

# **THERMOHYDRODYNAMIC ANALYSIS OF MISALIGNED JOURNAL BEARINGS**

**A THESIS**

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for the award of the degree  
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**By :**

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**July, 1998**

## CERTIFICATE

Certified that the thesis entitled “ **THERMOHYDRODYNAMIC ANALYSIS OF MISALIGNED JOURNAL BEARINGS** ” which is being submitted by **Shri Sukhwant Singh Banwait** in fulfillment of the requirements for the award of the degree of **Doctor of Philosophy in Mechanical Engineering**, to Thapar Institute of Engineering and Technology (A Deemed University), PATIALA, INDIA is a record of candidate's own work carried out by him under our supervision and guidance. The matter embodied in this thesis has not been submitted in part or full to any other University or Institute for the award of any degree.



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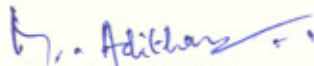
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## ABSTRACT

Hydrodynamic journal bearings are used to support rotating shafts of machines such as compressors, turbogenerators, pumps, etc. under imposed operating conditions which comprises speed and load. These bearings are normally designed using data developed with the assumptions that they operate with isoviscous lubricants and are aligned. Oil viscosity is a strong function of oil temperature and varies throughout the film. This, in turn, influences the prediction of various bearing performance characteristics. The temperature of the lubricant is largely controlled by the speed of the journal. Since there is a tendency to design rotors to operate at high speeds for optimum performance, the assumption of constant temperature (and hence constant viscosity) appears to be doubtful. Further, due to manufacturing tolerances, deflection of journal and bearing support, asymmetric bearing load, etc., journal bearings may quite often operate at the misaligned condition. The geometry of clearance space between the journal and the bush influences the bearing performance significantly. The clearance geometry depends largely on the amount of misalignment present between the axes of bearing and journal. Therefore, for a more accurate prediction of bearing performance, it is essential to perform a complete thermohydrodynamic (THD) analysis of misaligned journal bearings. The present study has been planned with these forethoughts.

Although the theory of thermohydrodynamic (THD) lubrication developed quite significantly during the last few decades, it remained confined mainly to the problems of aligned journal bearings. The published literature on the THD investigations on misaligned journal bearings is scant. Even the isothermal literature on misaligned journal bearings is generally limited to the computation of static and dynamic performance characteristics of circular bearing geometries (i.e. plain and two-axial-groove) only. Therefore, there is a need to investigate the THD effects in misaligned circular and non-circular journal bearings with more realistic mathematical models and boundary conditions for the physical problems involved. This work is an attempt to bridge this gap present in the existing literature.

The work contained in this study is broadly divided into five parts. The first part deals with the validity of the results. In the second part, the static and dynamic performance

characteristics, isopressure curves, pressure profiles, isotherms and temperature profiles are studied for different bearing configurations viz. plain journal bearing, two-axial-groove journal bearing, elliptical journal bearing and three-lobe journal bearing respectively. The third part consist of the study of transient motion trajectories followed by the comparative study of vital parameters of journal bearings. The last part deals with the effect of operating parameters on static and dynamic characteristics.

The generalised Reynolds equation is used to represent the flow of lubricant in the clearance space of journal bearings. The clearance space geometry between bearing pads/lobes and journal is expressed by including the effect of bi-planar misalignment. In the thermal (THD) analysis, the temperature distributions in the fluid-flow field and the bush-housing assembly are computed using the three-dimensional energy and Fourier heat conduction equation, respectively. A one-dimensional heat conduction equation for journal has been solved. Viscosity of the lubricant has been expressed as function of temperature using conventional exponential form. Heat transfer between bush/journal and oil in supply grooves is considered.

Finite element method is employed for solving the Reynolds equation and finite difference method with Successive Over Relaxation is employed for solving the energy and heat conduction equations. The computational procedure used to obtain the performance characteristics is an iterative one. In the global iteration scheme, the viscosity of oil is computed using the previous iteration temperatures. The Reynolds equation is then solved to obtain the converged pressure field. The temperature field for the lubricant, bush-housing and journal is then established for the computed film pressures which, in turn, gives the modified values for lubricant viscosity for the solution of Reynolds equation in the next iteration. Suitable weighting factors have been used to improve the rates of convergence of various iteration loops.

The performance characteristics of misaligned journal bearings are reported as functions of load. These characteristics include eccentricity ratio, attitude angle, maximum fluid-film pressure, minimum fluid-film thickness, side flow, mean journal temperature, maximum bush temperature, sixteen stiffness coefficients, sixteen damping coefficients, threshold speed and whirl frequency ratio. The results for aligned and misaligned journal

bearings are given for three different values of bi-planar misalignment ratios. To examine the combined effects of misalignment and THD, isopressure curves and pressure profiles are plotted for the aligned and misaligned journal bearings. Also, the isotherms and temperature profiles are plotted for oil and bush at different locations for all bearing configurations operating with misalignment.

The analysis reveals that performance characteristics are highly influenced by the inclusion of thermal effects. The THD results for the performance characteristics of misaligned journal bearings show that these characteristics are significantly altered by misalignment, particularly, when the load is heavy and/or misalignment is large. These studies indicate appreciable reduction in the bearing load capacity by the misalignment. The minimum fluid-film thickness is reduced drastically when the bearings operate with misalignment. The effect of misalignment is to make the distribution of bearing pressure asymmetric and at the same time, it changes the maximum bearing pressure values. At higher load values, misalignment reduces the side flow. The misalignment significantly improves the stability margin of all journal bearing systems.

The journal temperature and maximum bush temperature, each, increases with the increase in misalignment. For a misaligned bearing, the journal temperature varies significantly along the axial length of the journal. Hence, the journal cannot be assumed as a lumped thermal element in the analysis of misaligned bearings. The bush-fluid interface temperature at all load values increases with the misalignment in all the journal bearings studied. There is considerable variation in bush-fluid interface temperature along the circumferential and axial direction. This variation further increases with the increase in the value of load. The isotherms indicate that the heat flows into the journal from the hot regions of oil and out of the journal in the cold regions of the oil. They also show that due to misalignment, the maximum temperature region does not remain in the central plane but shifts away from the central plane. The maximum bush temperature occurs in the vicinity of hottest region of oil-film. In the high temperature region of the bush, most of the heat transfers directly out of the bush outer surface to the surroundings. It is also observed that heat transfer takes place not only in radial direction in the bush but also in the circumferential and axial directions. Hence, a two-dimensional thermal analysis is not

sufficient for the study of misaligned journal bearings and a complete three-dimensional analysis is desired.

The study of transient motion trajectories of journal-centre show that all bearings are relatively more stable when operating under misaligned condition and the relative stability is even better when the operating load is higher.

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## NOMENCLATURE

**Note :** A symbol with bar indicates a dimensionless quantity.

a	viscosity coefficient
$A_1, A_2, A_3$	coefficients of the heat conduction equation for journal (equ. (3.29))
$\bar{A}^e$	area of eth element of discretised flow domain
b	viscosity coefficient
$B_{xx} \ B_{xz}$ $B_{zx} \ B_{zz}$	damping coefficients associated with force and displacement, (N s / m)
$\bar{B}_{xx} \ \bar{B}_{xz}$ $\bar{B}_{zx} \ \bar{B}_{zz}$	$(B_{xx}, B_{xz}, B_{zx}, B_{zz}) / (\mu_0 R^4 / c^3)$
$B_{xyx} \ B_{xyz}$ $B_{zly} \ B_{zly}$	damping coefficients associated with force and rotation, (N s / rad)
$\bar{B}_{xyx} \ \bar{B}_{xyz}$ $\bar{B}_{zly} \ \bar{B}_{zly}$	$(B_{xyx}, B_{xyz}, B_{zly}, B_{zly}) / (\mu_0 R^5 / c^3)$
$B_{\gamma_x \gamma_x} \ B_{\gamma_x \gamma_z}$ $B_{\gamma_z \gamma_x} \ B_{\gamma_z \gamma_z}$	damping coefficients associated with moment and rotation, (N-m s/ rad)
$\bar{B}_{\gamma_x \gamma_x} \ \bar{B}_{\gamma_x \gamma_z}$ $\bar{B}_{\gamma_z \gamma_x} \ \bar{B}_{\gamma_z \gamma_z}$	$(B_{\gamma_x \gamma_x}, B_{\gamma_x \gamma_z}, B_{\gamma_z \gamma_x}, B_{\gamma_z \gamma_z}) / (\mu_0 R^6 / c^3)$
$B_{\gamma_x x} \ B_{\gamma_x z}$ $B_{\gamma_z x} \ B_{\gamma_z z}$	damping coefficients associated with moment and displacement, (N s)
$\bar{B}_{\gamma_x x} \ \bar{B}_{\gamma_x z}$ $\bar{B}_{\gamma_z x} \ \bar{B}_{\gamma_z z}$	$(B_{\gamma_x x}, B_{\gamma_x z}, B_{\gamma_z x}, B_{\gamma_z z}) / (\mu_0 R^5 / c^3)$
c	radial clearance, (m)
$\bar{c}$	$c / R$
$C_L$	contraction coefficient (equ. (3.15))
$C_{pair}, C_p$	specific heat of air and lubricant respectively, (N.m / Kg °C)

$C_1, C_2, C_3,$ $C_4, C_5, C_6$	coefficients of the heat conduction equation for bush-housing (equ.(3.25))
$D$	journal diameter, (m)
$\bar{D}_e$	non-dimensional Dissipation number, = $\mu \omega_j / (C_p \rho T_r c^2)$
$e$	eccentricity in aligned bearings, (m)
$e_0$	midplane eccentricity in misaligned bearings, (m)
$\nu$	whirl frequency, (rad / s)
$\bar{\nu}$	whirl frequency ratio = $\nu / \omega_j$
$g$	acceleration due to gravity, (m/ s <sup>2</sup> )
$h$	fluid-film thickness, (m)
$\bar{h}$	$h / c$
$\bar{h}_{min}$	minimum fluid-film thickness
$h_{ab}, h_{aj}$	convective heat transfer coefficient of bush-housing and journal respectively, (W / m <sup>2</sup> °C)
$h_b$	convective heat transfer coefficient of bush-housing (for solid to fluid), (W / m <sup>2</sup> °C)
$K_1, K_2, K_3,$ $K_4, K_5$	coefficients of the energy equation (equ.(3.22))
$k_a, k_f, k_b, k_j$	thermal conductivity of air, lubricant, bush-housing and journal respectively, (W / m °C)
$L$	length of the bearing, (m)
$M_c$	critical mass of the journal, (Kg)
$\bar{M}_c$	$M_c / (\mu_0 R^4 / (\omega_j c^3))$
$M, M_x, M_z$	total fluid-film moment and components in x-axis and z-axis respectively, (N-m)
$\bar{M}, \bar{M}_x, \bar{M}_z$	$(M, M_x, M_z) / (\mu_0 \omega_j R^5 / c^2)$

$N_i$	shape function for $i$ th node of a parabolic element in discretized fluid domain
$p$	fluid-film pressure, (Pa)
$\bar{p}$	$p / (\omega_j \mu_o R^2 / c^2)$
$\bar{p}_{\max}$	maximum fluid-film pressure
$\bar{P}_e$	non-dimensional inverse Peclet Number = $k_f / (C_p \rho \omega_j c^2)$
$Q_s$	side flow, ( $m^3/s$ )
$\bar{Q}_s$	$Q_s / (\omega_j c R^4)$
$R$	journal radius, (m)
$r$	radial coordinate of the bearing,
$\bar{r}$	$r / R$
$R_1, R_2$	inner and outer radii of the bush-housing, (m)
$\bar{R}_1, \bar{R}_2$	$R_1, R_2 / R$
$S_{xx}, S_{xz}$ $S_{zx}, S_{zz}$	stiffness coefficients associated with force and displacement, (N / m)
$\bar{S}_{xx}, \bar{S}_{xz}$ $\bar{S}_{zx}, \bar{S}_{zz}$	$(S_{xx}, S_{xz}, S_{zx}, S_{zz}) / (\mu_o \omega_j R^4 / c^3)$
$S_{x\gamma_x}, S_{x\gamma_z}$ $S_{z\gamma_x}, S_{z\gamma_z}$	stiffness coefficients associated with force and rotation, (N / rad)
$\bar{S}_{x\gamma_x}, \bar{S}_{x\gamma_z}$ $\bar{S}_{z\gamma_x}, \bar{S}_{z\gamma_z}$	$(S_{x\gamma_x}, S_{x\gamma_z}, S_{z\gamma_x}, S_{z\gamma_z}) / (\mu_o \omega_j R^5 / c^3)$
$S_{\gamma_x \gamma_x}, S_{\gamma_x \gamma_z}$ $S_{\gamma_z \gamma_x}, S_{\gamma_z \gamma_z}$	stiffness coefficients associated with moment and rotation, (N-m / rad)
$\bar{S}_{\gamma_x \gamma_x}, \bar{S}_{\gamma_x \gamma_z}$ $\bar{S}_{\gamma_z \gamma_x}, \bar{S}_{\gamma_z \gamma_z}$	$(S_{\gamma_x \gamma_x}, S_{\gamma_x \gamma_z}, S_{\gamma_z \gamma_x}, S_{\gamma_z \gamma_z}) / (\mu_o \omega_j R^6 / c^3)$

$S_{\gamma_x x}, S_{\gamma_x z}$ $S_{\gamma_z x}, S_{\gamma_z z}$	stiffness coefficients associated with moment and displacement, (N-m / m)
$\frac{\bar{S}_{\gamma_x x}}{S_{\gamma_x x}}, \frac{\bar{S}_{\gamma_x z}}{S_{\gamma_x z}}$	$( S_{\gamma_x x}, S_{\gamma_x z}, S_{\gamma_z x}, S_{\gamma_z z} ) / ( \mu_0 \omega_j R^5 / c^3 )$
t	time, (s)
$\bar{t}$	$t \cdot \omega_j$ , non-dimensional time
$T_a$	ambient temperature, (°C)
$T_b$	bush-housing temperature, (°C)
$T_f$	fluid-film temperature, (°C)
$T_j$	journal surface temperature, (°C)
$T_m$	mean temperature across fluid-film thickness, (°C)
$T_r$	reference temperature, (°C)
$T_s$	supply temperature of the lubricant, (°C)
$\frac{\bar{T}_a}{\bar{T}_j}, \frac{\bar{T}_b}{\bar{T}_s}, \frac{\bar{T}_f}{\bar{T}_s}$	$( T_a, T_b, T_f, T_j, T_s ) / T_r$
u, v, w	component of lubricant velocity in circumferential, axial and radial directions respectively, (m / s)
$\bar{u}, \bar{v}, \bar{w}$	$u, v, \frac{w}{c} / ( R \omega_j )$
$W_x, W_z$	fluid-film reactions in x and z directions, (N)
W	$\sqrt{ ( W_x^2 + W_z^2 ) } \equiv W_z$ for an external vertical load, (N)
$\bar{W}, \bar{W}_x, \bar{W}_z$	$W, W_x, W_z / ( \mu_0 \omega_j R^4 / c^2 )$
x y z	coordinate axis in the horizontal(circumferential), axial and vertical(radial) direction with origin at geometric centre of bearing
X, Z	horizontal and vertical components of disturbed journal centre from its steady equilibrium position, (m)

$X_{jo}, Z_{jo}$	$(e \sin\phi, -e \cos\phi)$ coordinates of steady state equilibrium position of journal centre of aligned bearings, (m)
$X_{jo}, Z_{jo}$	$(e_0 \sin\phi_0, -e_0 \cos\phi_0)$ coordinates of steady state equilibrium position of journal centre of misaligned bearings at bearing midplane, (m)
$\bar{X}, \bar{Z},$ $\bar{X}_{jo}, \bar{Z}_{jo}$	$(X, Z, X_{jo}, Z_{jo}) / c$
$X_{Li}, Z_{Li}$	lobe centre coordinates for $i$ th lobe, (m)
$\bar{X}_{Li}, \bar{Z}_{Li}$	$X_{Li}, Z_{Li} / c$
$\dot{X}, \dot{Z}$	horizontal and vertical components of lateral velocity of disturbed journal, (m / s)
$\bar{\dot{X}}, \bar{\dot{Z}}$	$(\dot{X}, \dot{Z}) / (c \omega_j)$
$y$	axial coordinate, (m)
$z$	coordinates across fluid-film thickness, (m)
$\bar{z}$	$z / h$
$\alpha$	angular coordinate
$\beta$	$y / R$
$\Gamma_\alpha, \Gamma_\beta$	boundary of eth element along $\alpha$ and $\beta$ directions respectively
$\varepsilon$	$e / c$ , eccentricity ratio
$\varepsilon_0$	$e_0 / c$ , midplane eccentricity ratio at steady state equilibrium position of misaligned bearings
$\mu_{air}, \mu$	viscosity of the air and lubricant respectively, (N.s / m <sup>2</sup> )
$\mu_0$	reference viscosity of the lubricant, (N.s / m <sup>2</sup> )
$\bar{\mu}$	$\mu / \mu_0$
$\rho_{air}, \rho$	mass density of the air and lubricant respectively, Kg / m <sup>3</sup>
$\phi$	attitude angle, (deg)

$\varphi_0$	midplane attitude angle at steady state equilibrium position of misaligned bearings, (deg)
$\lambda$	aspect ratio, = $\pm L / D$
$\omega_j$	angular speed of journal, (rad / s)
$\Omega$	threshold speed, (rad / s)
$\bar{\Omega}$	$\Omega / \sqrt{(g / c)}$
$\gamma_x, \gamma_z$	misalignment parameters - angular rotation of journal axis in horizontal and vertical plane at steady state, (rad)
$\Delta\gamma_x, \Delta\gamma_z$	rotational perturbation of journal axis in horizontal and vertical plane respectively from its steady state position, (rad)
$\frac{\bar{\gamma}_x, \bar{\Delta\gamma}_x}{\gamma_z, \Delta\gamma_z}$	$(\gamma_x, \Delta\gamma_x, \gamma_z, \Delta\gamma_z) / \bar{c}$
$\Delta\dot{\gamma}_x, \Delta\dot{\gamma}_z$	angular velocity components of disturbed journal axis in horizontal and vertical planes, (rad / s)
$\bar{\Delta\dot{\gamma}}_x, \bar{\Delta\dot{\gamma}}_z$	$(\Delta\dot{\gamma}_x, \Delta\dot{\gamma}_z) / (\bar{c} \omega_j)$

## MATRICES AND VECTORS

[ A ]	global fluidity matrix for pressure field
[ B ]	global right-hand side matrix for pressure field
{ $\bar{p}$ }	nodal pressure vector for discretized flow domain
{ $\bar{Q}$ }	nodal flow vector for discretized flow domain

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# CHAPTER - I

## INTRODUCTION

Although the theory of hydrodynamic lubrication was developed in the late nineteenth century, the hydrodynamic journal bearings have been used to support rotating or stationary shafts since early ages. These bearings find their use in most of the high speed rotating machines such as turbogenerators, grinders, pumps, turbines and engines. If designed properly, they possess long life and excellent damping capabilities. Satisfactory operation of the rotating machine depends a great deal on the performance of the bearing systems supporting their rotating parts. It has often been noticed that the rotating machines fail due to malfunctioning of their bearings. Therefore, the design of hydrodynamic bearing systems is vital for the safe running of the rotating machines.

In general, the hydrodynamic journal bearings are designed using empirical relations and design data developed under simplifying assumptions. Amongst the most common assumptions used for the development of design data of journal bearings are (i) viscosity of the lubricant is constant throughout, i.e. the journal bearing operate under isothermal condition, and (ii) there is no misalignment between the journal and the bearing axes. Modern bearings are required to operate at high speeds and/or under heavy load conditions. This leads to substantial variation of lubricant temperature in the circumferential, axial and radial directions. Besides, heavy load on shaft may result in misalignment. Manufacturing or installation errors may also cause some degree of misalignment. Under these conditions, the above said assumptions are questionable. Failure of several bearings also points in this direction. Therefore, it is desirable to include the thermal and misalignment effects in the analysis and design of journal bearings. The present work has been undertaken with these forethoughts.

Ever since, O. Reynolds proposed his theory of hydrodynamic lubrication, extensive theoretical investigations have been undertaken by researchers. These works may broadly be classified as :

(a) Isothermal (IHD) analysis : which is carried out considering temperature of the bearing and the lubricant to be constant. Thus, viscosity of the lubricant is assumed constant in the solution of governing Reynolds equation. This viscosity is determined using the operating temperature of the bearing. When bearing surfaces are assumed flexible, the study is referred to as Elastohydrodynamic(EHD).

(b) Thermohydrodynamic(THD) analysis : In this type of analysis, the temperature of the lubricant is assumed to vary along one, two or all the three directions, namely, circumferential, radial and axial. The viscosity, which depends largely on temperature, varies accordingly. The pressure and temperature fields are found using iterative solutions of the governing equations. The bearing surfaces are considered rigid in the THD analysis. The analysis which considers thermal and elastic effects, both, is sometimes called thermoelastohydrodynamic(TEHD) analysis.

The present work is limited to the THD analysis of aligned and misaligned journal bearings. The temperature has been assumed to vary in all the three directions and a more general bi-planar misalignment has been considered for the theoretical analysis. The study has been carried out on circular and non-circular journal bearings to investigate the thermal and misalignment effects on these geometries. Exact problem definition is given in the next chapter.

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# CHAPTER - II

## LITERATURE SURVEY

A brief review of the work undertaken by various researchers / investigators for the analysis of hydrodynamic journal bearings are presented in sections 2.1 to 2.3 under the following grouping :

- (i) Early Works
- (ii) Isothermal (IHD) studies, and
- (iii) Thermohydrodynamic (THD) studies

### 2.1 EARLY WORKS

The hydrodynamic film lubrication was effectively used even before it was understood fully. Hydrodynamic journal bearings are in use since the early stages of development of machinery. In 1883, the Russian engineer N. Petrov[1] was the first to establish theoretically the friction effect of viscous shearing of an intervening fluid film. He explained the functional relationship between the frictional force and the bearing parameters.

In the same year, Bouchamp Tower [2-3] reported the results of a series of experiments which eventually led to the development of hydrodynamic lubrication theory by Osborne Reynolds [4]. Nearly two decades later in 1904, the German physicist Sommerfeld [5] provided further theoretical extensions to the journal bearing lubrication problem. In 1918, Rayleigh[6] investigated the effect of lubricant film shape on performance characteristics of a bearing using calculus of variations. In the early 50s, the Reynolds equation was solved by Ocvirk [7] for an infinitely long and narrow journal bearings.

### 2.2 ISOTHERMAL STUDIES

The studies made under the assumptions of constant oil viscosity are referred to as IHD studies. A brief review of the literature on aligned and misaligned journal bearings under IHD conditions are presented in the following sub-sections :

### 2.2.1 Aligned Journal Bearings

Based on the Reynolds equation and assumptions of rigid bush / liner and constant oil viscosity throughout the film, several studies [8-15] are available on the performance characteristics of circular and non-circular aligned bearings. These studies are devoted to the analysis of static and dynamic performance characteristics of bearings operating in laminar and turbulent flow regimes.

Maik [16] made a theoretical comparative study of performance characteristics of various forms of two-lobe journal bearings. He predicted that two-lobe configuration can provide consistently good dynamic performance over a wide range of load conditions.

Significant contributions to the literature on hydrodynamic instability were made by Flack et al. [17] and Lanes et al. [18-19]. Flack et al. [17] carried out the studies for rigid and flexible rotors following the linearised as well as the non-linear approaches considering rigid bearing surfaces.

Lanes et al. [18-19] observed discrepancies between experimental and computed values of the threshold speed for three-lobe journal bearings. They found the experimental values to be higher. It was attributed to the possible errors in the theoretical calculations.

Chandrawat and Sinhasan [20] have presented algorithms for two solution techniques, namely the Gauss-Siedel iterative scheme and the linear complementarity problem (LCP) approach, for finding the positive pressure region in a hydrodynamic journal bearing. The two techniques are compared through the results and the computer time for plain journal bearing operating in laminar flow regimes. The LCP is reported to be far superior, when closer tolerances are required on pressure ( $< 0.1\%$ ), otherwise both the techniques are equally efficient.

Choy et al. [21] have performed the nonlinear transient and frequency response analysis of a hydrodynamic journal bearing.

Kostrzewsky et al. [22] performed experiments to study the steady state operating characteristics and dynamic coefficients of a two-axial groove journal bearing. The four stiffness and four damping coefficients were determined as functions of Sommerfield

number for two fixed shaft speeds, with estimated uncertainties for each. The results agreed with the uncertainty of the data for the speed, load and temperature ranges.

Taylor et al [23,24] determined the static and dynamic characteristics of a three-lobe journal bearing with a preload factor of 0.75. They tested three operating speeds. The results indicated that the maximum pressure occurred circumferentially near the point of minimum oil film. The non-dimensionalised data at all the three speeds were independent of any given Sommerfeld number, thus, the Reynolds number (which depends on speed) had little influence for the range of conditions tested.

Elrod and Vijayaraghavan [25] presented a stability analysis for liquid-lubricated journal bearings incorporating the effect of displacement perturbations of the cavitation boundaries. They reported that these displacements are of disturbance magnitude and should be included in a stability analysis.

### **2.2.2 Misaligned Journal Bearings**

Early work on misaligned journal bearings [26-29] was mainly confined to the experimental investigation of oil-film pressure distribution in axial direction. McKee and McKee [26] experimentally observed that the pressure distribution of misaligned journal bearings loses axial symmetry. Cowlin [27] listed misalignment as one of the principal causes for deviation of bearing performance from that predicted by the hydrodynamic theory. Piggott [28] showed that misalignment can reduce load carrying capacity by as much as 40 percent. Buske [29] showed that the hydrodynamic pressure pattern in a bearing is very sensitive to misalignment.

A first analytical study for the pressure profiles of a full journal bearing with axes skew was first carried out by Walther and Sassenfeld [30]. An experimental study of misaligned bearings was pursued by Dubois, Mabie and Ocvirk [31]. They found that for characterization of a spatially misaligned bearing, three parameters are necessary: the eccentricity ratio and two angular displacements in two angular directions. This approach has some short comings. Firstly, the data cannot be interpreted in terms of a geometrical parameter and secondly, it does not specify the boundaries and boundary conditions clearly.

In a subsequent theoretical study, Khrisanova [32] derived a geometric expression for film thickness with reference to components of skew in vertical and horizontal planes

and solved the Reynolds equation assuming  $180^\circ$  film extent and the boundaries as straight and inclined at an angle “gamma” at the beginning and at the end of the positive pressure zone in circumferential direction. This approach suffered from many short comings. Numerical solutions of Reynolds equation for a full or half journal bearing under misaligned conditions are given by Smalley and McCallion [33] and Stockley and Donaldson [34]. Smalley and McCallion presented design charts relating to the influence of the misaligning couple. Stockley and Donaldson analysed the effect of misalignment in a  $180^\circ$  partial bearing in which the misalignment was restricted to an axial tilt as the result of shaft bending. These studies too, assumed complex boundary conditions to identify positive pressure region of oil-film. Moreover, these studies were carried out for short bearings only ( $L/D \leq 1$ ) and influence of the presence of axial oil groove(s) was ignored.

Asanabe et al. [35] made an analytical and experimental attempt to study the static performance characteristics of a two-axial groove misaligned bearing but their work was restricted to misalignment in vertical plane only. Moreover, Sommerfeld boundary conditions were used for the solution of the Reynolds equation.

Bannister [36] reported theoretical and experimental investigations into the influence of non-linearity and misalignment on eight oil film force coefficients for a  $120^\circ$  partial journal bearing with a length-to-diameter ratio of one.

Several investigators [37-42] solved the Reynolds equation by employing the Gauss Elimination method or Gauss-Siedel relaxation procedure to establish extent of positive pressure region. These approaches predicted a more logical boundary for the positive pressure zone.

Pinkus and Bupara [37] published considerable theoretical data of different grooved bearings, but they used the eccentricity,  $\epsilon$  and the angle between the center line and the vertical coordinate as independent variables. Their load direction is a variable dependent on eccentricity and the misalignment ratio. Their vertical misalignment does not mean that the journal axis is tilted in the plane parallel with the load, and their horizontal grooves are not in the plane perpendicular to the load. This variable load direction makes their data's application and comparison rather difficult.

Pafelias and Broniarek [38] computed a set of sixteen stiffness and sixteen damping coefficients to study the bearing-system dynamics with general misalignment in the journal bearings. Reason and Siew [39] presented a numerical solution for the three-dimensional pressure distribution effects on the performance of a journal bearing with spatial misalignment.

Ikeuchi et al. [40] fixed the load direction and solved the attitude angles at different eccentricities for a circumferentially grooved bearings. They only presented the misalignment effects on load capacities and attitude angles. Buckholz and Lin [41] studied the misalignment effect on load for Non-Newtonian lubricants.

Jakeman [42] studied the performance of a misaligned journal bearing and worked out force and moment oil film dynamic coefficients for a stern tube bearings. In a subsequent study, Jakeman [43] developed a non-linear oil film response model for the dynamically misaligned stern tube bearing for the purpose of conducting lateral vibration analysis of marine propeller shafting.

Abdel-Latif and Mokhtar [44] explored the effects of both bearing roughness in the longitudinal and transverse directions together with journal misalignment on the bearing behavior, especially if the bearing runs at the limit of full film lubrication.

Vijayaraghavan and Keith [45, 46] modified the method of Pinkus and Bupara [37] by considering the mass continuity at both start and end edges of oil film and published considerable data on static characteristics of the grooved and ungrooved bearings. They incorporated grid transformation and adaptation techniques into a finite difference method to consider cavitation and starvation effects.

Choy et al. [47] have depicted the misalignment effect on one stiffness coefficient and critical mass of an ungrooved bearing at one set of misalignment parameters under some load conditions. They [48] further studied the nonlinear effects in a hydrodynamic journal bearing with special attention to the effects of journal axial misalignment. The study predicted that at light loads, journal axial misalignment results in a double hump in the fluid film pressure distribution and disappears as the load increases. At higher load condition, the peak fluid film pressure decreases with misalignment at a lightly misaligned condition and increases with misalignment at a highly misaligned condition.

Sharda et al. [49] reported the EHD performance characteristics of a misaligned journal bearing. They observed that the bush flexibility improves the bearing load capacity and the stability of the bearing.

Yu and Keith [50] developed a boundary element cavitation algorithm to predict cavitation in journal bearings with axially variable clearance. The results for aligned and misaligned journal bearings are compared with those obtained using the finite difference method.

Qiu and Tieu [51] presented all static and dynamic characteristics of a horizontally grooved bearing under different eccentricity and misalignment conditions and compared them with the available experimental data. The effect of misalignment on all bearing characteristics and on the stability of the rotor-bearing system are analyzed.

### **2.3 THERMOHYDRODYNAMIC STUDIES**

The study of thermal effects in journal bearings has received considerable attention in last few decades. A large number of investigations on the thermal effects in journal bearings have been reported in literature. A brief review of the thermal literature on hydrodynamic journal bearing are reviewed in the following sub-sections under the heading of aligned and misaligned journal bearings :

#### **2.3.1 Aligned Journal Bearings**

One of the earliest analytical works is reported by Kingsbury [52] who outlined a graphical solution method for solving three equations : Newton's law of viscous shear across the fluid film, a viscosity-temperature relation; and equation for heat generation and heat flow in the fluid film.

One of the earliest experimental works is that of Musket and Morgan[53] who performed heat balance experiments for a journal bearing. They reported that it takes a long time (4 to 6 hours in some cases) to reach temperature equilibrium in a journal bearing.

Hagg [54] studied the influence of speed and viscosity on the shear stress of a journal bearing. He provided a graph for estimating frictional power losses in a journal bearing operating under high-speed conditions.

Clayton and Wilkie [55] experimentally obtained temperature profile in both radial and circumferential directions along with the temperature gradient in the bush under steady-state condition. Other early experimental work was performed by Hamilton et al. [56] who investigated a method of predicting heat dissipation in journal bearings.

Cole [57] measured bush temperature distribution in radial and circumferential directions and reported discrepancies between actual operating conditions and results of the constant viscosity assumption. He showed that only about one-half of the heat generated in the lubricant film of the bearing was carried away by the lubricant flow. Burr[58] continued this work and concluded that heat dissipated from a rotating shaft may be much more than from a stationary shaft and from the bearing housings.

The significance of including variation of viscosity across the film thickness was first reported by Zienkiewicz [59] in the year 1958, who discovered that it is responsible for an appreciable change in load-carrying capacity in bearings and therefore, should not be neglected in theoretical studies.

Tipei and Nica [60] conducted experiments to study the influence of viscosity variation on the operating conditions of finite journal bearing.

Dowson et al. [61], conducted a major experimental work and found that heat flow patterns in the bush are a combination of both radial flow and a significant amount of circumferential flow traveling from the hot region in the vicinity of minimum film thickness to the cooler region near the oil inlet. Dowson's test results also showed that cyclic variation in shaft surface temperature is small and shaft can, therefore, be treated as an isothermal component. They also inferred that shaft temperature remains within a few degrees of the mean temperature of bush inner surface. Lastly, the experiments indicated that the axial temperature gradients within the bush are negligible.

Much more sophisticated analytical work was done by Nica [62] who obtained a temperature distribution for a finite journal bearing considering viscosity variation. He presented the results for film temperature for some specific cases which compared well with the earlier experimental results of Tipei and Nica [60]. The short comings of this work was the assumption of constant viscosity across the film and the half-Sommerfeld condition used.

McCallion et al. [63] suggested decoupling of the energy equation from that of the Reynolds equation by neglecting pressure gradients in the film. Appropriate equations were derived and solved numerically by the finite difference method. The results compared favourably with the experimental work of Dowson [61].

Majumdar and Saha [64] also assumed a constant viscosity across film thickness and experimented with a bearing of length-to-diameter ratio of one and compared the load capacity with isothermal solution of Raimondi et al. [8]. They found a 35 % reduction in the load capacity at eccentricity ratio = 0.4.

Gardner and Ulshmid [65] carried out an experimental work under turbulent conditions and determined the journal temperatures under various operating conditions for two journal bearing applications.

Safar and Szeri [66] extended the thermohydrodynamic theory for the bearing from laminar to turbulent regime. Assuming that the heat transfer in the bush is in the radial direction only, they integrated the Laplace heat-conduction equation in the solid and applied the boundary condition of matching the temperature and heat flux at that interface. This boundary condition significantly simplifies the computations since it removes the necessity of iteration on that boundary. However, the method has limitation in determining the location and magnitude of the maximum bush temperature.

Following the earlier experimental work of Majumdar and Saha [64], Majumdar[67] numerically solved the Energy and Reynolds equations by neglecting heat conduction in the bush. It was reported that the extent of the film and also the load capacity decreases if one includes the density and viscosity variation in the lubricant as a function of temperature.

Sugnami and Szeri [68] found that under laminar conditions, the shaft temperature remains constant at approximately the discharge oil temperature: whereas, under turbulent conditions, an insulated shaft assumption yielded results that were in closer agreement with the experimental work of Gardner and Ulshmid [65].

Pinkus and Bupara [69] described a method of including variable viscosity in bearing analysis by the use of a simple energy equation uncoupled from the Reynolds equation. The relevant adiabatic solutions are made independent of the specifics of the kind of lubricant used and of its initial conditions. Performance results such as load, friction, maximum

temperature, flow, etc. are given for the two-axial-groove journal bearing covering three  $L/D$  ratios. The effect of variable viscosity on the performance of misaligned bearings is also examined.

Boncompain et al. [70] considered variation of viscosity across oil film and solved the 3-D energy equation in fluid domain and 2-D heat conduction equation for bush. They analyzed dynamic characteristics of a finite journal bearing under laminar conditions. Thermal effects were found to cause a large change in stiffness and damping coefficients. Ferron et al. [71] followed a similar procedure but they solved 3-D energy equation and 3-D heat conduction equation and computed a mixing temperature by performing a simple energy balance between recirculating oil and supply oil at the inlet. Good agreement was found with measurements reported for pressure and temperature, but a large discrepancy was noted between the predicted and measured values of eccentricity ratios.

Mitsui and Yamada [72] performed a thermohydrodynamic (THD) analysis in which thermal conductivity and specific heat were assumed to be constant while viscosity was assumed to vary as a function of temperature. The transverse velocity term was neglected. The two-dimensional Laplace heat-conduction equation was solved by an infinite series approximation. The energy equation was solved using the Crank-Nicholson's method and the Reynolds equation by the successive over-relaxation method. They found that both journal velocity and lubricant viscosity have a profound effect on the temperature fields and the load-carrying capacity.

Tonnesen and Hansen [73] reported experimental study of thermal effects in hydrodynamic journal bearings operating under steady-state conditions. Both viscosity and oil inlet geometry were found to have a significant effect on the operating temperatures. The shaft temperature was found to increase with decreasing loads when a high-viscosity lubricant was used. Moreover, the oil flow governed by the inlet pressure and / or the inlet geometry was found to have a pronounced influence on the onset of instability leading to the conclusion that these factors must be considered in the design of bearings.

Knight and Barrett [74] developed an approximate solution technique for multilobe journal bearings. Their energy equation was a simple ordinary differential equation based on a constant axial temperature and constant temperature across the film. They neglected the

heat conduction in lubricant and bush and assumed that all the heat generated is removed by the side leakage. In their latter work [75] the effect of heat transfer between the fluid film and its surrounding was also included in an approximate way. The viscosity across the film was assumed to be constant and the heat exchanged in the oil supply grooves was not considered. They found that the supply pressure has pronounced effects on the operating eccentricities, temperature, flow rate and dynamic characteristics of the bearing studied.

Lund et al [76,77] presented an approximate analysis for solving the 3-D energy equation for journal bearing film, coupled with the heat conduction equation for the bearing sleeve. The analysis approximates the temperature profile across the film thickness by a fourth order polynomial while the circumferential temperature variation is expressed in terms of a Fourier series expansion. Heat exchange in the supply groove were considered in this work. To test the validity of the theory, experiments were performed for two-axial groove journal bearing over a range of loads and speeds. A fair agreement was reported between the theoretical and their experimental work.

Gethin and Medwell [78] experimentally investigated the thermohydrodynamic behaviour of a high speed cylindrical bore journal bearing fed by two-axial-grooves. They highlighted the complex temperature patterns which are generated in the bush.

Gethin [79] described a theoretical investigation into the effect of thermoplastic bush deformation and journal thermal expansion on the behavior of a cylindrical bore bearing fed by two-axial grooves. The main conclusion which follows from this work is that thermoelastic bush deformation is an important factor in predicting bearing performance for a given load.

Boncompain et al. [80] solved the 3-D energy equation in fluid domain and 2-D heat conduction equation for bush. They extended their earlier work [70 - 71] in order to take into account the reversed flow at the inlet and the recirculating heat flux across the cavitation zone. Thermoelastic displacements of the shaft and the bush are also taken into account. Theoretical results are then compared with the experimental data.

Khonsari and Beaman [81] investigated thermohydrodynamic effects in journal bearings operating under steady-state loading and formulated an analytical model for finite journal bearings. The model includes correction factors for the cavitation effects and the

mixing of the recirculating oil and supply oil at the inlet. They compared their results with those of Dowson et al [61]. Results show that the oil-bush interface temperature drops slightly in the vicinity of the inlet followed by a rapid rise in the circumferential direction and drop in the cavitation region. The computed mixing temperature is very close to the minimum bush temperature obtained experimentally indicating the cooling effect of the inlet oil propagating into the solid.

Heshmat and Pinkus [82] offered a conceptual and experimental investigation of the mechanism of mixing in the oil groove, in terms of the relevant flows and temperatures. Results of tests aimed at measuring the mixing inlet temperatures as a function of a range of operating conditions and bearing sizes are correlated in terms of an appropriate mixing function. Empirical equations based on the above correlation for the determination of inlet temperatures are then given for both thrust and journal bearings.

Khonsari [83,84] presents an extensive survey of the work pertaining to thermal effects in hydrodynamic bearings. It includes discussions on theoretical, computational and experimental works pertaining to heat effects in journal bearings published in various technical journals.

Braun et al. [85] coupled the Reynolds equation with the energy equation to perform a rather complete analysis for a cryogenic journal bearing. To correctly account for the changes in the fluid viscosity, the energy equation was coupled with the shaft and bearing heat conduction equations. The effects of pressure and temperature on the density, viscosity, and load-carrying capacity were further discussed.

Singh et al. [86] studied thermoelastohydrodynamic (TEHD) effects for an elliptical bearing. The 3-D momentum and continuity equations and 3-D elasticity equations were solved to obtain pressure in the lubricant flow field and deformation in the bearing liner. The energy equation and Fourier heat conduction equation were used to obtain the temperature distribution in the fluid film and the bearing body. They used an approach similar to the one proposed by Lund et al. [76] for thermal analysis but limited thermal solution to the positive pressure region only.

Sinhasan et al. [87] presented TEHD analysis of a two-axial-groove journal bearing using finite element method to solve the governing equations. The conventional 3-D

energy equation was modified to account for the flow of lubricant to the cavitation region but it does not consider the heat exchange in the supply grooves.

Paranjpe and Han [88] developed a THD analysis of finite journal bearings based on a fully conservative finite-volume formulation for accurate prediction of the bearing performance characteristics. Sample calculations for automotive bearing show that the oil supply pressure and supply configuration (oil feed arrangement) significantly affect the bearing performance.

Gethin and El-Deihi [89] presented a thermal model for comparison between predicted and measured performance trends of a twin axial groove journal bearing. The analysis confirms the sensitivity of global bearing performance with respect to loading direction.

Tripp and Melodick [90] presented a series of measurements taken with experimental setup using a commercially representative lubricant and a non-Newtonian lubricant. Analysis of the measurements show that the oil film thickness in the hydrodynamic regime of lubrication with non-Newtonian oils is dominated by the shear-dependent viscosity and the elastic effects of these oils do not make a significant contribution to the oil film load bearing capacity.

Sheeja and Prabhu [91] carried out a theoretical and experimental investigations for a plain journal bearing considering the influence of thermal effects on Non-Newtonian lubrication. The investigations reveal that the thermal effect is more pronounced than the non-Newtonian effect on the dynamic characteristics.

Fitzgerald and Neal [92] contributed to the stock of experimental thermal data for axial-groove journal bearings. They also discussed the consequences of neglecting the effect of bush heat transfer in the prediction of operating temperature.

Paranjpe and Han [93] developed a THD analysis of steadily loaded journal bearings that included mass conserving cavitation, 3-D energy equation in the oil film, heat conduction in the bushing and journal, and mixing of hot re-circulating oil with the supply oil. The journal temperature is assumed constant and is computed by satisfying “no-net-heat-flux” condition. Predictions of the analysis compared well with the measurements reported by Dowson et al. [61]. They [94] further developed a transient THD analysis for

dynamically loaded journal bearings that included mass conserving cavitation and 3-D energy equation in the oil film with moving grids in the oil film. The journal is treated as a lumped thermal element. They considered the thermal transients in the oil film and found that the oil film temperatures vary considerably over time and space.

Gethin [95] presented a numerical investigation into the effect of temperature boundary conditions and viscous dissipation on the behaviour of two- and three-lobe bearings. For two-lobe bearing, setting the viscous generation to zero in the cavitated film gave the best agreement with the experiment. The model was also used successfully for the three-lobe bearing confirming its general applicability.

### **2.3.2 Misaligned Journal Bearings**

Some work has been reported in the area of misaligned thermohydrodynamic journal bearings as discussed below :

Barun et al. [96] analyzed the effects of misalignment on the energy dissipation mechanism and presented an analysis that coupled the various equations involved. The journal temperature was assumed to be constant. The effects of misalignment on pressure and fluid temperature were analyzed for isothermal, adiabatic and arbitrary( i.e. finite heat transfer coefficient) fluid film boundary conditions. Simultaneous solutions were obtained by finite difference method and the results were presented for various tilt angles. The model was found to useful in predicting hot-spots in a journal bearing.

Safar, Mokhtar and Peekent [97] undertook adiabatic analysis of a journal bearing for maximum allowable misalignment with a length-to-diameter ratio of one. The Reynolds and energy equations were solved simultaneously using finite difference method. The results showed that the bearing behaviour is significantly affected by journal misalignment.

Jang et al. [98] presented an adiabatic solution for a misaligned journal bearing operating with non-Newtonian lubricant.

Chun and Lalas [99] carried out a parametric study of the effects of oil inlet temperature and pressure in half-circumferential / axial grooved journal bearings under aligned and misaligned conditions. The effects of changes of the wall temperature and the L/D ratio have been examined. The wall temperature and heat transfer rate have been found to be of secondary importance to the mixing effectiveness at the groove and the resulting

final mixture temperature. The variability of the temperature, has been shown to smooth out the peaks of both pressure and friction during the misaligned conditions.

## 2.4 PRESENT WORK

The survey presented above indicates that, although the investigations on thermohydrodynamic (THD) lubrication attracted a large number of researchers during the past more than three decades, the studies remained confined mainly to the problems of aligned journal bearings. The published literature on the THD analysis of misaligned journal bearings is rather scant. Even the published literature on the isothermal studies on misaligned journal bearings is generally limited to the circular bearing geometries. Studies on the effect of misalignment in the case of noncircular bearing geometries, such as elliptical and three-lobe bearings, have also been scant. Therefore, a need has been felt to investigate the thermohydrodynamic effects in misaligned circular and noncircular bearing configurations with more realistic mathematical models and boundary conditions for the physical problems involved.

The work contained in this work pertains to four bearing configurations, namely, circular plain, circular two-axial-groove, elliptical and three-lobe. Static and dynamic performance characteristics together with temperature and pressure profiles, isotherms for liquid and solid domains and other thermal characteristics of these bearing geometries, Figs. 2.1(a) to (d) have been studied and appropriate conclusions drawn.

The generalised Reynolds equation is used to represent the flow of lubrication in the clearance space of the journal bearing. The film thickness is expressed by including bi-planar misalignment. A suitable viscosity-temperature relation has been used to express viscosity of oil as a function of temperature. For the thermal analysis, the temperature distributions in the fluid-flow field (liquid domain) and the bush-housing assembly are respectively represented by the three-dimensional energy and Fourier heat conduction equations. Because of rotation and high thermal conductivity of the material, the journal temperature distribution is assumed to vary only along its length with convective heat transfer from axial ends. Appropriate boundary conditions have been used for the solution of all governing

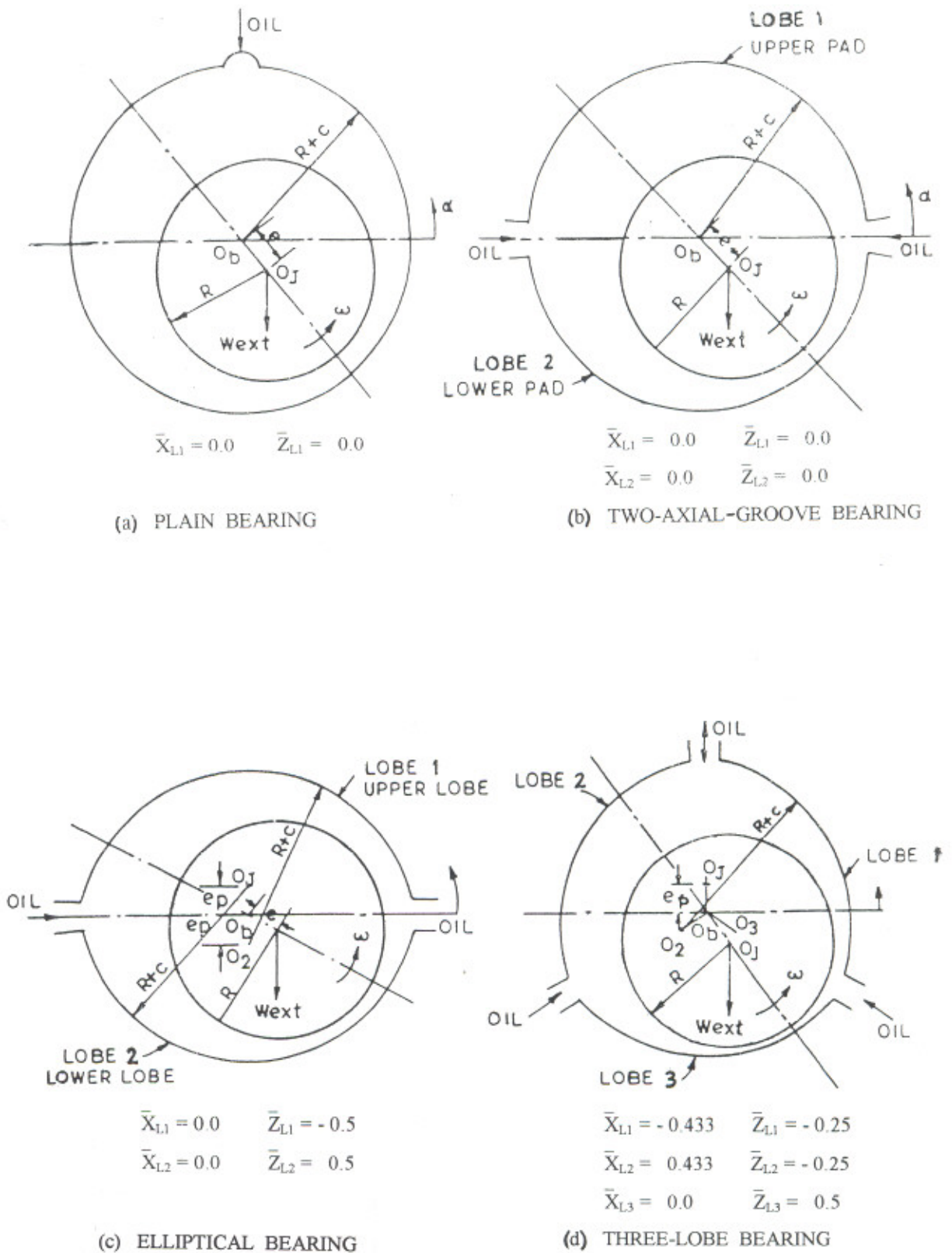


Fig. 2.1 JOURNAL BEARING GEOMETRIES

equations. Mixing of recirculating hot oil with the cool supply oil has been considered. Heat transfer from solid-to-liquid and liquid-to-solid in the oil supply grooves is included.

Finite element method is employed for solving the Reynolds equation whereas finite difference method with successive over relaxation (SOR) technique has been used to obtain solution of the energy and heat conduction equations. The overall computational procedure used is an iterative one. Initially a temperature field is assumed. Viscosity obtained from this field is used to solve the Reynolds equation. The pressure field from the solution of Reynolds equation is used as input for the solution of energy equation. Converged temperature fields for the liquid and solid(bush and journal) domains are obtained from the solutions of energy and heat conduction equations in nested loops. These temperature fields are then used to modify viscosity field for the solution of Reynolds equation. Iterations are carried out until a converged solution of pressure and temperature fields are obtained. Suitable weighting factors have been used to improve the rates of convergence of various iteration loops.

Various performance characteristics of all journal bearing configurations studied in this work have been presented as functions of load. These characteristics include mid-plane eccentricity ratio, attitude angle, minimum fluid-film thickness, side flow, sixteen stiffness and sixteen damping coefficients, instability threshold speed and whirl frequency ratio. Maximum bush temperature and mean journal temperature have also been plotted as functions of load. Pressure and temperature profiles and isotherms have been presented for specified loads for all types of bearings. Results for aligned bearings have been given for ready reference. Before presenting the results, however, the analysis, the boundary conditions and the developed computer programme have been adequately validated by comparing the present results with the published experimental and the theoretical results. Linearised journal-centre motion trajectories have been drawn to carry out stability analysis.

Some of the important conclusions drawn from this study are

- Misalignment reduces the load carrying capacity and the value of minimum fluid-film thickness for all types of bearings.
- Misalignment improves the stability margin.
- Misalignment makes the bearings to run hotter.

- For the study of misaligned bearings it is essential to use a three-dimensional thermal analysis whereas a two-dimensional analysis is adequate for the study of the aligned bearings.

## 2.5 ORGANISATION OF THESIS

The thesis has been presented in six chapters. A brief detail of each of the six chapters are given below :

In chapter I, importance of journal bearings, introduction to various types of analyses and significance of the present work have been covered.

Chapter II presents a survey of literature starting from the earliest works. Although the survey encompasses the works on isothermal, elastohydrodynamic, thermohydrodynamic and thermoelastohydrodynamic analyses, a more emphasis has been given to the thermal studies. Research papers on aligned as well as misaligned bearings have been surveyed and the gaps in the literature are clearly brought out. In the light of the prevailing gaps in literature, the problem for the present work has been defined.

Chapter III deals with the details of mathematical modelling and analysis. In this chapter, various equations with boundary conditions are given. Mathematical relations for the computation of static and dynamic performance characteristics are also given in this chapter.

Computational experiment used in the present work is described in chapter IV. The algorithms for the solution of problem are presented in flow chart form for better understanding.

Results and discussions are given in chapter V. The chapter first deals with the validity of the present work and then the results for various types of bearings are presented and discussed.

In chapter VI main conclusions drawn from the present work are given. A brief outline of the future work is also given in this chapter.

## CHAPTER - III

### MATHEMATICAL MODELLING AND ANALYSIS

#### 3.1 INTRODUCTION

In this chapter, the mathematical formulation for THD analysis of hydrodynamic journal bearings, in general, and misaligned hydrodynamic journal bearings, in particular, have been dealt with. In the succeeding sections of this chapter, representative mathematical models for the flow field, bush-housing assembly and journal with appropriate boundary conditions are described. System equations for the discretised solution domains using finite element method / finite difference method have been obtained. The analysis is presented in non-dimensional form.

In general, a thermohydrodynamic analysis involves the solutions of the following coupled equations

- i) Generalised Reynolds equation for pressure field in fluid domain.
- ii) Energy equation for fluid temperature field.
- iii) Heat conduction equations for bush and journal temperature fields.

The solution of the Reynolds equation requires the viscosity of lubricating oil, which is computed using the temperature field obtained from the solution of energy equation. The solution of energy equation needs the pressure field established from the solution of Reynolds equation. Further, the energy equation for fluid domain and heat conduction equations for bush and journal need to be solved simultaneously to establish the temperature fields for the fluid, the bush and the journal.

Thus, the overall solution of a THD problem involves handling the above coupled equations. The converged solutions of these equations yield the pressure field, the temperature fields for the fluid, the bush and the journal. The viscosity field is obtained using established temperature field for the fluid and a suitable viscosity-temperature relationship.

In the present work, the Reynolds equation is solved using finite element method whereas, the energy and heat conduction equations are solved using finite difference

method. The expressions for static and dynamic performance characteristics of a journal bearing system are also given in this Chapter. Using the linearized approach, in which the motion of the journal centre is restricted within a close proximity of the static equilibrium position, Routh's criteria is applied to the resulting characteristic equation to derive the expressions for critical mass, instability threshold speed and whirl frequency ratio and also to trace the journal-centre motion trajectories for a free translatory whirl.

### 3.2 HYDRODYNAMIC ANALYSIS

#### 3.2.1 Generalized Reynolds Equation

The flow of lubricant in the clearance space of each lobe / pad of a journal bearing is governed by the following generalized non-dimensional modified Reynolds equation :

$$\frac{\partial}{\partial \alpha} \left( \bar{h}^3 \cdot \bar{F}_2 \frac{\partial \bar{p}}{\partial \alpha} \right) + \frac{\partial}{\partial \beta} \left( \bar{h}^3 \cdot \bar{F}_2 \frac{\partial \bar{p}}{\partial \beta} \right) = \frac{\partial}{\partial \alpha} \left( \bar{h} - \bar{h} \frac{\bar{F}_1}{\bar{F}_0} \right) + \frac{\partial \bar{h}}{\partial \bar{t}} \quad \text{----- (3.1)}$$

The non-dimensional viscosity functions  $\bar{F}_0$ ,  $\bar{F}_1$  and  $\bar{F}_2$  in equations (3.1) are defined by the relations :

$$\bar{F}_0 = \int_0^1 \frac{1}{\bar{\mu}} d\bar{z}; \quad \bar{F}_1 = \int_0^1 \frac{\bar{z}}{\bar{\mu}} d\bar{z}; \quad \bar{F}_2 = \int_0^1 \frac{\bar{z}}{\bar{\mu}} \left( \bar{z} - \frac{\bar{F}_1}{\bar{F}_0} \right) d\bar{z} \quad \text{----- (3.2)}$$

#### 3.2.2 Fluid-Film Thickness

The fluid-film thickness in non-dimensional form for any lobe in equation (3.1) is given by the following relation :

$$\begin{aligned} \bar{h} = 1 - \{ \bar{X}_{j0} - \bar{X}_{Li} + \bar{X} + \beta ( \bar{\gamma}_x + \Delta \bar{\gamma}_x ) \} \cos \alpha \\ - \{ \bar{Z}_{j0} - \bar{Z}_{Li} + \bar{Z} + \beta ( \bar{\gamma}_z + \Delta \bar{\gamma}_z ) \} \sin \alpha \end{aligned} \quad \text{----- (3.3)}$$

where, for an aligned bearing,

$\bar{\gamma}_x = \bar{\gamma}_z = 0$  and  $\Delta \bar{\gamma}_x = \Delta \bar{\gamma}_z = 0$  and the equation reduces to :

$$\bar{h} = 1 - ( \bar{X}_{j0} - \bar{X}_{Li} + \bar{X} ) \cos \alpha - ( \bar{Z}_{j0} - \bar{Z}_{Li} + \bar{Z} ) \sin \alpha \quad \text{----- (3.4)}$$

For a bearing operating under a steady-state condition, the perturbation coordinates  $\bar{X}$ ,  $\bar{Z}$ ,  $\Delta \bar{\gamma}_x$  and  $\Delta \bar{\gamma}_z$  are each equal to zero.

The value of  $\frac{\partial \bar{h}}{\partial \bar{t}}$  is obtained by differentiating the expression for  $\bar{h}$  partially with respect to non-dimensional time,  $\bar{t}$ .

### 3.2.3 Viscosity - Temperature Relation

In equation (3.2), the non-dimensional viscosity of the lubricant,  $\bar{\mu}$  ( $= \mu / \mu_0$ ) is obtained from the relation [77] :

$$\mu / \rho = 10^{-6} [\exp ( b ( T + 273.12 )^{-a} ) - 0.8] \quad \text{----- (3.5)}$$

where a and b are the viscosity coefficients and are obtained by specifying values of  $\mu$  at two different temperatures and  $\rho$  is the density of the lubricant. For isothermal (IHD) studies, the viscosity is assumed to be constant.

### 3.2.4 Boundary Conditions for Reynolds Equation

The boundary conditions used for solving the generalised Reynolds equation for pressure distribution,  $\bar{p}$  are :

(i) At the bearing side boundaries i.e. ( for  $\beta = \pm \lambda$  )

$$\bar{p} = 0 \quad \text{----- (3.6a)}$$

(ii) On the supply groove boundaries

$$\bar{p} = \bar{p}_s = 0 \quad \text{----- (3.6b)}$$

here,  $\bar{p}_s$  is non-dimensional supply pressure (assumed 0 in the present work)

(iii) At the trailing edge of the positive pressure region : The following two conditions must be satisfied simultaneously.

$$\bar{p} = 0 \quad \text{----- (3.6c)}$$

$$\frac{\partial \bar{p}}{\partial \alpha} = 0 \quad \text{----- (3.6d)}$$

In the present work, these boundary conditions are satisfied by using the linear complementarity approach described by Chandrawat et al. [20].

### 3.2.5 Finite Element Formulation of Reynolds Equation

The entire lubricant flow domain is discretised using two-dimensional, 4-noded linear, isoparametric finite elements. For the study of a misaligned bearing, full axial length is discretised while for aligned bearings only half of the length is considered taking advantage of axial symmetry.

The pressure distribution  $\bar{p}^e$  over an element is represented as :

$$\bar{p}^e = \sum N_i \cdot \bar{p}_i, \quad i = 1, 2, \dots, 4 \quad \text{----- (3.7)}$$

where  $N_i$  is the shape function for the  $i$ th node of an element and  $\bar{p}_i$  is the corresponding nodal pressure.

Using orthogonality condition of Galerkin's method [100], equation (3.1) yields the following system equations for the discretised fluid domain.

$$[A] \{ \bar{p} \} = \{ B \} + \{ \bar{Q} \} \quad \text{----- (3.8)}$$

where  $[A]$  is the assembled coefficient matrix,  $\{ B \}$  is the assembled column vector consisting of static and dynamic terms and  $\{ \bar{Q} \}$  is the nodal flow vector.  $\{ \bar{p} \}$  is the nodal pressure vector.

The relations for computing the coefficients of the matrices  $[A]$ ,  $\{ B \}$  and  $\{ \bar{Q} \}$  in equation (3.8) for the  $e$ th element are given below :

For the  $m$ th iteration, the equation (3.1) yields,

$$[a_{ij}^e]^m = \int_{\bar{A}_e} \bar{h}^3 \bar{F}_2^{m-1} \left( \frac{\partial N_i}{\partial \alpha} \frac{\partial N_j}{\partial \alpha} + \frac{\partial N_i}{\partial \beta} \frac{\partial N_j}{\partial \beta} \right) d \bar{A}; \quad i = 1, 2, \dots, 4 \quad \text{----- (3.9a)}$$

$$\{b_i^e\}^m = \int_{\bar{A}_e} \left[ \left( \bar{h} - \bar{h} \frac{\bar{F}_1^{m-1}}{\bar{F}_0^{m-1}} \frac{\partial N_i}{\partial \alpha} \right) - N_i \frac{\partial \bar{h}}{\partial t} \right] d \bar{A}; \quad i = 1, 2, \dots, 4 \quad \text{----- (3.9b)}$$

$$\{ \bar{q}_i^e \}^m = \int_{\Gamma_{\beta^e}} N_i \left[ \bar{h} \bar{F}_2^{m-1} \frac{\partial \bar{p}}{\partial \alpha} - \bar{h} \left( 1 - \frac{\bar{F}_1^{m-1}}{\bar{F}_0^{m-1}} \right) \right] \partial \beta + \int_{\Gamma_{\alpha^e}} N_i \bar{h} \bar{F}_2^{m-1} \frac{\partial \bar{p}}{\partial \beta} \partial \alpha; \quad i = 1, 2, \dots, 4 \quad \text{----- (3.9c)}$$

### 3.3 THERMAL ANALYSIS

In this section, the energy equation for the lubricant flow field, the heat conduction equations for the bush-housing assembly and the journal with their respective boundary conditions are presented.

#### 3.3.1 Energy Equation

The energy equation in lubrication is generally obtained by incorporating the usual thin-film approximation into the generalized energy equation. This equation is, however, valid only in the positive-pressure region. With the inception of cavitation, the fluid-film

splits into a large number of streamers which are separated by air/gas streams. In the present work it is assumed that the lubricant and gas streamers are distributed in such a manner that the lubricating film consists of the uniformly distributed streams of oil and air/gas and its properties may be represented by the effective fluid properties, defined later in this section. Thus, the three-dimensional energy equation, given below, is applied to the entire fluid domain (i.e. lubrication and cavitation regions) to obtain the lubricant temperature field :

$$\bar{h}^2 \left( \bar{u} \frac{\partial \bar{T}_f}{\partial \bar{\alpha}} + \bar{v} \frac{\partial \bar{T}_f}{\partial \bar{\beta}} + \frac{\bar{w}}{\bar{h}} \frac{\partial \bar{T}_f}{\partial \bar{z}} \right) = \bar{D}_e \cdot \bar{\mu} \left[ \left( \frac{\partial \bar{u}}{\partial \bar{z}} \right)^2 + \left( \frac{\partial \bar{v}}{\partial \bar{z}} \right)^2 \right] + \bar{P}_e \frac{\partial^2 \bar{T}_f}{\partial \bar{z}^2} \quad \text{----- (3.10)}$$

The operator  $\frac{\partial}{\partial \bar{\alpha}}$  is introduced to change the shape of the film into a rectangular field and is given by :

$$\frac{\partial}{\partial \bar{\alpha}} = \frac{\partial}{\partial \alpha} - \frac{\bar{z}}{\bar{h}} \left( \frac{\partial \bar{h}}{\partial \alpha} \right) \frac{\partial}{\partial \bar{z}} \quad \text{----- (3.11)}$$

The substitution from equation (3.11) into equation (3.10) yields

$$\bar{h}^2 \left[ \bar{u} \frac{\partial \bar{T}_f}{\partial \alpha} + \bar{v} \frac{\partial \bar{T}_f}{\partial \beta} + \frac{1}{\bar{h}} \left( \bar{w} - \bar{z} \bar{u} \frac{\partial \bar{h}}{\partial \alpha} \right) \frac{\partial \bar{T}_f}{\partial \bar{z}} \right] = \bar{D}_e \cdot \bar{\mu} \left[ \left( \frac{\partial \bar{u}}{\partial \bar{z}} \right)^2 + \left( \frac{\partial \bar{v}}{\partial \bar{z}} \right)^2 \right] + \bar{P}_e \frac{\partial^2 \bar{T}_f}{\partial \bar{z}^2} \quad \text{----- (3.12)}$$

The values of non-dimensional effective inverse Peclet number ( $\bar{P}_e$ ) and Dissipation number ( $\bar{D}_e$ ) are as follows :

$$\bar{P}_e = \frac{k_{fe}}{C_{pe} * \rho_e * \omega_j * c^2} \quad \text{----- (3.13a)}$$

$$\bar{D}_e = \frac{\mu_e * \omega_j}{C_{pe} * \rho_e * T_r * c^2} \quad \text{----- (3.13b)}$$

In computing effective inverse Peclet number ( $\bar{P}_e$ ) and Dissipation number ( $\bar{D}_e$ ), effective values of fluid properties ( $k_{fe}$ ,  $C_{pe}$ ,  $\rho_e$  and  $\mu_e$ ) are used, given by the following relations :

$$k_{fe} = k_f C_L + k_{air} (1 - C_L) \quad \text{----- (3.14a)}$$

$$C_{pe} = C_p C_L + C_{pair} (1 - C_L) \quad \text{----- (3.14b)}$$

$$\rho_e = \rho C_L + \rho_{air} (1 - C_L) \quad \text{----- (3.14c)}$$

$$\mu_e = \mu_f C_L + \mu_{air} (1 - C_L) \quad \text{----- (3.14d)}$$

The contraction coefficient,  $C_L$ , in the above relations is defined as follows :

$$C_L = \int_0^1 (\bar{u} \bar{h})_{te} d\bar{z} / \int_0^1 (\bar{u} \bar{h}) d\bar{z} \quad \text{----- (3.15)}$$

The subscript  $te$  refers to the trailing edge of positive-pressure region of the fluid-film.  $C_L$  is unity in the positive-pressure region.

The values of the non-dimensional velocities  $\bar{u}$  and  $\bar{v}$  have been found using the following relations :

$$\bar{u} = \bar{h}^2 \frac{\partial \bar{p}}{\partial \alpha} \left[ \int_0^{\bar{z}} \frac{\bar{z}}{\bar{\mu}} d\bar{z} - \frac{\bar{F}_1}{\bar{F}_0} \int_0^{\bar{z}} \frac{d\bar{z}}{\bar{\mu}} \right] + \frac{1}{\bar{F}_0} \int_0^{\bar{z}} \frac{d\bar{z}}{\bar{\mu}} \quad \text{----- (3.16)}$$

$$\bar{v} = \bar{h}^2 \frac{\partial \bar{p}}{\partial \beta} \left[ \int_0^{\bar{z}} \frac{\bar{z}}{\bar{\mu}} d\bar{z} - \frac{\bar{F}_1}{\bar{F}_0} \int_0^{\bar{z}} \frac{d\bar{z}}{\bar{\mu}} \right] \quad \text{----- (3.17)}$$

The non-dimensional velocity,  $\bar{w}$  is obtained by solving the following continuity equation :

$$\frac{\partial \bar{u}}{\partial \alpha} + \frac{\partial \bar{v}}{\partial \beta} + \frac{1}{\bar{h}} \frac{\partial \bar{w}}{\partial \bar{z}} = 0 \quad \text{----- (3.18)}$$

Due to numerical uncertainties, however, direct computation of the velocities using equation (3.18) induces difficulties in the convergence process. Thus to determine the radial velocity component ( $\bar{w}$ ), the continuity equation (3.18) is differentiated partially with respect to  $\bar{z}$  which gives a second order differential equation :

$$\frac{\partial^2 \bar{w}}{\partial \bar{z}^2} + \bar{h} \frac{\partial}{\partial \bar{z}} \left( \frac{\partial \bar{u}}{\partial \alpha} + \frac{\partial \bar{v}}{\partial \beta} \right) - \frac{\partial}{\partial \bar{z}} \left( \bar{z} \frac{\partial \bar{u}}{\partial \bar{z}} \frac{\partial \bar{h}}{\partial \alpha} \right) = 0 \quad \text{----- (3.19)}$$

This equation is then integrated using the finite differences with the following boundary conditions :

$$\bar{w} = 0 \quad \text{at} \quad \bar{z} = 0 \quad \text{----- (3.20a)}$$

$$\bar{w} = \frac{\partial \bar{h}}{\partial \alpha} \quad \text{at} \quad \bar{z} = 1 \quad \text{----- (3.20b)}$$

### 3.3.1.1 Boundary Conditions for the Energy Equation

(i) On the fluid-journal interface ( $\bar{z} = 1$ ) :

$$\bar{T}_f = \bar{T}_j \quad \text{----- (3.21a)}$$

(ii) On the fluid-bush interface ( $\bar{z} = 0$ ) :

$$\bar{T}_f = \bar{T}_b \quad \text{----- (3.21b)}$$

(iii) Temperature distribution at the inlet edge of the bearing pad :

The following two alternative boundary conditions for temperature distribution have been examined.

#### *Boundary Conditions 1 (BC 1)*

The temperature is assumed to vary quadratically with  $\bar{z}$ , i.e.

$$\bar{T}_f = a_0 + a_1 \bar{z} + a_2 \bar{z}^2 \quad \text{----- (3.21c)}$$

The coefficients of the above equation,  $a_0$ ,  $a_1$  and  $a_2$  are obtained by using equation (3.21a) and (3.21b) together with relation (3.30) and (3.32) given later.

#### *Boundary Conditions 2 (BC 2)*

The temperature is given by the following cubic polynomial :

$$\bar{T}_f = a_0 + a_1 \bar{z} + a_2 \bar{z}^2 + a_3 \bar{z}^3 \quad \text{----- (3.21d)}$$

In order to obtain the values of the coefficients of this equation, the conditions defined by equations (3.21a), (3.21b), (3.30) and (3.32) are employed as in the case of BC 1. But, however,  $\bar{T}_b$  in equation (3.21b) is assumed to be equal to supply temperature  $\bar{T}_s$  and  $\frac{\partial \bar{T}_f}{\partial \bar{z}}$  is taken as zero consistent with the boundary condition at the inlet of pad for the bush, discussed later.

#### **3.3.1.2 Finite Difference Formulation of Energy Equation**

The system equations for the entire fluid-domain is obtained as follows based on the nodes nomenclature given in Fig. 3.1 :

$$\begin{aligned} \bar{T}_{f,i,j,k} = & K_1 \bar{T}_{f,i-1,j,k} + K_2 \bar{T}_{f,i,j,k-1} + K_3 \bar{T}_{f,i,j,k+1} \\ & + K_4 \bar{T}_{f,i,j-1,k} + K_5 \bar{T}_{f,i,j+1,k} \end{aligned} \quad \text{----- (3.22)}$$

where  $K_1$ ,  $K_2$ ,  $K_3$ ,  $K_4$  and  $K_5$  are the coefficients of the equation, the values of which are evaluated using grid geometry and values of velocity components, fluid-film thickness,  $\bar{D}_e$  and  $\bar{P}_e$ .

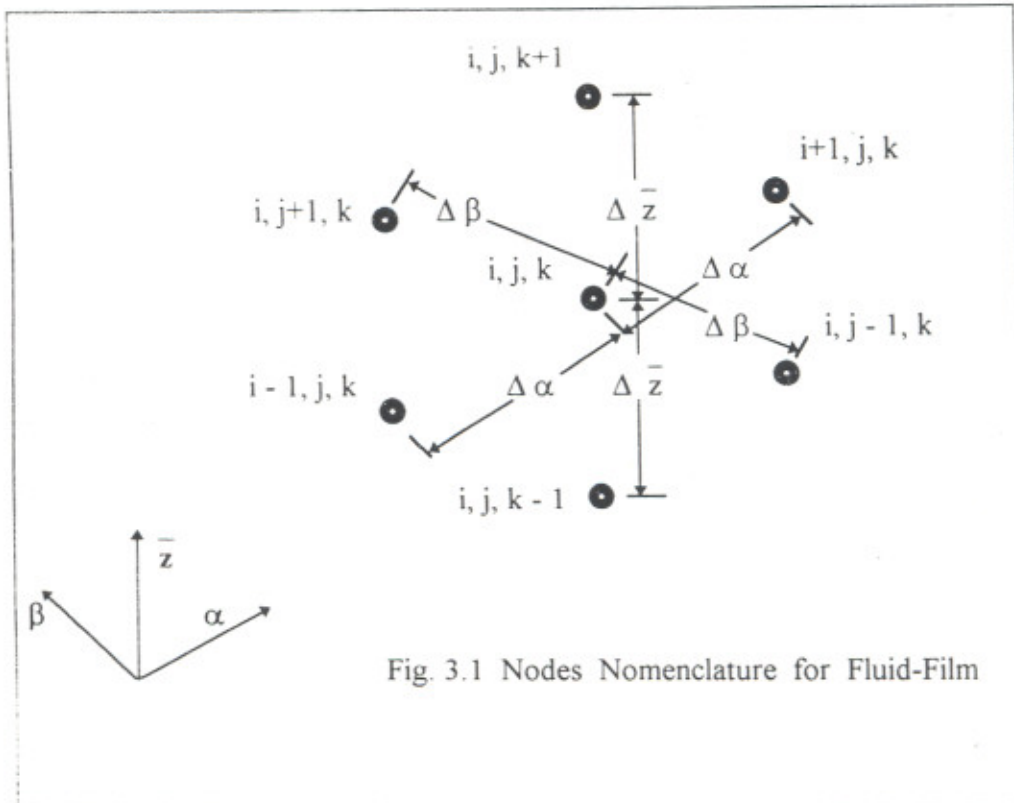


Fig. 3.1 Nodes Nomenclature for Fluid-Film

### 3.3.2 Heat Conduction Equation for Bush-Housing

The Fourier heat conduction equation in the non-dimensional cylindrical coordinate form, given below, has been solved for the temperature distribution in the bush and the housing, both of which have been assumed to be cylindrical and possessing the same thermal properties.

$$\frac{\partial^2 \bar{T}_b}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{T}_b}{\partial \bar{r}} + \frac{\partial^2 \bar{T}_b}{\partial \beta^2} + \frac{1}{\bar{r}^2} \frac{\partial^2 \bar{T}_b}{\partial \alpha^2} = 0 \quad \text{----- (3.23)}$$

#### 3.3.2.1 Boundary Conditions for Heat Conduction Equation

The following boundary conditions have been used for the solution of heat conduction equation (3.23)

(i) On the fluid-bush interface ( $\bar{z} = 0, \bar{r} = \bar{R}_1$ ):

Continuity of heat flux gives

$$k_b \cdot \left( \frac{\partial \bar{T}_b}{\partial \bar{r}} \right) \Big|_{\bar{r} = \bar{R}_1} = - \frac{k_f}{\bar{c} \cdot \bar{h}} \left( \frac{\partial \bar{T}_f}{\partial \bar{z}} \right) \Big|_{\bar{z} = 0} \quad \text{----- (3.24a)}$$

(ii) On the outer part of the bush-housing ( $\bar{r} = \bar{R}_2$ ):

The free convection and radiation hypothesis gives

$$\left( \frac{\partial \bar{T}_b}{\partial \bar{r}} \right) \Big|_{\bar{r} = \bar{R}_2} = - \frac{h_{ab} \cdot R}{k_b} (\bar{T}_b \Big|_{\bar{r} = \bar{R}_2} - \bar{T}_a) \quad \text{----- (3.24b)}$$

(iii) On the lateral faces of the bearing ( $\beta = \pm \lambda$ ):

The free convection and radiation hypothesis gives

$$\left( \frac{\partial \bar{T}_b}{\partial \beta} \right) \Big|_{\beta = \pm \lambda} = - \frac{h_{ab} \cdot R}{k_b} (\bar{T}_b \Big|_{\beta = \pm \lambda} - \bar{T}_a) \quad \text{----- (3.24c)}$$

(iv) At the outlet edge of bearing pad, free convection from bush to oil in the supply groove

gives

$$\left( \frac{\partial \bar{T}_b}{\partial \alpha} \right) \Big|_{\alpha = \alpha_e} = - \frac{h_{fb} \cdot R}{k_b} (\bar{T}_b - \bar{T}_s) \quad \text{----- (3.24d)}$$

where  $\alpha_e$  = circumferential coordinate of the outlet edge of bearing pad

(v) At the inlet edge of the bearing pad ( $\alpha = \alpha_i$ ); two alternative boundary conditions consistent with boundary conditions for fluid-film have been used.

Thus, with BC 1: at the oil supply point on the outer surface

$$\bar{T}_b \Big|_{\bar{r} = \bar{R}_2} = \bar{T}_s \quad \text{----- (3.24e)}$$

In addition, a free convection of heat between oil and solid has been assumed giving

$$\left( \frac{\partial \bar{T}_b}{\partial \alpha} \right) \Big|_{\alpha = \alpha_i} = - \frac{h_{fb} \cdot R}{k_b} (\bar{T}_b - \bar{T}_s) \quad \text{----- (3.24f)}$$

where  $\alpha_i$  = circumferential coordinate of the inlet edge of bearing pad

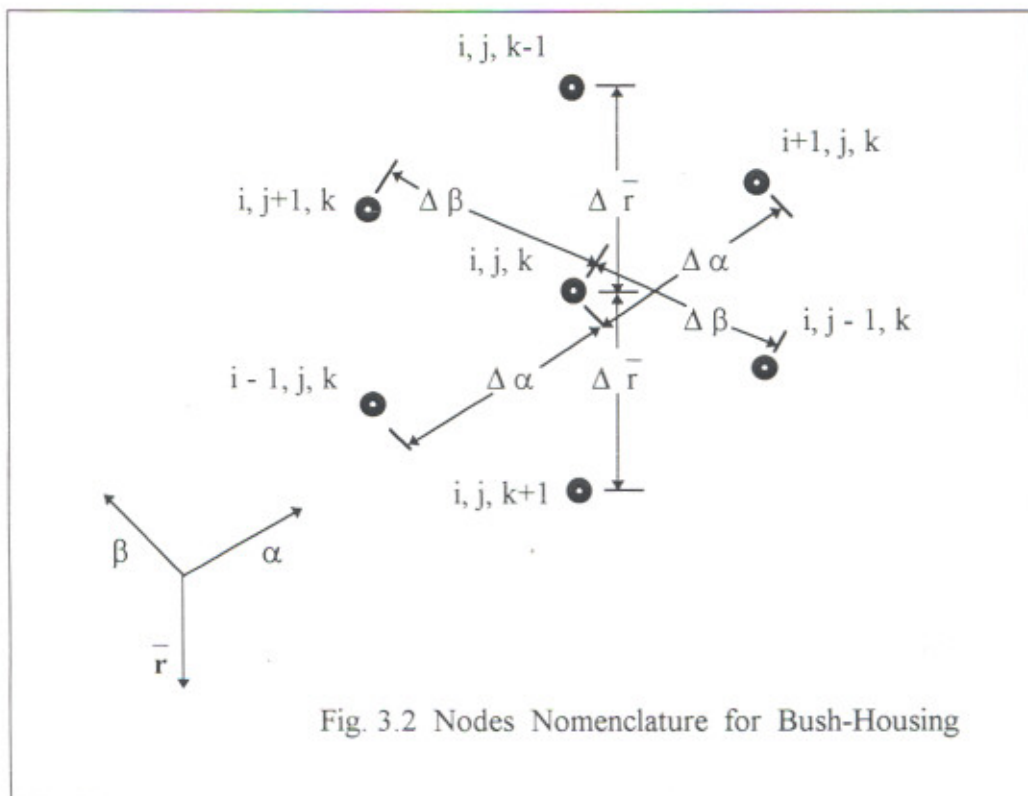
with BC 2,  $\bar{T}_b = \bar{T}_s$  for all values of  $\bar{r}$ . ----- (3.24g)

### 3.3.2.1 Finite Difference Formulation

The system equations for the entire discretised bush-housing assembly is formulated as below based on the nodes nomenclature given in Fig. 3.2 :

$$\begin{aligned} \bar{T}_{b_{ij,k}} = & C_1 \bar{T}_{b_{i,j,k+1}} + C_2 \bar{T}_{b_{i,j,k-1}} + C_3 \bar{T}_{b_{i,j+1,k}} \\ & + C_4 \bar{T}_{b_{i,j-1,k}} + C_5 \bar{T}_{b_{i+1,j,k}} + C_6 \bar{T}_{b_{i-1,j,k}} \end{aligned} \quad \text{----- (3.25)}$$

where  $C_1, C_2, C_3, C_4, C_5$  and  $C_6$  are the coefficients determined from the grid geometry in the case of all internal nodes. For nodes on the boundary, appropriate boundary conditions have been applied assuming heat transfer in all the three directions in order to compute the values of coefficients  $C_i$ s.



### 3.3.3 Heat Conduction Equation for the Journal

For the purpose of establishing the journal temperature field, the following assumptions have been made :

- (i) Heat is conducted by the journal only in the axial direction.
- (ii) Journal temperature does not vary in radial or circumferential direction at any section.
- (iii) Heat is flowing out of the journal from its axial ends.

The steady-state unidirectional heat conduction equation used is :

$$k_J \cdot \left( \frac{\partial^2 \bar{T}_J}{\partial y^2} \right) \Delta y \cdot A_J + \Delta q = 0 \quad \text{----- (3.26a)}$$

where  $\Delta q$  = heat input to the element =  $q \cdot \Delta y$

$q$  = heat input per unit length

$\Delta y$  = length of the element,

and  $A_J$  = cross-sectional area of the journal =  $\pi \cdot R^2$

Equation (3.26a) reduces to the following non-dimensional form :

$$\pi \cdot \left( \frac{\partial^2 \bar{T}_J}{\partial \beta^2} \right) + \bar{q} = 0 \quad \text{----- (3.26b)}$$

The non-dimensional heat input to journal,  $\bar{q}$  is given by

$$\bar{q} = - \frac{k_r}{k_j \cdot \bar{c}} \left[ \int_0^{2\pi} \frac{1}{h} \left( \frac{\partial \bar{T}_r}{\partial \bar{z}} \right) d\alpha \right] - \frac{\bar{c} \cdot h_{fj} \cdot R}{k_j} (\bar{T}_j - \bar{T}_m) \cdot \alpha_G \quad \text{----- (3.27)}$$

where  $\alpha_G$  is the groove angle in radians.

In equation (3.27), the second term on right hand side represents the heat transfer between the journal and the lubricant in oil groove.

#### 3.3.3.1 Boundary Conditions for Heat Conduction Equation for Journal

The above equation have been solved with the following boundary condition :

At axial ends, i.e.  $\beta = \pm \lambda$

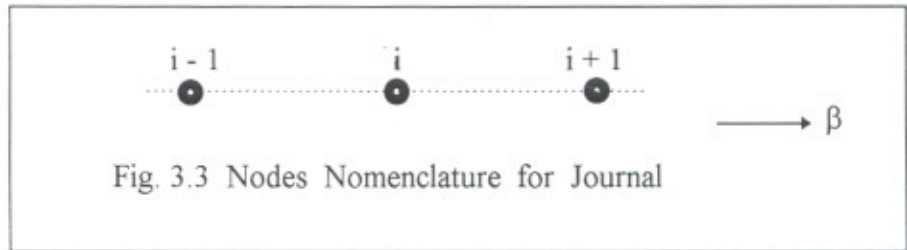
$$\left( \frac{\partial \bar{T}_J}{\partial \beta} \right)_{|\beta = \pm \lambda} = - \frac{h_{aj} \cdot R}{k_j} (\bar{T}_{j|\beta = \pm \lambda} - \bar{T}_a) \quad \text{---- (3.28)}$$

#### 3.3.3.1 Finite Difference Formulation

The system equation for the entire discretised journal is formulated as below based on the nodes nomenclature given in Fig. 3.3 :

$$\bar{T}_{j,i} = A_1 \bar{T}_{j,i-1} + A_2 \bar{T}_{j,i+1} + A_3 \quad \text{----- (3.29)}$$

where  $A_1$ ,  $A_2$  and  $A_3$  are the coefficients determined from the grid geometry in the case of all internal nodes. For nodes on the boundary, appropriate boundary conditions have been applied assuming heat transfer in two directions in order to compute the values of coefficients  $A_i$ 's.



### 3.3.4 Mixing in the Oil Groove

Hot recirculating oil mixes with incoming cold oil from oil supply. The equation used for computing the mean temperature,  $\bar{T}_m$  is based on the overall energy balance [71]. Thus

$$\bar{Q} \cdot \bar{T}_m = \bar{T}_r \cdot \bar{Q}_r + \bar{T}_s \cdot \bar{Q}_s \quad \text{----- (3.30)}$$

and for a unit length of bearing

$$\bar{Q} = \int_0^1 (\bar{h} \cdot \bar{u}) \Big|_{\alpha_i} d\bar{z} \quad \text{----- (3.31a)}$$

$$\bar{Q}_r = \int_0^1 (C_L \cdot \bar{h} \cdot \bar{u}) \Big|_{\alpha_e} d\bar{z} \quad \text{----- (3.31b)}$$

$$\bar{T}_r \cdot \bar{Q}_r = \int_0^1 (C_L \cdot \bar{h} \cdot \bar{u} \cdot \bar{T}_f) \Big|_{\alpha_e} d\bar{z} \quad \text{----- (3.31c)}$$

$$\bar{Q}_s = \bar{Q} - \bar{Q}_r \quad \text{----- (3.31d)}$$

The mean temperature,  $\bar{T}_m$  is related to the assumed temperature distribution,  $\bar{T}_f(\bar{z})$  across the fluid-film at the inlet of the bearing pad as below :

$$\bar{T}_m = \int_0^1 \bar{T}_f(\bar{z}) \cdot d\bar{z} \quad \text{----- (3.32)}$$

### 3.4 PERFORMANCE CHARACTERISTICS

The relations for computing static and dynamic performance characteristics are given as follows :

#### 3.4.1 Static Performance Characteristics

These characteristics include mid-plane eccentricity ratio, attitude angle and side flow. Pressure and temperature fields obtained are used in the following relations to compute values of various parameters.

##### (a) Bearing Load and Moment

The fluid-film reaction components in the x and z directions are :

$$\begin{Bmatrix} \bar{W}_x \\ \bar{W}_z \end{Bmatrix} = \int_{-\lambda}^{\lambda} \int_0^{2\pi} \bar{p} \begin{Bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{Bmatrix} d\alpha d\beta \quad \text{----- (3.33)}$$

For a vertical load support,  $\bar{W}_x = 0$  and  $\bar{W}_z = \bar{W}$ , load supported by the bearing.

In addition to the force components, there will be unbalance moment components about x- and z- axis in a misaligned journal bearing. The dimensionless moment components acting on the journal are given by :

$$\begin{Bmatrix} \bar{M}_x \\ \bar{M}_z \end{Bmatrix} = \int_{-\lambda}^{\lambda} \int_0^{2\pi} \beta \bar{p} \begin{Bmatrix} \cos(\alpha) \\ -\sin(\alpha) \end{Bmatrix} d\alpha d\beta \quad \text{----- (3.34)}$$

For an aligned bearing  $\bar{M}_x = \bar{M}_z = 0$ .

##### (b) Steady-State Equilibrium Position (Eccentricity Ratio and Attitude Angle)

The steady-state equilibrium position of a journal is usually expressed in terms of eccentricity ratio ( $\epsilon_0$ ) and attitude angle ( $\phi$ ). The values of eccentricity ratio and attitude angle are established during the iterative solution of a journal bearing problem satisfying the condition of vertical load support for any specified load ( $\bar{W}$ ).

##### (c) Side Flow ( $\bar{Q}_s$ )

The side flow parameter is given by the relation :

$$Q_s = - \int_0^{2\pi} \frac{h^3}{\mu} \left( \frac{\partial \bar{p}}{\partial \beta} \right)_{\beta=\lambda} d\alpha - \int_0^{2\pi} \frac{h^3}{\mu} \left( \frac{\partial \bar{p}}{\partial \beta} \right)_{\beta=-\lambda} d\alpha \quad \text{----- (3.35)}$$

### 3.4.2 Dynamic Performance Characteristics

For a misaligned journal bearing system, the dynamic performance characteristics are expressed in terms of sixteen stiffness and sixteen damping coefficients. The values of stiffness and damping coefficients are evaluated by assuming that the disturbed position of the journal centre remains in close proximity of its static equilibrium position so that changes in temperature, and hence, fluid viscosity are small and negligible in comparison to their steady-state values.

The dynamic coefficients are computed to account for changes in force and moment arising from lateral and angular displacement and velocity perturbations. Thus, thirty-two such coefficients are required to define the oil-film forces and moments produced by a dynamically misaligned bearing. In the case of an aligned bearing, there are no moment components, and therefore, only eight dynamic coefficients are sufficient to represent its dynamic behaviour. These coefficients are defined below.

The expressions for dynamic coefficients are given by :

$$\begin{aligned}
 \begin{bmatrix} \delta \bar{W}_x \\ \delta \bar{W}_z \\ \delta \bar{M}_x \\ \delta \bar{M}_z \end{bmatrix} &= \begin{bmatrix} -\partial \bar{W}_x / \partial \bar{X} & -\partial \bar{W}_x / \partial \bar{Z} & -\partial \bar{W}_x / \partial \bar{\gamma}_x & -\partial \bar{W}_x / \partial \bar{\gamma}_z \\ -\partial \bar{W}_z / \partial \bar{X} & -\partial \bar{W}_z / \partial \bar{Z} & -\partial \bar{W}_z / \partial \bar{\gamma}_x & -\partial \bar{W}_z / \partial \bar{\gamma}_z \\ -\partial \bar{M}_x / \partial \bar{X} & -\partial \bar{M}_x / \partial \bar{Z} & -\partial \bar{M}_x / \partial \bar{\gamma}_x & -\partial \bar{M}_x / \partial \bar{\gamma}_z \\ -\partial \bar{M}_z / \partial \bar{X} & -\partial \bar{M}_z / \partial \bar{Z} & -\partial \bar{M}_z / \partial \bar{\gamma}_x & -\partial \bar{M}_z / \partial \bar{\gamma}_z \end{bmatrix} \begin{bmatrix} \bar{X} \\ \bar{Z} \\ \bar{\gamma}_x \\ \bar{\gamma}_z \end{bmatrix} \\
 &+ \begin{bmatrix} -\partial \bar{W}_x / \partial \dot{\bar{X}} & -\partial \bar{W}_x / \partial \dot{\bar{Z}} & -\partial \bar{W}_x / \partial \dot{\bar{\gamma}}_x & -\partial \bar{W}_x / \partial \dot{\bar{\gamma}}_z \\ -\partial \bar{W}_z / \partial \dot{\bar{X}} & -\partial \bar{W}_z / \partial \dot{\bar{Z}} & -\partial \bar{W}_z / \partial \dot{\bar{\gamma}}_x & -\partial \bar{W}_z / \partial \dot{\bar{\gamma}}_z \\ -\partial \bar{M}_x / \partial \dot{\bar{X}} & -\partial \bar{M}_x / \partial \dot{\bar{Z}} & -\partial \bar{M}_x / \partial \dot{\bar{\gamma}}_x & -\partial \bar{M}_x / \partial \dot{\bar{\gamma}}_z \\ -\partial \bar{M}_z / \partial \dot{\bar{X}} & -\partial \bar{M}_z / \partial \dot{\bar{Z}} & -\partial \bar{M}_z / \partial \dot{\bar{\gamma}}_x & -\partial \bar{M}_z / \partial \dot{\bar{\gamma}}_z \end{bmatrix} \begin{bmatrix} \dot{\bar{X}} \\ \dot{\bar{Z}} \\ \dot{\bar{\gamma}}_x \\ \dot{\bar{\gamma}}_z \end{bmatrix}
 \end{aligned}$$

----- (3.36)

or,

$$\begin{bmatrix} \delta \bar{W}_x \\ \delta \bar{W}_z \\ \delta \bar{M}_x \\ \delta \bar{M}_z \end{bmatrix} = \begin{bmatrix} \bar{S}_{xx} & \bar{S}_{xz} & \bar{S}_{xy} & \bar{S}_{zy} \\ \bar{S}_{zx} & \bar{S}_{zz} & \bar{S}_{zy} & \bar{S}_{zy} \\ \bar{S}_{xyx} & \bar{S}_{xyy} & \bar{S}_{yxx} & \bar{S}_{yxy} \\ \bar{S}_{zyx} & \bar{S}_{zyy} & \bar{S}_{zyx} & \bar{S}_{zyy} \end{bmatrix} \begin{bmatrix} \bar{X} \\ \bar{Z} \\ \bar{\gamma}_x \\ \bar{\gamma}_z \end{bmatrix} + \begin{bmatrix} \bar{B}_{xx} & \bar{B}_{xz} & \bar{B}_{xy} & \bar{B}_{zy} \\ \bar{B}_{zx} & \bar{B}_{zz} & \bar{B}_{zy} & \bar{B}_{zy} \\ \bar{B}_{xyx} & \bar{B}_{xyy} & \bar{B}_{yxx} & \bar{B}_{yxy} \\ \bar{B}_{zyx} & \bar{B}_{zyy} & \bar{B}_{zyx} & \bar{B}_{zyy} \end{bmatrix} \begin{bmatrix} \bar{\dot{X}} \\ \bar{\dot{Z}} \\ \bar{\dot{\gamma}}_x \\ \bar{\dot{\gamma}}_z \end{bmatrix} \quad \text{----- (3.37)}$$

Equation (3.36) is combined with the equations (3.33) and (3.34) to convert the load and moment components into pressure derivatives for determining the values of dynamic coefficients.

### 3.5 STABILITY ANALYSIS

The journal of a hydrodynamic bearing, if disturbed from its static equilibrium position, experiences changes in the hydrodynamic forces and moments acting on it. This disturbs the equilibrium of the journal and induces free translatory and conical whirl in the journal. The dynamic behaviour of a journal bearing system under these conditions can be studied by integrating the resulting nonlinear equations of motion for the disturbed journal.

The technique of linearization, as described below, is found to be quite good to deal with this type of problem, especially when the disturbance is confined to close proximity of the static equilibrium position.

#### 3.5.1 Linearized Equation of Motion and Stability Margin

The equations of disturbed motion of the journal for free translatory whirl are as follows

$$[ \bar{M}_J ] \begin{Bmatrix} \bar{X} \\ \bar{Z} \end{Bmatrix} = \begin{Bmatrix} \delta \bar{W}_x ( \bar{X}, \bar{Z}, \bar{\dot{X}}, \bar{\dot{Z}} ) \\ \delta \bar{W}_z ( \bar{X}, \bar{Z}, \bar{\dot{X}}, \bar{\dot{Z}} ) \end{Bmatrix} \quad \text{----- (3.38)}$$

where

$[ \bar{M}_J ]$  is the diagonal mass matrix;

$\ddot{\bar{X}}, \ddot{\bar{Z}}$  are the components of acceleration in x and z-directions;

$\delta \bar{W}_x, \delta \bar{W}_z$  are the components of the out of balance force on the journal in x and z-directions; respectively.

$\delta \bar{W}_x$  and  $\delta \bar{W}_z$  depend on the instantaneous position and velocity of the journal centre and are usually non-linear functions of  $\bar{X}, \bar{Z}, \dot{\bar{X}}$  and  $\dot{\bar{Z}}$ . However, within close proximity of the equilibrium position of the journal, hydrodynamic forces may be assumed to be linear functions of  $\bar{X}, \bar{Z}, \dot{\bar{X}}$  and  $\dot{\bar{Z}}$ .

Stiffness and damping coefficients, computed using the equations (3.36 - 3.37), may now be considered invariant in the small proximity of the equilibrium position and the equation (3.36) can be written as :

$$\bar{M}_J \ddot{\bar{X}} + \bar{B}_{xx} \dot{\bar{X}} + \bar{B}_{xz} \dot{\bar{Z}} + \bar{S}_{xx} \bar{X} + \bar{S}_{xz} \bar{Z} = 0 \quad \text{----- (3.39a)}$$

$$\bar{M}_J \ddot{\bar{Z}} + \bar{B}_{zx} \dot{\bar{X}} + \bar{B}_{zz} \dot{\bar{Z}} + \bar{S}_{zx} \bar{X} + \bar{S}_{zz} \bar{Z} = 0 \quad \text{----- (3.39b)}$$

The characteristic equation for the linearized equation of motion is a quadratic polynomial equation of the following form :

$$\sigma^4 + A_1 \sigma^3 + A_2 \sigma^2 + A_3 \sigma + A_4 = 0 \quad \text{----- (3.40)}$$

where  $\sigma$  is a complex variable, and

$A_1, A_2, A_3, A_4$  are functions of the journal mass,  $\bar{M}_J$  and the dynamic coefficients.

Using the characteristic equation (3.40) and Routh's criteria, the stability margin of the journal bearing system, in terms of critical mass  $\bar{M}_C$ , is obtained. The journal speed at the threshold of instability of the journal bearing system is called the instability threshold speed or simply threshold speed and is determined by :

$$\bar{\Omega} = \frac{\Omega}{\omega_n} = \sqrt{(\bar{M}_C / \bar{W})} \quad \text{----- (3.41)}$$

and

$$\bar{M}_C = \frac{K_1}{\bar{v}^2} \quad \text{----- (3.42)}$$

where  $\bar{v}$  is defined as the whirl frequency ratio.

$$K_1 = \frac{\bar{S}_{xx} \bar{B}_{zz} + \bar{S}_{zz} \bar{B}_{xx} - \bar{S}_{xz} \bar{B}_{zx} - \bar{S}_{zx} \bar{B}_{xz}}{\bar{B}_{xx} + \bar{B}_{zz}} \quad \text{----- (3.43)}$$

$$\bar{v}^2 = \frac{(\bar{S}_{xx} - K_1)(\bar{S}_{zz} - K_1) - \bar{S}_{xz} \cdot \bar{S}_{zx}}{\bar{B}_{xx} \bar{B}_{zz} - \bar{B}_{xz} \bar{B}_{zx}} \quad \text{----- (3.44)}$$

and

$$\omega_n = \sqrt{(g/c)} \quad \text{----- (3.45)}$$

$v$  is called whirl frequency ratio of the journal bearing system for free translatory whirl.

A journal bearing system is asymptotically stable when the journal mass  $\bar{M}_J$  is less than  $\bar{M}_C$ . Likewise, a system is asymptotically stable when operating speed of the journal is less than the threshold speed,  $\Omega$ . A negative value of  $\bar{M}_C$  indicates that the journal bearing system will be stable for all the values of journal mass  $\bar{M}_J$ . Similarly, a negative value of  $\bar{v}^2$  implies the absence of whirl.

### 3.5.2. Journal Centre-Motion Trajectory

Equations (3.39) can be integrated to give the displacement and velocity components of instantaneous journal centre position. The locus of the instantaneous journal centre position, as the bearing responds to a given initial disturbance, can be traced by repetitively integrating the governing equations of motion. This locus is known as the journal centre motion trajectory. For simplicity, it will be referred to as "linear trajectory".

At any time  $\bar{t}$ , the equation (3.39) is written as :

$$[\bar{M}_J] \begin{Bmatrix} \bar{\ddot{X}} \\ \bar{\ddot{Z}} \end{Bmatrix}_{\bar{t}} = \begin{Bmatrix} \delta \bar{W}_x \\ \delta \bar{W}_z \end{Bmatrix}_{\bar{t}} = \begin{bmatrix} \bar{S}_{xx} & \bar{S}_{xz} & \bar{B}_{xx} & \bar{B}_{xz} \\ \bar{S}_{zx} & \bar{S}_{zz} & \bar{B}_{zx} & \bar{B}_{zz} \end{bmatrix} \begin{Bmatrix} \bar{X} \\ \bar{Z} \\ \bar{\dot{X}} \\ \bar{\dot{Z}} \end{Bmatrix}_{\bar{t}} \quad \text{----- (3.46)}$$

where the subscript  $\bar{t}$  refers to the instantaneous values and the stiffness and damping coefficients ( $\bar{S}_{xx}$ ,  $\bar{S}_{xz}$ ,  $\bar{S}_{zx}$ ,  $\bar{S}_{zz}$ ,  $\bar{B}_{xx}$ ,  $\bar{B}_{xz}$ ,  $\bar{B}_{zx}$ ,  $\bar{B}_{zz}$ ) are evaluated at the static equilibrium position of the journal centre.

The second order equations (3.46) is written in the first order form by defining  $\bar{X} = \bar{X}_1$ ,  $\bar{Z} = \bar{X}_2$ ,  $\bar{\dot{X}} = \bar{X}_3$ ,  $\bar{\dot{Z}} = \bar{X}_4$  as :

$$\begin{aligned}
 \begin{pmatrix} \bar{X}_1 \\ \bar{X}_2 \\ \bar{X}_3 \\ \bar{X}_4 \end{pmatrix}_{\bar{t}} &= \begin{pmatrix} \bar{X}_3 \\ \bar{X}_4 \\ \delta \bar{W}_x / \bar{M}_J \\ \delta \bar{W}_z / \bar{M}_J \end{pmatrix}_{\bar{t}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\bar{S}_{xx} / \bar{M}_J & -\bar{S}_{xz} / \bar{M}_J & -\bar{B}_{xx} / \bar{M}_J & -\bar{B}_{xz} / \bar{M}_J \\ -\bar{S}_{zx} / \bar{M}_J & -\bar{S}_{zz} / \bar{M}_J & -\bar{B}_{zx} / \bar{M}_J & -\bar{B}_{zz} / \bar{M}_J \end{bmatrix} \begin{pmatrix} \bar{X}_1 \\ \bar{X}_2 \\ \bar{X}_3 \\ \bar{X}_4 \end{pmatrix}_{\bar{t}} \\
 & \text{----- (3.47)}
 \end{aligned}$$

Equations (3.47) are numerically integrated using the fourth order Runge-Kutta method to obtain the displacement and velocity components of the journal at time  $\bar{t} + \delta \bar{t}$  in terms of the known values at time  $\bar{t}$ . This gives a linear trajectory.

\*\*\*\*\*

# CHAPTER - IV

## SOLUTION PROCEDURE

### 4.1. INTRODUCTION

The governing system equations developed in the previous chapter for the thermohydrodynamic analysis of journal bearings have been solved using a personal computer of the type "Pentium PC (166 MHz)". The computer program is written in FORTRAN-77 and it requires a computer memory of about 500 kB. This chapter presents the strategy adopted to find the matched solutions of the lubrication (Reynolds), energy and heat conduction equations to establish fluid-film pressure field. Also, the solution scheme for plotting linearised journal-centre motion trajectories is discussed.

Since the solution procedure involves various iterative stages, the convergence criterion for each stage is also given in this chapter. The methods adopted to achieve a faster rate of convergence of the direct iteration schemes for the coupled problems have also been discussed

The developed program is general and can be used to obtain IHD as well as THD solutions for a specified eccentricity ratio or a given vertical load.

### 4.2. SOLUTION SCHEME FOR MAIN PROGRAM

The pressure field, attitude angle and components of fluid-film reaction ( $\bar{W}_x$ ,  $\bar{W}_z$ ) satisfying vertical load support for a specified eccentricity ratio/given vertical load are established using three nested loops as shown in Fig. 4.1. Outside the outer loop, all the temperatures are set equal to the reference temperature and the non-dimensional viscosity functions are then calculated. The Reynolds equation is solved and attitude angle is modified to satisfy the condition of vertical load support in the inner loop (index J). The middle loop (index K) is used to obtain thermal solution. This loop can, however, be skipped by setting the value of index NTEMP to 0. The value of the eccentricity ratio required for the given load  $\bar{W}$  is found in the outer loop (index I). This loop may also be skipped by setting  $NWZ = 0$  for solving the problem for the specified eccentricity ratio.

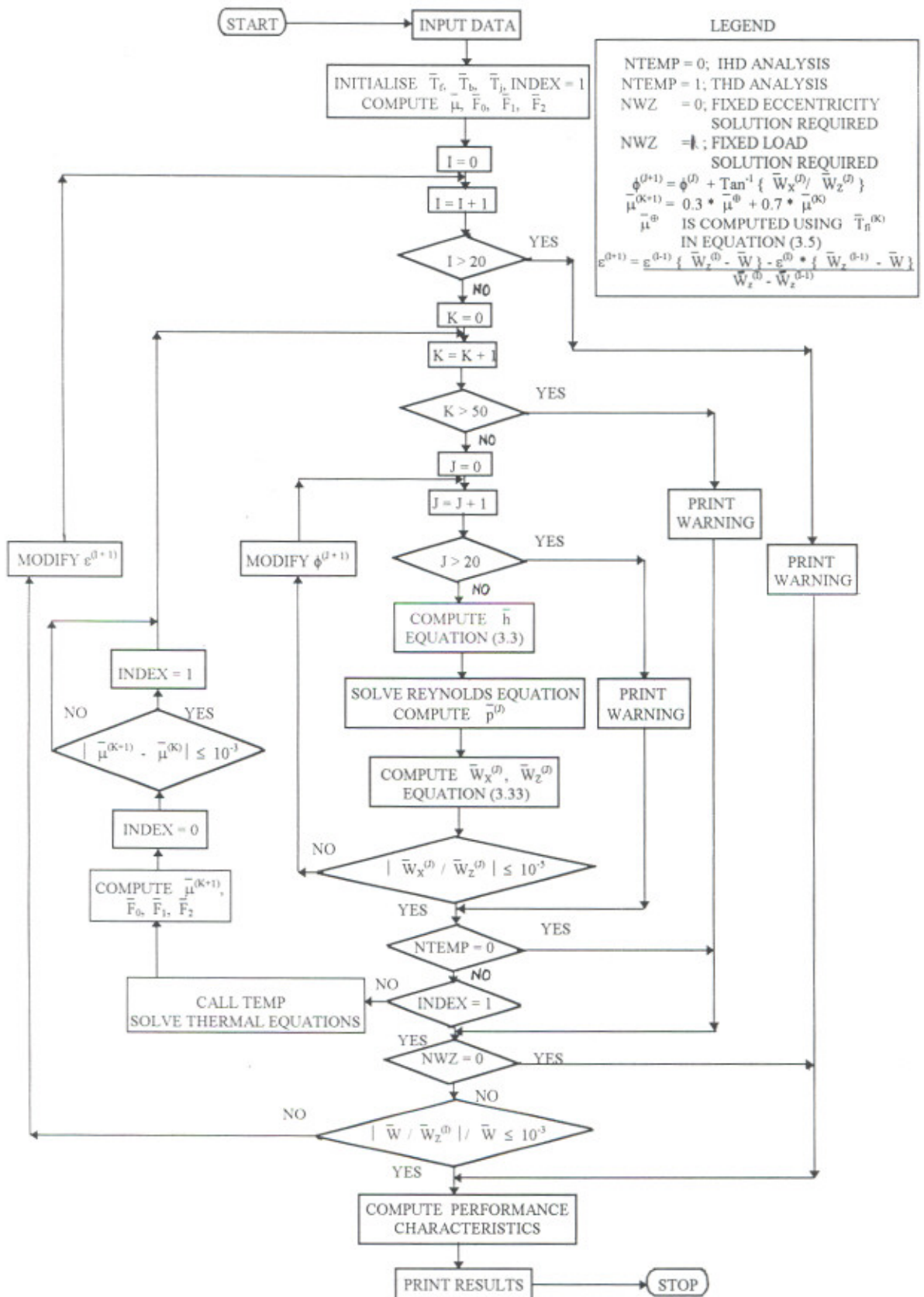


Fig. 4.1 SOLUTION SCHEME FOR MAIN PROGRAM

The solution of Reynolds equation is obtained by using Linear Complementarity Problem (LCP) approach described by Chandrawat et al.[20] in this inner loop. For termination of this inner loop, two criteria have been used. The first criterion is concerned with the convergence of solution according to which the horizontal component of film reaction must become diminishingly small in comparison with the vertical component. In the  $J$ th iteration, the horizontal ( $\bar{W}_x^{(J)}$ ) and vertical ( $\bar{W}_z^{(J)}$ ) components of the fluid film reactions are computed using equation (3.33). When the condition

$$| \bar{W}_x^{(J)} / \bar{W}_z^{(J)} | \leq 0.00001 \quad \text{----- (4.1)}$$

is satisfied, the solution is considered to have converged and the iterations are terminated otherwise the attitude angle,  $\phi$  is modified using the equation (4.2) given below :

$$\phi^{(J+1)} = \phi^{(J)} + \tan^{-1} ( \bar{W}_x^{(J)} / \bar{W}_z^{(J)} ) \quad \text{----- (4.2)}$$

The loop is also terminated when the number of iterations exceeds 20.

When a thermohydrodynamic analysis is to be performed, the index NTEMP is set equal to 1. This allows the control to pass to the middle loop. For finding the solution of energy and heat conduction equations, the subroutine TEMP is called in this loop with the iteration number of the middle loop as its argument. This subroutine returns the computed temperature fields for lubricant, bush and journal. Oil viscosity is then computed using the following relation {equation (3.5)} :

$$\bar{\mu}^{(K+1)} = Wt_v \cdot \frac{e^{b \cdot \{T_f^{(K)} + 273.12\}^{-a} - 0.8}}{\mu_o} + (1 - Wt_v) \cdot \bar{\mu}^{(K)} \quad \text{----- (4.3)}$$

In the above relation  $Wt_v$  is the weighting factor and its value as 0.3 is found to be quite good for the solution of all problems considered in this work. The middle loop is terminated when the following condition is satisfied at every node of the fluid-film :

$$| \bar{\mu}^{(K+1)} - \bar{\mu}^{(K)} | \leq 0.001 \quad \text{----- (4.4)}$$

As noted earlier, the outer loop is used to modify the eccentricity ratio for getting the solution corresponding to a given load. When  $NWZ = 1$ , this loop is executed and when the following convergence criteria is satisfied, the loop is terminated.

$$| \bar{W} - \bar{W}_Z^{(I)} | / \bar{W} \leq 0.001 \quad \text{----- (4.5)}$$

If the convergence is not achieved and the value of loop index  $I$  is less than 20, the eccentricity is modified using the following relations :

When  $I \geq 2$  ;

$$\epsilon^{(I+1)} = \{ \epsilon^{(I-1)} ( \bar{W}_Z^{(I)} - \bar{W} ) - \epsilon^{(I)} ( \bar{W}_Z^{(I-1)} - \bar{W} ) \} / ( \bar{W}_Z^{(I)} - \bar{W}_Z^{(I-1)} ) \quad \text{----- (4.6a)}$$

When  $I = 1$  ;

$$\epsilon^{(2)} = \epsilon^{(1)} + 0.01, \quad \text{for } \bar{W}_Z^{(1)} < \bar{W} \quad \text{----- (4.6b)}$$

$$\epsilon^{(2)} = \epsilon^{(1)} - 0.01, \quad \text{for } \bar{W}_Z^{(1)} > \bar{W} \quad \text{----- (4.6c)}$$

### 4.3 SOLUTION SCHEME FOR SUBROUTINE TEMP

The solution scheme for subroutine TEMP is shown in Fig. 4.2. In this subroutine, the solutions of the energy equation for fluid-film and heat conduction equations for the bush-housing assembly and the journal are obtain. The solution scheme is once again an iterative one using two loops. In the inner loop, the solutions of energy equation for fluid-film and heat conduction equation for the bush-housing assembly is obtained to yield temperature fields for these domains corresponding to a specified journal temperature field. Using the established fluid-film temperatures in the inner loop, the journal temperature field is computed in the outer loop. On the termination of the outer loop, the subroutine returns the established temperature fields for fluid-film, bush-housing assembly and the journal.

Pressure field established in the inner loop and the iteration number of the middle loop (called INDEX in this section) of the main program are used as input to subroutine TEMP. Using the pressure field, velocity components {equations (3.16), (3.17) and (3.18)} are computed at each node of the discretised fluid-film. Contraction coefficients,  $C_L$  are then obtained using these velocity components. These values are held constant during the execution of the two loops referred above.

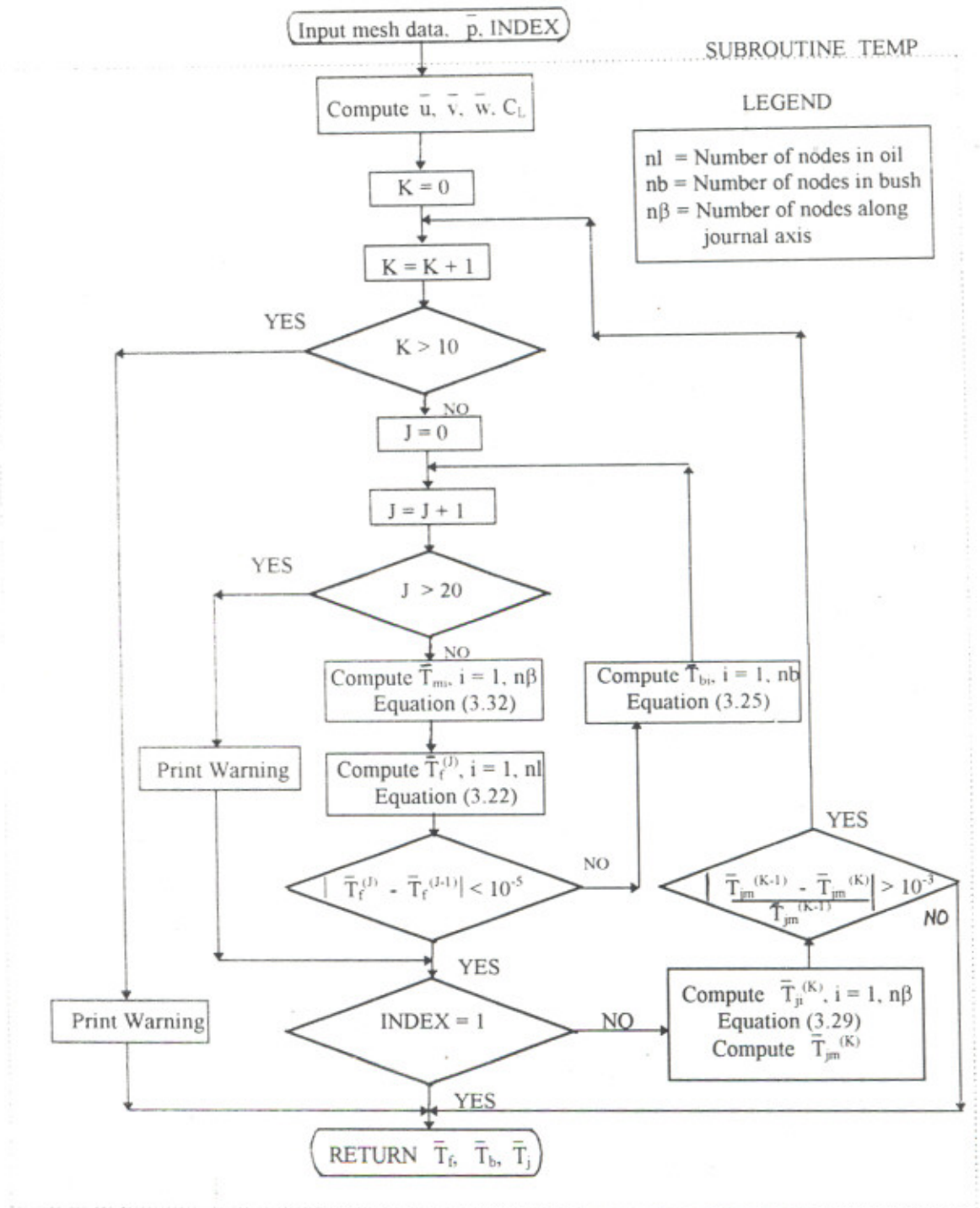


Fig. 4.2 SOLUTION SCHEME FOR SUBROUTINE TEMP



In the inner loop ( index J), fluid-film temperatures are computed at every node using equation (3.22). A check is then made for convergence of the loop according to the following criterion at every node :

$$| \bar{T}_f^{(J)} - \bar{T}_f^{(J-1)} | \leq 0.00001 \quad \text{----- (4.7)}$$

If this condition is satisfied at every node, the inner loop is terminated otherwise the heat conduction equation for the bush-housing assembly, equation (3.25), is solved using the Successive Over Relaxation approach. In the solution of heat conduction equation, the temperature in the  $m$  th iteration is computed using a over relaxation factor,  $W_{OR} = 1.9105$  as follows :

$$\bar{T}_b^{(m)} = (1 - W_{OR}) \cdot \bar{T}_b^{(m-1)} + W_{OR} \cdot \bar{T}_b^{(m)} \quad \text{----- (4.8)}$$

where  $\bar{T}_b^{(m)}$  on the right hand side is computed using equation (3.25).

Since the temperature gradient of the fluid-film at the bush-fluid interface acts as an input for the solution of heat conduction equation, its weighted value, defined below, is used to obtain a converged solution :

$$\left( \frac{\partial \bar{T}_f^{(J)}}{\partial \bar{z}} \right)_{\text{weighted}} = W_{t1} \cdot \left( \frac{\partial \bar{T}_f^{(J)}}{\partial \bar{z}} \right) + (1 - W_{t1}) \cdot \left( \frac{\partial \bar{T}_f^{(J-1)}}{\partial \bar{z}} \right)_{\text{weighted}} \quad \text{----- (4.9)}$$

The value of weight  $W_{t1} = 0.2$  was used and  $\left( \frac{\partial \bar{T}_f^{(0)}}{\partial \bar{z}} \right)$  was assumed zero at every node on the interface.

The modified bush temperature field,  $\bar{T}_b^{(J)}$  was then used to recompute the fluid-film temperatures in the  $(J + 1)$  th iteration and the iterations were continued till, either the converged solution was obtained as per criterion {equation (4.7)} or value of J exceeds 20.

On the termination of the inner loop, the control enters the outer loop (index K). In this loop, a check is first made for the value of INDEX. If the INDEX = 1, the outer loop is skipped i.e. journal temperature is not modified. This step greatly improves the overall convergence of the scheme for the THD solution. When INDEX > 1, the journal temperature distribution is computed using equation (3.29) with a over relaxation factor of 1.9105, same as used for the bush<sub>h</sub>-housing assembly. For the convergence of this loop, a criterion based on the mean journal temperature is used as below :

$$\left| \frac{\bar{T}_{jm}^{(K)} - \bar{T}_{jm}^{(K-1)}}{\bar{T}_{jm}^{(K-1)}} \right| \leq 0.001 \quad \text{----- (4.10)}$$

where

$$\bar{T}_{jm}^{(K)} = \sum_{i=1}^{n\beta+1} \bar{T}_{ji}^{(K)} / (n\beta + 1)$$

$$\bar{T}_{jm}^{(K-1)} = \sum_{i=1}^{n\beta+1} \bar{T}_{ji}^{(K-1)} / (n\beta + 1)$$

This loop is also terminated when the value of K exceeds 10 in the event the solution is not converged.

#### 4.4 SOLUTION SCHEME FOR LINEAR TRAJECTORIES

A trajectory of a journal centre motion can be obtained for a given initial disturbance by numerically integrating the governing equations of motion using fourth-order Runge-Kutta method. This particular method is preferred for the present investigation owing to its accuracy and self starting nature. The scheme for the trajectories is shown in Fig. 4.3.

For a given external load, the values of stiffness coefficients (  $\bar{S}_{xx}$ ,  $\bar{S}_{xz}$ ,  $\bar{S}_{zx}$ ,  $\bar{S}_{zz}$  ), and damping coefficients (  $\bar{B}_{xx}$ ,  $\bar{B}_{xz}$ ,  $\bar{B}_{zx}$ ,  $\bar{B}_{zz}$  ) and critical mass (  $\bar{M}_C$  ) are first determined after establishing the static equilibrium position and pressure field by using the scheme for main program, Fig. 4.1. These values are given as input for the determination of trajectories. In addition, the journal mass,  $\bar{M}_J$ , the time interval  $\delta \bar{t}$  and the initial values of disturbance (  $\bar{X}_o$ ,  $\bar{Z}_o$ ,  $\bar{\dot{X}}_o$ ,  $\bar{\dot{Z}}_o$  ), denoted respectively by  $\bar{X}_1$ ,  $\bar{X}_2$ ,  $\bar{X}_3$ , and  $\bar{X}_4$ , are also given as input. The solution of equation (3.47) then yields the point on the trajectory.

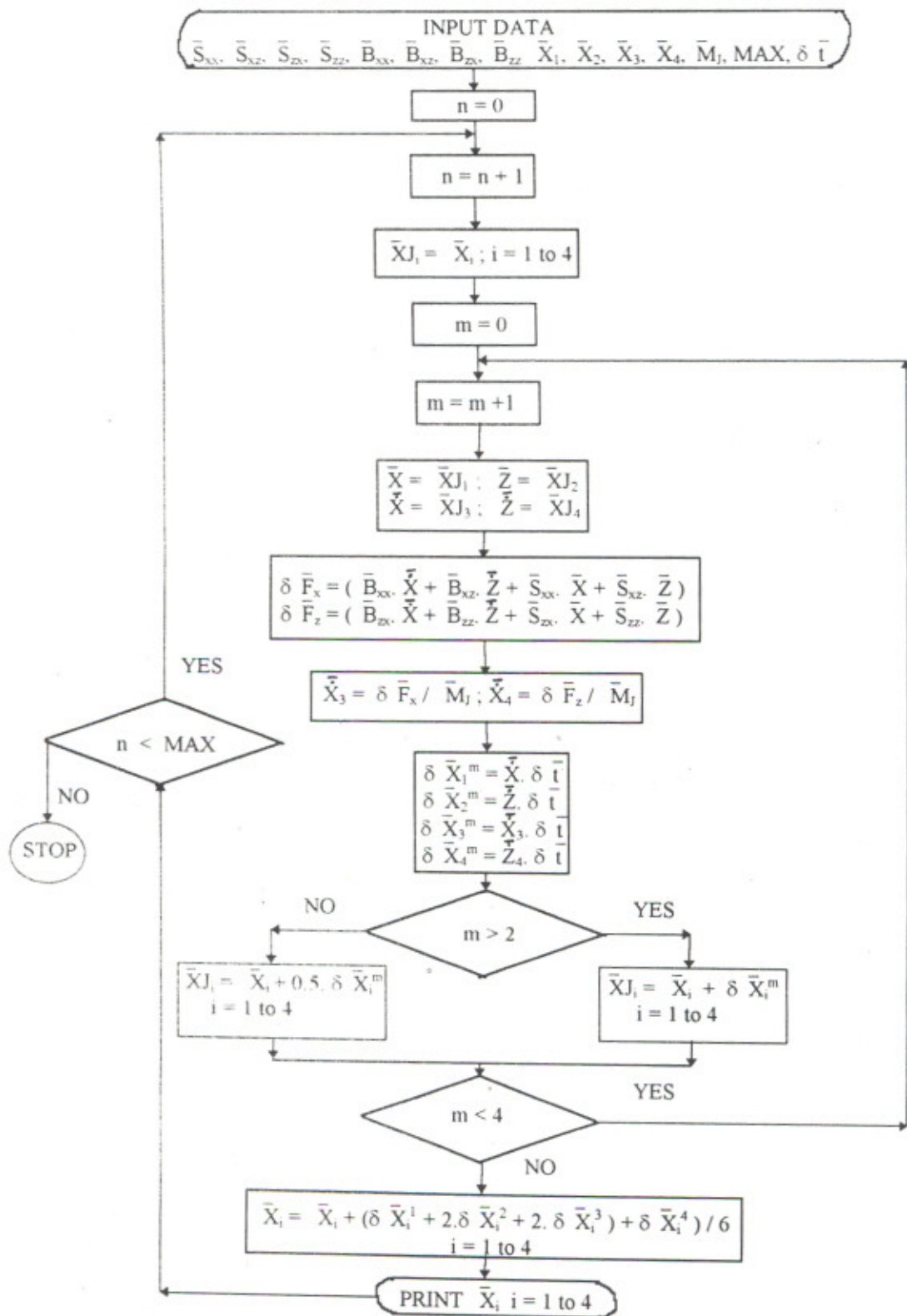


Fig. 4.3 SOLUTION SCHEME FOR LINEAR TRAJECTORIES

## CHAPTER - V

### RESULTS AND DISCUSSION

#### 5.1 INTRODUCTION

This chapter presents the results obtained using the analysis and solution algorithms discussed in the previous chapters. The chapter has been organized in the following manner: first of all the analysis and computer programs have been validated by comparing the present results with the published theoretical / experimental results; then the results for four journal bearing configurations, namely, plain, two-axial-groove, elliptical, and three-lobe, are presented and discussed; next the journal center trajectories for transient response of some bearing configurations obtained from the linearized analysis have been presented; and in the end some parametric studies have been presented. In general, the presented results consist of (i) static performance characteristics (ii) Stiffness coefficients (iii) Damping coefficients (iv) Stability parameters (v) iso-pressure curves and pressure profiles and (vi) isotherms and temperature profiles. In the analysis, the lubricant has been assumed to be thermo-viscous and the bearings are considered to be operating in the laminar-flow regime. As the present work concerns mainly with the study of the effect of misalignment on the performance of the above types of journal bearings operating under thermohydrodynamic lubrication conditions, the results have been presented in such a manner so as to bring out this effect clearly. However, for ready reference results for aligned bearings have also been given.

#### 5.2 VALIDITY OF RESULTS

It is already mentioned in Chapter II that the THD analyses of misaligned bearings has been scantily reported in literature. Therefore, to establish the validity of the approach of present study following strategy has been adopted :

- IHD performance characteristics of aligned two-axial-groove, elliptical and three-lobe journal bearings have been compared with the published results.
- IHD performance characteristics of misaligned plain and two-axial-groove journal bearings have been compared with the published results.

- The THD performance characteristics of aligned plain journal bearing and two-axial-groove journal bearing have been compared with the published experimental / theoretical results.
- Some parameters from the THD analysis of misaligned plain journal bearing have been compared with the available published results.

These comparisons are presented and discussed in the following sub-sections.

### 5.2.1 Validity of IHD Results

The static and dynamic performance characteristics of three bearing configurations, namely, two-axial-groove, elliptical and three-lobe, operating with iso-viscous lubricant have been obtained and compared with the published results of Lund and Thomsen [15] in Table 5.1. The comparison of results is quite good.

For the validity of results of misaligned journal bearings, the results for plain journal bearing with three different set of values of misalignment parameters (defined in Fig. 5.1) are compared with the published data of Jakeman [42] in Table 5.2. Likewise, the results for a two-axial-groove journal bearing are compared with the published data of Qiu and Tieu[51] in Tables 5.3 to 5.5. These results indicate very good agreement.

### 5.2.2 Validity of THD Results <sup>1</sup>

As noted earlier, the main objective of the present work is to carry out THD analyses of various types of bearing configurations operating under misaligned conditions. Therefore, to validate the present work a more appropriate approach would have been to conduct experimental investigations on these bearings and use the experimental results for validating the theoretical results from the present analysis. Since the experimental work was outside the scope of the present work, a detailed comparison has been made of the THD results for aligned plain and two-axial-groove journal bearings and that for the misaligned plain journal bearing. As a complete three-dimensional analysis has been carried out, it is expected that this approach will adequately validate the thermal analysis for aligned as well as misaligned journal bearings.

In the following, these comparisons are presented and discussed. However, before a detailed analysis of these comparisons is presented, it is considered necessary to discuss the two different thermal boundary conditions examined in this work.

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<sup>1</sup> Paper given at ref.[102] was published from this part of the work.

Table 5.1 Comparison of Static and Dynamic Performance Characteristics of Two-Axial-Groove, Elliptical and Three-Lobe Journal Bearings  
 $L/D = 1$

	Two-Axial-Groove		Elliptical		Three-Lobe	
	1*	2*	1*	2*	1*	2*
$\varepsilon$	0.650	0.650	0.250	0.250	0.246	0.246
$\phi$	39.6	39.720	88.4	88.28	59.5	59.460
$\bar{W}$	5.8084	5.895	2.8095	2.842	3.4743	3.517
$\bar{S}_{xx}$	7.61	8.489	2.27	2.330	7.10	7.351
$\bar{S}_{xz}$	1.86	1.356	9.53	9.748	9.37	9.637
$\bar{S}_{zx}$	- 20.78	-19.689	- 12.26	- 12.817	- 15.29	- 15.792
$\bar{S}_{zz}$	20.64	20.338	19.62	19.752	18.89	19.028
$\bar{B}_{xx}$	9.27	8.430	10.61	10.970	16.18	16.636
$\bar{B}_{xz} = \bar{B}_{zx}$	- 8.33	- 10.139	7.29	7.247	- 2.24	- 2.568
$\bar{B}_{zz}$	42.77	40.617	37.66	39.049	38.16	39.393

1\* : Present Work

2\* : Reference [15]

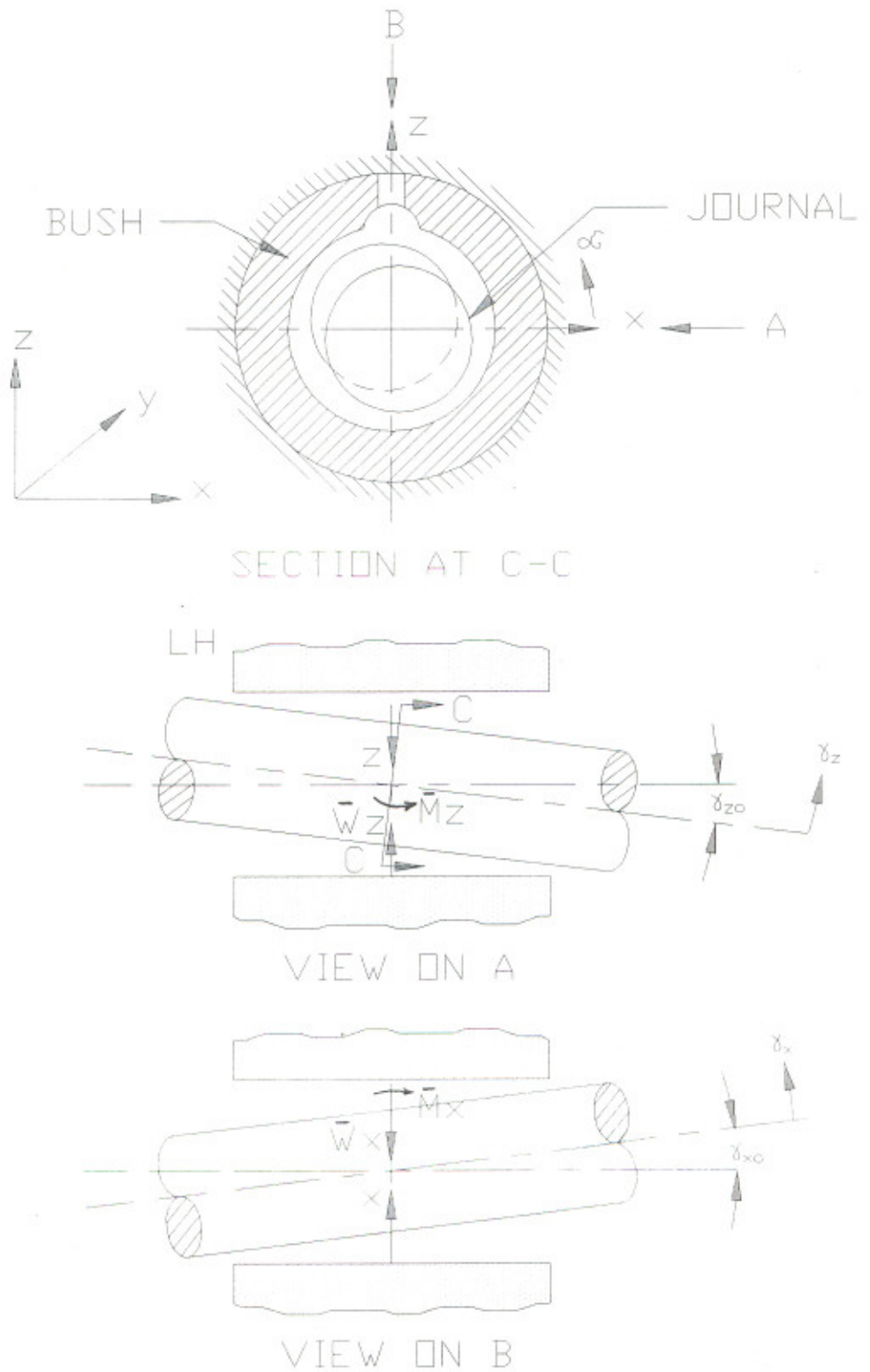


Fig. 5.1 GEOMETRICAL PARAMETER FOR PLAIN MISALIGNED JOURNAL BEARING

Table 5.2 Comparison of dimensionless oil-film parameters of a misaligned plain journal bearing - IHD analysis  
 Bearing  $150^\circ$  Partial arc and  $L/D = 0.5$

$\varepsilon$ $Y_x$ $Y_z$	0.9		0.8		0.6	
	0.0		0.5		0.5	
	0.0		0.0		-0.5	
	1*	2*	1*	2*	1*	2*
W	9.99	10.05	3.88	3.97	1.32	1.42
$\phi$ , deg	22.10	22.10	25.70	25.29	34.70	32.83
$\bar{S}_{xx}$	15.15	15.52	7.48	7.92	2.27	2.81
$\bar{S}_{xz}$	-11.08	-11.10	-3.04	-3.51	-0.47	-0.94
$\bar{S}_{zz}$	-54.74	-55.76	-19.81	-20.84	-5.76	-6.97
$\bar{S}_{zz}$	137.59	139.14	33.71	35.53	7.67	9.43
$\bar{S}_{xy}$	0.0	0.0	1.36	1.56	0.48	0.74
$\bar{S}_{yx}$	0.0	0.0	-1.37	-1.66	-0.26	-0.58
$\bar{S}_{zy}$	0.0	0.0	-2.98	-3.31	-1.07	-1.60
$\bar{S}_{yz}$	0.0	0.0	4.11	4.19	1.34	2.21
$\bar{S}_{yx}$	0.0	0.0	1.41	1.59	0.52	0.72
$\bar{S}_{xz}$	0.0	0.0	-1.45	-1.69	-0.35	-0.53
$\bar{S}_{zx}$	0.0	0.0	-2.98	-3.45	-1.14	-1.60
$\bar{S}_{yz}$	0.0	0.0	3.70	4.86	1.54	2.15
$\bar{S}_{xy}$	0.68	0.68	0.50	0.59	0.17	0.27
$\bar{S}_{yz}$	-0.58	-0.58	-0.34	-0.44	-0.09	-0.20
$\bar{S}_{zx}$	-2.17	-2.16	-1.04	-1.21	-0.35	-0.56
$\bar{S}_{yz}$	6.39	6.43	1.85	2.12	0.51	0.82
$\bar{B}_{xx}$		7.08	4.55	4.62	1.75	2.01
$\bar{B}_{xz}$	6.68	-18.08	-7.33	-7.36	-2.19	-2.62
$\bar{B}_{zx}$	-16.44	-17.38	-7.33	-7.40	-2.19	-2.60
$\bar{B}_{zz}$	-16.44	103.47	33.57	34.33	9.65	10.99
	99.40					
$\bar{B}_{xy}$		0.0	0.75	0.77	0.30	0.38
$\bar{B}_{yx}$	0.0	0.0	-1.47	-1.64	-0.53	-0.73
$\bar{B}_{zy}$	0.0	0.0	-1.61	-1.54	-0.56	-0.70
$\bar{B}_{yz}$	0.0	0.0	2.82	1.67	1.43	0.76
	0.0					
$\bar{B}_{yx}$		0.0	0.75	0.76	0.30	0.40
$\bar{B}_{xz}$	0.0	0.0	-1.61	-1.55	-0.56	-0.72
$\bar{B}_{zx}$	0.0	0.0	-1.47	-1.67	-0.53	-0.76
$\bar{B}_{yz}$	0.0	0.0	2.82	3.0	1.43	1.88
	0.0					
$\bar{B}_{xy}$		0.27	0.27	0.28	0.11	0.15
$\bar{B}_{yz}$	0.27	-0.70	-0.44	-0.46	-0.16	-0.24
$\bar{B}_{zx}$	-0.68	-0.68	-0.44	-0.46	-0.16	-0.24
$\bar{B}_{yz}$	-0.68	3.38	1.27	1.34	0.45	0.61
	3.35					

1\* : Present work

2\* : Reference [42]; data from this reference has been converted into present non-dimensional form.

Table 5.3 : Comparison of Static Performance Characteristics of Two- Axial-Groove Journal Bearing - IHD analysis  
 $L / D = 1$  with  $20^\circ$  grooves

Load, $\bar{W}$	Minimum film thickness, $\bar{h}_{min}$				Attitude Angle, $\phi$ (Degrees)				Side Flow, $\bar{Q}_s$			
	$\bar{\gamma}_x = 0.0$		$\bar{\gamma}_x = 0.2$		$\bar{\gamma}_x = 0.0$		$\bar{\gamma}_x = 0.2$		$\bar{\gamma}_x = 0.0$		$\bar{\gamma}_x = 0.2$	
	$\bar{\gamma}_z = 0.0$		$\bar{\gamma}_z = 0.0$		$\bar{\gamma}_z = 0.0$		$\bar{\gamma}_z = 0.0$		$\bar{\gamma}_z = 0.0$		$\bar{\gamma}_z = 0.0$	
	1*	2*	1*	2*	1*	2*	1*	2*	1*	2*	1*	2*
0.45	0.89	0.89	0.695	0.70	75.4	75.0	70.1	68.5	0.125	0.13	0.133	0.13
0.64	0.85	0.85	0.656	0.66	70.5	70.0	66.5	65.5	0.171	0.18	0.172	0.18
1.27	0.72	0.73	0.546	0.54	59.7	60.0	57.3	57.0	0.290	0.30	0.285	0.28
2.55	0.55	0.55	0.395	0.40	50.1	50.0	48.3	49.0	0.435	0.44	0.425	0.42
3.82	0.45	0.45	0.305	0.30	44.9	45.0	43.2	43.0	0.515	0.48	0.503	0.48
6.37	0.33	0.34	0.204	0.21	38.9	40.0	37.1	37.0	0.603	0.52	0.585	0.50
12.73	0.20	0.21	0.102	0.10	31.6	32.0	29.4	30.0	0.691	0.55	0.666	0.53
19.10	0.15	0.16	0.062	-	27.9	27.5	25.3	26.0	0.727	0.56	0.696	0.55

Table 5.4 : Comparison of Stability Parameters of Two-Axial-Groove Journal Bearing - IHD analysis  
 $L / D = 1$  with  $20^\circ$  grooves

Load, $\bar{W}$	Whirl Frequency Ratio, $\bar{\nu}$				Threshold Speed, $\bar{\Omega}$			
	$\bar{\gamma}_x = 0.0$		$\bar{\gamma}_x = 0.2$		$\bar{\gamma}_x = 0.0$		$\bar{\gamma}_x = 0.2$	
	$\bar{\gamma}_z = 0.0$		$\bar{\gamma}_z = 0.0$		$\bar{\gamma}_z = 0.0$		$\bar{\gamma}_z = 0.0$	
	1*	2*	1*	2*	1*	2*	1*	2*
0.45	0.500	0.50	0.497	0.50	2.091	2.10	2.191	2.2
0.64	0.501	0.50	0.499	0.50	2.097	2.10	2.163	2.2
1.27	0.506	0.505	0.502	0.50	2.108	2.10	2.169	2.2
2.55	0.508	0.51	0.495	0.49	2.110	2.10	2.211	2.3
3.82	0.495	0.49	0.472	0.48	2.162	2.15	2.327	2.4
6.37	0.445	0.45	0.394	0.40	2.406	2.40	2.838	2.8

1\* : Present Work

2\* : Reference [51]; results from this reference has been converted into present non-dimensional form.

Table 5.5 : Comparison of Dynamic Performance Characteristics of Two-Axial-Groove Journal Bearing - IHD analysis  
 $L/D = 1$  with  $20^\circ$  grooves

(i) Stiffness Coefficients

Load, $\bar{W}$	$\bar{S}_{xx}$				$\bar{S}_{xz}$				$\bar{S}_{zx}$				$\bar{S}_{zz}$			
	$\bar{y}_x = 0.0$ $\bar{y}_z = 0.0$		$\bar{y}_x = 0.2$ $\bar{y}_z = 0.0$		$\bar{y}_x = 0.0$ $\bar{y}_z = 0.0$		$\bar{y}_x = 0.2$ $\bar{y}_z = 0.0$		$\bar{y}_x = 0.0$ $\bar{y}_z = 0.0$		$\bar{y}_x = 0.2$ $\bar{y}_z = 0.0$		$\bar{y}_x = 0.0$ $\bar{y}_z = 0.0$		$\bar{y}_x = 0.2$ $\bar{y}_z = 0.0$	
	1*	2*	1*	2*	1*	2*	1*	2*	1*	2*	1*	2*	1*	2*	1*	2*
	0.45	1.50	1.50	1.74	1.40	-2.89	-3.0	-3.14	-3.1	9.75	9.8	10.54	10.4	1.54	1.5	1.50
0.64	1.52	1.50	1.69	1.50	-2.16	-2.2	-2.31	-2.3	7.27	7.4	7.67	7.6	1.56	1.6	1.51	1.5
1.27	1.59	1.60	1.67	1.52	-1.39	-1.45	-1.42	-1.4	4.67	4.75	4.84	5.0	1.67	1.7	1.70	1.7
2.55	1.55	1.55	1.59	1.51	-0.82	-0.8	-0.79	-0.8	3.60	3.6	3.71	3.8	2.14	2.2	2.24	2.2
3.82	1.49	1.50	1.55	1.50	-0.51	-0.55	-0.47	-0.5	3.37	3.35	3.48	3.5	2.67	2.8	2.79	2.8
6.37	1.44	1.45	1.55	1.52	-0.17	-0.2	-0.10	-0.15	3.36	3.4	3.51	3.6	3.62	3.5	3.81	3.8
12.73	1.41	1.47	1.62	1.60	0.34	0.30	0.54	0.5	3.67	3.7	4.02	4.0	5.81	5.7	6.29	6.1
19.10	1.46	1.50	1.77	1.75	0.60	0.60	1.02	1.0	4.14	4.2	4.68	4.6	7.68	7.7	8.67	8.4

(ii) Damping Coefficients

Load, $\bar{W}$	$\bar{B}_{xx}$				$\bar{B}_{xz} = \bar{B}_{zx}$				$\bar{B}_{zz}$			
	$\bar{y}_x = 0.0$ $\bar{y}_z = 0.0$		$\bar{y}_x = 0.2$ $\bar{y}_z = 0.0$		$\bar{y}_x = 0.0$ $\bar{y}_z = 0.0$		$\bar{y}_x = 0.2$ $\bar{y}_z = 0.0$		$\bar{y}_x = 0.0$ $\bar{y}_z = 0.0$		$\bar{y}_x = 0.2$ $\bar{y}_z = 0.0$	
	1*	2*	1*	2*	1*	2*	1*	2*	1*	2*	1*	2*
	0.45	5.90	6.15	6.57	6.5	1.53	1.50	1.85	1.5	19.57	19.50	20.98
0.64	4.50	4.60	4.91	4.9	1.59	1.55	1.82	1.6	14.63	14.70	15.34	15.1
1.27	3.11	3.00	3.24	3.1	1.82	1.75	1.86	1.75	9.50	9.70	9.78	9.9
2.55	2.22	2.30	2.25	2.3	1.86	1.80	1.82	1.8	7.41	7.50	7.53	7.6
3.82	1.78	1.80	1.81	1.8	1.78	1.75	1.75	1.75	6.95	7.00	7.05	7.0
6.37	1.39	1.40	1.46	1.5	1.72	1.65	1.71	1.7	6.90	7.05	6.99	7.0
12.73	0.96	1.05	1.05	1.1	1.55	1.60	1.58	1.65	7.32	7.50	7.52	7.5
19.10	0.87	0.85	0.95	0.9	1.64	1.65	1.63	1.7	8.14	8.10	8.28	8.3

1\* : Present Work

2\* : Reference [51]; results from this reference has been converted into present non-dimensional form.

**5.2.2.1 Selection of Thermal Boundary Conditions :** As mentioned above, two different thermal boundary conditions have been examined in this work. In the first of these boundary conditions (BC1), the temperature of lubricant at the inlet of a bearing lobe/pad is assumed to vary as a parabolic function of distance from the fluid-bush interface. The coefficients of the describing polynomial are determined by assuming fluid temperature at the fluid-bush interface to be equal to the bush temperature and that at the fluid-journal interface equal to the journal temperature. The third condition requires the determination of mean fluid temperature after accounting for mixing of re-circulating oil with the cool supply oil and then equating it to the mean of the assumed temperature distribution over the film thickness. For the determination of temperature field for the bush, heat conduction equation is solved after prescribing oil supply temperature at the bearing outer surface on the inlet edge of the lobe / pad.

The second thermal boundary conditions (BC2) assumes the bush temperature constant equal to the supply temperature at the radial inlet edge of lobe / pad, whereas the fluid temperature is assumed to vary as a cubic function of distance from the fluid-bush interface. The coefficients of the describing polynomial are obtained by assuming the temperature of the lubricant equal to that of bush at the bush-fluid interface, and that of the journal at the fluid-journal interface. The third condition assumes the continuity of heat flux at the bush-fluid interface and for the fourth condition, a mean temperature is obtained as in the case of BC1.

Fig. 5.2 shows the variation of maximum bush temperature obtained using the two boundary conditions with eccentricity ratio for two speeds (2000 and 4000 r.p.m.). It is seen from this figure that with BC2, the bush temperature distribution has a cup shape which is quite inconsistent when compared with the experimental results for the range of eccentricity ratio considered. Results from the first boundary conditions (BC1) do not show this trend. Also, the experiments indicate that the bush temperature at the inlet of the pad is higher than the supply temperature (Fig. 5.3), which is not possible with BC2. Therefore, the first boundary conditions (BC1) are more appropriate and have been used in this work throughout.

**5.2.2.2 Plain Journal Bearing :** Results obtained for a plain journal bearing are compared with the published experimental and theoretical works of Ferron et al [71], Figs. 5.2 to 5.5. The present results were obtained for two sets of data for thermal properties of bush-housing assembly. The operating conditions shown in Table 5.6 and the viscosity-temperature relation

**Table 5.6 Operating Conditions**

---

Journal Radius, $R$	= 0.05	m
External Bearing Radius, $R_2$	= 0.10	m
Bearing Length, $L$	= 0.08	m
Radial Clearance, $c / R$	= 0.0029	
Lubricant Viscosity at 40 °C	= 0.0277	Pa.s
Viscosity Coefficients,	$k_0$	= 3.287
	$k_1$	= 3.064
	$k_2$	= 0.777
Lubricant Density, $\rho$	= 860	Kg / m <sup>3</sup>
Lubricant Specific Heat, $C_p$	= 2000	J / Kg. °C
Thermal Conductivity, $k_f$	= 0.13	W / m °C
Bush-Housing Thermal Conductivity, $k_b$	= 250	W / m °C
Journal Thermal Conductivity, $k_j$	= 50	W / m °C
Convection Heat Transfer Coefficient, $h_{fb}$	= 80	W / m °C
Inlet Lubricant Temperature, $T_s$	= 40	°C
Ambient Temperature, $T_a$	= 40	°C
Reference Temperature, $T_r$	= 40	°C
Oil Groove Angle	= 18°	

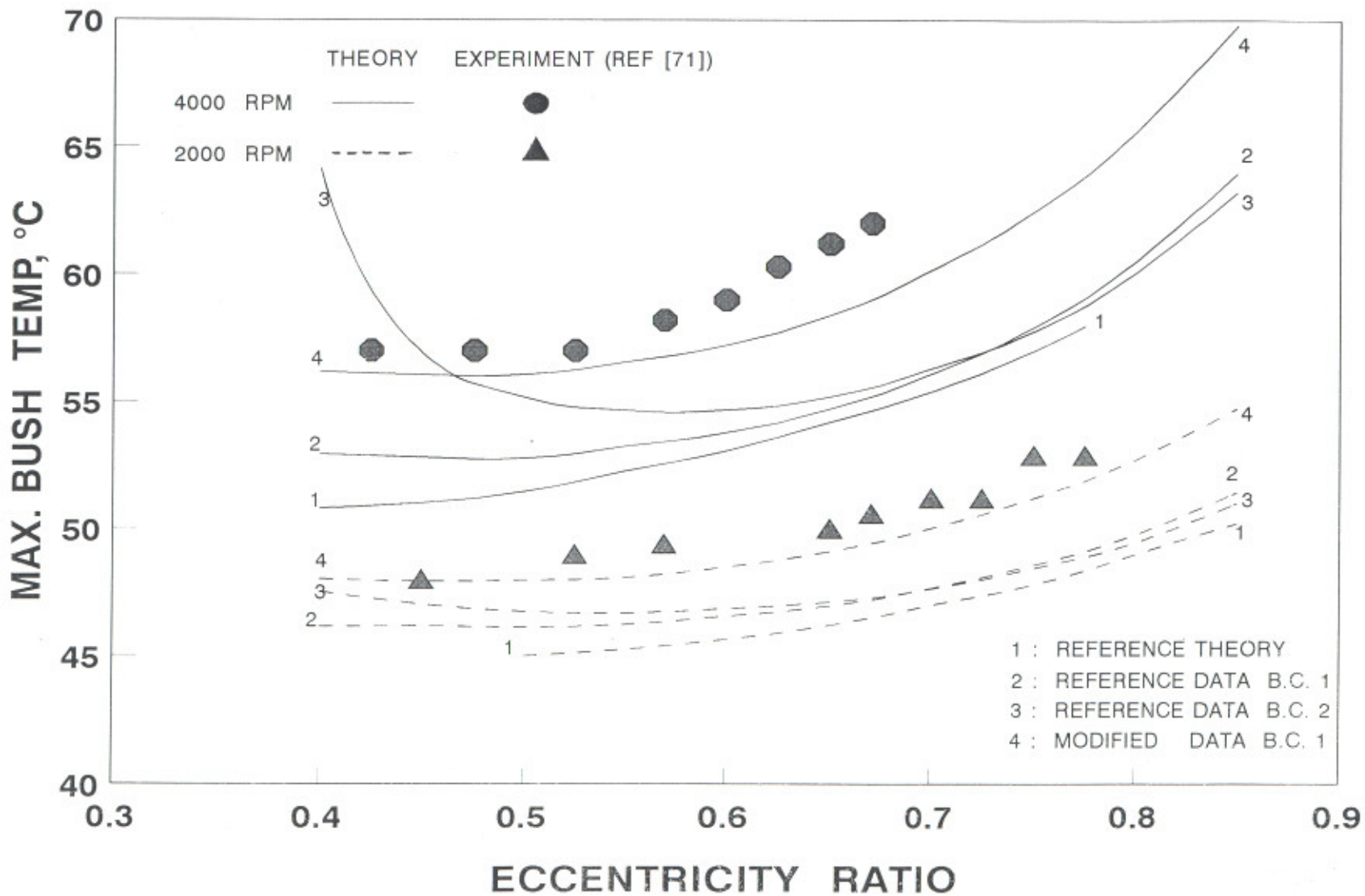


Fig. 5.2 COMPARISON OF RESULTS FOR PLAIN JOURNAL BEARING (MAXIMUM BUSH TEMPERATURE)

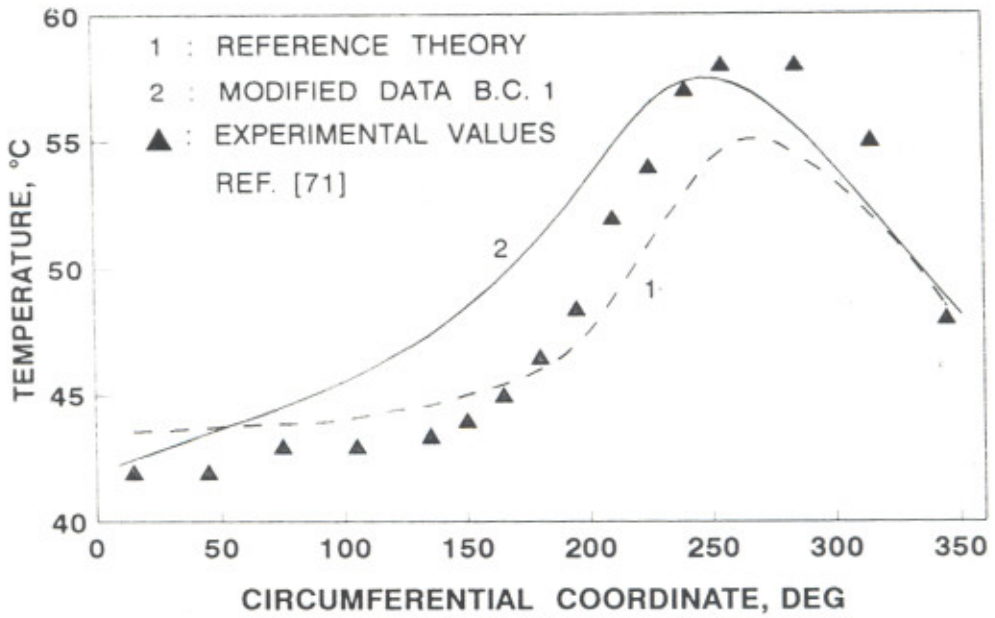


Fig. 5.3 TEMPERATURE VARIATION IN MID-PLANE AT 4000 RPM AND UNDER 6000 N

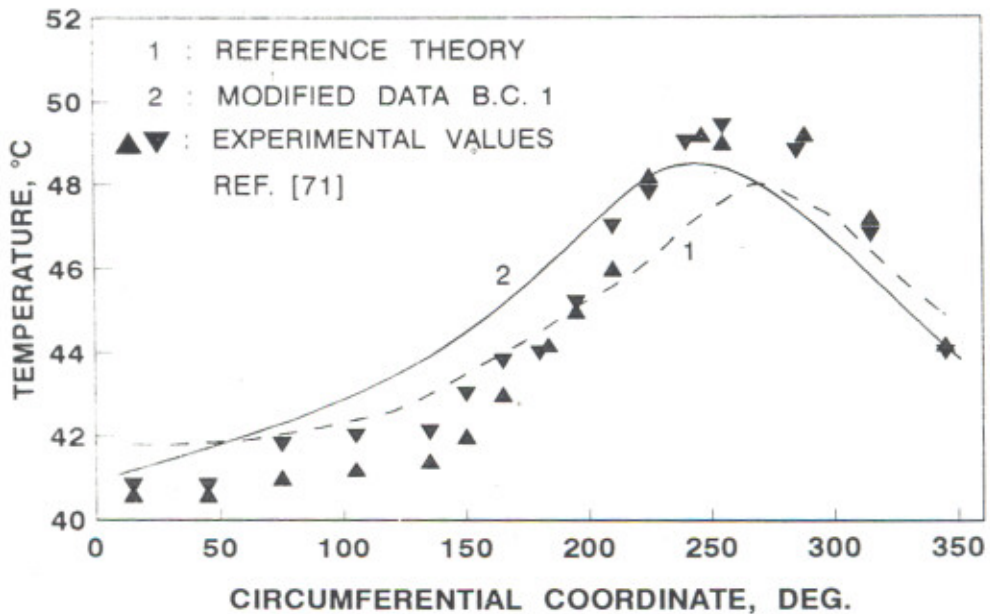


Fig. 5.4 TEMPERATURE VARIATIONS IN MID-PLANE AT 2000 RPM AND UNDER 4000 N

used in taking the present results were chosen identical to those employed by Ferron et al. in their theoretical work. These figures indicate that

- The present results are closer to the published experimental data with reference data as well as the modified data. With the modified data, however, the difference between bush temperatures as predicted by the experiment and from the present work is significantly less, Figs. 5.2 to 5.4. It may be noted here that the modified thermal properties are quite possible as several other researchers have used these properties closer to the chosen ones.
- The bearing load capacity at a given eccentricity ratio as indicated by theory is on the higher side, Fig. 5.5. Boncompain et al [80] also obtained similar results but their values being still higher.
- The computed maximum bush temperature is lower for both speeds considered, Fig.5.2. It may be noted that the modified data give very close values of the maximum bush temperature when compared with the experimental results at both the speeds. Similar trend was also observed at other speeds but not presented here.
- The affect of free convection of heat from the bush at outlet of pad to fluid in the supply groove is quite important. A value of  $h_{fb} = 1500 \text{ W / m}^2 \text{ }^\circ\text{C}$  have been used in the present work which is found to be quite appropriate. This value of  $h_{fb}$  has been chosen by an iterative method by simulating the experimental conditions. The effect of  $h_{fb}$  is found to be more pronounced in the cavitation zone i.e. after the location of maximum bush temperature and a higher value of  $h_{fb}$ , as expected, gives more drop in the bush temperature near the trailing edge of the pad / lobe.
- It has been observed that the maximum bush temperature occurs at / or near the trailing edge of the positive-pressure region. The experimental results also indicate this trend.
- Although results for maximum pressures as well as pressure distribution are not presented here, the values of pressure and the extent of positive-pressure region were found to be very close to the respective experimental values given in the reference for all values of eccentricity ratio, speeds and loads. Infact, the difference was so small that the experimental and theoretical values nearly overlapped and primarily that is the reason for not presenting these results here.
- The journal temperature distribution has been obtained along the axial length by neglecting the radial variation of the temperature. This assumption is quite satisfactory because of high

thermal conductivity of the journal material. The computed journal temperature along the length of the journal varies by about one degree for an aligned bearing as seen in the course of the present work. For misaligned bearings this variation is much higher, especially in the case of an elliptical or a three-lobe bearing. Therefore, for the sake of convenience, the journal temperature has been presented by taking a mean over the length. Fig. 5.6 shows the variation of mean journal temperature against load for two speeds (2000 r.p.m. and 4000 r.p.m.) and the two sets of bush thermal properties. As expected, the journal temperature at higher speed is higher at a given load. Also, the journal temperature is higher when the bush thermal conductivity ( $k_b$ ) is lesser as the journal has to transfer more amount of heat now.

**5.2.2.3 Two-Axial-Groove Journal Bearing :** A two-axial-groove bearing differs from a plain bearing in the manner of oil supply. Two supply grooves are provided in this case as against only one in a plain bearing. Thus for the purpose of thermal studies, a two-axial-groove bearing will represent a multi-lobe configuration and the comparison obtained for such a bearing can easily be extended for a multi-lobe configuration. Accordingly, the THD results obtained for a two-axial-groove journal bearing are compared with those published by Lund and Tonnesen [77] in Figs 5.7(a) to (d). Table 5.7 gives the data used in the present work as well as in the above reference. For numerical discretization, 37 divisions in the circumferential direction in each lobe, 10 divisions each in the axial and across the thickness of the oil film have been used. The first boundary conditions (BC1) are used here.

Fig. 5.7(a) shows the variation of mean journal temperature as a function of load for two values of speed. It is seen from this figure that the mean journal temperatures obtained in the present work are slightly on higher side when compared with the experimental results. In comparison, the theoretical results of Lund et al. are on the lower side and differ significantly at the higher speed.

Fig. 5.7(b) shows the results for the maximum bush temperature versus load for two speeds. The results with the present analysis are quite comparable with the referred experimental values. In fact these results are more consistent as compared to the theoretical results from the reference.

The circumferential temperature profile for the bush temperature at two different speeds (3500 r.p.m. and 5000 r.p.m. ) for a given load of 5600 N are shown in Fig. 5.7(c) and Fig. 5.7(d). The theoretical results from the reference show large discrepancies in the upper lobe,

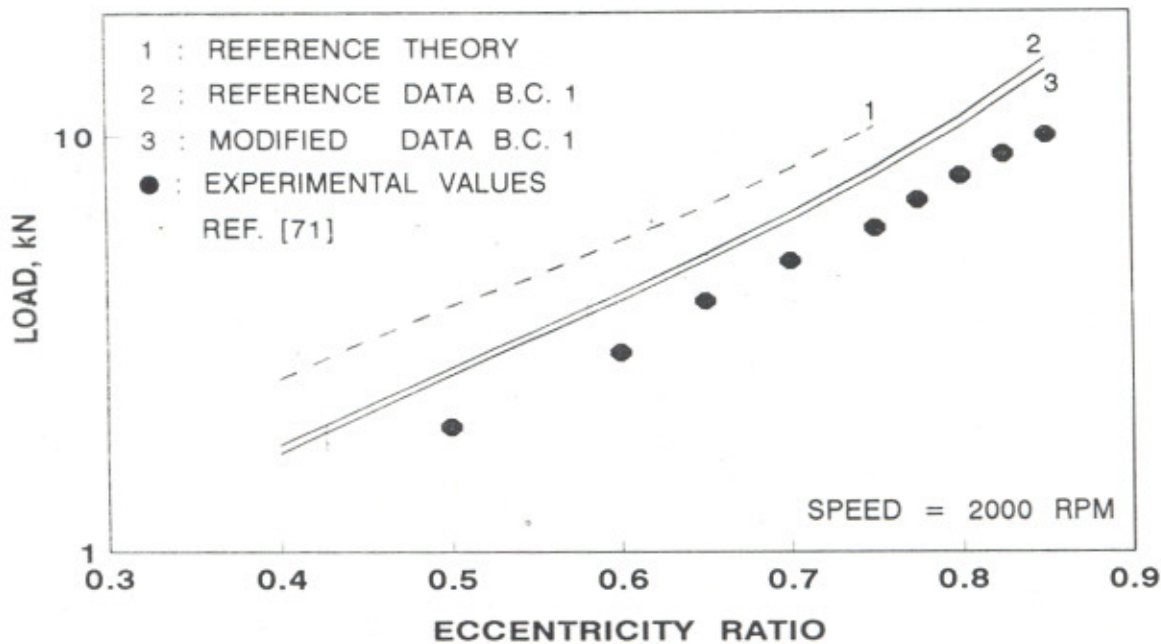


Fig. 5.5 COMPARISON OF RESULTS FOR PLAIN JOURNAL BEARING

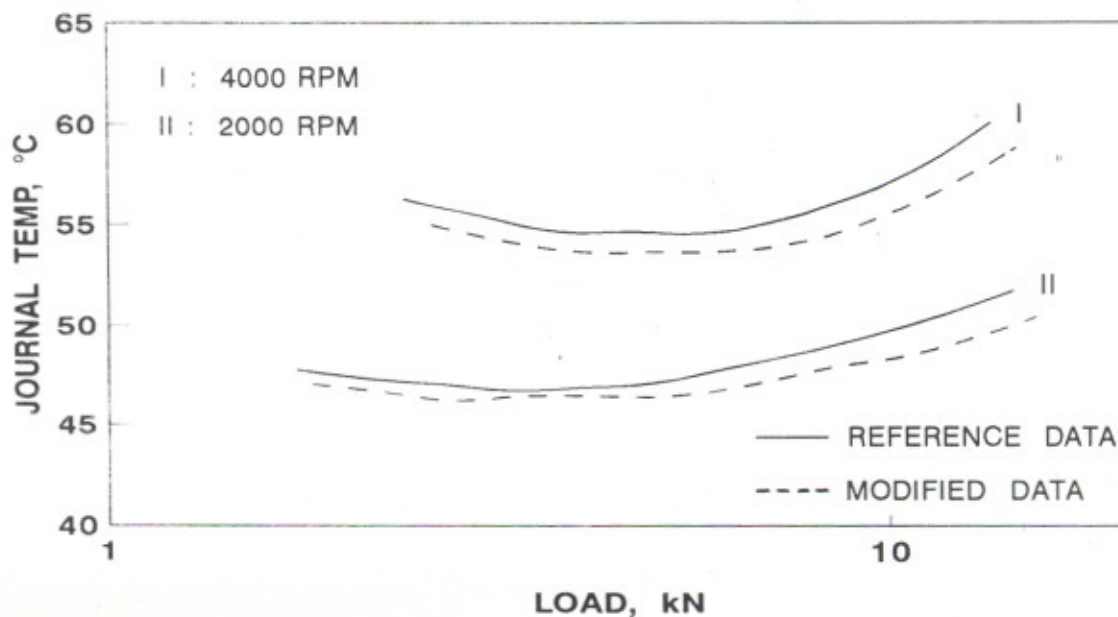
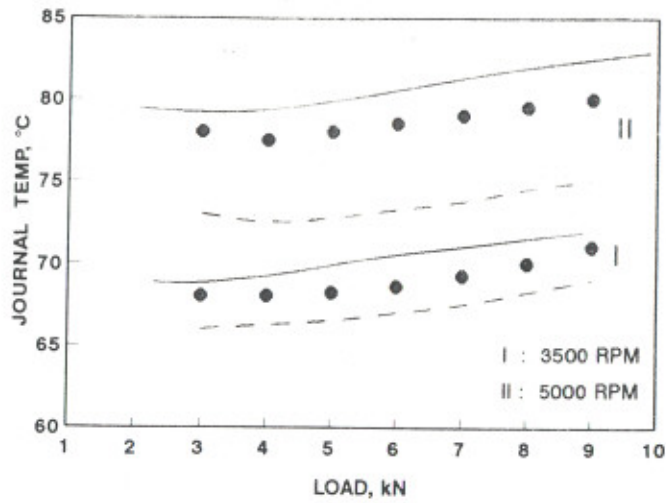
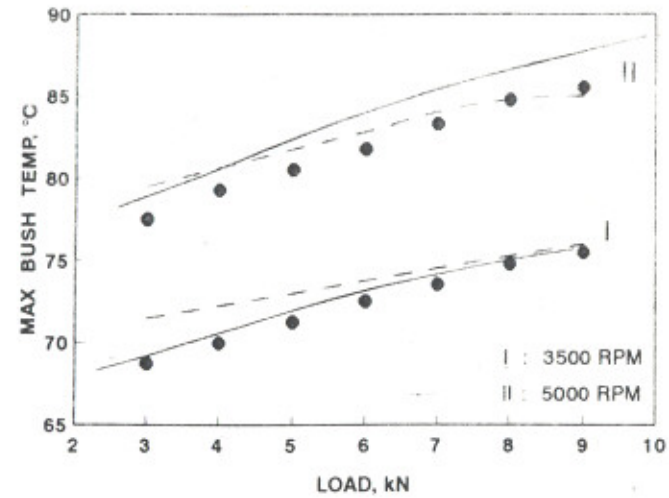


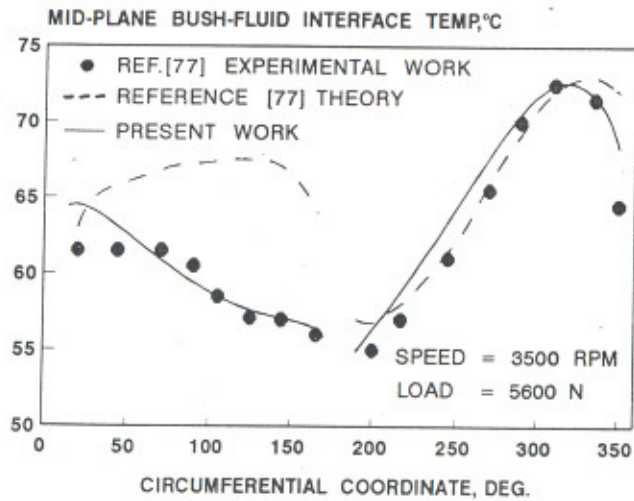
Fig. 5.6 PLAIN JOURNAL BEARING - VARIATION OF JOURNAL TEMPERATURE



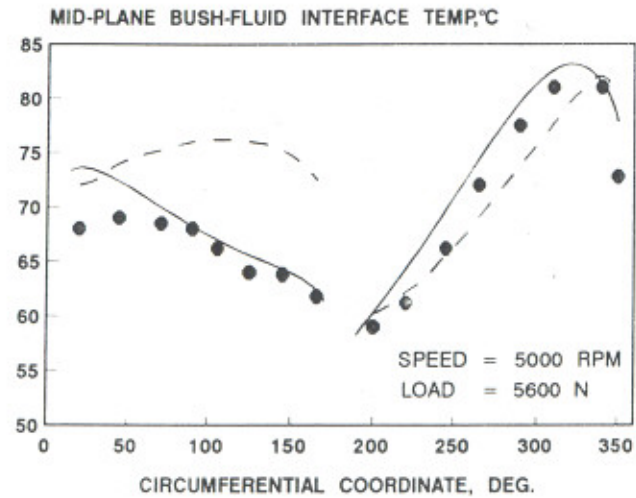
(a)



(b)



(c)



(d)

Fig. 5.7 COMPARISON OF RESULTS FOR TWO-AXIAL GROOVE JOURNAL BEARING

**Table 5.7 Data for Calculation**

---

Bearing Dimensions :	Diameter, $D = 0.1 \text{ m}$	Length, $L = 0.05 \text{ m}$
	Radial Clearance, $c = 68.6 \text{ }\mu\text{m}$	at 3500 RPM
		$66.8 \text{ }\mu\text{m}$ at 5000 RPM
	Groove Angle = $20^\circ$	
Oil Properties :	Kinematic Viscosity, = $21.5 * 10^{-6} \text{ m}^2 / \text{s}$ at $50^\circ\text{C}$	
	$6.6 * 10^{-6} \text{ m}^2 / \text{s}$ at $90^\circ\text{C}$	
	Mass Density, $\rho = 850 \text{ Kg} / \text{m}^3$	
	Specific Heat, $C_p = 2000 \text{ J} / \text{Kg } ^\circ\text{C}$	
	Thermal Conductivity, $k = 0.15 \text{ W} / \text{m } ^\circ\text{C}$	
	Oil Supply Temperature, $T_s = 50^\circ\text{C}$	
	Oil Supply Pressure : $0.12 \text{ MPa}$ ( $\alpha = 90^\circ$ )	
	$0.15 \text{ MPa}$ ( $\alpha = 270^\circ$ )	
Bearing Sleeve :	Outer Radius = $0.1 \text{ m}$	
	Thermal Conductivity, $k_b = 50 \text{ W} / \text{m } ^\circ\text{C}$	
	Ambient Temperature, $T_a = 20^\circ\text{C}$	
	Outer Surface Heat Transfer Coefficient = $50 \text{ W} / \text{m}^2 \text{ } ^\circ\text{C}$	
	Groove Wall Heat Transfer Coefficient = $100 \text{ W} / \text{m}^2 \text{ } ^\circ\text{C}$	
Shaft :	Diameter = $0.1 \text{ m}$	
	Thermal Conductivity, $k_j = 50 \text{ W} / \text{m } ^\circ\text{C}$	
	Surface Heat Transfer Coefficient = $50 \text{ W} / \text{m}^2 \text{ } ^\circ\text{C}$	
	Ambient Temperature, $T_a = 20^\circ\text{C}$	

where the measured and computed temperatures differ by more than 12 °C. The present results are very close to the experimental results over both the lobes except a small region near the inlet edge of the upper lobe. The bush temperature distribution in the positive pressure region in the lower lobe shows very good agreement between the calculated values and measured values, the maximum deviation being only about 2.5 °C.

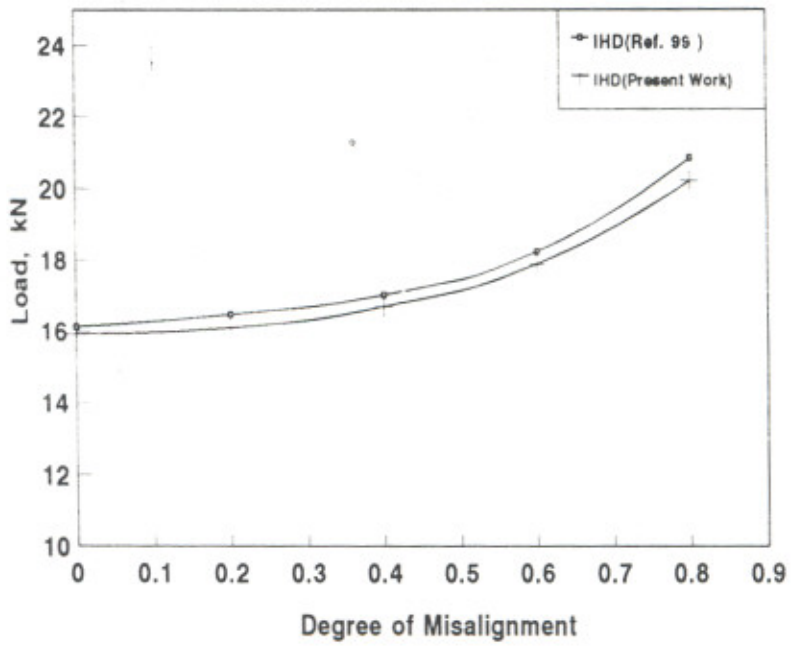
**5.2.2.4 Plain Misaligned Journal Bearing :** The available THD results for a misaligned plain journal bearing are scant. Chun and Lalas[99] published a few results of misaligned plain journal bearing which were obtained using an approximation for the heat transfer in journal and bush. A comparison of some results have been made with their results in Figs. 5.8(a) to 5.8(b). The operating conditions shown in table 5.8, the viscosity-temperature relation and the degree of misalignment used in taking the present results were chosen identical to those employed by Chun and Lalas in their theoretical work. For the purpose of this comparison, the expression for fluid-film thickness was modified to account for misalignment in the plane of the eccentricity vector as employed by Chun and Lalas. The results of the computations are compared for isothermal and THD conditions, as shown in Figs. 5.8(a) to 5.8(b). A good agreement is seen between the present and published theoretical results. The difference in the present and published results may be attributed to the approximate thermal analysis for the bush and the journal employed by Chun and Lalas.

### **5.2.3 Concluding Remarks on Validation**

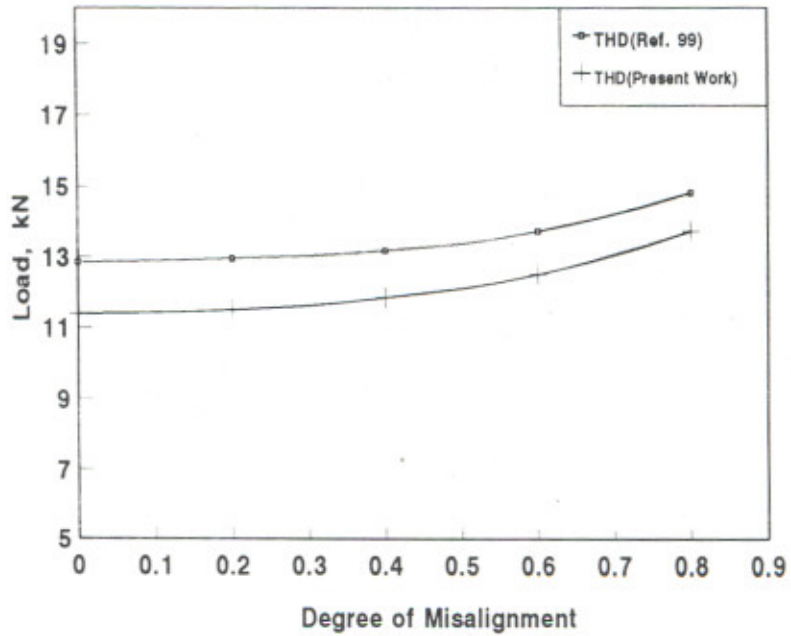
Based on the comparisons provided in this section, it may be said that the analysis and the developed programs are adequately validated for the study of misaligned journal bearings operating under thermohydrodynamic lubrication conditions.

## **5.3. STEADY STATE THD ANALYSIS OF MISALIGNED JOURNAL BEARINGS**

A three-dimensional thermal analysis has been carried out in the present work as it is expected to give more accurate results especially when the bearings operate under misalignment conditions. Studies of performance characteristics of plain, two-axial-groove, elliptical and three-lobe journal bearings have been performed by using geometry and operating characteristics shown in Table 5.9.



(a)



(b)

Ref. [99]; results from this reference has been converted into dimensional form.

Fig. 5.8 Load Vs. Degree of Misalignment

**Table 5.8 Operating Conditions**

---

Bearing Diameter, $2R$	= 0.10	m
Bearing Length, $L$	= 0.08	m
Radial Clearance, $c / R$	= 0.0029	
Eccentricity Ratio	= 0.80	
Speed of the Journal, $N$	= 2000	R.P.M.
Lubricant Viscosity at 40 °C	= 0.0277	Pa.s
Temperature Viscosity Coefficient	= 0.0033	°C <sup>-1</sup>
Lubricant Density, $\rho$	= 860	Kg / m <sup>3</sup>
Lubricant Specific Heat, $C_p$	= 2000	J / Kg. °C
Thermal Conductivity, $k_f$	= 0.13	W / m °C
Bush-Housing Thermal Conductivity, $k_b$	= 250	W / m °C
Journal Thermal Conductivity, $k_j$	= 50	W / m °C
Convection Heat Transfer Coefficient of Lubricant to the bush, $h_{fb}$	= 50	W / m °C
Convection Heat Transfer Coefficient of air to the bush, $h_{ab}$	= 5	W / m °C
Convection Heat Transfer Coefficient of Lubricant to the journal, $h_{fj}$	= 130	W / m °C
Oil Groove Angle	= 17.1°	
Inlet Lubricant Temperature, $T_s$	= 40	°C
Ambient Temperature, $T_a$	= 45	°C
Reference Temperature, $T_r$	= 40	°C

**TABLE 5.9 Operating Characteristics**

Journal Radius, R	=	50	mm
Journal Speed, N	=	3500	r.p.m.
External Bearing Radius, R <sub>2</sub>	=	100	mm
Bearing Length, L	=	100	mm
Radial Clearance, c/R	=	0.003	
Lubricant Viscosity	at 50 °C	=	21.5 * 10 <sup>-6</sup> m <sup>2</sup> /s
	at 90 °C	=	6.6 * 10 <sup>-6</sup> m <sup>2</sup> /s
Lubricant Density, ρ	=	860	Kg / m <sup>3</sup>
Lubricant Specific Heat, C <sub>p</sub>	=	2000	J / Kg. °C
Lubricant Thermal Conductivity, k <sub>f</sub>	=	0.13	W / m °C
Bush-Housing Thermal Conductivity, k <sub>b</sub>	=	250	W / m °C
Journal Thermal Conductivity, k <sub>j</sub>	=	50	W / m °C
Convection Heat Transfer Coefficient, h <sub>ab</sub> ( from bush to air )	=	50	W / m °C
Convection Heat Transfer Coefficient, h <sub>fb</sub> ( from bush to oil )	=	1500	W / m °C
Inlet Lubricant Temperature, T <sub>s</sub>	=	40	°C
Ambient Temperature, T <sub>a</sub>	=	30	°C
Reference Temperature, T <sub>r</sub>	=	40	°C
Groove Angle	=	20	°

The static and dynamic performance characteristics of misaligned journal bearing are presented and discussed in this section. For ready reference, however, the performance characteristics of aligned bearings have also been given. In general, these characteristics have been presented as functions of load. All static performance characteristics except eccentricity ratio are presented in dimensional form whereas the dynamic performance characteristics have been given in the non-dimensional form. A conversion table from non-dimensional to dimensional form is given as Appendix I.

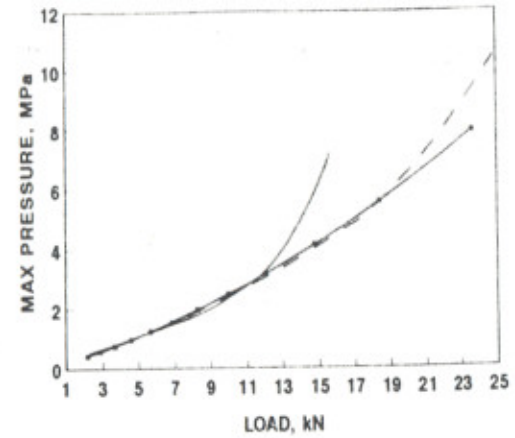
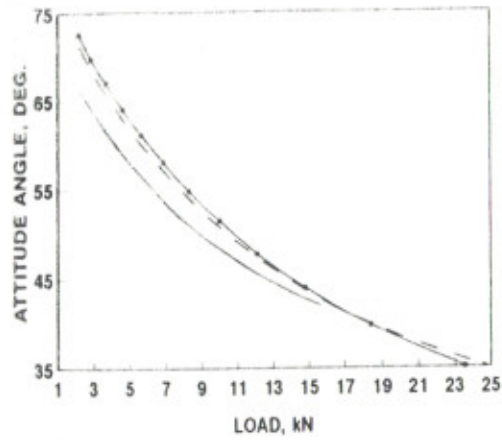
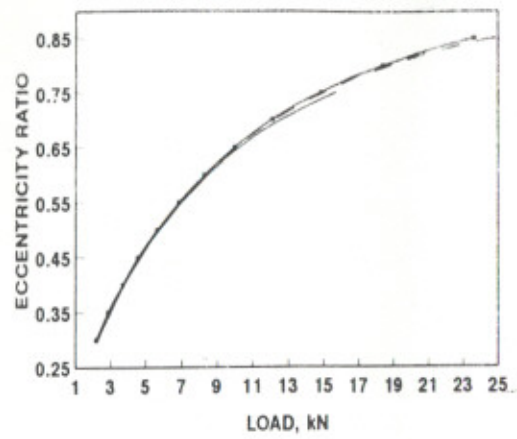
The static performance characteristics include eccentricity ratio ( $\epsilon_o$ ), attitude angle ( $\phi$ ), maximum pressure ( $p_{max}$ ), minimum fluid-film thickness ( $h_{min}$ ), side flow ( $Q_s$ ), mean journal temperature and maximum bush temperature. Dynamic performance characteristics are presented in terms of sixteen stiffness coefficients ( $\bar{S}_{xx}, \bar{S}_{xz}, \bar{S}_{zx}, \bar{S}_{zz}, \bar{S}_{xyx}, \bar{S}_{xyy}, \bar{S}_{zyx}, \bar{S}_{zyz}, \bar{S}_{yxx}, \bar{S}_{yxy}, \bar{S}_{yzz}, \bar{S}_{yzy}, \bar{S}_{zxx}, \bar{S}_{zxz}, \bar{S}_{zxy}, \bar{S}_{zyz}$ ), sixteen damping coefficients ( $\bar{B}_{xx}, \bar{B}_{xz}, \bar{B}_{zx}, \bar{B}_{zz}, \bar{B}_{xyx}, \bar{B}_{xyy}, \bar{B}_{zyx}, \bar{B}_{zyz}, \bar{B}_{yxx}, \bar{B}_{yxy}, \bar{B}_{yzz}, \bar{B}_{yzy}, \bar{B}_{zxx}, \bar{B}_{zxz}, \bar{B}_{zxy}, \bar{B}_{zyz}$ ), instability threshold speed and whirl frequency ratio. To facilitate logical conclusions from this analysis, the results are presented for two different values of bi-planar misalignment ratios. In addition to these characteristics, isopressure curves, pressure profiles, isotherms and temperature profiles for liquid and solid domains have also been presented. In the present work, a pre-load factor of 0.5 has been used in the study of elliptical and three-lobe journal bearings.

### 5.3.1 Static Performance Characteristics

Various static performance characteristics of a plain, two-axial-groove, elliptical and three-lobe journal bearing are shown in Figs. 5.9 to 5.12. In the following, salient features of these characteristics are discussed.

**5.3.1.1 Eccentricity Ratio :** It is observed from the results presented in Figs. 5.9(a), 5.10(a), 5.11(a) and 5.12(a) that the eccentricity ratio ( $\epsilon_o$ ) shows a decreasing trend as misalignment is increased in all the bearing systems. This effect is mainly caused due to the decrease in the film thickness caused by misalignment on one side of the mid-plane which results in the increase in fluid-film pressure substantially and hence increase in load at a given eccentricity.

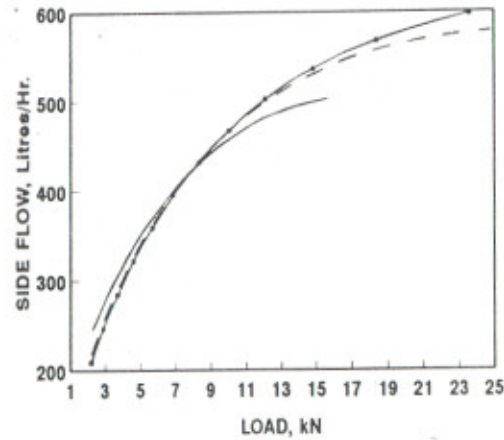
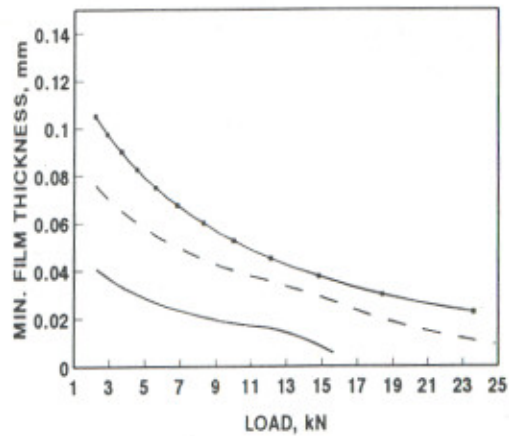
**5.3.1.2 Attitude Angle :** Figs. 5.9(b), 5.10(b), 5.11(b) and 5.12(b) shows that for all the bearing systems, the attitude angle ( $\phi$ ) decreases with the increase in misalignment. The effect is more pronounced at higher values of misalignment ratio. For a vertical load support, this trend is expected to improve the stability of the journal bearing system.



(a)

(b)

(c)



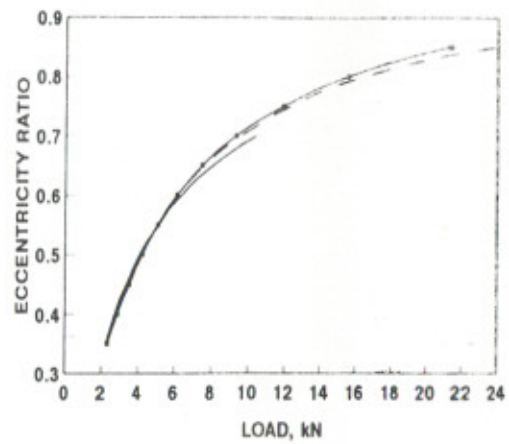
LEGEND :

- $\bar{\gamma}_x = \bar{\gamma}_z = 0.0$
- - -  $\bar{\gamma}_x = \bar{\gamma}_z = 0.2$
- — —  $\bar{\gamma}_x = \bar{\gamma}_z = 0.4$

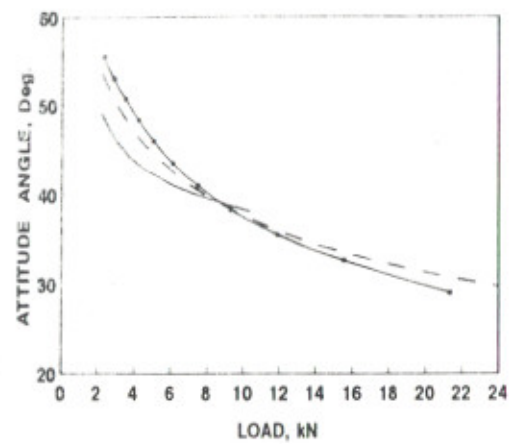
(d)

(e)

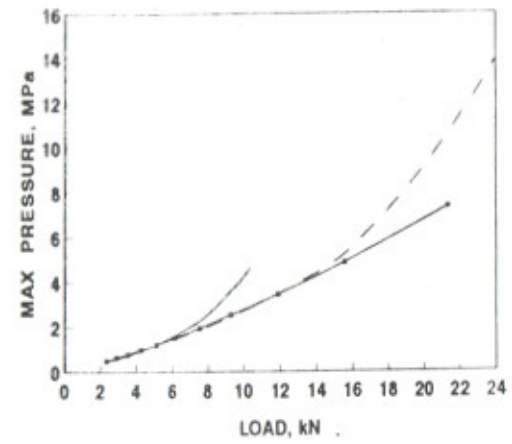
Fig. 5.9 PLAIN JOURNAL BEARING - STATIC PERFORMANCE CHARACTERISTICS  
(  $\epsilon_0$ ,  $\phi$ ,  $p_{\max}$ ,  $h_{\min}$ ,  $Q_s$  )



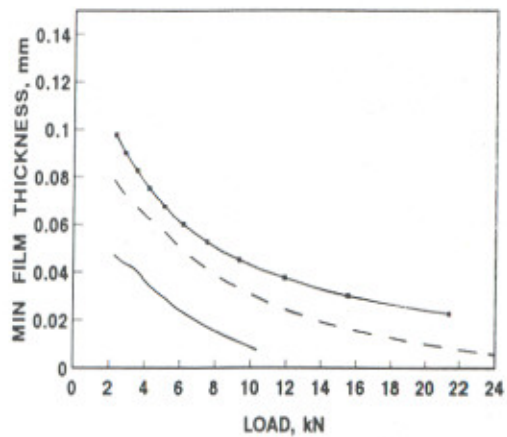
(a)



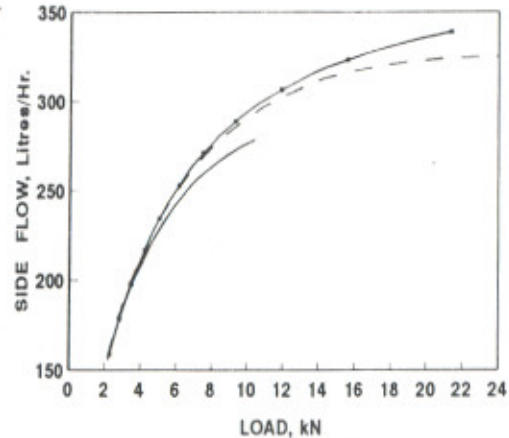
(b)



(c)



(d)



(e)

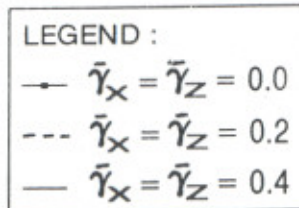
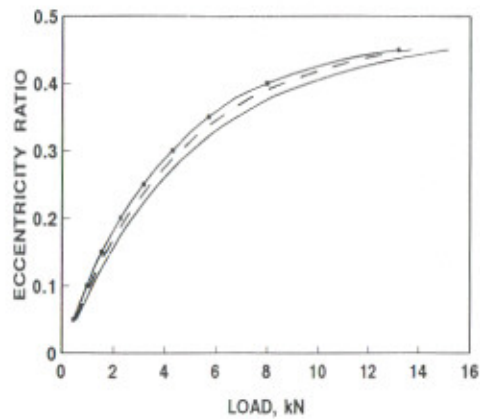
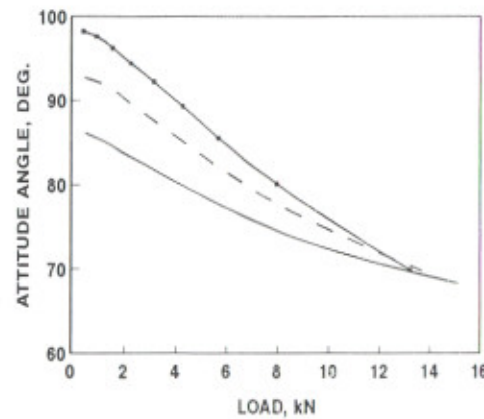


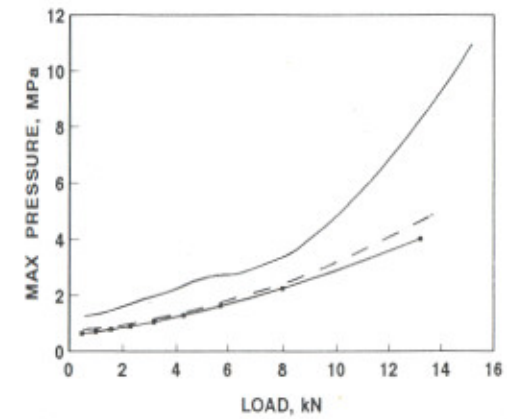
Fig. 5.10 TWO-AXIAL-GROOVE JOURNAL BEARING - STATIC PERFORMANCE CHARACTERISTICS  
 ( $\epsilon_0$ ,  $\phi$ ,  $p_{\max}$ ,  $h_{\min}$ ,  $Q_s$ )



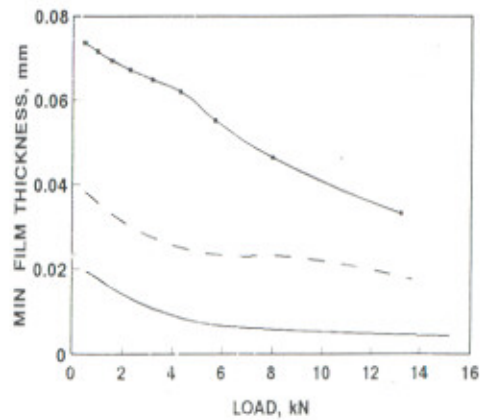
(a)



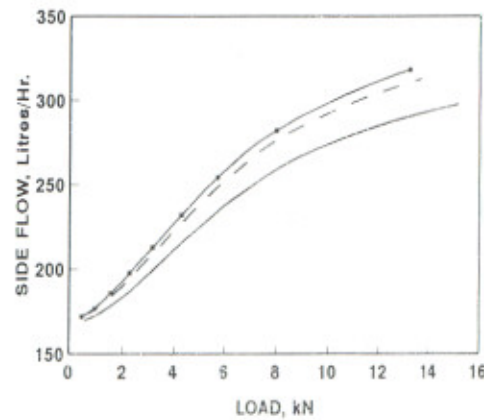
(b)



(c)



(d)



(e)

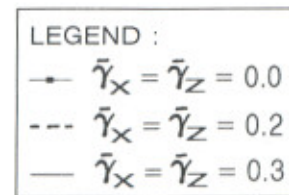
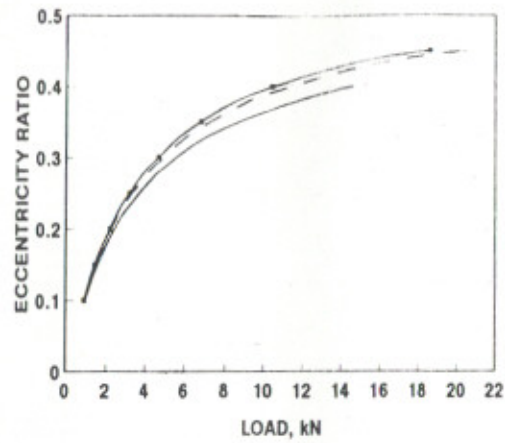
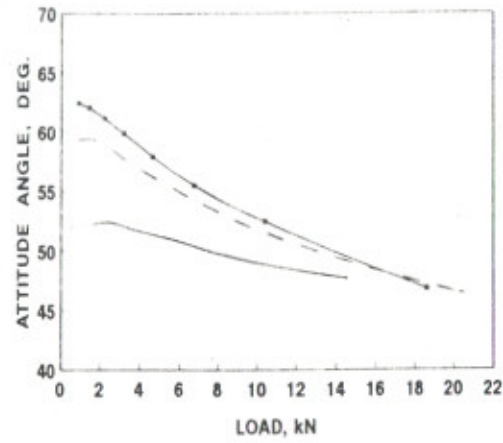


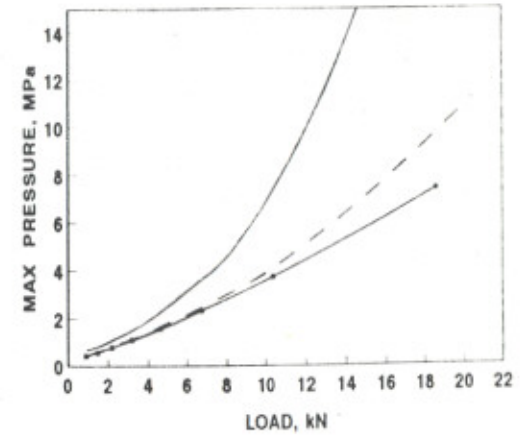
Fig. 5.11 ELLIPTICAL JOURNAL BEARING - STATIC PERFORMANCE CHARACTERISTICS  
 ( $\epsilon_0$ ,  $\phi$ ,  $p_{\max}$ ,  $h_{\min}$ ,  $Q_s$ )



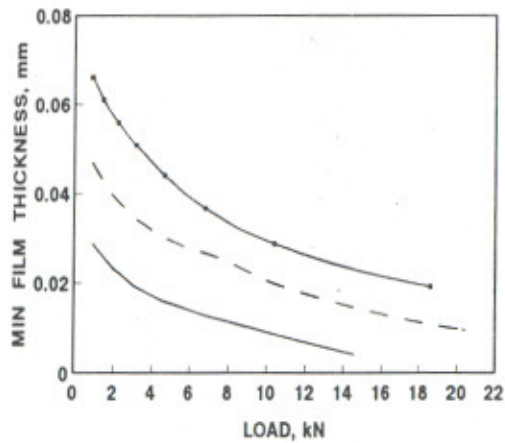
(a)



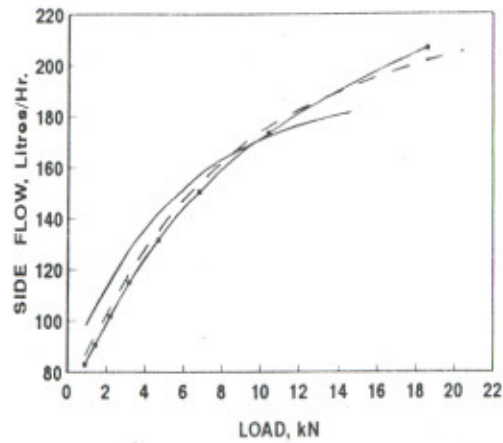
(b)



(c)



(d)



(e)

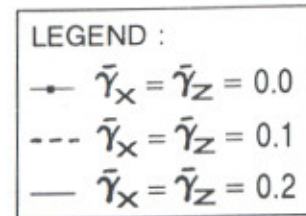


Fig. 5.12 THREE-LOBE JOURNAL BEARING - STATIC PERFORMANCE CHARACTERISTICS  
 (  $\epsilon_0$ ,  $\phi$ ,  $p_{max}$ ,  $h_{min}$ ,  $Q_s$  )

70

**5.3.1.3 Maximum Fluid-Film Pressure:** At higher load values, the values of maximum fluid-film pressures are significantly higher in a misaligned plain and two-axial-groove bearing as compared to an aligned bearing, Figs. 5.9(c) and 5.10(c). At moderate loads, however, the maximum fluid-film pressures are slightly lower than the corresponding values for an aligned bearing.

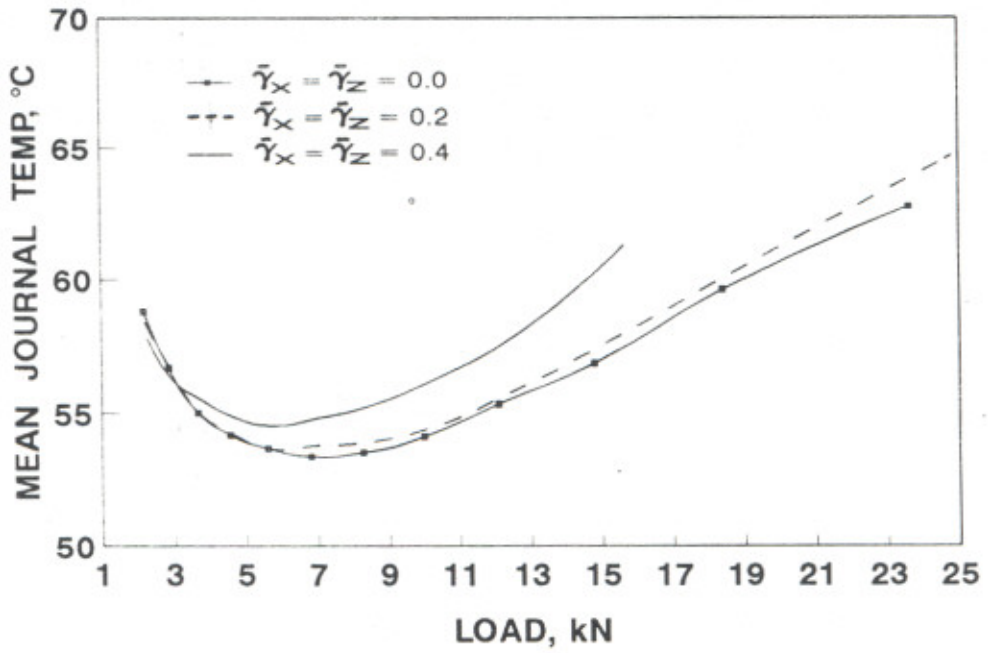
For elliptical and three-lobe bearings, it is observed from Figs. 5.11(c) and 5.12(c) that the maximum pressure ( $p_{\max}$ ) increases significantly with the increase in misalignment for all values of load.

**5.3.1.4 Minimum Fluid-film Thickness :** The effect of misalignment is quite adverse on fluid-film thickness. It is seen that for all the bearing systems, as the misalignment increases, the minimum fluid-film thickness decreases for all values of load, Figs. 5.9(d), 5.10(d), 5.11(d) and 5.12(d). This may lead to eventual failure of the journal bearing system due to metal-to-metal contact and resulting wear.

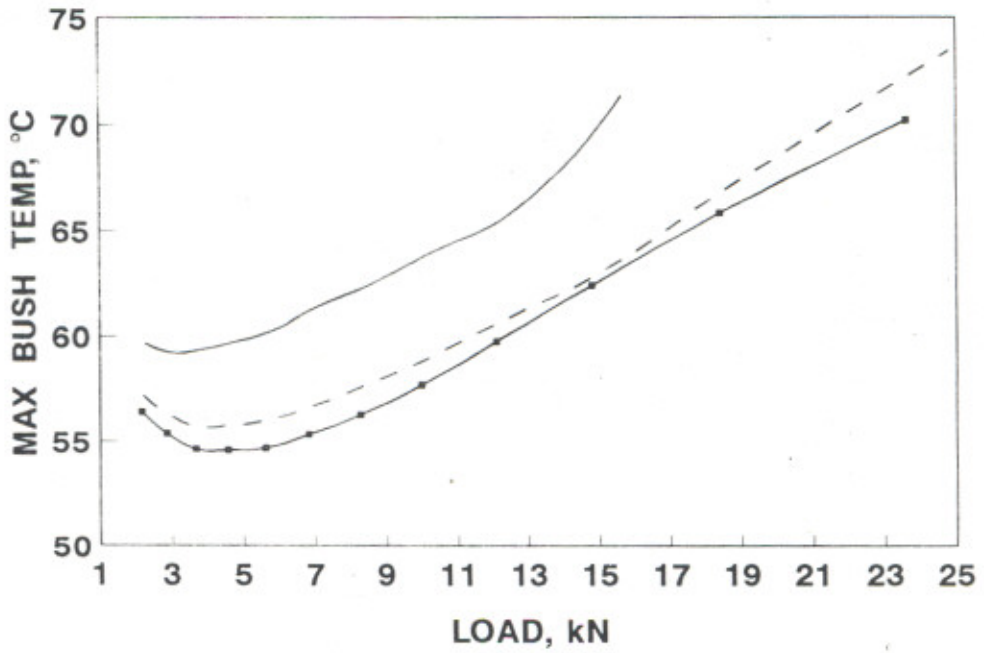
**5.3.1.5 Side Flow :** Figs. 5.9(e), 5.10(e) and 5.12(e) reveals that at light loads, the effect of increasing misalignment leads to increased side flow ( $Q_s$ ) whereas at higher load values, the trend is reversed for all bearings except elliptical journal bearing where increased misalignment leads to increased side flow ( $Q_s$ ) at all load values, Fig. 5.11(e).

**5.3.1.6 Mean Journal Temperature and Maximum Bush Temperature :** The mean journal temperature and the maximum bush temperature both, are shown in Figs. 5.13 to 5.16. Figs. 5.13(a), 5.14(a), 5.15(a) and 5.16(a) reveals that at heavy load, an increase in the misalignment leads to higher mean journal temperature which is probably caused by the reduction in side flow requiring greater amount of heat generated due to viscous friction now being transferred out of the bearing by the solids. Figs. 5.13(b) and 5.14(b) which shows the maximum bush temperatures, also confirms this observation.

It is a well known fact that the bearing load capacity is limited by a specified value of the minimum fluid-film thickness and / or a maximum allowable bush temperature. The bearing is expected to fail whenever these limits are violated. With reference to the present work, if we limit the maximum bush temperature at 70 °C (say) in case of plain journal bearing, the load capacity reduces from 23.6 kN for an aligned bearing to only 15.2 kN for a misaligned bearing with  $\bar{\gamma}_x = \bar{\gamma}_z = 0.4$ , Fig. 5.13(b). This shows a decrease of about 35.6 % in the load capacity due to misalignment. Likewise, if the value of minimum fluid-film thickness should not reduce

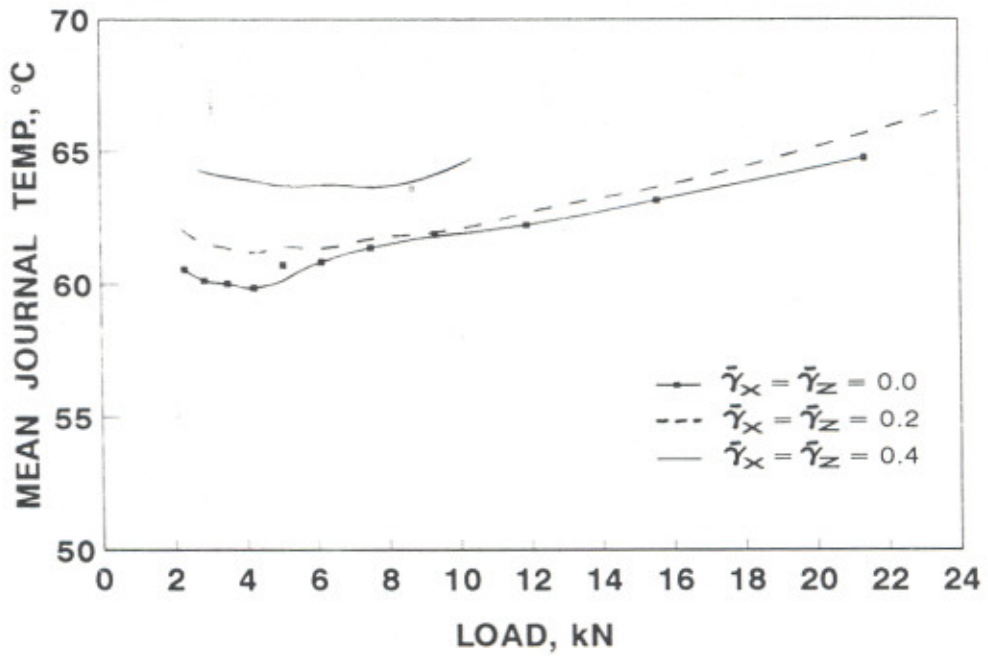


(a)

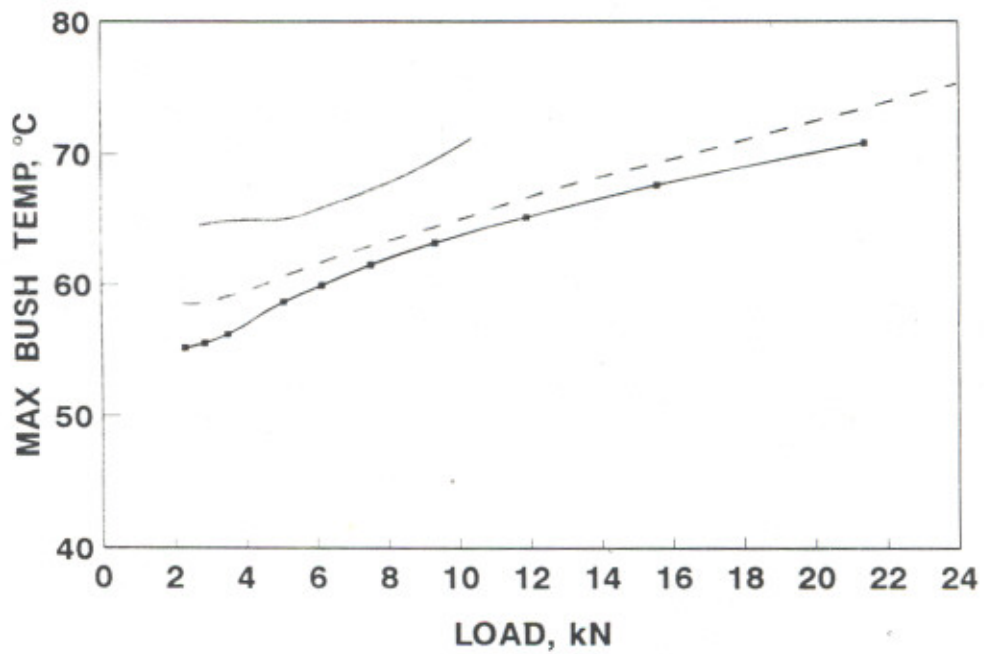


(b)

Fig. 5.13 PLAIN JOURNAL BEARING - STATIC PERFORMANCE CHARACTERISTICS (  $T_J$  AND  $T_B$  )

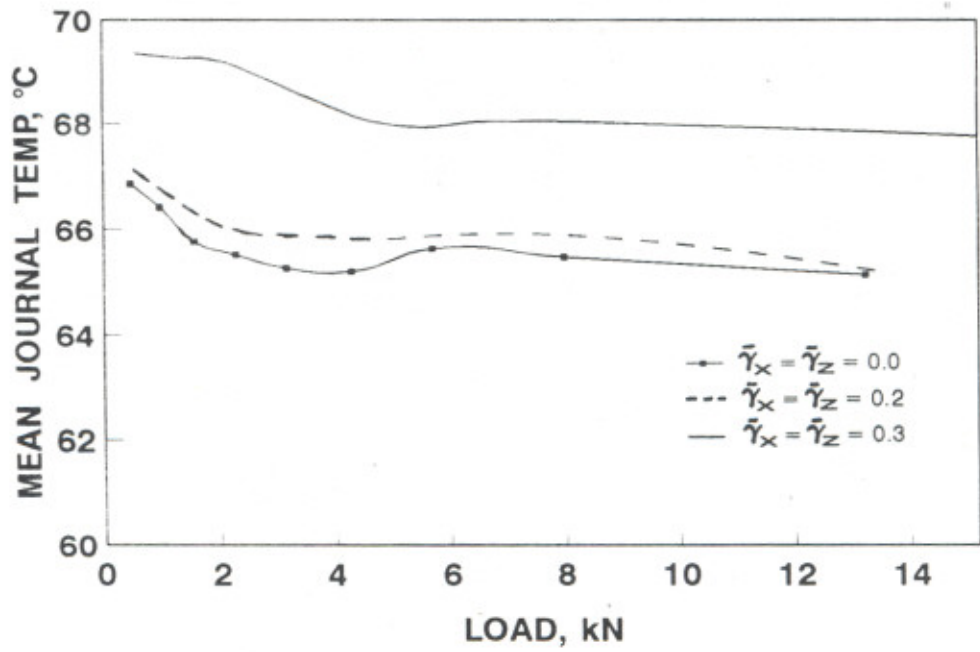


( a )

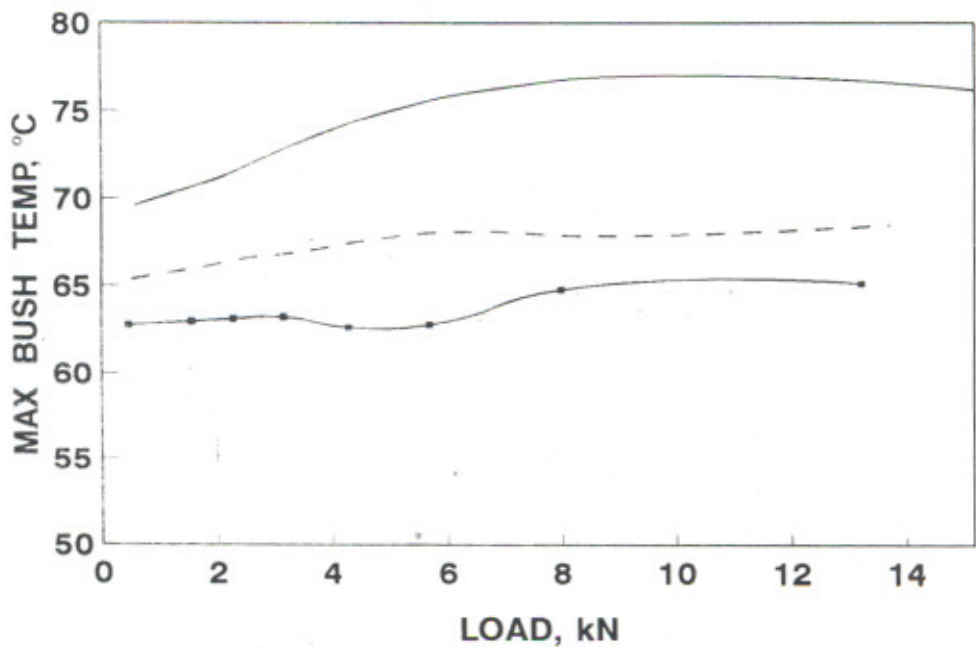


( b )

Fig. 5.14 TWO-AXIAL-GROOVE JOURNAL BEARING - STATIC PERFORMANCE CHARACTERISTICS (  $T_J$  AND  $T_B$  )

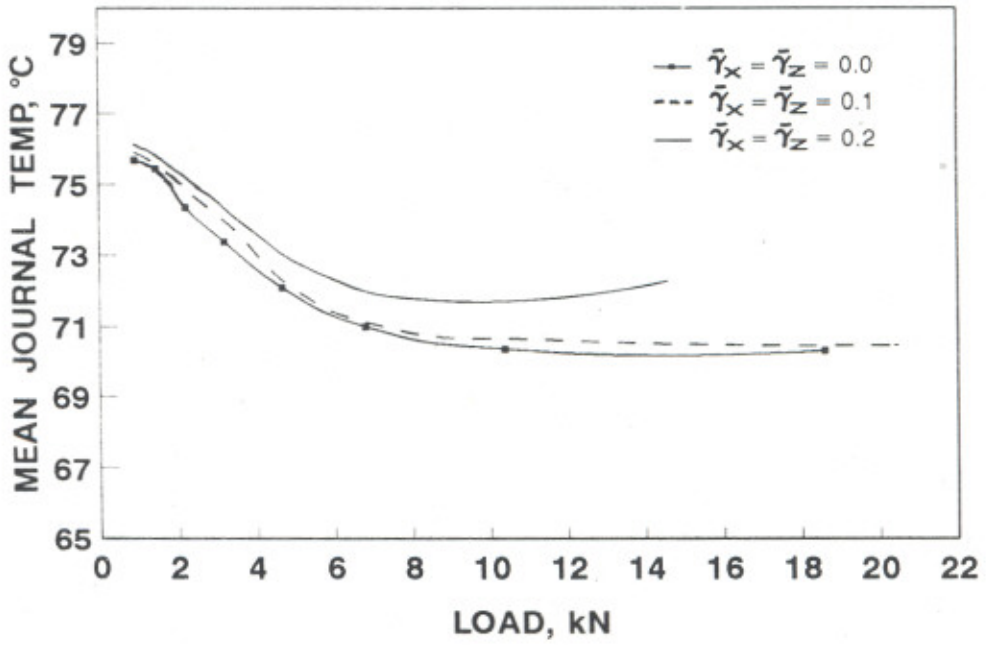


(a)

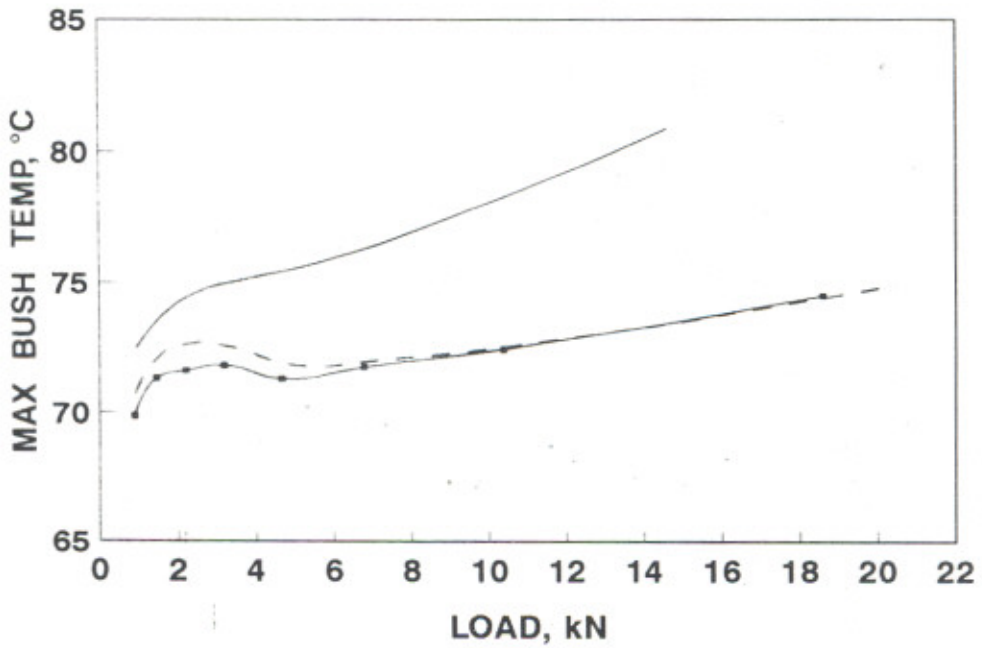


(b)

Fig. 5.15 ELLIPTICAL JOURNAL BEARING - STATIC PERFORMANCE CHARACTERISTICS (  $T_J$  AND  $T_B$  )



(a)



(b)

Fig. 5.16 THREE-LOBE JOURNAL BEARING - STATIC PERFORMANCE CHARACTERISTICS (  $T_J$  AND  $T_B$  )

below 0.02 mm (say), the bearing load capacity reduces from nearly 27 kN to 9 kN in the misaligned bearing showing a reduction of 66.7 % in the load capacity of the plain journal bearing, Fig. 5.9(d).

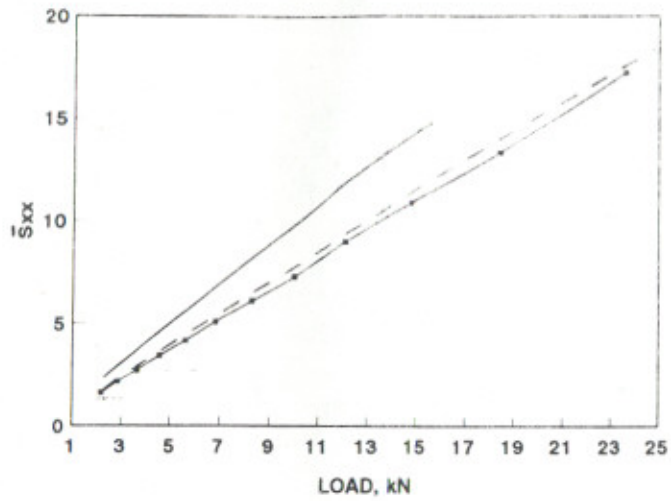
Figs. 5.15(a) and 5.16(a) indicates that for elliptical and three-lobe bearings, the mean journal temperature slightly reduces as the load supported by the bearing increases and the trend is similar for aligned and misaligned bearings.

The maximum bush temperature, however, increases with the increase in load. For example, at a load value of 10 kN, the maximum bush temperature is 65.6 °C in case of an aligned elliptical journal bearing whereas for bearing running with a misalignment of  $\bar{\gamma}_x = \bar{\gamma}_z = 0.2$ , it is approximately 68 °C and rises to 77 °C for a misalignment of  $\bar{\gamma}_x = \bar{\gamma}_z = 0.3$ , Fig. 5.15(b).

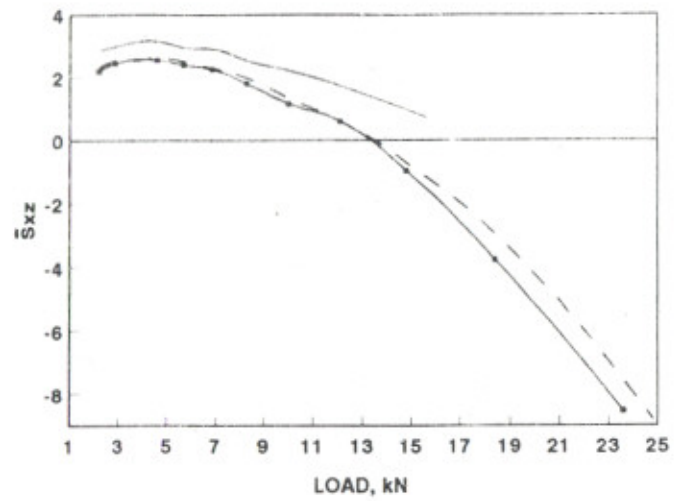
### 5.3.2 Stiffness Coefficients

The stiffness coefficients for the four bearing systems are plotted in Figs. 5.17 to 5.28. In general, the values of  $\bar{S}_{xx}$  {Figs. 5.17(a), 5.20(a), 5.23(a), 5.26(a)},  $\bar{S}_{xz}$  {Figs. 5.17(b), 5.20(b), 5.23(b), 5.26(b)} and  $\bar{S}_{zz}$  {Figs. 5.17(d), 5.20(d), 5.23(d), 5.26(d)} at any load are found to increase with misalignment for all the bearing systems studied. The stiffness coefficient,  $\bar{S}_{zx}$  {Figs. 5.17(c), 5.20(c)} shows no significant change due to misalignment in case of circular bearings where as in case of non-circular bearings,  $\bar{S}_{zx}$  {Figs. 5.23(c), 5.26(c)} increases with misalignment at any load value. The stiffness coefficients,  $\bar{S}_{xx}$  {Figs. 5.17(a), 5.20(a), 5.23(a), 5.26(a)} and  $\bar{S}_{zz}$  {Figs. 5.17(d), 5.20(d), 5.23(d), 5.26(d)} increases with the increase in load values where as the stiffness coefficients,  $\bar{S}_{xz}$  {Figs. 5.17(b), 5.20(b), 5.23(b), 5.26(b)} and  $\bar{S}_{zx}$  {Figs. 5.17(c), 5.20(c), 5.23(c), 5.26(c)} decreases with the increase in the load values.

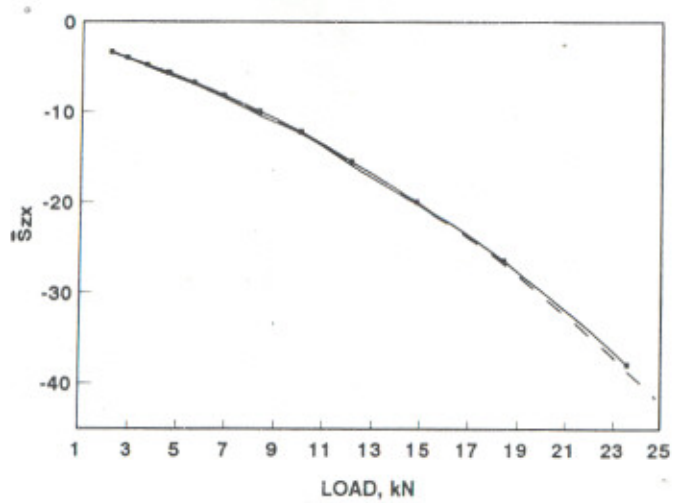
Likewise, the figures show that the values of stiffness coefficients,  $\bar{S}_{xyx}$  {Figs. 5.18(a), 5.21(a), 5.24(a), 5.27(a)},  $\bar{S}_{yxx}$  {Figs. 5.18(e), 5.21(e), 5.24(e), 5.27(e)},  $\bar{S}_{yxx}$  {Figs. 5.19(c), 5.22(c), 5.25(c), 5.28(c)},  $\bar{S}_{y_x y_z}$  {Figs. 5.19(d), 5.22(d), 5.25(d), 5.28(d)},  $\bar{S}_{y_z y_x}$  {Figs. 5.19(f), 5.22(f), 5.25(f), 5.28(f)}, increase with the increase in misalignment. In all these cases, the effect is more pronounced at higher values of misalignment ratio. Figures reveal that the values of stiffness coefficients  $\bar{S}_{xy_z}$  {Figs. 5.18(b), 5.21(b), 5.27(b)},  $\bar{S}_{zr_z}$  {Figs. 5.18(d), 5.21(d), 5.24(d), 5.27(d)},  $\bar{S}_{y_x z}$  {Figs. 5.18(f), 5.21(f), 5.27(f)},  $\bar{S}_{z_x}$  {Figs. 5.19(a), 5.22(a)},  $\bar{S}_{z_z}$  {Figs.



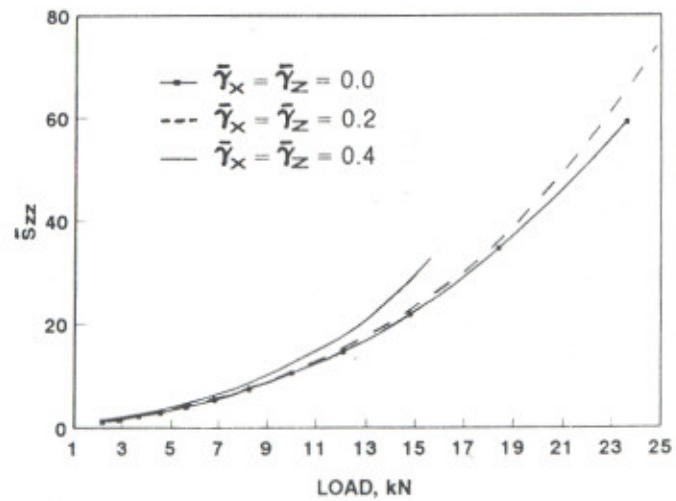
(a)



(b)

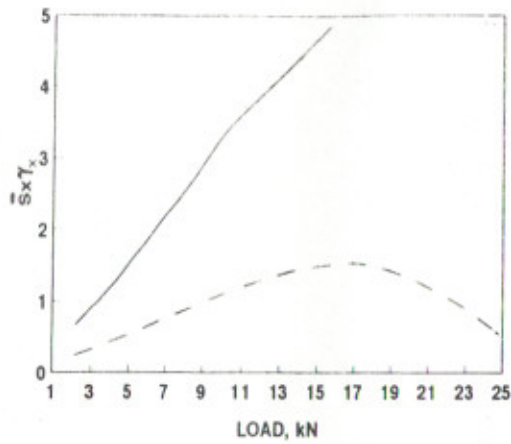


(c)

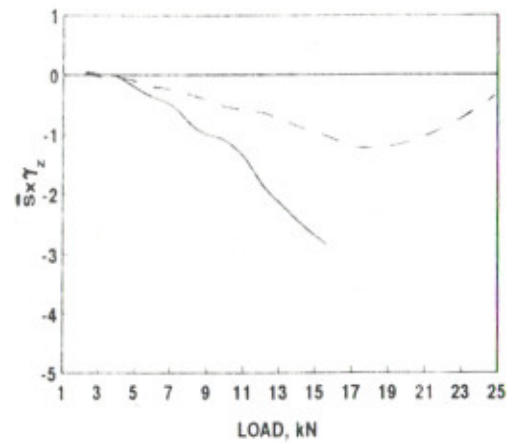


(d)

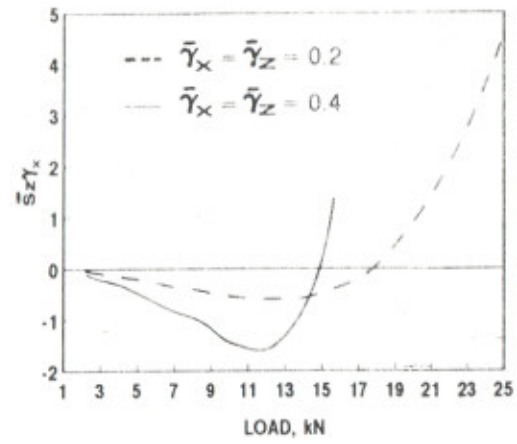
Fig. 5.17 PLAIN JOURNAL BEARING - STIFFNESS COEFFICIENTS  
( $\bar{S}_{xx}$ ,  $\bar{S}_{xz}$ ,  $\bar{S}_{zx}$ ,  $\bar{S}_{zz}$ )



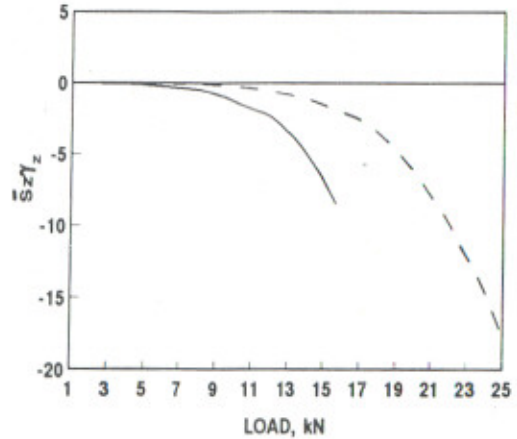
(a)



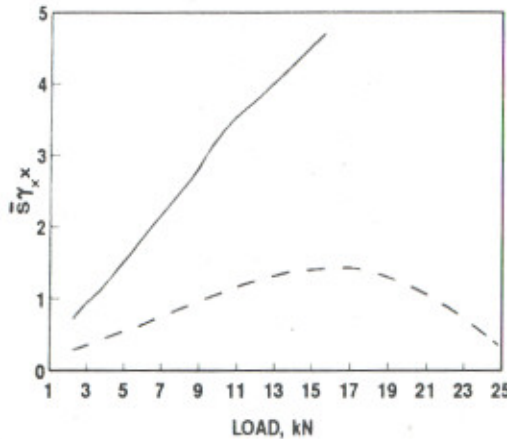
(b)



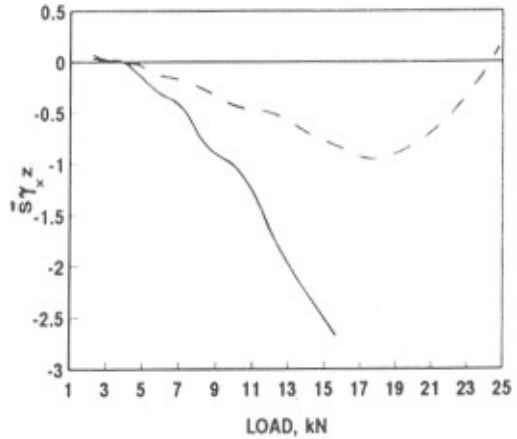
(c)



(d)

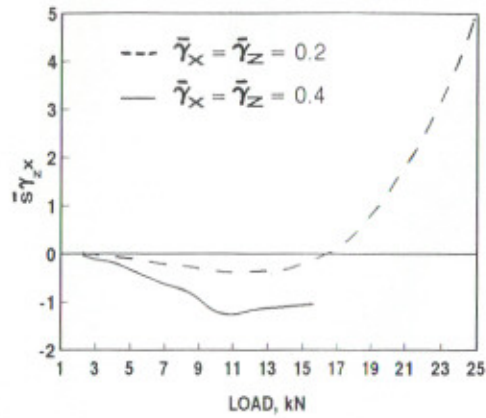


(e)

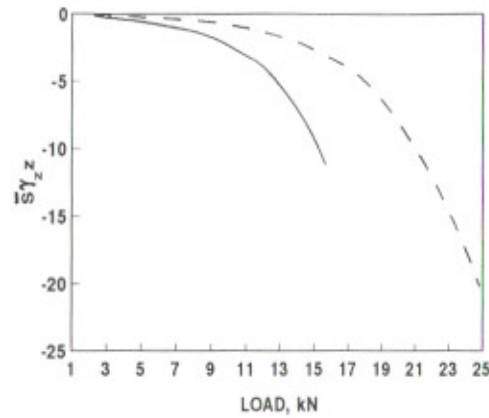


(f)

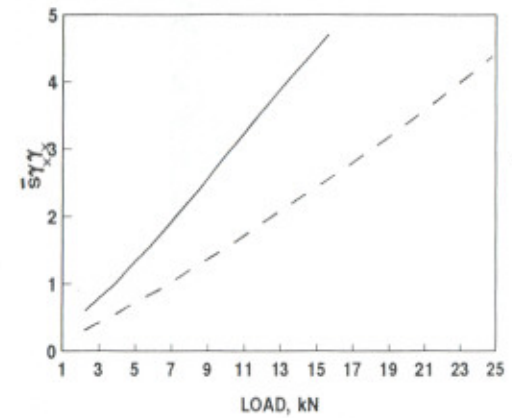
Fig. 5.18 PLAIN JOURNAL BEARING - STIFFNESS COEFFICIENTS  
 $(\bar{S}_{x\gamma_x}, \bar{S}_{x\gamma_z}, \bar{S}_{z\gamma_x}, \bar{S}_{z\gamma_z}, \bar{S}_{\gamma_x x}, \bar{S}_{\gamma_x z})$



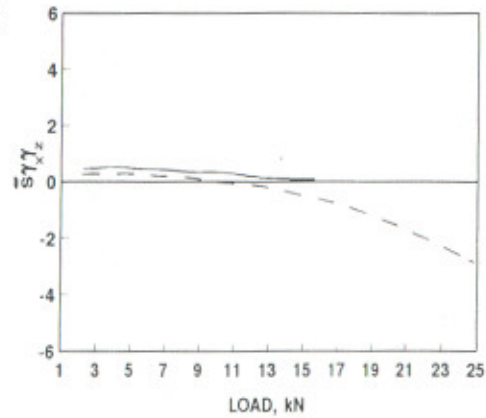
(a)



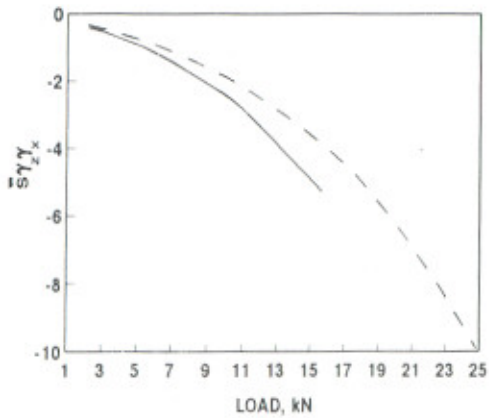
(b)



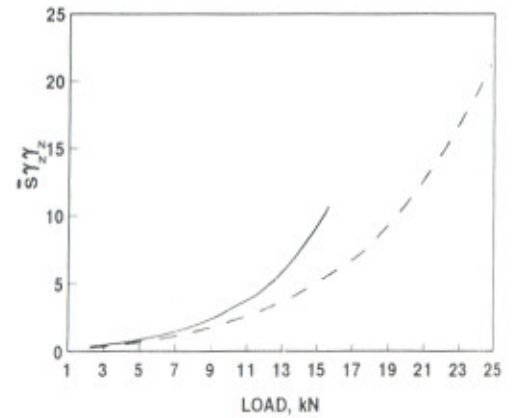
(c)



(d)



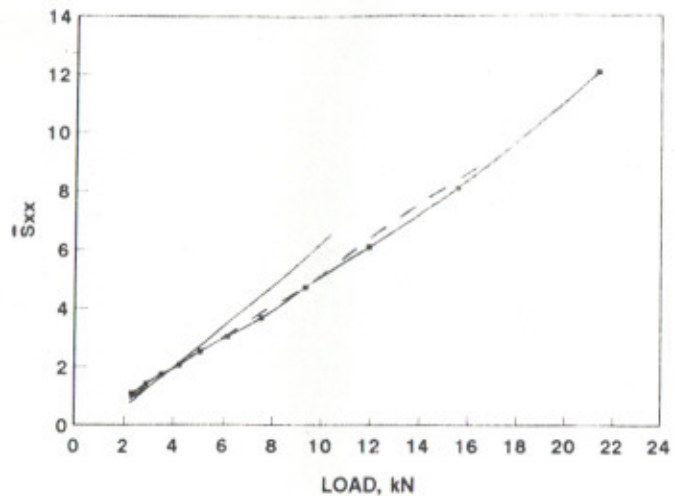
(e)



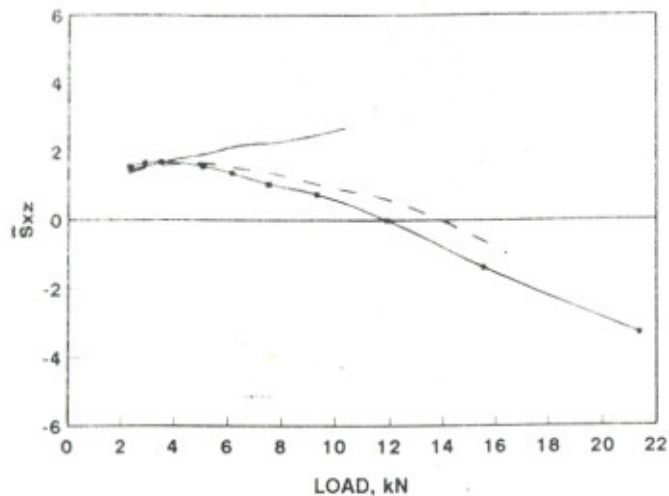
(f)

Fig. 5.19 PLAIN JOURNAL BEARING - STIFFNESS COEFFICIENTS

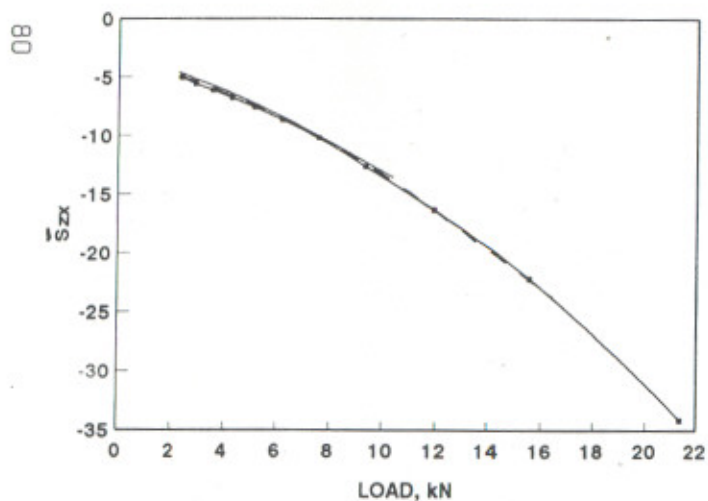
$$(\bar{S}_{\gamma_z x}, \bar{S}_{\gamma_z z}, \bar{S}_{\gamma_x x}, \bar{S}_{\gamma_x z}, \bar{S}_{\gamma_z x}, \bar{S}_{\gamma_z z})$$



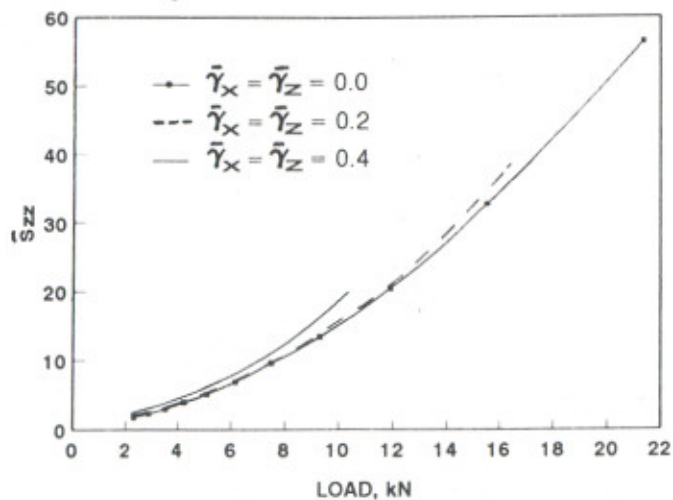
(a)



(b)

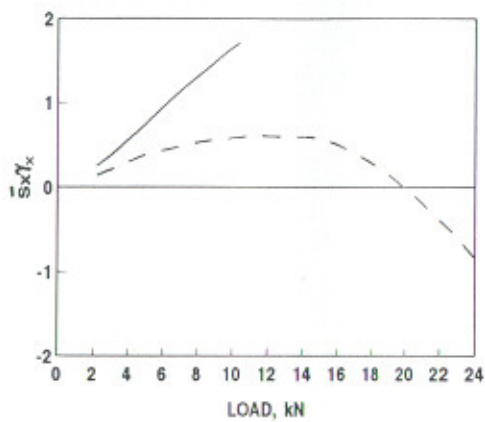


(c)

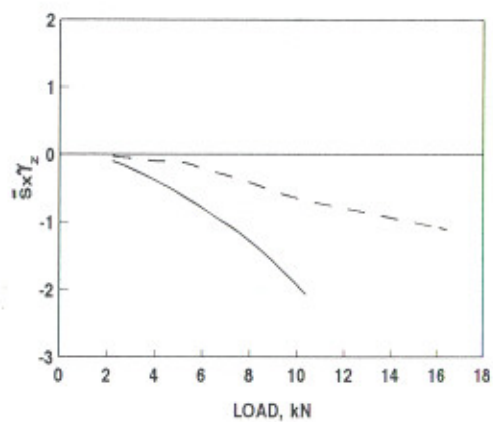


(d)

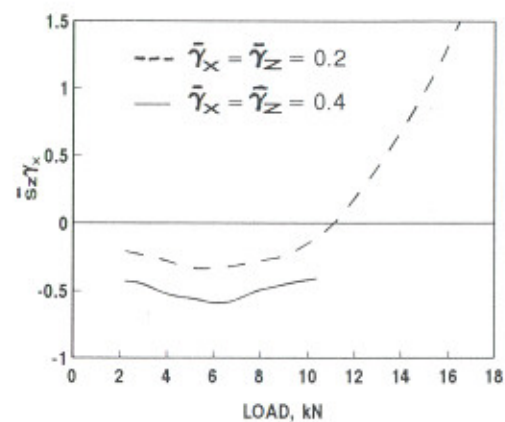
Fig. 5.20 TWO-AXIAL-GROOVE JOURNAL BEARING - STIFFNESS COEFFICIENTS  
(  $\bar{S}_{xx}$ ,  $\bar{S}_{xz}$ ,  $\bar{S}_{zx}$ ,  $\bar{S}_{zz}$  )



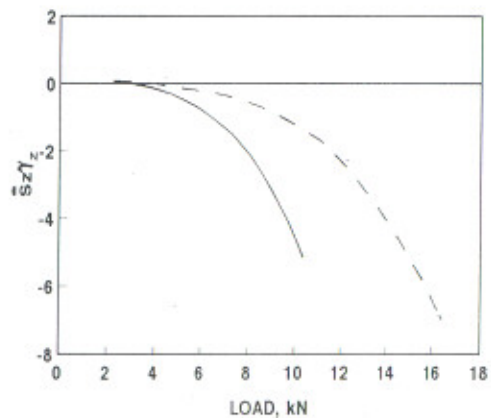
(a)



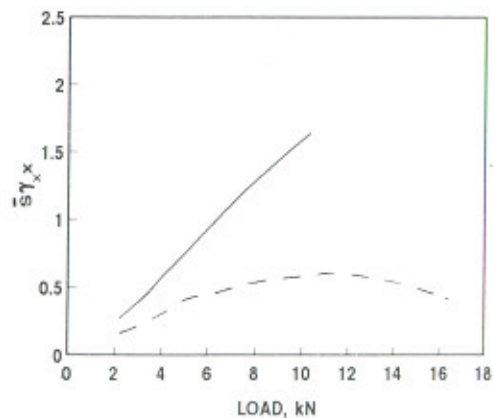
(b)



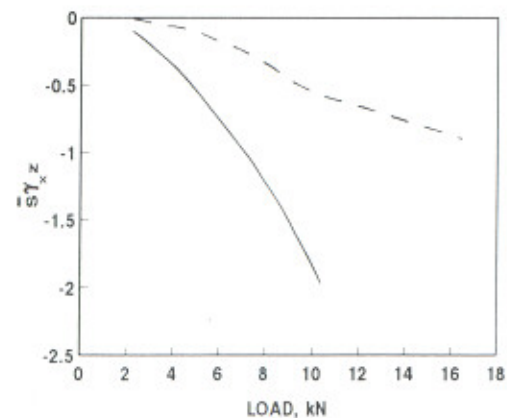
(c)



(d)



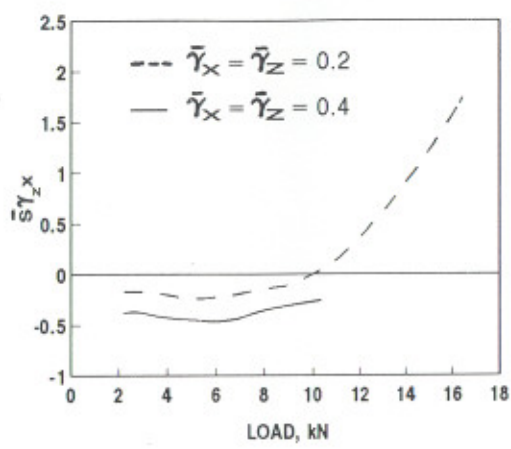
(e)



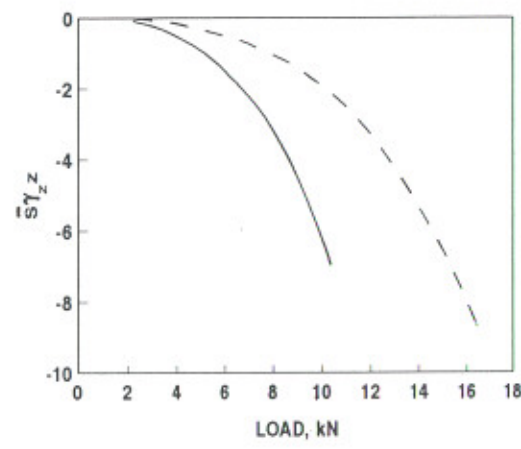
(f)

Fig. 5.21 TWO-AXIAL-GROOVE JOURNAL BEARING - STIFFNESS COEFFICIENTS

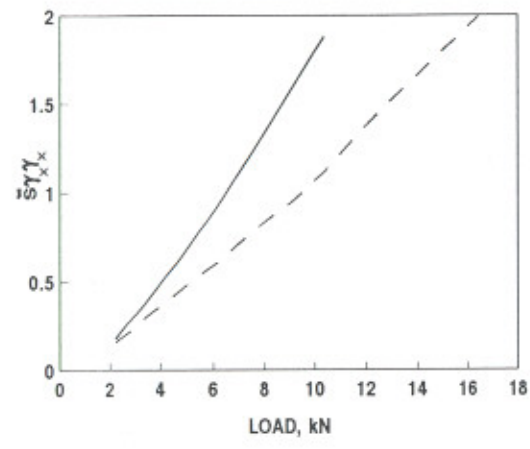
$$(\bar{S}_x\gamma_x, \bar{S}_x\gamma_z, \bar{S}_z\gamma_x, \bar{S}_z\gamma_z, \bar{S}_\gamma x, \bar{S}_\gamma z)$$



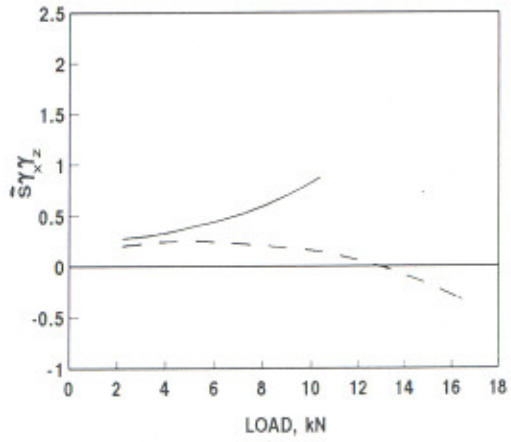
(a)



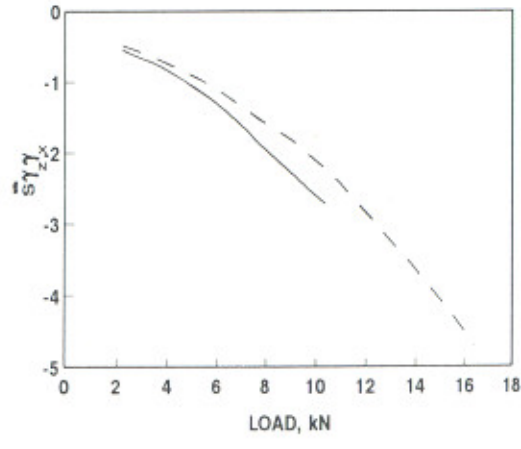
(b)



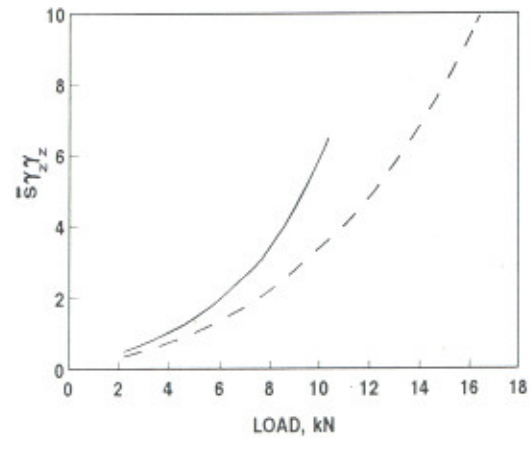
(c)



(d)



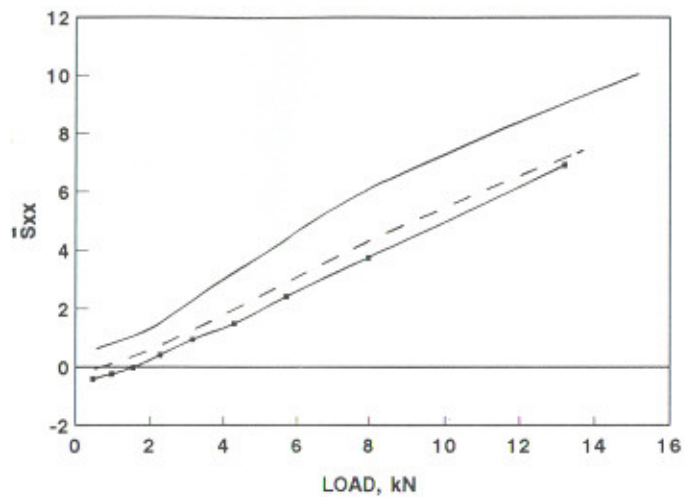
(e)



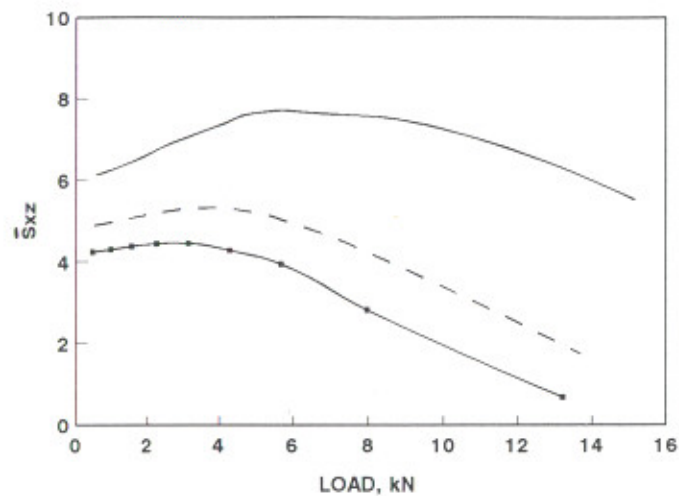
(f)

Fig. 5.22 TWO-AXIAL-GROOVE JOURNAL BEARING - STIFFNESS COEFFICIENTS

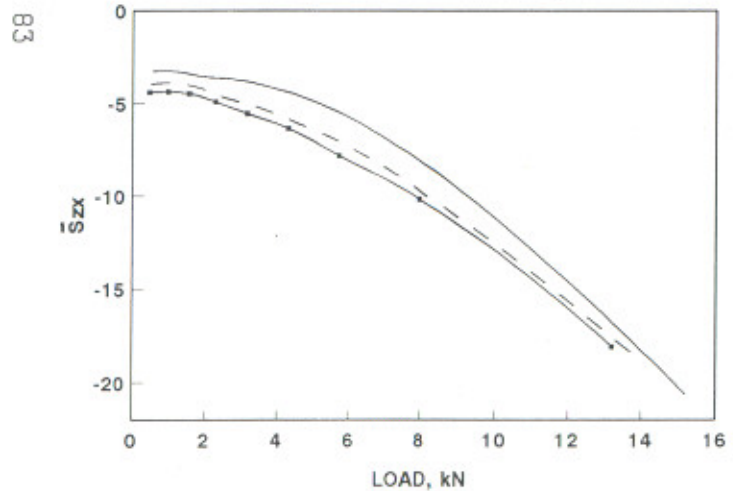
$$(\bar{S}_{\gamma_z}, \bar{S}_{\gamma_z}, \bar{S}_{\gamma_x\gamma_x}, \bar{S}_{\gamma_x\gamma_z}, \bar{S}_{\gamma_z\gamma_x}, \bar{S}_{\gamma_z\gamma_z})$$



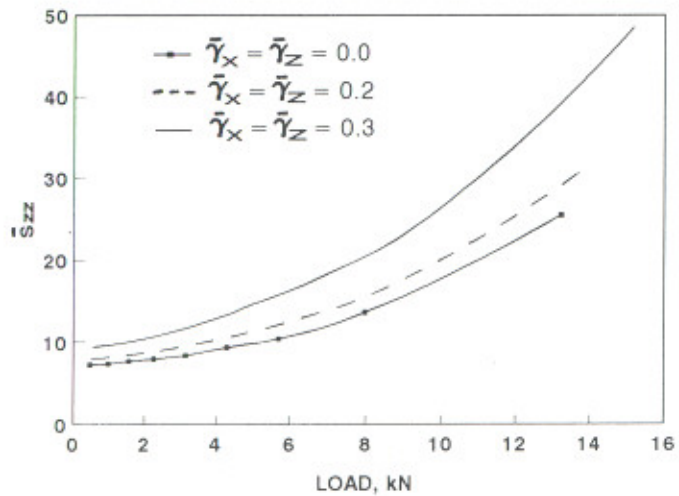
(a)



(b)



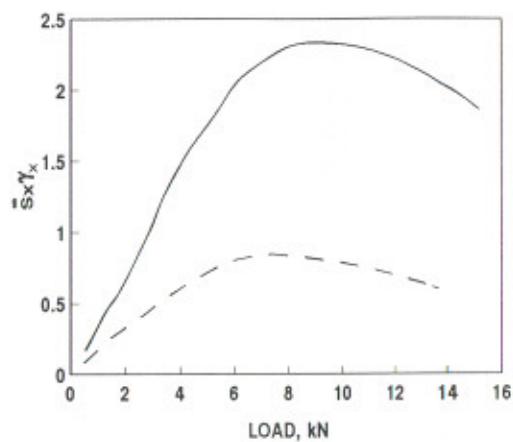
(c)



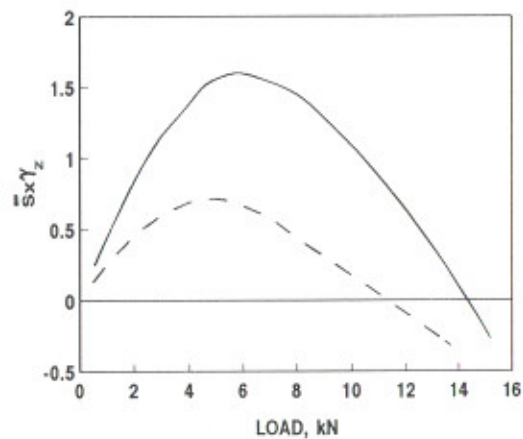
(d)

Fig. 5.23 ELLIPTICAL JOURNAL BEARING - STIFFNESS COEFFICIENTS

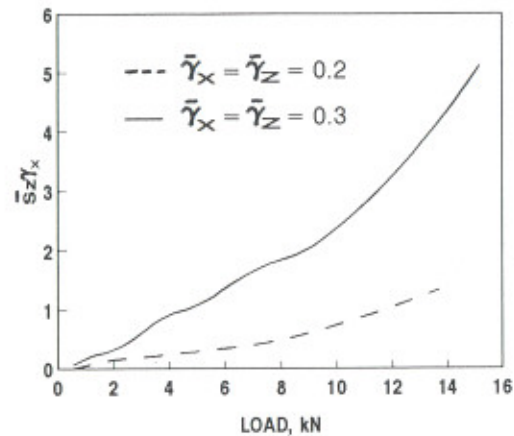
(  $\bar{S}_{xx}$ ,  $\bar{S}_{xz}$ ,  $\bar{S}_{zx}$ ,  $\bar{S}_{zz}$  )



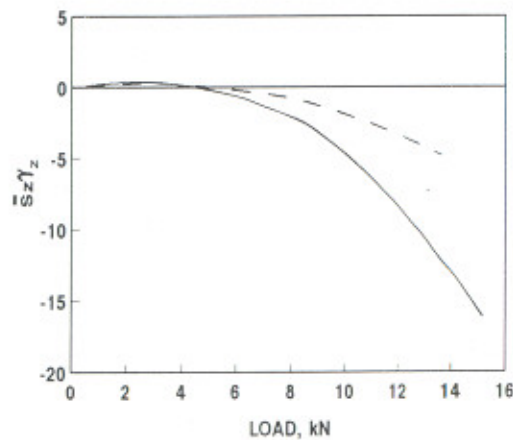
(a)



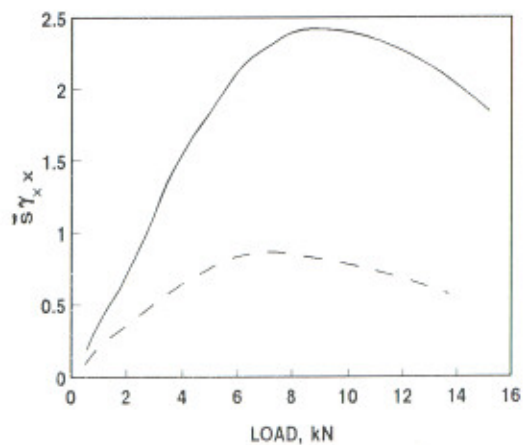
(b)



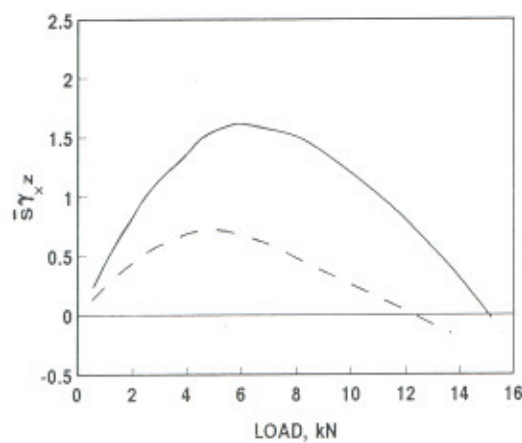
(c)



(d)



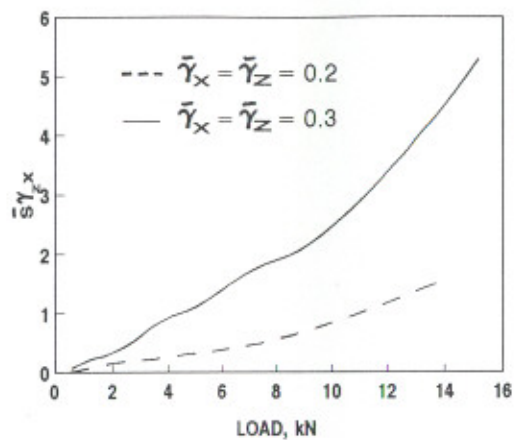
(e)



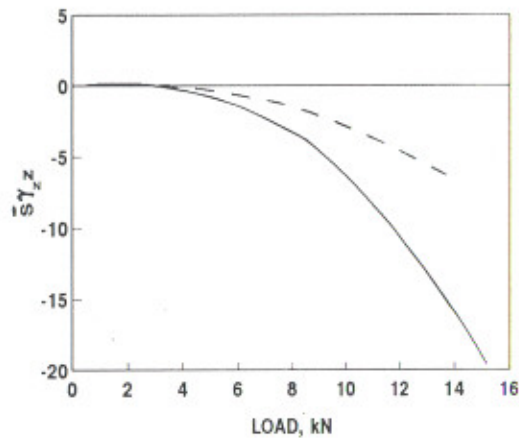
(f)

Fig. 5.24 ELLIPTICAL JOURNAL BEARING - STIFFNESS COEFFICIENTS

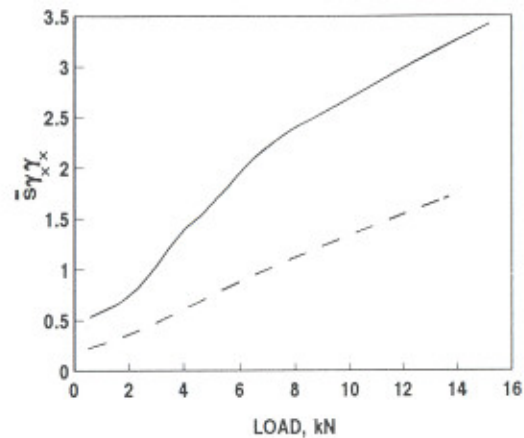
( $\bar{S}_{x\gamma_x}$ ,  $\bar{S}_{x\gamma_z}$ ,  $\bar{S}_{z\gamma_x}$ ,  $\bar{S}_{z\gamma_z}$ ,  $\bar{S}_{\gamma_x x}$ ,  $\bar{S}_{\gamma_x z}$ )



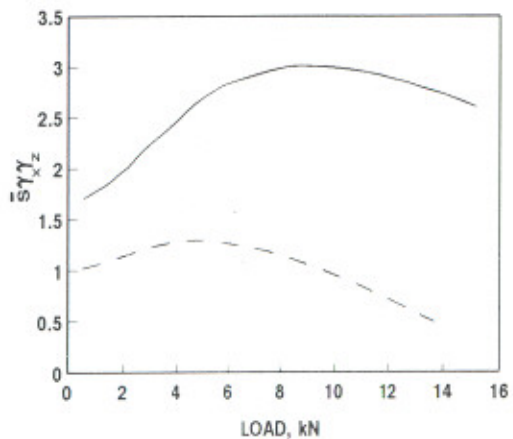
(a)



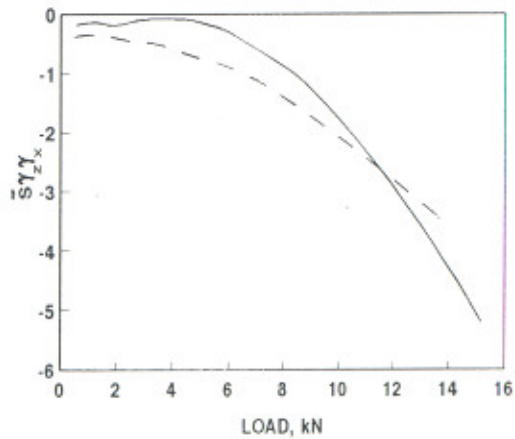
(b)



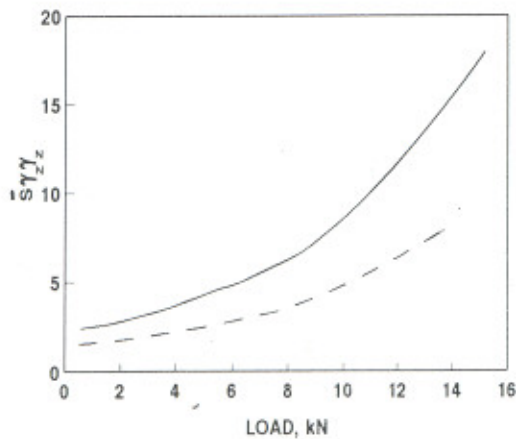
(c)



(d)



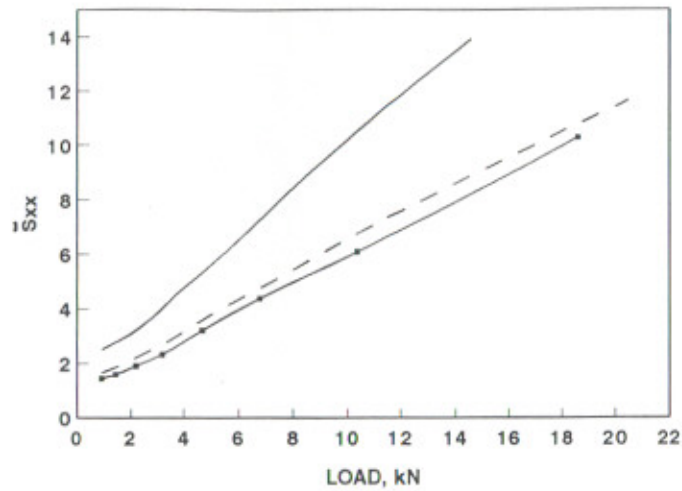
(e)



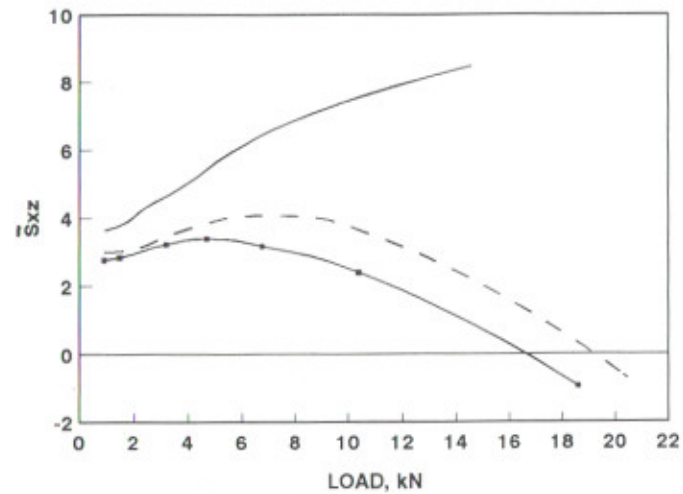
(f)

Fig. 5.25 ELLIPTICAL JOURNAL BEARING - STIFFNESS COEFFICIENTS

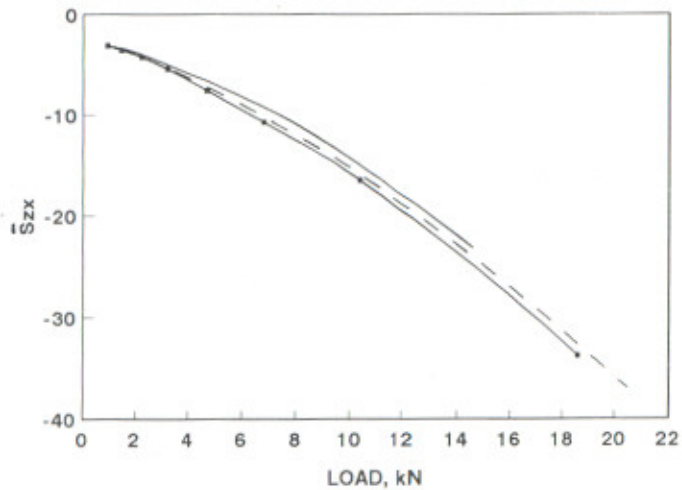
$$(\bar{s}\gamma_{z,x}, \bar{s}\gamma_{z,z}, \bar{s}\gamma_{x,x}, \bar{s}\gamma_{x,z}, \bar{s}\gamma_{z,x}, \bar{s}\gamma_{z,z})$$



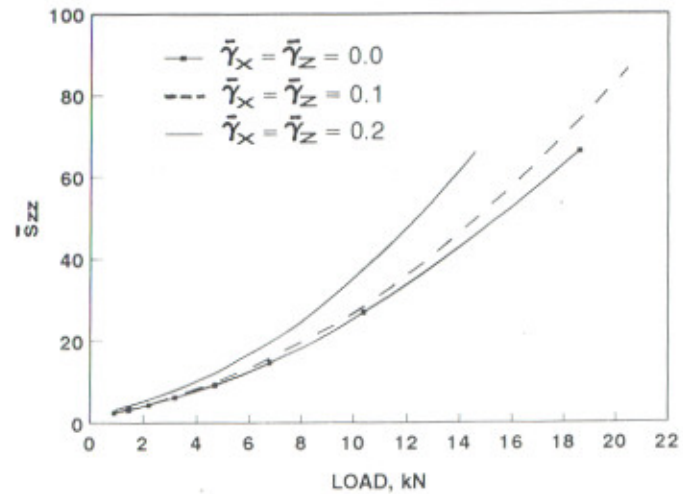
(a)



(b)

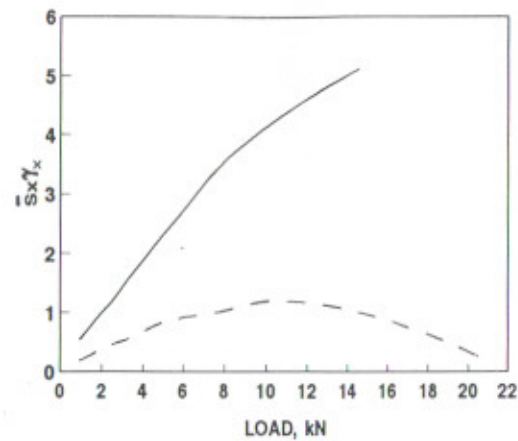


(c)

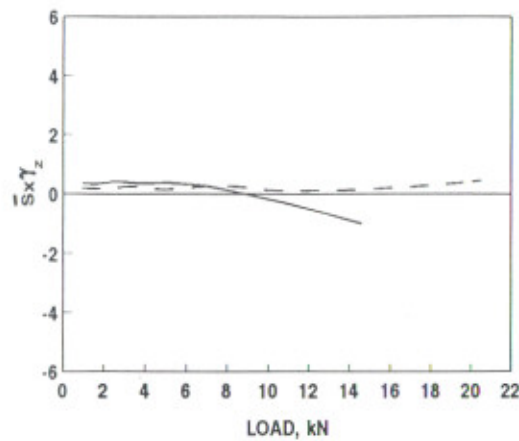


(d)

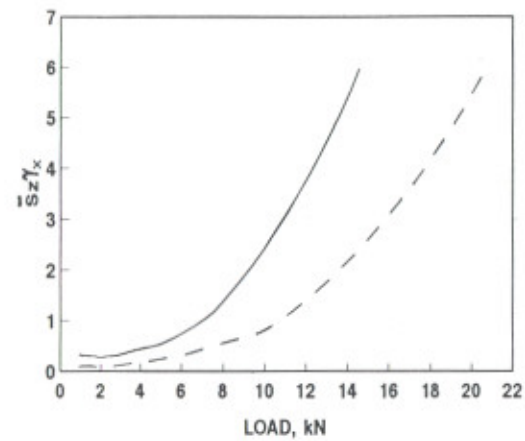
Fig. 5.26 THREE-LOBE JOURNAL BEARING - STIFFNESS COEFFICIENTS  
( $\bar{S}_{xx}$   $\bar{S}_{xz}$   $\bar{S}_{zx}$   $\bar{S}_{zz}$ )



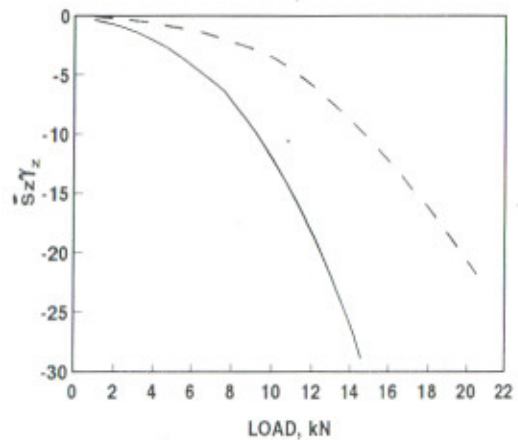
(a)



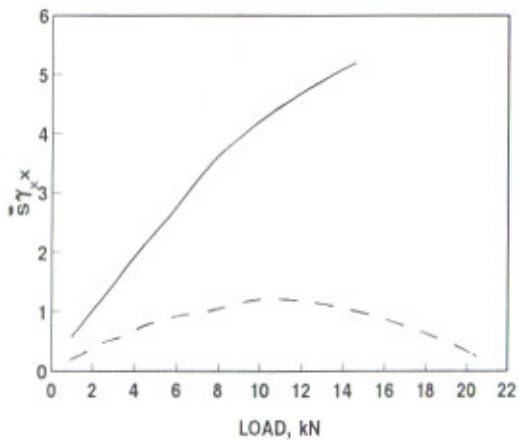
(b)



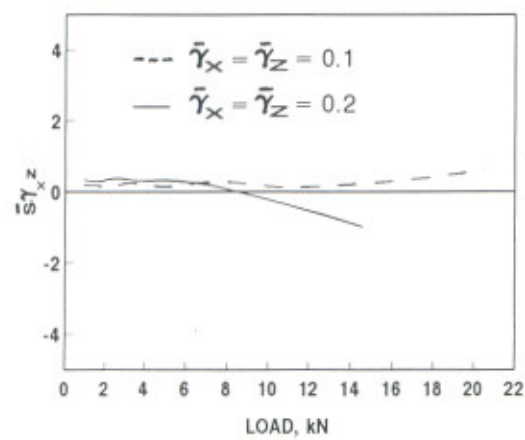
(c)



(d)



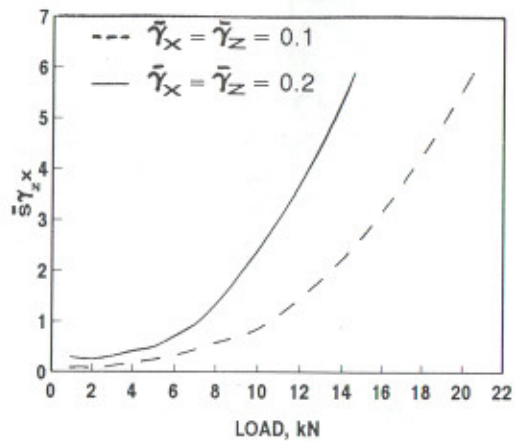
(e)



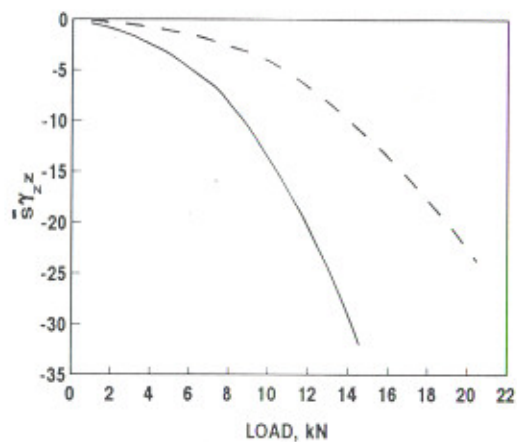
(f)

Fig. 5.27 THREE-LOBE JOURNAL BEARING - STIFFNESS COEFFICIENTS

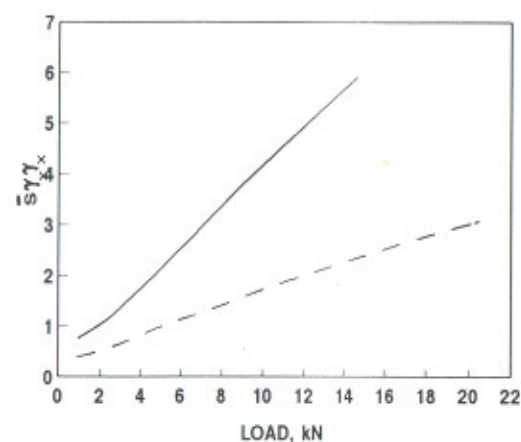
$$(\bar{S}_x\gamma_x, \bar{S}_x\gamma_z, \bar{S}_z\gamma_x, \bar{S}_z\gamma_z, \bar{S}_x\gamma_x, \bar{S}_x\gamma_z)$$



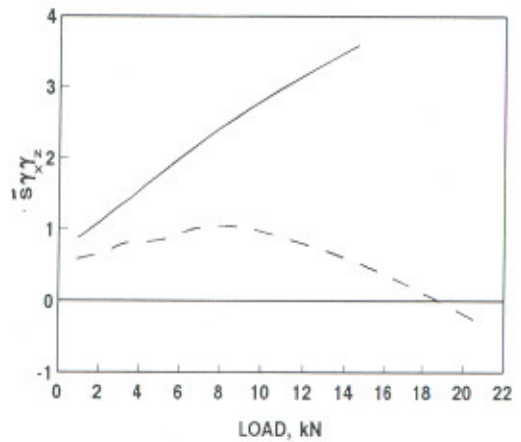
(a)



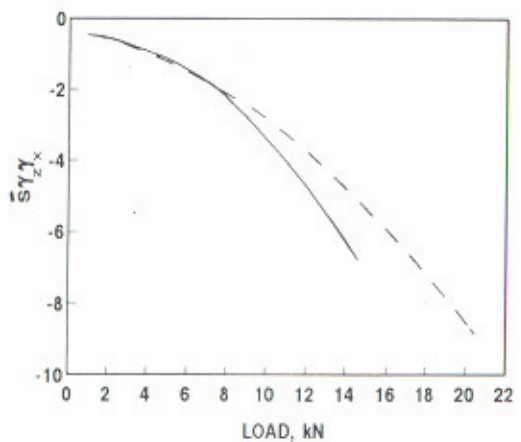
(b)



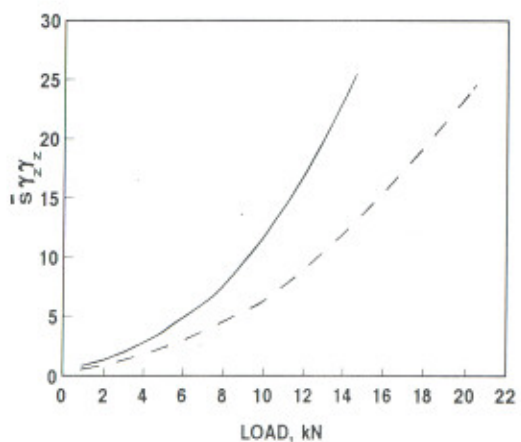
(c)



(d)



(e)



(f)

Fig. 5.28 THREE-LOBE JOURNAL BEARING - STIFFNESS COEFFICIENTS

$$(\bar{S}_{\gamma_x}, \bar{S}_{\gamma_z}, \bar{S}_{\gamma_x \gamma_x}, \bar{S}_{\gamma_x \gamma_z}, \bar{S}_{\gamma_z \gamma_x}, \bar{S}_{\gamma_z \gamma_z})$$

5.19(b), 5.22(b), 5.25(b), 5.28(b)}, and  $\bar{S}_{z\gamma_x}$  {Figs. 5.19(e), 5.22(e)}, decrease with the increase in misalignment. In the case of plain journal bearing and two-axial-groove journal bearing, the stiffness coefficients  $\bar{S}_{x\gamma_z}$ ,  $\bar{S}_{z\gamma_z}$ ,  $\bar{S}_{\gamma_x z}$ ,  $\bar{S}_{\gamma_z z}$  and  $\bar{S}_{\gamma_z \gamma_x}$  have negative values in the entire range of load and their absolute values are increased by the misalignment. Figs. 5.18(c) and 5.21(c) reveal that as the misalignment increases,  $\bar{S}_{z\gamma_x}$  decreases at light loads but increases at higher load values for plain and two-axial-groove journal bearings whereas for elliptical and three-lobe journal bearings, the stiffness coefficient,  $\bar{S}_{z\gamma_x}$  increases with the increase in misalignment, {Figs. 5.24(c) and 5.27(c)}.

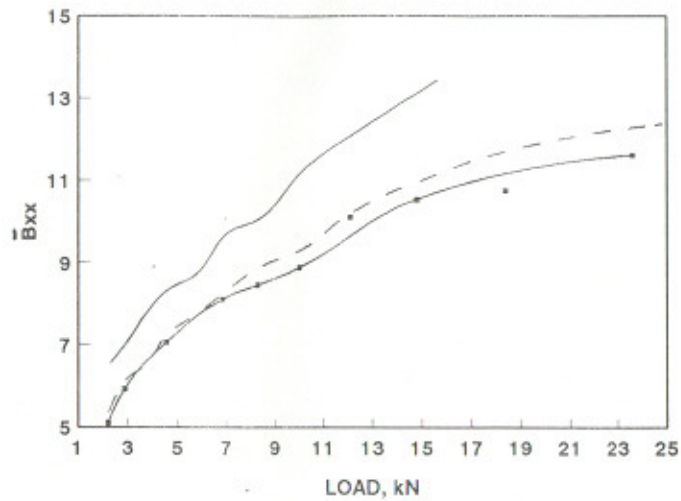
It has been noticed that in the case of non-circular bearings,  $\bar{S}_{\gamma_z x}$  {Figs. 5.25(a), 5.28(a)} and  $\bar{S}_{\gamma_z \gamma_x}$  {Figs. 5.25(e), 5.28(e)} increase with the increase in the value of misalignment. In the case of elliptical journal bearing,  $\bar{S}_{x\gamma_z}$  {Fig. 5.24(b)} and  $\bar{S}_{\gamma_x z}$  {Fig. 5.24(f)} show a rising trend with the increase in value of misalignment.

### 5.3.3 Damping Coefficients

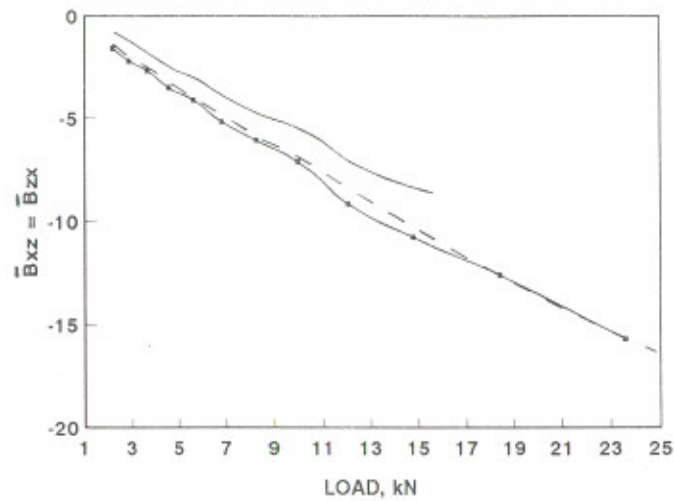
The damping coefficients are presented for all the bearing systems in Figs. 5.29 to 5.40. It is observed from the figures that the damping coefficients  $\bar{B}_{xx}$  {Figs. 5.29(a), 5.32(a), 5.35(a), 5.38(a)},  $\bar{B}_{xz}$  (=  $\bar{B}_{zx}$ ) {Figs. 5.29(b), 5.32(b), 5.35(b), 5.38(b)},  $\bar{B}_{zz}$  {Figs. 5.29(c), 5.32(c), 5.35(c), 5.38(c)},  $\bar{B}_{x\gamma_x}$  (=  $\bar{B}_{\gamma_x x}$ ) {Figs. 5.30(a), 5.33(a), 5.36(a), 5.39(a)},  $\bar{B}_{\gamma_x \gamma_x}$  {Figs. 5.31(a), 5.34(a), 5.37(a), 5.40(a)},  $\bar{B}_{\gamma_x \gamma_z}$  (=  $\bar{B}_{\gamma_z \gamma_x}$ ) {Figs. 5.31(b), 5.34(b), 5.37(b), 5.40(b)} and  $\bar{B}_{\gamma_z \gamma_z}$  {Figs. 5.31(c), 5.34(c), 5.37(c), 5.40(c)} for all the bearing systems increase with the increase in misalignment. This increase is more at higher value of misalignment ratio. The damping coefficients  $\bar{B}_{x\gamma_z}$  (=  $\bar{B}_{\gamma_z x}$ ) {Figs. 5.30(b), 5.33(b), 5.39(b)},  $\bar{B}_{z\gamma_x}$  (=  $\bar{B}_{\gamma_x z}$ ) {Figs. 5.30(c), 5.33(c), 5.39(c)} and  $\bar{B}_{z\gamma_z}$  (=  $\bar{B}_{\gamma_z z}$ ) {Figs. 5.30(d), 5.33(d), 5.36(d), 5.39(d)} are reduced with the increase in misalignment.

In the case of elliptical journal bearing, the damping coefficient  $\bar{B}_{x\gamma_z}$  (=  $\bar{B}_{\gamma_z x}$ ) {Fig. 5.36(b)} and  $\bar{B}_{z\gamma_x}$  (=  $\bar{B}_{\gamma_x z}$ ) {Fig. 5.36(c)} show a rising trend with the increase in misalignment.

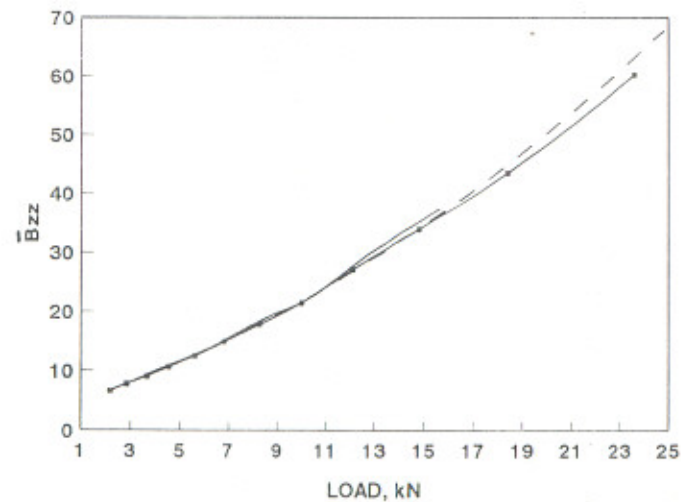
The damping coefficients,  $\bar{B}_{xx}$  {Figs. 5.29(a), 5.32(a), 5.35(a), 5.38(a)},  $\bar{B}_{zz}$  {Figs. 5.29(c), 5.32(c), 5.35(c), 5.38(c)},  $\bar{B}_{x\gamma_x}$  (=  $\bar{B}_{\gamma_x x}$ ) {Figs. 5.30(a), 5.33(a), 5.36(a), 5.39(a)},  $\bar{B}_{\gamma_x \gamma_x}$  {Figs. 5.31(a), 5.34(a), 5.37(a), 5.40(a)} and  $\bar{B}_{\gamma_z \gamma_z}$  {Figs. 5.31(c), 5.34(c), 5.37(c), 5.40(c)} increase with the rise in load values. Whereas  $\bar{B}_{xz}$  (=  $\bar{B}_{zx}$ ) {Figs. 5.29(b), 5.32(b), 5.35(b),



(a)



(b)



(c)

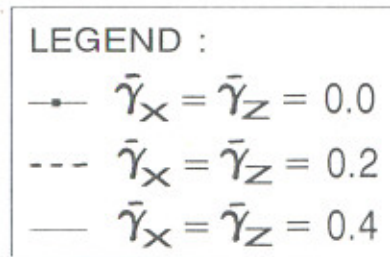
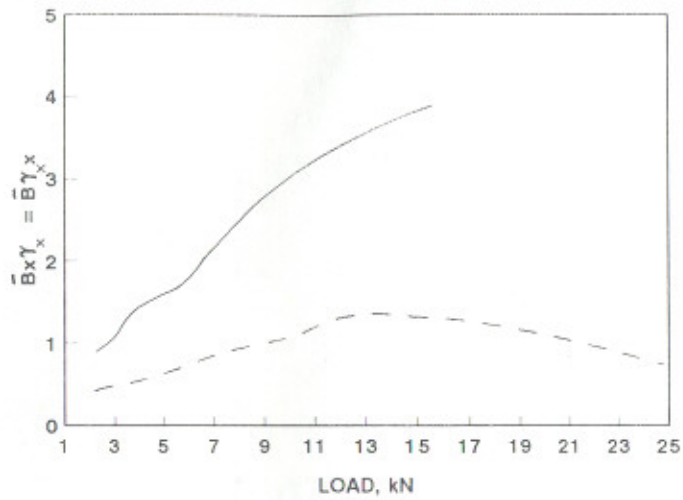
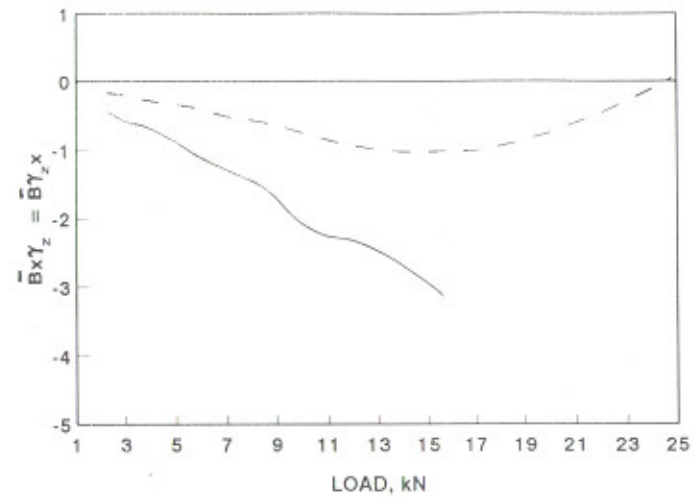


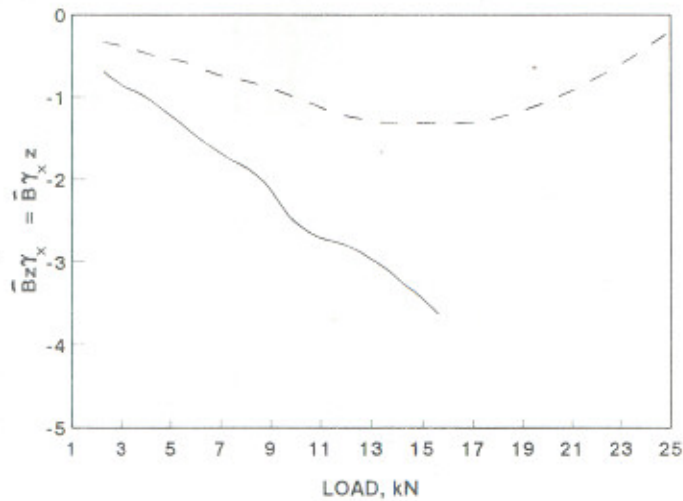
Fig. 5.29 PLAIN JOURNAL BEARING - DAMPING COEFFICIENTS  
(  $\bar{B}_{xx}$ ,  $\bar{B}_{xz}$ ,  $\bar{B}_{zx}$ ,  $\bar{B}_{zz}$  )



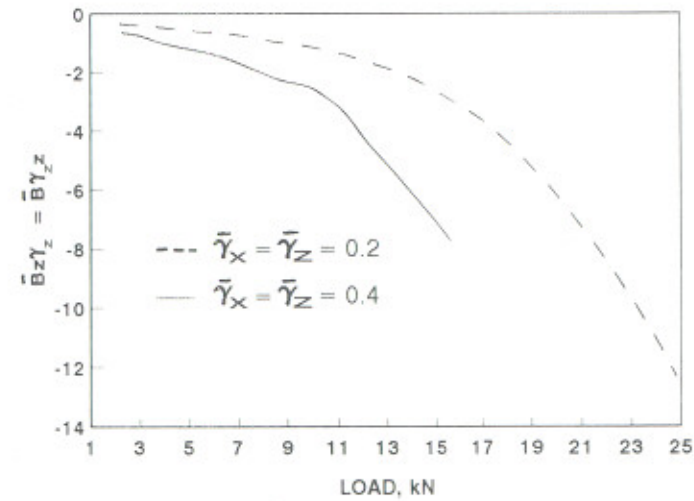
(a)



(b)



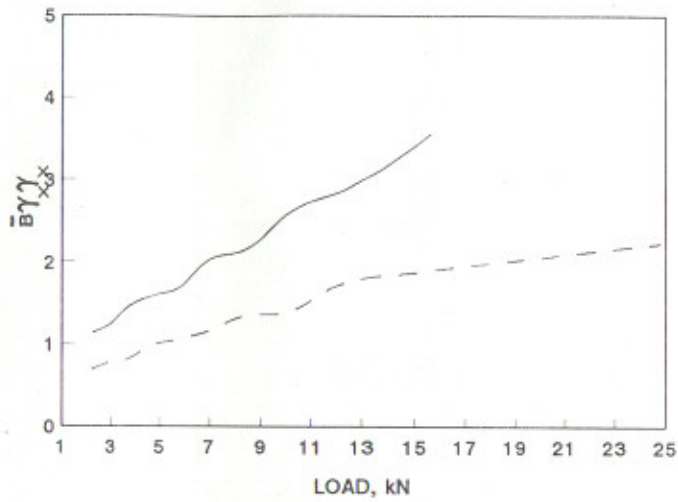
(c)



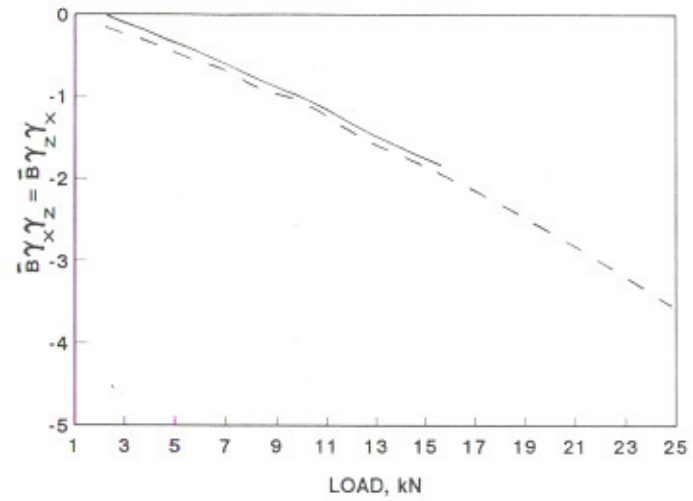
(d)

Fig. 5.30 PLAIN JOURNAL BEARING - DAMPING COEFFICIENTS

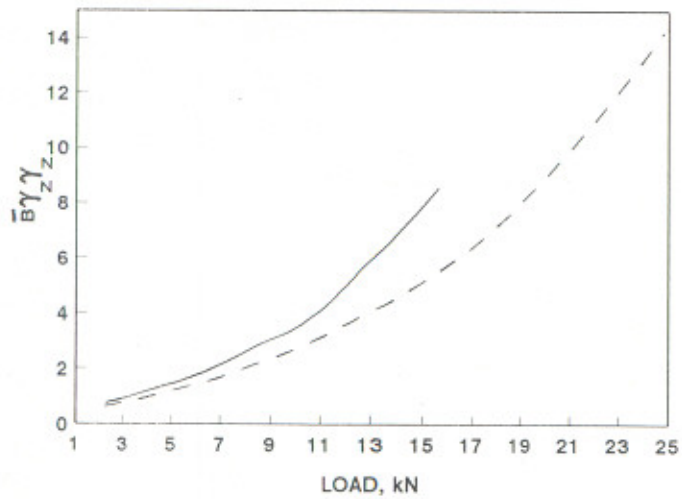
( $\bar{B}_x \bar{\gamma}_x$ ,  $\bar{B}_x \bar{\gamma}_z$ ,  $\bar{B}_z \bar{\gamma}_x$ ,  $\bar{B}_z \bar{\gamma}_z$ ,  $\bar{B}_x \bar{\gamma}_x$ ,  $\bar{B}_z \bar{\gamma}_x$ ,  $\bar{B}_x \bar{\gamma}_z$ ,  $\bar{B}_z \bar{\gamma}_z$ )



(a)



(b)



(c)

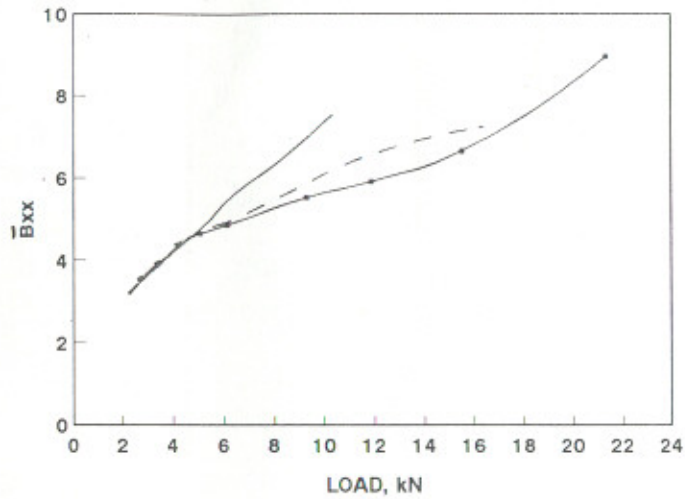
LEGEND :

---  $\bar{\gamma}_x = \bar{\gamma}_z = 0.2$

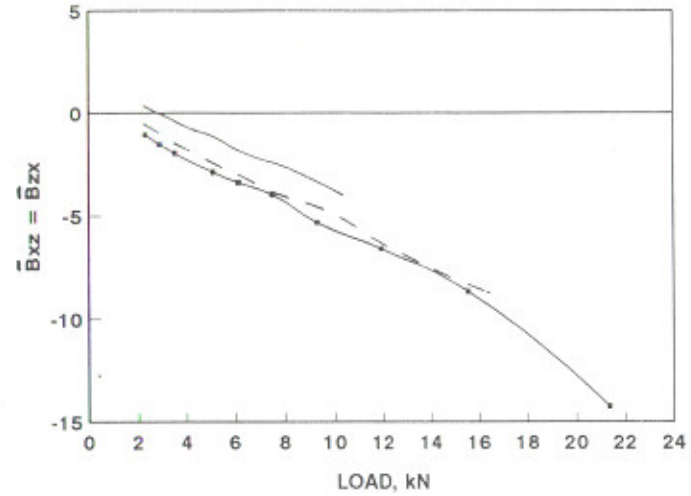
—  $\bar{\gamma}_x = \bar{\gamma}_z = 0.4$

Fig. 5.31 PLAIN JOURNAL BEARING - DAMPING COEFFICIENTS

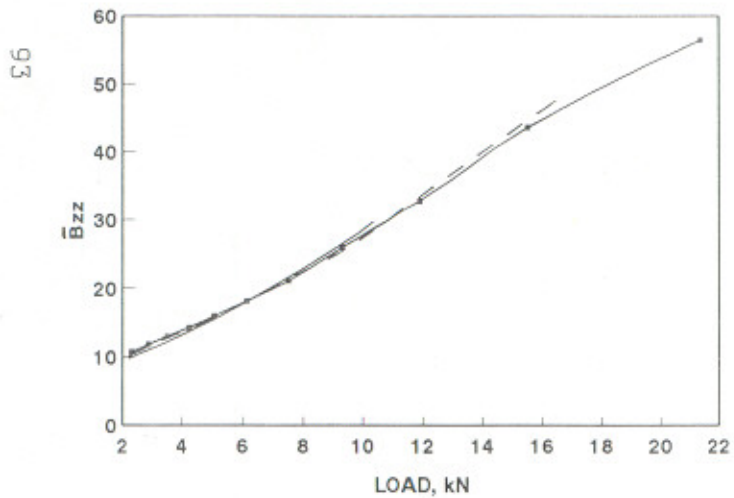
( $\bar{\gamma}_{x\dot{x}}$ ,  $\bar{\gamma}_{x\dot{z}}$ ,  $\bar{\gamma}_{z\dot{x}}$ ,  $\bar{\gamma}_{z\dot{z}}$ )



(a)



(b)



(c)

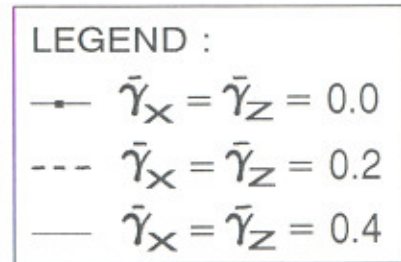
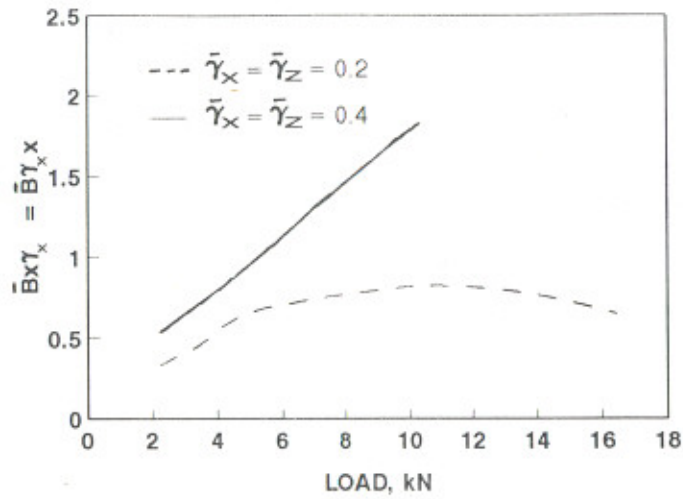
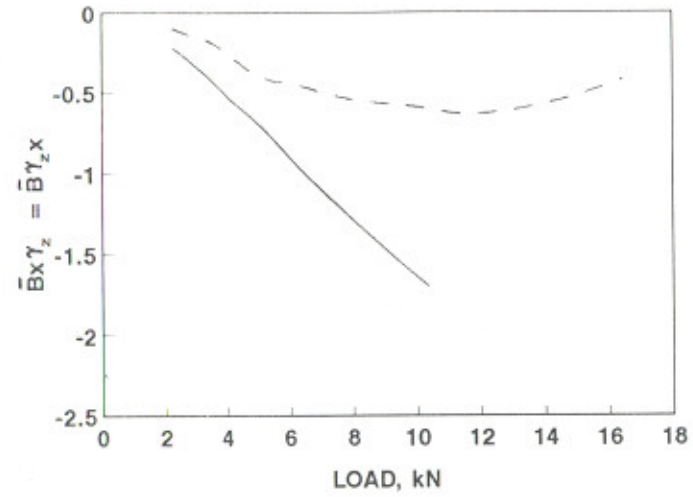


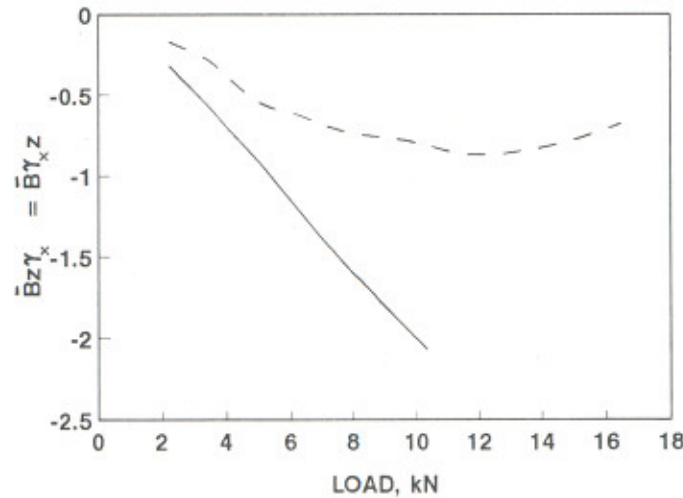
Fig. 5.32 TWO-AXIAL-GROOVE JOURNAL BEARING - DAMPING COEFFICIENTS  
(  $\bar{B}_{xx}$ ,  $\bar{B}_{xz}$ ,  $\bar{B}_{zx}$ ,  $\bar{B}_{zz}$  )



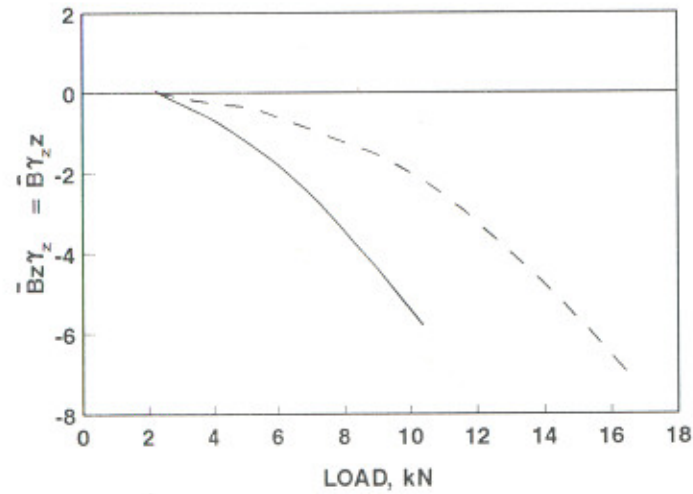
(a)



(b)



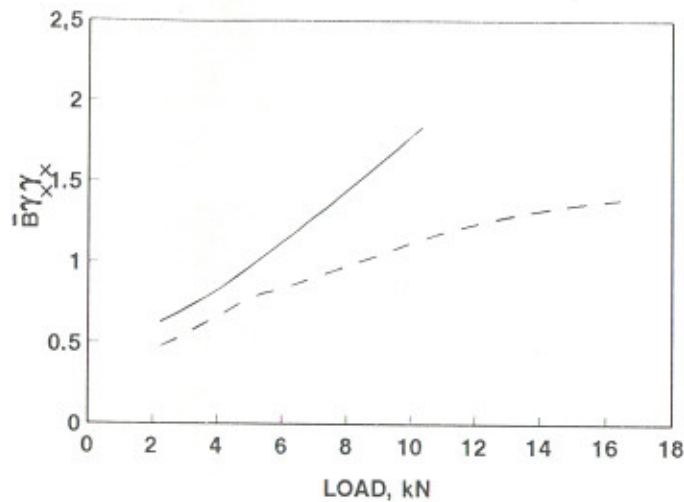
(c)



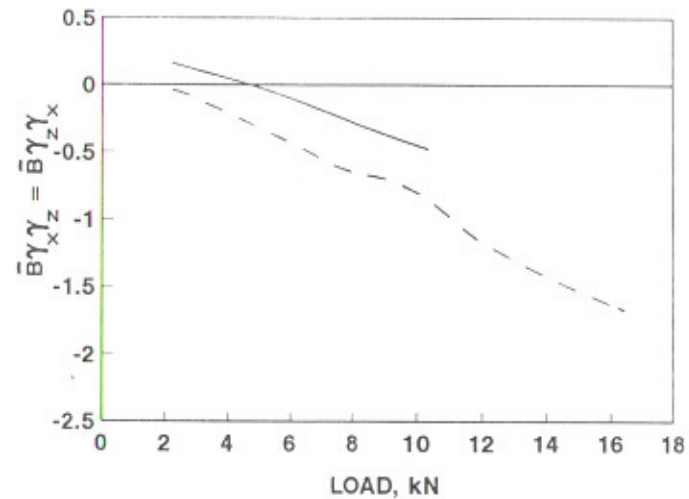
(d)

Fig. 5.33 TWO-AXIAL-GROOVE JOURNAL BEARING - DAMPING COEFFICIENTS

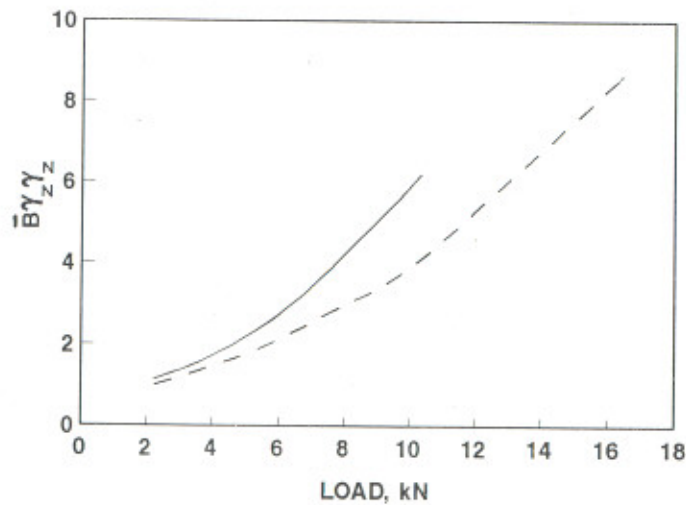
( $\bar{B}_x \gamma_x$   $\bar{B}_x \gamma_z$   $\bar{B}_z \gamma_x$   $\bar{B}_z \gamma_z$   $\bar{B} \gamma_x^x$   $\bar{B} \gamma_x^z$   $\bar{B} \gamma_z^x$   $\bar{B} \gamma_z^z$ )



(a)



(b)



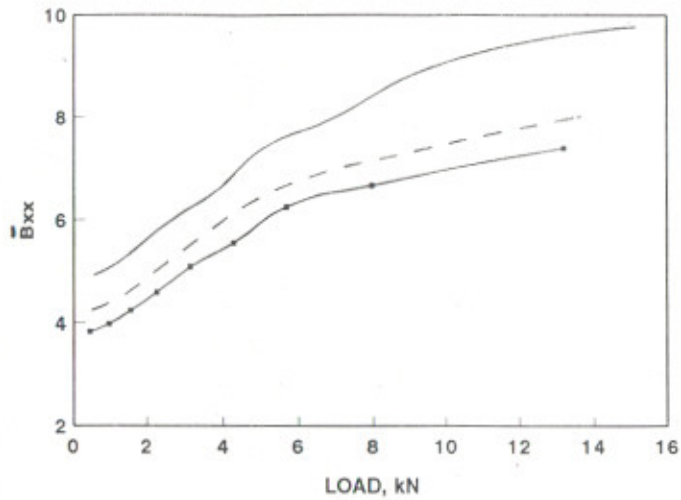
(c)

LEGEND :

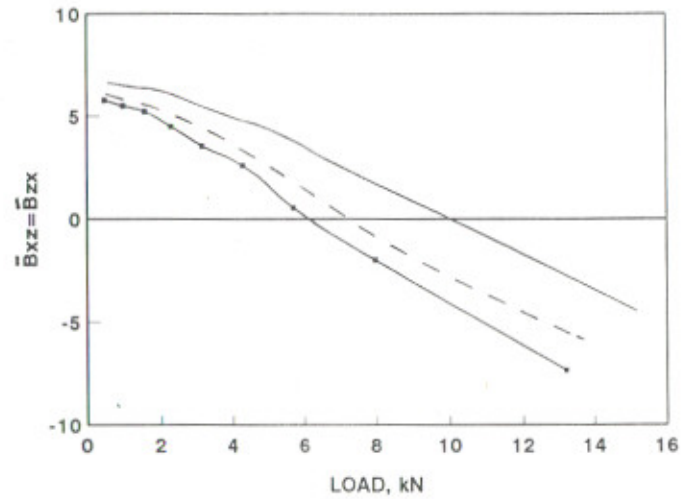
---  $\bar{\gamma}_x = \bar{\gamma}_z = 0.2$

—  $\bar{\gamma}_x = \bar{\gamma}_z = 0.4$

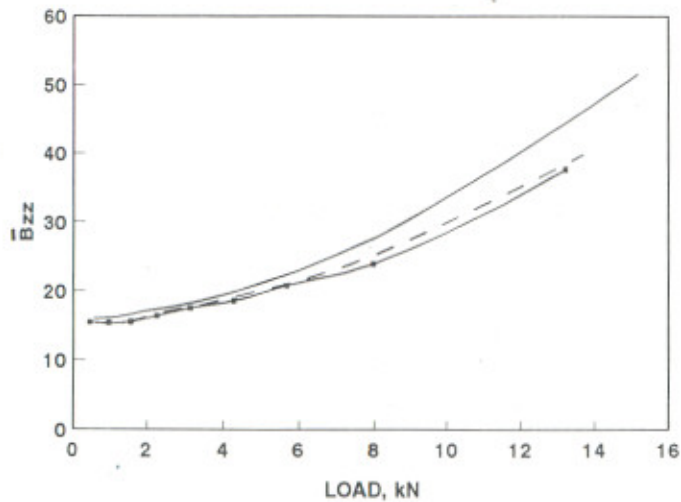
Fig. 5.34 TWO-AXIAL-GROOVE JOURNAL BEARING - DAMPING COEFFICIENTS  
 $(\bar{\gamma}_{x'x'}, \bar{\gamma}_{x'z'}, \bar{\gamma}_{z'x'}, \bar{\gamma}_{z'z'})$



(a)



(b)



(c)

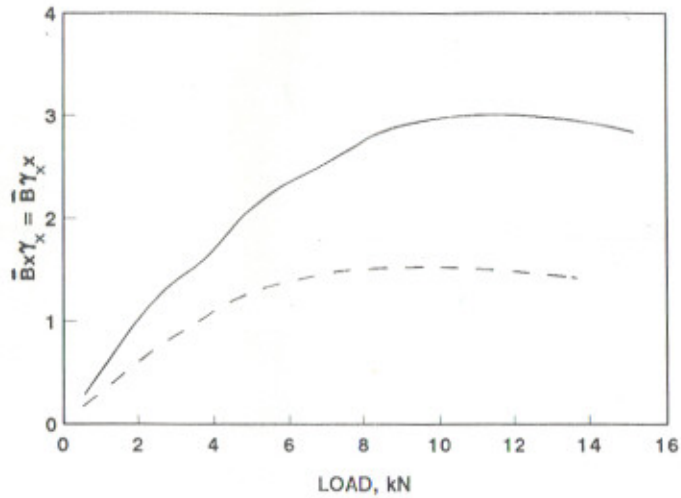
LEGEND :

—•—  $\tilde{\gamma}_x = \tilde{\gamma}_z = 0.0$

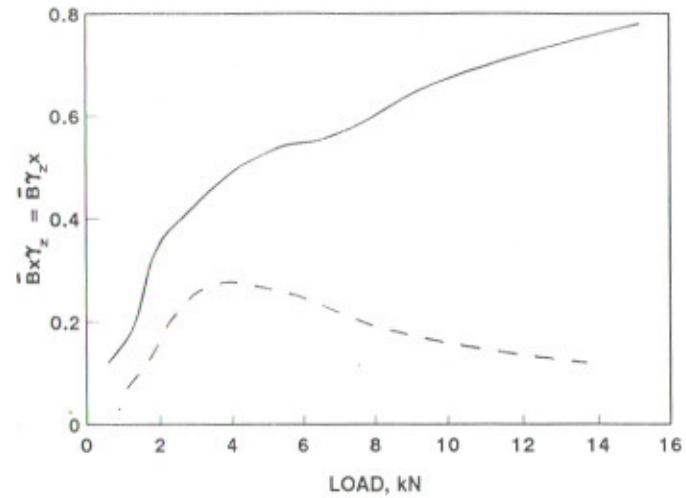
- - -  $\tilde{\gamma}_x = \tilde{\gamma}_z = 0.2$

—  $\tilde{\gamma}_x = \tilde{\gamma}_z = 0.3$

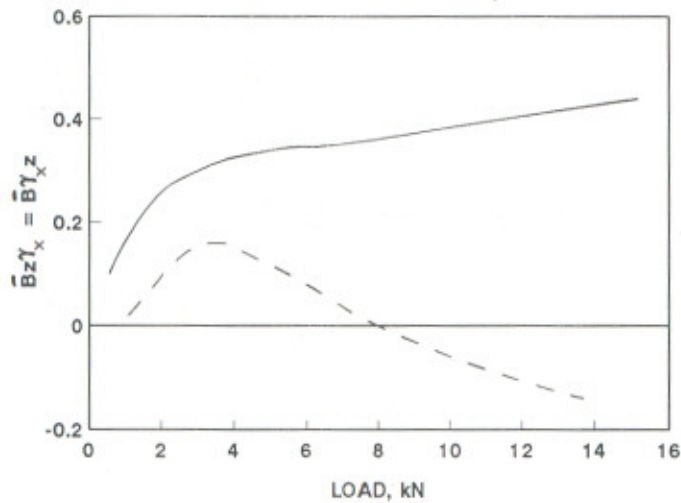
Fig. 5.35 ELLIPTICAL JOURNAL BEARING - DAMPING COEFFICIENTS  
(  $\bar{B}_{xx}$ ,  $\bar{B}_{xz}$ ,  $\bar{B}_{zx}$ ,  $\bar{B}_{zz}$  )



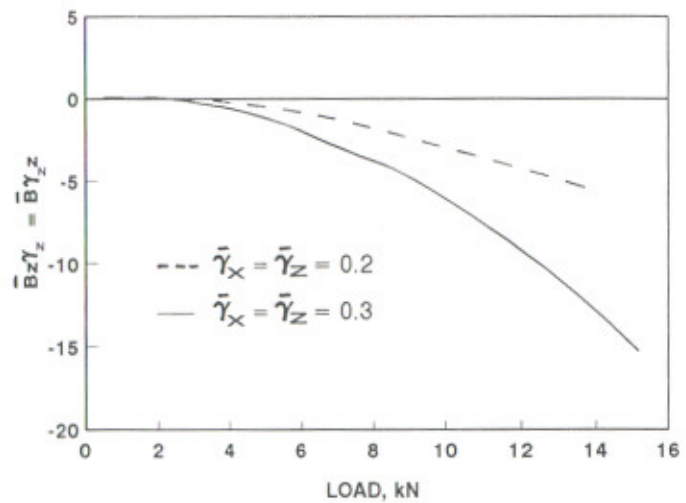
(a)



(b)



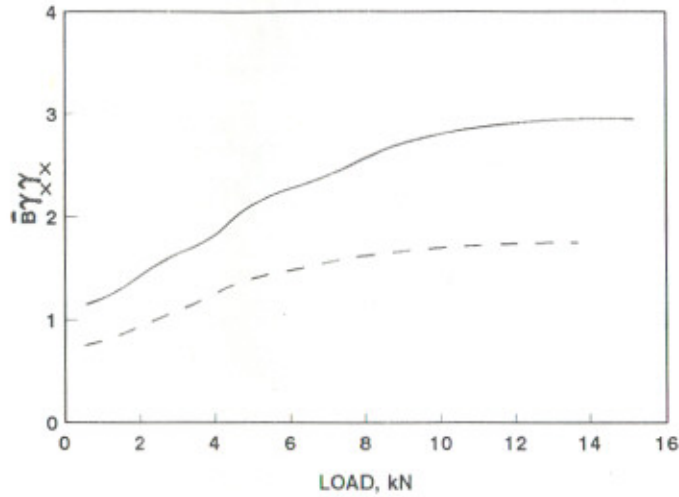
(c)



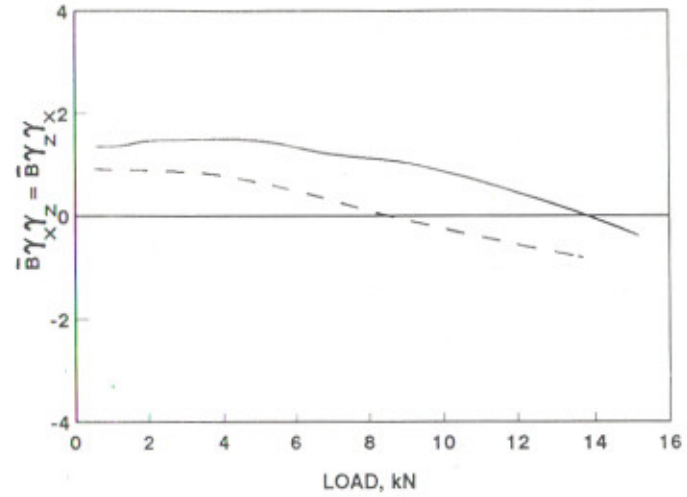
(d)

Fig. 5.36 ELLIPTICAL JOURNAL BEARING - DAMPING COEFFICIENTS

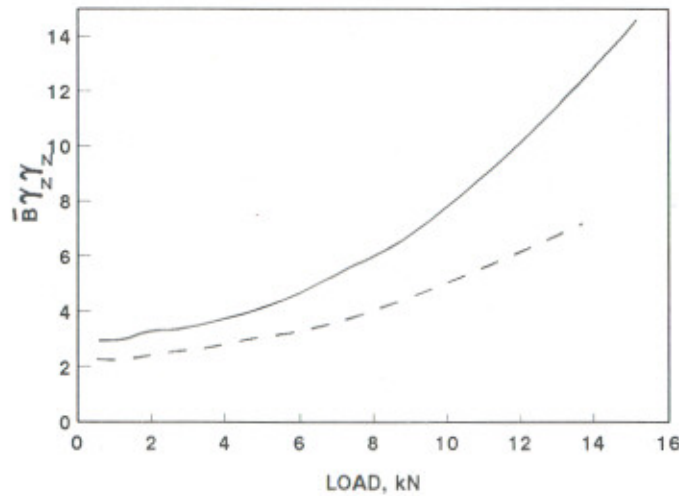
( $\bar{B}_x \bar{\gamma}_x$ ,  $\bar{B}_x \bar{\gamma}_z$ ,  $\bar{B}_z \bar{\gamma}_x$ ,  $\bar{B}_z \bar{\gamma}_z$ ,  $\bar{B} \bar{\gamma}_x^x$ ,  $\bar{B} \bar{\gamma}_z^x$ ,  $\bar{B} \bar{\gamma}_x^z$ ,  $\bar{B} \bar{\gamma}_z^z$ )



(a)



(b)



(c)

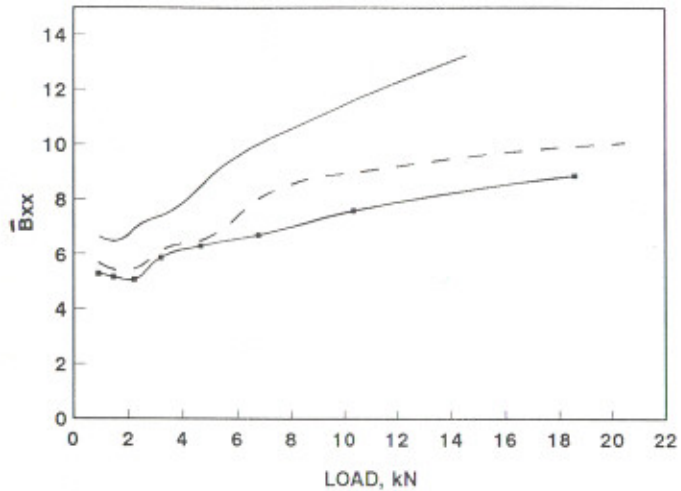
LEGEND :

---  $\bar{\gamma}_x = \bar{\gamma}_z = 0.2$

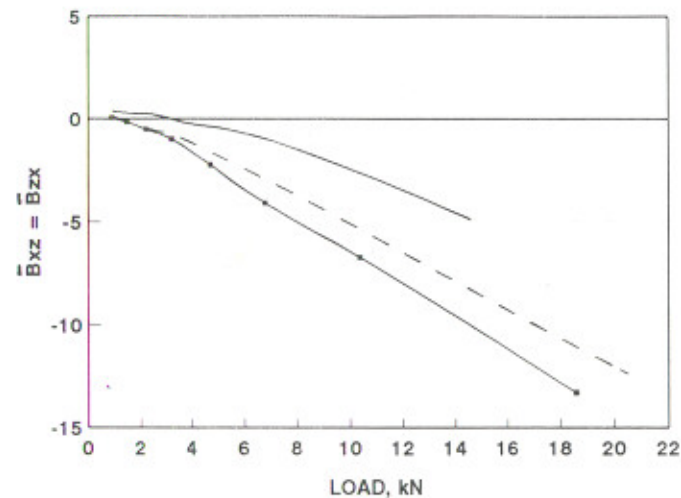
—  $\bar{\gamma}_x = \bar{\gamma}_z = 0.3$

Fig. 5.37 ELLIPTICAL JOURNAL BEARING - DAMPING COEFFICIENTS

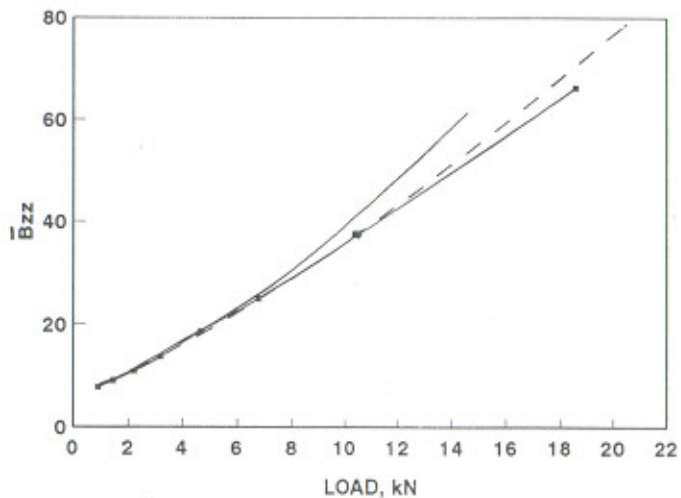
( $\bar{\gamma}_{x_x}, \bar{\gamma}_{x_z}, \bar{\gamma}_{z_x}, \bar{\gamma}_{z_z}$ )



(a)



(b)

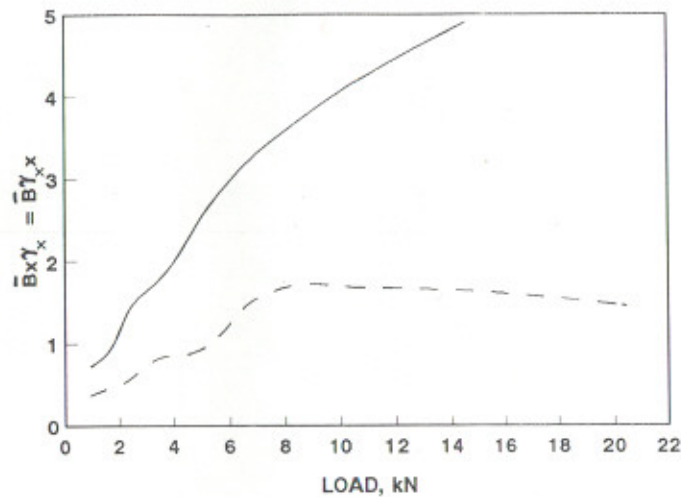


(c)

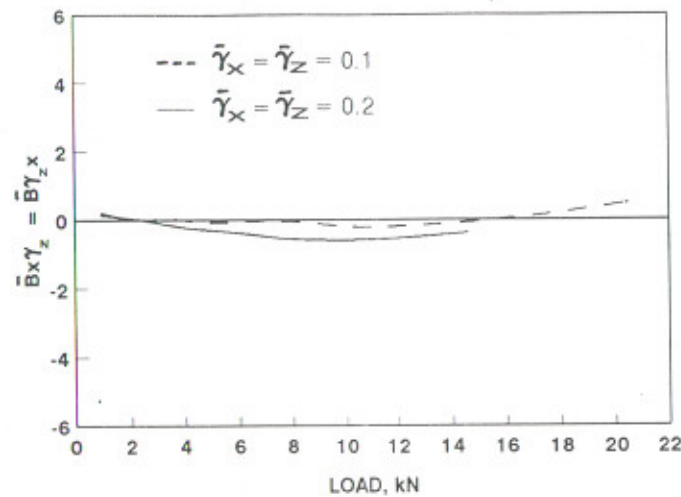
LEGEND :

- $\bar{\gamma}_x = \bar{\gamma}_z = 0.0$
- - -  $\bar{\gamma}_x = \bar{\gamma}_z = 0.1$
- $\bar{\gamma}_x = \bar{\gamma}_z = 0.2$

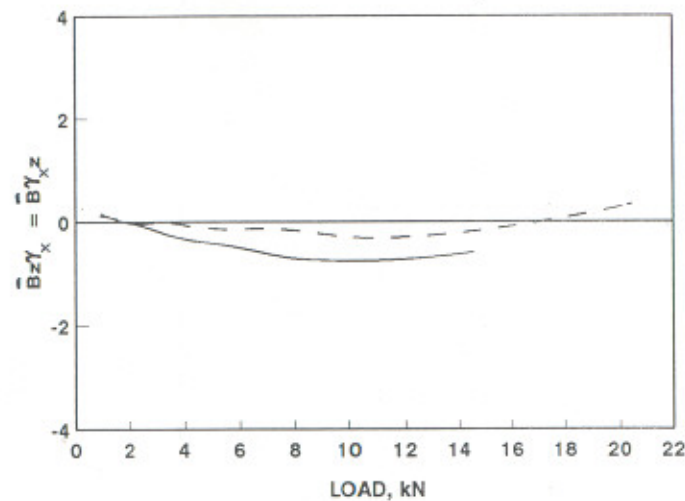
Fig. 5.38 THREE-LOBE JOURNAL BEARING - DAMPING COEFFICIENTS  
(  $\bar{B}_{xx}$ ,  $\bar{B}_{xz}$ ,  $\bar{B}_{zx}$ ,  $\bar{B}_{zz}$  )



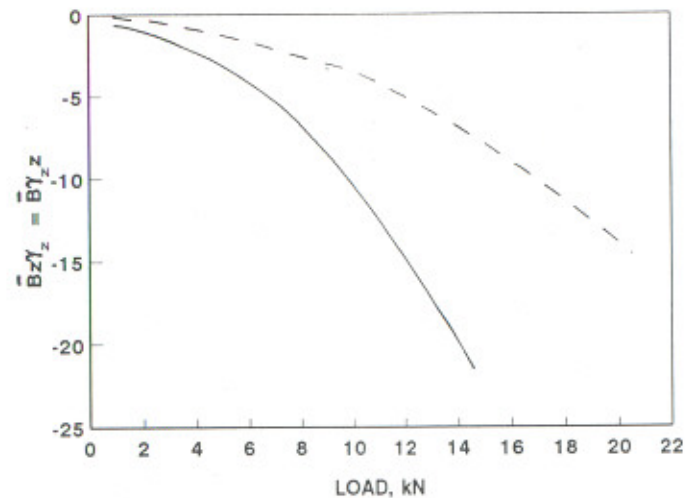
(a)



(b)



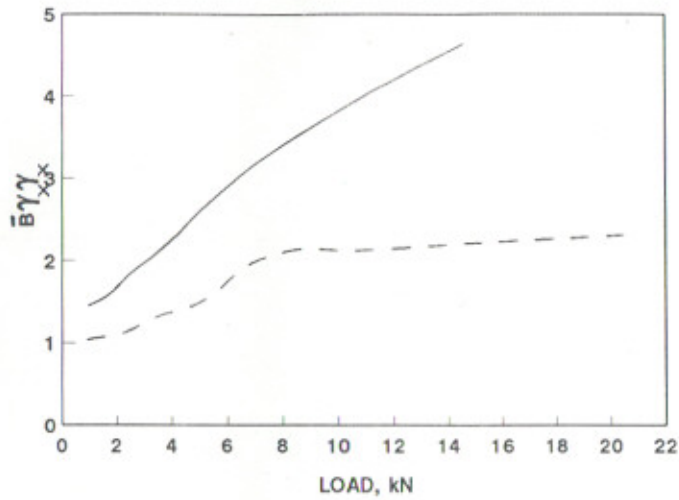
(c)



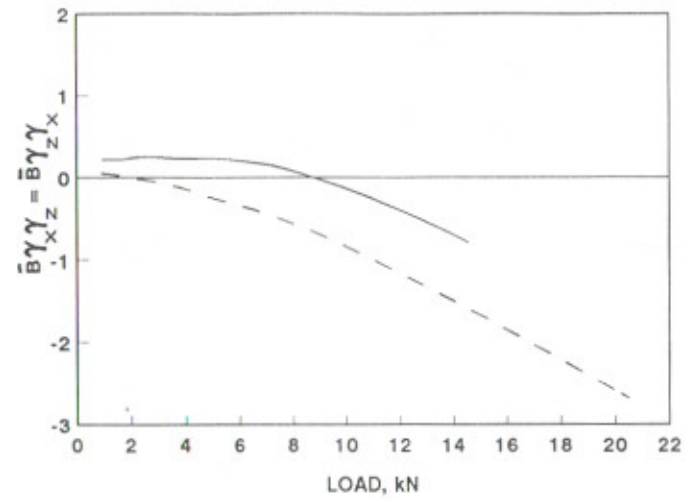
(d)

Fig. 5.39 THREE-LOBE JOURNAL BEARING - DAMPING COEFFICIENTS

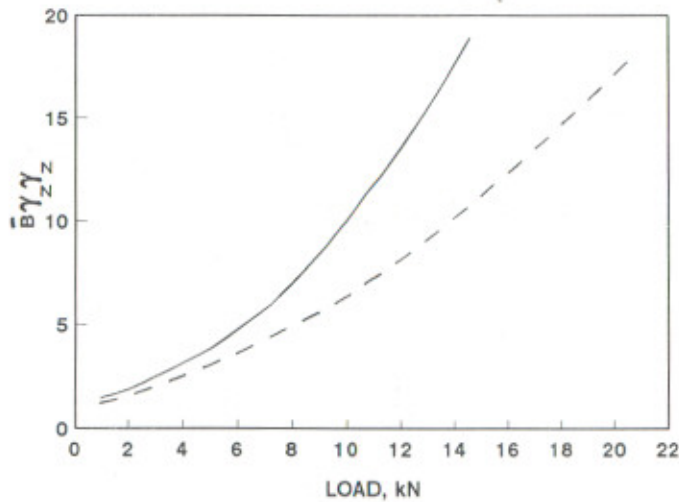
( $\bar{B}_{xx} \gamma_x$ ,  $\bar{B}_{xz} \gamma_z$ ,  $\bar{B}_{zx} \gamma_x$ ,  $\bar{B}_{zz} \gamma_z$ ,  $\bar{B}' \gamma_x x$ ,  $\bar{B}' \gamma_z z$ )



(a)



(b)



(c)

LEGEND :

---  $\bar{\gamma}_x = \bar{\gamma}_z = 0.1$

—  $\bar{\gamma}_x = \bar{\gamma}_z = 0.2$

Fig. 5.40 THREE-LOBE JOURNAL BEARING - DAMPING COEFFICIENTS  
 $(\bar{\gamma}_{x_x}, \bar{\gamma}_{x_z}, \bar{\gamma}_{z_x}, \bar{\gamma}_{z_z})$

5.38(b)},  $\bar{B}_{z\gamma_z} (= \bar{B}_{\gamma_z z})$  {Figs. 5.30(d), 5.33(d), 5.36(d), 5.39(d)} and  $\bar{B}_{\gamma_x \gamma_z} (= \bar{B}_{\gamma_z \gamma_x})$  {Figs. 5.31(b), 5.34(b), 5.37(b), 5.40(b)} decrease with the rise in load values.

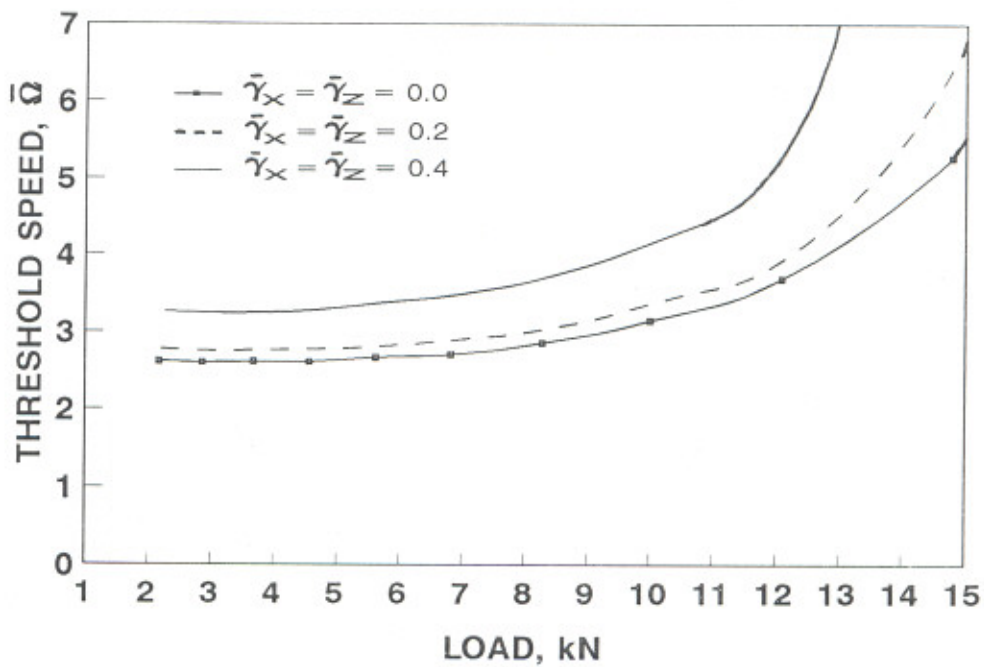
In the case of plain journal bearing and two-axial-groove journal bearing, the damping coefficients  $\bar{B}_{x\gamma_z} (= \bar{B}_{\gamma_z x})$  {Figs. 5.30(b) and 5.33(b)} and  $\bar{B}_{z\gamma_x} (= \bar{B}_{\gamma_x z})$  {Figs. 5.30(c) and 5.33(c)} decrease with the increase in load values. Whereas in case of elliptical journal bearing, at light loads, the damping coefficients  $\bar{B}_{x\gamma_z} (= \bar{B}_{\gamma_z x})$  {Fig. 5.36(b)} and  $\bar{B}_{z\gamma_x} (= \bar{B}_{\gamma_x z})$  {Fig. 5.36(c)}, increase with the increase in the load value but at lower values of misalignment and at high load values,  $\bar{B}_{x\gamma_z} (= \bar{B}_{\gamma_z x})$  and  $\bar{B}_{z\gamma_x} (= \bar{B}_{\gamma_x z})$  show a decreasing trend. The damping coefficients,  $\bar{B}_{x\gamma_z} (= \bar{B}_{\gamma_z x})$  {Fig. 5.39(b)} and  $\bar{B}_{z\gamma_x} (= \bar{B}_{\gamma_x z})$  {Fig. 5.39(c)} are almost constant in the entire range of load values, in case of both aligned and misaligned three-lobe journal bearing.

### 5.3.4 Stability Parameters

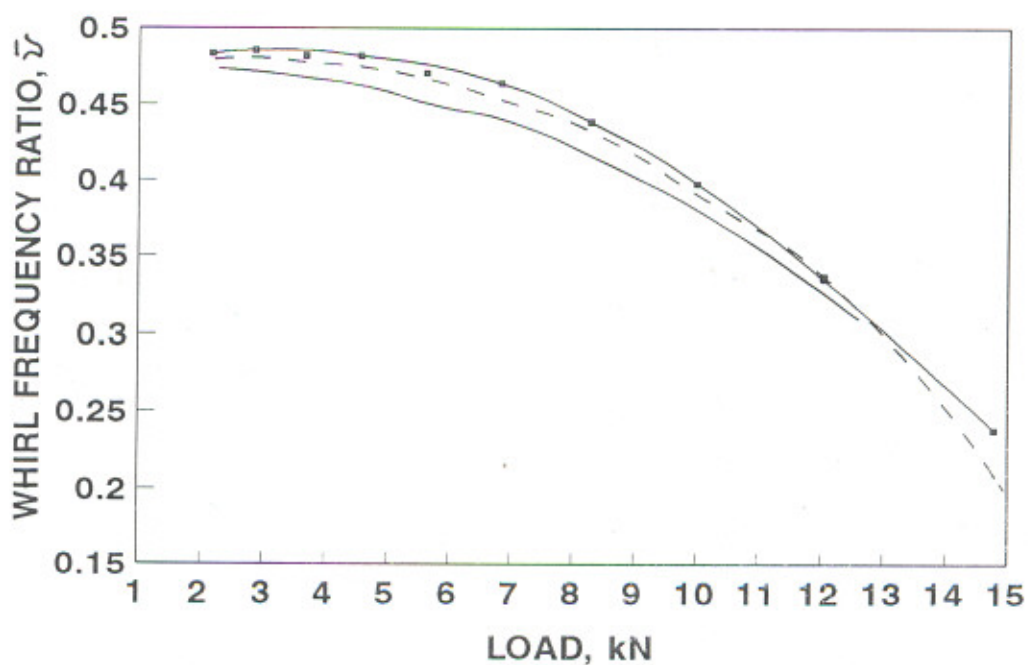
The values of instability threshold speed ( $\bar{\Omega}$ ) and whirl frequency ratio ( $\bar{\nu}$ ) are shown in Figs. 5.41 to 5.44 for all the bearing systems. The figures show that an increase in the misalignment significantly improves stability margin defined by  $\bar{\Omega}$ . The tendency to whirl after a disturbance is also reduced by misalignment. These effects are quite important at light loads where the journal bearing system has a tendency to become unstable in case of plain and two-axial-groove journal bearing. As an example, in case of plain journal bearing at 4 kN load,  $\bar{\Omega}$  increases to 3.3 with the misalignment ratio of  $\bar{\gamma}_x = \bar{\gamma}_z = 0.4$  from 2.65 for an aligned bearing showing an improvement in stability margin by about 23 %, Fig. 5.41(a).

For the aligned elliptical and three-lobe bearings, the values of threshold speed are first reduced and then increased as the load supported by the bearing is increased {Figs. 5.43(a) and 5.44(a)}. The both types of bearings tend to become infinitely stable for some value of load. The trends for the misaligned bearings are also same. In the later cases, however, the values of  $\bar{\Omega}$  at each load are higher and the values of load at which the two journal bearing systems become infinitely stable are lower compared to the corresponding values for aligned bearings.

Whirl frequency ratio reduces with the increase in bearing load, {Figs. 5.41(b), 5.42(b), 5.43(b) and 5.44(b)}. In case of plain journal bearing and two-axial-groove bearing, whirl frequency ratio reduces with the increase in misalignment. For elliptical bearing, this

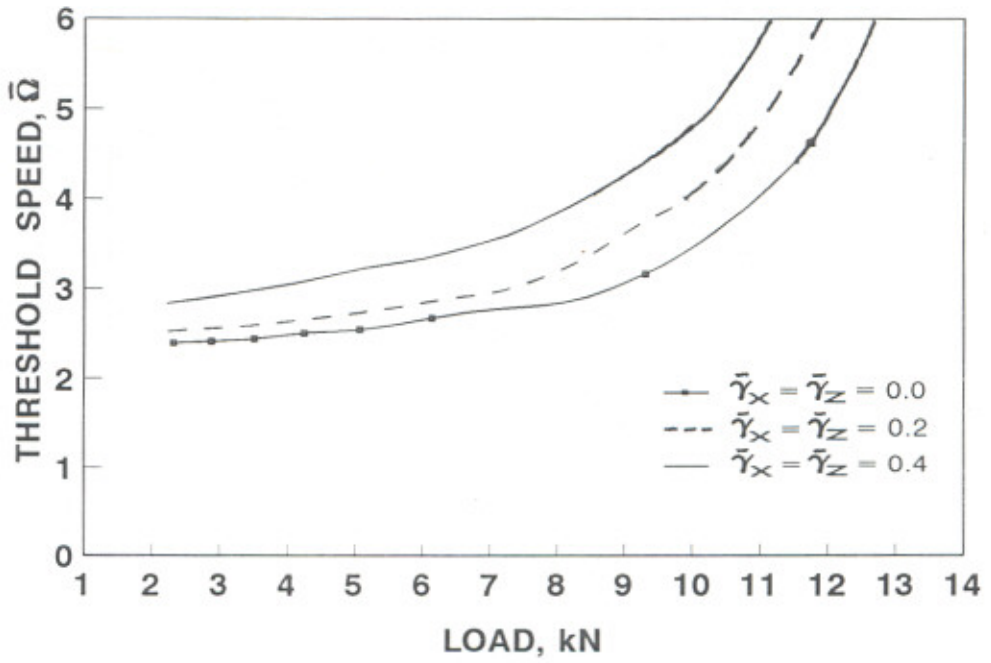


(a)

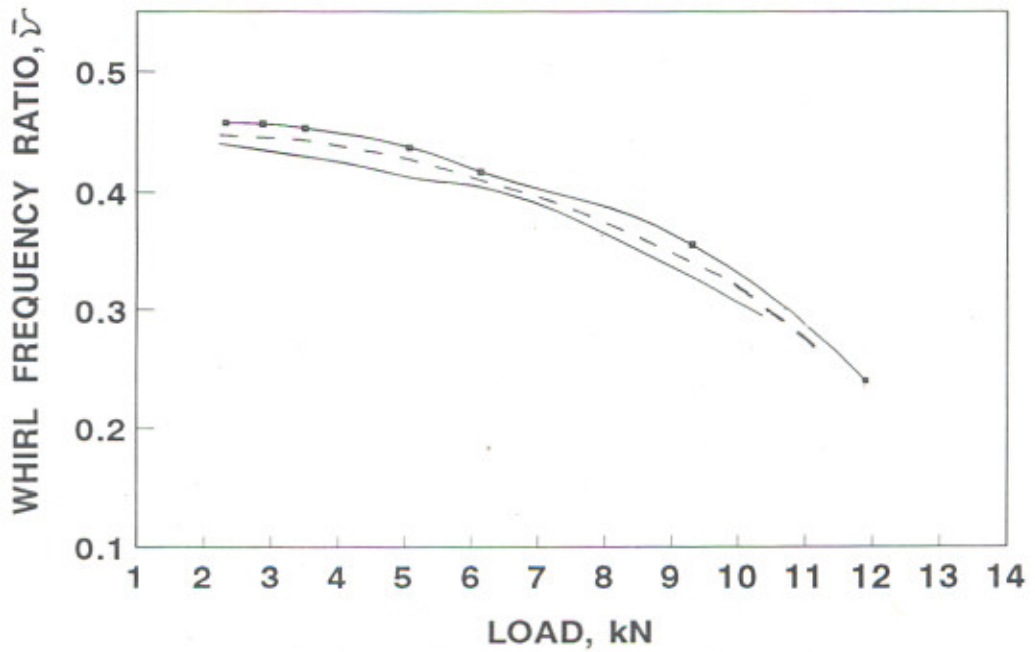


(b)

Fig. 5.41 PLAIN JOURNAL BEARING - STABILITY PARAMETERS

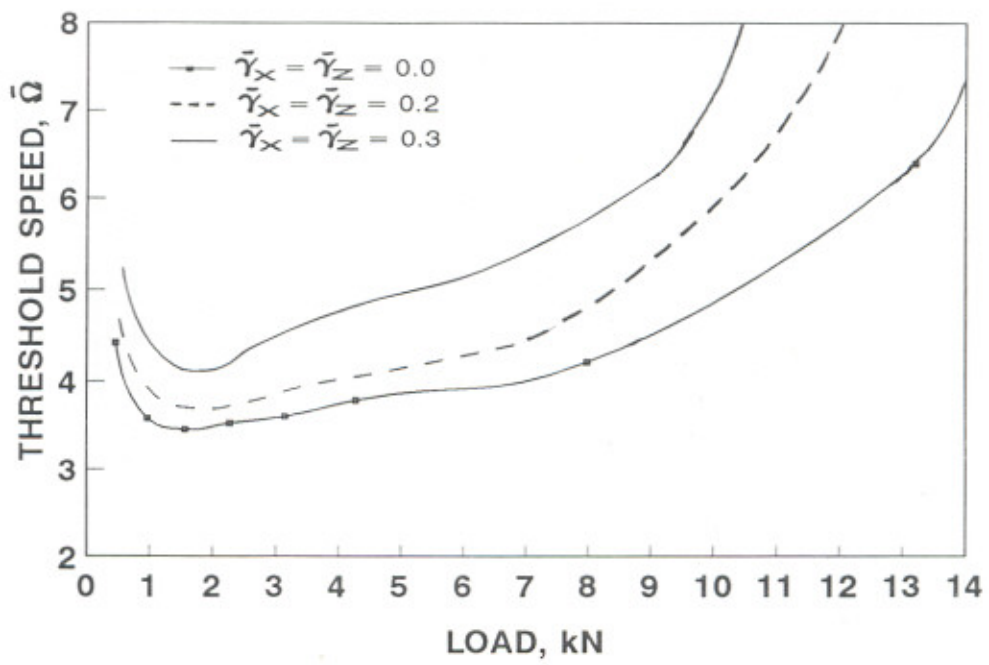


( a )

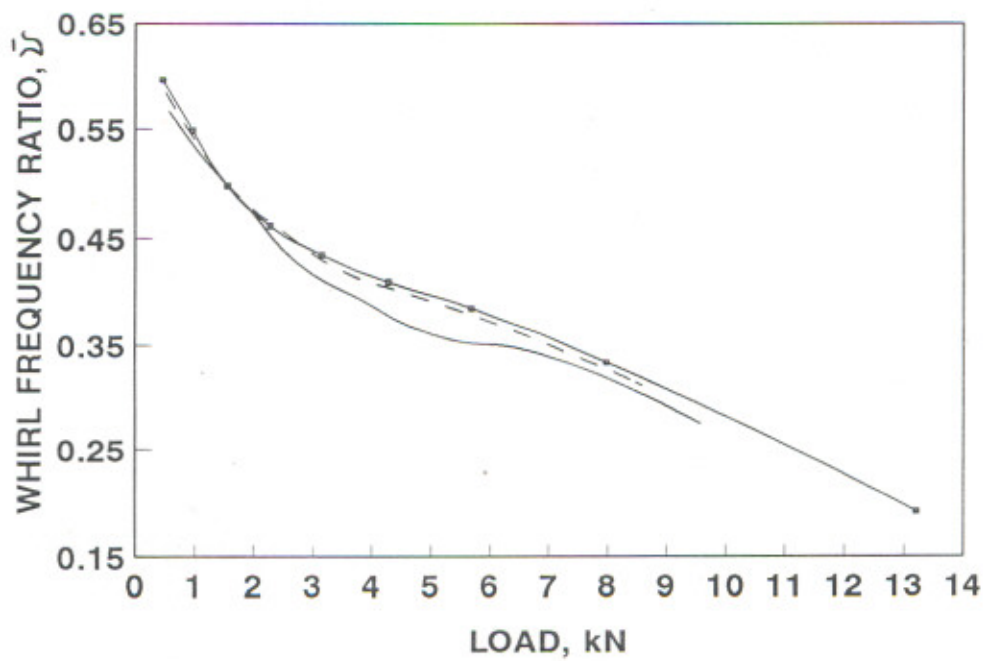


( b )

Fig. 5.42 TWO-AXIAL-GROOVE JOURNAL BEARING - STABILITY PARAMETERS

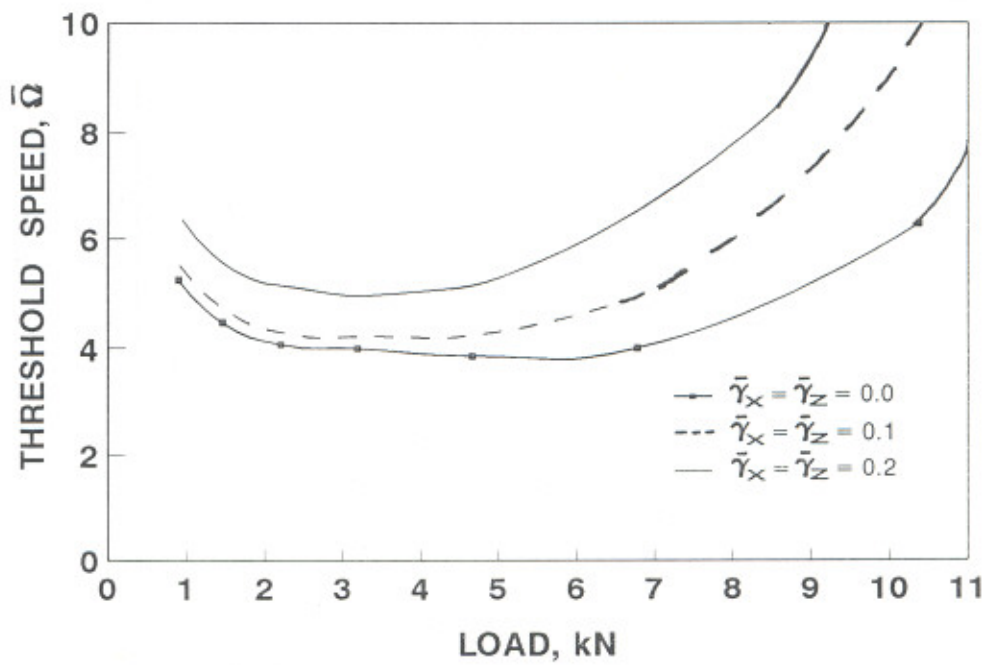


( a )

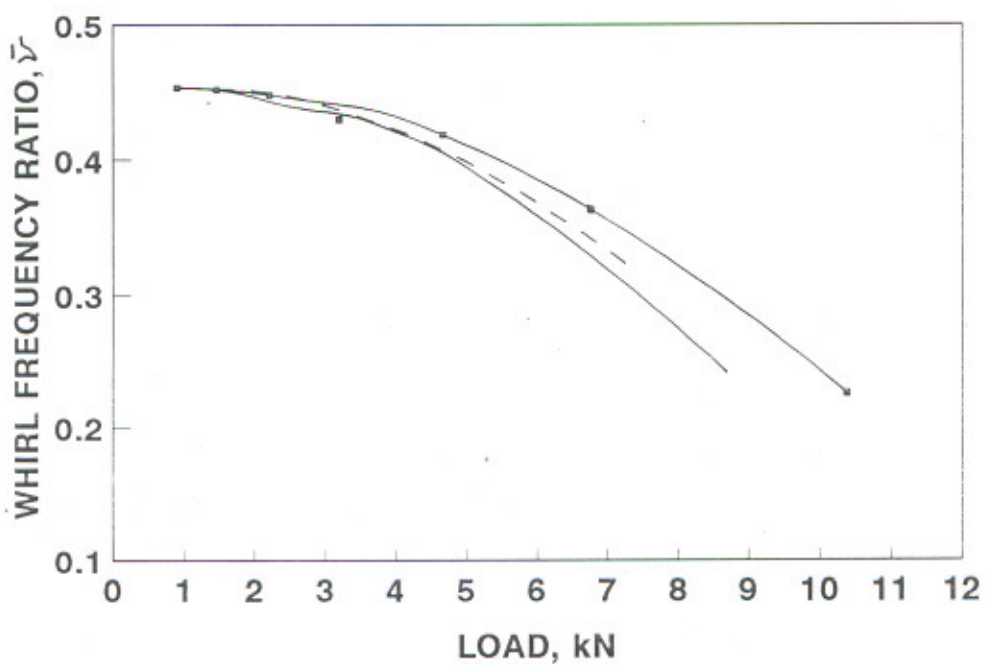


( b )

Fig. 5.43 ELLIPTICAL JOURNAL BEARING - STABILITY PARAMETERS



(a)



(b)

Fig. 5.44 THREE-LOBE JOURNAL BEARING - STABILITY PARAMETERS

parameter,  $\bar{v}$  is not effected much by misalignment where as for three-lobe bearings, the misalignment reduces the whirl frequency ratio only at higher loads.

### 5.3.5 Isopressure Curves and Pressure Profiles

Figs. 5.45, 5.46, 5.47 and 5.48 shows the isopressure curves for the aligned and misaligned bearings each operating at 10 kN load. It is found that the effect of misalignment is to make the distribution of fluid-film pressure asymmetric in all bearings.

The pressure profiles have been plotted at circumferential (section  $A_1 - A_1$ ) and axial (section  $B_1 - B_1$ ) planes which pass through the point of maximum fluid-film pressure in respective cases, Fig. 5.49. The circumferential and the axial coordinates corresponding the maximum pressure points are expressed as  $\alpha_p$  and  $\beta_p$  respectively.

The pressure profiles, in case of plain journal bearing for a light load (2 kN) and heavy loads (10 kN and 15 kN) are shown in Figs. 5.50 to 5.51 for two different values of misalignment ratios. Likewise, the Figs. 5.52 to 5.53 show the pressure profiles for two-axial-groove bearing for a light load (2 kN) and heavy load (10 kN) at two different values of misalignment ratios. It is seen from these figures that the maximum pressure in both cases is slightly reduced with the misalignment. There is not much change in the location of maximum pressure point.

Figs. 5.51 and 5.53 present the pressure profiles for plain journal bearing and two-axial-groove journal bearing with more severe misalignment,  $\bar{\gamma}_x = \bar{\gamma}_z = 0.4$ , again for two values of load. The maximum pressures in the cases of misaligned bearings are much higher than those for the respective pressures for the aligned bearings. Also, it is noticed that the location of maximum pressure shifts significantly with misalignment, both, circumferentially as well as axially.

The pressure profiles for elliptical and three-lobe bearings at a light (2 kN) and a heavy load (10 kN) are shown in Figs. 5.54 and 5.55 respectively. It is observed from these figures that for both values of load (i) the values of maximum fluid-film pressures are increased and (ii) location of maximum pressure does not lie in the mid-plane in the case of the misaligned bearings.

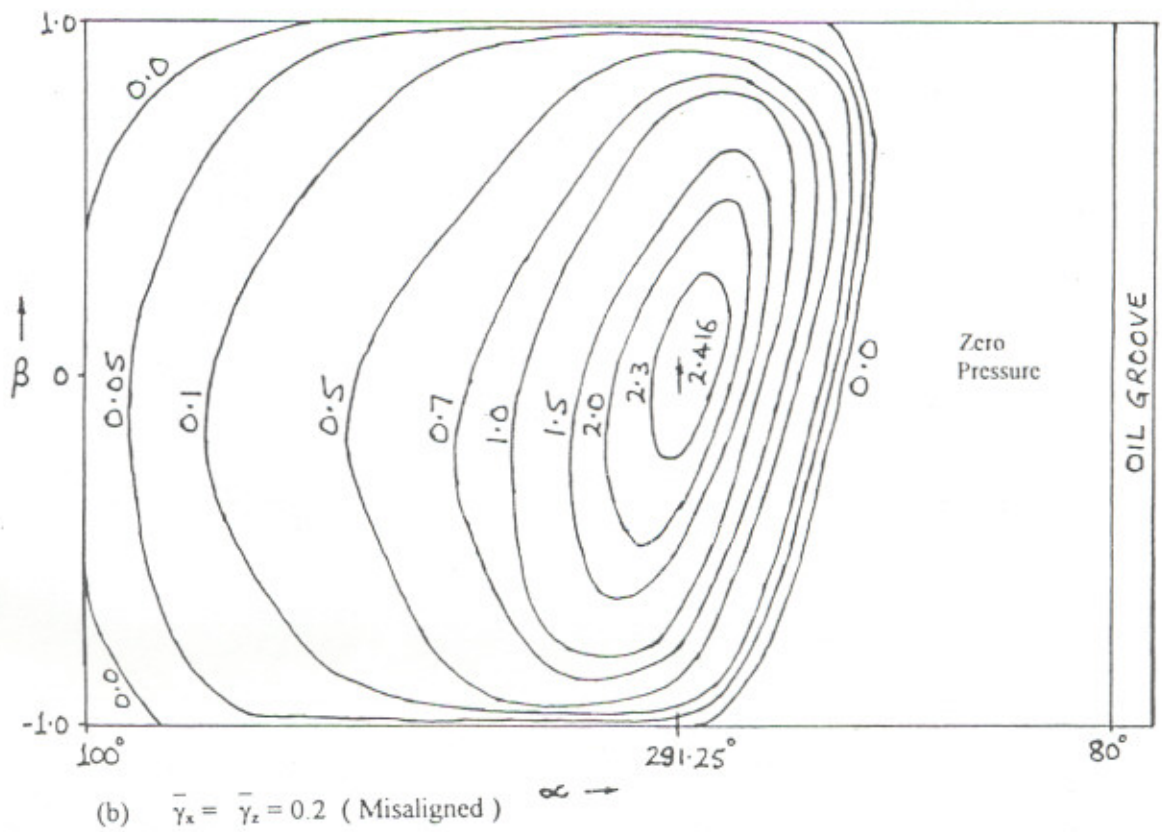
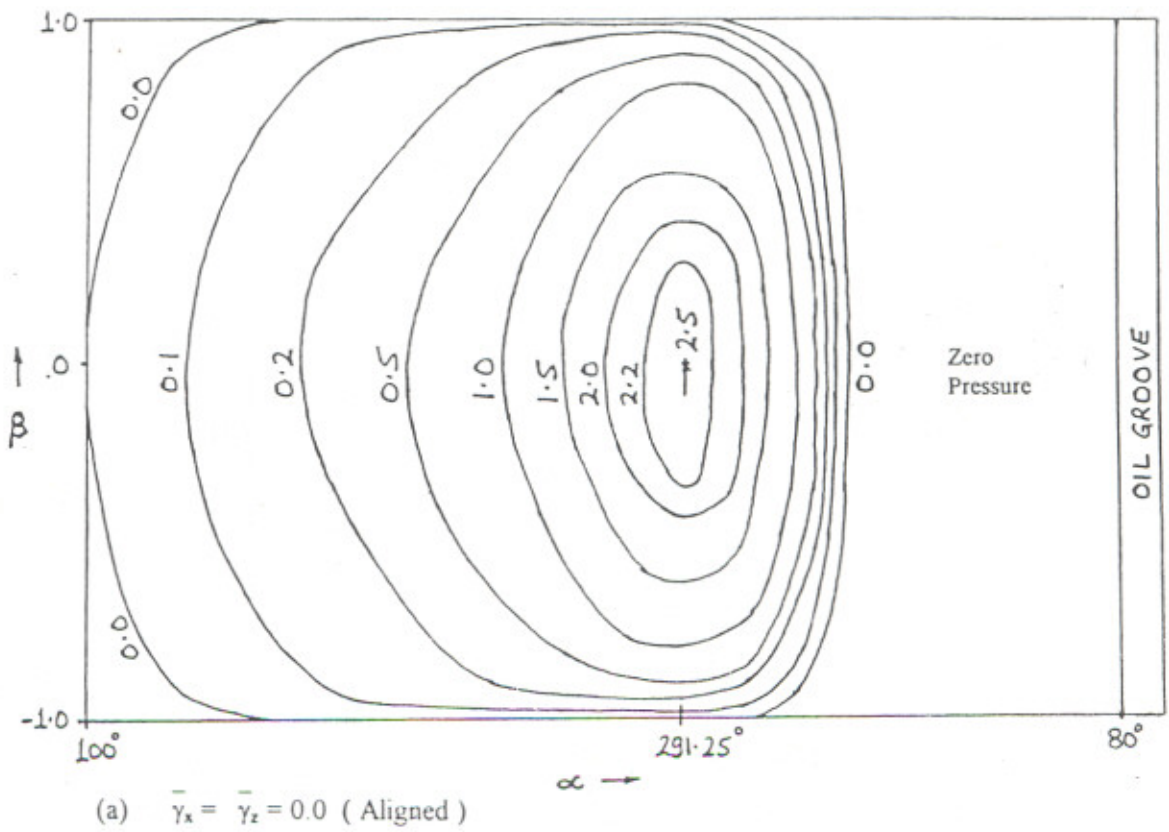
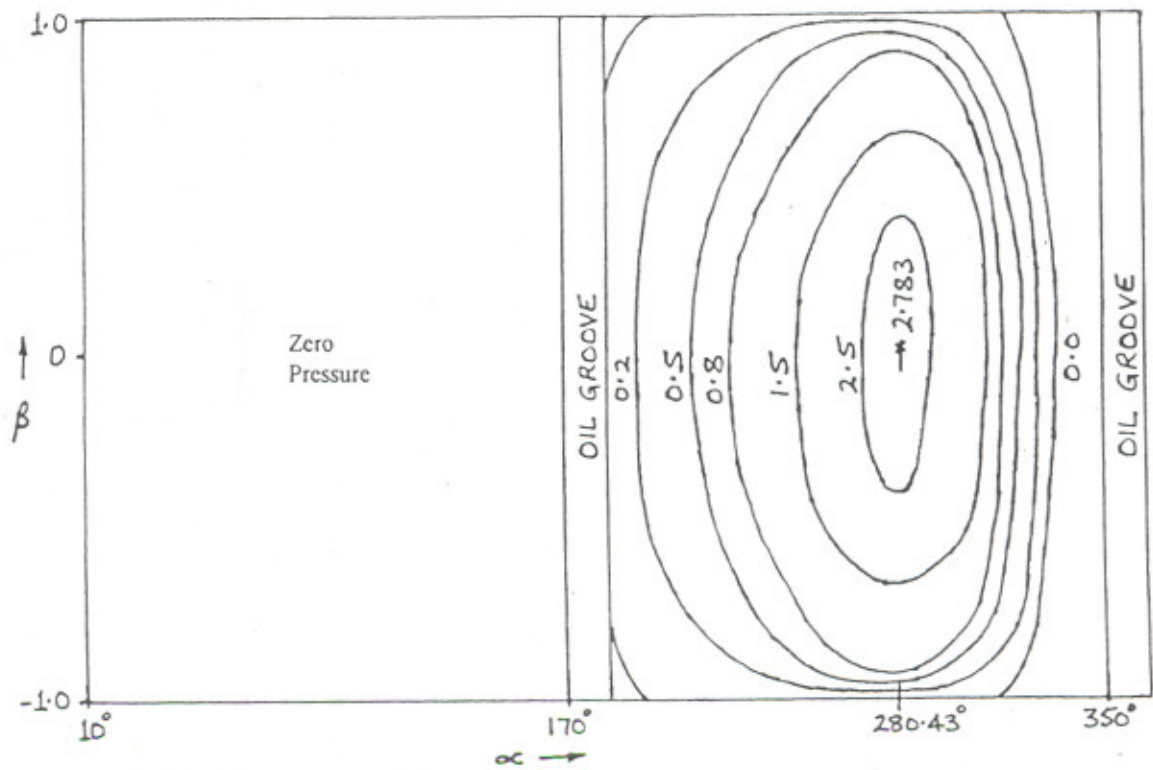
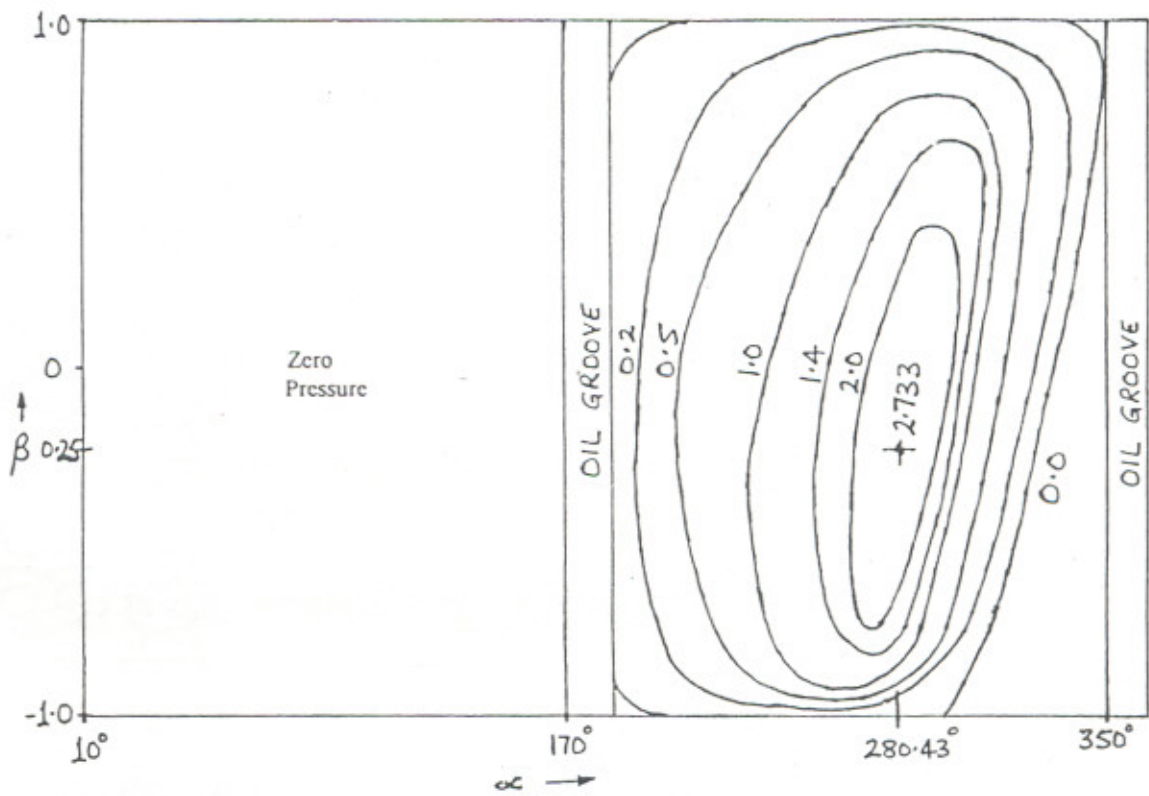


Fig. 5.45 PLAIN JOURNAL BEARING: ISOPRESSURE CURVES  
(LOAD = 10 kN)



(a)  $\bar{\gamma}_x = \bar{\gamma}_z = 0.0$  ( Aligned )



(b)  $\bar{\gamma}_x = \bar{\gamma}_z = 0.2$  ( Misaligned )

Fig. 5.46 TWO-AXIAL-GROOVE JOURNAL BEARING : ISOPRESSURE CURVES ( LOAD = 10 kN )

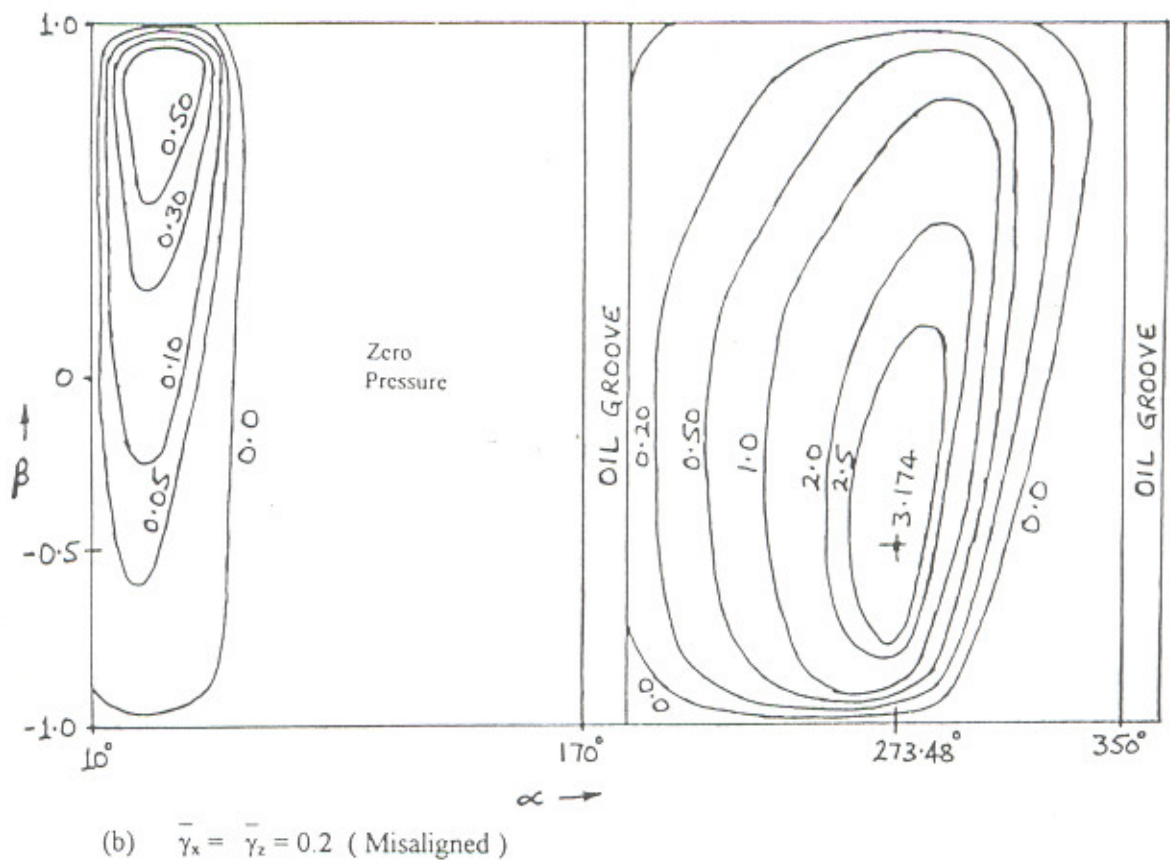
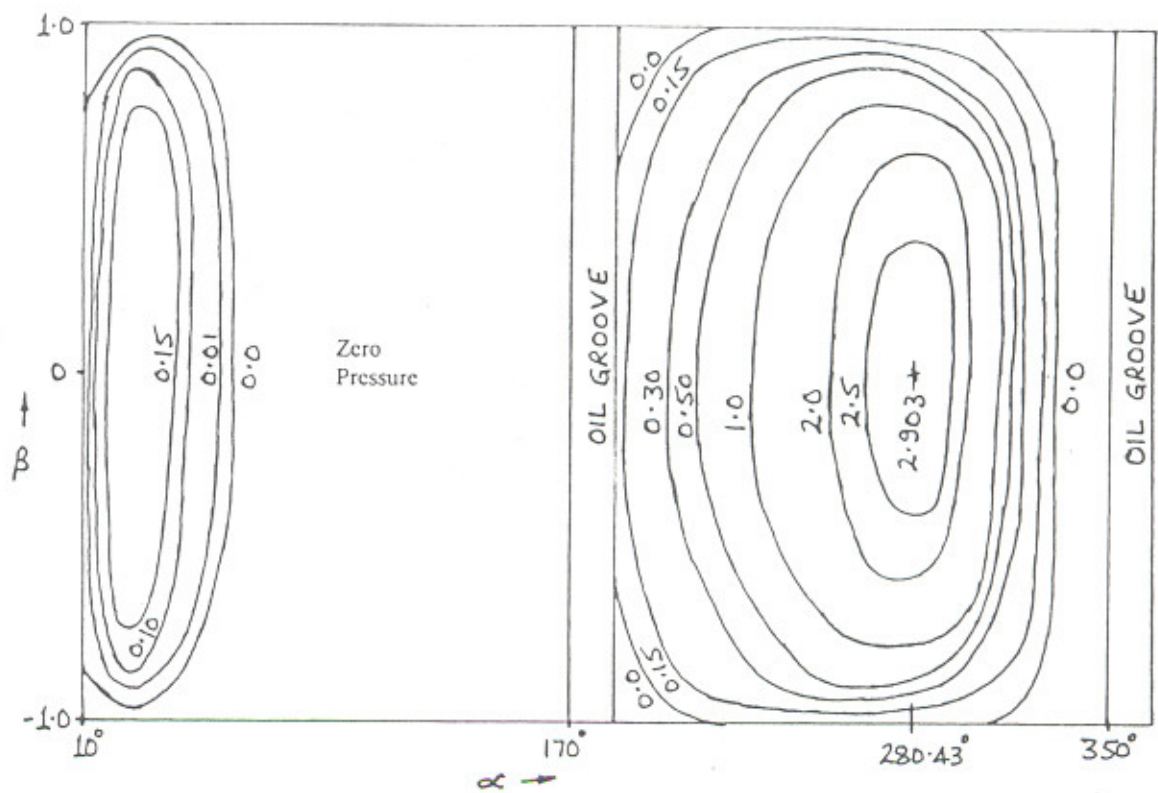


Fig 5.47 ELLIPTICAL JOURNAL BEARING : ISOPRESSURE CURVES ( LOAD = 10 kN )



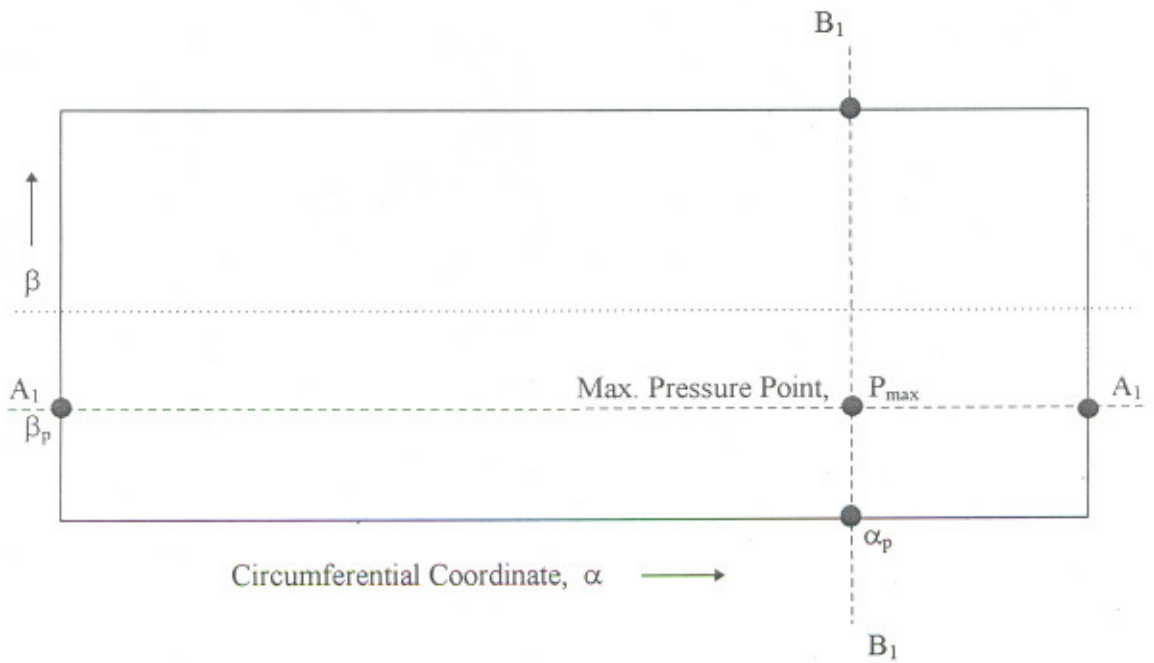
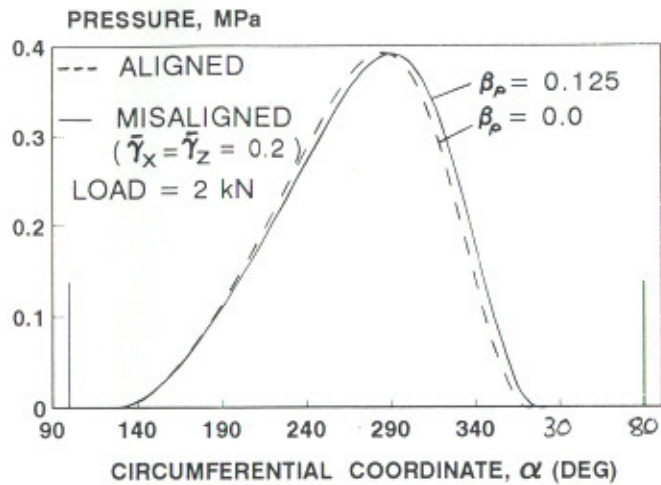
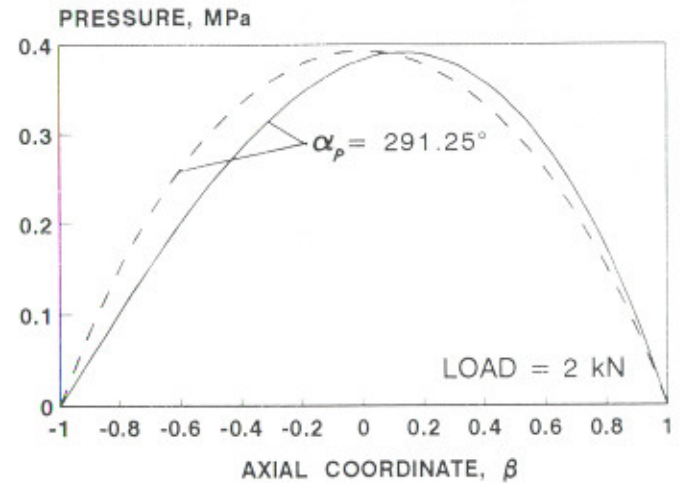


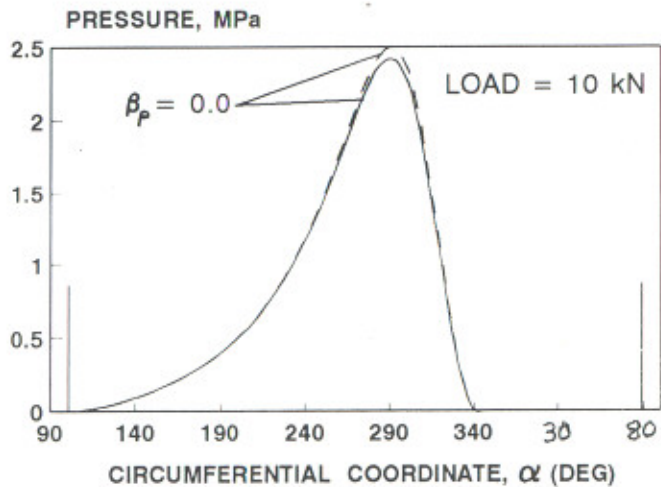
Fig. 5.49 Unwrapped Journal Bearing showing the Coordinate System and the Position of Maximum Pressure Point



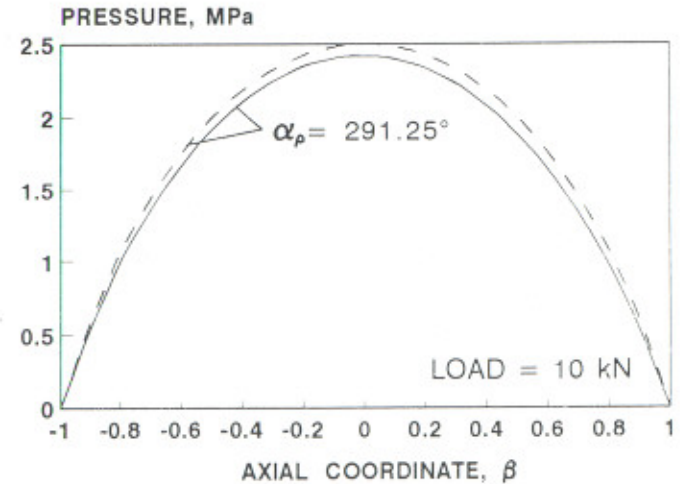
(a) SECTION AT  $A_1 - A_1$



(b) SECTION AT  $B_1 - B_1$

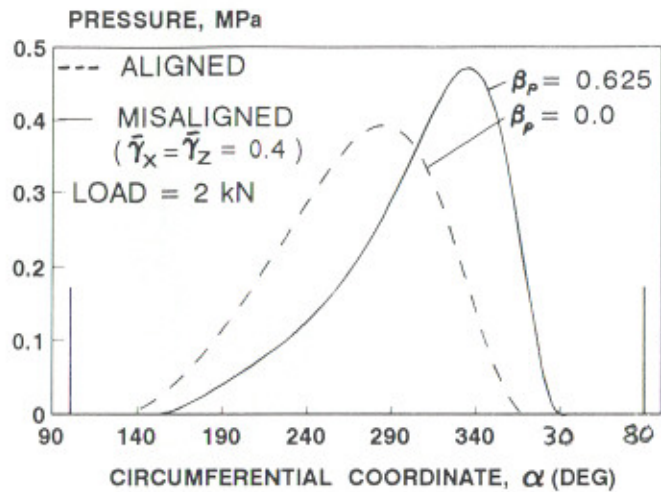


(c) SECTION AT  $A_1 - A_1$

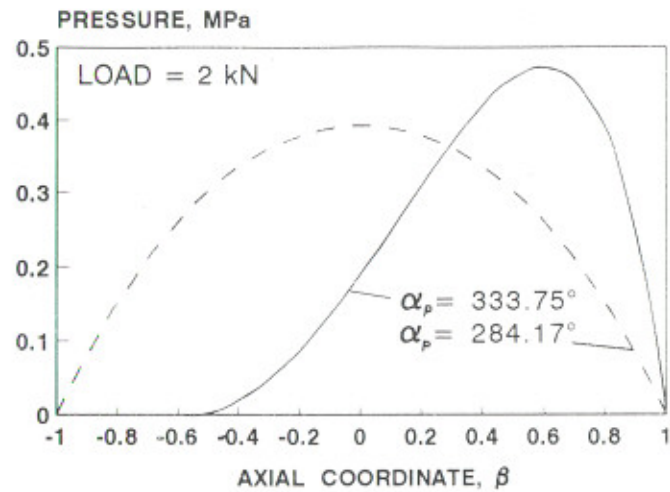


(d) SECTION AT  $B_1 - B_1$

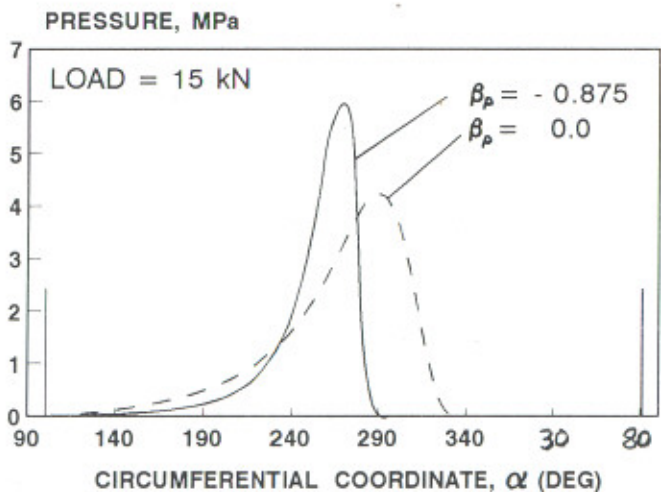
Fig. 5.50 PLAIN JOURNAL BEARING - PRESSURE PROFILES



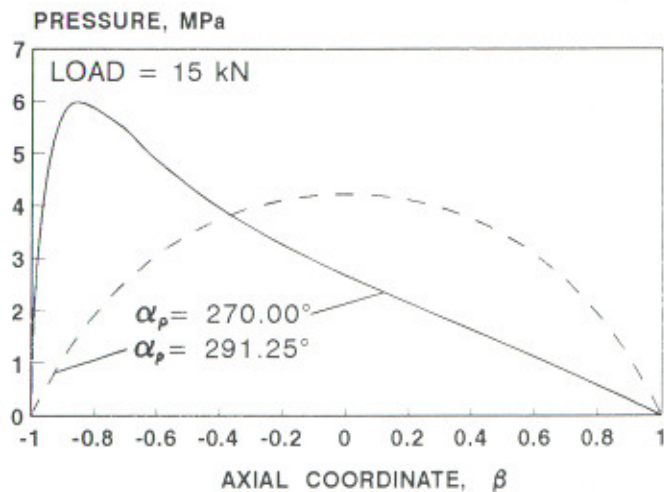
(a) SECTION AT A<sub>1</sub>-A<sub>1</sub>



(b) SECTION AT B<sub>1</sub>-B<sub>1</sub>

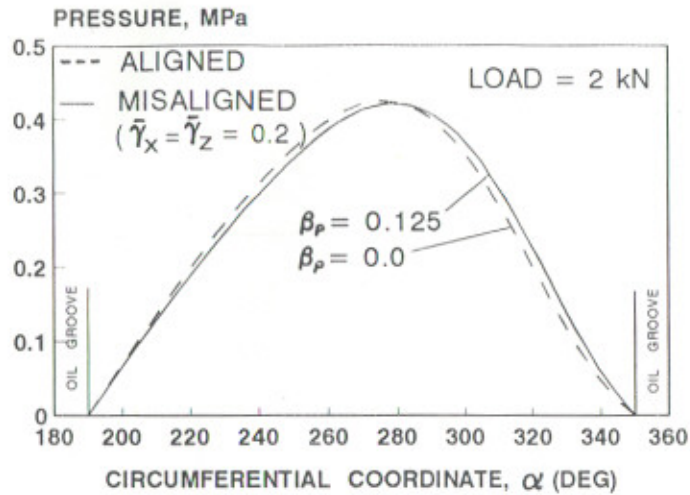


(c) SECTION AT A<sub>1</sub>-A<sub>1</sub>

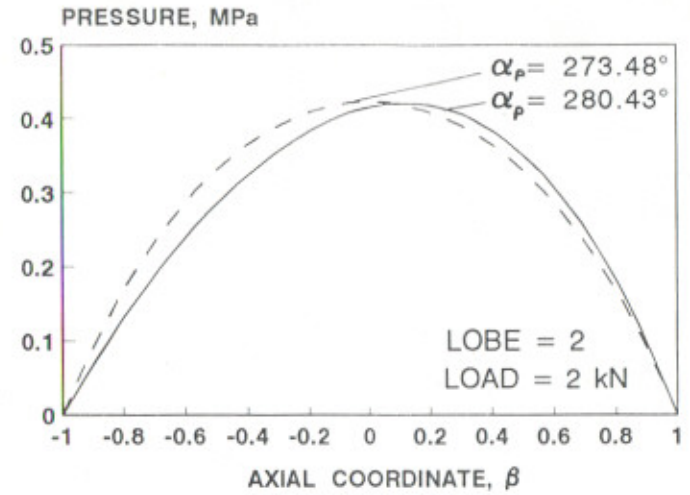


(d) SECTION AT B<sub>1</sub>-B<sub>1</sub>

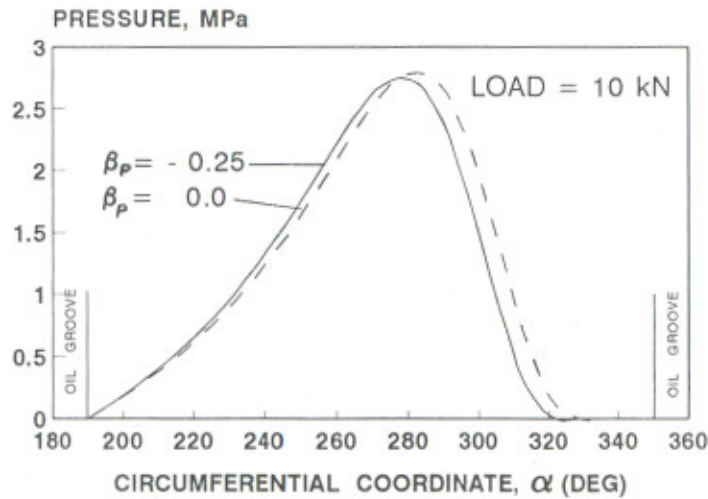
Fig. 5.51 PLAIN JOURNAL BEARING - PRESSURE PROFILES



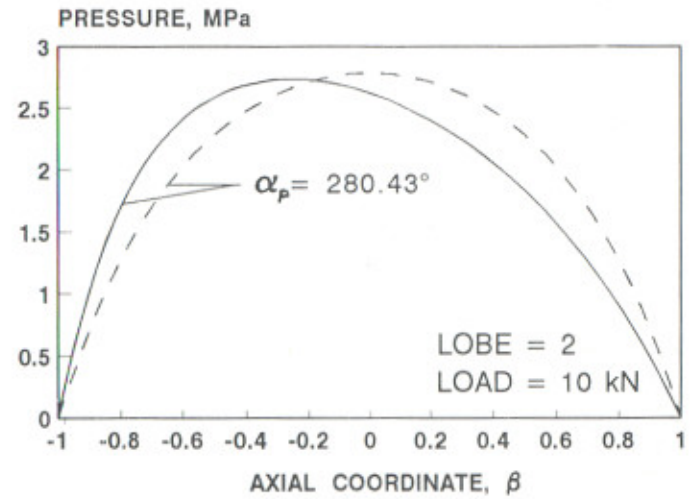
(a) SECTION AT  $A_1 - A_1$



(b) SECTION AT  $B_1 - B_1$

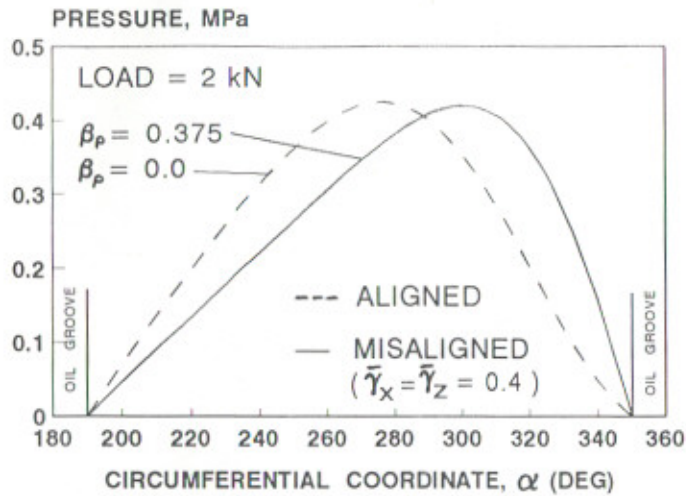


(c) SECTION AT  $A_1 - A_1$

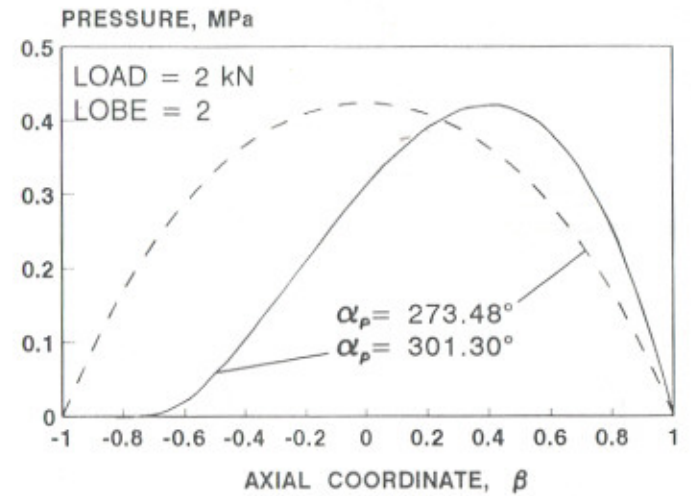


(d) SECTION AT  $B_1 - B_1$

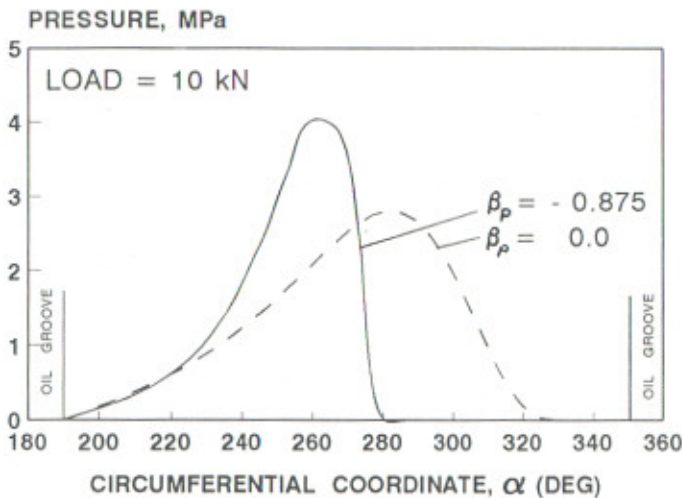
Fig. 5.52 TWO-AXIAL-GROOVE JOURNAL BEARING - PRESSURE PROFILES



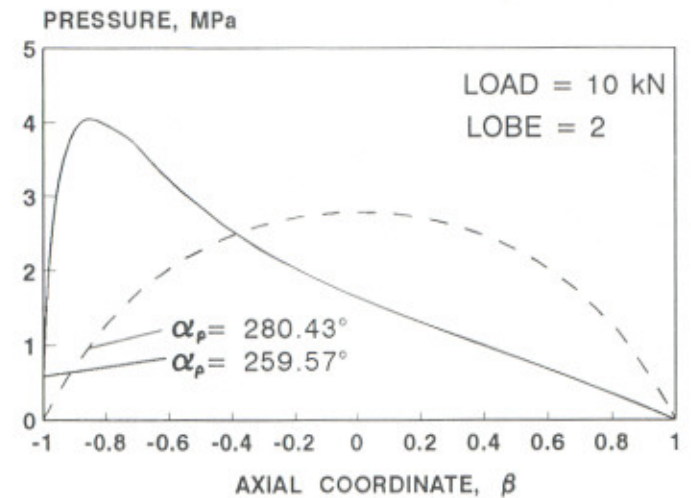
(a) SECTION AT A<sub>1</sub>-A<sub>1</sub>



(b) SECTION AT B<sub>1</sub>-B<sub>1</sub>

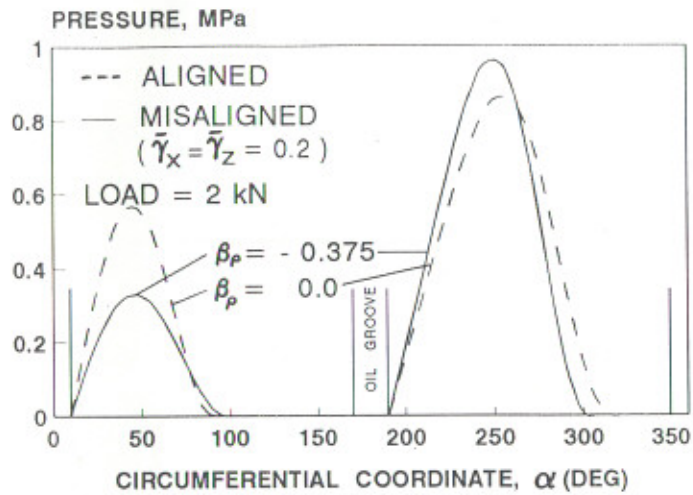


(c) SECTION AT A<sub>1</sub>-A<sub>1</sub>

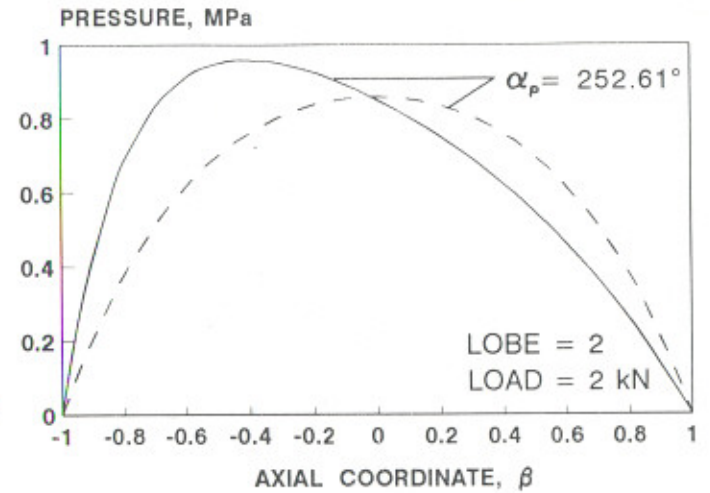


(d) SECTION AT B<sub>1</sub>-B<sub>1</sub>

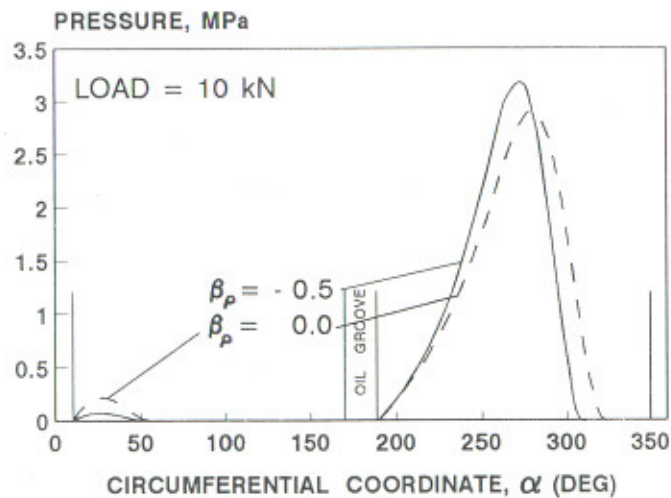
Fig. 5.53 TWO-AXIAL-GROOVE JOURNAL BEARING - PRESSURE PROFILES



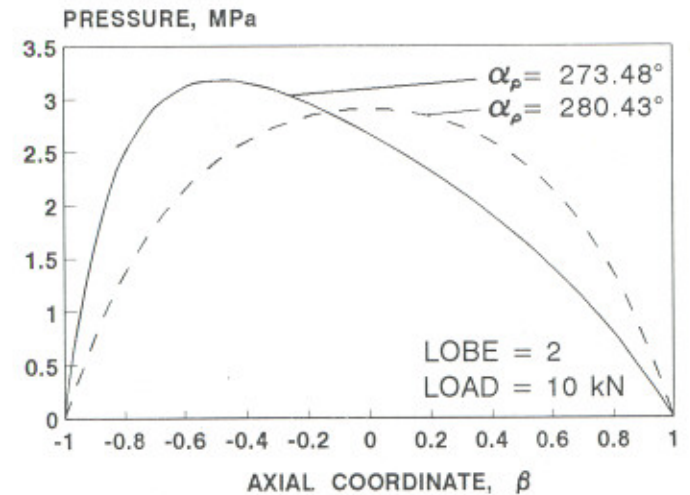
(a) SECTION AT  $A_1 - A_1$



(b) SECTION AT  $B_1 - B_1$

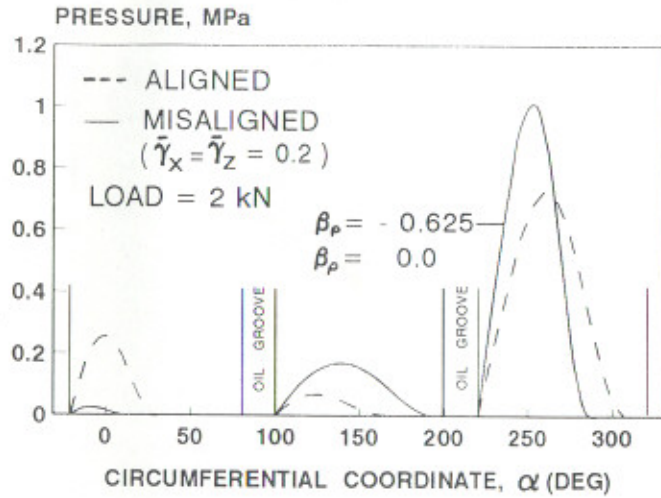


(c) SECTION AT  $A_1 - A_1$

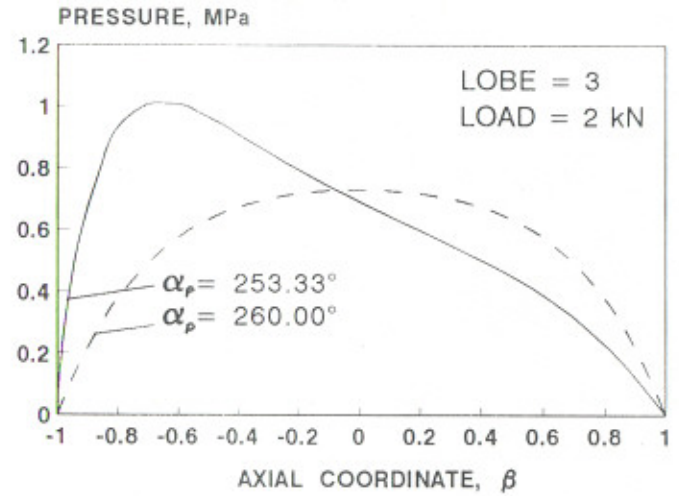


(d) SECTION AT  $B_1 - B_1$

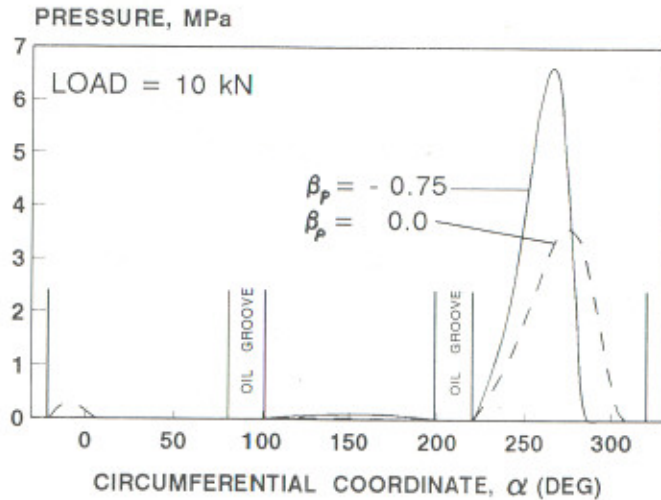
Fig. 5.54 ELLIPTICAL JOURNAL BEARING - PRESSURE PROFILES



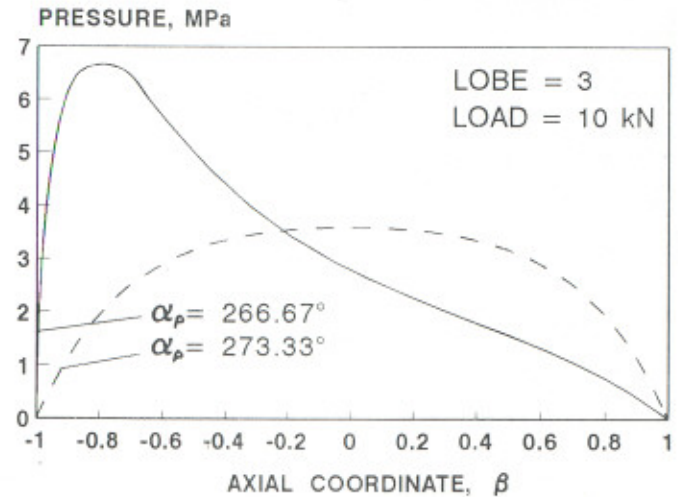
(a) SECTION AT  $A_1 - A_1$



(b) SECTION AT  $B_1 - B_1$



(c) SECTION AT  $A_1 - A_1$



(d) SECTION AT  $B_1 - B_1$

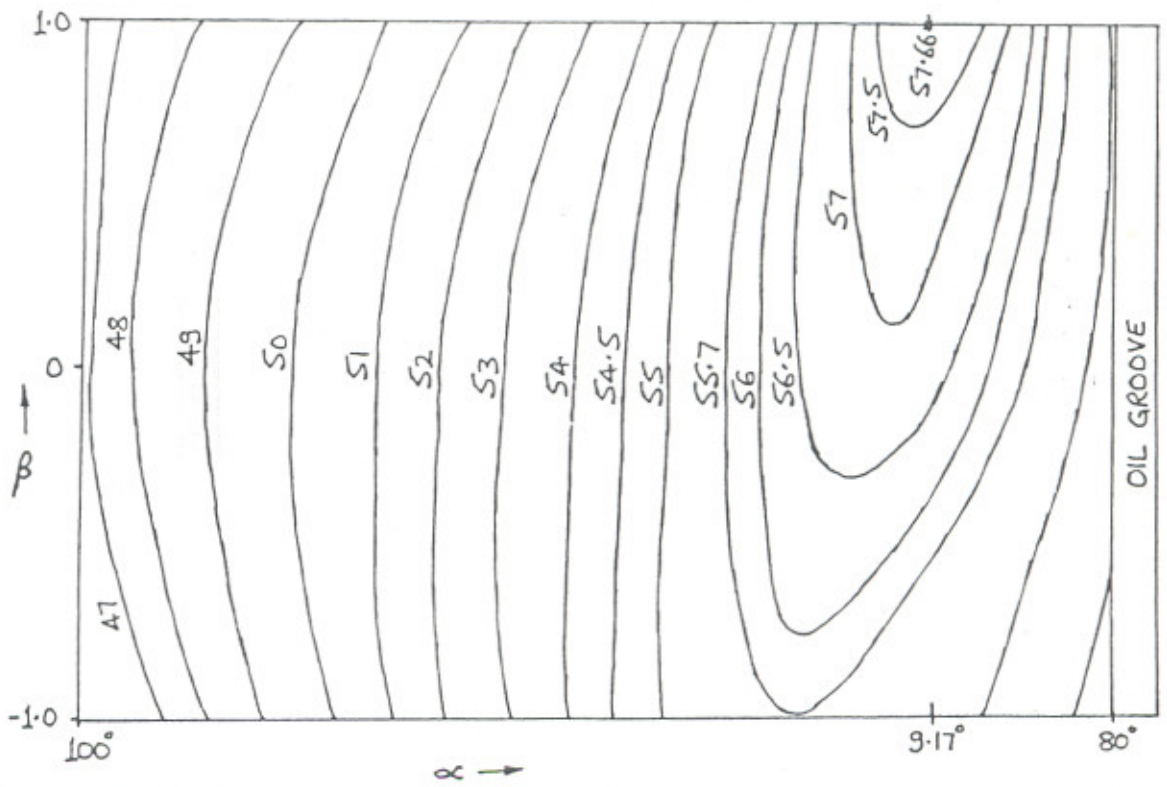
Fig. 5.55 THREE-LOBE JOURNAL BEARING - PRESSURE PROFILES

### 5.3.6 Isotherms and Temperature Profiles

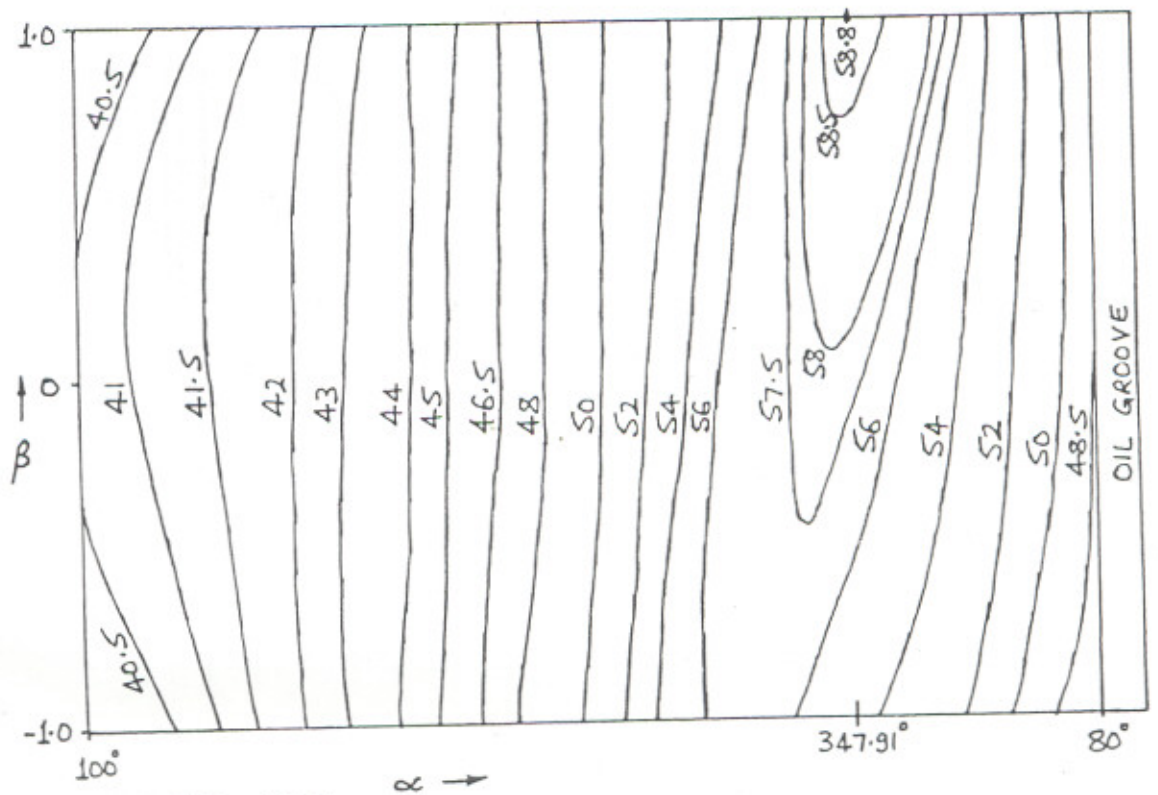
Figs. 5.56, 5.57, 5.58 and 5.59 show the bush-fluid interface isotherms for the four bearing configurations at two values of load ( 2 kN and 10 kN) and  $\bar{\gamma}_x = \bar{\gamma}_z = 0.2$ . The temperature difference occurring along circumferential direction and axial direction, and the maximum temperature in the bush-fluid isotherms has been summarised in table 5.10 for all the bearing geometries. It is revealed that at both the loads, there is considerable variation in temperature along the circumferential and axial directions indicating heat transfer in these directions. The temperature variation in axial direction is more in the case of non-circular bearings as compared to the circular bearings and this variation increases with the increase in the value of load. The trend for other values of misalignment was also found to be similar, although not reported here.

The oil and bush isotherms for the bearing systems are shown in Figs. 5.60, 5.61, 5.62 and 5.63 for the four bearings each operating with a load = 10 kN and misalignment of  $\bar{\gamma}_x = \bar{\gamma}_z = 0.2$ . These isotherms have been plotted at three circumferential planes for each bearing. From these isotherms it is observed that heat flows into the journal from the hot region in the oil-film and out of the journal in the cold region of the oil-film. There is considerable variation in temperature across the thickness of the oil-film and the bush. The isotherms show that due to misalignment, expectedly, the maximum temperature zone does not remain in the central plane ( $\beta = 0$ ) but shifts away from the central plane. The bush isotherms indicate that the maximum bush temperature occurs in the vicinity of the hottest region of oil-film. In the high temperature region of the bush, most of the heat transfers directly out of the bush outer surface to the surroundings. It is also observed from these figures that heat transfer takes place not only in radial direction in the bush but also in the circumferential and axial directions.

The interface temperature profiles have been plotted at circumferential (section  $A_2 - A_2$ ) and axial (section  $B_2 - B_2$ ) planes which pass through the point of maximum interface temperature in respective cases, Fig. 5.64. The coordinates of the maximum temperature on the interface are designated as  $\alpha_t$  and  $\beta_t$  as shown in the figure.



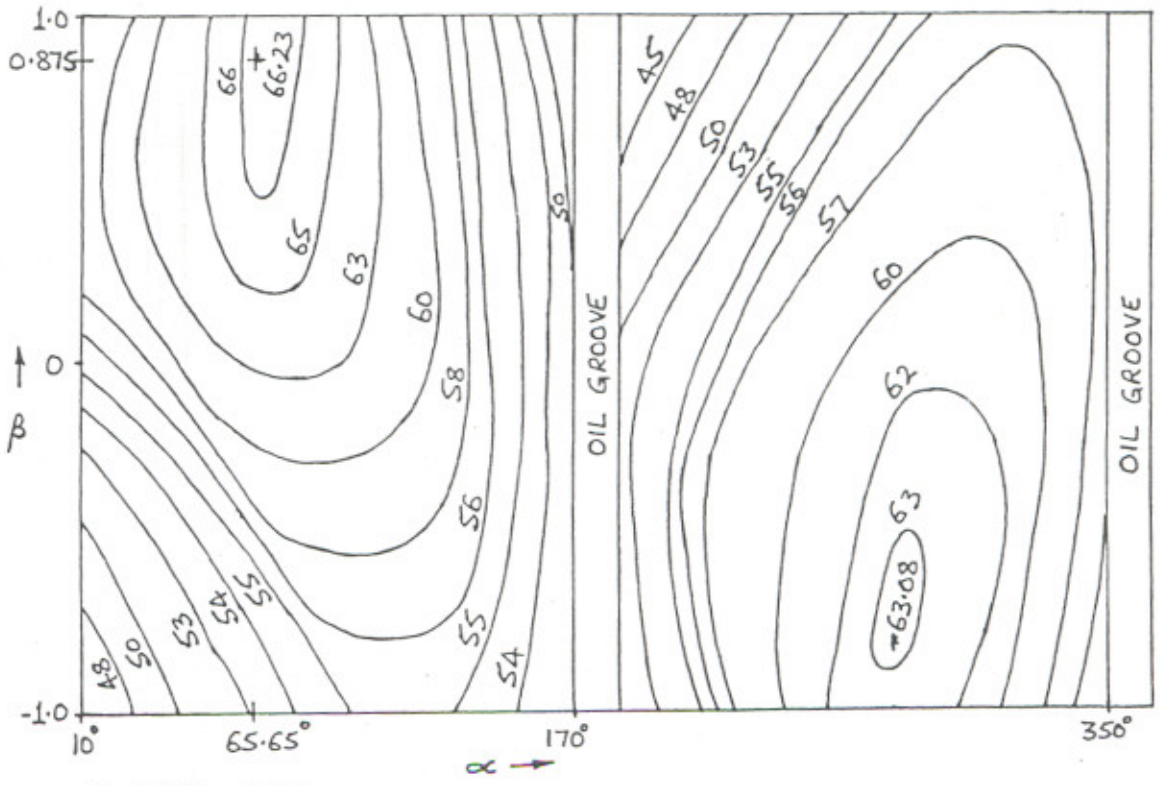
(a) LOAD = 2 kN



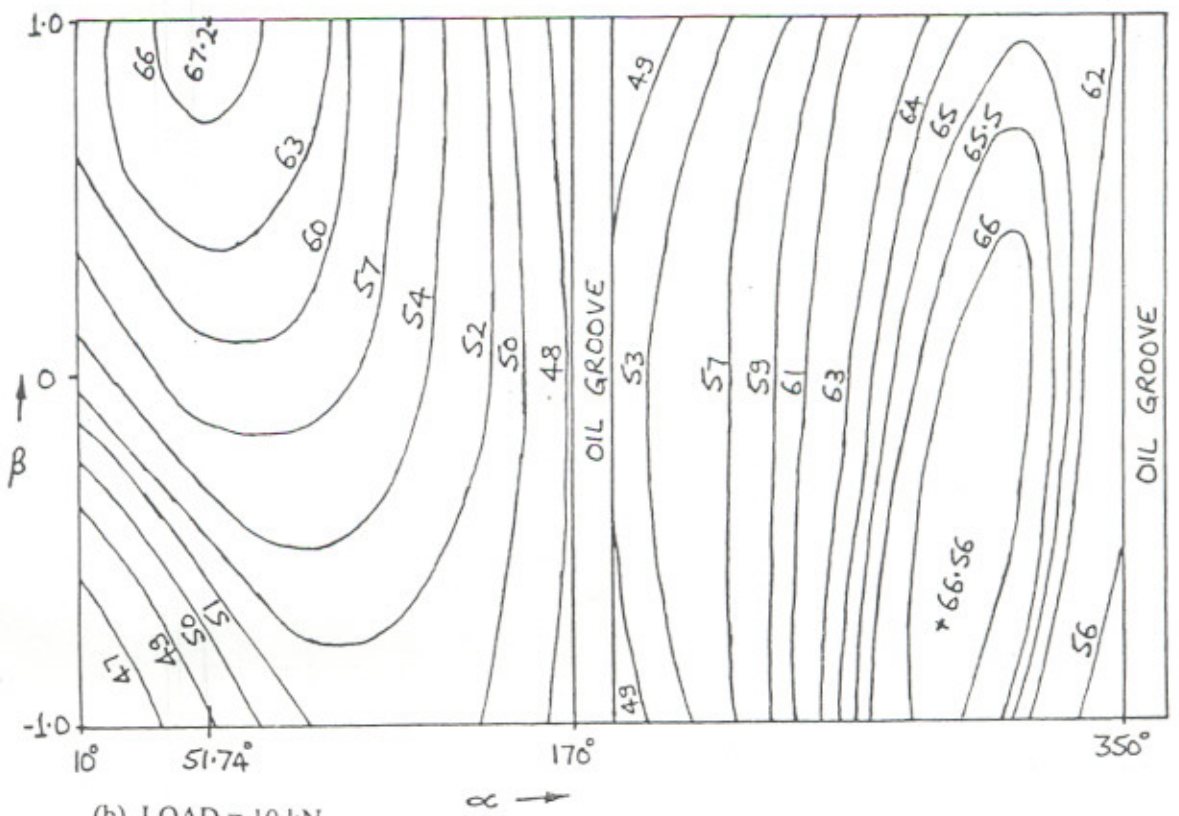
(b) LOAD = 10 kN

Fig. 5.56 PLAIN JOURNAL BEARING; BUSH-FLUID INTERFACE ISOTHERMS  
( $\gamma_x = \gamma_z = 0.2$ )





(a) LOAD = 2 kN



(b) LOAD = 10 kN

Fig. 5.8 ELLIPTICAL JOURNAL BEARING: BUSH-FLUID INTERFACE ISOTHERMS ( $\gamma_x = \gamma_z = 0.2$ )

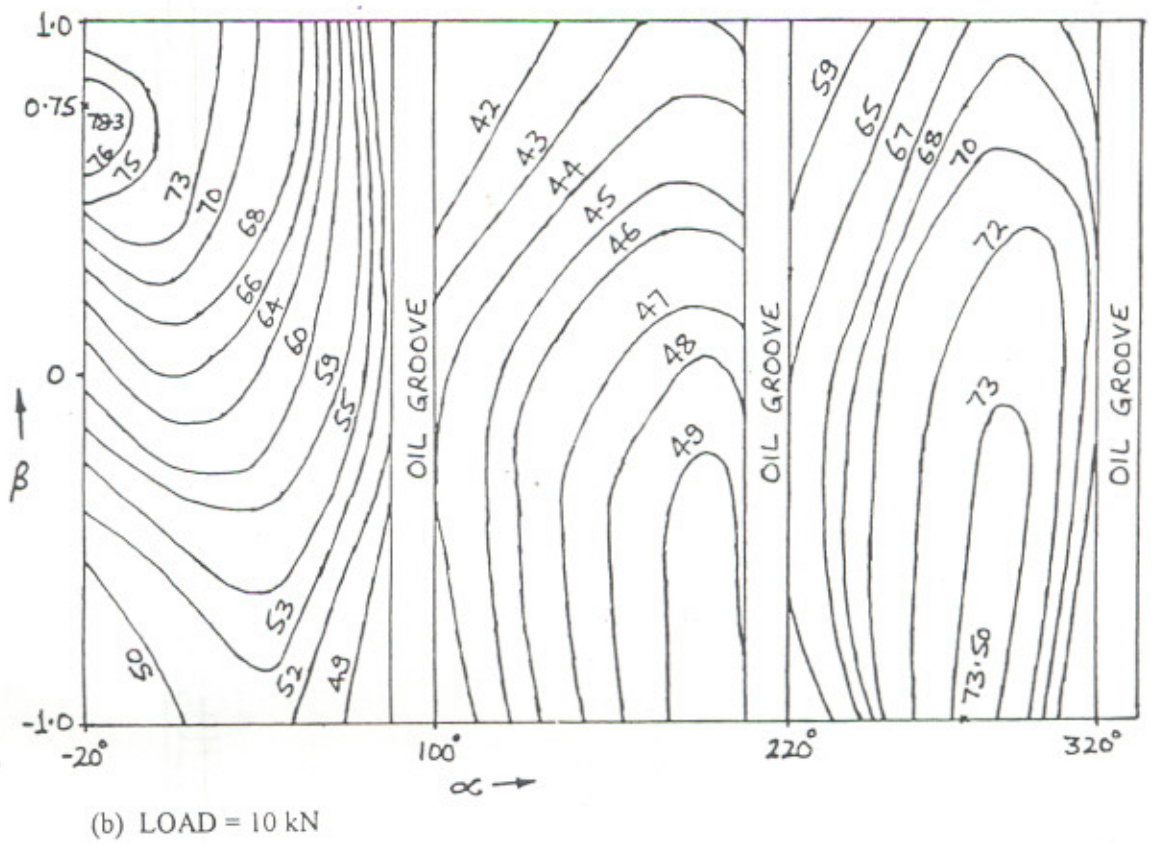
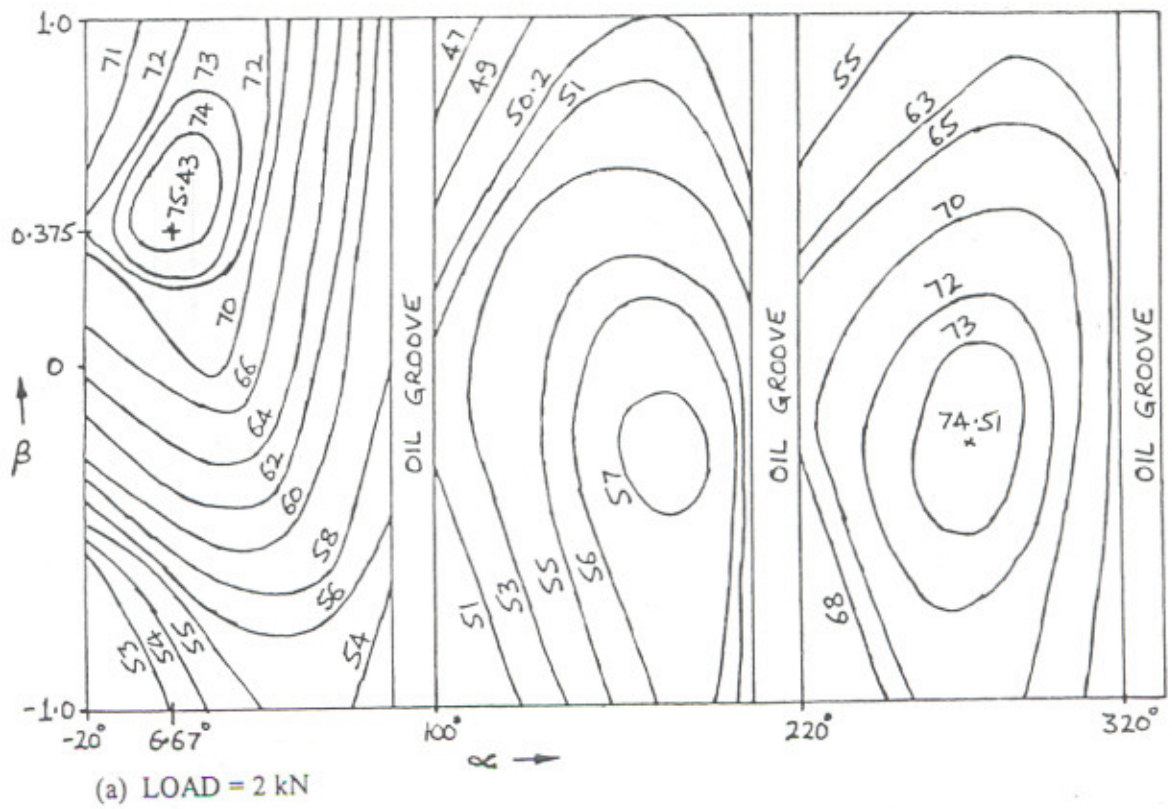


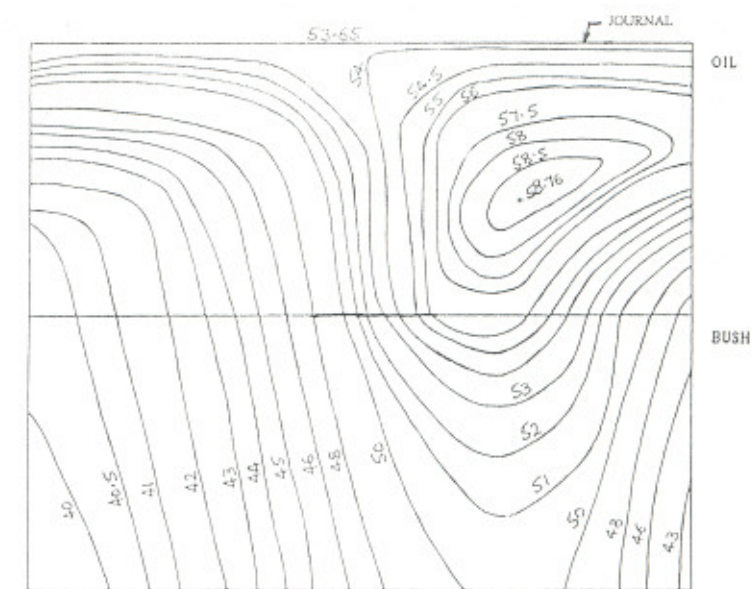
Fig. 5.9 THREE-LOBE JOURNAL BEARING : BUSH-FLUID INTERFACE  
ISOTHERMS ( $\bar{\gamma}_x = \bar{\gamma}_z = 0.2$ )

Table 5.10 Summary of Bush-Fluid Interface Isotherms

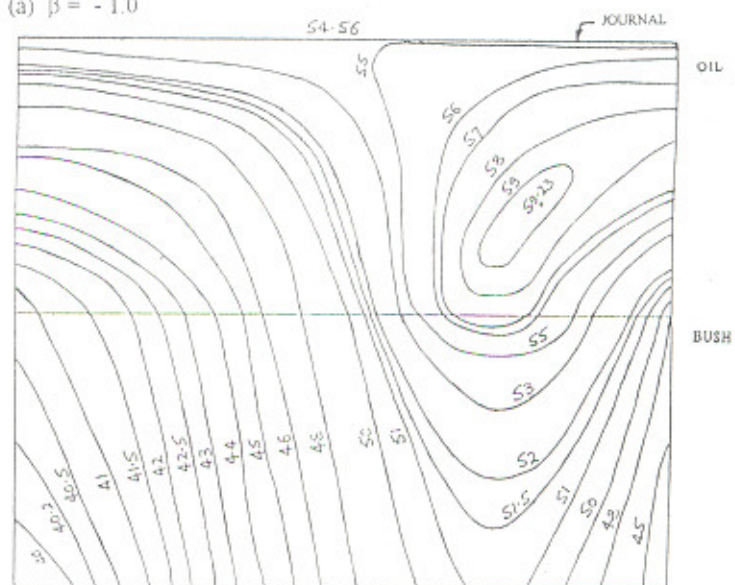
( Misalignment ratio,  $\bar{\gamma}_x = \bar{\gamma}_z = 0.2$  )

Type of Bearing	Load, kN	Lobe*	Temperature Difference, °C		Maximum Temperature, °C
			Circumferentially	Axially	
Plain	2	1	11.6	3.4	57.6
	10	1	18.8	4.3	58.8
Two-Axial-Groove	2	1	5.6	2.8	54.33
		2	10.0	6.8	59.30
	10	1	8.7	7.5	52.86
		2	18.2	4.0	64.54
Elliptical	2	1	16.0	12.2	66.23
		2	11.5	8.0	63.08
	10	1	19.2	18.2	67.20
		2	13.0	3.5	66.56
Three-Lobe	2	1	10.0	22.0	75.43
		2	6.0	7.0	57.44
		3	5.0	15.5	74.51
	10	1	20.3	29.3	78.33
		2	5.0	6.0	49.75
		3	6.0	6.5	73.50

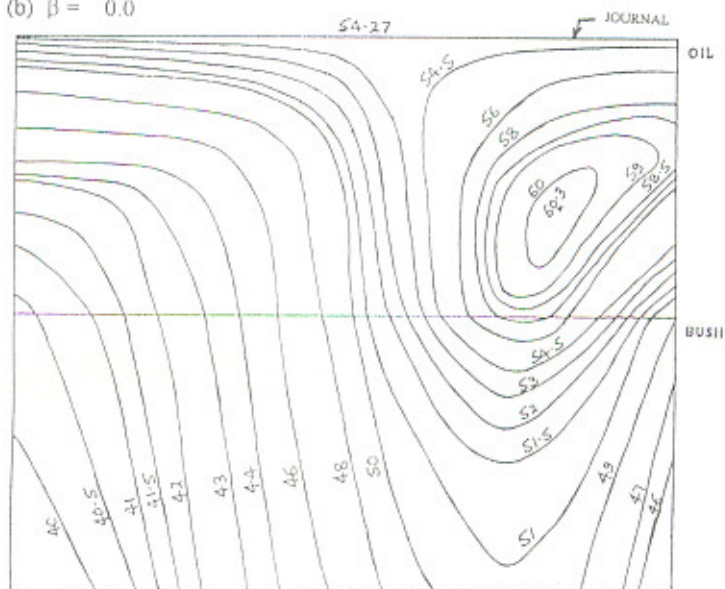
\* lobe nomenclature is given in Fig. 2.1.



(a)  $\beta = -1.0$



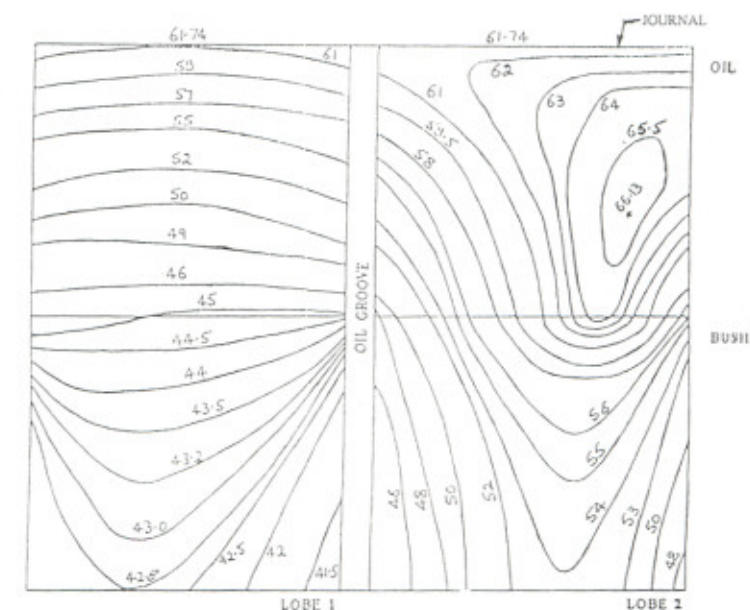
(b)  $\beta = 0.0$



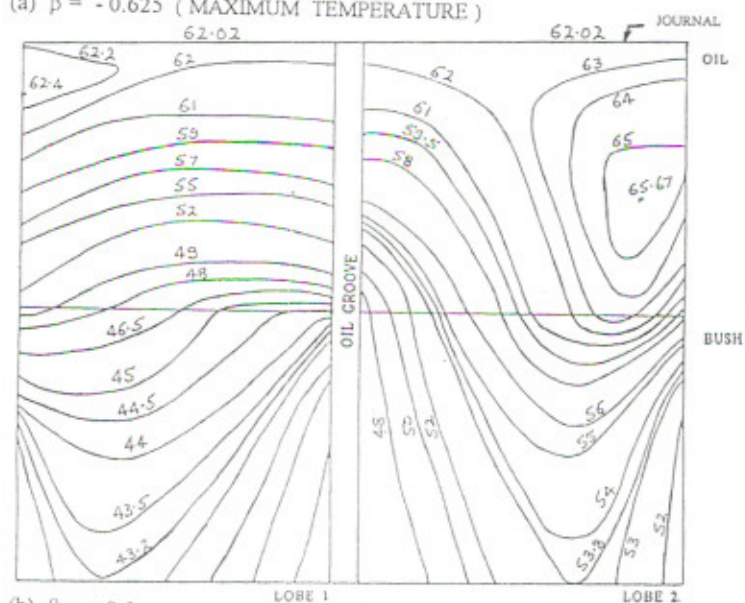
(c)  $\beta = 1.0$  (MAXIMUM TEMPERATURE)

LOAD = 10 kN  
 $(\bar{\gamma}_x = \bar{\gamma}_z = 0.2)$

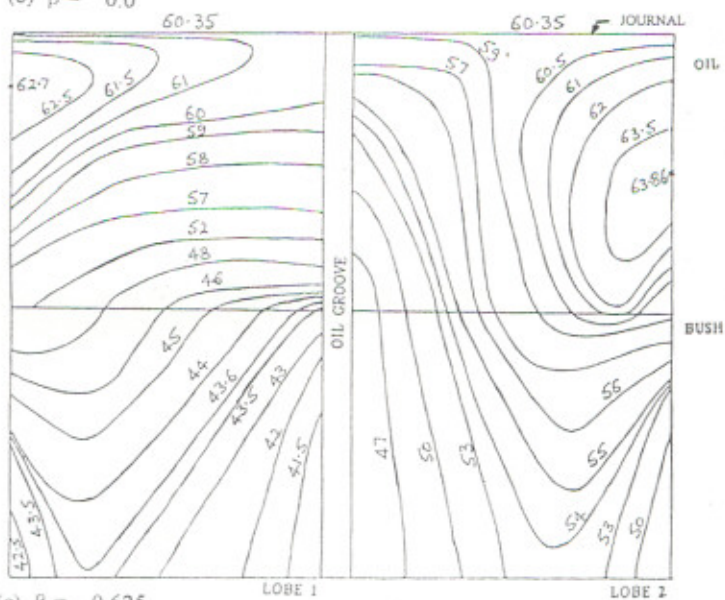
Fig. 5.60 PLAIN JOURNAL BEARING: ISOTHERMS



(a)  $\beta = -0.625$  (MAXIMUM TEMPERATURE)



(b)  $\beta = 0.0$



(c)  $\beta = 0.625$

LOAD = 10 kN  
 $(\bar{\gamma}_x = \bar{\gamma}_z = 0.2)$

Fig. 5.61 TWO-AXIAL-GROOVE JOURNAL BEARING : ISOTHERMS

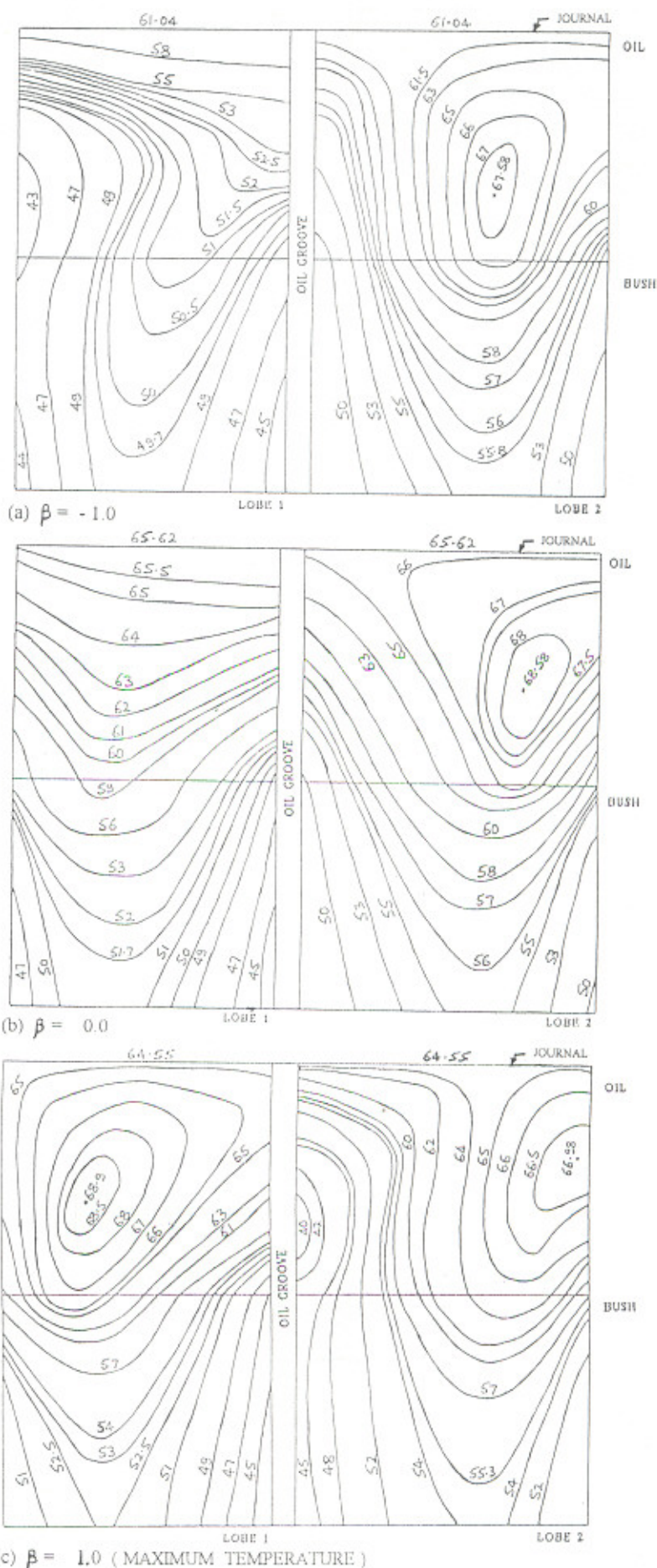


Fig. 5.62 ELLIPTICAL JOURNAL BEARING : ISOTHERMS

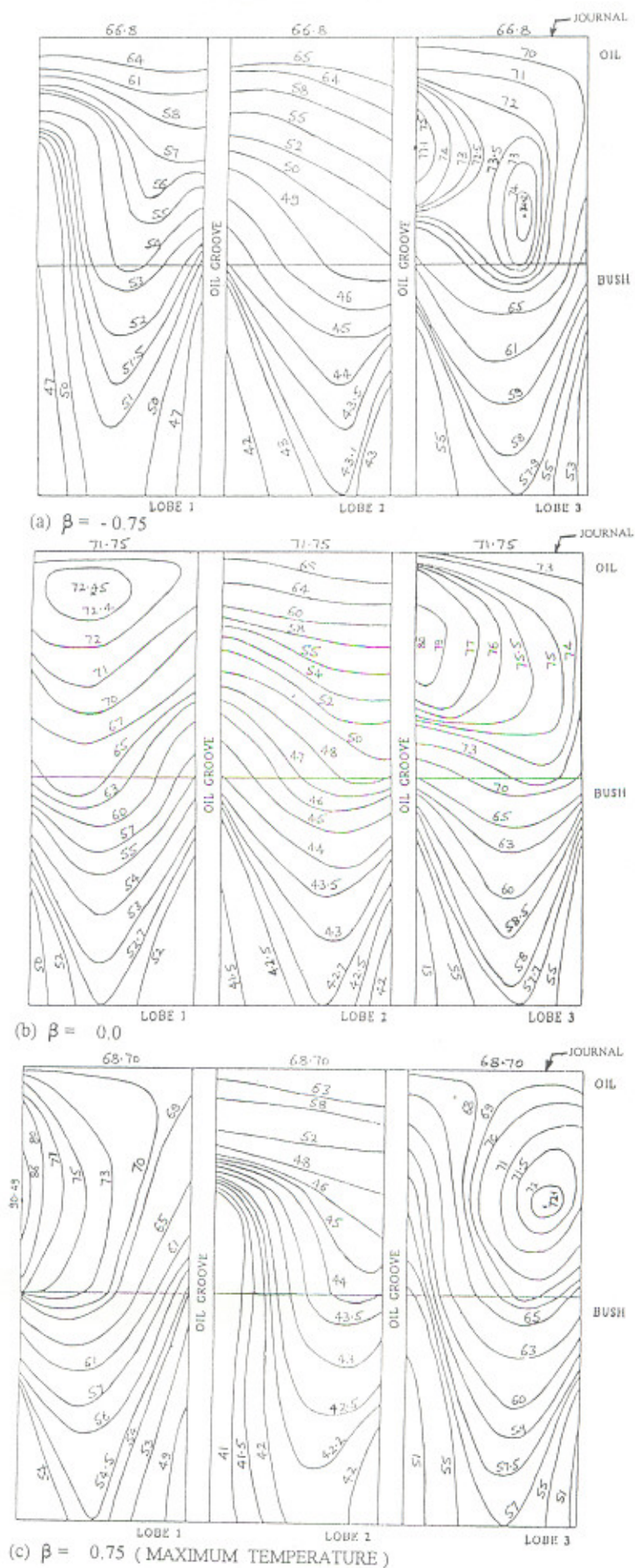


Fig. 5.63 THREE-LOBE JOURNAL BEARING: ISOTHERMS

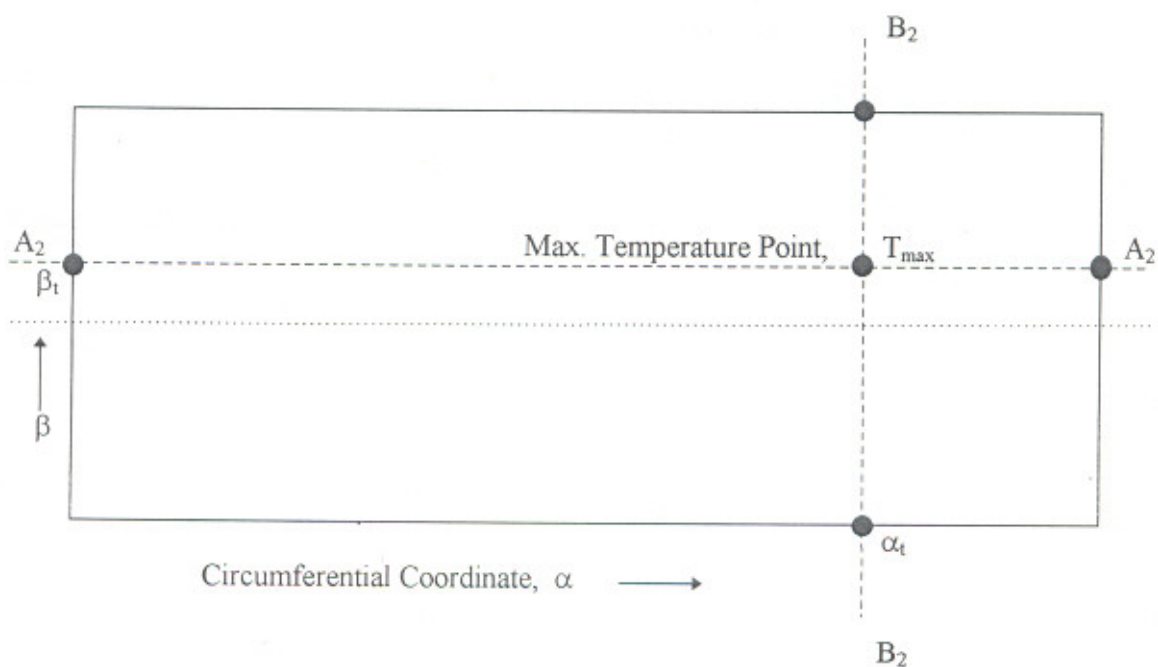


Fig. 5.64 Unwrapped Journal Bearing showing the Coordinate System and the Position of Maximum Temperature Point

The bush-fluid interface temperature profiles for aligned and misaligned journal bearings are shown in Figs. 5.65 to 5.70. These figures reveal that the maximum interface temperature is higher in the case of a misaligned bearing and the axial temperature profiles, indicate that the axial variation of temperature is quite significant in the case of misaligned bearings especially for non-circular bearing configurations.

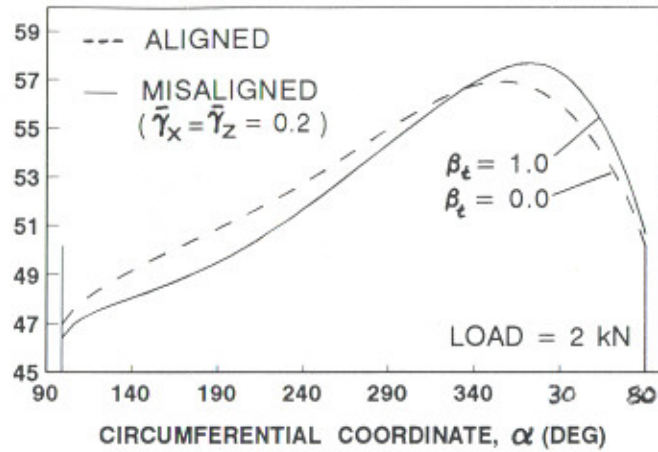
The Figs. 5.65(a), 5.66(a), 5.67(a) and 5.68(a) reveals that in the case of plain journal bearing and lobe 1 of two-axial-groove journal bearing supporting a light load (2 kN), the temperature is higher at the inlet of edge of the bearing pad, which could be the result of mixing of relatively higher amount of hot recirculating oil with the cool supply oil at lower values of eccentricity ratios. For heavy loads, the operating eccentricity is higher and hence the amount of cool supply oil will be relatively much more than the recirculating oil. Thus the temperature after mixing will be lesser.

It is observed that at the light load, the difference of temperature across the bush ends in misaligned plain journal bearing and two-axial-groove journal bearing is respectively 2.7 °C and 7 °C, {Figs. 5.65(b) and 5.67(b)} whereas for an aligned bearing it is only about 0.4 °C and 0.7 °C respectively. At heavier load, this difference is even higher and increases further with the increase in misalignment. Figs. 5.66(a) to 5.66(d) and 5.68(a) to 5.68(d) show the bush-fluid interface temperature profiles of aligned and misaligned journal bearings at the higher values of misalignment ratio ( $\bar{\gamma}_x = \bar{\gamma}_z = 0.4$ ) for light and a heavy load. The trends in this case are also similar to those in Figs. 5.65 and 5.66 except that the difference in temperature between aligned and misaligned bearing cases is much higher.

The bush-fluid interface temperature profiles for elliptical journal bearing and three-lobe journal bearing are shown in Figs. 5.69 and 5.70. Figs. 5.69(a) and 5.70(a) reveal that at light load, misalignment leads to significant rise (7 °C) in bush-fluid interface temperature in the lobe 1. Fig. 5.69(b) shows that at light load, the axial variation of temperature increases from 3 °C to 13 °C due to misalignment in case of elliptical journal bearing where as in three-lobe journal bearing {Fig. 5.70(b)}, the axial variation of temperature increases from 5.0 to 22.3 °C.

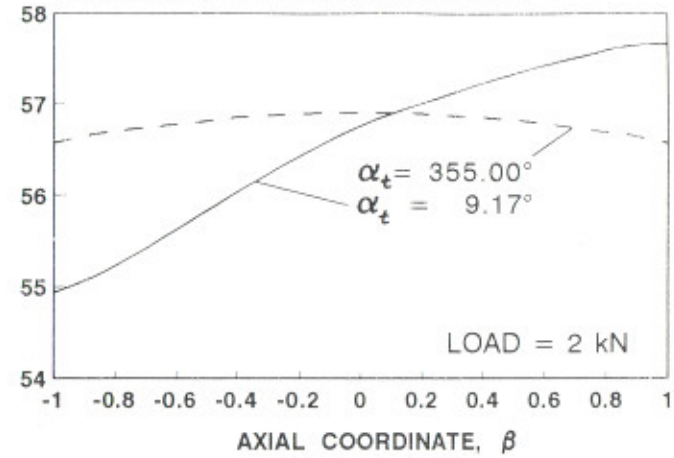
Table 5.11 summaries the observations of temperature profiles studied. These results indicate that the assumption of negligible change in temperature across the axial length of bearing, generally used for the study of aligned bearings, is not quite valid for the analysis of misaligned bearings as it may lead to serious inaccuracies.

BUSH-FLUID INTERFACE TEMPERATURE, °C



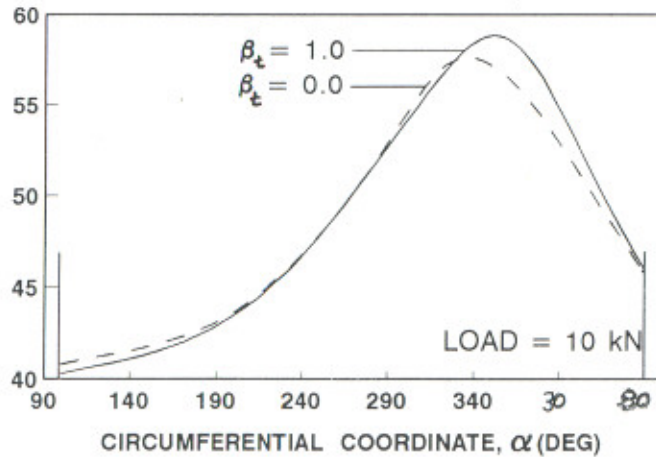
(a) SECTION AT  $A_2 - A_2$

BUSH-FLUID INTERFACE TEMPERATURE, °C



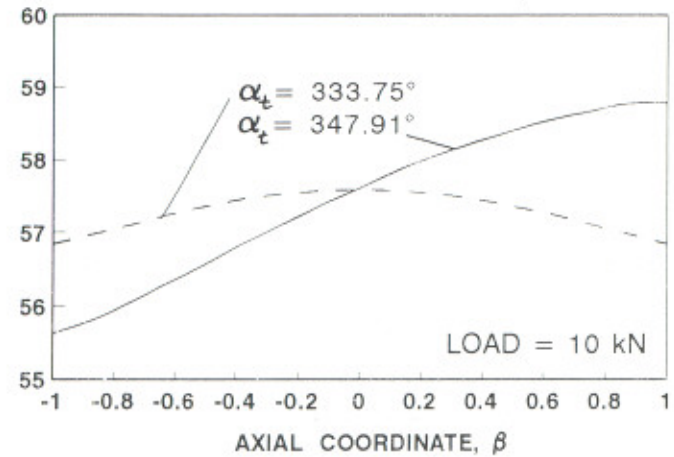
(b) SECTION AT  $B_2 - B_2$

BUSH-FLUID INTERFACE TEMPERATURE, °C



(c) SECTION AT  $A_2 - A_2$

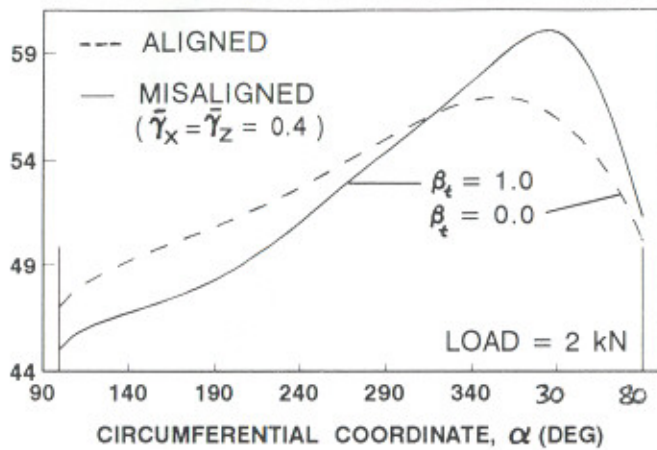
BUSH-FLUID INTERFACE TEMPERATURE, °C



(d) SECTION AT  $B_2 - B_2$

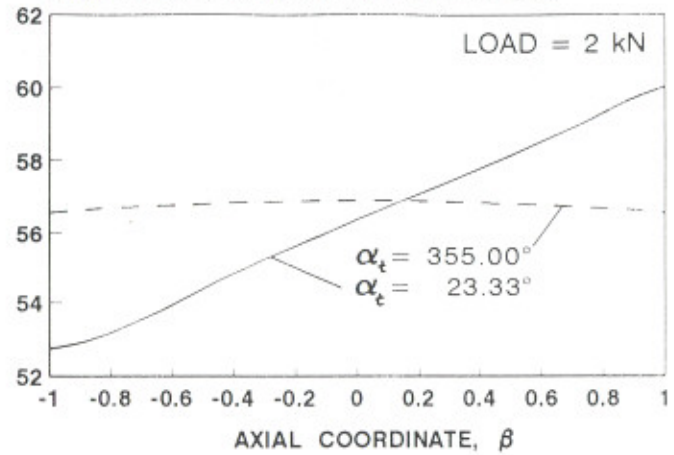
Fig. 5.65 PLAIN JOURNAL BEARING - TEMPERATURE PROFILES

BUSH-FLUID INTERFACE TEMPERATURE, °C



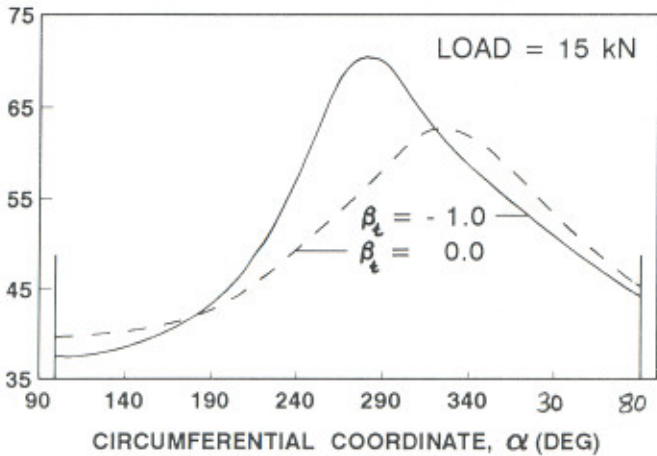
(a) SECTION AT  $A_2 - A_2$

BUSH-FLUID INTERFACE TEMPERATURE, °C



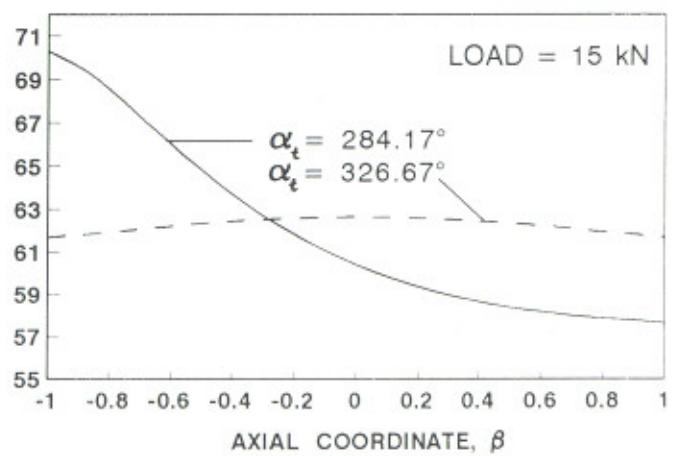
(b) SECTION AT  $B_2 - B_2$

BUSH-FLUID INTERFACE TEMPERATURE, °C



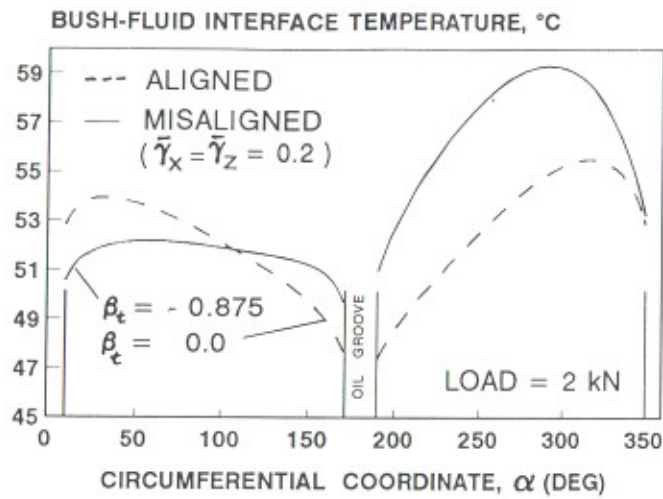
(c) SECTION AT  $A_2 - A_2$

BUSH-FLUID INTERFACE TEMPERATURE, °C

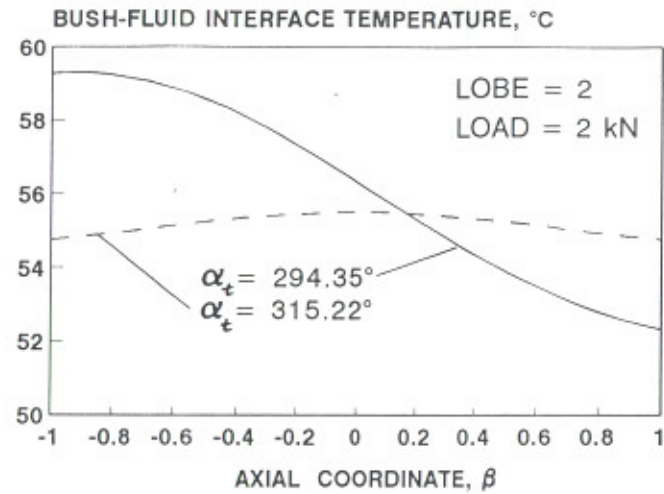


(d) SECTION AT  $B_2 - B_2$

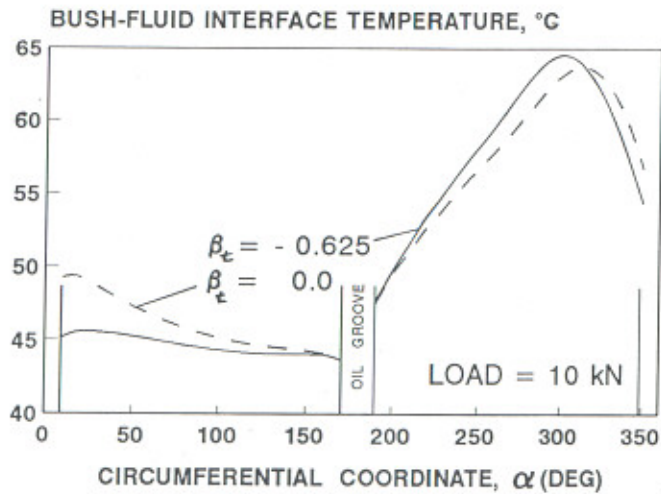
Fig. 5.66 PLAIN JOURNAL BEARING - TEMPERATURE PROFILES



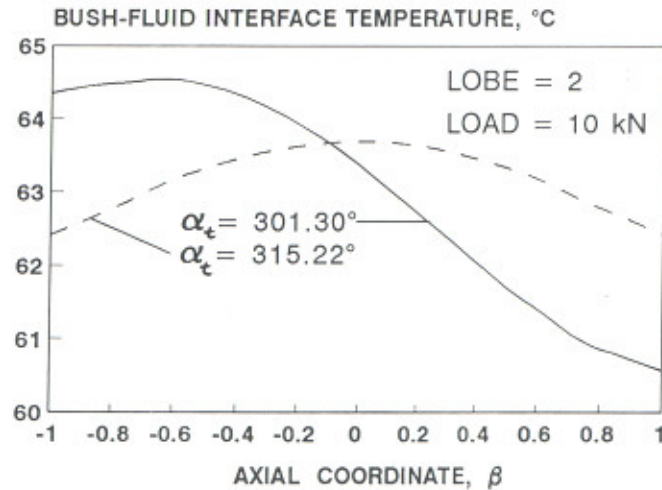
(a) SECTION AT  $A_2 - A_2$



(b) SECTION AT  $B_2 - B_2$



(c) SECTION AT  $A_2 - A_2$



(d) SECTION AT  $B_2 - B_2$

Fig. 5.67 TWO-AXIAL-GROOVE JOURNAL BEARING - TEMPERATURE PROFILES

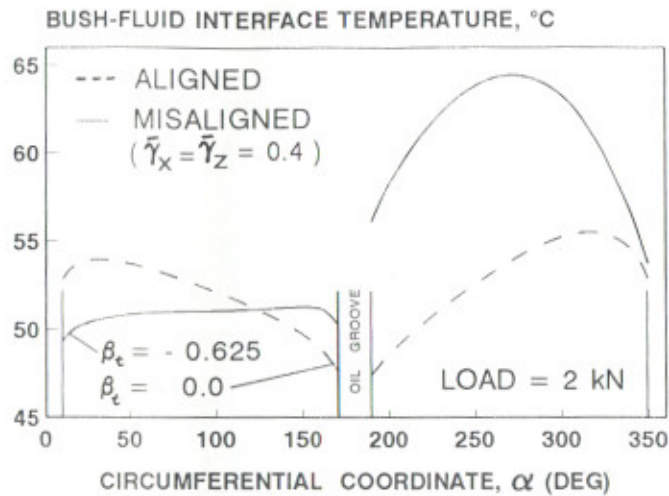
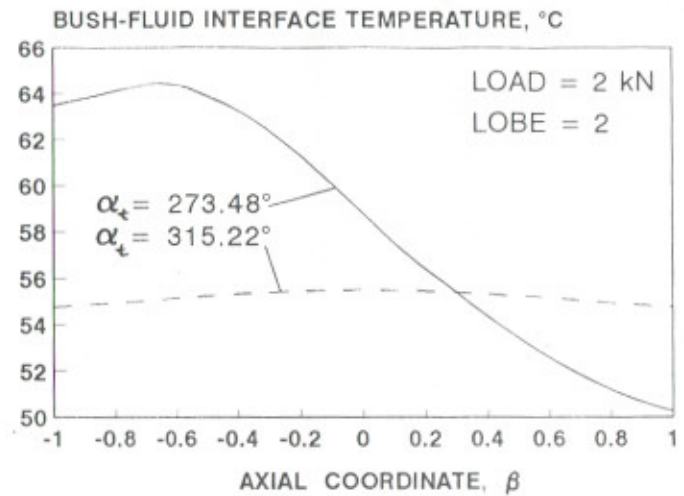
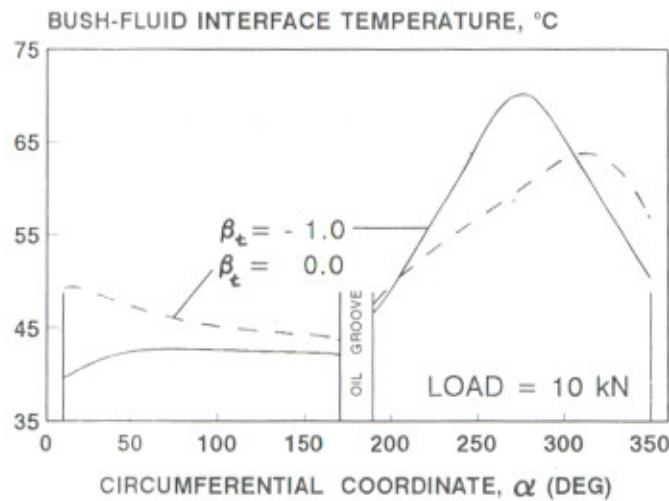
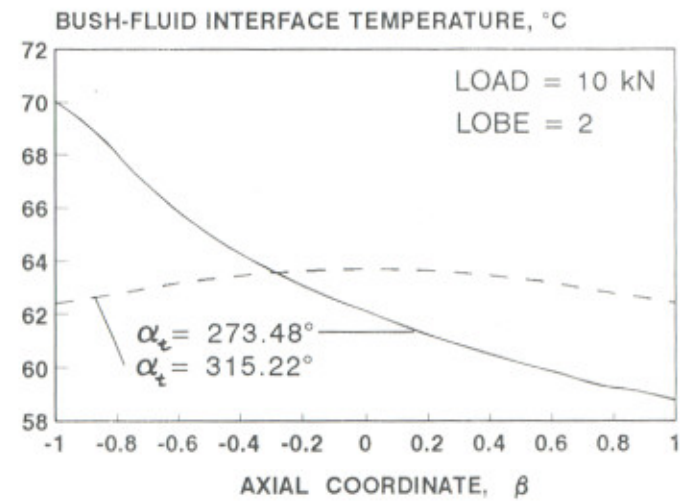
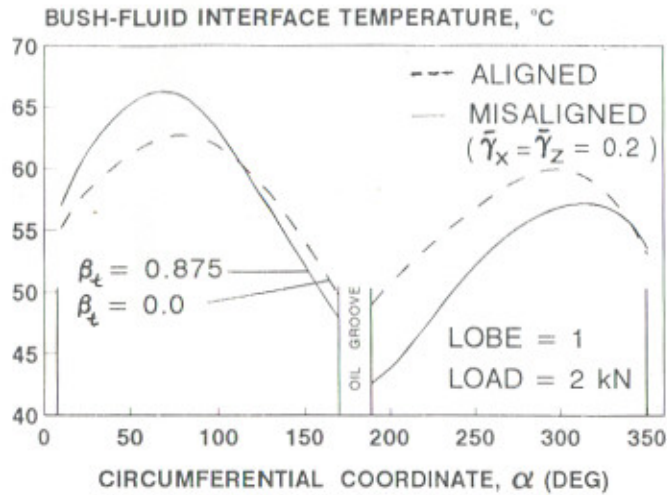
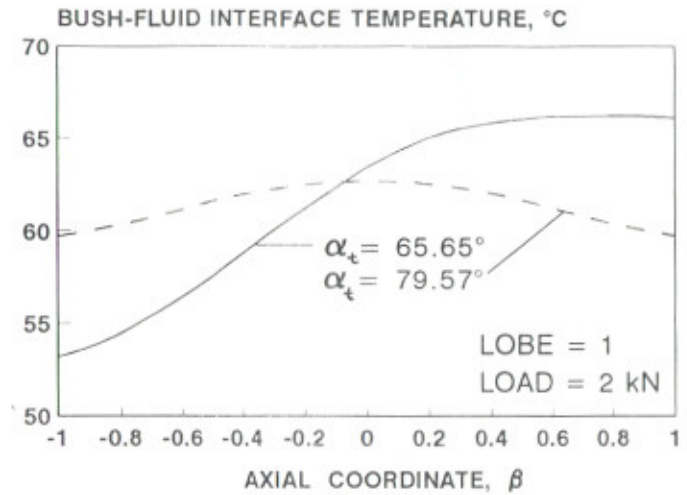
(a) SECTION AT  $A_2 - A_2$ (b) SECTION AT  $B_2 - B_2$ (c) SECTION AT  $A_2 - A_2$ (d) SECTION AT  $B_2 - B_2$ 

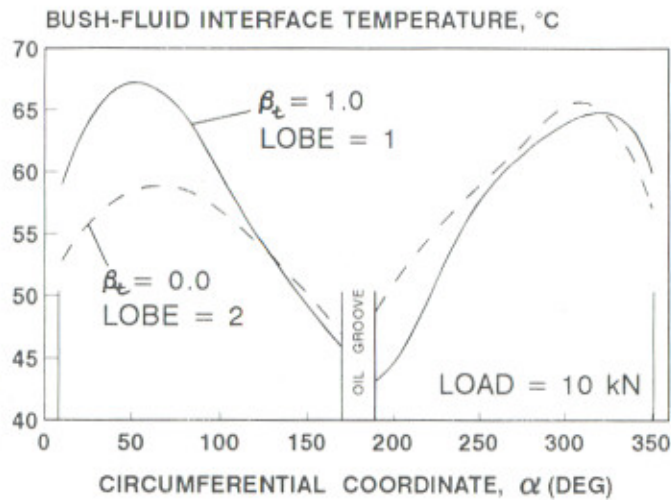
Fig. 5.68 TWO-AXIAL-GROOVE JOURNAL BEARING - TEMPERATURE PROFILES



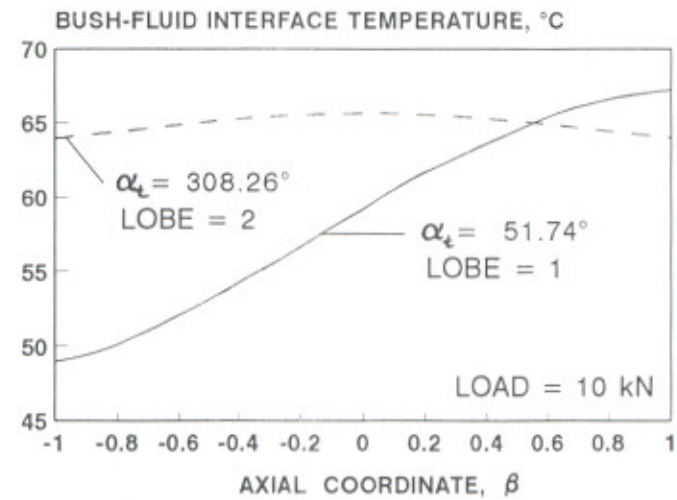
(a) SECTION AT  $A_2 - A_2$



(b) SECTION AT  $B_2 - B_2$



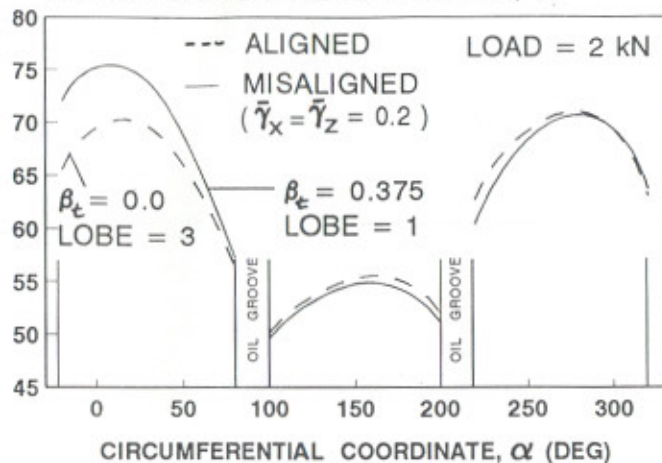
(c) SECTION AT  $A_2 - A_2$



(d) SECTION AT  $B_2 - B_2$

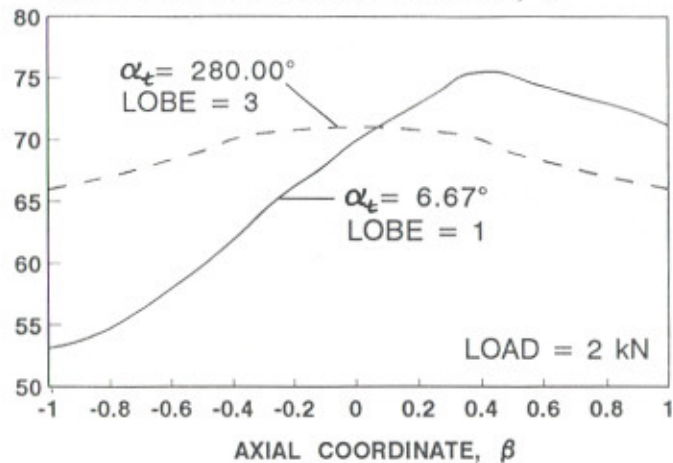
Fig. 5.69 ELLIPTICAL JOURNAL BEARING - TEMPERATURE PROFILES

BUSH-FLUID INTERFACE TEMPERATURE, °C



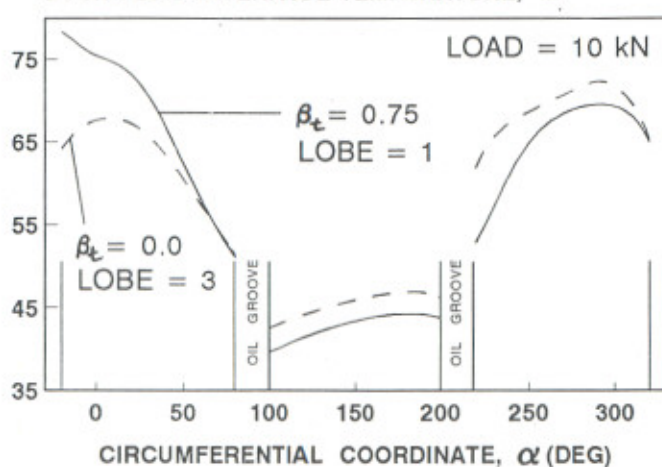
(a) SECTION AT  $A_2 - A_2$

BUSH-FLUID INTERFACE TEMPERATURE, °C



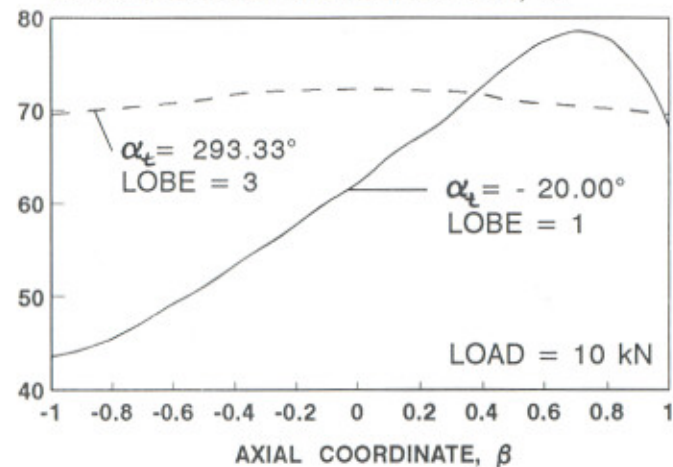
(b) SECTION AT  $B_2 - B_2$

BUSH-FLUID INTERFACE TEMPERATURE, °C



(c) SECTION AT  $A_2 - A_2$

BUSH-FLUID INTERFACE TEMPERATURE, °C



(d) SECTION AT  $B_2 - B_2$

Fig. 5.70 THREE-LOBE JOURNAL BEARING - TEMPERATURE PROFILES

Table 5.11 Summary of Temperature Profiles

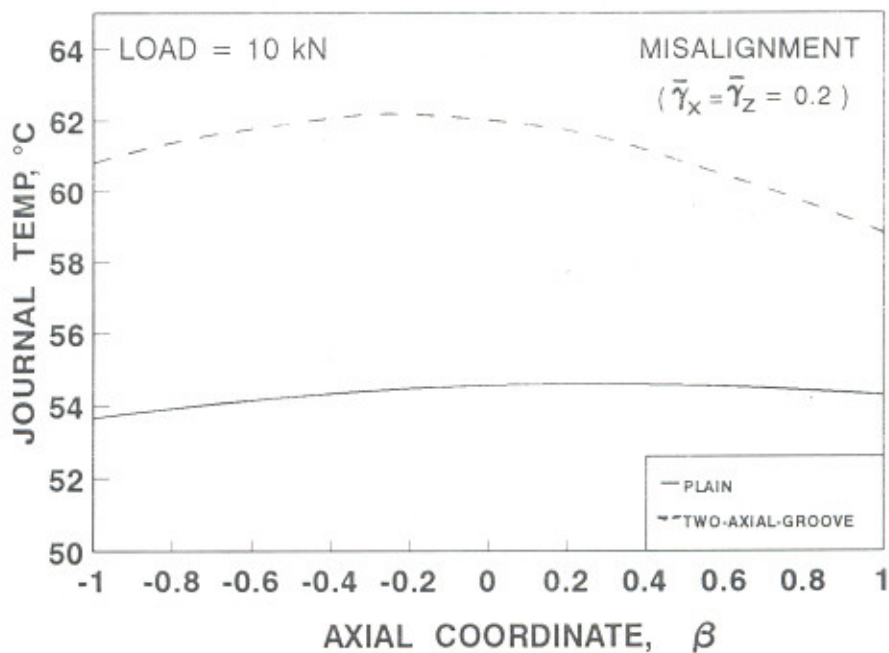
Type of Journal Bearing	Load, kN	Misalignment, $\bar{\gamma}_x = \bar{\gamma}_z$	Maximum Temperature, °C	Axial Temperature Difference, °C (Maximum)
Plain	2.0	0.0	56.9	0.4
		0.2	57.7	2.7
		0.4	60.0	7.3
	10	0.0	57.6	0.8
		0.2	58.8	3.2
	15	0.0	62.8	1.0
0.4		70.2	12.7	
Two-Axial-Groove	2.0	0.0	55.5	0.7
		0.2	59.3	7.0
		0.4	64.4	14.2
	10.0	0.0	63.7	1.3
		0.2	64.5	4.0
		0.4	69.6	11.3
Elliptical	2.0	0.0	62.7	3.0
		0.2	66.2	13.0
	10.0	0.0	65.6	1.7
		0.2	67.2	18.2
Three-lobe	2.0	0.0	71.0	5.0
		0.2	75.4	22.3
	10.0	0.0	72.3	2.7
		0.2	78.3	34.7

Figs. 5.71(a) and 5.71(b) show that the axial variation of journal temperature for a load of 10 kN and misalignment of  $\bar{\gamma}_x = \bar{\gamma}_z = 0.2$  for the four bearing geometries studied. The figures clearly show that due to misalignment the journal temperature varies significantly along the axial length of the journal and the temperatures at the two ends of the journal are different. The difference between the maximum journal temperature and minimum journal temperature is more in the case of non-circular bearings{Fig. 5.71(b)} as compared to the circular bearings, Fig. 5.71(a). Hence, the journal cannot be assumed as a lumped thermal element in the analysis of misaligned bearings.

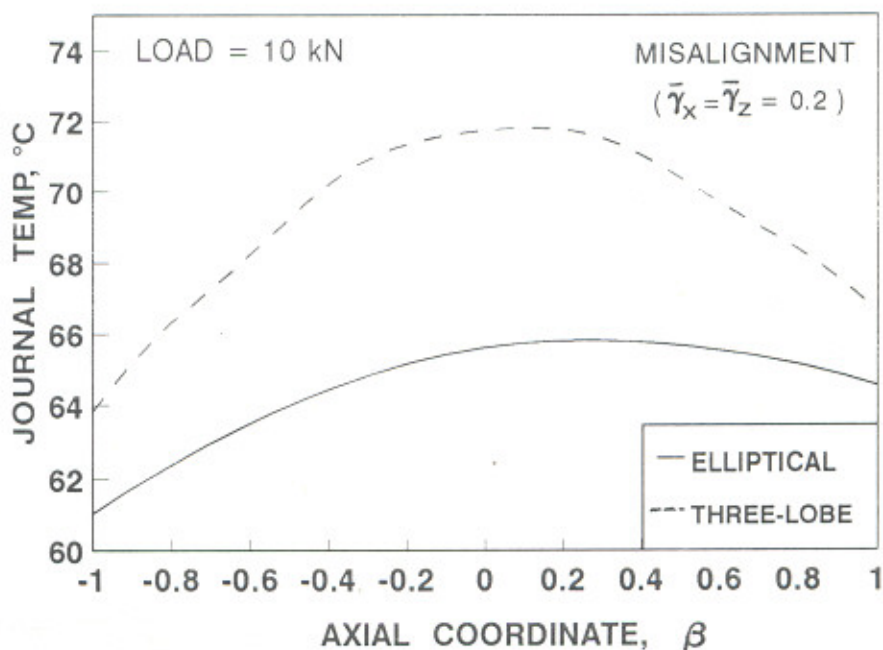
#### 5.4 TRANSIENT MOTION TRAJECTORIES

In the present work, the effect of misalignment on the dynamic response of four journal bearing systems has been studied via the transient motion trajectories of journal-centre obtained using the linearised equations of motion for free translatory whirl.

In order to bring out the effect of misalignment clearly, the transient motion trajectories of journal-centre are drawn by keeping the value of journal mass,  $\bar{M}_J$  same for the aligned and the misaligned journal bearings. For a given initial disturbance, trajectories have been traced until the final position of the journal centre reaches within specified distance from the static equilibrium position. The time taken during this period is recorded. A journal bearing system for which the journal centre after initial disturbance takes lesser time to reach desired position is relatively more stable than the one which takes more time. The trajectories so obtained are given in Figs. 5.72 to 5.79 for the four bearings. Table 5.12 gives the time recorded for each trajectory and the effect of misalignment is presented in terms of a parameter “% diff.” defined below the table. It can be clearly seen that all bearings are relatively more stable when operating under misalignment. Also the relative stability is even better when the operating load is higher.



( a )



( b )

Fig. 5.71 JOURNAL TEMPERATURE PROFILES

$$\bar{x}_0 = \bar{z}_0 = 0.0005$$

SCALE : 1 Div. = 0.00025 Units

$$\bar{\dot{x}}_0 = \bar{\dot{z}}_0 = 0.0$$

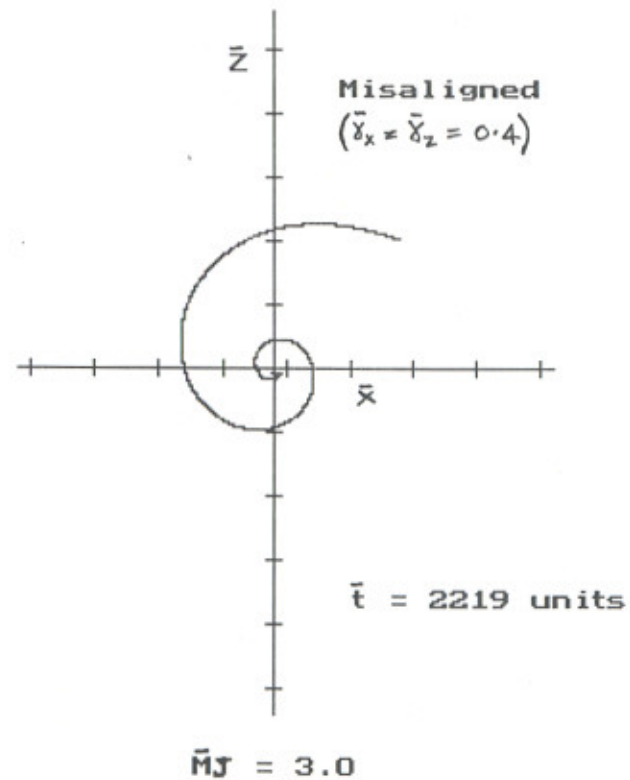
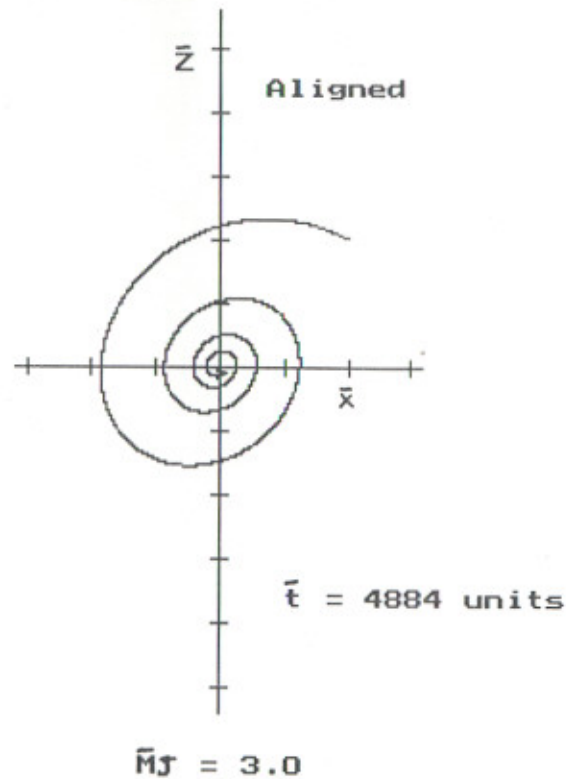


Fig. 5.72 : Linear Trajectories for Plain Journal Bearing - (Load = 2 kN)

$$\bar{x}_0 = \bar{z}_0 = 0.0005$$

SCALE : 1 Div. = 0.00025 Units

$$\bar{\dot{x}}_0 = \bar{\dot{z}}_0 = 0.0$$

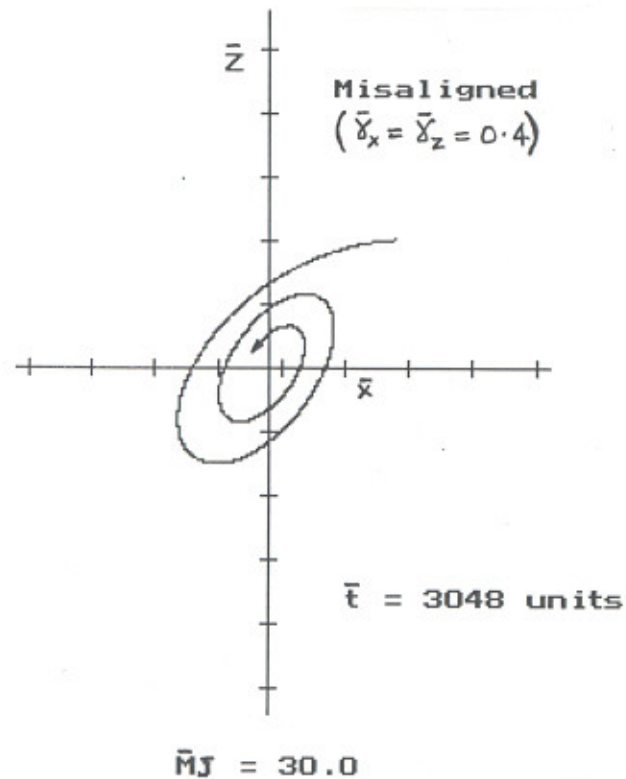
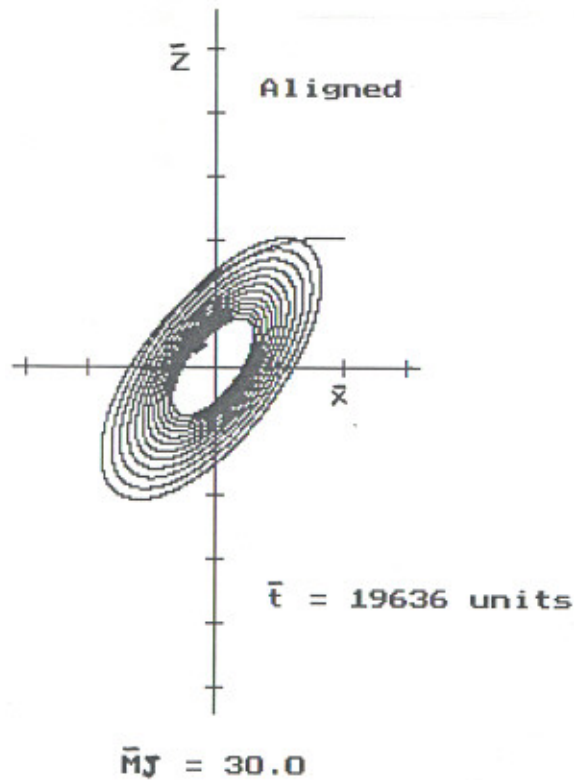


Fig. 5.73 : Linear Trajectories for Plain Journal Bearing - (Load = 10 kN)

$$\bar{x}_o = \bar{z}_o = 0.0005$$

SCALE : 1 Div. = 0.00025 Units

$$\bar{\dot{x}}_o = \bar{\dot{z}}_o = 0.0$$

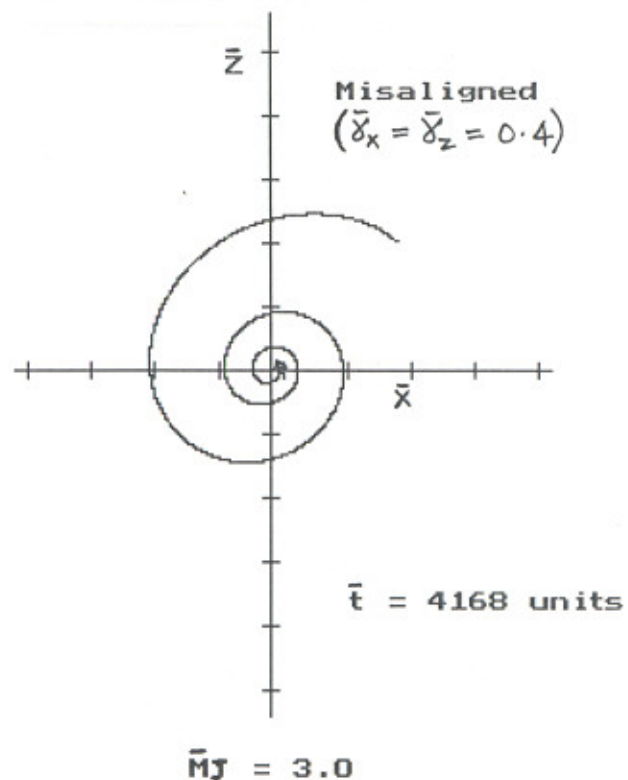
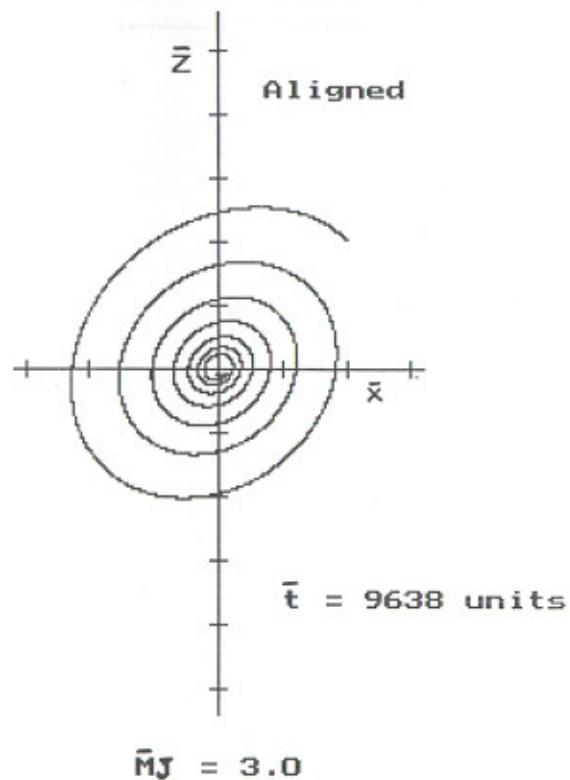


Fig. 5.74 : Linear Trajectories for Two-Axial-Groove Journal Bearing - (Load = 2 kN)

$$\bar{x}_0 = \bar{z}_0 = 0.0005$$

SCALE : 1 Div. = 0.00025 Units

$$\bar{x}'_0 = \bar{z}'_0 = 0.0$$

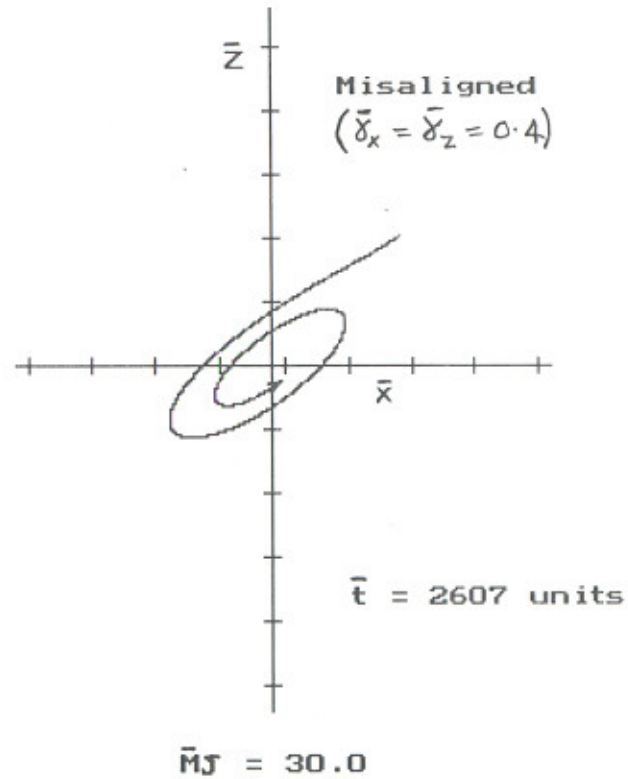
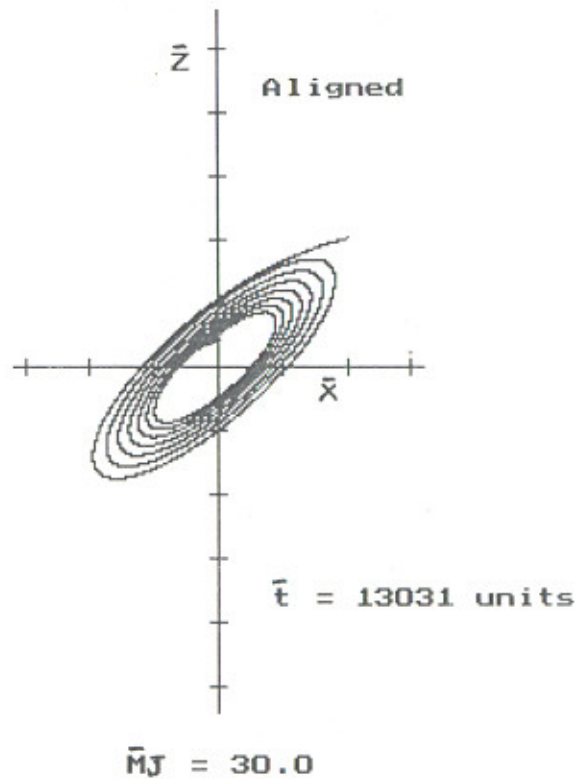


Fig. 5.75 : Linear Trajectories for Two-Axial-Groove Journal Bearing - (Load = 10 kN)

$$\bar{x}_o = \bar{z}_o = 0.0005$$

SCALE : 1 Div. = 0.00025 Units

$$\bar{\dot{x}}_o = \bar{\dot{z}}_o = 0.0$$

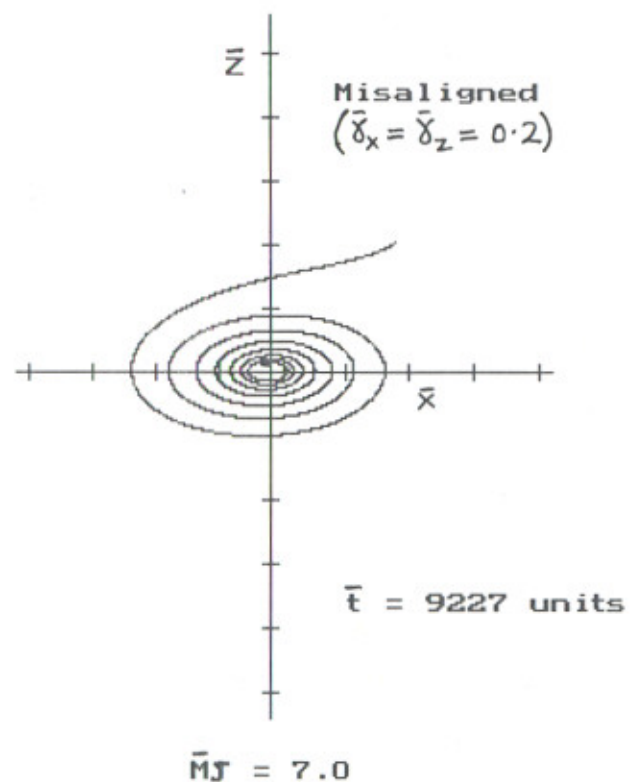
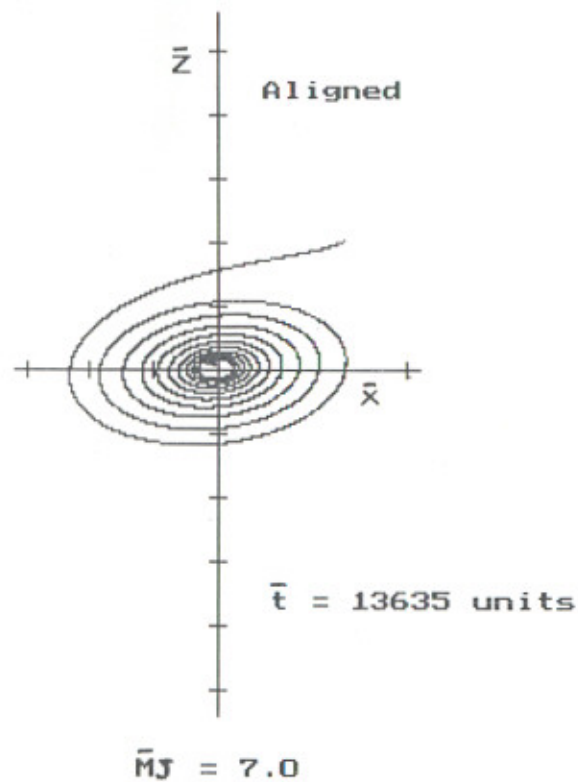


Fig. 5.76 : Linear Trajectories for Elliptical Journal Bearing - (Load = 2 kN)

$$\bar{x}_0 = \bar{z}_0 = 0.0005$$

SCALE : 1 Div. = 0.00025 Units

$$\bar{\dot{x}}_0 = \bar{\dot{z}}_0 = 0.0$$

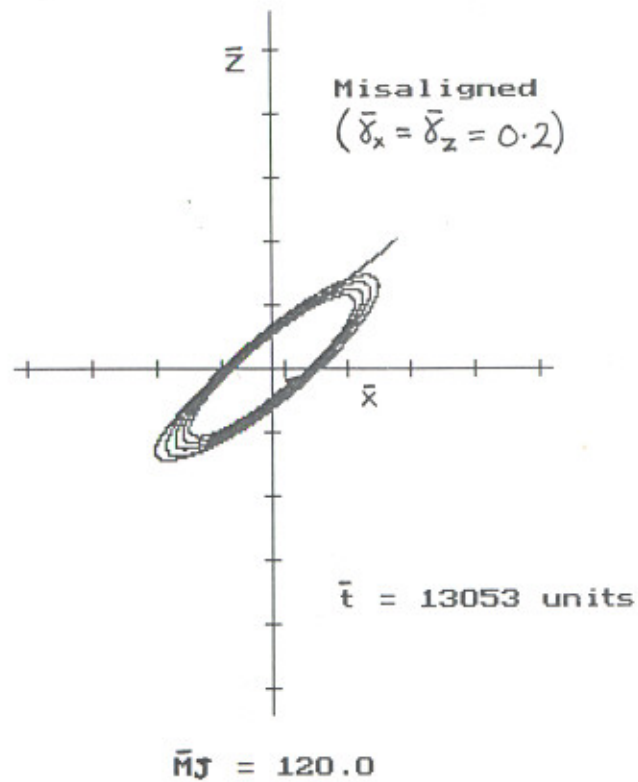
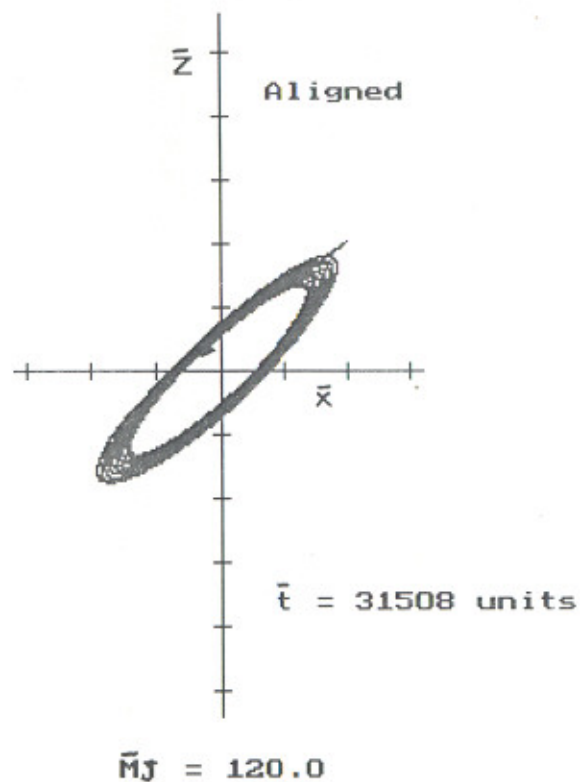
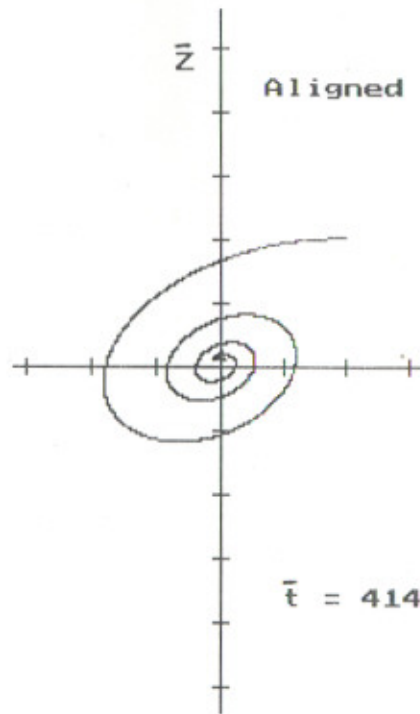


Fig. 5.77 : Linear Trajectories for Elliptical Journal Bearing - (Load = 10 kN)

$$\bar{x}_o = \bar{z}_o = 0.0005$$

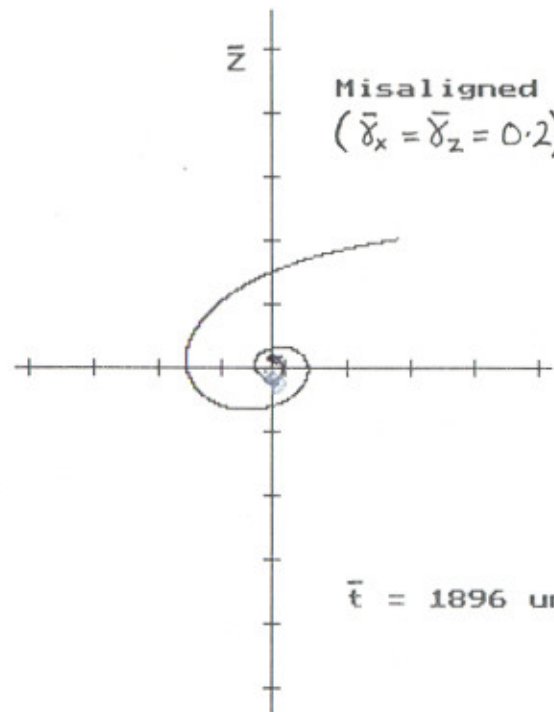
$$\bar{\dot{x}}_o = \bar{\dot{z}}_o = 0.0$$

SCALE : 1 Div. = 0.00025 Units



$$\bar{t} = 4148 \text{ units}$$

$$\bar{M}_J = 7.0$$



$$\bar{t} = 1896 \text{ units}$$

$$\bar{M}_J = 7.0$$

Fig. 5.78 : Linear Trajectories for Three-Lobe Journal Bearing - (Load = 2 kN)

$$\bar{X}_0 = \bar{Z}_0 = 0.0005$$

SCALE : 1 Div. = 0.00025 Units

$$\bar{X}'_0 = \bar{Z}'_0 = 0.0$$

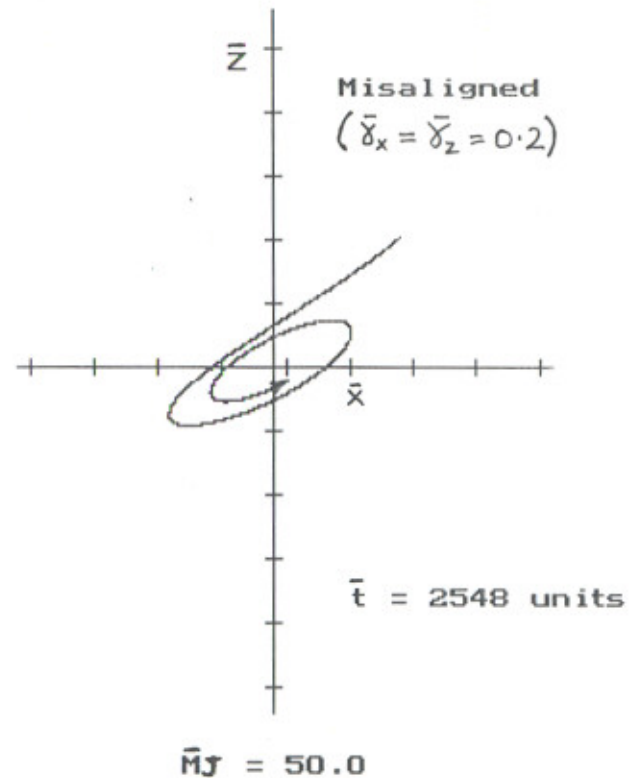
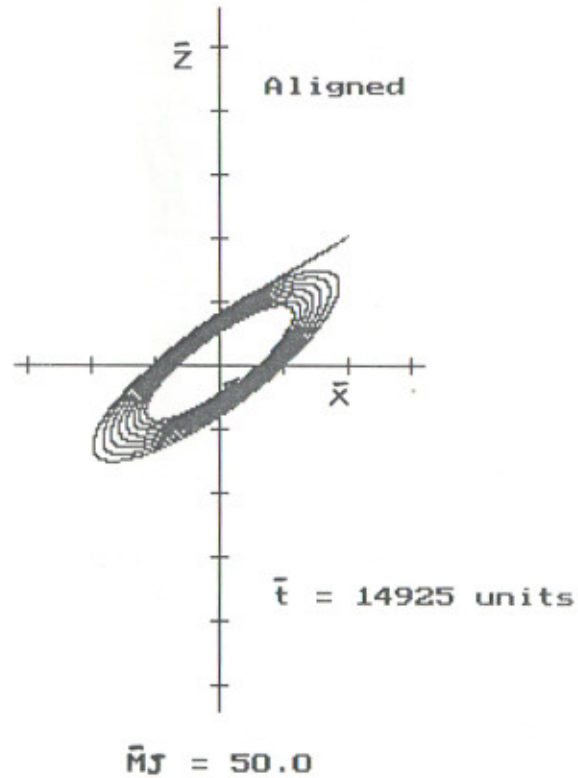


Fig. 5.79 : Linear Trajectories for Three-Lobe Journal Bearing - (Load = 10 kN)

Table 5.12 Comparison of Time Units Required for Reaching a Stable Position

$$\bar{X} = \bar{Z} = 0.005$$

$$\bar{X} = \bar{Z} = 0.0$$

$$d \bar{t} = 0.01$$

Type of Journal Bearing	Load, kN	Critical Mass, $\bar{M}_C$		Journal Mass, $\bar{M}_J$	Time (units)		% dif.*
		Aligned	Misaligned		Aligned	Misaligned	
Plain	2.0	4.91	7.73	3.0	4884	2219	54.57
	10.0	34.12	65.40	30.0	19636	3048	84.48
Two-Axial-Groove	2.0	4.03	5.53	3.0	9638	4168	56.75
	10.0	39.50	136.52	30.0	13031	2607	80.00
Elliptical	2.0	8.72	9.74	7.0	13635	9227	32.33
	10.0	148.08	180.17	120.0	31508	13053	58.57
Three-Lobe	2.0	12.00	19.46	7.0	4148	1896	54.29
	8.0	59.67	142.40	50.0	14925	2548	82.93

$$* \% \text{ dif.} = \frac{\text{time units for aligned bearing} - \text{time units for misaligned bearing}}{\text{time units for aligned bearing}} * 100$$

## 5.5 COMPARATIVE STUDY OF SOME VITAL PARAMETERS OF MISALIGNED JOURNAL BEARINGS

The failure of journal bearings generally occurs due to one or more of the following parameters reaching some critical values :

- (i) Minimum fluid-film thickness ( $h_{\min}$ )
- (ii) Stability parameters ( $\bar{\Omega}$  and  $\bar{v}$ )
- (iii) Bush and Journal Temperatures ( $T_b$  and  $T_j$ )

### 5.5.1 Minimum Fluid-Film Thickness ( $h_{\min}$ )

Table 5.13 shows the comparative values of minimum fluid-film thickness ( $h_{\min}$ ) for two values of load for the four types of journal bearings. It is observed that due to misalignment of  $\bar{\gamma}_x = \bar{\gamma}_z = 0.2$ , the minimum fluid-film thickness changes drastically. The % change in  $h_{\min}$ , at light load is between 20 % to 30 % for plain journal bearing and two-axial groove journal bearing. At higher load values (20 kN), the % change in  $h_{\min}$  is between 40 % and 60 % in case of plain journal bearing and two-axial groove journal bearing. For elliptical and three-lobe journal bearing, the % change in  $h_{\min}$  is more than 50 % at the light load. At a heavy load, however, the % change in  $h_{\min}$  decreases slightly in elliptical journal bearing but further increases in the three-lobe journal bearing.

### 5.5.2 Stability Parameters ( $\bar{\Omega}$ and $\bar{v}$ )

Table 5.14(a) shows the % change in the values of threshold speed ( $\bar{\Omega}$ ) due to misalignment ( $\bar{\gamma}_x = \bar{\gamma}_z = 0.2$ ) at various load capacities. At a light load, the increase in values of threshold speed ( $\bar{\Omega}$ ) is about 5 % each for plain, two-axial-groove and elliptical journal bearings where as it is 27 % in the case of the three-lobe journal bearing. At higher load values, the % change in  $\bar{\Omega}$  is on higher side as compared to the values at light load. The magnitude of change being more in case of three-lobe journal bearing.

The % change in the values of whirl frequency ratio, ( $\bar{v}$ ) due to misalignment ( $\bar{\gamma}_x = \bar{\gamma}_z = 0.2$ ) at various load values are shown in table 5.14 (b). It is observed from this table that the change in whirl frequency ratio due to misalignment is only marginal. It is seen that the whirl frequency ratio is reduced by the misalignment.

Table 5.13 Comparative Study of Minimum Film Thickness in Journal Bearings

Type of Journal Bearing	Load, kN	Minimum film thickness, in mm		% Change
		Aligned	Misaligned	
Plain	2	0.10690	0.07751	27.49
	10	0.05256	0.04005	23.80
	20	0.02729	0.01636	40.05
Two-Axial-Groove	2	0.10137	0.08068	20.41
	10	0.04276	0.03081	27.94
	20	0.02414	0.00971	59.77
Elliptical (Preload Factor = 0.5)	2	0.06794	0.03136	53.84
	10	0.04055	0.02335	42.42
Three-Lobe (Preload Factor = 0.5)	2	0.05730	0.02336	59.23
	10	0.02947	0.00868	70.54

Table 5.14 Comparative Study of Stability Parameters in Journal Bearings

(a) Threshold Speed,  $\bar{\Omega}$ 

Type of Journal Bearing	Load, kN	Threshold Speed, $\bar{\Omega}$		% Change
		Aligned	Misaligned	
Plain	2	2.6178	2.7811	+ 6.23
	10	3.1110	3.4160	+ 9.80
	20	*	*	-
Two-Axial-Groove	2	2.3855	2.5006	+ 4.82
	10	3.2577	4.9228	+ 51.11
	20	*	*	-
Elliptical (Preload Factor = 0.5)	2	3.4818	3.6797	+ 5.68
	10	6.4166	7.0779	+ 10.31
Three-Lobe (Preload Factor = 0.5)	2	4.0834	5.2019	+ 27.39
	10	5.8491	22.6078	+ 286.52

(b) Whirl Frequency Ratio,  $\bar{\nu}$ 

Type of Journal Bearing	Load, kN	Whirl Frequency Ratio, $\bar{\nu}$		% Change
		Aligned	Misaligned	
Plain	2	0.4836	0.4796	- 0.83
	10	0.4029	0.3855	- 4.32
	20	**	**	-
Two-Axial-Groove	2	0.4581	0.4506	- 1.64
	10	0.3464	0.3250	- 6.18
	20	**	**	-
Elliptical (Preload Factor = 0.5)	2	0.4782	0.4781	- 0.02
	10	0.2820	0.2671	- 5.32
Three-Lobe (Preload Factor = 0.5)	2	0.4553	0.4467	- 1.89
	10	0.2419	0.0921	- 61.92

\* Negative critical mass - the bearing is stable under all conditions.

\*\* Negative value for  $\bar{\nu}^2$  - absence of whirl under all conditions.

### 5.5.3 Bush and Journal Temperatures ( $T_b$ and $T_j$ )

Table 5.15(a) shows the maximum temperature occurring in the bush and the axial temperature difference across the bush ends for both the aligned and misaligned (  $\bar{\gamma}_x = \bar{\gamma}_z = 0.2$  ) bearings at different load values.

With the increase in load value, the maximum bush temperature increases both in the aligned and misaligned bearing. Due to the misalignment, there is a significant rise in maximum bush temperature in all the journal bearings studied. Higher load values further increases the bush temperature.

The axial temperature difference across the bush ends in misaligned bearings is more as compared to the aligned bearings. This difference is quite significant and it increases with the increase in load. Hence, the axial heat flow in bush cannot be neglected in misaligned journal bearings.

Table 5.15(b) shows the temperature change occurring in the values of mean journal temperature due to misalignment at different load values. It is observed that misalignment has little effect on mean journal temperature.

## 5.6 EFFECT OF OPERATING PARAMETERS ON STATIC AND DYNAMIC CHARACTERISTICS<sup>2</sup>

The most significant of all operating parameters affecting the bearing performance are found to be radial clearance, oil viscosity and rotating speed of the journal. This section presents the effect of these parameters on the performance characteristics of a plain journal bearing.

### 5.6.1 Static Performance Characteristics

The static performance characteristics have been obtained for a constant vertical load of 10 kN on the plain journal bearing. The operating conditions used are given in table 5.16.

**5.6.1.1 Effect of Radial Clearance :** The static performance characteristics of plain journal bearing for four different values of radial clearance ratio (0.001 to 0.004) and two lubricating oils (Oil 1 and Oil 2) are shown in Figs. 5.80(a) to 5.80(e). It is observed from Fig. 5.80(a) that as the radial clearance increases the eccentricity ratio also increases. This increase is quite high in lesser viscous lubricant (Oil 2).

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<sup>2</sup> : Paper given at ref.[103] was published from this part of the work.

Table 5.15 Comparative Study of Temperature Variation in Different Bearings

(a) Bush

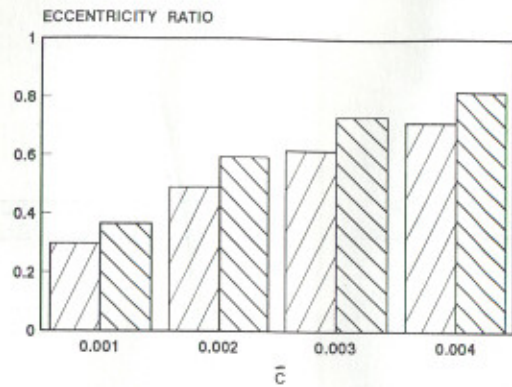
Type of Journal Bearing	Load, kN	Max. Temp. in Bush, °C		Temp. Variation, °C	Axial Temp. Diff. Across Bush Ends, °C		Temp. Variation, °C
		Aligned	Misaligned		Aligned	Misaligned	
Plain	2	56.90	57.66	0.76	0.35	3.43	3.08
	10	57.59	58.78	1.19	0.78	4.37	3.59
	20	67.42	68.65	1.23	1.39	6.52	5.13
Two-Axial-Groove	2	55.50	59.30	3.8	0.74	9.34	8.6
	10	63.69	64.54	0.85	1.27	9.50	8.23
	20	69.86	72.60	2.74	1.68	11.19	9.51
Elliptical (Preload Factor = 0.5)	2	62.69	66.23	3.54	4.78	13.44	8.66
	10	65.64	67.20	1.56	4.08	18.76	14.68
Three-Lobe (Preload Factor = 0.5)	2	71.01	75.43	4.42	8.46	18.02	9.56
	10	72.28	78.33	6.05	7.76	25.61	17.85

(b) Journal

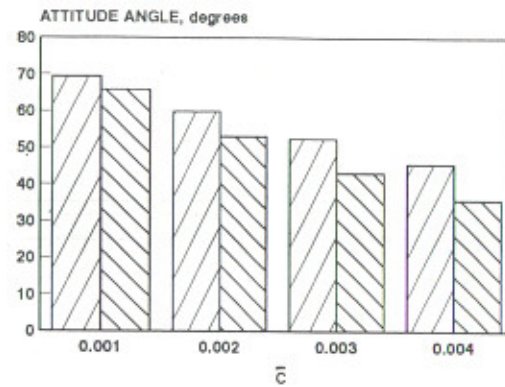
Type of Journal Bearing	Load, kN	Mean Journal Temp., °C		Temp. Variation, °C
		Aligned	Misaligned	
Plain	2	59.63	59.35	0.28
	10	54.06	54.56	0.50
	20	60.79	61.41	0.62
Two-Axial-Groove	2	61.97	63.18	1.21
	10	61.89	62.02	0.13
	20	64.13	65.20	1.07
Elliptical (Preload Factor = 0.5)	2	64.05	65.89	1.84
	10	64.17	65.62	1.45
Three-Lobe (Preload Factor = 0.5)	2	73.89	75.26	1.37
	10	70.51	71.75	1.24

**Table 5.16    Operating Conditions**

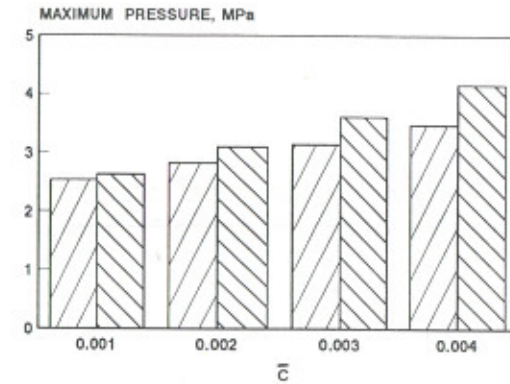
Journal Radius, R	= 0.05	m
External Bearing Radius, R <sub>2</sub>	= 0.1	m
Bearing Length, L	= 0.08	m
Radial Clearance, CBAR	= 0.003	(for different N)
Speed of the Journal, N	= 3000	R.P.M. (for different CBAR)
Properties of Lubricants		
<u>Oil 1 (SAE 30) :</u>		
Kinematic Viscosity at 40 °C	= 0.08137	Pa.s
Kinematic Viscosity at 100 °C	= 0.00979	Pa.s
Density, ρ	= 840	Kg / m <sup>3</sup>
Specific Heat, C <sub>p</sub>	= 2083	J / Kg. °C
Thermal Conductivity, k <sub>f</sub>	= 0.14	W / m °C
<u>Oil 2 (ISO VG 32) :</u>		
Kinematic Viscosity at 40 °C	= 0.0340	Pa.s
Kinematic Viscosity at 100 °C	= 0.0048	Pa.s
Density, ρ	= 846	Kg / m <sup>3</sup>
Specific Heat, C <sub>p</sub>	= 2000	J / Kg. °C
Thermal Conductivity, k <sub>f</sub>	= 0.13	W / m °C
Bush-Housing Thermal Conductivity, k <sub>b</sub>	= 250	W / m °C
Journal Thermal Conductivity, k <sub>j</sub>	= 50	W / m °C
Convection Heat Transfer Coefficient, h <sub>ab</sub>	= 80	W / m °C
Oil Groove Angle	= 20 <sup>0</sup>	
Inlet Lubricant Temperature, T <sub>s</sub>	= 40	°C
Ambient Temperature, T <sub>a</sub>	= 40	°C
Reference Temperature, T <sub>r</sub>	= 40	°C



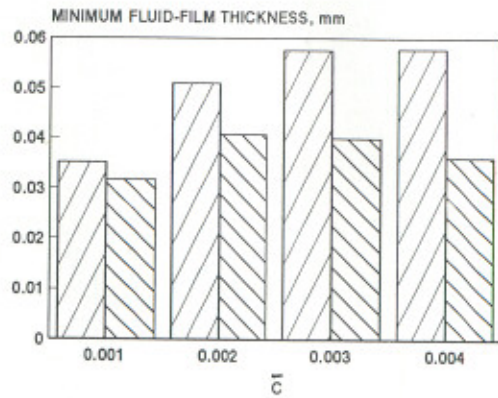
(a)



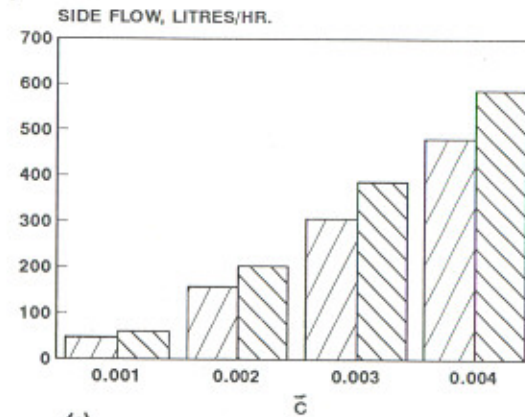
(b)



(c)



(d)



(e)



Fig. 5.80 PLAIN JOURNAL BEARING - STATIC PERFORMANCE CHARACTERISTICS WITH DIFFERENT  $\bar{c}$  VALUES

Fig. 5.80(b) shows that the attitude angle decreases with the increase in radial clearance. For the same value of radial clearance, the higher the viscosity of the lubricant, higher is the attitude angle. This difference increases with the increase in radial clearance.

The maximum pressure rises as the radial clearance of the bearing increases, Fig. 5.80(c). At lower values of radial clearance, the difference between the maximum pressure for the two lubricants is not significant. At higher values of the radial clearance, however, the maximum pressure with lesser viscosity oil is significantly higher than that for the higher viscosity oil (Oil 1).

For oil 1 (more viscous), the value of minimum fluid-film thickness is found to be least when the value of radial clearance ratio ( $\bar{c}$ ) is 0.001 and it is found to increase as  $\bar{c}$  is increased, Fig. 5.80(d). For oil 2 on the other hand, it is least when  $\bar{c} = 0.001$ , becomes maximum with  $\bar{c} = 0.002$  and then decreases as  $\bar{c}$  is further increased. Since it is always desirable to have as high a value of minimum fluid-film thickness as possible, the value for  $\bar{c}$  may be selected between 0.002 and 0.003 for any type of oil.

Fig. 5.80(e) reveals that higher radial clearance leads to more side flow. However, higher is the value of viscosity, smaller is the side flow.

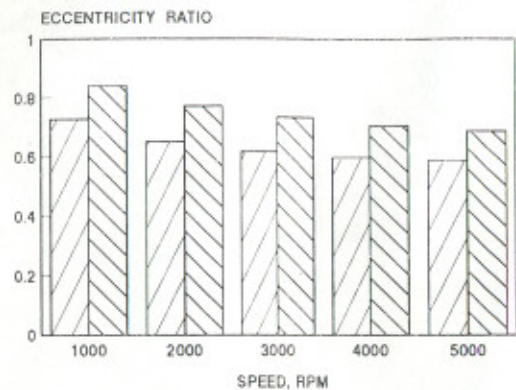
**5.6.1.2 Effect of Journal Speed** The static performance characteristics of plain journal bearing for five values of journal speed (1000 r.p.m. to 5000 r.p.m.) and two lubricating oils are shown in Figs. 5.81(a) to 5.81(e). It is observed from Fig. 5.81(a) that as the journal speed increases the eccentricity ratio decreases. The reduction in eccentricity ratio is more in more viscous lubricant (Oil 1).

Fig. 5.81(b) shows that the attitude angle increases with the increase in journal speed. For the same value of journal speed, the attitude angle is higher when higher viscosity oil is used.

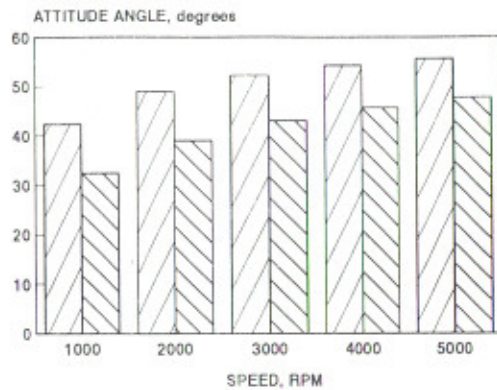
The maximum pressure decreases as the journal speed of the bearing increases, Fig. 5.81(c). Less viscous lubricant gives higher maximum pressure values.

The minimum fluid-film thickness increases as the journal speed increases, Fig. 5.81(d). For the same value of journal speed, the more viscous lubricant yields higher value of minimum fluid-film thickness.

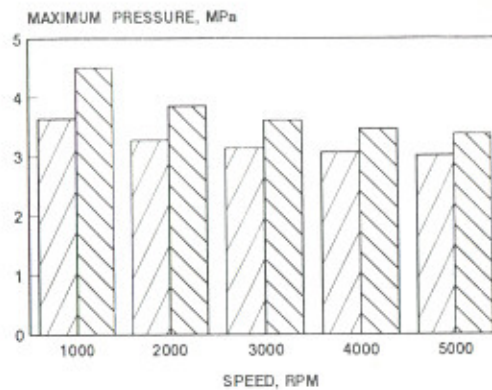
Fig. 5.81(e) reveals that higher journal speed leads to more side flow. The side flow is, however, reduced with increase in viscosity.



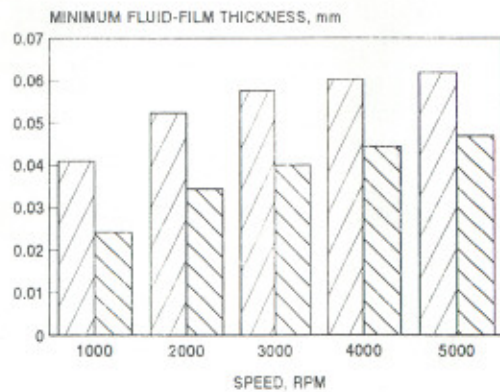
(a)



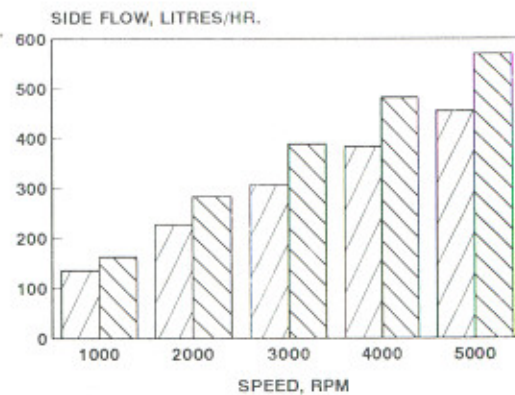
(b)



(c)



(d)



(e)

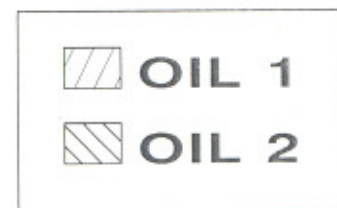


Fig. 5.81 PLAIN JOURNAL BEARING - STATIC PERFORMANCE CHARACTERISTICS WITH DIFFERENT JOURNAL SPEEDS

## 5.6.2 Dynamic Performance Characteristics

**5.6.2.1 Stability Parameters :** The stability parameters (instability threshold speed and whirl frequency ratio) of plain journal bearing with different values of radial clearance ( $\bar{c}$ ) and two lubricating oils are shown in Figs. 5.82(a) - 5.82(b) and with different values of journal speed and two lubricating oils in Figs. 5.83(a) - 5.83(b).

Fig. 5.82(a) shows that stability improves on either side of a particular value of radial clearance for both oils. Also, whereas the instability threshold speed with both lubricating oils is practically the same at  $\bar{c} = 0.001$ , its value corresponding to lesser viscosity oil becomes infinity for  $\bar{c} = 0.004$  while for the other oil it is much lower. Fig. 5.82(b) indicates reduction of whirl with the increase in the radial clearance for both oils.

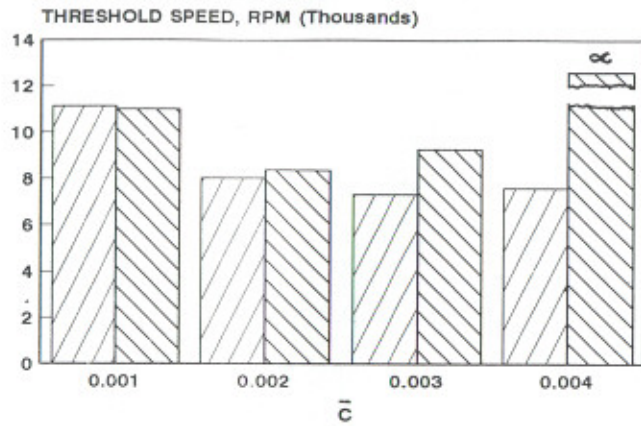
Fig. 5.83(a) shows that the stability of the bearing system deteriorates as the speed is increased. Likewise, tendency to whirl also increases with the increase in speed.

## 5.6.3 Thermal Analysis

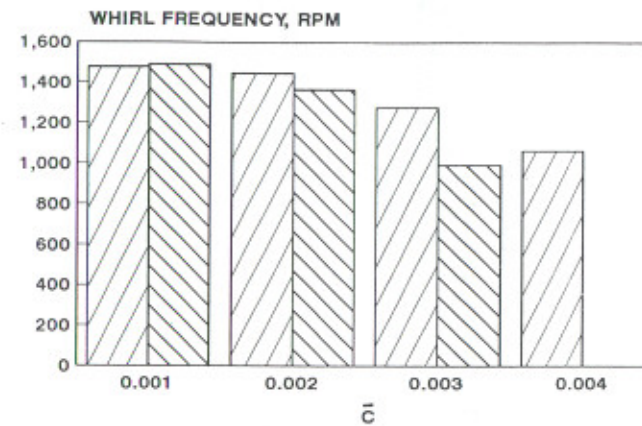
Figs. 5.84(a) - 5.84(b) shows the variation of bush and journal temperatures with radial clearance, respectively. Both these figures reveal that as the radial clearance increases, bush and journal temperatures are reduced. The same trend is exhibited by both oils used. The lower viscosity oil, however, results in marginally lower temperatures.

Figs. 5.85(a) - 5.85(b) shows the variation of bush and journal temperatures with speed, respectively. These figures show that the bush and the journal, both, run hotter as the speed of the journal is increased. Similar trend is obtained for the two oils. The lower viscosity oil, however, results in marginally lower temperatures.

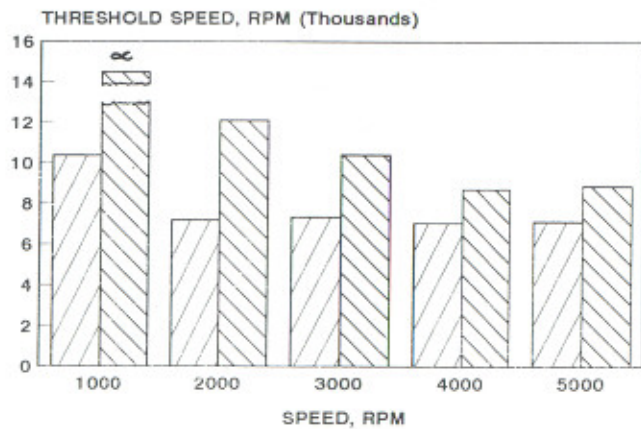
Hence, the radial clearance ratio ( $\bar{c}$ ) between 0.002 and 0.003 may be selected from the point of view of minimum fluid-film thickness. The stability of the journal bearing improves on either side of a particular value of radial clearance and the whirl frequency is reduced with the increase in the radial clearance. In general, low viscosity oil is good from stability and operating temperatures point of view but from the minimum fluid-film thickness criteria, a higher viscosity oil is desirable.



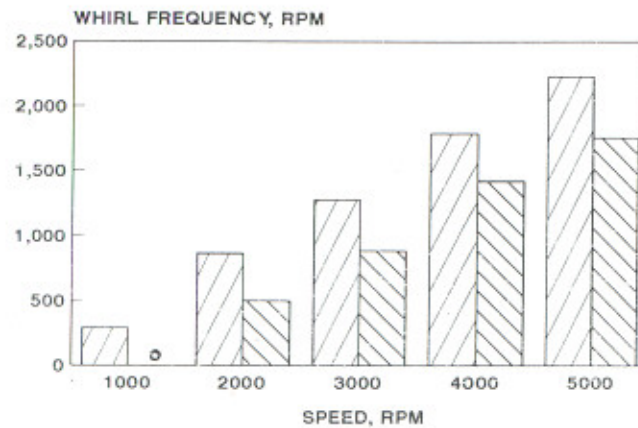
(a)



(b)

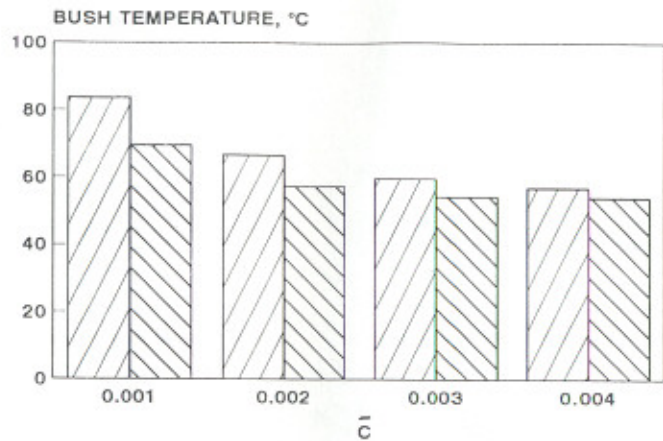
Fig. 5.82 PLAIN JOURNAL BEARING - STABILITY PARAMETERS WITH DIFFERENT  $\bar{c}$  VALUES

(a)

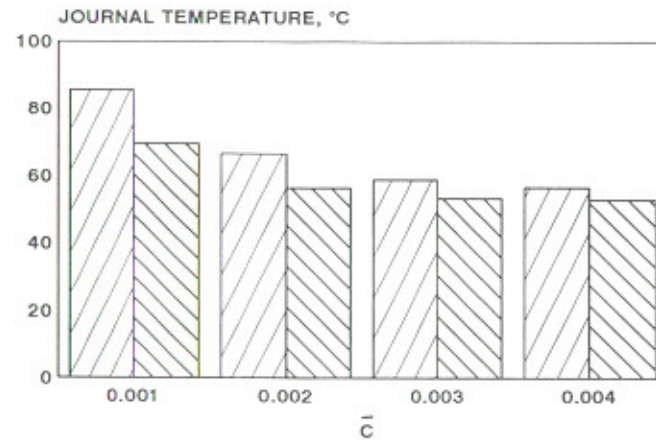


(b)

Fig. 5.83 PLAIN JOURNAL BEARING - STABILITY PARAMETERS WITH DIFFERENT JOURNAL SPEEDS

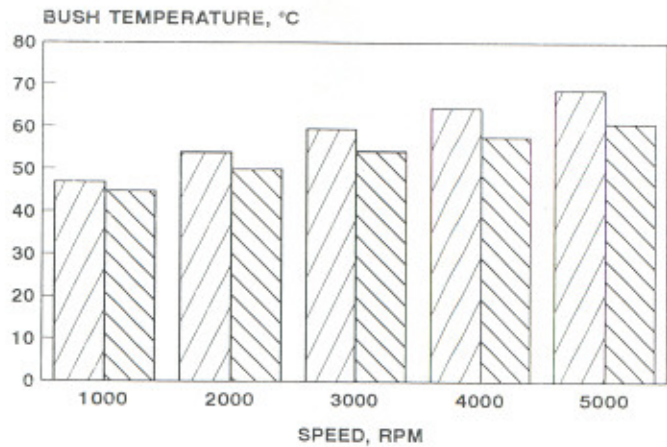


(a)

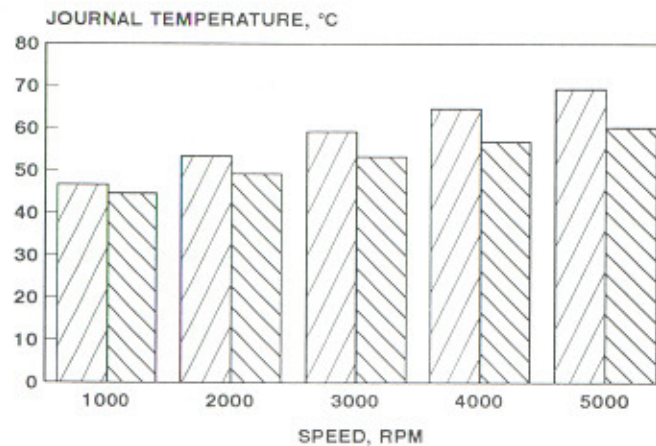


(b)

Fig. 5.84 PLAIN JOURNAL BEARING - VARIATION OF TEMPERATURES WITH DIFFERENT  $\bar{c}$  VALUES



(a)



(b)

Fig. 5.85 PLAIN JOURNAL BEARING - VARIATION OF TEMPERATURES WITH DIFFERENT JOURNAL SPEEDS

## CHAPTER - VI

### CONCLUSIONS AND CLOSURE

#### 6.1 CONCLUSIONS

Based on the results and discussion presented in the preceding chapter, the following conclusions can be drawn for the journal bearing configurations studied :

1. The boundary conditions defined as BC1 in this work are more appropriate.
2. Heat transfer from the oil in supply groove to solids and from solid to oil must be considered for more accurate simulation of the actual conditions.
3. In general, theory predicts less bush temperature than the experimental values especially when the thermohydrodynamic properties of bush / journal material are not exactly known.
4. The mid-plane eccentricity ratio decreases with the increase in misalignment at all load values for all the bearing systems.
5. The attitude angle decreases with the increase in misalignment for all the bearing systems. The effect is more pronounced at higher values of misalignment ratio.
6. In general, for all types of bearings, the maximum fluid-film pressure increases with the increase in misalignment.
7. For all the journal bearings, the minimum fluid-film thickness decreases as the misalignment is increased for all values of load. This may eventually lead to failure of the journal bearing systems due to metal-to-metal contact.
8. At light loads, the effect of increasing misalignment leads to increased side flow in all except elliptical bearings, whereas at higher load values, the trend is reversed. In elliptical journal bearing, increased misalignment leads to increased side flow at all load values.
9. An increase in the misalignment leads to higher mean journal temperature in all the bearing systems.

10. The maximum bush temperature increases with the increase in load for all bearing systems( for aligned as well as misaligned bearings). The maximum bush temperature further rises due to misalignment in all the journal bearings studied.
11. The misalignment significantly affects the magnitude of stiffness and damping coefficients.
12. The misalignment in bearings significantly improves the stability margin defined by the threshold speed.
13. In all bearing geometries operating with misalignment, bush temperature varies considerably along the axial length particularly in non-circular bearings and this variation further increases with the increase in load. Hence, a two-dimensional thermal analysis (ignoring axial variation of temperature) is not sufficient for the study of misaligned bearings. A complete three-dimensional analysis is suggested for this purpose.
14. The isotherms indicate that heat transfer takes place not only in radial direction in the bush but also in the circumferential and axial directions (Therefore, a three-dimensional analysis must be conducted).
15. The journal temperature varies significantly along the axial length of the journal and the temperatures at the two ends of the journal are different in the case of misaligned bearings. Hence, the journal cannot be assumed as a lumped thermal element in the analysis of misaligned bearings.
16. The transient motion trajectories of journal-centre show that all bearings are relatively more stable when operating under misalignment and the relative stability is even better when the operating load is higher.
17. The radial clearance ratio between 0.002 and 0.003 is recommended from the point of view of minimum fluid-film thickness.
18. In general, low viscosity oil is good from stability and operating temperatures point of view but from the minimum fluid-film thickness criteria, a higher viscosity oil is desirable.

## 6.2 CLOSURE

The THD investigations for misaligned journal bearings presented in this thesis indicates that bearing misalignment has an appreciable effect on static and dynamic

performance characteristics of plain, two-axial-groove, elliptical and three-lobe journal bearings.

Thermal / Elastic distortions of bush and journal may be vital in some cases for a more accurate modelling of journal bearing problem. This aspect has not been considered in the present work. However, there is a need for further work in this regard.

Further, only laminar flow conditions have been assumed in the present work. Since thermal effects are expected to be more pronounced with turbulent flow conditions, future investigations should include superlaminar flow conditions also.

The present work deals with the Newtonian lubricants. The nonlinear behaviour of non-Newtonian lubricants combined with the TEHD and misalignment effects would present an even more complex problem which could be a topic for future research.

There are many problems which are not fully understood and explained while accounting for the THD analysis; for example, the back flow of oil, mixing in oil grooves, dynamic response of the journal bearing system during starting and stopping with misalignment and non-linear behaviour of the journal bearing system during disturbance, to name a few. It is, therefore, expected that future investigations will be addressed more comprehensively at these problems.

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\*Published / Presented from the present work.

## APPENDIX - I

$\frac{\bar{B}_{xx}}{\bar{B}_{zx}}, \frac{\bar{B}_{xz}}{\bar{B}_{zz}}$	$(B_{xx}, B_{xz}, B_{zx}, B_{zz}) / 50570.37$
$\frac{\bar{B}_{xyx}}{\bar{B}_{zyx}}, \frac{\bar{B}_{xyz}}{\bar{B}_{zyz}}$	$(B_{xyx}, B_{xyz}, B_{zyx}, B_{zyz}) / 2528.5185$
$\frac{\bar{B}_{\gamma_x \gamma_x}, \bar{B}_{\gamma_x \gamma_z}}{\bar{B}_{\gamma_z \gamma_x}, \bar{B}_{\gamma_z \gamma_z}}$	$(B_{\gamma_x \gamma_x}, B_{\gamma_x \gamma_z}, B_{\gamma_z \gamma_x}, B_{\gamma_z \gamma_z}) / 126.4259$
$\frac{\bar{B}_{\gamma_x x}, \bar{B}_{\gamma_x z}}{\bar{B}_{\gamma_z x}, \bar{B}_{\gamma_z z}}$	$(B_{\gamma_x x}, B_{\gamma_x z}, B_{\gamma_z x}, B_{\gamma_z z}) / 2528.5185$
$\bar{c}$	$c / 0.05$
$\bar{v}$	$v / 366.5191$
$\bar{h}$	$h / 0.00015$
$\bar{M}_c$	$M_c / 137.9747$
$\bar{M}, \bar{M}_x, \bar{M}_z$	$(M, M_x, M_z) / 139.0126$
$\bar{p}$	$p / 1112100.40$
$\bar{Q}_s$	$Q_s / 3.4361 * 10^{-7}$
$\bar{r}$	$r / 0.05$
$\bar{R}_1, \bar{R}_2$	$R_1, R_2 / 0.05$
$\frac{\bar{S}_{xx}, \bar{S}_{xz}}{\bar{S}_{zx}, \bar{S}_{zz}}$	$(S_{xx}, S_{xz}, S_{zx}, S_{zz}) / 18535006.0$
$\frac{\bar{S}_{xyx}, \bar{S}_{xyz}}{\bar{S}_{zyx}, \bar{S}_{zyz}}$	$(S_{xyx}, S_{xyz}, S_{zyx}, S_{zyz}) / 926750.32$
$\frac{\bar{S}_{\gamma_x \gamma_x}, \bar{S}_{\gamma_x \gamma_z}}{\bar{S}_{\gamma_z \gamma_x}, \bar{S}_{\gamma_z \gamma_z}}$	$(S_{\gamma_x \gamma_x}, S_{\gamma_x \gamma_z}, S_{\gamma_z \gamma_x}, S_{\gamma_z \gamma_z}) / 46337.507$
$\frac{\bar{S}_{\gamma_x x}, \bar{S}_{\gamma_x z}}{\bar{S}_{\gamma_z x}, \bar{S}_{\gamma_z z}}$	$(S_{\gamma_x x}, S_{\gamma_x z}, S_{\gamma_z x}, S_{\gamma_z z}) / 926750.32$

$\bar{t}$	$t * 366.5191$
$\frac{\bar{T}_a, \bar{T}_b, \bar{T}_f, \bar{T}_j, \bar{T}_s}{\bar{T}_j, \bar{T}_s}$	$(T_a, T_b, T_f, T_j, T_s) / 40.00$
$\bar{u}, \bar{v}, \bar{w}$	$u, v, w / 18.3260$
$\bar{W}, \bar{W}_x, \bar{W}_z$	$W, W_x, W_z / 2780.2510$
$\frac{\bar{X}, \bar{Z}, \bar{X}_{jo}, \bar{Z}_{jo}}{\bar{X}_{jo}, \bar{Z}_{jo}}$	$(X, Z, X_{jo}, Z_{jo}) / 0.00015$
$\bar{X}_{Li}, \bar{Z}_{Li}$	$X_{Li}, Z_{Li} / 0.00015$
$X, Z$	$(X, Z) / 0.0550$
$\epsilon$	$e / 0.00015$
$\bar{\mu}$	$\mu / 0.027308$
$\lambda$	$= \pm 1.00$
$\bar{\Omega}$	$\Omega / 255.7342$
$\frac{\bar{\gamma}_x, \bar{\Delta\gamma}_x, \bar{\gamma}_z, \bar{\Delta\gamma}_z}{\bar{\gamma}_z, \bar{\Delta\gamma}_z}$	$(\gamma_x, \Delta\gamma_x, \gamma_z, \Delta\gamma_z) / 0.003$
$\bar{\Delta\dot{\gamma}}_x, \bar{\Delta\dot{\gamma}}_z$	$(\Delta\dot{\gamma}_x, \Delta\dot{\gamma}_z) / 1.0996$

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$\bar{t}$	$t * 366.5191$
$\bar{T}_a, \bar{T}_b, \bar{T}_f,$ $\bar{T}_j, \bar{T}_s$	$(T_a, T_b, T_f, T_j, T_s) / 40.00$
$\bar{u}, \bar{v}, \bar{w}$	$u, v, w / 18.3260$
$\bar{W}, \bar{W}_x, \bar{W}_z$	$W, W_x, W_z / 2780.2510$
$\bar{X}, \bar{Z},$ $\bar{X}_{jo}, \bar{Z}_{jo}$	$(X, Z, X_{jo}, Z_{jo}) / 0.00015$
$\bar{X}_{Li}, \bar{Z}_{Li}$	$X_{Li}, Z_{Li} / 0.00015$
$X, Z$	$(X, Z) / 0.0550$
$\epsilon$	$e / 0.00015$
$\bar{\mu}$	$\mu / 0.027308$
$\lambda$	$= \pm 1.00$
$\bar{\Omega}$	$\Omega / 255.7342$
$\bar{\gamma}_x, \bar{\Delta\gamma}_x,$ $\bar{\gamma}_z, \bar{\Delta\gamma}_z$	$(\gamma_x, \Delta\gamma_x, \gamma_z, \Delta\gamma_z) / 0.003$
$\bar{\Delta\dot{\gamma}}_x, \bar{\Delta\dot{\gamma}}_z$	$(\Delta\dot{\gamma}_x, \Delta\dot{\gamma}_z) / 1.0996$

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