THEORETICAL STUDY ON LOVE WAVE PROPAGATION IN IRREGULAR EARTH'S STRUCTURE

Dissertation submitted in partial fulfillment of the requirements for the award of the degree of

Master of Science

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Submitted by

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CERTIFICATE

This is to certify that the thesis entitled **"Theoretical Study On Love Wave Propagation In Irregular Earth's Structure"**, being presented in partial fulfillment of the requirements for the award of the degree of Master of Science in the School of Mathematics,Thapar Institute of Engineering and Technology, Patiala, is a bonafide work carried out under the supervision of **Dr. Pramod Kumar Vaishnav.**

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

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This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

20/06/2018

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Date:

Place

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Nomenclature

F	Body force per unit volume
H, h	Thickness of the layer
$\sigma_1(r,t)$	Force density distribution function
$\delta(x)$	Dirac-delta function
Ω	Cubical dilation
ρ	Density of the medium
$u_i = (u, v, w)$	Displacement components
$\tau_{ij}, S_{ij}, \sigma_{ij}$	Stress components
e_{ij}	Strain components
x_i	Cartesian co-ordinates
t	Time parameter

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Abstract

Theoretical study on Love wave propagation in irregular Earth's structure has been discussed in the present thesis. For the propagation of Love wave, we used some particular mediums of the Earth with the effect of several parameters on the phase velocity of Love wave. The standard dispersion relation of Love wave has been obtained in particular cases in both problems. The parametric effect on the phase velocity of Love wave has been demonstrate graphically. This thesis entitled "Theoretical study on Love wave propagation in irregular Earth's structure" is carried out for the study of Love wave propagation in detail. The thesis contains three major chapters in addition to a chapter References at the end. **Chapter 1** express the introduction for the relevant problems. It gives the explanation of studies in the field of theoretical seismology. In **chapter 2**, we considered the Love wave propagation in initially stressed isotropic medium lying over initially stressed orthotropic half-space with rectangular irregularity at the interface of layer and half-space. The effect of depth of irregularity and initial stresses explained graphically. In **chapter 3**, the propagation of love wave in reinforced medium lying over non-homogeneous anisotropic half-space with parabolic interface of layer and half-space has been discussed. The significant effect of depth of parabolic irregularity and reinforced parameters on the phase velocity of Love wave has been observed graphically. It has been observed that the phase velocity of Love wave decreases in the presence of irregular interface. The present study is helpful for seismologist, geologist and engineers to find the exact location of earthquake.

Chapter 1

Introduction

Seismology refers to the scientific study of seismic waves and earthquakes which gives us the information about the Earth's interior, geology of the Earth and the physics behind the earthquake. Seismic waves travel through Earth's interior. Earthquakes are one of the most devastating natural hazards that cause great loss of life. The term seismology is derived from two Greek words, 'seismos' meaning shaking, and 'logos' meaning science or treatise. Everyday, about fifty strong earthquakes are felt locally,from which several of these produce different seismic waves and that are measured with sensitive instruments anywhere around the world. Seismology is the primitive means through which scientists are able to study about the deep interior of Earth, which is impossible through direct observations.

An interesting position is occupied by Seismology in more general fields of Earth science and geophysics. It provides appealing theoretical problems which includes study of elastic wave propagation in complex media, also it can be simply applied as a way to investigate distinct areas of interest. Seismology is directed by investigations, and enhancement in means and data possibility had route to breakthrough in seismology's theory and understanding of the Earth's structure.

1.1 Earth's structure

The Earth's interior is categorized into main three layers; crust, mantle and core . Below the Earth's surface , the core of the Earth is around 2,900 *km* (1,800 miles) . It consist of heavy ball of the elements like nickel and iron . The core of the Earth is further categorized into two layers , that are - the inner core and the outer core layer . The center of Earth is called as inner core and it is solid and about 1,250 *km* (780 miles) thick . The metal is always molten because of the high temperature of the outer core , but the inner core cannot melt due to its great pressures even if the temperature reaches 3700°C . The thickness of the outer core is about 2,200 *km* (1370 miles) . The outer core spins around the inner core because of the Earth's rotates and which results in the Earth's magnetism . The mantle is commonly categorized into several layers based upon radial discontinuities in the seismic velocity structure (Anderson, 1989) which include the asthenosphere , lithosphere , transition zone and lower mantle . The asthenosphere underlies the lithosphere and is weak and flows plastically under geologic stresses . The lithosphere-asthenosphere boundary is typically identified by seismologists in the upper mantle as a negative velocity gradient . The outermost layer is the crust that ranges from 5-70 *km* in depth . The oceanic crust are the thin parts underlying ocean basins (5-10 *km*) and also are composed of dense iron magnesium silicate rocks .The categories of rock of the crust are as- sial and sima. Hard and soft rocks of the crust exhibits strong anisotropy (reinforcement) . It's accepted now that the Earth is anisotropic and this anisotropy is sometimes hard to measure and to interpret from the mineralogical and seismological points of view . The reasons are multiple origins of anisotropy , the complexity of the phenomenon's involved and the multiple scales that must be considered . In an anisotropic medium the physical properties vary as a function of direction .

1.2 Seismic waves and their classifications:

Seismic waves are those waves of energy that travel through the Earth's layers . Volcanic eruptions , large landslides , earthquakes and large scale man-made explosions are result of seismic waves.

Mainly Seismic waves are categorized into major two types - body waves and surface waves.Body waves are those which travel through the interior of the body , and surface waves are those waves which travel at the Earth's surface. So, seismic waves are transferring the energy from one medium to another medium within the Earth's layer. Simple harmonic motions is carried out by particles of the medium . Also the seismic energy is transmitted as complex set of wave motions.

The exact location within the Earth is called focus of an earthquake whereas Epicenter is the surface of the Earth straight above the focus . Earthquake shaking is generated through two principal categories of seismic waves, i.e., body waves and surface waves.

1) Body waves:

Body waves are those waves that travel through the interior of the Earth or any body and there paths are controlled by the material properties in terms of density and modulus with higher frequency than surface waves. The density and modulus vary according to composition , temperature and phase. The effect of the body waves is similar to the effect of refraction of light waves. Body waves are categorized as follows namely, P (primary) waves and S (secondary) waves;

i) P waves: P or primary waves are longitudinal waves in which direction of motion of particle is the along or opposite to the direction of wave propagation which travel faster than surface waves. Inside the Earth, the travelling speeds of p waves from about 6 *km*/sec in surface rock to about 10.4 *km*/sec near the Earth's core. The velocity drops to about 8 *km*/sec ,as the waves enter the core. It increases to about 11 *km*/sec near the centre of the

Earth. It increases to about 11 *km*/sec near the Earth's centre.

ii) S waves: S waves or secondary waves are transverse in nature. After the faster-moving P-waves the S-waves arrive at seismograph stations and displace the ground normal to the direction of propagation. The velocity of S wave is a function of shear modulus (μ) and density (ρ). As liquid and gases do not support shear stresses so S waves can only travel through solids . Secondary waves are slower than Primary waves. The speeds are typically around 60% of that of P-waves in any given material. Classification of Shear waves in two groups: Horizontally polarized shear wave (SH) and vertically polarized shear wave (SV).

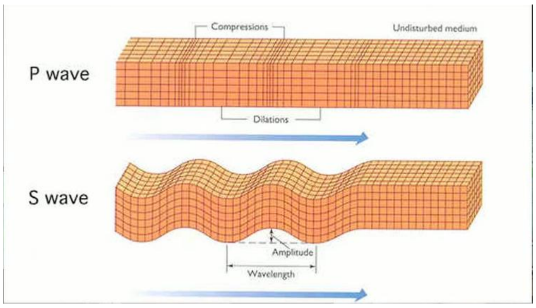

Figure 1.1: P and S wave propagation.

2) Surface wave:

Seismic surface waves are those waves that travel along Earth's surface. Surface waves are classified as a form of mechanical surface waves. The surface waves get diminished as they get further from the surface. They travel more slowly than primary and secondary waves. Following are the categories of surface waves:

i) Love waves: Love waves causes the horizontal shifting of the Earth during an earthquake. Love waves are the surface seismic waves . In 1911, a British seismologist, A.E.H Love first discovered an important surface wave named as Love wave in his honour. There is no particle motion in the vertical plane . The particle motion takes place only in the horizontal plane and also transverse to the direction of propagation . Love wave propagate in the presence of superficial layer of finite thickness lying over semi-infinite medium which exist only if the layer's shear velocity is less than half-space velocity .

ii) Rayleigh waves:

In 1885, Lord Rayleigh (British physicist) observed the motion of plane waves in an elastic half-space, and predicted the existence of surface waves. These waves are elliptically polarized in plane which is determined by the normal to the surface and by the direction of propagation. The Rayleigh wave velocity is 0.9 times (approx) the Love wave velocity. These waves are generated due to the interaction of P and SV waves with the free surface. They can exist in a homogeneous half-space as well as in a layered one .

Chapter 2

Dispersion relation of Love wave in isotropic medium (under initial stresses) lying over an orthotropic half-space with irregular interface

2.1 Objective

Love wave propagation in superficial layer (isotropic medium under initial stressed) which is lying over an orthotropic half-space (associated with initial stresses) is discussed in this chapter. The interface of layer and half-space is taken irregular. The significant effect of rectangular irregularity on the propagation of Love wave has been observed graphically. The presence of initial stresses in the generalized dispersion relation approve the significant effect of these parameters on Love wave propagation. The obtained dispersion relation is in closed form and which is in reconciliation with the classical Love wave dispersion relation.

2.2 Mathematical structure of the problem

We considered an initially stressed isotropic layer with finite thickness H over an orthotropic (initially stressed) half-space. The rectangular irregularity is taken at the interface of layer and half-space. Depth of irregularity is taken h and length is $2l$. Along the x-axis , the Love wave is propagating and z-axis is taken vertically downward to the direction of wave propagation , as in Fig. (2.1). The upper surface of the isotropic layer is stress free $\left(\sigma_{ij} = 0\right)$. The shape of irregularity at the interface is taken as $z = \varepsilon f(x)$, where

$$f(x) = \begin{cases} 0; & |x| > l \\ 2l; & |x| \leq l \end{cases}, \quad \texttt{and} \quad \varepsilon = h/2l << 1. \tag{2.2.1}$$

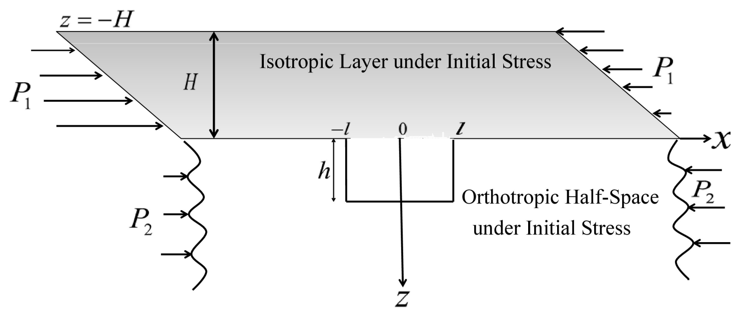

Figure 2.1: Geometry of the problem.

2.3 Dynamics and solution of isotropic medium

The governing equations of motion for Love wave propagation in (initially stressed) semi-infinite medium , within absence of body forces are

$$\left.\begin{aligned} &\frac{\partial \sigma_{11}}{\partial x} + \frac{\partial \sigma_{12}}{\partial y} + \frac{\partial \sigma_{13}}{\partial z} - P_1\left(\frac{\partial w_z}{\partial y} - \frac{\partial w_y}{\partial z}\right) = \rho_1 \frac{\partial^2 u}{\partial t^2}, \\ &\frac{\partial \sigma_{21}}{\partial x} + \frac{\partial \sigma_{22}}{\partial y} + \frac{\partial \sigma_{23}}{\partial z} - P_1 \frac{\partial w_z}{\partial x} = \rho_1 \frac{\partial^2 v}{\partial t^2}, \\ &\frac{\partial \sigma_{31}}{\partial x} + \frac{\partial \sigma_{32}}{\partial y} + \frac{\partial \sigma_{33}}{\partial z} - P_1 \frac{\partial w_y}{\partial x} = \rho_1 \frac{\partial^2 w}{\partial t^2}, \end{aligned}\right\} \tag{2.3.1}$$

where, σ_{ij} are the incremental stress components, along x, y and z-directions the u_1, v_1 and w_1 are displacement components in semi-infinite medium respectively. Also , the rotational components that are w_x, w_y and w_z along x, y, z directions, ρ_1 is the density of isotropic materials and P_1 is the initial stress associated with isotropic layer. The stress-strain relations in the isotropic medium are given by Hooke's law as

$$\sigma_{ij} = \lambda \Delta \delta_{ij} + 2\mu_1 e_{ij}, \tag{2.3.2}$$

where λ, μ_1 are Lame's constants, and e_{ij} are the strain components.

For propagation of Love wave along x-direction, we will have $u_1 = 0 = w_1$ and $v_1 = v_1(x,z,t)$ and the strain components will be

$$e_{11} = 0, e_{12} = \frac{1}{2}\frac{\partial v_1}{\partial x}, e_{13} = 0, e_{23} = \frac{1}{2}\frac{\partial v_1}{\partial z}, e_{22} = 0, e_{33} = 0, \tag{2.3.3}$$

where u_1, v_1 and w_1 are representing displacement components in the isotropic medium along x, y and z directions.

The change in volume per unit volume is called the cubical dilation Δ . For the small strain $\Delta = e_{11} + e_{22} + e_{33}$. The stress components will be obtained as

$$\sigma_{11} = 0, \sigma_{12} = \mu_1 \frac{\partial v_1}{\partial x}, \sigma_{22} = 0, \sigma_{13} = 0, \sigma_{23} = \mu_2 \frac{\partial v_1}{\partial z}, \sigma_{33} = 0. \tag{2.3.4}$$

Using stress-strain relations and standard Love wave conditions in the governing equation of motion (2.3.1), we have

$$\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial z^2} - \frac{P_1}{2\mu_1}\frac{\partial^2 v_1}{\partial x^2} = \frac{1}{c_1^2}\frac{\partial^2 v_1}{\partial t^2}, \tag{2.3.5}$$

where $c_1 = \sqrt{\frac{\mu_1}{\rho_1}}$ is the shear wave velocity.

Let us consider the harmonic solution of the Eq. (2.3.5) along x-direction as $v_1(x,z,t) = v_1^*(z)e^{ik(x-ct)}$, where c is representing phase velocity of Love wave and k is representing wave number. Thus, Eq. (2.3.5) takes the form

$$\frac{d^2 v_1^*}{dz^2} + k^2 m_1^2 v_1^* = 0, \tag{2.3.6}$$

where $m_1 = \sqrt{\frac{c^2}{c_1^2} - \xi_1 - 1}$, $\xi_1 = \frac{P_1}{2\mu_1}$ and $c_1 = \sqrt{\frac{\mu_1}{\rho_1}}$ is shear wave velocity.

The solution of Eq. (2.3.6) is given as

$$v_1^*(x,z,t) = A_1 e^{ikm_1 z} + A_2 e^{-ikm_1 z}, \tag{2.3.7}$$

where these A_1, A_2 are arbitrary constants. Displacement in the isotropic medium is obtained as

$$v_1(x,z,t) = \left(A_1 e^{ikm_1 z} + A_2 e^{-ikm_1 z}\right) e^{ik(x-ct)} \tag{2.3.8}$$

2.4 The displacement equation in an orthotropic half-space

Let u_2, v_2, and w_2 be representing the displacement components in the $x-, y-$, and z-directions, respectively. Using conventional Love wave conditions, i.e., $u_2 = 0, w_2 = 0$, and $v_2 = v_2(x,z,t)$ in governing equations of motion, in absence of body forces , the non vanishing equation of motion can be written as

$$\frac{\partial \tau_{21}}{\partial x} + \frac{\partial \tau_{22}}{\partial y} + \frac{\partial \tau_{23}}{\partial z} - P_2 \frac{\partial w_z'}{\partial x} = \rho_2 \frac{\partial^2 v_2}{\partial t^2}, \tag{2.4.1}$$

where v_2 is representing displacement component in orthotropic half-space . Also , w_z' is the rotational component along z-direction. Here the incremental stress components are τ_{ij} and the density of the material in the half-space be ρ_2 .

The stress-strain relation for the orthotropic half-space is

$$\begin{aligned}
\tau_{11} &= B_{11}e_{11} + B_{12}e_{22} + B_{13}e_{33}, \\
\tau_{12} &= 2Q_3 e_{12}, \\
\tau_{22} &= B_{21}e_{11} + B_{22}e_{22} + B_{23}e_{33}, \\
\tau_{23} &= 2Q_1 e_{23}, \\
\tau_{33} &= B_{31}e_{11} + B_{32}e_{22} + B_{33}e_{33}, \\
\tau_{31} &= 2Q_2 e_{31},
\end{aligned} \tag{2.4.2}$$

where the incremental normal elastic coefficient are B_{ij} , Q_i are the shear moduli. e_{ij} be strain components.

Now using stress-strain relations in Eq. (2.4.1). The equation of motion in orthotropic half-space take the form

$$\frac{\partial}{\partial x}\left(Q_3\frac{\partial v_2}{\partial x}\right)+\frac{\partial}{\partial z}\left(Q_1\frac{\partial v_2}{\partial z}\right)-P_2\frac{\partial}{\partial x}\left(\frac{1}{2}\frac{\partial v_2}{\partial x}\right)=\rho_2\frac{\partial^2 v_2}{\partial t^2}, \tag{2.4.3}$$

$$\left(Q_3-\frac{P_2}{2}\right)\frac{\partial^2 v_2}{\partial x^2}+Q_1\frac{\partial^2 v_2}{\partial z^2}=\rho_2\frac{\partial^2 v_2}{\partial t^2}, \tag{2.4.4}$$

Taking solution of equation of motion as $v_2 = v_2^*(z)e^{ik(x-ct)}$, where k and c be the wave number and phase velocity of Love wave respectively. By using above solution, Eq. (2.4.3) takes the form

$$\frac{d^2 v_2^*}{dz^2}-m_2^2 v_2^*=0, \tag{2.4.5}$$

here $m_2 = k\sqrt{\frac{Q_3}{Q_1}-\xi_2-\frac{c^2}{c_2^2}}$, $\xi_2 = \frac{P_2}{2Q_1}$ and $c_2 = \sqrt{\frac{Q_1}{\rho_2}}$ is shear wave velocity. The solution of Eq. (2.4.5) is obtained as

$$v_2^* = A_3 e^{-m_2 z}+A_4 e^{m_2 z}. \tag{2.4.6}$$

where A_3 and A_4 are arbitrary constants. The displacement in orthotropic half-space is obtain as

$$v_2 = \left(A_3 e^{-m_2 z}+A_4 e^{m_2 z}\right)e^{ik(x-ct)}. \tag{2.4.7}$$

We are interested in the solution of the Eq. (2.4.7) which is bounded and vanish as $z\to\infty$. i.e.,

$$v_2 = A_3 e^{-m_2 z}e^{ik(x-ct)}. \tag{2.4.8}$$

2.5 Boundary conditions

Upper surface of the isotropic the layer is traction free stress free hence the shearing stress component vanishes there i.e.,

$$\mu_1\frac{\partial v_1}{\partial z}=0, \quad at\ \ z=-H.$$

The displacement components and the stress components are continuous at interface of layer and the half-space i.e $z = \varepsilon f(x)$,

$$v_1(z) = v_2(z), \quad \mu_1 \frac{\partial v_1}{\partial z} = Q_1 \frac{\partial v_2}{\partial z}.$$

2.6 Dispersion relation

A relation between phase velocity and wave numbers is called dispersion relation of Love wave. We obtain following phase velocity equations by using boundary conditions:

$$A_1 e^{-ikm_1 H} - A_2 e^{ikm_1 H} = 0, \tag{2.6.1}$$

$$A_1 ik\mu_1 m_1 e^{ikm_1 \varepsilon f(x)} - A_2 ik\mu_1 m_1 e^{-ikm_1 \varepsilon f(x)} = -A_3 m_2 Q_1 e^{-m_2 \varepsilon f(x)}, \tag{2.6.2}$$

$$A_1 e^{ikm_1 \varepsilon f(x)} + A_2 e^{-ikm_1 \varepsilon f(x)} = A_3 e^{-m_2 \varepsilon f(x)}. \tag{2.6.3}$$

Eliminating arbitrary constants from phase velocity equations as

$$\begin{vmatrix} e^{-ikm_1 H} & -e^{ikm_1 H} & 0 \\ ik\mu_1 m_1 e^{ikm_1 \varepsilon f(x)} & -ik\mu_1 m_1 e^{-ikm_1 \varepsilon f(x)} & m_2 Q_1 e^{-m_2 \varepsilon f(x)} \\ e^{ikm_1 \varepsilon f(x)} & e^{-ikm_1 \varepsilon f(x)} & -e^{-m_2 \varepsilon f(x)} \end{vmatrix} = 0,$$

which reduced to

$$-ikm_1 \mu_1 \left\{ e^{ikm_1 (H + \varepsilon f(x))} - e^{-ikm_1 (H + \varepsilon f(x))} \right\} - m_2 Q_1 \left\{ e^{ikm_1 (H + \varepsilon f(x))} - e^{-ikm_1 (H + \varepsilon f(x))} \right\} = 0. \tag{2.6.4}$$

The generalized dispersion relation of Love wave is obtained as follows:

$$\tan \left[kH(1 + h/H) \sqrt{\frac{c^2}{c_1^2} - \xi_1 - 1} \right] = \frac{Q_1}{\mu_1} \frac{\sqrt{\frac{Q_3}{Q_1} - \xi_2 - \frac{c^2}{c_2^2}}}{\sqrt{\frac{c^2}{c_1^2} - \xi_1 - 1}}. \tag{2.6.5}$$

2.7 Validation of the problem

1. Case-I: $h = 0$ i.e.,If the interface of half-space and layer is regular , Eq. (2.6.5) reduced to

$$\tan\left[kH\sqrt{\frac{c^2}{c_1^2}-\xi_1-1}\right]=\frac{Q_1}{\mu_1}\frac{\sqrt{\frac{Q_3}{Q_1}-\xi_2-\frac{c^2}{c_2^2}}}{\sqrt{\frac{c^2}{c_1^2}-\xi_1-1}}, \tag{2.7.1}$$

which is Love wave dispersion relation in plain boundary surfaces.

2. Case-II: If the superficial layer is initially stress free (i.e., $\xi_1 = 0$), Eq. (2.7.1) converted to

$$\tan\left[kH\sqrt{\frac{c^2}{c_1^2}-1}\right]=\frac{Q_1}{\mu_1}\frac{\sqrt{\frac{Q_3}{Q_1}-\xi_2-\frac{c^2}{c_2^2}}}{\sqrt{\frac{c^2}{c_1^2}-1}}, \tag{2.7.2}$$

3. Case-III: If the orthotropic half-space is initially stressed free with rigidity μ_2 (i.e., $Q_1 \to Q_3 \to \mu_2$ and $\xi_2 = 0$), Eq. (2.7.2) takes the form of standard Love wave dispersion relation as

$$\tan\left[kH\sqrt{\frac{c^2}{c_1^2}-1}\right]=\frac{\mu_2}{\mu_1}\frac{\sqrt{1-\frac{c^2}{c_2^2}}}{\sqrt{\frac{c^2}{c_1^2}-1}}. \tag{2.7.3}$$

Eq. (2.7.3) represents the dispersion relation of Love wave in the isotropic medium lying over an homogeneous half-space as Love(1911).

2.8 Numerical Results

The rigidities and density of the isotropic and orthotropic mediums are taken from Gubbins (1990) as:

(a) For the orthotropic half-space,

$Q_1 = 5.82 \times 10^{10} N/m^2, \quad Q_3 = 3.99 \times 10^{10} N/m^2, \quad \rho_1 = 4.5 \times 10^3 kg/m^3.$

(b) For the isotropic layer,

$$L = 0.1387 \times 10^{10} N/m^2, \quad N = 0.2774 \times 10^{10} N/m^2, \quad \rho_2 = 3.4 \times 10^3 kg/m^3.$$

The effect of depth of rectangular irregularity and initial stresses on the phase velocity of Love wave has been observed graphically as:

Fig. (2.2) demonstrate effect of (ξ_1) i.e., initial stress associated with isotropic layer in presence of irregularity on the Love wave's phase velocity. The curves i.e., Curve 1, curve 2, curve 3 and curve 4 are plotted for initial stress values ,that are , $\xi_1 = 0.2, 0.4, 0.6$ and 0.8 in the presence of depth $(h/H = 0.2)$ of irregularity and initial stress $\xi_2 = 0.2$. It's been observed that phase velocity of Love wave increases as the value of (ξ_1) increases (i.e., value of initial stress). The effect of initial stress is more visible for the large wave numbers.

Fig. (2.3) demonstrate the effect of (ξ_1) i.e., initial stress associated with isotropic layer in absence of irregularity on the Love wave's phase velocity. It's been noticed that phase velocity of Love wave increases rapidly as the value of (ξ_1) i.e., initial stress increases in absence of irregular interface. We concluded that the nature of phase velocity is same in both cases but the phase velocity increases in different way.

Fig. (2.4) deals with the effect of initial stress ξ_2 associated with orthotropic half-space in presence of irregular interface on the Love wave's phase velocity . The curves i.e., Curve 1, curve 2 and curve 3 are plotted for initial stresses values ,that are , $\xi_2 = 0.2, 0.4$ and 0.6 in the presence of depth $(h/H = 0.2)$ of irregularity and initial stress $\xi_1 = 0.2$. It is observed that the initial stress's presence in orthotropic half-space decreases the Love wave's phase velocity . Due to presence of irregular interface the Love wave's phase velocity decreases.

Fig. (2.5) demonstrate the effect of ξ_2 i.e, initial stress , in absence of irregular interface on the phase velocity of Love wave. It is noticed that Love wave's phase velocity decreases rapidly in absence of irregular interface. The phase velocity of Love wave is being affected significantly with presence of irregularity .

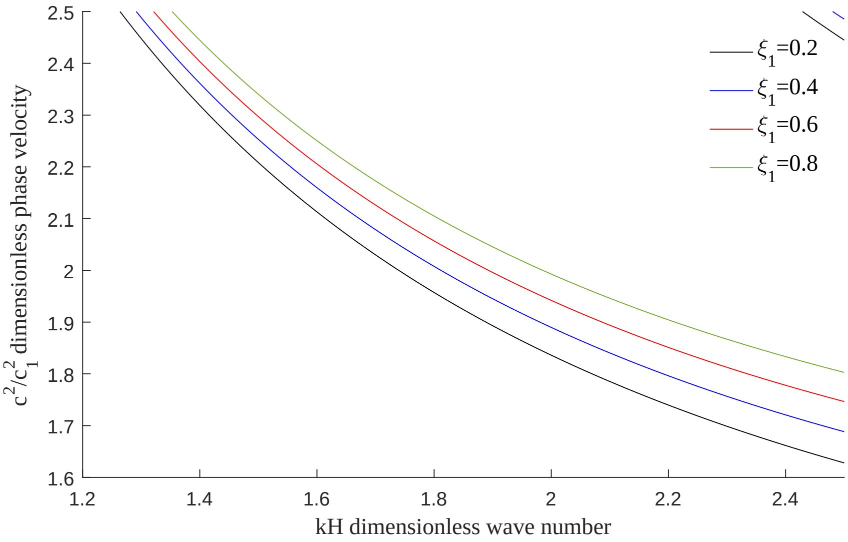

Figure 2.2: Non-dimensional phase velocity c^2/c_1^2 versus dimensionless kH i.e., wave number with initial stress's effect $\xi_1 = 0.2$, 0.4, 0.6, 0.8 in the presence of irregularity

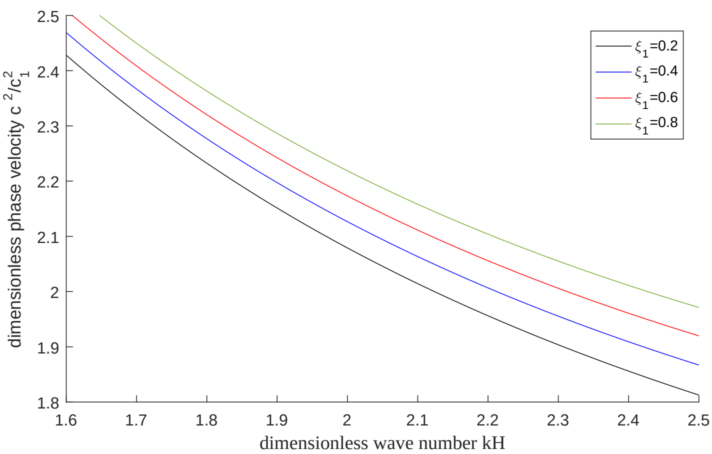

Figure 2.3: Non-dimensional phase velocity c^2/c_1^2 versus dimensionless kH i.e., wave number with initial stress's effect $\xi_1 = 0.2$, 0.4, 0.6, 0.8 in the absence of irregularity

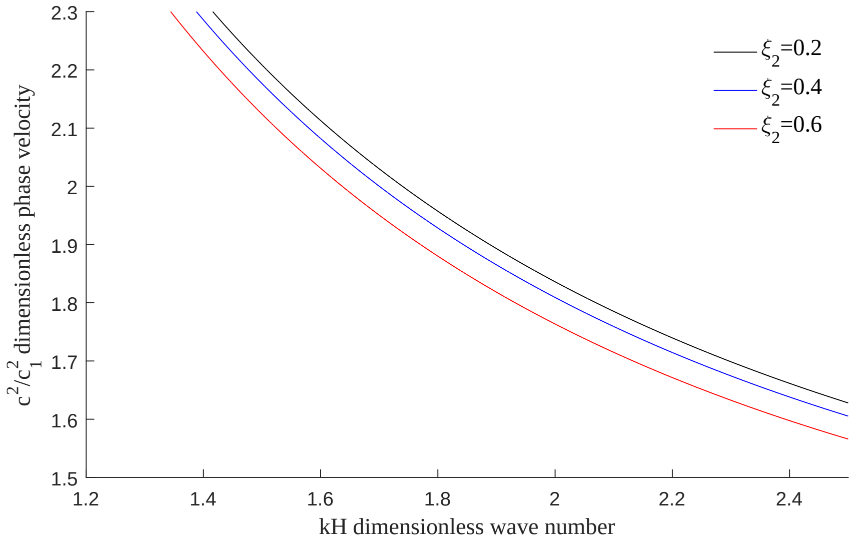

Figure 2.4: Non-dimensional phase velocity c^2/c_1^2 versus dimensionless kH i.e., wave number with initial stress's effect $\xi_2 = 0.2$, 0.4, 0.6 in the presence of irregularity

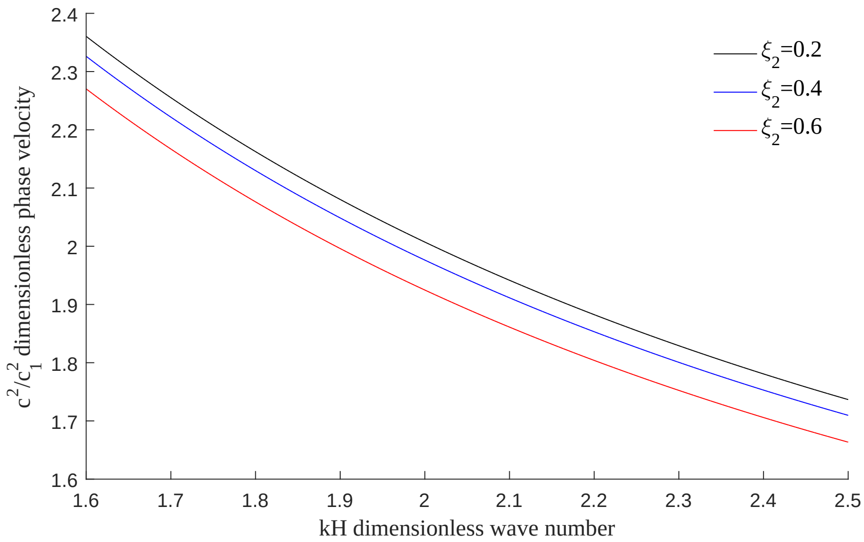

Figure 2.5: Non-dimensional phase velocity c^2/c_1^2 versus dimensionless kH i.e., wave number with initial stress's effect $\xi_2 = 0.2$, 0.4, 0.6 in the absence of irregularity.

Fig. (2.6) represents the effect of depth of irregularity on Love wave's phase velocity in the presence of initial stresses. The curves from curve 1, curve 2 and curve 3 are plotted for $h/H = 0.2$, 0.4 and 0.6 with $\xi_1 = 0.2 = \xi_2$. The phase velocity of Love wave is affected significantly due to the presence of irregularity at interface of layer and half-space . It is noticed that Love wave's phase velocity decreases most rapidly as value of depth of irregularity increases.

Fig. (2.7) deals with the effect of depth of irregularity in absence of initial stresses on the Love wave's phase velocity. Here in the figure, curve 1, curve 2 and curve 3 are plotted for the numerically small value of depth of irregularity $(h/H = 0.01, 0.03, 0.05)$. The Love wave's phase velocity decreases as numerical value of depth of irregularity increases. In this case we considered numerically small values of depth of irregularity for more visibility.

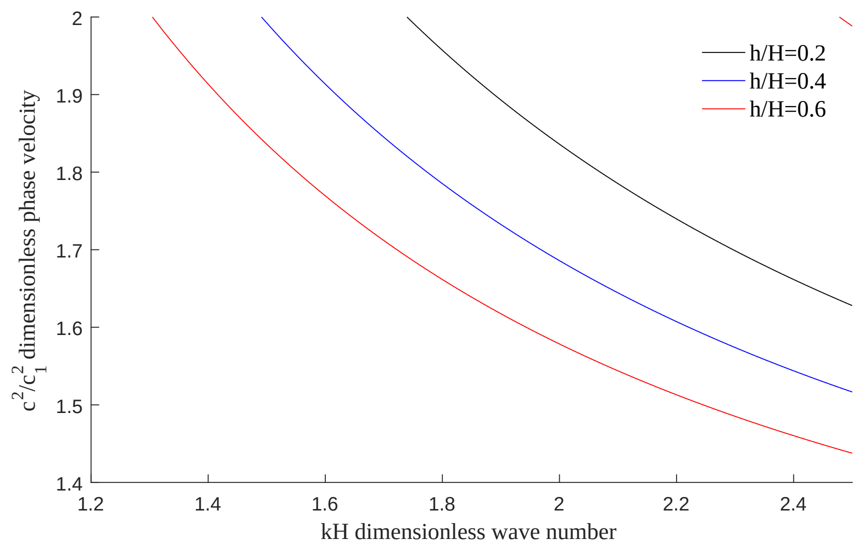

Figure 2.6: Non-dimensional phase velocity c^2/c_1^2 versus dimensionless kH i.e., wave number with effect of depth of irregularity $h/H = 0.2$, 0.4, 0.6 in the presence of initial stresses.

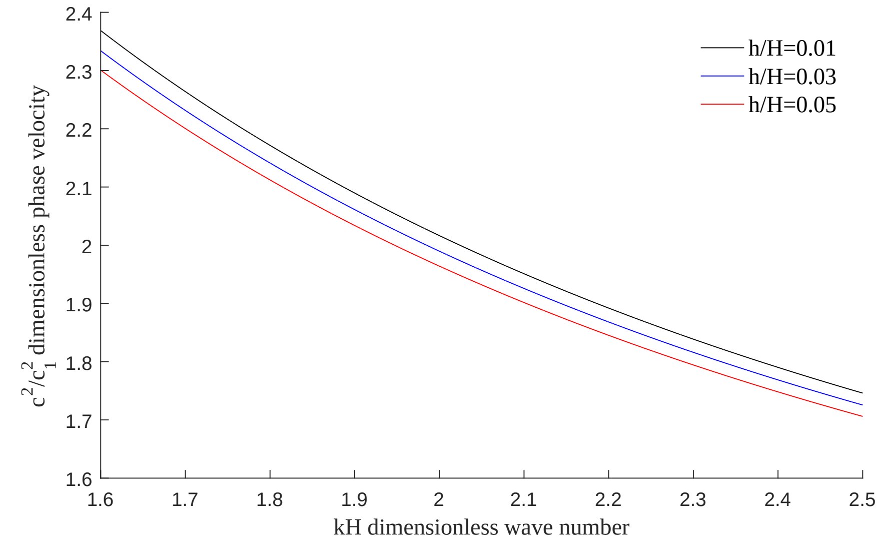

Figure 2.7: Non-dimensional phase velocity c^2/c_1^2 versus dimensionless kH i.e., wave number with effect of depth of irregularity $h/H = 0.01,\ 0.03,\ 0.05$ in the absence of initial stresses.

2.9 Conclusions

The Dispersion relation of Love wave in isotropic medium (under initial stresses) which is lying over an orthotropic half-space with irregular interface is being derived analytically. In particular cases , the standard dispersion relation of Love wave is obtained. It is observed that the presence of depth of irregularity at interface of layer and half-space affected the phase velocity of Love wave significantly. The phase velocity of Love wave decreases rapidly as depth of the irregularity increases. Effect of initial stresses has been noticed in the presence and absence of irregularity. The presence of initial stresses affected the Love wave's phase velocity significantly. The speed of Love wave increases as value of initial stress (ξ_1) increase, whereas, phase velocity of the Love wave decreases as value of initial stress (ξ_2) increases. The parametric effect of initial stresses on the phase velocity remains same in the presence of irregularity.

Chapter 3

Dynamic effect of parabolic irregularity and inhomogeneity on the Love wave propagation in reinforced medium lying over anisotropic half-space

3.1 Objective

In this chapter, we considered fiber-reinforced layer of finite thickness (H) over anisotropic non-homogeneous half-space with parabolic irregular interface at layer and half-space for Love wave propagation , where parabolic irregularity of depth h is taken at the interface of the layer and half-space. The directional rigidities and density vary quadratically with respect to the depth z in the anisotropic non-homogeneous half-space. The generalized dispersion relation of Love wave has been obtained in the presence of depth of irregularity, reinforced parameters and heterogeneity. The significant effect of depth of irregularity, heterogeneity and reinforced parameters on the Love wave's phase velocity is observed graphically. We obtained standard dispersion relation of Love wave in particular cases.

3.2 Mathematical formulation of the problem

For the Love wave propagation, the fiber-reinforced medium (layer) is taken over anisotropic non-homogeneous half-space with irregular interface of half-space and layer. The parabolic irregularity having depth h is taken at the interface of the layer and anisotropic half-space, H corresponds the finite thickness of reinforced layer. The origin of Cartesian co-ordinates is seized at irregular interface of layer and half-space. Here the Love wave propagates on $x-$direction and the $z-$axis is taken vertically downward to the direction of wave propagation. The non-homogeneity in anisotropic half-space is taken as $N = N(1+\alpha z)^2, L = L(1+\alpha z)^2$ and $\rho = \rho(1+\alpha z)^2$, where the heterogeneity parameter is α. At the interface , equation of parabolic irregularity is taken as

$$z = \varepsilon f(x), \;\; \texttt{where} \;\; f(x) = \begin{cases} h\left(1 - \frac{x^2}{l^2}\right) \;\; \texttt{for} \;\; |x| \leq l \\ 0 \;\; \texttt{for} \;\; |x| > l \end{cases}$$

and $\varepsilon = \frac{h}{2l}$ and $\varepsilon << 1$.

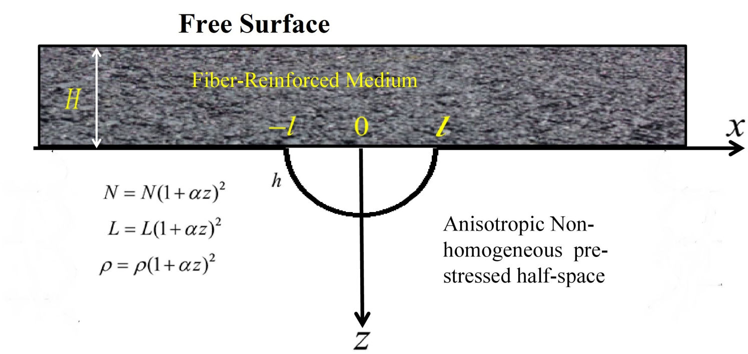

Figure 3.1: Geometry of the problem.

3.3 Displacement in fiber-reinforced medium

The stress-strain relation with preferred direction a is (Belfield et al. 1983) for a transversely isotropic linear elastic material

$$\tau_{ij}^* = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha\left(a_k a_m e_{km}\delta_{ij} + e_{kk}a_i a_j\right) + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki})$$
$$+\beta a_k a_m e_{km} a_i a_j, \tag{3.3.1}$$

for all $m,i,k,j,= 1,2,3$, where e_{ij} are the strain components, τ_{ij}^* = stress components, δ_{ij} = Kronecker delta, $\overrightarrow{a} = (a_1,a_2,a_3)$ are the preferred direction of reinforcement snd $a_1^2 + a_2^2 + a_3^2 = 1$. The coefficients λ, α and β are elastic constants and μ_T are transverse shear moduli , μ_L are the longitudinal shear moduli in preferred direction.

The direction of reinforcement may taken as $(a_1,0,a_3)$, since, Love wave is propagating along $x-$direction. The conventional Love waves conditions are

$$u_1 = 0 = w_1, \text{and } v_1 = v_1(x,z,t),\ \frac{\partial}{\partial y} \equiv 0. \tag{3.3.2}$$

Displacement throughout $y-$direction is the solution of following equation:

$$\frac{\partial \tau_{12}^*}{\partial x} + \frac{\partial \tau_{22}^*}{\partial y} + \frac{\partial \tau_{23}^*}{\partial z} = \rho_1 \frac{\partial^2 v_1}{\partial t^2}, \tag{3.3.3}$$

where ρ_1 is the density of reinforcement. Substituting stress-strain relation and Love wave conditions in above equation, Eq. (3.3.3) reduced to

$$p_\mu \frac{\partial^2 v_1}{\partial x^2} + 2q_\mu \frac{\partial^2 v_1}{\partial x \partial z} + r_\mu \frac{\partial^2 v_1}{\partial z^2} = \rho_1 \frac{\partial^2 v_1}{\partial t^2}, \tag{3.3.4}$$

where $p_\mu = \mu_T + (\mu_L - \mu_T)a_1^2$, $q_\mu = a_1 a_3(\mu_L - \mu_T)$, $r_\mu = \mu_T + (\mu_L - \mu_T)a_3^2$. Let us consider the solution of Eq. (3.3.4) as

$$v_1(x,z,t) = v_1^*(z)e^{ik(x-ct)}, \tag{3.3.5}$$

here $v_1^*(z)$ is the solution of following equation

$$r_\mu \frac{d^2 v_1^*}{dz^2} + 2ikq_\mu \frac{dv_1^*}{dz} + k^2(\rho_1 c^2 - p_\mu)v_1^*(z) = 0, \tag{3.3.6}$$

The solution of Eq. (3.3.6) is obtained analytically as

$$v_1^*(z) = A_1 e^{ikS_1 z} + A_2 e^{ikS_2 z}, \tag{3.3.7}$$

where A_2 and A_1 are associated as arbitrary constants.

Therefore, displacement in reinforced medium is obtained as

$$v_1(z) = \left(A_1 e^{ikS_1 z} + A_2 e^{ikS_2 z}\right) e^{ik(x-ct)}, \tag{3.3.8}$$

where $S_1 = \frac{1}{r_\mu}\left\{-q_\mu + M\right\}$, $S_2 = -\frac{1}{r_\mu}\left\{q_\mu + M\right\}$ and $M = \sqrt{q_\mu^2 + r_\mu\left(\rho_1 c^2 - p_\mu\right)}$.

3.4 Displacement in anisotropic non-homogeneous half-space

We considered a non-homogeneous anisotropic half-space below the reinforced layer for the Love wave's propagation. In this half-space absence of body forces, governing equation of motion are is given as

$$N\frac{\partial^2 v_2}{\partial x^2} + \frac{\partial}{\partial z}\left(L\frac{\partial v_2}{\partial z}\right) = \rho_2 \frac{\partial^2 v_2}{\partial t^2} \tag{3.4.1}$$

Assuming, solution of Eq. (3.6.1) as

$$v_2 = V_2(z) e^{ik(x-ct)} \tag{3.4.2}$$

Substituting Eq. (3.6.2) into Eq. (3.6.1), we obtained

$$\frac{d^2 V_2}{dz^2} - \frac{1}{L}\frac{dL}{dz}\frac{dV_2}{dz} + \frac{k^2}{L}(c^2\rho_2 - N)V_2 = 0 \tag{3.4.3}$$

Now, variations in the rigidities and density are taken as

$$N = N(1+\alpha z)^2, L = L(1+\alpha z)^2, \rho_2 = \rho_2(1+\alpha z)^2, \tag{3.4.4}$$

where α is inhomogeneity parameter. Using Eq. (3.6.3) into Eq. (3.4.4), we obtained

$$\frac{d^2 V_2}{dz^2} - S_3^2 V_2 = 0, \tag{3.4.5}$$

where $S_3^2 = \frac{k^2}{L}(N - c^2\rho_2)$

The solution to (6) may be assumed as

$$V_2 = Ae^{S_3 z} + Be^{-S_3 z} \tag{3.4.6}$$

Thus the solution for the non-homogeneous, anisotropic half-space may be taken as

$$v_2 = V_2(z)e^{ik(x-ct)} = \frac{V_2}{\sqrt{L}}e^{ik(x-ct)} = \frac{Ae^{-S_3 z}}{\sqrt{L}(1+\alpha z)}e^{ik(x-ct)} \tag{3.4.7}$$

The displacement in non-homogeneous anisotropic half-space is given by Eq. (3.6.4).

3.5 Boundary conditions

1. The upper surface of the reinforced layer is considered stress free; that is

$$r_\mu \frac{\partial v_1}{\partial z} + q_\mu \frac{\partial v_1}{\partial x} = 0,$$

   at $z = -H$.

2. The stress components and displacement components are continuous at irregular interface $z = \varepsilon f(x)$; that is

$$r_\mu \frac{\partial v_1}{\partial z} + q_\mu \frac{\partial v_1}{\partial x} = L(1+\alpha z)^2 \frac{\partial v_2}{\partial z},$$

$$v_1(z) = v_2(z)$$

3.6 Dispersion relation

By using boundary conditions, We obtained the following phase velocity equation as

$$A_1(r_\mu S_1 + q_\mu)e^{-ikS_1 H} + A_2(r_\mu S_2 + q_\mu)e^{-ikS_2 H} = 0, \tag{3.6.1}$$

$$A_1 ik(r_\mu S_1 + q_\mu)e^{ikS_1 \varepsilon f(x)} + A_2 ik(r_\mu S_2 + q_\mu)e^{ikS_2 \varepsilon f(x)}$$

$$= -A_3\sqrt{L}\{\alpha + S_3(1+\alpha\varepsilon f(x))\}e^{-S_3\varepsilon f(x)}, \tag{3.6.2}$$

$$A_1 e^{ikS_1\varepsilon f(x)} + A_2 e^{ikS_2\varepsilon f(x)} = \frac{A_3}{\sqrt{L}}\frac{e^{-S_3\varepsilon f(x)}}{(1+\alpha\varepsilon f(x))}. \tag{3.6.3}$$

Eliminating arbitrary constants A_1, A_2 and A_3 from Eqs. (3.6.1), (3.6.2) and (3.6.3) as

$$\begin{vmatrix} (r_\mu S_1 + q_\mu)e^{-ikS_1H} & (r_\mu S_2 + q_\mu)e^{-ikS_2H} & 0 \\ ik(r_\mu S_1 + q_\mu)e^{ikS_1\varepsilon f(x)} & ik(r_\mu S_2 + q_\mu)e^{ikS_2\varepsilon f(x)} & \sqrt{L}\{\alpha + S_3(1+\alpha\varepsilon f(x))\}e^{-S_3\varepsilon f(x)} \\ e^{ikS_1\varepsilon f(x)} & e^{ikS_2\varepsilon f(x)} & -\frac{1}{\sqrt{L}}\frac{e^{-S_3\varepsilon f(x)}}{(1+\alpha\varepsilon f(x))}. \end{vmatrix} = 0,$$

Solution of above determinant is

$$-\frac{ikM}{(1+\alpha\varepsilon f(x))}\left\{e^{ik\left\{\frac{M}{r_\mu}(H+\varepsilon f(x))\right\}} - e^{-ik\left\{\frac{M}{r_\mu}(H+\varepsilon f(x))\right\}}\right\}$$

$$-L(\alpha + S_3(1+\alpha\varepsilon f(x)))\left\{e^{ik\left\{\frac{M}{r_\mu}(H+\varepsilon f(x))\right\}} + e^{-ik\left\{\frac{M}{r_\mu}(H+\varepsilon f(x))\right\}}\right\} = 0.$$

The closed from of dispersion relation is obtained as

$$\tan\left[\frac{k(H+\varepsilon f(x))\sqrt{q_\mu^2 + r_\mu(\rho_1 c^2 - p_\mu)}}{r_\mu}\right]$$

$$= \frac{L(1+\alpha\varepsilon f(x))\left\{\alpha + k\sqrt{\frac{1}{L}(N - c^2\rho_2)}(1+\alpha\varepsilon f(x))\right\}}{k\sqrt{q_\mu^2 + r_\mu(\rho_1 c^2 - p_\mu)}} \tag{3.6.4}$$

Eq. (3.6.4) is obtained generalized dispersion relation of Love wave in assumed Earth model. The presence of depth of irregularity, reinforced parameters and heterogeneity parameter approves the significant effect of these parameters on Love wave's phase velocity.

3.7 Validation of the problem

Case-I: Substitute $\mu_T \to \mu_L \to \mu_1$ in Eq. (3.6.4). The mathematical terms in Eq. (3.6.4) reduced as $p_\mu \to \mu_1$, $r_\mu \to \mu_1$ and $q_\mu \to 0$, we obtained the dispersion relation as

$$\tan\left[k(H+\varepsilon f(x))\sqrt{\frac{c^2}{c_1^2} - 1}\right]$$

$$= \frac{L(1+\alpha\varepsilon f(x))\left\{\frac{\alpha}{k}+\sqrt{\frac{1}{L}(N-c^2\rho_2)}(1+\alpha\varepsilon f(x))\right\}}{\mu_1\sqrt{\frac{c^2}{c_1^2}-1}} \tag{3.7.1}$$

Eq. (3.7.1) is Love wave's dispersion relation in presence of irregularity and heterogeneity parameters.

Case-II: Taking isotropic properties in half-space i.e., $N \to L \to \mu_2$. Eq. (3.7.1) reduced to

$$\tan\left[k(H+\varepsilon f(x))\sqrt{\frac{c^2}{c_1^2}-1}\right] = \frac{L(1+\alpha\varepsilon f(x))\left\{\frac{\alpha}{k}+\sqrt{1-\frac{c^2}{c_2^2}}(1+\alpha\varepsilon f(x))\right\}}{\mu_1\sqrt{\frac{c^2}{c_1^2}-1}} \tag{3.7.2}$$

The obtained dispersion relation approves the effect of heterogeneity and irregularity on the phase velocity of Love wave.

Case-III: For the standard Love wave dispersion, considering isotropic homogeneous half-space with plain surface i.e., $\alpha \to 0$, $\varepsilon \to 0$, $h \to 0$. Eq. (3.7.2) reduced to standard Love wave dispersion relation as

$$\tan\left\{kH\sqrt{\frac{c^2}{c_1^2}-1}\right\} = \frac{\mu_2}{\mu_1}\frac{\sqrt{1-\frac{c^2}{c_2^2}}}{\sqrt{\frac{c^2}{c_1^2}-1}}. \tag{3.7.3}$$

Eq. (3.7.3) represents the standard dispersion relation of Love wave in isotropic homogeneous layer and half-space as Love (1911).

3.8 Numerical computation and discussions

For the dynamic effect of parabolic irregularity and heterogeneity on the Love wave propagation in reinforced medium, we represents the numerical data from Gubbins (1990) as follows:

(a) For the reinforced layer, the rigidities and density are ,

$\mu_L = 0.707\times10^{10}N/m^2$, $\mu_T = 0.35\times10^{10}N/m^2$, $\rho = 1.6\times10^3kg/m^3$.

(b) For the inhomogeneous anisotropic half-space, the rigidity and density are

$L = 0.1387 \times 10^{10} N/m^2$, $N = 0.2774 \times 10^{10} N/m^2$ and $\rho_1 = 3.4 \times 10^3 kg/m^3$.

In the graphical representation of dispersion relation, the dimensionless phase velocity (c^2/c_1^2) is taken on $y-$axis and dimensionless wave number (kH) is taken on $x-$axis. The term of parabolic irregularity in the dispersion relation has been explained in dimensionless form as:

$$\varepsilon \alpha h \left(1 - \frac{x^2}{l^2}\right) = \varepsilon \left(\frac{\alpha}{k}\right) kH \left(\frac{h}{H}\right) \left(1 - \frac{x^2}{l^2}\right).$$

The significant effect of depth of parabolic irregularity and inhomogeneity parameters is explained in several cases graphically as follows:

Fig. (3.2) demonstrate effect of reinforced medium on Love wave's phase velocity in the presence of irregular interface. The Love wave's phase velocity decreasing slightly with increasing the reinforced parameters a_1 and a_3(i.e., $a_1^2 = 0.25, 0.35, 0.45, 0.55$ and $a_3^2 = 0.75, 0.65, 0.55, 0.45$, such that $a_1^2 + a_2^2 + a_3^2 = 1$). In this case, we take the fixed value of heterogeneity parameter $(\alpha = 0.2)$ and depth $(h/H = 0.2)$ of the irregularity.

Fig. (3.3) demonstrate effect of reinforced parameters on the phase velocity of Love wave. In this figure, we take the reverse values of the reinforced parameters (i.e., a_1^2 is decreasing and a_3^2 is increasing with reinforced condition $a_1^2 + a_2^2 + a_3^2 = 1$) and other parameters considered same as fig. (3.2). It is observed that, Love wave's phase velocity increases with decrease in reinforce parameter a_1.

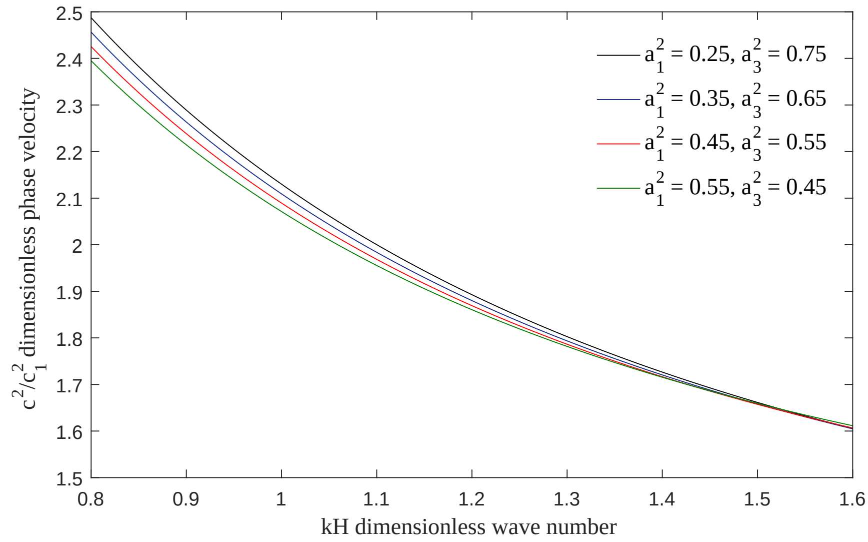

Figure 3.2: Non-dimensional phase velocity c^2/c_1^2 versus dimensionless kH i.e., wave number, with effect of reinforced parameters $(a_1^2, a_3^2) = (0.25, 0.75)$, $(0.35, 0.65)$, $(0.45, 0.55)$, $(0.55, 0.45)$

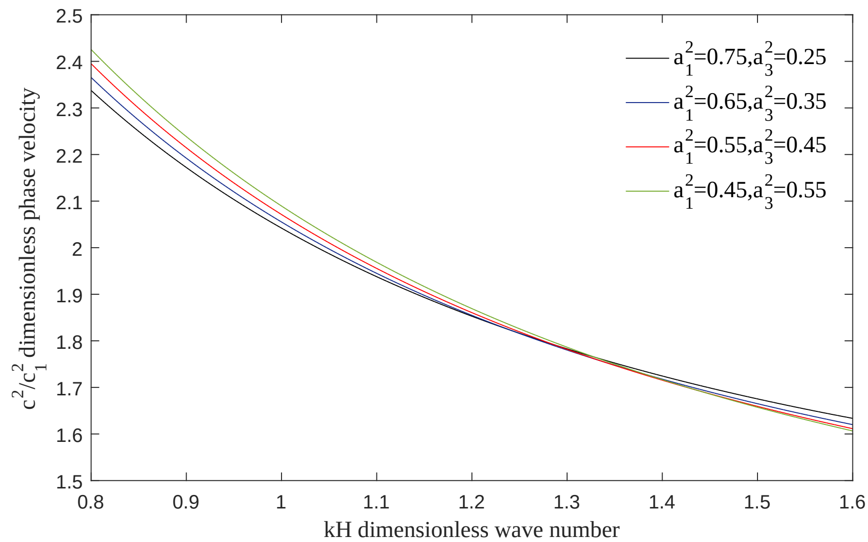

Figure 3.3: Non-dimensional phase velocity c^2/c_1^2 dimensionless kH i.e., wave number, with effect of reinforced parameters $(a_1^2, a_3^2) = (0.75, 0.25)$, $(0.65, 0.35)$, $(0.55, 0.45)$, $(0.45, 0.35)$

Fig. (3.4) depicts the effect of reinforced parameters on Love wave's phase velocity. Here in this case, numerical changes reinforced parameters are (a_1^2, a_3^2)=$(1.0, 0.0)$, $(0.0, 1.0)$, and other parameters taken with fixed value same as fig. (3.2). It is now noticed that the Love wave's phase velocity increases rapidly in this appropriate case.

Fig. (3.5) is showing the effect of heterogeneity parameter α on dispersion curve of propagation Love wave. Here , we considered the effect of heterogeneity parameter in the presence of depth ($h/H = 0.2$) of irregularity on Love wave's phase velocity. Also, presence of heterogeneity in anisotropic half-space affected the phase velocity of Love wave. The phase velocity increases slightly with the increaes in value of heterogeneity parameter (α) .

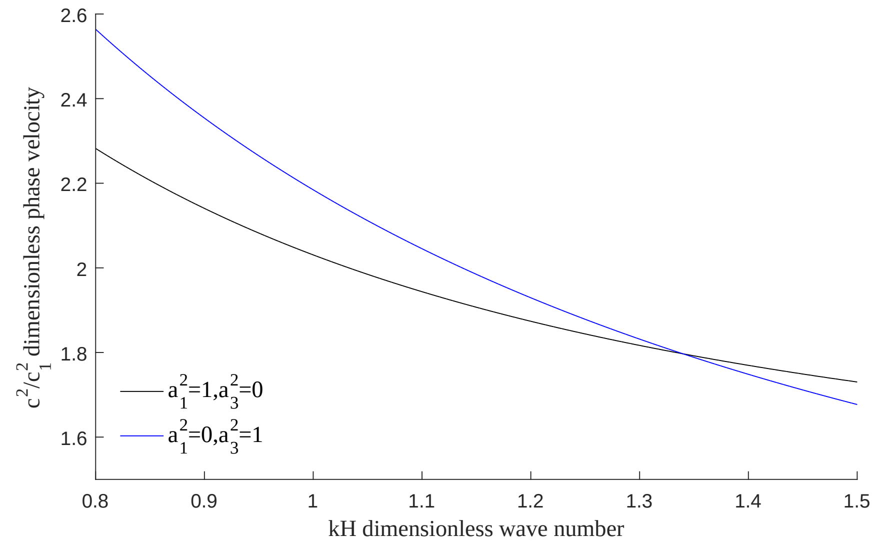

Figure 3.4: Non-dimensional phase velocity c^2/c_1^2 versus dimensionless kH i.e., wave number,with effect of reinforced parameters $(a_1^2, a_3^2) = (1.0, 0.0), (0.0, 1.0)$

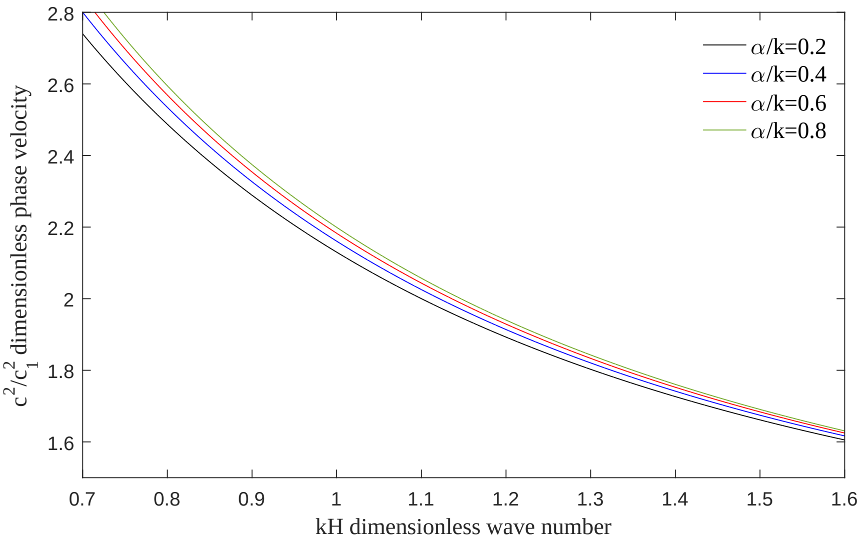

Figure 3.5: Non-dimensional phase velocity c^2/c_1^2 versus dimensionless kH i.e., wave number with effect of heterogeneity parameter $\alpha = 0.2, 0.4, 0.6, 0.8$

Fig. (3.6) represents effect of depth of irregularity on Love wave's phase velocity in reinforced medium. It is observed that the presence of irregularity at interface of reinforced layer and half-space affected speed of Love wave in anisotropic non-homogeneous half-space. The Love wave's phase velocity decreases rapidly with increases in depth of irregularity at interface ($z = \varepsilon f(x)$). The curves i.e., curve 1, curve 2 and curve 3 are plotted for $h/H = 0.1, 0.5 and 0.9$ and other parameters taken same as fig. (3.2).

Fig. (3.7) demonstrate effect of multiplier ε on Love wave's phase velocity.The curves i.e., curve 1, curve 2, curve 3 and curve 4 are plotted for $\varepsilon = 0.2, 0.4, 0.6$ and 0.8. Here in figure, heterogeneity parameter, reinforced parameters and depth of irregularity are taken as $a = 0.2$, $a_1^2 = 0.25$, $a_3^2 = 0.75$ and $h/H = 0.2$ respectively. It is observed that Love wave's phase velocity decreases moderately as value of ε increases.

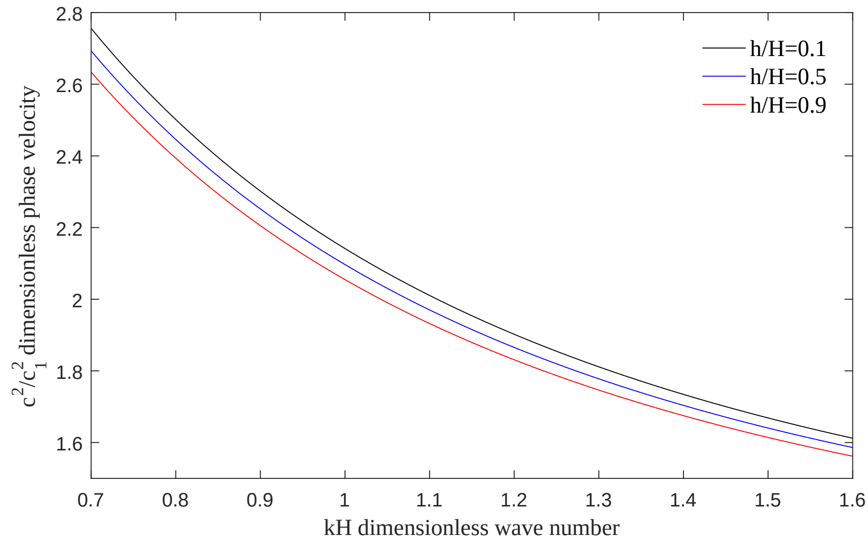

Figure 3.6: Non-dimensional phase velocity c^2/c_1^2 versus dimensionless kH i.e., wave number, with the effect of depth of iregularity $h/H = 0.1, 0.5, 0.9$

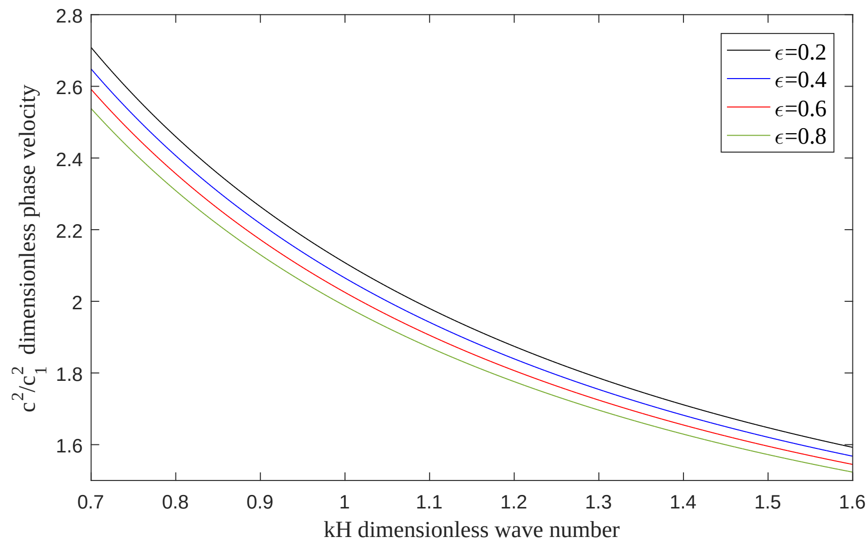

Figure 3.7: Non-dimensional phase velocity c^2/c_1^2 versus dimensionless kH i.e., wave number, with the effect of multiplie of iregularity $\varepsilon = 0.2, 0.4, 0.6, 0.8$.

3.9 Conclusions

The generalized dispersion relation of the Love wave propagation in reinforced medium is derived analytically. The dynamic effect of depth of irregularity and heterogeneity parameter on Love wave's phase velocity in reinforced medium is observed graphically. It has been observed that at the interface of layer and half-space in the presence of irregularity, it significantly affected the Love wave's phase velocity in anisotropic non-homogeneous half-space . An interesting changes have been noticed in the nature of phase velocity in presence of irregularity: the presence of irregularity reflect the effect of reinforced parameters on Love wave's phase velocity. Vaishnav et al. (2017) have been observed that the phase velocity of Love wave decreases in both cases (**case-(i):** numerical value of a_1^2 **increases** and a_3^2 decreases, **case-(ii):** numerical value of a_1^2 decreases and a_3^2 **increases**). But in the present study, the reinforced parameters have opposite effect on the phase velocity of Love wave due to irregular interface of layer and half-space as we can see in fig. (3.2) and fig. (3.3). The depth of irregularity has an impact on the dispersion curve, the phase velocity of Love wave decreases moderately as the thickness (h) of irregularity increases as shown in fig. (3.6). It also has been noticed that the phase velocity of Love wave decreases as the numerical value of α increases. The high impact of reinforced parameters have been observed in fig. (3.4). In this case, the random presence of a_1^2 and a_3^2 increases the phase velocity curves rapidly. The parameter ε play an important role to observe the behavior of Love wave in irregular Earth's structure. The phase velocity of Love wave also decreases as the numerical value of ε increases. The present study is helpful for the seismologist and geologist for the investigation of Earth's interior with several possibilities of minerals.

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