EFFECTS OF CORIOLIS FORCE ON THE EQUILIBRIUM STRUCTURES OF ROTATIONALLY AND TIDALLY DISTORTED STELLAR MODELS

Thesis submitted in partial fulfillment of the requirement for
The award of the degree of
Masters of Science
In
Mathematics and Computing

Submitted by
Nidhi Jain
Roll no.- 30703013

Under
the guidance of
Dr. Mahesh Kumar Sharma

JULY 2009
School of Mathematics and Computer Applications
Thapar University
Patiala-147004 (PUNJAB)
INDIA

DEDICATED
TO MY PARENTS
AND
MY SUPERVISOR
DR. MAHESH KUMAR SHARMA
FOR SPENDING
THEIR PRESENT
TO MAKE
MY FUTURE

CERTIFICATE

*I hereby certify that the work which is being presented in the thesis entitled "***Effects of coriolis force on the equilibrium structures of rotationally and tidally distorted stellar models***" in partial fulfillment of the requirements for the award of degree of Master of Science, School of Mathematics and Computer Applications, Thapar University, Patiala is an authentic record of my own work carried out under the supervision of Dr. Mahesh Kumar Sharma.*

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

Nidhi Jain
(Nidhi Jain)
Reg.no. 30703013

This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

(Dr. Mahesh Kumar Sharma)
Assistant Professor
SMCA, Thapar University
Patiala.

Countersigned by:

10.7.09
Dr. S.S. Bhatia
(Professor & Head)
School of Mathematics & Computer Applications
Thapar University, Patiala.

RK Sharma
Dr. R.K. Sharma 16/7
Dean of Academic Affairs
Thapar University
Patiala.

ACKNOWLEDGEMENTS

I wish to express my deep sense of gratitude to Dr. M.K.Sharma, Assistant Professor, SMCA, Thapar University Patiala for his meticulous guidance, constant encouragement and moral support all through the pursuance of this work. I am fortunate to have had an opportunity to work under his supervision.

I wish to express my sincere thanks to Dr. S.S.Bhatia, Head, SMCA, Thapar University Patiala for their valuable support and encouragement given to me throughout this work. This thesis could not have been in its present state without their suggestions.

It is difficult to express in words my feeling of gratitude to Dr. A.K.Lal, Associate Professor, SMCA, Thapar University Patiala as a PG coordinator as well as for his affection and generous help which made it possible to complete this work.

I would also like to extend a word of thanks to Mr. Ankush Pathania, Associate lecturer, SMCA, Thapar University Patiala who supported me with a continuous understanding, valuable support during the completionof this work.

I also take this opportunity to express my sincere thanks to the staff of SMCA Thapar University Patiala for providing me constant encouragement.

I owe my parents for their continuous support and blessings. Special appreciations are due to my loving sister Monika and brother Harish for their love and care bestowed upon me.

Dated: 11-Aug-09 (NIDHI JAIN)

ABSTRACT

The present thesis deals with the problems of determining the effects of rotation and/or tidal distortions on the equilibrium structure of gaseous spheres in the presence of coriolis force. Such a study has practical relevance in astrophysics where it is expected to help in understanding the problems of stellar stability and stellar variability of rotating stars as well as of stars in binary and multiple systems.

This thesis consists of two chapters. Chapter one is introductory in which a brief explanation of astrophysical significance of the theoretical study of the problem of determining the effects of rotation and/or tidal distortions on the equilibrium structures of gaseous spheres and averaging technique of Kippenhahn and Thomas has been discussed. Certain results obtained by Mohan, Saxena and Agarwal and a brief survey of literature have been also presented in this chapter. In chapter two, effect of coriolis force on the equilibrium structures of rotationally and tidally distorted stellar models is presented and applied to the Prasad and Roche model of stars. Analysis of results have also been discussed in this chapter.

CONTENTS

CHAPTER – I

INTRODUCTION

This chapter is introductory in nature. In section 1.1 we first explain in brief the astrophysical significance of the theoretical study of the problem of determining the effects of rotation and/or tidal distortions on the equilibrium structures of gaseous spheres. A brief survey of the literature available on the subject is presented in Section 1.2. In section 1.3, we present the fundamental system of differential equations which governs the equilibrium structure of a gaseous sphere in hydrostatic and thermal equilibrium. Kippenhahn and Thomas (12) averaging technique to determine the equilibrium structures of rotationally and tidally distorted gaseous spheres has been discussed in section 1.3.1. The concepts of Roche equipotentials and Roche limits are then introduced in section 1.3.2. Certain results obtained by Kopal (14) for the Roche equipotentials are also presented in this subsection. Mohan, Saxena and Agarwal's (31) approach for determining the equilibrium structures of rotationally and tidally distorted gaseous spheres have been discussed in section 1.4. A brief summary of the work presented in the succeeding chapter, is finally presented in section 1.5.

1.1 ASTROPHYSICAL SIGNIFICANCE OF THE PROBLEM OF DETERMINING THE EFFECTS OF ROTATIONAL AND TIDAL DISTORTIONS ON THE EQUILIBRIUM STRUCTURES OF GASEOUS SPHERES

The theoretical model of a star is essentially a self gravitating gaseous sphere in hydrostatic and thermal equilibrium. Theoretical studies of the problems of the equilibrium structure of gaseous spheres have often been carried out to understand the nature of the internal structures, responsible for various observed phenomena of the stars. Whereas some of the stars are observed as single stars others are observed in groups of two or more stars.

Observations also show that some of the stars are rotating about their axes of rotation. This rotation may be a solid body rotation or a differential rotation. Many of the stars in binary and multiple systems are also known to be rotating about their axes as well as revolving around each other. Thus if we consider the equilibrium model of a single star which is not rotating as a gaseous sphere, the equilibrium model of a rotating star will be rotationally distorted gaseous sphere. Similarly, the equilibrium model of a non-rotating star appearing in a binary or a multiple system will be a tidally distorted gaseous sphere and a rotationally and tidally distorted gaseous sphere if the star is rotating as well.

Some of the stars are observed as variable stars in which brightness of the stars vary with time. In some of these variable stars, the variations in luminosity are periodic. It is reasonable to assume in the case of such regular variable stars that these are pulsating gaseous spheres in which the variation in luminosity is being caused by the periodic contraction and expansion of the gaseous mass. The regular variable stars gained importance in astrophysics when it was discovered that there exists a definite relation between the periods of pulsation and the luminosities of such stars. This relationship has been utilized to determine the distance of stars. Such investigations are also expected to help us in better understanding the nature of the internal structure of the stars.

Theoretical investigation of the effects of rotation and tidal forces on the equilibrium structures of gaseous spheres needs a deep study. Analytic studies of the problems of rotating stars and stars in binary systems have engaged the attention of astrophysicist since long with a view to analyze and understand the observational behavior of such stars. A large number of stars observed in the sky are rotating stars or binary stars. In a binary system of stars the two stars normally rotate about their own axis as well as revolve about their common center of mass. In a majority of binary stars, one star called primary, is generally more massive compared to its companion star. Contact binaries have also been observed in which outermost surfaces of the two stars just touch each other. Thus, the study of rotationally and tidally distorted stars are expected to help in better understanding of problems of stellar stability.

1.2 BRIEF SURVEY OF THE LITERATURE.

Most of the theoretical studies about the equilibrium structures of the stars have been carried out in literature by assuming the star to be an undistorted spherical gaseous sphere. Extensive literature is now available on this subject (see for instance Chandrasekhar (5), Kippenhahn and Weigert (13), Kopal (15,17).

The brightness of some stars varies with time. These are called variable stars. In some of these variable stars, the variations in luminosity are periodic. An important class of regular variable stars is Cepheid variables. The regular variable stars, gained importance in astrophysics in the year 1912, when Miss Leavitt discovered that there exists a definite relation between the periods of pulsation and the luminosities of such stars and the relationship could be utilized to determine the distance of these stars. In most of the theoretical studies of such stars, the variable star is represented by a gaseous sphere, both in hydrostatic and thermal equilibrium.

Some of the variable stars are also observed to be rotating stars. In such cases the rotation of the star will effect the equilibrium structure. However, if the pulsating star is a member of the binary or a multiple system, then its equilibrium structure will also get affected by the rotational and tidal forces.The theoretical investigation related to the problems of determining the equilibrium structures and stability of rotating and self gravitating objects, possibly begun with the work of Newton. He was the first to realize the importance of the law of gravitation for explaining the figures of celestial bodies. Later on Maclaurin, Clairaut, Laplace, Legendre, Jacobi etc. contributed ideas, necessary for the development of the general theory of rotating bodies.

Edward Arthur Milne developed a technique for constructing the first detailed model for a slowly rotating star in pure radiative equilibrium in the year 1923. Authors such as James (11), Endal and Sofia (8), Kopal (16), Geroyannis and Valvi (10) have also investigated the problems of equilibrium structures of rotating stars. Some of the stars have solid body rotation while some of the stars are observed to be rotating differentially. In such type of stars different parts of the star are rotating about the axis of rotation with different angular velocities. Problems of differentially rotating stellar models have also been studied in literature.Authors such as Geroyannis and Antonakopoulus (9), Mohan et al (27,28,29) have also analyzed the problems of differentially rotating stars.

Equilibrium structures of stars which appear in binary and multiple systems are likely to be affected by both the rotational as well as the tidal effects of the companion stars. Attempts have been made in literature to determine the effects of rotation and tidal distortions on the equilibrium structure of the stars in binary and multiple systems. In a series of papers Chandrasekhar (2,3,4) developed a first order analysis which he applied to the study of the rotational problem, the tidal problem and the binary star problem. The method, however, was found unsuitable when the separation between the components is only a few times the undisturbed radius of the primary. Kippenhahn and Thomas (12) suggested a practical way of analyzing the effects of rotation and tidal distortions on the equilibrium structures of stars by approximating the actual equipotentials surfaces of the star by Roche equipotentials.

Chan and Chau (1) developed a method which allows an efficient and accurate investigation of the structure and evolution of a rotationally and tidally distorted star in binary systems. Lal et al (21) have discussed the equilibrium structures of rotationally and tidally distorted primary component of binary systems taking into account the effect of mass variation inside the star. Lal et al (22) studied the validity of series expression being used for determining the position of a point on a Roche equipotential in case of rotating stars and stars in binary systems. Deupree and Karkas (7) studied the structure and evolution of binary stars using the two-dimensional stellar structure algorithm.

Kopal and Ali (18) studied the integrability of the Roche coordinates. The simple hypothesis of the pulsating model of a regular variable star is made all the more complicated by the fact that some of the variable stars are observed to be rotating stars or stars in binary or multiple systems. Most of the authors have studied pulsations of stars having solid body rotation. However, there are several variable stars which are suspected to be rotating differentially. Lal and Mohan (20) have presented a method for computing the equilibrium structures and various physical parameters of a primary component of the binary system. Mohan and Saxena (24) used the Kippenhahn and Thomas (12) averaging technique in conjunction with

Kopal's results on Roche equipotentials to determine the combined effects of rotation and tidal distortions on the equilibrium structures of the of the stars.

Lal (19) studied in detail the equilibrium structures of differentially rotating stellar models. Lal et al (20) applied this technique to study the equilibrium structures of differentially rotating and tidally distorted white dwarf models of stars. Lal, Pathania and Mohan (35) have tried to determine the effect of coriolis force on the shapes of Roche equipotential surfaces of rotating stars and stars in binary systems. They have obtained the equations of Roche equipotential which takes into account the effect of coriolis force besides the centrifugal and gravitationl forces. Sharma (36) have also studied the equilibrium structures and eigen frequencies of Prasad model, Roche model and composite models without taking into account the effect of coriolis force. Lal and Pathania (34) have modified Kopal's expression for Roche equipotential to incorporate the effects of coriolis force and then they have applied this modified expression for the Roche equipotential to compute the equilibrium structures and shapes of polytropic models of rotating stars and stars in binary system.

Whereas the properties of equilibrium structures of undistorted gaseous sphere have been investigated in detail in literature, the effect of rotation and tidal distortions on the equilibrium structures of gaseous spheres has still not been fully understood. For instance comments have generally been made that the approaches based on Roche equipotential for computing the equilibrium structures of stars in binary systems carry out their studies in fixed frame of reference and do not account for coriolis force which is expected to arise in such cases when rotating frames of reference are used. In the present work we have addressed ourselves to the analytic and computational studies of problems related to this field.

1.3 BASIC EQUATIONS DETERMINING THE EQUILIBRIUM STRUCTURE OF A GASEOUS SPHERE

The general problem of determining the effects of rotation and / or tidal distortions on the equilibrium structure of realistic models of stars is quite complex. No general approach is as yet available which can determine exactly the combined effects of rotation and tidal distortions on the equilibrium structure of gaseous spheres. Most of the attempts in this direction have generally tried to investigate some particular aspects of this problem in certain approximate ways. In its present state, the problem of determining the effects of rotation and tidal distortions on the structure, stability of gaseous spheres thus far has not been satisfactorily solved. Keeping in view its importance in astrophysics, there is still need for further investigations in this direction.

The system of basic equations which governs the equilibrium structure of a gaseous sphere in hydrostatic and thermal equilibrium, especially as they pertain to the problems of the equilibrium structure of stellar models, are by now well established in literature. Let P and ρ denote the pressure and the density respectively at a point distant r from the center of the sphere and $M(r)$ the mass contained in a sphere of radius r. Also let T be the temperature at a point distant r from the center of the sphere, $L(r)$ the net amount of energy crossing a spherical surface of radius r per second and ε the rate of energy generation from thermo – nuclear processes per gram per second, then the equation of conservation of mass states is

$$\frac{dM(r)}{dr}=4\pi r^2\rho \tag{1.1}$$

The equation of hydrostatic equilibrium gives

$$\frac{dp}{dr}=-G\frac{M(r)}{r^2}\rho \tag{1.2}$$

where G is the gravitational constant.

The luminosity equation is

$$\frac{d\,L(r)}{dr}=4\pi r^2\rho\varepsilon \tag{1.3}$$

and the energy transport equation states

$$\frac{dT}{dr}=-\frac{3\kappa}{4ac}\frac{\rho}{T^3}\frac{L(r)}{4\pi r^2} \tag{1.4}$$

in case of radiative equilibrium and for convective equilibrium,

$$\frac{dT}{dr}=(1-\frac{1}{\gamma})\frac{T}{P}\frac{dP}{dr} \tag{1.5}$$

where κ is the mass absorption coefficient of the gas, c is the velocity of light, a is the Stefan – Boltzmann constant and γ is an adiabatic exponent which is equal to the ratio of specific heats at constant pressure and volume (for a perfect non degenerate particle gas in the absence of radiation ($\gamma=1$).

In addition to the above four differential equations we also require three more explicit relations which characterize more specifically the behavior of the interior gas. They are the equation of state, the equation for the mass absorption for energy generation by thermonuclear processes. These equations may be formally represented by

$$P=P(\rho,T,\text{chemical composition}) \tag{1.6a}$$

$$\kappa=\kappa(\rho,T,\text{chemical composition}) \tag{1.6b}$$

and

$$\varepsilon=\varepsilon(\rho,T,\text{chemical composition}) \tag{1.6c}$$

Equations (1.1 – 1.5) are to be satisfied in every layer of the gaseous sphere. In addition certain boundary conditions are also to be satisfied. It is obvious from the definitions of $M(r)$ and $L(r)$, that at the center we have

$$r=0 \,:\, M(r)=0,\, L(r)=0 \tag{1.7a}$$

$$r = R\text{: } M(r)=M \text{ and } L(r) = L \tag{1.7b}$$

where R is the radius of the gaseous sphere, M the total mass and L the total energy radiated by the gaseous sphere. The pressure and the temperature at the outermost surface of a gaseous sphere, taken as the representative model of star, are often approximated by what are generally known as zero boundary conditions. Under these conditions we take

$$r=R : P=0 \text{ and } T=0 \tag{1.7c}$$

Conditions (1.7c) are reasonably accurate for most of the theoretical stellar models. If moreover, more accurate computations are to be done then we may replace (1.7c) by

$$r=R : P=P_s \text{ and } T=T_s \tag{1.7d}$$

where P_s is determined from the pressure of the upper atmospheric layers and T_s is determined from the effective temperature of the star.

In problems where the thermal properties of the model are either not important or are not to be investigated, the equilibrium structure of the gaseous sphere may be determined by solving equations (1.1 – 1.2) using some suitable equation of state together with boundary conditions

At the center $r=0 : M(r) = 0$ (1.8a)

At the surface $r=R : M(r) = M,\; P = 0 \text{ or } P_s,\; \rho=0 \text{ or } \rho_s$ (1.8b)

A number of the theoretical as well as numerical studies regarding the equilibrium structures of gaseous spheres, particularly those which have particular reference to the problems of the equilibrium structures of the stars are available in literature (Chandrasekhar (5), Kippenhahn and Weigert (13)).

1.3.1 AVERAGING TECHNIQUE OF KIPPENHAHN AND THOMAS

To study the effects of rotation and tidal distortions on the equilibrium structure of gaseous spheres, Kippenhahn and Thomas (12) developed the concept of topologically equivalent spherical surfaces corresponding to actual equipotential surfaces of a rotationally and tidally distorted model. They define on these equivalent spherical surfaces, quantities such as $\bar{f}, \bar{g}$ etc. which denote certain averages of the quantities f, g respectively on the actual equipotential surfaces. If ψ denotes the total potential (gravitation, rotation and tidal forces) of a rotationally and tidally distorted model at an arbitrary point $P(x, y, z)$ then $\psi(x, y, z)$ = constant is an equipotential surface. Let V_ψ be the volume enclosed by the equipotential surface ψ= constant and S_ψ the surface area of this equipotential surface. For any function $f(x, y, z)$ they define $\bar{f}$ as its mean value over the equipotential surfaces ψ = constant by the relation

$$\bar{f} = \frac{1}{S_\psi} \int_{\psi = const.} f \, d\sigma \tag{1.9}$$

where $d\sigma$ denotes the surface element of the equipotential surface ψ = constant. Clearly $\bar{f}$ is a function of equipotential surface ψ only and can be obtained as equation (1.9) for each equipotential surface ψ = constant. Kippenhahn and Thomas also define a variable r_ψ in analogy with the radius of sphere by the relation

$$V_\psi = \frac{4}{3} \pi r_\psi^3 \tag{1.10}$$

Also by definition

$$S_\psi = \int_{\psi=\text{constant}} d\sigma \tag{1.11}$$

Obviously, in general, S_ψ is not equal to $4\pi r_\psi^2$. Kippenhahn and Thomas (12) define a function $g\,(x, y, z)$ by the relation

$$g = \frac{d\psi}{dn} \tag{1.12}$$

This g corresponds to the force of gravity of a sphere. The distance dn between two neighboring surfaces ψ = constant and $\psi + d\psi$ = constant is, in general, not constant (i.e. not same at all points of the surface). They used (1.12) to compute the mean values $\bar{g}$ and $\overline{g^1}$ with the help of relations

$$\left.\begin{aligned} \bar{g} &= \frac{1}{S_\psi} \int_{\psi=const} \frac{d\psi}{dn} d\sigma \\ \overline{g^{-1}} &= \frac{1}{S_\psi} \int_{\psi=const} \left(\frac{d\psi}{d\,n}\right)^{-1} d\sigma \end{aligned}\right\} \tag{1.13}$$

Both $\bar{g}$ and $\overline{g^{-1}}$ are functions of ψ alone and represent the value of g and g^{-1} respectively over the topologically equivalent spherical surface. The volume dV_ψ between the surface ψ = constant and $\psi + d\psi$ = constant is given by

$$dV_\psi = \int_{\psi=const} dn\ d\sigma = \int_{\psi=const} \left(\frac{d\psi}{dn}\right)^{-1} dn \ = S_\psi\, \overline{g^{-1}}\, d\psi \tag{1.14}$$

Kippenhahn and Thomas(12) also defined nondimensional parameters u, v and w as

$$u = \frac{S_\psi}{4\pi r_\psi^2},\ v = \frac{\bar{g}\, r_\psi^2}{GM_\psi},\ w = \frac{\overline{g^{-1}}\, GM_\psi}{r_\psi^2} \tag{1.15}$$

where M_ψ is the mass enclosed by equipotential surface ψ=constant.

We may thus regard the equipotential surface ψ = constant to be topologically equivalent to a sphere of radius r_ψ for which various functions are defined by the above relations. (It may be noticed that if ψ is the gravitational potential of a sphere then the surface ψ = constant is spherical surface with $r_\psi = r$ for which u =1 and $g = GM_\psi / r_\psi^{\,2}$ is constant on these spheres and therefore v and w are constants and equal to 1).

Equations (1.9) to (1.15) are purely mathematical definitions, which have been applied by Kippenhahn and Thomas (12) to gravitational fields of gaseous spheres distorted by rotational and tidal forces. In hydrostatic equilibrium the equipotential surfaces are also surface of equipressure and equidensity. Therefore, on an equipotential surface the pressure P_ψ and the density ρ_ψ are also constant. Using these concepts, Kippenhahn and Thomas (12) obtained the equations governing the equilibrium structure of a rotationally and tidally distorted stellar model in the following manner

From equation (1.10) the mass dM_ψ between the equipotential surface ψ = constant and $\psi + d\psi$ = constant is given by

$$dM_\psi = dV_\psi\, \rho_\psi = 4\pi\, r_\psi^2 \rho_\psi\, dr_\psi \tag{1.16}$$

Thus, we get

$$\frac{dM_\psi}{dr_\psi} = 4\pi\, r_\psi^2 \rho_\psi \tag{1.17}$$

From equation (1.14) and (1.16) we have

$$d\psi = \frac{d\psi}{dV_\psi} dV_\psi = (\frac{dV_\psi}{d\psi})^{-1} \frac{dM_\psi}{\rho_\psi} = \frac{dM_\psi}{S_\psi\, \overline{g^{-1}}\, \rho_\psi} \tag{1.18}$$

Using relations (1.15), we get

$$d\psi = \frac{G\, M_\psi\, dM_\psi}{4\pi\, r_\psi^4 \rho_\psi\, u\, w} \tag{1.19}$$

The conditions for hydrostatic equilibrium, $dP_\psi / d\psi = -\rho_\psi$, can now be written with equation (1.15) in the form

$$\frac{dP_\psi}{dM_\psi} = -\frac{G\, M_\psi}{4\pi\, r_\psi^4} f_p \tag{1.20}$$

where

$$f_P = \frac{1}{u\,w} = \frac{4\pi r_\psi^4}{G M_\psi} \frac{1}{S_\psi \overline{g^{-1}}}$$

The factor f_P is a function of ψ only. If ψ is known, the equipotential surface can be determined, and then consequently values of $S_\psi, r_\psi, \overline{g}$ and $\overline{g^{-1}}$ for each equipotential surface can be obtained simply from the geometry of the equipotentials. The mass M_ψ which depends on the density distribution ρ_ψ can be determined by integrating the equation (1.17). Similarly the other structure equations derived by Kippenhahn and Thomas (12), which includes the effects of rotation and tidal distortions on the equilibrium structure of gaseous spheres are as follows.

For chemically homogenous spheres, the nuclear energy generation rate ε depends only upon density ρ_ψ and the temperature T_ψ and are, therefore, constant on equipotential surface.

Thus, if L_ψ is the energy which passes per second through the equipotential surface $\psi =$ constant, then

$$\frac{dL_\psi}{dM_\psi} = \varepsilon \tag{1.21}$$

Using equation (1.17), it can be written as

$$\frac{dL_\psi}{dr_\psi} = 4\pi r_\psi^2 \rho_\psi\, \varepsilon \tag{1.22}$$

If the energy is transported by radiation, then the energy transport equation is

$$F_\psi = -\frac{4acT_\psi^3}{3\kappa} \frac{d\psi}{dn} \frac{dT_\psi}{dM_\psi} \frac{4\pi r_\psi^4\, u\, w}{G M_\psi} \tag{1.23}$$

where F_ψ is the radiative flux on the equipotential surface $\psi =$constant. By integrating F_ψ over the equipotential surface ψ = constant, we get

$$L_\psi = \int_{\psi = constant} F_\psi \, d\sigma$$

$$= -\frac{4acT_\psi{}^3}{3\kappa} \frac{dT_\psi{}^3}{dM_\psi} u\,w \frac{4\pi r_\psi{}^4}{GM_\psi} \int_{\psi = constant} \left(\frac{d\psi}{dn}\right) d\sigma \tag{1.24}$$

$$= -\frac{64\pi^2 acT_\psi{}^3 r_\psi{}^4}{3\kappa} u^2\, v\, w \frac{dT_\psi}{dM_\psi}$$

so that

$$\frac{dT_\psi}{dM_\psi} = -\frac{3\kappa L_\psi}{64\pi^2 acT_\psi{}^3 r_\psi{}^4} f_T \tag{1.25}$$

Using equation (1.16), this equation can be expressed as

$$\frac{dT_\psi}{dr_\psi} = -\frac{3\kappa \rho_\psi L_\psi}{16\pi a c T_\psi^3 r_\psi^2} f_T \tag{1.26}$$

where

$$f_T = \frac{1}{u^2\, v\, w}$$

Equations (1.17), (1.20), (1.21) and (1.25) which are the four basic equations governing the equilibrium structure of a gaseous sphere distorted by rotational and tidal forces may now be collected together and written as.

$$\frac{dM_\psi}{dr_\psi} = 4\pi r_\psi^2 \rho_\psi \tag{1.27a}$$

$$\frac{dP_\psi}{dM_\psi} = -\frac{GM_\psi}{4\pi r_\psi^4} f_p \tag{1.27b}$$

$$\frac{dL_\psi}{dM_\psi} = \varepsilon \tag{1.27c}$$

and

$$\frac{dT_\psi}{dM_\psi} = -\frac{3\kappa L_\psi}{64\pi^2 acT_\psi{}^3 r_\psi^4} f_T \tag{1.27d}$$

where

$$f_P = \frac{1}{u\,w} \qquad \text{and} \qquad f_T = \frac{1}{u^2\, v\, w}.$$

These reduce to the normal equations used for determining the equilibrium structures of spherical models of stars when distortion parameters u, v, w are set one each. The boundary conditions which the above equations has to satisfy are

$$M_\psi = 0, \quad L_\psi = 0 \tag{1.28a}$$

at the center $r_\psi = 0$

$$M_\psi = M_0, L_\psi = L_{\psi S}$$

$$P_\psi = 0, T_\psi = 0 \ or \ P_\psi = P_{\psi s}, T_\psi = T_{\psi s}$$

at the free surface $r_\psi = R_\psi$ (1.28b)

where M_0 is the total mass of the model and $L_{\psi s}, P_{\psi s}, T_{\psi s}$ are the values of L_ψ, P_ψ, T_ψ respectively, on the outermost equipotential surface.

1.3.2 ROCHE EQUIPOTENTIAL

Roche equipotentials have often been used to analyze the problems of rotationally and tidally distorted models of stars. In order to introduce the concept of Roche equipotential, we assume two components of a close binary system known as primary and secondary star. The primary star is supposed to be much more massive than the secondary which is assumed as a point mass causing tidal effects on the more massive primary component. Both the components of binary system are assumed to be rotating about their axes as well as revolving about their common center of mass. Following Kopal (14), Mohan and Singh (25,26) certain results on Roche equipotential which are of practical interest to the present study, are summarized below:

Suppose that M_0 and M_1 are the masses of the two components of a close binary system separated by distance *D*. Further suppose that the primary component of this system of mass M_0 is much larger than its companion star of mass $M_1 (M_0 \geq M_1)$ which can be regarded as a point mass. Next suppose that the position of the two components is referred to a rectangular system of Cartesian coordinates with origin at the center of gravity of mass M_0, the $x-$ axis along the line joining the mass centers of two components and $z-$axis perpendicular to the plane of the orbit of the two components (Fig. 1.1). Then the total potential ψ due to the gravitational and disturbing force acting at an arbitrary point $P(x, y, z)$, which is not inside any of these two gaseous spheres is given by:

$$\psi = \frac{GM_0}{r} + \frac{GM_1}{r_2} + \frac{\Omega^2}{2}[(x - \frac{M_1 D}{M_0 + M_1})^2 + y^2] \tag{1.29}$$

where $r^2 = x^2 + y^2 + z^2$ and $r_2^2 = (D - x)^2 + y^2 + z^2$ represent the squares of the distances of *P* from the center of gravity of the two components, Ω denotes the angular velocity of rotation of the system about an axis perpendicular to the *xy*- plane and passing through the center of gravity of the system and *G* the constant of gravitation.

The first, second and third term on the right hand side of equation (1.29) respectively represent the potential which arises due to the mass M_0 of the primary component, the disturbing potential of its companion of mass M_1 and the potential arising from the centrifugal force respectively. Equation (1.29) strictly holds at points which are outside the components of binary system. In case we assume Roche model for the primary (In Roche model it is assumed that the total mass of a star is concentrated at its center and this point mass is surrounded by an evanescent envelope in which density varies inversely as the square of the distance from its center) and a point mass for the secondary component, equation (1.29) holds everywhere. Also if assume that the angular velocity Ω is identical with Keplerian angular velocity, that is,

$$\Omega^2 = \Omega_k^2 = G\frac{M_0 + M_1}{D^3} \tag{1.30}$$

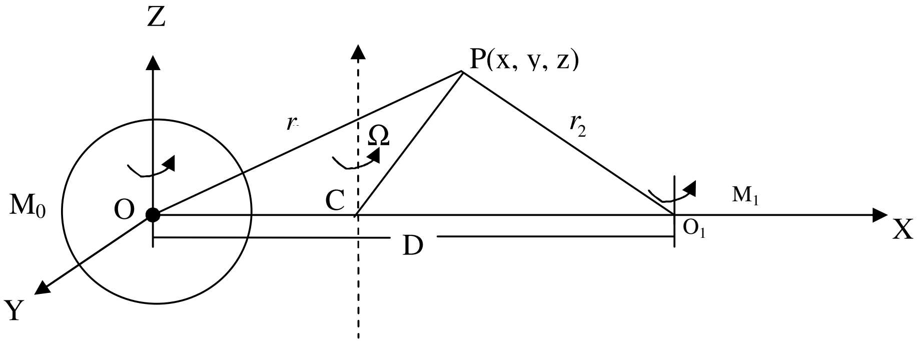

Fig 1.1: Axis of Reference for binary system

then we get a relation of the type

$$n = \frac{q+1}{2} \tag{1.31}$$

Equation (1.29) can be expressed in nondimensional form as

$$\psi^* = \frac{1}{r^*} + q[\frac{1}{\sqrt{1 - 2\lambda r^* + r^{*2}}} - \lambda r^*] + n r^{*2}(1 - \nu^2) \tag{1.32}$$

where

$$\psi^* = \frac{D\psi}{GM_0} - \frac{M_1^{\,2}}{2M_0(M_0 + M_1)}$$

is the nondimensional form of total potential ψ and $r^* = r/D$ is nondimensional form of r. Also $\lambda = \sin\theta\cos\phi$, $\mu = \sin\theta\sin\phi$ and $\nu = \cos\theta\,(r, \theta, \phi$ being the polar spherical coordinate of the point *P*). Moreover,

$$q = \frac{M_1}{M_0} \tag{1.33}$$

is a nondimensional parameter representing the ratio of mass of the secondary over primary and $2n$ represents the square of the normalized angular velocity Ω. The equation (1.29) reduces to the potential of a purely rotating spherical model if $q=0$. For $n=0$, it reduces to the potential of a non-rotating spherical model distorted by the tidal effects of the companion alone.

The surfaces generated by setting ψ = constant on the left hand side of equation (1.29) are referred to as Roche equipotentials. Roche equipotentials in nondimensional form may be represented by ψ^* = constant where ψ^* is same as defined in equation (1.32). The form of Roche-equipotential depends entirely upon

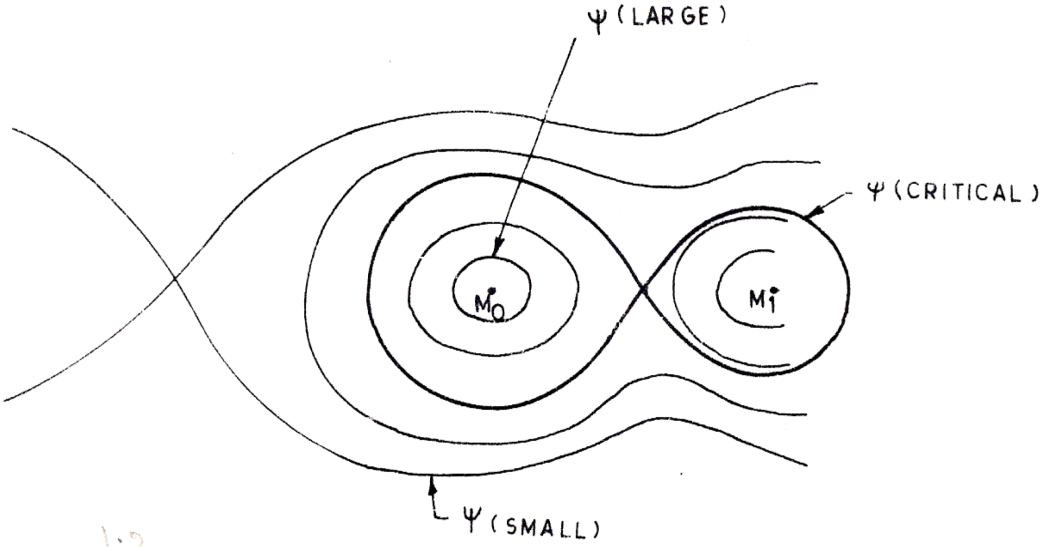

Fig 1.2 Roche equipotential surfaces (two dimensional)

the values of ψ . If ψ is large the corresponding equipotentials consist of two separate ovals, closed around each of the two mass points (Fig. 1.2). For specified values of M_0, M_1, Ω and D the right hand side of equation (1.29) can be large only if r and r_2 becomes small. Therefore, large values of ψ correspond to equipotentials which differ little from spheres surrounding each of the two mass centers. With decreasing values of ψ, these spherical equipotential surfaces become oval shaped and get elongated in the direction of the center of gravity of the system until for a certain critical value of ψ, which is characteristic of each mass ratio, both oval shaped surfaces unite at a single point on the x-axis to form a dumbbell like configuration. These limiting values of ψ are called Roche limits. For certain mass ratios Kopal (14) computed the numerical values of Roche limits in the case of synchronous binary stars for values of q ranging from zero to one.

Defining a non-dimensional variable r_0 by the relation

$$r_0 = \frac{1}{\psi^* - q} \tag{1.34}$$

Kopal (14) has also shown that on the surface of Roche equipotentials, (r, θ, ϕ) are connected through the relation

$$r^* = r_0 [1 + C_3 r_0^3 + C_4 r_0^4 + C_5 r_0^5 + C_6 r_0^6 + C_7 r_0^7 + C_8 r_0^8 + C_9 r_0^9 + ...] \tag{1.35}$$

where

$$\begin{aligned} C_3 &= q P_2 + n(1 - v^2),\ \mathrm{C}_4 = \mathrm{q\,P}_3,\ \mathrm{C}_5 = \mathrm{q\,P}_4 \\ C_6 &= q\, p_5 + 3C_3^{\ 2},\ \mathrm{C}_7 = q P_6 + 7\, q\, C_3^{\ 2} P_3 \\ C_8 &= q P_7 + 8q C_3 P_4 + 4q^2 P_3^2 \\ C_9 &= q P_8 + 9\, q\, C_3 P_5 + 9\, q^2 P_3 P_4 \end{aligned}$$

Here, $P_j = P_j(\lambda)$ are the Legendre polynomials and terms upto second order of smallness in n and q have been retained in equation (1.35). This relation helps to obtain the shape of a Roche equipotentials ψ = constant.

The volume enclosed by the equipotential surface ψ^* = constant is given by

$$V_\psi = \frac{2}{3} \int_{-1}^{1} \int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}} \frac{r^3}{\mu} d\lambda\, dv \tag{1.36}$$

Kopal (14) has shown that the explicit expression of V_ψ in terms of r_0 defined by equation (1.34), can be represented as

$$V_\psi = \frac{4}{3}\pi D^3 r_0^3 [1 + 2n r_0^3 + (\frac{12}{5} q^2 + \frac{8}{5} n q + \frac{32}{5} n^2) r_0^6 + \frac{15}{7} q^2 r_0^8 + 2q^2 r_0^{10} + ...] \qquad (1.37)$$

where terms up to second order of smallness in n and q are retained.

Following the approach of Kopal (14), Mohan and Singh(25,26) obtained the explicit expressions for the surface area S_ψ and the values of averages or parameters r_ψ, $\overline{g}$ and $\overline{g^{-1}}$ on the Roche equipotential ψ = constant. These are

$$S_\psi = 2\int_{-1}^{1} \int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}} \frac{r^2}{\mu} d\lambda \, d\nu$$
$$= 4\pi D^2 r_0^2 [1 + \frac{4n}{3} r_0^3 + (\frac{7}{5} q^2 + \frac{14}{15} n q + \frac{56}{15} n^2) r_0^6 + \frac{9}{7} q^2 r_0^8 + \frac{11}{9} q^2 r_0^{10} + ...] \qquad (1.38)$$

$$r_\psi = (\frac{3V_\psi}{4\pi})^{1/3}$$
$$= D r_0 [1 + \frac{2n}{3} r_0^3 + (\frac{4}{5} q^2 + \frac{8}{15} n q + \frac{76}{45} n^2)\, r_0^6 + \frac{5}{7} q^2 r_0^8 + \frac{2}{3} q^2 r_0^{10} + ...] \qquad (1.39)$$

$$\overline{g} = \frac{2}{S_\psi} \int_{-1}^{1} \int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}} (\frac{d\psi}{d n}) \frac{r^2}{\mu} d\lambda \, d\nu$$
$$= \frac{G M_\psi}{D^2 r_0^2} [1 - \frac{8n}{3} r_0^3 - (3q^2 + 2 n q + \frac{40}{9} n^2) r_0^6 - \frac{51}{14} q^2 r_0^8 - \frac{13}{3} q^2 r_0^{10} + ...] \qquad (1.40)$$

and

$$\overline{g^{-1}} = \frac{2}{S_\psi} \int_{-1}^{1} \int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}} (\frac{d\psi}{d n})^{-1} \frac{r^2}{\mu} d\lambda \, d\nu$$
$$= \frac{D^2 r_0^2}{G M_\psi} [1 + \frac{8n}{3} r_0^3 + (\frac{31}{5} q^2 + \frac{62}{15} n q + \frac{584}{45} n^2) r_0^6 + \frac{101}{14} q^2 r_0^8 + \frac{25}{3} q^2 r_0^{10} + ...] \qquad (1.41)$$

Inverting the relation (1.39) they also obtain

$$r_0 = r_\psi^* [1 - \frac{2n}{3} r_\psi^{*3} - [\frac{4}{5} q^2 + \frac{8}{15} n q - \frac{4}{45} n^2] r_\psi^{*6} - \frac{5}{7} q^2 r_\psi^{*8} - \frac{2}{3} q^2 r_\psi^{*10} - ...] \tag{1.42}$$

where $r_\psi^* = r_\psi / D$, r_ψ^* being the nondimensional form r_ψ. In all the above expressions terms upto second order of smallness in n and q have been retained.

1.4 MOHAN, SAXENA AND AGARWAL’S APPROACH FOR COMPUTING THE EFFECTS OF ROTATIONAL AND TIDAL DISTORTIONS ON THE EQUILIBRIUM STRUCTURES OF ROTATIONALLY AND TIDALLY DISTORTED GASEOUS SPHERES

Mohan, Saxena and Agarwal (32) used the concept of Roche equipotentials proposed by Kopal in conjunction with Kippenhahn and Thomas’s averaging approach to explicitly obtain equations governing the equilibrium structures of rotationally and/or tidally distorted stars and applied these to analyze the problems of rotating stars and stars in binary systems.

In order to determine the inner structure of a rotationally and tidally distorted gaseous sphere, the system of equations (1.27) has to be integrated numerically subject to the boundary conditions (1.28) specified therein. Therefore, the evaluation of the actual equipotential surface of a rotationally and tidally distorted gaseous sphere is complicated. Kippenhahn and Thomas (12) proposed that for evaluation of the distortion parameters u, v, w, f_P, f_T etc., the actual equipotential surface may be replaced by Roche equipotential surface (It may be noted that this approximation is reasonably valid for most of the models of the actual stars. In fact as far back as 1933, Chandrasekhar had shown that for stars whose central density bears to the mean density a ratio of 100 or more, the Roche model of a rotating configuration will represent the actual equipotential surfaces of the star within an error of less than one percent).

Once the equipotential surfaces of a rotationally and tidally distorted star are approximated by the Roche equipotentials, the results obtained by Kopal (14,15) and Mohan and Singh (25,26) may be used to evaluate explicitly the values of the distortion parameters u, v, w, f_P and f_T appearing in stellar structure equations (1.20) and (1.26). Using equations (1.15), (1.20), (1.26) and (1.37 – 1.42) the explicit expressions of the distortions parameters u, v, w, f_P and f_T on the equipotential surface as obtained by Mohan et al (27,32) are

$$u = 1 - (\frac{1}{5}q^2 + \frac{2}{15}n\,q + \frac{4}{45}n^2)\,r_\psi^{*6} - \frac{1}{7}q^2\,r_\psi^{*8} - \frac{1}{9}q^2\,r_\psi^{*10} + \ldots \tag{1.43a}$$

$$v = 1 - \frac{4n}{3}r_\psi^{*3} - (\frac{7}{5}q^2 + \frac{14}{15}\,n\,q + \frac{68}{45}\,n^2)\,r_\psi^{*6} - \frac{31}{14}q^2\,r_\psi^{*8} - 3q^2 r_\psi^{*10} - \ldots \tag{1.43b}$$

$$w = 1 + \frac{4n}{3}\,r_\psi^{*3} + (\frac{23}{5}q^2 + \frac{16}{15}n\,q + \frac{212}{45}n^2)\,r_\psi^{*6} + \frac{81}{14}q^2\,r_\psi^{*8} + 7\,q^2\,r_\psi^{*10} + \ldots \tag{1.43c}$$

$$f_P = 1 - \frac{4n}{3}\,r_\psi^{*3} - (\frac{22}{5}q^2 + \frac{44}{15}n\,q + \frac{128}{45}n^2)\,r_\psi^{*6} - \frac{79}{14}q^2\,r_\psi^{*8} - \frac{62}{9}q^2 r_\psi^{*10} - \ldots \tag{1.43d}$$

and

$$f_T = 1 - (\frac{14}{5}q^2 + \frac{28}{15}n\,q + \frac{56}{45}n^2)\,r_\psi^{*6} - \frac{46}{14}q^2\,r_\psi^{*8} - \frac{34}{9}q^{\,2} r_\psi^{*10} - \ldots \tag{1.43e}$$

where $r_\psi^{\bullet} = r_\psi / D$ is the nondimensional form of r_ψ and terms upto second order of smallness in n and q are retained.

The values of M_ψ, P_ψ, L_ψ etc. on the various equipotential surfaces of a rotationally and tidally distorted gaseous spheres may now be obtained by solving the system of differential equations (1.27) with boundary conditions

(1.28) and using the values of distortion parameters f_P and f_T as given in (1.43).

It may be noted that while approximating the equipotential surfaces of a rotationally and tidally distorted model by Roche equipotentials, the structure of the star is not approximated by the structure of a Roche model. In the case of no distortion *(n=q=0),* equation (1.43) gives $u = v = w = f_P = f_T = 1$ and the system of differential equations (1.27) reduce to the equations governing the equilibrium structure of the original undistorted star and not of the Roche model.

Stellar structure equations can be now used to integrate the system of differential equation (1.27) governing the equilibrium structure of a rotationally and tidally distorted gaseous sphere. At every step, the values of the distortion parameters u, v, w, f_P and f_T have to be computed using (1.43).

In case the thermal properties are not considered important and only hydrostatic equilibrium of a rotationally and tidally distorted gaseous spheres is to be investigated then we need only to integrate equation (1.17) and (1.20) subject to the boundary conditions

At the center $r_\psi = 0$

$$M_\psi = 0 \tag{1.44a}$$

and at the free surface $r_\psi = R_\psi$

$$\begin{aligned} &M_\psi = M_0, P_\psi = 0 \\ &\rho_\psi = 0 \ or \ P_\psi = P_{\psi s}, \rho_\psi = \rho_{\psi s} \end{aligned} \tag{1.44b}$$

In case the star is being distorted by rotational forces alone (or tidal forces alone) we may set $q=0$ $(n=0)$ in (1.43) and still use the above approach to determine the equilibrium structure of corresponding purely rotationally distorted or purely tidally distorted model. For obtaining the structure of the primary component of a synchronous binary system we may set $n = (q+1)/2$.

Mohan and Saxena (24) found it more convenient to work with r_0 in place of M_ψ or r_ψ as independent variable by using (1.34) which is connected with variable r_ψ through relation (1.42). Saxena(39) expressed the system of differential equations governing the equilibrium structure of a rotationally and tidally distorted stellar model as

$$\frac{dM_\psi}{dr_0} = 4\pi D^3 \rho_\psi \, r_0^2 f_1 , \tag{1.45a}$$

$$\frac{dP_\psi}{dr_0} = -\frac{GM_\psi}{Dr_0^2} \rho_\psi f_2 , \tag{1.45b}$$

$$\frac{dL_\psi}{dr_0} = 4\pi \varepsilon D^3 \rho_\psi \, r_0^2 f_1 , \tag{1.45c}$$

and

$$\frac{dT_\psi}{dr_0} = -\frac{3\kappa L_\psi}{16\pi D ac T_\psi^3} \frac{\rho_\psi}{r_0^2} f_3 . \tag{1.45d}$$

Here f_1, f_2 and f_3 are functions of n, q and r_0 incorporating the effects of rotation and tidal distortions on the equilibrium structure equations of a stellar model. The explicit expressions for these distortion parameters as given by Saxena (39) are

$$f_1 = 1 + 4n r_0^3 + \left(\frac{36}{5} q^2 + \frac{24}{5} n q + \frac{96}{5} n^2\right) r_0^6 + \frac{55}{7} q^2 \, r_0^8 + \frac{26}{3} q^2 \, r_0^{10} + \dots \tag{1.46a}$$

$$f_2 = 1 - \left(\frac{2}{5} q^2 + \frac{4}{15} n q + \frac{16}{15} n^2\right) r_0^6 - \frac{9}{14} q^2 \, r_0^8 - \frac{8}{9} q^2 \, r_0^{10} + \dots \tag{1.46b}$$

and

$$f_3 = 1 + \frac{4n r_0^3}{3} + \left(\frac{6}{5} q^2 + \frac{4}{5} n q + \frac{224}{45} n^2\right) r_0^6 + \frac{24}{14} q^2 \, r_0^8 + \frac{20}{9} q^2 \, r_0^{10} + \dots \tag{1.46c}$$

In these above expressions terms upto second order of smallness in n, q and upto r_0^{10} in r_0 are retained. The boundary conditions governing the system of differential equations (1.45) are:

At the center $r_0 = 0,$

$$M_\psi = 0, \; L_\psi = 0 \tag{1.47a}$$

and at the free surface $r_0 = r_{0s}$

$$\begin{aligned} &M_\psi = M_0, \; L_\psi = L_{\psi s} \\ &P_\psi = 0, \; \rho_\psi = 0, \; T_\psi = 0 \;\; or \;\; P_\psi = P_{\psi s}, \; \rho_\psi = \rho_{\psi s}, T_\psi = T_{\psi s} \end{aligned} \tag{1.47b}$$

where r_{0s} being the value of r_0 at the free surfaces.

In fact

$$r_{0s} = \frac{1}{\psi_s^* - q} \tag{1.48}$$

where ψ_s^* is the nondimensional form of the total potential ψ on the outermost equipotential surface of a rotationally and tidally distorted stellar model. In the case of no distortion, the above equations reduce to the usual equations governing the equilibrium structure of an undistorted gaseous sphere.

1.5 THE PRESENT WORK

In case of stars in binary systems besides gravitational and centrifugal forces, coriolis force also comes into picture. While computing the effects of rotational and tidal distortions on the equilibrium structures of rotating stars and stars in binary systems, Mohan and Saxena (24) and Mohan et al (27,32) and Singh and Sharma (37,38) approach does not explicitly accounts for the effects of coriolis force. In the present thesis, an attempt made by Lal et al (19,20) to study the effect of coriolis force on the equilibrium structures of rotationally and tidally distorted stellar models has been briefly presented.

Keeping this in view, the structural analysis of rotationally and tidally distorted Prasad model and Roche model in the presence of coriolis force have been analyzed in the present study. Analysis of results based on the present study has also been discussed.

CHAPTER – II

EFFECTS OF CORIOLIS FORCE ON THE EQUILIBRIUM STRUCTURES OF ROTATIONALLY AND TIDALLY DISTORTED STELLAR MODELS.

It is thus evident that theoretical investigations of determining the effects of rotation and tidal forces on the equilibrium structures of gaseous spheres can help in better understanding the observed phenomena of the rotating stars and stars in binary and multiple systems. Such studies are also expected to help in better understanding the problems of stellar stability in general and the problems of stellar variability of rotating stars and stars in binary or multiple systems in particular.

Kopal (14) introduced the concept of Roche equipotentials to analyze the problems of rotating stars and stars in binary systems. Since then several authors such as Kopal (16,17,18), Mohan and Singh (25,26), Mohan et al (27,29) and Lal et al (21) have used this concept to analyze the problems of rotationally and/or tidally distorted stars. In this approach Roche approximation for the inner structure of a star is used to obtain an expression for the potential of a rotating star and star in a binary system. This approach, accounts for the effects of gravitational and centrifugal forces. However, it does not explicitly take into account the effect of coriolis force.

In the present chapter we briefly discuss the expression for Roche equipotential of the primary component of a binary system in a rotating frame of reference which explicitly accounts for the effects of coriolis force besides gravitational and centrifugal forces. The expression for the Roche equipotential surface of a rotationally and tidally distorted star and the expressions for volume, surface area, radial distance, $\bar{g}$ and $\overline{g^{-1}}$ which incorporates the effect of coriolis force in addition to the centrifugal and gravitational force as given by Lal et al(19,20) are briefly discussed in section 2.1. In section 2.1.1 equilibrium structures of rotationally and tidally distorted stars incorporating the effect of coriols force has been discussed.

Section 2.2 and 2.3 contains the equilibrium structures of rotationally and tidally distorted Prasad model and Roche model respectively. Analysis of results are presented in section 2.4.

2.1 ROCHE EQUIPOTENTIALS OF ROTATIONALLY AND TIDALLY DISTORTED STELLAR MODELS IN THE PRESENCE OF CORIOLIS FORCE

A rotating star is a star rotating about an axis passing through its center. In case of a binary system of stars we have two stars which are rotating about their axes passing through their centers as well as revolving about their common center of mass. In the case of most of the observed binary stars, one star (usually called the primary component) is more massive than the other star (called the secondary component).

In a binary system following Kopal (14), let M_0 and M_1 be the masses of the two components separated by distance *D*. The primary component of mass M_0 of this binary system is supposed to be much massive than its companion star of mass M_1 $(M_0 >> M_1)$ which for all practical purposes is regarded as a point mass. The primary is supposed to have its normal configuration. However, its inner structure is approximated by Roche model for the purpose of computation of the potential of the system. (In case of Roche model it is assumed that the total mass of a star is concentrated at its center and this point mass is surrounded by an evanescent envelope in which density varies inversely as the square of the distance from its center). This approximation is reasonably valid for majority of the realistic stars of main sequence and post main sequence stages .

Now suppose that the position of the two components of such a binary system

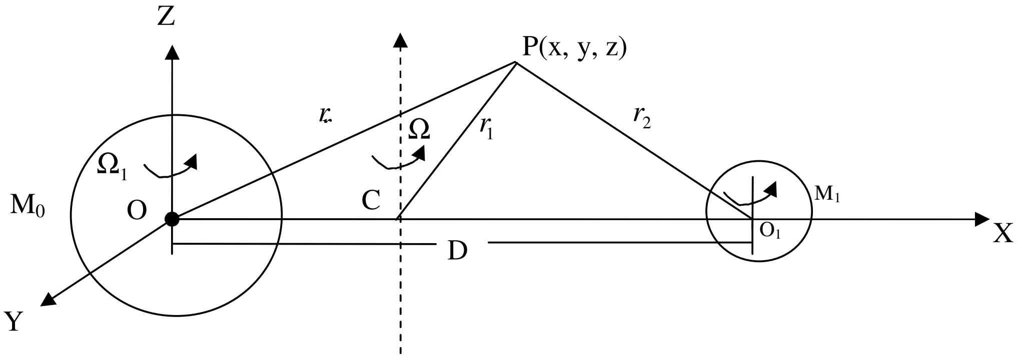

Fig. 2.1: Axis of Reference for a binary system

is referred to a rectangular system of cartesian coordinates with origin at the center of gravity of the primary star of mass M_0, $x-$ axis along the line joining the mass centers of the two stars, and $z-$axis perpendicular to the plane of the orbit of the two components (Fig 2.1).In the figure $r = \sqrt{x^2 + y^2 + z^2}$, $r_2 = \sqrt{(D-x)^2 + y^2 + z^2}$ and $r_1 = \sqrt{(x-d_1)^2 + y^2 + z^2}$, represent the distances of a point P(x, y, z) from the centers of gravity of the primary star with center at O, secondary star with center at O_1 and the center of gravity C ($(d_1, 0, 0)$ where $d_1 = M_1 D/(M_0 + M_1)$) of the system respectively. Let Ω denote the angular velocity of revolution of the system about a line parallel to z- axis which passes through the center of gravity C of the system and is perpendicular to the xy - plane. Also let Ω_1 be the angular velocity of rotation of the primary about z – axis.

In a binary system, the primary star (which is of interest to us in our present study) is rotating about axis OZ with angular velocity Ω_1 as well as revolving about an axis parallel to z – axis passing through the common center of mass C with angular velocity Ω. The point P when it is inside the primary will experience the effects of coriolis force besides the gravitational and centrifugal forces. Using the concepts of classical dynamics, for the system described above (Fig 2.1), the total potential at a point P inside the primary component, which experiences the effect of coriolis force besides the gravitational and centrifugal forces given by Lal et al (35)

$$\psi = \frac{GM_0}{r} + \frac{GM_1}{r_2} + \frac{1}{2}(\vec{\Omega}\times\vec{r}_1)\cdot(\vec{\Omega}\times\vec{r}_1) + \vec{V}\cdot(\vec{\Omega}\times\vec{r}_1) \tag{2.1}$$

Here $\vec{V}$ is the velocity of particle of unit mass at point P(x, y, z) with respect to a rotating frame of reference, rotating with the angular velocity of system. The first two terms in equation (2.1) correspond to the gravitational potential which arises due to the primary and secondary component of binary system and third term is due to centrifugal force. These three terms are same as earlier obtained by Kopal (14) in his studies to the problems of Roche Model and its applications to close binary system. The fourth term $\vec{V}\cdot(\vec{\Omega}\times\vec{r}_1)$ represents the contribution of the coriolis force to the potential at point P, where $\vec{V}$ is the tangential component of velocity of this particle in the rotating frame of reference. Points inside the rotating star will be subjected to coriolis force even when such a point is not having any external velocity because of the differences in the velocity of the rotation of the primary and angular velocity of revolution of the nonsynchronous binary system. (This difference of course vanishes in case of synchronous binary stars where velocity of rotation is same as that of revolution). In the earlier studies carried out on Roche equipotentials by Kopal (14) and Mohan and Saxena (24), the contribution of this last term $\vec{V}\cdot(\vec{\Omega}\times\vec{r})$, which arises on account of coriolis force, has been neglected. In the present study we have tried to incorporate its effect on the subsequent studies.

If we write $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ to represent the position vector of the point P(x, y, z) inside the star rotating about its axis with angular velocity $\vec{\Omega}_1$, the term $\vec{V}\cdot(\vec{\Omega}\times\vec{r}_1)$ can be simplified as follows. In the rotating frame of reference, the point P(x, y, z) rotates in a circle of radius PL with angular velocity $\Omega_P = \Omega_1 - \Omega$ (Fig 2.2).

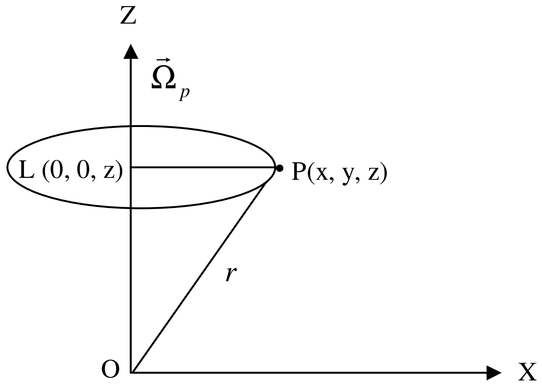

Fig. 2.2: Rotation of point P about z- axis

We thus have

$$\vec{V} = \vec{\Omega}_P \times \overrightarrow{LP} \tag{2.2}$$

where

$$\vec{\Omega}_P = (\Omega_1 - \Omega)\hat{k} \tag{2.3}$$

and

$$\overrightarrow{LP} = x\hat{i} + y\hat{j} \tag{2.4}$$

Using (2.3) and (2.4) in (2.2), we get

$$\vec{V} = -(\Omega_1 - \Omega)(y\hat{i} - x\hat{j})\,. \tag{2.5}$$

Also

$$\vec{\Omega}\times\vec{r}_1 = -\Omega(y\hat{i} - (x - d_1)\hat{j}) \tag{2.6}$$

so the term $\vec{V}\cdot(\vec{\Omega}\times\vec{r}_1)$ in (2.1) becomes

$$\vec{V}\cdot(\vec{\Omega}\times\vec{r}_1) = (\Omega\Omega_1 - \Omega^2)(x^2 + y^2 - xd_1) \tag{2.7}$$

which is the contribution to potential at point P(x, y, z) inside the star due to coriolis force (For a point P outside the primary star this will be zero unless P has some external velocity). Using equation (2.7) in equation (2.1), we get the modified expression for potential at a point P inside the star in cartesian form as

$$\psi = \frac{GM_0}{r} + \frac{GM_1}{r_2} + \frac{\Omega^2}{2}[(x-d_1)^2 + y^2] + (\Omega\Omega_1 - \Omega^2)(x^2 + y^2 - xd_1) \tag{2.8}$$

Following Kopal (14), equation (2.8) may be expressed in nondimensional form as

$$\begin{aligned}\psi^* = \frac{1}{r^*} + \frac{q}{\sqrt{1-2\lambda r^* + r^{*2}}} + \frac{\Omega^{*2}}{2}[r^{*2}(1-\nu^2) \ + \ d_1^{*2} - 2\lambda r^* d_1^*] \\ + \ (\Omega_P^{*2} - \Omega^{*2})\,[r^{*2}(1-\nu^2) - \lambda r^* d_1^*]\end{aligned} \tag{2.9}$$

where

$$\psi^* = \frac{D\psi}{GM_0},\ \Omega^{*2} = \frac{\Omega^2 D^3}{GM_0},\ \Omega_P^{*2} = \frac{D^3(\Omega\Omega_1)}{GM_0} \text{ and } d_1^* = \frac{M_1}{M_0 + M_1} \tag{2.10}$$

In the above expressions $r^* = r/D$ is nondimensional form of r, and $\lambda = \sin\theta\cos\varphi$, $\mu = \sin\theta\sin\varphi$, $\nu = \cos\theta$ (r, θ, φ being the polar spherical coordinates of the point *P*). Moreover, $q = M_1/M_0$ is a nondimensional parameter representing the ratio of the mass of the secondary component over the primary component (we assume $q << 1$).

Equation (2.9) is the most general expression of the potential at point P which incorporates the effect of coriolis force in addition to centrifugal and gravitational forces. If we assume that the angular velocity Ω is identical with Keplerian angular velocity Ω_K where $\Omega_k^2 = G(M_0 + M_1)/D^3$ then in terms of the nondimensional variables used by us, we get a relation

$\Omega^{*2} = 2n = (q+1)$.Using this relation, equation (2.9) can be written in more simplified form as

$$\psi^{**} = \frac{1}{r^*} + q[\frac{1}{\sqrt{1-2\lambda r^* + r^{*2}}} - \alpha\lambda r^*] + \beta n r^{*2}(1-\nu^2) \tag{2.11}$$

where

$$\alpha = \sqrt{n_1/n}\,,\ \beta = 2\sqrt{n_1/n} - 1\,,\ \Omega_P^{*2} = 2n_1 \text{ and } \psi^{**} = \frac{D\psi}{GM_0} - \frac{M_1^{\ 2}}{2M_0(M_0+M_1)} \tag{2.12}$$

For binary system rotating synchronously, the angular velocity due to rotation and revolution are same, that is, $\vec{\Omega}_1 = \vec{\Omega}$. Hence there will be no explicit term for coriolis force. In such cases equation (2.9) reduces to

$$\psi^{**} = \frac{1}{r^*} + q[\frac{1}{\sqrt{1-2\lambda r^* + r^{*2}}} - \lambda r^*] + n r^{*2}(1-\nu^2) \tag{2.13}$$

This expression (2.13) is same as earlier obtained by Kopal (14).

Also there is no Coriolis force in pure tidal case and therefore expression for ψ^{**} in this case is identical to its earlier expression obtained by Kopal (14) and can be obtained directly from (2.13) by putting $n=0$.

In case of pure rotation also, coriolis force is not generated as there is no revolution of the center of the star and hence no rotating frame of reference. In such a case the expression for Roche equipotential for a purely rotating star which is not subject to tidal effects of the companion star becomes

$$\psi^* = \frac{1}{r^*} + n r^{*2}(1-\nu^2) \tag{2.14}$$

which can be obtained from (2.13) by setting $q=0$ or from (2.11) by setting $q=0, \beta=1$. This expression is same as earlier discussed by Kopal (14) for pure rotating stars.

Thus on explicit inclusion of coriolis force, expression for Roche equipotential gets modified from the earlier one obtained by Kopal (14) only in the case of nonsynchronous binaries. In case of synchronous binaries, purely rotating and purely tidally distorted stars there is no change in it.Adopting an approach similar to the one adopted by Kopal (14) and Mohan et al (31), we may define a nondimensional variable r_0 by the relation

$$r_0 = \frac{1}{\psi^{**} - q} \tag{2.15}$$

Following Kopal (14), variables (r,θ,ϕ) on the surfaces of the modified Roche equipotentials (2.11) can be shown to be connected through the relation

$$\begin{aligned} r = r_0\,[1 &+ \lambda qt\, r_0^2 + a_0 r_0^3 + (qP_3 + 2\lambda^2 q^2 t^2)\, r_0^4 + (q\,P_4 + 5a_0\lambda qt)\, r_0^5 + (qP_5 + 3a_0{}^2 + 6\lambda q^2 tP_3)\, r_0^6 \\ &+ (qP_6 + 7\,a_0 qP_3 + 7\lambda q^2 tP_4)\, r_0^7 + (qP_7 + 8\,a_0 q\,P_4 + 8\lambda q^2 tP_5 + 4q^2\,P_3{}^2)\, r_0^8 \\ &+ (q\,P_8 + 9\,a_0 qP_5 + 9\lambda q^2 tP_6 + 9\,q^2 P_3\,P_4)\, r_0^9 + (q\,P_9 + 10\,a_0 q\,P_6 + 10\lambda q^2 tP_7 \\ &+ 5q^2\{P_4{}^2 + 2\,P_3\,P_5\})\, r_0^{10} + \cdots] \end{aligned} \tag{2.16}$$

where $a_0 = q\,P_2 + \beta n(1-\nu^2)$, $t = 1-\alpha$ and $P_j = P_j\,(\lambda)$ denote Legendre polynomials. As earlier terms upto second order of smallness in n, n_1 and q and terms upto $r_0{}^{10}$ in r_0 are retained in (2.16). Relation (2.16) incorporates the effect of coriolis force and can be used to obtain the shapes of various Roche equipotential surfaces ψ^{**} = constant.

Again following Kopal (14) expressions for volume V_ψ, surface area S_ψ and value of radial distance r_ψ of a point on the equipotential surface ψ^{**} = constant inside the primary can be shown to be

$$V_\psi = \frac{4\pi}{3} D^3\, r_0^3 [1 + 2(\beta n)\, r_0^3 + 3q^2 t^2 r_0^4 + (\frac{12}{5} q^2 + \frac{32}{5}(\beta n)^2 + \frac{8}{5}(\beta n)\, q)\, r_0^6 + \frac{15}{7} q^2\, r_0^8 + 2q^2 r_0^{10} + ...] \tag{2.17}$$

$$S_\psi = 4\pi D^2 r_0^2 [1 + \frac{4}{3}(\beta n)\, r_0^3 + \frac{5}{3} q^2 t^2\, r_0^4 + (\frac{7}{5} q^2 + \frac{56}{15}(\beta n)^2 + \frac{14}{15}(\beta n) q) r_0^6 + \frac{9}{7} q^2\, r_0^8 + \frac{11}{9} q^2 r_0^{10} + ...] \tag{2.18}$$

and

$$r_\psi = D\, r_0\, [1 + \frac{2}{3}(\beta n)\, r_0^3 + q^2 t^2 r_0^4 + (\frac{4}{5} q^2 + \frac{76}{45}(\beta n)^2 + \frac{8}{15}(\beta n)\, q)\, r_0^6 + \frac{5}{7} q^2\, r_0^8 + \frac{2}{3} q^2\, r_0^{10} + ...] \tag{2.19}$$

Inverting (2.19) we have

$$r_0 = r_\psi^* [1 - \frac{2}{3}(\beta n)\, r_\psi^{*3} - q^2 t^2 r_\psi^{*4} - (\frac{4}{5} q^2 - \frac{4}{45}(\beta n)^2 + \frac{8}{15}(\beta n) q)\, r_\psi^{*6} - \frac{5}{7} q^2 r_\psi^{*8} - \frac{2}{3} q^2 r_\psi^{*10} + ...] \tag{2.20}$$

where $r_\psi^* = r_\psi / D$, r_ψ^* being the nondimensional form of r_ψ. Similarly using equation(1.13) of chapter I, the explicit expressions for the values of $\overline{g}$ and $\overline{g^{-1}}$, in the presence of coriolis force, at points inside the primary are given as

$$\overline{g} = \frac{GM_\psi}{D^2 r_0^2} [1 - \frac{8}{3}(\beta n) r_0^3 - 2q^2 t^2 r_0^4 - (2\,q^2 + \frac{28}{9}(\beta n)^2 + \frac{4}{3}(\beta n)\, q)\, r_0^6 - \frac{15}{7} q^2\, r_0^8 - \frac{7}{3} q^2\, r_0^{10} + ...] \tag{2.21}$$

$$\overline{g^{-1}} = \frac{D^2\, r_0^{\,2}}{G\, M_\psi} [1 + \frac{8}{3}(\beta n) r_0^3 + 5q^2 t^2 r_0^4 + (\frac{26}{5} q^2 + \frac{524}{45}(\beta n)^2 + \frac{52}{15}(\beta n)\, q) r_0^6 + \frac{40}{7} q^2\, r_0^8 + \frac{19}{3} q^2\, r_0^{10} + ...] \tag{2.22}$$

where $t = 1 - \alpha$.

As in earlier studies in obtaining the above expressions, terms upto second order of smallness in $n, n_1,\ q$ and terms upto r_0^{10} in r_0 have been retained. In absence of the effect of coriolis force ($\alpha=\beta=1$) these reduce to expressions earlier obtained by Mohan et al (31).

2.1.1 EQUILIBRIUM STRUCTURES OF ROTATIONALLY AND TIDALLY DISTORTED STARS INCORPORATING THE EFFECT OF CORIOLIS FORCE

Following the approach presented in section 1.4 of chapter I and using the modified expressions for Roche equipotential and other parameters obtained in section 2.1. which incorporate the effects of coriolis force besides gravitational and centrifugal forces, Lal et al (34,35) derived the values of distortions parameters u, v, w, f_p and f_t as follows:

$$u=[1-\frac{1}{3}q^2t^2r_\psi^{*4}-(\frac{1}{5}q^2+\frac{4}{45}(\beta n)^2+\frac{2}{15}(\beta n)q)\,r_\psi^{*6}-\frac{1}{7}q^2r_\psi^{*8}-\frac{1}{9}q^2r_\psi^{*10}-...] \qquad (2.23)$$

$$v=[1-\frac{4}{3}(\beta n)r_\psi^{*3}-(\frac{2}{5}q^2+\frac{8}{45}(\beta n)^2+\frac{4}{15}(\beta n)q)r_\psi^{*6}-\frac{5}{7}q^2r_\psi^{*8}-q^2r_\psi^{*10}-...] \qquad (2.24)$$

$$\begin{aligned} w=[1&+\frac{4}{3}(\beta n)\,r_\psi^{*3}+3q^2t^2r_\psi^{*4}+(\frac{18}{5}q^2+\frac{152}{45}(\beta n)^2+\frac{12}{5}(\beta n)\,q\,)\,r_\psi^{*6} \\ &+\frac{30}{7}q^2r_\psi^{*8}+5\,q^2r_\psi^{*10}+...] \end{aligned} \qquad (2.25)$$

$$f_P=[1-\frac{4}{3}(\beta n)\,r_\psi^{*3}-\frac{8}{3}q^2t^2r_\psi^{*4}-(\frac{17}{5}q^2+\frac{68}{45}(\beta n)^2+\frac{34}{15}(\beta n)q)\,r_\psi^{*6}-\frac{29}{7}q^2r_\psi^{*8}-\frac{44}{9}q^2r_\psi^{*10}-...] \qquad (2.26)$$

$$f_T=[1-\frac{7}{3}q^2t^2r_\psi^{*4}-(\frac{14}{5}q^2+\frac{56}{45}(\beta n)^2+\frac{28}{15}(\beta n)q)\;r_\psi^{*6}-\frac{23}{7}q^2r_\psi^{*8}-\frac{34}{9}q^2r_\psi^{*10}-...] \qquad (2.27)$$

where $r_\psi^*=r_\psi/D$ is the nondimensional form of r_ψ and terms up to second order of smallness in $n, n_1,\ q$ and upto r_ψ^{10} in r_ψ are retained. These reduce to expressions earlier obtained in Mohan et al (31) by setting $\alpha=\beta=1$. The values of $P_\psi,\ \rho_\psi,\ L_\psi$ etc. on the various equipotential surfaces of a rotationally and tidally distorted gaseous sphere may now be obtained by solving the system of differential equations (1.27) subject to the boundary conditions (1.28) and using the values of the distortion factors f_P and f_T as given in equations (2.26-2.27).

It may be noted that approximating the equipotential surfaces of a rotationally and tidally distorted model by Roche equipotentials, the structure of the star is not approximated by the structure of a Roche model. This is evident from the fact that in the case of no distortion ($n = n_1 = q = 0$), equations (2.23-2.27) yields $u = v = w = f_P = f_T = 1$ and the system of differential equations (1.27) reduce to the equations governing the equilibrium structure of original undistorted star and not of the undistorted Roche model.

Usual numerical methods, which are used for solving the stellar structure equations, can be applied here to integrate the system of given differential equations which govern the equilibrium structures of rotationally and tidally distorted gaseous spheres. However, at each step the values of the distortion parameters u, v, w, f_P and f_T has to be computed using equations (2.23-2.27).

In case the thermal properties are not considered important and only hydrostatic equilibrium of a rotationally and tidally distorted gaseous sphere is to be investigated then we need only to integrate equations (1.27a) and (1.27b) subject to the boundary conditions

At the center $r_\psi = 0$:

$$M_\psi = 0 \tag{2.28a}$$

and at the free surface $r_\psi = R_\psi$:

$$M_\psi = M_0,\ P_\psi = 0,\ \rho_\psi = 0 \ or \ P_\psi = P_{\psi s},\ \rho_\psi = \rho_{\psi s} \tag{2.28b}$$

In expressions (2.23-2.27) terms up to second order of smallness in n, n_1 and q have only been retained. Therefore, our above analysis is valid for the rotationally and tidally distorted models in which the distorting forces causing rotational and tidal distortions are not too large.

For computational work, sometimes it is found more convenient to work with r_0 in place of r_ψ or M_ψ as the independent variable. Expression for r_0 is given by (2.15) and is connected with variable r_ψ through the relation (2.20). By using these relations in equations (1.27), the system of differential equations governing the equilibrium structure of a rotationally and tidally distorted model which incorporates the effects of coriolis force besides the gravitational and centrifugal forces can be expressed as

$$\frac{dM_\psi}{dr_0} = 4\pi D^3 \rho_\psi r_0^2 f_1 \tag{2.29a}$$

$$\frac{dP_\psi}{dr_0} = -\frac{GM_\psi}{Dr_0^2} \rho_\psi f_2 \tag{2.29b}$$

$$\frac{dL_\psi}{dr_0} = 4\pi \varepsilon D^3 \rho_\psi r_0^2 f_1 \tag{2.29c}$$

$$\frac{dT_\psi}{dr_0} = -\frac{3\kappa L_\psi \rho_\psi}{16\pi Dac T_\psi^3 r_0^2} f_3 \tag{2.29d}$$

with

$$r_0 = \frac{1}{\psi^{**} - q}$$

Here f_1, f_2, f_3 are certain functions of distortion parameters n, n_1, q and r_0 and incorporate the effects of coriolis force in addition to centrifugal and gravitational forces on the equilibrium structure equations of rotationally and tidally distorted stellar model (In case of no distortion $f_1 = f_2 = f_3 = 1$). Explicit expressions for distortion parameters $f_1 = \frac{dr_\psi}{dr_0}\frac{r_\psi^2}{D^3}$, $f_2 = \frac{f_p}{r_\psi^2}\frac{dr_\psi}{dr_0}D$, $f_3 = \frac{f_t}{r_\psi^2}\frac{dr_\psi}{dr_0}D$ and r_0 when terms up to second order of smallness in n, n_1, q and upto r_0^{10} in r_0 are retained, are given as

$$f_1 = [1 + 4(\beta n)\, r_0^{\,3} + 7q^2 t^2 r_0^{\,4} + (\frac{36}{5}q^2 + \frac{96}{5}(\beta n)^2 + \frac{24}{5}(\beta n)\, q) r_0^{\,6} + \frac{55}{7}q^2 r_0^{\,8} + \frac{26}{3}q^2 r_0^{\,10} + ...] \tag{2.30a}$$

$$f_2 = [1 + \frac{1}{3}q^2\, t^2 r_0^{\,4} + (\frac{3}{5}q^2 + \frac{4}{15}(\beta n)^2 + \frac{2}{5}(\beta n) q) r_0^{\,6} + \frac{6}{7}q^2 r_0^{\,8} + \frac{10}{9}q^2 r_0^{\,10} + ...] \tag{2.30b}$$

$$f_3 = [1 + \frac{4}{3}(\beta n)\, r_0^{\,3} + \frac{2}{3}q^2 t^2 r_0^{\,4} + (\frac{6}{5}q^2 + \frac{224}{45}(\beta n)^2 + \frac{4}{5}(\beta n) q) r_0^{\,6} + \frac{12}{7}q^2 r_0^{\,8} + \frac{20}{9}q^2 r_0^{\,10} + ...] \tag{2.30c}$$

$$r_0 = r_\psi^* \;\; [1 - \frac{2}{3}(\beta n)\, r_\psi^{*3} - q^2 t^2\, r_\psi^{*4} - (\frac{4}{5}q^2 - \frac{4}{45}(\beta n)^2 + \frac{8}{15}(\beta n) q)\, r_\psi^{*6} - \frac{5}{7}q^2 r_\psi^{*8} - \frac{2}{3}q^2 r_\psi^{*10} - ...] \tag{2.30d}$$

where r_ψ^* is the nondimensional value of the radius of topologically equivalent spherical surface. Effects of Coriolis force appear in these expressions through α and β. The boundary conditions given in equation (1.28) now becomes

At the center $r_0 = 0$:

$$M_\psi = 0, \;\; L_\psi = 0 \tag{2.31a}$$

and at the free surface $r_0 = r_{0s}$:

$$\begin{gathered} M_\psi = M_0, \;\; L_\psi = L_{\psi s} \\ P_\psi = 0 \; or \; P_{\psi s}, \;\; T_\psi = 0 \; or \; T_{\psi s} \end{gathered} \tag{2.31b}$$

Here M_0 is the total mass of the model and $L_{\psi s}, P_{\psi s}, T_{\psi s}$ are the values of L_ψ, P_ψ, T_ψ respectively, on the outermost equipotential surface, ψ^{**} = constant.

At the free surface, $r_0 = r_{0s}$ where $r_{0s} = \frac{1}{\psi_s^{**} - q}$ (2.32)

ψ_s^{**} being the nondimensional value of the total potential ψ^{**} on the outermost equipotential surface of the rotationally and tidally distorted stellar model.

2.2 EQUILIBRIUM STRUCTURE OF ROTATIONALLY AND TIDALLY DISTORTED PRASAD MODEL:

In this section ,we discuss the equilibrium structure of rotationally and tidally distorted Prasad model. If we assume that the primary component of binary system behaves as Prasad model and rotating about its axis, then its equilibrium structure will be distorted by rotation as well as tidal effects of the companion. In Prasad model, density varies according to the law

$$\rho = \rho_c (1 - \frac{r^2}{R^2})$$

ρ_c being the density at the centre and r the distance of an element from the center of the model of radius R. This model was first studied by Prasad(40). Singh and Sharma (37,38) determined the effects of rotation and tidal distortions on the equilibrium structures of Prasad and Roche model without taking into account the effect of coriolis force.

Let r_ψ denote the radius of the topologically equivalent spherical model which corresponds to an equipotential surface $\psi =$ constant of the rotationally and tidally distorted Prasad model. Also, let R_ψ be the value of r_ψ on the equipotential surface ψ = constant of the rotationally and tidally distorted model. Following this approach, as discussed in previous section, r_ψ is given as

$$r_\psi = D\, r_0 [1 + \frac{2}{3}(\beta n)\, r_0^3 + q^2 t^2 r_0^4 + (\frac{4}{5} q^2 + \frac{76}{45}(\beta n)^2 + \frac{8}{15}(\beta n)\, q)\, r_0^6 + \frac{5}{7} q^2\, r_0^{\,8} + \frac{2}{3} q^2\, r_0^{\,10} + ...] \qquad (2.33)$$

So, the density distribution law of rotationally and tidally distorted Prasad model is given as

$$\rho_\psi = \rho_c(1-\frac{r_\psi^{\ 2}}{R_\psi^{\ 2}}) \tag{2.34}$$

On substituting the value of r_ψ from equation (2.33) to (2.34), we get

$$\begin{aligned}\rho_\psi = \rho_c[1-\frac{\mathrm{r}_0^{\ 2}}{R_\psi^{\ 2}}\{1+\frac{4}{3}(\beta n)\mathrm{r}_0^{\ 3}+2(\mathrm{q}^2(1\text{-}\alpha)^2)\mathrm{r}_0^{\ 4}+(\frac{8}{5}q^2+\frac{172}{45}(\beta n)^2+\frac{16}{15}(\beta n)q)\mathrm{r}_0^{\ 6}\\ +(\frac{10}{7}q^2)\mathrm{r}_0^{\ 8}+(\frac{4}{3}q^2)\mathrm{r}_0^{\ 10}\}]\end{aligned} \tag{2.35}$$

On substituting value of ρ_ψ from (2.35) in (2.29a) and integrating w.r.t. r_0 and using the fact that M_ψ =0 at center r_0=0 we get

$$\begin{aligned}M_\psi = \frac{4\pi D^3 r_0^{\ 3}\rho_c}{3}[1-\frac{3r_0^{\ 2}}{5R_\psi^{\ 2}}+2(\beta n)r_0^{\ 3}+3(\mathrm{q}^2(1\text{-}\alpha)^2)r_0^{\ 4}\text{-}\frac{2(\beta n)r_0^{\ 5}}{R_\psi^{\ 2}}+(\frac{36}{5}q^2+\frac{96}{5}(\beta n)^2\\ +\frac{24}{5}(\beta n)q-\frac{9(\mathrm{q}^2(1\text{-}\alpha)^2)}{R_\psi^{\ 2}})\frac{r_0^{\ 6}}{3}+(\frac{55}{7}q^2\text{-}\frac{44}{5}\frac{q^2}{R_\psi^{\ 2}}\text{-}\frac{1276}{45}\frac{(\beta n)^2}{R_\psi^{\ 2}}\text{-}\frac{88}{15}(\beta n)q)\frac{3}{11}r_0^{\ 8}\\ +(\frac{26}{3}q^2\text{-}\frac{65}{7}\frac{q^2}{R_\psi^{\ 2}})\frac{3r_0^{\ 10}}{13}\text{-}2\frac{q^2}{R_\psi^{\ 2}}r_0^{\ 12}\}]\end{aligned} \tag{2.36}$$

Similarly, on substituting value of ρ_ψ from (2.35) and value of M_ψ from (2.36) in (2.29b) and integrating w.r.t. r_0 we get

$$
\begin{aligned}
P_\psi = \frac{-G4\pi D^2 r_0{}^2 \rho_c{}^2}{3}[K + \frac{1}{2} - \frac{2r_0{}^2}{5R_\psi{}^2} + \frac{2}{5}(\beta n)r_0{}^3 + \{\frac{1}{3}\frac{(\mathrm{q}^2(1\text{-}\alpha)^2)}{R_\psi{}^2} + \frac{3}{5}\frac{1}{R_\psi{}^4} + 3(\mathrm{q}^2(1\text{-}\alpha)^2)\}\frac{r_0{}^4}{6} - \frac{16(\beta n)r_0^5}{21R_\psi{}^2} \\
+\{\frac{3}{5}\frac{q^2}{R_\psi{}^2} + \frac{4}{15}\frac{(\beta n)^2}{R_\psi{}^2} + \frac{2}{5}\frac{(\beta n)q}{R_\psi{}^2} - \frac{61(\mathrm{q}^2(1\text{-}\alpha)^2)}{6R_\psi{}^2} - \frac{(\mathrm{q}^2(1\text{-}\alpha)^2)}{5R_\psi{}^4} + \frac{36}{15}q^2 + \frac{96}{15}(\beta n)^2 + \frac{24}{15}(\beta n)q)\}\frac{r_0{}^6}{8} \\
+\frac{4}{45}\frac{(\beta n)}{R_\psi{}^4}r_0{}^7 + (\frac{1441}{385}\frac{q^2}{R_\psi{}^2} - \frac{8000}{495}\frac{(\beta n)^2}{R_\psi{}^2} - \frac{506}{165}\frac{(\beta n)q)}{R_\psi{}^2} - \frac{9}{25}\frac{q^2}{R_\psi{}^4} - \frac{12}{75}\frac{(\beta n)^2}{R_\psi{}^4} - \frac{6}{25}\frac{(\beta n)q)}{R_\psi{}^4} + \frac{21(\mathrm{q}^2(1\text{-}\alpha)^2)}{30R_\psi{}^4} + \\
\frac{165}{77}q^2)\frac{1}{10}r_0{}^8 + (\frac{26}{3}q^2 - \frac{66}{7}\frac{q^2}{R_\psi{}^2})\frac{3r_0{}^9}{11} + \{\frac{-74}{63}\frac{q^2}{R_\psi{}^2} + \frac{384}{75}\frac{(\beta n)^2}{R_\psi{}^4} + \frac{22}{25}\frac{(\beta n)q)}{R_\psi{}^4} + \frac{141}{175}\frac{q^2}{R_\psi{}^4}\}\frac{1}{12}r_0{}^{10} - \\
\frac{30q^2}{13R_\psi{}^2}r_0{}^{11} + \{\frac{-22}{9}\frac{q^2}{R_\psi{}^2} + \frac{74}{105}\frac{q^2}{R_\psi{}^4}\}\frac{1}{14}r_0{}^{12} + \frac{22}{15}\frac{q^2}{(16R_\psi{}^4)}r_0{}^{14}]
\end{aligned}
\tag{2.37}
$$

where K is a constant of integration whose value may be calculated by using boundary condition say P_ψ =0 at $r_0 = r_{0s}$ .This yields

$$
\begin{aligned}
K = -\frac{1}{2} + \frac{2\mathrm{r}_{0s}{}^2}{5R_\psi{}^2} - \frac{2}{5}(\beta n)\mathrm{r}_{0s}{}^3 - \{\frac{1}{3}\frac{(\mathrm{q}^2(1\text{-}\alpha)^2)}{R_\psi{}^2} + \frac{3}{5}\frac{1}{R_\psi{}^4} + 3(\mathrm{q}^2(1\text{-}\alpha)^2)\}\frac{\mathrm{r}_{0s}{}^4}{6} + \frac{16(\beta n)\mathrm{r}_{0s}{}^5}{21R_\psi{}^2} \\
-\{\frac{3}{5}\frac{q^2}{R_\psi{}^2} + \frac{4}{15}\frac{(\beta n)^2}{R_\psi{}^2} + \frac{2}{5}\frac{(\beta n)q}{R_\psi{}^2} - \frac{61(\mathrm{q}^2(1\text{-}\alpha)^2)}{6R_\psi{}^2} - \frac{(\mathrm{q}^2(1\text{-}\alpha)^2)}{5R_\psi{}^4} + \frac{36}{15}q^2 + \frac{96}{15}(\beta n)^2 + \frac{24}{15}(\beta n)q)\}\frac{\mathrm{r}_{0s}{}^6}{8} \\
-\frac{4}{45}\frac{(\beta n)}{R_\psi{}^4}\mathrm{r}_{0s}{}^7 - (\frac{1441}{385}\frac{q^2}{R_\psi{}^2} - \frac{8000}{495}\frac{(\beta n)^2}{R_\psi{}^2} - \frac{506}{165}\frac{(\beta n)q)}{R_\psi{}^2} - \frac{9}{25}\frac{q^2}{R_\psi{}^4} - \frac{12}{75}\frac{(\beta n)^2}{R_\psi{}^4} - \frac{6}{25}\frac{(\beta n)q)}{R_\psi{}^4} + \frac{21(\mathrm{q}^2(1\text{-}\alpha)^2)}{30R_\psi{}^4} + \\
\frac{165}{77}q^2)\frac{1}{10}\mathrm{r}_{0s}{}^8 - (\frac{26}{3}q^2 - \frac{66}{7}\frac{q^2}{R_\psi{}^2})\frac{3\mathrm{r}_{0s}{}^9}{11} - \{\frac{-74}{63}\frac{q^2}{R_\psi{}^2} + \frac{384}{75}\frac{(\beta n)^2}{R_\psi{}^4} + \frac{22}{25}\frac{(\beta n)q)}{R_\psi{}^4} + \frac{141}{175}\frac{q^2}{R_\psi{}^4}\}\frac{1}{12}\mathrm{r}_{0s}{}^{10} + \\
\frac{30q^2}{13R_\psi{}^2}\mathrm{r}_{0s}{}^{11} - \{\frac{-22}{9}\frac{q^2}{R_\psi{}^2} + \frac{74}{105}\frac{q^2}{R_\psi{}^4}\}\frac{1}{14}\mathrm{r}_{0s}{}^{12} - \frac{22}{15}\frac{q^2}{(16R_\psi{}^4)}\mathrm{r}_{0s}{}^{14}]
\end{aligned}
\tag{2.38}
$$

Effects of Coriolis force appear in these expressions through α and β.

2.3 EQUILIBRIUM STRUCTURE OF ROTATIONALLY AND TIDALLY DISTORTED ROCHE MODEL:

In this section, we determine the effects of rotation and tidal distortion on the equilibrium structures of Roche model. A Roche model is a model in which entire mass of the star is assumed to be concentrated at its centre which is surrounded by an evanescent envelope in which density varies inversely as the square of the distance from its center. Density distribution ρ is given by

$$\rho = \in r^{-\alpha}$$

where α is some positive real number (α=2 for Roche model). Also, on the equipotential surface ρ is constant. Therefore, we can suppose that the density ρ_ψ on the surface of topologically equivalent sphere of radius r_ψ is given by

$$\rho_\psi = \in r_\psi^{-\alpha}$$

Let r_ψ denote the radius of the topologically equivalent spherical model which corresponds to an equipotential surface ψ= constant of this rotationally and tidally distorted Roche model. Also, let R_ψ be the value of r_ψ on the equipotential surface ψ = constant of this rotationally and tidally distorted model. Following this approach, as discussed above, r_ψ is given as

$$r_\psi = D\, r_0 [1 + \frac{2}{3}(\beta n) r_0^3 + q^2 t^2 r_0^4 + (\frac{4}{5} q^2 + \frac{76}{45}(\beta n)^2 + \frac{8}{15}(\beta n) q) r_0^6 + \frac{5}{7} q^2 r_0^8 + \frac{2}{3} q^2 r_0^{10} + \ldots] \tag{2.39}$$

So, the density distribution law of rotationally and tidally distorted Roche model is given as

$$\rho_\psi = \frac{\in}{r_\psi^2} \tag{2.40}$$

On substituting the value of r_ψ from equation (2.39) to (2.40), we get

$$\rho_\psi = \frac{\in}{\mathrm{r}_0{}^2}[1-\frac{4}{3}(\beta n)\mathrm{r}_0{}^3-2(\mathrm{q}^2(1-\alpha)^2)\mathrm{r}_0{}^4-(\frac{8}{5}q^2+\frac{92}{45}(\beta n)^2+\frac{16}{15}(\beta n)q)\mathrm{r}_0{}^6 -(\frac{10}{7}q^2)\mathrm{r}_0{}^8-(\frac{4}{3}q^2)\mathrm{r}_0{}^{10}] \qquad (2.41)$$

On substituting value of ρ_ψ from (2.41) in (2.29a) of previous section and integrating w.r.t. r_0 we get

$$M_\psi = M_0 + 4\pi D^3 r_0 \in [1+\frac{2}{3}(\beta n)r_0{}^3+(\mathrm{q}^2(1-\alpha)^2)r_0{}^4+(\frac{28}{5}q^2+\frac{532}{45}(\beta n)^2+\frac{56}{15}(\beta n)q)\frac{r_0{}^6}{7}+ (\frac{5}{7}q^2)r_0{}^8+(\frac{2}{3}q^2)r_0{}^{10}] \qquad (2.42)$$

where

$$M_0 = \frac{4}{3}\pi R_\psi{}^3\bar{\rho}[1-\frac{3\in}{\bar{\rho}(3-\alpha)R_\psi{}^2}] \qquad (2.43)$$

Similarly, on substituting value of ρ_ψ from (2.41) and value of M_ψ from equation (2.42) in (2.29b) and integrating w.r.t. r_0 we get

$$P_\psi = \in [K+\frac{1}{3\mathrm{r}_0{}^3}+\frac{4}{3}(\beta n)\log(\mathrm{r}_0)+\frac{5}{3}(\mathrm{q}^2(1-\alpha)^2)\mathrm{r}_0 -(q^2-\frac{512}{45}(\beta n)^2+\frac{2}{3}(\beta n)q)\frac{\mathrm{r}_0{}^3}{3}+ (\frac{4}{7}q^2)\frac{\mathrm{r}_0{}^5}{5}+(\frac{2}{3}q^2)\frac{\mathrm{r}_0{}^7}{7}] \qquad (2.44)$$

Where K is the constant of integration. Now to evaluate K, we use the boundary condition that $P_\psi = 0$, at the free surface ($r_0 = r_{0s}$).

$$K = -[\frac{1}{3\mathrm{r}_{0s}{}^3}+\frac{4}{3}(\beta n)\log(\mathrm{r}_{0s})+\frac{5}{3}(\mathrm{q}^2(1-\alpha)^2)\mathrm{r}_{0s}-(q^2-\frac{512}{45}(\beta n)^2+\frac{2}{3}(\beta n)q)\frac{\mathrm{r}_{0s}{}^3}{3}+ (\frac{4}{7}q^2)\frac{\mathrm{r}_{0s}{}^5}{5}+(\frac{2}{3}q^2)\frac{\mathrm{r}_{0s}{}^7}{7}] \qquad (2.45)$$

2.4. ANALYSIS OF RESULTS:

Using expressions obtained in above section, the graphs have been drawn to see the effect of coriolis force on mass and pressure for different values of rotational parameter n, n_1 and tidal parameter q, as the radius varies from centre to surface.

The results show that in both the cases of rotationally and tidally distorted Prasad and Roche model , in the presence of coriolis force, the mass increases from centre to surface. In the case of Prasad model, fig. 2.3 shows that if the angular velocity of rotation n_1 increases and n, q are fixed then the increase in mass is negligible near the centre but as we move near to the surface the mass increases, however in case of Roche model fig. 2.4 the trend is similar but the mass varies almost linearly from centre to surface. Fig. 2.5 show that in Prasad model if the parameter n_1 and q are fixed, with increase in the value of n the increase in the mass is negligible, but in case of Roche model fig. 2.6 significant change can be seen near the surface.

In case of Prasad model in the presence of coriolis force the pressure decreases from centre to near the surface but as we move near to the surface, it increases rapidly.In fig. 2.7 for Prasad model if the angular velocity of rotation n_1 increases, and n and q are fixed,the decrease in pressure from centre towards surface is negligible but near the surface there is an increase in the pressure. Fig. 2.8 show that if the parameter n_1 and q are fixed, with increase in the value of n, the trend is similar as in fig. 2.7. In case of Roche model fig. 2.9 the pressure decreases more rapidly near the centre and it is almost negligible towards surface for all the cases considered. It can be seen from fig. 2.9 that the effect of variation in parameters n and n_1 is negligible.

Fig. 2.10 and fig. 2.12 show the effect of coriolis force on mass of rotationally and tidally distorted Prasad model and Roche model respectively. In Prasad model as well as in Roche model with the inclusion of coriolis force there is an increase in the mass as we move towards the surface.In case of Prasad model fig. 2.11 shows that with the inclusion of coriolis force, the pressure increase is more very near to the surface. However, in case of Roche model fig 2.13 the effect of coriolis force on pressure is negligible.

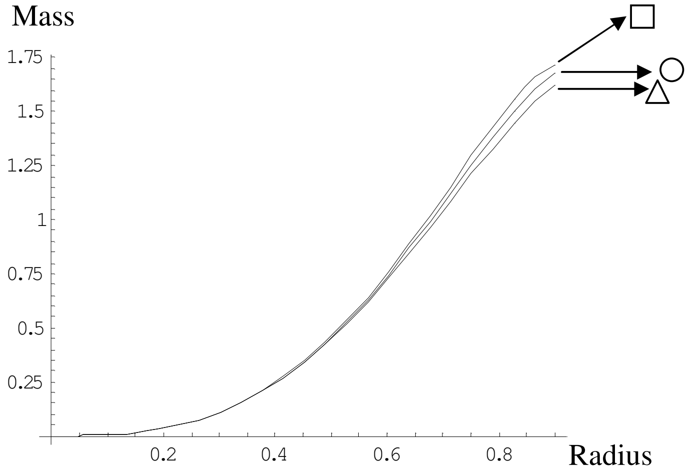

Fig. 2.3:Graphs of radius vs mass for Prasad model in which angular velocity of rotation n_1 varies and n and q remains fixed.

□ : n =0.1, n_1=0.15, q =0.1 ; ○: n =0.1, n_1=0.1, q =0.1;

△: n =0.1, n_1=0.05, q =0.1

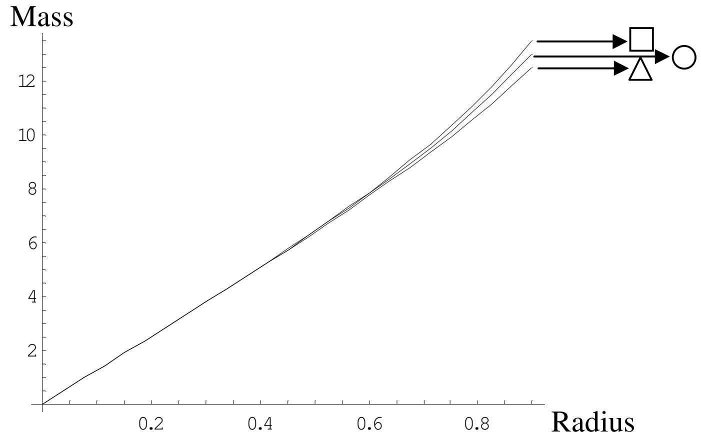

Fig. 2.4: Graphs of radius vs mass for Roche model in which angular velocity of rotation n_1 varies and n and q remains fixed.

□ : n =0.1, n_1=0.15, q =0.1 ; ○: n =0.1, n_1=0.1, q =0.1;

△ : n =0.1, n_1=0.05, q =0.1

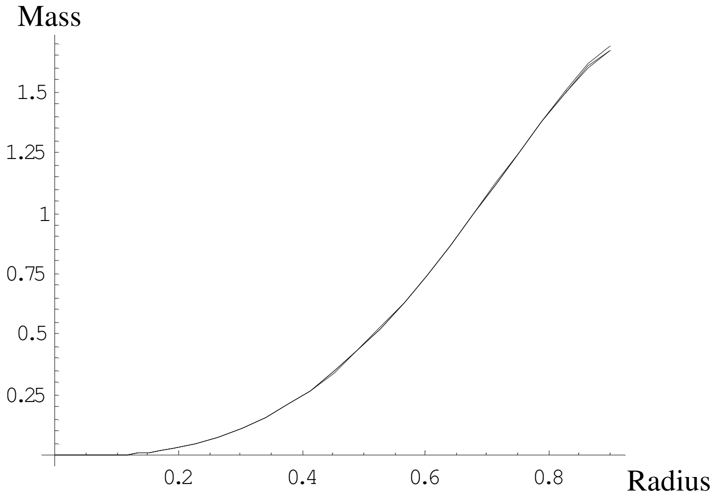

Fig. 2.5: Graphs of radius vs mass for Prasad model in which angular velocity of revolution n varies and n_1 and q remains fixed.

(a) n =0.01, n_1 =0.1, q =0.1; (b) n =0.05, n_1 =0.1, q =0.1;
(c) n =0.15, n_1 =0.1, q =0.1

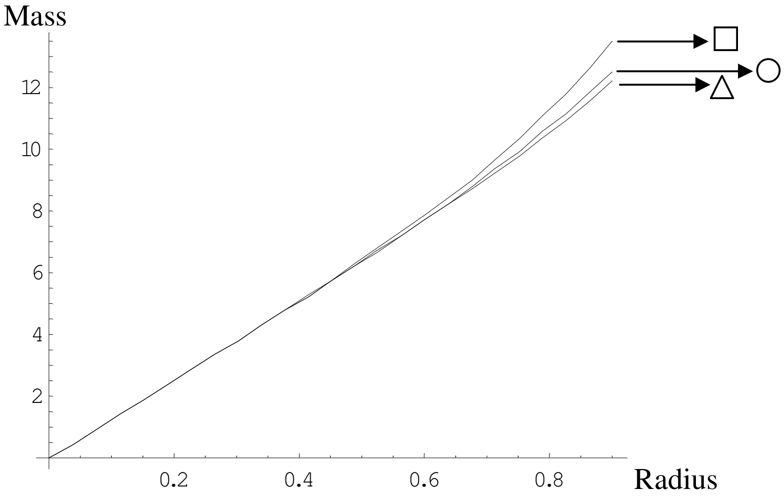

Fig. 2.6: Graphs of radius vs mass for Roche model in which angular velocity of revolution n varies and n_1 and q remains fixed.

□ : n =0.15, n_1 =0.1, q =0.1 ; ○: n =0.05, n_1 =0.1, q =0.1;
△: n =0.01, n_1 =0.1, q =0.1

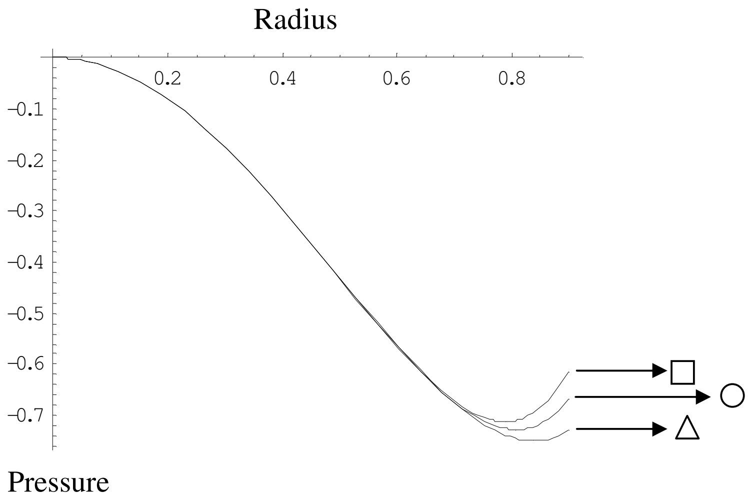

Fig. 2.7:Graphs of radius vs pressure for Prasad model in which angular velocity of rotation n_1 varies and n and q remains fixed.

□ : n =0.1, n_1=0.15, q =0.1 ; ○ : n =0.1, n_1=0.1, q =0.1;

△ : n =0.1, n_1=0.05, q =0.1

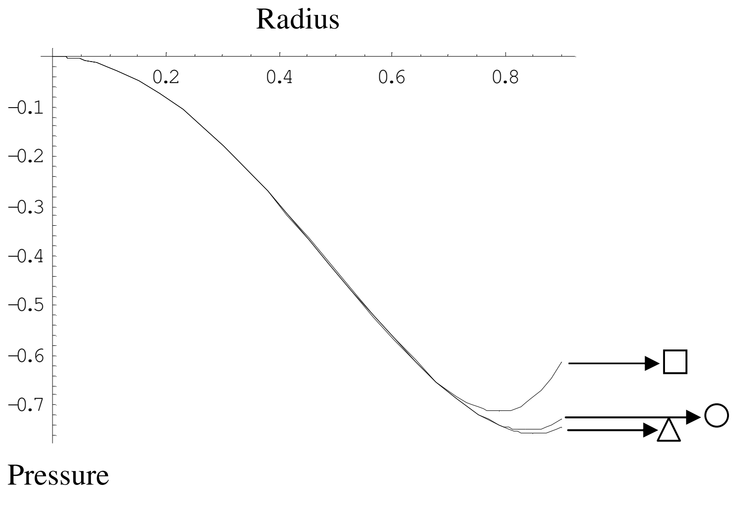

Fig. 2.8: Graphs of radius vs pressure for Prasad model in which angular velocity of revolution n varies and n_1 and q remains fixed.

□ : n =0.15, n_1=0.1, q =0.1 ; ○: n =0.05, n_1=0.1, q =0.1;

△: n =0.01, n_1=0.1, q =0.1

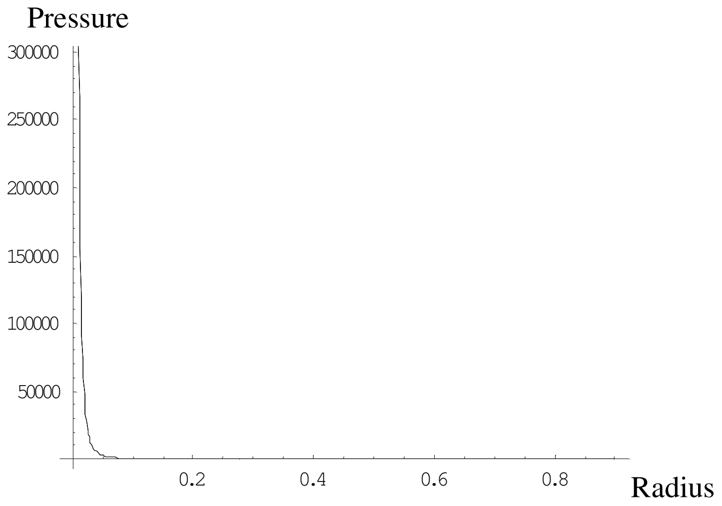

Fig. 2.9: Graphs of radius vs pressurre for Roche model
for all the cases considered

(a) n =0.15, n_1 =0.1, q =0.1 (a) n =0.1, n_1 =0.15, q =0.1
(b) n =0.05, n_1 =0.1, q =0.1 (b) n =0.1, n_1 =0.05, q =0.1
(c) n =0.01, n_1 =0.1, q =0.1 (c) n =0.1, n_1 =0.1, q =0.1

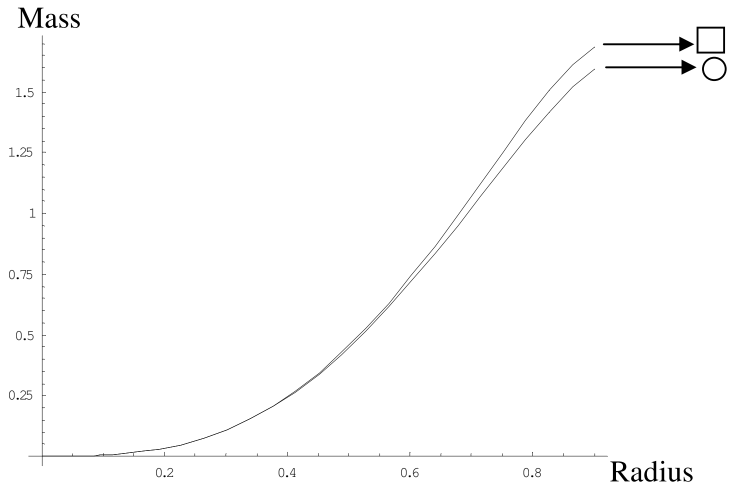

Fig. 2.10: Effect of coriolis force on mass for Prasad model
□ : q =0.1, n =0.01, n_1 =0.1; ○: q =0.1, n =0.01;

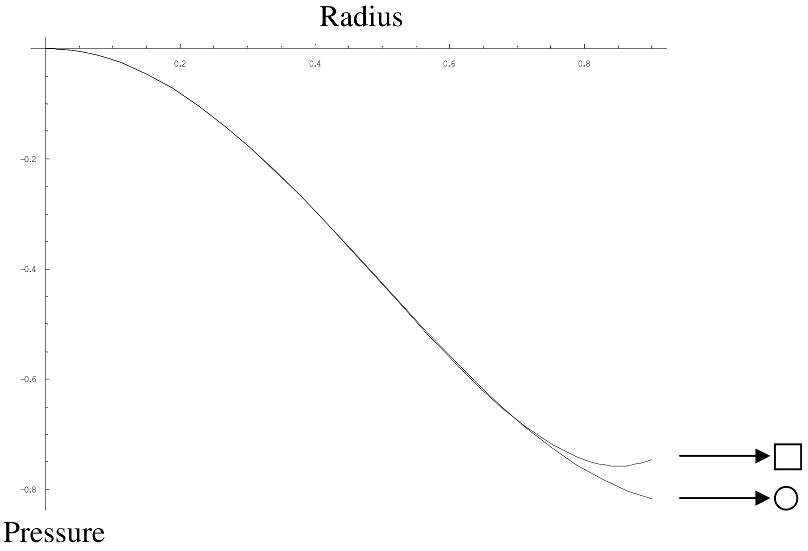

Fig. 2.11: Effect of coriolis force on pressure for Prasad model
□: q =0.1, n =0.01, n_1 =0.1;　　○ : q =0.1, n =0.01;

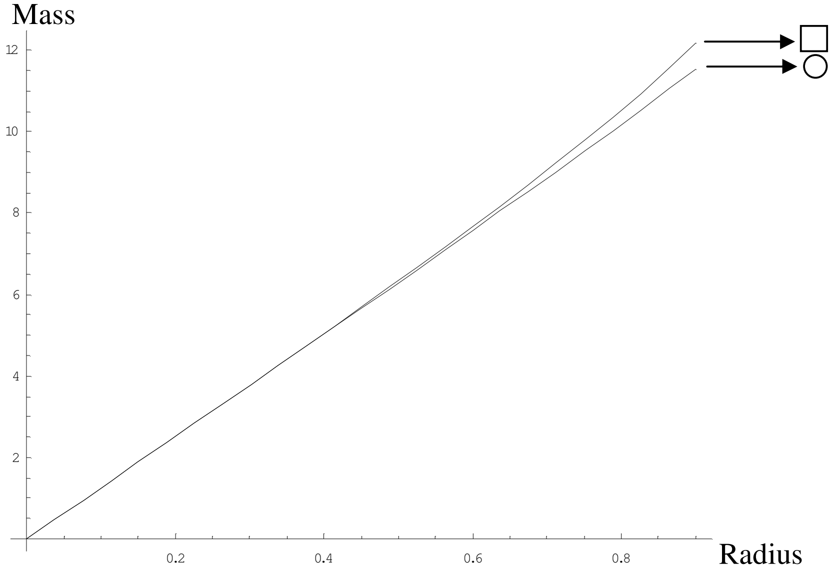

Fig. 2.12: Effect of coriolis force on mass for Roche model
□: q =0.1, n =0.01, n_1 =0.1;　　○ : q =0.1, n =0.01;

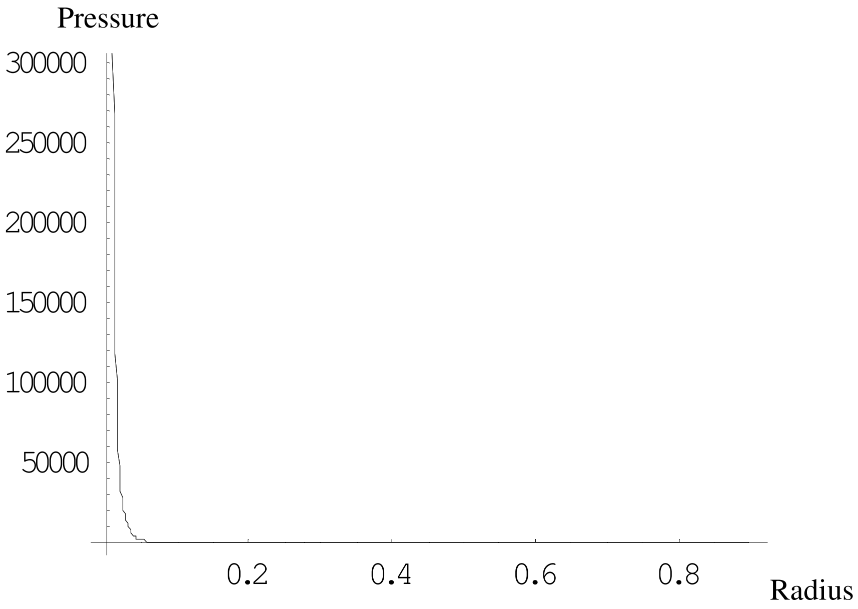

Fig. 2.13: Effect of coriolis force on pressure for Roche model
(a) q =0.1, n =0.01, n_1=0.1; (b) q =0.1, n =0.01;

BIBLIOGRAPHY

1. Chan, K. L. and Chau, W. Y., Structure and evolution of very close binary systems. I. structure equations including rotational and tidal distortions and calculations for 2, 1 and 0.6 Zero-age main sequence stars, Astrophysical Journal, 233, 950 – 960, 1979.

2. Chandrasekhar, S., The equilibrium of distorted polytropes (I) The rotational problem, Monthly Notices of Royal Astronomical Society, 93, 390 - 406,1933.

3. Chandrasekhar, S., The equilibrium of distorted polytropes (II) The tidal problem, Monthly Notices of Royal Astronomical Society, 93, 449 - 461, 1933.

4. Chandrasekhar, S., The equilibrium of distorted polytropes (III) The double star problem, Monthly Notices of Royal Astronomical Society, 93, 462 - 471,1933.

5. Chandrasekhar, S., An Introduction to the study of Stellar Structure, The University of Chicago Press, Chicago, 1939.

6. Cox, J. P. and Giuli, R. T., Principle of stellar structure, Vol. I and II,Gordon and Breach, New York, 1968.

7. Deupree, R. G. and Karakas, A. I., The structure of close binaries in two dimensions, Astrophysical Journal, 633, 418 – 423, 2005.

8. Endal, A. S. and Sofia, S., The evolution of rotating stars. I. Method and exploratory Calculations for 7 star, Astrophysical Journal, 210, 184–198, 1976.

9. Geroyannis, V. S. and Antonakopoulus, G. A., Structure distortion of polytropic stars by differential rotation: Third order perturbation theory,Astrophysics and Space Science, 74, 97 – 110, 1981.

10. Geroyannis, V. S. and Valvi, F. N., Third-order perturbation theory: structure parameters of uniformly rotating polytropic stars and improvement and accuracy of the results, Astrophysical Journal Supplement Series, 62, 363 – 371, 1986.

11. James, R. A., The structure and stability of rotating gas masses, Astrophysical Journal, 140, 552 – 582, 1964.

12. Kippenhahn, R. and Thomas, H. C., A simple method for the solution of the stellar structure equation including rotation and tidal forces, Stellar Rotation,Proceedings of IAU Colloq 4, Ohio State Univ, Columbus, ed. Slettebak, A., D., Gordon and Breach Science Publishers, 1970.

13. Kippenhahn, R. and Weigert, A., Stellar Structure and Evolution, Springer –Verlag publ., 1990.

14. Kopal, Z., Roche Model and Its Application to Close Binary Systems,Advances in Astronomy and Astrophysics, 9, 1 – 65, 1972.

15. Kopal, Z., Dynamics of close Binary Systems, D. Reidel Publ. Co.,Dordrecht, Holland, 1978.

16. Kopal, Z., Effects of rotation on Internal Structure of the Stars, Astrophysics and Space Science, 93, 149 – 175, 1983.

17. Kopal, Z., The Roche problem and its significance for double star astronomy,Kluwer Academic Publishers, Dordrecht, Netherland, 1989.

18. Kopal, Z. and Ali, A. K. M. Sekender., On the integrability of the Roche coordinates, Astrophysics and Space Science, 11, 423 – 429, 1971.

19. Lal, A. K., Structure oscillations and stability of differentially rotating and tidally distorted stellar models, PhD Thesis, Roorkee University, Roorkee, India, 1993.

20. Lal, A. K., Mohan, C. and Singh, V. P., Equilibrium structures of differentially rotating and tidally distorted white dwarf models of stars, Astrophysics and space Science, 301, 51 – 60, 2006.

21. Lal A. K., Saini, S., Mohan, C. and Singh, V. P., Equilibrium structures of rotationally and tidally distorted polytropic models of stars, Astrophysics and Space Science, 306, 165 – 169, 2006.

22. Lal, A. K., Saini, S., Mohan, C. and Singh, V. P., On the validity of series expansion being used for the position of a point on a Roche equipotential,Astrophysics and Space Science, 310, 317 – 320, 2007.

23. Mohan, C. and Agarwal, S. R., Effects of rotation and tidal distortions on the periods of adiabatic oscillations of composite models of stars, Astrophysics and Space Science, 129, 73 – 94, 1987.

24. Mohan, C. and Saxena, R. M., Effects of rotation and tidal distortions on the structure of polytropic stars, Astrophysics and Space Science, 95, 369 – 381,1983.

25. Mohan, C. and Singh, V. P., Use of Roche coordinates in the problems of small oscillations of rotationally distorted stellar models, Astrophysics and Space Science, 54, 293 – 304, 1978.

26. Mohan, C. and Singh, V. P., Use of Roche Coordinates in the problems of small oscillations of tidally distorted stellar models, Astrophysics and Space Science, 56, 109 – 117, 1978.

27. Mohan, C., Lal, A. K. and Singh, V. P., Equilibrium structure of differentially rotating polytropic models of the stars, Astrophysics and Space Science, 193, 69 – 85, 1992.

28. Mohan, C., Lal, A. K., and Singh, V. P., Equilibrium structure of stars obeying a generalized differential rotation law, Astrophysics and Space Science, 215, 111 – 130, 1994.

29. Mohan, C., Lal, A. K. and Singh, V. P., Equilibrium structures of differentially rotating primary components of binary stars, Astrophysics and Space Science, 254, 97 – 109, 1997.

30. Mohan, C., Lal, A. K. and Singh, V. P., Effects of differential rotation on the eigenfrequencies of small adiabatic barotropic modes of oscillations of the stars, Indian J pure appl. Math, 29 (2), 199 – 220, 1998.

31. Mohan, C., Saxena, R. M. and Agarwal, S. R., Equilibrium structures of rotationally and tidally distorted stellar models, Astrophysics and Space Science, 163, 23 – 39, 1990.

32. Mohan, C., Saxena, R. M. and Agarwal, S. R., Eigenfrequencies of small adiabatic barotropic models of oscillations of rotationally and tidally - distorted stars, Astrophysics and Space Science, 178, .89 – 105, 1991.

33. Tassoul, J. L., Theory of Rotating Stars, Princeton University Press, Princeton, 1978.

34. Lal A. K, Pathania . A, Mohan . C Effects of Corilis force on the equilibrium structures of rotating stars and stars in binary systems, Astrophysics and space sci.,315157-165, 2008.

35. Lal A. K, Pathania . A , Mohan . C Effects of Coriolis force on the shapes of rotating stars and stars in binary systems , Astrophysics and space sci. , 319 45-53,2009.

36. Sharma M.K. 'Effects of differential rotation and tidal distortion on the structure and oscillations of stars' Ph.D. thesis University of Roorkee 1977.

37. Singh V.P. and Sharma M.K. "Effects of differential rotations on the periods of small adiabatic oscillations of stellar models", Indian Journal of Pure and Appl. Math.,26(1),69,1995.

38. Singh V.P. and Sharma M.K. "Effects of differential rotations on the periods of small adiabatic oscillations of stellar models", Indian Journal of Pure and Appl. Math.,27(1),121,1996.

39. Saxena R.M., "The effects of rotation and tidal distortions on the structure and periods of small oscillations of gaseous spheres", Ph.D. thesis, Roorkee Univ., India,1984.

40. Prasad C., "Radial Oscillations of a stellar model", M.N.R.A.S., London, Vol.109(1), p.103, 1949.