PROPAGATION OF SH WAVES IN AN ELASTIC LAYERED MEDIA

Thesis submitted in partial fulfillment of the requirements

for the award of the degree of
Masters of Science
in

Mathematics and Computing

Submitted by

Vikas Vats

Roll no. – 301103020

under

the guidance of

Dr. Abhishek Kumar Singh and Dr. Satish Kumar Sharma

to the

School of Mathematics and Computer Applications
Thapar University
Patiala-147004 (Punjab)
INDIA
June 2013

DEDICATED
TO
GOD, MY PARENTS, MY TEACHERS AND MY FRIENDS

CERTIFICATE

*I hereby certify that the work which is being presented in this thesis entitled "**Propagation of SH waves in an elastic Layered Media**" which is being submitted for the award of degree of Master of Sciences in Mathematics and Computing from School of Mathematics and Computer Applications, Thapar University, Patiala, is an authentic record of my own work carried out under the supervision of **Dr. Abhishek Kumar Singh and Dr. Satish Kumar Sharma.***

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

Vikas vats

This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

Dr. Abhishek Kumar Singh
(Assistant Professor)
SMCA, Thapar University
Patiala.

Dr. Satish Kumar Sharma
(Assistant Professor)
SMCA, Thapar University
Patiala.

Countersigned by:
Dr. Rajesh Kumar
(Associate Professor & Head)
School of Mathematics and Computer Applications
Thapar University
Patiala.

Dr. S.K. Mohapatra
Dean of Academic Affairs
Thapar University
Patiala.

ACKNOWLEDGEMENT

It gives me immense pleasure to acknowledge my sincere gratitude to my academic supervisor, Dr. Abhishek Kumar Singh and Dr. Satish Kumar Sharma, School of Mathematics and Computer Applications, Thapar University, Patiala, for their constant help, encouragement and support throughout the course of this work.

I would like to extend my special thanks to Dr. Rajesh Kumar, Associate Professor and Head, School of Mathematics and Computer Applications, Thapar University, Patiala, for providing all necessary facilities in the department and encouraging me directly and indirectly for successful completion of thesis work. He also extended his help when I was in difficulties.

I express my deepest gratitude to my parents, my uncle Mr. Deshraj Vats, my brothers, my friends and most importantly my senior Tanupreet Kaur for unconditional support in all spheres of my life.

VIKAS VATS

Vikas Vats

Abstract

This thesis aims to study the propagation of SH wave in a monoclinic layer lying over an isotropic half-space and propagation of SH wave in double isotropic layer lying over an isotropic half-space. The dispersion relation has been obtained for both the problems. Variation of the phase velocity with the wave number has been depicted by means of graphs. Moreover, for the double layer problem effect of thickness ratio on the dispersion curve has been studied.

There are two main types of waves generated during earthquakes, body waves (P-waves, S-waves) and surface waves, which include Love waves and Rayleigh waves. Surface waves are generated by the constructive interference of incident P and S -waves arriving at the free surface and propagating parallel to the surface. The amplitude of surface waves decreases with increasing depth and are affected by lateral variations in structure. Another property that surface waves exhibit is dispersion, i.e., the velocity of a wave on the surface is dependent on its frequency (or period).

Love waves are a type of surface wave formed by the constructive interference of multiple reflections of SH waves at the free surface. Love waves are faster than Rayleigh waves and therefore arrive before them on a seismogram. The particle motion for Love waves is parallel to the surface but perpendicular to the direction of propagation and found on the transverse record of a rotated seismogram. Because Love waves need a low-velocity layer over a half-space to exist, they are always dispersive.

Dispersion is the apparent surface-wave velocity that depends on the period and reflects the velocity variation with depth. Dispersion appears on a seismogram as different periods arriving at different times. In general, short period surface waves, which sample rocks closer to the surface, travel slower than long period waves. Long periods are sensitive to faster velocities found deeper in the Earth. Love waves exhibit dispersion and are used to estimate shear-velocity variations in the crust and upper mantle.

Chapter-1 deals with the study of basic concepts and equation of motion of elastic medium. In this chapter, the importance of problems of elastic wave propagation has been highlighted and the brief outlines of the anisotropic elasticity have been presented.

Chapter-2 deals with the study of Love wave propagation in an homogeneous isotropic layer lying over homogeneous isotropic half space. The dispersion relation has been found in closed form.

Chapter-3 deals with the propagation of SH wave in a monoclinic layer lying over an isotropic homogeneous half-space. The dispersion relation is found in the closed form and matched with the classical Love wave equation as a particular case. It is observed that phase velocity is depending on the wave number and the thickness of the layer. Graphical illustration is being made for the study.

Chapter-4 deals with the propagation of SH wave in the double isotropic homogeneous layer lying over an isotropic homogeneous half-space. The dispersion relation is found in closed form. It is observed that phase velocity is depending on the wave number, thickness of the uppermost layer and thickness ratio of the layers. Some of the important peculiarities have been traced out through graphs.

CONTENTS

CHAPTER-1

INTRODUCTION

1.1 Basic Definitions

Continuum mechanics is a branch of mechanics that deals with kinematics and mechanical behaviour of continuous matter. If a body contains a sufficiently large number of molecules, so that the distance distances between two neighbouring molecules are negligible in comparison with the dimensions of the body, then the body is said to be the continuous body and it behaves in accordance with the law of mechanics. To simplify mathematical analysis, it is convenient to disregard the actual discrete molecular structure of the body, and treat the matter as uniformly and continuously distributed in the region of space without voids occupied by the body. There are mainly two themes to perform the study of mechanics of continuous matter: (i) Derivation of fundamental equations and (ii) Derivation of constitutive relations. Fundamental equations for continuous body are based upon universal laws of physics such as conservation of mass, momentum and principle of energy etc., while the constitutive relations characterize the material properties of the matter, e.g., mechanical and thermal properties. These equations are in fact, the key points around which the various studies in the field of continuum mechanics proceed. Mathematically, the fundamental equations of the continuum mechanics are developed in two separate but essentially equivalent formulations. One, the integral or global form, derives from a consideration of the basic principles being applied to a finite volume of the material. The other, a differential or field approach, leads to equations resulting from the basic principles being applied to a very small (infinitesimal) element of volume. Under the continuum assumption, the field quantities such as density and volume which reflect the mechanical or kinematical properties of continuum bodies are expressed mathematically as continuous functions or at worst as piecewise continuous functions of space and time variable. Mathematical theory of continuous media is built upon the basic concepts of stress, motion and deformation, the law of conservation of mass, linear momentum, moment of momentum, energy and on the constitutive relations.

The problems of wave propagation through continuous bodies have been a subject of keen interest since long. The theory of wave propagation in elastic solids was developed during 19^{th} and 20^{th} centuries by Poisson (1829), Kelvin (1863), Rayleigh (1877, 1885, 1912), Stoneley (1924), Stokes (1924), Love (1944), Biot (1965) and many others. Wave is a mode of energy transfer from one place to another, in a medium, often with little or no permanent displacement of the particles of the medium, i.e., little or no associated mass transport; instead there are oscillations around almost fixed positions. The mechanical waves require a medium to travel, while the electromagnetic waves can travel through the vacuum. Thus, in wave propagation, the particles in the medium do not change their original positions but they oscillate about their mean positions and usually periodic in nature with a finite velocity, e.g., water waves, sound waves, elastic waves. It is interesting to note that all wave motions have two important characteristics in common. First, energy is propagated to distant point. Second, the disturbance travels through the medium without giving the medium, as a whole, any permanent displacement. Each successive particle of the medium performs a motion similar to its predecessors but later in time returns to its original.

1.2 Stress and Strain relations

There are two types of forces acting on a continuous body (i) internal forces and (ii) external forces. Internal forces are the forces of interaction between the constituent particles of a continuum. External forces are those generated by the external agencies. Note that the internal forces act even when no external forces are applied on the body. The deformation in the continuum is mainly due to external forces. These forces are (i) Body forces and (ii) Surface forces. Body forces are proportional to the mass contained in the volume element of the body, i.e., they act on all volume elements distributed continuously throughout the body, e.g., gravitational force. As they act on volume element, therefore these forces are also known as volume forces. Surface forces are those forces which act upon and are distributed in some fashion over a surface element of the body, regardless of whether that element is a part of the bounding surface, or an arbitrary element of surface within the body, e.g., hydrostatic pressure on the surface of a submerged body.

Continuum solids such as rocks at normal temperature when subjected to external loads get deformed and when external forces are removed, these continuum solids return to their original shape and size. This property exhibited by a continuum is called elastic property and the continuum is called the elastic body. In fact, when the external forces are applied on the continuous body, the relative positions of its constituent particles get altered and the body is called the strained body. The change in the relative position of the particles is known as deformation. At this stage of deformation, the particles resist to change their positions but it is the external force which makes them to change their positions up to some extent. Thus, when the external forces are withdrawn, these particles at once regain their original shape and size. The elastic property of a continuum body depends on the strength of resistance to this type of deformation. Greater the resistance of a body to deform, the more elastic it is said to be. That is why steel is more elastic than rubber.

The forces per unit area set up inside the body to resist deformation are called stresses. The deformation of the body accompanying stress is called strain. Thus stress and strain occur together. The strain set up in a body in such a way that there is a change in volume but no change in shape, is called dilatation. There are two kinds of dilatation: compression, in which volume is reduced; and rarefaction, in which the volume is increased. The second type of elastic deformation is a change of shape without a change in volume and is called shear. Consider a surface element, ΔS situated either in the interior or on the boundary of a medium, and let the force acting on this surface element be $T\Delta S$. We have

$$\underline{\lim}_{\Delta s\to 0}\frac{T\Delta s}{\Delta s}=T\left(x_i;v\right) \qquad (1.2.1)$$

where the vector T is called stress vector and represents the surface force per unit area of the surface element acting at the point $\left(x_i\right)$ whose orientation is specified by a unit normal vector, υ. The stress force depends not only on the position of the surface element but also on the orientation of the surface element. The state of stress at any point of a medium is completely characterized by the nine quantities, called stress tensors, τ_{ij} (see sokolnikoff, 1956). In more precise form, if T^{υ} be the stress vector acting at a point of a surface to which υ is normal, then the stress tensor can be written as

$$T^{\upsilon}=\tau_{ij}\upsilon_j \qquad \left(i,\, j=1,2,3\right) \qquad (1.2.2)$$

where, τ_{ij} is the j[th] component of the stress vector acting on a surface element to which x_i axis is normal. The relation of the strain tensor e_{ij} with the components of displacement vector $u=\left(u_1,u_2,u_3\right)$ for a continuous deformable medium, is given by (see sokolnikoff, 1956)

$$e_{ij} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right] \tag{1.2.3}$$

In the classical theory of elasticity, the relation between the stress components, τ_{ij} and the strain components, e_{ij} for an elastic solid continuum is given by Hooke's law. It states that, within the elastic limit, the stress is a linear function of strain. That is

$$\tau_{ij} = c_{ijkl}e_{kl} \qquad (i, j, k, l = 1, 2, 3) \tag{1.2.4}$$

where c_{ijkl} are the elastic constants or elastic moduli, which characterize the elastic properties of the body. These constants are 81 in number. If the elastic constants vary from point to point of the medium, i.e., the elastic constants are the functions of the position, the body is said to be elastically non homogeneous or inhomogeneous. On the other hand if the elastic constants are same for all points of the medium, then the body is called elastically homogeneous.

1.3 Equation of motion for an elastic solid

Considered a continuous solid medium, every portion of which is contained within the volume *V* and bounded by a closed surface *S*. The resultant force acting on the matter within *V* (combine both forces, i.e., body force and surface force) must be equal to the inertia force. We shall calculate the x_i - component of this force. Suppose *F* is the body force per unit volume, T^{υ} be the surface force per unit area and $\rho(x_1, x_2, x_3)$ be the mean density of the medium. The component of the force of inertia acting on the mass contained within the volume element *dV* which is bounded by the surface element *dS* is $\rho \frac{\partial^2 u_i}{\partial t^2} dV$ .Then, we have,

$$\int_V F_i dV + \int_S T_i^{\upsilon} dS = \int_V \rho \frac{\partial^2 u_i}{\partial t^2} dV \tag{1.3.1}$$

Making use of the relation (1.2.2) and applying Gauss Divergence theorem, the second term in the left hand side of the equation (1.3.1) becomes

$$\int_S \tau_{ji} \upsilon_j dS = \int_V \tau_{ji,i} dV \tag{1.3.2}$$

where $\tau_{ji,j}$ denotes the derivative of τ_{ji} with respect to x_j of the co-ordinate system.

Using (1.3.2) into equation (1.3.1), we have

$$\int_V \left(F_i + \tau_{ji,j} - \rho \frac{\partial^2 u_i}{\partial t^2} \right) dV = 0 \tag{1.3.3}$$

Since the region V of integration is arbitrary and the integrand of the equation (1.3.3) is continuous, it gives (see Rudin, 1976: pp. 138)

$$\tau_{ji,j} + F_i = \rho \frac{\partial^2 u_i}{\partial t^2} \qquad (i, j = x, y, z) \tag{1.3.4}$$

This is the equation of motion for a homogeneous elastic body, in the presence of body forces. For a homogeneous isotropic elastic medium, the coefficient c_{ijkl} can be expressed by only two elastic constants λ and μ called Lame’s parameters and the stress-strain relation

(Hooke’s law) is given by

$$\tau_{ij} = \lambda \delta_{ij} e_{kk} + 2\mu e_{ij} \tag{1.3.5}$$

where δ_{ij} is Kronecker delta.

Inserting (1.2.3) and (1.3.5) into (1.3.4), the equation of motion in the absence of body force in vector form is given by

$$(\lambda + \mu)\nabla\nabla.u + \mu\nabla^2 u = \rho \frac{\partial u^2}{\partial t^2} \tag{1.3.6}$$

Using the vector identity given below

$$\nabla^2 u = \nabla\nabla.u - \nabla \times (\nabla \times u) \tag{1.3.7}$$

into equation (1.3.6), we have

$$\left(\lambda+2\mu\right)\nabla\nabla.u+\mu\nabla\times\left(\nabla\times u\right)=\rho\frac{\partial u^{2}}{\partial t^{2}} \tag{1.3.8}$$

Now we shall show that this equation of motion leads to two wave equations, one representing the dilatational wave and another one representing the distortional wave. They are obtained respectively by taking divergence and curl of equation (1.3.8) separately. First, taking the divergence of equation (1.3.8) and using the vector identity $\nabla\times\nabla\phi=0$,we have

$$\nabla^{2}\left(\nabla.u\right)=\frac{1}{c_{L}^{2}}\frac{\partial^{2}\left(\nabla.u\right)}{\partial t^{2}},\quad c_{L}^{2}=\frac{\lambda+2\mu}{\rho} \tag{1.3.9}$$

Equation (1.3.9) represents the dilatational wave in the displacement form propagating with the phase speed c_{L} . Again, taking the curl of equation (1.3.8) and using the vector identity $\nabla\times\nabla\phi=0$,we have

$$-\mu\nabla\times\nabla\times\left(\nabla\times u\right)=\rho\frac{\partial\left(\nabla\times u\right)}{\partial t^{2}} \tag{1.3.10}$$

Using the vector identity in (1.3.7) and $\nabla.\left(\nabla\times u\right)=0$, the equation (1.3.10) can be reduced as

$$\nabla^{2}\left(\nabla\times u\right)=\frac{1}{c_{T}^{2}}\frac{\partial^{2}\left(\nabla\times u\right)}{\partial t^{2}}\quad ,c_{T}^{2}=\frac{\mu}{\rho} \tag{1.3.11}$$

Equation (1.3.11) represents the distortional wave in terms of displacement. Equations (1.3.9) and (1.3.11) can be represented in terms of potentials due to Helmholtz's theorem, which states that any vector field is uniquely separable into a divergence-free part (solenoidal part) and a curl-free part (irrotational part) as

$$u=\nabla\phi+\nabla\times\psi \qquad \nabla.\psi=0 \tag{1.3.12}$$

where the function ϕ is called the scalar potential of u and ψ is called its vector potential.

We have $\nabla.u=\nabla^{2}\phi$ and $\nabla\times u=\nabla\times\nabla\times\psi=\nabla\nabla.\psi-\nabla^{2}\psi=-\nabla^{2}\psi$, and since $\nabla.\psi=0$.

Substituting equation (1.3.12) into (1.3.9) and (1.3.11), we have

$$\nabla^2\phi = \frac{1}{c_L^2}\frac{\partial^2\phi}{\partial t^2} \tag{1.3.13}$$

$$\nabla^2\psi = \frac{1}{c_T^2}\frac{\partial^2\psi}{\partial t^2} \tag{1.3.14}$$

These equations represent the wave motions in terms of potentials for a homogeneous isotropic elastic body. Since $\nabla\times\nabla\phi = 0$, the motion is curl-free or irrotational and $\nabla .u \neq 0$, there are volume changes in the material, the equation (1.3.13) represents the dilatational motion which is also called compressional. As the velocity of this wave depend both on the bulk and shear moduli, it is associated with shearing as well as compression. Since $\nabla .(\nabla\times\psi) = 0$, the motion represented by equation (1.3.14) is purely shear without any volume change and hence, this equation represents the distortional motion.

1.4 Nature of elastic waves

The wave travelling through an elastic solid with finite velocity is known as elastic wave. As shown above that in a homogeneous isotropic elastic medium, there can travel two types of waves with different velocities. We shall now show that one is longitudinal in nature and other is transverse in nature. Consider a plane displacement wave propagating with phase speed c in the direction of unit propagation vector *p* in the form (D'Alembert solution)

$$u = d.f(r.p - ct), \tag{1.4.1}$$

where *r is* the position vector and *d* is the unit vector in the direction of motion. Substituting (1.4.1) into equation (1.3.6), we obtain

$$\left[\mu d + (\lambda+\mu)(p.d)\,p - \rho c^2 d\right] f''(r.p - ct) = 0$$

which is equivalent to

$$(\mu - \rho c^2)d + (\lambda+\mu)(p.d)\,p = 0 \tag{1.4.2}$$

Since the vectors p and d, are two different unit vectors, therefore, equation (1.4.2) can be satisfied only in two ways: either $d = \pm p$ or $d.p = 0$.

If $d = \pm p$, then we have $d.p = \pm 1$ and equation (1.4.2) gives

$$c = c_L = \sqrt{\frac{\lambda + 2\mu}{\rho}} \tag{1.4.3}$$

This shows that motion of the particles are along or parallel to the direction of propagation. Thus, the wave travelling with phase speed c_L and represented by the equation (1.3.13) is a longitudinal wave. If $d \neq p$, then both the terms in equation (1.4.2) must vanish separately. This would give $d.p = 0$, and

$$c = c_T = \sqrt{\frac{\mu}{\rho}} \tag{1.4.4}$$

This shows that the motion of particles is perpendicular to the direction of wave propagation. Thus, the wave travelling with phase speed c_T and represented by the equation (1.3.14) is a transverse wave.

1.5 Seismic waves

The study of elastic wave propagation through the earth medium is concerned with the subject of Seismology. There are reasonable grounds in the open literature that Earth medium is composed of different types of materials maybe isotropic or anisotropic, homogeneous or inhomogeneous and elastic or viscoelastic in nature. Although the materials in the earth medium are not perfectly elastic, but they can be best modelled with the theory of elasticity.

The elastic waves travelling through the earth medium are known as Seismic waves in Seismology, they may be caused due to sudden break of the rocks or explosion. These waves can be divided in two kinds: (i) Body waves and (ii) Surface waves. The waves which can travel deep into the earth medium are called body waves. These waves are known as P- waves (Primary waves) and S- waves (Secondary waves) in Seismology. The P- wave which is longitudinal in nature, associate with pulling or pushing of the particles in the same direction in which the energy is travelling. They are the fastest elastic waves and can propagate through solid rocks and fluids like water or the liquid layers of the earth.

The S- wave is transverse in nature, i.e., the particle motion is perpendicular to the direction of wave propagation. This wave can be polarised into vertical and horizontal directions, which are known as shear vertical (or SV-) wave and shear horizontal (or SH-) wave respectively. The velocity of the body wave depends not only on the elastic property of the medium but also on the density of the medium. The propagation of the SV-wave in the elastic medium through a surface boundary is often, associated with P- waves. If a train of P- /SV- wave comes across a surface boundary, the phenomena of reflections involve P- and SV- waves. But, if a train of SH- wave strikes at a boundary, only the reflection of SH- wave takes place. Those waves, which can propagate in the neighbourhood of free surface of a body or along the interfaces between half spaces or layer over a half space are known as surface waves and the strength, amplitude of these waves decrease exponentially with increasing in distance from the boundary surface. There are mainly three types of surface waves: Rayleigh waves, Stoneley waves and Love waves.

Rayleigh waves are the surface waves that travel along the stress free boundary of an elastic half space such that the disturbance is largely confined to the neighbourhood of the free boundary of the half space. It was introduced by Rayleigh (1887) and hence, called Rayleigh waves. These waves are the result of superposing longitudinal and transverse waves. Thus, Rayleigh wave is a not a new wave; it is a combination of vibrations due to longitudinal and transverse waves. During the propagation of Rayleigh waves, the surface particle motion is found to be counter clockwise elliptical (retrograde), which changes from retrograde at the surface to prograde (clockwise elliptical) at depth, passing through a node at which there is no horizontal motion. Stoneley waves are those surface waves, which can propagate along the interface between liquid-solid or solid half spaces. They are non dispersive in nature. These waves can travel along the solid interface when their elastic properties are nearly same.

Love (1911) found that certain type of shear wave can travel in a layer lying over an elastic half space, provided the phase speed of the wave lies between the phase speed of shear wave in the layer and that of in the half space. It is the fastest surface wave and moves the ground side to side in a horizontal plane parallel to the earth’s surface but at the right angle to the direction of propagation.

1.6 Applications of elastic waves

Elastic waves have numerous applications in various fields. The subject of elastic wave propagation and their phenomena of reflection and transmission from a boundary surface are of great practical importance in the field of Seismology, Earthquake engineering and Geophysics. They give valuable information about the interior of the material body. In geophysics, they are helpful not only in the exploration of internal structure of earth but also in the exploration of valuable materials buried inside the earth like minerals, metals, hydrocarbons and petroleum, etc. The method of wave propagation used in the exploration of oils, minerals, crystals and others is one of the best suitable methods because it is cheap and less time consuming.

The body waves (P- and S- waves) are used in earthquake engineering for predicting earthquake in the dynamic response of soils and man- made structures. For locating the epicentre of an earthquake, seismic signals are recorded by seismograms at different seismic stations. On each of these seismic stations, the S-P time intervals are measured. These S-P time intervals are used to determine the distance that the waves have travelled from the origin to that station. The actual location of the earthquake's epicentre will be on the perimeter of a circle drawn around the recording stations. The radius of this circle is the unknown epicentral distance. One S-P time measurement will produce one epicentral distance, the distance from which the waves will come out. Three stations are needed in order to "triangulate" the location and using the seismic signals, the epicentre of the earthquake is determined. This is one of the best methods to know the epicentre of the earthquake.

In geophysics, the wave propagation technique is used for the estimation of the earth's internal composition. The body waves have been sent from one station and then, the signals are received in other station and examined. These signals give indirect information about the internal structure of the earth. To know about the deep interior of earth materials, the seismic body waves are very useful. It is proved in the literature that seismic S-waves cannot travel through the interior of the core (the innermost part of the earth). This had led to the conclusion that the earth core is composed of material which is non viscous liquid like. Thus, the earth core is believed to be in liquid form at high temperature (which makes the liquid of the core almost a non-viscous) and harder than the solid. Some notable books containing the literature of wave propagation are Ewing et al. (1957), Brekhovskikh (1960), Achenbach (1976), Graff (1991), Udias (1999), Aki and Richards (2002).

1.7 Anisotropic elastic solids

An elastic body which contains an internal structure (such as crystals) so that the elastic properties of the material are not same in all directions at a point of the body, is said to be anisotropic or aeolotropic elastic body. The variation of properties for purely elastic solids, containing crystals, can be fully described by a fourth order tensor c_{ijkl} of material constants. These materials constants are 81 in number. Since $\tau_{ij} = \tau_{ji}, c_{ijkl}e_{kl} = c_{jikl}e_{kl}$ and equation (1.2.4) reduces to

$$\tau_{ij} = c_{ijkl}e_{kl}; \quad c_{ijkl} = c_{jikl} \tag{1.7.1}$$

Now, c_{ijkl} can be written as the sum of symmetric and skew symmetric tensors, i.e., $c_{ijkl} = c'_{ijkl} + c''_{ijkl}$, where c'_{ijkl} and c''_{ijkl} are respectively the symmetric and skew symmetric with respect to suffixes *k* and *l*, defined as

$$c'_{ijkl} = \frac{1}{2}\left(c_{ijkl} + c_{ijlk}\right) = c'_{ijlk} \text{ and } c''_{ijkl} = \frac{1}{2}\left(c_{ijkl} - c_{ijlk}\right) = c''_{ijlk}$$

Equation (1.7.1) can be written in the form as

$$\tau_{ij} = c'_{ijkl}e_{kl} + c''_{ijkl}e_{kl} \tag{1.7.2}$$

But, $c''_{ijkl}e_{kl} = c''_{ijlk}e_{lk}$ (interchanging the dummy indices *k* and *l*)

$= -c_{ijkl}e_{lk}$ (since $c''_{ijkl} = -c''_{ijlk}$)

$= -c''_{ijkl}e_{kl}$ (since $e_{kl} = e_{lk}$)

Hence, $c''_{ijkl}e_{kl} = 0$

Equation (1.7.2) reduces to

$$\tau_{ij} = c'_{ijkl} e_{kl}; \quad c'_{ijkl} e_{kl} = c'_{ijkl} e_{lk} \tag{1.7.3}$$

Combining equations (1.7.1) and (1.7.3), we have

$$\tau_{\mathrm{ij}} = c_{ijkl} e_{kl}; \tag{1.7.4}$$

where $c_{ijkl} = c_{jikl} = c_{ijlk} = c_{jilk}$ because of the symmetry of τ_{ij} and e_{kl}. Hence, c_{ijkl} has 36 distinct elastic coefficients instead of 81. Introducing the notations

$$\tau_{11} = \tau_1,\ \tau_{22} = \tau_2,\ \tau_{33} = \tau_3,\ \tau_{23} = \tau_4,\ \tau_{31} = \tau_5,\ \tau_{12} = \tau_6,$$

$$e_{11} = e_1,\ e_{22} = e_2,\ e_{33} = e_3,\ 2e_{23} = e_4,\ 2e_{31} = e_5,\ 2e_{12} = e_6,$$

$$c_{1111} = c_{11},\ c_{1122} = c_{12},\ c_{1133} = c_{13},\ c_{1123} = c_{14},\ c_{1131} = c_{15},\ c_{1112} = c_{16}, \text{etc.},$$

the compact form of equation (1.7.4) can be written as

$$\tau_i = c_{ij} e_j, \qquad (i, j = 1, 2....,6) \tag{1.7.5}$$

The number of elastic constants can be reduced further to twenty one, to ascertain the existence of strain energy density function W. This function is defined by

$$W = \frac{1}{2} c_{ij} e_i e_j, \tag{1.7.6}$$

such that

$$\frac{\partial W}{\partial e_i} = \tau_i \tag{1.7.7}$$

One can see that

$$\frac{\partial W}{\partial e_k} = \frac{1}{2} c_{ki} e_i + \frac{1}{2} c_{ik} e_i \tag{1.7.8}$$

Now, from equations (1.7.5), we have $\tau_k = c_{ki}e_i$, and hence, from (1.7.7) $\frac{\partial W}{\partial e_k} = c_{ki}e_i$.

Owing to (1.7.8), this yields $c_{ki} = c_{ik}$ or $c_{ij} = c_{ji}$, that is c_{ij} is symmetric. This reduces the number of elastic constants to twenty one. These twenty one independent elastic constants in the stress-strain relation (Hooke's law) corresponds to general anisotropic medium. As the stress components, τ_i and strain components, e_i depend on the choice of the co-ordinate systems, the elastic coefficients, c_{ij}, in general, depend on the reference frame. When the elastic coefficients, c_{ij} remain invariant under a given transformation of co-ordinates for a medium then it is called elastically symmetric. The elastic symmetry imposes restrictions on these coefficients for a particular type of anisotropic symmetry.

A material body which is elastically symmetric with respect to one plane has 13 elastic coefficients to represent the stress-strain relation and such a medium is known as anisotropic medium with monoclinic symmetry or monoclinic medium. Thus, in this type of anisotropy symmetry, the elastic coefficients reduces to 13 from 21. Lithium tantalate, lithium neobate, etc. show the monoclinic elastic symmetry.

A material body, which has elastic symmetry with respect to two mutually perpendicular planes, has 9 elastic coefficients to represent the stress-strain relation and such a medium is known as anisotropic medium with orthorhombic symmetry or orthotropic medium. Clearly, if there are two orthogonal planes of elastic symmetry in a material, then the third orthogonal plane is automatically a plane of elastic symmetry. Thus in such a medium, there are three mutually orthogonal planes of elastic symmetry. Wood is an example of such an orthotropic anisotropic medium.

If a material body exhibits the same elastic properties with respect to each of three orthogonal planes of symmetry, then the strain-energy density function would remain invariant when the co-ordinates axes are permuted cyclically. In such a medium, there are only 3 elastic coefficients to represent the elastic properties of the medium and such a medium is known as anisotropic medium with cubic symmetry or cubic medium. If a material body shows the elastic symmetry with respect to an axis of a cartesian co-ordinate system, the strain energy function remains unaltered, when the system is rotated about the axis through any angle. Thus, when a material body has an axis of elastic symmetry, it has two planes of elastic symmetry at right angle to each other. Such an elastic symmetry is represented by 5 elastic coefficients and is known as transversely isotropic material. In this symmetry, the three planes of three dimensional system have one plane symmetry which is anisotropic in nature and the other two planes are isotropic in nature. A fibre-reinforced medium, a perfectly conducting self-reinforced elastic medium and a prestressed elastic medium are some examples of anisotropic medium.

A fibre-reinforced composite material with the reinforcement distributed continuously in concentric circles is a material of locally transversely isotropic with the circumferential direction as the preferred direction coinciding with fibre directions. The fibres may be continuous, in which each fibre extends through a body from one boundary to another, or discontinuous, but in the discontinuous case, the length of the fibre must be large as compared to its diameter. Such a material medium is known as fibre-reinforced medium. In such composites, the fibres are usually arranged in parallel straight lines. The idea of continuous theory in the fibre reinforced material is developed by Adkins and Rivlin (1955), Adkins (1956), Spencer (1972, 1974) and Maugin (1981) on the theory of large deformations of elastic materials reinforced by inextensible cords. An example of circumferential reinforcement, for which the fibres are arranged in concentric circles, giving strength and stiffness in the tangential direction. This produces a material, which is potentially useful for supporting a radially applied pressure or for reinforcement of a hole in a plate. Such a composite material is then locally transversely isotropic, with the direction of the axis of transverse isotropy now which is not constant, but everywhere directed along the tangents to circles in which the fibres lie. Thus, elastic fibre–reinforced composite materials, typical carbon fibre-epoxy resin composites are not just anisotropic but also strongly an isotropic in the sense that modulus for extension in the fibre direction greatly exceeds the moduli for extension in the transverse direction and for shear in the fibre or transverse direction. Fibres have an excellent potential to improve the mechanical properties of rapid-setting materials, and could be used effectively to improve the performance of repairs. The behaviour of fibre-reinforced rapid setting materials is similar to that of Portland cement fibre-reinforced concrete. A fibre medium in which the body force play an important role in a uniformed motion of the particles is known as self-reinforced fibre medium. This material is reinforced by strong fibres. In a perfectly conducting fibre-reinforced elastic medium, the motion of the particles create electric current and hence, a Lorentz body force is generated by the movement of the particles and acts as a body force in the particles motion. This type of medium is a good example of self-reinforced fibre medium.

A state of initial stress in a deformable medium induces mechanical properties which depend mainly on the magnitude of the stress and are quite distinct from those associated with the rigidity of the material itself. Such a medium, which changes the state of the medium due to some physical or mechanical loads, is defined as prestressed medium. The presence of an initial stress may increase or decrease the over all rigidity of an elastic structure. In a rod under axial compression, the initial stress produces a decrease in lateral stiffness. For increasing values of the compression, this decrease will overcome the natural bending rigidity of the rod, producing an instability known as buckling. On the other hand, a cable hanging under its own weight is under an initial tension which increases its rigidity. In geophysical applications, the state of the initial stress in the earth is a result of a slow but highly irreversible process of a viscous or plastic nature. Rapid deformation may be approximate to elastic anisotropy in nature. In the rapid elastic deformations in earth, the initial stress is associated with a slow process of creep due to viscous and plastic deformation.

The wave propagation in crystalline media plays a very interesting role in geophysics and also in ultrasonic and signal processing. Monoclinic medium is an example of such medium. The propagation of seismic waves in various media with different geometries were studied by many authors. Some of the recent notable works were done by Chattopadhyay et al (2009a, 2009b, 2010a, 2010b, 2010c, 2011a, 2011b, 2011c, 2011d, 2011e, 2011f, 2012a, 2012b, 2012c, 2013).

CHAPTER-2

PROPAGATION OF LOVE WAVE IN AN ISOTROPIC LAYER LYING OVER AN ISOTROPIC HALF-SPACE

Love wave is a kind of surface wave which is horizontally polarized shear wave. For the propagation of Love wave, a superficial layer is needed. The particle motion in these waves is transverse and is confined to the horizontal plane. The simplest model in which Love wave can propagate consists of a homogeneous isotropic layer on a homogeneous isotropic half space. Love waves are dispersive, i.e., their velocities are dependent on frequency.

2.1 Formulation of the problem

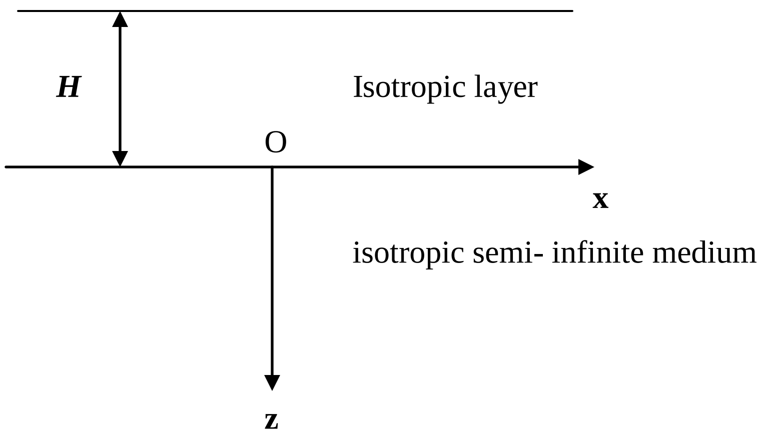

Figure 2.1: Geometry of the problem

Let the density and rigidity of the layer and isotropic semi-infinite medium are μ_1, ρ_1 and μ_1, ρ_1 respectively. The thickness of the layer is *H*. Let the consider x-axis along the direction of the wave propagation and *z*-axis is taken vertically downwards.

Let *u*, *v*, *w* are displacements along *x*, *y*, *z* axis respectively.

For love wave propagation, we have

$$u = w = 0 \text{ and } v = v(x, z, t). \tag{2.1.1}$$

At first we look for the equation governing the propagation of Love wave in homogeneous isotropic elastic medium. Since the equations of motion for a homogeneous isotropic elastic solid in the absence of body forces are

$$\tau_{ij,j} = \rho u_i \qquad (i, j = 1, 2, 3)$$

In component form, above equation can be written as

$$\frac{\partial}{\partial x}\tau_{xx}+\frac{\partial}{\partial y}\tau_{yx}+\frac{\partial}{\partial z}\tau_{zx}=\rho\frac{\partial^2 u}{\partial t^2}, \tag{2.1.2}$$

$$\frac{\partial}{\partial x}\tau_{xy}+\frac{\partial}{\partial y}\tau_{yy}+\frac{\partial}{\partial z}\tau_{zy}=\rho\frac{\partial^2 v}{\partial t^2} \tag{2.1.3}$$

and

$$\frac{\partial}{\partial x}\tau_{xz}+\frac{\partial}{\partial y}\tau_{yz}+\frac{\partial}{\partial z}\tau_{zz}=\rho\frac{\partial^2 w}{\partial t^2}. \tag{2.1.4}$$

Hooke's law in isotropic medium gives

$$\tau_{ij}=\lambda\Delta\delta_{ij}+2\mu\varepsilon_{ij}, \tag{2.1.5}$$

where λ,μ are Lame's constant and Δ is cubical dilation.

Since,

$$\varepsilon_{ij}=\frac{1}{2}\left[\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right] \tag{2.1.6}$$

Using equations (2.1.1), (2.1.2), (2.1.3), (2.1.4), (2.1.5) and (2.1.6), we get the only non vanishing equation as

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial z^2}=\frac{1}{\beta^2}\frac{\partial^2 v}{\partial t^2}, \tag{2.1.7}$$

where $\beta^2=\frac{\mu}{\rho}$ is the shear wave speed.

If (u_1,v_1,w_1) and (u_2,v_2,w_2) are the displacement components for the layer and the half space, then the equations of motion for propagation of Love wave in layer and half space are given by

$$\frac{\partial^2 v_1}{\partial x^2}+\frac{\partial^2 v_1}{\partial z^2}=\frac{1}{\beta_1^2}\frac{\partial^2 v_1}{\partial t^2}, \tag{2.1.8}$$

$$\frac{\partial^2 v_2}{\partial x^2}+\frac{\partial^2 v_2}{\partial z^2}=\frac{1}{\beta_2^2}\frac{\partial^2 v_2}{\partial t^2}, \tag{2.1.9}$$

where $\beta_i^2=\frac{\mu_i}{\rho_i} \quad (i=1,2)$.

The boundary conditions are as follows:

(i) The upper surface is stress free,

$$\tau_{yz} = \mu_1 \frac{\partial v_1}{\partial z} = 0 \qquad \text{at } z = -H, \tag{2.1.10}$$

(ii) The displacements are continuous at common interface,

$$v_1 = v_2 \qquad \text{at } z = 0, \tag{2.1.11}$$

(iii) The stress is continuous at common interface,

$$\mu_1 \frac{\partial v_1}{\partial z} = \mu_2 \frac{\partial v_2}{\partial z} \qquad \text{at } z = 0. \tag{2.1.12}$$

2.2 Solution of the problem

We can assume the solution of (2.1.8) and (2.1.9) as

$$v_1(x,z,t) = V_1(z)e^{ik(x-ct)} \tag{2.2.1}$$

$$v_2(x,z,t) = V_2(z)e^{ik(x-ct)} \tag{2.2.2}$$

where k is the wave number and c is the velocity of love wave.

In view of (2.2.1), equation (2.1.8) gives,

$$\frac{d^2V_1}{dz^2} + s_1^2 V_1 = 0, \tag{2.2.3}$$

where $s_1^2 = k^2\left(\frac{c^2}{\beta_1^2} - 1\right)$

Solution of equation (2.2.3) as

$$V_1 = A\cos s_1 z + B\sin s_1 z, \tag{2.2.4}$$

where A and B are arbitrary constants.

In view of (2.2.2), equation (2.1.9) gives,

$$\frac{d^2V_2}{dz^2} + s_2^2 V_2 = 0 \tag{2.2.5}$$

where $s_2^2 = k^2\left(1 - \frac{c^2}{\beta_2^2}\right)$.

Keeping in mind that love wave dies out with increase in depth, so we can write the solution of the equation (2.2.5) as

$$V_2 = Ce^{-s_2 z}, \tag{2.2.6}$$

where C is an arbitrary constant.

With the help of above obtained values boundary condition (2.1.10) gives

$$A\sin s_1 H + B\cos s_1 H = 0. \tag{2.2.7}$$

Using boundary condition (2.1.11), we get

$$A = C \ . \tag{2.2.8}$$

Using boundary condition (2.1.12), we get

$$\mu_1 s_1 B = -\mu_2 s_2 C. \tag{2.2.9}$$

Eliminating A, B and C from (2.2.7), (2.2.8) and (2.2.9), we get

$$\tan\left[kH\sqrt{\frac{c^2}{\beta_1^2}-1}\right] = \frac{\mu_2\sqrt{1-\frac{c^2}{\beta_2^2}}}{\mu_1\sqrt{\frac{c^2}{\beta_1^2}-1}}. \tag{2.2.10}$$

This is the dispersion relation for the propagation of Love wave in an isotropic layer lying over a homogeneous isotropic half space. Which also known as Love wave equation.

Validity $\frac{c^2}{\beta_2^2} < 1$ and $\frac{c^2}{\beta_1^2} > 1$

Thus $\beta_1 < c < \beta_2$

Hence, for propagation of Love wave, velocity of wave must be greater than the phase velocity of layer and less than phase velocity of lower medium.

CHAPTER-3

PROPAGATION OF SH WAVES IN A REGULAR MONOCLINIC LAYER LYING OVER AN ISOTROPIC HALF-SPACE

In the present problem we have considered the propagation of SH wave in a regular monoclinic layer over an isotropic semi-infinite medium.

3.1 Formulation of the problem

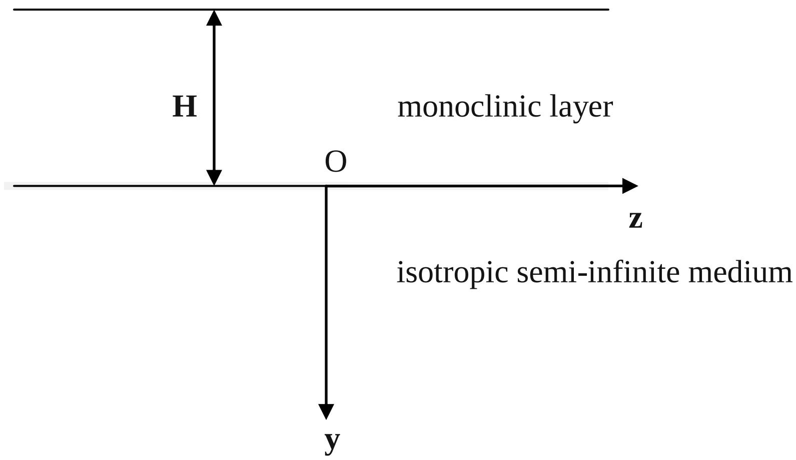

Figure 3.1: Geometry of the problem.

Let us consider $\rho_{i,} u_i$, (i = 1,2) as the densities and displacements in monoclinic layer (of thickness H) and semi-infinite isotropic medium respectively.

Assuming z-axis along the interface of the layer and semi-infinite medium and y-axis is taken vertically downwards.

First we will deduce the equation of motion for propagation of SH wave in monoclinic layer.

For monoclinic layer, we have following strain-displacement relation as

$$S_1 = \frac{\partial u}{\partial x}, S_2 = \frac{\partial v}{\partial y}, S_3 = \frac{\partial w}{\partial z}, S_4 = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}, S_5 = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, S_6 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}, \quad (3.1.1)$$

where u, v, w are displacements along x, y, z axis respectively and S_i (i =1,2,....6) are strain components.

The stress-strain relations for a rotated y-cut quartz plate, which exhibits monoclinic symmetry with x being the diagonal axes are

$$
\begin{aligned}
T_1 &= C_{11}S_1 + C_{12}S_2 + C_{13}S_3 + C_{14}S_4, \\
T_2 &= C_{12}S_1 + C_{22}S_2 + C_{23}S_3 + C_{24}S_4, \\
T_3 &= C_{13}S_1 + C_{23}S_2 + C_{33}S_3 + C_{34}S_4, \\
T_4 &= C_{14}S_1 + C_{24}S_2 + C_{34}S_3 + C_{44}S_4, \\
T_5 &= C_{55}S_5 + C_{56}S_6, \\
T_6 &= C_{56}S_5 + C_{66}S_6,
\end{aligned}
\tag{3.1.2}
$$

where

$T_i\,(i=1,2,...,6)$ are stress components and $C_{ij} = C_{ji}\ (i, j = 1, 2, ..., 6)$ are medium constants.

The equations of motion in the absence of body forces are

$$
\frac{\partial T_1}{\partial x} + \frac{\partial T_6}{\partial y} + \frac{\partial T_5}{\partial z} = \rho \frac{\partial^2 u}{\partial t^2},
$$

$$
\frac{\partial T_6}{\partial x} + \frac{\partial T_2}{\partial y} + \frac{\partial T_4}{\partial z} = \rho \frac{\partial^2 v}{\partial t^2}, \tag{3.1.3}
$$

And

$$
\frac{\partial T_5}{\partial x} + \frac{\partial T_4}{\partial y} + \frac{\partial T_3}{\partial z} = \rho \frac{\partial^2 w}{\partial t^2}.
$$

For SH waves propagating in the *z*-direction with the displacement only in the *x*-direction, we have

$$
u = u(y, z, t),\ v = 0, w = 0. \tag{3.1.4}
$$

Using equations (3.1.1) and (3.1.4), we get

$$
S_1 = 0, S_2 = 0, S_3 = 0, S_4 = 0, S_5 = \frac{\partial u}{\partial z}, S_6 = \frac{\partial u}{\partial y}. \tag{3.1.5}
$$

With the help of equation (3.1.5) equation (3.1.2) gives

$$
T_1 = T_2 = T_3 = T_4 = 0,\ T_5 = C_{55}\frac{\partial u}{\partial z} + C_{56}\frac{\partial u}{\partial y} \text{ and } T_6 = C_{56}\frac{\partial u}{\partial z} + C_{66}\frac{\partial u}{\partial y}. \tag{3.1.6}
$$

Using equations (3.1.6) and (3.1.4) in equation (3.1.3), we obtain the non-vanishing equation of motion as

$$
\mathrm{C}_{66}\frac{\partial^2 u_1}{\partial y^2} + 2\mathrm{C}_{56}\frac{\partial^2 u_1}{\partial y \partial z} + \mathrm{C}_{55}\frac{\partial^2 u_1}{\partial z^2} = \rho_1 \frac{\partial^2 u_1}{\partial t^2}. \tag{3.1.7}
$$

In view of equations (3.1.4), (2.1.5), (2.1.6) equation of motion (2.1.2) to (2.1.4) gives the non-vanishing equations of motion for propagation of SH wave in lower semi-infinite medium as

$$\frac{\partial^2 u_2}{\partial y^2}+\frac{\partial^2 u_2}{\partial z^2}=\frac{1}{\beta_2{}^2}\frac{\partial^2 u_2}{\partial t^2}, \tag{3.1.8}$$

where

$\beta_2=\sqrt{\frac{\mu_2}{\rho_2}}$ is shear wave speed lower semi-infinite medium.

The boundary conditions are as follows:

(i) The upper surface is stress free,

$$C_{56}\frac{\partial u_1}{\partial z}+C_{66}\frac{\partial u_1}{\partial y}=0 \qquad \text{at } y=-H, \tag{3.1.9}$$

(ii) The stress is continuous at common interface,

$$C_{56}\frac{\partial u_1}{\partial z}+C_{66}\frac{\partial u_1}{\partial y}=\mu_2\frac{\partial u_2}{\partial y} \qquad \text{at } y=0, \tag{3.1.10}$$

(iii) The displacements are continuous at common interface,

$$u_1=u_2 \qquad \text{at } y=0. \tag{3.1.11}$$

3.2 Solution of the problem

We can assume that the solution of (3.1.7) and (3.1.8) as

$$u_1(y,z,t)=U_1(y)e^{ik(z-ct)} \tag{3.2.1}$$

$$u_1(y,z,t)=U_2(y)e^{ik(z-ct)} \tag{3.2.2}$$

where k is wave number and c is velocity of SH wave.

Using equation (3.2.1) in (3.1.7), we get

$$U_1=(A\cos Ty+B\sin Ty)e^{-Py}, \tag{3.2.3}$$

where

$P=ik\frac{C_{56}}{C_{66}}$, $T=k\sqrt{\left(\frac{C_{56}}{C_{66}}\right)^2-\frac{C_{55}}{C_{66}}+\frac{c^2}{\beta_1^2}}$, $\beta_1^2=\frac{C_{66}}{\rho_1}$, A and B are arbitrary constants

Using equation (3.2.2) in (3.1.8), we get

$$U_2=De^{-s_2 y}, \tag{3.2.4}$$

where

$s_2^2 = k^2\left[1-\frac{c^2}{\beta_2^2}\right]$ and D is an arbitrary constant.

With the help of obtained values boundary condition (3.1.9) gives

$$\tan TH = \frac{C_{56}ik + C_{66}\left(\frac{B}{A}T - P\right)}{C_{56}\frac{B}{A}ik - C_{66}\left(T + P\frac{B}{A}\right)} \tag{3.2.5}$$

Using boundary condition (3.1.10), we get

$$C_{56}Aik + C_{66}\left(BT - AP\right) = -\mu_2 D s_2 \tag{3.2.6}$$

Using boundary condition (3.1.11), we get

$$A = D \tag{3.2.7}$$

Eliminating A, B and D from (3.2.5), (3.2.6) and (3.2.7), we get

$$\tan\left\{Hk\sqrt{\left(\frac{C_{56}}{C_{66}}\right)^2 - \frac{C_{55}}{C_{66}} + \frac{c^2}{\beta_1^2}}\right\} = \frac{\mu_2 s_2}{C_{66}k\sqrt{\left(\frac{C_{56}}{C_{66}}\right)^2 - \frac{C_{55}}{C_{66}} + \frac{c^2}{\beta_1^2}}} \tag{3.2.8}$$

which is the dispersion relation for propagation of SH waves in a monoclinic layer lying over an isotropic semi-infinite medium.

3.3 Particular case

When $C_{56} = 0, C_{55} = C_{66} = \mu_1$ then dispersion relation (3.2.8) becomes

$$\tan\left\{kH\sqrt{\frac{c^2}{\beta_1^2} - 1}\right\} = \frac{\mu_2 s_2}{\mu_1 s_1},$$

where $s_1^2 = k^2\left[\frac{c^2}{\beta_2^2} - 1\right]$

which is classical Love wave equation.

3.4 Numerical example and discussion

For the graphical representation of phase velocity of SH wave propagating in a monoclinic layer lying over an isotropic half space, we take the following data :

(i) For monoclinic layer (Tiersten, 1969)

$C_{55} = 0.94\times10^{11}\ N/m^2,\ C_{56} = -0.11\times10^{11}\ N/m^2,$

$C_{66} = 0.93\times10^{11}\ N/m^2,\ \ \rho_1 = 7,450\ Kg/m^3$

(ii) For isotropic homogeneous semi-infinite medium (Gubbins, 1990)

$\mu_2 = 6.54\times10^{10}\ N/m^2,\ \rho_2 = 3,409\ Kg/m^3$

Figure 3.2 represent the variation in dimensionless phase velocity $\left(c/\beta_1 \right)$ against dimensionless wave number (kH) for propagation of SH waves in a regular monoclinic layer lying over an isotropic half-space. Graph reveals that, phase velocity decreases with increase in wave number.

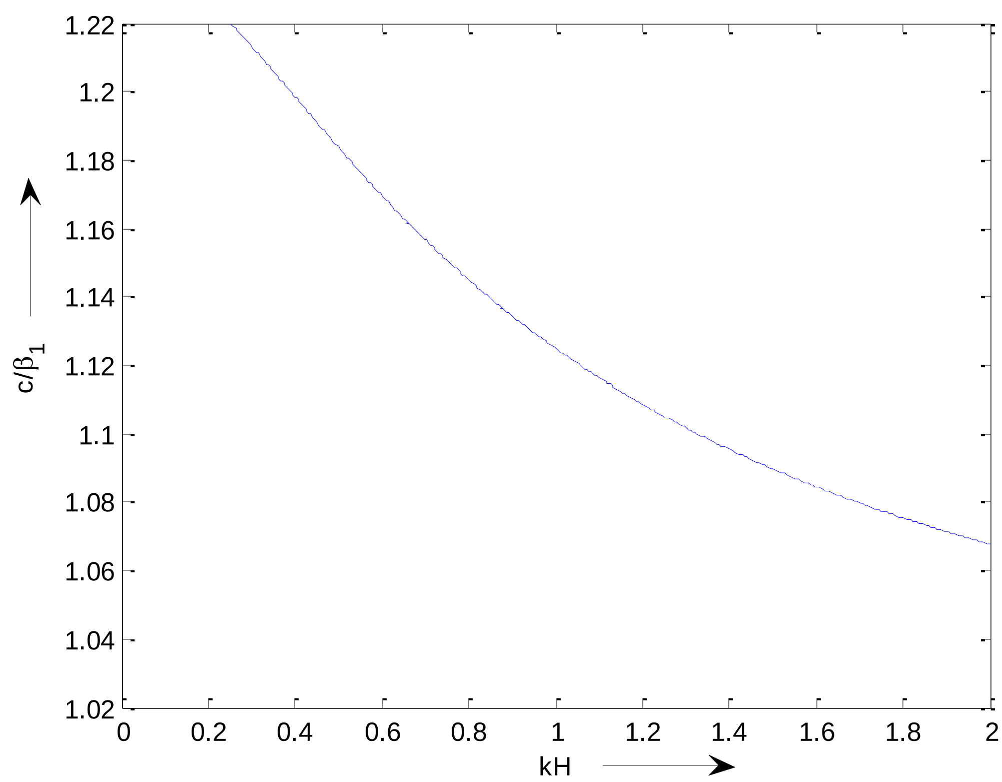

Figure 3.2: Variation of the dimensionless phase velocity against dimensionless wave number

CHAPTER-4

PROPAGATION OF SH WAVE IN DOUBLE ISOTROPIC LAYERS LYING OVER AN ISOTROPIC HALF-SPACE

In the present problem we have considered the propagation of SH wave in two different isotropic layers lying over homogeneous isotropic half-space.

4.1 Formulation of the problem

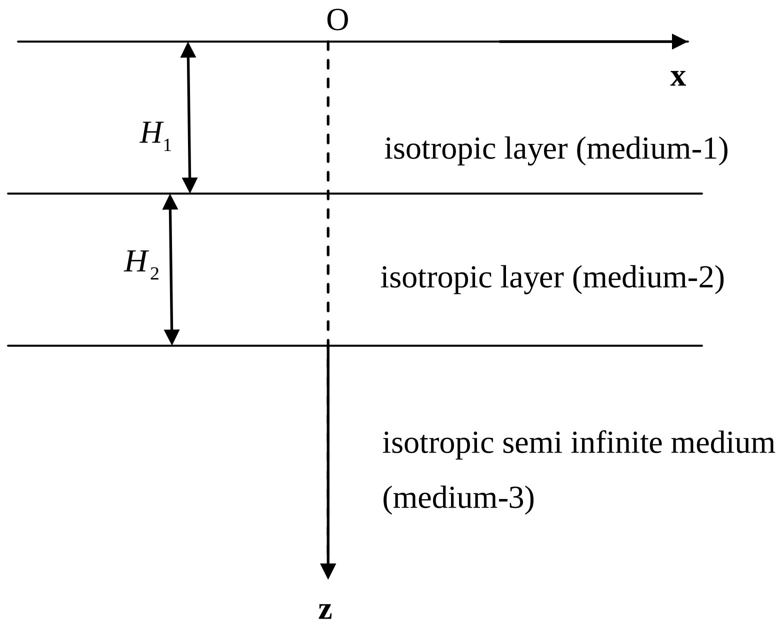

Figure 4.1: Geometry of the problem

Let the density and rigidity of medium-1, medium-2 and medium-3 are $\rho_1, \mu_1;\ \rho_2, \mu_2$ and ρ_3, μ_3 respectively. The thickness of the first layer (medium-1) and second layer (medium-2) are H_1 and H_2 respectively.

Let us consider *x*-axis along the direction of the wave propagation and *z*-axis is taken vertically downwards. Let *u*, *v*, *w* are displacements along *x*, *y*, *z* axis respectively.

For SH wave propagation we have,

$$u = w = 0 \text{ and } v = v(x, z, t). \tag{4.1.1}$$

If $(u_1, v_1, w_1), (u_2, v_2, w_2)$ and (u_3, v_3, w_3) are the displacements for the medium-1, medium-2 and medium-3 respectively, then equations of motion for propagation of SH wave in medium 1, 2 and 3 are respectively given by

$$\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial z^2} = \frac{1}{\beta_1^2}\frac{\partial^2 v_1}{\partial t^2}, \tag{4.1.2}$$

$$\frac{\partial^2 v_2}{\partial x^2}+\frac{\partial^2 v_2}{\partial z^2}=\frac{1}{\beta_2^2}\frac{\partial^2 v_2}{\partial t^2} \tag{4.1.3}$$

and

$$\frac{\partial^2 v_3}{\partial x^2}+\frac{\partial^2 v_3}{\partial z^2}=\frac{1}{\beta_3^2}\frac{\partial^2 v_3}{\partial t^2}, \tag{4.1.4}$$

where $\beta_i^2=\frac{\mu_i}{\rho_i}(i=1,2,3)$.

The boundary conditions are given as follows:

(i) $\mu_1\frac{\partial v_1}{\partial z}=0$ at z=0, (4.1.5)

(ii) $\mu_1\frac{\partial v_1}{\partial z}=\mu_2\frac{\partial v_2}{\partial z}$ at $z=H_1$, (4.1.6)

(iii) $v_1=v_2$ at $z=H_1$, (4.1.7)

(iv) $\mu_2\frac{\partial v_2}{\partial z}=\mu_3\frac{\partial v_3}{\partial z}$ at $z=H_1+H_2$, (4.1.8)

(v) $v_2=v_3$ at $z=H_1+H_2$. (4.1.9)

4.2 Solution of the problem

We can assume that the solution of equations (4.1.2), (4.1.3) and (4.1.4) as

$$v_1(x,z,t)=V_1(z)e^{ik(x-ct)} \tag{4.2.1}$$

$$v_2(x,z,t)=V_2(z)e^{ik(x-ct)} \tag{4.2.2}$$

$$v_3(x,z,t)=V_3(z)e^{ik(x-ct)} \tag{4.2.3}$$

where k is wave number and c is velocity of SH wave.

Using equation (4.2.1) in (4.1.2), we get

$$V_1=A\cos s_1 z+B\sin s_1 z, \tag{4.2.4}$$

where

$s_1^2=k^2\left[\frac{c^2}{\beta_1^2}-1\right]$, A and B are arbitrary constants.

Using equation (4.2.2) in (4.1.3), we get

$$V_2 = C\cos s_2 z + D\sin s_2 z\,, \tag{4.2.5}$$

where

$s_2^2 = k^2\left[\dfrac{c^2}{\beta_2^2} - 1\right]$, C and D are arbitrary constants.

Using equation (4.2.3) in (4.1.4), we get

$$V_3 = Ee^{-s_3 z}\,, \tag{4.2.6}$$

where

$s_3^2 = k^2\left[1 - \dfrac{c^2}{\beta_3^2}\right]$ and E is an arbitrary constant

Now, with the help of above obtained values boundary condition (4.1.5) gives

$$B = 0 \tag{4.2.7}$$

Using boundary condition (4.1.6), we get

$$\mu_1 s_1\left(-A\sin s_1 H_1 + B\cos s_1 H_1\right) = \mu_2 s_2\left(-C\sin s_2 H_1 + D\cos s_2 H_1\right) \tag{4.2.8}$$

Using boundary condition (4.1.7), we get

$$A\cos s_1 H_1 + B\sin s_1 H_1 = C\cos s_2 H_1 + D\sin s_2 H_1 \tag{4.2.9}$$

Using boundary condition (4.1.8), we get

$$\mu_2 s_2\left(-C\sin s_2\left(H_1 + H_2\right) + D\cos s_2\left(H_1 + H_2\right)\right) = -\mu_3 s_3 Ee^{-s_3\left(H_1 + H_2\right)} \tag{4.2.10}$$

Using boundary condition (4.1.9), we get

$$C\cos s_2\left(H_1 + H_2\right) + D\sin s_2\left(H_1 + H_2\right) = Ee^{-s_3\left(H_1 + H_2\right)} \tag{4.2.11}$$

Eliminating A, B, C, D and E from equations (4.2.7), (4.2.8), (4.2.9), (4.2.10) and (4.2.11), we get

$$\tan s_2 H_2 = \frac{\mu_2 s_2\left(\mu_3 s_3 - \mu_1 s_1 \tan s_1 H_1\right)}{\left(\mu_2 s_2\right)^2 + \mu_1 s_1 \mu_3 s_3 \tan s_1 H_1}\,. \tag{4.2.12}$$

which is the dispersion relation for the propagation of SH waves in two isotropic layers lying over isotropic half space.

3.3 Numerical example and discussion

For the graphical representation of phase velocity of SH wave propagating in two different isotropic layers lying over homogeneous isotropic half space, we take the following data :

(i) For isotropic layer (medium-1) (Gubbins, 1990)

$$\mu_1 = 6.34\times10^{10}\ {}^{N}\!/_{m^2},\ \ \rho_1 = 3{,}364\ {}^{Kg}\!/_{m^3}$$

(ii) For isotropic layer (medium-2) (Gubbins, 1990)

$$\mu_2 = 7.84\times10^{10}\ {}^{N}\!/_{m^2},\ \ \rho_2 = 3{,}535\ {}^{Kg}\!/_{m^3}$$

(iii) For isotropic half space (medium-3) (Gubbins, 1990)

$$\mu_3 = 7.1\times10^{10}\ {}^{N}\!/_{m^2},\ \ \rho_3 = 3{,}321\ {}^{Kg}\!/_{m^3}$$

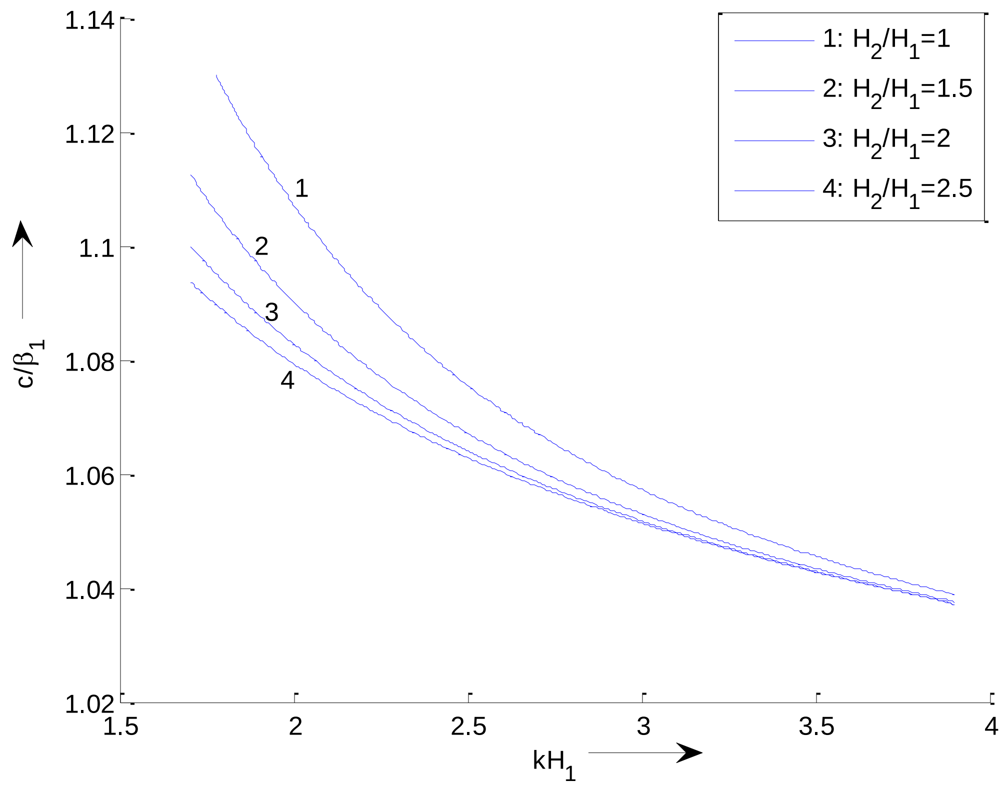

Figure 4.2: Variation of the dimensionless phase velocity against dimensionless wave number

Figure 4.2 represent the variation in dimensionless phase velocity $\left(c/\beta_1 \right)$ against dimensionless wave number $\left(kH_1 \right)$ for different values of thickness ratio of the layers for the propagation of SH waves in double isotropic homogeneous layers lying over isotropic homogeneous half space. It is evident for the graph that phase velocity decreases with increase in the wave number. It is also observe that phase velocity decreases with increase thickness ratio of the layers.

REFERENCES

- Achenbach, J.D., 1976. *Wave Propagation in Elastic Solids*. North-Holland Publishing Company, Amsterdam, New York, Oxford.
- Aki, K. and Richards, P.G., 2002. *Quantitative Seismology: Theory and Methods. (2nd ed.),* University Sceince Books, Sausalito, California.
- Biot, M.A., 1965. *Mechanics of Incremental Deformations*. Wiley, New York.
- Brekhovskikh, L.M., 1960. *Waves in Layered Media*., Academic Press, New York.
- Chattopadhyay, A., Gupta, S. and Singh, A.K. and Sahu, S.A., 2011c. G-type seismic wave in magnetoelastic monoclinic layer, *Applied Mathematics*, 2(2), 145–154.
- Chattopadhyay, A., Gupta, S. and Singh, A.K. and Sahu, S.A., 2011d. Effect of point source, self-reinforcement and heterogeneity on the propagation of magnetoelastic shear wave, *Applied Mathematics,* 2(3), 271-282.
- Chattopadhyay, A., Gupta, S. and Singh, A.K. and Sahu, S.A., 2011f. G-type seismic waves in low velocity magnetoelastic monoclinic layer, *Proceedings of International Seminar on Recent Advances in Geoscience*, January 11-13, 2011, organized by Indian School of Mines, Dhanbad, 256-258.
- Chattopadhyay, A., Gupta, S. and Singh, A.K., 2009b. Influence of gravity on the propagation of SH waves in a magnetoelastic self-reinforced media, *International Journal of Mechanics and Solids*, 4(1), 71-83.
- Chattopadhyay, A., Gupta, S. and Singh, A.K., 2010c. The dispersion of shear wave in multilayered magnetoelastic self-reinforced media, *International Journal of Solids and Structures*, 47, 1317-1324.
- Chattopadhyay, A., Gupta, S. and Singh, A.K., 2012b. Torsional surface waves in a self-reinforced medium over a heterogeneous half space, *International Journal of Geomechanics*, 12(2), 193–197.
- Chattopadhyay, A., Gupta, S., Sahu, S.A. and Singh, A.K., 2010a. Dispersion of shear waves in an irregular magnetoelastic self-reinforced layer sandwiched between two isotropic half-spaces, *International Journal of Theoretical and Applied Mechanics*, 5(1), 27-45.
- Chattopadhyay, A., Gupta, S., Sahu, S.A. and Singh, A.K., 2011a. Dispersion equation of magnetoelastic shear waves in an irregular monoclinic layer. *Applied Mathematics and Mechanics*, 32(5), 571-586.

- Chattopadhyay, A., Gupta, S., Singh, A.K. and Sahu, S.A., 2009a. Propagation of shear waves in an irregular magnetoelastic monoclinic layer sandwiched between two isotropic half-spaces, *International Journal of Engineering, Science and Technology*, 1(1), 228–244.
- Chattopadhyay, A., Gupta, S., Singh, A.K. and Sahu, S.A., 2010b. Propagation of SH waves in an irregular non homogeneous monoclinic crustal layer over a semi-infinite monoclinic medium, *International Journal of Applied Mathematical Sciences*, 4(44), 2157-2170.
- Chattopadhyay, A., Sahu, S.A. and Singh, A.K., 2013. Dispersion of SH waves in an irregular Non Homogeneous Self-reinforced Crustal Layer over a Semi-infinite Self-reinforced medium, *Journal of Vibration and Control*, 19 (1), 109-119.
- Chattopadhyay, A., Sahu, S.A., Singh, A.K., 2012c. Dispersion of G-type seismic wave in magnetoelastic self-reinforced layer, *International Journal of Applied Mathematics and Mechanics*, 8(9), 79-98.
- Chattopadhyay, A., Singh, A.K., 2011b. G-type seismic waves in fibre reinforced media, *Meccanica*, 47(7), 1775-1785.
- Chattopadhyay, A., Singh, A.K., 2011e. Effect of point source and heterogeneity on the propagation of magnetoelastic shear wave in a monoclinic medium, *International Journal of Engineering, Science and Technology*, 3(2), 68-83.
- Chattopadhyay, A., Singh, A.K., 2012a. Propagation of magnetoelastic shear waves in an irregular self-reinforced layer, *Journal of Engineering Mathematics*, 75, 139-155.
- Ewing, W.M., Jardetsky, W.S. and Press, F., 1957. *Elastic Waves in Layered Media*. McGraw-Hill, New York.
- Graff, K.F., 1991. *Wave Motion Elastic Solids.* (Oxford engineering science series), Dover Publications; New ed.
- Gubbins, D., 1990. *Seismology and Plate Tectonics*. Cambridge University press, Cambridge.
- Kelvin, L., 1863. On the rigidity of the earth. *Phil. Trans. Roy. Soc. Lond.* A, 153, 573-582.
- Love, A.E.H., 1944. *A Treatise on the Mathematical Theory of Elasticity. (4th ed.)* Dover Publications, New York.
- Maugin, G.A., 1981. Wave motion in magnetisable deformable solids. *Int. J. Engng. Sci.*, 19(3), 321-388.

- Poisson, S.D. 1829. Memoire sur I'equilibre et le movement des corps e'lastiques. *Mem. D.I'Acad. Roy. d. Sci. I'Inst., France*, 8, 357-570.
- Rayleigh, L. 1877. On progressive waves. *Proc. Lond. Math. Soc.*, S1-9, 21-26.
- Rayleigh, L. 1885. On waves propagation on the plane surface of an elastic solid. *Proc. Lond. Math. Soc.*, 17, 4-11.
- Rayleigh, L. 1912. On the propagation of waves through a stratified medium, with special reference to the question of reflection. *Proc. Roy. Soc. Lond.* A, 86(586), 207-226.
- Rudin, W., 1976. *Principles of Mathematical Analysis*. McGraw-Hill International Edition, Mathematical series.
- Sokolnikoff, I.S., 1956. *Mathematical Theory of Elasticty*. McGraw-Hill, New York.
- Spencer, A.J.M.,1972. *Deformation of Fibre-reinforced Material.* Oxford University, United Kingdom.
- Spencer, A.J.M.,1974. Boundary layers in highly anisotropic plane elasticity. *Int. J. Solids Structs.,* 10(10), 1103-1123.
- Stokes, C.G., 1924. On the dynamical Theory of diffraction. *Trans. Camp. Phil. Soc.*, 9, *Stoke's Math. and Phys.*, Papers 2, Cambridge.
- Stoneley, R., 1924. Elastic waves at the surface of separation of two solids. *Proc. Roy. soc. Lond.* A, 106(738), 416-428.
- Udias, A., 1999. *Principles of Seismology*. Cambridge University Press, Cambridge.