

A New Method for Voltage Stability Analysis of Radial Distribution Networks

Thesis submitted in partial fulfillment of the requirements for the award of degree of

**Master of Engineering
in
Power Systems & Electric Drives**



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CERTIFICATE

I hereby certify that the work which is being presented in the thesis entitled, "A New Method for Voltage Stability Analysis for Radial Distribution Networks", in partial fulfillment of the requirements for the award of degree of Master of Engineering in *Power Systems & Electric Drives* submitted in Electrical & Instrumentation Engineering Department of Thapar University, Patiala, is an authentic record of my own work carried out under the supervision of *Dr. Smarajit Ghosh, Prof. & Head, EIED*.

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

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Arminster Singh

Dedicated to My Parents

ABSTRACT

In this Thesis work, an attempt has been made for voltage stability analysis of electric power distribution system. A new formula for computing the voltage stability index (VSI) of all nodes of any distribution network is proposed in this thesis work. The node having the minimum value of voltage stability index is the most sensitive node. Using three different examples, 29-node, 33-node and 69-node radial distribution network, it has been shown that the node having the minimum value of voltage magnitude becomes the most sensitive node using the proposed method which is also the end node. The critical values of total real power load (TPL) and total reactive power load (TQL) for constant power, constant current, constant impedance, composite as well as exponential load modelling is derived out for the sub-station voltage of 1.0 pu, 1.025 pu and 1.050 pu respectively for 69-node radial distribution network. The system will actually collapse beyond the computed values of TPL and TQL because load-flow based methods have a tendency to collapse before the exact critical values of TPL and TQL. The superiority of the proposed method has been established by comparing with other methods available in literature.

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LIST OF SYMBOLS

VSI	:	Voltage Stability Index
NB	:	Total no. of Nodes
LN1	:	Total no. of Branches
jj	:	Branch no. i.e., $jj = 1,2,3,\dots, LN1$
m1	:	IS(jj) be the Sending–end Node of Branch-jj
m2	:	IR(jj) be the Receiving –end Node of Branch-jj
ISS(jj)	:	IS(jj) for all jj
IRR(jj)	:	IR(jj) for all jj
V(m1)	:	Voltage of Sending – end Node of Branch – jj
V(m2)	:	Voltage of Receivng – end Node of Branch – jj
R(jj)	:	Resistance of Branch – jj
X(jj)	:	Reactance of Branch – jj
Z(jj)	:	Impedance of Branch – jj
I(jj)	:	Current through the Branch – jj
I _r (jj)	:	Real Component of I(jj)
I _m (jj)	:	Imaginary Component of I(jj)
PL(m2)	:	Active Power Load at Node m2
QL(m2)	:	Reactive Power Load at Node m2
IL(m2)	:	Load Current at Node m2
LP(jj)	:	Real Power Loss of Branch – jj
LQ(jj)	:	Reactive Power Loss of Branch – jj
DVMAX	:	Maximum Voltage Difference
VV(m2)	:	Magnitude of Voltage Difference
TPL	:	Total Real power Load
TQL	:	Total Reactive Power Load

Modern power grids are extremely complex and widespread. Surges in power lines can cause massive network failures and permanent damage to multimillion-dollar equipment in power generation plants. After electricity is generated at power plants it has to get to the customers that use the electricity. The transmission and distribution system delivers electricity from the generating site (electric power plant) to residential, commercial, and industrial facilities.

The electricity first goes to a transformer at the power plant that boosts the voltage up to 400 kVA for transmission through extra-high voltage (EHV) transmission lines. When electricity travels long distances, it is better to have it at higher voltages since the electricity can be transferred more efficiently at high voltages. High voltage transmission lines carry electricity long distances to a substation. At transmission substations a reduction in voltage occurs for distribution to other points in the system through high voltage (HV) transmission lines. Further voltage reductions for commercial and residential customers take place at distribution substations, which connect to the primary distribution network.

Transformers are a crucial link in the electric power distribution system. Utility transformers are high-voltage distribution transformers typically used by utilities to step down the voltage of electricity going into their customers' buildings. Distribution transformers are one of the most widely used elements in the electric distribution system. They convert electricity from the high voltage levels in utility transmission systems to voltages that can safely be used in businesses and homes. Distribution transformers are either mounted on an overhead pole or on a concrete pad. Most commercial and industrial buildings require several low-voltage transformers to decrease the voltage of electricity received from the utility to the levels used to power lights, computers, and other electric-operated equipment.

Transformers consist of two primary components: a core made of magnetically permeable material; and a conductor, or winding, typically made of a low resistance material such as copper or aluminum. The conductors are wound around a magnetic core to transform current from one voltage to another. Liquid insulation material or air surrounds the transformer core and conductors to cool and electrically insulate the

transformer. Many different distribution transformer designs are available to utilities, depending on the loading patterns and needs of the end-user. Transformer engineers modify transformer design and vary material depending upon the needs of a particular utility (cost of energy, capacity, etc.). Distribution system is introduced at first followed by black out, voltage stability and voltage collapse.

1.1 Distribution System

The part of power system which distributes electric power for local use is known as distribution system.

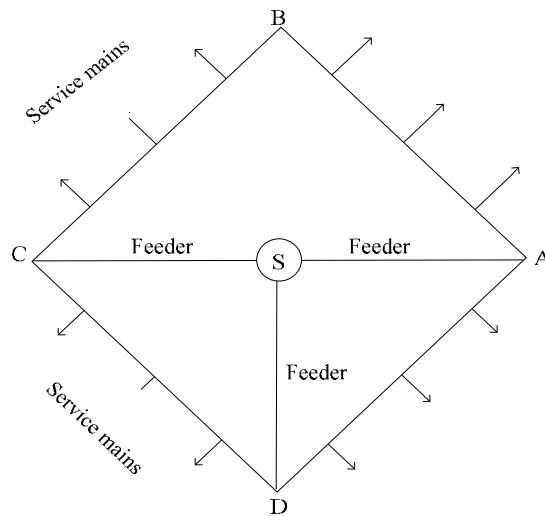


Figure 1.1 Single Line Diagram of a Typical Low Tension Distribution System

In general, the distribution system is the electrical system between the sub-station fed by the transmission system and the consumers' meters. It generally consists of feeders, distributors and the service mains. Figure 1.1 shows the single line diagram of a typical low tension distribution system.

- (i) **Feeders:** A feeder is a conductor, which connects the sub-station (or localized generating station) to the area where power is to be distributed. Generally, no tappings are taken from the feeder so that the current in it remains the same throughout. The main consideration in the design of a feeder is the current carrying capacity.
- (ii) **Distributor:** A distributor is a conductor from which tappings are taken for supply to the consumers. In Figure 1.1, AB, BC, CD, and DA

are the distributors. The current through a distributor is not constant because tapping are taken at various places along its length. While designing a distributor, voltage drop along its length is the main consideration since the statutory limit of voltage variations is $\pm 10\%$ of rated value at the consumer's terminals.

- (iii) **Service mains:** A service mains is generally a small cable which connects the distributor to the consumer's terminals.

1.2 Classification of Distribution Systems

A distribution system may be classified according to:

- (i) **Nature of current:** According to nature of current, distribution system may be classified as (a) d.c. distribution system and (b) a.c. distribution system. Now-a-days a.c. system is universally adopted for distribution of electric power as it is simpler and more economical than direct current method.
- (ii) **Type of construction:** According to type of construction, distribution system may be classified as (a) overhead system and (b) underground system. The overhead system is generally employed for distribution as it is 5 to 10 times cheaper than the equivalent underground system. In general, the underground system is used at places where overhead construction is impracticable or prohibited by the local laws.
- (iii) **Scheme of connection:** According to scheme of connection, the distribution system may be classified as (a) radial system, (b) ring main system and (c) inter-connected system. Each scheme has its own advantages and disadvantages.

1.3 A.C. Distribution

Nowadays electrical energy is generated, transmitted and distributed in the form of alternating current. One important reason for the widespread use of alternating current in preference to direct current is the fact that alternating voltage can be conveniently changed in magnitude by means of a transformer. Transformer has made it possible to transmit a.c. power at high voltage and utilize it at a safe

potential. High transmission and distribution voltages have greatly reduced the current in the conductors and the resulting line losses.

There is no definite line between transmission and distribution according to voltage or bulk capacity. However, the down sub-station is fed by the transmission system and the consumers' meters. The a.c. distribution system is classified into (i) primary distribution system and (ii) secondary distribution system.

- (i) **Primary Distribution System:** It is part of a.c. distribution system, which operates at voltages somewhat higher than general utilization and handles large blocks of electrical energy than the average low-voltage consumer uses. The voltage used for primary distribution depends upon the amount of power to be conveyed and the distance of the sub-station required to be fed. The most commonly used primary distribution voltages are 22 kV, 6.6 kV and 2.2 kV. Due to economic considerations, primary distribution is carried out by 3-phase, 3-wire system.

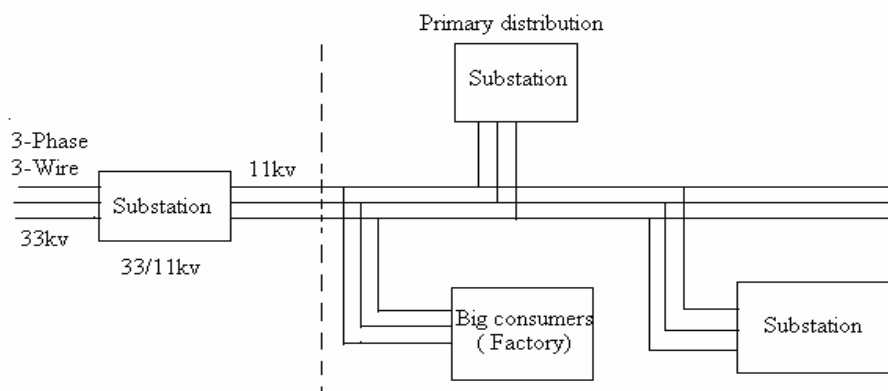


Figure 1.2 Primary Distribution System

Figure 1.2 shows a typical primary distribution system. Electric power from the generating station is transmitted at high voltage to the sub-station located in or near the city. At this sub-station, voltage is stepped down to 11kV with the help of step-down transformer. Power is supplied to various sub-stations for distribution or to big consumers at this voltage. This forms the high voltage distribution or primary distribution.

- (ii) **Secondary Distribution System:** It is that part of a.c. distribution system that includes the range of voltages at which the ultimate consumer utilizes the electrical energy delivered to him. The secondary distribution employs 400/230 V, 3-phase, 4-wire system. Figure 1.3 shows a typical secondary distribution system.

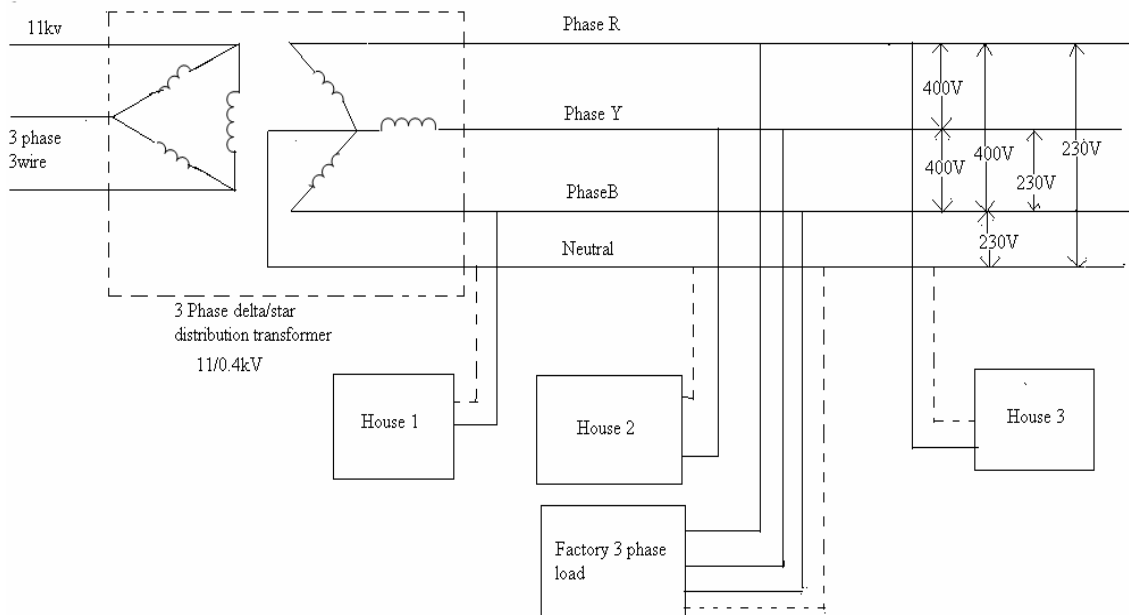


Figure 1.3 Secondary Distribution System

The primary distribution circuit delivers power to various sub-stations, called distribution sub-stations. The sub-stations are situated near the consumer's localities and contain step-down transformers. At each distribution sub-station, the voltage is stepped down to 400 V and power is delivered by 3-phase, 4-wire a.c. system. The voltage between any two phases is 400 V and between any phase and neutral is 230. The single phase domestic loads are connected between any one phase and the neutral whereas 3-phase 400 V motor loads are connected across 3-phase lines directly.

1.4 Connection Scheme of Distribution System

All distribution of electrical energy is done by constant voltage system. In practice, the following distribution circuits are generally used:

- (i) **Radial System:** In this system, separate feeders radiate from a single sub-station and feed the distributors at one end only. Figure 1.4 (a) shows a single line diagram of a radial system for d.c. Distribution where a feeder OC supplies a distributor AB at point A. Obviously, the distributors are fed at one point only i.e. point A in this case. Figure 1.4 (b) shows a single line diagram of radial system for a.c. distribution. The radial system is employed only when power is generated at low voltage and the sub-station is located at the centre of load. This is the simplest distribution circuit and has the lowest initial cost. However, it suffers from the following drawbacks:
- The end of the distributor nearest to the feeding point will be heavily loaded.
 - The consumers are dependent on a single feeder and single distributor. Therefore, any fault on the feeder or distributor cuts off supply to the consumers who are on the side of the fault away from the sub-station.
 - The consumers at the distant end of the distributor would be subjected to serious voltage fluctuations when the load on the distributor changes.

Due to these limitations, this system is used for short distances only.

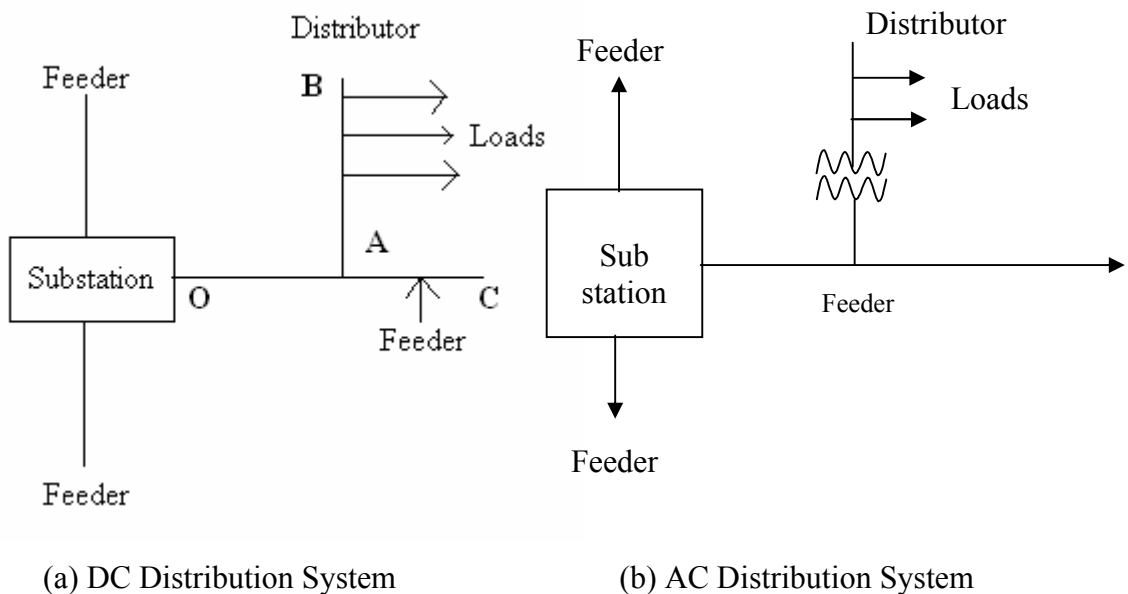


Figure 1.4 Single Line Diagram of Radial System

- (ii) **Ring Main System:** In this system, the primaries of distribution transformers form a loop. The loop circuit starts from the sub-station bus-bars, makes a loop through the area to be served, and returns to the sub-station. Figure 1.5 shows the single line diagram of ring main system for a.c. Distribution where sub-station supplies to the closed feeder LMNOPQRS and Q of the feeder through distribution transformers.

The ring main system has the following advantages:

- (a) There are less voltage fluctuations at consumer's terminals
 - (b) The system is very reliable as each distributor is fed via two feeders.
- In the event of fault on any section of the feeder, the continuity of supply is maintained. For example, suppose that fault occurs at any point F of section SLM of the feeder. Then section SLM of the feeder can be isolated for repairs and at the same time continuity of supply is maintained to all the consumers via the feeder SRQPONM.

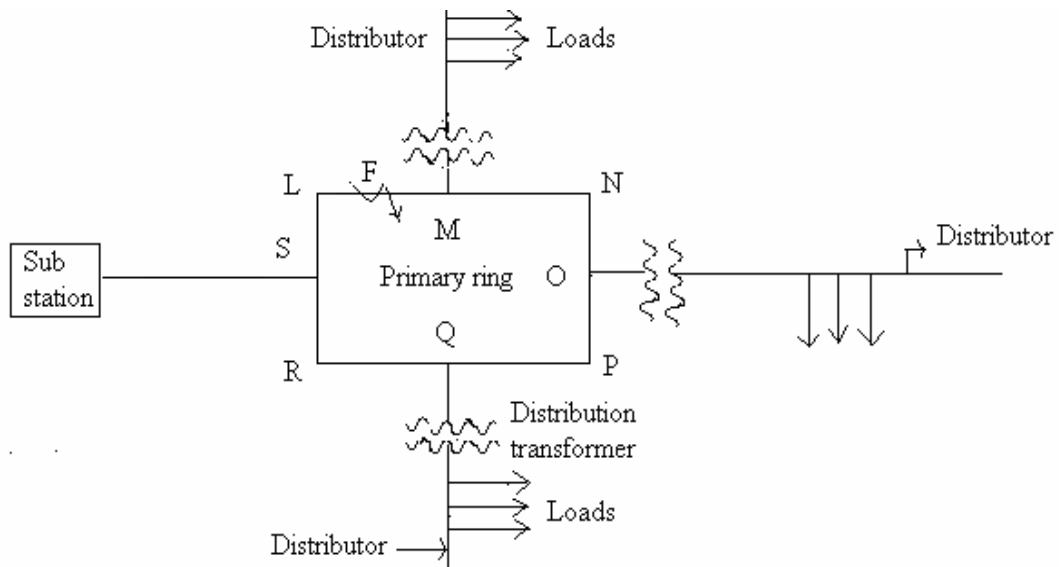


Figure 1.5 Ring Main Systems

- (iii) **Interconnected System:** When the feeder ring is energized by two or more than two generating stations or sub stations, it is called interconnected system. Figure 1.6 shows the single line diagram of interconnected system where the closed feeder ring ABCD is supplied by two sub-stations S_1 and S_2 at points D and C respectively. Distributors are

connected to points O, P, Q and R of the feeder ring through distribution transformers.

The interconnected system has the following advantages:

- (a) It increases the service reliability.
- (b) Any area fed from one generating station during peak load hours can be fed from the other generating station. This reduces reserve power capacity and increases efficiency of the system.

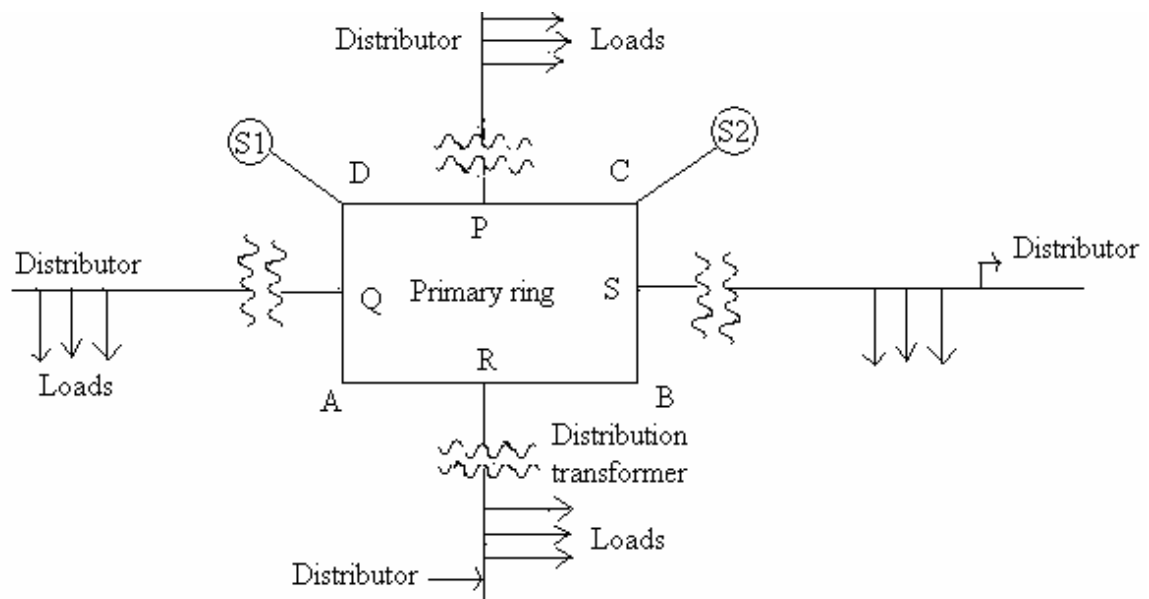


Figure 1.6 Interconnected Systems

1.5 Requirement of a Distribution System

A considerable amount of effort is necessary to maintain an electric power supply within the requirements of various types of consumers. Some of the requirements of a good distribution system are: proper voltage, availability of power on demand, and reliability

- (i) **Proper Voltage:** One important requirement of a distribution system is that voltage variations at consumers' terminals should be as low as possible. The changes in voltage are generally caused due to the variation of load on the system. Low voltage causes loss of revenue, inefficient lighting and possible burning out of motors. High voltage causes lamps to burn out permanently and may cause failure of other appliances. Therefore, a good distribution system should ensure that the voltage variations at consumers' terminals are within

permissible limits. The statutory limit of voltage variations is $\pm 10\%$ of the rated value at the consumers' terminals. Thus, if the declared voltage is 230 V, then the highest voltage of the consumer should not exceed 244 V while the lowest voltage of the consumer should not be less than 216 V.

- (ii) **Availability of Power Demand:** Power must be available to the consumers in any amount that they may require from time to time. For example, motors may be started or shut down, lights may be turned on or off, without advance warning to the electric supply company. As electrical energy cannot be stored, therefore, the distribution system must be capable of supplying load demands of the consumers. This necessitates that operating staff must continuously study load patterns to predict in advance those major load changes that follow the known schedules.
- (iii) **Reliability:** Modern industry is almost dependent on electric power for its operation. Homes and office buildings are lighted, heated, cooled and ventilated by electric power. This calls for reliable service. Unfortunately electric power, like everything else that is man-made, can never be absolutely reliable. However, the reliability can be improved to a considerable extent by (a) inter-connected system, (b) reliable automatic control system and (c) providing additional reserve facilities.

1.6 Design Considerations in Distribution System

Good voltage regulation of a distribution network is probably the most important factor responsible for delivering good service to the consumers. For this purpose, design of feeders and distributors requires careful consideration.

- (i) **Feeders:** A feeder is designed from the point of view of its current carrying capacity while the voltage drop consideration is relatively not important. It is because voltage drop in a feeder can be compensated by means of voltage regulating equipment at the sub-station.
- (ii) **Distributors:** A distributor is designed from the point of view of the voltage drop in it. It is because a distributor supplies power to the consumers and there is a statutory limit of voltage variations at the consumer terminal ($\pm 10\%$ of rated value). The size and length of the

distributor should be such that voltage at the consumer's terminals is within the permissible limits.

1.7 Blackout

A blackout is a condition where a major portion or all of an electrical network is de-energized with much of the system tied together through closed breakers. Any area whose tie-lines to the high voltage grid cannot support reasonable contingencies is a candidate for a blackout. System separations are possible at all loading levels and all times in the year. Changing generation patterns, scheduled transmission outages, and rapid weather changes among other reasons can all lead to blackouts. Separations due to dynamic instability are typically initiated by multiple contingencies such as loss of corridors, several transmission circuits, several generating units, or delayed fault clearing.

The system just prior to a blackout may not be dynamically unstable but in an overloaded condition. At such loadings, the collapse may come about due to damage to thermally overloaded facilities, or circuits contacting underlying facilities or vegetation. When an overloaded facility trips, other facilities will increase their loadings and may approach their thermal capabilities or relay trip settings.

The system restoration sequence and timing will be directly impacted by the various sizes, types, and state of operation of the system generating units prior to the blackout. After a system has blacked out, the system operators perform a survey of the system status. Circuit breaker positions will not provide a reliable indication of faulted versus non-faulted equipment. Breakers can be found in the closed position, but the associated transmission facility is faulted. If the system blackout is storm-initiated, this condition is quite possible. The storm can continue to damage equipment after the system is de-energized. Also, equipment with neutral connections, such as reactors, transformers, and capacitors, may be locked out from the neutral over current conditions during system shutdown. These facilities may be in perfectly serviceable condition. Most relay systems will remain reliable and secure during restoration, provided there is adequate fault current available to activate the relaying. The most questionable relay reliability issues come from re-closing relays.

1.8 Definition and Classification of Voltage Stability

Power system stability is defined as characteristics for a power system to remain in state of equilibrium after a disturbance as reported by Cusum *et al.* [1]. Traditionally, the stability problem has been the rotor angle stability, i.e. maintaining synchronous operation. Instability may also occur without loss of synchronism, in which case the concern is the control and stability voltage reported by Kundur [2].

1.8.1 Voltage Stability: The voltage stability is the ability of a power system to maintain steady acceptable voltage at all buses in the system at normal operating condition and after being subjected to a disturbance. Power system is voltage stable if voltage after a disturbance is close to voltage at normal operating condition. A power system becomes unstable when voltages uncontrollably decrease due to outage of equipment (generator, line, transformer, bus bar, etc.), increment of load, decrement of production and/or weakening of voltage control.

1.8.2 Voltage Instability: Voltage instability stems from the load dynamics to restore power consumption beyond the capability of the combined transmission and generation system.

1.8.3 Voltage Collapse: Voltage collapse is the process by which voltage instability leads to the loss of voltage in a significant part of the system. This condition results from reactive losses significantly exceeding the reactive resources available to supply them. Circuits loaded above surge impedance loadings and reduced output of shunt capacitors as voltages decline can lead to accelerating voltage drops. Voltage collapse can look like both a steady-state problem with time to react and a problem where no effective operator intervention is possible. It is very hard to predict the area that will be affected or electrically isolated from the grid.

1.9 Classification of Power System Stability

Power system stability is classified as rotor angle and voltage stability. A classification of power system stability based on time scale and driving force criteria is shown in Table 1.1. The driving forces for an instability mechanism are named generator-driven and load driven.

Table 1.1 Classification of Power System Stability

Time scale	Generator-driven		Load-driven	
Short-term	Rotor angle stability		Short term voltage stability	
	Small-signal	transient		
Long-term	Frequency stability		Long –term voltage stability	
			Small disturbance	Large disturbance

The rotor angle stability is divided into small-signal and transient stability. The small-signal stability is present for small disturbances in the form of undamped electromechanical oscillations. The transient stability is due to lack of synchronizing torque and is initiated by large disturbances. The time frame is called short-term time scale, because the dynamics typically last for a few seconds.

The voltage problem is load-driven as described above. The voltage stability may be divided into short and long-term voltage stability according to time scale of load components dynamics. Short-term voltage stability is characterized by components such as induction motor, excitation of synchronous generators and electronically controlled devices such as HVDC and static var compensator. The time scale of short-term voltage stability is same as rotor angle stability. The modelling and the analysis of these problems are similar. The distinction between rotor angle and short-term voltage instability is sometimes difficult, because most practical voltage collapse includes some elements of both voltage and angle instability.

When short-term dynamics have died out some time after the disturbance, the system enters a slower time frame. The dynamics of the long-term time scale lasts for several minutes. Two types of stability problems emerge in the long term time scale: frequency and voltage problems. Frequency problems may appear after a major

disturbance resulting in power system islanding. Frequency instability is related to the active power imbalance between generators and loads. An island may be either under or over-generated when the system frequency either declines or rises.

The analysis of long-term voltage stability requires detailed modelling of long-term dynamics. The long-term voltage stability is characterized by scenarios such as load recovery by the action of on-load tap changer or through load self-restoration, delayed corrective control actions such as shunt compensation switching or load shedding. The long-term dynamics such as response of power plant controls, boiler dynamics and automatic generation control also effect long-term voltage stability. The modelling of long-term voltage stability requires consideration of transformer on-load tap changers, characteristics of static loads, manual control actions of operators, and automatic generation control.

For purpose of analysis, it is sometimes useful to classify voltage stability into small and large disturbances. Small disturbance voltage stability considers the power systems ability to control voltages after small disturbances, e.g. change in load. The analysis of small disturbance voltage stability is done in steady state. In that case power system can be liberalized around an operating point and the analysis is typically based on eigen value and eigen vector technique. Large disturbances voltage stability can be studied by using non-linear time domain simulations in the short term time frame and load-flow analysis in the long term time frame. The voltage stability is, however, a single problem on which a combination of both linear and non-linear tools can be used.

1.10 Causes of Voltage Instability

The following are the causes of voltage instability:

1. The load on transmission line is too high.
2. The voltage sources are too far from the load centers.
3. The source voltages are too low.
4. There is insufficient load reactive compensation.

1.11 Literature Survey on Voltage Stability Analysis of Power Distribution Systems

Engineers have long been struggling with developing voltage stability criteria for their systems shown by Taylor [3]. Sole reliance on either P–V or V–Q analysis is not sufficient to assess voltage stability and proximity to voltage collapse. Each analysis is needed to confirm the results of the other (i.e., P–V analysis is needed to confirm the results of V–Q analysis and vice versa). Member systems may use either method for general voltage stability evaluation, contingency screening etc. But voltage stability margins must be demonstrated by both P–V and V–Q analysis.

In addition to P–V and V–Q analysis, full long–term dynamic simulation, fast dynamic simulation by Cusum *et al.* [1], nodal analysis by Gao *et al.* [4], Kundur [2] and security–constrained optimal power flow analysis by Merrit *et al.* [5] are valuable tools for providing insights into the voltage instability and collapse phenomenon.

Voltage stability is a property of power distribution system that enables it to stay in a state of equilibrium voltage under normal operating condition and the system also returns to an acceptable state of equilibrium voltage after a disturbance. If the power consumption from the system goes beyond its capability, a sequence of events accompanying voltage instability results in a low acceptable voltage profile of the distribution networks. Unlike transmission networks, distribution networks are radial in nature. The distribution networks have high R/X ratio compared to the transmission networks, and hence are ill–conditioned in nature. The voltage stability index of distribution networks is usually different from that of transmission networks because the latter have $X \gg R$. During derivation of voltage stability index of distribution networks, X and R are equally significant and are generally both taken into account. The distribution networks experience voltage collapse beyond certain critical loading conditions because the modern power distribution networks are constantly being faced with an ever–growing load demand. The system voltage stability is system’s capability to keep acceptable voltages in all buses in normal conditions after disturbances.

Literature survey shows that a few works have been done on voltage stability analysis of power distribution system. Ajjarapu *et al.* [6] presented the earlier works on voltage stability analysis of transmission system. Brownell *et al.* [7] provided the recordings of increased load demand of a system and showed its voltage collapse. They also proposed urgent compensation of reactive power. Jasmon and Lee [8] proposed a voltage stability analysis of radial distribution networks. They reduced the whole network by its single line diagram that is valid only at the derived operating point. They had put voltages of all nodes equal to 1.0 pu to simplify the derivation of voltage stability index. This method is unable to handle changing load pattern. Using Thevenin's theorem, Chebbo *et al.* [9] suggested a method to study the voltage collapse. Rahman *et al.* [10] proposed a method to study the voltage collapse using Thevenin's theorem. They suggested a voltage stability index. Gubina *et al.* [11] proposed a method to study voltage stability analysis of radial distribution networks reducing the system model to its single line equivalent. Chakravorty *et al.* [12] proposed a voltage stability index to identify the most sensitive node of the network. They handled the composite load using power convergence and used the load-flow technique proposed by Das *et al.* [14]. They have shown that the critical loading for constant current load is maximum. Ranjan *et al.* [13] suggested a new voltage stability index to identify the most sensitive node of the network. They assumed the equality of magnitude of voltage for sending-end node and receiving-end node of each branch while deriving voltage stability index. They have shown that critical loading for constant impedance load is maximum. They have compared the result for critical loading with the result obtained by Chakravorty *et al.* [12]. They were silent regarding the result of critical loading for composite load modelling. For solving radial distribution network S.Ghosh and D.Das [15] presented a load-flow technique based on nodes beyond branches, which is very efficient computationally. Mesut E. Baran *et al.* [16] proposed a method which is used for loss reduction and load balancing of distribution system. Mesut E. Baran *et al.* [17] also presented a new formulation of the power flow equations of radial distribution networks. By using this they tried to determine the optimal size of the capacitors placed on the nodes of radial distribution system so that the real power losses are minimized. Zechun Hu *et al.* [18] introduces a new method for obtaining a maximum loading point (MLP) of electric power system based on load-flow method with optimum multiplier in rectangular coordinates.

1.12 Objectives of the Thesis Work

The thesis work endeavors to derive a new expression of voltage stability index (VSI) and its applications in planning of power distribution system. The objectives are divided into the following:

- To derive a new expression of VSI to be computed for all nodes of the distribution networks.
- To identify the most sensitive nodes of the distribution networks.
- To compute the critical values of total real power load (TPL) and total reactive power load (TQL) of the system.

1.13 Scope of the Thesis Work

Jasmon and Lee [8] while deriving the expression for VSI had reduced the whole network into its single line equivalent that is valid at the operating point and put the voltage magnitude 1.0 pu for all nodes that led wrong results.

Ranjan *et al.* [13] while deriving the expression for VSI had reduced the whole network into its single line equivalent indirectly at the operating point and assumed the magnitude of sending–end voltage of each branch is equal to that of the magnitude of receiving–end voltage that led wrong results.

A new expression for VSI is derived for electric power distribution networks to be computed for all nodes. With the help of derived expression for VSI the most sensitive nodes of the networks are identified. The critical values of TPL and TQL are also computed. The critical values of TPL and TQL of the network selected by utility have also been computed, beyond which voltage collapse will occur.

1.14 Organization of the Thesis Work

Chapter 1 has presented the introduction of distribution system, black out, voltage stability, voltage collapse, and literature survey, objectives of the research, scope of the research and organization of the research.

Chapter 2 presents a new expression of VSI to be computed for all nodes of the distribution networks. The node having minimum value of voltage stability index

is the most sensitive node that is more prone to voltage collapse compared to other nodes due to change of its load. Three different types of radial distribution networks have been selected. The most sensitive node is the end node having the minimum voltage in all the three cases using the proposed method. The critical loading for 69–node radial distribution network has been computed by the proposed method, and has been compared with the methods [12,13].

Chapter 3 presents the overall conclusions and future scope of research work.

References present the list of previous papers published by researchers in voltage stability analysis of power distribution system.

Appendix – A shows the line data and load data of 29–node radial distribution network available in [15].

Appendix – B shows the line data and load data of 33–node radial distribution network available in [16].

Appendix – C shows the line data and load data of 69–node radial distribution network available in [17].

Appendix – D shows biography of candidate.

Voltage stability is a property of power distribution system that enables it to stay in a state of equilibrium voltage under normal operating condition and the system also returns to an acceptable state of equilibrium voltage after a disturbance. If the power consumption from the system goes beyond its capability, a sequence of events accompanying voltage instability results in a low acceptable voltage profile of the distribution networks. Unlike transmission networks, distribution networks are radial in nature. The distribution networks have high R/X ratio compared to the transmission networks, and hence are ill-conditioned in nature. The voltage stability index of distribution networks is usually different from that of transmission networks because the latter have $X \gg R$. During derivation of voltage stability index of distribution networks, X and R are equally significant and are generally both taken into account. The distribution networks experience voltage collapse beyond certain critical loading conditions because the modern power distribution networks are constantly being faced with an ever-growing load demand. The system voltage stability is system's capability to keep acceptable voltages in all buses in normal conditions after disturbances. Literature survey has already been introduced in **Chapter 1 (Art.1.11)**.

The main aim of this thesis work is to propose a new voltage stability index formula without any assumption to compute the voltage stability index of all nodes of the network without reducing the network and also to identify the exact most sensitive node of the radial distribution network. The critical loading conditions of 69-node radial distribution network [17] is found out for constant power, constant current, constant impedance, composite and exponential load modellings for 1.0, 1.025 and 1.05 pu respectively. The superiority of the proposed method has been compared with the other methods [12,13]. The load-flow proposed by Ghosh *et al.* [15] is used in this thesis work.

2.1 Assumption

The three phase radial distribution networks are assumed to be balanced and can be represented by their single line diagram.

2.2 Derivation of Voltage Stability Index

Figure 2.1 shows the single line diagram of a radial distribution network. Table 2.1 shows the branch number, sending-end node and receiving-end node of

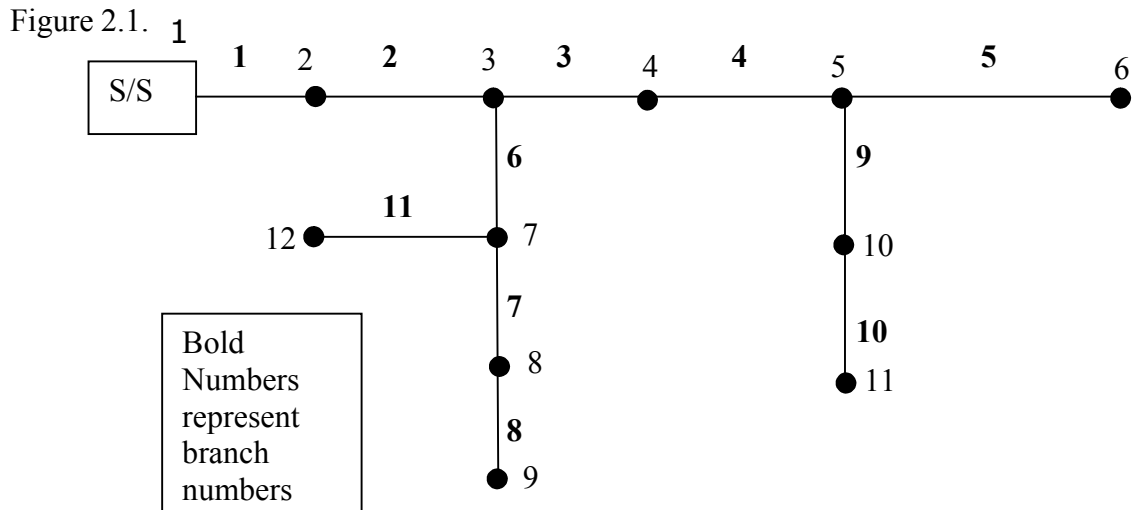


Figure 2.1 Single Line Diagram of Radial Distribution Network

Table 2.1 Branch number (jj), Sending-end node ($m1 = IS(jj)$), Receiving-end node ($m2 = IR(jj)$) and nodes beyond branches 1, 2, 3, ..., 10 of Figure 2.1

Branch Number (jj)	Sending end $m1 = IS(jj)$	Receiving end $m2 = IR (jj)$
1	1	2
2	2	3
3	3	4
4	4	5
5	5	6
6	3	7
7	7	8
8	8	9
9	5	10
10	10	11
11	7	12

Figure 2.2 shows a branch of Figure 2.1. For branch- jj , $m1$ is the sending-end node and $m2$ is the receiving-end node.

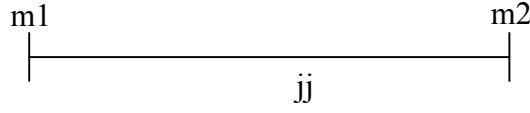


Figure 2.2 Single Line Diagram

Current through the branch–jj is expressed by

$$\frac{|V(m1)|\angle\delta(m1) - |V(m2)|\angle\delta(m2)}{|Z|\angle\theta} = I(jj) = \sum_{m'2} \frac{P(m'2) - jQ(m'2)}{V^*(m'2)}$$

where m'2 are set of the receiving–end nodes beyond the branch–jj.

$$\text{i.e., } \frac{|V(m1)|\angle\delta(m1) - |V(m2)|\angle\delta(m2)}{|Z|\angle\theta} = \frac{P(m2) - jQ(m2)}{V^*(m2)} + \sum_{m'2 \neq m2} \frac{P(m'2) - jQ(m'2)}{V^*(m'2)}$$

$$\text{i.e., } \frac{|V(m1)|\angle\delta(m1) - |V(m2)|\angle\delta(m2)}{|Z|\angle\theta} = \frac{P(m2) - jQ(m2)}{V^*(m2)} + (a + jb)$$

$$\text{where } a + jb = \sum_{m'2 \neq m2} \frac{P(m'2) - jQ(m'2)}{V^*(m'2)} = \sum_{k > jj} [I_r(k) + jI_m(k)]$$

i.e.,

$$[|V(m1)|\angle\delta(m1) - |V(m2)|\angle\delta(m2)]V^*(m2) = [P(m2) - jQ(m2)]|Z|\angle\theta + (a + jb)V^*(m2)|Z|\angle\theta$$

i.e.,

$$|V(m2)|^2 - |V(m1)||V(m2)|\angle(\delta_1 - \delta_2) + (a + jb)V^*(m2)|Z|\angle\theta + [P(m2) - jQ(m2)]|Z|\angle\theta = 0$$

$$\text{i.e., } |V(m2)|^2 - |V(m1)||V(m2)|\angle(\delta_1 - \delta_2) + (c + jd)V^*(m2) + (e + jf) = 0 \quad (2.1)$$

$$\text{where } c + jd = (a + jb)|Z|\angle\theta = \sum_{k > jj} [I_r(k) + jI_m(k)][R(jj) + jX(jj)]$$

$$= \left[\sum_{k > jj} I_r(k) R(jj) - \sum_{k > jj} I_m(k) X(jj) \right] + j \left[\sum_{k > jj} I_r(k) X(jj) + \sum_{k > jj} I_m(k) R(jj) \right] \quad (2.2)$$

$$\text{and } e + jf = [P(m2) - jQ(m2)][R(jj) + jX(jj)]$$

$$= P(m2)R(jj) + Q(m2)X(jj) + j[P(m2)X(jj) - Q(m2)R(jj)] \quad (2.3)$$

Putting $\delta_1 - \delta_2 \approx 0$ and also neglecting angle δ_2 , we have from Eq. (2.1),

$$\left[|V(m2)|^2 - [|V(m1)| - c] |V(m2)| + e \right] + j[d|V(m2)| + f] = 0 \quad (2.4)$$

Equating real and imaginary parts of Eq. (2.4), we have

$$|V(m2)|^2 - [|V(m1)| - c]|V(m2)| + e = 0 \quad (2.5)$$

$$d|V(m2)| + f = 0 \quad (2.6)$$

The roots of Eq. (2.5) will be real if

$$[|V(m1)| - c]^2 - 4e \geq 0 \quad (2.7)$$

$$\text{where } c = \sum_{k>jj} I_r(k) R(jj) - \sum_{k>jj} I_m(k) X(jj) \quad (2.8)$$

$$\text{and } e = P(m2)R(jj) + Q(m2)X(jj) \quad (2.9)$$

From Eq. (2.7), we have

$$[|V(m1)| - c]^2 - 4e \geq 0$$

$$\text{i.e., } [|V(m1)| - c]^2 \geq 4e$$

$$\text{i.e., } [|V(m1)| - c]^4 \geq 16e^2$$

$$\text{i.e., } [|V(m1)| - c]^4 - 16e^2 \geq 0$$

$$\text{Let } L(m2) = [|V(m1)| - c]^4 - 16e^2 \quad (2.10)$$

be the voltage stability index (VSI) of the node m2. Therefore, the voltage magnitude of the sending-end node m1 of branch-jj must be known to calculate the voltage stability index (VSI) of the receiving-end node m2 of branch-jj. The load-flow result of the network must be known prior to computation of voltage stability index. Here $c = 0$ for the end nodes because $I_r(k) = 0$ and $I_m(k) = 0$. The load-flow proposed by Ghosh *et al.* [15] is used in this thesis work.

The voltage at any receiving-end node (m2) of branch-jj is expressed by

$$V(m2) = V(m1) - I(jj)Z(jj) \quad (2.11)$$

$$\text{i.e., } V(m2) = V(m1) - I(jj) [R(jj) + j X(jj)] \quad (2.12)$$

$$\text{where } m1 = IS(jj) \quad (2.13)$$

$$\text{and } m2 = IR(jj) \quad (2.14)$$

The load current of any receiving-end node $m2 = IR(jj)$ of branch-jj is expressed by

$$IL(m2) = \frac{PL(m2) - jQL(m2)}{V^*(m2)} \quad (2.15)$$

The real and reactive power losses of branch-jj are expressed by

$$LP = |I(jj)|^2 R(jj) \quad (2.16)$$

$$\text{and } LQ = |I(jj)|^2 X(jj) \quad (2.17)$$

respectively.

The current through branch–jj is the sum of all load currents of all nodes beyond branch–jj i.e.,

$$I(jj) = \sum_{i=1}^{N(jj)} IL\{IE(jj, i)\} \quad (2.18)$$

where $N(jj)$ is the total number of nodes beyond branch jj and $IE(jj, i)$ is the receiving-end node beyond branch–jj. The algorithm for identification of nodes beyond each branch–jj is available in ref.[15]. Table 2.2 shows the nodes beyond each branch of Figure 2.1.

Table 2.2 Nodes beyond Each Branch of Figure 2.1

Branch Number (jj)	Nodes beyond Branch–jj	Total Number of Nodes beyond Branch–jj
1	2,3,4,5,6,7,8,9,10,11,12	11
2	3,4,5,6,7,8,9,10,11,12	10
3	4,5,6,10,11	5
4	5,6,10,11	4
5	6	1
6	7,8,9,12	4
7	8,9	2
8	9	1
9	10,11	2
10	11	1

To determine the nodes beyond all the branches the IDENT software proposed by Ghosh and Das [15] is used and it is explained in Art. 2.3.

2.3 Identification of Nodes beyond All the Branches (IDENT Software) [15]

Before identifying the nodes beyond all the branches, the following variables are defined at first:

ic is the node count (identifies the number of nodes beyond a particular branch),

$IK(ic)$ is the node identifier (helping to identify nodes beyond all the branches),

$N(jj)$ is the total number of nodes beyond branch jj ,

$IE(jj, ic + 1)$ is the receiving-end node and

$IE(jj, ic + 1)$ is explained below.

Let us consider the first branch in Figure 2.1 (Table 2.1), i.e. $jj = 1$; the receiving-end node of branch-1 is 2, i.e. $IR(jj) = IR(1) = 2$. Therefore, $IE(jj, ic + 1) = IE(1, ic + 1)$ will help to identify all the nodes beyond branch-1. This will help to compute the exact current flowing through branch-1. Similarly, for branch-2, i.e. $jj = 2$; the receiving-end node of branch-2 is 3, i.e. $IR(jj) = IR(2) = 3$. Therefore, $IE(jj, ic + 1) = IE(2, ic + 1)$ will identify all the nodes beyond branch-2 that will help to compute the exact current flowing through branch-2. Before identification of nodes beyond a particular branch, 'ic' has to be reset to zero. For identification of nodes beyond a particular branch, 'ic' will be incremented by 1. For $jj = 1$ (first branch of Figure 2.1), $IR(jj) = IR(1) = 2$; we check whether $IR(1) = IS(i)$ or not for $i = 2, 3, 4, \dots, LN1$. It is seen that $IR(1) = IS(2) = 2$, the corresponding receiving-end nodes are $IR(2) = 3$. Therefore, $IE(1, 1) = 2, IE(1, 2) = 3$. There should not be any repetition of nodes while identifying nodes beyond a particular branch. From the above discussion, it is seen that node 2 is connected to node 3. First this IDENT software will check whether node 3 appears in the left-hand column of Table 2.2. It is seen that node 3 is connected to nodes 4 and 7. Therefore, $IE(1, 3) = 4$ and $IE(1, 4) = 7$. Next the algorithm will check whether nodes 4 and 7 appear in the left-hand column of Table 2.1. It is seen that the node 4 is connected to node 5 and node 7 is connected to node 8. Therefore, $IE(1, 5) = 5$ and $IE(1, 6) = 8$. The proposed logic will thereafter again check whether nodes 5 and 8 are connected to any other nodes. This process will continue unless all nodes are identified beyond branch-1. The nodes beyond branch-1 are as shown in Table 2.2. The total current flowing through branch-1 is equal to the sum of the load currents of all nodes beyond branch-1. For $jj = 2$ (second branch of Figure 2.1; Table 2.1), $IR(jj) = IR(2) = 3$, we check whether $IR(2) = IS(i)$ or not for $i = 3, 4, \dots, LN1$. It is seen that $IR(2) = IS(3) = 3$ and $IR(2) = IS(6) = 3$. The corresponding receiving-end nodes are $IR(3) = 4$ and $IR(6) = 7$. Therefore, $IE(2, 1) = 3$ and $IE(2, 2) = 7$. Again node 3 is connected to nodes 4 and 7. The proposed logic will identify the nodes that are connected to nodes 4 and 7. It will check whether node 4 and node 7 appear in the left-hand column of Table 2.1 or not.

It is seen that node 4 is connected to node 5 and node 7 is connected to node 8. Therefore, $IE(2, 3) = 5$ and $IE(2,4) = 8$. The proposed logic will check whether nodes 5 and 8 are connected to any other nodes or not. This process will continue unless all nodes are identified beyond branch-2. The nodes beyond branch-2 are shown in Table 2.2. Similarly it is necessary to consider the receiving-end node of branch-3, branch-4,..., branch-LN1 (= 11) in Figure 2.1 and, in a similar way to that discussed above, the nodes are to be identified beyond the rest of branches. The nodes beyond each branch of Figure 2.1 are shown in Table 2.2. If the receiving-end node of any branch in Figure 2.1 is an end node of a particular lateral, the total current of this branch is equal to the load current of this node. For example, consider node 6 in Figure 2.1 (branch-5, Table 2.1); this is an end node. Therefore, the branch current $I(5)$ is equal to the load current of node 6 only. Similarly, 9, 11 and 12 are end nodes of Figure 2.1. The proposed computer logic will identify all the end nodes automatically. The algorithm of IDENT software is presented below.

- Step -1 : Start
- Step -2 : Read sending-end and receiving-end nodes and total number of nodes and branches.
- Step -3 : $jj = 1$
- Step -4 : $k = jj + 1$
- Step -5 : Set $ic = 0$ and $id = 0$
- Step -6 : $i = k$
- Step -7 : $nc = 0$
- Step -8 : If $IR(jj) \neq IS(i)$, go to Step - 18 .
- Step -9 : If $ic = 0$, go to Step - 16 .
- Step -10 : $in = 1$
- Step -11 : If $IR(i) = IE(jj, ic+1)$, go to Step -13.
- Step -12 : $nc = 1$
- Step -13 : $in = in + 1$
- Step -14 : If $in < ic$, go to Step - 11
- Step -15 : If $nc = 1$, go to Step-18 else go to Step- 17
- Step -16 : $IE(jj, ic+1) = IR(jj)$

- Step -17 : $ic = ic + 1$, $IK(ic) = i$, $IE(jj, ic+1) = IR(jj)$,
 $N(jj) = ic+1$
- Step -18 : $i = i + 1$
- Step -19 : If $ic \leq LN1$, go to Step - 7
- Step -20 : If $ic \neq 0$, go to Step - 25
- Step -21 : $id = id + 1$
- Step -22 : If $id > ic$, go to Step - 26
- Step -23 : $IR(jj) = IE(jj, id)$ and $K = IK(id) + 1$.
- Step -24 : If $id \leq ic$, go to Step -6 else go to Step-26 .
- Step -25 : $IE(jj, ic+1) = IR(jj)$,
 $N(jj) = ic+1$.
- Step -26 : $jj = jj + 1$.
- Step -27 : If $jj \leq LN1 - 1$, go to Step-4
- Step -28 : $IE(LN1) = IR(LN1)$.
- Step -29 : Stop

2.4 Algorithm for Computation of VSI

The algorithm for computation the value of VSI at all nodes and identification of most sensitive node is presented below.

- Step -1 : Read the system data
- Step -2 : Set $V(i) = 1.0 + j 0.0$ for all i i.e., $i = 1, 2, \dots, NB$
Set $VV(i) = V(i)$ for all i i.e., $i = 1, 2, \dots, NB$
- Step -3 : Set $ISS(jj) = IS(jj)$ and $IRR(jj) = IR(jj)$ for
 $jj = 1, 2, 3, \dots, LN1$
- Step -4 : Set iteration count $k = 1$
- Step -5 : Set $kMAX = 100$ (say)
- Step -6 : Set $DVMAX = 0.0$ and $\epsilon = 0.00001$
- Step -7 : Identify the nodes beyond each branch using the IDENT software available in ref.[15].
- Step -8 : Compute load currents $IL(m2)$ for all the node $m2$ i.e., for $m2 = 2, 3, 4, \dots, NB$ using Eq. (2.15).

- Step-9 : Compute the current through each branch i.e., $I(jj)$ for all jj i.e., $jj = 1,2,3,\dots, LN1$ using Eq. (2.18).
- Step -10 : Set $jj = 1$
- Step -11 : Set $m1 = ISS (jj)$ and $m2 = IRR (jj)$. Compute receiving-end voltage $V (m2)$ for all $m2$ using Eq. (2.11).
- Step -12 : Compute the absolute change in voltage at node $m2$ i.e., $DV(m2) = ABS(|V(m2)| - |VV(m2)|)$
- Step -13 : $jj = jj + 1$
- Step -14 : If $jj < LN1$, go to Step-11, otherwise go to Step-15
- Step -15 : Find max value of $DV(m2)$ from $DV(m2)$ for $m2 = 2,3,4,\dots, NB$.
- Step -16 : $DVMAX = DV(m2)$
- Step -17 : If $DVMAX < \epsilon$ go to Step-21 else go to Step-8.
- Step -18 : $k = k + 1$
- Step -19 : Set $VV (m2) = V (m2)$ for $m2 = 2,3,\dots, NB$.
- Step -20 : If $k < kMAX$, go to Step-8, otherwise go to Step-29.
- Step -21 : Print "Solution has converged"
- Step-22 : Compute voltages of each node.
- Step-23 : Check the node point. If it is end node, $c = 0$ else calculate c using Eq.(2.8).
- Step-24 : Compute $IL(m2)$ of all $m2$ ($m2 = 2,3, 4,\dots, NB$) for all using Eq. (15) for all load modellings and $I(jj)$ for all jj i.e., $jj = 1, 2,3,\dots, LN1$ using Eq. (18). Also calculate real and reactive power losses for each branch- jj i.e., $jj = 1,2,3,\dots, LN1$ using Eq. (2.16) and Eq.(2.17) respectively.
- Step-25 : Compute the voltage stability index of all nodes using Eq. (2.10).
- Step-26 : Identify the node of the network having minimum voltage.
- Step-27 : Identify the node of the network having minimum VSI and print the results and go to Step-29
- Step-28 : Print "Solution has not converged"
- Step-29 : Stop

2.5 Load Modelling

A balanced load that can be represented either as constant power, constant current, constant impedance or as an exponential load is considered here. The general expression of load is shown below.

$$P(m2) = P_n [a_0 + a_1 V(m2) + a_2 V^2(m2) + a_3 V^{e1}(m2)] \quad (2.19)$$

$$Q(m2) = Q_n [b_0 + b_1 V(m2) + b_2 V^2(m2) + b_3 V^{e1}(m2)] \quad (2.20)$$

where, P_n and Q_n are nominal real and reactive power respectively and $V(m2)$ is the voltage at node $m2$.

For all the loads, Eq.(19) and Eq.(20) are modeled as

$$a_0 + a_1 + a_2 + a_3 = 1.0 \quad (2.21)$$

$$b_0 + b_1 + b_2 + b_3 = 1.0 \quad (2.22)$$

For constant power (CP) load $a_0 = b_0 = 1$ and $a_i = b_i = 0$ for $i = 1, 2, 3$. For constant current (CI) load $a_1 = b_1 = 1$ and $a_i = b_i = 0$ for $i = 0, 2, 3$. For constant impedance (CZ) load $a_2 = b_2 = 1$ and $a_i = b_i = 0$ for $i = 0, 1, 3$. Composite load modelling is combination of CP, CI and CZ. For exponential load $a_3 = b_3 = 1$ and $a_i = b_i = 0$ for $i = 0, 1, 2$ and $e1$ and $e2$ are 1.38 and 3.22 respectively.

2.6 Computation of the Critical Loading

To compute the critical values of TPL and TQL of the network, the most sensitive node of the network is identified at first. The real power load and reactive power load of this node are increased by 0.1 times of its previous value in each step and the load-flow is run. The load-flow based methods for computation of critical loading have tendency of premature collapse. The algorithm for computation of critical values of TPL (MW) and TQL (MVar) is presented below.

- Step -1 : Set $A1 = 1.0$ and $k = 1$
- Step -2 : Read system data
- Step -3 : Set $kMAX = 100$
- Step -4 : $PL(m2) = PL(m2) * A1$ and $QL(m2) = QL(m2) * A1$
- Step -5 : Identify the nodes beyond each branch using the IDENT software available in [15] discussed in **Art. 2.3**.
- Step -6 : Compute the voltage of each node using the load-flow

algorithm [15] discussed in **Art. 2.3**.

- Step-7 : Compute $L(m_2)$ at each node using Eq. (15) for $m_2 = 2, 3, 4, \dots, NB$.
- Step-8 : Identify the most sensitive node i.e., the node having the minimum value of $L(m_2)$.
- Step-9 : $A_1 = A_1 + 0.1$ and $k = k + 1$
- Step-10 : If $k < k_{MAX}$, go to Step - 4, otherwise go to Step-12
- Step-11 : Print the result i.e., value of $L(m_2)$ of most sensitive node and TPL and TQL in MW and MVA respectively.
- Step-12 : Stop

2.7 Examples

Three examples are selected to demonstrate the effectiveness of the proposed method for selection of the most sensitive node. The first example is a 29-node radial distribution network shown in Figure 2.3. Base values are 11 kV and 100 MVA. Line data and load data for 29-node radial distribution network are available in [15] shown in Appendix-A (Table A1 and A2 respectively). Table 2.3 shows the voltage stability index of all nodes of 29-node radial distribution network for constant power, constant current, constant impedance, composite and exponential load modelling. Table 2.4 shows the load-flow solution of 29-node radial distribution network for constant power, constant current, constant impedance, composite and exponential load modellings for sub-station voltage 1.0 pu

The second example is a 33-node radial distribution network shown in figure 2.4. Base values are 12.66 kV and 100 MVA. Line data and load data for 33-node are available in [16] shown in Appendix-B (Table B1 and B2 respectively) nodes and branches have been renumbered. Table 2.5 shows the voltage stability index of all nodes of 33-node radial distribution network for constant power, constant current, constant impedance, and composite and exponential load modellings. Table 2.6 shows the load-flow solution of 33-node radial distribution network for constant power, constant current, constant impedance, composite and exponential load modelling for sub-station voltage 1.0 pu.

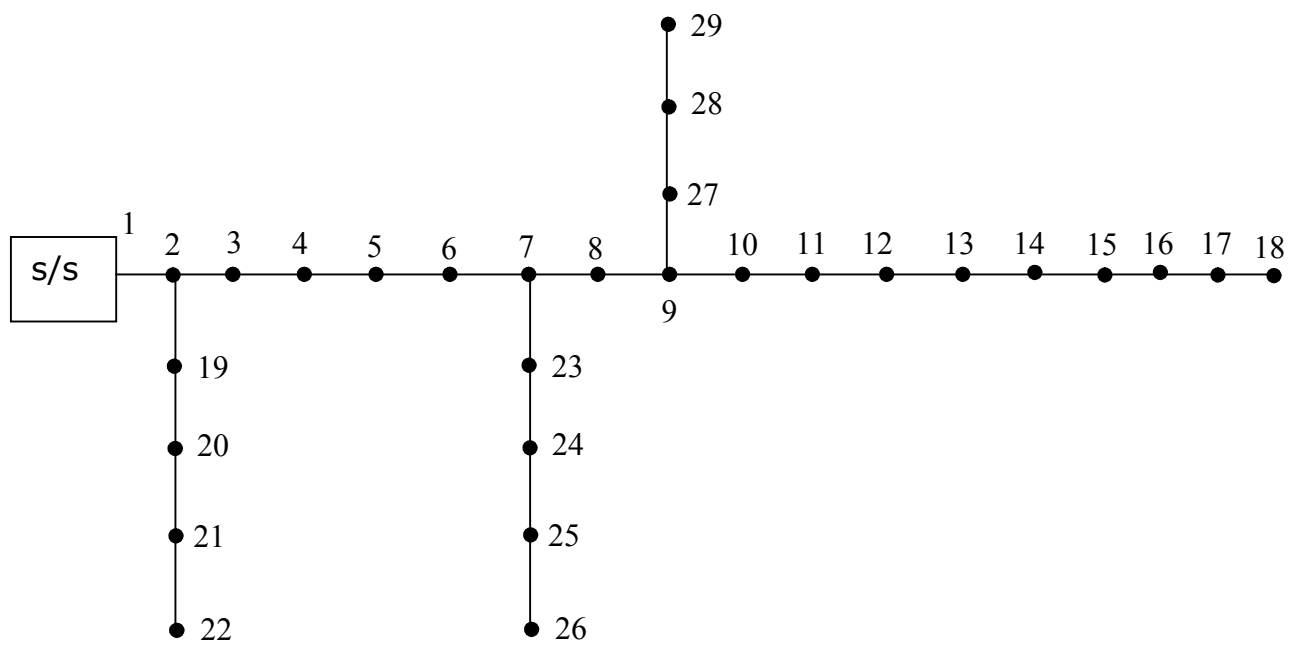


Figure 2.3 29-Node Radial Distribution Network [15]

Table 2.3 Stability Index of all nodes of 29–node radial distribution network for CP, CI, CZ, Composite and Exponential load modelling

Node Number	Stability Index of all nodes of 29–node radial distribution network for				
	Constant Power (CP) load modelling	Constant Current (CI) load modelling	Constant Impedance (CZ) load modelling	Composite load modelling	Exponential Modelling
2	0.969926	0.979019	0.982143	0.978224	0.985019
3	0.782771	0.844234	0.864600	0.838561	0.863199
4	0.627984	0.732480	0.767758	0.722533	0.760651
5	0.553883	0.678829	0.721527	0.666725	0.711616
6	0.491039	0.632712	0.681958	0.618774	0.672985
7	0.351630	0.524429	0.587428	0.506582	0.573019
8	0.282871	0.468608	0.538403	0.448779	0.520658
9	0.247230	0.438252	0.511570	0.417442	0.493490
10	0.197417	0.393888	0.472025	0.371727	0.452698
11	0.152397	0.350716	0.432968	0.327412	0.411123
12	0.127749	0.325326	0.409699	0.301445	0.386123
13	0.115846	0.312519	0.397908	0.288384	0.374201
14	0.092798	0.286343	0.373564	0.261739	0.348445
15	0.077353	0.267447	0.355786	0.242574	0.329531
16	0.069235	0.256912	0.345787	0.231923	0.318936
17	0.064037	0.249935	0.339135	0.224883	0.311942
18	0.059620	0.243791	0.333246	0.218691	0.305658
19	0.809008	0.862170	0.879585	0.857258	0.875211
20	0.789435	0.842881	0.860780	0.837945	0.856188
21	0.774242	0.827920	0.846211	0.822962	0.841387
22	0.770288	0.824028	0.842421	0.819063	0.837540
23	0.639390	0.740347	0.774295	0.730749	0.765706
24	0.616561	0.717805	0.752618	0.708165	0.743503
25	0.605810	0.707181	0.742404	0.697521	0.732993
26	0.362715	0.533054	0.594792	0.515530	0.578515
27	0.345228	0.515138	0.577832	0.497554	0.560777
28	0.338974	0.508691	0.571718	0.491088	0.554384
29	0.336969	0.506620	0.569754	0.489011	0.552336

Table 2.4 Voltage magnitude (pu) of each node for 29–node radial distribution network for CP, CI, CZ, Composite and Exponential load modelling

Node Number	Voltage magnitude (pu) of each node for				
	Constant Power (CP) load modelling	Constant Current (CI) load modelling	Constant Impedance (CZ) load modelling	Composite load modelling	Exponential Modelling
1	1.000000	1.000000	1.000000	1.000000	1.000000
2	0.948951	0.964126	0.968934	0.962754	0.967704
3	0.894888	0.928188	0.938599	0.925174	0.935946
4	0.865602	0.909556	0.923165	0.905575	0.919680
5	0.846871	0.897851	0.913544	0.893232	0.909507
6	0.776896	0.855104	0.878752	0.848010	0.872607
7	0.732658	0.829374	0.858183	0.820601	0.850674
8	0.710823	0.816937	0.848316	0.807312	0.840136
9	0.673279	0.796032	0.831863	0.784901	0.822536
10	0.627748	0.771201	0.812454	0.758199	0.801714
11	0.599103	0.755927	0.800588	0.741718	0.788687
12	0.586586	0.749365	0.795514	0.734620	0.783100
13	0.554399	0.732805	0.782780	0.716658	0.769050
14	0.528687	0.719810	0.772836	0.702526	0.758049
15	0.513709	0.712318	0.767118	0.694367	0.751710
16	0.503706	0.707357	0.763341	0.688958	0.747513
17	0.494217	0.702689	0.759793	0.683862	0.743557
18	0.491045	0.701132	0.758611	0.682162	0.742239
19	0.943040	0.958577	0.963606	0.957173	0.962300
20	0.938105	0.953953	0.959175	0.952522	0.957802
21	0.936840	0.952769	0.958041	0.951331	0.956651
22	0.935731	0.951730	0.957047	0.950286	0.955640
23	0.886417	0.920711	0.931654	0.917608	0.928803
24	0.882252	0.917041	0.928250	0.913893	0.925295
25	0.880305	0.915327	0.926662	0.912158	0.923660
26	0.766776	0.847382	0.872033	0.840063	0.865505
27	0.763072	0.844559	0.869580	0.837157	0.862909
28	0.761900	0.843667	0.868804	0.836238	0.862087
29	0.761410	0.843293	0.868480	0.835854	0.861742

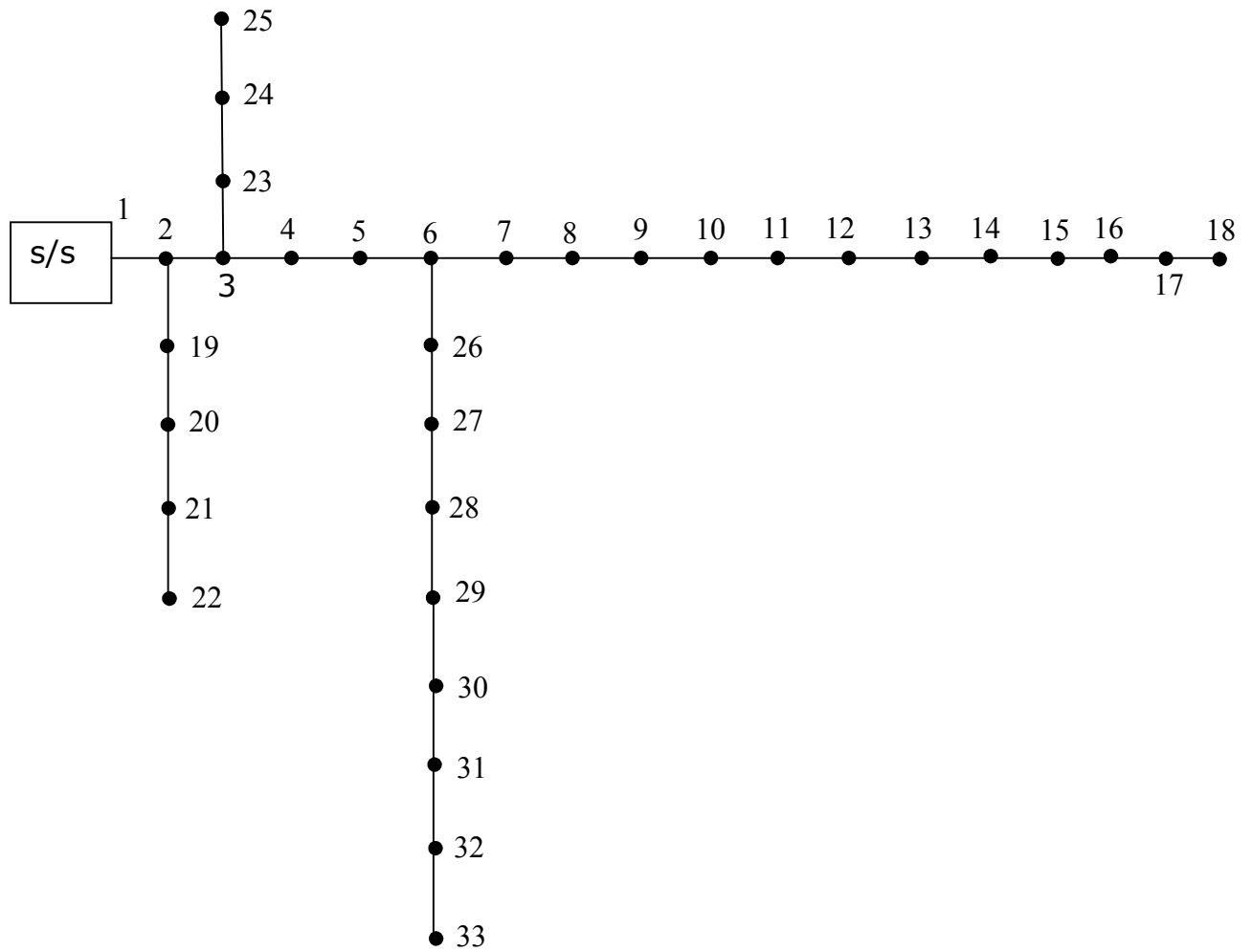


Figure 2.4 33-Node Radial Distribution Network [16]

Table 2.5 Stability Index of all nodes of 29–node radial distribution network for CP, CI, CZ, Composite and Exponential load modelling

Node Number	Stability Index of all nodes of 29–node radial distribution network for				
	Constant Power (CP) load modelling	Constant Current (CI) load modelling	Constant Impedance (CZ) load modelling	Composite load modelling	Exponential Modelling
2	0.997323	0.997486	0.997620	0.997468	0.997757
3	0.977568	0.978926	0.980032	0.978774	0.980606
4	0.926240	0.930424	0.933858	0.929956	0.933868
5	0.898201	0.904233	0.909174	0.903557	0.908995
6	0.869402	0.877321	0.883805	0.876432	0.883491
7	0.807407	0.819390	0.829205	0.818037	0.829026
8	0.799734	0.812170	0.822362	0.810766	0.822230
9	0.780270	0.793949	0.805166	0.792402	0.804879
10	0.759990	0.775011	0.787333	0.773308	0.786792
11	0.745826	0.761777	0.774867	0.759966	0.773864
12	0.742852	0.759001	0.772254	0.757167	0.771201
13	0.734291	0.751023	0.764756	0.749121	0.763845
14	0.717926	0.735753	0.750390	0.733723	0.749094
15	0.711376	0.729637	0.744632	0.727555	0.743324
16	0.707211	0.725753	0.740981	0.723638	0.739601
17	0.702225	0.721107	0.736615	0.718952	0.735228
18	0.697493	0.716693	0.732465	0.714500	0.730979
19	0.987708	0.988321	0.988825	0.988253	0.988755
20	0.983357	0.984001	0.984534	0.983929	0.984484
21	0.971528	0.972256	0.972870	0.972176	0.972791
22	0.969242	0.969988	0.970619	0.969906	0.970535
23	0.930451	0.934202	0.937292	0.933785	0.936935
24	0.916301	0.920417	0.923837	0.919962	0.923445
25	0.894957	0.899612	0.903520	0.899101	0.902952
26	0.811134	0.822882	0.832504	0.821556	0.832110
27	0.803766	0.816005	0.826030	0.814623	0.825697
28	0.779532	0.793392	0.804749	0.791822	0.805517
29	0.747716	0.763670	0.776753	0.761857	0.777722
30	0.732427	0.749383	0.763294	0.747454	0.763925
31	0.719625	0.737398	0.751987	0.735374	0.752779
32	0.709228	0.727682	0.742835	0.725578	0.743407
33	0.706711	0.725328	0.740618	0.723205	0.741174

Table 2.6 Voltage magnitude (pu) of each node for 33–node radial distribution network for CP, CI, CZ, Composite and Exponential load modelling

Node Number	Voltage magnitude (pu) of each node for				
	Constant Power (CP) load modelling	Constant Current (CI) load modelling	Constant Impedance (CZ) load modelling	Composite load modelling	Exponential Modelling
1	1.000000	1.000000	1.000000	1.000000	1.000000
2	0.997032	0.997186	0.997312	0.997169	0.997294
3	0.982939	0.983904	0.984695	0.983796	0.984578
4	0.975457	0.976943	0.978156	0.976777	0.977985
5	0.968061	0.970079	0.971723	0.969853	0.971495
6	0.949661	0.953028	0.955760	0.952650	0.955603
7	0.946175	0.949789	0.952720	0.949383	0.952650
8	0.941332	0.945305	0.948526	0.944858	0.948347
9	0.935064	0.939532	0.943150	0.939029	0.942898
10	0.929416	0.934337	0.938317	0.933782	0.938006
11	0.928557	0.933547	0.937583	0.932984	0.937251
12	0.927057	0.932170	0.936305	0.931594	0.935936
13	0.920945	0.926567	0.931108	0.925932	0.930677
14	0.918679	0.924491	0.929183	0.923834	0.928756
15	0.917267	0.923199	0.927987	0.922529	0.927540
16	0.915899	0.921949	0.926830	0.921265	0.926361
17	0.913873	0.920098	0.925118	0.919393	0.924649
18	0.913266	0.919543	0.924606	0.918833	0.924130
19	0.996504	0.996661	0.996791	0.996644	0.996772
20	0.992926	0.993111	0.993267	0.993091	0.993246
21	0.992222	0.992413	0.992574	0.992392	0.992553
22	0.991584	0.991781	0.991947	0.991759	0.991926
23	0.979353	0.980419	0.981300	0.980301	0.981168
24	0.972682	0.973941	0.974995	0.973803	0.974842
25	0.969357	0.970718	0.971862	0.970569	0.971698
26	0.947731	0.951247	0.954097	0.950852	0.953941
27	0.945167	0.948882	0.951893	0.948465	0.951739
28	0.933728	0.938346	0.942081	0.937826	0.942110
29	0.925510	0.930782	0.935042	0.930187	0.935209
30	0.921952	0.927511	0.932000	0.926884	0.932184
31	0.917791	0.923695	0.928459	0.923028	0.928631
32	0.916876	0.922856	0.927681	0.922180	0.927855
33	0.916592	0.922596	0.927440	0.921917	0.927619

The third example is a 69–node radial distribution network shown in figure 2.5 Base values are 12.66 kV and 100 MVA. Line data and load data for 69–node radial distribution network are available in ref. [17] shown in Appendix–C (Table C1 and C2 respectively) nodes and branches have been renumbered. Table 2.7, Table 2.8, Table 2.9, Table 2.10 and Table 2.11 show the voltage stability index and voltage magnitude of all nodes of 69–node radial distribution network for constant power, constant current, constant impedance, composite and exponential load modelling respectively for sub-station voltage 1.0 pu

Table 2.12 shows the most sensitive node and minimum voltage of 29–node, 33–node and 69–node radial distribution network for constant power, constant current, constant impedance, composite and exponential load modelling for sub-station voltage 1.0 pu From Table 2.12, the most sensitive node is the end node which is also the node having the minimum voltage.

Table 2.13 shows the real and reactive power losses of 29–node, 33–node and 69–node radial distribution network for constant power, constant current, constant impedance, composite and exponential load modelling for sub-station voltage 1.0 pu.

Table 2.14 shows total real and reactive power load of 29–node, 33–node and 69–node radial distribution network for constant power, constant current, constant impedance, composite and exponential load modelling for sub-station voltage 1.0 pu.

Table 2.15 shows the comparison of critical loading by the proposed method with the methods [12,13] for constant power, constant current, constant impedance, composite and exponential load modelling for substation voltage of 1.0 pu, 1.025 pu and 1.050 pu respectively. The critical loading is maximum for constant impedance load whereas it is minimum for constant power load obtained by the proposed method. The critical loading for composite load model is lower than the constant current load using the proposed method. The critical loading using the method [12] is maximum for constant current while it by the method [13] is for constant impedance.

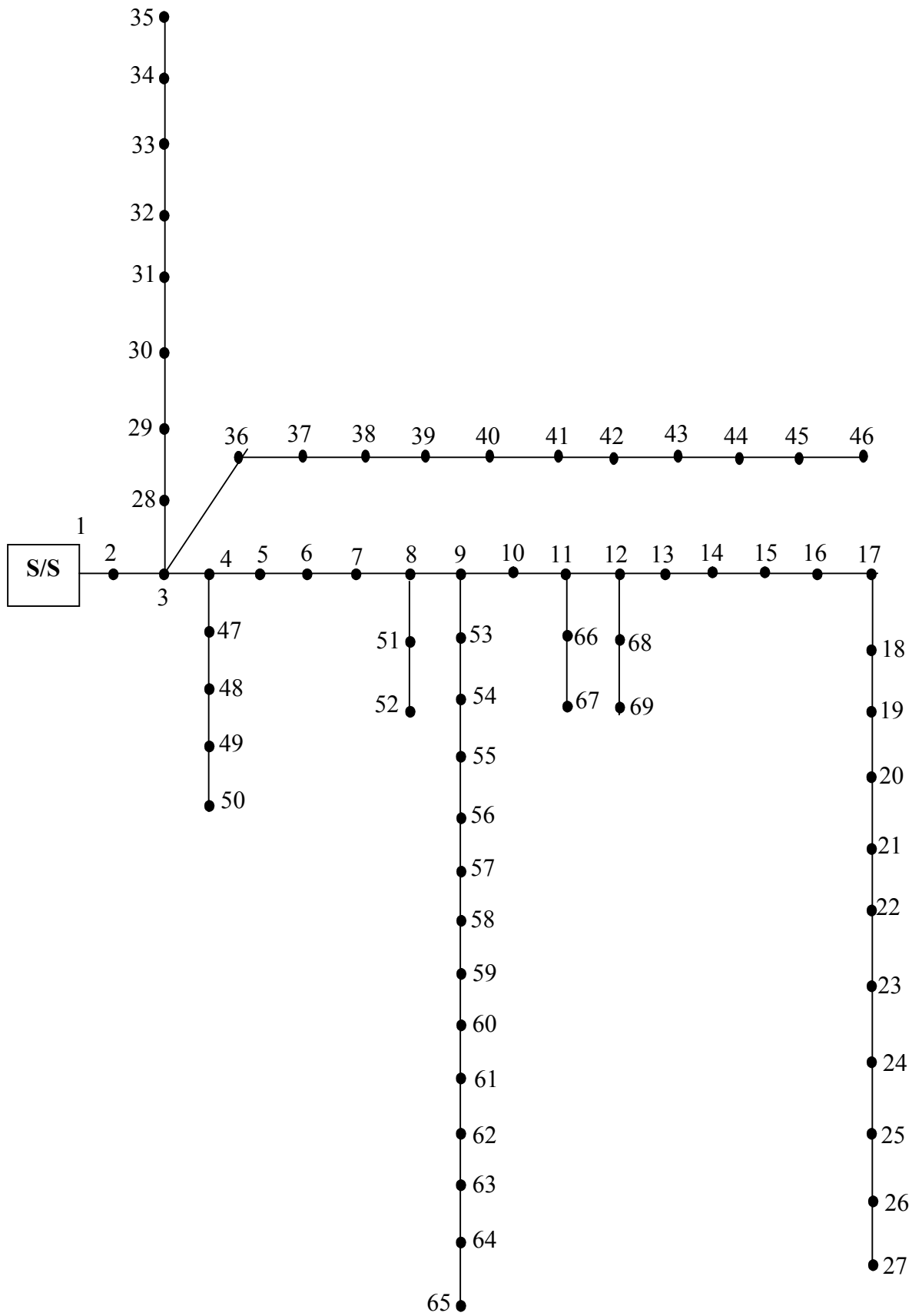


Figure 2.5 69–Node Radial Distribution Network [17]

Table 2.7 Voltage magnitude (pu) of and stability index of each node for 69–node radial distribution network for CP load modelling

Node Number	Voltage magnitude (pu)	Stability Index	Node Number	Voltage magnitude (pu)	Stability Index
1	1.000000	-----	36	0.999919	0.999701
2	0.999967	0.999916	37	0.999747	0.999316
3	0.999933	0.999788	38	0.999589	0.998707
4	0.999839	0.999553	39	0.999543	0.998289
5	0.999021	0.997900	40	0.999541	0.998170
6	0.990087	0.986897	41	0.998843	0.996956
7	0.980796	0.951732	42	0.998551	0.994867
8	0.978580	0.923243	43	0.998512	0.994153
9	0.977447	0.916709	44	0.998504	0.994047
10	0.972450	0.909492	45	0.998405	0.993938
11	0.971349	0.893694	46	0.998405	0.993636
12	0.968191	0.888902	47	0.999789	0.999230
13	0.965269	0.876880	48	0.998545	0.996267
14	0.962373	0.866351	49	0.994704	0.989220
15	0.959505	0.855973	50	0.994160	0.978981
16	0.958972	0.847306	51	0.978545	0.917031
17	0.958093	0.845263	52	0.978535	0.916901
18	0.958084	0.842609	53	0.974660	0.910105
19	0.957619	0.842278	54	0.971418	0.899356
20	0.957321	0.840757	55	0.966944	0.886287
21	0.956839	0.839791	56	0.962576	0.870106
22	0.956833	0.838212	57	0.940102	0.843569
23	0.956761	0.838145	58	0.929042	0.774212
24	0.956604	0.837888	59	0.924765	0.742544
25	0.956435	0.837281	60	0.919740	0.728603
26	0.956365	0.836778	61	0.912343	0.713809
27	0.956346	0.836557	62	0.912054	0.692633
28	0.999926	0.999719	63	0.911666	0.691656
29	0.999854	0.999595	64	0.909766	0.690445
30	0.999733	0.999326	65	0.909191	0.685039
31	0.999712	0.998917	66	0.971293	0.890210
32	0.999605	0.998767	67	0.971292	0.890021
33	0.999349	0.998294	68	0.967861	0.878592
34	0.999013	0.997340	69	0.967860	0.877509
35	0.998946	0.996059			

Table 2.8 Voltage magnitude (pu) and stability index of each node for 69–node radial distribution network for CI load modelling

Node Number	Voltage magnitude (pu)	Stability Index	Node Number	Voltage magnitude (pu)	Stability Index
1	1.000000	-----	36	0.999923	0.999714
2	0.999968	0.999920	37	0.999751	0.999330
3	0.999936	0.999799	38	0.999593	0.998722
4	0.999848	0.999579	39	0.999547	0.998304
5	0.999084	0.998030	40	0.999545	0.998185
6	0.990754	0.987766	41	0.998848	0.996973
7	0.982090	0.954930	42	0.998556	0.994887
8	0.980026	0.928279	43	0.998518	0.994174
9	0.978972	0.922150	44	0.998509	0.994068
10	0.974154	0.915307	45	0.998411	0.993960
11	0.973093	0.899997	46	0.998410	0.993659
12	0.970055	0.895360	47	0.999798	0.999264
13	0.967257	0.883736	48	0.998560	0.996317
14	0.964483	0.873590	49	0.994741	0.989309
15	0.961737	0.863587	50	0.994200	0.979124
16	0.961227	0.855230	51	0.979991	0.922461
17	0.960384	0.853259	52	0.979982	0.922334
18	0.960376	0.850701	53	0.976424	0.916034
19	0.959931	0.850382	54	0.973460	0.906156
20	0.959646	0.848916	55	0.969374	0.894139
21	0.959185	0.847985	56	0.965387	0.879253
22	0.959179	0.846463	57	0.944874	0.854815
23	0.959110	0.846398	58	0.934779	0.790695
24	0.958960	0.846151	59	0.930873	0.761289
25	0.958799	0.845566	60	0.926290	0.748310
26	0.958732	0.845081	61	0.919543	0.734581
27	0.958713	0.844868	62	0.919280	0.714779
28	0.999929	0.999732	63	0.918927	0.713868
29	0.999858	0.999608	64	0.917198	0.712742
30	0.999737	0.999340	65	0.916676	0.707702
31	0.999715	0.998931	66	0.973037	0.896619
32	0.999609	0.998781	67	0.973037	0.896434
33	0.999353	0.998309	68	0.969736	0.885384
34	0.999018	0.997356	69	0.969735	0.884329
35	0.998950	0.996076			

Table 2.9 Voltage magnitude (pu) and stability index of each node for 69–node radial distribution network for CZ load modelling

Node Number	Voltage magnitude (pu)	Stability Index	Node Number	Voltage magnitude (pu)	Stability Index
1	1.000000	-----	36	0.999925	0.999725
2	0.999969	0.999923	37	0.999754	0.999341
3	0.999939	0.999807	38	0.999596	0.998733
4	0.999854	0.999598	39	0.999550	0.998316
5	0.999134	0.998131	40	0.999548	0.998197
6	0.991284	0.988432	41	0.998852	0.996987
7	0.983120	0.957456	42	0.998561	0.994905
8	0.981176	0.932297	43	0.998522	0.994193
9	0.980186	0.926498	44	0.998514	0.994087
10	0.975523	0.919960	45	0.998416	0.993979
11	0.974496	0.905088	46	0.998415	0.993678
12	0.971564	0.900587	47	0.999805	0.999291
13	0.968873	0.889316	48	0.998573	0.996360
14	0.966206	0.879510	49	0.994775	0.989390
15	0.963565	0.869839	50	0.994237	0.979259
16	0.963074	0.861761	51	0.981142	0.926801
17	0.962264	0.859855	52	0.981133	0.926676
18	0.962256	0.857382	53	0.977825	0.920762
19	0.961829	0.857073	54	0.975078	0.911571
20	0.961555	0.855656	55	0.971296	0.900382
21	0.961112	0.854756	56	0.967607	0.886522
22	0.961106	0.853285	57	0.948637	0.863714
23	0.961040	0.853223	58	0.939299	0.803846
24	0.960896	0.852984	59	0.935687	0.776298
25	0.960741	0.852418	60	0.931450	0.764107
26	0.960677	0.851950	61	0.925213	0.751252
27	0.960659	0.851744	62	0.924969	0.732586
28	0.999932	0.999743	63	0.924644	0.731730
29	0.999860	0.999619	64	0.923049	0.730674
30	0.999740	0.999350	65	0.922567	0.725933
31	0.999718	0.998942	66	0.974443	0.901804
32	0.999611	0.998792	67	0.974442	0.901623
33	0.999356	0.998321	68	0.971254	0.890910
34	0.999021	0.997368	69	0.971253	0.889881
35	0.998954	0.996089			

Table 2.10 Voltage magnitude (pu) and stability index of each node for 69–node radial distribution network for Composite load modelling

Node Number	Voltage magnitude (pu)	Stability Index	Node Number	Voltage magnitude (pu)	Stability Index
1	1.000000	-----	36	0.999922	0.999713
2	0.999968	0.999920	37	0.999751	0.999329
3	0.999936	0.999798	38	0.999592	0.998721
4	0.999847	0.999576	39	0.999546	0.998303
5	0.999077	0.998014	40	0.999544	0.998184
6	0.990680	0.987655	41	0.998848	0.996971
7	0.981946	0.954560	42	0.998556	0.994885
8	0.979865	0.927716	43	0.998517	0.994172
9	0.978803	0.921544	44	0.998509	0.994066
10	0.973966	0.914659	45	0.998410	0.993958
11	0.972901	0.899302	46	0.998410	0.993657
12	0.969851	0.894648	47	0.999797	0.999260
13	0.967040	0.882983	48	0.998558	0.996311
14	0.964254	0.872798	49	0.994737	0.989299
15	0.961495	0.862756	50	0.994196	0.979109
16	0.960983	0.854368	51	0.979830	0.921857
17	0.960136	0.852390	52	0.979821	0.921730
18	0.960128	0.849822	53	0.976228	0.915369
19	0.959681	0.849501	54	0.973232	0.905392
20	0.959395	0.848030	55	0.969103	0.893254
21	0.958932	0.847095	56	0.965073	0.878221
22	0.958925	0.845568	57	0.944345	0.853522
23	0.958856	0.845503	58	0.934143	0.788838
24	0.958706	0.845255	59	0.930197	0.759189
25	0.958543	0.844667	60	0.925565	0.746103
26	0.958476	0.844180	61	0.918746	0.732258
27	0.958457	0.843967	62	0.918479	0.712299
28	0.999929	0.999731	63	0.918122	0.711380
29	0.999857	0.999607	64	0.916374	0.710244
30	0.999736	0.999338	65	0.915846	0.705162
31	0.999715	0.998930	66	0.972846	0.895913
32	0.999608	0.998780	67	0.972845	0.895728
33	0.999352	0.998307	68	0.969531	0.884639
34	0.999017	0.997354	69	0.969530	0.883582
35	0.998950	0.996074			

Table 2.11 Voltage magnitude (pu) and stability index of each node for 69–node radial distribution network for Exponential load modelling

Node Number	Voltage magnitude (pu)	Stability Index	Node Number	Voltage magnitude (pu)	Stability Index
1	1.000000	-----	36	0.999926	0.999730
2	0.999970	0.999927	37	0.999755	0.999346
3	0.999940	0.999813	38	0.999597	0.998739
4	0.999857	0.999614	39	0.999551	0.998322
5	0.999146	0.998230	40	0.999549	0.998203
6	0.991237	0.989025	41	0.998854	0.996994
7	0.983011	0.957827	42	0.998562	0.994912
8	0.981052	0.932012	43	0.998524	0.994199
9	0.980054	0.926041	44	0.998516	0.994093
10	0.975348	0.919583	45	0.998417	0.993985
11	0.974312	0.904462	46	0.998417	0.993685
12	0.971351	0.899962	47	0.999808	0.999304
13	0.968629	0.888613	48	0.998580	0.996391
14	0.965931	0.878701	49	0.994793	0.989450
15	0.963260	0.868928	50	0.994256	0.979329
16	0.962764	0.860684	51	0.981018	0.926333
17	0.961945	0.858766	52	0.981009	0.926207
18	0.961936	0.856243	53	0.977671	0.920467
19	0.961505	0.855947	54	0.974897	0.911225
20	0.961227	0.854510	55	0.971077	0.900030
21	0.960780	0.853597	56	0.967352	0.886035
22	0.960773	0.852106	57	0.947998	0.863937
23	0.960707	0.852045	58	0.938466	0.802208
24	0.960561	0.851804	59	0.934777	0.773734
25	0.960404	0.851236	60	0.930440	0.761351
26	0.960339	0.850758	61	0.924133	0.748091
27	0.960321	0.850549	62	0.923887	0.729190
28	0.999933	0.999748	63	0.923558	0.728335
29	0.999862	0.999624	64	0.921944	0.727271
30	0.999741	0.999355	65	0.921457	0.722465
31	0.999719	0.998947	66	0.974258	0.901124
32	0.999613	0.998797	67	0.974258	0.900941
33	0.999357	0.998325	68	0.971039	0.890131
34	0.999022	0.997373	69	0.971038	0.889090
35	0.998955	0.996093			

Table 2.12 Stability Index of the most sensitive node and magnitude of minimum voltage of 29–node, 33–node and 69–node radial distribution network for CP, CI, CZ, Composite and Exponential load modelling for substation voltage of 1.0

Type of load modelling	29–node radial distribution network		33–node radial distribution network		69–node radial distribution network	
	Stability Index of most sensitive node (Node No. = 18)	Magnitude of Minimum Voltage in pu (Node No. = 18)	Stability Index of most sensitive node (Node No. = 18)	Magnitude of Minimum Voltage in pu (Node No. = 18)	Stability Index of most sensitive node (Node No. = 65)	Magnitude of Minimum Voltage in pu (Node No. = 65)
Constant Power	0.059620	0.491045	0.697493	0.913266	0.685039	0.909191
Constant Current	0.243791	0.701132	0.716693	0.919543	0.707702	0.916676
Constant Impedance	0.333246	0.758611	0.732465	0.924606	0.725933	0.922567
Composite load	0.218691	0.682162	0.714500	0.918833	0.705162	0.915846
Exponential load	0.305658	0.742239	0.730979	0.924130	0.722465	0.921457

Table 2.13 Real power loss and Reactive Power Loss of 29–node, 33–node and 69–node radial distribution network for CP, CI, CZ, Composite and Exponential load modelling for substation voltage of 1.0 pu

Type of load modelling	29–node radial distribution network		33–node radial distribution network		69–node radial distribution network	
	Real power loss (kW)	Reactive power loss (kVAr)	Real power loss (kW)	Reactive power loss (kVAr)	Real power loss (kW)	Reactive power loss (kVAr)
Constant Power	867.98	346.33	202.52	135.12	224.93	102.13
Constant Current	334.08	134.72	176.51	117.51	191.44	87.76
Constant Impedance	228.09	92.22	156.77	104.18	167.11	77.30
Composite load	371.51	149.63	179.34	119.43	195.11	89.34
Exponential load	251.89	101.75	157.33	104.55	168.05	77.70

Table 2.14 Total Real power load and Reactive Power Load of 29–node, 33–node and 69–node radial distribution network for CP, CI, CZ, Composite and Exponential load modelling for substation voltage of 1.0 pu

Type of load modelling	29–node radial distribution network		33–node radial distribution network		69–node radial distribution network	
	Real power load (kW)	Reactive power load (kVAr)	Real power load (kW)	Reactive power load (kVAr)	Real power load (kW)	Reactive power load (kVAr)
Constant Power	1929.99	1079.99	3714.99	2300.00	3801.89	2692.59
Constant Current	1615.49	907.65	3543.34	2181.05	3633.45	2573.05
Constant Impedance	1456.20	820.15	3400.46	2082.75	3495.96	2475.52
Composite load	1654.25	928.83	3562.45	2194.40	3651.92	2586.17
Exponential load	1565.45	686.19	3493.52	1962.48	3583.79	2352.71

Table 2.15 Comparison of critical loadings for CP, CI, CZ, Composite and Exponential load modelling obtained by the proposed method and by the methods [12] and [13]

Type of load	S/S Voltage (pu)	Critical loading condition											
		Proposed Method				Method in ref. [13]				Method in ref. [12]			
		TPL (MW)	TQL (MVA _r)	SI _{min} = SI ₆₅	V _{min} = V ₆₅ (pu)	TPL (MW)	TQL (MVA _r)	SI _{min} = SI ₆₅	V _{min} = V ₆₅ (pu)	TPL (MW)	TQL (MVA _r)	SI _{min} = SI ₆₅	V _{min} = V ₆₅ (pu)
Constant Power (CP)	1.000	12242.08	8670.17	0.047185	0.462556	13.304	9.429	0.03810	0.29544	12.212	8.654	0.0491	0.4708
	1.025	12861.79	9109.06	0.055758	0.482388	14.344	10.102	0.06377	0.37887	12.767	9.047	0.0728	0.5194
	1.050	13498.23	9559.80	0.057811	0.486662	14.454	10.237	0.08979	0.41941	13.422	9.537	0.0756	0.5244
Constant Current (CI)	1.000	17468.20	12360.58	0.044151	0.455106	16.809	11.986	0.07366	0.38094	15.051	10.656	0.1044	0.5728
	1.025	18352.34	12986.20	0.048739	0.466494	17.583	12.708	0.08821	0.41712	15.812	11.199	0.1152	0.5826
	1.050	19257.05	13626.37	0.053727	0.477999	18.297	13.019	0.09214	0.42482	16.594	11.752	0.1269	0.5969
Constant Impedance (CZ)	1.000	25817.59	18279.58	0.042500	0.451028	18.005	12.757	0.08862	0.41901	14.055	9.954	0.2195	0.6845
	1.025	27130.80	19209.41	0.046561	0.461424	18.916	13.403	0.09311	0.42949	14.764	10.458	0.2423	0.7016
	1.050	28463.30	20152.81	0.051723	0.473728	19.850	14.065	0.09771	0.43997	15.492	10.974	0.2669	0.7188
Composite Load	1.000	15229.20	10779.20	0.047605	0.463741	-----	-----	-----	-----	14.651	10.377	0.0745	0.5224
	1.025	16084.99	11384.84	0.052484	0.475194	-----	-----	-----	-----	15.468	10.956	0.0818	0.5349
	1.050	16968.12	12009.81	0.057706	0.486598	-----	-----	-----	-----	16.244	11.506	0.0929	0.5521
Exponential Load	1.000	23199.77	11149.70	0.051879	0.474306	18.158	8.523	0.07861	0.39425	<i>14.39</i>	<i>8.045</i>	<i>0.2079</i>	<i>0.6786</i>
	1.025	24237.20	12177.63	0.057694	0.487080	19.628	9.866	0.07912	0.39549	<i>15.67</i>	<i>9.063</i>	<i>0.2199</i>	<i>0.6954</i>
	1.050	25308.03	13276.79	0.063769	0.499429	20.377	10.708	0.08561	0.41142	<i>16.84</i>	<i>10.098</i>	<i>0.2409</i>	<i>0.7010</i>

3.1 Conclusions

A new formula of voltage stability index is proposed in this thesis work to compute the most sensitive node of radial distribution network without reducing the network. In this proposed method, the most sensitive node and the node having the minimum voltage are identical that have been demonstrated by three examples 29–node [15], 33–node [16] and 69–node [17] radial distribution network for constant power, constant current, constant impedance, composite and exponential load modelling. The critical loadings of the 69–node radial distribution network [17] have been found out for constant power, constant current, constant impedance, composite and exponential load modelling for substation voltage of 1.0 pu, 1.025 pu and 1.050 pu and the results obtained by the proposed method has been compared with the methods [12,13] available in literature. The comparison shows that the critical loading by the proposed method is superior to that obtained by methods [12,13]. The critical loading obtained by the proposed method is maximum for constant impedance load and minimum for constant power load.

3.2 Future Scope of Work

After carrying this work in voltage stability analysis of distribution systems, the following guidelines seem to be worth pursuing in this area:

1. Fuzzy voltage stability analysis.
2. Voltage stability analysis using bifurcation technique.

References

- [1] T.V. Cuseum and R. Mailhot, "Validation of a Fast Voltage Stability Analysis Method on Hydro-Quebec System", IEEE Transactions on Power Systems, Vol. 12, No. 1, pp. 282–292, 1997.
- [2] P. Kundur, "Power System Stability and Control", McGraw Hill, New York, Vol. 1, 1994
- [3] C.W. Taylor, "Power System Voltage Stability", McGraw Hill, New York, Vol. 2, 1994
- [4] B. Gao, G.K. Morison and P. Kundur, "Voltage Stability Evaluation Using Modal Analysis", IEEE Transactions on Power Systems, Vol.7, No.4, pp.1529 –1542, 1992.
- [5] W.C. Merrit, C.H. Taylor, R.C. Burchett and H.H. Happ, "Security Constrained Optimization – a Case Study", IEEE Transactions on Power Systems, Vol.3, No.3, pp.970 –977, 1988.
- [6] V. Ajarapu and C. Christy, "The Continuation of Power Flow: A Tool for Steady State Voltage Stability", IEEE Transactions on Power Systems, Vol.7, No.1, pp. 416–423, 1992.
- [7] G. Brownell and H. Clark, "Analysis and Solutions for Bulk System Voltage Instability" , IEEE Computer Applications in Power, Vol. 2, No. 3, pp. 31–35, 1989.
- [8] G.B. Jasmon and L.H.C.C. Lee, "Distribution Network Reduction for Voltage Stability Analysis and Load Flow Calculations", International Journal of Electrical Power and Energy Systems, Vol.13, No. 1, pp. 9 –13, 1991.
- [9] A.M. Chebbo, M.J.H. Sterling and M.R. Irving, "Reactive Power Dispatch Incorporating Voltage Stability", Proceedings IEE Part C (GTD), Vol. 139, No. 3, pp. 253 –260, 1992.
- [10] T.K.A. Rahman and G.B. Jasmon, "A New Technique for Voltage Stability Analysis in a Power System and Improved Load Flow Algorithm for Distribution Network", International Conference on Energy Management and Power Delivery, Proceedings of EMPD, Vol. 2, pp. 21–23, 1995.

- [11] F. Gubina and B. Strmcnik, “ A Simple Approach to Voltage Stability Assessment in Radial Networks”, IEEE Transactions on Power Systems, Vol. 12, No. 3, pp. 1121 –1128, 1997.
- [12] M. Chakravorty and D. Das, “Voltage Stability Analysis of Radial Distribution Networks”, International Journal of Electrical Power and Energy Systems, Vol. 23, No. 2, pp. 129–135, 2001.
- [13] R. Ranjan, B. Venkatesh and D. Das, “Voltage Stability Analysis of Radial Distribution Networks”, International Journal of Electric Power Components and Systems, Vol. 31, No. 2, pp. 501–11, 2004.
- [14] D. Das, D.P. Kothari and A. Kalam, “Simple and Efficient Method for Load Flow Solution of Radial Distribution Networks”, International Journal of Electrical Power & Energy Systems, Vol. 17, No. 5, pp. 335–346, 1995.
- [15] S. Ghosh and D. Das, “Method for Load–Flow Solution of Radial Distribution Networks”, IEE Part C (GTD), Vol. 146, No. 6, pp. 641 – 648, 1999.
- [16] M.E. Baran and F.F. Wu, “Network Reconfiguration in Distribution Systems for Loss Reduction and Load Balancing”, IEEE Transactions on Power Delivery; Vol. 4, No. 2, pp. 1401 – 1407, 1989.
- [17] M.E. Baran and F.F. Wu, “Optimal Sizing of Capacitors Placed on a Radial Distribution System”, IEEE Transactions on Power Delivery, Vol. 4, No. 1, pp. 735 – 743, 1989.
- [18] Zechun Hu and Xifan Wang, “Efficient Computation of Maximum Loading Point by Load Flow Method with Optimal Multiplier”, IEEE Transactions on Power Delivery, Vol. 23, No. 2, pp. 804 – 806, 2008.

APPENDIX A

Table A.1 Line Data of 29–Node Radial Distribution Network [15]

Branch Number	Sending end Node	Receiving end Node	Branch resistance (Ω)	Branch reactance (Ω)
1	1	2	0.151116	0.147893
2	2	3	0.427420	0.418330
3	3	4	0.477870	0.467677
4	4	5	0.427420	0.418303
5	5	6	2.050099	0.580464
6	6	7	0.641129	0.627455
7	7	8	0.477870	0.467677
8	8	9	0.623066	0.609777
9	9	10	1.244177	0.518680
10	10	11	1.798056	0.509101
11	11	12	2.274380	0.643967
12	12	13	0.927017	0.625792
13	13	14	1.798056	0.509101
14	14	15	0.675810	0.661396
15	15	16	0.983608	0.410053
16	16	17	1.190166	0.496164
17	17	18	0.453347	0.443678
18	2	19	1.798056	0.509101
19	19	20	1.356999	0.565714
20	20	21	2.050099	0.580464
21	21	22	0.927017	0.625792
22	7	23	1.190166	0.496164
23	23	24	1.356999	0.565714
24	24	25	1.919086	0.800040
25	25	26	3.061977	0.866967
26	9	27	5.085669	1.439954
27	27	28	6.150296	1.741392
28	28	29	2.274380	0.643967

Table A.2 Load Data of 29–Node Radial Distribution Network [15]

Node Number	PL (kW)	QL (kVAr)
1(S/S)	0.0	0.0
2	37.50	33.07
3	12.00	10.58
4	18.75	16.54
5	37.50	33.07
6	12.00	10.58
7	18.75	16.53
8	12.00	10.58
9	47.25	41.67
10	37.50	33.07
11	37.50	33.07
12	12.00	10.58
13	12.00	10.58
14	18.75	16.53
15	37.50	33.07
16	47.25	41.67
17	75.00	66.14
18	37.50	33.07
19	37.50	33.07
20	37.50	33.07
21	37.50	33.07
22	18.75	16.53
23	18.75	16.53
24	75.00	66.14
25	47.25	41.67
26	18.75	16.53
27	18.75	16.53
28	18.75	16.53
29	37.50	33.07

BASE kV = 11 and BASE MVA = 100

APPENDIX B

Table B.1 Line Data of 33–Node Radial Distribution Network [16]

Branch Number	Sending end Node	Receiving end Node	Branch resistance (Ω)	Branch reactance (Ω)
1	1	2	0.0922	0.0470
2	2	3	0.4930	0.2511
3	3	4	0.3660	0.1864
4	4	5	0.3811	0.1941
5	5	6	0.8190	0.7070
6	6	7	0.1872	0.6188
7	7	8	0.7114	0.2351
8	8	9	1.0300	0.7400
9	9	10	1.0040	0.7400
10	10	11	0.1996	0.0650
11	11	12	0.3744	0.1238
12	12	13	1.4680	1.1550
13	13	14	0.5416	0.7129
14	14	15	0.5910	0.5260
15	15	16	0.7463	0.5450
16	16	17	1.2890	1.7210
17	17	18	0.7320	0.5740
18	2	19	0.1640	0.1565
19	19	20	1.5042	1.3554
20	20	21	0.4095	0.4784
21	21	22	0.7089	0.9373
22	3	23	0.4512	0.3083
23	23	24	0.8980	0.7091
24	24	25	0.8960	0.7011
25	6	26	0.2030	0.1034
26	26	27	0.2842	0.1447
27	27	28	1.0590	0.9337
28	28	29	0.8042	0.7006
29	29	30	0.5075	0.2585
30	30	31	0.9744	0.9630
31	31	32	0.3105	0.3619
32	32	33	0.3410	0.5302

Table B.2 Load Data of 33–Node Radial Distribution Network [16]

Node Number	PL (kW)	QL (kVAr)
1(S/S)	0.0	0.0
2	100.0	60.0
3	90.0	40.0
4	120.0	80.0
5	60.0	30.0
6	60.0	20.0
7	200.0	100.0
8	200.0	100.0
9	60.0	20.0
10	60.0	20.0
11	45.0	30.0
12	60.0	35.0
13	60.0	35.0
14	120.0	80.0
15	60.0	10.0
16	60.0	20.0
17	60.0	20.0
18	90.0	40.0
19	90.0	40.0
20	90.0	40.0
21	90.0	40.0
22	90.0	40.0
23	90.0	50.0
24	420.0	200.0
25	420.0	200.0
26	60.0	25.0
27	60.0	25.0
28	60.0	20.0
29	120.0	70.0
30	200.0	600.0
31	150.0	70.0
32	210.0	100.0
33	60.0	40.0

BASE kV = 12.66 and BASE MVA = 100

APPENDIX C

Table C.1 Line Data of 69–Node Radial Distribution Network [17]

Branch Number	Sending–end	Receiving–end	Resistance (Ω)	Reactance (Ω)
1	1	2	0.0005	0.0012
2	2	3	0.0005	0.0012
3	3	4	0.0015	0.0036
4	4	5	0.0251	0.0294
5	5	6	0.3660	0.1864
6	6	7	0.3811	0.1941
7	7	8	0.0922	0.0470
8	8	9	0.0493	0.0257
9	9	10	0.8190	0.2707
10	10	11	0.1872	0.0619
11	11	12	0.7114	0.2351
12	12	13	1.0300	0.3400
13	13	14	1.0440	0.3450
14	14	15	1.0580	0.3496
15	15	16	0.1966	0.0650
16	16	17	0.3744	0.1238
17	17	18	0.0047	0.0016
18	18	19	0.3276	0.1083
19	19	20	0.2106	0.0696
20	20	21	0.3416	0.1129
21	21	22	0.0140	0.0046
22	22	23	0.1591	0.0526
23	23	24	0.3463	0.1145
24	24	25	0.7488	0.2475
25	25	26	0.3089	0.1021
26	26	27	0.1732	0.0572
27	3	28	0.0044	0.0108
28	28	29	0.0640	0.1565
29	29	30	0.3978	0.1315
30	30	31	0.0702	0.0232
31	31	32	0.3510	0.1160
32	32	33	0.8390	0.2816
33	33	34	1.7080	0.5646
34	34	35	1.4740	0.4873
35	3	36	0.0044	0.0108
36	36	37	0.0640	0.1565
37	37	38	0.1053	0.1230
38	38	39	0.0304	0.0355
39	39	40	0.0018	0.0021
40	40	41	0.7283	0.8509
41	41	42	0.3100	0.3623

Continued...

42	42	43	0.0410	0.0478
43	43	44	0.0092	0.0116
44	44	45	0.1089	0.1373
45	45	46	0.0009	0.0012
46	4	47	0.0034	0.0084
47	47	48	0.0851	0.2083
48	48	49	0.2898	0.7091
49	49	50	0.0822	0.2011
50	8	51	0.0928	0.0473
51	51	52	0.3319	0.1114
52	9	53	0.1740	0.0886
56	53	54	0.2030	0.1034
53	54	55	0.2842	0.1447
54	55	56	0.2813	0.1433
55	56	57	1.5900	0.5337
56	57	58	0.7837	0.2630
57	58	59	0.3042	0.1006
58	59	60	0.3861	0.1172
59	60	61	0.5075	0.2585
60	61	62	0.0974	0.0496
61	62	63	0.1450	0.0738
62	63	64	0.7105	0.3619
63	64	65	1.0410	0.5302
64	11	66	0.2012	0.0611
65	66	67	0.0047	0.0014
67	12	68	0.7394	0.2444
68	68	69	0.0047	0.0016

Table C.2 Load Data of 69–Node Radial Distribution Network [17]

Node Number	PL(kW)	QL(kVAr)	Node Number	PL(kW)	QL(kVAr)
1	00.00	00.00	36	26.00	18.55
2	00.00	00.00	37	26.00	18.55
3	00.00	00.00	38	00.00	00.00
4	00.00	00.00	39	24.00	17.00
5	00.00	00.00	40	24.00	17.00
6	2.600	2.200	41	1.200	1.000
7	40.40	30.00	42	00.00	00.00
8	75.00	54.00	43	6.000	4.300
9	30.00	22.00	44	00.00	00.00
10	28.00	19.00	45	39.22	26.30
11	145.0	104.0	46	39.22	26.30
12	145.0	104.0	47	00.00	00.00
13	8.000	5.000	48	79.00	56.40
14	8.000	5.500	49	384.7	274.0
15	00.00	00.00	50	384.7	274.0
16	45.50	30.00	51	40.50	28.30
17	60.00	35.00	52	3.600	2.700
18	60.00	35.00	53	4.350	3.500
19	00.00	00.00	54	26.40	19.00
20	1.000	00.60	55	26.00	17.20
21	114.0	81.00	56	00.00	00.00
22	5.000	3.500	57	00.00	00.00
23	00.00	00.00	58	00.00	00.00
24	28.00	20.00	59	100.0	72.00
25	00.00	00.00	60	00.00	00.00
26	14.00	10.00	61	1244.0	888.0
27	14.00	10.00	62	32.00	23.00
28	26.00	18.60	63	00.00	00.00
29	26.00	18.60	64	227.0	162.0
30	00.00	00.00	65	59.00	42.00
31	00.00	00.00	66	18.00	13.00
32	00.00	00.00	67	18.00	13.00
33	14.00	10.00	68	28.00	20.00
34	19.50	14.00	69	28.00	20.00
35	6.000	4.000			

BASE kV = 12.66 and BASE MVA = 100

APPENDIX D

Biography of Candidate

Personal Information

Name Arminder Singh
D.O.B. 01/11/1984
Address House No. 317, ward no. 3
 College back side, Sobha Singh Nagar
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Academic Qualification

M.E in Power System and Electrical Drives from Thapar University, securing 7.92 CGPA.

B.Tech in Electrical Engg. from Sri Sukhmani Institute of Engg. & Tech., Derabassi Affiliated to PTU, Jalandhar in June 2006 securing 79.2%.

Higher Secondary from Punjab Education Board in 2001 securing 63.56 %.

High School from Punjab Education Board in 1999 securing 75.38%.

Campus Placements

TRIDENT Group

Ballarpur Industries Ltd.

Paper Communicated

S.Ghosh and A. Singh, “A New Method for Voltage Stability Analysis of Radial Distribution Networks”, communicated to International Journal of Electrical Power and Energy Systems, **Paper Number : IJEPES-D-08-00108.**