DISSERTATION

On

Bond Graph Modeling and Control of Segway Using Principle of Inverted Pendulum System

Submitted in partial fulfillment of the requirement for the award of degree

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IN

CAD/CAM Engineering

Submitted by

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DECLARATION

*I hereby declare that work done in the thesis report entitled, "**Bond Graph Modeling and Control of Segway using Principle of Inverted Pendulum System**" submitted towards partial fulfillment of requirement for award of **Master of Engineering degree** in **CAD/CAM Engineering** in **Mechanical Engineering Department** of **Thapar University, Patiala**, is an authentic record of work carried out by me under the supervision and guidance of **Dr. Tarun Kumar Bera**, **Associate Professor and Dr. Ashish Singla**, **Assistant Professor** of **Mechanical Engineering Department, Thapar University, Patiala**.*

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This is to certify that above declaration made by the student concerned is correct to the best of my knowledge and belief.

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ABSTRACT

In the area of dynamics and control theory, inverted pendulum system is an interesting and classical control problem which is used as a standard for testing various new control strategies and algorithms. Inverted pendulum system is an excellent experimental platform to carry out research in the area of control theory and robotics. It is an under-actuated, nonlinear and unstable system. The control designing for such system is very difficult task. A personal transporter vehicle, called segway (Two wheeled inverted pendulum system) is also based on the stabilization principle of inverted pendulum system. The concept of segway is similar to the problem of balancing a broomstick on the end of the index finger. This thesis work presents the bond graph modeling and control of segway using the principle of inverted pendulum system. The work describes the bond graph model of inverted pendulum system with a nonlinear heuristic controller used as a swing up control and the PI controller as a stabilization control. The experimental validation of bond graph model of inverted pendulum system is given by comparing the bond graph simulation results with the experimental results. The bond graph model of segway is developed and control is developed using a PI controller. The bond graph simulation results for the forward and backward motion of segway are presented as pitch angle, speed of the vehicle with time. For the turning of the vehicle, Ackermann steering mechanism is adopted. The results of turning motion of the segway are also presented as yaw angle response of main vehicle body with time.

Keywords: Inverted pendulum, segway, heuristic controller, PI controller, bond graph.

LIST OFABBREVIATIONS

ABS	Antilock braking system
C	Compliance element
DOF	Degree of freedom
GY	Gyrator element
GLIP	Googoltech linear inverted pendulum
I	Inertial element
PI	Proportional integral
R	Resistive element
Se	Source of effort
Sf	Source of flow
SIMO	Single input multiple output
TF	Transformer element
TWIP	Two wheeled inverted pendulum

NOMENCLATURE

a	Distance of the axle form the vehicle cg
Cg	Center of gravity
D	Friction coefficient
F	Force
g	Acceleration due to gravity
G_i	Integral gain
G_p	Proportional gain
h	Handle of the Segway
H	Height of the vehicle cg from the suspension reference point
I	Moment of inertia
J	Polar moment of inertia
K	Stiffness
L	Length
m	Mass of the pendulum
M	Mass of the vehicle body
r_w	Radius of wheel
R	Damping
S_{time}	Time of steering
S_{return}	Steer return timing
V	Velocity
Vtv	Velocity to voltage conversion gain
x, y, z	Displacements in the three directions
$\dot{x}, \dot{y}, \dot{z}$	Velocities in three directions
θ_{st}	Steering input
θ_{max}	Maximum angle rotation
θ , $\dot{\theta}$	Angular displacement, angular velocity
τ	Torque
μ_m	Motor torque constant

SUBSCRIPTS

bx, by, bz	x, y , z direction of vehicle body
B	Vehicle body
L	Left wheel
Md	Motor damping
P	Pendulum
R	Right wheel
S	Suspension
Seg	Segway
wx, wy, wz	x, y, z directions of wheel
W	Wheel
x, y, z	Direction of structure

LIST OF FIGURES

LIST OF TABLES

TABLE OF CONTENT

Chapter 1 Introduction

Modeling and simulation plays an integral role in the area of engineering design process. The system with accurate mathematical description provides the easy way to study the system accurately and quickly. Modeling and simulation of a physical system helps in understanding the dynamical behavior of the system. Modeling and simulation helps in getting the data and information about any system that how it will behave without testing it actually in real life. Simulation of various systems before their actual production is a very important part of the design process. Modeling and simulation helps in reducing the cost, increasing the quality and safety of the system. Today many complex systems are controlled by powerful algorithms which continuously monitor the system surroundings and simulate the system on the go. This simulation allows the algorithms to predict and hence, control the system in a better manner with more accuracy. Most of the control algorithms have mathematical equations which can only be solved numerically and not analytically.

1.1 Background and Motivation

Under-actuated systems are those mechanical systems which cannot be commanded to follow the arbitrary path in the configuration space. This is due to the reason that such types of systems have less number of actuated forces than its degree of freedom. The pendulum whose center of mass lies above its pivot point is known as inverted pendulum. It is an under-actuated, nonlinear and unstable system. In case of inverted pendulum system, there is one input force to move the cart horizontally back and forth and the system has two degree of freedoms. One is the linear motion of the cart along horizontal and other one is the angular movement of pendulum about its pivot point. Since we have control only on the position of the cart along the track but we are not having any direct control for the angular motion of the pendulum.

Inverted pendulum system is an excellent experimental platform to carry out research in the area of control theory and robotics to promote the development of new control algorithm. Such controls and algorithms can be applied into different fields like artificial intelligence, robot control technology, underwater robotics, missiles interception control, rocket launching, aerospace engineering, earthquake resistant building design and various industrial applications.

The main control objective in case of inverted pendulum is the swinging the pendulum up from downward equilibrium position to unstable inverted position and make it stabilize at the unstable equilibrium position. This can be obtained either by giving torque to the pivot point or moving the pivot point forward and backward in the horizontal direction as a part of feedback system. These control strategies used for the cart pendulum is same to the problem of balancing a broomstick on the end of index finger. The most relevant example of inverted pendulum is a human being because a human with upright body have to make constant adjustments to maintain the balance while walking or running. Suppose if a person is in standing position and then he suddenly lean into forward direction. The person will be out of balance at that stage and he will probably fall on his face. At that time, his brain knows that he is going out of balance and then brain commands a signal to step forward and stop the falling. If the person continues to lean forward then he will always put one step forward instead of falling. The same concept of cart pendulum system has been applied to segway (Two wheeled Inverted pendulum system) as shown in the Fig. 1.1.

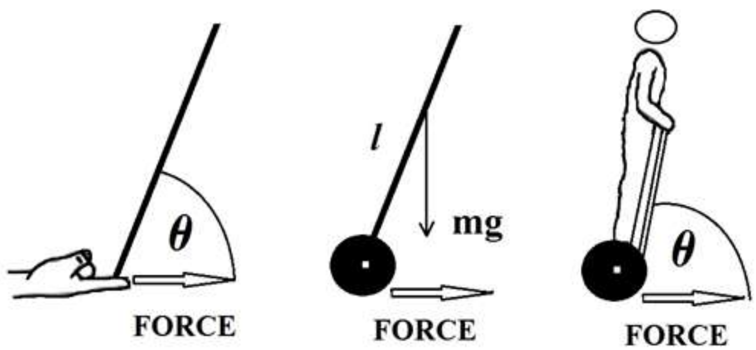


Fig. 1.1 Concept of balancing a broomstick on index finger applied to segway

1.2 Bond Graph Modeling

Bond-graph model is an efficient tool for modeling engineering systems. The technique is just like the block diagram technique and signal flow graph technique .but the arcs in case of the bond graph model denote the bi-directional energy exchange. The bond graph model is composed of single port, double port as well as multi port elements. Bond graph modeling technique is a graphical representation of the dynamical behavior of any system. This technique was developed by Prof. H.M. Paynter at MIT in 1959. The basic idea of bond graph modeling is conservation of energy for developing a generalized technique that is capable of representing systems from different domains. The physical system can be of any domains (mechanical, electrical, thermodynamic, hydraulic and chemical). Bond graph model can be generated by using seven types of elements. These elements are I, C, R, SE, SF, TF and GY. These elements are interacted to the system basically represented by the two junctions, 1-junction and 0-junction.1-junction represents the effort sum junction and 0-junction represents the flow sum junction. Any physical system can be represented in bond graph by these elements and junctions using some algorithm. The elements of the bond graph behave differently with the different systems. The power that is flowing in a system is represented by product of two components i.e. one is the effort and other is flow. Each bond symbolizes the flow of power form one element to another element of the system either in the form of a flow or an effort associated with it. The parameters representations of various types of system's flow variable and the effort variable are listed in Table 1.1.

Table 1.1 Flow and effort variables in different energy domains

Systems	**Effort (e)**	**Flow (f)**
Mechanical	Torque (N-m^2)	Angular velocity (rad/s)
	Force (N)	Velocity (m/s)
Electrical	Voltage (V)	Current (A)
Thermal	Temperature (K)	Entropy change rate (W/K)
	Pressure (N/m^2))	Volume change rate (m^3/s)
Hydraulic	Pressure (N/m^2))	Volume flow rate (m^3)/s)
Chemical	Chemical potential (Nm/mol)	Mass flow rate (mol/s)
	Entropy (Q/K)	Mass flow rate (kg/s)
Magnetic	Magnetic-motive force (em)	Magnetic flux rate (Wb/s)

1.2.1 Parts of bond graph model

The basic elements that are used to represent a physical system using bond graph are I, C, R, Se, Sf, TF and GY. Bond in the bond graph is defined as a line connecting all the elements. The power flow is represented by a bond in the bond graph which may be a flow information or effort information. Bond graph model can be classified as follows

- Single port passive elements (inertial element (I), compliance element (C) and resistive element (R)).

- Single port active elements or source elements (source of effort (Se) and source of flow (Sf))
- Multi-port conversion elements (gyrator (GY) and transformer (TF)).
- Junction related elements (effort sum junction (1) and flow sum junction (0)).

1.2.2 Single port elements

The single port elements are those which are connected to the system only on one point. These elements are of two types, active and passive elements. The elements which are the power source for the system are known as active elements. The elements which interact with the power within the system are known as passive elements. Single port elements are defined as follows

- Inertial element: It is represented by the character 'I' in the bond graph. It is an energy storing element. It gives the relationship between flow and effort in such a manner that the integration of effort value gives the flow. The inertial element graphically represented as shown in the Fig. 1.2.

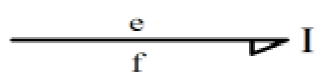


Fig. 1.2 Bond graph representation of inertial element

$$F = m\ddot{x} = \frac{df}{dt} \times m \quad (1.1)$$

$$f = \frac{1}{m}\int_{\infty}^{t} e dt \quad (1.2)$$

$$= k + \frac{1}{m}\int_{\infty}^{t} \mathrm{edt} \quad (1.3)$$

From Eq. (1.1–1.3), it is observed that the integral causality is observed if the output from the element is flow and the input to the element is effort.

- Compliance element: It is represented by a character 'C' in the bond graph. It is also used as an energy storing element. The integral causality for this element is only preferable. The graphical representation of compliant element as shown in Fig. 1.3.

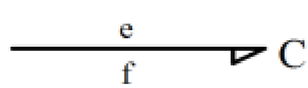


Fig. 1.3 Bond Graph representation of compliance element

From Eq. (1.4) and (1.5), it is observed that the integral causality is observed if the output from the element is effort and the input to the element is flow.

- Resistive element: It is represented by ‘R’ character in the bond graph. Unlike I and C elements, R-element is an energy dissipater. The graphical representation of resistive element as shown in Fig. 1.4.

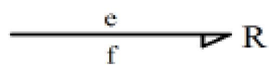


Fig. 1.4 Bond Graph representation of resistive element

- Source of effort: Source of effort is represented by ‘Se’ in the bond graph. It gives power to the system and in the form of external effort acting on the system. The example of source of effort in mechanical system is force. The graphical representation is shown in Fig. 1.5.

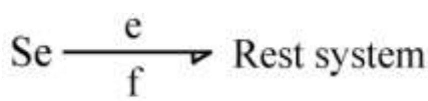


Fig. 1.5 Representation of source of effort in bond graph

- Source of flow: It is represented by ‘Sf’. It gives power to the system and in the form of external flow to system. The example of source of effort in mechanical system is force. The graphical representation is shown in Fig. 1.6.

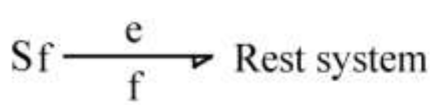


Fig. 1.6 Bond Graph representation of source of flow element

1.2.3 Basic 2-port elements

The elements which are connected to two points on the system are 2-port elements. These are converters basically which redistribute amount of flow and effort, between the two points that they are connected to. The basic 2-port elements are ‘transformer and the gyrator’. The symbolic representation of transformer is ‘TF’ and gyrator is ‘GY’.

- Transformer: Transformer does not create nor destroys energy. They redistribute the amount of flow and effort information. The transformer element magnifies the flow from input side to provide us with the flow on the output side of the element.

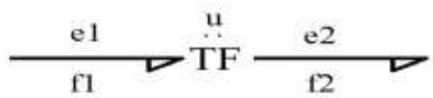


Fig. 1.7 Bond Graph representation of transformer element

$$f_2 = u.\ f_1 \tag{1.7}$$

$$e_1.\ f_1 = e_2.\ f_2 \tag{1.8}$$

$$\therefore e_2 = \frac{1}{u} e_1 \tag{1.9}$$

From Eq. (1.7–1.9), it is observed that as power in always conserved, there is also a relation between the input and output effort. As the transformer converts flow to flow and effort to effort the causal direction is not changed when the power flows through a transformer.

- Gyrator: Gyrator does not create nor destroys energy. It redistributes the amount of flow and effort information. The gyrator element magnifies the flow from input side to provide us with the effort on the output side of the element.

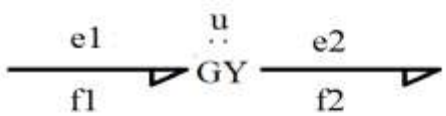


Fig. 1.8 Bond Graph representation of gyrator element

$$e_2 = u.\ f_1 \tag{1.10}$$

$$e_1.\ f_1 = e_2.\ f_2 \tag{1.11}$$

$$e_1 = u.\ f_2 \tag{1.12}$$

From Eq. (1.10–1.12), it is observed that as power in always conserved, so that the output flow and input effort are connected to each other.. As the gyrator converts effort to flow and vice versa. The causal direction is changed when the power flows through a gyrator.

***1**.2.4 Junction elements*

The junctions are of two types, 0-junction and 1-junction. The junctions represent points on the system where other elements can connected. Power is distributed among the connected elements but the nature of distribution depends on the type of junction.

- 1-Junction: 1 junction represents equality of flow. Suppose power comes to a 1- junction through a bond 'a' and leaves the junction through 'b' and 'c'.

$$p_a = p_b + p_c \tag{1.13}$$

$$e_a f_a = e_b f_b + e_c f_c \tag{1.14}$$

As flows are equal

$$e_a = e_b + e_c \tag{1.15}$$

From Eq. (1.13–1.15), it is observed that for power to be conserved, 1-junction is also an effort sum junction.

- 0-Junction: 0-junction represents equality of effort. Suppose power comes to a 0- junction through a bond 'a' and leaves the junction through 'b' and 'c'.

$$p_a = p_b + p_c \tag{1.16}$$

$$e_a f_a = e_b f_b + e_c f_c \tag{1.17}$$

As efforts are equal

$$f_a = f_b + f_c \tag{1.18}$$

From Eq. (1.16–1.18), it is observed that for power to be conserved, 0-junction is also a flow sum junction.

1.3 Inverted Pendulum System

An inverted pendulum is a type of pendulum whose center of mass lies above its pivot point. The problem of inverted pendulum is the combined research area of control theory, robotics and computer control. The inverted pendulum system is high order, unstable, highly coupled, multi variable system that is why it is categorized as a nonlinear control system. It has wide applications into different fields like artificial intelligence, robot control technology, underwater robotics, missiles interception control, rocket launching, aerospace engineering, earthquake resistant building design and various industrial applications. In this type of system, there are two types of equilibrium conditions. One is stable equilibrium condition where the pendulum is hanging in vertically downward direction and other one is the unstable equilibrium condition which is the vertically upward position of the pendulum. The main control objective is to swing the pendulum up from downward equilibrium position to unstable upward equilibrium position and make it stabilize at this unstable equilibrium position.

1.4 Segway

Segway is a two wheeled inverted pendulum system. This device is a self balancing human transporter and looks like a high technological scooter. It works on the principle of lean steer mechanism. The configuration of Segway consists of two wheels and an Inverted pendulum as a handle attached to the vehicle body. To move forward or backward the rider just has to lean the pendulum forward or backward. The segway consists of a control system, series of sensors and a motor system.

Fig. 1.9 Two wheeled inverted pendulum system, segway [1]

1.4 Contribution of the Thesis

The contributions of the thesis are as follows

- Bond graph model of cart inverted pendulum system.

- Control of cart inverted pendulum system with heuristic controller as a swing up control and PI controller as stabilization control.
- Validation of bond graph simulations results with experimental results of inverted pendulum system.
- Bond graph model of segway (two wheeled inverted pendulum system) with PI controller.
- Bond graph simulation results for forward and backward motion of Segway as pitch, speed response are presented.

1.5 Organization of the Thesis

This thesis is organized in five chapters, which are introduced as follows:

Chapter 1 introduces to the basics of bond graph modeling technique and its various elements. The basic introduction about inverted pendulum system and concept of segway are also discussed. The chapter concludes with the discussion on contribution and organization of the thesis.

Chapter 2 deals with the summary of the literature survey performed for the completion of the thesis. The literature review was done in three areas i.e., (i) modeling and control of cart pendulum system, (ii) modeling and control of segway and (iii) bond graph technique and applications. The literature gap and objectives of the present work are also discussed.

Chapter 3 deals with the inverted pendulum system's bond graph modeling and control. Firstly the brief introduction of experimental setup and experimental results of inverted pendulum system are discussed. Then the bond graph Modeling and control of inverted pendulum system is discussed and then validations of simulation results through bond graph with the experimental results are presented.

Chapter 4 deals with the bond graph modeling and control of segway. The bond graph simulation results for the forward and backward motion of segway as pitch, speed response are presented.

Chapter 5 concludes the thesis and suggests scope for future research.

The literature review is done for having the complete knowledge of the area of thesis work. The literature review is done in three areas i.e., (i) modeling and control of the inverted pendulum system, (ii) modeling and control techniques of the segway (two wheeled inverted pendulum system) and (iii) bond graph modeling technique and its applications. The literature gap and objective of the thesis is also discussed in this chapter.

2.1 Literature Survey

2.1.1 Literature survey on inverted pendulum system modeling and control

The inverted pendulum is the most important mechanical systems studied in control theory which consists of a cart on which a pendulum rod is mounted. The balancing of an inverted pendulum on the inverted position by the movement of a cart along the horizontal direction is a classical control problem [2]. Inverted pendulum system is an ideal experimental setup for research purpose and for teaching and testing various control theories and control algorithms and apply them to high technology areas such as robotics, aeronautics. Thus, the knowledge of inverted pendulum control has also been widely used in the study of stabilization of space vehicle as a test apparatus [3].

The inverted pendulum is an under actuated mechanical system having two degrees of freedom. One is the linear movement of the cart along the horizontal track and other one is the angular movement of the pendulum about the pivot point. There is only one actuation force, a control input voltage from the DC motor to drive the cart along the horizontal track. The main characteristic of the system is that the system is an open loop system, highly unstable system as we have to stabilize the angular position of the pendulum to the unstable equilibrium position (inverted position). The dynamics of the cart pendulum contains non-linear terminology so that it is categorized as a highly non linear system. The mathematical model of cart pole system is developed using Newtonian or Euler-Lagrange equations [2–4].

The different types of inverted pendulum are defined as such: a single linear inverted pendulum, a parallel double inverted pendulum, serial double inverted pendulum and so on. An inverted pendulum system consists of a pendulum rod which is freely pivoted and connected to a cart which is actuated with the help of motor. When the cart is stationary means there is no actuation force acting on the cart, the pendulum will be in the downward position which is the stable equilibrium position for the pendulum. If the disturbance is applied to the pivot point or to the cart, the pendulum is displaced from its stable equilibrium position [5].

The downward equilibrium position is known as pendant position. The control objective in case of cart and pendulum system is to take the pendulum from the pendant position to the inverted position by means of some controller and then stabilize the pendulum at the unstable inverted position. The control design for the Inverted pendulum system is a very challenging task. This can be achieved by moving the cart backward and forward along the horizontal track with in finite track length. As the pendulum is reached to the upright position then the cart is also required to bring to the reference home position [6].

The stabilization control and the swing-up control of a cart pole system is a common and basic experimentation in most control laboratories all over the world .The main objective here is to take the pendulum to the unstable equilibrium position that is the upright position and maintains the pendulum over there despite of the small disturbances. And the cart is the only mean which can provide the actuation to the pendulum. To drive the pendulum from downward position to the inverted position, necessary energy is required to be injected to the system. The appropriate amount of energy must be injected to the system so that the sum of the potential energy and the rotational kinetic energy of the pendulum become equals to the potential energy at the upright or inverted position [7].

Various control algorithms have been proposed. Many researchers have developed various control methods and algorithms for the inverted pendulum control. The various controls are: sliding- mode control, feedback control, feed-forward control, robust control, genetic algorithm, hybrid control and so on. The effectiveness of a swing up and stabilization control depends upon the feedback linearization and the consideration of total energy shaping. The different types of controllers are used as a stabilization control for the inverted pendulum system. Such controllers are Proportional, Integral and Derivative (PID) [7], State Feedback Controllers [2–4, 9], LQR controller [8].

The energy based controller is used for swinging the pendulum up from the downward position to the inverted unstable position and after that the control is switched from swing up to the stabilization control after the pendulum has reached near to the upright position. The stabilization control is a basically type of linear feedback control mechanism. The control based on the total energy shaping is very efficient and effective way for swinging up the pendulum. The of cart pole system's behavior is totally dependent on the on parameter which is the maximum pivot point's acceleration of the pendulum rod [9].

The dynamics of any system can be studied by modeling the behavior of the system. The model of the complete system is simplified and decomposed into smaller module to predict its behaviour. The bond graph modeling is done for the Quanser Rotary Inverted Pendulum and simulation is done via 20sim modeling and simulation. The bond graph modeling for the Inverted pendulum system can be obtained by the use of feedback linearizing technique which is a non linear control methodology and provides an alternative solution to the existing classical linear control methodology. The non linear control law is implemented if all the states of the systems can be measured [10].

The logic based non linear heuristic controller design that determines the direction and time moment at which the cart should move in the forward or backward direction along the track so that there will be increase in pendulum's swing angle. The other one is the energy controller design which adds the required amount of energy into the cart pendulum system in order to achieve the optimal energy state corresponds to the inverted position. Both the controllers are capable of swinging the pendulum up but energy controller is more reliable and robust. The linear quadratic regulator controller is used for the stabilization control [11].

The pendulum can be swung up to the unstable equilibrium position with the help of energy control law**.** An "energy well" is defined within the track of the cart so that the cart is prevented to go outside the track. When the energy of the pendulum becomes equal the optimal energy corresponds to the inverted position then it is said to be in "cruise" mode. After this mode stabilization controller is activated. The control method is developed using the Lyapunov functions approach. The control for the swinging of the Inverted pendulum system is also developed using energy control method based on the Lyapunov function. It is one of the effective control method used for the control of the under actuated system. The energy control method is an easy way to control as compared to other control strategies. The principle used in case of energy control is that the controlled input from the controller is provided to such direction so that the pendulum's mechanical energy increases. Lyapunov function is designed on the basis of mechanical energy [12–14].

2.1.2 Literature survey on modeling and control of segway

Inverted pendulum systems have been used as a subject of number of studies in automatic control. The configuration which contains inverted pendulum is mounted on the two wheels is known as segway or high technology personal transporter robot. The designing of such robots is a challenging task. It includes selection of various drive mechanisms, sensors and microprocessors for signal processing and control. This type of robot is equipped with two actuation drive, a gyroscopic sensor and an accelerometer for measuring pitch angle and pitch angular velocity. Control algorithms and signal processing are distributed among the three microprocessors, one for each drive and one is responsible for the stabilizing control [15].

The mechanical model of a monocycle is designed by attaching an inverted pendulum by a hinge to a cylinder. The forward motion, rolling without slipping on a horizontal surface of the model is studied. With the help of electric drive, pendulum is rotated around the cylinder and the cylinder's rolling motion is also driven through the electric drive. The control input is the voltage. The pendulum is stabilized at the unstable equilibrium position. The mechanical system studied has two degrees of freedom and one control input. The system is stabilized with the help of feedback gain and delay [16].

MBS dynamics modeling software tool Matlab SimMechanics is used for the modeling and control of wheeled robot. The system is controlled by feedback linearization technique using LQR Controller. The dual accelerometer is used as pitch sensor. The influence of noise sensors and delays also discussed [17]. For safety and reliability purpose, antilock braking system, traction controls are used in the modern autonomous vehicles. The bond graph modeling is done for the four wheel vehicle which consists of several energy domains. The model is simulated to check the performance of ABS system under different operating conditions [18].

The most challenging problem in balancing a two-wheeled robotic machine arises when the load carried by the machine changes its position along the body of vehicle. The problems also arise due to the size of the load. Two types of control techniques used to control the system. One is the proportional derivative control and other one is the fuzzy logic control to balance the vehicle. Simulations are presented on the effect of changing the level and time duration of the disturbances and changing the speed during the balancing of vehicle [19].

The stabilization based on energy controller for a wheeled Inverted pendulum is developed with the help of Lagrangian mechanics. The actuation of the two wheels in the same directions generates the forward and backward motion. Opposite wheel velocities can cause the turning of the vehicle around the vertical axis. By using the linearized models, various control laws have been applied to the wheeled inverted pendulum [20, 21].

The two wheeled inverted pendulum system is implemented and designed with the compensation of the friction. Friction is one of the critical factors in case of self balancing vehicle that affects its performance. The friction parameters are identified via system's dynamics. Sliding mode control is designing is done independently for yaw and pitch angle response. Gyroscopic sensor with Kalman filter is used to measure the pitch angle of the vehicle. The dynamics of the model is derived with the help of Lagrangian mechanics [22].

2.1.3 Literature survey on bond graph modeling technique

Bond graph modeling is a technique that is used to model any physical system. This modeling technique represents the dynamics of a system. With the help of bond graph modeling technique the system from different types of energy domains like electrical, thermal, mechanical etc. can be modeled. Bond graph technique is especially useful if the system consists of sub–systems from different energy domains. Any system can be represented by predefined elements which define the behavior of different components present the system [23, 24].

The mathematical modeling and simulation of a system by using these mathematical equations under different conditions is an important part of the design process of any engineering system. If the system seems to be very complicated and complex then it is very difficult to derive the mathematical equations by using the traditional methods. These kinds of systems can be easily handled by using the bond graph modeling technique. In this technique, bonds are to be drawn connecting different elements in the system and differential equations describing the system can be found by analyzing one component at a time. The process can be automated and can be programmed very easily [25].

The systems which generally consist of mechanical systems and an electronic controller are known as mechatronics system. Bond graph technique can be used for designing, fault estimation, identification and diagnosis of such systems. The design process of the mechatronic system consists of utilizing various tools for controller design such as drawing the signal flow graph. The bond graph technique the whole system is analyzed systematically; the signal flow graph can be drawn directly and easily from the bond graph. Hence, bond graph is a powerful tool for analyzing mechatronic systems [23, 26–29].

To model a complex thermodynamic system, using the conventional bond graph technique is very difficult task. For such systems, convection bond graphs are drawn to represent the system. In a convection bond graph, two independent effort variables were used for simulating the system, instead of one effort variable in a conventional bond graph. The two efforts are represented by drawing a dashed line parallel to the solid line in the bond graph [30].

Only some specific essential information of a system is required to solve the system using bond graph technique. This information must be conveyed in such a manner, so that it can follow some algorithm to derive the differential equations describing the system. So, the main idea of the bond graph technique is to formulate some kind of language, which can convey the essential characteristics of the system, to the computer, for pre-defined algorithms to be applied on it [31].

2.2 Literature Gap

After extensive literature survey relating to inverted pendulum system, segway and bond graph modeling, it is observed that many different types of mechatronics and mechanical systems have been successfully simulated using the bond graph approach and different types of swing up and stabilization controls have been used for the control of inverted pendulum system. But there is very less work done in the area of inverted pendulum's modeling and control in bond graph. Bond graph modeling technique has also not applied to the modeling and simulations of two wheeled inverted pendulum robot or segway.

2.3 Objective of the Current Work

It is observed from the literature survey that the bond graph technique is very useful in modeling various multi-body systems and it is very useful in simulating various conditions for various control algorithms for mechanical and mechatronic systems. The objectives of the thesis are as follows:

- To develop bond graph model of linear inverted pendulum system
- To design swing up control of inverted pendulum using heuristic controller
- To design stabilization control of inverted pendulum using PI controller
- To validate inverted pendulum's bond graph simulation results with experimental results
- To develop bond graph model and control of segway

An inverted pendulum is a type of pendulum whose centre of mass lies above its pivot point. The problem of inverted pendulum is the combined research area of control theory, robotics and computer control.

3.1 Mathematical Model of GLIP Setup

The mathematical model of inverted pendulum system (Fig. 3.1) can be obtained using Euler-Lagrangian approach. The linear inverted pendulum system is simplified as a system consists of a cart and a pendulum rod, mounted on the cart. The air resistance and other friction factors are not taken into consideration while deriving the equations of motions. The system's parameters are defined in Table 3.1.

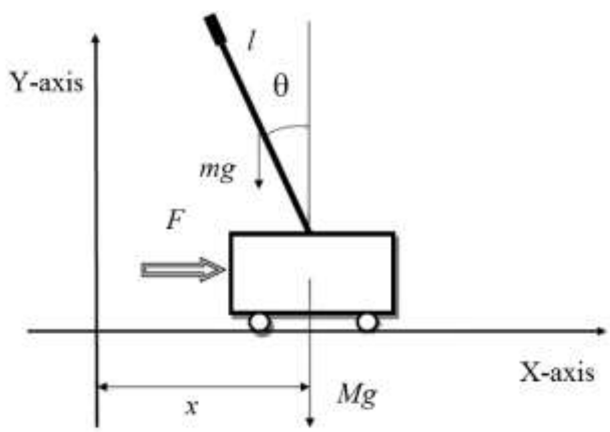


Fig 3.1 Schematic diagram of a cart with an inverted pendulum

Table 3.1 Definition of system parameters

M	cart mass
m	pendulum rod mass
b	friction coefficient of cart
I	rod inertia
x	cart position
F	actuation force acting on the cart
θ	angle of the pendulum with vertically upward position
l	distance from rod mass centre to the rod axis rotation centre

Since this system having two degree of freedom, the two generalized coordinates are required to represent the position of the system in space at some particular instant. x, θ are the generalized coordinates. The kinematics of the cart pendulum system is given as follows

$$X = \begin{bmatrix} x_p \\ y_p \end{bmatrix} = \begin{bmatrix} x - l \sin \theta \\ l \cos \theta \end{bmatrix} \tag{3.1}$$

$$\dot{X} = \begin{bmatrix} \dot{x}_p \\ \dot{y}_p \end{bmatrix} = \begin{bmatrix} \dot{x} - l \sin \dot{\theta} \\ l \cos \dot{\theta} \end{bmatrix} \tag{3.2}$$

The kinetic energy of the complete system is given as the sum of kinetic energy of the rod of pendulum and cart's kinetic energy which is given by

$$KE_{total} = KE_{cart} + KE_{pendulum} \tag{3.3}$$

The cart's kinetic energy is defined as

$$KE_{cart} = \frac{1}{2} M \dot{x}^2 \tag{3.4}$$

The pendulum rod's kinetic energy is given as:

$KE_{pendulum}$= Pendulum rod's translational kinetic energy + Pendulum rod's rotational kinetic energy) (3.5)

Let's assume that x_p = x-axis coordinate of pendulum rod mass centre and y_p = y-axis coordinate of pendulum rod mass centre

$$x_p = x - l \sin \theta \tag{3.6}$$

$$y_p = l \cos \theta \tag{3.7}$$

The translational kinetic energy of the pendulum rod is $\frac{1}{2} m\left[\left(\frac{dx_p}{dt}\right)^2 + \left(\frac{dy_p}{dt}\right)^2\right]$ and rotational kinetic energy is $\frac{1}{2} I \dot{\phi}^2$. The total kinetic energy of the pendulum rod is given as

$$KE_{pendulum} = \frac{1}{2} m\left[\left(\frac{dx_p}{dt}\right)^2 + \left(\frac{dy_p}{dt}\right)^2\right] + \frac{1}{2} I \dot{\theta}^2 \tag{3.8}$$

So, the total kinetic energy of the system is given by

$$KE_{\text{total}} = KE_{\text{cart}} + KE_{\text{pendulum}} \tag{3.9}$$

$$KE_{\text{total}} = = \frac{1}{2}(M+m)\dot{x}_p{}^2 + \frac{1}{2}ml^2\dot{\theta}^2 - ml\, cos\dot{\theta}\, x_p \tag{3.10}$$

In this system, only the pendulum is heaving the potential energy. The potential energy of the complete system is given as

$$PE_{\text{total}} = KE_{\text{cart}} + KE_{\text{pendulum}} \tag{3.11}$$

$$= 0 + (-mgl\, cos\, \theta) \tag{3.12}$$

Now, the Lagrangian coefficient is given by

$$L = KE - PE \tag{3.13}$$

$$L = \frac{1}{2}(M+m)\dot{x}_p{}^2 + \frac{1}{2}ml^2\dot{\theta}^2 - ml\, cos\, \dot{\theta}\, x_p - (-mg\, l\, cos\, \theta) \tag{3.14}$$

To derive the damping force of the system, Rayleigh dissipation function is used which is given as

$$R = \frac{1}{2}b\dot{x}^2 \tag{3.15}$$

Due to the two generalized coordinates, there are two equations of motion of the system. Therefore Lagrangian equations for the system are given as

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} + \frac{\partial R}{\partial \dot{x}} = F \tag{3.16}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\, \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} + \frac{\partial R}{\partial\, \dot{\theta}} = 0 \tag{3.17}$$

These equations can be calculated by putting the value of Lagrangian and Rayleigh's dissipation function in the above equations. The system equations of motion are given as

$$(M+m)\, \ddot{x} - ml\, cos\theta\, \ddot{\theta} + ml\, sin\, \theta\dot{\theta}^2 + b\ddot{x} = F \tag{3.18}$$

$$(I+ml^2)\, \ddot{\theta} - mgl\, sin\, \theta - ml\, cos\, \theta\, \ddot{x} = 0 \tag{3.19}$$

These equations are non linear in nature. To linearize the system, take sin $\phi = \phi$ *and* $\cos\phi = 1$, Neglecting the square terms we get

$$(M+m)\ddot{x} - ml\ddot{\theta} + ml\theta\ddot{\theta} + b\ddot{x} = F \tag{3.20}$$

$$(I+ml^2)\ddot{\theta} - mgl\theta = ml\ddot{x} \tag{3.21}$$

3.2 Bond Graph Model

Inverted pendulum is an assembly of thin pendulum rod attached at bottom to the moving cart. The cart is having linear motion along the horizontal track with the help of control input voltage from the DC motor. The complete model of inverted pendulum system consists of three sub systems. The first component is the cart along with the pendulum. The second one is the DC motor that is used to control the wheels of the cart. The third part is the controller part for swinging up and stabilizing control. The diagram of cart pendulum is shown in the Fig. 3.2. There are two important locations that can be considered as a point of interest. One is the centre of the mass of pendulum rod and other one is the point where the pendulum rod is attached to the cart. The coordinates of these points are measured from the fixed reference.

The bond graph model is developed with the help of kinematic relationships at different points in this system. First of all, the cart's movement is considered in the horizontal direction with the help of DC motor. The cart has only one velocity component which is in the horizontal direction. The point at which the pendulum rod is connected to the cart will be of same horizontal velocity as that of the cart. But, the pendulum rod has angular motion about the point where it is pinned to the cart. Therefore, the centre of mass of the pendulum rod will have both horizontal as well as vertical velocity components. The horizontal component of the velocity is the sum of the horizontal velocity at the pivot point and the trigonometric function of the pendulum rod's angular displacement. The vertical component will only be the function of angular position. The relationships between the velocities are given as

$$\dot{x}_p = \dot{x} - l\sin\dot{\theta} \tag{3.22}$$

$$\dot{y}_p = l\cos\dot{\theta} \tag{3.23}$$

The bond graph model shown in Fig. 3.2 is developed according to the above equations of motions (3.22–3.23). The various kinds of velocity parameters associated with each bond are shown in the bond graph. The lines with the half arrow at their ends are the power bonds represents either effort or flow and the ends with the complete arrow represents the information bonds (sensor) either measuring rate of flow or effort. The 1 junction represents the effort sum junction and the 0 junction represents the flow sum junction. The causal sign (straight line at the end of arrow) denotes the direction of effort. The Bond graph model of inverted pendulum system in Fig. 3.2 consists of three subsystems, cart pendulum, motor and PI controller. The symbol $1_{\dot{x}}$ in the bond graph denoted the cart's linear velocity component in the horizontal direction. The cart's mass (I: *M*), friction coefficient of the cart (R : R_c) and a flow detector element ((Df : $\dot{x}$) which is used to measure the position of the cart by integrating the velocity component) are connected to the linear velocity component of cart. $1_{\dot{x}_p}$ and $1_{\dot{y}_p}$ denote the horizontal, vertical component of rod mass center, respectively. The pendulum mass (I: *m*) is connected to these both velocity components. The weight of the pendulum mass (Se: *-mg*) which is acting in the vertical downward direction and stiffness of the pendulum rod (C: K_r) is connected to the vertical velocity component of pendulum mass center. The symbol $1\dot{t}_h$ denotes the pendulum's angular velocity. The rotary inertia of rod (I: *J*), rotary stiffness (C: K_p), air resistance (R: R_p) and a flow detector ((Df: $\dot{\theta}$) which gives the angular position of the inverted pendulum by integrating the angular velocity component) are connected to the angular velocity component. The cart and pendulum are connected via the two transformer elements to form the equations of motion. The $u_1 = l \sin \theta$, $u_2 = l \cos \theta$ are modulus of the transformer. The motor is used to provide actuation force to the cart which is represented as an effort to the linear velocity component of the cart. The motor resistance (R: R_m), motor torque constant (m), rotor inertia of motor (I: J_m), gear ratio (g), motor damping (R:R_d) are the parameters of the motor represented in the bond graph model (Fig. 3.2). The flow detector element (Df: w_m) is used for the measurement of motor shaft's angular velocity. The motor shaft's angular velocity is converted into linear velocity with the help of the transformer element heaving modulus r which is the radius of the belt. The belt stiffness (C: K_S), belt resistance (R: R_s) are represented in the model. The model in bond graph for PI controller is discussed in the controller Section 3.3.

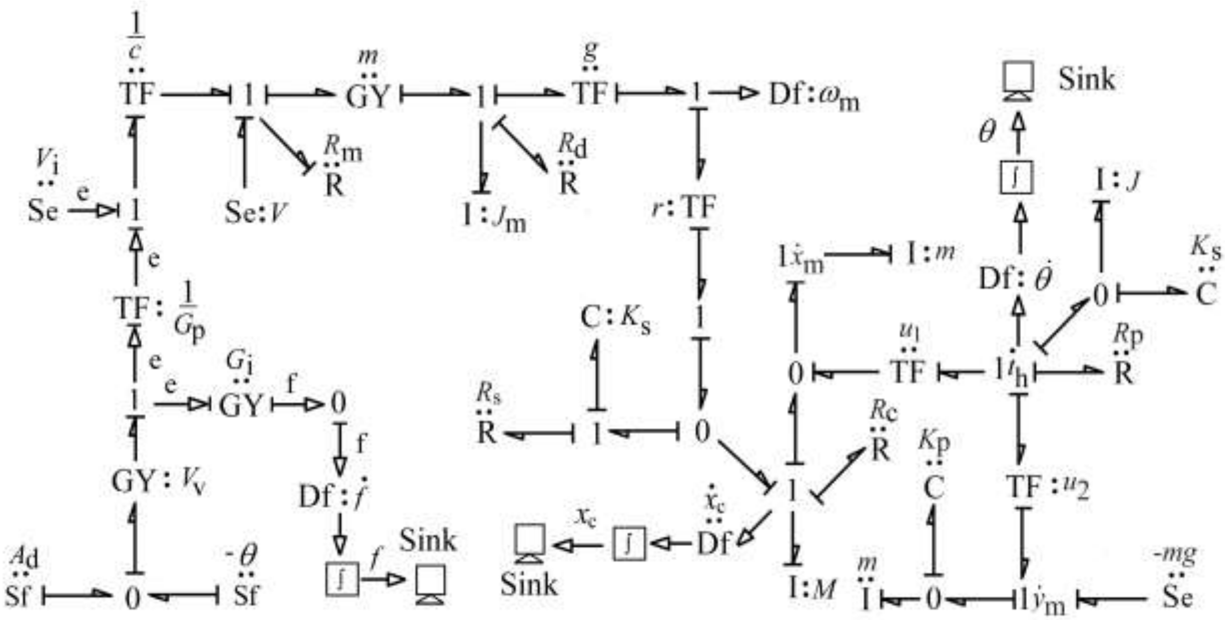


Fig. 3.2 Bond graph model of inverted pendulum system

The parameters required for bond graph simulation are given in Table 3.2.

Table 3.2 Parameter values

Symbol	**Parameters**	**Parameter values**
M	mass of the cart	1.096 kg
m	mass of pendulum	0.109 kg
J	rotary inertia of rod	0.0034 kg m^2
l	length of the rod of pendulum	0.25 m
R_c	Coefficient of friction of cart	0.1 Ns/m
R_p	air resistance	0 Ns/m
K_r	rod stiffness	10^6 N/m
K_p	rotational stiffness of rod	10^6 N/m
mo	motor constant	0.1 Nm/A
R_d	Damping	0.03 Ns/m
J_m	motor inertia	0.00014 kg m^2
R_S	resistance loss	0.1 Ns/m
K_S	belt stiffness	10^6 N/m
g	gear ratio	1
R_m	motor resistance	2.5 Ns/m
r	belt radius	0.0195 m
v	control voltage	7.5 V
g	acceleration due to gravity	9.8 m/ s^2
V_v	velocity to voltage convertor	33

G_p	proportional gain	0.121
G_i	integral gain	1.9
u_1	modulus of transformer	$l \sin \theta$
u_2	modulus of transformer	$l \cos \theta$

3.3 Controller Design

In case of cart pendulum system, two specific tasks are needed to be controlled. The first one is the swinging up of the pendulum rod to upright position and this is the unstable equilibrium position from the downward equilibrium position with the help of a specific controller. As the pendulum is reached near to the unstable equilibrium, the second task is the stabilization of the pendulum near to the inverted position with the help of state feedback controller. The control scheme (Fig. 3.3) used for swing up is heuristic controller and for stabilizing control the controller used is proportional-integral (PI) control.

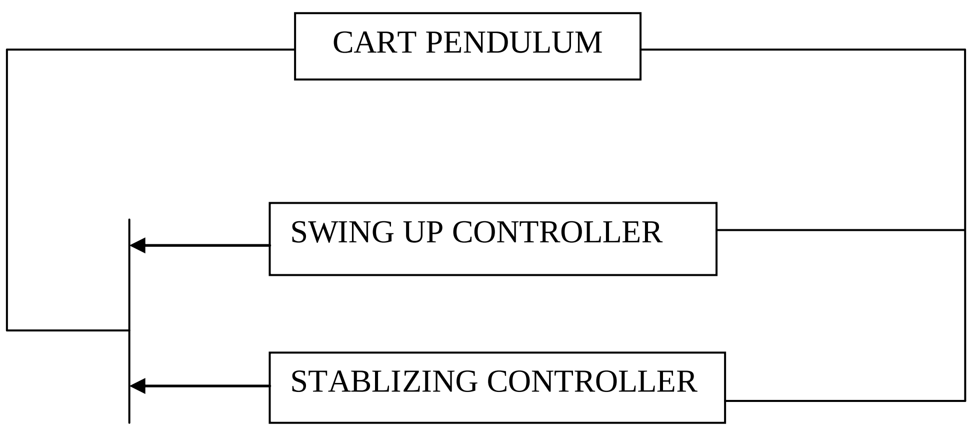


Fig. 3.3 Controller design for inverted pendulum

3.3.1 Heuristic controller

Heuristic controller is a type of logic based non-linear controller design that determines the direction and time moment at which the cart should move in the forward or backward direction along the track so that there will be increase in pendulum's swinging angle. The direction and time moment at which the cart should move depend upon the state feedback. The heuristic controller applies a constant voltage gain to move the cart in the forward as well as backward direction. This logic based controller design totally depends upon the angle of the pendulum which is one of the states that can be measured with the help of sensor or feedback control. This control method will change the cart's direction at the instant when the pendulum's angular position crosses the downward position during its swing. So, the downward position is the optimal time moment to pump energy to the inverted pendulum system by changing the direction of cart. As the pendulum rod crosses the downward equilibrium position, the cart must move in the opposite direction to the direction of pendulum swing at that instant. This control scheme is very much effective in swinging the pendulum up from downward equilibrium position to unstable upright position by moving the cart backward and forward repeatedly with the help of constant voltage gain.

Heuristic controller is basically a type of energy controller which pumps the appropriate amount of energy to the pendulum by giving the maximum acceleration to the cart in the horizontal direction so that the pendulum is able to swing up. The swinging of the rod of pendulum to the inverted position from the downward equilibrium position can be done by adding the amount of energy to the system. The desired value of energy can be achieved by using feedback control. By using the sign conditions of the acceleration of the cart, the energy of the pendulum can be controlled. This can be obtained by satisfying a specific mathematical condition i.e. the total mechanical energy of the inverted pendulum system.

Our main objective here is to take the pendulum to the upright position and maintain the pendulum over there despite of the small disturbances. The cart is the only mean which can provide the actuation to the pendulum. The appropriate amount of energy must be injected to the system so that the sum of the potential energy and the rotary kinetic energy of pendulum become equals to the potential energy of the pendulum at the upright position. This means that initially the rotational kinetic energy of the pendulum system must be increased whatever may be the value of the translational kinetic energy. There is no important role of the translational kinetic energy as the translation and rotation of the pendulum are decoupled. Let H denotes the sum of the potential energy and the rotational kinetic energy and it is given as

$$H = \frac{1}{2} I\dot{\theta}^2 + mgl \tag{3.24}$$

where I is the rotary inertia of the rod about the hinge and θ is the angular displacement of the rod from vertical axis. Now, taking the derivative of the H with respect to time we get

$$\frac{dH}{dt} = I\dot{\theta}\ddot{\theta} + mgl\,(-\sin\theta)\,\dot{\theta} \tag{3.25}$$

Since $$(I+ml^2)\ \ddot{\theta} - mgl \sin\theta - ml \cos\theta\ \ddot{x} = 0 \tag{3.26}$$

$$\ddot{\theta} = \frac{mgl \sin\theta - ml \cos\theta\ \ddot{x}}{(I+ml^2)} \tag{3.27}$$

Now, putting the value of $\ddot{\theta}$ from the Eq. (3.27) in Eq. (3.25) we get

$$\frac{dH}{dt} = -ml\ \ddot{x} \cos\theta\ \dot{\theta} \tag{3.28}$$

In order to increase energy, the value of the derivative must be greater than or equal to zero.

$$\frac{dH}{dt} \geq 0 \tag{3.29}$$

It implies that $$-\ ml\ \ddot{x}\ \cos\theta \dot{\theta}\ >= 0 \tag{3.30}$$

The above equation signifies that in order to increase the energy function, the acceleration of the cart, $\ddot{x}$ and the pendulum's angular velocity $\dot{\theta}$ must be of opposite sign. This logic based control design is totally dependent on the angular displacement of the pendulum and its angular velocity. There will be a time moment, when the pendulum will reach to a point where the cart's movement will not be able to add more energy to the pendulum system. This condition arises when the pendulum's angular position becomes horizontal. At that situation, the cart's movement will takes away the energy from the pendulum. But, still it has to build more energy to the pendulum. So, if such condition arises at that moment we will stop the cart's movement and allow the pendulum to simply return to the downward position. When the pendulum crosses the downward position once again, logic based controller will be again able to add more energy to the pendulum by the cart's movement until the pendulum approaches to the upright position. There is a direct relation between the magnitude of the voltage gain and the time taken by the pendulum to swing up. The reliability and the performance of this controller are completely dependent on the voltage gain.

3.3.2 PI controller

PI controller is used as a stabilizing control for the cart inverted pendulum system. The PI controller is used to catch and hold the inverted pendulum at the inverted position. PI controller stands for proportional integral control which is a control loop mechanism which depends upon the feedback. This controller continuously calculates the difference between the desired value and the feedback value and generates a control force to minimize this error difference. In case of the inverted pendulum system, the desired value is the difference between inverted angular position and feedback value obtained with the help of feedback sensor used for the measurement of angular displacement of the pendulum. The mathematical expression for PI controller is given as

$$u(t) = k_p e(t) + k_i \int_0^t e(t)\, dt \tag{3.31}$$

Where k_p and k_i are the proportional and integral gains, respectively. The bond graph model for PI controller is shown in Fig. 3.4. There are two sources of flow that are added to the flow sum junction. One flow is the desired value that is the inverted angular position and the other one is the negative value of feedback angular value measured by the sensor. The error between these flows is converted to the voltage with the help of gyrator. Now according to the voltage error signal, a control force is generated by the PI controller to minimize the voltage error signal. In this bond graph representation, G_p is the proportional gain and G_i is the integral gain.

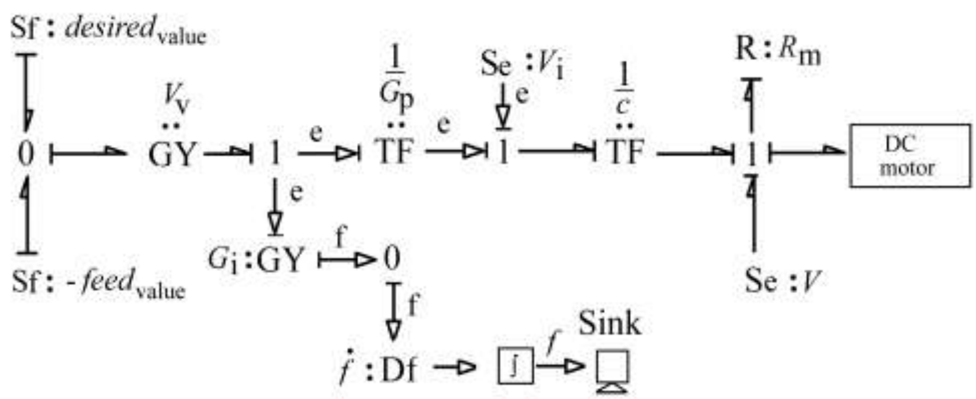
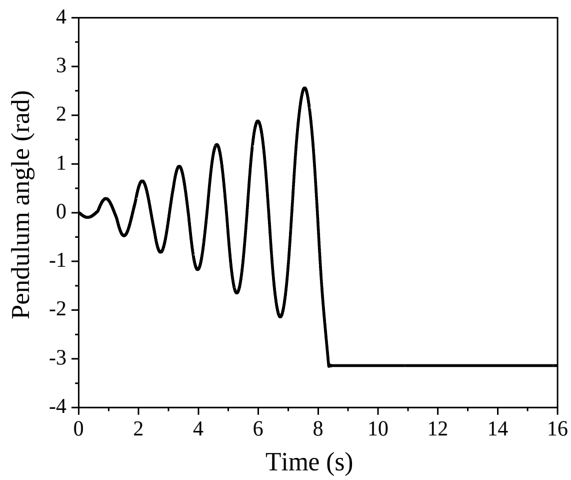


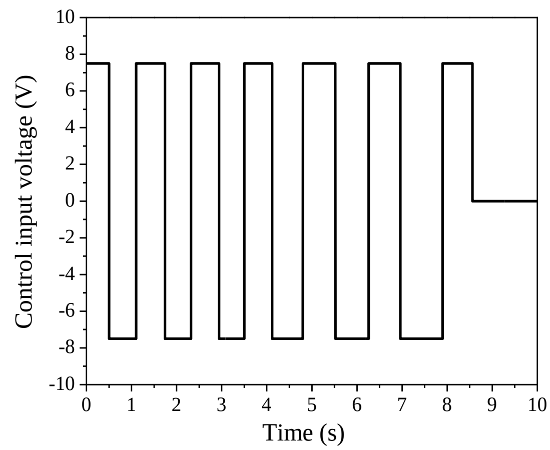
Fig 3.4 Bond graph model for PI controller

3.4 Simulation Results

The bond graph simulation results of the cart inverted pendulum system are presented in Fig. 3.5. The first plot is given for the pendulum angle with the time for the swinging up and the stabilizing control of cart pendulum system. Pendulum angle takes approximately about 8.5 seconds for stabilization with six increasing swings of the pendulum as shown in Fig. 3.5(a). The second plot is for the control input voltage applied for the actuation of the cart with time. The control input voltage alternates from -7.5V–7.5V during the swing up control as shown in Fig. 3.5(b). The third plot is for the cart’s position with time during swing up and the stabilization control as shown in Fig. 3.5(c).



(a)



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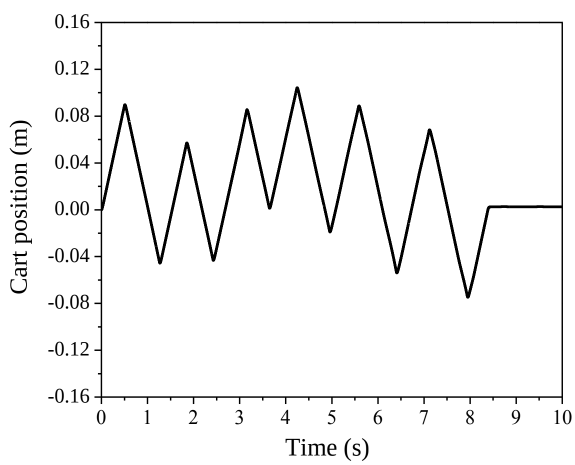


(c)

Fig. 3.5 Plot of (a) pendulum angle, (b) control input voltage and (c) cart position from simulation results

3.5 Experimental Setup Description

Inverted Pendulum system is an ideal experimental setup (Fig. 3.6) for research purpose and for teaching and testing various control theories and control algorithms and use them as an application to high technological areas (robotics, aerospace research). The linear inverted pendulum system developed by Googol tech, Hong Kong and it is known as Googoltech Linear Inverted Pendulum (GLIP). It is an open control and modular platform.

Fig. 3.6 Experimental setup of GLIP

The overall design is composed of three sub-systems, mechanical system, feedback system and controller interface. Mechanical system design basically consists of cart pendulum system, track and mechanism used to drive the cart. The drive mechanism is basically a motor with a toothed wheel mounted onto the shaft that is used to pull a chain to which the cart is connected. The motion of cart is limited to horizontal direction. The track is in the form of U channel. Feedback network consists of sensor and encoders to read the position of cart and pendulum.

3.5.1 Technical specifications

MATLAB Windows2000 with Simulink as a user interface to model, performing analysis and simulations to improve control system and to evaluate control performance directly. Sliding guide bars and the synchronization gear belt are made up of precision stainless steel, industrial grade material respectively. Limit switches, motor, anti-collision buffer device and unique structure design for safety and suitability. The hardware platform of the setup is based on DSP-based motion controller. Few specifications are given in Table 3.3.

Table 3.3 Specifications

Series	GLIP 2001
Valid travel distance	720 mm
Maximum motion velocity	6000 mm/s
Weight	15 kg
Volume	1000mm × 220mm × 150mm

3.5.2 System parameters

The system parameters obtained from manual are given in Table 3.4.

Table 3.4 System Parameters

Parameter	**Symbol**	**Value**
Mass of the cart	M	1.096 kg
Mass of the pendulum rod	m	0.109 kg
Coefficient of friction of the cart	b	0.1 Ns/m
Distance from the rod mass centre to rod rotation centre	l	0.25 m
Inertia of pendulum rod	I	0.0034 kg m^2

3.5.3 Block diagram model of experimental setup

The block diagram model of inverted pendulum system is shown in the Fig. 3.7. The system is a single input and multiple output (SIMO) system by its nature. The voltage act as a control input for the system and linear cart displacements and angular position of pendulum are the outputs from the system. Here, the control objective is the stabilization of the angle of the pendulum to the inverted unstable equilibrium position and it is a very challenging task to do. The main characteristic of the system is that the system is highly unstable system due to stabilization of the angular position of the pendulum to the inverted position. The dynamics of inverted pendulum contains some non-linear terms so that it is categorized as a non linear system. The inverted pendulum system is also an under actuated system due to only one actuation force (DC motor) for driving the cart in horizontal direction and having two DOF.

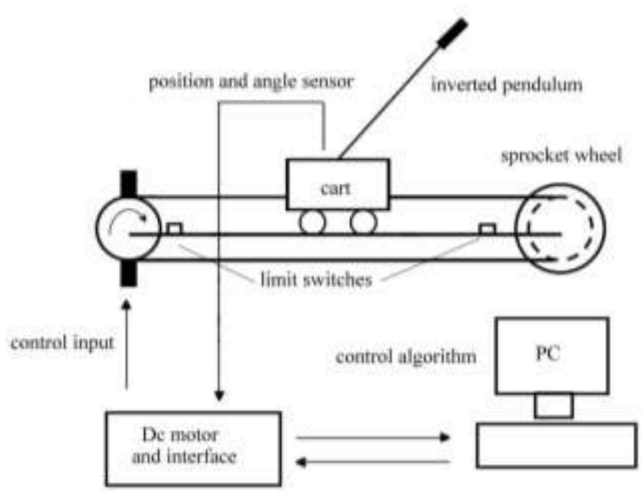
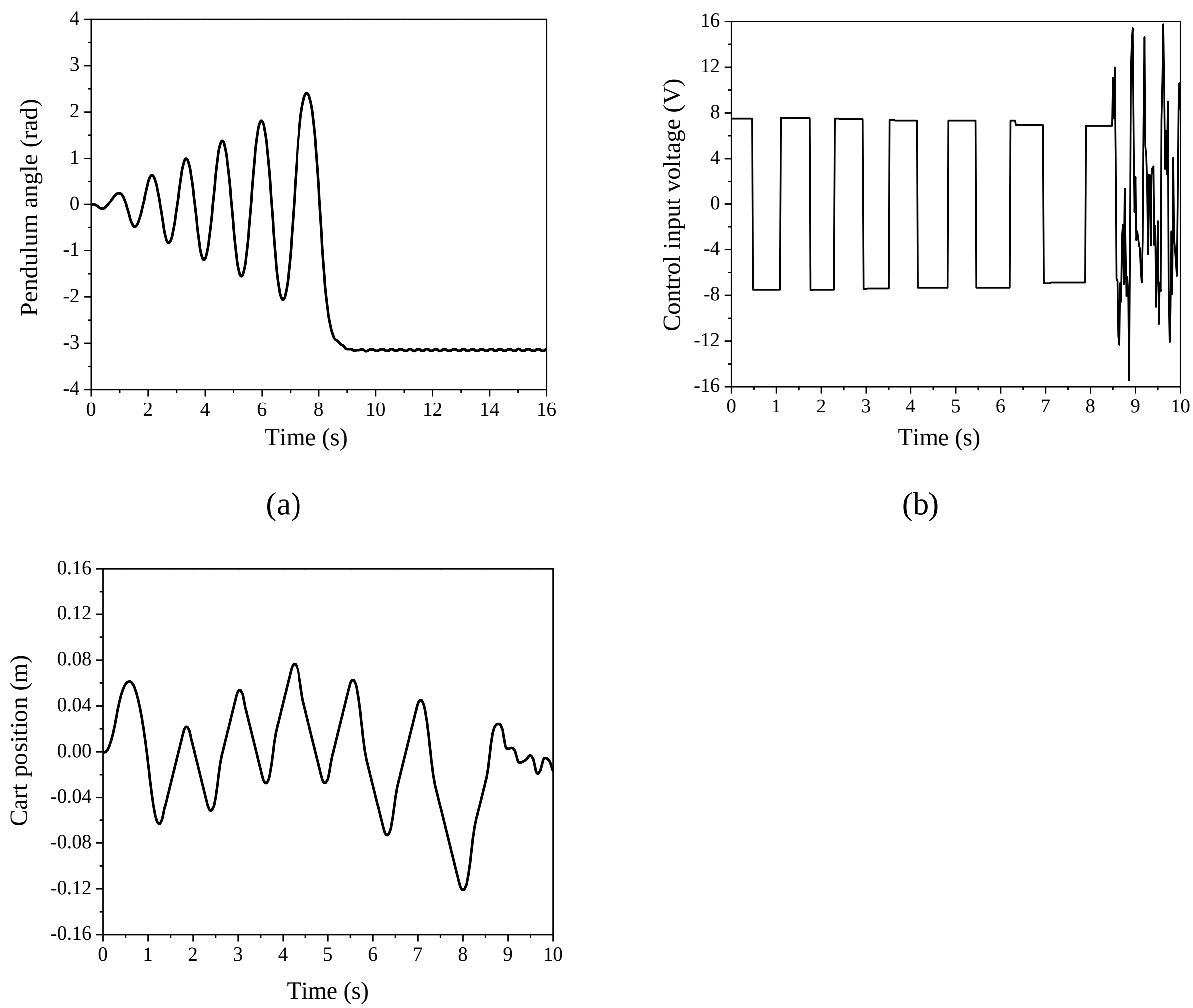


Fig. 3.7 Block diagram of inverted pendulum system

3.5.4 Experimental results

The experimental results for the inverted pendulum system are presented in Fig. 3.8. The first plot is given for the pendulum angle with the time for the swinging up and the stabilizing control of inverted pendulum system. The pendulum angle takes approximately about 8.5 s for stabilization with six increasing swings of the pendulum as shown in Fig. 3.8(a). The second plot is for the control input voltage applied for the actuation of the cart with time. The control input voltage alternates from 7.5–7.5 V during the swing up control as shown in the Fig. 3.8(b). The third plot is for the cart’s position with time during swing up and the stabilization control as shown in the Fig. 3.8(c).

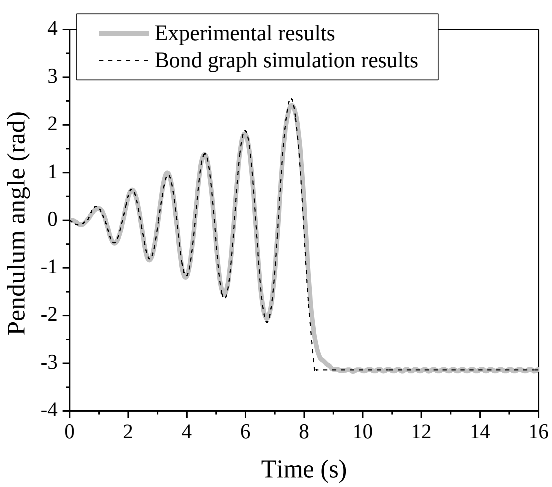


(c)

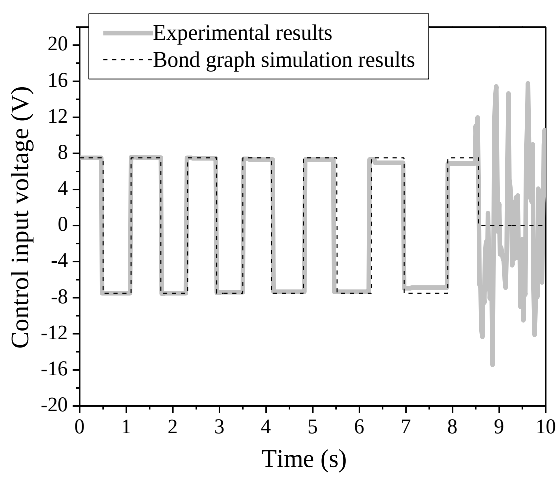
Fig. 3.8 Plot of (a) pendulum angle, (b) control input and (c) cart position from experimentation

3.6 Validation of Results

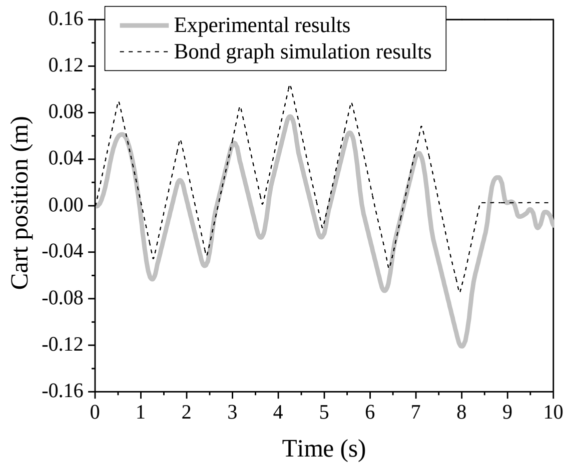
The validation of results for inverted pendulum system is given by comparing the bond graph simulation results with the experimental results. The first plot is for the pendulum angle with time as shown in the Fig. 3.9(a). The second plot is for the control input voltage with time as shown in the Fig. 3.9(b) and the third plot is for the cart position with time as shown in the Fig. 3.9(c). For the plot of pendulum angle with the time, results from the bond graph simulation is almost same as that of experimentation. In both the results, the pendulum angle is reaching to the inverted position in six increasing swings. The stabilization time is almost the same for the both results which is approximately about 8.5 seconds. The control input voltage plot with time from both the experimentation as well as bond graph simulation is also the same for the swing up control. For the swing up control, the control input voltage varies between -7.5 V to 7.5 V in both the cases. In the plot of cart position with time, the results from the bond graph simulation and experimentation are not exactly matching with each other but approximately both the results are following the same pattern. In the real time experimentation, there are various resisting factors which affect the cart's movement along the tack like friction factors related to track, air resistances which are not considered in modelling the behavior of the system in bond graph.



(a)



(b)



(c)

Fig. 3.9 Comparison between experimental result and simulation result of (a) pendulum angle with time, (b) control input voltage with time and (c) cart position with time

The segway (vehicle) is composed of an inverted pendulum attached to the vehicle body which is connected to the two wheels through an axle. There is suspension system located between the wheel axle and vehicle body. The suspension system is modeled as a parallel spring damper system. The principle behind the working of this two wheeled scooter, segway (Fig. 4.1) is the stabilization control principle of the inverted pendulum system.

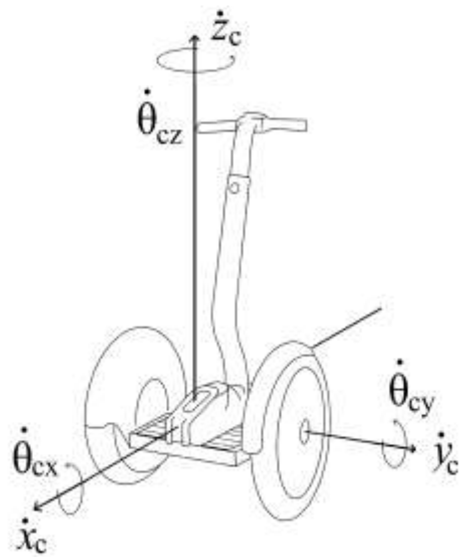


Fig. 4.1 Model of personal transporter vehicle, segway

4.1 Word Bond Graph of Segway

The modeling of the two wheeled inverted pendulum system or Segway is done by the combination of five sub models which are main vehicle body, wheel, inverted pendulum, differential and suspension system. The Fig. 4.2 represents the whole system, segway in word bond graph form where the bonds that are represented with the help of two parallel lines denoted the multi bonds between two modules. In Fig 4.2, at the interface of these subsystems, flow variables are marked. The generalized effort variables (forces and torques for linear and angular velocity respectively) are not mentioned in the Fig. 4.2 for maintaining the clarity of the figure. The suspension system is used to connect the left and right wheels to the main body of the vehicle. The differential is connected to the main body of the vehicle and the two wheels with the help of scalar bonds. Likewise, the inverted pendulum is also connected to the main vehicle body via scalar bonds. The nomenclatures used in the modeling of segway are given in Table 4.1.

Table 4.1 Nomenclature Used in the Modeling of segway

Nomenclature	**Description**
a	Distance of the axle form the vehicle cg
Cg	Center of gravity
D	Friction coefficient
F	Force
G	Acceleration due to gravity
G_p	Proportional gain
G_i	Integral gain
H	Height of the vehicle cg from the suspension reference point
h	Handle of the Segway
J	Polar moment of inertia
I	Moment of inertia
K	Stiffness
L	Length
M	Mass of the pendulum
M	Mass of the vehicle body
R	Damping
r_w	Radius of wheel
S_{time}	Time of steering
S_{return}	Steer return timing
V	Velocity
Vtv	Velocity to voltage conversion gain
x, y, z	Displacements in the three directions
$\dot{x}$, $\dot{y}$, $\dot{z}$	Velocities in three directions
θ_{st}	Steering input
θ_{max}	Maximum angle rotation
θ , $\dot{\theta}$	Angular displacement, angular velocity
τ	Torque
μ_1	Coefficient of friction
μ_m	Motor torque constant
Subscript	
B	Vehicle body
bx, by, bz	x, y , z direction of vehicle body
L	Left wheel
Md	Motor damping
P	Pendulum
R	Right wheel
S	Suspension
Seg	Segway
x, y, z	Direction of structure
W	Wheel
wx, wy, wz	x, y, z directions of wheel

4.1.1 Main vehicle body

The model of segway is shown in Fig. 4.1.The vehicle is assumed to be symmetrical about the longitudinal axis. The suspension system is taken as a simple linear spring damper system which is used as energy dissipation as well as energy storing element at the both ends of the main body of the vehicle which allows the wheel to rotate freely about the axle. The main vehicle body's motion can be expressed as the three linear displacements about the three body fixed coordinate axis and rotational motion of the body is given as yaw, pitch and roll motion. The Newton-Euler equations are used to model the main body of the vehicle. These equations contains attached body fixed axis which are aligned to the inertia principal axis.

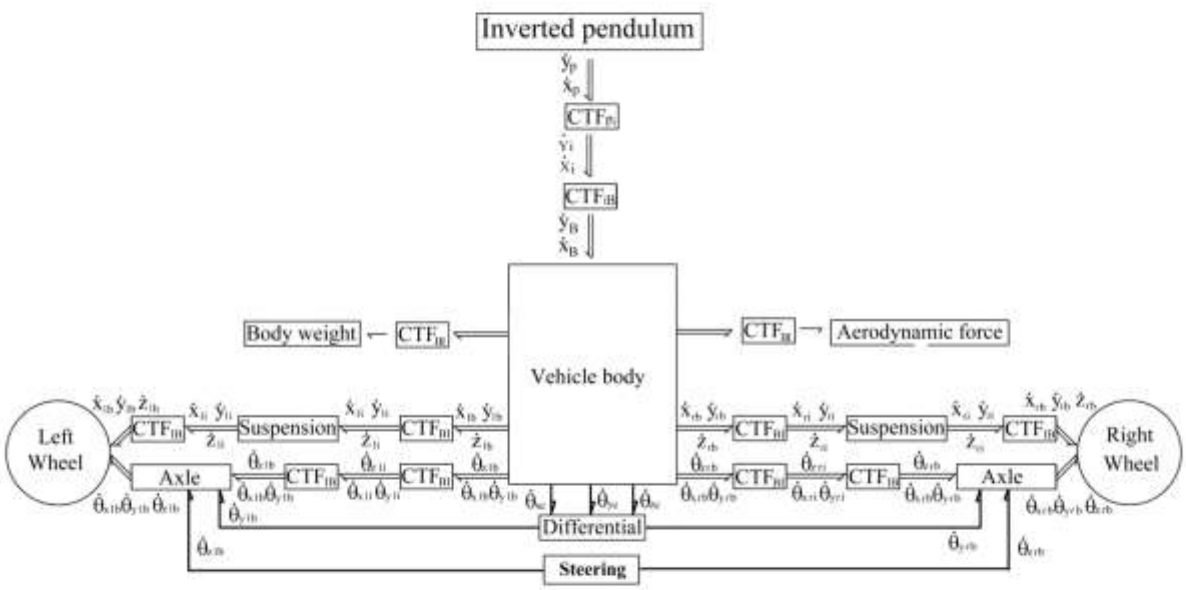


Fig. 4.2 Word bond graph model of segway

The modeling of the main body of segway (vehicle body) is done as a rigid body having six degrees of freedom (yaw, roll, pitch, heave, sway and surge motion). The rigid body's motion can be given relative to a coordinate system which is rotating as well as translating with it. It is assumed that the local coordinate system frame which is connected at the mass center of the body is aligned with the inertial principal axes. The three linear displacements and rotations about the body fixed *x*, *y* and *z* axes describe the motion of the vehicle. The orientation of the vehicle is given by the yaw, pitch and roll angles. The Newton-Euler equations [18] are given as

$$\sum F_x = m_c \ddot{x}_c + m_c (\dot{z}_c \dot{\theta}_{cy} - \dot{y}_c \dot{\theta}_{cz}) \quad (4.1)$$

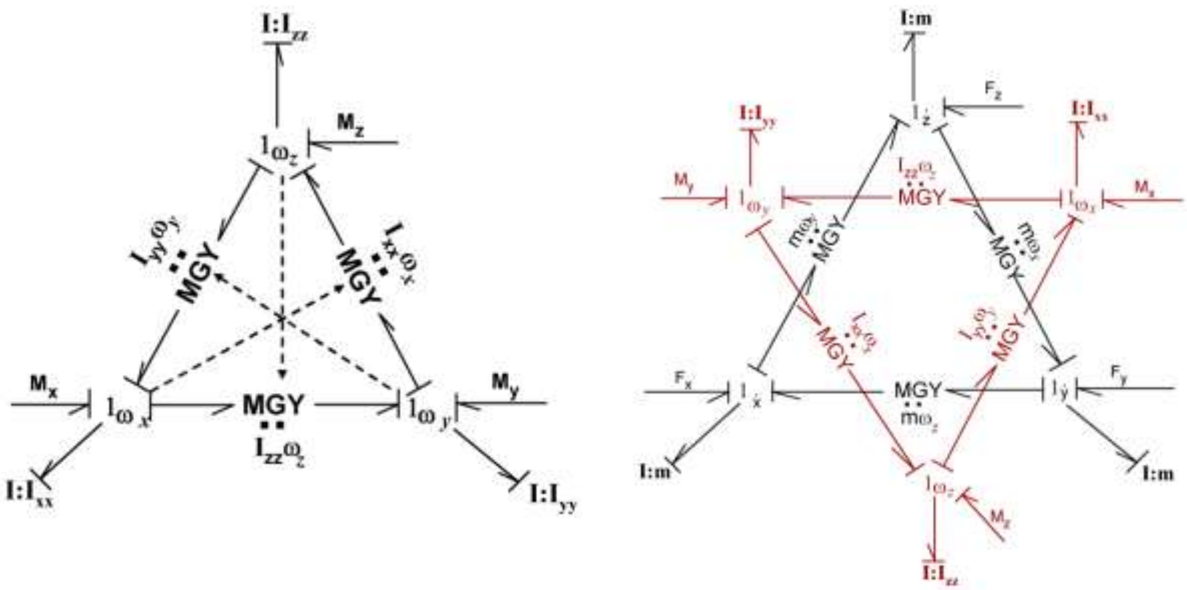
$$\sum F_y = m_c \ddot{y}_{cb} + m_c \ (\dot{x}_c \dot{\theta}_{cz} - \dot{z}_c \dot{\theta}_{cx}) \tag{4.2}$$

$$\sum F_z = m_c \ddot{z}_c + m_c \ (\dot{y}_c \dot{\theta}_{cx} - \dot{x}_c \dot{\theta}_{cy}) \tag{4.3}$$

$$\sum M_x = J_{cx} \ddot{\theta}_{cx} - \dot{\theta}_{cy}\dot{\theta}_{cz}(J_{cy} - J_{cz}) \tag{4.4}$$

$$\sum M_y = J_{cy} \ddot{\theta}_{cy} - \dot{\theta}_{cz}\dot{\theta}_{cx}(J_{cz} - J_{cx}) \tag{4.5}$$

$$\sum M_z = J_{cz} \ddot{\theta}_{cz} - \dot{\theta}_{cx}\dot{\theta}_{cy}(J_{cx} - J_{cy}) \tag{4.6}$$



(a) (b)

Fig. 4.3 Bond graph model for (a) Euler Junction Structure, (b) Newton-Euler equations [18]

The first three Eq. (4.1–4.3) are the Newton's equations where the pseudo forces are appearing with the use of the non inertial frame. The rests of the Eq. (4.4–4.6) which accounts for gyroscopic moment are the Euler's equations. The velocities (linear or angular) used in the above equations are seen from the momentarily frame aligned to the body fixed principal axes. The bond graph representations of these equations are known as Euler Junction Structure (EJS) shown in the Fig. 4.3 (a). Assume a rigid body having mass m, moments of inertia about the principal axes (*x*, *y* and *z*) at the mass center with origin as I_{xx}, I_{yy} and I_{zz}, angular velocities as ω_x, ω_y and ω_z, external moments as M_x, M_y and M_z, external force components as F_x, F_y and F_z. Newton's equations are modeled using the gyrator ring structure. The bond graph representation for Newton-Euler equations are given in the Fig. 4.3 (b).

Figure 4.3 (b) is considered as a basic building unit for the modeling of the whole system in bond graph. Now further step in the modeling is to compute the velocities at different points on the rigid body, transform them to the inertial coordinate system and physical constraint implementations at chosen points. The kinematic constraint also accounts the resulting moments and forces from those constraints in the bond graph model. For three linear velocities of left suspension reference point (whose body fixed coordinates are x_1, y_1, z_1) in the moving system of axes, the equations can be written as

$$\dot{x}_1 = \dot{x}_c + z_1\dot{\theta}_{cy} - y_1\dot{\theta}_{cz} \tag{4.7}$$

$$\dot{y}_1 = \dot{y}_c + x_1\dot{\theta}_{cz} - z_1\dot{\theta}_{cx} \tag{4.8}$$

$$\dot{z}_1 = \dot{z}_c + y_1\dot{\theta}_{cx} - x_1\dot{\theta}_{cy} \tag{4.9}$$

Same type of expressions can be written for the left suspension's angular velocities and likewise, same expression can be written for right suspension. The transformation of velocity from the moving frame to the inertial frame can be written using successive multiplication of the rotary matrices as

$$\begin{Bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{Bmatrix} = \mathbf{T}_{\psi,\theta,\varphi} \begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} \omega_X \\ \omega_Y \\ \omega_Z \end{Bmatrix} = \mathbf{T}_{\psi,\theta,\varphi} \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} \tag{4.10}$$

where

$$\mathbf{T}_{\psi,\theta,\varphi} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \tag{4.11}$$

ψ, θ and ϕ are the *Z-Y-X* Euler angles form called Cardan angles. These coordinate transformations are represented in bond graph by coordinate transformation block or CTF. The similar type of function structure is used in bond graph to represent the velocities transformations from inertial frame to body fixed frame. The angles represented in equation (4.11) are known as Euler angles. The transformation to Euler angle rates from the body fixed angular velocities is given as below

$$\begin{Bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \cos\theta\sin\phi \\ 0 & -\sin\phi & \cos\theta\cos\phi \end{bmatrix}^{-1} \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} = \begin{bmatrix} 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi / \cos\theta & \cos\phi / \cos\theta \end{bmatrix} \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} \quad (4.12)$$

Such transformation in bond graph is given in Fig. 4.4.

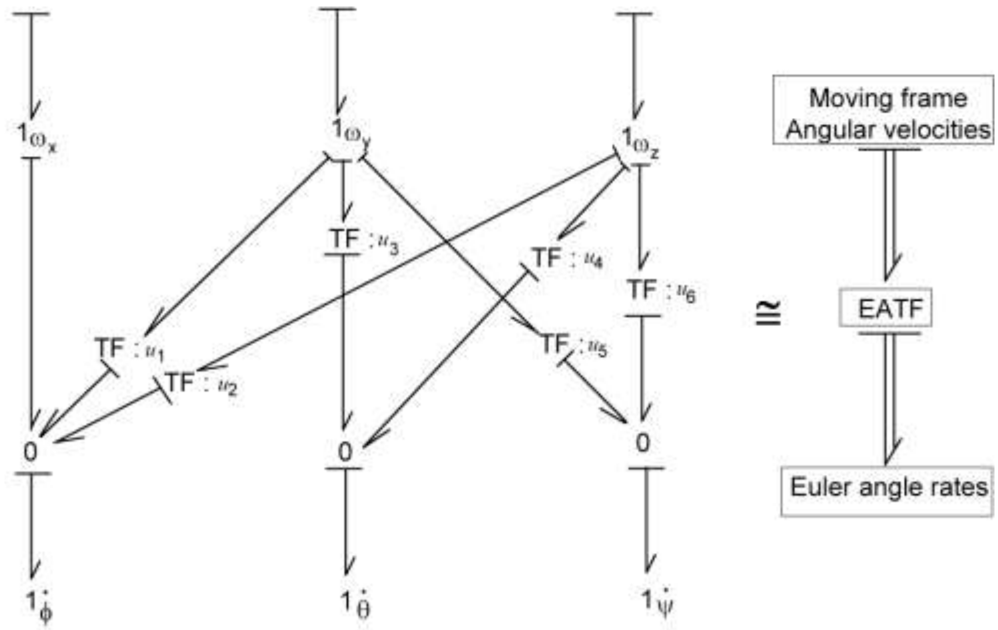


Fig. 4.4 Coordinate transformation from moving angular velocities to Euler angle rates [18]

Figure 4.7 represents the models of main vehicle body and transformation of velocities (linear to angular) to the suspension reference points. Three moments and forces (three set each) act upon the body. First set contains the aerodynamic forces and weight of the body which is in the inertial frame and act in non-inertial frame on the vehicle main body model through coordinate transformation. Secondly, torque from the motor which is in the wheel frame is transformed two times, firstly from the wheel body fixed frame to the inertial frame and then to vehicle body fixed frame to act upon the body. Third one are the forces and moments of suspension acting in the inertial frame are transformed to get forces in the body fixed direction. Then these forces are multiplied with the moment arms. Both sides force on the body are to be added to obtain the total suspension force at the body's center of gravity. The bond graph coordinate transformation block for coordinate transformation from inertial to moving frame is shown in Fig. 4.5 and from moving frame to inertial frame is shown in Fig. 4.6.

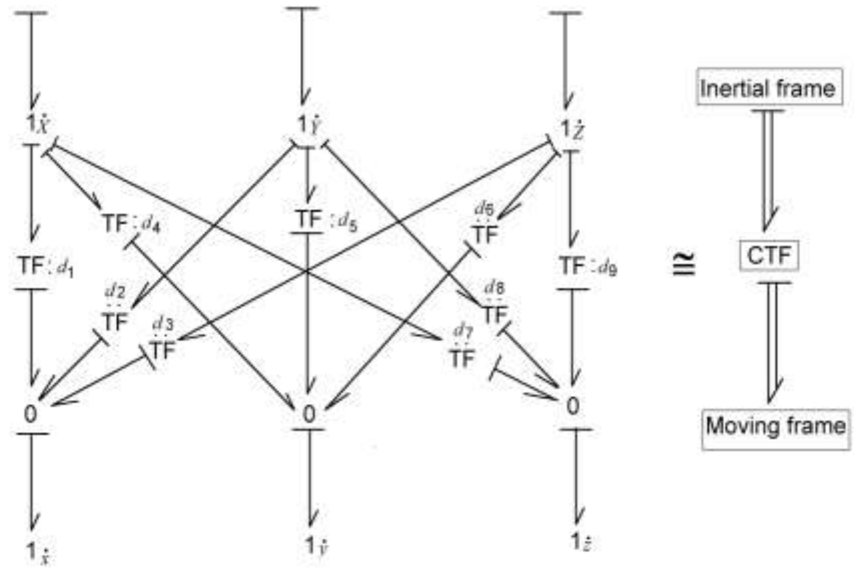


Fig. 4.5 Coordinates transformations block from inertial frame to moving frame [18]

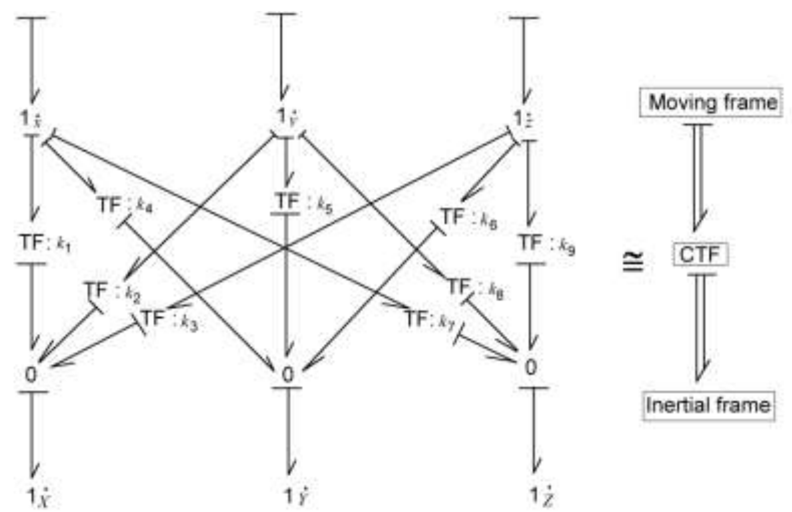


Fig. 4.6 Coordinates transformations block from moving frame to inertial frame [18]

4.1.2 Bond graph modeling of inverted pendulum system

The bond graph modeling for inverted pendulum system which is attached to the vehicle main body is shown in the Fig. 4.7. The detailed discussion about the bond graph modeling of Inverted pendulum system is given in Chapter 3 under Section 3.3. The inverted pendulum system is connected to the main body of the vehicle through the two coordinate transformation blocks. The first coordinae transformation is from the moving frame (inverted pendulum) to the inertial frame as shown in the Fig. 4.9 (b). The second coordinate transformation is from inertial frame to moving frame (vehicle body) as shown in the Fig. 4.9 (a). The angle θ shown in the bond graph of inverted pendulum in Fig. 4.8 represents the pitch angle which is to be controlled by the PI controller.

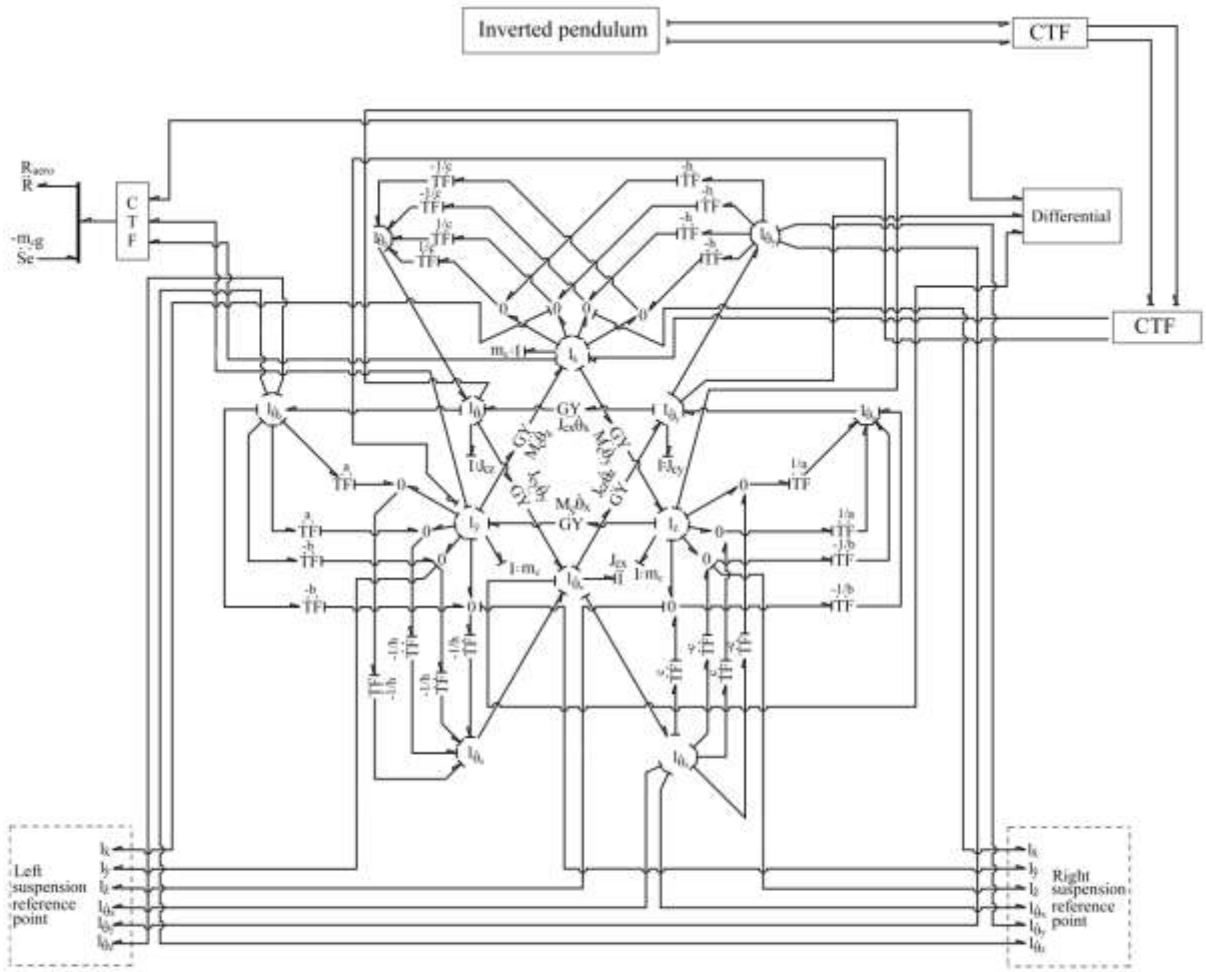


Fig. 4.7 Bond graph modeling of main vehicle body and attached components

4.1.3 Bond graph modeling of wheel

The bond graph modeling for the wheel is shown in the Fig. 4.10. It is modeled as a rigid body having six degrees of freedoms. There are six input ports in the bond graph modeling of wheel. Three effort inputs are coming for the suspension and torque is coming from the differential. The wheel is having both linear and angular velocities. The contact point between the wheel and road is not a fixed point. It changes as per the wheel rotation about the axle. The wheel is rotated about the axle axis and it does not change its orientation in the inertial frame. There is a rotational symmetry between wheel and the axle about an axis. The gravity force and the normal contact force between the wheel and road always act in the inertial Z-axis. The weight of the wheel is acting in z direction which can be given by source of effort (Se: -w_{m}g). The mass of the wheel is represented as I: w_{m} and r_{w} is the radius of the wheel. The traction force or tractive force which is used to generate the motion between the wheel body and tangential surface is given by Se: F_{t}.The polar moment of inertia is represented by J_{wx} ,J_{wy} and J_{wz} in *x*, *y*, *z* directions respectively. The three flow detectors ((Df : $\dot{f}_1$), (Df : $\dot{f}_2$), (Df : $\dot{f}_3$)) are used to measure the velocities of the wheel about *x*, *y*, *z* directions, respectively.

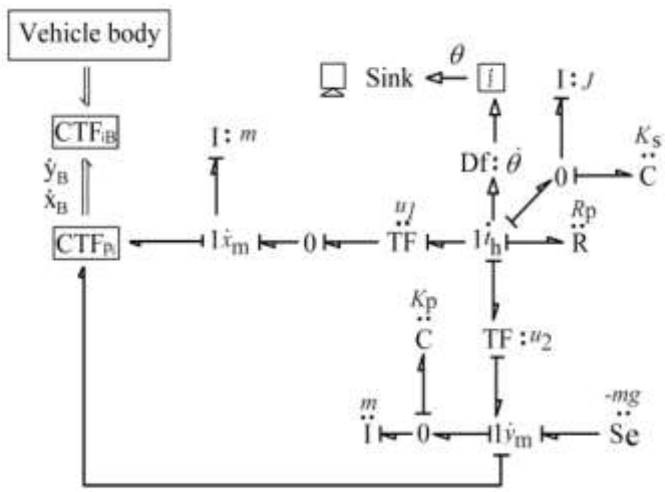
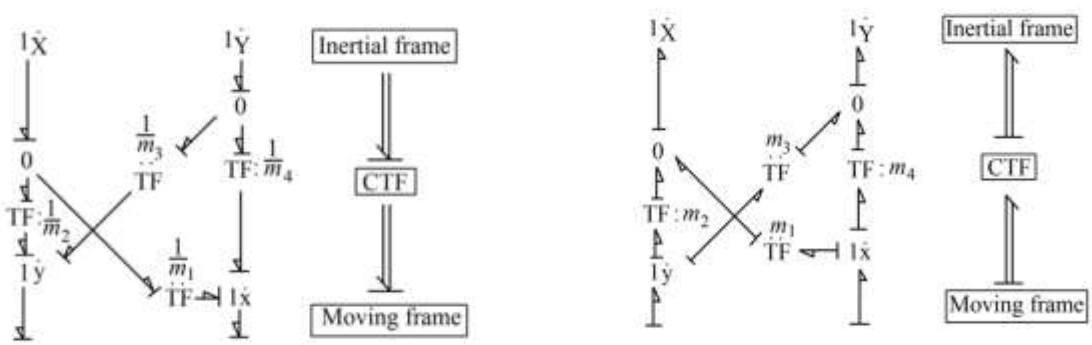


Fig 4.8 Bond graph model for inverted pendulum



(a) (b)

Fig. 4.9 Bond graph model of CTF block (a) moving frame to inertial frame and (b) inertial frame to moving frame

4.1.4 Bond graph model of suspension

The bond graph modeling for suspension system which is located between the wheel axle and vehicle body is given below in the Fig. 4.11. The suspension system is modeled as a parallel spring damper system. In the bond graph model, the stiffness (C: K_{high}), damping (R: R_{high}) in *x*, *y* directions and stiffness (C: K_z) and damping (R: R_z) in z direction are connected to *0* junction between the flow input from the vehicle body and effort output to the wheel. There are three flow input ports from the main vehicle body and three effort output ports to the wheel.

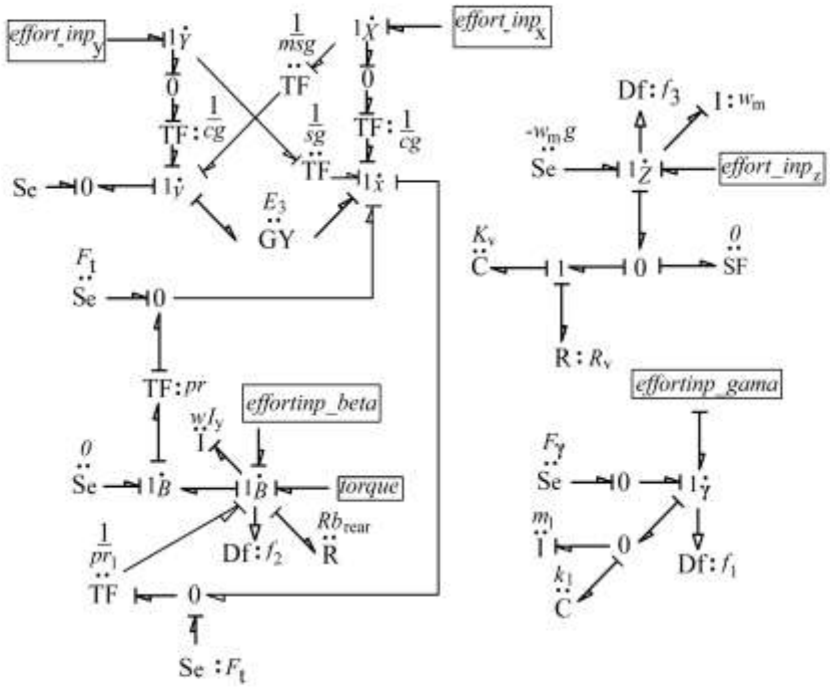


Fig. 4.10 Bond graph model for a wheel

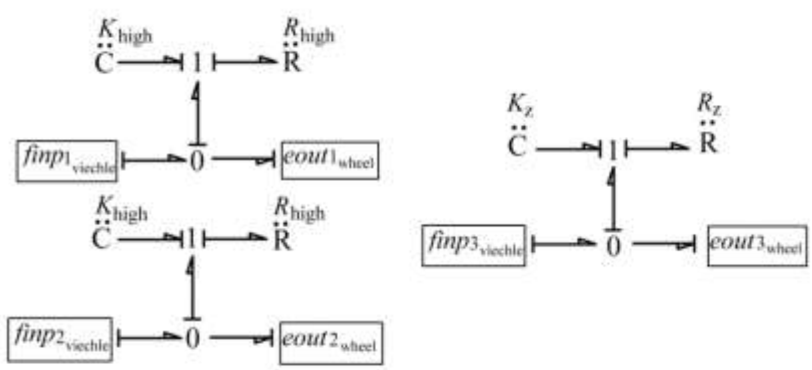


Fig. 4.11 Bond graph model for suspension system

4.1.5 Bond graph model of differential

The bond graph modeling for differential is shown in the Fig. 4.12 (a).The differential consists of a DC motor with a Proportional integral control (shown in the Fig. 4.12 (b)). The PI controller is used a stabilization controller for balancing the pitch angle θ (shown in Fig. 4.8) of inverted pendulum attached to the main vehicle body of segway. The desired value in case of PI controller is the inverted position of the pendulum and feedback value is the value of the pitch angle measured by the sensor. The bond graph model for PI controller is shown in Fig. 4.12 (b). There are two sources of flow that are added to the flow sum junction. One flow is the desired value that is the inverted angular position and the other one is the negative value of feedback angular value measured by the sensor. The error between these flows is converted to the voltage with the help of gyrator. Now according to the voltage error signal, a control force is generated by the PI controller to minimize the voltage error signal. In this bond graph representation, $G_p$is the proportional gain and G_i is the integral gain.

In the bond graph modeling of differential (Fig. 4.12 (a)), there are two output ports from the *0* junction providing the equal torque to both the left and right wheels. There are three input ports from the vehicle body to the differential and these inputs are transformed with the help of coordinate transformation block CTF as shown in Fig. 4.6 from moving frame to inertial frame.

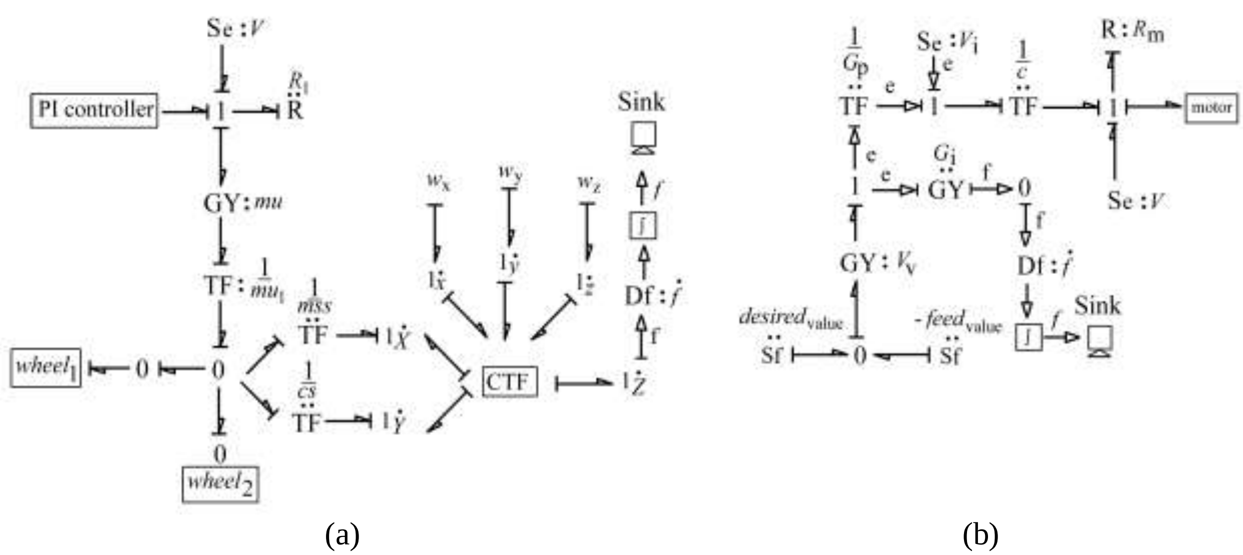


Fig. 4.12 Bond graph modeling for (a) differential and (b) PI controller

4.1.6 Bond graph modeling of steering

The steering is provided for the turning motion of the vehicle. The bond graph modeling for steering is given in the Fig. 4.13. Steering is basically a moment which has applied to the axles of the two wheels about z-axis causing the change in the yaw angle of the wheel (also the relative torque on the body of the vehicle. The rotation rate of left and right wheel about the z axis are denoted by $1_{\dot{\delta}_1}$and $1_{\dot{\delta}_2}$ junction in the bond graph model. The modulus of two transformers u_1 and u_2 are determined by the Ackermaan's formulae [22] given as

$$\dot{\delta}_1 = \left[\frac{(a+b)\,cos^2\,\theta_1 + c\,tan\,\theta_1 cos^2\,\theta_1}{(a+b)\,cos^2\,\theta_{st} - c\,tan\,\theta_{st} cos^2\,\theta_{st}}\right]\dot{\delta} \tag{4.13}$$

$$\dot{\delta}_2 = \left[\frac{(a+b)\,cos^2\,\theta_2 + c\,tan\,\theta_2 cos^2\,\theta_2}{(a+b)\,cos^2\,\theta_{st} - c\,tan\,\theta_{st} cos^2\,\theta_{st}}\right]\dot{\delta} \tag{4.14}$$

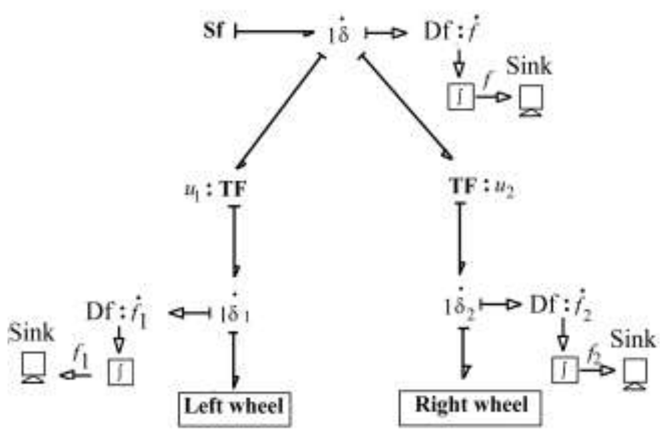


Fig. 4.13 Bond graph model for steering

Table 4.2 Bond graph simulation parameters for segway

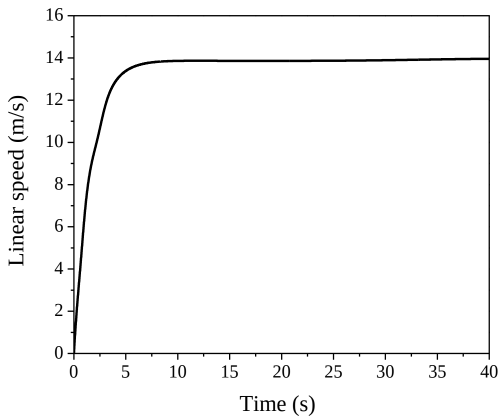
Subsystems	**Parameter values**			
Vehicle body	M = 400 kg	J_{bx}= 260 kg m^2	J_{by}= 1110 kg m^2	J_{bz}=1370 kg m^2
	a =0.9 m	h = 0.1 m		
Suspension	K_s = 1e^5 N/m	R_s = 1.6 e^5 N s/m	K_z = 1e^9 N/m	R_z = 2 e^5 N s/m
Wheel	m_w =15 kg	J_{wx}=0.1 kg m^2	J_{wy}=0.1 kg m^2	J_{wz}=0.1 kg m^2
	r_w = 0.3 m	K_w = 1e^6 N/m	D_x =7000	D_y =2000
Pendulum	I_p= 1 kg m^2	K_p = 1e^1 N/m	m_p =5 kg	K_{p1} = 1e^6 N/m
	l = 0.5 m	H = 11 N	R_p = 10 N s/m	
Motor	μ_m = 1	R_m = 2.5 N s/m	R_{md} = 0.03 N s/m	J_m=0.0014 kg m^2
PI controller	G_i =1.22	G_p = 1	*vtv* = 100	
Steering	θ_{max} =0.25 rad	θ_{st} =0.025 rad	t_{steer} = 25 s	t_{return} = 50 s

4.2 Bond Graph Simulation Results of Segway

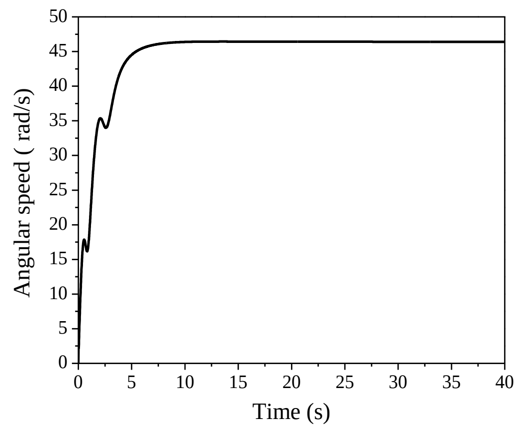
The bond graph simulation results for the segway are given here by simulating the various conditions.

Case 1: Response of the pitch angle, linear speed of vehicle and angular speed of wheel with time, when a constant positive voltage is applied to the motor

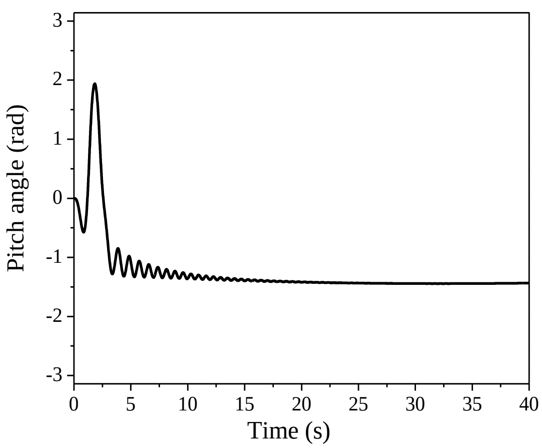
The results are simulated by providing a constant positive voltage of 200 V (without control) to the motor which produces torque to the wheels and vehicle starts moving in the forward direction. The plot for linear speed of the vehicle is shown in Fig. 4.14 (a), indicates that the vehicle accelerates in the forward direction and attains a constant speed of 13.5 m/s after 5 sec. It will continue to move in the forward direction until the constant voltage is provided to it. The plot for the angular speed of wheel is also shown in Fig. 4.14 (b) which shows that the wheel attains a constant angular velocity of 45.02 rad/s after 5 sec. The third plot, Fig. 4.14 (c) shows the behavior of the pitch angle with time when the constant positive voltage from the motor is provided to the wheels. Since, there is no stabilization control for pitch angle in this case, the vehicle accelerates, there are some oscillations in the pitch angle about the reference position (inverted position) and when the vehicle attains a constant speed in the forward direction, the pitch angle also comes to the rest at 1.44 radian from the reference position in the backward direction due to the inertia.



(a)



(b)

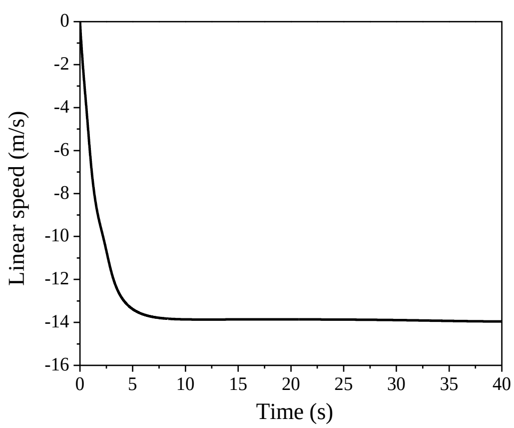


(c)

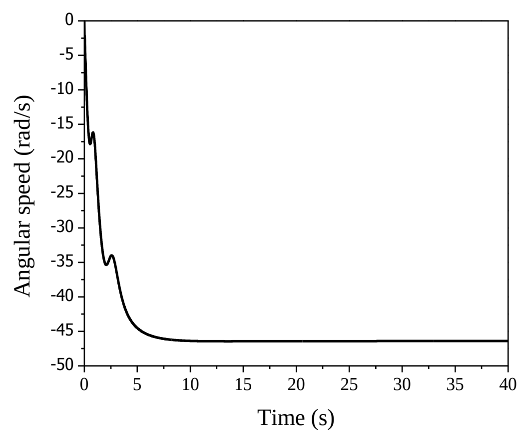
Fig. 4.14 Plot for (a) pitch angle, (b) linear speed of the vehicle and (c) angular speed of the wheel of vehicle with time for constant positive voltage from motor

Case 2: Response of the pitch angle, linear speed of vehicle and angular speed of wheel with time, when a constant negative voltage is applied from the motor

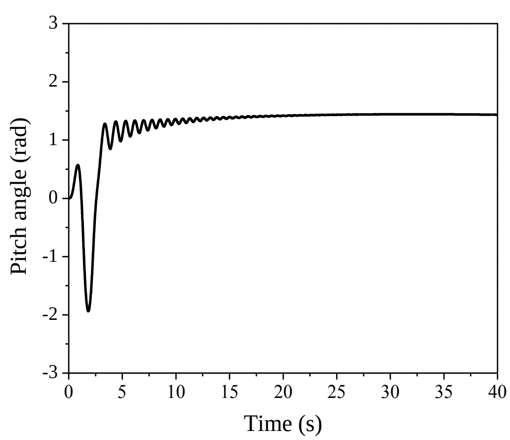
The results are simulated by providing a constant positive voltage of -200 V (without control) from the motor to the two wheels of the vehicle which produce torque in the wheels and vehicle starts moving in the backward direction. The plot for linear speed of the vehicle is shown in the Fig. 4.15 (a), indicates that the vehicle accelerates in the backward direction and attains a constant speed of 13.5 m/s after 5 sec. It will continue to move in the backward direction until the constant voltage is provided to it. The plot for the angular speed of wheel is also shown in the Fig. 4.15 (b) which shows that the wheel attains a constant angular velocity of 45.02 rad/s after 5 sec. The third plot, Fig. 4.15 (c) shows the behavior of the pitch angle with time when the constant negative voltage from the motor is provided to the wheels. Since, there is no stabilization control for pitch angle in this case. As the vehicle accelerates, there are some oscillations in the pitch angle about the reference position (inverted position) and when the vehicle attains a constant speed in the backward direction, the pitch angle also comes to the rest at 1.44 rad from the reference position in the forward direction due to the inertia.



(a)



(b)



(c)

Fig. 4.15 Plot for (a) pitch angle, (b) linear speed of the vehicle and (c) angular speed of the wheel of vehicle with time for constant negative voltage from motor

Case 3: Response of the pitch angle, linear speed and angular speed of vehicle with time, when a constant force is provided to handle in the forward direction

The bond graph simulation results for the segway as pitch, speed response are presented in Fig. 4.16, when a constant force is provided to the handle in the forward direction. . A constant force of 11 N is provided in the forward direction to the handle (inverted pendulum) of segway. As the force is provided to the handle in the forward direction, the pitch angle is displaced for its reference position (inverted position). For the vehicle, the control is designed in such a manner that the vehicle will not move in the forward or backward direction to balance the vehicle due to change in the pitch angle until the pitch angle is displaced through 15° from its reference position. Up to the time equals to 0.8 sec, the vehicle will not move and the linear and angular velocities of the vehicle will be zero. When the pitch angle is displaced through 15° in the forward direction from the reference position then the control part (PI controller) will activated and will measure the change in pitch angle and apply an effort or voltage signal to minimize the change in the pitch angle which in results provide a torque from motor to the two wheels and vehicle start moving in the forward direction. As the force will be continue in the forward direction, the vehicle will continue to move in the forward direction. Now after time equals to 1 sec, the force from the handle is removed and PI Controller will try to take the handle to the reference position (inverted position). But due to inertia the pitch angle is displaced in backward direction due to forward motion of segway. And then due to this change in the pitch angle, Segway moves in the backward direction to minimize the change in pitch angle and finally vehicle comes to the rest after t = 4 sec and handle will come to its reference position after a little bit oscillations about the reference position in t = 7 sec. The plot for pitch angle with time (Fig. 4.16 (a)), plot for linear speed of the vehicle with time (Fig. 4.16 (b)) and plot for angular speed of the wheel with time (Fig. 4.16 (c)) are shown below:

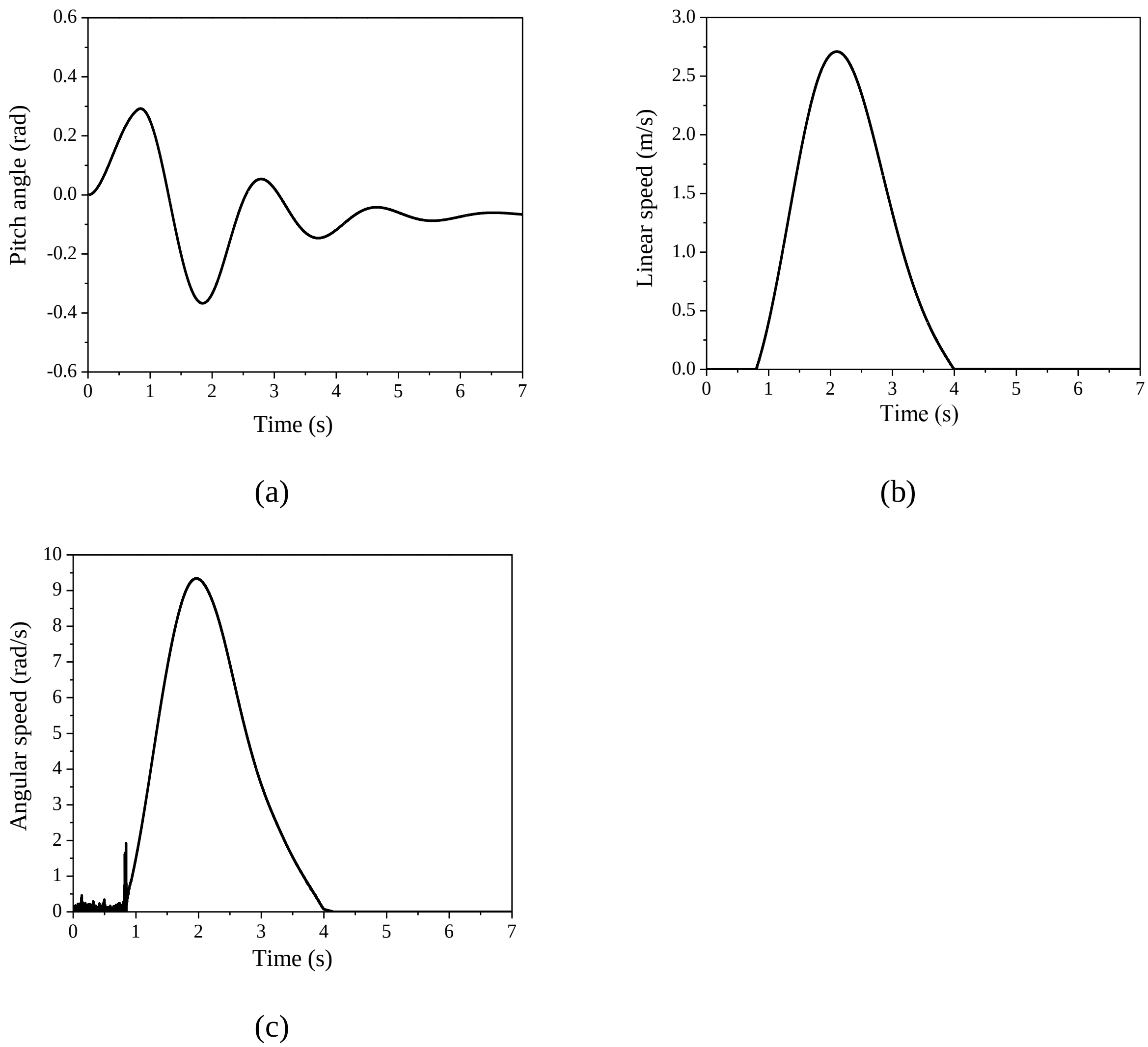
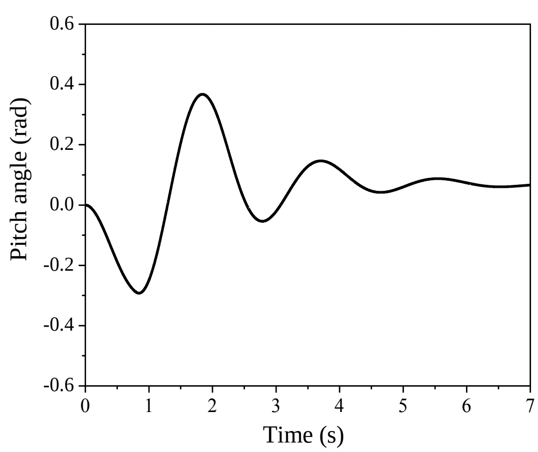


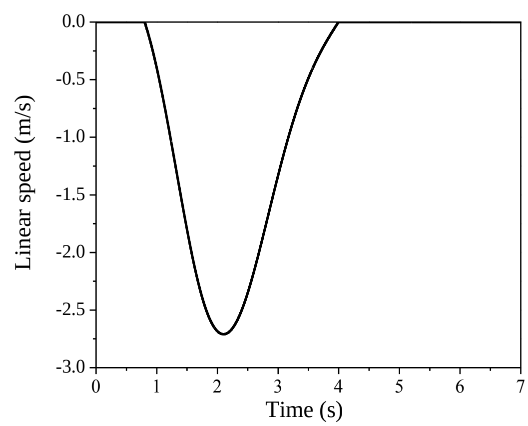
Fig. 4.16 Plot for (a) pitch angle, (b) linear speed of the vehicle and (c) angular speed of the wheel of vehicle with time for forward handle force

Case 4: Response of the pitch angle, linear speed and angular speed of vehicle with time, when a constant force is provided to handle in the backward direction

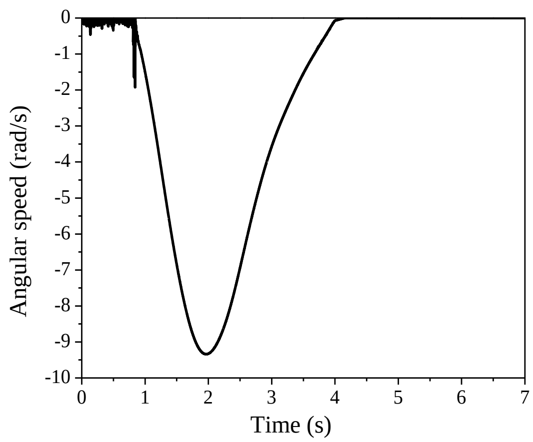
The bond graph simulation results for the Segway as pitch, speed response are presented in Fig. 4.17, when a constant force is provided to the handle in the backward direction. A constant force of 11 N is provided in the backward direction to the handle (inverted pendulum) of segway. As the force is provided to the handle in the forward direction, the pitch angle is displaced for its reference position (inverted position). For the vehicle, the control is designed in such a manner that the vehicle will not move in the forward or backward direction to balance the vehicle due to change in the pitch angle until the pitch angle is displaced through 15° from its reference position. Up to the time equals to 0.8 sec, the vehicle will not move and the linear and angular velocities of the vehicle will be zero. When the pitch angle is displaced through 15° in the backward direction from the reference position then the control part (PI controller) will activated and will measure the change in pitch angle and apply an effort or voltage signal to minimize the change in the pitch angle which in results provide a torque from motor to the two wheels and vehicle start moving in the backward direction. As the force will be continue in the backward direction, the vehicle will continue to move in the forward direction. Now after time equals to 1 sec, the force from the handle is removed and PI Controller will try to take the handle to the reference position (inverted position). But due to inertia the pitch angle is displaced in forward direction due to backward motion of Segway. And then due to this change in the pitch angle, Segway moves in the forward direction to minimize the change in pitch angle and finally vehicle comes to the rest after t = 4 sec and handle will come to its reference position after a little bit oscillations about the reference position in t = 7 sec. The plot for pitch angle with time (Fig. 4.17 (a)), plot for linear speed of the vehicle with time (Fig. 4.17 (b)) and plot for angular speed of the wheel with time (Fig. 4.17 (c)) are shown below:



(a)



(b)



(c)

Fig. 4.17 Plot for (a) pitch angle, (b) linear speed of the vehicle and (c) angular speed of the wheel of vehicle with time for backward handle force

Case 5: Response of the pitch angle, linear speed of vehicle, yaw angle of the vehicle and yaw angle of the vehicle body, when steering is provided to the vehicle

The simulation results are shown here in which a constant positive voltage from the motor of 200 V is provided to the two wheels of the vehicle and vehicle is moving in the forward direction with a linear speed of 13.5 m/s. Then after t = 25 s, a steering input of 0.025 radian is provided to the axle of wheel about the z axis of the wheel. The maximum angle of rotation is limited to the 0.25 rad. The first plot, Fig. 4.18 (a) shows the linear speed of the vehicle with time. This plot shows that after t = 25 s, there is decrease in the vehicle speed as the turning motion starts and also there is change in the yaw angle of the wheel (Fig. 4.18 (b)). As the yaw angle of the wheel reaches to the 0.25 radian, wheels stops turning further and the vehicle moves with a constant speed. After t = 50 s, the steer is returned back. As the steer is returned back, the speed of the vehicle increased up to the earlier speed of 13.5 m/s. The yaw angle of the wheel also comes to the initial position before steering. The variation of the yaw angle of the vehicle is shown in the Fig. 4.18 (c). There are very small changes in the pitch angle during the steering motion as shown in the Fig. 4.18 (d).

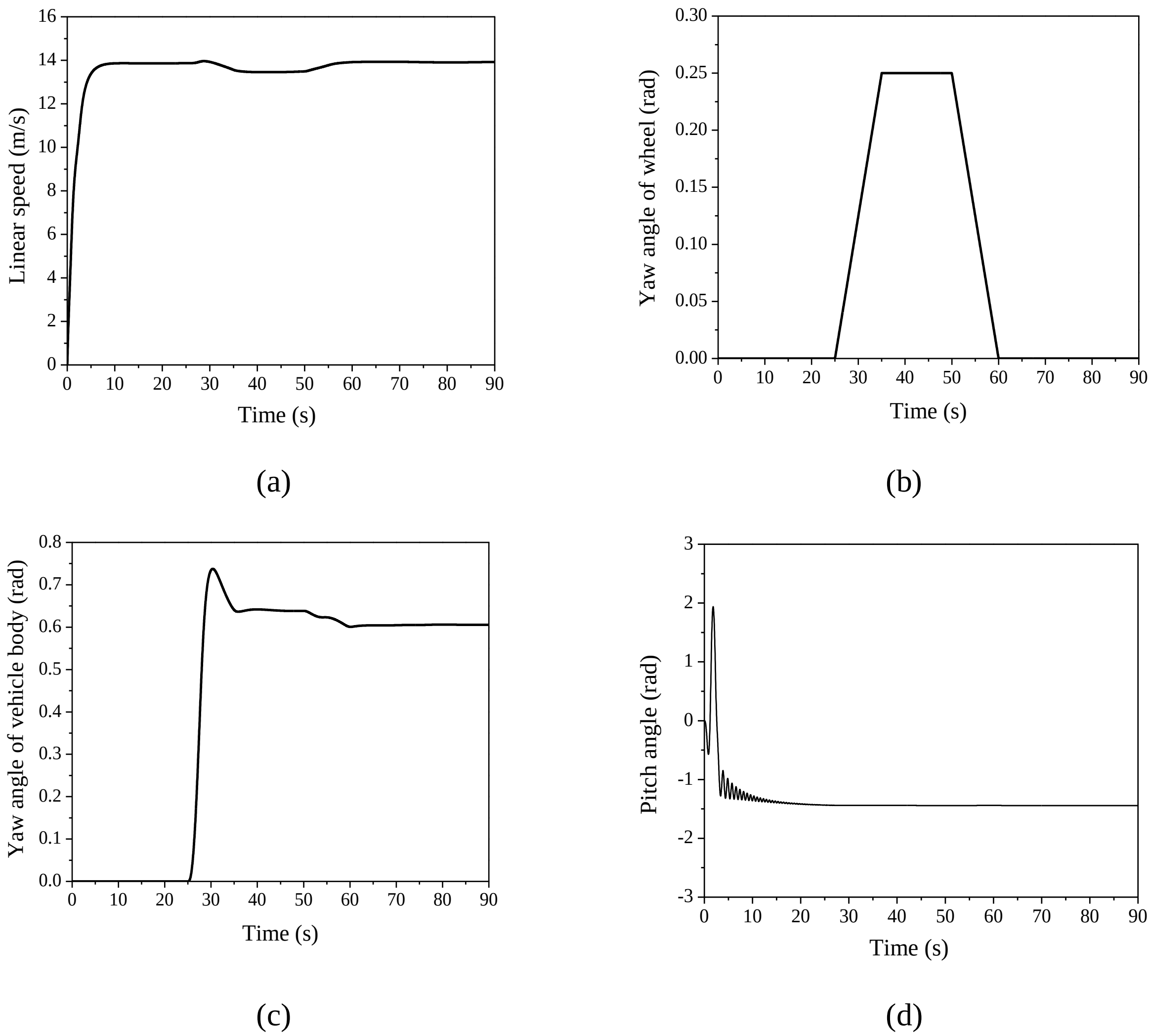


Fig. 4.18 Plot for (a) linear speed of vehicle, (b) yaw angle of the wheel and (c) yaw angle of the vehicle body and (d) pitch angle of vehicle with time when steering is provided

5.1 Conclusions

The objective of this thesis is to develop the bond graph model and control for the linear inverted pendulum system and its application in segway. The conclusions of the thesis are as follows:

- Bond graph modeling technique is very efficient tool for modeling any physical and mechanical system, and simulating various kinds of conditions for the same.
- The logic based Heuristic controller is very much capable of swinging up the pendulum from the downward equilibrium position to the upright, unstable equilibrium position by the use of voltage as a control input.
- The proportional integral (PI) control is very much effective control technique for stabilizing the pendulum rod at the inverted unstable equilibrium position.
- The bond graph simulation results for the swinging up and stabilizing control of cart pendulum system is also validated through the experimental results.
- After the successful modeling and control of linear inverted pendulum system through bond graph, the application of the inverted pendulum system i.e. segway is also modeled in bond graph domain and control system is designed on the basis of the stabilizing control of inverted pendulum system.
- The proportional integral (PI) control is very much capable of measuring the change in the pitch angle from the reference position (inverted position) and generates a control input signal for forward and backward motion of the segway to minimize that change.

5.2 Scope of Future Work

Based on the work done in this thesis, the following work is suggested for the future work:

- The Heuristic controller and PI controller may be replaced by other controller for swinging up and stabilizing control to get better accuracy and results.
- The various new control strategies and algorithms can be tested for the inverted pendulum system using bond graph model which may be applicable to various practical application in aeronautics and robotics.

- The modeling and control of segway can be improved by using advanced control schemes and methods.

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