \geq T(\bar{x})$ with strict inequality holding in at least one of the conditions out of (i) and (ii). Here, $(x_{ij} = 1, 2, \dots, M; j = 1, 2, \dots, N)$ and $\bar{y} = (y_j : j = 1, 2, \dots, N)$. A discussion about efficient solution can be found in the works of Ignizio (1982), Steuer (1986), Prakash, Balaji and Tuteja (1999), Prakash and Gupta (2006), Prakash, Madhusudan and Kunal (2007), Prakash, Kumar, Prasad and Gupta (2008).

2.2 Formulation of the Problem

Suppose that there are M ration shops, N potential warehouse sites, K is the maximum number of sites, which can be selected from among the N potential sites for locating warehouses at them. The M ration shops are to be clustered upto K sites in such a way that each shop is assigned to a unique site which is selected for the task. There is no

restriction on the number of ration shops to be clustered to a selected site. Let c_{ij} and t_{ij} ($i = 1, 2, \dots, M; j = 1, 2, \dots, N$) units be the cost and time respectively of meeting requirements of ration shop i from site j . Let c_j units be the setup cost of a warehouse at the site j and B units be the budgetary amount allocated for the setup of warehouses. Let x_{ij} be the variable assuming value 0 or 1 according as ration shop i is not assigned or assigned to site j , and y_j be the variable assuming the value 0 or 1 according as potential site j is not selected or selected for locating a warehouse at it. Let C and T denote the total cost and the duration respectively of meeting requirements of all the ration shops from their assigned warehouses. The mathematical formulation of this problem is as follows. The two objective functions which are sought to be minimized are

$$C = \sum_{i=1}^M \sum_{j=1}^N c_{ij} x_{ij} \quad (2.1)$$

$$T = \max\{t_{ij} x_{ij} : i = 1, 2, \dots, M; j = 1, 2, \dots, N\} \quad (2.2)$$

Constraints of the problem are

$$\sum_{j=1}^N y_j \leq K \quad (2.3)$$

$$\sum_{j=1}^N x_{ij} = 1 \quad (i = 1, 2, \dots, M) \quad (2.4)$$

$$x_{ij} - y_j \leq 0 \quad (i = 1, 2, \dots, M; j = 1, 2, \dots, N) \quad (2.5)$$

$$\sum_{j=1}^N c_j y_j \leq B \quad (2.6)$$

$$x_{ij}, y_j = 0 \text{ or } 1 \quad (i = 1, 2, \dots, M; j = 1, 2, \dots, N) \quad (2.7)$$

Note that the objective functions C provided by Eq. (2.1) and T provided by Eq. (2.2) are not accorded priorities. The constraint (2.3) ensures that upto a maximum of K sites can be selected from among the N potential warehouse sites while the constraints (2.4) and (2.5) ensure that each ration shop is assigned to a unique site which is selected for locating a warehouse. And the constraint (2.6) ensures that the total setup cost of the warehouses does not exceed the allocated budgetary amount B .

It is required to obtain the set of efficient solutions of the problem given by (2.1)-(2.7). For the purpose of listing the efficient solutions, a solution $(\bar{X}^{(1)}, \bar{Y}^{(1)})$ shall be called the 1st efficient solution if it is the optimal solution of the problem with the minimization of C and T as the first and second prioritized objectives respectively. A solution $(\bar{X}^{(2)}, \bar{Y}^{(2)})$ shall be called the 2nd efficient solution if no efficient solution (\bar{X}, \bar{Y}) of the problem exists satisfying the conditions: (i) $C(\bar{X}^{(1)}) < C(\bar{X}) < C(\bar{X}^{(2)})$ and (ii) $T(\bar{X}^{(1)}) > T(\bar{X}) > T(\bar{X}^{(2)})$. The 3rd and subsequent efficient solutions are defined in the same way as is done for the 2nd efficient solution.

2.3 Solution Procedure

The problem formulated above is a binary integer nonlinear problem because the variables x_{ij} s and y_j s are binary integers and the objective function T given by Eq. (2.2) is nonlinear. The set of efficient solutions is obtained through an iterative heuristic algorithm incorporating tabu search requiring solutions of the prioritized bicriterion problems. The prioritized bicriterion problems are solved as such without being linearized and their total number is

only one more than that of the efficient solutions. Procedures to obtain the 1st, 2nd and the subsequent efficient solutions are explained below.

2.3.1 Procedure to Obtain 1st Efficient Solution

Note that the 1st efficient solution $(\bar{X}^{(1)}, \bar{Y}^{(1)})$ of the problem formulated above given by eqns. (2.1)-(2.7) is the optimal solution of the problem wherein the total cost C and the duration T of meeting entire requirements of the ration shops from their assigned warehouses, are minimized with the first and the second priorities respectively, subject to the constraints (2.3)-(2.7). The problem yielding the 1st efficient solution is designated as the 1st prioritized bicriterion problem of the formulated problem. The heuristic method proposed to solve the 1st bicriterion problem is a modification and extension of the heuristic method of Ignizio (1968, 1971) to solve a partial covering problem with a single objective. The proposed method consists of two phases. In the first phase, a solution of the 1st prioritized bicriterion problem is obtained using the add rule. In the second phase, the solution obtained in the first phase is improved successively using a combination of ‘add’ and ‘drop’ rules incorporating tabu search. In the first phase, we start with no warehouse sites selected and select the site wherefrom the total cost of meeting bulk requirements of all the ration shops is minimized without violating the budgetary constraint. In the case of a tie, we select the site wherefrom the maximum time among the ‘times’ of meeting bulk requirements of each ration shop is also minimized. And if there is a tie again, we arbitrarily select a site among the ‘tied’ ones. Next we select a second site which, in combination with the site already selected, minimizes

the total cost of meeting bulk requirements of all the ration shops without violating the budgetary constraint. Each ration shop is assigned to the site wherefrom the cost of meeting its requirement is minimized among the selected sites. In the case of a tie, we select the site which, in combination with the already selected site, also minimizes the maximum time among the ‘times’ of meeting bulk requirement of each ration shop from its assigned site. We then select a third site, which, in combination with the two sites already selected, minimizes the total cost of meeting bulk requirements of all the ration shops without violating the budgetary constraint. Ties can be handled in the way explained above. We continue to select sites using ‘add’ rule in this way. This process of selecting sites, using add rule, is terminated if either K sites have been selected or even if less than K sites have been selected but it is not possible to select a new site which would improve the solution. This leads us to a solution of the 1st prioritized bicriterion problem of the formulated problem which is termed as its iteration 0.

In the second phase, we invoke drop and add rules with tabu search for further improving the solution. The incorporation of tabu search allows search for global optimal solution in a wider region thereby increasing the possibility of arriving at the global optimal solution or close to it and, thus, leading to better results. Since we have already explained how tabu search does all this in Section 2.1, we shall not, therefore, discuss it here again in order to avoid repetition. Instead we shall concentrate only on explaining the algorithm consisting of drop and add rules with tabu search invoked for further improving the solution. Selection of sites using add rule in the way explained above, yields the 0th iterative solution $(\bar{X}_{(0)}^{(1)}, \bar{Y}_{(0)}^{(1)})$ for the 1st efficient solution $(\bar{X}^{(1)}, \bar{Y}^{(1)})$ and is also the incumbent solution for

it. The incumbent solution for the 1st efficient solution $(\bar{X}^{(1)}, \bar{Y}^{(1)})$ is the best solution obtained so far. For obtaining the 1st iterative solution $(\bar{X}_{(1)}^{(1)}, \bar{Y}_{(1)}^{(1)})$ from the 0th iterative solution $(\bar{X}_{(0)}^{(1)}, \bar{Y}_{(0)}^{(1)})$, we invoke drop and add rules with tabu search as follows. We form two separate tabu lists for drop and add rules. The tabu list for drop rule consists of two most recently selected sites appearing in the order of their selection. The two sites included in the tabu list will be prohibited from drop from the 0th iterative solution and the drop rule will drop only the first selected site. And the tabu list for the add rule includes the 1st site selected in the 0th iterative solution $(\bar{X}_{(0)}^{(1)}, \bar{Y}_{(0)}^{(1)})$ which will be prohibited from being selected in the next 1st iterative solution $(\bar{X}_{(1)}^{(1)}, \bar{Y}_{(1)}^{(1)})$. The add rule with its associated tabu list has to be implemented after invoking the drop rule with its tabu list resulting into the 1st iterative solution. Having obtained the 1st iterative solution $(\bar{X}_{(1)}^{(1)}, \bar{Y}_{(1)}^{(1)})$ for the 1st efficient solution $(\bar{X}^{(1)}, \bar{Y}^{(1)})$ of the formulated problem, we compare it with the incumbent solution. If the current solution $(\bar{X}_{(1)}^{(1)}, \bar{Y}_{(1)}^{(1)})$ is better than the incumbent solution, the incumbent solution changes to it and if the current solution is not better than the incumbent solution, then the incumbent solution does not change. We find the 2nd iterative solution $(\bar{X}_{(2)}^{(1)}, \bar{Y}_{(2)}^{(1)})$ proceeding almost similarly but effecting changes in the tabu lists. The tabu list for the drop rule used for obtaining 2nd iterative solution $(\bar{X}_{(2)}^{(1)}, \bar{Y}_{(2)}^{(1)})$ will now include the two most recent selected sites included in the 1st iterative solution $(\bar{X}_{(1)}^{(1)}, \bar{Y}_{(1)}^{(1)})$ and will appear in the order of their selection. Similarly, the tabu list for the add rule used for obtaining the 2nd iterative solution $(\bar{X}_{(2)}^{(1)}, \bar{Y}_{(2)}^{(1)})$ will now include the 1st site selected in the 1st iterative solution $(\bar{X}_{(1)}^{(1)}, \bar{Y}_{(1)}^{(1)})$. If the 2nd iterative solution $(\bar{X}_{(2)}^{(1)}, \bar{Y}_{(2)}^{(1)})$ is better than the incumbent solution, then the incumbent solution changes to it and if not then the incumbent solution does not

change. The 3^{rd} iterative solution $(\bar{X}_{(3)}^{(1)}, \bar{Y}_{(3)}^{(1)})$ and subsequent ones will be obtained in the same way as done in obtaining the 1^{st} and 2^{nd} iterative solutions. We change the incumbent solution to the current iterative solution if it is better than the incumbent solution, else the incumbent solution remains the same. We terminate this process of obtaining newer iterative solutions when we either revisit an already visited iterated solution or it is not possible to obtain a solution. When this happens, then the most recent incumbent solution provides the 1^{st} efficient solution $(\bar{X}^{(1)}, \bar{Y}^{(1)})$ of the formulated problem.

2.3.2 Procedure to Obtain 2^{nd} and Subsequent Efficient Solutions

The 2^{nd} efficient solution $(\bar{X}^{(2)}, \bar{Y}^{(2)})$ of the formulated problem is obtained by solving the problem obtained from the 1^{st} prioritized bicriterion problem after replacing c_{ij} s corresponding to the $t_{ij} \geq T(\bar{X}^{(1)})$ by an arbitrarily large positive number M . The problem thus obtained is similar to the 1^{st} prioritized bicriterion problem and is designated as the 2^{nd} prioritized bicriterion problem. The 2^{nd} prioritized bicriterion problem is solved by adopting the method for solving the 1^{st} prioritized bicriterion problem remembering that M is an arbitrarily large positive number. The 3^{rd} and the subsequent efficient solutions are obtained in the same way as is done to obtain the 2^{nd} efficient solution. This process of obtaining efficient solutions is terminated after encountering a prioritized bicriterion problem whose optimal solution has a variable x_{ij} at level 1 with which is associated the cost M indicating that it is no longer possible to obtain a new efficient solution with the lesser duration.

2.4 Numerical Problem

Now we illustrate the procedure explained in Section 2.3 by applying it to obtain the set of efficient solutions of a numerical problem which is obtained by taking $M = 5$, $N = 7$, $K = 3$, $B = 1400000$ and assigning numerical values to all the c_j s, c_{ij} s, t_{ij} s, in the problem formulated in Section 2.2. Table 2.1 provides tableau representation of the numerical problem.

Table 2.1: Tableau representation of the problem

Ration Shops	Warehouse Potential Sites						
	1	2	3	4	5	6	7
1	40	30	50	120	180	170	150
	8	2	11	6	10	9	7
2	70	80	130	130	170	140	20
	6	6	9	7	8	10	11
3	80	20	200	70	140	130	60
	8	9	5	8	11	12	10
4	60	10	70	80	30	160	210
	11	6	13	11	9	8	6
5	90	100	20	60	40	50	220
	10	13	8	12	6	6	14
Setup Cost (c_j)	1,00,000	3,00,000	7,00,000	8,00,000	4,00,000	2,00,000	5,00,000

In this Table, rows 1-5 correspond to ration shops and columns 1-7 correspond to potential warehouse sites. Marginal row entries depict setup costs c_j s of warehouses at the respective potential sites. Upper and lower entries of a cell (i, j) depict the units of cost c_{ij} and time t_{ij} respectively of meeting the bulk requirement of ration shop i from potential warehouse site j .

Following the procedure explained in Section 2.3.1 for obtaining the 1st efficient solution

$(\bar{X}^{(1)}, \bar{Y}^{(1)})$ of the numerical problem, we draw Table 2.2 showing the iterative solutions $(\bar{X}_{(r)}^{(1)}, \bar{Y}_{(r)}^{(1)})$ ($r = 0, 1, \dots, 6$) together with the variables x_{ij} s, y_j s at level 1, the total cost $C(\bar{x})$ and the duration $T(\bar{x})$ of meeting bulk requirements of all the ration shops from their assigned warehouses, tabu lists for add and drop rules and the incumbent solutions.

Table 2.2: The iterative solutions $(\bar{X}_{(r)}^{(1)}, \bar{Y}_{(r)}^{(1)})$ ($r = 0, 1, \dots, 6$)

Iterative Solution	Variables at level 1	Total Cost of meeting requirement	Duration of meeting requirement	Tabu List for drop rule	Tabu List for add rule	Incumbent Solution
$(\bar{X}_{(0)}^{(1)}, \bar{Y}_{(0)}^{(1)})$	x_{12}, x_{21}, x_{32} x_{42}, x_{53} y_2, y_3, y_1	$C(\bar{X}_{(0)}^{(1)}) = 150$	$T(\bar{X}_{(0)}^{(1)}) = 9$	$\{y_3, y_1\}$	$\{y_2\}$	$(\bar{X}_{(0)}^{(1)}, \bar{Y}_{(0)}^{(1)})$
$(\bar{X}_{(1)}^{(1)}, \bar{Y}_{(1)}^{(1)})$	x_{11}, x_{27}, x_{37} x_{41}, x_{53} y_3, y_1, y_7	$C(\bar{X}_{(1)}^{(1)}) = 200$	$T(\bar{X}_{(1)}^{(1)}) = 11$	$\{y_1, y_7\}$	$\{y_3\}$	$(\bar{X}_{(0)}^{(1)}, \bar{Y}_{(0)}^{(1)})$
$(\bar{X}_{(2)}^{(1)}, \bar{Y}_{(2)}^{(1)})$	x_{12}, x_{27}, x_{32} x_{42}, x_{51} y_1, y_7, y_2	$C(\bar{X}_{(2)}^{(1)}) = 170$	$T(\bar{X}_{(2)}^{(1)}) = 11$	$\{y_7, y_2\}$	$\{y_1\}$	$(\bar{X}_{(0)}^{(1)}, \bar{Y}_{(0)}^{(1)})$
$(\bar{X}_{(3)}^{(1)}, \bar{Y}_{(3)}^{(1)})$	x_{12}, x_{27}, x_{32} x_{42}, x_{55} y_7, y_2, y_5	$C(\bar{X}_{(3)}^{(1)}) = 120$	$T(\bar{X}_{(3)}^{(1)}) = 11$	$\{y_2, y_5\}$	$\{y_7\}$	$(\bar{X}_{(3)}^{(1)}, \bar{Y}_{(3)}^{(1)})$
$(\bar{X}_{(4)}^{(1)}, \bar{Y}_{(4)}^{(1)})$	x_{12}, x_{21}, x_{32} x_{42}, x_{55} y_2, y_5, y_1	$C(\bar{X}_{(4)}^{(1)}) = 170$	$T(\bar{X}_{(4)}^{(1)}) = 9$	$\{y_5, y_1\}$	$\{y_2\}$	$(\bar{X}_{(3)}^{(1)}, \bar{Y}_{(3)}^{(1)})$
$(\bar{X}_{(5)}^{(1)}, \bar{Y}_{(5)}^{(1)})$	x_{11}, x_{27}, x_{37} x_{45}, x_{55} y_5, y_1, y_7	$C(\bar{X}_{(5)}^{(1)}) = 190$	$T(\bar{X}_{(5)}^{(1)}) = 11$	$\{y_1, y_7\}$	$\{y_5\}$	$(\bar{X}_{(3)}^{(1)}, \bar{Y}_{(3)}^{(1)})$
$(\bar{X}_{(6)}^{(1)}, \bar{Y}_{(6)}^{(1)})$	x_{12}, x_{27}, x_{32} x_{42}, x_{51} y_1, y_7, y_2	$C(\bar{X}_{(6)}^{(1)}) = 170$	$T(\bar{X}_{(6)}^{(1)}) = 11$	$\{y_7, y_2\}$	$\{y_1\}$	$(\bar{X}_{(3)}^{(1)}, \bar{Y}_{(3)}^{(1)})$

Since the iterative solutions $(\bar{X}_{(2)}^{(1)}, \bar{Y}_{(2)}^{(1)})$ and $(\bar{X}_{(6)}^{(1)}, \bar{Y}_{(6)}^{(1)})$ are identical, the 6th iterative solution revisits the 2nd iterative solution indicating that the computation for obtaining the 1st efficient solution should be stopped. Note that $(\bar{X}_{(3)}^{(1)}, \bar{Y}_{(3)}^{(1)})$ being the most recent

incumbent solution is the best solution among all the iterative solutions and as such provides the 1st efficient solution of the numerical problem. The remaining efficient solutions $(\bar{X}^{(k)}, \bar{Y}^{(k)})$ ($k = 2, 3, 4$) are obtained following the procedure explained in Section 2.3.2. The set of all efficient solutions is shown in Table 2.3.

Table 2.3: Efficient Solutions of the numerical problem obtained incorporating tabu search

Efficient Solutions	Variables at Level 1	Total Cost of meeting requirements	Duration of meeting requirements
$(\bar{X}^{(1)}, \bar{Y}^{(1)})$	$x_{12}, x_{27}, x_{32}, x_{42}$ $x_{55}; y_2, y_5, y_7$	$C(\bar{X}^{(1)}) = 120$	$T(\bar{X}^{(1)}) = 11$
$(\bar{X}^{(2)}, \bar{Y}^{(2)})$	$x_{12}, x_{21}, x_{32}, x_{42}$ $x_{53}; y_1, y_2, y_3$	$C(\bar{X}^{(2)}) = 150$	$T(\bar{X}^{(2)}) = 9$
$(\bar{X}^{(3)}, \bar{Y}^{(3)})$	$x_{12}, x_{21}, x_{31}, x_{42}$ $x_{53}; y_1, y_2, y_3$	$C(\bar{X}^{(3)}) = 210$	$T(\bar{X}^{(3)}) = 8$
$(\bar{X}^{(4)}, \bar{Y}^{(4)})$	$x_{12}, x_{22}, x_{33}, x_{42}$ $x_{55}; y_2, y_3, y_5$	$C(\bar{X}^{(4)}) = 360$	$T(\bar{X}^{(4)}) = 6$

To facilitate a convenient, easy visual comparison of the efficient solutions obtained through the iterative heuristic algorithm with and without incorporating a tabu search scheme, we reproduce the efficient solutions of the numerical problem obtained through the iterative heuristic algorithm without incorporating a tabu search scheme from the paper by Praveena, Prasad and Prakash (1999) and show them in Table 2.4. From Tables 2.3 and 2.4, we find that the incorporation of a tabu search scheme results into increasing the number of efficient solutions than those obtained without its incorporation, thus leading to better results. In order to avoid repetition, we are not elaborating the idea here, because we have explained it in sufficient measure earlier.

Table 2.4: Efficient Solutions of the numerical problem obtained without incorporating tabu search

Efficient Solutions	Variables at Level 1	Total Cost of meeting requirements	Duration of meeting requirements
$(\bar{X}^{(1)}, \bar{Y}^{(1)})$	$x_{12}, x_{21}, x_{32}, x_{42}$ $x_{53}; y_1, y_2, y_3$	$C(\bar{X}^{(1)}) = 150$	$T(\bar{X}^{(1)}) = 9$
$(\bar{X}^{(2)}, \bar{Y}^{(2)})$	$x_{12}, x_{21}, x_{31}, x_{42}$ $x_{53}; y_1, y_2, y_3$	$C(\bar{X}^{(2)}) = 210$	$T(\bar{X}^{(2)}) = 8$
$(\bar{X}^{(3)}, \bar{Y}^{(3)})$	$x_{12}, x_{22}, x_{33}, x_{42}$ $x_{55}; y_2, y_3, y_5$	$C(\bar{X}^{(3)}) = 360$	$T(\bar{X}^{(3)}) = 6$

2.5 Closure

The incorporation of tabu search in a heuristic algorithm allows search for a global solution in a wider region compared to the case without incorporating tabu search, thereby increasing the possibility of arriving at the global solution or close to it. This is brought out through developing a heuristic algorithm incorporating a tabu search scheme and applying it to solve a numerical problem of selecting warehouse sites and clustering ration shops to them. Application of the heuristic algorithm incorporating a tabu search scheme to the numerical problem yields four efficient solutions as shown in Table 2.3 while only the last three efficient solutions were obtained without incorporating a tabu search scheme as was done previously by Praveena, Prasad and Prakash (1999). The model considered in this work is versatile. It can be adapted to new situations. It can deal with large size problems after computerization and is capable of incorporating different types of constraints and objective functions other than those considered in the model. This can be achieved if the terms warehouses and ration shops are stretched to include any supply and demand points.

REFERENCES

- [1] Ignizio, J.P., 1968. "A Method to Achieve Optimum Air Defense Sensor Allocation", M.S. Thesis, University of Alabama, Alabama.
- [2] Ignizio, J.P., 1971. "A Heuristic Solution to Generalized Covering Problems", Ph.D. Dissertation, Virginia Polytechnic Institute, Blacksburg.
- [3] Ignizio, J.P., 1982. "Linear programming in Single- and Multi-objective Systems", Prentice-Hall, Englewood Cliffs, New Jersey, pp. 374-376, 475-483.
- [4] Ignizio, J.P., Cavalier, T.M., 1994. "Linear Programming", Prentice-Hall, Englewood Cliffs, New Jersey, pp. 457-505.
- [5] Prakash, S., Balaji, B.V., Tuteja, D. 1999. "Optimizing dead mileage in urban bus routes through a nondominated solution approach", European Journal of Operational Research, Vol. 114, pp. 465-473.
- [6] Prakash, S., Gupta, A., 2006. "Nondominated solutions of a site selection problem with two objectives without assignment of priorities to them", News Bulletin of the Calcutta Mathematical Society, Vol. 29, No. 1-3 , pp. 3-8.
- [7] Prakash, S, Kumar, P., Prasad, B.V.N.S., Gupta, G., 2008. "Pareto optimal solutions of a cost-time trade-off bulk transportation problem", European Journal of Operational Research, Vol. 188, pp. 85-100.
- [8] Prakash, S., Madhusudan, A., Kunal, 2007. "Clustering offices of a company to

internet service providers”, Proceedings of the National Academy of Sciences, India, Section A, Vol. 77, No. 1, pp. 31-35.

- [9] Praveena, V., Prasad, B.V.N.S., Prakash, S., 1999. “Selection of warehouse sites for clustering ration shops to them with two objectives”, Proceedings of the Seminar on Recent Trends and Advances of Mathematics & Statistics in Engineering & Technology held at Indian School of Mines, Dhanbad during November 28-29, 1997, Narosa Publishing House, New Delhi, pp. 174-179.
- [10] Rardin, R.L., 1998. “Optimization in Operations Research”, Pearson Education (Singapore) Pte. Ltd., Indian Branch, New Delhi, pp. 15-16, 670-705.
- [11] Steuer, R.E., 1986. “Multiple Criteria Optimization: Theory, Computation, and Application”, Wiley, New York, pp. 138-159.

Chapter 3

A HEURISTIC FOR MULTI-OBJECTIVE BULK TRANSPORTATION PROBLEM WITH COST-TIME TRADE-OFFS

In this Chapter, a cost-time trade-off bulk transportation problem with the objectives to minimize the total cost and the duration of the bulk transportation problem without according priorities to them is considered. The entire requirement of each destination is to be met from a single source only; however a source can supply to any number of destinations subject to the availability of the commodity at it. The Extremum Difference Method (EDM) is applied to obtain the set of Pareto optimal solutions of this problem. This work provides an alternate and a simple heuristic procedure to obtain the solutions obtained by Prakash, Kumar, Prasad and Gupta (2008).

3.1 Preview

The classical transportation problem (TP) is one of the many well structured problems in operations research that has been extensively studied in literature. The TP is one of the subclass of the linear programming problems for which simple and practical computational procedures have been developed that take the advantage of the special structure of the problem. The TP is amongst the most important special linear programming problem - in terms of the frequency with which it appears in the applications and also in the simplicity of the procedures developed for its solutions. The importance of TP can also be gauged from the fact that this laid the foundation for further theoretical and algorithmic development of the minimal cost network flow optimization problems and the applicability in the real-life problem of distribution scheduling, etc.

There are different types of transportation problems and the simplest of them that is now standard in the literature was first presented by Hitchcock (1941), along with a constructive solution and, later independently, by Koopmans (1947). Because of this and the work done earlier by Hitchcock, the classical transportation problem is often referred as Hitchcock-Koopmans' transportation problem.

In the recent past, TP with a different single objective to minimize the duration of transportation has been studied by many researchers such as Sharma and Swarup (1977), Seshan and Tikekar (1980), Prakash (1982), Papamantou, Paparrizos and Samaras (2004), Sonia and Puri (2004), Sonia, Khandelwal and Puri (2008), etc. TP with multiple objectives has also been discussed by Prakash (1981), Purushotam, Prakash and Dhyani (1984), Prakash,

Agarwal and Shah (1988).

TP has been paid much attention and classified into several types of transmutations since it was first proposed by Hitchcock (1941). One of the variants of the TP is Bulk Transportation Problem (BTP) which differs from the TP that it stipulates that the total requirement of each destination has to be met from a single source only; however, a source can supply to any number of destinations subject to the availability of the commodity at the source. The BTP was first formulated by Maio and Roveda (1971) with the single objective to minimize the total transportation cost. Srinivasan and Thompson (1973) presented an algorithm consisting of the two phases to solve the problem. Verma and Puri (1996) has applied branch and bound method to minimize the cost of the bulk transportation problem. Murthy (1976) proposed a method based on the principle of lexicographic minimum to solve the bulk transportation problem. Prakash and Ram (1995) have considered a BTP with minimization of the total cost and the duration of bulk transportation as primary and secondary objectives respectively.

Prakash, Kumar, Prasad and Gupta (2008) have considered the same problem as that by Prakash and Ram (1995) with the alteration that the two objectives are not assigned priorities. They have developed two algorithms for obtaining the set of non-dominated solutions of the altered BTP, each requiring a sequence of single-objective bulk transportation problem to be solved. Both the algorithms employ a branch and bound method for solving the single-objective BTP. Each single-objective BTP is derived from the previous one with the exception of the first one. Both the algorithms make use of the previously available information as done in all the branch and bound methods. The two algorithms differ in

the sense that one of them uses the preemptive priority factors whereas the other does not. Moreover, the composition of the objective function and the criterion to stop the branch and bound method employed for solving the problem are different in the two algorithms. Even though the two algorithms are significantly different, yet the solutions obtained through both of them are identical.

3.2 Formulation of the Problem

Let there be m sources and n destinations. Let a_i ($i = 1, 2, \dots, m$) be the amounts of the units of the commodity available at the source i , b_j ($j = 1, 2, \dots, n$) be the amounts of units of the commodity required at each destination j , c_{ij} ($i = 1, 2, \dots, m; j = 1, 2, \dots, n$) be the unit cost of transportation of the commodity from source i to destination j , t_{ij} ($i = 1, 2, \dots, m; j = 1, 2, \dots, n$) be the unit time of transportation of the commodity from source i to destination j and x_{ij} ($i = 1, 2, \dots, m; j = 1, 2, \dots, n$) be the variables assuming the value 0 or 1 depending upon whether the requirement of the destination j is not met or met from the the source i ; where a_i, b_j, c_{ij}, t_{ij} are non-negative real numbers. Let C and T denote the total cost and the duration of the bulk transportation respectively. The entire requirement of each destination is to be met from one source only; however, a source can supply to any number of destinations subject to the availability of the commodity at the source. The transportation starts simultaneously. Thus, the mathematical formulation of the problem is as given below.

Determine x_{ij} s which minimize the two objective functions given by Eqns. (3.1) and (3.2),

without according priorities to them.

$$C = \sum_{j=1}^n \sum_{i=1}^m c_{ij} x_{ij} \quad (3.1)$$

$$T = \max\{t_{ij} x_{ij} : i = 1, 2, \dots, m; j = 1, 2, \dots, n\} \quad (3.2)$$

subject to the constraints

$$\sum_{j=1}^n b_j x_{ij} \leq a_i \quad (i = 1, 2, \dots, m) \quad (3.3)$$

$$\sum_{i=1}^m x_{ij} = 1 \quad (j = 1, 2, \dots, n) \quad (3.4)$$

$$x_{ij} = 0 \text{ or } 1 \quad (i = 1, 2, \dots, m; j = 1, 2, \dots, n) \quad (3.5)$$

It is intended to generate a set of efficient solutions of the problem provided by Eqns. (3.1)-(3.5). For the purpose of listing the efficient solutions of the cost-time trade-off bulk transportation problem formulated above, we denote the 1st efficient solution by $\bar{x}^{(1)}$, if it is the optimal solution of the formulated problem with the minimization of C as the first priority objective and that of T as the second priority objective. An optimal solution $\bar{x}^{(2)}$ is called the 2nd efficient solution, if there does not exist an efficient \bar{y} of the formulated problem satisfying the conditions (i) $C(\bar{x}^{(1)}) < C(\bar{y}) < C(\bar{x}^{(2)})$ and (ii) $T(\bar{x}^{(1)}) > T(\bar{y}) > T(\bar{x}^{(2)})$. The 3rd and subsequent efficient solutions are defined in the same way as done for the 2nd efficient solution. A discussion about the efficient solutions can be found in the works of Ignizio (1982), Steuer (1986), Prakash, Balaji and Tuteja (1999), etc.

3.3 Solution Procedure

The cost-time trade-off BTP formulated in Section 3.2 is an integer nonlinear problem. This is so because the objective function provided by Eq. (3.2) is nonlinear and the decision variables x_{ij} s assume the values 0 or 1. Kasana and Kumar (2004) have applied Extremum Difference Method (EDM) to the transportation problem and the generalized assignment problem and have obtained better results than those obtained by applying the Vogel's Approximation Method (VAM). The method is based on the principle that if an allocation is not made at the least cost cell of a row or column having the largest extremum difference, then the cost penalty will be higher for any other choices of rows and columns with other extremum differences. This causes the increase in the value of the objective function of the minimization type which is not desired. Here we apply the EDM to the said problem and obtain a set of efficient solutions that give more flexibility to the decision maker than that given by the solutions obtained by applying the techniques of Prakash, Kumar, Prasad and Gupta (2008) and that too with lesser computational effort. Procedure to obtain the set of efficient solutions is discussed below.

3.3.1 Procedure to Obtain 1st Efficient Solution

In the 1st efficient solution, the first priority is assigned to minimization of C provided by Eq. (3.1) and the second priority is assigned to the minimization of T provided by Eq. (3.2). Thus, the cost-time trade-off BTP is solved with priorities to obtain the 1st efficient solu-

tion of the BTP. The Tableau representation of the problem with m sources S_1, S_2, \dots, S_m with their respective capacities a_1, a_2, \dots, a_m and n destinations D_1, D_2, \dots, D_n with their respective demands b_1, b_2, \dots, b_n , together with the unit cost c_{ij} and time of transportation t_{ij} from i^{th} source to j^{th} destination is given in Table 3.1.

Table 3.1: Tableau representation of bulk transportation problem

Sources	Destinations				Availability
	D_1	D_2	\dots	D_n	
S_1	c_{11}	c_{12}	\dots	c_{1n}	a_1
	t_{11}	t_{12}	\dots	t_{1n}	
S_2	c_{21}	c_{22}	\dots	c_{2n}	a_2
	t_{21}	t_{22}	\dots	t_{2n}	
\vdots	\vdots	\vdots	\dots	\vdots	\vdots
S_m	c_{m1}	c_{m2}	\dots	c_{mn}	a_m
	t_{m1}	t_{m2}	\dots	t_{mn}	
Requirements	b_1	b_2	\dots	b_n	

Since the entire requirement of each destination D_j is to be met from a single source only, all the variables x_{ij} s corresponding to the cells in the column associated to D_j except one would be zero. Thus, a set consisting of one non-zero x_{ij} for each column would determine the solution of the 1st BTP provided it satisfies constraints (3.3)-(3.5). The algorithm to obtain the solution is explained hereafter.

Step I Obtain a row reduced matrix by deleting those rows wherein the availability a_i is less than the requirement b_j for all j . This is done since the requirement of a single destination is to be met from a single source only i.e. drop the source S_i if $a_i < b_j \forall j$.

Step II In the reduced matrix delete those cells (i, j) for which the availability a_i is less than the demand b_j i.e. drop the cells (i, j) if $a_i < b_j$.

Step III Calculate the cost penalty P_j ($j = 1, 2, \dots, n$) for each destination by EDM i.e. the difference of maximum and minimum costs. If the maximum and minimum cost of transportation to a destination j is c_{Mj} and c_{mj} respectively, then the cost penalty for destination j is given by $P_j = c_{Mj} - c_{mj}$. If the cost of transportation to a destination j from all the sources is same, then the penalty for destination j will be zero.

Step IV Select the destination k having penalty P_k where $P_k = \max\{P_j : j = 1, 2, \dots, m\}$. Let (i, k) be the least cost cell for selected destination k and allocate the amount b_k to the selected cell i.e. $x_{ik} = 1$. Drop the destination k from the table and update the availability of the source i from a_i to $a_i - b_k$. In case of tie of cost penalty, the first priority is given to the destination that has the least cost lower than that of the other destinations in tie and the second priority is given to the destination having lower duration of transportation as well. In case of tie at both these aspects, it is an arbitrary choice of the decision maker. If the demand of the selected destination cannot be satisfied at the least cost then do not allocate the destination.

Step V Apply the previous step till all the destinations have been considered. If the demands of all the destinations have been met by applying the above step, an optimal solution has been obtained and is designated the 1st efficient solution $\bar{x}^{(1)}$. Otherwise, there is atleast one destination whose demand has not been met. In that case apply the heuristic moves as explained in the next step.

Step VI Retrace from the final table of the problem in which the solution has not been obtained, i.e., the demand of all the destinations have not been met following the

constraints (3.3)-(3.5). Shift the assignment of the last destination allocated to some other source at the next least cost. Check whether a solution is obtained by applying Step IV. If still not, retrace to the penultimate table and shift the allocation of the destination made in that table to some other source with the next least cost. Check whether a solution is now obtained by applying Step IV. If not, apply this step again in the same backward approach till the solution is obtained.

The retracing approach has been considered since the allocations were made considering the decreasing penalties calculated by the EDM, i.e., keeping in view the largest penalties that would be incurred if the allocations are not made at the least costs. So the retracing approach allows us to obtain the solution at the minimum of the maximum penalties.

3.3.2 Procedure to Obtain 2^{nd} and Subsequent Efficient Solutions

Having, thus, obtained the 1^{st} efficient solution $\bar{x}^{(1)}$ of the 1^{st} cost-time trade-off BTP, the 2^{nd} efficient solution $\bar{x}^{(2)}$ is obtained by deleting all the cells (i, j) corresponding to the $t_{ij} \geq T(\bar{x}^{(1)})$. The resultant problem is designated the 2^{nd} BTP and is solved in the same manner as the 1^{st} BTP thereby obtaining the 2^{nd} efficient solution $\bar{x}^{(2)}$. Further, the 3^{rd} efficient solution is obtained by deleting those cells (i, j) , in the 2^{nd} BTP, that correspond to the $t_{ij} \geq T(\bar{x}^{(2)})$. Subsequent efficient solutions are obtained by proceeding exactly in the same way as for $\bar{x}^{(2)}$ and $\bar{x}^{(3)}$.

3.4 Numerical Problem

Here, we consider the same numerical problem as taken up by Prakash, Kumar, Prasad and Gupta (2008), given in Table 3.2 below with 4 sources and 5 destinations and the respective unit cost and duration of transportation from each source to each of the destinations.

Table 3.2: Costs and times of bulk transportation of numerical problem

Sources	Destinations					Availability
	D_1	D_2	D_3	D_4	D_5	
S_1	2 4	3 4	3 10	7 8	1 7	5
S_2	4 4	1 7	1 12	2 14	8 8	4
S_3	1 8	7 2	11 4	1 4	5 4	3
S_4	20 4	30 6	10 7	2 2	5 2	2
Requirement	3	3	2	2	1	

The upper and lower entries of a cell (i, j) depict the units of cost and time of bulk transportation from source S_i to destination D_j respectively. The marginal row and column depict the units of the commodity required at the destinations and available at the sources respectively. The numerical problem seeks to determine x_{ij} s which minimize the two-objective functions:

$$C = \sum_{j=1}^5 \sum_{i=1}^4 c_{ij} x_{ij} \quad (3.6)$$

$$T = \max\{t_{ij} x_{ij} : i = 1, 2, 3, 4; j = 1, 2, 3, 4, 5\} \quad (3.7)$$

without according priorities to them subject to the constraints (3.3)-(3.5) after assigning the

numerical values of all the parameters therein.

Since none of the sources have the availability less than the requirements of each of the destinations thus no row is dropped from the table; whereas the cells (4,1) and (4,2) are dropped since $a_4 < b_1$ and $a_4 < b_2$. It can be easily verified that the penalties using EDM for the destinations D_1 , D_2 , D_3 , D_4 and D_5 are 3, 6, 10, 6 and 7 respectively. Since the largest penalty corresponds to destination D_3 , so make an allocation in the cell (2,3) as it has the least cost for D_3 , thus, we get $x_{23} = 1$. Drop the destination D_3 and update the availability of source S_2 to 2 units to obtain Table 3.3.

Table 3.3: Tableau after first allocation

Sources	Destinations					Availability
	D_1	D_2	D_3	D_4	D_5	
S_1	2	3	3	7	1	5
	4	4	10	8	7	
S_2	×	×	1	2	8	2
			12	14	8	
S_3	1	7	11	1	5	3
	8	2	4	4	4	
S_4	×	×	10	2	5	2
			7	2	2	
Requirement	3	3	2	2	1	

It should be noted here that the cells (2,1) and (2,2) have been dropped from Table 3.3 since the supply of source S_2 is now less than the demands of destinations D_1 and D_2 . Recalculate the penalties using EDM from Table 3.3 and proceed in the same manner as above to obtain Table 3.4 and subsequently Table 3.5, wherein it can be seen that the destination D_2 is allocated to source S_1 and the destination D_1 is not allocated to any of the sources, i.e. a solution to the 1st BTP has not been obtained yet. The status of the

allocation so far is given in Table 3.6.

Table 3.4: Tableau after second allocation

Sources	Destinations					Availability
	D_1	D_2	D_3	D_4	D_5	
S_1	2	3	3	7	1	4
	4	4	10	8	7	
S_2	×	×	1	2	8	2
			12	14	8	
S_3	1	7	11	1	5	3
	8	2	4	4	4	
S_4	×	×	10	2	5	2
			7	2	2	
Requirement	3	3	2	2	1	

Table 3.5: Tableau after third allocation

Sources	Destinations					Availability
	D_1	D_2	D_3	D_4	D_5	
S_1	2	3	3	7	1	4
	4	4	10	8	7	
S_2	×	×	1	2	8	2
			12	14	8	
S_3	×	×	×	1	×	1
				4		
S_4	×	×	10	2	5	2
			7	2	2	
Requirement	3	3	2	2	1	

Table 3.6: Status of allocations

Destination	Source
D_1	S_1
D_2	Not Assigned
D_3	S_2
D_4	S_3
D_5	S_1

Now, we apply the heuristic moves as explained in the algorithm in Section 3.3.1. Starting from Table 3.5, rather than allocating at the cell (1, 2), make an allocation at the cell (1, 1); it can still be seen that the solution is not obtained, since in this case destination D_2 remains unallocated to any of the sources. Thus, retracing back to Table 3.4, wherein the allocation

was made in the cell (3, 4), if the allocation is made in the cell (4, 4), the reduced matrix, thus obtained, is given in Table 3.7.

Table 3.7: Tableau after applying heuristic

Sources	Destinations					Availability
	D_1	D_2	D_3	D_4	D_5	
S_1	2	3	3	7	1	4
	4	4	10	8	7	
S_2	×	×	1	2	8	2
			12	14	8	
S_3	1	7	11	1	5	3
	8	2	4	4	4	
S_4	×	×	10	2	5	0
			7	2	2	
Requirement	3	3	2	2	1	

After calculating the penalties of both the destinations in this table, the allocation is made in the cell (1, 2). Thus, the final allocation can be made in the cell (3, 1). Hence, keeping in view, all the constraints (3.3)-(3.5), all the destinations have been allocated to certain sources, thereby obtaining the 1st efficient solution $\bar{x}^{(1)}$ for which the variables x_{ij} s at level 1 are $x_{23}, x_{15}, x_{44}, x_{12}, x_{31}$; the total cost and the duration of the bulk transportation are $C(\bar{x}^{(1)}) = 1 + 1 + 2 + 3 + 1 = 8$ and $T(\bar{x}^{(1)}) = \max\{12, 7, 2, 4, 8\} = 12$ units respectively.

The remaining efficient solutions are obtained following the procedure explained in Section 3.3.2. For obtaining the 2nd efficient solution, we obtain the 2nd BTP by dropping those cells (i, j) for which the $t_{ij} \geq T(\bar{x}^{(1)}) = 12$. Subsequently, the allocations are made as in the case of the 1st BTP. Thus, the 2nd efficient solution $\bar{x}^{(2)}$ is obtained for which the variables x_{ij} s at level 1 are $x_{13}, x_{15}, x_{44}, x_{22}, x_{31}$; the total cost and the duration of the bulk transportation are $C(\bar{x}^{(2)}) = 3 + 1 + 2 + 1 + 1 = 8$ and $T(\bar{x}^{(2)}) = \max\{10, 7, 2, 7, 8\} = 10$ units respectively. Comparing the two solutions obtained it can be easily checked that the 2nd solution dominates

the 1st one. Thus, redesignate the 2nd efficient solution $\bar{x}^{(2)}$ as the 1st efficient solution $\bar{x}^{(1)}$.

For obtaining the 2nd efficient solution, we reduce the 1st BTP to the 2nd BTP by dropping the cells (i, j) for which the $t_{ij} \geq T(\bar{x}^{(1)}) = 10$ units. Applying the same procedure as above, we obtain the 2nd efficient solution $\bar{x}^{(2)}$ for which the variables at level 1 are given by $x_{11}, x_{22}, x_{43}, x_{34}, x_{15}$; the total cost and the duration of the bulk transportation are $C(\bar{x}^{(2)}) = 2 + 1 + 10 + 1 + 1 = 15$ and $T(\bar{x}^{(2)}) = \max\{4, 7, 7, 4, 7\} = 7$ units respectively.

Similarly, the 3rd efficient solution $\bar{x}^{(3)}$ is obtained by dropping the cells (i, j) for which the $t_{ij} \geq T(\bar{x}^{(2)}) = 7$. Applying the same procedure, we obtain the 3rd efficient solution $\bar{x}^{(3)}$ for which the variables at level 1 are given by $x_{21}, x_{12}, x_{33}, x_{44}, x_{35}$; the total cost and the duration of the bulk transportation are $C(\bar{x}^{(3)}) = 4 + 3 + 11 + 2 + 5 = 25$ $T(\bar{x}^{(3)}) = \max\{4, 4, 4, 2, 4\} = 4$.

To obtain the 4th Pareto optimal solution, we drop all the cells (i, j) in the 3rd BTP for which $t_{ij} \geq T(\bar{x}^{(3)}) = 4$ units. It can be seen that as a result, the destination D_3 cannot be allocated to any of the source and hence there exists no efficient solution of the numerical problem for which the duration of bulk transportation is less than 4 units and thus, the process of obtaining further efficient solutions is terminated. Thus, the numerical problem is found to have three efficient solutions. The set of efficient solutions of the numerical problem together with the variables x_{ij} s at level 1, total cost and the duration of the bulk transportation are shown in Table 3.8.

Table 3.8: Set of efficient optimal solutions of the numerical problem

Efficient Solution	Variables at Level 1	Total Cost	Total Duration
$\bar{x}^{(1)}$	$x_{13}, x_{15}, x_{22},$ x_{31}, x_{44}	$C(\bar{x}^{(1)}) = 8$	$T(\bar{x}^{(1)}) = 10$
$\bar{x}^{(2)}$	$x_{11}, x_{22}, x_{43},$ x_{34}, x_{15}	$C(\bar{x}^{(2)}) = 15$	$T(\bar{x}^{(2)}) = 7$
$\bar{x}^{(3)}$	$x_{21}, x_{12}, x_{33},$ x_{44}, x_{35}	$C(\bar{x}^{(3)}) = 25$	$T(\bar{x}^{(3)}) = 4$

3.5 Closure

The incorporation of heuristic algorithm allows search for optimal solution in a wider region compared to the case without incorporating the heuristic, thereby improving the chances of obtaining an optimal solution. This has been brought out through developing a heuristic algorithm and applying it to the numerical problem of bulk transportation problem. Application of the algorithm to the numerical problem yields the same three efficient solutions, as shown in Table 3.8, that were obtained by Prakash, Kumar, Prasad and Gupta (2008). The proposed algorithm reduces the computation work and still obtains the identical solutions as obtained by the two algorithms incorporating the branch and bound techniques given by Prakash, Kumar, Prasad and Gupta (2008).

REFERENCES

- [1] Hitchcock, F., 1941. "The distribution of a product from several sources to numerous locations", *Journal of Mathematical Physics*, Vol. 20, pp. 224-230.
- [2] Ignizio, J.P., 1982. "Linear programming in Single- and Multi-objective Systems", Prentice-Hall, Englewood Cliffs, New Jersey, pp. 374-376, 475-483.
- [3] Kasana, H.S., Kumar, K.D., 2004. "Introductory Operations Research: Theory and Applications", Springer-Verlag.
- [4] Koopmans, T.C., 1947. "Optimum utilization of the transport system", *Proceedings of International Statistical Conferences: Econometric*, Washington DC, pp. 136-145.
- [5] Maio, A.D., Roveda, C., 1971. "An all zero-one algorithm for a certain class of transportation problems", *Operations Research*, Vol. 19, pp. 1406-1418.
- [6] Murthy, M.S., 1976. "A bulk transportation problem", *OPSEARCH*, Vol. 13, pp. 143-155.
- [7] Papamanthou, C., Paparrizos, K., Samaras N., 2004. "Computational experience with exterior point algorithms for the transportation problems", *Applied Mathematics and Computation*, Vol. 158, pp. 459-475.
- [8] Prakash, S., 1981. "A transportation problem with objectives to minimize total cost and duration of transportation", *OPSEARCH*, Vol. 18, pp. 235-238.

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- [9] Prakash, S., 1982. "On minimizing the duration of transportation", Proceedings of the Indian Academy of Sciences-Mathematical Sciences, Vol. 91, pp. 53-57.
- [10] Prakash, S., Agarwal, A.K., Shah, S., 1988. "Nondominated solutions of cost-time trade-off transportation and assignment problems", OPSEARCH, Vol. 25, pp. 126-131.
- [11] Prakash, S., Balaji, B.V., Tuteja, D. 1999. "Optimizing dead mileage in urban bus routes through a nondominated solution approach", European Journal of Operational Research, Vol. 114, pp. 465-473.
- [12] Prakash, S., Kumar, P., Prasad, B.V.N.S., Gupta, G., 2008. "Pareto optimal solutions of a cost-time trade-off bulk transportation problem", European Journal of Operational Research, Vol. 188, pp. 85-100.
- [13] Prakash, S., Ram, P.P., 1995. "A bulk transportation problem with objectives to minimize the total cost and duration of transportation", The Mathematics Student, Vol. 64, pp. 206-214.
- [14] Purushotam, S., Prakash, S., Dhyani, P., 1984. "A transportation problem with minimization of duration and total cost of transportation as high and low priority objectives respectively", Bulletin of the Technical University of Istanbul, Vol. 37, pp. 1-11.
- [15] Seshan, C.R., Tikekar, V.G., 1980. "On Sharma-Swarup algorithm for time minimizing transportation problems", Proceedings of Indian Academy of Sciences-

Mathematical Sciences, Vol. 89, pp. 101-102.

- [16] Sharma, J.K., Swarup, K., 1977. "Time minimizing transportation problems", Proceedings of Indian Academy of Sciences-Mathematical Sciences, Vol. 86, pp. 513-518.
- [17] Sonia, Khandelwal, A., Puri, M.C., 2008. "Bilevel time minimizing transportation problem", Discrete Optimization, Vol. 5, No. 4, pp. 714-723.
- [18] Sonia, Puri, M.C., 2004. "Two level hierarchical time minimizing transportation problem", TOP, Vol. 12, No. 2, pp. 301-330.
- [19] Srinivasan, V., Thompson, G.L., 1973. "An algorithm for assigning uses to sources in a special class of transportation problems", Operations Research, Vol. 21, pp. 284-295.
- [20] Steuer, R.E., 1986. "Multiple Criteria Optimization: Theory, Computation, and Application", Wiley, New York, pp. 138-159.
- [21] Verma, V., Puri, M.C., 1996. "A branch and bound method for cost minimizing bulk transportation problem", OPSEARCH, Vol. 33, No. 3, pp. 145-161.

Chapter 4

MULTI-OBJECTIVE CHINESE POSTMAN PROBLEM WITH DISTANCE-TIME TRADE-OFFS

Transportation systems can be represented by graphs with travel weights accorded to each of the edges that represent the roads to be traveled. This chapter gives brief introduction of the Euler's path and the description of the Chinese postman problem. The said problem is then extended to multi-objective problem by considering multiple weights for each edge. Finally, this chapter presents an algorithm to solve this multi-objective problem and implements the same on a biobjective Chinese postman problem.

4.1 Preview

The eminent Swiss mathematician Leonhard Euler (1736) had studied the problem whether it is possible to trace a route of the city of Königsberg (Figure 4.1) by walking across all the seven bridges that connected the two islands in the river Pregel with the rest of the city on the adjacent shores, a route that would cross each bridge exactly once.

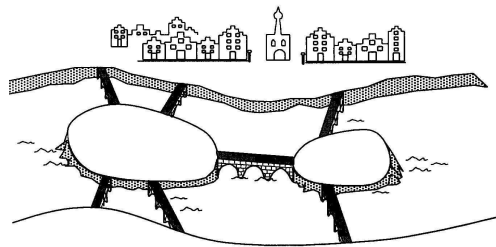


Figure 4.1: The map of Königsberg

In 1962, the Chinese mathematician Mei-Ko Kwan posed the Chinese Postman Problem where it is required that the postman has to cover his assigned segment before returning to the post office following the shortest walking distance. Though the two problems seem different, yet they have some commonalities. They both admit schematic representations based on the concept of *graphs*. A graph $G = (V, E)$ is defined as a relationship between the set V of *vertices* or *nodes* and the set E of *edges* or *arcs* wherein for each edge, two of the vertices are specified to be *joined* by this edge. The bridges of Königsberg may be considered as edges, each edge joining a pair of the four vertices which correspond to the two islands and the two shores (Figure 4.2).

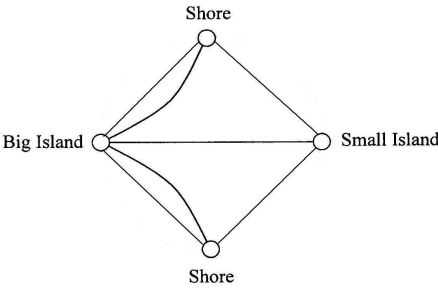


Figure 4.2: The bridges of Königsberg

The similarity between the two graph-theoretical problem formulation reaches deeper. Given any graph G , for one of its vertices, say v_0 , find an edge e_1 that joins the vertex v_0 to another vertex, say v_1 . The process is applied repeatedly to find a second edge e_2 which joins vertex v_1 to some vertex v_2 and so on. This process yields a sequence of vertices, where some may be revisited, where the successive pairs are joined by edges. The sequence of these edges is referred as *path* between the first and the last vertex (Figure 4.3).

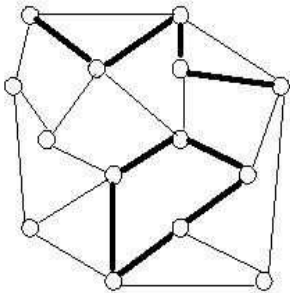


Figure 4.3: Open and closed paths in the graph

If all the pair of vertices in a graph can be connected by a path, then the whole graph is considered *connected*. A path in a graph is called *closed path* or *circuit* if it returns to its

starting point, else it is called *open path* (Figure 4.3).

Mei-Ko Kwan's Chinese Postman Problem is to determine, in a given connected graph, a minimal closed path that traverses every edge *at least once* to deliver mail on every assigned block. Euler also considers closed paths meeting all edges, but aims at characterizing all graphs which possess an *Euler tour*, a closed path that crosses each edge *exactly once*. Euler proved that they are precisely those connected graphs in which each vertex is of even *degree*, i.e. the number of edges incident on a vertex should be even. However it is very rare that every vertex in a road network is even. Since every road must be traced, some roads need to be retraced. Such retracing is referred as "deadheading". Thus the problem is to plan a route so that the total amount of deadheading is as small as possible.

Mei-Ko Kwan (1962) proposed a method that could be used to minimize the length of routes walked by mail carriers. Edmonds and Johnson (1973) solved the problem in undirected graph and directed graph in time. Orloff (1974) put forward the Rural postman problem, which is commonly associated with mail delivery in rural areas. Papadimitriou (1976) proposed that if the graph is a mixed graph, then the problem is NP-Hard. Another related problem was discussed by Tang and Xie (2000), in which it was assumed that some roads can be traversed once because of flood. Thimbleby (2003) has reviewed wide range of applications of the problem and also presents an executable code to solve it for directed multigraphs. Groves and Vuuren (2005) have studied Rural Postman Problem in which a subset of edges $E_r \subseteq E$ has to be traversed. Martínez (2005) has solved the windy postman problem to find the minimum cost of tour traversing all the edges of an undirected graph, where the cost depends on the direction of the edge. Mohan, Gendreau and Rousseau (2008)

have constructed an Eulerian tour of minimum expected length by considering a subset of edges that require service with their respective probabilities.

4.2 Definition of the Problem

The CPP is defined as: Given an undirected graph $G = (V, E, w)$, where each edge e_i is assigned a non-negative weight $w(e_i)$. The problem is to determine a minimum circuit which uses each edge atleast once.

In fact, if the graph is an Eulerian graph, we know that there exists an Eulerian circuit. On the contrary, if the graph is not an Eulerian graph, then the problem is mathematically defined as:

In a graph G , find a set of edges, $E_1 \subseteq E$ such that (1) “doubling” the edges of the set E_1 in the graph makes the graph an Eulerian one; (2) $Min w(E_1) = \sum_{e \in E_1} w(e)$.

If E_1 satisfies (1), it is called a feasible solution in graph G and when it satisfies both (1) and (2) it is called an optimal solution.

In this paper, this single objective CPP is extended to a multi-objective CPP by considering a set of linear objective functions, each minimizing the sum of the respective weights of all the arcs, without according any priority to them. Thus, the Multi-Objective Chinese Postman Problem (MOCPP) is defined as:

Given an undirected connected graph $G = (V, E, W)$, where V is the set of vertices, E is the set of edges and each edge $e_i \in E$ is assigned a vector of

non-negative weights $W(e_i) = (w_1(e_i), w_2(e_i), \dots, w_n(e_i))$. The problem is to determine a circuit, that minimizes all the objective functions each of which corresponds to sum of the respective weights of all the edges and uses each edge atleast once.

If the graph is not Eulerian, the problem is mathematically defined as: In a graph G , find a set of edges $E_1 \subseteq E$ such that (1) “doubling” the arcs of the set E_1 in the graph makes the graph an Eulerian one; (2) $Min W(E_1) = \sum_{e \in E_1} W(e)$, where $W(e) = (w_1(e), w_2(e), \dots, w_n(e))$ and the weights w_1, w_2, \dots, w_n are not accorded any priorities.

If E_1 satisfies (1), it is called a feasible solution in the graph G and when it satisfies both (1) and (2), then it is called an optimal solution.

4.3 Solution Procedure

The unique optimality has always been of great interest but due to the complexity of the real-life problems, i.e. more than one objective, and those too generally, of conflicting interests, the concept of non-dominated solution or efficient solution has gained importance.

A vector of decision variables $x^* \in \Omega$ for a k -objective problem is non-dominated or efficient if there does not exist another $x \in \Omega$ such that $f_i(x) \leq f_i(x^*)$ for all $i = 1, 2, \dots, k$ and $f_j(x) < f_j(x^*)$ for atleast one j . A discussion about the efficient solutions can be found in the works of Prakash (1981), Ignizio (1982), Sharma and Prakash (1986), Steuer (1986), Praveena, Prakash and Prasad (1999), Prakash, Kumar, Prasad and Gupta (2008).

A procedure is proposed to find a set of solutions giving first priority to the first objective function and the subsequent priorities to the subsequent objective functions where the n objective functions correspond to the respective n weights. The set may determine a unique optimality or generate set of efficient solutions. The solution procedure is broadly divided into three steps:

- To determine the set E_1 of edges required to be “doubled” so that the graph becomes Eulerian.
- To find the minimum weighted such set E_1 .
- To construct the Eulerian circuit from the graph obtained after doubling the edges of E_1 obtained above in the original graph.

4.3.1 Procedure to Determine the Set E_1 of Edges to be “doubled” to Convert the Graph into an Eulerian Graph

This step is performed only if the graph G is not an Eulerian graph. It is worthwhile to note here that the number of vertices having odd degree in a connected graph is always even. So create the partition of the vertices, having odd degree, into pairs and consider all the possible paths by joining these pairs. Determine the respective sum of all the weights along all these paths. Any one of these paths when doubled in the graph converts it into an Eulerian graph. Thus, any of these paths, referred as feasible solution, may constitute the set E_1 .

4.3.2 Procedure to Determine the Minimum Weighted Set E_1

In order to determine the optimal solutions from amongst the feasible solutions obtained in Section 4.3.1, the objective functions are prioritized. The first priority is given to the minimization of sum of $w_1(e)$ for all $e \in E_1$ and the second priority to the minimization of sum of $w_2(e)$ and so on for all $e \in E_1$. Thus if E_1 and E_2 are two sets of edges obtained in Section 4.3.1, then the priority is given to the set E_1 if $W(E_1) < W(E_2)$ which happens if $\sum_{e \in E_1} w_1(e) < \sum_{e \in E_2} w_1(e)$. However, if $\sum_{e \in E_1} w_1(e) = \sum_{e \in E_2} w_1(e)$, then the decision is taken whether $\sum_{e \in E_1} w_2(e) < \sum_{e \in E_2} w_2(e)$ or not. If so, then again E_1 is considered the minimal weighted set else E_2 . However, if values of all the objective functions for each of the weights of both the sets E_1 and E_2 are same, then either of the sets is considered as the minimal set for construction of the Eulerian circuit in the given graph.

4.3.3 Procedure to Determine the Eulerian Circuit

If the given graph G is already an Eulerian graph possessing the closed circuit then the circuit can be constructed directly by following Fleury's algorithm (1883) generating a unique optimal solution, otherwise the arcs of the set E_1 obtained above in Section 4.3.2 are doubled in the graph G , thereby, converting it into a multigraph having all vertices of even degree, and hence an Eulerian circuit. The Eulerian circuit is then obtained by following the Fleury's algorithm and the values of the objective function are obtained by simply adding the respective weights of each of the arcs. This generates one of the solutions, whereas the others may be generated by considering the other partitions of the odd vertices. Hence, the

problem possesses unique solution in case G is an Eulerian graph, else a set of non-dominated or efficient solutions and the choice of the solution in the latter case depends upon the choice of the priority of the decision maker.

4.4 Numerical Problem

Consider a graph as shown in Figure 4.4. The graph consists of 9 vertices and 12 arcs. Each

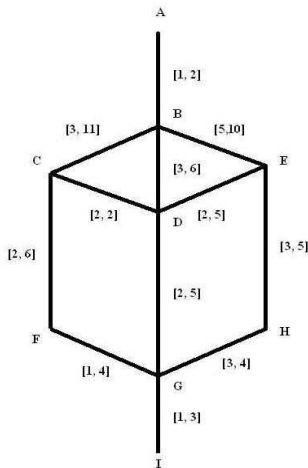


Figure 4.4: An undirected graph G with 4 odd vertices

of the arc is assigned a vector of non-negative weights, where the first weight corresponds to the distance between the respective vertices and the second weight corresponds to the time taken in travelling the distance between the respective vertices. It is required to find the Eulerian circuit of this bi-objective Chinese postman problem that minimizes the total distance travelled and also the total time taken to travel through all the arcs.

Step 1. This graph contains four vertices, viz. A, C, E and I, having odd degree. The following three partitions of these vertices in pairs are possible.

Consider the first partition where the odd vertices are partitioned into pairs (A, C) &

Table 4.1: Possible partitions of the odd vertices

Partition No.	Respective pairs of vertices
1	(A, C) & (E, I)
2	(A, E) & (C, I)
3	(A, I) & (C, E)

(E, I) . Since the graph G in Figure 4.4 is a connected undirected graph, so it is clear that there exists more than one path connecting A to C and E to I . For example, A and C are connected by the paths $A - B - C$, $A - B - D - C$, etc and the vertices E and I are connected by the paths $E - H - G - I$, $E - D - G - I$, etc. One path each of the pair of vertices in combination is the feasible solution set E_1 that on being doubled in the graph G converts it into an Eulerian graph.

Step 2. From amongst the feasible paths obtained in Step 1, the paths with the minimum weights for the respective vertices are shown in the Table 4.2 below:

Table 4.2: Minimum weighted paths		
Pair of Vertices	Minimum weighted Path	Weight on the path (w_1, w_2)
(A, C)	$A - B - C$	$(4, 13)$
(E, I)	$E - D - G - I$	$(5, 13)$
Total Weight		$(9, 26)$

It should be noted that the path that minimizes the total distance travelled among the pair of vertices is selected, and in case more than one paths have the same minimum distance travelled then the preference should be given to the one that minimizes the time duration of path. Hence, we get $E_1 = \{(A, B), (B, C), (E, D), (D, G), (G, I)\}$.

Step 3. Double the edges of the set E_1 in the graph G to obtain $G' = (V, E \cup E_1, W)$

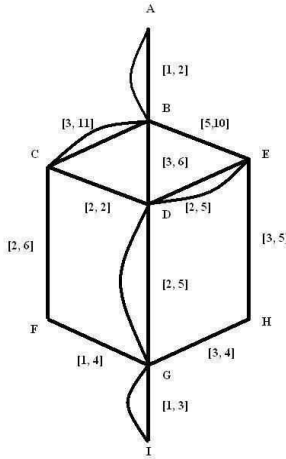


Figure 4.5: An undirected graph G' after doubling the arcs of E_1 in G

(Figure 4.5), where G' is now a multigraph in which all the vertices are of even degree and hence possesses an Eulerian circuit that can be obtained by Fleury’s algorithm by starting from any vertex. One such circuit is given by $A - B - C - F - G - I - G - D - E - H - G - D - E - B - D - C - B - A$. The weights of the doubled arcs remaining the same, the total weights i.e. the values of the objective function are given in Table 4.3 below.

Table 4.3: 1st efficient solution

Weights	Value of weights
$w_1(E \cup E_1)$	37 units
$w_2(E \cup E_1)$	89 units

Thus, an efficient solution corresponding the partition (A, C) and (E, I) has been obtained in which the total distance travelled is 37 units and total time duration is 89 units.

Similarly, considering the other two partitions as mentioned in Table 4.1 and applying the same procedure as explained in Section 4.3, two more efficient solutions can be found by obtaining the corresponding minimum weighted sets and doubling the arcs in graph G to

obtain a multigraph G' . Hence, the set of efficient solutions of the problem may be tabulated as in Table 4.4.

Table 4.4: Efficient solutions of the numerical problem

Weights \rightarrow Partitions \downarrow	$w_1(E \cup E_1)$	$w_2(E \cup E_1)$
$(A, C) \& (E, I)$	37 units	89 units
$(A, E) \& (C, I)$	38 units	88 units
$(A, I) \& (E, C)$	39 units	86 units

4.5 Closure

The Chinese postman problem finds applications in many areas wherever the edge traversal is studied, viz. a mail carrier delivering mail to buildings along the streets, a snowplow clearing snow from each lane of the streets, a police car making rounds through all the streets several times a day, a road sweeping machine must clean all the streets, a garbage collector visiting all the streets to collect the household refuse, etc. Due to the complexity of the real-life problems and the conflicting nature of the objectives involved, the problem was extended to a multi-objective one and studied from the perspective of obtaining a set of efficient solutions in case there does not exist a unique optimal solution. It was found that such a set exists and the final choice of the solution will depend on the choice of the decision maker as to which objective function is given the top priority by him/her.

REFERENCES

- [1] Edmonds, J., Johnson, E.L., 1973. “Matching, Euler Tours and the Chinese Postman Problem”, *Mathematical Programming*, Vol. 5, pp. 88-124.
- [2] Euler, L., 1736. “Solutio problematis ad geometriam situs pertinentis”, *Commentarii Academiae Scientiarum Imperialis Petropolitanae*, Vol. 8, pp. 128-140.
- [3] Fleury, 1883. “Deux problemes de geometrie de situation”, *Journal de mathematiques elementaires*, pp. 257-261.
- [4] Groves, G.W., Vuuren, J.H. van, 2005. “Efficient heuristics for the Rural Postman Problem”, *ORiON*, Vol. 21, No. 1, pp. 35-51.
- [5] Ignizio, J.P., 1982. “Linear Programming in Single- & Multi- Objective Systems”, Prentice-Hall, Englewood Cliffs, New Jersey, pp. 374-376, 475-483.
- [6] Kwan, M., 1962. “Graphics Programming Using Odd and Even Points”, *Chinese Mathematics*, Vol. 1, pp. 237-277.
- [7] Martínez, F.J.Z., 2005. “The Windy Postman Problem on Series-Parallel Graphs”, *Proceedings of European Conference on Combinatorics, Graph Theory and Applications*, pp. 161-166.
- [8] Mohan, S., Gendreau, M., Rousseau, J-M., 2008. “The Stochastic Eulerian tour Problem”, *Transportation Science*, Vol. 42, No. 2, pp. 166-174.

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- [9] Orloff, C.S., 1974. "A Fundamental Problem in Vehicle Routing", *Networks*, Vol. 4, pp. 35-64.
- [10] Papadimitriou, C.H., 1976. "On the Complexity of Edge Traversing", *Journal of A.C.M.*, Vol. 23, pp. 544-554.
- [11] Prakash, S., 1981. "A transportation problem with objectives to minimize total cost and duration of transportation", *OPSEARCH*, Vol. 18, pp. 235-238.
- [12] Prakash, S., Kumar, P., Prasad, B.V.N.S., Gupta, G., 2008. "Pareto optimal solutions of a cost-time trade-off bulk transportation problem", *European Journal of Operational Research*, Vol. 188, pp. 85-100.
- [13] Praveena, V., Prasad, B.V.N.S., Prakash, S., 1999. "Selection of warehouse sites for clustering ration shops to them with two objectives", *Proceedings of the Seminar on Recent Trends and Advances in Mathematics and Statistics in Engineering & Technology*, pp. 174-179.
- [14] Sharma, V., Prakash, S., 1986. "Optimizing dead mileage n urban bus routes", *Journal of Transportation Engineering*, Vol. 112, pp. 121-129.
- [15] Steuer, R.E., 1986. "Multiple Criteria Optimization: Theory, Computation and Application", John-Wiley & Sons, New York.
- [16] Tang, Z.Y., Xie, Z., 2000. "Postman Problems in Areas Flooded", *Communications on Applied Mathematics and Computing*, Vol. 14, No. 1, pp. 25-30.

- [17] Thimbleby, H., 2003. "The directed Chinese Postman Problem", *Journal of Software Practice and Experience*, Vol. 33, No. 11, pp. 1081-1096.

Chapter 5

EFFICIENT SOLUTIONS FOR MULTI-OBJECTIVE GENERALIZED ASSIGNMENT PROBLEM

The Multi-Objective Generalized Assignment Problem (MGAP) with two objectives, where one objective is linear and the other one is nonlinear, has been considered, with the constraints that a job is assigned to one and only one worker though he may be assigned more than one job depending upon the time available with him. An algorithm is proposed to find the set of efficient solutions of the problem, determining assignments of jobs to workers with two objectives without according priorities to them. The two objectives are to minimize the total cost of assignment as well as the duration of completion of all the jobs.

5.1 Preview

The term “assignment problem” was first coined by Votaw and Orden (1952) but the beginning of the development of the solution algorithms and variations of the assignment problem (AP) was the publication by Kuhn (1955) on the Hungarian method of its solution. Assignment problems involve optimally matching the elements of two or more sets, where the dimension of the problem refers to the number of sets of elements to be matched. When there are only two sets, they may be referred to as “tasks” and “agents”. Thus, for example, “tasks” may be jobs to be done and “agents” the people or machines that can do them. In its original version, the assignment problem involved assigning each job to a different worker, with each worker being assigned one job (a one-to-one assignment) according to a predefined objective function. Among its many applications, we mention the assignment of tasks to workers, of jobs to machines, of fleets of aircrafts to trips or the assignment of buses to routes. The AP is a special case of the transportation problem, which is a special case of the minimum cost flow problem, which in turn is a special case of a linear programming.

Over the past 50 years, many variations on the classical AP have been proposed. One of the variations of the classic AP is the model with side constraints. Caron, Hansen and Jaumard (1999) use a mathematical model for a variation of the classic AP in which there are m workers and n jobs, not every worker is qualified to do every job, and the objective is the utility maximization. Dell’Amico and Martello (1997) present a variation on the classic AP in which there are m workers and n jobs, but only k of the workers and jobs are to be assigned, where k is less than both m and n . They suggested that it can be used to solve

sub-problems in the problem of assigning time slots on a communications satellite being used to transmit information from m earth stations to n different earth stations. Yadavalli (2001) has worked on the constrained assignment problem. Punnen and Aneja (1995) have applied tabu search to solve the constrained assignment problem.

In most practical applications, each agent requires a quantity of some limited resource to process a given job. Therefore, the assignments have to be made taking into account the resource availability of each agent. The problem derived from the classic AP by taking into account these capacity constraints is known as the Generalized Assignment Problem (GAP). Thus, the GAP is an example of a one-to-many assignment problem that recognizes capacity limits.

The GAP, unlike assignment problem, does not have one-to-one correspondence between the jobs and the workers. The jobs are assigned to workers in such a way that each job is assigned to some worker though he may be assigned no job or more than one job subject to the availability of time at his disposal. The objective here is to minimize the total cost of assignment of jobs. Many researchers including Cattrysse and Van Wassenhove (1992), Amini and Racer (1995), Chu and Beasley (1997), Diaz and Fernandez (2001), Haddadi and Ouzia (2004) and Kasana and Kumar (2004) have proposed various methods including exact and approximate ones, for solving the GAP and its variants. Lourenço and Serra (2002) have applied heuristic techniques to solve the GAP.

It is a well known fact that multi-objective optimization problems have continued to attract a lot of attention due to the vast application. Evans (1984) has presented some reasons for the rapidly growing increase in interest of multi-objective mathematical programming and

also discussed advantages and disadvantages of a few general approaches towards multi-objective mathematical programming. Gandibleux and Freville (2000) have used the Tabu search heuristic on the two linear objectives to solve the 0-1 knapsack problem. Lourenço, Paixao and Portugal (2001) have presented multi-objective metaheuristics for solving the real-life crew scheduling problems in public transport companies. Zhang and Ong (2007) have proposed LP-based heuristic to solve the Biobjective Generalized Assignment Problem (BiGAP) wherein both the objective functions are linear.

In the present study, a BiGAP is considered in which both the objective functions are of minimization type performance measures, but one objective function is linear and the other non-linear. An algorithm based on the adaptive search heuristics has been proposed to solve this BiGAP without according priorities to the two objective functions, to obtain a set of non-dominated solutions. A detailed study about the non-dominated solutions can also be found in the works of Ignizio (1982), Steuer (1986), Prakash, Balaji and Tuteja (1999), Junker (2004), Prakash, Madhusudan and Kunal (2007), Prakash, Kumar, Prasad and Gupta (2008), etc.

5.2 Formulation of the Problem

Consider the case of m workers and n jobs. Let $c_{ij}(i = 1, 2, \dots, m; j = 1, 2, \dots, n)$ represent the cost of assigning the worker i to job j ; $t_{ij}(i = 1, 2, \dots, m; j = 1, 2, \dots, n)$ denote the time taken by the worker i to perform job j ; $x_{ij}(i = 1, 2, \dots, m; j = 1, 2, \dots, n)$ be the binary decision variable assuming the value 1 or 0 according as the worker i has been

assigned the job j or not and $b_i (i = 1, 2, \dots, m)$ be the time available with worker i . Let C and T denote the total cost and the duration of assignment of jobs respectively. The mathematical formulation of the problem is as follows.

Determine x_{ij} s which minimize the two objective functions:

$$C = \sum_{j=1}^n \sum_{i=1}^m c_{ij} x_{ij} \quad (5.1)$$

$$T = \max \left\{ \sum_{j=1}^n t_{ij} x_{ij} : (i = 1, 2, \dots, m) \right\} \quad (5.2)$$

subject to the constraints

$$\sum_{i=1}^m x_{ij} = 1 \quad j = 1, 2, \dots, n \quad (5.3)$$

$$\sum_{j=1}^n t_{ij} x_{ij} \leq b_i \quad i = 1, 2, \dots, m \quad (5.4)$$

$$x_{ij} = 0 \text{ or } 1 \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n \quad (5.5)$$

Here, Eqs. (5.1) and (5.2) represent the objective functions that are sought to be minimized. The two objective functions are not accorded any priority. While constraint (5.3) ensures that each job is assigned to exactly one worker, constraint (5.4) ensures that the assignments to each worker are made only according to the time available with the worker. It is required to find the set of non-dominated solutions of the problem provided by Eqs. (5.1)-(5.5). Here, it is important to note that a worker may be assigned no job, one job or more than one job.

5.3 Solution Procedure

The BiGAP is an integer nonlinear problem. This is so because the objective function provided by Eq. (5.2) is nonlinear and the decision variables x_{ij} s assume the integer value 0 or 1. A procedure is outlined to obtain the set of non-dominated solutions.

5.3.1 Procedure to Obtain 1st Efficient Solution

The 1st non-dominated solution is obtained by assigning the first priority to the objective function given by Eq. (5.1) and the second priority to the objective function given by Eq. (5.2). The algorithm is explained below:

Step 1. Calculate the least cost penalties for each job, i.e. the difference of least and second least costs, for various workers.

Step 2. Select the job with the largest penalty. In case of tie, select the job that has lower least cost. In case of tie on that count as well, select the job that has lower time duration corresponding to the least cost cell, else the job can be selected arbitrarily. Let the k^{th} job be selected.

Step 3. Select the least cost cell of the k^{th} job selected above. In case of tie in the least cost cells, select the one that has lower time duration in the least cost cells. In case of tie on the least time duration of the selected least cost cells as well, assign to the worker which has greater time available with him, otherwise assign arbitrarily. If the selected least cost cell (r, k) does not satisfy the time constraint leave the job unassigned, otherwise

make an assignment in the cell, i.e. assign job k to worker r and update the time available with him to $b_r - t_{rk}$. Drop the k^{th} job from further consideration.

Step 4. Repeat steps 2 and 3 till either all the jobs have been assigned to a unique worker or there arise a situation that some jobs are left unassigned because of the time constraints of the workers at their least costs. In the former case, a non-dominated solution is obtained, whereas in the latter case apply the next step.

Step 5. If some of the jobs are left unassigned then the job schedule of certain workers is considered for change i.e. allocation of combination of jobs is considered for moving, preferring the least increase in cost, if improvement in the same is not possible, depending upon the following situations that may occur.

Step 5.1. An allocation of worker r is considered to move from job j to job j' if

$$\sum_{s=1}^n t_{rs}x_{rs} < b_r, \quad (s \neq j)$$

where job j' is so chosen that

$$\sum_{s \neq j} c_{rs}x_{rs} < \sum_{s \neq j'} c_{rs}x_{rs}$$

if possible.

Step 5.2. A combination of assignments of worker r is considered to move from jobs

j & j' to job j^* if

$$\sum_{s \neq j, j'} c_{rs} x_{rs} < b_r$$

Once an assignment is shifted from the cell (r, s) , the cell is dropped for further considerations. Repeat the steps 5.1 and 5.2 till all the jobs have been assigned. The solution thus obtained will be termed the 1st non-dominated solution denoted by $\bar{X}^{(1)}$.

5.3.2 Procedure to Obtain 2nd and Subsequent Efficient Solutions

After having obtained the 1st non-dominated solution $\bar{X}^{(1)}$, the 2nd non-dominated solution $\bar{X}^{(2)}$ is obtained by introducing an additional constraint

$$\sum_{j=1}^n t_{ij} x_{ij} < T(\bar{X}^{(1)}) \quad \forall i \quad (5.6)$$

to the formulation of the problem, to restrict the aspiration level of the decision maker. The 2nd non-dominated solution is obtained by dropping all the cells (i, j) in which $t_{ij} \geq T(\bar{X}^{(1)})$ and assigning the jobs afresh by following the same procedure as explained in Section 5.3.1 or by moving the assignments from the table of 1st non-dominated solution in the following manner.

Step I Select the workers who are working for $T(\bar{X}^{(1)})$ amount of time, i.e., satisfy the

following equality

$$\sum_{j=1}^n t_{ij}x_{ij} = T(\bar{X}^{(1)})$$

Step II Shift the assignments of the jobs of worker to some other worker by assigning at the available least cost cells satisfying constraints (5.3)-(5.6) as explained in steps 5.1 and 5.2.

Step III Stop the procedure when all the jobs are assigned to some workers satisfying all the constraints (5.3)-(5.6).

The solution hence obtained is termed the 2^{nd} non-dominated solution denoted by $\bar{X}^{(2)}$. The 3^{rd} non-dominated solution may be obtained by following the same procedure as for 2^{nd} non-dominated solution just by replacing $T(\bar{X}^{(1)})$ by $T(\bar{X}^{(2)})$ and the same procedure can be extended to obtain the subsequent solutions as well. The procedure terminates at the n^{th} non-dominated solution $\bar{X}^{(n)}$, when it is not possible to obtain a non-dominated solution $\bar{X}^{(n+1)}$ where $T(\bar{X}^{(n+1)}) < T(\bar{X}^{(n)})$.

5.3.3 Some Additional Assignment Procedures

Some more non-dominated solutions can be found by following the assignment procedures explained below.

Case I Following the same penalties on cost, the assignments are made in the cells that have

least amount of time, satisfying the constraints (5.3)-(5.5). In case of tie in the cells for the least amount of time, the assignments are made in the cells that have lower cost corresponding to the least time. In case of tie on that count as well, the assignment is made arbitrarily. If the solution is not obtained, i.e. not all the jobs are assigned, then the procedure as explained in step 5 of the algorithm in Section 5.3.1 is applied. The solution should be compared with the non-dominated solutions already obtained for its dominance or non-dominance. The second and the subsequent non-dominated solutions can be obtained by following the same procedure as explained earlier.

Case II Calculate the least time penalties for all the jobs for various workers, i.e. the difference of least and second least time duration for all the jobs by various workers, and assign the jobs to the worker with least cost, following the same procedure as explained in Section 5.3.1 where the penalties are calculated on the time duration and obtain a set of non-dominated solutions.

Case III Using the same least time penalties for the all the jobs, assign the jobs at the least time duration satisfying the constraints (5.3)-(5.5). The ties are broken as explained in Case I above generating a set of non-dominated solutions.

It is noteworthy here that some of the solutions obtained using the above cases may dominate or get dominated themselves by the solutions obtained by procedure as explained in Sections 5.3.1 and 5.3.2.

5.4 Numerical Problem

Consider the problem of multi-objective GAP with $m = 4, n = 7$ as given in Table 5.1.

The first entry in each cell represents c_{ij} and the second represents t_{ij} .

Table 5.1: Numerical BiGAP with 4 workers and 7 jobs

J → W ↓	1	2	3	4	5	6	7	Available Time(b_i)
1	9 4	7 8	8 3	12 10	9 10	12 8	15 7	15
2	10 3	5 10	8 5	11 6	15 3	16 10	20 2	12
3	4 12	3 12	10 2	9 2	12 7	14 9	12 10	20
4	8 7	7 6	8 5	10 4	10 9	14 9	10 12	14
Penalty	4	2	0	1	1	2	2	

In order to obtain the 1st non-dominated solution of the said problem, the objective functions given by Eqs. (5.1) and (5.2) are assigned first and second priorities respectively. The penalties shown here are the least cost penalties for all the jobs. Apply the procedure as explained in Section 5.3.1 to assign the jobs strictly in the order 1, 2, 7, 6, 4, 5 and 3 which corresponds to assigning the jobs in decreasing order of penalties.

The largest penalty is 4 for job 1 and the least cost occurs for the worker 3 and since he has sufficient time available with him so the job is allocated to him and the time available with him is updated to 8 units. The second largest penalty is 2 that occurs for 3 jobs - job 2, job 6 and job 7. Since job 2 has the least cost out of these three jobs, so job 2 is considered for allocation. The least cost cell for job 2 occurs for worker 3 but the job cannot

be allocated to him since he does not have the required time to perform the job. So job 2 is left unassigned as yet. Next job 7 is considered for allocation since job 7 has least cost lower than that of job 6. The least cost for job 7 occurs for worker 4 who is allocated this job since he has the required time to perform the job. Thus, the time available with worker 4 is updated to 2 units. Thereafter job 6 is considered for allocation and is allocated to worker 1 whose available time is updated to 7 units. The next largest penalty is 1 that corresponds to two jobs - job 4 and job 5. There is a tie in the least costs for job 4 and job 5 but the preference is given to job 4 since the least cost cell i.e. (3, 4) for job 4 has lower time duration than that in the cell (1, 5) for job 5. Thus, job 4 is allocated then to worker 3, who has already been allocated job 1, and his available time is updated to 6 units. Now job 5 cannot be assigned at the least cost i.e. to the worker 1 since he does not have the required time available with him. Thus, job 5 is left unassigned as well. The last job under consideration is job 3 which has the same least cost for three workers - worker 1, worker 3 and worker 4. The job 3 is allocated to worker 1 since he has the least time duration as well.

Thus, so far job 2 and job 5 have not been assigned to any worker. So disregarding the least cost cells for these two jobs, calculate the cost penalties by considering the next two least costs. It can be seen that the penalties for both the jobs is 2. The preference is given to job 2 since it has the least cost lower than that of job 5. Thus, job 2 is allocated to worker 2 and his available time is updated to 2 units. The job 5 still remains unassigned since no worker has the required time available with him to perform the job. Thus, the status of assignment after applying steps 1 - 4 of algorithm given in Section 5.3.1 is given in Table 5.2.

Table 5.2: Initial assignment of jobs

J→ W↓	1	2	3	4	5	6	7	Working Duration
1			①			①		11
2		①						10
3	①			①				14
4							①	12

Since job 5 has not been assigned to any worker, select the least cost cell for the same i.e. the cell (1,5) that corresponds to worker 1. Now change the allocation of worker 1 so as to include the cell (1,5) in his allocation. Since the total time available with the worker is 15 units and he is already working for 11 units of time, thus some other allocation of worker 1 has to be shifted to the cell (1,5). The two possible options to move are (1,3) or (1,6). The preference is given in moving the assignment from (1,6) to (1,5), since that reduces the cost of assignment, even though the working duration of worker 1 increases to 15 units. This iteration leaves job 6 unassigned. Drop the cell (1,6) for further consideration of assignments.

Repeating the step 5 of the algorithm of Section 5.3.1, now on job 6, the available least cost cell is either (3,6) or (4,6). Since the least cost and the time duration is same in both these cells, the allocation is made in (3,6) since worker 3 has greater amount of time available with him. Now the assignment of worker 3 may be changed in 2 ways, i.e. moving the assignments of cell (3,1) or (3,4) to the cell (3,6). But the move of shifting assignment from (3,4) to (3,6) is not possible since it violates the time constraint for worker 3. Shift the assignment from (3,1) to (3,6), leaving job 1 unassigned, and update the time available

with worker 3 to 9 units. Thus, the second iteration of step 5 of the algorithm leaves the cell (3, 1) unavailable for further consideration. The third and the subsequent iterations of step 5 of the algorithm are given below in Table 5.3.

Table 5.3: Iterations of step 5 on numerical BiGAP

Iteration No.	Job to be assigned	Variables x_{ij} at level 1	Job left unassigned
3	1	$x_{13}, x_{15}, x_{22}, x_{34}, x_{36}, x_{41}$	7
4	7	$x_{13}, x_{15}, x_{22}, x_{34}, x_{37}, x_{41}$	6
5	6	$x_{13}, x_{15}, x_{22}, x_{34}, x_{37}, x_{46}$	1
6	1	$x_{11}, x_{13}, x_{22}, x_{34}, x_{37}, x_{46}$	5
7	5	$x_{11}, x_{13}, x_{22}, x_{34}, x_{37}, x_{45}$	6
8	6	$x_{11}, x_{13}, x_{26}, x_{34}, x_{37}, x_{45}$	2
9	2	$x_{11}, x_{13}, x_{26}, x_{32}, x_{34}, x_{45}$	7
10	7	$x_{11}, x_{13}, x_{17}, x_{26}, x_{32}, x_{34}, x_{45}$	—

Since all the jobs have been assigned to unique workers satisfying all the constraints, thus we have obtained the 1st non-dominated solution denoted by $\bar{X}^{(1)}$ and the corresponding assignments are given in Table 5.4 and the total cost and the duration of assignment are given in Table 5.5.

Table 5.4: Job assignment of 1st non-dominated solution of numerical BiGAP

J → W ↓	1	2	3	4	5	6	7	Working Duration
1	①		①				①	14
2						①		10
3		①		①				14
4					①			9

Table 5.5: 1st non-dominated solution of numerical BiGAP

Variables at Level 1	$x_{11}, x_{13}, x_{17}, x_{26}, x_{32}, x_{34}, x_{45}$
$C(\bar{X}^{(1)})$	70 units
$T(\bar{X}^{(1)})$	14 units

To obtain the 2nd non-dominated solution $\bar{X}^{(2)}$, the cells (i, j) wherein $t_{ij} \geq 14$ are dropped

for considerations. The solution may be obtained by following two procedures as explained in Section 5.3.2. Starting from the assignment table of 1st non-dominated solution, i.e. the Table 5.4, the assignments of jobs 1 and 3 are considered to move from existing worker to some other worker satisfying the constraints (5.3)-(5.6). The move results in 2nd non-dominated solution given in Table 5.6.

Table 5.6: 2nd non-dominated solution of numerical BiGAP

Variables at Level 1	$x_{11}, x_{13}, x_{26}, x_{27}, x_{32}, x_{44}, x_{45}$
$C(\bar{X}^{(2)})$	76 units
$T(\bar{X}^{(2)})$	13 units

Continuing the same procedure, the 3rd non-dominated solution is obtained and is given below in Table 5.7.

Table 5.7: 3rd non-dominated solution of numerical BiGAP

Variables at Level 1	$x_{11}, x_{13}, x_{26}, x_{27}, x_{34}, x_{35}, x_{42}$
$C(\bar{X}^{(3)})$	81 units
$T(\bar{X}^{(3)})$	12 units

The non-dominated solutions may also be obtained by allocating afresh in Table 5.1, satisfying the constraints (5.3)-(5.6) and following the same procedure as explained in Section 5.3.1 to obtain the 1st non-dominated solution. The solution obtained is given below in Table 5.8.

Table 5.8: 4th non-dominated solution of numerical BiGAP

Variables at Level 1	$x_{11}, x_{17}, x_{26}, x_{33}, x_{35}, x_{42}, x_{44}$
$C(X^{(4)})$	79 units
$T(X^{(4)})$	11 units

It is easy to check that the 4th non-dominated solution displayed in Table 5.8 dominates

the 3rd non-dominated solution given in Table 5.7. Hence the solution of Table 5.7 is rejected and that of Table 5.8 is redesignated the 3rd non-dominated solution. The procedure terminates here since it is not possible to assign the jobs and obtain some other non-dominated solution by following the procedures explained in Sections (5.3.1) and (5.3.2). Thus, the non-dominated solutions obtained so far are given below in Table 5.9.

Table 5.9: Set of non-dominated solutions of numerical BiGAP

	Cost of assignment	Duration of assignment
1 st non-dominated solution	70	14
2 nd non-dominated solution	76	13
3 rd non-dominated solution	79	11

However, some more solutions to the problem may be obtained by applying each of the three cases as explained in Section 5.3.3. Applying Case I of the said section, we obtain only one non-dominated solution, shown in Table 5.10.

Table 5.10: Set of solutions of numerical BiGAP by applying Case I of Section 5.3.3

	Cost of assignment	Duration of assignment
1 st non-dominated solution	83	8

Applying the procedure explained in Case II of Section 5.3.3, we obtain some more solutions that may dominate or get dominated by the solutions obtained earlier. The non-dominated solutions obtained in this case are given in Table 5.11.

Table 5.11: Set of solutions of numerical BiGAP by applying Case II of Section 5.3.3

	Cost of assignment	Duration of assignment
1 st non-dominated solution	84	9
2 nd non-dominated solution	74	11
3 rd non-dominated solution	72	12
4 th non-dominated solution	71	13
5 th non-dominated solution	66	15

Applying Case III of the Section 5.3.3, we obtain only one non-dominated solution, shown in Table 5.12 which is same as the solution obtained by applying Case I.

Table 5.12: Set of solutions of numerical BiGAP by applying Case III of Section 5.3.3

	Cost of assignment	Duration of assignment
1 st non-dominated solution	83	8

Moreover, it is easy to check that the solution wherein $T(\bar{X}^{(n)}) < 8$ is not possible since $t_{i6} \geq 8 \forall i$, i.e. job 6 cannot be assigned for duration less than 8 units. Hence the final set of non-dominated solutions in decreasing value of the first objective function given by Eq. (5.1) is given below in Table 5.13.

Table 5.13: Final set of non-dominated solutions of numerical BiGAP

	Cost of assignment	Duration of assignment
1 st non-dominated solution	83	8
2 nd non-dominated solution	74	11
3 rd non-dominated solution	72	12
4 th non-dominated solution	71	13
5 th non-dominated solution	70	14
6 th non-dominated solution	66	15

5.5 Closure

The main contribution of this work is the study of the bi-objective generalized assignment problem wherein one objective function is linear and the other one is non-linear. Both the objective functions, the linear one on the cost of the assignment of jobs and the non-linear one on the duration of the completion of the jobs, are minimized. The solution procedure incorporated moving the assignments of workers to restrict the aspiration criteria

of the decision maker so as to search the smaller neighbourhood for the possible set of non-dominated solutions giving thereby greater flexibility to the decision maker.

REFERENCES

- [1] Amini, M.M., Racer, M., 1995. "A hybrid heuristic for generalized assignment problem", *European Journal of Operational Research*, Vol. 87, No. 2, pp. 343-348.
- [2] Caron, G., Hansen, P., Jaumard, B., 1999. "The assignment problem with seniority and job priority constraints", *Operations Research*, Vol. 47, No. 3, pp. 449-453.
- [3] Cattrysse, D.G., Van Wassenhove, L.N., 1992. "A survey of the algorithms for the generalized assignment problem", *European Journal of Operational Research*, Vol. 60, No. 3, pp. 260-272.
- [4] Chu, P.C., Beasley, J.E., 1997. "A genetic algorithm for the generalized assignment problem", *Computers & Operations Research*, Vol. 24, No. 1, pp. 17-23.
- [5] Dell'Amico, M., Martello, S., 1997. "The k -cardinality assignment problem", *Discrete Applied Mathematics*, Vol. 76, No. 1-3, pp. 103-121.
- [6] Diaz, J.A., Fernandez, E., 2001. "A tabu search heuristic for the generalized assignment problem", *European Journal of Operational Research*, Vol. 132, No. 1, pp. 22-38.
- [7] Evans, G.W., 1984. "An overview of techniques for solving multiobjective mathematical programs", *Management Science*, Vol. 30, pp. 1268-1282.
- [8] Gandibleux, X., Freville, A., 2000. "Tabu search based procedure for solving the 0-1 multi-objective knapsack problem: the two objective case", *Journal of Heuristics*, Vol.

-
- 6, pp. 361-383.
- [9] Haddadi, S., Ouzia, H., 2004. "Effective algorithm and heuristic for the generalized assignment problem", *European Journal of Operational Research*, Vol. 153, No. 1, pp. 184-190.
- [10] Ignizio, J.P., 1982. "Linear programming in Single- and Multi-objective Systems", Prentice-Hall, Englewood Cliffs, New Jersey, pp. 374-376, 475-483.
- [11] Junker, U., 2004. "Preference based search and multi-criteria optimization", *Annals of Operations Research*, Vol. 130, pp. 75-115.
- [12] Kasana, H.S., Kumar, K.D., 2004. "Introductory Operations Research. Theory and Applications", Springer-Verlag, Berlin.
- [13] Kuhn, H.W., 1955. "The hungarian method for the assignment problem", *Naval Research Logistics Quarterly*, Vol. 2, No. 1-2, pp. 83-97.
- [14] Lourenço, H.R., Paixao, J.P., Portugal, R., 2001. "Multiobjective metaheuristics for the bus driver scheduling problem", *Transport Science*, Vol. 35, pp. 331-343.
- [15] Lourenço, H.R., Serra, D., 2002. "Adaptive approach heuristics for the generalized assignment problem", *Mathware & Soft Computing*, Vol. 9, pp. 209-234.
- [16] Prakash, S., Balaji, B.V., Tuteja, D., 1999. "Optimizing dead mileage in urban bus routes through a non-dominated solution approach", *European Journal of Operational Research*, Vol. 114, pp. 465-473.

-
- [17] Prakash, S., Kumar, P., Prasad, B.V.N.S., Gupta, G., 2008. "Pareto optimal solutions of a cost-time trade-off bulk transportation problem", *European Journal of Operational Research*, Vol. 188, pp. 85-100.
- [18] Prakash, S., Madhusudan, A., Kunal, 2007. "Clustering offices of a company to internet service providers", *Proceedings of the National Academy of Sciences, India*, Vol. 77(A), No. 1, pp. 31-35.
- [19] Punnen, A.P., Aneja, Y.P., 1995. "A tabu search algorithm for resource constrained assignment problem", *Journal of Operational Research Society*, Vol. 46, No. 2, pp. 214-220.
- [20] Steuer, R.E., 1986. "Multiple Criteria Optimization: Theory, Computation, and Application", Wiley, New York, pp. 138-159.
- [21] Votaw, D.F., Orden, A., 1952. "The personnel assignment problem", *Symposium on Linear Inequalities and Programming, SCOOP 10, US Air Force*, pp. 155-163.
- [22] Yadavalli, V.S.S., Venkataramana, V.V., Umasankar, C., 2001. "Constrained assignment problem: Lexi search approach", *International Journal of Management and Systems*, Vol. 17, No. 2, pp. 197-204.
- [23] Zhang, C.W., Ong, H.L., 2007. "An efficient solution to biobjective generalized assignment problem", *Advances in Engineering Software*, Vol. 38, pp. 50-58.

Chapter 6

MULTI-OBJECTIVE FIRE STATION LOCATION PROBLEM

The problem of selecting a fixed number of sites, from amongst a given number of potential fire station sites for clustering the areas to them subject to several constraints and two objectives is considered. One of the constraint is that each area is should be allocated to a unique fire station site selected for locating a fire station at it; and the capacity of each fire station site, i.e. the maximum number of areas that can be allocated to a fire station site, is also fixed. The two objective functions are to minimize the total operating cost and the duration to cover the areas from the selected fire station sites. A heuristic algorithm is proposed to find the set of efficient solutions of this problem.

6.1 Preview

Facility location, also known as location analysis, is a branch of Operations Research concerning with mathematical modelling an solution of problems of facilities so as to minimize transportation costs, avoid placing hazardous material near residential areas, outperform competitor's facilities, etc.

A number of case studies describing the application of facility location models to the strategic design of real-life have been discussed in past, showing the growing awareness and importance that practitioners are devoting to this area. The rapid evolution of computer and communications technology has made possible the optimization of facility location in real-world production-distribution systems. In some cases, however the problem size and complexity along with the management's wish to obtain "good" solutions in reasonable time have driven researchers to develop heuristic procedures. This chapter concerns with real-life application of the location of emergency facilities or services that are related to the public sector.

Demain, Fomin, Hajiaghayi and Thilikos (2005) have developed an algorithm for (k, r) -center in planar graphs where an input graph G has $\leq k$ vertices (called *centers*) such that every vertex of G is within distance $\leq r$ from some center and have conjectured that the same can be applied to the installation of the emergency services such as fire stations where the fire stations can be located at k locations and every building is to be covered within r city blocks. Melo, Nickel and Saldanha-da-Gama (2009) have presented a literature review of the facility location models in the context of supply chain management.

Emergency services include police patrol, ambulances, fire protection, towing and emergency repair of gas, electricity and water. Fire Service is conceived of as an organized public service having the primary objective of preventing fires from occurring and reducing the loss of life and property due to fires. Fire protection services and fire service related activities are conducted in relation to both the characteristics of the spatial environment as well as to the socio-economic and demographic characteristics of the population. Individually or in combination, spatial, social and economic factors, contribute to the incidence of fire and to the response capability of a fire protection service. These factors, to a large extent, determine the nature of fire hazards and influence the location of fire stations and the effective delivery of services.

Spatial considerations are primary to planning fire services, in order to determine the distribution of fire stations and their specific locations given social and economic needs and the existing communication and transportation networks. Non-spatial aspects, ranging from the nature of the fire to that of the fire service itself, its resource inputs, organizational set-up, and resource allocations and deployment policies for the fire service delivery, are also involved. While most features of the spatial environment, are relatively static, e.g. the urban layout, non-spatial aspects are continuously changing. Furthermore, the spatial and non-spatial environment which presents significant constraints on, and potential solutions for, effective and efficient urban fire protection.

Hogg (1968) have developed a technique which minimizes the total number of fire appliance journey times to fires for any given number of fire stations, thereby generating a set of solutions giving the best combination of r station sites from a set of n alternative sites,

where r varies from $n - 1$ to 1. Kolesar and Blum (1973) have developed an inverse square root function for the relation between average response distance and the number of locations at which response units are stationed in a region. The square-root response distance model is combined with response distance-response time relations to find the optimal resource allocations given the resource constraints and response time standards.

Plane and Hendrick (1977) had setup a hierarchical objective function for the set-covering problem which simultaneously minimizes the number of fire stations and the maximization of the number of fire stations within the minimum total number of stations, thereby reducing the annual costs for the fire companies in Denver. Brueckner (1981) has shown that the fire protection exhibits substantial publicness which suggests that increasing the community's population should not greatly reduce the level of fire protection, holding the suppression capacity fixed, thereby providing the fire services at a lower per capita cost in a larger community.

Sanli and Al-Tamimi (1990) have reviewed the principles affecting the performance of the service systems with regard to spatial relationships and communication patterns within cities and in particular the principles related to location and allocation of fire services. Badri, Mortagy and Alsayed (1998) have presented a multiple criteria modeling approach, through integer goal programming, to the fire-station location problem that involves conflicting objectives incorporating both the travel times and travel distances from stations to demand sites.

Liu, Huang and Chandramouli (2006) present an approach to suitably situating new fire stations, considering multiple objectives of maximizing the coverage of the routes, achieving

a reasonable distance between fire stations and maximizing the areas that can be served by fire stations within 6 minutes, using GIS and ANT algorithm. Guo and Fu (2007) have reviewed how the fire situation in China has become relatively stable in recent years and have also discussed the ongoing research in the areas of theory of fire dynamics, evaluation techniques for fire risk and fire and rescue services of urban areas, modernization of the fire department apparatus and equipment, human behaviour in fire among others.

Beraldi and Bruni (2009) have formulated and solved a probabilistic model for determining the optimal location of facilities in congested emergency systems. Iannoni, Morabito and Saydam (2009) have developed a method that can be used to make decisions regarding the optimal location of the ambulances on the highways and the coverage areas of the ambulances in order to minimize mean user response time or remedy an imbalance in ambulance workloads within the system.

In this chapter, we formulate the problem with the objectives to minimize the cost and the duration of covering the areas from the locations identified for setting up the fire stations at the least setup cost. Each potential location has a capacity to cover upto a fixed number of areas. The formulation of the multi-objective fire location problem is given hereunder.

6.2 Formulation of the Problem

Suppose there are M areas, N potential fire station locations, K is the number of number of sites where the fire stations are to be located. The M areas are to be clustered to K sites in such a way that each area is assigned to a unique site which is selected for the

task. Each selected potential location may cater upto a maximum of L areas. Let c_{ij} and t_{ij} ($i = 1, 2, \dots, M; j = 1, 2, \dots, N$) units be the cost and time respectively of covering the area i from location j . Let C_j be the setup cost of a fire station at the site j . Let x_{ij} be the decision variable assuming value 0 or 1 according as area i is not clustered or clustered to site j , and y_j be the variable assuming value 0 or 1 according as the potential site j is not selected or selected for locating a fire station at it. Let C and T denote the total cost and the total duration respectively of meeting requirements of all the areas from their assigned fire stations. The mathematical formulation of this problem is as follows. The two objective functions which are sought to be minimized are

$$C = \sum_{i=1}^M \sum_{j=1}^N c_{ij} x_{ij} \quad (6.1)$$

$$T = \max\{t_{ij} x_{ij} : i = 1, 2, \dots, M; j = 1, 2, \dots, N\} \quad (6.2)$$

Constraints of the problem are

$$\sum_{j=1}^N y_j = K \quad (6.3)$$

$$\sum_{i=1}^M x_{ij} y_j \leq L \quad (j = 1, 2, \dots, N) \quad (6.4)$$

$$\sum_{j=1}^N x_{ij} = 1 \quad (i = 1, 2, \dots, M) \quad (6.5)$$

$$x_{ij} - y_j \leq 0 \quad (i = 1, 2, \dots, M; j = 1, 2, \dots, N) \quad (6.6)$$

$$x_{ij}, y_j = 0 \text{ or } 1 \quad (i = 1, 2, \dots, M; j = 1, 2, \dots, N) \quad (6.7)$$

Note that the objective functions given by Eqs. (6.1) and (6.2) are not accorded any priorities. The constraint (6.3) ensures that K sites are selected for locating fire stations at them, whereas constraint (6.4) ensures that the capacity of each of the selected potential locations is at the most L areas. The constraints (6.5) and (6.6) ensure that each area is assigned to a unique site which is selected for locating a fire station.

It is required to obtain a set of efficient solutions of the problem given by (6.1)-(6.7). For the purpose of listing the efficient solutions, a solution $(\bar{X}^{(1)}, \bar{Y}^{(1)})$ shall be called the 1st efficient solution if it is the optimal solution of the problem with the minimization of C and T as the first and second prioritized objectives respectively. A solution $(\bar{X}^{(2)}, \bar{Y}^{(2)})$ shall be called the 2nd efficient solution if no efficient solution (\bar{X}, \bar{Y}) of the problem exists satisfying the conditions: (i) $C(\bar{X}^{(1)}) < C(\bar{X}) < C(\bar{X}^{(2)})$ and (ii) $T(\bar{X}^{(1)}) > T(\bar{X}) > T(\bar{X}^{(2)})$. The 3rd and subsequent efficient solutions are defined in the same way as is done for the 2nd efficient solution. A discussion about efficient solutions can be found in the works of Ignizio (1982), Steuer (1986), Prakash, Balaji and Tuteja (1999), Prakash, Madhusudan and Kunal (2007), Prakash, Kumar, Prasad and Gupta (2008).

6.3 Solution Procedure

The problem formulated above is a binary integer nonlinear problem because the variables x_{ij} s and y_j s are binary integers and the objective function T given by 6.1 is nonlinear. The set of efficient solutions is obtained through an iterative heuristic algorithm. The prioritized bi-criterion problems are solved as such i.e. without being linearized. Procedures to obtain

1^{st} , 2^{nd} and subsequent efficient solutions are explained below.

6.3.1 Procedure to Obtain 1^{st} Efficient Solution

Note that the 1^{st} efficient solution $(\bar{X}^{(1)}, \bar{Y}^{(1)})$ of the problem formulated above given by Eqns. (6.1)-(6.7) is the optimal solution of the problem wherein the total cost C and the duration T of covering the areas from their assigned fire stations, are minimized with the first and second priorities respectively, subject to the constraints (6.3)-(6.7). The problem yielding the 1^{st} efficient solution is designated as the 1^{st} prioritized bi-criterion problem. The procedure to obtain the 1^{st} efficient solution is given below.

Step I. Identify all the ${}^N C_K$ combinations of the potential locations.

Step II. Rank the combinations in accordance with their total setup cost. The least setup cost given the first rank.

Step III. Starting from the first ranked combination of the potential fire station locations, calculate the cost penalty for each of the selected potential location. The cost penalty is the minimum penalty that might be incurred in case the potential location is not associated with an area which corresponds to the least cost for the said location, i.e. the cost penalty CP_j for the location j is the positive difference between the least and second least cost associated with j^{th} location, i.e. $CP_j = c_{2j} - c_{1j}$, where c_{1j} and c_{2j} are the least and second least costs of the j^{th} destination. If the least cost occurs more than once in a column j , then the corresponding penalty $CP_j = 0$.

Step IV. Select the j^{th} destination with largest cost penalty CP_j . Make an allocation

at the least cost cell of the selected column j . In case of tie of the largest penalty between columns, select the column which has the least cost lower than the least of the other. In case of tie on the least cost count as well, select the column which has least amount of time associated with the least cost. In case both the least cost and the associated time are same for the tied columns, then it is an arbitrary choice for the decision maker. After allocating the area, drop it from further considerations.

Step V. Repeat Step III till $M - 1$ areas have been allocated to the selected potential locations keeping in view their capacities. The last area is to be allocated following the least cost rule and not on the basis of cost penalty. The area will be allocated to the least cost cell of the available locations, i.e. the potential location with available capacity. When all the areas have been allocated to unique selected potential fire station location, the first efficient solution $(\bar{X}^{(1)}, \bar{Y}^{(1)})$ has been found.

6.3.2 Procedure to Obtain 2^{nd} and Subsequent Efficient Solutions

The 2^{nd} efficient solution $(\bar{X}^{(2)}, \bar{Y}^{(2)})$ of the formulated problem is obtained by solving the problem obtained from the 1^{st} prioritized bicriterion problem after dropping all those cells (i, j) corresponding to $t_{ij} \geq T(\bar{X}^{(1)})$. The problem, thus obtained, is similar to the 1^{st} prioritized bicriterion problem and is designated as the 2^{nd} prioritized bicriterion problem. The 2^{nd} prioritized bicriterion is solved by adopting the method for solving the 1^{st} prioritized bicriterion problem. The 3^{rd} and the subsequent efficient solutions are obtained in the same way as done to obtain the 2^{nd} efficient solution. This process of obtaining the efficient solu-

tions is terminated after encountering a prioritized bicriterion problem where it is impossible to allocate atleast one area to one of the selected potential fire station location because of the unavailability of the least cost cells satisfying the time constraint.

The above mentioned procedures to obtain the efficient solutions should be implemented for all the ${}^N C_K$ combinations of the potential fire station locations to obtain different sets of efficient solutions for all the combinations. Thereafter, all the solutions should be compared to generate a final set of efficient solutions out of which the decision maker may choose the one depending on his/her priority of the decision parameters.

6.4 Numerical Problem

Table 6.1: Tableau representation of the problem

Areas	Fire Station Potential Sites				
	1	2	3	4	5
1	40	70	80	60	90
	2	6	8	11	10
2	30	80	20	10	100
	8	3	9	6	13
3	50	130	200	70	20
	11	9	3	13	8
4	120	130	70	80	60
	6	7	8	4	12
5	180	170	140	30	40
	10	8	11	9	2
6	170	140	130	160	50
	9	10	12	8	6
7	150	20	60	210	220
	7	11	10	4	14
Setup Cost (C_j)	1,00,000	8,00,000	7,00,000	3,00,000	4,00,000

Now we illustrate the procedure explained in Section 6.3 by applying it to obtain the set of efficient solutions of the numerical problem presented in Table 6.1 considering $M = 7$, $N = 5$, $K = L = 3$ and assigning numerical values to all the c_{ijs} , t_{ijs} , C_{js} in the problem formulated in Section 6.2. In Table 6.1, rows 1-7 correspond to areas and columns 1-5 correspond to potential fire station locations. Upper and lower entries of a cell (i, j) depict the units of costs c_{ij} and time t_{ij} respectively of covering the area i from the fire station location j .

Step I. Since three fire stations are to be located, thus, the combinations of the locations will be ${}^5C_3 = 10$ in number. We start the procedure by considering the combination that has the least setup cost. In the numerical problem being considered here, the combination of 3 potential fire station locations with the least setup cost is 1, 4 and 5 with a total setup cost as 8,00,000 units.

Step II. Calculate the cost penalties of the selected sites, i.e. the positive difference of the least and the second least costs of the selected locations. The penalties are 10, 20 and 20 respectively for the locations 1, 4 and 5. Since there is a tie in the largest penalty for locations 4 and 5, location 4 is given the first priority since the least cost for location 4 is lower than that for the location 5. Thus, the first allocation is made in the cell $(2, 4)$ i.e. area 2 is clustered to location 4.

Step III. Recalculate the cost penalties for the selected potential locations. The new penalties are 10, 30 and 20 for the locations 1, 4 and 5 respectively. Since the largest penalty again corresponds to the location 4, thus the allocation is made in the next least cost cell for location 4, i.e. in the cell $(5, 4)$. Hence area 5 is allocated to the location 4.

Step IV. Repeating the procedure of calculating the penalties and allocating the areas at the least cost cells corresponding to the largest penalties we obtain the following allocations. Note that the area 7 is allocated to the location 1 since the cost incurred to cover area 7 is

Table 6.2: Allocations of the areas to the locations 1,4 and 5

Area	Potential Location
1	1
2	4
3	5
4	4
5	4
6	5
7	1

least from location 1 amongst the selected potential sites. Thus, the first efficient solution has been obtained wherein the cost and the time to cover all the areas from the selected potential locations is given in the Table 6.3.

Table 6.3: 1st Efficient Solution

Cost to cover all the areas	380 units
Time to cover all the areas	9 units
Total Setup Cost	8,00,000 units

Step V. The second efficient solution of the problem for the same combination of the selected potential locations i.e. 1, 4 and 5 is obtained by dropping the cells (i, j) wherein the time $t_{ij} \geq T(\bar{X}^{(1)}) = 9$ units. The updated table of the problem is given in Table 6.4.

Step VI. In Table 6.4, it can be observed that there is only one choice for allocating the areas 1, 3 and 5; since in each of these three cases, two potential locations have been

Table 6.4: Tableau representation of the problem

Areas	Fire Station Potential Sites				
	1	2	3	4	5
1	40	70	80	-	-
	2	6	8	-	-
2	30	80	20	10	-
	8	3	9	6	-
3	-	130	200	-	20
	-	9	3	-	8
4	120	130	70	80	-
	6	7	8	4	-
5	-	170	140	-	40
	-	8	11	-	2
6	-	140	130	160	50
	-	10	12	8	6
7	150	20	60	210	-
	7	11	10	4	-
Setup Cost (C_j)	1,00,000	8,00,000	7,00,000	3,00,000	4,00,000

dropped because of the time constraint introduced in Step V above. Thus, areas 1, 3 and 5 are allocated to location 1, 5 and 5 respectively. For allocating the other areas to the fire stations, the same procedure of calculating the cost penalties and then allocating at the least cost cells is followed. The initial cost penalties now for locations 1, 4 and 5 are 10, 70 and 20 respectively. Since the largest penalty corresponds to location 4, thus an allocation is made in the cell (2, 4), i.e. the area 2 is allocated to location 4. Continuing the procedure as already explained, the allocations made are given in Table 6.5.

Thus, the second efficient solution has been obtained wherein the cost and the time to cover all the areas from the selected potential locations is given in the Table 6.6.

Step VII. Continuing the procedure of Steps V and VI to obtain the 3rd efficient solution, we drop the cells (i, j) wherein $t_{ij} \geq T(\bar{X}^{(2)}) = 8$ units. It can be observed that a solution

Table 6.5: Allocations of the areas to the locations 1,4 and 5 and $t_{ij} < 9$

Area	Potential Location
1	1
2	4
3	5
4	4
5	5
6	5
7	1

Table 6.6: 2nd Efficient Solution

Cost to cover all the areas	390 units
Time to cover all the areas	8 units
Total Setup Cost	8,00,000 units

to the numerical problem with the selected location sites 1, 4 and 5 with $t_{ij} < 8$ units is not possible since the area 3 cannot be allocated to any of the locations. Thus, considering the combination 1, 4 and 5 of potential fire station sites, it is not possible to find the 3rd efficient solution. Thus the two efficient solutions obtained using the combination 1, 4 and 5 of the potential fire station locations is given in Table 6.7.

Table 6.7: Efficient Solutions obtained with Locations 1, 4 and 5

	Cost	Duration	Setup Cost
1 st Efficient Solution	380	9	8,00,000
2 nd Efficient Solution	390	8	8,00,000

Step VIII. Since there are total of ${}^N C_K$ combinations, in this case 10 in number, the remaining are considered on the basis of the least setup cost of the the selected sites. Thereafter, the same procedure as applied above in Step I - VI is followed to generate a set of efficient solutions of the problem.

For instance, the next combination of the potential fire station locations considered is locations 1, 3 and 4, and the efficient solutions obtained, following the procedure of Steps I - VI, are given in Table 6.8 .

Table 6.8: Efficient Solutions obtained with Locations 1, 3 and 4

	Cost	Duration	Setup Cost
1st Efficient Solution	410	12	11,00,000
2nd Efficient Solution	470	11	11,00,000
3rd Efficient Solution	590	10	11,00,000
4th Efficient Solution	670	9	11,00,000

Similarly, the remaining combinations of the potential fire station sites are considered and after applying the same procedure, the possible efficient solutions of each of the combinations are given in Tables 6.9-6.16.

Table 6.9: Efficient Solutions obtained with Locations 1, 3 and 5

	Cost	Duration	Setup Cost
1st Efficient Solution	350	10	12,00,000
2nd Efficient Solution	390	9	12,00,000
3rd Efficient Solution	400	8	12,00,000

Table 6.10: Efficient Solutions obtained with Locations 1, 2 and 4

	Cost	Duration	Setup Cost
1st Efficient Solution	400	11	12,00,000
2nd Efficient Solution	580	10	12,00,000
3rd Efficient Solution	610	9	12,00,000

Table 6.11: Efficient Solutions obtained with Locations 1, 2 and 5

	Cost	Duration	Setup Cost
1st Efficient Solution	410	11	13,00,000
2nd Efficient Solution	580	8	13,00,000

Table 6.12: Efficient Solutions obtained with Locations 3, 4 and 5

	Cost	Duration	Setup Cost
1st Efficient Solution	320	11	14,00,000
2nd Efficient Solution	460	10	14,00,000
3rd Efficient Solution	480	8	14,00,000

Table 6.13: Efficient Solutions obtained with Locations 2, 4 and 5

	Cost	Duration	Setup Cost
1st Efficient Solution	280	11	15,00,000
2nd Efficient Solution	480	8	15,00,000

Table 6.14: Efficient Solutions obtained with Locations 1, 2 and 3

	Cost	Duration	Setup Cost
1st Efficient Solution	510	11	16,00,000
2nd Efficient Solution	720	10	16,00,000
3rd Efficient Solution	750	9	16,00,000

Table 6.15: Efficient Solutions obtained with Locations 2, 3 and 4

	Cost	Duration	Setup Cost
1st Efficient Solution	410	13	18,00,000
2nd Efficient Solution	470	12	18,00,000
3rd Efficient Solution	500	11	18,00,000
4th Efficient Solution	530	10	18,00,000
5th Efficient Solution	690	9	18,00,000
6th Efficient Solution	890	8	18,00,000

Table 6.16: Efficient Solutions obtained with Locations 2, 3 and 5

	Cost	Duration	Setup Cost
1st Efficient Solution	330	10	19,00,000

Comparing all the efficient solutions shown in Tables 6.7-6.16 for all the combinations of the potential fire station locations, the final set of efficient solutions, in the increasing order of cost of operation to cover the areas from the selected sites, is given in Table 6.17.

Table 6.17: Set of Efficient Solutions of the Numerical Problem

	Cost	Duration	Combination of Locations	Setup Cost
1st Efficient Solution	280	11	2,4,5	15,00,000
2nd Efficient Solution	330	10	2,3,5	19,00,000
3rd Efficient Solution	380	9	1,4,5	8,00,000
4th Efficient Solution	390	8	1,3,4	11,00,000

6.5 Closure

In this chapter, the general and complex real-life problem of locating a fixed number of fire stations at potential location sites, and allocating the areas to them, has been modelled as a biobjective problem. A heuristic procedure based on the cost penalties and the adaptive search techniques has been developed to solve the problem. Application of the heuristic procedure yields four solutions as shown in Table 6.17. The choice of the final solution depends on the priorities of the decision maker which, in this case, may be the civic administration and the priorities of the civic administrations may differ depending upon their respective budgets. The model considered here is dynamic. It can be employed on the problems of locating the ambulances in the cities and on the highways, police patrol vehicles and other emergency services that are related to the public sector.

REFERENCES

- [1] Badri, M.A., Mortagy, A.K., Alsayed, A., 1998. "A multi-objective model for locating fire stations", *European Journal of Operational Research*, Vol. 110, pp. 243-260.
- [2] Beraldi, P., Bruni, M.E., 2009. "A probabilistic model applied to emergency service vehicle location", *European Journal of Operational Research*, Vol. 196, pp. 323-331.
- [3] Brueckner, J.K., 1981. "Congested Public Goods: The Case of Fire Protection", *Journal of Public Economic*, Vol. 15, pp. 45-58.
- [4] Demaine, E.D., Fomin, F., Hajiaghayi, M., Thilikos, D., 2005. "Fixed-Parameter Algorithms for (k, r) -Center in Planar Graphs and Map Graphs", *Transactions on Algorithms*, Vol. 1, pp. 1-16.
- [5] Guo, T., Fu, Z., 2007. "The fire situation and progress in fire safety science and technology in China", *Fire Safety Journal*, Vol. 42, pp. 171-182.
- [6] Hogg, J.M., 1968. "The Siting of Fire Stations", *Operations Research*, Vol. 19, pp. 275-287.
- [7] Iannoni, A.P., Morabito, R., Saydam, C., 2009. "An optimization approach for ambulance location and the districting of the response segments on highways", *European Journal of Operational Research*, Vol. 195, pp. 528-542.
- [8] Ignizio, J.P., 1982. "Linear programming in Single- and Multi-objective Systems", Prentice-Hall, Englewood Cliffs, New Jersey, pp. 374-376, 475-483.

-
- [9] Kolesar, P., Blum, E.H., 1973. "Square Root Laws for Fire Engine Response Distances", *Management Science*, Vol. 19, pp. 1368-1378.
- [10] Liu, N., Huang, B., Chandramouli, M., 2006. "Optimal Siting of Fire Stations Using GIS and ANT algorithm", *Journal of Computing in Civil Engineering*, Vol. 20, pp. 361-369.
- [11] Melo, M.T., Nickel, S., Saldanha-da-Gama, F., 2009. "Facility location and supply chain management - A review", *European Journal of Operational Research*, Vol. 196, 401-412.
- [12] Plane, D.R., Hendrick, T.E., 1977. "Mathematical Programming and the Location of Fire Companies for the Denver Fire Department", *Operations Research*, Vol. 25, pp. 563-578.
- [13] Prakash, S., Balaji, B.V., Tuteja, D. 1999. "Optimizing dead mileage in urban bus routes through a nondominated solution approach", *European Journal of Operational Research*, Vol. 114, pp. 465-473.
- [14] Prakash, S, Kumar, P., Prasad, B.V.N.S., Gupta, G., 2008. "Pareto optimal solutions of a cost-time trade-off bulk transportation problem", *European Journal of Operational Research*, Vol. 188, pp. 85-100.
- [15] Prakash, S., Madhusudan, A., Kunal, 2007. "Clustering offices of a company to internet service providers", *Proceedings of the National Academy of Sciences, India, Section A*, Vol. 77, No. 1, pp. 31-35.

- [16] Sanli, I., Al-Tamimi, F., 1990. "The Spatial Distribution and Resource Allocation of Fire Safety Service System", *Journal of King Saud University, Architecture and Planning*, Vol. 2, pp. 23-41.
- [17] Steuer, R.E., 1986. "Multiple Criteria Optimization: Theory, Computation, and Application", Wiley, New York, pp. 138-159.

Chapter 7

CONCLUSIONS

In this Chapter, utility and future possible extensions of the work are indicated.

The real-life models considered in this work can be useful in different ways to a variety of people in doing their work optimally through saving money and time. People in industry, researchers and people from other walks of life can be benefited through the study of this work because it would open new ways of tackling problems encountered by them.

The models considered here are versatile. Large size problems can be tackled after computerization. The models can be adapted to new situations by making changes in the constraints and the objective functions. For instance, the warehouse problem can be adapted to a new situation if the terms warehouses and ration shops are stretched to include any supply and demand points. In a similar way, the Chinese postman problem can be adapted to reduce traveling expenses and time of salesman and courier services. The problems considered in Chapter 3, 5 and 6 of this work can also be adapted to new situations.

Future extensions of the present work are possible by making changes in the objective

function, constraints and then developing algorithms for solving new problems thus generated. There are many real-life multiobjective optimization problems whose solutions are not known at the present juncture. Further, there are many real-life optimization problems which are not even formulated and many problems even unknown at this point of time. This opens never ending vistas for researchers.

Appendix A

PUBLICATIONS OUT OF THE PRESENT WORK

List of Papers Published

1. Prakash, S., Sharma, M.K., Singh, A., 2009. "Selection of warehouse sites for clustering ration shops to them with two objectives through a heuristic algorithm incorporating tabu search", OPSEARCH, Vol. 46, No. 4, pp. 449-460.
2. Prakash, S., Sharma, M.K., Singh, A., 2009. "An efficient heuristic for multi-objective bulk transportation problem", Proceedings of the IEEE 39th International Conference on Computers & Industrial Engineering, DOI: 10.1109/ICCIE.2009.5223526, pp. 1005-1009.
<http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=5223526>.
3. Prakash, S., Sharma, M.K., Singh, A., 2009. "A heuristic for multi-objective chinese

postman problem”, Proceedings of the IEEE 39th International Conference on Computers & Industrial Engineering, DOI: 10.1109/ICCIE.2009.5223529, pp. 596-599.

<http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=5223529>.

List of Papers Communicated

1. Prakash, S., Sharma, M.K., Singh, A., “Non-dominated Solutions for Cost-Time Trade-Off Bulk Transportation Problem”, communicated to Yugoslav Journal of Operations Research.
2. Prakash, S., Sharma, M.K., Singh, A., “Pareto optimal solutions for Multi-Objective Generalized Assignment Problem”, communicated to South African Journal of Industrial Engineering.