

Load-Flow Analysis of Radial Distribution Networks

*Thesis submitted in partial fulfillment of requirement for the award of
degree of*

Master of Engineering
in
Power System and Electric Drives



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CERTIFICATE

I, hereby, certify that the thesis entitled “ **Load Flow Analysis of Radial Distribution Networks** ” in the partial fulfillment of the requirement for the degree of MASTER OF ENGINEERING in POWER SYSTEM AND ELECTRIC DRIVES, submitted in the department of Electrical and Instrumentation Engineering, Thapar University, Patiala is an authentic work carried out under the guidance of *Dr. SMARAJIT GHOSH*, Professor and Head of EIED, Thapar University and refers other researcher’s works, which are duly listed in the reference section.

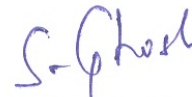
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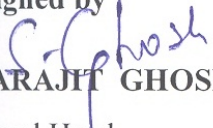
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ABSTRACT

In the thesis a simple method to solve the load-flow problem of radial distribution networks is reported. This algorithm easily computes the power flow through any branch exploiting the radial feature of the distribution networks. The proposed method finds the clusters of branches for feeders, laterals and sub laterals. Each clusters contains branches which are connected in a chain. If the receiving-end node of any branch (say 'a') has more than one outgoing branches. This branch (a) is marked 2. If the receiving end node of any branch (say 'b') has one outgoing branch, this branch (b) is marked 1. This method computes not only voltage magnitude but also computes voltage angles. The proposed method has been tested by two examples (33-node and 69- node radial distribution networks). The proposed method converged well for constant power, constant current, constant impedance, composite and exponential load modeling. The method has also been tested for different R/X ratio of every branch changing R of each branch in each time by a particular constant. The superiority of the proposed method is checked by it CPU time and memory requirement with other methods [11, 19, 25].

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LIST OF SYMBOLS

NB :	Total number of nodes.
LK :	Total number of branches.
jj :	Branch no. i.e., $jj = 1,2,3,\dots,LK$.
m1 :	Sending – end node.
m2 :	Receiving – end node.
IS(jj) :	Sending - end node of branch jj.
IR(jj) :	Receiving - end node of branch jj.
PL(m2) :	Real power load at node m2.
QL(m2) :	Reactive power load at node m2.
LP(jj) :	Real power loss at branch - jj.
LQ(jj) :	Reactive power loss at branch - jj.

Chapter 1

Introduction

1.1 Overview

The best distribution system is one that will supply cost-effectively and safely electric service to both present and future probable loads. The function of the electric power distribution system in a building or an installation site is to receive power at one or more supply points and to deliver it to the individual lamps, motors and all other electrically operated devices. The importance of the distribution system to the function of a building makes it almost imperative so that the best system can be designed and installed. In order to design the best distribution system, the system design engineer must have information concerning the loads and a knowledge of the various types of distribution systems those are applicable. The various categories of buildings have many specific design challenges, but certain basic principles are common to all. Such principles, if followed, will provide a soundly executed design.

Several factors to consider in modern power distribution systems result from two relatively recent changes. The first recent change is utility deregulation. The traditional dependence on the utility for problem analysis, energy conservation measurements and techniques and a simplified cost structure for electricity has changed. The second change is less obvious to the designer that will have an impact on the types of equipment and systems being designed. It is the diminishing quantity of qualified building electrical operators, maintenance departments and facility engineers.

Modern electric power technologies may be of use to the designer and building owner in addressing these new challenges. The advent of microprocessor devices (smart devices) into power distribution equipment has expanded facility owner's options and capabilities, allowing for automated communication of vital power system information (both energy data and system operation information) and electrical equipment control.

Load-flow studies are performed using computer software that simulates actual steady-state power system operating conditions enabling the evaluation of bus voltage profiles, real and reactive power flow and losses. Conducting load-flow study using multiple scenarios ensure that the power system is adequately designed to satisfy the performance criteria. A properly designed system helps contain initial capital investments and future operating costs. The benefits of power flow study in electrical power system decrease unexpected downtime, reduce operating and maintenance costs and get more capacity out of existing assets. Load-flow analysis required when (a) significant plant/load expansion occurs, (b) local generation is proposed to be added, (c) new utility feed installation, (d) large motor addition to the system, (e) new transformer installation etc. Unlike traditional circuit analysis, a power flow study usually uses the simplified notation such as a single-line diagram, per-unit system and focuses on various forms of AC power (i.e., real, reactive and apparent) rather than voltage and current. The great importance of power flow or load-flow studies is in the planning of the future expansion of power systems as well as in determining the best operation of existing systems. The principal information obtained from the power flow study is the magnitude and phase angle of the voltage at each bus and the real and reactive power flowing in each line.

The distribution systems are radial in nature whereas transmission systems are looped in nature. Transmission lines generally run substation to substation at very high voltage levels. The voltage level is typically 66 kV up to 765 kV. These circuits are usually "point to point" with sophisticated relay protection schemes on both ends. Distribution lines (feeders) are the ones that run through a neighbourhood serving individual customers along the way. Distribution voltages are in the range of 230V (for 1-phase) / 415 V (for 3-phase) to 33 kV. The voltage is stepped down for the customer's utilization through the use of transformers. Typical feeder loads are a few hundred amps. The transmission system is channelling the power from place to place, the distribution system is dividing the flow up and channelling it to the end users.

A diagram including the several components that are usually to be found in the makeup of such a system shown in Fig. 1.1. Particular attention is paid to those elements, which will make up the component under discussion about the distribution system. While the energy flow is obviously from the power generating plant to the consumer, it may be more

informative for our purposes to reverse the direction of observation and consider events from the consumer back to the generating source.

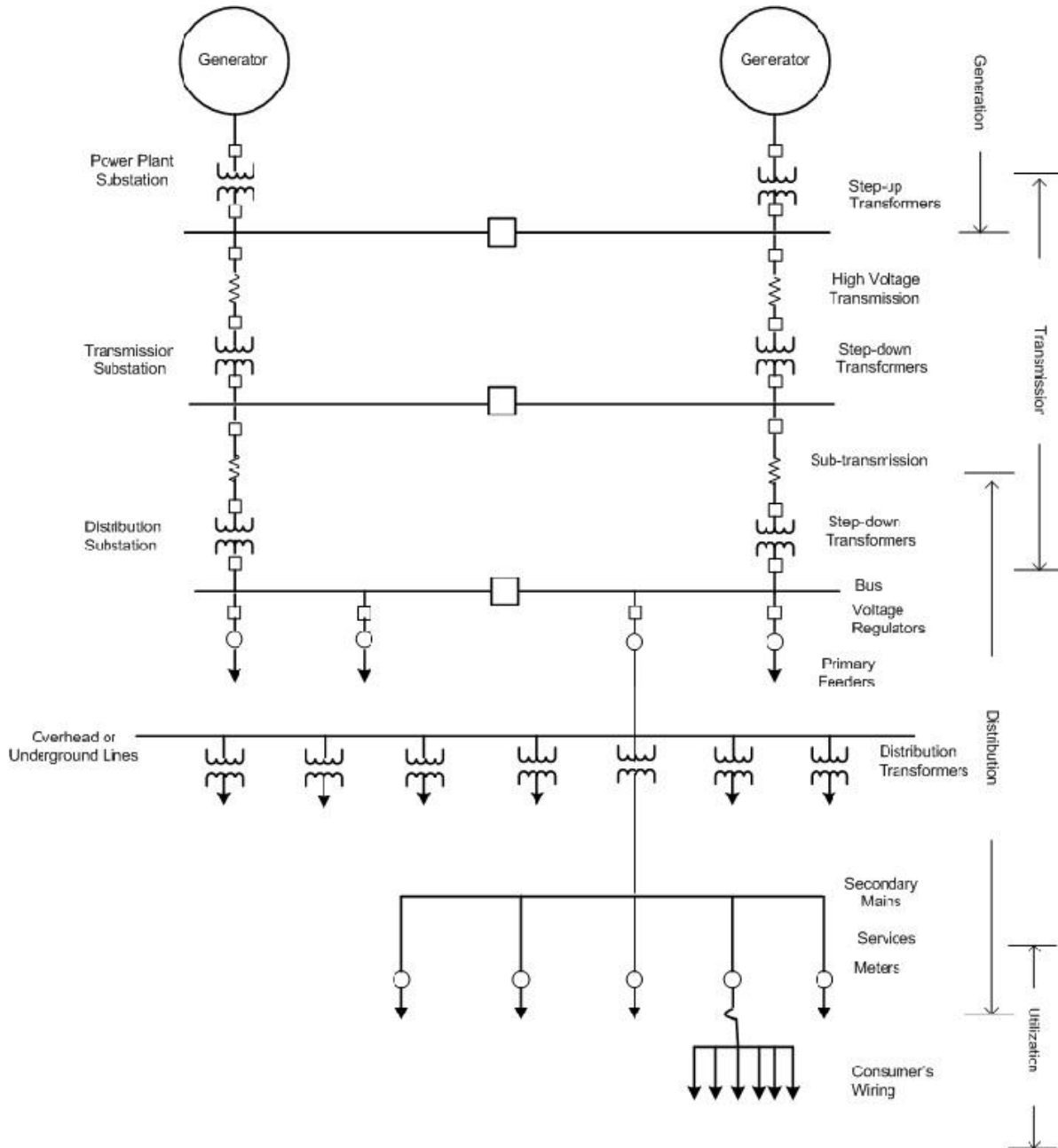


Figure 1.1 Single line diagram of a typical power system. [www.docstoc.com]

Energy is consumed by users at a nominal utilization voltage that may range generally (in India) from 220 V to 230 V and from 230 V and 440 V. For some large commercial and [3]

industrial users the nominal values are 440 V to 3.3 kV. It flows through a metering device that determines the billing for the consumer but, which may also serve to obtain data useful later for planning, design and operating purposes. The metering equipment usually includes a means of disconnecting the consumer from the incoming of supply, which should become necessary for any reason. The energy flows through conductors to the meter from the secondary mains (if any). These conductors are referred to as the consumer's service or sometimes also as the service drop. Several services are connected to the secondary mains those who serve as a path to the several services from the distribution transformers, which supply them.

1.2 Electricity Distribution

Electrical Distribution is the final stage in the delivery of electricity to end users. A distribution system's network carries electricity from the transmission system and delivers it to consumers. Typically, the network would include medium-voltage (less than 50 kV) power lines, electrical substations and pole-mounted transformers, lower-voltage (less than 1000 V) distribution wiring and sometimes electricity meters. So that the part of power system used for distribution of electric power for local use is known as a distribution system. In general, the distribution system is the electrical system between the substation fed by the transmission system and the consumer's meters.

1.2.1 Global Design of a Distribution Networks

The electric utility system is usually divided into three subsystems which are generation, transmission, and distribution. A fourth division, which sometimes is made, is sub transmission. However, the latter can really be considered as a subset of transmission since the voltage levels and protection practices are quite similar. The distribution system is commonly broken down into three components: distribution substation, distribution primary and secondary. At the substation level, the voltage is reduced and the power is distributed in smaller amounts to the customers. Consequently, one substation will supply many customers with power. Furthermore, most customers are connected to only one of the three phases in the distribution system. Therefore, the power flow on each of the lines is different and the system is typically 'unbalanced'. This characteristic needs to be accounted for in load-flow studies related to distribution networks.

(i) Feeders: A feeder is a conductor, which connects the sub-station (or localized generating station) to the area where power is to be distributed. Generally, no tappings are taken from the feeder so that the current in it remains the same throughout. The main consideration in the design of a feeder is the current carrying capacity.

(ii) Distributor: A distributor is a conductor from which tapping is taken to supply to the consumers. In Fig. 1.2 AB, BC, CD, and DA are the distributors. The current through a distributor is not constant because tapping is taken at various places along its length. While designing a distributor, voltage drop along its length is the main consideration since the statutory limit of voltage variations is $\pm 10\%$ of rated value of the consumer's terminals.

(iii) Service mains: A service mains are generally a small cable which connects the distributor to the consumer's terminals.

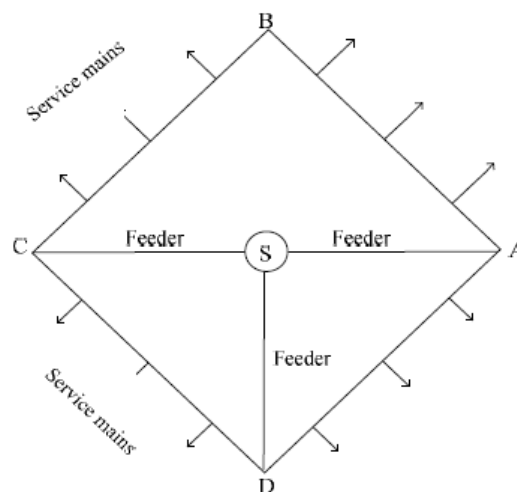


Figure 1.2 Diagram of a typical Low Tension Distribution System. [www.itu.dk]

1.2.2 Modern Distribution System

The modern distribution system begins as the primary circuit leaves the sub-station and ends as the secondary service enters the customer's meter socket. A variety of methods, materials, and equipment are used among the various utility companies, but the end result is similar. First, the energy leaves the sub-station in a primary circuit, usually with all three phases.

The most common type of primary is known as a **Wye (star) configuration** (so named because of the shape of a "Y") The Wye configuration includes 3 phases (represented by the

three outer parts of the "Y") and a neutral (represented by the centre of the "Y".) The neutral is grounded both at the substation and at every power pole.

The other type of primary configuration is known as **delta**. This method is older and less common. Delta is so named because of the shape of the Greek letter delta, a triangle. Delta has only 3 phases and no neutral. In delta there is only a single voltage, between two phases (phase to phase), while in Wye there are two voltages, between two phases and between a phase and neutral (phase to neutral). Wye primary is safer because if one phase becomes grounded, that is, makes connection to the ground through a person, tree, or other object, it should trip out the circuit breaker tripping similar to a household fuse cut-out system. In delta, if a phase makes a connection to ground, it will continue to function normally. It takes two or three phases to make a connection to the ground before the fuse cut-outs will open the circuit.

1.3 Requirement of a Distribution system

A considerable amount of effort is necessary to maintain an electric power supply within the requirements of various types of consumers. Some of the requirements of a good distribution system are: proper voltage, availability of power on demand, and reliability

1.3.1 Proper Voltage: One important requirement of a distribution system is that voltage variations at consumer's terminals should be as low as possible. The changes in voltage are generally caused due to the variation of load on the system. Low voltage causes loss of revenue, inefficient lighting and possible burning out of motors. High voltage causes the lamps to burn out permanently and may cause failure of other appliances. Therefore, a good distribution system should ensure that the voltage variations at the consumer's terminals are within permissible limits. The statutory limit of voltage variations is +10% of the rated value at the consumers' terminals. Thus, if the declared voltage is 230 V, then the highest voltage of the consumer should not exceed 244 V while the lowest voltage of the consumer should not be less than 216 V.

1.3.2 Availability of Power Demand: Power must be available to the consumers in any amount that they may require from time to time. For example, motors may be started or shut down, the lights may be turned on or off, without advance warning to the electricity supply company. As electrical energy cannot be stored, therefore, the distribution system must be

capable of supplying load demands of the consumers. This necessitates that operating staff must continuously study load patterns to predict in advance those major load changes that follow the known schedules.

1.3.3 Reliability: Modern industry is almost dependent on electric power for its operation. Homes and office buildings are lighted, heated, cooled and ventilated by electric power. This calls for reliable service. Unfortunately electric power, like everything else that is man-made, can never be absolutely reliable. However, the reliability can be improved to a considerable extent by (a) interconnected system, (b) reliable automatic control system and (c) providing additional reserve facilities.

1.4 Essential Parts of a Distribution System

Various types of distribution system have identical subsystems and components. These components can be connected and configured in various alternative ways depending upon the area covered, load density, type and importance of consumer, reliability and freedom from interruption desired, the cost of land and right of way available. Table 1.1 shows the essential parts of a distribution system.

Table 1.1 Essential Parts of a Distribution System

Sr. No.	Elements	Function	Remarks
1.	Sub transmission Circuits	To receive power from main bulk power receiving station and delivering power to the distribution substations.	1. 3 – phase 3 wire AC system at 50 Hz. 2. High voltage overhead lines 66kV/33kV. 3. Radial/loop/ring/mesh configurations.

2.	Distribution Substations	<ol style="list-style-type: none"> 1. To step down voltage received from sub transmission level. 2. To feed primary distribution circuits. 3. To arrange switching protection, metering, control. 	<ol style="list-style-type: none"> 1. Out door air insulated or indoor SF6 gas insulated. 2. Two voltage level buses. 3. Located near the load centre.
3.	Primary distribution system	To feed power to various distribution transformers through primary feeder.	<ol style="list-style-type: none"> 1. Radial modified radial, loop ring circuit. 2. High voltage for higher load densities.
4.	Distribution transformers	To step down the voltage to the secondary distribution level.	It steps down the voltage to 415V level. Distribution transformers are generally pole mounted, foundation mounted. The typical rating of transformer is 100 kVA to 500 kVA.
5.	Secondary distribution system	To feed the consumer.	<ol style="list-style-type: none"> 1. Overhead + Underground distribution lines. 2. Radial network. 3. 3 phase 4 wire system with grounded neutral. 4. Service mains and service network.

The classified distribution systems are as follows:

(i) Sub Transmission Circuits ; (ii) Distribution Substations ; (iii) Primary Distribution Circuit; (iv) Distribution Transformers ; (v) Secondary Distribution System.

1.5 AC Distribution System

In this era electrical energy is generated, transmitted and distributed in the form of alternating current. One important reason for the widespread use of alternating current in preference to direct current is the fact that alternating voltage can be conveniently changed in magnitude by means of a transformer. The transformer has made it possible to transmit alternating power at

high voltage and utilize it as a safe potential. High transmission and distribution voltages have greatly reduced the current in the conductors and the resulting line losses.

There is no definite line between transmission and distribution according to voltage or bulk capacity. However, the down sub-station is fed by the transmission system and the consumer's meters. The AC distribution system is classified into (i) primary distribution system and (ii) secondary distribution system.

1.5.1 Primary Distribution System: It is part of the AC distribution system, which operates at voltages somewhat higher than general utilization and handles larger blocks of electrical energy than the average low-voltage consumer uses.

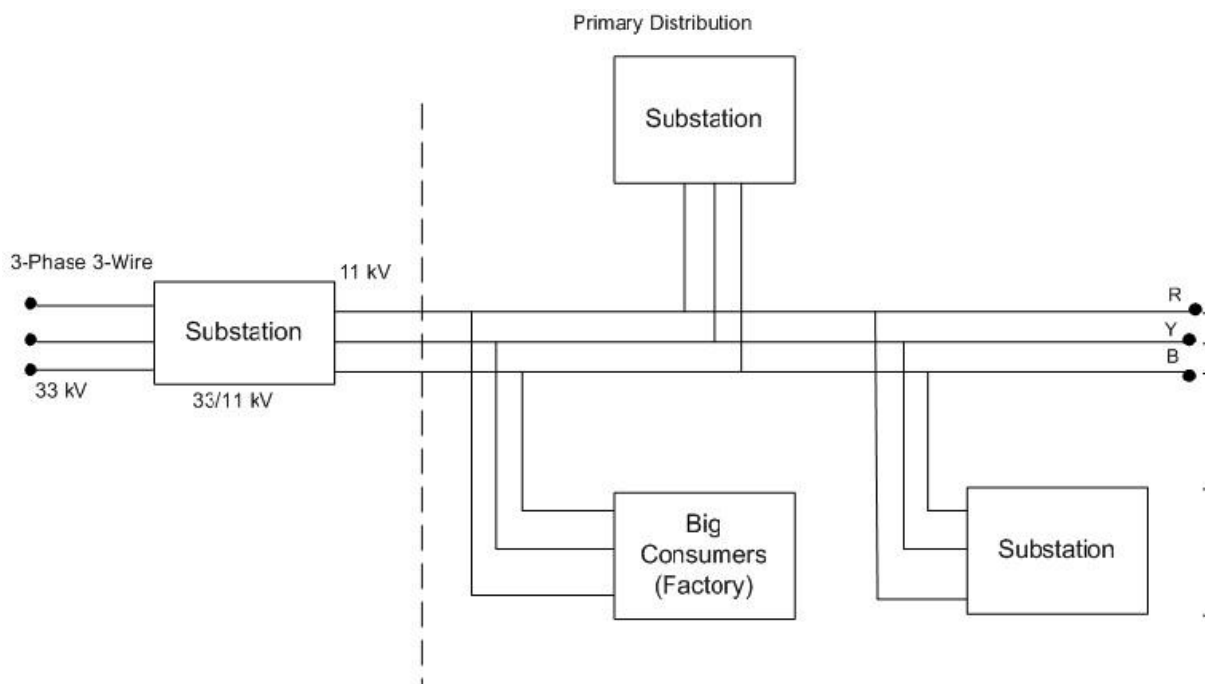


Figure 1.3 Primary Distribution Systems. [electric-bus.blogspot.com].

The voltage used for primary distribution depends upon the amount of power to be conveyed and the distance of the sub-station required to be fed. The most commonly used primary distribution voltages are 11 kV, 6.6 kV and 2.2 kV. Due to economic considerations, primary distribution is carried out by 3-phase, 3-wire system. Figure 1.3 shows a typical primary distribution system. Electric power from the generating station is transmitted at high voltage to the sub-station located in or near the city. At this sub-station, the voltage is stepped down to 11kV with the help of step-down transformer. Power is supplied to various sub-stations for

distribution or to big consumers at this voltage. This forms the high voltage distribution or primary distribution.

1.5.2 Secondary Distribution System: It is that part of the AC distribution system that includes the range of voltages at which the ultimate consumers utilizes the electrical energy delivered to them. The secondary distribution employs 400/230 V, 3-phase, 4-wire system. Figure 1.4 shows a typical secondary distribution system.

The primary distribution circuit delivers power to various sub-stations, called distribution substations. The sub-stations are situated near the consumer's localities and contain step-down transformers. At each distribution sub-station, the voltage is stepped down to 400 V and power is delivered by 3-phase, 4-wire AC system. The voltage between any two phases in 400 V and between any phase and neutral is 230 V.

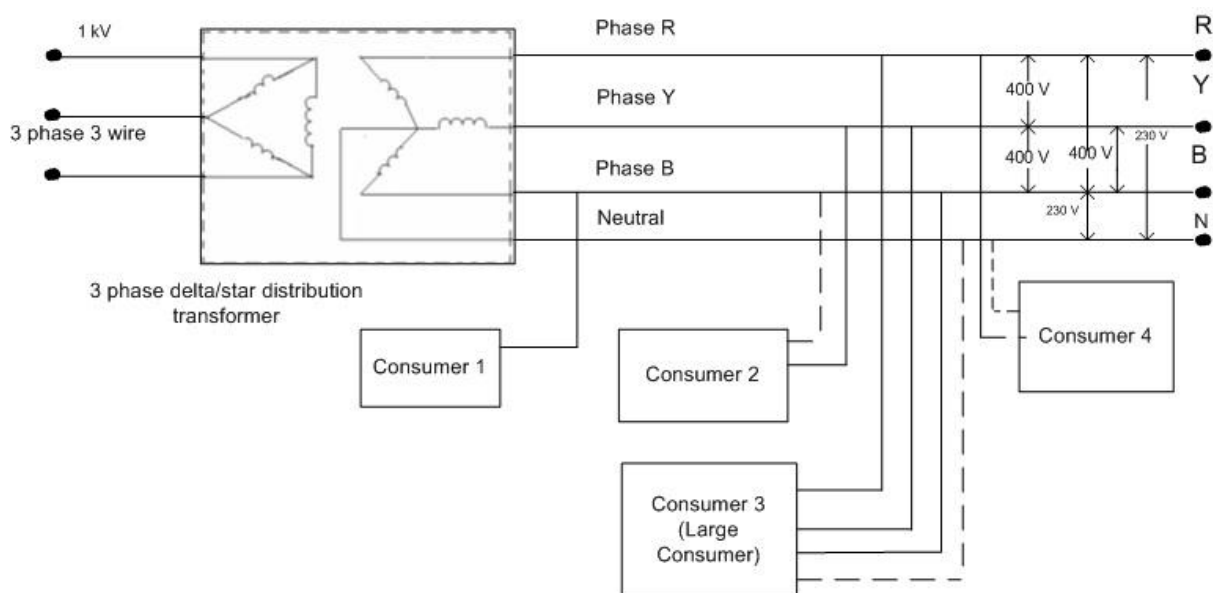


Figure 1.4 Secondary Distribution Systems. [electric-bus.blogspot.com].

1.6 Nominal Voltage Levels in Transmission and Distribution Systems

3-phase, 50Hz or 60 Hz, systems are universally used for generation, transmission and distribution of electrical power. The voltages are stepped at generating station for the purpose of long distance high power transmission. The voltages are stepped down to an appropriate level for sub-transmission level. The voltages are again stepping down at distribution substation and deliver to the consumer. Thus, the entire power system network has several

AC voltage levels obtained by means of power transformers located in substations and distribution system voltage levels are standardized.

1.7 Limits for Distribution Systems Permissible Steady State Voltage

During the load changes, the voltage drop in distribution circuits also changes resulting in variation in service voltage. Certain voltage variation is permissible. For each nominal system voltage, certain highest system voltage and certain lowest system voltage are specified by the standards as shown in Table 1.2. The distribution system is designed and operated to ensure that the supply voltage at service connection is within specified limits at no load to peak load.

Table 1.2 Reference value of voltage limit in distribution system

Class	System Voltage Nominal RMS	Permissible Highest System Voltage RMS	Permissible Lowest System Voltage RMS
LV (1Ph)	230 V	264 V	200 V
LV (3Ph)	400 V	457 V	347 V
MV (3Ph)	3.3 kV	3.6 kV	3 kV
	6.6 kV	7.2 kV	6 kV
	11 kV	12 kV	10.5 kV
	22 kV	24 kV	20 kV
	33 kV	36 kV	30 kV
HV (3Ph)	66 kV	72.5 kV	60 kV

1.8 Different Connection Schemes of a Distribution System

All distribution of electrical energy is done by constant voltage system. In practice, the following distribution circuits are generally used. According to connection scheme the distribution system has three types as given below

- (i) Radial System.
- (ii) Ring Main System.
- (iii) Interconnected system.

1.8.1 Radial System: In this system, separate feeders radiate from a single sub-station and feed the distributors at one end only. Figure 1.5 (a) shows a single line diagram of a radial system for DC Distribution where a feeder OC supplies a distributor AB at point A. Obviously, the distributors are fed at one point only i.e. point A in this case. Figure 1.5 (b) shows a single line diagram of the radial system for AC distribution. The radial system is employed only when power is generated at low voltage and the sub-station is located at the centre of load. This is the simplest distribution circuit and has the lowest initial cost. However, it suffers from the following drawbacks:

- (a) The end of the distributor nearest to the feeding point will be heavily loaded.
- (b) The consumers are dependent on a single feeder and single distributor. Therefore, any fault on the feeder or distributor cuts off the supply to the consumers who are on the side of the fault away from the sub-station.
- (c) The consumers at the distant end of the distributor would be subjected to serious voltage fluctuations when the load on the distributor changes. Due to these limitations, this system is used for short distances only.

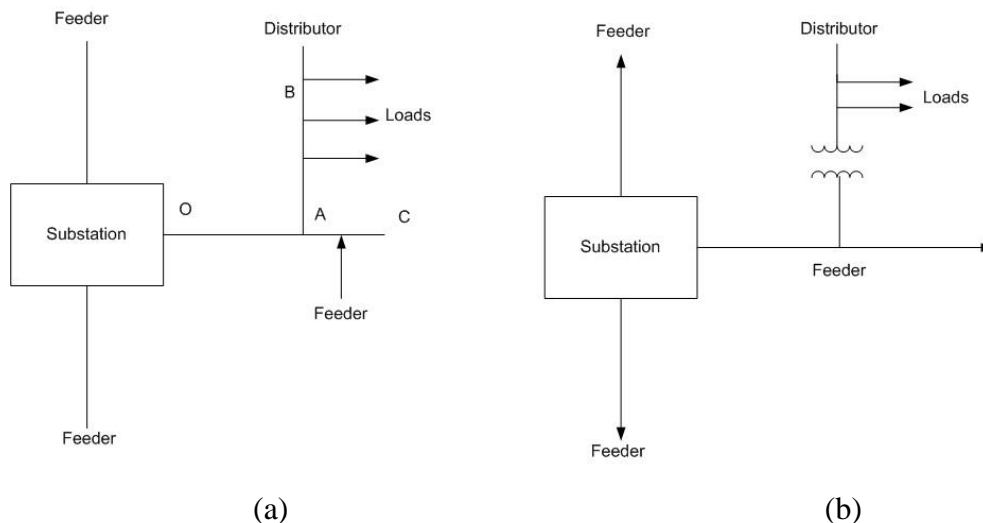


Figure 1.5 Single Line Diagram of Radial System [www. etap.com]

1.8.2 Ring Main System: In this system, the primaries of distribution transformers in loop form which starts from the sub-station bus bars, makes a loop through the area to be served,

and returns to the substation. Figure 1.6 shows the single line diagram of a ring main system for AC Distribution where sub-station supplies to the closed feeder LMNOPQRS and Q of the feeder through distribution transformers.

The ring main system has the following advantages:

- (a) There are less voltage fluctuations at consumer's terminals.
- (b) The system is very reliable as each distributor is fed via two feeders. In the event of a fault on any section of the feeder, the continuity of supply is maintained. For example, suppose that a fault occurs at any point F of section SLM of the feeder. Then section SLM of the feeder can be isolated for repairs and at the same time continuity of supply is maintained to all the consumers via the feeder SRQPONM.

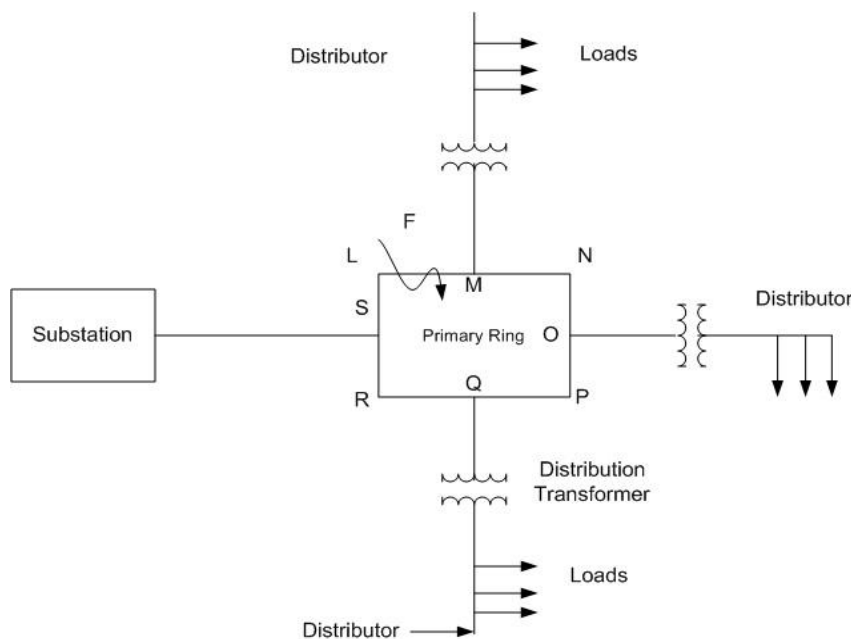


Figure 1.6 Ring Main Systems [www.ecmweb.com]

1.8.3 Interconnected System: When the feeder ring is energized by two or more than two generating stations or sub stations, it is called interconnected system. Figure 1.7 shows the single line diagram of interconnected system where the closed feeder ring ABCD is supplied by two sub-stations S_1 and S_2 at points D and C respectively.

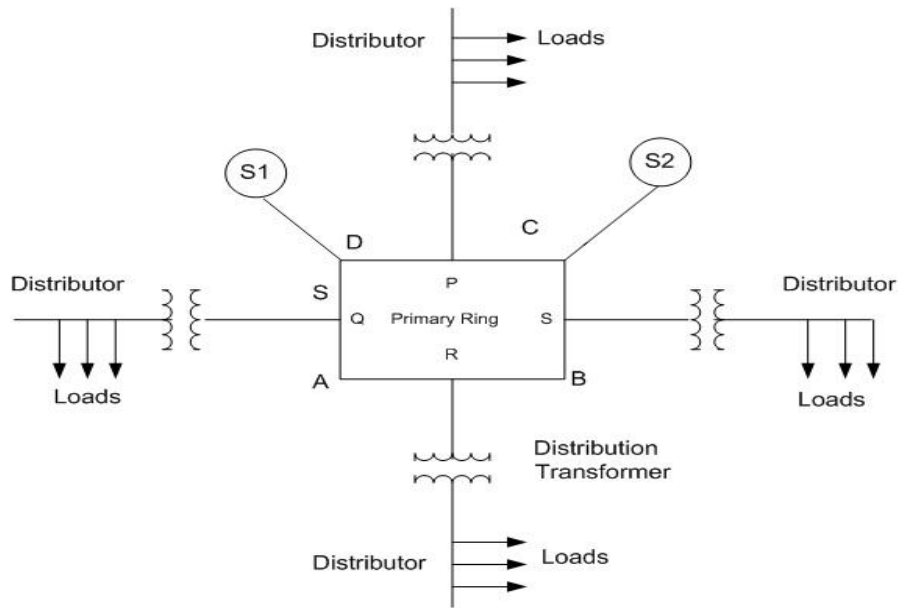


Figure 1.7 Interconnected Systems [www.arcadvisor.com]

The literature survey of the load-flow of the power distribution networks has been presented in **Chapter 2**.

Chapter 2

Literature Survey on Load-Flow Analysis of Power Distribution Systems

2.1 Literature review

In the literature, there are a number of efficient and reliable load-flow solution techniques, such as: Newton-Raphson's method, Fast Decoupled Load-flow method and Gauss-Seidel method. These techniques are well valid for transmission systems. The distribution systems are radial in nature (practically weakly meshed) whereas transmission systems are loop in nature. The distribution networks have a high R/X ratio compared to the transmission networks and hence are ill-conditioned in nature.

Stott and Alsac [1] first suggested a simple, very reliable and fast load-flow solution method with a wide range of practical application. It was useful for accurate or approximate off-line and on-line routine and contingency calculations for networks of any size and could be implemented efficiently on computers. In their paper, the detailed performance on a series of practical problems had been presented. The algorithm was simpler, faster and more reliable than Newton's method and had lower storage requirements for entirely in-core solutions. Correlations of general interest between the power-mismatch convergence criterion and actual solution accuracy were obtained.

Bose and Rajicic [2] proposed Fast Decoupled method, which was probably the most popular because of its efficiency. Its reliability for most transmission systems was very high but it had difficulties in convergence for systems with high ratios of branch resistance to reactance. Modifications, that retained the advantages of their method but could handle high R/X ratios, which were of great interest and certain compensation techniques had been used for their purpose. Both the series and parallel compensation techniques, however, gave mixed results and a new modification was presented here that performed better on several test systems.

Amerongen [3] presented a new version of the fast decoupled load-flow by which a broader range of problems in power systems could be solved. It was shown for normal test systems, so there was hardly any difference in the number of iterations. However, the new algorithm iterated faster if one or more problematic R/X ratio were present. In the paper the advantages of the new version were demonstrated by running on IEEE test systems with both uniformly and non-uniformly scaled resistances or reactances.

Baran and Wu [4] proposed the capacitor sizing problem for the determination of optimal capacitor placement in the radial distribution system to minimize the real power losses for a given load profile. Their problem formulation was in nonlinear form. They presented a new formulation of power flow equations in radial networks which were numerically robust and computationally efficient.

Renato Cespedes [5] mentioned a new method for the solution of load-flow in radial operated distribution networks. The method was based on an electric equivalent and in the elimination of the voltage phase angle in the equations to be solved, which permitted to obtain the exact solution working only with voltage magnitudes.

Nanda *et al.* [6] proposed a general purpose Fast Decoupled power flow model (GFDPF) that exhibited more or less best convergence properties of both well-behaved and ill-conditioned systems. Simple and efficient compensation technique was proposed to deal with g-limit enforcements associated with bus-type switching at voltage-controlled buses. The results demonstrated that the proposed GFDPF model exhibited more or less stable convergence behaviour for both well-behaved and ill-conditioned situations.

Das *et al.* [7] explained a novel method for load-flow technique for solving radial distribution networks by computing the total real and reactive power fed through any node. They had proposed an unique node, branch and lateral numbering scheme, which helped to evaluate exact real and reactive power loads fed through any node. Methods were based on the forward-backward sweep process for the solution of ladder networks. The proposed method could easily handle different types of load characteristics. Several Indian rural distribution networks had been successfully solved using the proposed method.

Shirmohammadi and Cheng [8] presented a three-phase power flow solution method for real-time analysis of primary distribution systems. The method proposed here was capable of addressing these modelling challenges while still maintaining a high execution speed required for real-time application in distributed automation systems. The paper also included test results from the application of a computer program developed based on the proposed method for large primary electric distribution systems.

Zimmerman and Chiang [9] proposed a novel power flow formulation and an effective solution method for general unbalanced radial distribution system. In their paper the authors exploited the radial structure and the decoupling numerical property of a distribution system to develop a Fast Decoupled Newton method for solving unbalanced distribution system problems. The modelling included unbalanced three phase, two phase and single phase branches of bus system, constant power, constant current, and constant impedance loads connected in star or delta formations, shunt capacitors, line charging capacitance, co-generators, switches and three phase transformers of various connection types. The proposed solution algorithm was evaluated on three-phase unbalanced 292-bus and 394-bus test systems with very promising results.

Zhang and Cheng [10] presented a modified Newton method for radial distribution systems in which the Jacobian matrix was in UDUT form, where U was a constant upper triangular matrix depending solely on system topology and D was a block diagonal matrix. With the formulation, the conventional steps of forming the Jacobian matrix, LU factorization and forward/backward substitution was replaced by back/forward sweeps on radial feeders with equivalent impedances. The proposed method could be applied to other applications, such as state estimation, solution of systems with loops, dispersed generators and three phase (unbalanced) representation.

Ghosh and Das [11] proposed a method, which involved only the evaluation of a simple algebraic expression of receiving-end voltages. They presented a load-flow technique based on nodes beyond branches, which was very efficient computationally. It had good and fast convergence characteristics. Loads in the formulation had been presented as constant power.

The speed requirement of the proposed method had also been compared with other existing methods and they had not reduced the network.

Bompard *et al.* [12] proposed a study on the convergence characteristics of the backward/forward sweep method, which was one of the most effective methods for the load-flow analysis of the radial distribution systems. In their research the convergence conditions and the evolution of the iterative process were investigated in detail for different load models, taking into account different line R/X ratio and different types of voltage-dependent loads.

Mok *et al.* [13] presented an efficient method of power flow analysis for solving balanced radial distribution systems. The radial distribution system was modelled as a series of interconnected single feeders. Using Kirchhoff's laws, a set of iterative power flow equations were developed to conduct the power flow studies. For the purpose of power flow study, the radial distribution system was modelled as a network of buses connected by distribution lines or switches connected to a voltage specified source bus. Each bus might also had a corresponding bus load, compensating load (shunt capacitor or inductor), lateral load and/or co-generator connected to it. This paper also compared the power flow results of a distribution system for the different voltage-dependent load models. The radial topology of distribution networks had been fully exploited such that a unique branch and node numbering scheme was utilized to achieve storage and computational economy.

Haque [14] proposed a very simple and efficient method of load-flow analysis for a general distribution system having a mesh network with more than one feeding node. The network conversion process created some break points and dummy buses. The characteristics of the original system were preserved by injecting appropriate complex power at the break points in the equivalent system. The injected powers were computed and updated during the iteration process through a reduced order impedance matrix. The performance of the proposed method was investigated on three different test systems with various modifications.

Aravindhbabu *et al.* [15] proposed a novel technique to obtain the solution of load-flow in radially operated distribution networks in which the loads could be represented by any model. The method was simple, easily programmable and based on the formation of a constant

sparse upper triangular matrix, which was for the determination of the bus voltages. The proposed method was tested through test results of IEEE 34 and IEEE 123 bus systems.

Augugliaro *et al.* [16] presented an efficient method for radial distribution network solution. The efficiency of the presented strategy made it suitable for distribution automation applications. It used a simple matrix representation for the network topology and branch current flow management.

Zhu and Tomsovic [17] analysed an adaptive distributed power flow solution method based on the compensation-based method. The numerical properties of the compensation-based power flow method was compared and analysed under different situations, such as load unbalance, sudden increase of single phase loads, degree of meshed loops, number of generator nodes and so on. They exploited in their research an adaptive compensation-based power flow method that was fast and reliable while maintaining necessary accuracy and appropriate for simulation of slow dynamics.

Mekhamer *et al.* [18] presented a new development for solving the load-flow problem for radial distribution feeders, without having to solve the well-known conventional load-flow equations. They exploited the table of solution variables such as voltage, injected powers at each node of the feeder and line losses. They compared their method with the original method from [4] and as well as the standard conventional load-flow methods.

Ranjan and Das [19] proposed a simple and efficient algorithm for solving problems in radial distribution network. In their research they mentioned simple algebraic recursive expression of voltage magnitude and all the data were stored in vector form. Their algorithm used basic principle of circuit theory and could be understood easily. They had reduced the network for the load-flow analysis of the networks.

Bhutad *et al.* [20] presented the distinguishing features of distribution system to the transmission system. In their research they proposed certain features of the distribution system which made it different and somewhat difficult to be analysed than to the transmission system. Analysing un-transposed lines, single-phase and two-phase laterals were some of those difficulties. They mentioned in their paper, the distribution system needed to be

analysed on the three-phase basis instead of the single-phase basis. Various load-flow methods for distribution systems were reviewed and hence applied on a sample 8-bus system. The performance of these methods were compared for various parameters.

Aravindhababu [21] proposed a new, robust, and fast technique to obtain the load-flow solution in distribution networks. The proposed method was based on the Newton-Raphson's technique using equivalent current-injection and rectangular coordinates. This method was simple, insensitive to R/X ratios of the distribution lines and used a constant Jacobian matrix. It was solved similar to Fast Decoupled power flow method .

Eminoglu and Hocaoglu [22] illustrated in their paper a simple and efficient method to solve the power flow problem in radial distribution systems. The proposed method considered voltage dependency of static loads and line charging capacitance. The method was based on the forward and backward voltage updating by using the polynomial voltage equation for each branch and backward ladder equation (Kirchhoff's Laws). Convergence ability and reliability of the method was compared with the Ratio-Flow method, which was based on classical forward-backward ladder method for different loading conditions, R/X ratios and different source voltage levels, under the wide range of exponents of loads.

Kumar and Selvan [23] proposed a simple approach to load-flow analysis of a radial distribution network. In their approach, computation of the branch current depended only on the current injected at the neighbouring node and the current in the adjacent branch. The approach started from the end nodes of sub lateral line, lateral line and main line and moved towards the root node during branch current computation. The node voltage evaluation began from the root node and moved towards the nodes located at the far end of the main, lateral and sub lateral lines. The proposed method found to be computationally efficient.

Sivanagaraju et al. [24] explained about distinctive load-flow solution technique for the analysis of weakly meshed distribution systems. The special topological characteristics of distribution networks had been fully exploited to make the solution possible. A branch-injection to branch-current matrix was formed (BIBC). This matrix was obtained by applying Kirchhoff's current law for the distribution network. Using the same matrix a solution for weakly meshed distribution network was proposed. Bus voltages were found by the forward -

sweep of the network. Test results of 33-bus and 69-bus systems were given to illustrate the performance of the presented method.

Ghosh and Sherpa [25] proposed a new and accurate method for load-flow solution for radial networks with minimum data preparation where the node and branch numbering in bus systems was not to be sequentially only like other methods. Their proposed method used the simple equation for voltage magnitude computation. Their method was capable to handle composite load modelling. Their method was effective and compared with other methods using examples through detailed study and different load-modelling pattern.

Wu and Zhang [26] suggested theoretical formulation of the forward/backward sweep with compensation power flow method. Subsequently, a novel solution of unbalanced three-phase power systems based on the loop-analysis method was developed. The proposed method had clear theory foundation and took full advantage of the radial (or weakly meshed) structure of distribution systems.

Eminoglu and Hocaoglu [27] proposed load-flow algorithms based on the forward/backward sweeps. Their convergence ability was quantitatively evaluated for different loading conditions, R/X ratios and sub-station voltage levels. Moreover, the effects of static load modelling on the convergence characteristics of algorithms were also investigated.

Ghosh [28] proposed a simple method for solving load-flow problem of radial distribution networks. In the paper the algorithm easily computed the power flow through any branch exploiting the radial feature of the distribution networks. The proposed algorithm was efficient for less data preparation and could handle arbitrary node numbering scheme very easily. The method was effective and easily computable.

Ghosh and Singh [29] proposed the effect of charging capacitance of the line had been incorporated into load-flow solution. A computer algorithm was developed in such a way that there was no need to adopt any sequential node numbering scheme for the solution and the angle of the receiving-end voltage was also computed along with the magnitude of the

voltage. The comparison of speed and memory requirement of the proposed method with the other methods had been verified to show its efficiency.

Hamouda and Zehar [30] presented an improved method to solve load-flow problem in balanced radial distribution systems with laterals. Their method was based on electric circuit laws with iteration and allowed the evaluation of both, voltage (rms) values and phase-angles. A simple technique of determining nodes beyond each branch was given through load-flow where speed convergence was increased by an appropriate choice of initial voltages. The method required a small number of iterations and less computational time.

Abul'Wafa [31] successfully analysed and solved network-topology-based method to the load-flow problem of radial distribution networks. The proposed technique was based on network graphical information and power flow equation formulation which was in matrix form to satisfy the need of distribution automation. In the technique a directed graph of a radial network represented by a nodes-by-nodes sparse matrix (S) allowed detection of the path of power flow from the reference node to the leaf end. The proposed method also allowed dynamic building of the two matrices: BIBC and BCBV matrix which were used to find out the load-flow solution.

2.2 Scope of the Research

The method proposed in the literature either nodes beyond a branch or to arrange the network with rigorous numbering scheme. The first case consumed huge memory because it needed two dimensional matrix (mxm). The second case was time consuming.

2.3 Objective of the Research

The thesis work endeavours to propose a new a new technique for load-flow analysis. The objectives are divided into the following:

- To use sequential numbering scheme.
- To reduce data preparation using the radial feature of distribution networks using only the source node of the feeder, lateral(s) and sub lateral(s).
- To check the loads-flow results using the constant power, constant current, constant impedance, composite and exponential loads.

- To identify the most sensitive nodes of the distribution networks.

2.4 Organization of the Thesis Work

Chapter 1 presents the introduction of the distribution system and load-flow analysis.

Chapter 2 presents a literature review on load-flow analysis, objective of the research, the scope of the research and organization of the thesis work.

Chapter 3 presents the load-flow analysis of radial distribution networks. The assumption, solution methodology, algorithms, examples and results and the conclusion.

Chapter 4 presents the summary of conclusions and the future scope of further research work.

References

Appendix – A Line data and load data of 33-node radial distribution network.

Appendix – B Line data and load data of 69-node radial distribution network.

Chapter-3

Load-Flow Analysis for Radial Distribution Network

Load-flow studies are performed using computer software that simulates actual steady-state power system operating conditions enabling the evaluation of bus voltage profiles, real and reactive power flow and losses. Conducting load-flow study using multiple scenarios ensure that the power system is adequately designed to satisfy the performance criteria. A properly designed system contain initial capital investments and future operating costs. The benefits of power flow study in electrical power system decrease unexpected downtime, minimize operating and maintenance costs and get more capacity out of existing assets. Load-flow analysis required when (a) significant plant/load expansion occurs, (b) local generation is proposed to be added, (c) new utility feed installation, (d) large motor addition to the system, (e) new transformer installation etc. Unlike traditional circuit analysis, a power flow study usually uses the simplified notation such as a single-line diagram, per-unit system and focuses on various forms of AC power (i.e., real, reactive and apparent) rather than voltage and current. The great importance of power flow or load-flow studies is in the planning and in the future expansion of power systems as well as in determining the best operation of existing systems. The principal information obtained from the power flow study is the magnitude and phase angle of the voltage at each bus and the real and reactive power flowing in each line. Literature survey has already been introduced in **Chapter 2**.

The main aim of this thesis work is to present a new load-flow technique for all nodes of the network without reducing the network.

3.1 Assumption

The three-phase radial distribution networks are assumed to be balanced and hence represented by their single-line diagram.

3.2 Load Modelling

Load modelling has a crucial role in voltage stability analysis of a distribution network system. Every load depends upon the voltage and frequency in the distribution system. A balanced load can be represented either as constant power, constant current, constant

impedance or as an exponential load. The method of load-flow analysis must have the capability to handle all types of load modelling. Equation (3.1) and (3.2) shows the load modelling.

$$P(m_2) = P_n[a_0 + a_1V(m_2) + a_2V^2(m_2) + a_3Ve1(m_2)] \quad (3.1)$$

$$Q(m_2) = Q_n[b_0 + b_1V(m_2) + b_2V^2(m_2) + b_3Ve1(m_2)] \quad (3.2)$$

where, P_n and Q_n are nominal real and reactive power respectively and $V(m_2)$ is the voltage at node m_2 .

For all the loads, Eq.(3.1) and Eq.(3.2) are modelled as

$$a_0 + a_1 + a_2 + a_3 = 1.0 \quad (3.3)$$

$$b_0 + b_1 + b_2 + b_3 = 1.0 \quad (3.4)$$

For constant power (CP) load $a_0 = b_0 = 1$ and $a_i = b_i = 0$ for $i = 1, 2, 3$. For constant current (CI) load $a_1 = b_1 = 1$ and $a_i = b_i = 0$ for $i = 0, 2, 3$. For constant impedance (CZ) load $a_2 = b_2 = 1$ and $a_i = b_i = 0$ for $i = 0, 1, 3$. Composite load modelling is combination of CP, CI and CZ. For exponential load $a_3 = b_3 = 1$ and $a_i = b_i = 0$ for $i = 0, 1, 2$ and $e1$ and $e2$ are 1.38 and 3.22 respectively [25].

3.3 Solution Methodology:

The following notations are used in this thesis work :

jj : Branch Number.

m1 : Sending – end node.

m2 : Receiving – end node.

IS(jj) : Sending - end node of branch jj.

IR(jj) : Receiving - end node of branch jj.

NB : Total number of nodes.

LK : Total number of branches.

$PL(m2)$: Real power load at node m2.

$QL(m2)$: Reactive power load at node m2.

$LP(jj)$: Real power loss at branch - jj.

$LQ(jj)$: Reactive power loss at branch - jj.

A single-line diagram of a radial distribution network is shown in Fig. 3.1 and Table 3.1 shows the branch number, sending end node and receiving end node of Fig. 3.1.

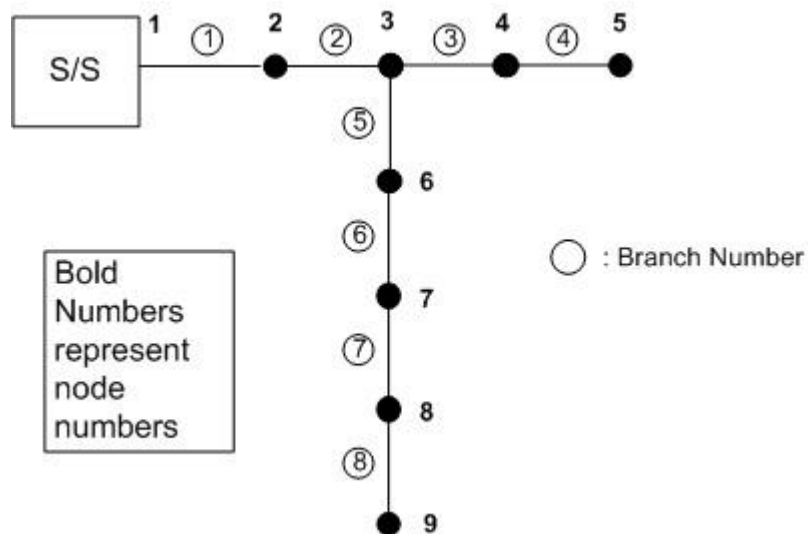


Figure 3.1 Single Line Diagram of Radial Distribution Network

Table 3.1 Branch number (jj), Sending end node ($m1 = IS (jj)$), Receiving end node ($m2 = IR (jj)$) and nodes beyond branches 1, 2, 3, ..., 10 of Fig. 3.1

Branch Number (jj)	Sending end $m1 = IS (jj)$	Receiving end $m2 = IR (jj)$
1	1	2
2	2	3

3	3	4
4	4	5
5	3	6
6	6	7
7	7	8
8	8	9

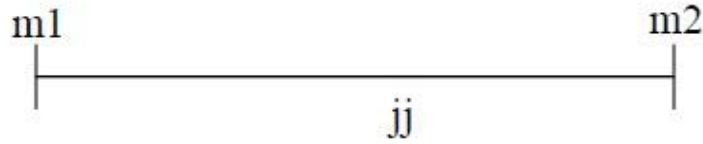


Figure 3.2 Single Line Diagram

From Fig. 3.2,

$$\bar{V}(m2) = \bar{V}(m1) - \bar{I}(jj)\bar{Z}(jj) \quad (3.5)$$

$$V(m2)\angle\delta_2 = V(m1)\angle\delta_1 - [I_{re}(jj) + jI_{im}(jj)][R(jj) + jX(jj)]$$

$$V(m2)\cos\delta_2 + jV(m2)\sin\delta_2 = V(m1)\cos\delta_1 + V(m1)\sin\delta_1 - [I_{re}(jj)R(jj) - I_{im}(jj)X(jj)] - j[I_{re}(jj)X(jj) + I_{im}(jj)R(jj)]$$

(3.6)

where $V(m2)$ is the magnitude and δ_2 is the angle of $\bar{V}(m2)$.

From Eq. (3.6), we have

$$V(m2)\cos\delta_2 = V(m1)\cos\delta_1 - A \quad (3.7)$$

$$V(m2)\sin\delta_2 = V(m1)\sin\delta_1 - B \quad (3.8)$$

where,

$$A = I_{re}(jj)R(jj) - I_{im}(jj)X(jj) \quad (3.9)$$

$$B = I_{re}(jj)X(jj) + I_{im}(jj)R(jj) \quad (3.10)$$

From Eq. (3.7) and Eq. (3.8),

$$\{V(m2)\}^2 = \{V(m1)\}^2 - 2[A \cos \delta_1 + B \sin \delta_1]V(m1) + (A^2 + B^2) \quad (3.11)$$

$$\text{i.e., } V(m2) = \sqrt{\{V(m1)\}^2 - 2[A \cos \delta_1 + B \sin \delta_1]V(m1) + (A^2 + B^2)} \quad (3.12)$$

From Eq. (3.7) and Eq. (3.8), we have

$$\tan \delta_2 = \frac{V(m1) \sin \delta_1 - B}{V(m1) \cos \delta_1 - A} \quad (3.13)$$

The real power loss of branch-jj is given by

$$LP(jj) = |I(jj)|^2 R(jj) \quad (3.14)$$

The reactive power loss of branch-jj is given by

$$LQ(jj) = |I(jj)|^2 X(jj) \quad (3.15)$$

To compute the current a unique software is proposed which forms an array of branches.

In the present method single dimensional arrays for each branch number, sending-end node and receiving-node are used instead of using the (m×n) matrix to reduce the memory elements.

From Fig.3.1 , the branch number 1 (jj=1) is the source branch and the receiving-end node of this branch is 2. A count j=1 is started. This is checked with the sending-end node of other branches and it is matched only for sending-end node of branch (jj=2), which is also 2. Hence they are matched. Therefore, branch-1 is connected with branch-2. The value of 'j' is increased by 1 i.e., $j_{new} = j_{old} + 1 = 2$. The receiving-end node (3) of the present branch (jj=2) is checked with the sending-end node of other remaining branches and it is matched for sending-end node of branch (jj=3), which is also 3. Hence this has matched. The value of 'j' is increased by 1 i.e., $j_{new} = j_{old} + 1 = 3$. The receiving-end node (4) of the present branch is

checked with the sending-end node of other remaining branches and it is matched for sending-end node of branch ($jj=4$), which is also 4. Hence this has matched. The value of 'j' is increased by 1 i.e., $j_{\text{new}} = j_{\text{old}} + 1 = 4$. The receiving-end node (5) of the present branch is checked with the sending-end node of other branches. This is not matched. The value of 'j' remains unaltered. Therefore, branches $jj=1, jj=2, jj=3$ and $jj=4$ are connected. But the branch $jj=5$ is not connected with this group. It will be in a separate cluster. The first cluster contains only (1,2,3,4) and the node 5 is clearly the end node of this branch and node 1 is the starting node of this cluster. Hence an array with size four is required to store these branches. For $jj=5$, the sending-end node is 3 and receiving-end node is 6. Here a count $k=1$ is started. Hence this receiving-end node (6) is checked with the sending-end node of other remaining branches and it is matched for sending-end node of branch $jj=6$, which is also 6. Therefore, $k_{\text{new}} = k_{\text{old}} + 1 = 2$. The receiving-end node (7) is checked with the sending-end node of other remaining branches and it is matched for sending-end node of branch $jj=7$, which is also 7. Therefore, $k_{\text{new}} = k_{\text{old}} + 1 = 3$. The receiving-end node (8) is checked with the sending-end node of other remaining branches and it is matched for sending-end node of branch $jj=8$, which is also 8. Therefore, $k_{\text{new}} = k_{\text{old}} + 1 = 4$. Since there are no branches further, receiving-end node (9) of other remaining branches and it is matched for sending-end node of branch $jj=8$ is the end node this cluster. The starting node of this cluster becomes 3. This cluster contains branches (5,6,7,8). Hence an array of size 4 is required. Hence two arrays are required to store two cluster as Fig. 3.1 has only one feeder and one lateral. With this example, the present method takes 4 and 4 size two independent single array (total 8 elements).

In order to club these two clusters, each branch is marked either 1 or 2. It is marked '1' when the receiving-end node of this branch i.e., the sending-end of its next one branch only. It is marked '2' when the receiving-end node of this branch i.e., the sending-end of its next more than one branch only. In Fig. 3.1, the branch 2 is only marked 2 and all other remaining branches are marked 1.

The method last in first out (LIFO) is used here for computation of current in each branch. The current taken by each load connected to each node is computed at first. The computation of current in each branch is done starting from end element of each cluster. For example, the branch current through $jj=4$ will be $I(jj=4) = IL(5)$. The branch current through $jj=3$ is the sum of branch current of $jj=4$ and load current at node $IR(jj)$ i.e.; $I(jj=3) = I(jj=4) + IL(4)$. For

$jj = 2$, the sending-end node is 3, which is also the receiving-end node of $jj = 5$. Hence $I(jj=2) = I(jj=3) + I(jj=5) + IL(3)$.

The following steps are followed to obtain the branches after every branch.

- Step 1 : Start
- Step 2 : Read the branch numbers $[IB(jj)]$, sending-end nodes $[IS(jj)]$ and receiving-end nodes $[IR(jj)]$ of branch jj . Each one is stored in respective single dimensional array.
- Step 3 : Store the first branch jj .
- Step 4 : For each jj , $IR(m2)$ is checked with $IS(m1)$ of $jj + 1$ for $jj = 1, 2, 3, \dots, LK$. If this matches, $jj+1$ is stored in the array. This branch is marked 1. Otherwise, go to step 4.
- Step 5 : If $IR(m2)$ of branch- $jj \neq IS(m1)$ of branch $jj+1$, the branch number $jj+1$ is not stored in this array. This $IS(m1)$ is again checked with the other $IR(m2)$ s and the required branch number for the matched $IR(m2)$ is marked 2.
- Step 5 : The step 4 is repeated for the branches $jj = jj+1$ to LK until $jj = LK$.
- Step 6 : STOP

The following steps are performed to compute the load-flow results.

- Step 1 : Start
- Step 2 : Read the line data, load data, base values, maximum number of iterations (ICMAX) and convergence constant ϵ .
- Step 3 : Put $V(i) = 1.0 + j0.0$ for $i = 1, 2, 3, \dots, NB$.
- Step 4 : Call the subroutine for identifying the branches after each branch.
- Step 5 : Iteration count (IC) = 1

- Step 6 : Incorporate the type of load.
- Step 7 : Compute the current of each branch.
- Step 8 : Compute voltage of each node using Eq.(3.12) and its angle by using Eq. (3.13).
- Step 9 : Compute the $\Delta V(i) = V(i)$ of previous iteration – $V(i)$ of present iteration for $i = 1,2,3,\dots, NB$.
- Step 10 : Obtain the min value of $\Delta V(i)$.
- Step 11 : If $\epsilon \leq 0.0001$, go to Step 14.
- Step 12 : $IC = IC + 1$
- Step 13 : If $IC \leq IC_{MAX}$, go to Step 7. Otherwise, go to Step 17.
- Step 14 : Print Converged.
- Step 15 : Compute total real power loss and reactive power losses using Eq. (3.14) and Eq. (3.15) for $jj = 1,2,3,4,\dots,NB$.
- Step 16 : Print the results and go to Step 18.
- Step 17 : Print “ NOT CONVERGED”.
- Step 18 : Stop

3.4 Examples

Two examples have been taken to demonstrate the effectiveness of the proposed method.

3.4.1 Example 1

The first example is a 33-node radial distribution network shown in Fig. 3.3. Base values are 12.66 kV and 100 MVA. Line data and load data for 33-node are available in [11] shown in Appendix-A (Table A1 and A2 respectively) nodes and branches have been renumbered. Sub-station voltage is 1.0 pu.

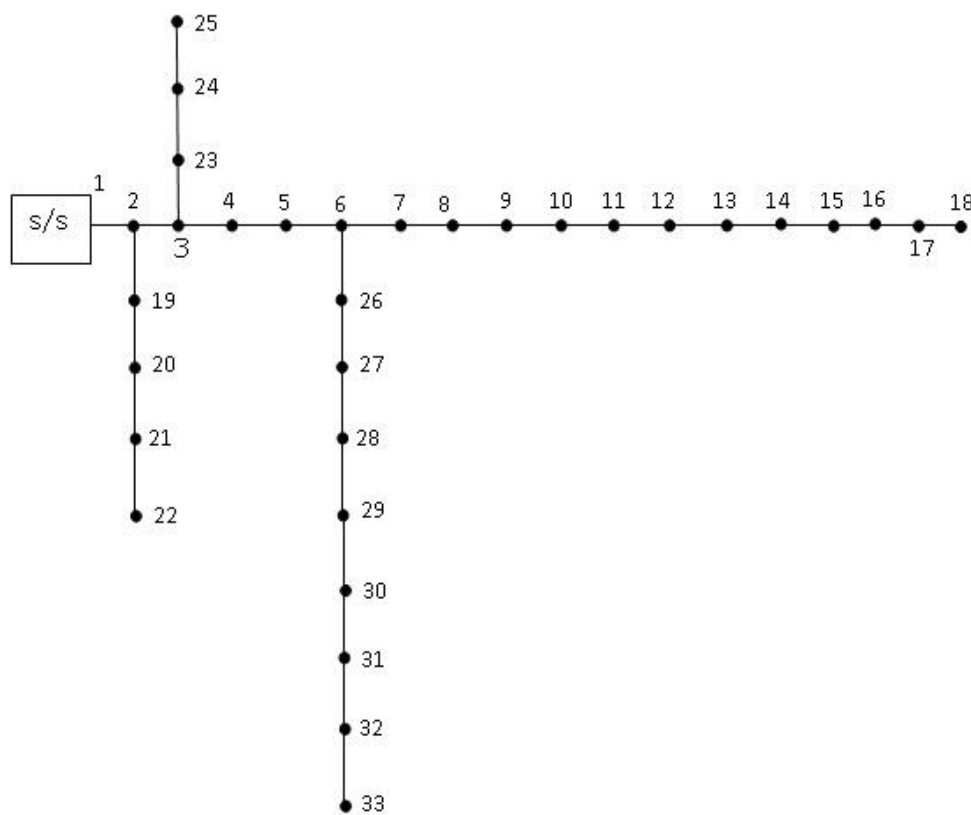


Figure 3.3 33-Node Radial Distribution Network. [11]

Table 3.2, Table 3.3, Table 3.4, Table 3.5 and Table 3.6 respectively show the real part of voltage, imaginary part of voltage, voltage magnitude and voltage angle of each of each node of Fig. 3.3 for constant power (CP), constant current (CI), constant impedance (CZ), composite (40% CP + 30% CI + 30% CZ) and exponential load modelling.

Table 3.7 shows the number of iterations, total real power loss (kW), total reactive power loss (kVAr), total real power load (kW), total reactive power load (kVAr), node number and minimum value of voltage (p.u.) for constant power, constant current, constant impedance, composite and exponential load modelling for 33-node radial distribution network.

Table 3.8 shows the R/X ratio of each branch of Fig. 3.3.

Table 3.9 shows the load-flow results for constant power load with different values of R/X. The starting value here is 1.1 and it has converged up to 4.7. The iteration number, total real power loss (kW), total reactive power loss (kVAr), minimum Voltage (p.u.).

Table 3.2 Real part of voltage, imaginary part of voltage, voltage magnitude and voltage angle of each node of Fig. 3.3 for Constant Power (CP) load modelling.

Node number	Real Part of the Voltage (p.u.)	Imaginary Part of the Voltage (p.u.)	Voltage magnitude (p.u.)	Voltage Angle (radian)
1	1.000000	0.000000	1.000000	0.000000
2	0.997032	0.000252	0.997032	0.000253
3	0.982938	0.001648	0.982939	0.001676
4	0.975454	0.002752	0.975458	0.002822
5	0.968054	0.003857	0.968061	0.003985
6	0.949659	0.002220	0.949662	0.002337
7	0.946175	-0.001591	0.946177	-0.001682
8	0.941333	-0.000990	0.941333	-0.001052
9	0.935063	-0.002176	0.935065	-0.002327
10	0.929413	-0.003255	0.929418	-0.003503
11	0.928553	-0.003135	0.928559	-0.003376
12	0.927055	-0.002944	0.927059	-0.003176
13	0.920937	-0.004392	0.920948	-0.004769
14	0.918664	-0.005642	0.918681	-0.006141
15	0.917248	-0.006236	0.917270	-0.006799
16	0.915878	-0.006599	0.915902	-0.007204
17	0.913842	-0.007816	0.913875	-0.008553
18	0.913234	-0.007963	0.913268	-0.008720
19	0.996504	0.000064	0.996504	0.000064
20	0.992926	-0.001097	0.992926	-0.001105
21	0.992221	-0.001432	0.992222	-0.001443
22	0.991583	-0.001783	0.991585	-0.001798
23	0.979353	0.001113	0.979353	0.001136
24	0.972682	-0.000401	0.972682	-0.000413
25	0.969357	-0.001139	0.969357	-0.001175
26	0.947728	0.002868	0.947733	0.003026
27	0.945162	0.003786	0.945169	0.004006
28	0.933716	0.005092	0.933730	0.005454
29	0.925491	0.006306	0.925512	0.006813
30	0.921920	0.007975	0.921955	0.008651
31	0.917770	0.006587	0.917794	0.007177
32	0.916857	0.006212	0.916879	0.006775
33	0.916575	0.006086	0.916595	0.006640

Table 3.3 Real part of voltage, imaginary part of voltage, voltage magnitude and voltage angle of each node of Fig. 3.3 for Constant Current (CI) load modelling.

Node number	Real Part of the Voltage (p.u.)	Imaginary Part of the Voltage (p.u.)	Voltage magnitude (p.u.)	Voltage Angle (radian)
1	1.000000	0.000000	1.000000	0.849999
2	0.657953	0.749318	0.997186	0.850231
3	0.648222	0.740185	0.983903	0.851537
4	0.642862	0.735626	0.976943	0.852591
5	0.637566	0.731138	0.970078	0.853657
6	0.627470	0.717316	0.953027	0.852110
7	0.627996	0.712545	0.949789	0.848386
8	0.624621	0.709543	0.945305	0.848965
9	0.621632	0.704481	0.939532	0.847792
10	0.618950	0.699920	0.934338	0.846714
11	0.618346	0.699399	0.933548	0.846829
12	0.617307	0.698481	0.932171	0.847012
13	0.614606	0.693389	0.926568	0.845558
14	0.614094	0.691068	0.924493	0.844307
15	0.613649	0.689736	0.923201	0.843708
16	0.613072	0.688576	0.921951	0.843339
17	0.612682	0.686444	0.920100	0.842115
18	0.612417	0.685937	0.919546	0.841963
19	0.657747	0.748800	0.996661	0.850043
20	0.656270	0.745373	0.993111	0.848883
21	0.656057	0.744628	0.992413	0.848548
22	0.655902	0.743923	0.991781	0.848196
23	0.646314	0.737224	0.980419	0.851012
24	0.643143	0.731387	0.973941	0.849510
25	0.641553	0.728494	0.970718	0.848771
26	0.625844	0.716371	0.951246	0.852742
27	0.623646	0.715152	0.948881	0.853642
28	0.615779	0.708030	0.938345	0.854972
29	0.609937	0.703085	0.930780	0.856221
30	0.606614	0.701636	0.927509	0.857904
31	0.605057	0.697936	0.923693	0.856559
32	0.604763	0.697080	0.922854	0.856193
33	0.604678	0.696810	0.922594	0.856070

Table 3.4 Real part of voltage, imaginary part of voltage, voltage magnitude and voltage angle of each node of Fig. 3.3 for Constant Impedance (CZ) load modelling.

Node number	Real Part of the Voltage (p.u.)	Imaginary Part of the Voltage (p.u.)	Voltage magnitude (p.u.)	Voltage Angle (radian)
1	1.000000	0.000000	1.000000	0.849999
2	0.658047	0.749409	0.997316	0.850220
3	0.648811	0.740751	0.984717	0.851467
4	0.643773	0.736487	0.978190	0.852469
5	0.638806	0.732300	0.971770	0.853482
6	0.629332	0.719406	0.955827	0.852083
7	0.629809	0.714931	0.952778	0.848614
8	0.626645	0.712138	0.948591	0.849170
9	0.623847	0.707440	0.943216	0.848107
10	0.621337	0.703211	0.938384	0.847133
11	0.620774	0.702730	0.937651	0.847243
12	0.619806	0.701882	0.936375	0.847418
13	0.617283	0.697176	0.931178	0.846103
14	0.616800	0.695029	0.929251	0.844961
15	0.616383	0.693798	0.928054	0.844416
16	0.615846	0.692727	0.926897	0.844083
17	0.615477	0.690759	0.925181	0.842968
18	0.615230	0.690293	0.924669	0.842831
19	0.657842	0.748894	0.996794	0.850034
20	0.656375	0.745492	0.993271	0.848883
21	0.656164	0.744754	0.992577	0.848551
22	0.656010	0.744054	0.991950	0.848202
23	0.646949	0.737869	0.981323	0.850959
24	0.643858	0.732193	0.975018	0.849504
25	0.642311	0.729383	0.971886	0.848791
26	0.627815	0.718531	0.954169	0.852676
27	0.625765	0.717405	0.951972	0.853519
28	0.618424	0.710820	0.942185	0.854797
29	0.612974	0.706254	0.935164	0.855988
30	0.609881	0.704925	0.932134	0.857560
31	0.608424	0.701506	0.928596	0.856338
32	0.608148	0.700715	0.927818	0.856004
33	0.608069	0.700465	0.927577	0.855892

Table 3.5 Real part of voltage, imaginary part of voltage, voltage magnitude and voltage angle of each node of Fig. 3.3 for Composite load modelling.

Node number	Real Part of the Voltage (p.u.)	Imaginary Part of the Voltage (p.u.)	Voltage magnitude (p.u.)	Voltage Angle (radian)
1	1.000000	0.000000	1.000000	0.846103
2	0.660854	0.746738	0.997169	0.846337
3	0.651019	0.737583	0.983796	0.847657
4	0.645595	0.733008	0.976777	0.848720
5	0.640234	0.728501	0.969852	0.849797
6	0.629992	0.714598	0.952649	0.848238
7	0.630504	0.709784	0.949383	0.844481
8	0.627086	0.706768	0.944859	0.845065
9	0.624049	0.701669	0.939029	0.843881
10	0.621322	0.697072	0.933783	0.842792
11	0.620709	0.696549	0.932985	0.842908
12	0.619656	0.695625	0.931594	0.843093
13	0.616907	0.690491	0.925933	0.841622
14	0.616380	0.688148	0.923835	0.840358
15	0.615925	0.686803	0.922530	0.839752
16	0.615337	0.685633	0.921266	0.839379
17	0.614934	0.683479	0.919395	0.838140
18	0.614664	0.682968	0.918835	0.837987
19	0.660646	0.746220	0.996644	0.846149
20	0.659154	0.742796	0.993091	0.844989
21	0.658939	0.742052	0.992392	0.844653
22	0.658780	0.741347	0.991759	0.844301
23	0.649093	0.734621	0.980301	0.847130
24	0.645890	0.728779	0.973803	0.845623
25	0.644283	0.725881	0.970569	0.844882
26	0.628348	0.713650	0.950851	0.848877
27	0.626123	0.712428	0.948464	0.849785
28	0.618153	0.705267	0.937824	0.851128
29	0.612235	0.700295	0.930185	0.852389
30	0.608873	0.698844	0.926881	0.854090
31	0.607286	0.695112	0.923026	0.852730
32	0.606986	0.694248	0.922178	0.852359
33	0.606899	0.693975	0.921915	0.852235

Table 3.6 Real part of voltage, imaginary part of voltage, voltage magnitude and voltage angle of each node of Fig. 3.3 for Exponential load modelling.

Node number	Real Part of the Voltage (p.u.)	Imaginary Part of the Voltage (p.u.)	Voltage magnitude (p.u.)	Voltage Angle (radian)
1	1.000000	0.000000	1.000000	0.841622
2	0.664363	0.743789	0.997296	0.841743
3	0.655379	0.734780	0.984593	0.842452
4	0.650522	0.730288	0.978008	0.843102
5	0.645741	0.725867	0.971527	0.843749
6	0.636863	0.712515	0.955653	0.841404
7	0.637501	0.707967	0.952694	0.837724
8	0.634369	0.705001	0.948393	0.838084
9	0.631683	0.700085	0.942944	0.836715
10	0.629283	0.695660	0.938051	0.835454
11	0.628727	0.695146	0.937297	0.835526
12	0.627771	0.694238	0.935982	0.835634
13	0.625373	0.689311	0.930721	0.833993
14	0.624961	0.687086	0.928797	0.832713
15	0.624567	0.685799	0.927579	0.832093
16	0.624050	0.684675	0.926400	0.831689
17	0.623743	0.682632	0.924685	0.830446
18	0.623509	0.682142	0.924165	0.830275
19	0.664156	0.743274	0.996774	0.841554
20	0.662673	0.739869	0.993248	0.840382
21	0.662460	0.739129	0.992555	0.840045
22	0.662303	0.738428	0.991928	0.839692
23	0.653532	0.731858	0.981183	0.841875
24	0.650476	0.726105	0.974858	0.840283
25	0.648947	0.723254	0.971714	0.839497
26	0.635423	0.711578	0.953995	0.841874
27	0.633481	0.710368	0.951799	0.842550
28	0.626689	0.703555	0.942193	0.843117
29	0.621648	0.698827	0.935311	0.843780
30	0.618736	0.697380	0.932295	0.845081
31	0.617397	0.693822	0.928746	0.843618
32	0.617151	0.693003	0.927970	0.843229
33	0.617084	0.692747	0.927734	0.843099

Table 3.7 Load-flow results of CP, CI, CZ, Composite and Exponential load modelling of Fig. 3.3

Type of Loads	Number of Iteration	Total Real Power Loss (kW)	Total Reactive Power Loss (kVAr)	Total Real Power Load (kW)	Total Reactive Power Load (kVAr)	Node Number	Minimum Value of Voltage (p.u.)
Constant Power	4	202.51	135.12	3715.00	2300.00	18	0.913268
Constant Current	4	176.51	117.51	3543.37	2181.08	18	0.919546
Constant Impedance	3	156.35	103.89	3394.41	2078.01	18	0.924669
Composite Load	4	179.34	119.43	3562.47	2194.42	18	0.918835
Exponential Load	3	156.96	104.31	3489.79	1956.15	18	0.924165

Table 3.8 R/X ratio of each branch of Fig. 3.3

Branch No.	R/X ratio
1	1.961702
2	1.963361
3	1.963519
4	1.963421
5	1.158416
6	0.302521
7	3.025946
8	1.391892
9	1.356757
10	3.024616
11	3.024232
12	1.270996
13	0.759714
14	1.123574
15	1.369358
16	0.748983
17	1.275261
18	1.047923
19	1.109783
20	0.855978
21	0.756321
22	1.463510
23	1.266394
24	1.277992
25	1.963249
26	1.964064
27	1.134197
28	1.147873
29	1.963249
30	1.011838
31	0.857972
32	0.643153

Table 3.9 Load-flow results for different R/X ratio of Fig. 3.3

Multiplying Factor of R	Iteration Number	Total Real Power Loss (kW)	Total Reactive Power Loss (kVAr)	Node Number	Minimum Voltage (p.u.)
1.1	5	223.83	136.53	18	0.906811
1.2	5	245.62	137.99	18	0.900261
1.3	5	267.91	139.50	18	0.893614
1.4	5	290.73	141.05	18	0.886865
1.5	5	314.09	142.66	18	0.880010
1.6	5	338.03	144.32	18	0.873044
1.7	6	362.65	146.06	18	0.865946
1.8	6	387.87	147.85	18	0.858738
1.9	6	413.77	149.71	18	0.851400
2.0	6	440.40	151.63	18	0.843926
2.1	6	467.80	153.63	18	0.836311
2.2	6	496.01	155.72	18	0.828544
2.3	7	525.20	157.92	18	0.820603
2.4	7	555.26	160.19	18	0.812503
2.5	7	586.33	162.57	18	0.804222
2.6	7	618.48	165.06	18	0.795748
2.7	7	651.81	167.67	18	0.787068
2.8	7	686.40	170.42	18	0.778167
2.9	7	722.36	173.31	18	0.769028
3.0	8	759.96	176.40	18	0.759598
3.1	8	799.10	179.64	18	0.749912
3.2	8	840.06	183.08	18	0.739920
3.3	8	883.03	186.74	18	0.729593
3.4	9	928.47	190.70	18	0.718865
3.5	9	976.27	194.92	18	0.707746
3.6	9	1026.94	199.46	18	0.696166
3.7	9	1080.86	204.38	18	0.684064
3.8	10	1138.82	209.80	18	0.671300
3.9	11	1201.31	215.77	18	0.657854
4.0	11	1268.95	222.35	18	0.643608
4.1	11	1343.04	229.72	18	0.628379
4.2	13	1425.89	238.23	18	0.611811
4.3	13	1518.85	248.01	18	0.593756
4.4	15	1626.89	259.79	18	0.573472
4.5	17	1756.35	274.42	18	0.550094
4.6	19	1921.65	293.97	18	0.521606
4.7	25	2168.37	325.03	18	0.481654

3.4.2 Example 2

The second example is a 69-node radial distribution network shown in Fig. 3.4 Base values are 12.66 kV and 100 MVA. Line data and load data for 69-node radial distribution network are available in [11] shown in Appendix-B (Table B1 and B2 respectively) nodes and branches have been renumbered. Substation voltage is 1.0 p.u.

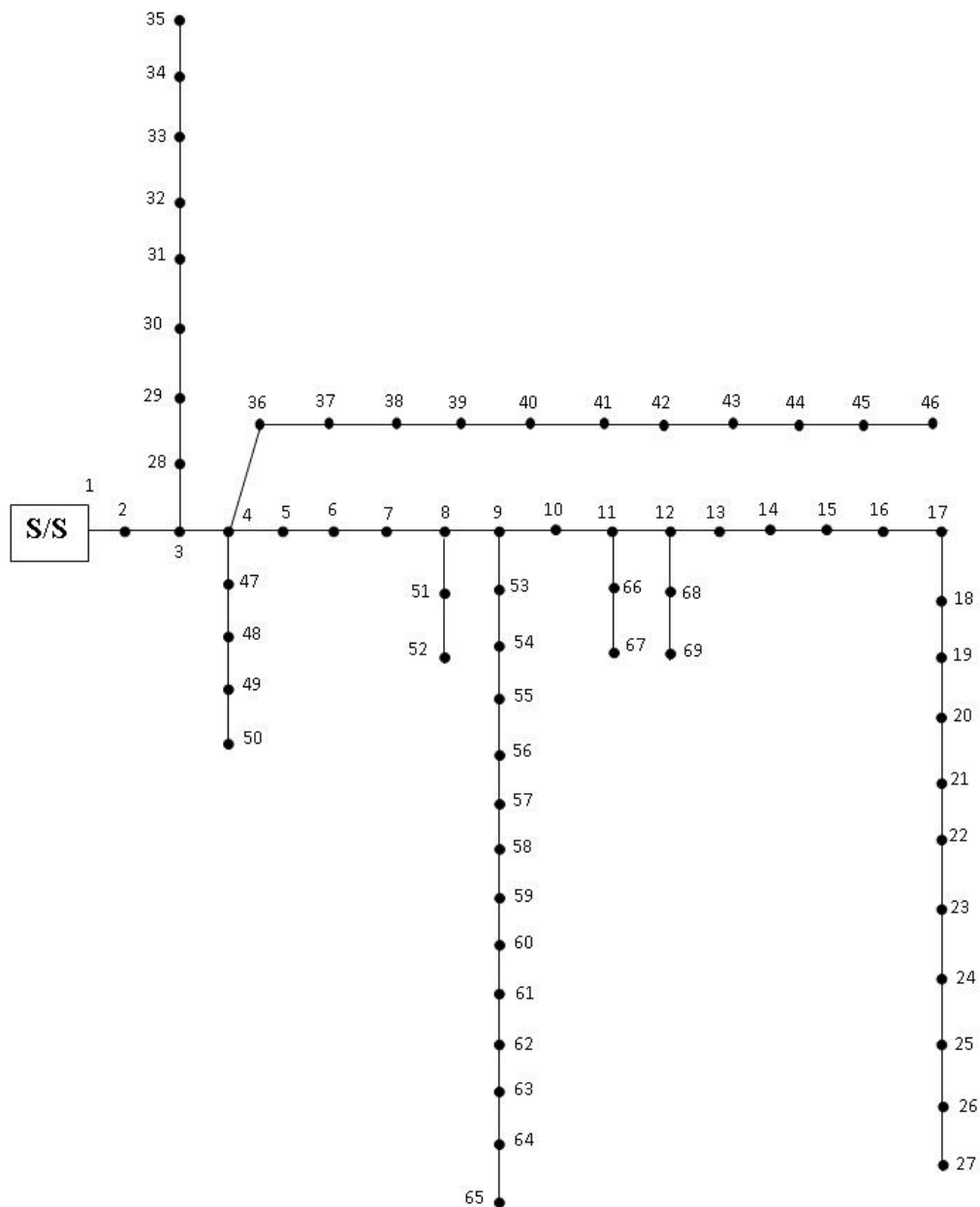


Figure 3.4 69–Node Radial Distribution Network. [11]

Table 3.10, Table 3.11, Table 3.12, Table 3.13 and Table 3.14 respectively show the real part of voltage, imaginary part of voltage, voltage magnitude and voltage angle of each of each node of Fig. 3.4 for constant power (CP), constant current (CI), constant impedance (CZ), composite (40% CP + 30% CI + 30% CZ) and exponential load modelling.

Table 3.15 shows the number of iterations, total real power loss (kW), total reactive power loss (kVAr), total real power load (kW), total reactive power load (kVAr), node number and minimum value of voltage (p.u.) for constant power, constant current, constant impedance, composite and exponential load modelling for 69-node radial distribution network.

Table 3.16 shows the R/X ratio of each branch of Fig. 3.4.

Table 3.17 shows the load-flow results for constant power load with different values of R/X. The starting value here is 1.2 and it has converged up to 3.6. The iteration number, total real power loss (kW), total reactive power loss (kVAr), minimum Voltage (p.u.).

Table 3.10 Real part of voltage, imaginary part of voltage, voltage magnitude and voltage angle of each node of Fig. 3.4 for Constant Power (CP) load modelling.

Node number	Real Part of the Voltage (p.u.)	Imaginary Part of the Voltage (p.u.)	Voltage magnitude (p.u.)	Voltage Angle (radian)
1	1.000000	0.000000	1.000000	0.000000
2	0.999967	-0.000021	0.999967	-0.000021
3	0.999933	-0.000043	0.999933	-0.000043
4	0.999839	-0.000103	0.999839	-0.000103
5	0.999021	-0.000323	0.999021	-0.000323
6	0.990087	0.000850	0.990088	0.000859
7	0.980794	0.002069	0.980796	0.002110
8	0.978578	0.002358	0.978581	0.002410
9	0.977445	0.002506	0.977448	0.002563
10	0.972443	0.003927	0.972451	0.004039
11	0.971341	0.004241	0.971350	0.004366
12	0.968178	0.005116	0.968192	0.005284
13	0.965253	0.005878	0.965271	0.006089
14	0.962352	0.006633	0.962375	0.006893
15	0.959478	0.007383	0.959507	0.007695
16	0.958944	0.007523	0.958974	0.007845
17	0.958063	0.007753	0.958094	0.008092
18	0.958054	0.007755	0.958085	0.008094
19	0.957588	0.007894	0.957621	0.008243
20	0.957289	0.007984	0.957322	0.008340
21	0.956806	0.008128	0.956841	0.008495
22	0.956799	0.008130	0.956834	0.008497
23	0.956727	0.008152	0.956762	0.008520
24	0.956571	0.008199	0.956606	0.008571
25	0.956401	0.008250	0.956437	0.008626
26	0.956331	0.008271	0.956367	0.008649
27	0.956312	0.008277	0.956348	0.008655
28	0.999926	-0.000047	0.999926	-0.000047
29	0.999854	-0.000093	0.999854	-0.000093
30	0.999733	-0.000056	0.999733	-0.000056
31	0.999712	-0.000049	0.999712	-0.000049
32	0.999605	-0.000016	0.999605	-0.000016
33	0.999349	0.000061	0.999349	0.000061
34	0.999013	0.000163	0.999013	0.000163
35	0.998946	0.000182	0.998946	0.000182
36	0.999919	-0.000052	0.999919	-0.000052
37	0.999747	-0.000164	0.999747	-0.000164
38	0.999589	-0.000206	0.999589	-0.000206
39	0.999543	-0.000218	0.999543	-0.000218
40	0.999541	-0.000219	0.999541	-0.000219
41	0.998843	-0.000410	0.998843	-0.000411
42	0.998551	-0.000491	0.998551	-0.000492
43	0.998512	-0.000501	0.998512	-0.000502

44	0.998504	-0.000504	0.998504	-0.000505
45	0.998405	-0.000536	0.998405	-0.000536
46	0.998405	-0.000536	0.998405	-0.000537
47	0.999789	-0.000134	0.999789	-0.000134
48	0.998545	-0.000916	0.998545	-0.000917
49	0.994699	-0.003329	0.994704	-0.003347
50	0.994153	-0.003671	0.994160	-0.003693
51	0.978543	0.002363	0.978546	0.002415
52	0.978533	0.002366	0.978536	0.002418
53	0.974657	0.002871	0.974662	0.002946
54	0.971413	0.003296	0.971419	0.003393
55	0.966938	0.003881	0.966946	0.004014
56	0.962567	0.004451	0.962578	0.004624
57	0.940043	0.010853	0.940105	0.011545
58	0.928941	0.014010	0.929047	0.015080
59	0.924643	0.015252	0.924769	0.016493
60	0.919591	0.016846	0.919745	0.018317
61	0.912175	0.017810	0.912348	0.019522
62	0.911884	0.017848	0.912059	0.019570
63	0.911495	0.017898	0.911671	0.019633
64	0.909590	0.018144	0.909771	0.019945
65	0.909014	0.018218	0.909197	0.020039
66	0.971284	0.004260	0.971294	0.004386
67	0.971284	0.004260	0.971293	0.004386
68	0.967848	0.005216	0.967862	0.005390
69	0.967847	0.005217	0.967861	0.005390

Table 3.11 Real part of voltage, imaginary part of voltage, voltage magnitude and voltage angle of each node of Fig. 3.4 for Constant Current (CI) load modelling.

Node number	Real Part of the Voltage (p.u.)	Imaginary Part of the Voltage (p.u.)	Voltage magnitude (p.u.)	Voltage Angle (radian)
1	1.000000	0.000000	1.000000	0.000000
2	0.999968	-0.000020	0.999968	-0.000020
3	0.999936	-0.000041	0.999936	-0.000041
4	0.999847	-0.000098	0.999847	-0.000098
5	0.999077	-0.000304	0.999077	-0.000304
6	0.990679	0.000809	0.990680	0.000816
7	0.981944	0.001965	0.981946	0.002001
8	0.979863	0.002239	0.979865	0.002285
9	0.978800	0.002378	0.978803	0.002430
10	0.973959	0.003756	0.973966	0.003857
11	0.972893	0.004060	0.972901	0.004173
12	0.969839	0.004906	0.969851	0.005059
13	0.967024	0.005640	0.967040	0.005832
14	0.964233	0.006368	0.964254	0.006604
15	0.961469	0.007091	0.961495	0.007375
16	0.960955	0.007225	0.960982	0.007519
17	0.960107	0.007447	0.960136	0.007756
18	0.960099	0.007449	0.960128	0.007758
19	0.959651	0.007583	0.959681	0.007901
20	0.959364	0.007669	0.959394	0.007994
21	0.958900	0.007808	0.958931	0.008142
22	0.958893	0.007810	0.958925	0.008145
23	0.958824	0.007831	0.958856	0.008167
24	0.958673	0.007876	0.958705	0.008216
25	0.958510	0.007925	0.958543	0.008268
26	0.958443	0.007946	0.958476	0.008290
27	0.958424	0.007951	0.958457	0.008296
28	0.999929	-0.000045	0.999929	-0.000045
29	0.999857	-0.000091	0.999857	-0.000091
30	0.999736	-0.000054	0.999736	-0.000054
31	0.999715	-0.000047	0.999715	-0.000047
32	0.999608	-0.000014	0.999608	-0.000014
33	0.999352	0.000063	0.999352	0.000063
34	0.999017	0.000165	0.999017	0.000165
35	0.998950	0.000183	0.998950	0.000184
36	0.999922	-0.000050	0.999922	-0.000050
37	0.999751	-0.000162	0.999751	-0.000162

38	0.999592	-0.000204	0.999592	-0.000204
39	0.999546	-0.000216	0.999546	-0.000216
40	0.999544	-0.000216	0.999544	-0.000217
41	0.998847	-0.000408	0.998848	-0.000408
42	0.998555	-0.000488	0.998556	-0.000489
43	0.998517	-0.000499	0.998517	-0.000500
44	0.998509	-0.000502	0.998509	-0.000502
45	0.998410	-0.000533	0.998410	-0.000534
46	0.998410	-0.000533	0.998410	-0.000534
47	0.999797	-0.000129	0.999797	-0.000129
48	0.998558	-0.000907	0.998558	-0.000909
49	0.994732	-0.003309	0.994737	-0.003326
50	0.994189	-0.003649	0.994196	-0.003670
51	0.979828	0.002244	0.979830	0.002290
52	0.979818	0.002247	0.979821	0.002293
53	0.976224	0.002720	0.976228	0.002786
54	0.973227	0.003117	0.973232	0.003203
55	0.969096	0.003664	0.969103	0.003781
56	0.965064	0.004196	0.965073	0.004348
57	0.944290	0.010136	0.944345	0.010734
58	0.934052	0.013064	0.934143	0.013986
59	0.930088	0.014217	0.930197	0.015284
60	0.925432	0.015694	0.925565	0.016957
61	0.918595	0.016593	0.918745	0.018062
62	0.918328	0.016629	0.918479	0.018105
63	0.917971	0.016675	0.918122	0.018164
64	0.916218	0.016905	0.916374	0.018449
65	0.915688	0.016974	0.915846	0.018534
66	0.972837	0.004079	0.972846	0.004193
67	0.972837	0.004079	0.972845	0.004193
68	0.969518	0.005004	0.969531	0.005161
69	0.969517	0.005004	0.969530	0.005162

Table 3.12 Real part of voltage, imaginary part of voltage, voltage magnitude and voltage angle of each node of Fig. 3.4 for Constant Impedance (CZ) load modelling.

Node number	Real Part of the Voltage (p.u.)	Imaginary Part of the Voltage (p.u.)	Voltage magnitude (p.u.)	Voltage Angle (radian)
1	1.000000	0.000000	1.000000	0.000000
2	0.999970	-0.000019	0.999970	-0.000019
3	0.999939	-0.000039	0.999939	-0.000039
4	0.999855	-0.000093	0.999855	-0.000093
5	0.999138	-0.000285	0.999138	-0.000286
6	0.991318	0.000744	0.991318	0.000750
7	0.983184	0.001813	0.983186	0.001844
8	0.981247	0.002066	0.981249	0.002106
9	0.980261	0.002195	0.980263	0.002239
10	0.975600	0.003520	0.975607	0.003608
11	0.974574	0.003812	0.974582	0.003912
12	0.971643	0.004623	0.971654	0.004758
13	0.968953	0.005324	0.968967	0.005494
14	0.966286	0.006018	0.966305	0.006228
15	0.963645	0.006707	0.963668	0.006960
16	0.963154	0.006835	0.963178	0.007097
17	0.962344	0.007046	0.962370	0.007322
18	0.962336	0.007049	0.962362	0.007324
19	0.961909	0.007176	0.961935	0.007460
20	0.961634	0.007259	0.961662	0.007548
21	0.961191	0.007391	0.961220	0.007689
22	0.961185	0.007393	0.961213	0.007691
23	0.961119	0.007413	0.961147	0.007713
24	0.960975	0.007456	0.961004	0.007759
25	0.960820	0.007503	0.960849	0.007809
26	0.960756	0.007522	0.960785	0.007829
27	0.960738	0.007528	0.960767	0.007835
28	0.999932	-0.000043	0.999932	-0.000043
29	0.999861	-0.000089	0.999861	-0.000089
30	0.999740	-0.000052	0.999740	-0.000052
31	0.999718	-0.000045	0.999718	-0.000045
32	0.999612	-0.000012	0.999612	-0.000012
33	0.999356	0.000065	0.999356	0.000065
34	0.999021	0.000167	0.999021	0.000167
35	0.998954	0.000185	0.998954	0.000185
36	0.999925	-0.000048	0.999925	-0.000048
37	0.999754	-0.000159	0.999754	-0.000159
38	0.999596	-0.000202	0.999596	-0.000202
39	0.999550	-0.000214	0.999550	-0.000214
40	0.999548	-0.000214	0.999548	-0.000214
41	0.998852	-0.000405	0.998852	-0.000406
42	0.998561	-0.000486	0.998561	-0.000486
43	0.998522	-0.000496	0.998522	-0.000497

44	0.998514	-0.000499	0.998514	-0.000500
45	0.998416	-0.000530	0.998416	-0.000531
46	0.998415	-0.000530	0.998415	-0.000531
47	0.999805	-0.000124	0.999805	-0.000124
48	0.998573	-0.000898	0.998573	-0.000899
49	0.994770	-0.003284	0.994775	-0.003302
50	0.994231	-0.003622	0.994237	-0.003644
51	0.981213	0.002071	0.981215	0.002110
52	0.981204	0.002074	0.981206	0.002114
53	0.977912	0.002504	0.977915	0.002560
54	0.975178	0.002862	0.975182	0.002935
55	0.971414	0.003355	0.971420	0.003454
56	0.967744	0.003835	0.967751	0.003962
57	0.948829	0.009215	0.948874	0.009712
58	0.939506	0.011868	0.939581	0.012632
59	0.935898	0.012912	0.935987	0.013796
60	0.931662	0.014250	0.931771	0.015294
61	0.925444	0.015059	0.925566	0.016271
62	0.925201	0.015091	0.925324	0.016309
63	0.924877	0.015133	0.925001	0.016360
64	0.923287	0.015339	0.923414	0.016612
65	0.922807	0.015400	0.922935	0.016687
66	0.974521	0.003830	0.974528	0.003930
67	0.974520	0.003830	0.974527	0.003931
68	0.971333	0.004718	0.971345	0.004857
69	0.971332	0.004718	0.971344	0.004857

Table 3.13 Real part of voltage, imaginary part of voltage, voltage magnitude and voltage angle of each node of Fig. 3.4 for Composite load modelling.

Node number	Real Part of the Voltage (p.u.)	Imaginary Part of the Voltage (p.u.)	Voltage magnitude (p.u.)	Voltage Angle (radian)
1	1.000000	0.000000	1.000000	0.000000
2	0.999968	-0.000020	0.999968	-0.000020
3	0.999936	-0.000041	0.999936	-0.000041
4	0.999848	-0.000097	0.999848	-0.000097
5	0.999084	-0.000301	0.999084	-0.000302
6	0.990754	0.000803	0.990755	0.000811
7	0.982090	0.001951	0.982092	0.001987
8	0.980026	0.002223	0.980028	0.002269
9	0.978972	0.002362	0.978975	0.002412
10	0.974150	0.003735	0.974157	0.003834
11	0.973088	0.004037	0.973096	0.004149
12	0.970047	0.004880	0.970059	0.005031
13	0.967245	0.005611	0.967261	0.005801
14	0.964467	0.006336	0.964488	0.006569
15	0.961716	0.007055	0.961742	0.007336
16	0.961205	0.007189	0.961232	0.007479
17	0.960361	0.007409	0.960390	0.007715
18	0.960353	0.007411	0.960381	0.007717
19	0.959907	0.007544	0.959937	0.007859
20	0.959621	0.007630	0.959652	0.007951
21	0.959159	0.007769	0.959191	0.008099
22	0.959153	0.007771	0.959184	0.008101
23	0.959084	0.007791	0.959116	0.008124
24	0.958934	0.007837	0.958966	0.008172
25	0.958772	0.007886	0.958804	0.008224
26	0.958705	0.007906	0.958738	0.008246
27	0.958686	0.007911	0.958719	0.008252
28	0.999929	-0.000045	0.999929	-0.000045
29	0.999858	-0.000090	0.999858	-0.000090
30	0.999737	-0.000053	0.999737	-0.000053
31	0.999715	-0.000047	0.999715	-0.000047
32	0.999609	-0.000014	0.999609	-0.000014
33	0.999353	0.000063	0.999353	0.000063
34	0.999018	0.000165	0.999018	0.000165
35	0.998950	0.000184	0.998950	0.000184
36	0.999923	-0.000050	0.999923	-0.000050
37	0.999751	-0.000161	0.999751	-0.000161
38	0.999593	-0.000203	0.999593	-0.000204
39	0.999547	-0.000216	0.999547	-0.000216
40	0.999545	-0.000216	0.999545	-0.000216
41	0.998848	-0.000407	0.998848	-0.000408
42	0.998556	-0.000488	0.998556	-0.000489
43	0.998517	-0.000499	0.998518	-0.000499

44	0.998509	-0.000501	0.998509	-0.000502
45	0.998411	-0.000533	0.998411	-0.000534
46	0.998410	-0.000533	0.998410	-0.000534
47	0.999798	-0.000129	0.999798	-0.000129
48	0.998559	-0.000906	0.998560	-0.000908
49	0.994735	-0.003306	0.994741	-0.003324
50	0.994193	-0.003646	0.994200	-0.003668
51	0.979991	0.002228	0.979993	0.002274
52	0.979982	0.002231	0.979984	0.002277
53	0.976424	0.002701	0.976428	0.002766
54	0.973459	0.003094	0.973464	0.003178
55	0.969372	0.003636	0.969379	0.003751
56	0.965384	0.004163	0.965393	0.004312
57	0.944836	0.010042	0.944890	0.010628
58	0.934709	0.012941	0.934799	0.013844
59	0.930789	0.014082	0.930895	0.015128
60	0.926184	0.015544	0.926314	0.016781
61	0.919423	0.016435	0.919570	0.017873
62	0.919159	0.016469	0.919306	0.017916
63	0.918805	0.016516	0.918953	0.017973
64	0.917072	0.016743	0.917225	0.018255
65	0.916549	0.016811	0.916703	0.018340
66	0.973033	0.004056	0.973041	0.004168
67	0.973032	0.004056	0.973040	0.004169
68	0.969727	0.004978	0.969740	0.005133
69	0.969726	0.004978	0.969739	0.005133

Table 3.14 Real part of voltage, imaginary part of voltage, voltage magnitude and voltage angle of each node of Fig. 3.4 for Exponential load modelling.

Node number	Real Part of the Voltage (p.u.)	Imaginary Part of the Voltage (p.u.)	Voltage magnitude (p.u.)	Voltage Angle (radian)
1	1.000000	0.000000	1.000000	0.000000
2	0.999970	-0.000021	0.999970	-0.000021
3	0.999941	-0.000041	0.999941	-0.000041
4	0.999858	-0.000099	0.999858	-0.000099
5	0.999150	-0.000330	0.999150	-0.000330
6	0.991261	0.000283	0.991261	0.000285
7	0.983054	0.000919	0.983055	0.000935
8	0.981100	0.001067	0.981100	0.001088
9	0.980104	0.001141	0.980105	0.001164
10	0.975399	0.002333	0.975401	0.002392
11	0.974362	0.002596	0.974366	0.002664
12	0.971400	0.003317	0.971406	0.003415
13	0.968677	0.003928	0.968685	0.004055
14	0.965978	0.004534	0.965989	0.004694
15	0.963305	0.005135	0.963319	0.005331
16	0.962808	0.005247	0.962823	0.005449
17	0.961989	0.005431	0.962004	0.005645
18	0.961980	0.005433	0.961996	0.005647
19	0.961548	0.005545	0.961564	0.005767
20	0.961271	0.005618	0.961287	0.005844
21	0.960823	0.005734	0.960840	0.005968
22	0.960816	0.005736	0.960833	0.005970
23	0.960749	0.005753	0.960767	0.005988
24	0.960604	0.005792	0.960621	0.006029
25	0.960447	0.005833	0.960464	0.006073
26	0.960382	0.005850	0.960400	0.006091
27	0.960364	0.005855	0.960382	0.006096
28	0.999934	-0.000046	0.999934	-0.000046
29	0.999862	-0.000091	0.999862	-0.000091
30	0.999741	-0.000054	0.999741	-0.000054
31	0.999720	-0.000048	0.999720	-0.000048
32	0.999613	-0.000015	0.999613	-0.000015
33	0.999357	0.000062	0.999357	0.000062
34	0.999022	0.000164	0.999022	0.000164
35	0.998955	0.000182	0.998955	0.000182
36	0.999927	-0.000050	0.999927	-0.000050
37	0.999755	-0.000162	0.999755	-0.000162
38	0.999597	-0.000204	0.999597	-0.000204
39	0.999552	-0.000216	0.999552	-0.000216
40	0.999549	-0.000217	0.999549	-0.000217
41	0.998854	-0.000409	0.998854	-0.000409
42	0.998563	-0.000490	0.998563	-0.000490
43	0.998524	-0.000500	0.998524	-0.000501

44	0.998516	-0.000503	0.998516	-0.000504
45	0.998418	-0.000535	0.998418	-0.000535
46	0.998417	-0.000535	0.998417	-0.000536
47	0.999809	-0.000130	0.999809	-0.000130
48	0.998580	-0.000909	0.998581	-0.000911
49	0.994789	-0.003314	0.994794	-0.003332
50	0.994251	-0.003655	0.994258	-0.003676
51	0.981066	0.001072	0.981066	0.001092
52	0.981057	0.001074	0.981057	0.001095
53	0.977728	0.001288	0.977729	0.001318
54	0.974964	0.001458	0.974965	0.001496
55	0.971156	0.001689	0.971157	0.001739
56	0.967442	0.001910	0.967444	0.001974
57	0.948101	0.005967	0.948120	0.006293
58	0.938568	0.007967	0.938602	0.008488
59	0.934876	0.008758	0.934917	0.009368
60	0.930534	0.009797	0.930586	0.010528
61	0.924243	0.010165	0.924299	0.010998
62	0.923998	0.010179	0.924054	0.011016
63	0.923669	0.010197	0.923725	0.011040
64	0.922060	0.010287	0.922117	0.011157
65	0.921574	0.010314	0.921631	0.011191
66	0.974308	0.002613	0.974312	0.002682
67	0.974307	0.002613	0.974311	0.002682
68	0.971087	0.003404	0.971093	0.003505
69	0.971086	0.003404	0.971092	0.003505

Table 3.15 Load-flow results of CP, CI, CZ, Composite and Exponential load modelling of Fig. 3.4

Type of Loads	Number of Iteration	Total Real Power Loss (kW)	Total Reactive Power Loss (kVAr)	Total Real Power Load (kW)	Total Reactive Power Load (kVAr)	Node Number	Minimum Value of Voltage (p.u.)
Constant Power	4	224.91	102.11	3801.89	2692.59	65	0.909197
Constant Current	3	195.11	89.33	3651.92	2586.17	65	0.915846
Constant Impedance	2	165.49	76.60	3476.67	2461.80	65	0.922935
Composite Load	3	191.44	87.76	3632.82	2572.60	65	0.916703
Exponential Load	2	166.88	77.19	3572.55	2336.40	65	0.921631

Table 3.16 R/X ratio of each branch of Fig. 3.4

Branch No.	R/X ratio
1	0.416667
2	0.416667
3	0.416667
4	0.853742
5	1.963519
6	1.963421
7	1.961702
8	1.964143
9	3.025490
10	3.024232
11	3.025946
12	3.029412
13	3.026087
14	3.026316
15	3.024616
16	3.024232
17	2.937500
18	3.024931
19	3.038961
20	3.025687
21	3.043478
22	3.024714
23	3.024454
24	3.025454
25	3.025465
26	3.027972
27	0.407407
28	0.408946
29	3.025095
30	3.025862
31	3.025862
32	2.979403
33	3.025151
34	3.024831
35	0.407407
36	0.408946
37	0.856098
38	0.856338
39	0.857143
40	0.855917

41	0.855644
42	0.857741
43	0.793103
44	0.793154
45	0.750000
46	0.404762
47	0.408545
48	0.408687
49	0.408752
50	1.961945
51	2.979354
52	1.963883
53	1.963249
54	1.964064
55	1.963015
56	2.979202
57	2.979848
58	3.023857
59	3.294369
60	1.963249
61	1.963710
62	1.964770
63	1.963249
64	1.963410
65	3.292962
66	3.357143
67	3.025368
68	2.937500

Table 3.17 Load-flow results for different R/X ratio of Fig. 3.4

Multiplying Factor of R	Iteration Number	Total Real Power Loss (kW)	Total Reactive Power Loss (kVAr)	Node Number	Minimum Voltage (p.u.)
1.2	5	278.03	105.01	65	0.893400
1.4	5	334.73	108.17	65	0.876909
1.6	5	395.61	111.66	65	0.859626
1.8	6	461.49	115.55	65	0.841416
2.0	6	533.18	119.90	65	0.822154
2.4	7	699.91	130.55	65	0.779559
2.6	8	799.31	137.27	65	0.755569
2.8	8	913.98	145.33	65	0.729122
3.0	9	1050.90	155.47	65	0.699228
3.2	11	1222.76	168.94	65	0.664128
3.4	14	1459.70	188.92	65	0.619805
3.6	24	1897.33	230.17	65	0.548907

Table 3.18 Comparison of relative CPU time and memory requirement of the proposed method with other existing methods for constant power load modelling.

Examples Methods	CPU Time		Memory Requirement	
	Example-1	Example-2	Example-1	Example-2
Proposed Method	1.00	1.00	1.00	1.00
Ghosh and Das [11]	1.70	2.45	2.69	4.61
Ranjan <i>et al.</i> [19]	1.62	2.02	2.68	4.53
Ghosh and Sherpa [25]	1.43	1.92	1.38	1.93

3.5 Conclusion

In the thesis work a new method for load-flow solution of radial distribution network is proposed in the thesis work. A number of clusters have been formed which depends on the number of feeders, laterals and sub laterals. There is an array for each cluster. Hence the memory requirement is reduced as compared to the method proposed by Ghosh and Das [11] and Ranjan et al. [19]. The proposed method has been tested with two examples (33-node and 69-node radial distribution networks) for constant power, constant current, constant impedance, composite and exponential load modelling in each case. Convergence for each of these load modelling in each case is satisfactory. The R/X ratio has been changed and convergence has been checked. The effectiveness of the proposed method has been checked with the other methods in terms of CPU time and memory requirement.

Chapter 4

Conclusions and Future Scope of Work

4.1 Conclusions

A new method for load-flow solution of radial distribution network is proposed in the thesis work. A number of clusters have been formed which depends on the number of feeders, laterals and sub laterals. There is an array for each cluster. Hence the memory requirement is reduced as compared to the method proposed by Ghosh and Das [11] and Ranjan et al. [19]. The proposed method has been tested with two examples (33-node and 69-node radial distribution networks) for constant power, constant current, constant impedance, composite and exponential load modelling in each case. Convergence for each of these load modelling in each case is satisfactory. The R/X ratio has been changed and convergence has been checked. The effectiveness of the proposed method has been checked with the other methods in terms of CPU time and memory requirement.

4.2 Future Scope of Work

After carrying this work in the load-flow analysis of distribution systems, the following guidelines seem to be worth pursuing this area:

- (i) Load-flow analysis for unbalanced network.
- (ii) Fuzzy load-flow analysis.
- (iii) Uncertainty in branch impedance.

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APPENDIX A

Table A.1 Line Data of 33-node Radial Distribution Network

Branch Number	Sending-end Node	Receiving-end Node	Branch resistance (Ω)	Branch reactance (Ω)
1	1	2	0.0922	0.0470
2	2	3	0.4930	0.2511
3	3	4	0.3660	0.1864
4	4	5	0.3811	0.1941
5	5	6	0.8190	0.7070
6	6	7	0.1872	0.6188
7	7	8	0.7114	0.2351
8	8	9	1.0300	0.7400
9	9	10	1.0040	0.7400
10	10	11	0.1996	0.0650
11	11	12	0.3744	0.1238
12	12	13	1.4680	1.1550
13	13	14	0.5416	0.7129
14	14	15	0.5910	0.5260
15	15	16	0.7463	0.5450
16	16	17	1.2890	1.7210
17	17	18	0.7320	0.5740
18	2	19	0.1640	0.1565
19	19	20	1.5042	1.3554
20	20	21	0.4095	0.4784
21	21	22	0.7089	0.9373
22	3	23	0.4512	0.3083
23	23	24	0.8980	0.7091
24	24	25	0.8960	0.7011
25	6	26	0.2030	0.1034
26	26	27	0.2842	0.1447
27	27	28	1.0590	0.9337
28	28	29	0.8042	0.7006
29	29	30	0.5075	0.2585
30	30	31	0.9744	0.9630
31	31	32	0.3105	0.3619
32	32	33	0.3410	0.5302

Table A.2 Load Data of 33-node Radial Distribution Network

Node Number	PL (kW)	QL (kVAr)
1(S/S)	0.0	0.0
2	100.0	60.0
3	90.0	40.0
4	120.0	80.0
5	60.0	30.0
6	60.0	20.0
7	200.0	100.0
8	200.0	100.0
9	60.0	20.0
10	60.0	20.0
11	45.0	30.0
12	60.0	35.0
13	60.0	35.0
14	120.0	80.0
15	60.0	10.0
16	60.0	20.0
17	60.0	20.0
18	90.0	40.0
19	90.0	40.0
20	90.0	40.0
21	90.0	40.0
22	90.0	40.0
23	90.0	50.0
24	420.0	200.0
25	420.0	200.0
26	60.0	25.0
27	60.0	25.0
28	60.0	20.0
29	120.0	70.0
30	200.0	600.0
31	150.0	70.0
32	210.0	100.0
33	60.0	40.0

BASE kV = 12.66 and BASE MVA = 100

APPENDIX B

Table B.1 Line Data of 69-node Radial Distribution Network

Branch Number	Sending-end	Receiving-end	Branch Resistance (Ω)	Branch Reactance (Ω)
1	1	2	0.0005	0.0012
2	2	3	0.0005	0.0012
3	3	4	0.0015	0.0036
4	4	5	0.0251	0.0294
5	5	6	0.3660	0.1864
6	6	7	0.3811	0.1941
7	7	8	0.0922	0.0470
8	8	9	0.0493	0.0257
9	9	10	0.8190	0.2707
10	10	11	0.1872	0.0619
11	11	12	0.7114	0.2351
12	12	13	1.0300	0.3400
13	13	14	1.0440	0.3450
14	14	15	1.0580	0.3496
15	15	16	0.1966	0.0650
16	16	17	0.3744	0.1238
17	17	18	0.0047	0.0016
18	18	19	0.3276	0.1083
19	19	20	0.2106	0.0696
20	20	21	0.3416	0.1129
21	21	22	0.0140	0.0046
22	22	23	0.1591	0.0526
23	23	24	0.3463	0.1145
24	24	25	0.7488	0.2475
25	25	26	0.3089	0.1021
26	26	27	0.1732	0.0572
27	3	28	0.0044	0.0108
28	28	29	0.0640	0.1565
29	29	30	0.3978	0.1315
30	30	31	0.0702	0.0232
31	31	32	0.3510	0.1160
32	32	33	0.8390	0.2816
33	33	34	1.7080	0.5646
34	34	35	1.4740	0.4873
35	3	36	0.0044	0.0108
36	36	37	0.0640	0.1565
37	37	38	0.1053	0.1230
38	38	39	0.0304	0.0355
39	39	40	0.0018	0.0021

40	40	41	0.7283	0.8509
41	41	42	0.3100	0.3623
42	42	43	0.0410	0.0478
43	43	44	0.0092	0.0116
44	44	45	0.1089	0.1373
45	45	46	0.0009	0.0012
46	4	47	0.0034	0.0084
47	47	48	0.0851	0.2083
48	48	49	0.2898	0.7091
49	49	50	0.0822	0.2011
50	8	51	0.0928	0.0473
51	51	52	0.3319	0.1114
52	9	53	0.1740	0.0886
56	53	54	0.2030	0.1034
53	54	55	0.2842	0.1447
54	55	56	0.2813	0.1433
55	56	57	1.5900	0.5337
56	57	58	0.7837	0.2630
57	58	59	0.3042	0.1006
58	59	60	0.3861	0.1172
59	60	61	0.5075	0.2585
60	61	62	0.0974	0.0496
61	62	63	0.1450	0.0738
62	63	64	0.7105	0.3619
63	64	65	1.0410	0.5302
64	11	66	0.2012	0.0611
65	66	67	0.0047	0.0014
67	12	68	0.7394	0.2444
68	68	69	0.0047	0.0016

Table B.2 Load Data of 69-node Radial Distribution Network

Node Number	PL(kW)	QL(kVAr)	Node Number	PL(kW)	QL(kVAr)
1	00.00	00.00	36	26.00	18.55
2	00.00	00.00	37	26.00	18.55
3	00.00	00.00	38	00.00	00.00
4	00.00	00.00	39	24.00	17.00
5	00.00	00.00	40	24.00	17.00
6	2.600	2.200	41	1.200	1.000
7	40.40	30.00	42	00.00	00.00
8	75.00	54.00	43	6.000	4.300
9	30.00	22.00	44	00.00	00.00
10	28.00	19.00	45	39.22	26.30
11	145.0	104.0	46	39.22	26.30
12	145.0	104.0	47	00.00	00.00
13	8.000	5.000	48	79.00	56.40
14	8.000	5.500	49	384.7	274.0
15	00.00	00.00	50	384.7	274.0
16	45.50	30.00	51	40.50	28.30
17	60.00	35.00	52	3.600	2.700
18	60.00	35.00	53	4.350	3.500
19	00.00	00.00	54	26.40	19.00
20	1.000	00.60	55	26.00	17.20
21	114.0	81.00	56	00.00	00.00
22	5.000	3.500	57	00.00	00.00
23	00.00	00.00	58	00.00	00.00
24	28.00	20.00	59	100.0	72.00
25	00.00	00.00	60	00.00	00.00
26	14.00	10.00	61	1244.0	888.0
27	14.00	10.00	62	32.00	23.00
28	26.00	18.60	63	00.00	00.00
29	26.00	18.60	64	227.0	162.0
30	00.00	00.00	65	59.00	42.00
31	00.00	00.00	66	18.00	13.00
32	00.00	00.00	67	18.00	13.00
33	14.00	10.00	68	28.00	20.00
34	19.50	14.00	69	28.00	20.00
35	6.000	4.000			

BASE kV = 12.66 and BASE MVA = 100