

COMPARATIVE STUDY OF INTEGER AND FRACTIONAL ORDER SLIDING MODE CONTROL

A Dissertation submitted in fulfillment of the requirements for the Degree

of

MASTER OF ENGINEERING

in

Electronic Instrumentation & Control Engineering

submitted by

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July, 2018

DECLARATION

I hereby certify that the work which is presented in this dissertation entitled, "**Comparative study of integer and fractional order sliding mode control**", in complete fulfillment of the requirements for the award of the degree of Master of Engineering in Electronic Instrumentation and Control Engineering, submitted to Electrical & Instrumentation Engineering Department of TIET, Patiala is as authentic record of my own work carried under the supervision of Ms. Ruchika Lamba. It refers others researcher's work which is duly listed in the reference section. The matter contained in this dissertation has not been submitted, neither in partially nor in full to any other degree to any other university or institute except as reported in the text and references.

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ACKNOWLEDGEMENT

This dissertation is the culmination of my two years journey in Thapar Institute of Engineering and Technology, Patiala, to procure my masters.

It is my sound honor to give felicitations to my life-guide, my father, as I owe this all spadework to you. You always helped me in harnessing my energy through spiritual and mystique preaches. I hyperbolically give eternal thanks to you. I am highly indebted to my mother for providing me constant motivation and support throughout this journey and also I am grateful to my sister Ojaswi, for long chatting with me over phone whenever I felt alone and exhausted of my work loads. I also give tribute to my cousin brothers Roshan and Ravi who taught me the fundamentals of Mathematics and Physics when I was studying in middle and high school. These members of my family always converted my obstacles into stepping stones.

This thesis is not only confined within the peripherals of Sliding mode control theory, Fractional controllers, Keyboards and MATLAB rather it is much more beyond these stuffs.

Staying 1600 Kilometers away from my lovely hometown Jamshedpur, invariably for two years alone perceived like I am navigating in vacuum. Sometimes it seemed to feel me like I am at my wits end. But still my guide Ms. Ruchika Lamba, Lecturer, EIED, role played as a light house of optimism that provided me a search light in order to carry my works and complete my thesis. I am extremely thankful and pay deep regards to her.

I also pay utmost gratitude to Dr. Swati Sondhi, Assistant Professor, EIED. She guided me the fundamentals of my work and taught me how to tackle problems associated with my thesis in a bold and direct way and always delivered hanker whenever I was on tenterhooks. Further I extend my gratitude to Dr. Sahaj Saxena, Assistant professor, EIED, who catalyzed my hindered works and frequently pulled me out away from quicksand of coding in MATLAB. He constantly channelized my spirit at grueling times in order to complete my thesis.

I am thankful to Dr. R.S. Kaler, HOD, EIED, and Dr. Nirbhowjap Singh, P.G. coordinator, EIED, for providing with adequate infrastructure in conducting the work. I also pay tribute to all staff members of EIED who helped me whenever I needed one.

I extend my acknowledgement to Ph.d. students of my department Ashutosh, Piyush, and Rohit. Being a senior they provided a good company and many a times helped me in my work.

I am also very thankful to my friends: Little, Akash, Abhinandan, Kamlesh, Ravi, Sunil, Yashdeep, Shoaib, Krishna and Sumit. They all helped me in every possible dimensions of my two years journey. I enjoyed their company and their talks of reminiscence.

Lastly, but above all I am extremely thankful to Almighty God. I have always being with the blazing light of Almighty God who protected me and provided me inner strengths to survive through thick and thin.

Nishant Chanchal

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LIST OF ACRONYMS

Symbol	Description
SMC	Sliding Mode Control
IOSMC	Integer Order Sliding Mode Control
FOSMC	Fractional Order Sliding Mode Control
IOSS	Integer Order Sliding Surface
FOSS	Fractional Order Sliding Surface
SM	Sliding Mode
FO	Fractional Order
PID	Proportional Integral and Derivative
FOPID	Fractional Order Proportional Integral and Derivative
VSCS	Variable Structure Control System
VSS	Variable Structure System
MIMO	Multi Input Multi Output
SISO	Single Input Single Output
LTI	Linear Time Invariant
ODE	Ordinary Differential equation
FDE	Fractional Differential Equation
RIP	Rotary Inverted Pendulum
NMPS	Non-Minimum Phase Systems
G-L	Grunwald-Letnikov
R-L	Riemann-Liouville

ABSTRACT

A comparative study of integer order sliding mode control (IOSMC) and fractional order sliding mode control (FOSMC) is outlined on two different systems. The numerical example of first system is subjected to non-linear sinusoidal disturbance while the second numerical example considered being a marginally stable system. Sliding mode control (SMC) theory is a category of variable structure control system (VSCS). SMC generates discontinuous control action that generates a high frequency ON or OFF states which forces the system state to converge to zero. However, this switching at high frequency produces chattering phenomenon that is supposed to be eliminated or minimized. The Lyapunov function is being considered that provides global asymptotic stability in finite time. The integer order sliding surface (IOSS) and fractional order sliding surface (FOSS) is designed for both the numerical examples representing two different systems and a control input law is determined such that it provides asymptotic convergence in finite amount of time span and is supposed to ensure zero steady state error. The fractional order (FO) is varied in between zero to one and various responses of both the systems are observed. For the system exposed to sinusoidal disturbance, FOSMC provides stability faster than the IOSMC scheme for values of FO that are close to one. In the case of marginal stable system, IOSMC adds stability to the system faster than the linear feedback control law. The performance of FOSMC on marginal stable system is not so better than compared to IOSMC method. The settling time of plant trajectories are large than produced by IOSMC. However, FOSMC performs better when values of FO are selected very close to one but responses are not so good when FO values are placed close to zero. Overall the simulation results prove the nucleus notion of SMC theory that provides finite time stability as well as robustness against external disturbances and perturbations and are insensitive to system parameters.

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Modern control engineering is a comparatively contemporary field of engineering that earned noteworthy in the beginning of second decade of the twentieth century. The industrial revolution that emerged in Great Britain marks a paramount transformation in history that led a change in manufacturing processes from primitive human hand driven methods to machineries [1] and this development led to a never broken and unending series of developments one after the other. Further, these machines were developed to regulate and control the processes in a desired way according to the requirements of the plant and even further modified into automatic machines that are now popularly called automated control systems.

The very first remarkable task in automatic control system was done by James Watt during the mid eighteenth century that was controlling the speed of the steam engine along the turns of the track [2]. After that, in the year 1922, automatic controllers for steering the ships was designed by Nicolas Minorsky and it was also proved that the systems described by a differential equations are useful in determining the system's stability [2]. Then Harry Nyquist designed the theory of determining the closed loop system's stability based on open loop response to steady state sinusoidal inputs which was rather simpler in determining the stability [2]. Evan's root locus approach, frequency response techniques and Bode plots were developed that helped in predicting the stability of most of the processes [2].

Since, the advancements in technology evolved, scientists around the world constructed robots which performed human tasks quite in a cognate manner as human hands and legs performed earlier to perform a particular task. These robots that were developed even got more complex in nature and also surplus amount of increments were observed in their degrees of freedom which in turn led to bag full of difficulties in determining their stability [3].

For several reasons, modern plants is constantly changing and updating their machineries. For example, the modern iron and steel industries must meet its requirements of producing certain amount of molten iron along with focusing on the quality of steel products within a given frame of time. So, definitely industries concentrated on robotics and installed robotic systems to perform industrial works and pace their production to face the cut throat competition in the market. So, these industries constantly keeps on changing and upgrading their systems and these systems are intricate in their performance and quiet difficult to control. Usually, these robotic systems are intricate because of multi inputs and multi outputs (MIMO). These systems no longer remained to be single input and single output (SISO) which is considered to be as classical control theory that has turned powerless over the time due to arrivals of latest technology and rather the usage of state space analysis is more focused now to analyze MIMO systems [2]. With the possibility of digital computers in 1960, time domain investigation and synthesis using state variables made possible to analyze the complexities of modern processes and plants [2].

The key engineering problem is how to predict the stability of such typical systems and how to stabilize them. Having such a rich theory to determine stability of a systems by using root locus method, frequency response analysis, Bode plots and Nyquist plots and also having well established Mathematics like Laplace transform and Fourier transform, and moreover having a popular and dominant PID controller, still it's not being easy to predict the stability of non-linear systems. One thing is far clear that these approaches to predict stability are mainly limited to linear and time invariant (LTI) systems.

Talking about PID controller which is broadly accepted since many decades and also it has proved its efficiency in many control processes but one drawback is about its parameters calculation using conventional design schemes are not reconcilable with most of the industrial controllers [4]. PID controllers appear in mainly two different forms: the first one is non-interacting or parallel form, and the second is interacting or series form and both of these forms are used in industries and thus the tuning parameter of PID is different for both types of the plants [4]. Apart from these disadvantages, PID controller also has certain numbers of restrictions. The major restriction among all is their linearity issue and therefore for non-linear systems the performance of PID controller varies. For instance, in temperature control systems

where a regular dynamic heating with non-resistant refrigeration is used, the overshoot produced may gradually be rectified and adjusted, it cannot be obligated downward [5, 6]. Also in coupled MIMO processes, through tuning a PID controller decoupling is not easily achievable and moreover this controller fails to provide optimization in a process which is the most needed requirement of modern industries [7]. Thus, it is difficult to control non-linear systems using PID controllers. However, implementing SMC technique it is possible to stabilize non-linear and unstable systems.

Sliding mode control is a non-linear control scheme that has discontinuous control action that pushes the system to slide along a cross section of the system's accustomed behavior [8]. SMC has an umbilical cord linked to variable structure systems (VSS) and therefore, in SMC one may intentionally vary its structure with the help of discontinuous control action that forces the phase trajectory to a stable hyper-plane [9, 10]. VSS found major place in control system design for non-linear systems, large scale systems, multi-dimensional systems, time-variant systems, MIMO systems and stochastic systems or in combinations of any of these systems [9, 11, 12, and 13]. The sliding mode technique have not been only limited to stabilization but also got extended to various control functions [11].

The SM controller possesses least sensitivity to the plant specifications as well as minimum perturbations to plant parameters and it also permits the decoupling of the original plant which was not possible with the help of PID controller and therefore, SMC decouples the actual plant into two constituents of lower dimension [14]. These are the reasons why SMC has got an exceptional place in modern control design techniques and so, today it is very popular in research community.

Now, when MIMO, time-variant and non-linear systems can be stabilized using SMC controller, then why it is not possible to further combine fractional calculus with SMC to analyze the performance of such systems. Moreover, to have a clear view, a comparison can be drawn between conventional SMC and fractional SMC on the basis of their performances on different systems.

The modern calculus that was developed by Newton and Leibnitz in seventeenth century brought new revolution in the entire history of mathematics [15, 16]. There is one commonly noticeable fact that there exist either n^{th} derivative or integral where n is usually an integer number.

But towards the end of seventeenth century Leibnitz received a letter from L' Hospital that questioned about what if n is a decimal number, i.e., a fraction rather than being an integer or what if n is a complex number i or it is in a form of an irrational number or quiet simply an any real or complex number [16]. So, that letter apparently asked about the existence of fractional calculus. These are the kinds of wonderful questions that mathematicians always pondered on. But mathematics has always been an elegant solution to most of the complex problems that existed and exists in this world. The answers to these questions were stuck in a limbo until Lacroix generalized the integer order.

A French mathematician Sylvestre Francois Lacroix in the year 1819 generalized the integer order case using gamma function [16]. And after this a never ending series of development was done in this field by Riemann and Liouville, Heaviside, Miller and Ross, Caputo, Grunwald and Letinkov, Oldham and Spanier and many more like them. The important thing to note about fractional calculus is that the fractional derivatives and integrals do not possesses local property or point property that means it possesses non-local distributed effects [16].

This mathematical tool helped in designing the FOSMC theory. The conventional IOSMC got transformed into FOSMC and thus resulted in FOSS.

In the last two decades IOSMC and FOSMC got popular and widely acceptable among scientists working in different field and also it has been one of the efficient controllers to analyze different types of control systems in various plants and processes.

1.2 OBJECTIVES OF THESIS

The marrow idea of the thesis is presented in brief.

1. To study the performance of integer order sliding mode control (IOSMC) and fractional order sliding mode control (FOSMC) on two different systems and analyze the behavior of the systems when they are subjected to these controllers.

2. To determine the integer order sliding surface (IOSS) and (FOSS) that forces the system trajectories to converge to zero.
3. To design a control input law based on both IOSMC and FOSMC that guarantees asymptotic stability under non-linear sinusoidal disturbance as well as to reduce the settling time.
4. To compare and interpret the results obtained by implementing IOSMC and FOSMC on both the systems.

1.3 ORGANIZATION OF THESIS

The thesis consists of following chapters that are succinctly outlined in this section of the anthologized work.

Chapter 1- Introduction:

The first chapter introduces the concise note on evolution of control theory that marked the development of automated systems. A brief note on the historical background on control systems is also presented. The fact from where the SMC theory stems out to the gradual advancements in FOSMC are discussed in this chapter.

Chapter 2- Literature survey:

The chapter is divided into three parts. The first section deals with the works of scholars who examined the behavior of IOSMC on various systems and analyzed its performance. The next section presents the implementation of fractional calculus to develop FO controllers. The last section provides works done by researchers to transform IOSMC into FOSMC.

Chapter 3- Sliding mode control:

A fundamental equation to approach a design of SM controller is concisely elaborated in this chapter. The design of sliding surface to guarantee stability and elementary concept of Lyapunov function are sketched in this chapter.

Chapter 4- Fractional calculus:

The chapter introduces with the birth of fractional calculus to its rise in development of FO controllers. Moreover, the fundamental preliminary mathematical equations to compute FO derivatives that bridge the gap of conventional SMC with FOSMC are discussed.

Chapter 5- Proposed methodology:

The design methodology of IOSS and FOSS on two different systems is outlined in this chapter. Further the determination of control input action is proposed in each case of controllers. The mathematical equations have been discussed in order to derive the control law by means of SMC theory.

Chapter 6- Results and discussions:

This chapter presents the outcome of the proposed methodology. The results of both the systems on implementing IOSMC and FOSMC on it are sketched. Moreover, the interpretations of the results obtained are discussed precisely in this chapter.

Chapter 7- Conclusion and future scope:

The chapter deduces a final remark on performance of IOSMC and FOSMC on both the systems and concludes with the core explanation of SMC theory and proves itself as an exemplar and suggestions for future scope.

CHAPTER 2

LITERATURE SURVEY

Sliding mode control is relatively a new domain evolved in control theory which gained attention by many researchers world-wide since 1970. It was developed by scientists of Russia and became popular among research community within a short span of time due to its attractive features.

Thereafter, many contributions in this field were made by several scientists. Most of the works was done after the year 1990. After this the work on SMC expanded throughout the world and this application was implemented on various types of models and systems. The associated works by several authors are discussed in this chapter.

2.1 LITERATURE REVIEW ON SMC

The work done by Utkin [17] in the year 1993, who is among one of the founders of VSS and SMC, deals with the fundamental ideas of basic mathematics and fundamental design characteristics of sliding mode theory as a principle operation mode. The major concepts of sliding mode that are discussed in this paper are its insensitive nature to parameter variations, reduction in order by the help of power converters. Also it is discussed that SMC can be used in non-acceptance of disturbance, decoupling of system and its straight forward execution by means of power converter. The capability of SMC methodology is proposed for adaptability of electric drives as well as practical aim of control theory. The control law and algorithms that are implemented on VSS and SMC are focused in this work.

This work has sketched the mathematical framework and design methodology of SMC and talks about its adaptation to multi-dimensional along with non-linear systems that works under uncertain conditions and has also outlined its application to control on various types of electric motors. The SMC methodology applied on electric drives has various practical usages in metal-trimming machine tools, also in the field robotics on problem of tracking position of robot joints

along with control of speed of manipulator joints of robots. It also found many applications in transport area like in speed control of cars and trams that are powered by battery, also in various process control methodology. The applications are widely expanded and can be observed in control of power electronic converters, also where system time is considered of discrete nature, the design techniques that are based on Lyapunov function, combustion engines as well as in aircrafts.

Ramirez [18] scrutinizes about how non-linear systems behaves on the applied dynamics of SMC. The effects of the differential algebraic equation method present in SMC on the non-linear SISO system are analyzed in this paper. The switching surfaces that depend on inputs including the time derivative of inputs with respect to time appear in natural way from fundamental differential algebraic equations that pertains to canonical forms of non-linear systems. The sliding surface used here usually show dynamically organized sliding regimes that are free from chattering where the input signal of highest derivative experiences all the type of ON-OFF discontinuities.

The results shown in this paper reveals the fact that this approach builds conceptual as well as experimental base for the further growth of SMC of non-linear systems.

Zhihong *et. al.* [19] developed a control strategy for rigid robotic manipulators. The paper throws light on a robust MIMO terminal SMC method for n-linked rigid robotic manipulators. This paper presents a MIMO terminal sliding plane variable vector that holds a relation with error dynamics of the system. The authors discussed that by utilizing MIMO terminal SMC along with a small number of structural characteristics of rigid robotic manipulators, the design of robust controller can be made possible such that in a finite amount of time output tracking error converges to zero and robustness of the system is still maintained under large system dynamics uncertainties. The authors had also shown that the large gain of terminal SMC can be notably minimized with respect to the linear SMC where the sampling intervals holds non-zero value. The authors have used to control a second order robotic system.

The results of this paper successfully show that the robustness of the system is established because of sliding mode controller but also due to its insensitive nature to disturbances and uncertainties in dynamics present in the system. Also chattering problem has been eliminated by

introducing boundary layer control law. The problem in practical implementation of this method is like most of the other control techniques, a perfect convergence is not possible where sampling interval has non-zero value. The other problem encountered in this paper is the need of non-linear electronic hardware.

Young *et. al.* [20] formulated a brief guide to SMC to perform, exercise and to conduct experiments for engineers working in control systems. This paper presents precise evaluation and estimation of the chattering problem and implementation of SM controller design solutions and offers a context of reference for future research in SMC.

The authors provide a frame for both in continuous time SMC along with discrete time SMC. The VSCS can be defined in continuous time domain that has a non-continuous control action. This again results in a continuous time domain feedback systems known as VSS that is determined by ordinary differential equations (ODE's) which possesses discontinuity in its right hand side. Moreover, this work elaborates the implementation of discrete time SMC is quiet simple because of adequate fast rate of sampling time and availability of microprocessor hardware. However, the crucial framework underlying the design of feedback system prevails to be in the continuous time domain.

The authors also present the concept of chattering occurrence. An aspect of switching alone can result in high frequency of oscillations that is referred to chattering. It is a type of action that swings back and forth in regular rhythm in the vicinity of sliding manifold. The author proposes the idea of suppression of this chattering can be achieved by implementing boundary layer conditions.

The paper highlights the design that promises robustness of SM controller in the existence of real engineering limitations like finite frequency of sliding and habitual relying dynamics of the system.

Chung *et. al.* [10] developed a general class of switching surface for SMC. The concept given by authors in this paper is on the relative degree of the sliding mode that is extended to more than one rather sticking on usually choosing relative degree as one in the case of conventional SMC. Firstly, the switching surface is selected in a way such that the state space model of the system, product of input matrices and output matrices is invertible. Secondly, the authors have developed

the switching surface in a manner such that the relative degree of transfer function is not restricted to one.

Somehow, there is certain disadvantage of this method. The non-linear function leads to high frequency limit cycle. The authors prove that the certainty for occurrence of a limit cycle for non-linear function for relative degree greater than two. Furthermore, this system holds inferior phase margin. But these demerits can be overcome through achieving smoothing of the control non-linearity in a specific boundary layer along with the saturation function that guarantees asymptotic stability of the entire system. Here, the thickness of boundary layer is selected suitably.

Huisseine *et. al.* [21] implemented SMC method to achieve robust control for a flexible link manipulator. The non-linear robust control is achieved by forming controller based on SMC technique. The control of flexible link manipulator is a challenging task due to its innate unstable dynamics performance of the position of end-effectors of manipulator. The potential of the control law developed in this paper worthy to achieve robustness. The non-acceptance of perturbations and at the same time reducing the effect of amplitude of disturbances is shown in this work.

The simulated results presented in this paper ensure the robustness and reductions of elastic vibrations as well as tracking performance. Finally, authors have shown that the potential of SMC leads to trajectory control under uncertain parameters conditions.

Bartolini *et. al.* [22] examined the implementation of second-order SMC to mechanical systems. The effective implementation of SMC to mechanical systems is not direct due to its sensitivity of such systems to chattering. But second order sliding mode can act against this chattering effect in order to minimize its effect and neutralizes it by restricting switching control to the upper derivatives of the control variable.

As a matter of fact this approach demands the presence of a several derivatives of time of switching variable. But in the availability of noise these time derivatives have some practical restrictions. This category of second order SMC provides finite amount of time to converge between the sliding control and sliding variable for systems whose relative degree is two. Indeed

this process works in minimizing the quota of differentiator levels in the controller as well as while working with non-modeled dynamics of an actuator.

The authors have presented a positive situations for tuning the parameters of the controller that have been found on the basis of assumptions based on the involvement of globally bounded uncertainties. The paper discusses over the formidable problems and solutions of control theory that is concerned with mechanical systems.

Kizmaz *et. al.* [23] developed SMC technique for suspended pendulum. The paper provides information about the use of SMC control algorithm due to its low order and its compatibility with most of the control systems. The system that is discussed in this paper possesses a sinusoidal function which is in reality is a non-linear function.

The designed controller for this system was tested and the simulation results were acquired for angles between zero and sixty degree, and the results prove the robustness of the SMC that yields an efficient control for this system but has large time constant.

Hirschorn [12] framed a generalized technique of SMC for multiple inputs of non-linear systems. The paper gives the key idea about how the control action reaches to zero while the state variables move towards the sliding surface and this is how the practical implementation of the SM controller should be on the multiple inputs non-linear systems.

The switching surface is considered to have co-dimensional sub-structures that are equivalent to the dimension of controlled vector fields. The design aspect of SM controller for such a non-linear system is not a routine task since linearization of these systems is not possibly controlled. The paper discusses that the trajectory of the states are continuously smooth curves and piecewise in nature because SM controller is being designed after sample and hold circuit that is applicable in digital control.

The author ensures to accomplish globally asymptotic stability of prevalent SM controller for homogeneous systems and also asymptotic stability must reach the surface in finite amount of time frame. At last, the paper dictates that the plant trajectory is required to develop gradually on the sliding surface and control variables must have a small crossing with the tangent space to the switching surface.

Cheng *et. al.* [24] developed the method of control allocation based on integral SMC scheme to control tolerance of faults in actuator of non-linear systems. Since the faults in actuators can be detected, the proposed method can bear several abnormal operations in actuator and at the same time continues to maintain control allocation. An important thing to note in this paper is the system state trajectories and faulty system appears to be matched when disturbances in the system is matched with the set point. Therefore, engineers can further estimate the act of matched non-deterministic faulty system according to the act of a chosen nominal system.

The authors have shown that the matched uncertainties and other parameters of disturbances are compensated by appropriately selecting sliding variables parameters. Hence, from the simulation results it is clear to predict the performances of the system for both normal as well as in abnormal situations. The proposed method in this paper can be implemented to maintain the attitude of spacecraft.

Alwi *et. al.* [25] proposes SMC design methodology to control fault tolerant of a civil aircraft. Like it was said in previous paper the faults in actuators can be easily noticed. Hence, a controller is implemented around the state feedback where the gain of the non-linear control term is permitted to expand at the onset of a fault which responds when fault occurs.

The proposed controller forces the sliding function to zero and as a result controller attempts to keep nominal tracking performance. Somehow, even if the overall breakdown of an actuator is observed, still a switch is present to provide a back up to an actuator and yet there is no change noticed in the linear part of the control law.

The authors have considered a span of real faults and using the non-linear aircraft model satisfying simulation results have been shown. Moreover, authors have also focused on sensor faults and have reconstructed the signal to make it free from error signals before controller uses it. Thus, reconfiguration of controller is not required.

Paz *et. al.* [26] proposed SMC for regulating a liquid level system. The work elaborated in this paper consists of challenges in controlling non-linear output using SMC scheme. The given liquid level system is interconnected that comprises of two tanks which is a standard industrial application. The task is to achieve the stability of the liquid level in both the tanks of the system via SMC technique. The motive of this work is to acquire the control system independent of

system parameters that are obtained from mathematical model of the plant by considering the error as the sliding surface and block the disturbances produced in the system that originates from environmental factors.

The authors have selected proper sliding surface that leads to keep away the use of observers or state estimators. Therefore, for this approach a straight forward measurement of the level of liquid in both the tanks is conducted solely by feedback state variable that reduces the computation than done in adaptive controllers. Moreover, this entire process was tested and efficiency of SMC was proved in real industrial surrounding with the experimental results shown in this paper.

Selvan *et. al.* [8] developed SMC technique for pressure process. In general, control of pressure parameter find applications in various plants like power plants, chemical and petro-chemical industries, iron and steel manufacturing plants and mainly in boilers. There are several controllers like PID, artificial intelligent controllers and fuzzy controllers. But in this paper an efficiency of SM controller is discussed.

SM controller is executed to regulate and to detect the pressure in the ongoing process. The authors suggest that control of the system excessively rely on sliding surfaces. But selecting an optimal sliding surface is not an easy task. However, a strong robustness can be achieved against external disturbances and uncertainties.

Yazici *et. al.* [27] developed a non-linear program based on SMC of an inverted pendulum. The criteria of SM controller can be found by suitably choosing the right objective function and limitations of the system.

The author used non-linear programming to find the control law that directs the non-linear state of this dynamic system in the direction of the stable sub-space in the state space and then further move towards the specific neighborhood of the sub-space. Thus the control law moves towards the origin. The solution to the problem only consists of handling algebraic differential equations and finally the results discussed in this paper shows that this approach stabilizes the inverted pendulum.

Kui *et. al.* [28] developed SMC technique to control non-linearity of three-dimensional (3-D) rigid pendulum. The 3-D rigid pendulum model is equivalent to the model of geo-stationary space craft. Basically, it is the reduced model that resembles the dynamics of geo-stationary spacecraft. The system comprises of rigid body held by a firm and frictionless fulcrum and possesses three degrees of rotation that reacts on by a fixed gravitational force. The SMC scheme is used for controlling non-linear attitude of this pendulum kept at any random position.

The authors elaborate the division of the system into two sub systems. Lyapunov function is used to find the overall control input law that consists of identical control components of both the sub systems, that guarantees each the sub system evolve on their individual switching surfaces.

The authors comment on the merits of rapid response speed and large steady state accuracy of SM controller that can be found in the simulation results provided in this paper. The authors also prove that there is no impact of system disturbances on the controller. Results proves that the implementation of this controller practically successes in producing a desired result in controlling non-linear behavior of the 3-D rigid pendulum.

Wai *et. al.* [29] proposed SMC method to adaptively stabilize and to track the control for an inverted pendulum fixed on the top of a movable cart wheel. The mechanism of the system consists of a dual axis inverted pendulum involving dynamics of an actuator too possesses large non-linearity. To design an appropriate controller needs much effort and skill and is not usually straightforward that perceives real time stabilization as well as error free tracking control for all time.

The simulation results in this paper reveal that proper balance of an inverted pendulum along with appropriate control of cart position is achieved. Furthermore, the experimental results also prove that this control strategy results in a high performance dynamic characteristics as well as strong robustness is also being observed with respect to the parameter variations and several initial states.

Banarjee *et. al.* [30] proportional SM controller for linear one stage inverted pendulum. The authors talked about the complexity of inherent non-linearity involved in an inverted pendulum and also about the environmental external disturbances present in this system. Although, the system is highly non-linear the SM controller is robust enough to control its non-linearity. The

paper illuminates the design guidelines of SMC and its features. This work focuses on the mathematical modeling of the proportional SM controller.

The results discussed in this paper reveal transparency about the robustness of the controller under various perturbations and uncertainties. Moreover by tuning the proportional gain maintains the robustness of the system and also decreases the settling time substantially that improves the performance of the system. However, switching at high frequency generates chattering that needs to be minimized.

Khanesar *et. al.* [31] developed SMC strategy to establish stability of RIP. The author dictates the nature of RIP as non-minimum phase system (NMPS), non-linear and unstable system. The system consists of one input and two outputs. The input of the system is the voltage provided to the rotor which regulates the pendulum. The outputs of the system are pendulum angle and rotor angle. The authors laid the mathematical modeling of rotary inverted pendulum (RIP) thoroughly. The motive of this work is to control both pendulum and rotor angle using SM controller. Since there are two outputs to be controlled, the two sliding surfaces need to be mathematically designed for controlling each output faithfully.

The paper mathematically proves that feedback linearization method fails to control and stabilize the dynamics of RIP model. On the basis of the desired performance a method of designing two switching surfaces are carried out. The Candidates' Lyapunov function is being considered that focuses more on the proper regulation of pendulum than the control of rotor. The successful mathematical design of switching surfaces is briefly mentioned in this work.

The simulation results show that the system converges to zero in finite time and asymptotic stability has been confirmed. Both rotor and pendulum angles have been controlled successfully and stability is achieved. However, the chattering problem was solved by the help of considering saturation function.

In almost every paper discussed by now clearly observed about the problem and cause of chattering phenomenon that essentially needs to be either reduced or eliminated. Several works were carried to reduce or eliminate chattering. Among several works conducted one work is being discussed here on how to overcome effect of chattering.

Loh *et. al.* [32] mentioned the technique to reduce chattering in SMC and to provide improvement in the outputs of the non-linear systems. The authors proposed an algorithm for the SMC of distinctive non-linear systems. The method is derived from the relation involved between speed and distance during the reaching phase. Moreover, this method is based on the relation between state variable and state velocity of the second order non-linear systems. The authors have developed the speed control connection in reaching phase. The result in this paper elaborates the better performance than the prior works and considerable reduction can be observed in chattering effect.

All the papers discussed above are based on conventional SMC that inherently appears in its integer order form. Researchers and scholars have used IOSMC to control variety of plants and processes and also proved the efficiency of traditional SMC.

2.2 LITERATURE REVIEW ON FRACTIONAL ORDER CONTROLLERS

The FO calculus gained radical attention among the scientists and researchers and several researches and experiments have been conducted on various systems. Some of the works are presented here.

Zennir *et. al.* [33] designed the robust fractional multi-controller for an inverted pendulum system. The inverted pendulum is fixed on a movable cart. The paper describes the inverted pendulum system as non-linear, multi variable and unstable system. The authors present the guidelines of multi-controller implementation to control and also elaborate operation of both linear and non-linear mathematical model of the system. The action of fractional order PID (FOPID) controller with the concept of approximation is proposed in this paper.

From the simulated results it can be concluded that use of FOPID enhances the robustness. However, there is a requirement of large optimum estimation to further reduce the order and selection of the parameters present in the FOPID. The results demonstrate the capability of this technique by tuning the controller parameters.

Sondhi *et. al.* [6] carried a survey on fractional order controller and its applications. The applications of FO based controller deals diversely on the forth coming domains of engineering

like process control, robotics and power electronics. The pros and cons of conventional PID controller are briefly discussed in this paper.

The crucial obstacle faced by traditional PID controller is their non-linearity. Therefore, for non-linear systems the performance of traditional PID controller varies. To solve this problem, conventional PID controller is being modified by the means of fractional calculus.

The authors discussed the mathematical preliminaries of fractional calculus. The Riemann-Liouville (R-L) definition and Grunwald-Letnikov (G-L) definition used to calculate non-integral functional derivatives and the formation of new mathematical expression for modified PID controller are provided in this paper. The implementation of FOPI controller is conducted on a SISO linear system. Here, the authors excluded the derivative control action on the system. The satisfying simulation results were acquired using step function for both traditional PID and FOPID controller. Moreover, these results were distinguished by the help of simulink model given in the paper.

Singh [34] proposed a method for stabilizing the equilibrium position which is unstable in the under-activated RIP system by means of implementing FO controller. The parameters present in the integer type PID controller are formed on the basis of pole placement technique. The basic PID controller is considered and converted into FO. The fractional parameters found in resulting FOPID controller is tuned by frequency domain analysis by means of Nyquist plot.

The paper presents the mathematical foundation of RIP and mathematical base of FOPID controller. The simulation results are demonstrated that clearly evident the robustness of the inverted pendulum in the existence of uncertainties and disturbances. Gain margin, phase margin as well as sensitivity of the system are being calculated that proves the robustness of the system. Moreover, from the result displayed in the paper clearly evidences the faithful tracking of set point is achieved. The inclusion of fractional calculus on integer type conventional PID controller enhanced the potential of overall resulting controller.

Mahani *et. al.* [35] has shown the performance assessment of linear non-minimum phase control systems with the help of fractional order partial pole-zero cancellation method. The real zeroes occurring at the right half part (RHP) of the S-plane effectively causes restrictions on the performance of NMPS. The paper dictates that the implementation of feedback cannot eliminate

such restrictions on NMPS due to the fact that zeroes in RHP of the S-plane fails to get cancelled by the poles of a controller that are unstable. It is because these cancellations further produce inherent instability in the system.

Therefore, the author proposed fractional calculus concept to conduct partial cancellation of zeroes present in RHP of the S-plane with RHP poles without affecting the inherent stability of the system. The results were compared with and without fractional order and it were shown that the under-shoot as well as the settling time is revamped in the resulting system.

Moreover, the optimum criteria of integral of absolute error multiplied by time is also comparatively improved and utilizing this optimum criteria the operation of closed loop systems are comparatively refined. The interesting thing to note in this paper is that the poles and zeroes are still being used to decide the stability of FO transfer function.

Narang *et. al.* [36] established continuous time model recognition of FO representations having time delays. Performing mathematical models of actual physical structures that consists of large temporary memories and large multi-dimensional form by the means of dynamic models having FO has radically been a topic of interest. Due to this fact, several identification methods in frequency domain as well as in time domain have been flourished to model such systems. The author provided the basic mathematical R-L definition and G-L definition involved in fractional calculus. A description of prototype SISO based FO transfer function in the realm of Laplace transform is also provided in this paper.

The paper presents the parameters consist in such a system in continuous time domain accompanied with approximation of time delays that are involved in plants' processes having time delays. In this work, a linear filter is brought in operation for concurrent approximation of overall parameters present in the model and to compare FO system models with time delays. The simulation results were successfully carried out in the presence of noise.

2.3 LITERATURE REVIEW ON FOSMC

The successful implementation of fractional calculus on various systems was observed by researchers and mathematicians. The diversity in applications of fractional calculus can be widely seen. Even popular traditional PID controller was transformed into fractional domain to

enhance its performance. Therefore, the non-integral calculus was implemented on SMC theory. The conventional IOSS was being transformed into FOSS and better results have been observed so far.

Several works has been carried out in FOSMC. The different classes of system have been placed under this test and better results have been observed. Here, some of the works are described in brief.

Valerio [37] introduced fractional calculus on SMC. The author elaborated that the FOSMC can be either implemented on processes where system is in FO or directly in SM controller where the dynamics of the switching surface holds FO. The term fractional corresponds to the incorporation of derivatives that is a fraction rather than usually being an integer with respect to time in the differential equation that dominates the system dynamics. This corresponds to fractional exponents of S and $j\omega$ in the Laplace domain and Fourier domain respectively.

The author gives a brief description on the formation of fractional derivatives and also presented G-L definition and R-L definition to tackle with fractional derivatives. In this paper SMC is discussed where the sliding surface holds a FO. The method of transforming FOSS for a SISO system is elaborated in the paper.

The author considered the plant dynamics and implemented the FOSS on the plant. From the simulated results it can be figured out that behavior of fractional dynamics has substantial robustness properties. The simulation was carried out using the G-L definition. To avoid chattering saturation function is being considered.

Hosseintia *et. al.* [38] deals with the idea of forming stable nature of switching systems. The order of switching system is in fraction. The paper generalizes the conventional Lyapunov theory to FO to deal with the constraints for the stability of switching systems and effective results are shown via simulation.

Efe [39] proposed an approach of FOSMC with reaching law. The reaching law method is embraced to attain control over the system. The important point to be observed in this paper is the representation of SMC by FO also has a same ground basis in logic for IOSMC.

The attractor characteristic of sliding manifold is studied by considering a condition of the stable switching sub-space. The adequate parameters and its suggestions for stability are presented in this paper.

The reaching law tends the FOSS in the closed loop which generates an attracting sliding surface sub-space that corresponds to zero. In the beginning of the paper basic Caputo definition to calculate FO derivatives is denoted.

The author used candidates' Lyapunov function that is differentiated holding fractional number of times, i.e., 0.5 times differentiation using the Leibniz law of derivative for performing simulation. A method to convert IOSS into FOSS is provided in this paper. The simulation results obtained signifies the strong adaptation of SMC with fractional calculus for non-linear dynamical systems while preserving the robustness of SM controller. However, the author mentioned the disadvantage of using the Leibniz rule for differentiating candidates' Lyapunov function since it generates infinite number of terms.

Zhao [40] implemented FO on fast terminal SMC strategy for a category of dynamical systems having uncertainties. The author discussed the R-L fractional derivative definition and also elaborated properties of fractional derivatives and integrals. These properties discussed in this paper are semi-group properties and composition rules.

The integer sliding surface is transformed into fractional surface and then the control input law is being established using the Lyapunov function to secure switching condition. Furthermore, to establish a stability of the closed loop system in finite amount of time is being guaranteed. Also, to terminate the chattering phenomenon caused in the system the control law is further modified using signum function.

The author tested this technique on three different systems considering the same FO as 0.9. The three different systems demonstrated in this paper are magnetic bearing system, inverted pendulum on a translating cart wheel and two link manipulator system. The simulation results prove the robustness and effectiveness of the demonstrated methodology. A cautious selection of tuning parameter is necessary to achieve perfect simulation result.

Bouarroudj *et. al.* [41] imposed FO to design SM controller for a non-linear system. A non-linear SISO system is being considered. The authors have introduced a new variable 'Z' transferred from one surface to other.

The paper introduces a brief note on fractional calculus where generalized fractional order mathematics in terms of gamma function is presented along with R-L, G-L and Caputo definition. The imposition of non-integral order calculus on integer SM controller is being thoroughly discussed in this work. The control input law has been derived using the concept of both the sliding surfaces and the intermediate variable 'Z' in the case of integer as well as fractional surfaces.

The non-linear system considered in this paper is an inverted pendulum on a movable cart. For faithful tracking of error vector and to guarantee the asymptotic stability candidates' Lyapunov function is chosen. Moreover, to reduce chattering effect saturation function is being considered. The simulation is performed and a comparison is drawn between traditional SMC and FOSMC. It can be observed from the paper that FO based SM controller converges faster than the conventional SM controller.

CHAPTER 3

SLIDING MODE CONTROL

3.1 INTRODUCTION

SMC theory is derived from a class of VSS. The principal and unique trait of VSS is that the changes can take place within the structure of the system itself at the time when transient process is going on [42]. The structure of a VSS can also be changed in a manner conforming to some pre-allocated laws or algorithms of structural change [10, 11, and 42]. When these structural changes happen and as a result the sort of a structure formed are not decided by any fixed program rather it is decided as per the present value of the error signal and error signal derivatives [42]. The structural change also known as switching takes place in accordance with the state of the system and therefore the transfer function or the gain of the system can be overall changed [11]. SMC also guarantee finite time convergence [43]. This attractive feature positions VSCS in a maverick category of control systems and also distinguishes from programmed controllers.

In the year 1960, VSCS was first proposed by S.V. Emelyanov, U. Itkis, Vladimir Utkin and B. Drazenovick in The Soviet Union [13]. Since this control theory was proposed in Russian language it was only limited in Russia until English translation of this work was conducted. And after that this control theory grabbed the attention of many researchers and scientists world-wide.

VSS contains an arrangement of continuous subsystems known as structures that possesses a specific switching law and thus the control actions remained no longer continuous in nature rather it has been discontinuous functions of the system state and also if there are disturbances present in the system is also discontinuous along with the source input [17].

As it is known so far that the structural changes take place at a particular state which further cause to bring discontinuities in the surface in the phase planes and these discontinuities in the surface is commonly known as switching surface which if fulfills the requirement of having positive attraction then that surface can be referred to sliding or switching surface [11].

SMC is defined as a non-continuous state function that results in either a high frequency on or high frequency off state [9, 10, 17, 20, and 26]. It features a feedback control rule and a directive of decision which is designated as a switching function [26]. Therefore, SMC uses high speed control action such that the state trajectory of the non-linear plant is forced onto a particular user selected surface in the state space and there after it preserves the state trajectory of that non-linear plant on this surface for the following time [9, 10, 31 and 44].

Some authors also describe sliding surface as a switching surface because there is a discontinuity observed on this sliding surface [44]. In SMC there are two types of gain that must be noted in the feedback path where one gain corresponds to the state trajectory of the plant above the sliding surface and other gain corresponds to the state trajectory of the plant below the sliding surface [31, 44]. Varying the structure using discontinuous control law forces the phase trajectory to hyper-plane that is considered to be stable [9, 10]. The essential framework is the design of sliding surface on which the plants' trajectory gradually evolves [9, 13, and 45].

The convergence or stability to the sliding surface is determined by considering a Lyapunov function [31, 43, and 45]. Importantly, stability is a described in terms of Lyapunov stability [31, 45]. Lyapunov stability determines the system trajectory of the plant can be placed somewhere close to origin [45].

The preliminary concept of stability is defined as, if there is an equilibrium state let's say $X = 0$ is referred to stable if for any value of $R > 0$, there exists, $r > 0$ in such a way that if $\|X(0)\| < r$, then $\|X(t)\| < R \quad \forall t \geq 0$, else there will be an unstable equilibrium point [45].

The plant's system trajectory claims to have two phases [46]:

1. Reaching Phase: It is the phase where plant's system trajectory reaches the sliding manifold,
2. Sliding phase: It is the phase where system trajectory gradually develops on the sliding manifold or in other words it starts to slide on the sliding manifold.

There are three major steps in designing the sliding mode controller [44]:

1. Fundamental step is to design the sliding surface.

2. To design the control input that maintains the trajectory of the system on the sliding surface.
3. And the last step is to sketch the condition that reduces chattering.

3.2 SLIDING SURFACE DESIGN

The fundamental step is the design of sliding surface that guarantees stability of the plant.

Consider a non-linear plant system with single input defined [30, 31, 43, 45, 47, and 48] as,

$$x^{(n)} = f(X, t) + b(X)u \quad (3.1)$$

Where x is referred to a scalar quantity and also is the output of the system. It is also known as state variables of the systems. u is also a scalar quantity which is a controlled input to a plant.

X is the state vector of form given as,

$$X = \begin{bmatrix} x \\ \dot{x} \\ \cdot \\ \cdot \\ \cdot \\ x^{(n-1)} \end{bmatrix} \quad (3.2)$$

Where $f(X, t)$ is an unknown non-linear function of state variable x and time t . However the stretch of lack of exactness on this unknown non-linear function is upper bounded by a continuous function of X which is known and in a same way $b(X)$ is also an unknown non-linear function of state variable x and time t but range of exactness on this unfamiliar non-linear function is upper bounded by a familiar continuous function of X [30, 45 and 49].

The control problem that needs to be solved for the state vector X is to route a particular time varying state in the presence of lack of exactness on $f(X, t)$ and $b(X, t)$ [45] is given as,

$$X_d = \begin{bmatrix} x_d \\ \dot{x}_d \\ \cdot \\ \cdot \\ \cdot \\ x_d^{(n-1)} \end{bmatrix} \quad (3.3)$$

Applying a finite control input u to the model tracking is made to be achieved such that the initial desired state $X_d(0)$ is supposed be [45] as shown below,

$$X_d(0) = X(0) \quad (3.4)$$

Consider the tracking error to be e in the variable x which is a state variable [30, 45] shown as,

$$e = X - X_d = \begin{bmatrix} x - x_d \\ \dot{x} - \dot{x}_d \\ \cdot \\ \cdot \\ \cdot \\ \dot{x} - \dot{x}_d^{(n-1)} \end{bmatrix} \quad (3.5)$$

The above equation is known as the tracking error vector.

Now, in the given state space define a time varying surface $S(X, t)$ [31, 45, and 47] given as,

$$S(X, t) = \left(\frac{d}{dt} + \lambda \right)^{n-1} e = 0 \quad (3.6)$$

where λ is defined as a constant that must be stringently positive and n is defined as an order of the system.

The sliding surface represented in (3.6) occurs inherently in its integer order. Important point to note here is $S(X, t) = 0$ is not a vector quantity rather it is a scalar [45].

Now, for a second order system the value of $n=2$, then from (3.6), the resulting time varying surface obtained is given as,

$$S = \frac{d}{dt}e + \lambda e \quad (3.7)$$

Now, if e is position error then \dot{e} is a velocity error.

Therefore, the conclusion can be drawn that the time varying surface is a weighted sum of positional as well as velocity error [45].

Here, $S \equiv 0$ depicts a linear differential equation and $e \equiv 0$ is a particular solution to this ODE for the proposed initial condition $X_d(0) = X(0)$.

Also solution to difficulty of tracking $X = X_d$ for all the time is being identical to that of staying on the time varying surface $S(X, t) \forall t > 0$ and hence, maintaining the scalar equation $S(X, t)$ at zero for all the time is the solution to the difficulty of tracking the n -dimensional vector X_d [10, 45].

The most important characteristic of SM controller is its robustness its insensitive nature to disturbances and uncertainties in dynamics present in the system and therefore, sliding surface is also called invariant [9, 13, 20, 45, 47, 50, 51, and 52].

This can be proved by the following equation [45] given as,

$$\frac{1}{2} \frac{d}{dt} S^2 \leq -\eta |S| \quad (3.8)$$

where η is defined as stringently positive constant and the term S^2 is a measure of squared distance of output phase trajectory to the surface.

Consider the phase space shown in Figure 3.1 which consists of different phase trajectories moving in plane. The phase space is a plot between X versus \dot{X} i.e. position versus velocity where position and velocity are function of time t .

From (3.8) reveals the fact that the phase output trajectory may start from any point in the phase space must hit the line $S(t) = 0$ as shown in Figure 3.1. The line $S(t) = 0$ has a slope of λ .

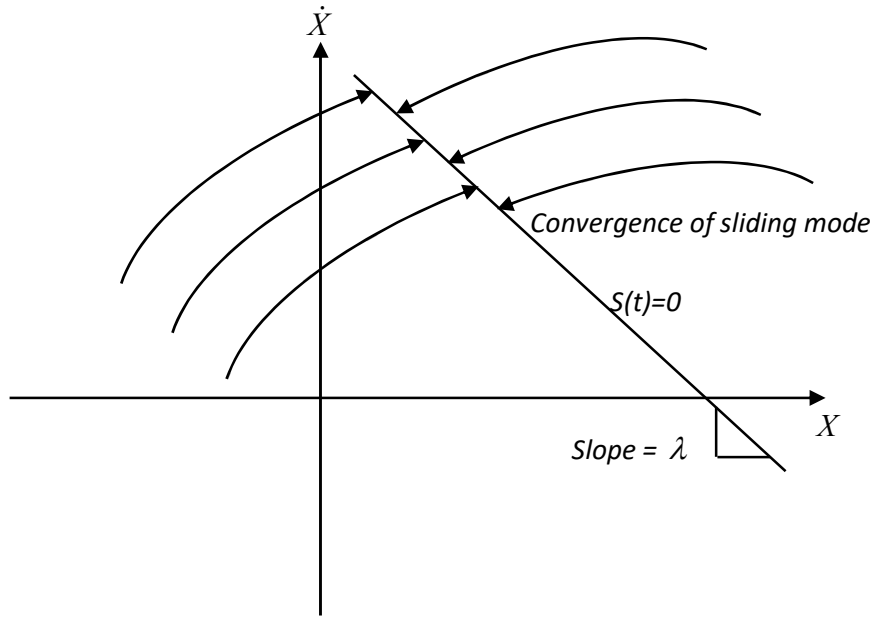


Figure 3.1: Geometrical significance of sliding surface equation

Figure 3.1 portrays different output trajectories point towards the line $S(t) = 0$. This line $S(t) = 0$ is called sliding surface and the behavior of various trajectories on this sliding surface is known as sliding mode [45].

The terms S^2 and $-\eta$ present in (3.8) implies that the output trajectory will terminate at the line $S(t) = 0$ and will always remain on this surface for all time [45].

To make certain that the output trajectory should deliberately develop gradually on the sliding surface, at every single point on the sliding surface, the range of the controlled vector fields are considered to have an insignificant crossing with the tangent space to the $S(t) = 0$ [12, 45].

Since the output trajectory always stays on this surface, sliding surface is also said to be an invariant set and this is the reason that once the output trajectory reaches the surface $S(t) = 0$ will always remain on this surface irrespective of disturbances [45]. This proves that SMC is insensitive to disturbances or tolerates dynamic uncertainties.

3.3 CHATTERING

The major drawback found in SMC is the chattering phenomenon. An aspect of switching alone can result in a non-decaying oscillatory element that have fixed amplitude and frequency that is

known as chattering [9, 20, 43, 53, and 54]. An aspect of switching alone can result in a non-decaying oscillatory element that have fixed amplitude and frequency that is known as chattering [20]. Since switching takes place at very high frequency that causes systems to produce a large frequency of oscillations in the vicinity of sliding manifold [9, 20, and 53]. If chattering is not minimized then this problem will cause inaccurate control, large heat dissipation in power circuits and may damage dynamical mechanical components involved in the system due to increase in friction [54]. The signum function is considered to reduce chattering effect [31, 43].

Mathematically signum function with respect to time is defined below as,

$$y = \text{sign}(t) = \begin{cases} 1; & \text{if } t > 0 \\ 0; & \text{if } t = 0 \\ -1; & \text{if } t < 0 \end{cases} \quad (3.9)$$

The plot of signum function with respect to time is given below in Figure 3.2:

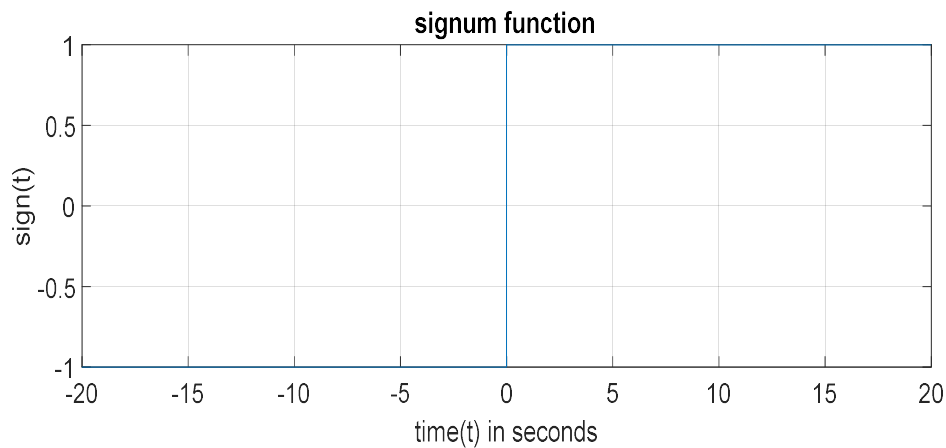


Figure 3.2: Plot of signum function with respect to time

CHAPTER 4

FRACTIONAL CALCULUS

4.1 INTRODUCTION

Fractional calculus is more than three century old concept of calculus and most fundamental formulation in fractional calculus was done in nineteenth century [16]. The conventional integer order derivatives and integrals are generalized into non-integer order by means of gamma function [16, 55, 56 and 57]. In mathematics, the term fractional calculus is defined for any arbitrary order and it takes part in analysis of differentiation and integration of any random order [58].

The geometrical significance of a derivative in conventional calculus is the tangent to a curve of a particular function, i.e., it is a point property since the tangent drawn to a curve passes through a unique point. But in case of fractional calculus derivatives and integrals do not possess local property or point property that means it possesses non-local distributed effects [16, 55, and 56]. The integer order derivative is being calculated by using factorials. The generalized factorial definition in terms of gamma function was developed in the year 1811 by French mathematician Adrien Marie Legendre [59] which laid the base stone in development of fractional calculus.

4.2 MATHEMATICAL PRELIMINARIES OF FRACTIONAL CALCULUS

Some of the basic mathematic preliminaries of fractional order derivatives are presented in this section:

4.2.1 Gamma function

Mathematically, it is defined [60] as,

$$\Gamma(m+1) = \int_0^{\infty} e^{-x} x^m dx \quad (4.1)$$

where $m > -1$.

4.2.2 Lacroix definition

In the year, 1819 Lacroix used gamma function in place factorial to calculate fractional order n^{th} derivatives [16] given as,

$$\frac{d^n g(x)}{dx^n} = \frac{\Gamma(m+1)}{\Gamma(m-n+1)} x^{(m-n)} \quad (4.2)$$

Where m is a positive integer and $m \geq n$.

4.2.3 Grunwald-Letnikov (G-L) derivative definition

The G-L definition to compute fractional derivative [16, 55, and 56] is expressed as,

$${}_a D_t^\alpha g(x) = \lim_{h \rightarrow 0} \frac{1}{h^\alpha} \sum_{j=0}^{\lceil \frac{x-a}{h} \rceil} (-1)^j \binom{\alpha}{j} g(x-jh) \quad (4.3)$$

Where α represents the order of derivative, $\binom{\alpha}{j}$ represents the binomial coefficient and a represents initial value. This method provides fine approximation for calculating derivatives for low values of h [57].

4.2.4 Riemann-Liouville (R-L) derivative definition

The definition of R-L derivative [16, 55, 56, and 57] is given as,

$${}_k D_t^\alpha g(x) = \frac{1}{\Gamma(p-\alpha)} \left(\frac{d}{dt} \right)^p \int_k^t \frac{g(\tau)}{(x-\tau)^{\alpha-p+1}} d\tau \quad (4.4)$$

where α is any real number lies between $(p-1) \leq \alpha < p$ and p is an integer.

4.2.5 Caputo derivative definition

The Caputo derivative [16, 55, and 56] is expressed as,

$${}_k D_t^\alpha g(x) = \frac{1}{\Gamma(p-\alpha)} \int_k^t \frac{g^{(p)}(\tau)}{(x-\tau)^{\alpha-p+1}} d\tau \quad (4.5)$$

where α is any real number lies between $(p-1) \leq \alpha < p$ and p is an integer.

4.2.6 Linearity property

The linearity property of fractional order derivative can be expressed as,

$${}_k D_t^\alpha (g(x) + h(x)) = {}_k D_t^\alpha g(x) + {}_k D_t^\alpha h(x) \quad (4.6)$$

4.3 DEVELOPMENT OF FRACTIONAL ORDER CONTROLLERS

Up until the end of nineteenth century there was slow pace of development in fractional calculus. After the year 1900, a rapid enhancement can be found in this domain and several mathematicians formulated the formula to calculate non-integral orders [61]. Later, these developments helped to turn many controllers into fractional order that traditionally existed in their integer form.

The mathematics of fractional calculus helped to construct fractional order control systems with controllers whose order got transformed from an integer to fractional numbers. The combination of fractional calculus with traditional SMC produced FOSMC where order of switching surface deals in fractional orders [37, 40, 41, and 55]. The resulting fractional controller helped to produce desired output for various complex systems. The implementation of FOSMC is being used to control attitude of spacecraft and is also validates the system stability and finite time convergence [62]. FOSMC have been used to regulate the single link flexible manipulator and making the system insensitive to external parametric uncertainties and disturbances [63]. Systems like inverted pendulum and quad-rotor unmanned aerial vehicle have been successfully controlled and stabilized using FOSMC [64, 65].

The FOSS is represented [40, 41, and 55] as,

$$S(x,t) = D^\alpha \dot{e} + \lambda e \quad (4.7)$$

where α is the order of derivative, and $\alpha \in (0,1)$.

CHAPTER 5

PROPOSED METHODOLOGY

5.1 INTRODUCTION

Generally, in control theory the mathematical model of real systems is framed and then controlled input law is calculated in order to stabilize the dynamics of systems [2, 43]. However, there are mismatches that prevail invariably in between the mathematical model of the system and the real system [43]. These mismatches are nothing but errors between actual and mathematical model that are predominantly generated by means of unprecedented extraneous or environmental disturbances [43]. Moreover, if the system dynamics is non-linear and unstable then stabilizing such systems turns out to be increasingly arduous [2]. Therefore, to bridge the gap of errors in between mathematical model and an actual system a control law has to be determined that yields the desired output of the closed loop systems even when the systems are exposed to external uncertainties and disturbances [43].

Here, two numerical examples of the system are considered on which the performance of IOSMC and FOSMC have been experimented.

5.2 OBJECTIVES OF THE PROPOSED METHODOLOGY

The objectives of the methodology are as follows:

1. To lay a design of IOSMC in order to stabilize the systems cited in both the examples. The most important part of IOSMC design is to develop a sliding surface. The trajectory of the systems must unfold on this designed sliding surface and once the trajectory gets unfold on the surface must remain obstinate for all the time on this surface even when there are extraneous disturbances are present.
2. A most important part of the methodology is to design a control input law which is the heart and soul of any system to be controlled in a desired manner. This control law should guarantee

asymptotic stability of the system. The control input law has been proposed using the equation of sliding surface in both the examples.

3. Moving further, the next step is to use mathematics of fractional calculus to design fractional order based SMC. Again applying the same concept of designing sliding surface and proposing control input law in order to analyze the performance of the systems to varying fractional orders.

4. Lastly, a comparative analysis of performance of integer type conventional SMC as well as FOSMC for the examples has been outlined.

5.3 NUMERICAL EXAMPLES

Here, two numerical examples of the system are considered on which the performance of SMC and FOSMC have been experimented. In both the examples a common Lyapunov function is considered [31, 41, 43, and 45] given as,

$$V = \frac{1}{2} S^2 \quad (5.1)$$

In order to provide globally finite time asymptotic stability, the derivative of Lyapunov function is supposed to be negative [31, 41, 43, and 45] as shown below,

$$\dot{V} = S\dot{S} < 0 \quad (5.2)$$

The assignment is to design a feedback control action u that guarantees asymptotic stability, i.e., this control action is supposed to force both x_1 and x_2 state variables and also supposed to attain a finite time convergence [43, 45] in both the examples considered here. In other words, a finite settling time should be available. A settling time is that time instant where the output of the system stays steady. The initial state of the state variables are assumed [43] as, $x_1(0) = 1$ and $x_2(0) = -2$ in both the numerical examples.

5.3.1 Example 1

Consider a SISO system [43] given in the following equations,

$$\dot{x}_1 = x_2 \quad (5.3)$$

$$\dot{x}_2 = u + f(x_1, x_2, t) \quad (5.4)$$

where x_1 and x_2 are state variables, $f(x_1, x_2, t)$ is the disturbance to the plant, and the control input law is given by u .

Firstly, assume an ideal condition that there is no disturbance present at all, i.e., $f(x_1, x_2, t) = 0$.

Now, consider a linear feedback control force [43] given as,

$$u = -k_1 x_1 - k_2 x_2 \quad (5.5)$$

where $k_1, k_2 > 0$.

Considering the values of $k_1 = 3$ and $k_2 = 4$ [43] and with the initial conditions stated above in section 5.3, the simulation result with this control law is portrayed in Figure 5.1.

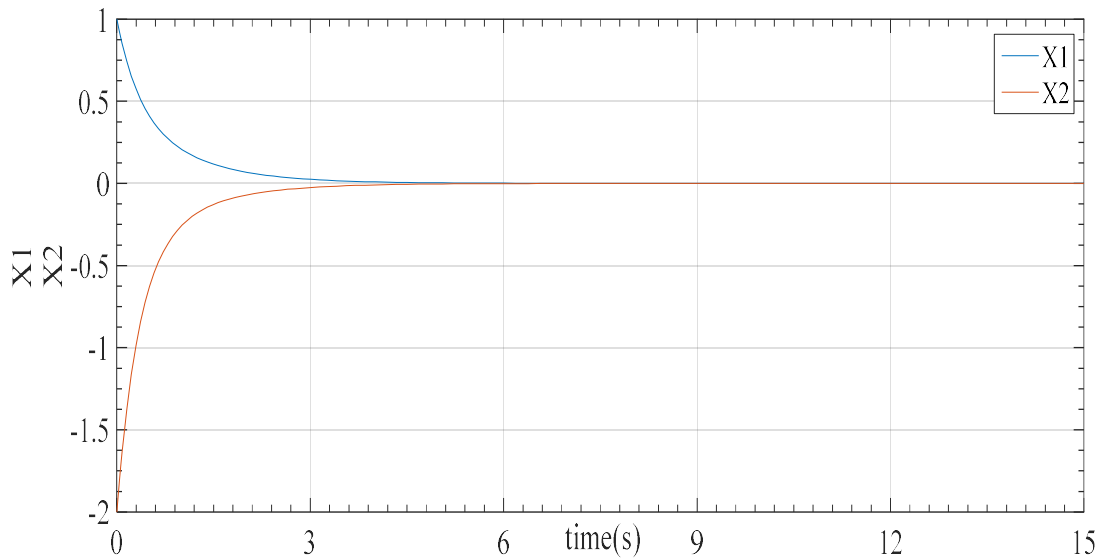


Figure 5.1: Response of state variables with respect to time under zero disturbances using linear feedback controller.

From Figure 5.1, the settling time T_{s1} for x_1 and the settling time T_{s2} for x_2 are 5.4 seconds respectively with zero steady state error and the conclusion is the system is found to be stable under zero disturbances.

Secondly, suppose at this stage where the system is stable suddenly, from somewhere a disturbance is caused, i.e., $f(x_1, x_2, t) = \sin(2t)$ [43]. Now, examine with the same control action and interpret the results.

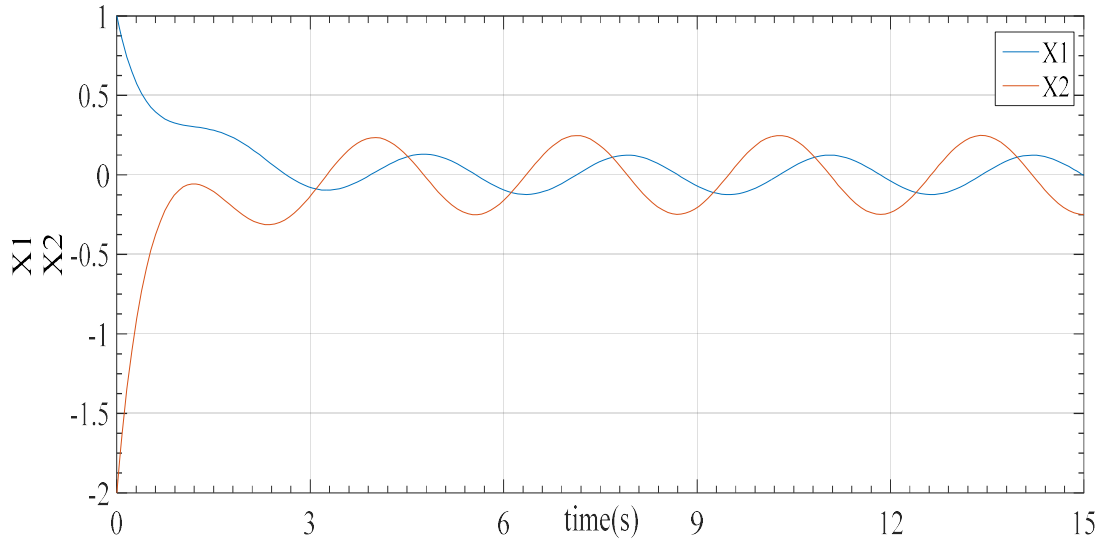


Figure 5.2: Response of state variables with respect to time with sinusoidal disturbance using linear feedback controller.

The Figure 5.2 shows both the state variables have not attained steady state. One thing is far clear that the disturbance is non-linear since it contains sine term. As, a matter of fact, the trajectories of the state variable starts following the sinusoidal disturbance. Therefore, the system turned unstable in presence of non-linear disturbance and hence, the control action turns out to be acquiescent against the sinusoidal disturbance and stabilizing the system becomes an uphill task.

Now, the question arises how to guarantee the stability of such a system? The candid answer is to implement an influencing controller to design a control input law such that it guarantees asymptotic stability. However, designing such a control law is a hard nut to crack.

A. Design of IOSMC based control law

The SM controller is being used to frame the control input action. The cardinal task is to design a sliding surface. From (3.7), (5.3) and (5.4) the designed switching surface is given below as,

$$S = x_2 + \lambda x_1 \tag{5.6}$$

In order to guarantee stability for all the time under disturbance, the resulting sliding surface in (5.6) is equated to zero. Therefore,

$$x_2 + \lambda x_1 = 0 \quad (5.7)$$

Differentiating (5.7) with respect to time, results as,

$$\dot{x}_2 + \lambda \dot{x}_1 = 0 \quad (5.8)$$

Substituting the terms of differential state variables \dot{x}_1 and \dot{x}_2 given in (5.3) and (5.4) in (5.8) results in the following equation shown as,

$$u + f(x_1, x_2, t) + \lambda x_2 = 0 \quad (5.9)$$

Therefore,

$$u = -f(x_1, x_2, t) - \lambda x_2 \quad (5.10)$$

Here, the terms appearing in left hand side of (5.10) is multiplied with a discontinuous function of sliding surface S , i.e., signum function of S , $\text{sgn}(S)$. Therefore, the resulting equation is given as,

$$u = -f(x_1, x_2, t) \text{sgn}(S) - \lambda x_2 \text{sgn}(S) \quad (5.11)$$

Furthermore, in order to reduce chattering effect the term $-\rho \text{sgn}(S)$ is added to the above control law u , where $\rho > 0$. The resulting control law is given in following equation as,

$$u = -f(x_1, x_2, t) \text{sgn}(S) - \lambda x_2 \text{sgn}(S) - \rho \text{sgn}(S) \quad (5.12)$$

This control law not only provides asymptotic stability but also results in finite time convergence.

B. Design of FOSMC based control law

Earlier in previous section the switching surface was differentiated in integer order. Now, from (4.7) the sliding surface can be differentiated in fractional values. Thus, using (4.7) the resulting FOSS formed is given below as,

$$S = D^\alpha x_2 + \lambda x_1 \quad (5.13)$$

Again, the sliding surface is equated to zero to provide an asymptotic stability in finite amount of time. Therefore, the resulting expression is as,

$$D^\alpha x_2 + \lambda x_1 = 0 \quad (5.14)$$

Differentiating (5.14) with respect to time, the resulting equation is shown below:

$$D^\alpha \dot{x}_2 + \lambda \dot{x}_1 = 0 \quad (5.15)$$

Substituting the terms of differential state variables of (5.3) and (5.4) in (5.15) results in the following equation given as,

$$D^\alpha (f(x_1, x_2, t) + u) + \lambda x_2 = 0 \quad (5.16)$$

Using the linearity property given in (4.6), (5.16) can be expressed as,

$$D^\alpha f(x_1, x_2, t) + D^\alpha u + \lambda x_2 = 0 \quad (5.17)$$

Again, (5.17) can be rewritten as:

$$D^\alpha u = -D^\alpha f(x_1, x_2, t) - \lambda x_2 \quad (5.17)$$

Here, the terms appearing in left hand side of (5.17) is multiplied with a discontinuous function of sliding surface S , i.e., signum function of S , $\text{sgn}(S)$. Therefore, the resulting equation is given as,

$$D^\alpha u = -D^\alpha f(x_1, x_2, t) \text{sgn}(S) - \lambda x_2 \text{sgn}(S) \quad (5.18)$$

In order, to reduce chattering effect, the same strategy is followed as in the integer order SMC.

Hence, the modified control law is given as,

$$D^\alpha u = -D^\alpha f(x_1, x_2, t) \text{sgn}(S) - \lambda x_2 \text{sgn}(S) - \rho \text{sgn}(S) \quad (5.19)$$

The above stated fractional order control force provides asymptotic stability and finite time convergence for certain values of order α .

5.3.2 Example 2

Consider a SISO system given in following equation (5.20) and (5.21):

$$\dot{x}_1 = x_2 \quad (5.20)$$

$$\dot{x}_2 = -x_1 + u \quad (5.21)$$

Firstly, assume that there is no control law, i.e., $u = 0$ and then check the behavior of the system.

The plot below depicts the behavior of the system.

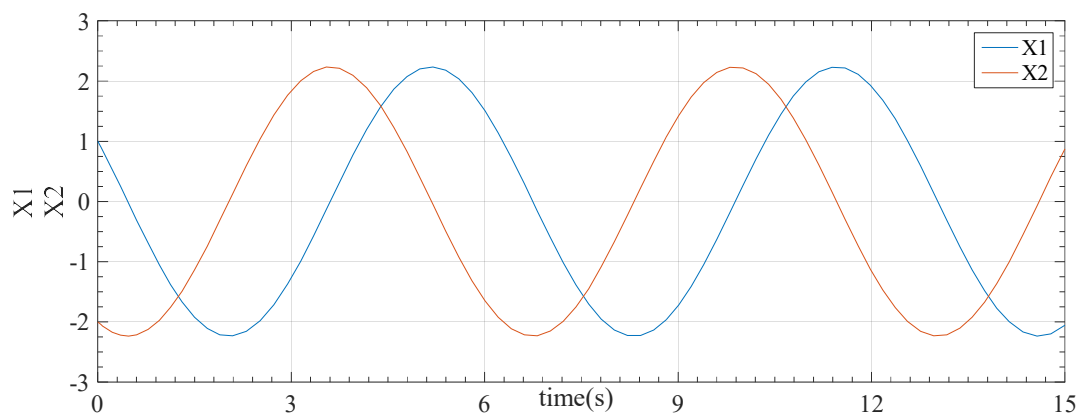


Figure 5.3: Response of state variables with respect to time having no control law.

The Figure 5.3 depicts that the system is marginally stable having constant amplitude of oscillations for all the time and settling time of both the state variables is infinite. Now, consider the same feedback control law given in (5.5) and check the behavior of the system.

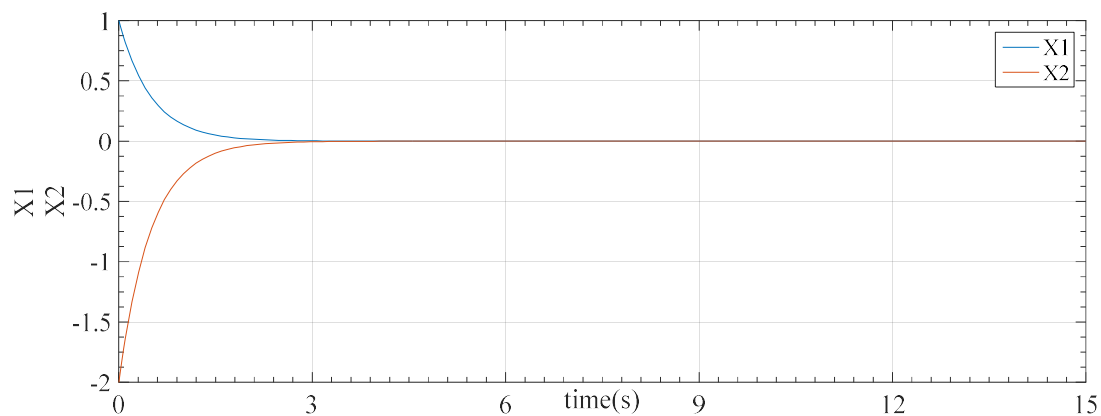


Figure 5.4: Response of state variables with respect to time with linear feedback controller.

From the Figure 5.4 it is concluded that the linear feedback control law stabilizes the marginal stable system. The settling time T_{s1} for x_1 and the settling time T_{s2} for x_2 are 3.4 seconds respectively with zero steady state error.

Now, if the system is forced to stability by above control action, then what is the need to implement SMC theory? Genuinely, SMC can be used to further minimize the settling time of the system.

A. Design of integer order SMC and FOSMC based control law

Following exactly the same strategy proposed in previous section for numerical example 1, control law is being calculated.

The integer order SMC based control input action is given as,

$$u = x_1 \operatorname{sgn}(S) - \lambda x_2 \operatorname{sgn}(S) - \rho \operatorname{sgn}(S) \quad (5.22)$$

The FOSMC based control input action is provided as,

$$D^\alpha u = D^\alpha x_1 \operatorname{sgn}(S) - \lambda x_2 \operatorname{sgn}(S) - \rho \operatorname{sgn}(S) \quad (5.23)$$

The control law above shown in (5.22) guarantees the stability of the system and also settling time is achieved earlier than it was in the case of linear control law. The above stated fractional order control force in (5.23) provides asymptotic stability and finite time convergence for certain values of order α .

The results and discussions of numerical example 1 and 2 are presented in the next section.

CHAPTER 6

RESULTS AND DISCUSSIONS

The entire simulation process is carried out using MATLAB software (version- 2015 A). The ODE's equations involved in SMC are simulated using 'Ode45' command and the FDE's involved in FOSMC are simulated by means of 'Fde12' command in MATLAB. The MATLAB processes 'Ode 45' command using Runge-Kutta method and 'Fde 12' command is processed using the definition of Caputo derivative.

6.1 RESULTS OF EXAMPLE 1

The parameters of controller are selected as $\lambda = 3$ and $\rho = 5$.

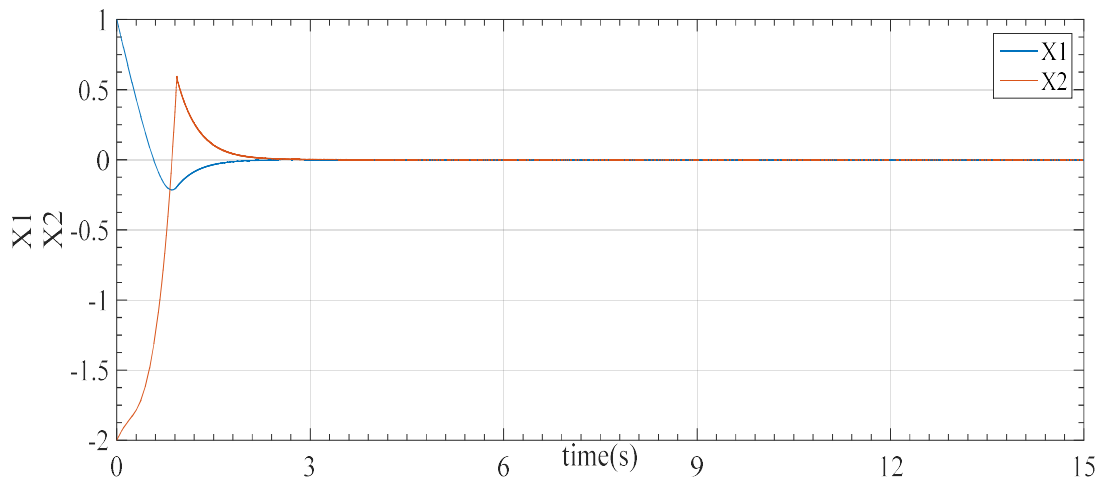


Figure 6.1: Response of state variables with respect to time using integer order SMC.

Figure 6.1 provides the result that is obtained using IOSMC. Both the state variables x_1 and x_2 achieve steady state in 2.77 seconds in the presence of non-linear sinusoidal disturbance. Moreover, the steady state error is found to be zero for both the state variables and also no chattering is observed. The proposed control successfully provides stability to the system.

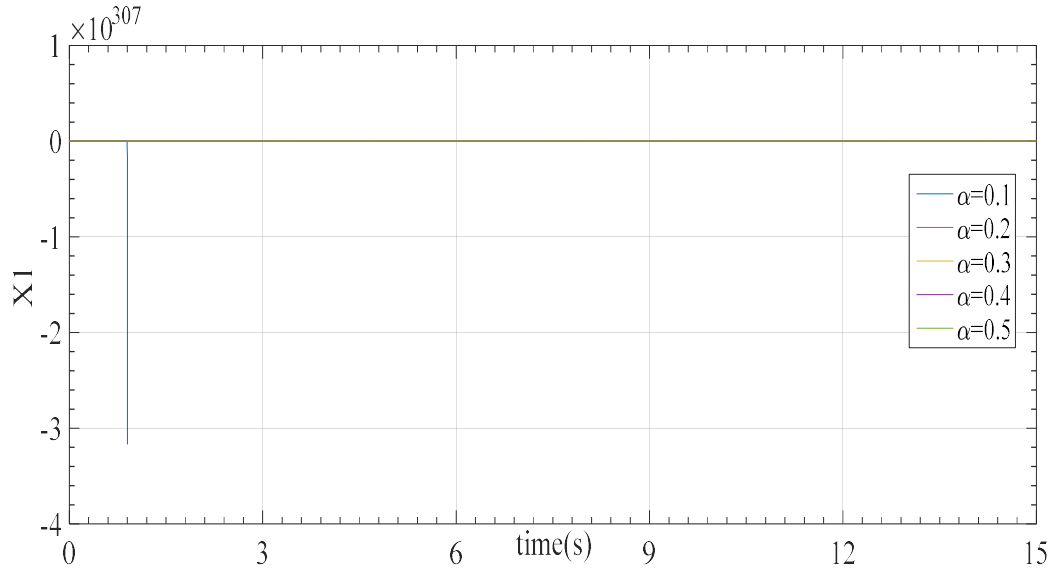


Figure 6.2: Response of state variable x_1 with respect to time using FOSMC for $\alpha = 0.1$ to 0.5 .

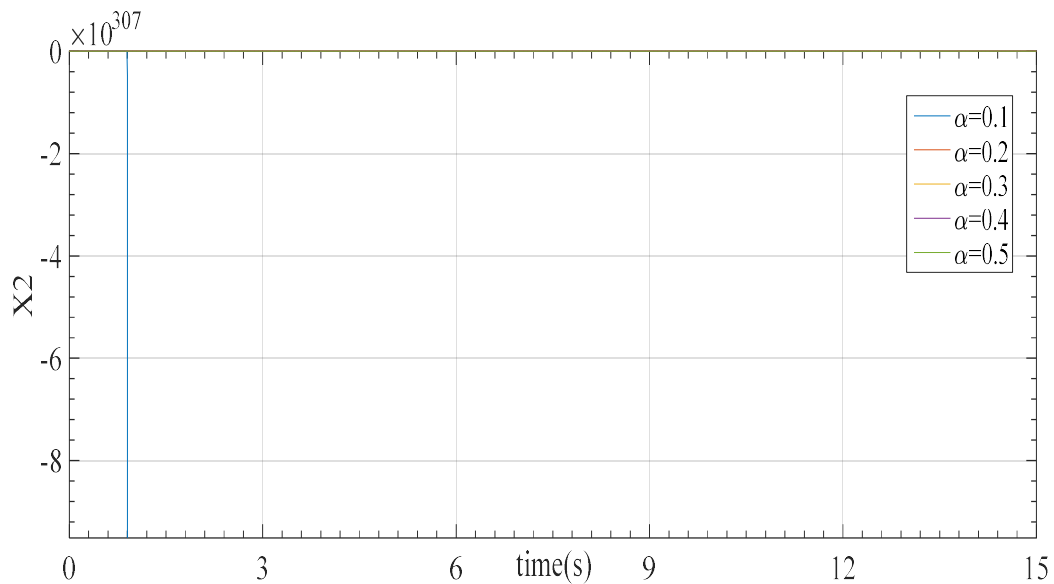


Figure 6.3: Response of state variable x_2 with respect to time using FOSMC for $\alpha = 0.1$ to 0.5 .

Figure 6.2 and Figure 6.3 depicts the behavior of x_1 and x_2 respectively as time goes on when α is varied from 0.1 to 0.5 in the case of FOSMC based controller. The system produces unstable behavior in this range of α since x_1 and x_2 produces unbounded response.

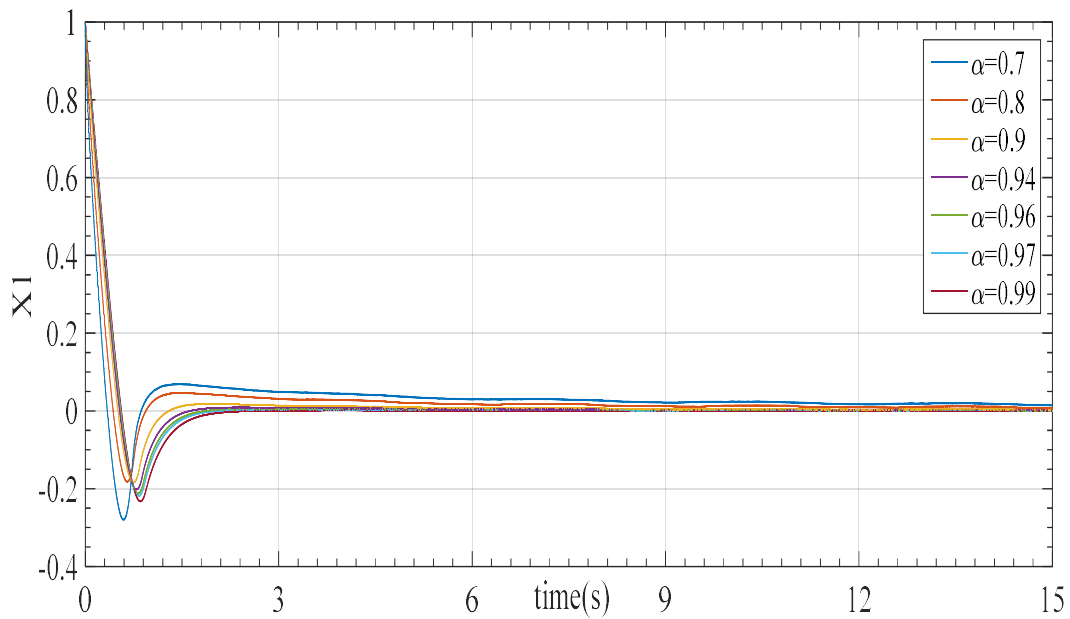


Figure 6.4: Response of state variable x_1 with respect to time using FOSMC for various values of α .

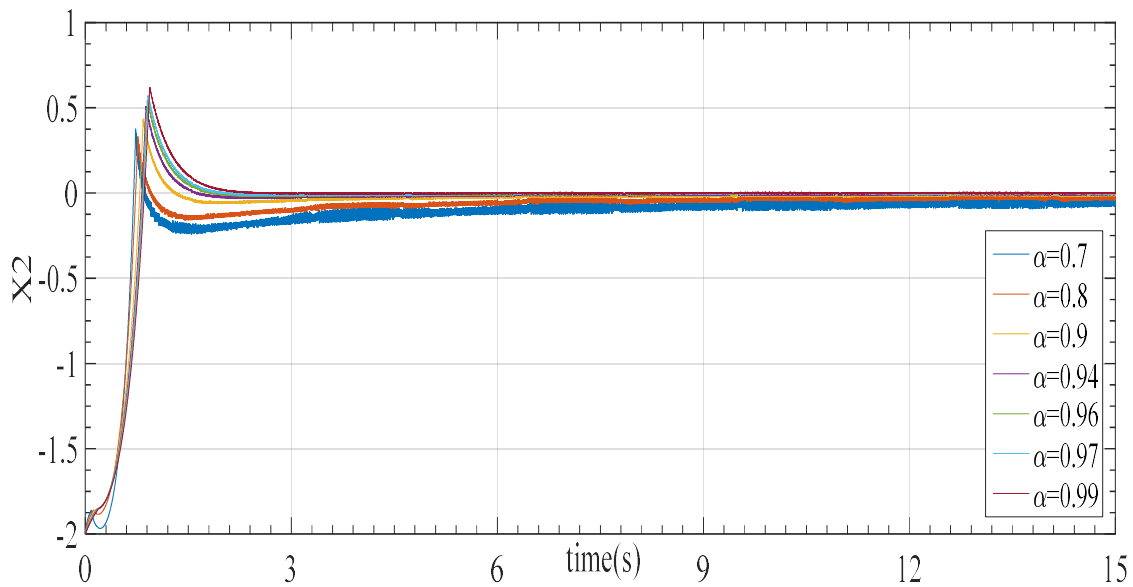


Figure 6.5: Response of state variable x_2 with respect to time using FOSMC for various values of α .

Figure 6.4 and Figure 6.5 depict the behavior of x_1 and x_2 respectively for the case of FOSMC type controller when order of fractions is varied randomly after 0.5. The values of α considered are 0.7, 0.8, 0.9, 0.94, 0.96, 0.97, and 0.99. The Table 6.1 shown below provides the settling time

for state variables x_1 and x_2 and their respective steady state error values for different fractional orders.

Table 6.1: settling time and steady state error values for varying α

α	T_{s1} (s)	T_{s2} (s)	e_{ss1}	e_{ss2}
0.7	9.1	9.5	0.01	0.02
0.8	6	6.5	0.01	0.02
0.9	5.9	6.3	0.0025	0.0025
0.94	4.5	5.5	0	0
0.96	2.9	3.2	0	0
0.97	2.5	3	0	0
0.99	2.4	2.4	0	0

6.2 RESULTS OF EXAMPLE 2

The parameters of controller are selected as $\lambda = 1.5$ and $\rho = 2$.

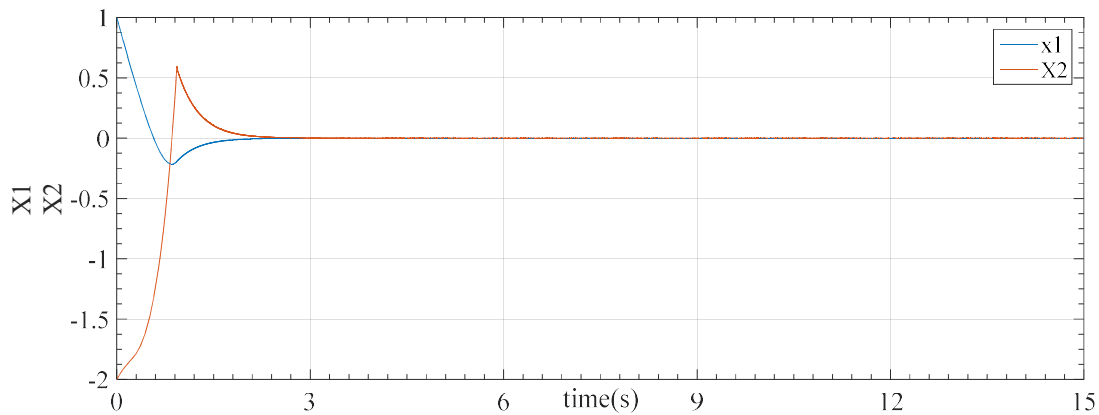


Figure 6.6: Response of state variables with respect to time using integer order SMC.

Figure 6.6 shown is the result that is obtained using IOSMC. The state variables x_1 and x_2 achieve steady state in 2.6 seconds with the proposed SMC based control law. Furthermore, the steady state error is zero for both the variables and no chattering is observed.

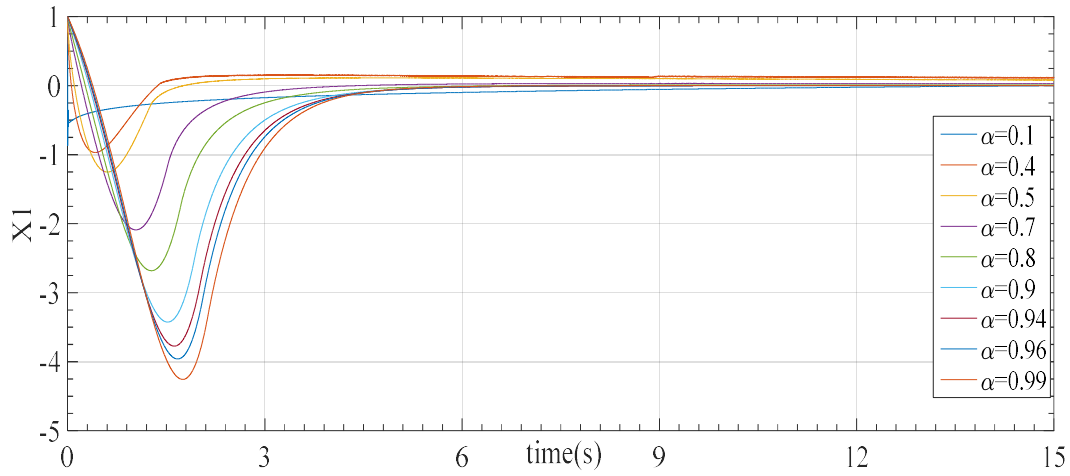


Figure 6.7: Response of state variable x_1 with respect to time using FOSMC for various values of α .

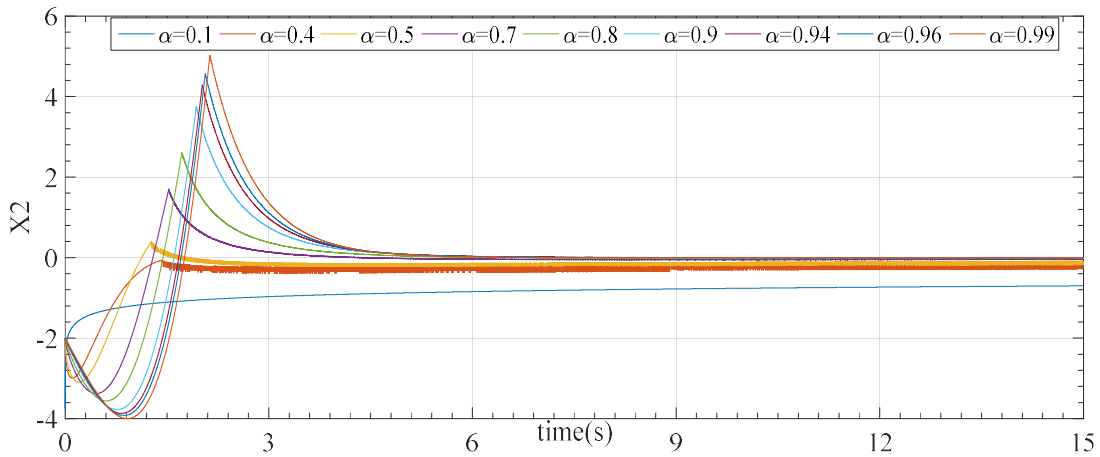


Figure 6.8: Response of state variable x_2 with respect to time using FOSMC for various values of α .

Figure 6.7 and Figure 6.8 portrays the performance of x_1 and x_2 respectively for FOSMC type controller when order of fractions is varied randomly from 0.1 to 0.99. The values of α considered are 0.1, 0.4, 0.5, 0.7, 0.8, 0.9, 0.94, 0.96, and 0.99. The Table 6.2 shown below provides the settling time for state variables x_1 and x_2 along with their respective steady state error values for different fractional orders.

Table 6.2: settling time and steady state error values for varying α

α	T_{s1} (s)	T_{s2} (s)	e_{ss1}	e_{ss2}
0.1	13.4	14	0	0.7
0.4	9	9	0.1	0.3
0.5	8.6	9	0.05	0.3
0.7	5	5.2	0.05	0.07
0.8	6.3	6	0.0015	0.0015
0.9	8	8	0	0
0.94	7.5	7.5	0	0
0.96	7.3	7.3	0	0
0.99	6.4	6.4	0	0

6.3 DISCUSSIONS

In the numerical example 1, the IOSMC based control law successfully delivers asymptotic stability in finite time of 2.77 seconds producing zero steady state error for both the state variables in the presence of non-linear sinusoidal disturbance while at the same time linear feedback control law provided instability. Also, no chattering effect is observed. In the case of FOSMC, discussing stability is not straight forward. Varying α from 0.1 to 0.5, the system behavior is unstable. Varying α from 0.5 up till 0.9 makes the system attain stability in finite time which is greater than the settling time in the case of IOSMC. Moreover it also produces steady state error and chattering in this range of α . Varying α beyond 0.9, settling time goes on decreasing, steady state error approaches zero and reduction in chattering can be observed. At $\alpha = 0.97$ and 0.99 , the settling time got reduced to 2.5 and 2.4 seconds respectively which is 15.4% less than the settling time provided by IOSMC. Moreover, steady state error is null and chattering is absent at this values of α . Therefore, increasing α minimizes settling time, steady state error along with chattering and renders better results at values of α close to one.

In numerical example 2, IOSMC provides finite time convergence in 2.6 seconds which is 30.76% less than as in the case of linear feedback controller with zero steady state error. In the case of FOSMC, it fails to reduce the settling time than in the case of IOSMC. However, for all

values of α considered here it provides stability. But for values of α close to zero and in the mid values between zero and one produces steady state error and chattering too. Increasing FO value beyond 0.9 produces better results with zero steady state error and no chattering. However, the settling time is still large than in the case of IOSMC.

In both the numerical examples increasing α reduces settling time, steady state error and chattering effect. For the values of α close to one provides better results.

CHAPTER 7

CONCLUSION AND FUTURE SCOPE

7.1 CONCLUSION

The comparative study of IOSMC and FOSMC has been claimed for two numerical examples representing two different systems are considered here. Two control laws have been designed for each numerical example. The performances of both the systems are put to test under IOSMC and FOSMC scheme. From the results of both the examples it can be concluded that the control law determined successfully handles the stability and guarantees finite time convergence with the proposed control input action. Both IOSS and FOSS controllers are able to deal with non-linear disturbances while linear feedback control law proved enfeeble against this disturbance and produced instability. The performance of FOSMC is better than IOSMC when non-linear sinusoidal disturbance affects the system stability since FOSMC reduces the settling time by 15.4%. However, there are certain limitations with FOSMC based controller. For marginal stable system behavior of IOSMC is dominant than FOSMC since settling time is large in case of FOSMC. Furthermore, the capability of FOSMC minimizes when values of FO are close to zero and also when these values are selected in the middle range of zero and one since it produces steady state error and chattering too. However, FOSMC responds better when FO values are placed close to one. For the marginal stable system IOSMC reduces the settling time by 30.76% than produced by linear control law.

Hence, the crux of SMC theory is that it provides asymptotic finite time stability and validates to be robust enough in the presence of extraneous uncertainties and disturbances as well as also possesses minimum sensitivity to the system parameters and proves its quintessence among various controllers.

7.2 FUTURE SCOPE

The suggestions for future scope can be the implementation of IOSMC and FOSMC in various research fields like robotics manipulators, power converters, and unmanned aerial vehicles and moreover, for such systems that are subjected to high non-linear disturbances.

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