STUDY OF RAYLEIGH LAMB WAVE PROPAGATION IN ELASTIC AND MICROPOLAR SOLIDS

Thesis submitted in partial fulfilment of the requirements

for the award of the degree of

Masters of Science

in

Mathematics and Computing

Submitted by

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to the

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July 2012

DEDICATED

TO

GOD, MY PARENTS AND MY TEACHERS

CERTIFICATE

I hereby certify that the work which is being presented in this thesis entitled "Study of Rayleigh-Lamb wave propagation in Elastic and Micropolar solids" which is being submitted for the award of degree of Master of Sciences, School of Mathematics and Computer Applications, Thapar University, Patiala, is an authentic record of my own work carried out under the supervision of ***Dr. Satish Kumar Sharma and Dr. Abhishek Kumar Singh.***

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

Gurneel Kaur Sohi

This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

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ACKNOWLEDGEMENT

It gives me immense pleasure to acknowledge my sincere gratitude to my academic supervisors, Dr. Satish Kumar Sharma and Dr. Abhishek Kumar Singh, School of Mathematics and Computer Applications, Thapar University, Patiala, for their constant help, encouragement and support throughout the course of this work.

I would like to extend my special thanks to Dr. S.S. Bhatia, Professor and Head, School of Mathematics and Computer Applications, Thapar University, Patiala, for providing help and necessary facilities in the department and directly and indirectlyencouraged me to be a successful one and also helped me when I was in difficulties.

I express my deepest gratitude to my parents, my sister and my friend Anu Punj for their unconditional support in all spheres of my life.

Gurneel Kaur Sohi

ABSTRACT

The Classical theory of Elasticity is unable to explain the behaviour of the materials having complex micro structures such as polymers, porous media, fibers etc. To model such materials Eringen (1964) gave a theory which assumes that every particle of a material may do micro rotation in addition to macro translation. Researchers (Fischer
(1982)), Fatemi et al.(2002) have even applied this theory to model the complex structures of bones.

Lamb waves (1917), which propagate in solid plates with free boundaries, are elastic perturbations for which displacements occur both in the direction of wave propagation and perpendicular to the plane of plate. These waves are commonly used in ultrasonic Non Destructive Testing applications including material characterization of elastic plates (W.P.Rogers(1995)), bonding inspection (Wu and Liu(1999)), coating inspection (Lee and Cheng(2001)), defect inspection (Wu and Liu(1999)) and thickness measurement of thin films (W.P.Rogers(1995)).

In the present work, a mathematical analysis has been done to study Lamb waves in Elastic and Micropolar plates.

The thesis comprises three chapters. Brief outline of the research work presented chapter wise in this thesis is as follows:

Chapter 1 – This chapter includes the discussion of Classical Theory of Elasticity which deals with the materials in which molecular structure is neglected. The chapter also includes a brief history of Elasticity, shortcomings of Classical theory of Elasticity and development of Micropolar Elasticity. Different types of waves, their propagation and various applications are also discussed.

Chapter 2 – It deals with the propagation of Rayleigh Lamb waves in homogeneous isotropic elastic plate. The secular equations which govern the propagation of plate waves are derived and discussed. The variation of Phase velocity with respect to the wave number has been studied for both symmetric and anti symmetric modes. Here I have reviewed the work carried by Graff (1991) in his book, “Wave motion in elastic solids”.

Chapter 3 – Rayleigh Lamb waves are studied in the context of Micropolar theory of Elasticity. Effects of micropolarity have been observed on symmetric and anti symmetric modes. Results for elastic plates have been deduced as a particular case. In this chapter following papers have been reviewed under different conditions.

(1). Sharma J. and Kumar S. (2009), "Lamb Waves in thermoelastic solid plates immersed in liquid with varying temperature", Meccanica, 44: 305-319 in the absence of liquid loading and thermal effects.

(2). Kumar R. and Partap G. (2006), "Rayleigh Lamb waves in Micropolar Isotropic Elastic Plate", Applied Mathematics and Mechanics, 26(8): 1049-1059 in a rectangular plate.

CONTENTS

List of Symbols

σ_{ij}	Stress tensor
e_{ij}	Strain tensor
m_{ij}	Couple stress tensor
δ_{ij}	Kronecker's delta
ε_{ijk}	Permutation symbol
λ	Lame's constant
μ	Lame's constant
$\vec{u}$	Displacement vector
d	Thickness of the plate
ρ	Density of the material
ω	Frequency
ξ	Wave number
c	Phase velocity
$\vec{\phi}$	Micro rotation vector
q,ψ	Potential functions
α, β, γ, K	Micropolar constants

Chapter 1

INTRODUCTION

1.1 Classical Theory Of Elasticity

Classical Theory of Elasticity explains the deformation of a body in terms of symmetric tensors of stress and strain. In this model, the material points are simply geometric points and molecular structure of material is ignored. This theory gave similar type of results for materials like steel, aluminium, concrete etc. It establishes a mathematical model which shall be practically important in applications to architecture, engineering, and all other useful arts in which the material of construction is solid.

1.1.1 Historical Introduction

The first Mathematician to consider the nature of the resistance of solids to rupture was Galileo. Although he treated solids as inelastic, not being in possession of any law connecting the displacements produced with the forces producing them, or of any physical hypothesis capable of yielding such a law, yet his enquiries gave the direction which was subsequently followed by many investigators. He determined the resistance of a beam, one end of which is built into a wall, when the tendency to break it arises from its own or an applied weight; and he concluded that the beam tends to turn about an axis perpendicular to its length, and in the plane of the wall. This problem, and, in particular, the determination of this axis, is known as Galileo's problem.

In the history of the theory started by the question of Galileo, undoubtedly the two great landmarks are the discovery of Hooke's Law (1678) and the formulation of the general equations by Navier (1821).

Hooke gave in 1678 the famous law of proportionality of stress and strain which bears his name, in the words "Ut tension sic vis; that is, Power of any spring is in the same proportion with the tension therefore". By "spring" Hooke means, as he proceeds to explain, any "springy body" and by "tension" what we should now call "extension" or more generally, "strain". This law forms the basis of the mathematical theory of Elasticity.

The generalized Hooke's Law states that the nine components of stress at any point of an elastic solid are functions of nine components of strain at that point i.e.,

$$\tau_{xx} = f_1(e_{xx}, e_{yy}, e_{zz}, e_{yz}, e_{xz}, e_{xy}, e_{zy}, e_{zx}, e_{yx})$$
$$\tau_{yy} = f_2(e_{xx}, e_{yy}, e_{zz}, e_{yz}, e_{xz}, e_{xy}, e_{zy}, e_{zx}, e_{yx})$$
$$----$$
$$\tau_{yz} = f_4(e_{xx}, e_{yy}, e_{zz}, e_{yz}, e_{xz}, e_{xy}, e_{zy}, e_{zx}, e_{yx})$$
$$----$$
$$\tau_{yx} = f_9(e_{xx}, e_{yy}, e_{zz}, e_{yz}, e_{xz}, e_{xy}, e_{zy}, e_{zx}, e_{yx})$$

Hence, we can write

$\tau_{ij} = c_{ijkl}\, e_{kl}$ where i, j, k, l= 1, 2, 3

In most general case, τ_{ij} and e_{kl} will have nine components each and c_{ijkl} will have 81 components.

Considering the symmetry of stress and strain components, the number of independent components reduces from 81 to 36.

If $c_{ij} = c_{ji}$ then, independent components reduce from 36 to 21. It is further possible to reduce the number of independent constants by considering the crystal symmetry. Therefore, in a isotropic material, the number of independent elastic constants reduce to two. Therefore, Hooke's Law in isotropic medium is given as

$$\tau_{ij} = \lambda\theta\delta_{ij} + 2\mu e_{ij}$$

where λ and μ are Lame's Constants, δ_{ij} is Kronecker's delta.

In the interval between the discovery of Hooke's Law (1678) and that of the general differential equations of Elasticity by Navier (1821), the attention of those mathematicians who occupied themselves with our science was chiefly directed to the solution and extension of Galileo's problem, and the related theories of the vibrations of bars and plates, and the stability of column. The first investigation of any importance is that of the elastic line or elastic by James Bernoulli in 1705, in which the resistance of a bent rod is assumed to arise from the extension and contraction of its longitudinal filaments, and the equation of the curve assumed by the axis is formed.

The most important work of the period for the general mathematical theory is the physical discussion of elasticity by Thomas Young. This naturalist besides defining his modulus of elasticity, was the first to consider shear as an elastic strain. During the first period in the history of elasticity (1638-1820) these various investigations of special problems were being made, there was a cause at work which was to lead to wide generalizations. This cause was physical speculation concerning the constitution of bodies. In the eighteenth century came the Newtonian conception of material bodies. Newton regarded his “molecules” as possessed of finite sizes and definite shapes, but his successors gradually simplified them into material points. The most definite speculation of this kind is that of Boscovich, for whom the material points were nothing but persistent centres of force. To this order of ideas belong the Laplace’s theory of capillarity and Poisson’s first investigation of the equilibrium of an “elastic surface”.

Navier was the first to investigate the general equations of equilibrium and vibration of elastic solids. He set out from the Newtonian conception of the constitution of bodies, and assumed that the elastic reactions arise from vibrations in the intermolecular forces which result from changes in the molecular configuration. He regarded the molecules as material points, and assumed that the force between two molecules, whose distance is slightly increased, is proportional to the product of the increment of the distance and some function of the initial distance. His method consists in forming an expression for the component in any direction of all the forces that act upon a displaced molecule, and thence the equations of motion of the molecule. The equations are thus obtained in terms of the displacements of the molecule. The material is assumed to be isotropic, and the equations of equilibrium and vibration contain a single constant of the same nature as Young’s modulus.

Cauchy and Poisson (1822) were the two mathematicians who got attracted to the theory of elasticity and in particular the problem of the transmission of waves through an elastic medium. By the autumn of 1822 Cauchy had discovered most of the elements of the pure theory of elasticity. He had introduced the notion of stress at a point determined by the tractions per unit of area across all plane elements through the point. He had shown that the stress is expressible by means of six component stresses, and also by means of three purely normal tractions across a certain triad of planes which cut each other at right angles-the “principal planes of stress” and expressed the state of strain near a point in terms of six components of stain, and also in terms of the extensions of a certain triad of lines which are at right angles to each other-the “principal axes of strain”. Cauchy obtained his stress-strain relationships for isotropic

materials by means of two assumptions, viz: (1) that the relations in question are linear (2) that the principal planes of stress are normal to the principal axes of strain.
The historical details of Elasticity may be found in the texts by Love (1944) and Sokolnikoff (1956).

1.2 Micropolar Elasticity

Within the elastic limits, some materials e.g., steel, aluminium, concrete etc. are found to exhibit results fairly coinciding with those of experimentally observed. However, in some materials e.g., fibrous, polymers, asphalts and crystals remarkable discrepancies are observed between the experimental results and those obtained using classical elasticity. These discrepancies are mainly because of the dominance of atomic structures of the material neglected in classical elasticity. Modern engineering structures are often made up of materials possessing an internal structure. Polycrystalline materials, materials with fibrous or coarse grain structure come in this category. Classical elasticity is inadequate to represent the behaviour of such materials. The analysis of such materials requires incorporating the theory of oriented media. For this region, micropolar theories were developed by Eringen (1966, 1976) for elastic solids, fluid and further for nonlocal polar fields. A micropolar continuum is a collection of inter connected particles in the form of small rigid bodies undergoing both translational and rotational motion. A comprehensive work in the micropolar theory of elastic solid and nonlocal continuum field theory is given by Eringen (1999, 2001).
The theory of micropolar continua, introduced in the mid 1960es by A.C. Eringen (1964, 1966, 1976, 1999), is a non-classical field theory for materials (solids and fluids) with a certain kind of microstructure. In classical continuum mechanics a material particle is assigned a certain position, irrespective of orientation, within a material body at a certain time instant. Micropolar material particles can additionally be oriented. In other words, to each micropolar material particle an additional object is assigned, called "director", which defines the orientation of the material particle. In the theory of micropolar continua, contrary to other microcontinuum field theories (Eringen's microstretch and micromorphic continua), this director is rigid, i.e., is used to describe only the microrotation of the material particles.
The micropolar theory of elasticity or micropolar elasticity incorporates a local rotation of points as well as the translation assumed in classical elasticity: and a couple stress (a torque per unit area) as well as the force stress (force per unit area). The idea of a couple stress can

be traced to Voigt (1887) during the early development of the theory of elasticity. Early theoretical work was done by the Cosserat brothers (1909), by Mindlin (1965) and by Nowacki (1969). Eringen incorporated micro-inertia (which allows incorporation of dynamic effects) and renamed Cosserat elasticity micropolar elasticity.

In the isotropic Cosserat solid or micropolar continuum, there are six elastic constants, in contrast to the classical elastic solid in which there are two, and the uniconstant material in which there is one. Micropolar elasticity may be viewed as a particular manifestation of nonlocality but is not equivalent to the general nonlocal elasticity. In multiscale modelling, micropolar effects are expected to appear as the consequence of the largest structural elements in the material. Multiscale modelling does not, however, allow prediction of micropolar effects in complex composites such as those of biological origin because the properties of all constituents are not well characterized. Research, mostly experimental, is presented in aspects of composite materials, micromechanics, cellular solids, and biological materials which can be understood via micropolar elasticity.

The constitutive relations are given as

$$\sigma_{ij} = \lambda u_{k,k}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + K\left(u_{j,i} - \varepsilon_{ijk}\phi_k\right)$$

$$m_{ij} = \alpha\phi_{k,k}\delta_{ij} + \beta\phi_{j,j} + \gamma\phi_{j,i} \qquad i, j = 1,2,3$$

in which σ_{ij} is the force stress (which is symmetric tensor in classical elasticity but is asymmetric here), m_{ij} is the couple stress (or moment per unit area), $\vec{u}$ is the displacement vector and ε_{ijk} is the permutation symbol.

1.2.1 Applications

According to Eringen (1966) the range of possible materials to be modelled by micropolar theory is very wide. It encompasses e.g., in Eringen's own words, "anisotropic fluids, liquid crystals with rigid molecules, rigid suspensions, magnetic fluids, cloud with dust, muddy fluids, biological fluids, animal blood with rigid cells, chopped fiber composites, bones, concrete with sand". Although disputable from a logical point of view, this enumeration shows how wide is the range of intended applications of micropolar theory. In the case of micropolar solids it is useful to distinguish two classes of materials, those with periodic microstructure and those with random microstructure.

The application of micropolar theory to solids with periodic microstructure (or "structure" in general) is extensive and rather successful. It ranges from natural materials with lattice structure (crystals) to man-made composite materials and engineering structures such as infinite-fiber composites, sandwich structures, grid structures, trusses and honeycombs. Examples of the successful application of micropolar theory to these materials are the calculation of micropolar elastic moduli of KNO3 and other crystals (especially ferroelectric ones) exhibiting polar phenomena by Askar (1972), Fischer-Hjalmars (1981) and the numerical analysis of steel-concrete grid structures in civil engineering, performed by Bažant and Christensen (1972).

The application of micropolar theory to solids with random microstructure, natural or man-made (chopped fiber composites, platelet composites, particulate composites, porous materials, foams, bone) although useful in principle, is very complicated in practice.

The micropolar continuum theory can be used for the mechanical analysis of cancellous bone and the extraction of micropolar elastic moduli for cancellous bone by a micromechanical analysis. Experimental evidence of micropolar elasticity in human compact bone has been reported by Lakes and co-workers (1982, 1986). The existence of micropolar effects or cortical bone in quasi-static bending has been demonstrated in 1982. The issue of application of higher order continuum theories to mechanical analysis of bone has been addressed by Fatemi et al. (2002). In 2002, a simplified two-dimensional bone-prosthesis configuration was analysed using a micropolar based FE formulation. Results of this analysis showed that the stress and strain intensities in the bone-prosthesis interface are different from those predicted by classical elasticity.

Yang and Lakes have applied the theory of micropolar solids for the bone analysis. Couple stress theory was studied and it was observed that couple stress theory, which admits an internal moment per unit area as well as the usual force per unit area, is a generalization of classical elasticity. Experimentally, demonstration of the existence of couple stress by measuring the effect of size on apparent stiffness of compact bone in quasi-static torsion is done. From these measurements, the characteristic length for bone in couple stress theory is obtained.

In the Journal of Biomechanics, Yang and Lakes experimentally studied micropolar and couple stress elasticity in bone in bending (Yang and Lakes(1982)). In this study, size effects in quasistatic bending of compact bone are observed. The effects are consistent with micropolar elasticity. From these, evaluation of two non classical elastic constants is possible.

In an article under the title "Experimental methods for study of Cosserat elastic solids and other generalized continua" (1995), R. S. Lakes explained that the behaviour of solids can be represented by a variety of continuum theories. For example, Cosserat elasticity allows the points in the continuum to rotate as well as translate, and the continuum supports couple per unit area as well as force per unit area. Experimental methods for determining the six Cosserat elastic constants of an isotropic elastic solid, or the six cosserat relaxation functions of a Cosserat viscoelastic solid are examined. Consideration of other generalized continuum theories (including micromorphic elasticity in which points rotate and deform, Cowin's void theory (theory of voids) in which points dilate and nonlocal elasticity) was done. Ways of experimentally discriminating among various generalized continuum representations are presented. The applicability to Cosserat elasticity to cellular solids and fibrous composite materials is considered as is the application of related generalized continuum theories.

According to the study done by Nakamura S. and Lakes (1995), distributions of stress and strain in composite and cellular materials can differ significantly from the predictions of classical elasticity. For example, concentration of stress and strain around holes and cracks is consistently less than classical predictions. Generalized continuum theories such as micropolar elasticity offer improved predictive power. In this article, a two dimensional finite element analysis is used which takes into account the extra degrees of freedom, to treat the problem of localized end loads acting upon a strip. The rate of decay of strain energy becomes slower in a two dimensional strip as the micropolar characteristic length is increased. For the strip geometry, a Cosserat solid exhibits slower stress decay than a classical solid.

1.3 General aspect of wave propagation

The effect of a sharply applied localized disturbance in a medium soon transmits or spreads to other parts of the medium. This simple fact forms a basis for study of the fascinating subject known as wave propagation. The manifestation of this phenomenon are familiar to everyone in forms such as the transmission of sound in air, the spreading of ripples on a pond of water, the transmission of seismic tremors in the earth, or the transmission of radio waves. These and many other examples could be cited to illustrate the propagation of waves through gaseous, liquid and solid media and free space.

The physical basis for the propagation of a disturbance ultimately lies in the interaction of the discrete atoms of the solid. Investigations along such lines are more atuned to physics than mechanics, however, in solid and liquid mechanics, the medium is regarded as continuous, so that properties such as density or elastic constants are considered to be continuous functions representing averages of microscopic quantities. In case of a solid, two distinct types of action will be possible in a wave. In one case, the solid will transmit tensile and compressive stress and the motion of particles will be in the direction of the wave motion. In addition, the solid may transmit shear stress, and the motion of the particles is transverse to the direction of propagation.

The outward propagation of waves from a disturbance is one aspect of wave motion. Inevitably, the waves encounter and interact with boundaries. In this area, the behaviour of waves in solids differ considerably from that of fluids. In a single solid wave, be it compression or shear, will generally produce both compression and shear waves on striking a boundary, whereas acoustic and electromagnetic waves will only generate waves of their own type. It is the continual propagation and reflection of waves in a bounded solid that brings about the state of static equilibrium. Speaking in these terms, every process of loading solid is a dynamic process involving the propagation and reflection of waves. However, if the rate of onset of the load is slow compared with many transit times of waves within the solid, static equilibrium effectively prevails and wave effects are of no consequence. It is only when loading rate are comparable with transit times of waves that the mechanics of wave propagation must be considered.

The propagation of waves in solids may be divided roughly into three categories. The first is elastic waves, where the stresses in the material obey Hooke’s law. The other two main categories, visco-elastic waves, where viscous as well as elastic stresses act, and plastic waves in which the yield stress of the material is exceeded.

The propagation of waves through an elastic solid medium requires investigation of a different character from those concerned with normal modes of vibration. In case of an isotropic medium Poisson and Ostrogradsky adopted methods which involve a synthesis of solutions of simple harmonic type and obtained a solution expressing the displacement at any time in terms of the initial distribution of displacement and velocity. The investigation was afterwards conducted in a different fashion by Stokes, who showed that Poisson’s two waves are waves of irrotational dilatation and waves of equivoluminal distortion, the latter involving rotation of the elements of the medium, Cauchy and Green discussed the propagation of plane waves through a crystalline medium and obtained equations for the velocity of propagation in terms of the direction of the normal normal to wave-front.

In general the plane surface has three sheets; when the medium is isotropical the sheets are spheres, and two of them are coincident. Blanchet extended poisson’s results to the case of a

crystalline medium. Christoffel discussed the advance through the medium of a surface of discontinuity. At any instant, the surface separates two portions of the medium in which the displacements are expressed by different formulae; and Christoffel showed that the surface moves normally to itself with a velocity which is determined, at any point, by the direction of the normal of the surface, according to the same law as holds for plane waves propagated in that direction. Besides the waves of dilatation and distortion which can be propagated through an isotropic solid body Lord Rayleigh has investigated a third type which can be propagated over the surface. The velocity of waves of this type is less than that of either of the other two.
In a micropolar solid (Cosserat and Cosserat(1909))with the micro inertia incorporated in the micropolar formulation, dispersion of shear waves is expected to occur. In particular, the wave speed of plane dilatational waves in an unbounded micropolar elastic medium is independent of frequency as in the classical case, that is, no dispersion. The speed of shear waves depends on frequency in a micropolar solid. A new kind of wave associated with the micro rotation is predicted to occur in micropolar solids. The difficulty with using waves to explore micropolar effects is that wave speed depends on frequency both from geometrical dispersion and from viscoelastic dispersion. It is not so easy to separate these effects in real dissipative materials.

1.3.1 Applications of wave phenomenon

The practical applications of wave phenomena surely go back to the early history of man. The shaping of stone implements, for example, consists of striking sharp, carefully placed blows along the edges of a flint. Propagation of wave phenomena has found applications in various fields like Siesmology, structure analysis, Non destructive evaluation etc.

The general subject of waves in the earth covers many interesting propagation phenomena. Earthquakes generate waves that may travel thousands of miles. Study of the propagation of such waves or tremors artificially produced have provided the most knowledge on the interior construction of the earth. Waves in the earth generated by blast are of concern from the standpoint either of blast detection or the protection of underground structures. Other aspect of waves in the earth pertains to Oil and Gas exploration. By studying the reflection of waves from underground discontinuities, it is possible to locate possible oil-bearing deposits.

The behaviour of structural materials under loads severe enough to cause permanent damage is an area of very great interest. Studies in this area generally fall in the category of an elastic wave propagation. Some of the techniques used in the study of these properties use elastic waves, such as waves in a high-strength steel rod, to dynamically load-test specimens of weaker materials. Most of the applications in this area are in various aspects of military and space technology. Another area in the study of structures involving wave phenomena is that of crack propagation or the interaction of dynamic stress fields with existing cracks, voids, or inclusions in a material.

By studying the propagation, reflection and attenuation of ultrasonic pulses, it is possible to determine many fundamental properties of materials such as elastic constants and damping characteristics. The field of non destructive testing makes wide use of ultrasonics to detect deflects by the reflection of pulse energy from the defect much in the manner of underwater sonar detection.

1.4 Waves in Infinite Medium

Two basic types of waves, dilatational and distortional, can propagate in an infinite medium, with each being characterized by a specific velocity. Furthermore, these wave types can exist independent or uncoupled from one another. Dilatational waves are also called irrotational and primary (P) waves. The rotational waves are also equi-voluminal, distortional and secondary (S) waves. The P and S wave designations have arisen in seismology, where they are also occasionally designated as the 'push' and 'shake waves'. Other respective designations are frequently used are Longitudinal and shear waves. These are body waves and can travel into the deep of the medium. Besides these body waves, there occurs surface waves, which can travel near the boundary surface of a medium and goes on diminishing with the distance away from the boundary surface. There are three types of surface waves: Rayleigh wave, Lamb wave and Love wave.

In classical elasticity, plane waves in unbounded medium propagate without dispersion (the wave speed is independent of frequency) for shear waves and dilatational waves.

In micropolar elasticity, dilatational waves propagate non-dispersively, i.e., with velocity independent of frequency in an unbounded isotropic micropolar elastic medium as in classical case. Shear waves propagate dispersively in micropolar solids. A new kind of wave associated with the micro rotation is predicted to occur in micropolar solids.

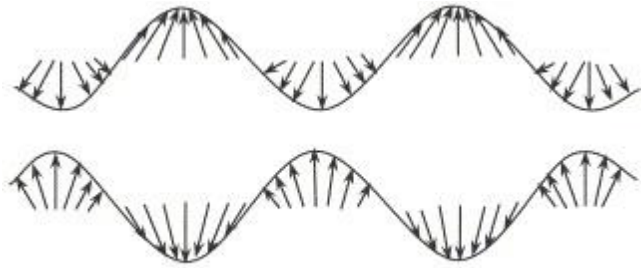
1.5 Lamb Waves

Lamb waves propagate in solid plates. They are elastic waves whose particle motion lies in the plane that contains the direction of wave propagation and the plane normal (the direction perpendicular to the plate). Lamb waves are guided waves that exist in thin walled structures. Because this type of wave can travel long distance with little attenuation, they have been studied intensively for structural health monitoring, especially in the past few decades.

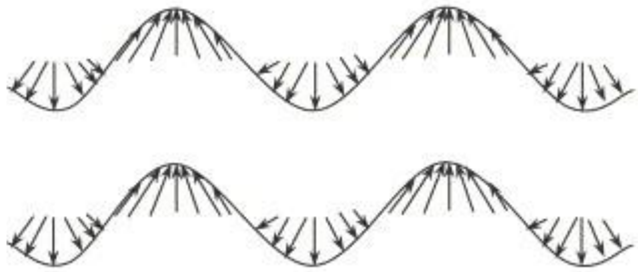
Early studies of wave propagation in plates were carried out by Rayleigh (1945) and Lamb (1917). The Rayleigh-Lamb theory applies to the propagation of continuous, straight crested waves in infinite plates with free surfaces. Rayleigh-Lamb frequency equations give the relationship between phase velocity and wave number. The displacement and stress distribution functions can be obtained after the Rayleigh-Lamb frequency equations are solved. Goodman (1952) studied the problem of circular crested waves, and found that the Rayleigh-Lamb frequency equations hold for this case too. Although the displacement and stress distribution of circular crested wave are initially different, these waves converge to the straight crested wave rapidly after a short distance (2~3 wavelength).

Lamb's characteristic equations indicate the existence of two entire families of sinusoidal wave modes in infinite plates of width d. This stands in contrast with the situation in unbounded media where there are just two wave modes, the longitudinal wave and the transverse or shear wave. As in Rayleigh waves which propagate along single free surfaces, the particle motion in Lamb waves is elliptical with its x and z components depending on the depth within the plate(Krautkramer(1990)). In one family of modes, the motion is symmetrical about the mid thickness plane. In the other family, it is antisymmetric. The phenomenon of velocity dispersion leads to a rich variety of experimentally observable waveforms when acoustic waves propagate in plates.

When Lamb waves are transmitted, particles move in one of two different ways. If the particle motion is symmetric to the mid surface, it is called symmetric Lamb wave (S0, S1, S2, …). If the particle motion is anti-symmetric with respect to the mid surface, it is called anti-symmetric Lamb waves (A0, A1, A2, …).



symmetric Lamb mode



anti-symmetric Lamb mode

(Ref: www.google/images.com)

Furthermore, a number of modes exist for each type of the Lamb waves. Lamb wave can be generated in a plate with free boundaries with an infinite number of modes for both symmetric and anti-symmetric displacements within the layer. The symmetric modes are also called longitudinal modes because the average displacement over the thickness of the plate or layer is in the longitudinal direction. The anti-symmetric modes are observed to exhibit average displacement in the transverse direction and these modes are also called flexural modes. The infinite number of modes exists for a specific plate thickness and acoustic frequency which are identified by their respective phase velocities. Symmetric and anti-symmetric Lamb waves have different phase and group velocities, as well as distribution of particle displacement and stress through the plate thickness.

The normal way to describe the propagation characteristics is by the use of dispersion curves based on the plate mode phase velocity as a function of the product of frequency times thickness. The dispersion curves are normally labelled as S0, A0, S1, A1 and so forth, depending on whether the mode is symmetric or anti-symmetric.

Dispersion curves-graphs that show relationships between wave velocity, wavelength and frequency in dispersive systems-can be presented in various forms. The form that gives the greatest insight into the underlying physics has angular frequency on the y-axis and wave number on x-axis. The form used by Viktorov(1967), that brought Lamb waves into practical use, has wave velocity on y-axis and the thickness/wavelength ratio, on the x-axis. The most practical formof all, for which cresit is due to J. and H. Krautkramer as well as to Floyd Firestone has wave velocity on y-axis and frequency-thickness product, on the x-axis.

1.5.1 The zero order modes

The symmetric and anti symmetric zero order modes have "nascent frequencies" of zero. They are the only modes that exist over the entire frequency spectrum from zero to indefinitely high frequencies. These two modes are the most important because

(a) they exist at all frequencies and
(b) in most practical situations they carry more energy than the higher order modes.

1.5.2 Higher order modes

As the frequency is raised, the higher order wave modes make their appearance in addition to the zero order modes. The first few higher order modes can be distinctly observed under favourable experimental conditions. Under less favourable conditions they overlap and cannot be distinguished.

The higher order Lamb modes are characterised by nodal planes within the plate, parallel to the plate surfaces. Each of these modes exists only above a certain frequency which can be called its "nascent frequency". At its nascent frequency, each of these modes has an infinite phase velocity and a group velocity of zero. In the high frequency limit, the phase and group velocities of all these modes converge to the shear wave velocity. Because of these convergences, the Rayleigh and Shear velocities (which are very close to one another) are of major importance in thick plates.

J. and H. Krautkramer have pointed out (1990) that Lamb waves can be conceived as a system of longitudinal and shear waves propagating at suitable angles across and along the plate. These waves reflect and mode-convert and combine to produce a sustained, coherent wave pattern. For this coherent wave pattern to be formed, the plate thickness has to be just right relative to the angles of proportion and wavelengths of the underlying longitudinal and shear waves; this requirement leads to the velocity dispersion relationships.

Chapter 2

RAYLEIGH LAMB WAVE PROPAGATION IN ELASTIC PLATE

2.1 INTRODUCTION

Lamb (1917) was the first to investigate the problem of wave propagation in an elastic plate of uniform material. Since then the term 'Lamb wave' has been used to refer to an elastic disturbance propagating in a solid plate with free boundaries. Thin plate theory (Leissa 1973) is a simplified version that fails to accurately incorporate dynamic response when the sample is thick compared to a wavelength. In contrast, thick plate theory (Lamb 1917) usually incorporates all the dynamics of the plate and is normally used when the sample is of the order of a wavelength of energy in the structure. The effect of internal friction on the propagation of plane waves in an elastic medium may also be considered owing to the fact that dissipation accompanies vibrations in solid media due to the conversion of elastic energy to heat energy . Several mathematical models have been used to accommodate the energy dissipation in vibrating solids, where it is observed that internal friction produces attenuation and dispersion and hence the effect of the elastic nature of material medium in the process of wave propagation cannot be neglected. In order to illustrate and compare the theoretical results in various situations, the numerical solution is carried out for aluminium-epoxy material by employing the functional iteration method and the corresponding dispersion curves for symmetric and skew-symmetric wave modes are presented graphically.

2.2 FORMULATION OF THE PROBLEM

We consider an infinite homogeneous isotropic elastic plate of thickness 2d. We take the origin of the coordinate system (x, y, z) on the middle surface of the plate. The xy plane is chosen to coincide with the middle surface and the z axis normal to it along the thickness. The surfaces are assumed to be stress free.

The basic governing equations of generalized elasticity in the absence of body forces and heat sources are

$$(\lambda+\mu)\nabla\left(\nabla.\vec{u}\right)+\mu\nabla^2\vec{u}=\rho\ddot{\vec{u}} \tag{2.2.1}$$

where $\vec{u} = (u, v, w)$ is the displacement vector, ρ is the density. The dot notation is used to denote time differentiation. λ and μ are Lame's parameters.

The constitutive relations are given by

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij}$$

where $e_{ij} = (u_{i,j} + u_{j,i})/2$ i, j=1, 2 is the strain tensor.

We define the quantities

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}, \quad x' = \frac{\omega^* x}{c_1}, \quad z' = \frac{\omega^* z}{c_1}, \quad t' = \omega^* t, \quad u' = \frac{\rho\omega^* c_1}{\upsilon T_0} u, \quad w' = \frac{\rho\omega^* c_1}{\upsilon T_0} w, \quad \delta^2 = \frac{c_2^2}{c_1^2},$$

$$\sigma'_{ij} = \frac{\sigma_{ij}}{\upsilon T_0}, \quad c_1^2 = \frac{\lambda + \mu}{\rho}, \quad c_2^2 = \frac{\mu}{\rho} \tag{2.2.2}$$

The governing equations in the non dimensional form can be written as

$$\left(1-\delta^2\right)\left(u_{,xx} + w_{,xz}\right) + \delta^2\left(u_{,xx} + u_{,zz}\right) = \ddot{u} \tag{2.2.3}$$

$$\delta^2\left(v_{,xx} + v_{,zz}\right) = \ddot{v} \tag{2.2.4}$$

$$\left(1-\delta^2\right)\left(u_{,zx} + w_{,zz}\right) + \delta^2\left(w_{,xx} + w_{,zz}\right) = \ddot{w} \tag{2.2.5}$$

BOUNDARY CONDITIONS

The boundaries of the plate are assumed to be stress free. Therefore, the non dimensional mechanical boundary conditions are given by

$$\sigma_{zz} = \sigma_{xz} = 0 \qquad at \quad z = \pm d$$

2.3 SOLUTION OF THE PROBLEM

In order to solve the problem, we introduce the potential functions q and ψ through the relations

$$u = \frac{\partial q}{\partial x} + \frac{\partial \psi}{\partial z} \tag{2.3.1}$$

$$w = \frac{\partial q}{\partial z} - \frac{\partial \psi}{\partial x} \tag{2.3.2}$$

in equations (2.2.3). We obtain,

$$\nabla^2 q = \ddot{q} \tag{2.3.3}$$

$$\delta^2 \nabla^2 \psi = \ddot{\psi} \tag{2.3.4}$$

Upon using (2.3.1) and (2.3.2) in the constitutive relation, we get,

$$\sigma_{zz} = \left[-(\lambda + 2\mu)\alpha'^2 - \lambda \alpha' \xi^2 \right] (A \sin \alpha' z + B \cos \alpha' z) - 2\mu i \xi \beta' (E \cos \beta' z - F \sin \beta' z) \tag{2.3.5}$$

$$\sigma_{xz} = 2i\xi\alpha' \mu (A \cos \alpha' z - B \sin \alpha' z) + \mu \left(\xi^2 - \beta'^2 \right) (E \sin \beta' z + F \cos \beta' z) \tag{2.3.6}$$

We take the solution of the form

$$q = q(z) e^{i\xi(x-ct)}$$

$$\psi = \psi(z) e^{i\xi(x-ct)} \tag{2.3.7}$$

where $c = \frac{\omega}{\xi}$ is the non dimensional phase velocity and ω and ξ are the non dimensional circular frequency and wave number, respectively. Upon using the equation (2.3.7) in equations (2.3.3), we get,

$$\nabla^2 \left(q(z) e^{i\xi(x-ct)} \right) = \frac{\partial^2}{\partial t^2} q(z) e^{i\xi(x-ct)}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \left(q(z) e^{i\xi(x-ct)} \right) = \frac{\partial^2}{\partial t^2} [q(z) e^{i\xi(x-ct)}]$$

$$-\xi^2 q(z) e^{i\xi(x-ct)} + \frac{\partial^2}{\partial z^2} q(z) e^{i\xi(x-ct)} = -\xi^2 c^2 q(z) e^{i\xi(x-ct)}$$

$$q = (A \sin \alpha' z + B \cos \alpha' z) e^{i\xi(x-ct)} \text{ where } \alpha'^2 = \xi^2 (c^2 - 1) \tag{2.3.8}$$

Upon using (2.3.7) in (2.3.4), we get,

$$\delta^2 \nabla^2 \left(\psi(z) e^{i\xi(x-ct)} \right) = \frac{\partial^2}{\partial t^2} \psi(z) e^{i\xi(x-ct)}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \left(\psi(z) e^{i\xi(x-ct)} \right) = \frac{\partial^2}{\partial t^2} \psi(z) e^{i\xi(x-ct)}$$

$$\delta^2 (-\xi^2 \psi(z) e^{i\xi(x-ct)} + \frac{\partial^2}{\partial z^2} \psi(z) e^{i\xi(x-ct)}) = -\xi^2 c^2 \psi(z) e^{i\xi(x-ct)}$$

$$\psi = (E \sin \beta' z + F \cos \beta' z) e^{i\xi(x-ct)} \text{ where } \beta'^2 = \xi^2 (\frac{c^2}{\delta^2} - 1) \tag{2.3.9}$$

The displacements are obtained from equation (2.3.1) and (2.3.2) as

$$u=\left[i\xi(A\sin\alpha' z+B\cos\alpha' z)+\beta'(E\cos\beta' z-F\sin\alpha' z)\right]e^{i\xi(x-ct)}$$

$$w=\left[\alpha'(A\cos\alpha' z-B\sin\alpha' z)-i\xi\beta'(E\sin\beta' z+F\cos\beta' z)\right]e^{i\xi(x-ct)}$$

2.4 DERIVATION OF SECULAR EQUATIONS

Invoking the boundary conditions at the surface $z=\pm d$ of the plate and using equations (2.3.8) and (2.3.9), we obtain the system of four simultaneous linear equations as below:

$$p(As_1+Bc_1)+q(Ec_2-Fs_2)=0$$

$$p(-As_1+Bc_1)+q(Ec_2+Fs_2)=0$$

$$f(Ac_1-Bs_1)+r(Es_2+Fc_2)=0$$

$$f(Ac_1+Bs_1)+r(-Es_2+Fc_2)=0$$

where $p=(\lambda+\mu)\alpha'^2+\lambda\xi^2, q=2i\beta'\xi\mu, c_1=\cos\alpha' z, c_2=\cos\beta' z, s_1=\sin\alpha' z, s_2=\sin\beta' z$
$f=2i\xi\alpha', r=\left(\xi^2-\beta'^2\right)$

Consider first, the case of symmetric waves. Following system of two homogeneous equations for the constants B and E are obtained.

$$\begin{bmatrix} p\cos\alpha' d & q\cos\beta' d \\ -f\sin\alpha' d & r\sin\beta' d \end{bmatrix}\begin{bmatrix} B \\ E \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Since the above system of equations is homogeneous, the determinant of coefficients has to vanish, which results in the frequency equation. Thus,

$$p\,r\cos\alpha' d\sin\beta' d+q\,f\cos\beta' d\sin\alpha' d=0$$

or the determinant can be written a

$$\frac{\tan\beta' d}{\tan\alpha' d}=\frac{-4\xi^2\alpha'\beta'}{\left(\xi^2-\beta'^2\right)^2} \qquad \textit{where } \alpha'^2=\xi^2\left(c^2-1\right), \beta'^2=\xi^2\left(\frac{c^2}{\delta^2}-1\right) \tag{2.4.1}$$

Similarly, for anti-symmetric modes, the equations for the constants A and F are

$$\begin{bmatrix} p\sin\alpha' d & -q\sin\beta' d \\ f\cos\alpha' d & r\cos\beta' d \end{bmatrix}\begin{bmatrix} A \\ F \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

which gives the Rayleigh-Lamb frequency equation for the propagation of anti-symmetric waves in a plate.

$$\frac{\tan\alpha' d}{\tan\beta' d}=\frac{-4\xi^2\alpha'\beta'}{\left(\xi^2-\beta'^2\right)^2} \qquad \textit{where } \alpha'^2=\xi^2\left(c^2-1\right), \beta'^2=\xi^2\left(\frac{c^2}{\delta^2}-1\right) \tag{2.4.2}$$

The equations (2.4.1) and (2.4.2) agree with that of Graff (1991).

2.5 NUMERICAL RESULTS AND DISCUSSION

With the view of illustrating the theoretical results obtained in the preceding sections and comparing these in the context of theory of elasticity, we now present some numerical results. The material chosen for this purpose is aluminium-epoxy composite, the physical data for which is given as (1998) $\varepsilon = 0.073, \lambda = 7.59\times10^{10}\,Nm^{-2}, \mu = 1.89\times10^{10}\,Nm^{-2}, \rho = 2.19\times10^{3}\,kgm^{-3}$, $\omega^{*} = 4.36\times10^{11}\,/\sec$.

Lamb waves exhibit velocity dispersion; that is, their velocity of propagation depends on the frequency (or wavelength), as well as on the elastic constants and density of the material. This phenomenon is central to the study and understanding of wave behaviour in plates.

After computing the roots and using these in various relations, the secular equation are solved to obtain the phase velocity by using iteration method. In order to achieve the desired level of accuracy, the sequence of iteration is made to converge after sampling it over 100 sample values.

The non dimensional phase velocity of symmetric and anti symmetric modes of wave propagation are computed for various values of non dimensional wave number from the secular equation for different boundary conditions. The corresponding dispersion curves for Rayleigh Lamb type modes are presented in Figures 1 and 2.

The zero order symmetrical mode travels at the "plate velocity" in the low wave number regime where it is properly called the "extensional mode". In this regime the plate stretches in the direction of propagation and contracts correspondingly in the thickness direction. From the figures, it is observed that in the case of lowest symmetric mode, the phase velocity remains constant with the variation in wave number.

The zero order anti symmetric mode is highly dispersive in the low wave number regime where it is properly called the "flexural mode". The phase velocity of lowest anti symmetric mode is zero at vanishing wave number and increases to become closer to the Rayleigh wave velocity at higher values of wave number.

The phase velocity of higher modes of propagation, symmetric and skew symmetric, attains quite large values at vanishing wave number, which decreases to become asymptotically close to the shear wave velocity. The magnitudes of velocity of higher symmetric and skew symmetric modes are observed to develop at a rate, which is approximately n times the magnitude of the velocity of the first mode (n=0). These numerically computed results are found to be in agreement with the corresponding analytical results and their trends are similar to those reported by Graff (1991) and Achenbach (1973).

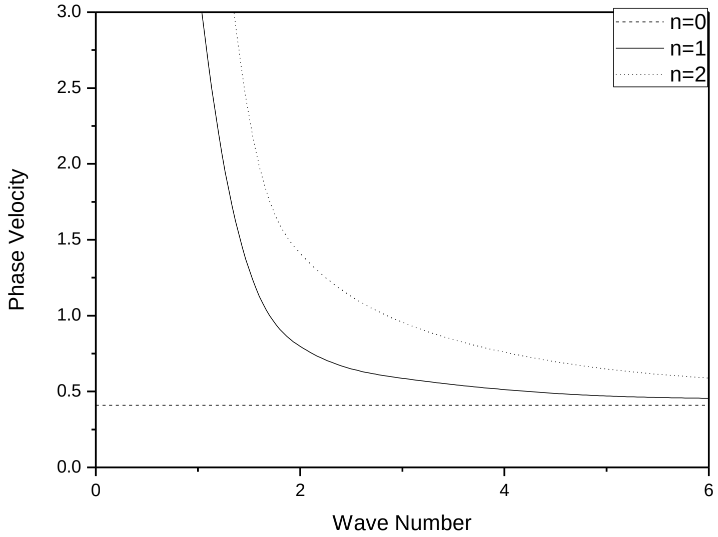


Figure 1: Dispersion curves for Symmetric mode of wave propagation

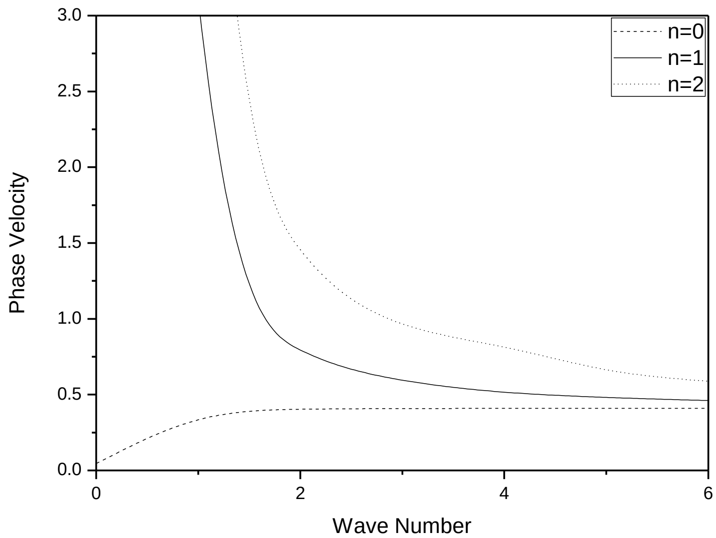


Figure 2: Dispersion curves for Skew symmetric mode of wave propagation

Chapter 3

RAYLEIGH-LAMB WAVES IN HOMOGENEOUS ISOTROPIC MICROPOLAR ELASTIC PLATE

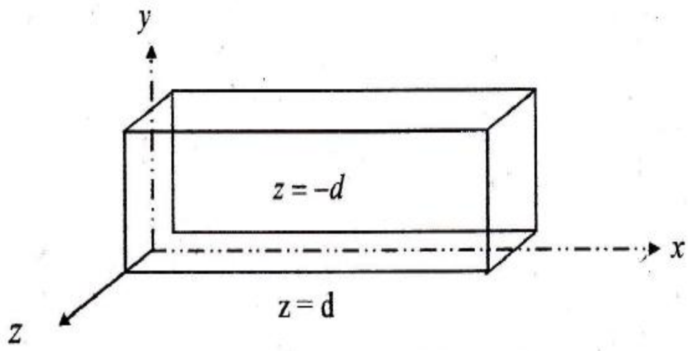
3.1 INTRODUCTION

The analysis of waves in thin plates was initiated by Lamb (1917) and in recognition, the solutions of the equations are termed as Lamb Waves. Due to theoretical and practical importance, many problems of waves and vibration of micropolar elasticity have been investigated by different researchers. Propagation of monochromatic waves in an infinite micropolar elastic plate has been studied by Nowacki and Nowacki (1969). Benerji and Sengupta (1977) discussed propagation of waves in a micropolar elastic layer immersed in an infinite liquid. Rao and Rao (1983) and Rao (1988) investigated the problems of longitudinal wave propagation in micropolar wave guide. Rao and Reddy (1993) studied the Rayleigh type propagation in a micropolar cylindrical surface. Kumar and Singh (1996) investigated propagation of waves in a micropolar generalized thermoelastic layers with stretch. Sharma and Kumar (2009) investigated the propagation of waves in cylindrical plates under fluid loading.

The propagation of waves in micropolar elastic plate is investigated. The secular equations for the micropolar elastic plate and mathematical conditions for symmetric and skew-symmetric wave mode propagation in completely separate terms are derived. The secular equation for elastic plate has been deduced as particular case from the derived secular equation. Finally, in order to illustrate the analytical development, the numerical solution is carried out for aluminium-epoxy composite material. The dispersion curves for symmetric and skew-symmetric wave modes, are computed numerically and presented graphically. The theory and numerical computations are found to be in close agreement.

3.2 FORMULATION OF THE PROBLEM

We consider an infinite homogeneous isotropic micropolar plate of thickness 2d initially undisturbed. We take the origin of the coordinate system (x, y, z) on the middle surface of the plate. The x-y plane is chosen to coincide with the middle surface of the plate and z- axis normal to it along the thickness of the plate. The surfaces z = $\pm$ d are assumed to be stress free boundaries of the plate.



(Ref: Pathania et al. (2007))

The basic governing equations of micropolar elasticity (Eringen (1966), Sharma et el.(2009)) in the absence of body forces and heat sources are

$$(\lambda+\mu)\nabla\left(\nabla.\vec{u}\right)+(\mu+K)\nabla^2\vec{u}+K\nabla\times\vec{\phi}=\rho\frac{\partial^2\vec{u}}{\partial t^2} \tag{3.2.1}$$

$$(\alpha+\beta+\gamma)\nabla\left(\nabla.\vec{\phi}\right)-\gamma\nabla\times\left(\nabla\times\vec{\phi}\right)+K\nabla\times\vec{u}-2K\vec{\phi}=\rho j\frac{\partial^2\vec{\phi}}{\partial t^2} \tag{3.2.2}$$

The constitutive relations are given by

$$\sigma_{ij}=\lambda u_{k,k}\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right)+K\left(u_{j,i}-\varepsilon_{ijk}\phi_k\right)$$

$$m_{ij}=\alpha\phi_{k,k}\delta_{ij}+\beta\phi_{j,j}+\gamma\phi_{j,i}$$

where i, j, k = 1, 2, 3 (3.2.3)

where $\vec{u}=(u_1,u_2,u_3)$ is the displacement vector , $\vec{\phi}$ is micro-rotation vector, λ,μ,γ,K are material constants, ρ is the density, σ_{ij} and m_{ij} are respectively the stress tensor and couple stress tensor and δ_{ij} is Kronecker's delta.

For 2-dimensional problems, we take

$$\vec{u} = (u, 0, w)$$
$$\vec{\phi} = (0, \phi, 0)$$

We have defined the quantities

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}, \qquad x' = \frac{\omega^* x}{c_1}, \qquad z' = \frac{\omega^* z}{c_1},$$

$$t' = \omega^* t, \qquad u' = \frac{\rho \omega^* c_1}{\upsilon T_0} u, \qquad w' = \frac{\rho \omega^* c_1}{\upsilon T_0} w$$

$$\delta^2 = \frac{c_2^2}{c_1^2}, \qquad \phi' = \frac{\rho c_1^2}{\upsilon T_0} \phi, \qquad \sigma'_{ij} = \frac{\sigma_{ij}}{\upsilon T_0},$$

$$m'_{ij} = \frac{c_1}{\omega^* \upsilon T_0} m_{ij}, \; c_1^2 = \frac{\lambda + \mu + K}{\rho}, \qquad c_2^2 = \frac{\mu + K}{\rho}$$

$$\delta_1^2 = \frac{c_4^2}{c_1^2}, \qquad c_3^2 = \frac{K^*}{\rho C_e \omega^*}, \qquad c_4^2 = \frac{\gamma}{\rho J},$$

$$p = \frac{K}{\rho c_1^2}, \qquad \delta^* = \frac{K c_1^2}{\gamma \omega^{*2}} \tag{3.2.4}$$

The equations (3.2.1), (3.2.2) in non dimensional form can be written as

$$\left(1 - \delta^2\right)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z}\right) + \delta^2 \nabla^2 u - p \frac{\partial \phi}{\partial z} = \frac{\partial^2 u}{\partial t^2} \tag{3.2.5}$$

$$\left(1 - \delta^2\right)\left(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial z^2}\right) + \delta^2 \nabla^2 w + p \frac{\partial \phi}{\partial x} = \frac{\partial^2 w}{\partial t^2} \tag{3.2.6}$$

$$\nabla^2 \phi + \delta^* \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) - 2\delta^* \phi = \frac{1}{\delta_1^2} \frac{\partial^2 \phi}{\partial t^2} \tag{3.2.7}$$

3.3 SOLUTION OF THE PROBLEM

In order to solve the problem, we introduce the potential functions q and ψ through the relations

$$u = \frac{\partial q}{\partial x} + \frac{\partial \psi}{\partial z} \tag{3.3.1}$$

$$w = \frac{\partial q}{\partial z} - \frac{\partial \psi}{\partial x} \tag{3.3.2}$$

Inserting (3.3.1), (3.3.2) in (3.2.5) - (3.2.7) we obtain

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right) q = 0 \tag{3.3.3}$$

$$\left(\nabla^2 \psi - \frac{1}{\delta^2}\ddot{\psi}\right) = \frac{p}{\delta^2}\phi \tag{3.3.4}$$

$$\nabla^2 \phi - 2\delta^* \phi - \frac{1}{\delta_1^2}\ddot{\phi} + \delta^* \nabla^2 \psi = 0 \tag{3.3.5}$$

Upon using equations (3.3.1), (3.3.2) in the constitutive relations (3.2.3), (3.2.4), we obtain

$$\sigma_{zz} = \ddot{q} - 2\left(\delta^2 - \frac{p}{2}\right)\left(q_{,xx} + \psi_{,xz}\right) \tag{3.3.6}$$

$$\sigma_{zx} = \ddot{\psi} + \left(2\delta^2 - p\right)\left(q_{,xz} - \psi_{,xx}\right) \tag{3.3.7}$$

$$m_{zy} = \frac{\lambda}{\rho c_1^2}\frac{\partial \phi}{\partial z} \tag{3.3.8}$$

We take the solution of the form

$$q = f(z) e^{i\xi(x-ct)}$$

$$\psi = g(z) e^{i\xi(x-ct)}$$

$$\phi = w(z) e^{i\xi(x-ct)} \tag{3.3.9}$$

where, $c = \frac{\omega}{\xi}$ is the non-dimensional phase velocity , ω and ξ are respectively the non dimensional circular frequency and wave number.

Using (3.3.9) in equations (3.3.3),

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right) f(z) e^{i\xi(x-ct)} = 0$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) f(z) e^{i\xi(x-ct)} = \frac{\partial^2}{\partial t^2} f(z) e^{i\xi(x-ct)}$$

$$-\xi^2 f(z) e^{i\xi(x-ct)} + \frac{\partial^2}{\partial z^2} f(z) e^{i\xi(x-ct)} = -\xi^2 c^2 f(z) e^{i\xi(x-ct)}$$

$$f(z) = A\cos m_1 z + B\sin m_1 z \qquad \text{where} \qquad m_1 = \xi^2(c^2 - 1)$$

$$q = (A\cos m_1 z + B\sin m_1 z) e^{i\xi(x-ct)} \tag{3.3.10}$$

Using (3.3.9) in (3.3.4) and (3.3.5), we get

$$\left(\nabla^2 - 2\delta^* + \frac{\omega^2}{\delta_1^2}\right)\phi + \delta^* \nabla^2 \psi = 0 \tag{3.3.11}$$

$$\frac{p}{\delta^2}\phi - \left(\nabla^2 + \frac{\omega^2}{\delta^2}\right)\psi = 0 \tag{3.3.12}$$

Solving the above two equations, we get

$$\psi = [(A' \cos m_2 z + B' \sin m_2 z) + (C' \cos m_3 z + D' \sin m_3 z)] e^{i\xi(x-ct)}$$

where $m_2 = \xi^2(a_2 c^2 - 1), m_3 = \xi^2(a_3 c^2 - 1)$ (3.3.13)

$$\phi = \frac{\delta^2}{p}\left[\left(\beta'^2 - m_2^2\right)\left(A' \cos m_2 z + B' \sin m_2 z\right) + \left(\beta'^2 - m_3^2\right)\left(C' \cos m_3 z + D' \sin m_3 z\right)\right] e^{i\xi(x-ct)}$$

where $\beta'^2 = \xi^2\left(\frac{c^2}{\delta^2} - 1\right)$ (3.3.14)

The displacements u and w are

$$u = \left[i\xi\left(A\cos m_1 z + B\sin m_1 z\right) + \left(-A' m_2 \sin m_2 z + B' m_2 \cos m_2 z - C' m_3 \sin m_3 z + D' m_3 \cos m_3 z\right)\right] e^{i\xi(x-ct)}$$

$$w = \left[\left(-m_1 A\sin m_1 z + m_1 B\cos m_1 z\right) - i\xi\left(A' \cos m_2 z + B' \sin m_2 z + C' \cos m_3 z + D' \sin m_3 z\right)\right] e^{i\xi(x-ct)}$$

BOUNDARY CONDITIONS

We consider the following mechanical boundary conditions

$$\sigma_{zz} = \sigma_{zx} = m_{zy} = 0 \qquad at \quad z = \pm d$$

3.4 DERIVATION OF SECULAR EQUATIONS

Invoking the boundary conditions at the surface $z = \pm d$ of the plate and using (3.3.12) - (3.3.14) we obtain the system of six simultaneous linear equations as below

$$P\left[Ac_1 + Bs_1\right] + Q\left[(A' s_2 - B' c_2)m_2 + (C' s_3 - D' c_3)m_3\right] = 0$$

$$P\left[Ac_1 - Bs_1\right] + Q\left[(-A' s_2 - B' c_2)m_2 + (-C' s_3 - D' c_3)m_3\right] = 0$$

$$Q\left[\left(-As_1 + Bc_1\right)m_1\right] + P\left[A' c_2 + B' s_2 + C' c_3 + D' s_3\right] = 0$$

$$Q\left[\left(As_1 + Bc_1\right)m_1\right] + P\left[A' c_2 - B' s_2 + C' c_3 - D' s_3\right] = 0$$

$$f_2\left(-A' s_2 + B' c_2\right)m_2 + f_3\left(-C' s_3 + D' c_3\right)m_3 = 0$$

$$f_2\left(A' s_2 + B' c_2\right)m_2 + f_3\left(C' s_3 + D' c_3\right)m_3 = 0$$

where $P = \beta'^2 - \xi^2 - \frac{p\xi^2}{\delta^2}, Q = -2i\xi\left(1 - \frac{p}{2\delta^2}\right), f_i = \beta'^2 - m_i^2$

$c_i = \cos m_i d, s_i = \sin m_i d$

The above system of equations have non-trivial solution if the determinant of the coefficient of amplitudes $\left[A, B, A', B', C', D'\right]^T$ vanishes.

$$\begin{vmatrix} Pc_1 & Ps_1 & Qs_2m_2 & -Qc_2m_2 & Qs_3m_3 & -Qc_3m_3 \\ Pc_1 & -Ps_1 & -Qs_2m_2 & -Qc_2m_2 & -Qs_3m_3 & -Qc_3m_3 \\ -Qs_1m_1 & Qc_1m_1 & Pc_2 & Ps_2 & Pc_3 & Ps_3 \\ Qs_1m_1 & Qc_1m_1 & Pc_2 & -Ps_2 & Pc_3 & -Ps_3 \\ 0 & 0 & -f_2s_2m_2 & f_2c_2m_2 & -f_3s_3m_3 & f_3c_3m_3 \\ 0 & 0 & f_2s_2m_2 & f_2c_2m_2 & f_3s_3m_3 & f_3c_3m_3 \end{vmatrix} = 0$$

After some reductions and manipulations, the secular equation is obtained as

$$\left[\frac{T_1}{T_2}\right]^{\pm 1} - \left[\frac{T_1}{T_3}\right]^{\pm 1} \frac{\left(\beta'^2 - m_2^2\right)m_2}{\left(\beta'^2 - m_3^2\right)m_3} = \frac{-4\xi^2\left(1-\frac{p}{2\delta^2}\right)^2}{\left(\beta'^2 - \xi^2 + \frac{p\xi^2}{\delta^2}\right)^2} m_1 m_2 \frac{\left(\beta'^2 - m_3^2\right) - \left(\beta'^2 - m_2^2\right)}{\left(\beta'^2 - m_3^2\right)}$$

where $T_i = \tan m_i d = \frac{s_i}{c_i}$ (3.4.1)

Here the superscript +1 refers to skew symmetric and -1 refers to symmetric modes. Equation (3.4.1) is the secular equation for the propagation of modified micropolar elastic waves in the plate. These equations can be recognized as Rayleigh-Lamb equations for symmetric and anti-symmetric waves in an infinite rectangular plate in micropolar elastic solid.

3.5 PARTICULAR CASE : Elastic Plate

In the absence of micropolarity effect (K=0=p), we have

$$m_2^2 = \beta'^2$$

And consequently, the secular equation (3.4.1) reduces to

$$\left[\frac{\tan m_1 d}{\tan \beta' d}\right]^{\pm 1} = \frac{-4\xi^2 m_1 \beta'}{\left(\xi^2 - \beta'^2\right)^2} \qquad where\ m_1{}^2 = \xi^2\left(c^2 - 1\right), \beta'^2 = \xi^2\left(\frac{c^2}{\delta^2} - 1\right)$$

which is same as derived in Chapter 2.

3.6 NUMERICAL RESULTS AND DISCUSSION

With the view of illustrating the theoretical results obtained in the preceding sections and comparing these in the context of theory of elasticity, we now present some numerical results. The material chosen for this purpose is aluminium-epoxy composite, the physical data for which is given as(1998) $\varepsilon = 0.073, \lambda = 7.59\times10^{10}\,Nm^{-2}, \mu = 1.89\times10^{10}\,Nm^{-2}, \rho = 2.19\times10^{3}\,kgm^{-3}$, $K = 0.0149\times10^{10}\,Nm^{-2}, j = 0.196\times10^{-4}\,m^{2}, \gamma = 0.268\times10^{6}\,N, \omega^{*} = 4.36\times10^{11}/\sec$.

The non dimensional phase velocity and attenuation coefficient of symmetric and skew symmetric modes of wave propagation are computed for various values of non dimensional wave number from the secular equation. The numerically computed values of phase velocity and attenuation are shown graphically in Figures 3 and 4 for different modes (n=0 to n=2). The phase velocity of lowest symmetric mode (n=0) remains constant with the variation in wave number whereas the phase velocity of lowest skew symmetric mode varies at lower wave number and becomes constant at higher wave number. The phase velocity of higher modes of wave propagation, symmetric and skew symmetric attains quite large values at vanishing wave number, which sharply slashes down to become steady with increasing wave number. It is observed that the phase velocities of different modes of wave propagation start from large values at vanishing wave number and then exhibit strong dispersion until the velocity flattens out to the value of the micropolar Rayleigh wave velocity of the material at higher wave numbers. The reason for this asymptotic approach is that for short wavelengths (or large wave number), the material plate behaves like a thick slab and hence the coupling between upper and lower boundary surfaces is reduced and as a result the properties of symmetric and skew symmetric waves become more and more similar.

The phase velocity for symmetric and skew symmetric modes in micropolar plate and elastic plate are compared in Figures 5 and 6. The solid curves correspond to micropolar elastic plate (MEP) and the dashed curves refers to elastic plate (EP). It is observed that for symmetric mode n=1, the values of phase velocity are smaller in micropolar than in elastic plate for wave number less than 3 and for wave number greater than 3, phase velocities in MEP and EP are nearly same. I case of symmetric mode n=2, for wave number lying between 1.5 and 3.0, phase velocity of EP is more than in case of MEP and phase velocity profiles coincide in respect of EP and MEP for wave number less than 1.5 and greater than 3.0.

For skew symmetric modes of wave propagation, we observe the following: (a) for mode n=1 and wave number less than 1.0, the phase velocity in elastic is more than in micropolar elastic and for wave number lying between 1.0 and 2.0, the phase velocities in MEP and EP are nearly same. For wave number lying between 1.0 and 2.0, phase velocity for MEP is more than in EP, and for wave number greater than 4.0 phase velocities in MEP and EP are nearly same (b) for modes n=0, 2, phase velocity profiles for micropolar elastic and elastic almost coincide.

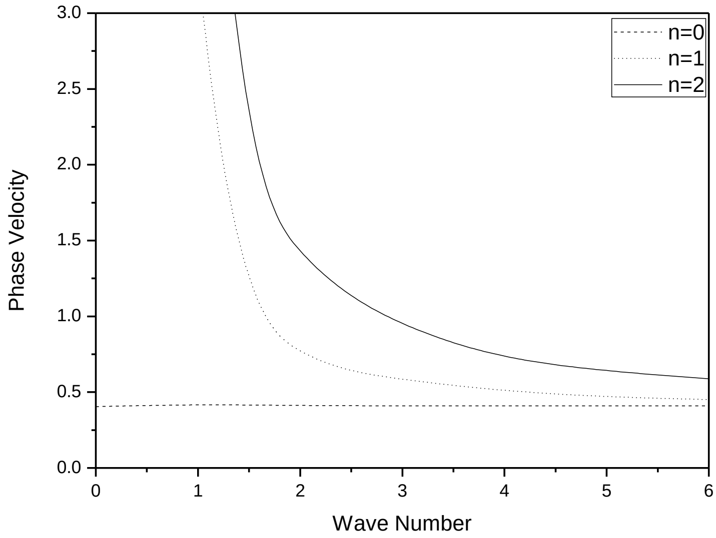


Figure 3 : Dispersion curves for Symmetric wave mode propagation

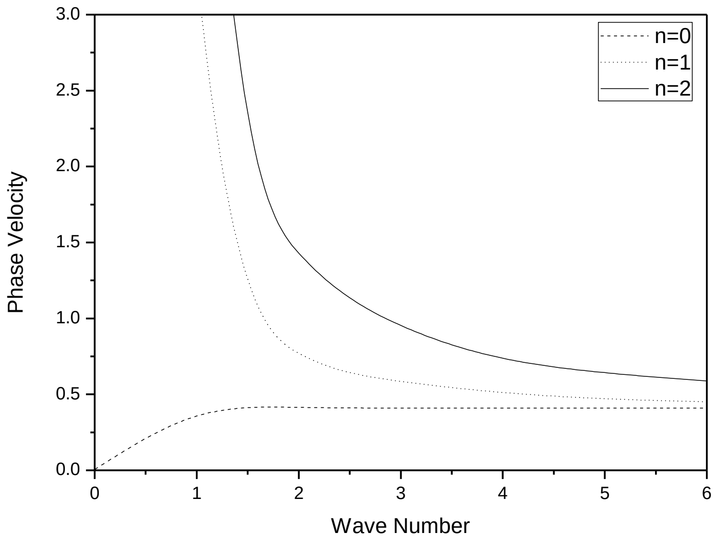


Figure 4 : Dispersion curves for Skew symmetric wave mode propagation

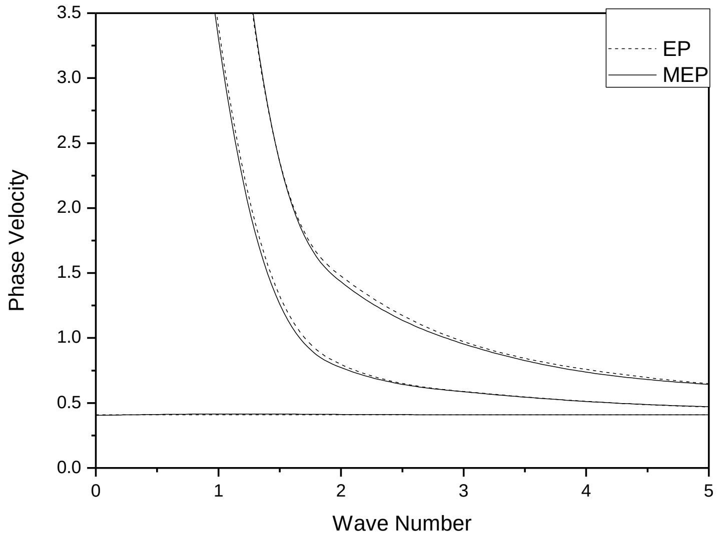


Figure 5 : Phase velocity profile for symmetric modes of wave propagation

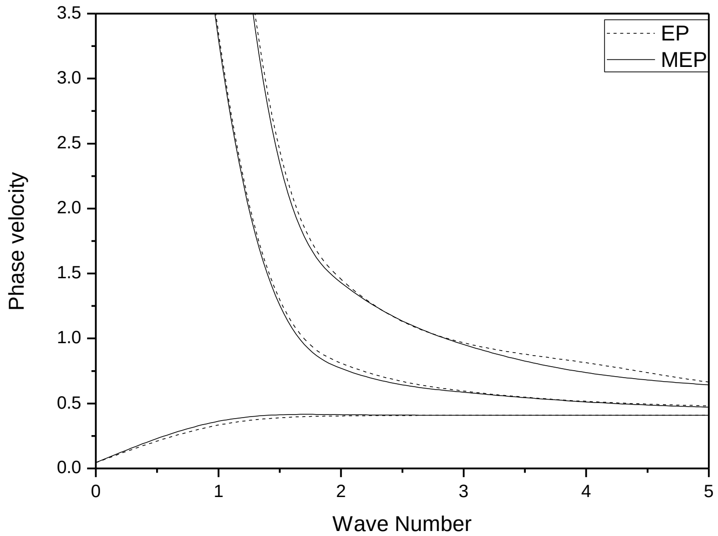


Figure 6 : Phase velocity profile for anti symmetric modes of wave propagation

CONCLUSION AND APPLICATIONS

Rayleigh Lamb waves have been studied in the context of both Classical and Micropolar theories of Elasticity. Dispersion curves for symmetric and anti symmetric modes of wave propagation are obtained. Significant effects of micropolarity have been observed on the modes.

Lamb waves are useful in ultrasonic pipe erosion/corrosion monitoring (Pei et al.(1995)) and for ultrasonic bone assessment by Lee and Yoon (2003). R.S.Lakes (1995) and Fatemi (2002) have applied micropolar model for bone analysis. Extensive developments in the applications of Lamb waves provides a foundation for the inspection of many industrial products in aerospace, pipe, and transportation. Guided waves like Rayleigh and Lamb waves have great potential for NDE. Unlike the more traditional ultrasonic scans, they allow the inspection of comparatively large areas on short timescales. They can be used in isotropic and anisotropic materials equally.

Seeing the wide range application of Micropolar materials and Lamb waves from structural analysis to bone analysis, further extension of our model may be useful in these fields.

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