ACCELERATED TARGET DETECTION USING FRACTIONAL FOURIER TRANSFORM

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DECLARATION

I hereby declare that the work, which is being presented in the dissertation, entitled as "**Accelerated Target Detection Using Fractional Fourier Transform (FrFT)**" by me in partial fulfilment of the requirements for the award of degree of Master of Engineering in Wireless Communication submitted at Electronics and Communication Engineering Department of Thapar University, Patiala, is an authentic record of my own work carried out under the guidance of **Dr. Sanjay Kumar (Assistant Professor)**, Electronics and Communication Department and refers other researcher's work which are duly listed in reference section.

The matter presented in this dissertation has not been submitted in any other University/Institute for the award of degree.

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ABSTRACT

The word “RADAR” was originally an acronym, for “Radio Detection and Ranging.” Today, the technology is so common that the word has become a Standard English noun.

In early days, the radar functionality was confined to detecting the target and determining the range. As the advancement of science and technology, there has been a remarkable evolution in the radar technology. Modern communication systems and signal processing has been playing a vital role for detecting the targets with a background of active clutters. Thus, traditional techniques have been playing a key role in order to achieve modern techniques in radar. The nonlinearities that arise in radar cause large interference by affecting the small desired signal and it is difficult to resolve the target from the interference. Some Time-Frequency analysis methods can be implemented in the radar, so that the parameters of the targets can be estimated with good accuracy.

Linear frequency modulated (LFM) signal is widely used for radar system, acoustic communication and sonar system. In a noisy environment detection and estimation of the LFM signal are extremely important, and they gain considerable attention in recent years. Radar transmitted signal is modulated as LFM signal due to the relative motion between radar and target, and nonlinearity exists due to the acceleration of target. Nonlinearity in the signal makes the spectrum aberrant. Target detection using a conventional FFT (Fast Fourier Transform) decreases the performance due to nonlinearity. The aberrant spectrum contains the information of radial acceleration. In the field of military, radar did not provide acceleration information because in the early time aircrafts have low mobility. In these days with the help of advancement in technology, the mobility of aircrafts have increased, therefore the effect of acceleration on signal spectrum of FFT could not be neglected.

This dissertation work is concerned with the study of RADAR signals processing with the ultimate goal of parameter estimation of Accelerated Radar Target. The technique is based on the Fractional Fourier Transform (FrFT), which is better suited for radar applications. The parameter estimation accuracy of system is also analysed with different values of signal to noise ratio (SNR).

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LIST OF ABBREVIATIONS

AF	Ambiguity Function
A/D	Analog To Digital
CSA	Chirp Scaling Algorithm
DFT	Discrete Fourier Transform
DSP	Digital Signal Processing
EEMD	Ensemble Empirical Mode Decomposition
FFT	Fast Fourier Transform
FIR	Finite Impulse Response
FrFT	Fractional Fourier Transform
GMTI	Ground Moving Target Indicator
HAF	High Order Ambiguity Function
IIR	Infinite Impulse Response
ISAR	Inverse Synthetic Aperture Radar
LFM	Linear Frequency Modulated
LPI	low Probability OF Intercept
MAC	Multiplier Accumulator
MCT	Maximum Chirplet transform
MTD	Moving Target Detection
NLS	Non Linear Least Squared
PWVD	Polynomial Wigner Ville Distribution
RMSE	Root Mean Square Error
SLAR	Side Looking Airborne Radar
STFT	Short Time Fourier Transform

STFrFT	Short Time Fractional Fourier Transform
SFFT	Simplified Fractional Fourier Transform
TF	Time Frequency
TFD	Time Frequency Distribution
TFM	Time Frequency Method
WT	Wigner Transform
WD	Wigner Distribution
WVD	Wigner Ville Distribution
WVHT	Wigner Ville Hough Transform

CHAPTER-1

INTRODUCTION

Over the past several decades, the field of Digital Signal Processing has been significantly contributing to the different areas of human endeavours in one way or the other. While conventional signal processing by and large expects stationary behaviour of the signal during the window of observation, it is worthwhile to note that, most of the man-made and natural signals are non-stationary in nature and hence time-frequency methods are more suitable than conventional Fourier based signal processing techniques.

Radar has been shown to be very efficient in object detection and has been used in many applications since World War II. In a radar system, a transmitter sends an electromagnetic signal and detects the echo of the reflected signal from a target. From this reflected signal, important information about the object can be extracted.

The transform is a method to convert a signal from one domain to another domain for extracting some other information contained in the signal which cannot be extracted from the signal in first domain. For example, investigating the utility of Fourier transform on the extraction of information contained in the signal. First the time-domain representation of a signal gives the information about signal's amplitude variation with respect to time but it tends to obscure information about frequency components present in the signal. When Fourier transform is applied to this signal, the resultant transformed signal in frequency domain gives the information about the frequency components present in the signal along with the amplitude associated with each frequency component. One of the important families of transforms is 'Integral Transform'. Actually, integral transform is an operator used to transform a signal into its equivalent form with the help of a 'kernel' function by integrating the kernel multiplied signal. The integration process involved in transformation has conferred the name as 'Integral Transform'. Mathematically, the transform of signal $x(t)$ from t-domain to s-domain can be expressed as:

$$X(s) = \int_{-\infty}^{\infty} x(t)\, K(s,t) dt \tag{1}$$

where, the transformed signal is given as $X(s)$ and $K(s,t)$ is known as kernel function associated with the respective transform. The family of integral transform constitutes many important transforms like: Fourier transform (FT), fractional Fourier transform (FrFT), Laplace transform, Hartley transform, Mellin transform, Hilbert transform, Hankel transform etc.

Over the past several years, with the remarkable innovation in technology, the field of digital signal processing has been also significantly contributing to the different areas of human endeavours in one way or the other. While conventional Fourier based signal processing by and large expects stationary behaviour of the signal during the window of observation, it is worthwhile to note that, the nature of most of the natural and man-made signals are non-stationary and hence time-frequency methods are more suitable than conventional signal processing techniques.

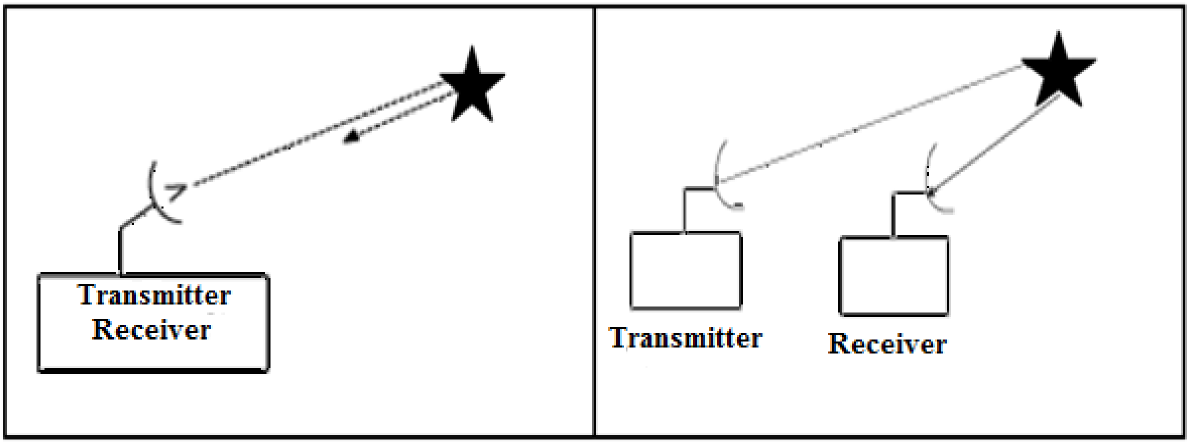
1.1 RADAR HISTORY

In the late nineteenth century, Heinrich Hertz demonstrated that radio waves could go through different type of materials, reflecting part of the transmitted signal. In the early twentieth century, radio waves became an interesting research topic, and scientists tried to find a practical use for radio waves for object detection. Several researchers focused on developing an innovative system to transmit and receive radio waves that would provide useful information about an object.

Radar theory became very important during the World War II, when ships and airplanes were navigated by using radars. Radars were also used to detect enemy objects. In the 1950’s, after the war was over, a new application for radar systems was found. Imaging radar was initially developed for military purposes and was known as Side Looking Airborne Radar (SLAR). A few years after the data acquisition, civilians were allowed to access the classified data for geological and natural resource studies.

1.2 RADAR PRINCIPLE

A radar system includes a transmitter and a receiver. The transmitter sends a radio wave signal. When this signal encounters an object, some portion of it is reflected back to the radar. From the reflected energy of the signal, important information about the target can be extracted. The two types of radar are monostatic radar and bistatic radar [1]. In monostatic radar, the transmitters and receivers are physically very close, while in bistatic radar, the transmitter and receiver are separated by a longer distance (see Figure 1.1).



(a) (b)

Figure 1.1: Radar transmitter and receiver (a) Monostatic and (b) Bistatic Radar [1]

1.3 DIGITAL SIGNAL PROCESSING

Signal processing is a technique that we can use to gather data from the real world and make sense of it. Our brain works as a kind of signal processor. Our sensors collect external stimuli and send the information to our brain, where it is interpreted and used to trigger an appropriate response. For some time, engineers have adapted this idea to develop electronic systems able to extract and process real world signals and turn them into useful data. Most of the signals encountered in the field of science and engineering are functions of a continuous variable such as time or space. Until World War II, analog methods played a dominant role in signal processing. The development of the theory of sampled data systems began in 1940's, which lead to the development of digital signal processing. Eventually, due to the advances in integrated technology, achievements in software engineering and improved algorithms in numerical analysis, the field of DSP experienced rapid expansion. There are several advantages in going for the digital processing of analog signals. These include consistency, accuracy, flexibility, predictability and realization of new algorithms. The emergence of dedicated DSP technology brought processors that were better optimized for signal processing calculations when compared with standard microprocessors [2].

A real world signal is a continuously varying analog signal and this is converted into digital signal by A/D converters, as required by the DSP processors. The continuous analog signal is sampled at Nyquist rate to avoid aliasing. After this, various DSP algorithms are used as required by the application system. Digital filters are used to achieve the desired frequency and phase responses. There are two basic types of digital filters, the finite impulse response (FIR) and infinite impulse response (IIR) filters.

In simple terms, they work as networks of single sample delays and MAC operations. Adaptive filtering allows filter coefficient to be updated while the system is operational. Correlation techniques are used to match two or more signals for detection and delay measurement between them. Algorithms for interpolation and decimation are also widely used. Conventional signal processing methods obtain strength from Fourier techniques named after *Jean Baptiste Joseph Fourier* (1768-1830). It transforms the signal in the time or spatial domain to the frequency domain, in which many characteristics of the signal are revealed. The advent of the Fourier transform algorithm (FFT), has boosted the proliferation of Fourier techniques, by virtue of its speed of implementation [2].

1.4 TIME-FREQUENCY (TF) ANALYSIS

In literature, there are three ways to explore the information about any signal.

- *Time domain representation:* Any signal can be described naturally as a function of time. It gives the information about amplitude variation with respect to time. But it tends to obscure information about frequency, because it assumes that the two variables time and frequency are mutually exclusive and orthogonal.
- *Frequency domain representation:* Any practical signal can be represented in the frequency domain by its Fourier transform. The Fourier transform is in general a complex quantity. Its magnitude is called the magnitude spectrum and phase is defined as the phase spectrum [3]. The square of the magnitude spectrum is the energy spectrum and shows signal energy distribution over the frequency domain. But the magnitude spectrum tells about frequencies that are present in the signal, not the "time of arrival" of those frequencies. Therefore, frequency domain representation hides the information about timing, as FT of a signal does not mention the variable time.
- *Time-frequency representation:* As time and frequency domain representations are inadequate to give all the information posses by the signal, an obvious solution is to seek a representation of the signal as a 'two-variable' function or distribution whose domain is two-dimensional time-frequency space. Its constant-time cross-

section shows the frequencies present at any time and constant-frequency cross-section shows the times at which those frequencies are present [4, 5]. Such a representation is called time-frequency distribution (TFD). Similarly, the plane in which signal is analyzed is defined as time-frequency plane.

Fourier transformation maps one-dimensional time domain signal into a one-dimensional frequency domain signal, i.e., the signal spectrum. Although, the Fourier transform provides the signal's spectral content, it fails to indicate the time location of the spectral components, which is important, for example, when we consider non-stationary or time-varying signals. In order to describe these signals, time-frequency representations are used. A time-frequency representation maps one-dimensional time domain signal into a two-dimensional function of time and frequency.

1.5 UTILITY OF TF ANALYSIS

Time-frequency analysis is a powerful tool which may be used in signal detection, characterization, and processing [6]. Understanding the Doppler frequency shift induced in SAR signal returns is essential in appreciating the utility of TF analysis for GMTI applications. Due to radar platform motion, each scatter on the ground reflects an echo with a Doppler shift proportional to the projection of the platform velocity along the line-of sight (the line passing through the radar and the scattering element) [7]. At broadside, this line-of-sight velocity is zero, and thus all stationary scatterers have a zero Doppler centroid (when observed by side-looking radar). If a scatterer is moving however, an additional Doppler shift is introduced which may vary from pulse to pulse, changing the Doppler centroid and Doppler rate. The echo from the ground will possess a certain Doppler bandwidth proportional to the antenna beam-width for each pulse, whereas the target has a narrow bandwidth for each pulse although its mean Doppler frequency varies through time (i.e. is non-stationary) [7].

In order to focus a target (i.e. perform azimuth compression) to obtain high azimuthal resolutions, one requires accurate knowledge of the relative motion between the radar and the target [7]. However, in some applications, this relative motion is not known to a sufficient accuracy (such as in airborne systems with poor inertial sensors) or the information is not available (such as when the target is moving). In these cases, one can estimate the motion-induced phase shift directly by integrating the instantaneous frequency estimated within the TF domain over time. This phase shift can then be used in a matched filter to achieve a focused image of the target.

Another advantage of TF analysis is that one can determine the instantaneous frequency without making any assumptions regarding its modulation through time. Conventional auto-focusing techniques compensate only linear and quadratic phase shifts, whereas the TF approach allows estimation and compensation regardless of the phase structure [8]. While the conventional techniques may be sufficient to focus a target moving with constant velocity (possessing a nearly parabolic range-history), an accelerating target with non-zero along-track acceleration or time-varying across track acceleration will have a significant cubic term in its range-history, and will benefit from TF focusing methods.

One of the most important applications of TF analysis is estimating a signal's instantaneous Doppler frequency, particularly in the presence of white noise, since it allows exploitation of the different frequency behaviors between signal and noise [8]. Although the target has an extended Doppler bandwidth due to radar-target motion during the synthetic aperture, its instantaneous bandwidth is much smaller, such that a point target has zero instantaneous bandwidth [8]. Conversely, white noise has a large instantaneous bandwidth, and therefore a TF transform will concentrate signal energy along the target's instantaneous frequency, while dispersing noise amongst many frequencies.

1.6 TIME FREQUECY ANALYSIS METHODS (TFM)

TFMs are used to analyze a signal in time and frequency domains simultaneously. A straight forward extension of the conventional Fourier transform, called Short-Time Fourier transform (STFT) attempts to bring out the evolutionary nature of the signals, both in time and frequency. Other than STFT, TFMs have been largely limited to academic research because of the complexity of the algorithms and the limitations in computing power. TFMs are mainly of two categories:

(i) Linear TFMs such as STFT, WT, FrFT

(ii) Quadratic TFMs, also called Energy Distributions such as WVD, Cohen class.

In contrast with the Linear TFMs, which decompose the signal on elementary components, the purpose of the Quadratic TFMs is to deal out the energy of the signal over the two variables viz. time and frequency. Among the Quadratic TFMs, WVD is the simplest and the most powerful, in representation and characterization.

1.6.1 Short-Time Fourier Transform (STFT)

Short-Time Fourier transform (STFT) is known to be the first TFM that was applied in practical systems like speech processing systems, order tracking, ISAR imaging, to name a few applications.

Fourier analysis becomes inadequate when the signal contains non-stationary or transitory characteristics like transients, trends etc. In an effort to correct this, Dennis Gabor [9] adapted the Fourier transform to analyze small sections of the signal at a time. In order to introduce time-dependency in the Fourier transform, a simple and intuitive solution consists in pre-windowing the signal to be analyzed *x(t)* around a particular time *t*, calculating its Fourier transform, and doing that for each time instant *t*. The resulting transform called the Short-Time Fourier transform, is therefore defined as:

$$STFT(\tau, f) = \langle x, g_{t,f} \rangle = \int x(t) g_{t,f}{}^{*}(t) dt = \int x(t) g(t-\tau)\, e^{-i2\pi f t} dt \qquad (2)$$

where, $g(t)$ is a short time analysis window, localized around $t = 0$ and $f = 0$. Because multiplication by the relatively short window $g(t-\tau)$ effectively suppresses the signal outside a neighbourhood around the analysis time point $t = \tau$, the STFT is a local spectrum of the signal $x(t)$. This relation expresses that the total signal can be decomposed as a weighted sum of elementary waveforms $g_{t,f}(t) = g(t-\tau)e^{i2\pi f t}$. These waveforms are obtained from the window *g(t)* by a translation in time and a translation in frequency. The corresponding group of translation in both time and frequency is called Weyl-Heisenberg group.

A time-localized Fourier transform performed on the signal within the window as shown in Figure 1.2. Subsequently, the window is removed along the time, and another transform is performed. The signal segment within the window function is assumed to be stationary. As a result, the STFT decomposes a time signal into a 2D time-frequency domain, and variations of the frequency within the window function are revealed.

While the STFT's compromise between time and frequency information can be useful, the drawback is that once a particular size is chosen for the time window, it remains the same for all frequencies. The time resolution of the STFT is proportional to the effective duration of the analysis window $g(t)$. Similarly, the frequency resolution of the STFT is proportional to the effective bandwidth of the analysis window $g(t)$.

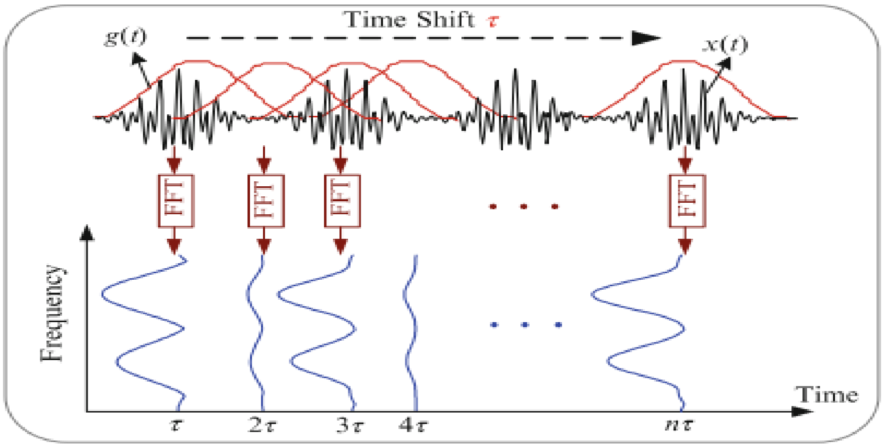


Figure 1.2: Time localized Fourier transform [9].

Consequently, for the STFT, we have a trade-off between the time and frequency resolutions. On one hand, a good time resolution requires a short window $g(t)$. On the other hand, a good frequency resolution requires a narrow-band filter i.e. a long window $g(t)$. This is the major drawback of STFT.

1.6.2 Fractional Fourier Transform (FrFT)

Namias introduced Fractional Fourier Transform [10] in the field of quantum mechanics for solving some classes of differential equations efficiently. Later, Ozaktas *et al.* [11] came up with the discrete implementation of FrFT. Since then, a number of applications of FrFT have been developed, mostly in the field of optics. However, it remains relatively unknown in acoustics.

Little need to be said of the importance and ubiquity of the ordinary Fourier transform in many diverse areas of science and engineering. As a generalization of the ordinary Fourier transform, the FrFT is only richer in theory and more flexible in applications, but not more costly in applications. Therefore, the transform is likely to have something to offer in every area in which Fourier transforms and related concepts are used.

The FrFT is basically a time- frequency distribution. It provides us with an additional degree of freedom (order of the transform p), which in most cases results in significant gains over the classical Fourier transform. With the development of FrFT and related concepts, we see that the ordinary frequency domain is merely a special case of a continuum of fractional Fourier domains. Every property and application of the ordinary Fourier transform becomes a special case of the FrFT. So in every area in which Fourier transforms and frequency domain concepts are used, there exists the potential for improvement by using the FrFT.

FrFT is most likely to improve the solutions to problems where chirps signals are involved. Chirp signals are not compact in time or frequency domain. They appear as inclined lines in the T-F plane and there exists an order for which such a signal is compact. The relationship between the optimum transform order '*p*' relation is used to calculate the optimal order for a sampled linear chirp signal with known chirp rate '*k*'. Conversely, it can be used to estimate chirp rate, given the optimum FrFT order.

Another advantage is that FrFT can be implemented with the same computational complexity as FFT. Ozaktas *et al*. [11,12] have come up with a discrete implementation of Fractional Fourier Transform. Like Cooley-Tukey's FFT, this efficient algorithm computes FrFT in O(NlogN) time which is about the same time as the ordinary FFT. Hence, in applications where FrFT replaces ordinary Fourier transform for performance improvement, no additional implementation cost will occur.

1.6.3 Wavelet Transform (WT)

The wavelet transform is similar to the Fourier transform (or much more to the windowed Fourier transform) with a completely different merit function. The main difference is that: Fourier transform decomposes the signal into sines and cosines, i.e. the functions localized in Fourier space; in contrary the wavelet transform uses functions that are localized in both the real and Fourier space. Generally, the wavelet transform can be expressed by the following equation:

$$\mathrm{wt}(s.\tau) = \langle x, \Psi_{s,\tau} \rangle = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} x(t) \Psi^{*}\left(\frac{t-\tau}{s}\right) dt \qquad (3)$$

where τ shifts time, s modulates the width (not frequency), and Ψ (t) is mother wavelet. By comparing the signal with a set of functions obtained from the scaling and shift of a base wavelet, it is realized as shown in Figure 1.3.

Continuous Wavelet Transform is a transform by which signals can be modeled as a linear combination of translations and dilations of a simple oscillatory function of finite duration called a mother wavelet *ψ(t)*. It provides very good spectral resolution at low frequencies at the expense of temporal resolution and very good temporal resolution at high frequencies at the expense of spectral resolution. This distinct feature of the Wavelet Transform makes it suitable for analyzing non-stationary acoustic signals. Wavelet transforms have been widely applied to the problem of transient detection and processing, primarily because the transform basis functions provide good time localization and it involves the tracking of local transform maxima across analysis scales.

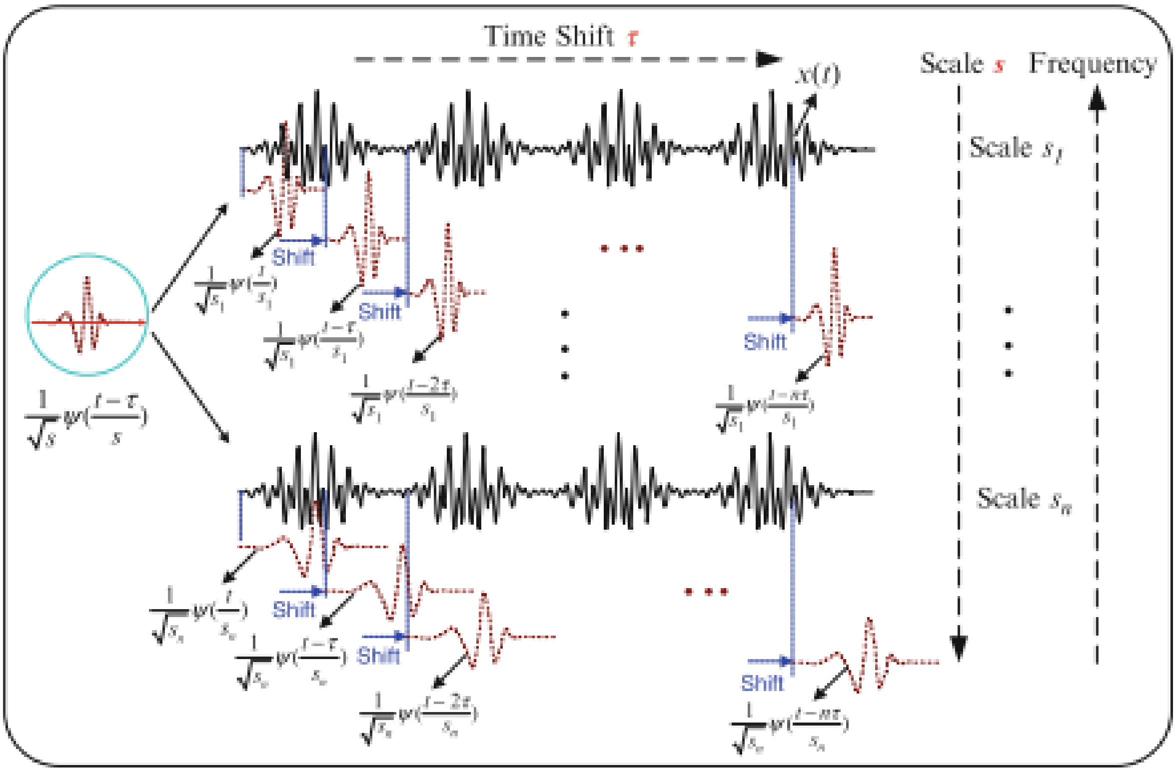


Figure 1.3: Wavelet transform with different scaling shift of basic wavelet [9].

To overcome the problems of redundancy and computational load, Mallat's filter bank implementation called discrete Wavelet transform is now widely used. According to multi scale filtering structure, Wave packet transform can divide the entire time-frequency plane into subtle tilings, while the classical WT can only find its finer analysis for lower-band only. Hence Discrete Wave packet transform is more competent to handle wide-band and high-frequency narrow band signals like transients. As a tool to process data from multiple channels, even this transform is computationally intensive. However, Win Sweldon's Lifting based implementation is a practical solution for the fast implementation of Wavelet and Wave packet transforms.

1.6.4 Wigner Ville Distribution

The Wigner-Ville distribution (WVD) is one member of the Cohen class which is a simple yet powerful tool to analyze the Doppler history of SAR signals [13]. Wigner originally developed the distribution for use in quantum mechanics in 1932, and it was introduced for signal analysis by Ville sixteen years later [14]. To obtain the Wigner-Ville distribution at a particular time, we add up pieces made from the product of the signal at a past time multiplied by the signal at a future time [14]. The continuous WVD of a signal is derived as [13]:

$$WVDs\,(t, f) = \int_{-\infty}^{+\infty} \exp(-j2\pi f\tau)\, s\left(t-\frac{\tau}{2}\right) s * \left(t-\frac{\tau}{2}\right) d\tau \quad (4)$$

The WVD can be regarded as the TF distribution offering the best resolution in the form of delta-pulses along the instantaneous frequency of a signal [13]. Additionally, the lack of smoothing maximally conserves the information content of the signal. The WVD is always real-valued, preserves time and frequency shifts, and satisfies the marginal properties. A more thorough description of the properties of the WVD is offered in [7, 13, 6].

One disadvantage is that problems arise in using the WVD for signals consisting of multiple components. Since it is a non-linear transformation, the WVD signal is not simply the sum of the WVD of each part. For instance, given a signal composed of two parts s_1 and s_2 such that

$$s(t) = s_1(t) + s_2(t) \quad (5)$$

the spectrum of s is the sum of the Fourier transforms of each component:

$$s = s_1 + s_2 \quad (6)$$

However, the energy density (which is related to the WVD of the signal) is not the sum of the energy densities of each part [6]:

$$|s|^2 = (|s_1| + |s_2|)^2 \quad (7)$$

$$= |s_1|^2 + |s_2|^2 + 2\mathrm{R}\ \{|s_1| * |s_2|\} \qquad (8)$$

$$\neq |s_1|^2 + |s_2|^2 \qquad (9)$$

where the $\mathrm{R}\{\cdot\}$ operation retains the real component of its argument.

The non-linearity of the WVD emphasizes the need to remove all clutter contributions to the signal prior to computing the TF transform. If clutter is not removed, even if the signal occupies a bandwidth well-separated from the clutter, the WVD cross-terms may obscure the target signal [7]. If the clutter is removed but the processed signal data contains multiple moving targets, cross-terms between these signals will still be present in the WVD. Generally, detection and tracking of the instantaneous frequency for multiple targets is completed by combining the WVD with the Hough transform [8]. The Hough transform is typically used for detecting straight lines in noisy imagery, although it may also be used to find higher-order polynomials (such as parabolas) traced out by accelerating targets in the time-frequency domain.

1.6.5 Ambiguity Function (AF)

In pulsed radar and sonar signal processing, an ambiguity function is a two-dimensional function of time delay and Doppler frequency $\chi(\tau, f)$ showing the distortion of returned pulse due to the receiver matched filters [14]. Commonly, but not exclusively, used in pulse compression radar due to the Doppler shift of the return from a moving target. The ambiguity function is determined by the properties of the pulse and the matched filter, and not any particular target scenario. Many definitions of the ambiguity function exist, some are restricted to narrowband signals and others are suitable to describe the propagation delay and Doppler relationship of wideband signals. Often the definition of the ambiguity function is given as the magnitude squared of other definitions [15].

For a given complex baseband pulse $s(t)$, the narrowband ambiguity function is given by:

$$\chi(\tau, f) = \int_{-\infty}^{\infty} s(t)s^*(t-\tau)\, e^{-i2\pi f t} dt \qquad (10)$$

Note that for zero Doppler shift $(f = 0)$ this reduces to the autocorrelation of $\mathrm{s(t)}$. A more concise way of representing the ambiguity function consists of examining the one-dimensional zero-delay and zero-Doppler "cuts", that is, $\chi(0, f)$ and $\chi(\tau, 0)$, respectively.

The matched filter output as a function of a time (the signal one would observe in a radar system) is a delay cut, with constant frequency given by the target's Doppler shift:

$$\chi(\tau, f_D) \tag{10}$$

The ambiguity function plays a key role in the field of time–frequency signal processing, as it is related to the Wigner–Ville distribution by a 2-dimensional Fourier transform. This relationship is fundamental to the formulation of other time–frequency distributions: the bilinear time–frequency distributions are obtained by a 2-dimensional filtering in the ambiguity domain (that is, the ambiguity function of the signal). This class of distribution may be better adapted to the signals considered [16].

Moreover the ambiguity distribution can be seen as the short-time Fourier transform of a signal using the signal itself as the window function. This remark has been used to define an ambiguity distribution over the time-scale domain instead of the time-frequency domain.

1.7 DISSERTATION ORGANIZATION

This dissertation includes five chapters. An outline of each chapter is given below:

Chapter 1st gives an introduction of radar system, digital signal processing and time-frequency analysis techniques.

Chapter 2nd is dedicated to the literature survey. The research papers which are relevant to this thesis are discussed here.

Chapter 3rd presents a study of linear chirp signal, fractional Fourier Transform and its properties. Also it discussed discrete implementation of FrFT and its desirable features for radar target detection.

Chapter 4th presents the different algorithms and mathematical model used in radar target detection and also includes meaningful results of radar target detection and its parameter estimation, estimated value will be studied with different parameters.

Chapter 5th concludes this dissertation, summarizing the major results and offering suggestions for future work on this topic.

CHAPTER-2

LITERATURE SERVEY

In order to start the dissertation, the first step is to study the research papers that have been published by other researchers. The papers that are related to this title are chosen and studied. With the help of this literature review, it gives more clear understanding to write this dissertation.

The topic of time-frequency methods is one of the modern DSP tools for non-stationary signal processing. Like all fields and particularly emerging ones, it has a plethora of different motivations. Many applications are reported in the fields of speech and image processing, communications, radar etc.

The application of Fractional Fourier Transform (FrFT) in parameters estimation of radar echo is the latest topic of interest. Many Time–Frequency methods are proposed including FrFT in the field of radar signal processing.

Namias introduced Fractional Fourier Transform in the field of quantum mechanics for solving some classes of differential equations efficiently [10]. Since then, a number of applications of Fractional Fourier Transform have been developed, mostly in the field of optics. The motivation behind the proposed method is the ability of FrFT to process chirp signals better than the conventional Fourier Transform. FrFT is basically a time-frequency distribution, a parameterized transform with parameter a, related to the chirp rate. It provides us with an additional degree of freedom (order of the transform), which in most cases results in significant gains over the classical Fourier transform. It is well known that in sonar systems, chirp processing can be applied in a number of areas. Some FrFT applications are reported in radars.

Ozaktas *et al.* [11, 22] have come up with a discrete implementation of Fractional Fourier Transform. Like Cooley-Tukey's FFT, this efficient algorithm computes FrFT in O(NlogN) time which is about the same time as the ordinary FFT. Hence, in applications where FrFT replaces ordinary Fourier transform for performance improvement, no additional implementation cost will occur.

Candan *et al.* [24] gives a satisfactory definition of the discrete FrFT that is fully consistent with the continuous transform. This definition has the same relation with the DFT as the continuous FrFT has with the ordinary continuous Fourier Transform.

Almeida [25] has interpreted FrFT as a rotation in the time-frequency plane. This paper describes its relationship with other TFMs such as WVD, AF, STFT and spectrogram, which support's the FrFT's interpretation as a rotational operator.

Ozaktas *et al.* [4] proposed filtering method in fractional Fourier domains may enable significant reduction of MSE compared to ordinary Fourier domain filtering. This reduction comes at essentially no additional computational cost because of the availability of the efficient algorithm for computing FrFT.

Jozef *et al.* [26] have developed an original method for constructing the TFM from the squared magnitudes of their FrFT outputs, using alpha-norm minimization by Renyi entropy maximization. In radar target identification problems, the target is assumed to have rigid body motion. But in real-world situations, a target may have rotating part beside the main body, like a helicopter with a rotor or a ship with scanning radar. Then, it is difficult to extract motion information (Doppler) using conventional techniques. Another scenario is maneuvering targets, such as aircrafts and missiles, where the Doppler frequencies are time- varying. TFMs like adaptive Chirplet representation have shown potential in these two radar applications.

Capus [17, 18] *et al.* have proposed the short-time implementation of FrFT. STFT variants of FrFT can be implemented in two ways, depending on how the optimum alpha is chosen. The optimum alpha can be selected for the whole data block, or one for each processing block length. These implementations show improvements in time-frequency resolutions with bat signals, linear and non-linear chirps. Individual chirps in a mixture of chirps can be extracted using FrFT by a filtering and reconstruction technique. Both linear as well as non linear chirps can be extracted by this method.

Sun *et al.* [27] have employed FrFT in radar signal processing. FrFT is applied in airborne SAR for detection of slow moving ground targets. For airborne SAR, the echo from a ground moving target can be regarded approximately as a chirp signal and FrFT is a way to concentrate the energy of a chirp signal. Unlike WVD, FrFT is a linear operator and do not suffer from cross terms. Moreover, to solve the problem whereby weak targets are shadowed by the side lobes of strong ones, a new filtering technique called CLEAN is used, thereby detecting strong and weak moving targets iteratively.

Djuric *et al.* [28] proposed the problem of the parameter estimation of chirp signals. Several closely related estimators are proposed whose main characteristics are simplicity, accuracy, and ease of on-line or off-line implementation. For moderately high signal-to-noise ratios they are unbiased and attain the Cramer-Rao bound. The Monte Carlo simulations verify the expected performance of the estimators. The approaches they have proposed for joint estimation of frequency rate, frequency and phase and frequency rate alone are simple, accurate, and achieve the Cramer-Rao bound for signal-to-noise ratios higher than 8 dB.

Boashash *et al.* [29] proposed the generalization of the WVD in order to effectively process nonlinear polynomial FM signals. A class of polynomial WVD's (PWVD's) that give optimal concentration in the time-frequency plane for FM signals with a modulation law of arbitrary polynomial form are defined. PWVD of nonlinear polynomial FM signals produce a row of delta functions along its IF law in the t - f plane. The expected values of these PWVD's are the Fourier transforms of some particular higher order moments and/or cumulates.

Daponte *et al.* [30] proposed an echo detection techniques based on time-frequency signal analysis for measuring of thickness in thin multilayer structures. These techniques are shown to provide high-resolution signal characterization in a time-frequency space, and good noise rejection performance. The experimental analysis was carried out by first emulating signals with different SNR's and noise bandwidths. Traditional techniques fail when the SNR decreases, whereas the time-frequency signal analysis achieves satisfactory performances.

Boashash *et al.* [31] proposed the correct use of the Wigner Distribution (WD) for time-frequency signal analysis requires use of the analytic signal. This version, often referred to as the Wigner-Ville Distribution (WVD), is straight forward to compute, does not exhibit any aliasing problem, and introduces no frequency artefacts. The problems introduced by the use of the Wigner Distribution with a real signal are clarified. It contains essentially the same information as the Wigner Distribution, but does not exhibit low-frequency artefacts produced in the real WD.

Besson *et al.* [32] proposed a method for the problem of estimating the parameters of a chirp signal observed in multiplicative noise, i.e., whose amplitude is randomly time-varying. Two methods for solving this problem are presented. First, an unstructured nonlinear least-squares approach (NLS) is proposed. The second approach combines the principle behind the high-order ambiguity function (HAF) and the NLS approach. Simulation results were presented that attested to the validity of the theoretical analysis. The NLS estimator was shown to provide slightly better performance than the HAF-based estimator.

Salemian *et al.* [33] introduced the principle behind the pulse compression radar. Pulse compression is an important signal processing technique used in radar system to reduce the peak power of a radar pulse by increasing the length of pulse without sacrificing the range resolution associated with a shorter pulse. The problem of the signal losses in a compression filter has been analyzed and explained. After simulation we find that use of poly phase code in small and medium range and use NLFM and weighted LFM for long range.

Gal *et al.* [34] addressed the problem of estimating the chirp signals embedded in Gaussian noise. The proposed method is based on a model of the signal phase as a polynomial. This approach offers the opportunity to represent these signals by an adequate state space model and to apply standard Kalman filtering procedures in view to estimate the parameters of chirp signals. Procedure simulations were made on linear chirp sinusoids with time-varying amplitude and are consistent with the theoretical approach.

Du *et al.* [21] proposed the use of fractional Fourier Transformation (FrFT) to estimate radial acceleration from radar echo. The acceleration estimation formula was deduced firstly, and then its estimation flow was given out. Differences in anti noise interference capability between FrFT, WVD-HT (Wigner-Ville Distribution and Hough Transformation) and WVD (Wigner-Ville Distribution) in estimating chirp signal parameters were analyzed. The algorithm to estimate radial acceleration parameters by FRFT was brought forward, analyzing qualitatively that FrFT is capable of estimating radial motion parameters excellently.

Jocab *et al.* [35] proposed detailed evaluation of a detector based on FrFT for detecting chirps. The motivation behind the proposed evaluation is the inherent ability of

FrFT detector ias compared to the conventional Fourier transform. Detection performance in white Gaussian noise as well as 1/f has been studied.

Geroleo *et al.* [36] proposed that the Wigner-Ville Hough transform (WVHT) is suboptimal in the detection and parameter estimation of linear frequency-modulated (LFM) continuous wave (LFMCW) low probability of intercept (LPI) radar waveforms because they are composed of concatenated LFM pulses. The new algorithm, called the periodic WVHT (PWVHT), significantly outperforms the WVHT for LFMCW signals.

Tao *et al.* [37] proposed short-time fractional Fourier transform (STFrFT) to locate the fractional Fourier domain (FrFD)-frequency contents which are required in some applications. It displays the time and FrFD-frequency information jointly in the short-time fractional Fourier domain (STFrFD). The time-FrFD-bandwidth product (TFBP) is defined to measure the resolvable area and the STFrFD support. It displays the chirp signal with high concentration and no cross terms, thus it plays a powerful role in the 2-D analysis of this kind of signal.

Zhang *et al.* [38] proposed analysis and processing of chirp pulses using the matched fractional Fourier transform (FrFT). The method for side lobe suppression using the matched FRFT is also proposed. For the chirp pulse, we can use the FrFT to complete the pulse compression with matched transform angle. By the resolution performance analysis, we can see that the matched FrFT can achieve the same resolution ability for the time delay and the Doppler parameters. Moreover, to suppress the side lobe level of the matched FrFT for the chirp pulse, we can add the weighting function in the transform kernel directly.

Millioz *et al.* [39] proposed a maximum chirplet transform (MCT), a simplification of the chirplet transform. A detection of the relevant maximum chirplets is proposed based on iterative masking, an iterative detection followed by window subtraction that does not require the recomputation of the spectrum. The detected chirps have been then gathered back into the FMCW signals constituting the analyzed signal, using a criterion based on the time-frequency proximity of the starting and ending points of the detected chirps.

Lin *et al.* [20] proposed a new method for the detection and parameter estimation of multicomponent LFM signals based on the fractional Fourier transform. For the optimization in the fractional Fourier domain, an algorithm based on Quasi-Newton method is proposed which consists of two steps of searching, leading to a reduction in computation without loss of accuracy. And for multicomponent signals, we further propose a signal separation technique in the fractional Fourier domain which can effectively suppress the interferences on the detection of the weak components brought by the stronger components.

Ma *et al.* [40] proposed the joint estimated radial velocity and time delay based Fractional Fourier Transform. They are closed related with the location of maximum FrFT spectrum of echo. At last, from very promising simulations, we can demonstrate the feasibility of such a new approach and its performance. Especially in underwater target case, in which the target range is very small, the search order can be limited in small range. And using the fast FRFT computation, the FrFT based is expected to estimate the radial velocity and range of target in practice.

Wang *et al.* [41] proposed a novel approach for the detection and parameter estimation of weak linear frequency modulated (LFM) signal based on Stochastic Resonance (SR). By correlating the segmented equal length LFM signals, the aperiodic LFM signal is transformed into periodic signal which is then input into SR system to estimate the chirp rate of LFM signal. The proposed method can accurately estimate the chirp rate and the initial frequency of LFM in the condition of SNR $\geq$ -20dB. The achieved SNR of parameter estimation is much lower than that of time frequency analysis methods, such as FrFT, STFT and WV based methods, and it does not require a prior knowledge of parameter range of LFM signal.

Cristallini *et al.* [42] proposed an innovative scheme for moving target detection and high resolution focusing that exploits a bank of chirp scaling algorithms (CSA), each one matched to a different along track target velocity component. An efficient SAR-MTI processing technique has been proposed on the basis of a bank of focusing filters (based on the CSA) each one matched to a different possible at target velocity component. The bank of focusing filters has a positive effect both on target detection and imaging performance, has been demonstrated on an emulated data set.

Tao Ran *et al.* [43] proposed a method of radar moving target detection and estimation based on FrFT. The moving target detection (MTD) method based on FrFT was compared with the method based on WVD, FrFT based MTD method does not produce the cross-term in the case of multiple target, thus it simplify the handling process, improve detection results, and lower false alarm possibility.

Rong chen *et al.* [44] proposed a novel parameter estimation method of chirp signal. By analyzing the practical signal form, the application limitation of the existing method is presented and its estimation error is derived. It is prove that the starting time of the chirp in the observed window should be taken into account to estimate the parameter of a practical observed chirp signal. In addition the energy integrity and sampling duration also play important role in parameter estimation.

Kumar *et al.* [45] proposed a new FrFT based ambiguity function to estimate the delay and Doppler in the received target echo to overcome the large computational complexity of the previous method. The performance of purposed method in term of sensitivity is observed in the presence of AWGN noise.

Zhang *et al.* [46] proposed a pre-estimation algorithm (PEA) to estimate the approximate chirp ratio of multi–component linear frequency modulated signal. A simplified fractional Fourier transform (SFFT) is introduced to estimate the all parameters of multi-LFM signal. Then, a new fast method combining STFT with PEA and CLEAN technique is presented for multi–LFM signal detection and parameter estimation.

Manman *et al.* [47] proposed a novel method for the multi-component LFM signal filtering based on the short-time fractional Fourier transform (STFRFT). By choosing an optimal rotation angle and adjusting the window width, interferences and noises in the multi-component signals can be separated and suppressed efficiently in the short-time fractional Fourier domain. The STFRFT can not only retain the linear properties of the short time Fourier transform, but also reduce the impact of Gibbs effect. The chirp signal in STFRFD has high concentration and little cross terms. Compared with the one-dimensional FRFT filtering, the STFRFT works well in the multi-component LFM signal separation. This method integrates the FRFT with STFT, and by adjusting the width of the Gaussian window function high frequency resolution is obtained. The merits of STFRFT are that it is not only richer in theory and more flexible in application but the cost of implementation is also low.

Hao *et al.* [48] proposed the method of multi component LFM signal detection and parameter estimation based on EEMD–FrFT (Ensemble Empirical Mode

Decomposition–Fractional Fourier transform), and this method was that with the EEMD algorithm, from the frequency domain decompose the analyzable signal to narrow-bandwidth components, whose center frequency changed from high to low, then accurately estimate the parameter and detect the signal of each component out of the pseudo-component with FrFT. This method solved the problem of mode aliasing

Song *et al.* [49] proposed introduces two iterative interpolation algorithms for the parameter estimation of linear frequency modulation (LFM) signal using fractional Fourier transform (FrFT). The estimated parameter of an LFM signal can be obtained by locating the peak of the periodogram in the FrFT domain. Two interpolation algorithms were proposed to improve the accuracy of parameter estimation by employing the FrFT coefficients relative to the true parameters and applying interpolation algorithms iteratively to refine the parameter estimation approach. The proposed algorithms can utilize more information from FrFT results, thereby achieving improvements in either accuracy or efficiency.

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CHAPTER 3
FRACTIONAL FOURIER TRANSFORM AND THEORY

Chirps are signals which exhibit a change in instantaneous frequency with time (either linear or non-linear) and are of particular interest in sonar, radars, acoustic communications, seismic surveying, ultrasonic applications, etc. The potential of FrFT lies in its ability of FrFT to process chirp signals better than the conventional Fourier Transform. The transform absorbs the chirp parameters in its kernel by a parameter p.

Namias introduced Fractional Fourier Transform [10] in the field of quantum mechanics for solving some classes of differential equations efficiently. Later, Ozaktas et al [11] came up with the discrete implementation of FrFT. Since then, a number of applications of FrFT have been developed, mostly in the field of optics. However, it remains relatively unknown in signal processing.

Little need to be said of the importance and ubiquity of the ordinary Fourier transform in many diverse areas of science and engineering. As a generalization of the ordinary Fourier transform, the FrFT is only richer in theory and more flexible in applications, but not more costly in applications. Therefore, the transform is likely to have something to offer in every area in which Fourier transforms and related concepts are used. The FrFT is basically a time- frequency distribution. It provides us with an additional degree of freedom (order of the transform), which in most cases results in significant gains over the classical Fourier transform. With the development of FrFT and related concepts, we see that the ordinary frequency domain is merely a special case of a continuum of fractional Fourier domains. So in every area in which Fourier transforms and frequency domain concepts are used, there exists the potential for improvement by using the FrFT.

3.1 BASIC CONCEPT OF FRACTIONAL TRANSFORM

So far, we have seen the definitions of conventional Fourier Transform. Before formally defining the Fractional Fourier Transform, we want to know that “What is a fractional transform?” and “How can we make a transformation to be fractional?” First we see a transformation *T,* we can describe the transformation as following:

$$T\{f(x)\} = F(u) \tag{12}$$

where, f and F are two functions with variables x and u respectively. As seen, we can say that F is a T transform of f. Now, another new transform can be defined as below:

$$T^p\{f(x)\} = F_p(u) \tag{13}$$

We call T^p here the "p-order fractional T transform" and the parameter p is called the "fractional order". This kind of transform is called "fractional transform".

Which satisfy following constraints:

1. Boundary conditions:

$$T^0\{f(x)\} = f(u) \tag{14}$$

$$T^1\{f(x)\} = F(u) \tag{15}$$

2. Additive property:

$$T^p\{T^q\{f(x)\}\} = T^{p+q}\{f(x)\} \tag{16}$$

3.2 LINEAR CHIRP SIGNAL

A linear chirp signal, its phase and its instantaneous frequency are given by the following equations. Two parameters completely define a chirp namely the start frequency f_0 and slop of the chirp.

Chirp signal $= e^{j(kt^2+f_0t+c)}$ (17)

Phase $= kt^2 + f_0t + c$ (18)

Instantaneous frequency = $2k$t + f_0 (19)

where, $f_0, c\ and\ 2k$ are the starting frequency, initial phase and chirp rate or slope respectively.

3.3 OVERVIEW OF FrFT

The FRFT of given signal x(t) is considered as: [17,18]

$$\mathrm{X_p(t,u) = F^p[y(t)\,] = \int_{-\infty}^{\infty} x(t)\ K_\alpha(t,u)\ dt} \tag{20}$$

where, p is a real number and called the order of the FRFT, F^p $[\cdot]$ denotes the FRFT operator, and $K_p(t,u)$ is the kernel of the FRFT:

$$K_p(t,u) = \begin{cases} \sqrt{\frac{1-j\cot\alpha}{2\pi}} \exp\left(j\frac{t^2+u^2}{2}\cot\alpha - tu\csc\alpha\right), & \alpha \neq n\pi, \\ \delta(t-u), & \alpha = 2n\pi, \\ \delta(t+u), & \alpha = (2n+1)\pi, \end{cases}$$

(21)

where, $\alpha = p\,\frac{\pi}{2}$ (22)

FrFT computation can be interpreted as a sequence of steps viz. a multiplication by a chirp in one domain followed by a Fourier transform, then multiplication by a chirp in the transform domain and finally a complex scaling. So, chirps form the basis functions of FrFT.

There are various other definitions of the FrFT. Of all these, the definition given above is particularly desirable because of its many properties and the relation to the classical Fourier transform.

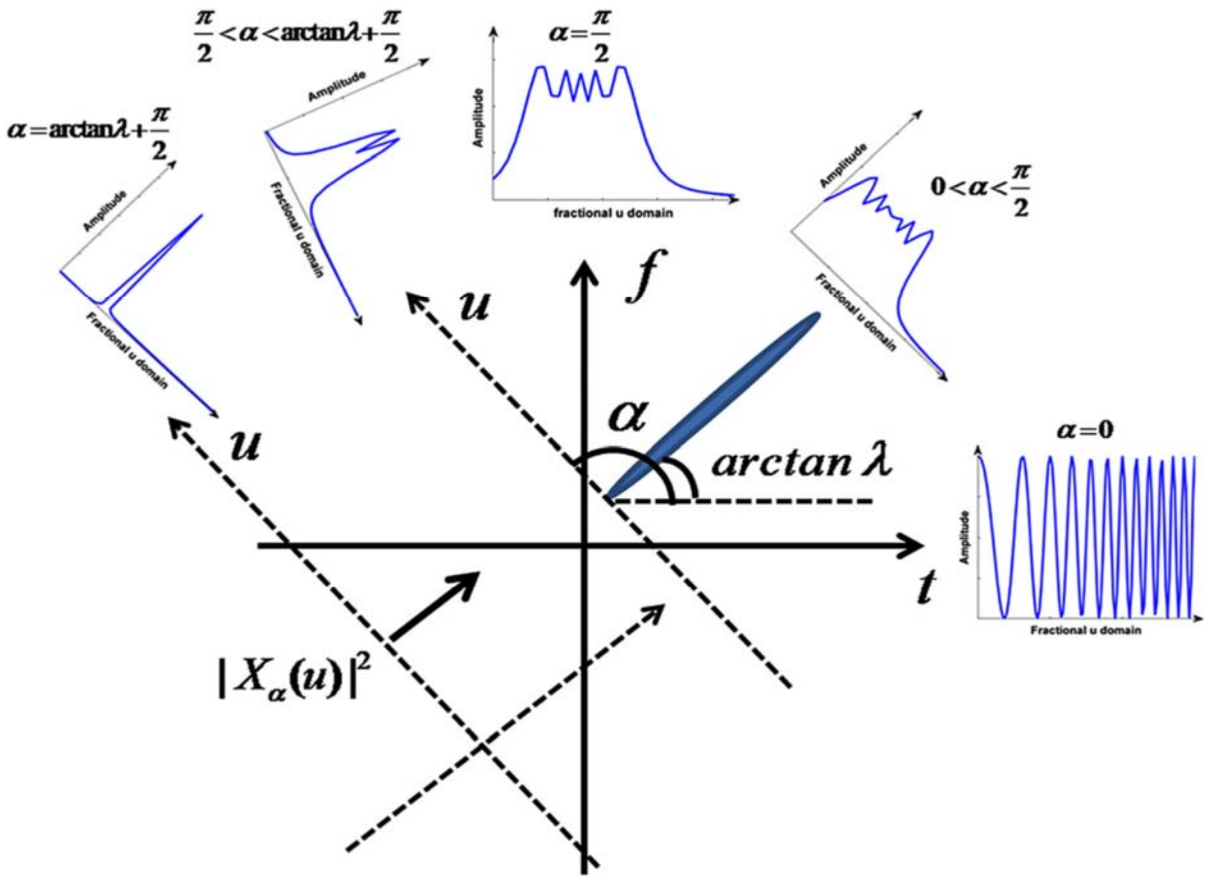

Figure 3.1 FrFT of the chirp signal *x (t)* with different order of transform [17].

As shown in figure 3.1, it is also interesting to note that this definition of the FrFT reduces to the classical FT when the order of the transformation p = 1. For p = 1 and -1, the transform corresponds to ordinary forward and inverse Fourier Transforms respectively where u and t represent frequency and time respectively.

3.4 TRANSFORM OPTIMIZATION

The FrFT parameter p is used to tune the transform to provide an optimal response to a given linear chirp signal. When the axis of rotation is matched to the chirp rate of the signal, the magnitude response of FrFT reaches its maximum. This procedure is known as transform optimization. The corresponding p is called the optimum p.

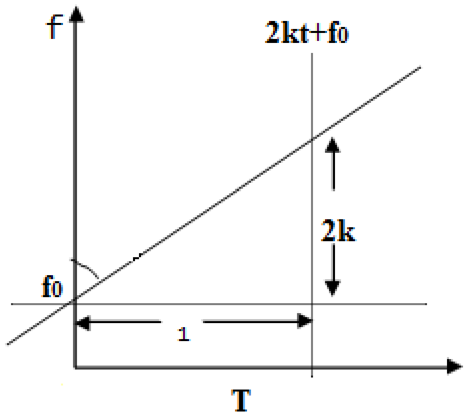

Figure 3.2: Relationship of Chirp rate and FrFT order [22].

Figure 3.2 shows the time-frequency plot of a chirp. There are two methods to describe chirp rate. The first is the quadratic phase parameter k in the algebraic definition of the linear chirp given in (17). It is also given as the optimum p parameter in the FrFT definition in (22). The relationship between the two is given as [17, 18]:

$$p_{opt} = \frac{2a}{\pi} = \frac{2}{\pi}\tan^{-1}\left(\frac{1}{2k}\right) \tag{23}$$

$$p_{opt} = \frac{2}{\pi}\tan^{-1}\left(\frac{{f_s}^2/N}{2k}\right) \tag{24}$$

The true relationship is dependent on the digital sampling scheme used and is given in (24), where f_s is the sampling frequency and N is the number of samples in the chirp signal. This relation is used to calculate the optimal order for a sampled linear chirp signal with known chirp rate k. Conversely, it can be used to estimate chirp rate, given the FrFT order p. The optimum FrFT order cannot be found analytically in general. So, a one dimensional search for p is necessary to find the optimum order, with which the chirp focuses well, i.e. On a given block of data, FrFT is done for different value of p, and we select the one that yield the maximum peak value.

3.5 FUNDAMENTAL PROPERTIES OF THE FRACTIONAL FOURIER TRANSFORM

In this section, we list some fundamental properties of FrFT, such as those for scaling, coordinate multiplication, and differentiation [19].

(1) Linearity

Let $\mathcal{F}^a$ denotes the ath order fractional transform operator. Then

$$\mathcal{F}^a[\sum_k b_k f_k(u)] = \sum_k b_k \mathcal{F}^a[f_k(u)] \tag{25}$$

(2) Integer order

When a is equal to an integer k, the ath order fractional Fourier transform is equivalent to the kth integer power of the ordinary Fourier transform, defined by repeated application. It means that

$$\mathcal{F}^k = (\mathcal{F})^k \tag{26}$$

Moreover, it has following relation

$$\mathcal{F}^2 = \mathcal{P}\ (parity\ operator) \tag{27}$$

$$\mathcal{F}^3 = \mathcal{F}^{-1} = (\mathcal{F})^{-1}\ \ (inverse\ transform\ operator) \tag{28}$$

$$\mathcal{F}^4 = \mathcal{F}^0 = \mathcal{I}\ \ (identity\ operator) \tag{29}$$

$$\mathcal{F}^j = \mathcal{F}^{j\ mod\ 4} \tag{30}$$

(3) Inverse

$$(\mathcal{F}^a)^{-1} = \mathcal{F}^{-a} \tag{31}$$

(4) Index additivity

$$\mathcal{F}^{a_1}\mathcal{F}^{a_2} = \mathcal{F}^{a_1+a_2} \tag{32}$$

(5) Commutativity

$$\mathcal{F}^{a_1}\mathcal{F}^{a_2} = \mathcal{F}^{a_2}\mathcal{F}^{a_1} \tag{33}$$

(6) Associativity

$$(\mathcal{F}^{a_1}\mathcal{F}^{a_2})\mathcal{F}^{a_3} = \mathcal{F}^{a_1}(\mathcal{F}^{a_2}\mathcal{F}^{a_3}) \tag{34}$$

(7) Parseval

$$\int f^*(u)g(u)du = \int f_a^*(u)g_a(u)du \tag{35}$$

(8) Time reversal

Let $\mathcal{P}$ denotes the parity operator. $\mathcal{P}[f(u)] = f(-u)$, then

$$\mathcal{F}^a\mathcal{P} = \mathcal{P}\mathcal{F}^a \tag{36}$$

$$\mathcal{F}^a[f(-u)] = f_a(-u) \tag{37}$$

(9) Transform of a scaled function

Let $\mathcal{M}(M)$ and $\mathcal{Q}(q)$ denotes the scaling and chirp multiplication operators, respectively. The definition of $\mathcal{M}(M)$ and $\mathcal{Q}(q)$ are as following.

$$\mathcal{M}(M)[f(u)] = |M|^{-\frac{1}{2}}f(\frac{u}{M}) \tag{38}$$

$$\mathcal{Q}(q)[f(u)] = e^{-i\pi qu^2}f(u) \tag{39}$$

Then,

$$\mathcal{F}^a\,\mathcal{M}(M) = \mathcal{Q}\left(-cot\left(\frac{1-cos^2\alpha'}{cos^2\alpha}\alpha\right)\right) \times \mathcal{M}\left(\frac{sin\alpha}{M\,sin\alpha'}\right)\mathcal{F}^{a'} \tag{40}$$

$$\mathcal{F}^a\left[|M|^{-\frac{1}{2}}f\left(\frac{u}{M}\right)\right] = \sqrt{\frac{1-i\,cot\alpha}{1-iM^2\,cot\alpha}}\,e^{i\pi u^2 cot\left(\frac{1-cos^2\alpha'}{cos^2\alpha}\alpha\right)} \times f_a\left(\frac{Mu\,sin\alpha'}{sin\alpha}\right) \tag{41}$$

Notice that the fractional Fourier transform of $f(\frac{u}{M})$ cannot be expressed as a scaled version of $f_a(u)$. Rather, the fractional Fourier transform of $f(\frac{u}{M})$ turns out to be a scaled and chirp modulated version of $f_{a'}(u)$ where $a \neq a'$ is a different order.

(10) Transform of a shifted function

Let $\mathcal{SH}(u_0)$ and $\mathcal{PH}(\mu_0)$ denotes the shift and the phase shift operators, respectively. The definition of $\mathcal{SH}(u_0)$ and $\mathcal{PH}(\mu_0)$ are as following:

$$\mathcal{SH}(u_0)[f(u)] = f(u + u_0) \tag{42}$$

$$\mathcal{PH}(\mu_0)[f(u)] = e^{i2\pi\mu_0 u} f(u) \tag{43}$$

Then,

$$\mathcal{F}^a \mathcal{SH}(u_0) = e^{i\pi u_0^2 sin\alpha cos\alpha}\, \mathcal{PH}(u_0 sin\alpha)\, \mathcal{SH}(u_0 cos\alpha)\mathcal{F}^a \tag{44}$$

$$\mathcal{F}^a[f(u+u_0)] = e^{i\pi u_0^2 sin\alpha cos\alpha} e^{i2\pi u u_0 sin\alpha} f_a(u + u_0 cos\alpha) \tag{45}$$

(11) Transform of a phase-shifted function

$$\mathcal{F}^a \mathcal{PH}(\mu_0) = e^{-i\pi\mu_0^2 sin\alpha cos\alpha} \mathcal{PH}(\mu_0 cos\alpha)\mathcal{SH}(-\mu_0 sin\alpha)\mathcal{F}^a$$

$$\mathcal{F}^a\left[e^{i2\pi\mu_0 u} f(u)\right] = e^{-i\pi\mu_0^2 sin\alpha cos\alpha} e^{i2\pi u u_0 cos\alpha} f_a(u - u_0 sin\alpha) \tag{46}$$

3.6 DISCRETE IMPLEMENTATION OF FrFT

A number of discrete implementations have been put forward. The most satisfactory ones, consistent with the important properties of index additivity, unitarity and reduction to DFT for unit order, are those implementations based on the discrete Hermite-Gaussian functions. To date, there is no fast algorithm for the exact computation of the discrete FrFT. However, a fast O (N log N) algorithm has been proposed, which calculates an approximation to the discrete samples of the FrFT with sufficient accuracy for many applications [12].

3.7 DESIRABLE FEATURES OF FrFT FOR ACTIVE AND INTERCEPT RADAR TARGET DETECTION

Chirps are not compact in the time or frequency domain. But, since chirps form the basis functions in FrFT, there exists an order for which it is compact in the FrFT domain. So, FrFT will improve solutions to problems where chirps signals are involved. Hence, FrFT is the ideal transform for processing chirp signals in active and intercept radar target detection. However, the algorithms for these two applications will be different. In the case of active Radar, the transmitted chirp signal is known a priori, and hence calculation of the optimum transform order p is straight forward. However, in the case of intercept sonar, the received waveform is unknown. So, to apply FrFT, a search algorithm has to be implemented to find the optimum transform order. For the problem of multiple chirps overlapping in time and frequency, an extraction algorithm will be required. All these additional challenges have been addressed successfully in the new technique, developed in the present thesis work and it will be detailed in Chapter 4.

CHAPTER-4
ACCELERATED TARGET DETECTION IN ACTIVE RADAR USING FrFT

Improving the detection performance in active radar can result in more target detection range. In this chapter, the potential of Fractional Fourier Transform (FrFT) in active radar processing for accelerated target detection is explored. The motivation behind the proposed method is the ability of FrFT to process chirp signals better than the conventional Fourier transform and also the preferred choice of chirp signal in active radar. The new scheme developed in this thesis, using FrFT is explained, followed by illustrative simulation results. In the simulations, the detection performances of the new method are plotted.

4.1 ACTIVE RADAR SCENARIO

In the simplest active radar system, a transmitter produces an electromagnetic (EM) pulse of short duration of the order of milliseconds. This pulse is transmitted through transducer array into the air medium, where the resulting EM wave propagates out at the speed of Light. A target in the path of this wave will reflect a portion of the energy back toward the same or another receiving array. The DOA algorithms like beam forming will bring out the bearing of the target, and also spatially filter the signal. The waveform of the received signal, obtained after spatial filtering, is the shifted and scaled version of the transmitted waveform, added with random noise. Since EM waves travel at a known speed, the elapsed time between the transmitted pulse and the received echo is a direct measure of the distance of the target being detected.

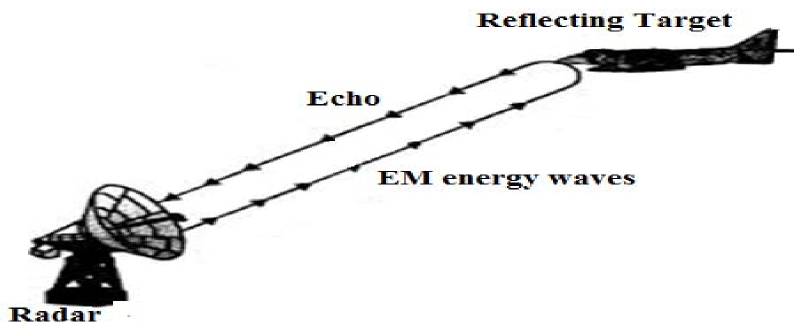

Figure 4.1: Active Radar systems.

Figure 4.1 illustrates a typical active radar scenario. Estimates of the space-time coordinates of the target are obtained by observing the effect of that target on the parameters of a transmitted signal namely delay and Doppler. In other words, the estimates of range and velocity can be obtained as a linear function of delay and Doppler measurements.

4.2 TIME FREQUENCY ANALYSIS OF CHIRP SIGNAL

Chirp signal or linear frequency modulated signal are characterize by starting frequency, initial phase and chirp rate. During the processing of chirp to detect the parameters of radar target the following parameter play an important role as shown in Figure 4.2.

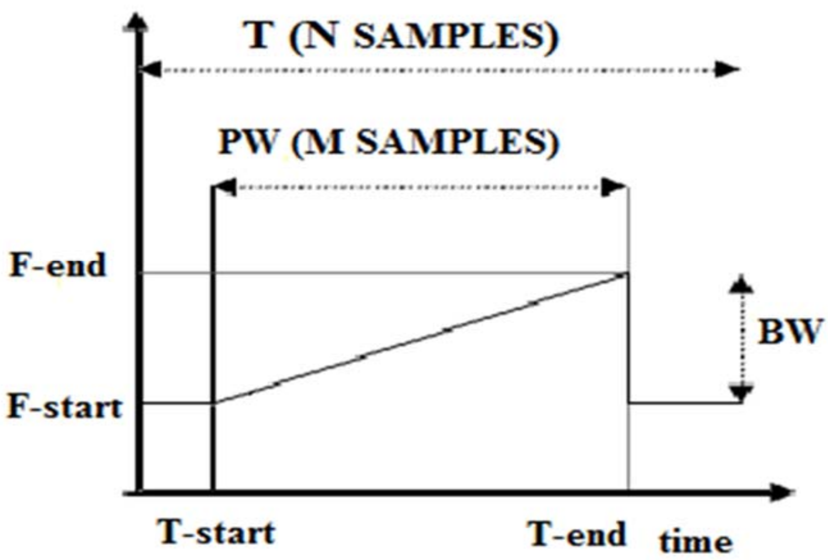

Figure 4.2 Time–Frequency plot of Chirp signal [40].

where,

T : processing duration (known)

N : Number of samples in T (known)

F_s : sampling rate (known), $N = T*f_s$

PW : Received Chirp pulse duration = M / f_s

M : samples in the received chirp pulse (unknown)

T-start : start sample number (unknown)

T-end : end sample number

F-start : start frequency of chirp

F-end : end frequency of chirp

BW : chirp bandwidth

p_{opt} : optimum p (unknown)

NMratio = N / M (unknown)

Fftres : fft resolution = f_s / N

FrFT is computed on cascading blocks of incoming data for a prefixed duration called processing duration. Ideally, processing duration should be equal to the chirp pulse duration. But, the duration of the received chirp pulse is unknown. Also, the transmissions from different emitters will be having different pulse widths. Since this information is not known to the receiver, there will always be a mismatch between the processing duration and pulse duration. In order to cater for this scenario, the processing duration may be fixed to a particular duration of *T* milliseconds and the received chirp is assumed to exist for this time duration *T* or for a duration lesser than *T*.

As mentioned in earlier chapters, (47) is used to calculate the optimal order for a linear chirp signal with known chirp rate of α. The true relationship is dependent on the digital sampling scheme used. With sampling frequency f_s, the total duration $T= f_s*N$ samples and it can be written as in (48). This relation is used to calculate the optimal order for a sampled linear chirp signal with known chirp rate k. Conversely, it can be used to estimate chirp rate, given the FrFT order.

$$p_{opt} = \frac{2a}{\pi} = \frac{2}{\pi}\tan^{-1}\left(\frac{1}{2k}\right) \quad (47)$$

$$p_{opt} = \frac{2a}{\pi} = \frac{2}{\pi}\tan^{-1}\left(\frac{f_s^{\,2}/N}{2k}\right) \quad (48)$$

In (48) assumes that the chirp has *N* samples, same as the processing sample length *M*. To cater for this mismatch, it is modified as in (45). This modified equation is used in all the simulations with M less than N.

$$p_{opt} = \frac{2a}{\pi} = \frac{2}{\pi}\tan^{-1}\left(\frac{f_s{}^2/N}{2k}\right) = \frac{2}{\pi}\tan^{-1}\left(\frac{f_s * M}{N * BW}\right) \quad (49)$$

Since slope $2k = \frac{BW}{PW}$ $\therefore \frac{1}{2k} = \frac{M/f_s}{BW}$

4.3 PARAMETER ESTIMATION CHIRP USING FrFT

A scheme for applying fractional Fourier Transform for chirp parameter estimation is developed in this dissertation. The interest here is to evaluate in detail the performance of FrFT for detection and analysis of chirps regardless of duration, frequency and bandwidth. The motivation behind the developed method is the ability of FrFT to process chirp signals better than the conventional Fourier Transform. But, in order to apply it for the radar application, the algorithm has to be adapted suitably.

In the time domain, chirp signal is defined as:

$$x(t) = e^{j(2\pi f_0 t + k_0 \pi t^2 + \phi_0)} \quad (50)$$

where, ϕ_0 represents initial phase, f_0 represents initial frequency, k_0 denotes the chirp-rate and $k_0 = B/T$, B represents frequency band, T represents time duration. In order to simplify derivation process, the initial phase is supposed to be ϕ_0= 0.

Applying (21) to (50), the corresponding FrFT expression of *x(t)* is:

$$X_p(u) = A_\alpha \int_{-T/2}^{T/2} exp\{j\pi[(k_0 + \cot\alpha)t^2 + 2(f_0 - u\csc\alpha)t + u^2\cot\alpha]\}\, dt \quad (51)$$

$$= A_\alpha \exp\,(j\pi u^2 \cot\alpha) \int_{-T/2}^{T/2} exp\{j\pi[(k_0 + \cot\alpha)t^2 + 2(f_0 - u\csc\alpha)t]\}\, dt \quad (52)$$

where, $A_\alpha = \frac{exp\left(-j\pi\frac{sgn(\sin\alpha)}{4} + j\alpha/2\right)}{|\sin\alpha|^{1/2}}$

when $k_0 + \cot\alpha = 0,$ (53)

$$X_p(u) = A_\alpha \exp\,(j\pi u^2 \cot\alpha).\mathrm{T}\frac{\sin[\pi(f_0 - u\csc\alpha)\mathrm{T}]}{\pi(f_0 - u\csc\alpha)\mathrm{T}} \quad (54)$$

$$\left|X_p(u)\right| = \frac{T}{\left|\sin\alpha^{1/2}\right|} Sa[\pi(f_0 - u\csc\alpha)T] \quad (55)$$

where, Sa (.) represents the sinc function.

As seen from (6), after applying the optimal order FrFT, most energy of the signal is concentrated in the bandwidth of $B_m = |2\sin\alpha/T|$ and $\left|X_p(u)\right|^2$ reaches the peak value if and only if $k_0 + \cot\alpha = 0$ and $f_0 - u\csc\alpha = 0$.
Based on the above property, the parameter estimation of x(t) can be operated as [20]:

$$\{\hat{p}, \hat{u}\} = \underset{p,u}{\operatorname{argmax}} \left|X_p(u)\right|^2 \quad (56)$$

$$\begin{cases} \hat{k} = -\cot(\hat{p}\pi/2) \\ \hat{f_0} = \hat{u}\csc(\hat{p}\pi/2) \end{cases} \quad (57)$$

However, before applying the fast discrete algorithm of FrFT presented in [11], it requires a dimensional normalization which means to define the corresponding analysis range and discrete resolution in the FrFT domain. Since the method of dimensional normalization presented in [11] is not applicable to the practical discrete signal, the engineering-oriented method, which is called as the discrete scaling transform method [22], is widely used. In this method, the sampling duration, which is denoted by t_d, and the sampling frequency f_s of the observed signal are directly selected to be the time width and the bandwidth respectively, Then the scaling parameter S and scaled coordinates $x = t/S$, $v = f$. S are introduced. By choosing $= \sqrt{t_d/f_s}$, the sampling interval of the observed signal is changed from $1/f_s$ to $1/\Delta x$, where $\Delta x = \sqrt{t_d f_s}$. After adopting the discrete scaling transforms method, (57) should be adjusted as [22]:

$$\begin{cases} \hat{k} = -\frac{f_s}{t_d}\cot(\hat{p}\pi/2) \\ \hat{f_0} = \frac{f_s}{t_d}\hat{u}\csc(\hat{p}\pi/2) \end{cases} \quad (58)$$

4.3.1 Optimum Order (p_{opt}) Estimation

In the case of the active radar, the transmitted chirp signal is known, and hence calculation of the optimum transform order is direct. However, in applications like intercept radar, the parameters of the received chirp pulse are unknown and so p_{opt} has to be estimated by some other means.

The optimum FrFT order cannot be found analytically in general. So, a one -

dimensional search for p is necessary to find the optimum order, with which the chirp focuses well. On a given block of data, FrFT is done for different values of p [0, 1] and the value of p that yields the maximum peak value is selected. The binary search implemented here generates the optimum p up to an accuracy of three decimals. For this, the FrFT peaks with p equal to 0.25 and 0.75 are compared first, from which one is able to infer whether p lies below or above 0.5. The p yielding the higher peak is taken as the optimum p in the first step. If the optimum value arrived at is 1, it indicates the echo to be a CW pulse. In such a situation, FFT processing will give equal performance and hence can be used in subsequent processing.

4.3.2 The flow of estimating optimum order of FrFT

The order of FrFT plays the main role to estimate the parameter of radar echo or LFM signal. The flow of estimating order of FrFT is given as:

Step1. Taking two point p_1 and p_2 in the range of FrFT order [0, 1], where the value of p_1 and p_2 are 0.25 and 0.75 respectively.

Step2. Calculate FrFT at order p_1 and p_2.

Step3. Select the FrFT order corresponding to maximum peak value as optimum order (p_{opt}).

Step4. Calculate the initial step size (Δp) using the FrFT order (p_{min}) corresponding to minimum peak value.

$$\Delta p = \frac{p_{min}}{2}$$

Step5. Initialize the counter i = 13.

Step6. Select new p_1 and p_2 as given below:

$$p_1 = p_{opt} - \Delta p$$

$$p_2 = p_{opt} + \Delta p$$

Step7. Repeat step 2 to step 4.

Step8. Repeat step 6 to step 7 until $p_1 = p_2$ or up to counter value 14.

Step9. Select the order of FrFT corresponding to maximum peak as a final optimum order for further processing of parameter estimation.

4.3.3 Flow chart of parameter estimation of chirp signal

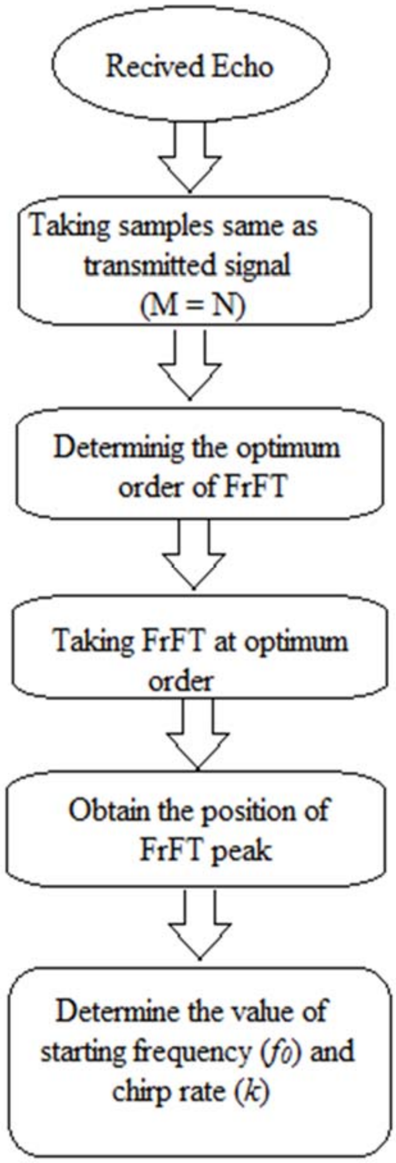

Figure 4.3: Flow chart of parameter estimation of chirp signal.

To detect the radar target, a constant frequency signal is transmitted by radar towards target. Target reflected signal (Linear frequency modulated signal) is processed using time-frequency technique to estimate its parameter. Length of received echo signal is assumed same as the processing length of receiver. Fractional Fourier Transform (FrFT) is applied to estimate the instantaneous frequency and chirp rate of received echo. Optimum order of FrFT is estimated as given in previous section. At this optimum order, FrFT gives an impulse in u-domain, with the help of optimum order (p_{opt}) and position of impulse (û), we can estimate the instantaneous frequency (f_0) and chirp rate (k).

4.3.4 SIMULATION RESULT

For this simulation, a chirp of 500 milliseconds is generated with a bandwidth (BW) of 900 Hz, start frequency of 100 Hz and sampling frequency 2000 Hz as shown in Figure 4.4.

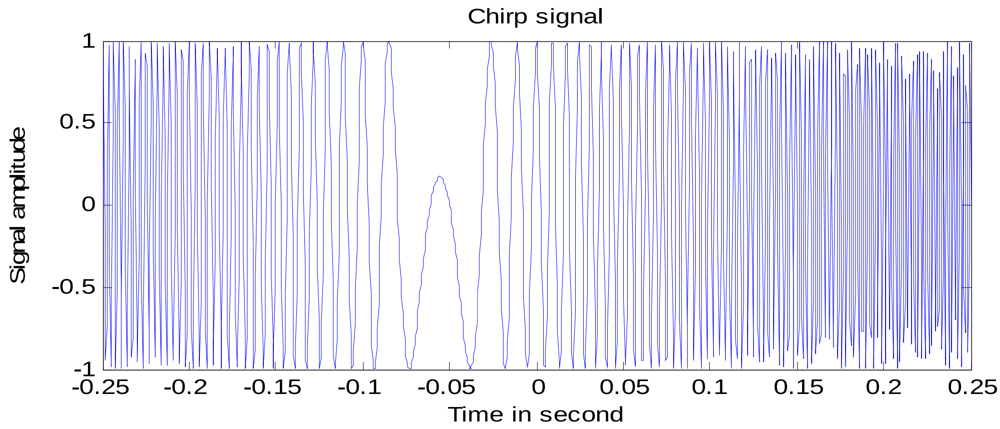

Figure 4.4: Real part of received noise free chirp signal.

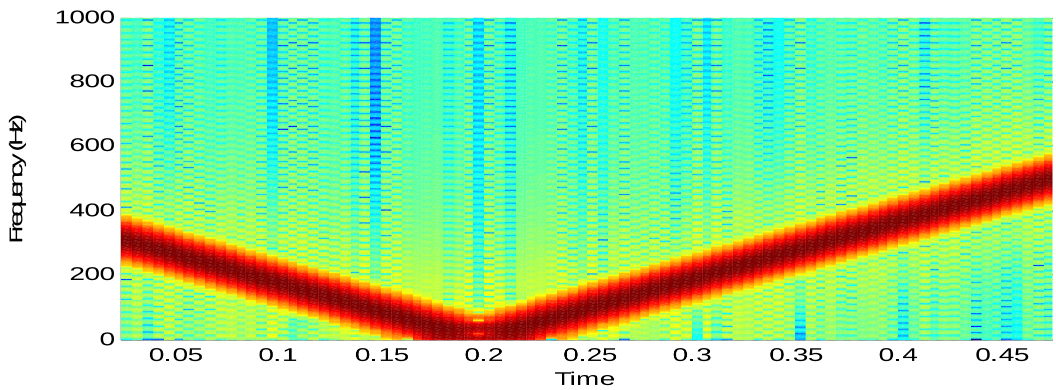

Figure 4.5: Spectrogram of received noise free chirp signal.

Figure 4.5 show the Time-Frequency relation of received signal, it clearly show that the frequency of received chirp signal linearly varies with time.

The processed data length N and the chirp samples M are kept same. Using (49), p_{opt} theoretically calculated as 0.7308.

The values of p in the search algorithm for 14 steps are given in Table 4.1. The corresponding FrFT peak values are also recorded. Estimated value of p corresponding to maximum FrFT peak is 0.7309, which is accurate to 3 decimals.

Table 4.1: Iterative computation of p value in search algorithm

Step	p_1	p_2	FrFT peak value	p_{opt}
1	0.2500	0.7500	3.5271	0.7500
2	0.6250	0.8750	1.8795	0.6250
3	0.5625	0.6875	2.6599	0.6875
4	0.6563	0.7188	4.4909	0.7188
5	0.7031	0.7344	7.2590	0.7344
6	0.7266	0.7422	6.9488	0.7266
7	0.7227	0.7305	11.8916	0.7305
8	0.7285	0.7324	9.8474	0.7324
9	0.7314	0.7334	11.8053	0.7314
10	0.7310	0.7319	12.0887	0.7310
11	0.7307	0.7312	12.0473	0.7308
12	0.7306	0.7308	12.0828	0.7309
13	0.7308	0.7309	12.0895	0.7309
14	0.7309	0.7309	12.0900	0.7309

The Table 4.1 shows that the optimum value of p with accuracy of 3 decimals achieved at 7^{th} step, the FrFT of the given signal with these seven values of p is given below in Figure 4.6.

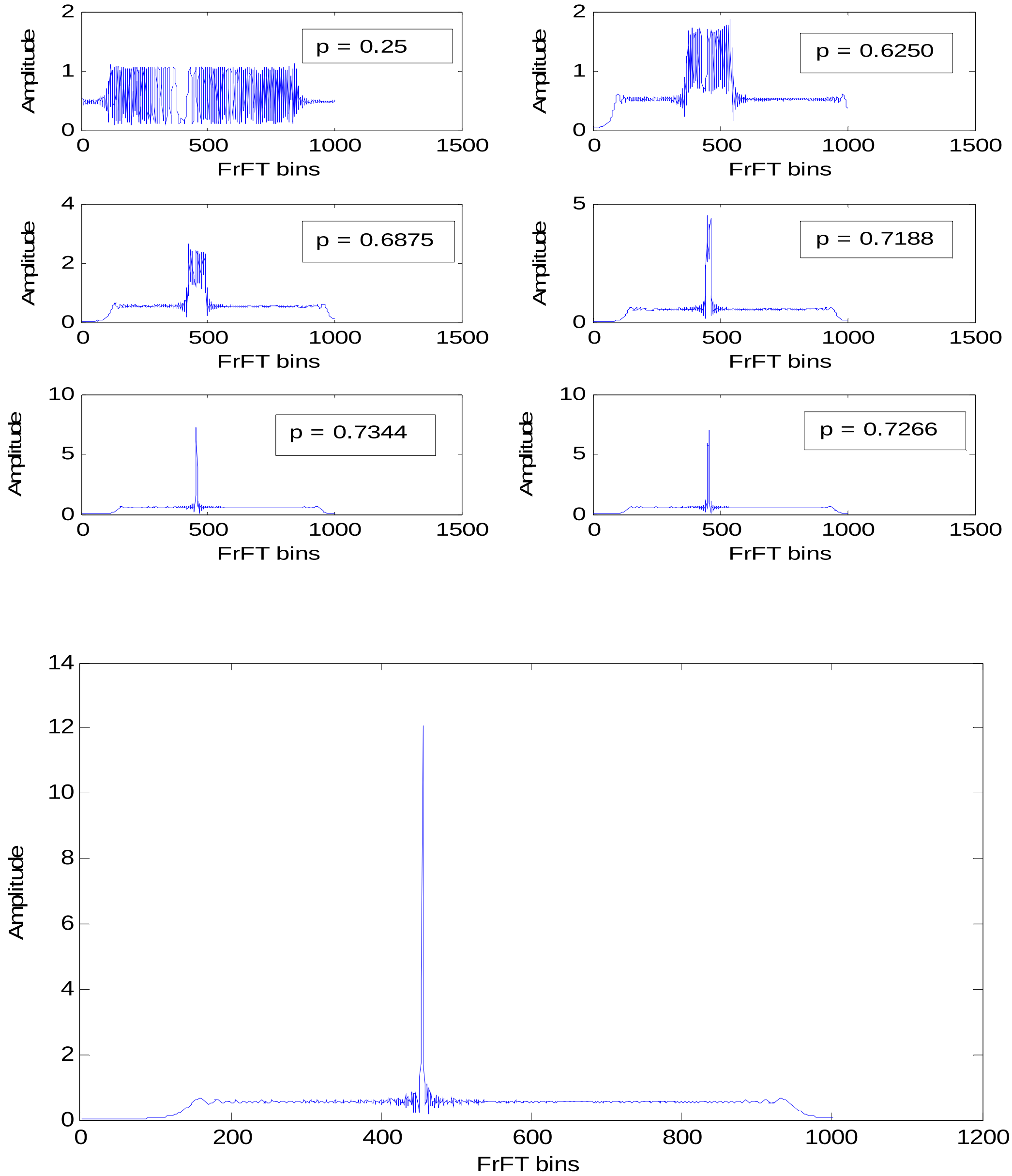

Figure 4.6: FrFT of given signal at different *p* values of search algorithm

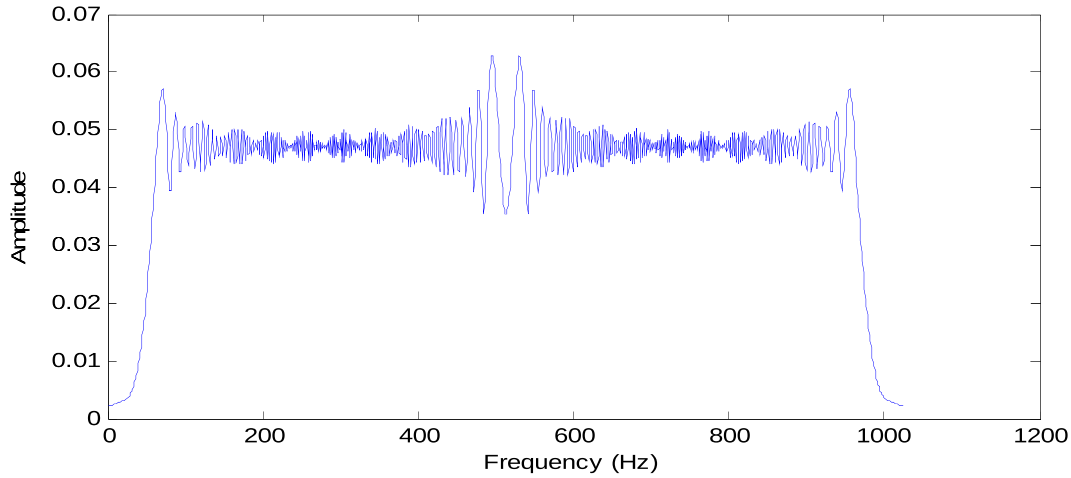

Figure 4.7: Double – Sided FFT of Chirp signal

Fractional Fourier transform of the chirp signal gives an impulse in the fractional domain as shown in Figure 4.3, the position of impulse is estimated and its value is 455. With the help of optimum order (*p*) and the position of impulse, chirp rate (k) and starting frequency (f_0) will be calculated (58) as shown in Table 4.2:

Table 4.2: Estimated and given value of chirp rate and starting frequency

Parameter	Given value	Estimated value
Chirp rate (k)	1800 Hz/sec.	1799.10 Hz/sec.
Starting Frequency (f_0)	100 Hz	100.87 Hz

4.4 ACCELERATION ESTIMATION OF RADAR TARGET USING FrFT

While radar transmits signal of fixed frequency, there is a linear relation between its phase and time. After transmitted signal is modulated by moving targets, nonlinear phase feature of radar echo signal is determined by relative motion relation between radar and target. Due to the effect of acceleration, radar target echo will be non-stationary. At this point the aberration will appear in signal spectrum. The method based on the typical FFT, may decrease the performance of target detection. The aberrant spectrum involves radial acceleration information. If it is available in signal procession, the acceleration will have an important influence on moving target tracking and classification [21].

In the field of military, radar did not provide acceleration information because in the early time aircrafts have low mobility. In these days with the help of advancement in technology, the mobility of aircrafts have increased, therefore the effect of acceleration on signal spectrum of FFT could not neglect.

In this section radial acceleration of the target is estimated using Fractional Fourier Transform (FrFT) for different time periods and different SNR values. To verify these characteristics, a mathematical model for target with uniform acceleration is deduced and then the computational formula is established. Finally simulations were conducted using MATLAB computational platform and the results are verified.

4.4.1 MATHEMATICAL MODEL OF ACCELERATED TARGET

The signal transmitted by radar can be considered as given below:

$$s(t) = A\cos[2\pi f_0 t + \emptyset_0] \qquad 0 \le t \le T \qquad (59)$$

where, A, f_0 and $\emptyset_0$ represent amplitude, starting frequency and phase of transmitted signal respectively.

There is a relative motion between target and radar. To simplify the solution, assume that the target is moving towards the radar. As shown in the given Figure 4.8, the target in the given time period is seen as linear motion of uniform acceleration.

When the transmitted signal hit the target, the received target echo is considered as:

$$R(t) = A\exp\left(j\left(2\pi(f_0 t + f_d t + k/2\ t^2) + \phi'_0\right)\right) \qquad 0 \le t \le T \qquad (60)$$

where, $f_d = \frac{2v}{\lambda_0}$ is the Doppler frequency, v is the velocity of target and λ_0 is the wavelength of echo signal.

It can be noticed that the square term in (60) is due to acceleration that varies as square with time.

The chirp rate or frequency modulated parameter *k* has relation with acceleration *a* as given below:

$$k = \frac{2a}{\lambda_0} \quad (61)$$

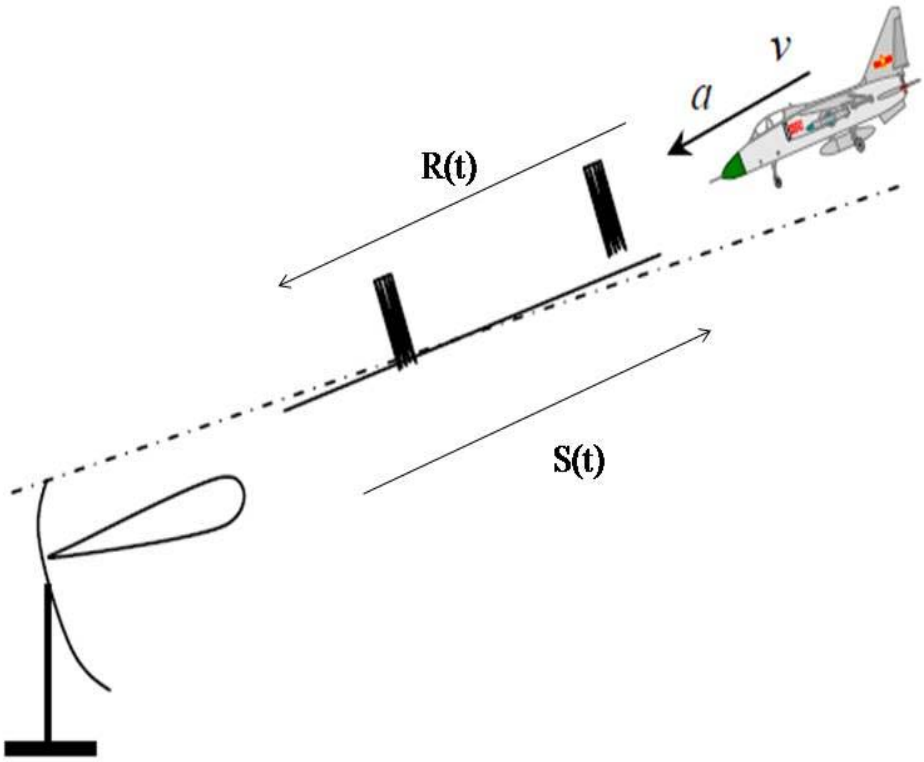

Figure: 4.8: Target moving towards the radar with uniform acceleration [21]

The instantaneous frequency of received echo signal is:

$$f_i(t) = \frac{1}{2\pi}\frac{d\phi}{dt} = f_0 + f_d + kt \quad (62)$$

The reflected signal R(t) from target add up with Additive White Gaussian Noise (AWGN) in the free space and the noisy signal is given by [23]:

$$y(t) = A\exp\left(j\left(2\pi(f_0 t + k/2\, t^2) + \phi'_0\right)\right) + w(t) \quad (63)$$

where, Additive White Gaussian noise is represented by $w(t)$

Applying the optimum order FrFT, the energy of received signal $y(t)$ is concentrated in the bandwidth of $B_m = |2\sin\alpha/T|$ and the peak value is $\left|X_p(u)\right|^2$, if and only if $k + \cot\alpha = 0$ and $f_0 - u\csc\alpha = 0$. As per the above property, estimation of the signal $y(t)$ parameters can be derived as [20]:

$$\{\hat{p},\ \hat{u}\} = \arg\max_{p,u}\left|Y_p(u)\right|^2 \tag{64}$$

The target acceleration estimation is based on the two dimensional search. According to (64) the peak position is at $\hat{u}$ and $\hat{p}$ is the estimated FrFT order. Using that value the acceleration estimation formula is derived as:

$$\hat{a} = \frac{\lambda f_s}{2T}\cot(\hat{\alpha}) \tag{65}$$

where, $\hat{\alpha} = \hat{p}\,\frac{\pi}{2}$

4.4.2 Flow chart of acceleration estimation of radar target

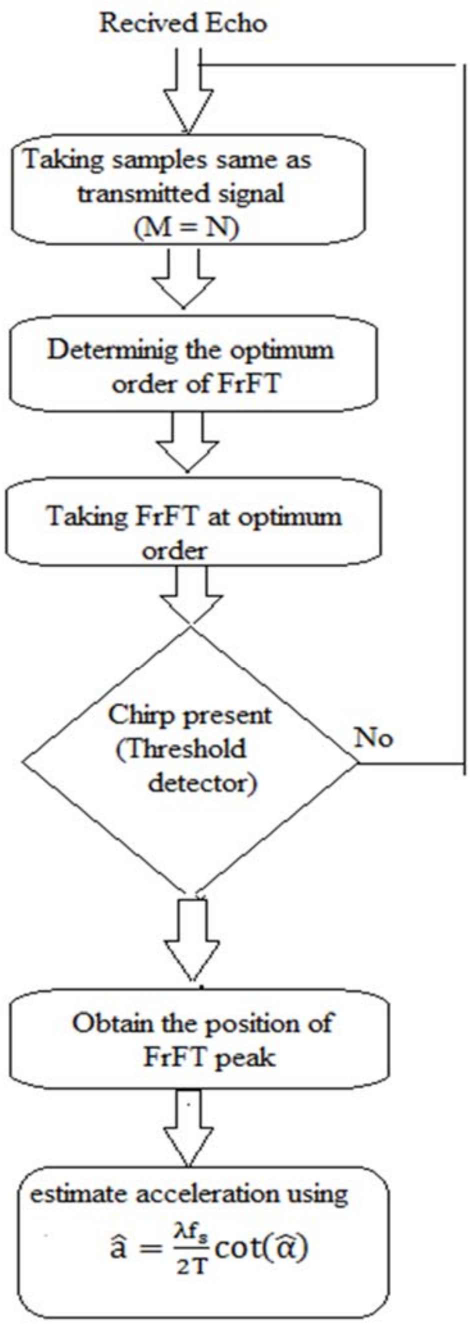

Figure 4.9: Flow chart of acceleration estimation of radar target

To detect the acceleration radar target, a constant frequency signal is transmitted by radar towards target. Target reflected signal is processed using FrFT to estimate its acceleration. Length of received echo signal is assumed same as the processing length of receiver. By estimating the optimum order and taking the FrFT of echo, threshold is applied to check that the target is present or not. After detecting the target, acceleration is estimated with the help of optimum order.

4.4.3. SIMULATION RESULTS AND ANALYSIS

4.4.3.1 Estimation of radial acceleration with different value of SNR

In term to estimate the parameter of accelerated target, it is considered that the initial speed of target is 45 m/s, acceleration is 200 m/s^2, the radar wavelength λ = 20 mm, duration of received signal is 100ms and the sampling frequency is 20 KHz.

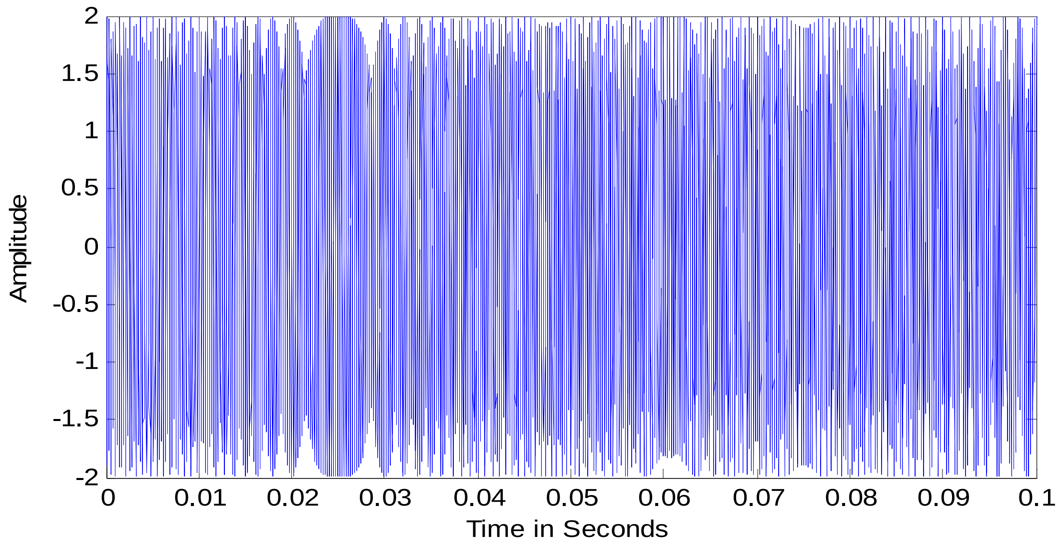

Figure 4.10: Radar received chirp signal without noise.

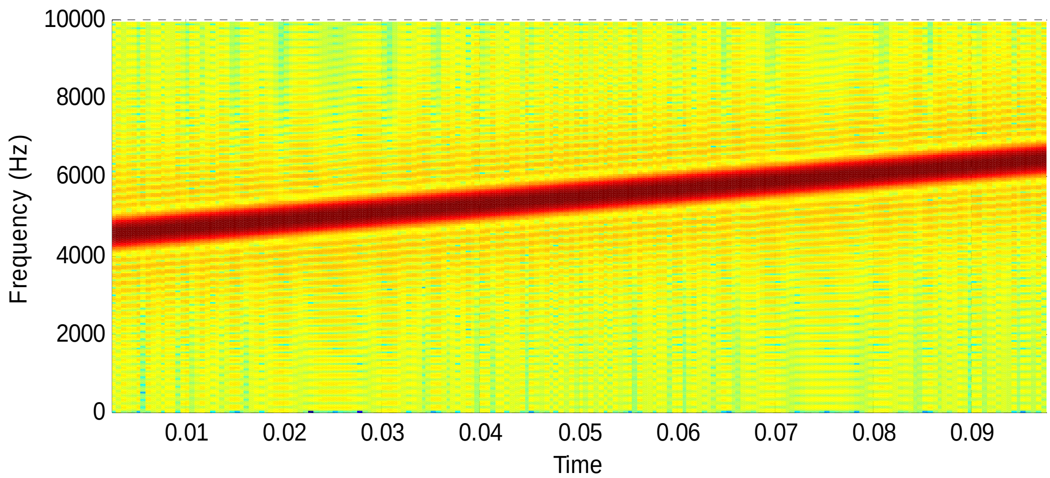

Figure 4.11: Spectrogram of received chirp signal.

Figure 4.11 show the Time-Frequency relation of received signal, it clearly show that the frequency of received chirp signal linearly varies with time.

The values of FrFT order (p) in the search algorithm for 14 steps are given in Table 4.3 for noise free chirp signal. The corresponding FrFT peak values are also recorded. Estimated value of p corresponding to maximum FrFT peak is 0.9369.

Table 4.3: Iterative computation of *p* value in search algorithm for noise free echo of accelerated target

Step	p_1	p_2	FrFT peak value	p_{opt}
1	0.2500	0.7500	2.2394	0.2500
2	-0.1250	0.6250	2.7628	0.6250
3	0.4375	0.8125	2.6494	0.8125
4	0.7188	0.9063	5.4121	0.9063
5	0.8594	0.9531	7.3171	0.9531
6	0.9297	0.9766	11.6342	0.9297
7	0.9180	0.9414	13.6043	0.9414
8	0.9355	0.9473	26.3541	0.9355
9	0.9326	0.9385	21.0407	0.9365
10	0.9370	0.9299	31.4545	0.9370
11	0.9363	0.9377	29.3492	0.9363
12	0.9459	0.9366	31.2576	0.9366
13	0.9365	0.9368	31.6098	0.9368
14	0.9367	0.9369	31.5990	0.9369

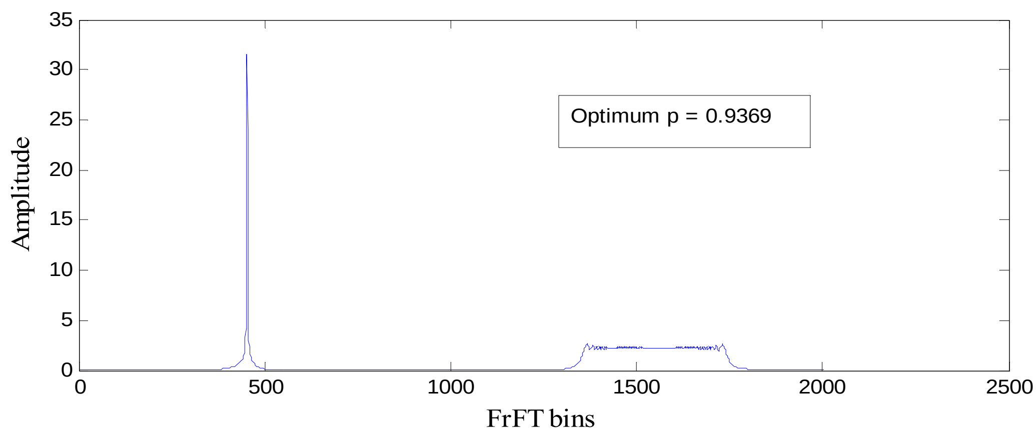

Figure 4.12: FrFT of the accelerated target echo at optimum order.

To check the performance of optimum order estimation algorithmic in the noisy environment, optimum order is estimated at different value of SNR. The estimated optimum order (p_{opt}) given in Table pp, and FrFT output corresponding to estimated order is shown in Figure 4.12.

Table 4.4: Estimated optimum order (p_{opt}) at different SNR values

SNR (dB)	-8	-4	-2	2	8
p_{opt}	0.9368	0.9369	0.9369	0.9368	0.9368

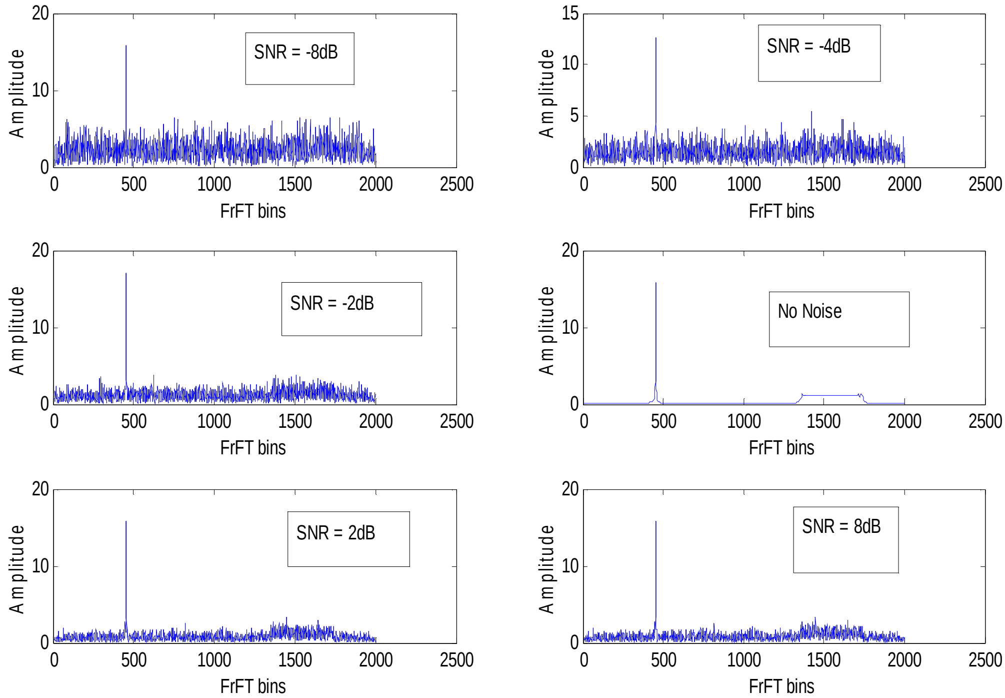

Figure 4.13: FrFT output of received echo at different values of SNR

Table 4.3 shows that the optimum order (p) of FrFT for radar target echo is 0.9369, the value of acceleration is calculated by putting this value of p in (65).

To check the performance of optimum order estimation algorithmic in the noisy environment, optimum order is estimated at different value of SNR. The FrFT output corresponding to estimated order is shown in Figure 4.13.

To estimate the acceleration in the noisy environment, where power of noise is more than the reflected signal, simulations are conducted when SNR varies from -15 to 0dB. After averaging 100 times, the result showed below in Table 4.5.

Table 4.5: Acceleration estimation for negative values of SNR

SNR(dB)	-15	-10	-8	-6	-4	-2	0
Acceleration (m/s^2)	203.67	197.00	196.98	201.243	196.98	200.85	199.21

Now SNR value is taken from 2 to 10dB and the simulation results is given below in Table 4.6.

Table 4.6: Acceleration estimation for positive values of SNR

SNR(dB)	2	4	6	8	10
Acceleration (m/s^2)	200.90	200.71	200.85	200.91	200.91

4.4.3.2 The result of estimation for different signal duration

To estimate the parameter of accelerated target at different signal duration, it is considered that the initial speed of target is 45 m/s, acceleration is 200 m/s^2, the radar wavelength $\lambda = 20$ mm, duration of received signal is 100ms and the sampling frequency is 20 KHz and the SNR is 10dB. The results of simulation are shown in Table 4.7.

Table 4.7: Estimation of Acceleration at different window size for received signal

T(s)	0.1	.09	.08	.07	.06	.05	.04	.03
Acceleration (m/s^2)	200.91	200.79	199.84	201.54	199.28	199.12	334.21	445
Peak Magnitude	16.71	2.10	1.76	1.50	1.28	1.35	1.20	1.20

In Table 4.7, it can be observed that the precision of measurement depend upon the duration of signal. The signal of longer duration has better precision than the signal of short duration. The amplitude of impulse peak also depend upon the signal duration, energy of shorter signal is less as compare to the longer duration signal therefore the peak amplitude is small.

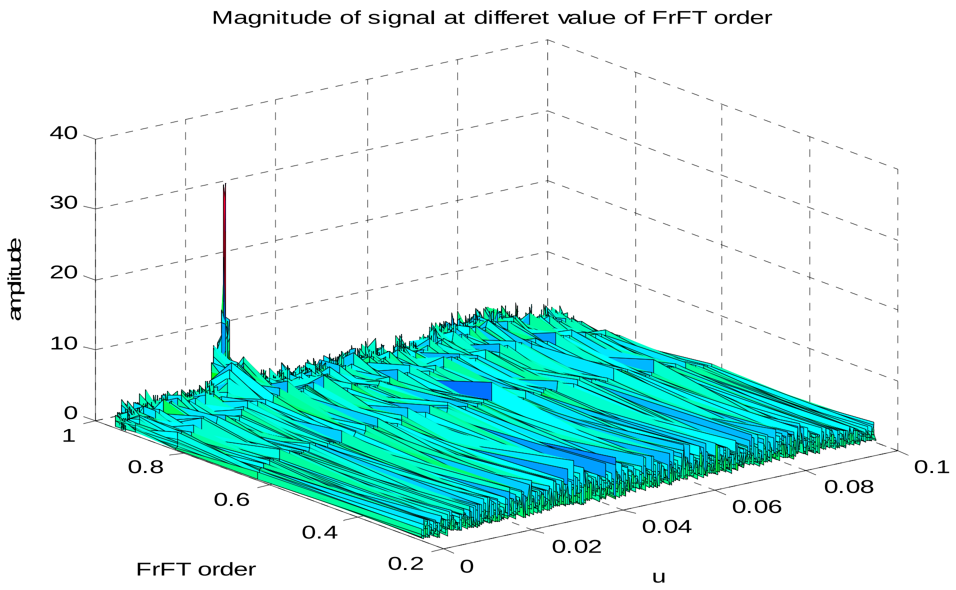

Figure: 4.14: Energy distribution of received signal at different order of FrFT.

From the simulation, received echo signal FrFT for the different order is shown in Figure 4.14 and the impulse is obtained at order 0.9368 i.e. energy concentrated as an impulse at this angle.

4.5 ACCURACY ANALYSIS

To evaluate the estimation error caused due to noise, firstly an observation signal is generated which is modelled as (63) with parameters A=1, f_0 = 200*Hz*, and k= 200 *Hz/s*. Then for the above given parameters, the estimation value is computed 100 times. Now the accuracy of these estimated parameters is given by root mean square error (RMSE), given as:

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{\xi} - \xi\right)^2} \qquad (66)$$

where, $\widehat{\xi_i}$ depicts the estimated value ξ in the i^{th} iteration, and *N* is the total number of iteration. The value of *N* is 100 in this simulation. The normalized value of RMSE is given as:

$$\text{Normalised RMSE} = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{\xi}-\xi\right)^2}}{Max(\hat{\xi}) - \min(\hat{\xi})} \tag{67}$$

Figure 4.15 shows the Normalized RMSE estimated parameters k (Chirp rate) and f_0 (Starting frequency) under various SNR.

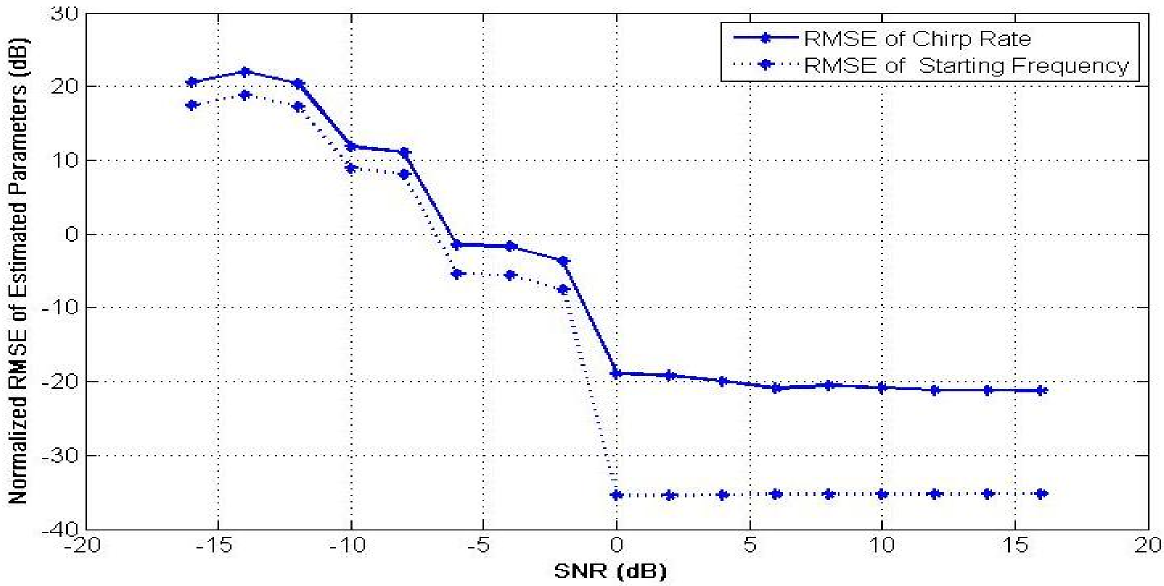

Figure 4.15: Normalized RMSE of the estimated chirp rate and initial frequency at different SNR values.

Parameter estimation mainly depends upon the FrFT optimum order, starting frequency and chirp rate are directly related with it. By estimating the Normalized RMSE of these parameters at different value of SNR, accuracy of system is analysed as shown in Table 4.8. Figure 4.15 show that the Normalized RMSE is decreased with increasing the SNR value and the value of Normalized RMSE at different value of SNR given in Table 4.8.

Table 4.8: Normalized RMSE at different values of SNR

SNR (dB)	Normalized RMSE of (f_0)	Normalized RMSE of (k)
-16	10^{18}	10^{21}
-10	10^{09}	10^{12}
-5	10^{5}	10^{9}
0	10^{-35}	10^{-19}
10	10^{-36}	10^{-21}

CHAPTER-5

CONCLUSION AND FUTURE SCOPE OF WORK

CONCLUSION

The dissertation focuses on analysis of linear frequency modulated (LFM) signals or chirp signals. As we have studied and found that LFM are widely used in information system such as radar, sonar, and communications. In these system to detect and estimate LFM signals is an important problem. In recent years, with progress in research of time-frequency analysis, many techniques based on time-frequency have been proposed to solve this problem.

The fractional Fourier is a newly developed time-frequency analysis tool and is becoming more and more attractive in signal processing, especially in processing of non stationary signal.

In this dissertation, we analysed the application of FrFT in radar signal processing. In the implementation of the method, Bisection method is used to simplify the peak detection in the fractional Fourier domains. The performance of this method is verified at different values of SNR and it is proved that there is no accuracy degradation. With the help of this method, starting frequency and chirp rate of the radar received signal was estimated. Simulations are conducted to verify the normalized root mean square error (RMSE) of these estimated value, it prove that the normalized RMSE is decreased with increasing the SNR.

In another section, the algorithm to estimate the parameters of radial accelerated target was brought forward. It is analysed that the FrFT is good enough to estimate the parameters. The analysis shows relation between estimated acceleration and duration of signal. The Accuracy of estimation is inversely proportional to the duration of received signal. Energy distribution of signal at different rotation angles is also shown.

Simulation results proved that this method will be able to improve the operational speed, control operational precision and give better anti-noise interference capability.

FUTURE SCOPE

The dissertation reports the results of the work carried out on the application of FrFT in parameter estimation of radar target. But this does not foreclose further work that can be carried out. Some of the possible areas for further studies are suggested below.

The algorithm proposed in this dissertation is based on the bisection method, which has less complexity but it consumes more time. So there is a need of improvement.

Practically the width of received echo will not be same as the processing length of radar, but here it is assume that the samples of received echo M are same as the number of samples in processing duration N. So there is also a need of improvement.

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