DQM FREE VIBRATION ANALYSIS OF COMPOSITE PLATES WITH ELASTICACALLY RESTARINED EDGES

A Thesis report submitted
in partial fulfillment of the requirements for
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IN

(CAD/CAM & ROBOTICS)

Submitted by:

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2008

at the feet of

My Mother……

CERTIFICATE

This is to certify that the thesis titled, **"DQM FREE VIBRATION ANALYSIS OF COMPOSITE PLATES WITH ELASTICALLY RESTRAINED EDGES"**, being submitted by **Mr. LAKHMI SINGH,** in partial fulfillment of the requirement for the award of degree of **MASTER OF ENGINEERING (CAD/CAM & ROBOTICS)** at **Mechanical Engineering Department**, **Thapar University, Patiala,** is a bonafide work carried out by him under our guidance and supervision and that no part of this thesis has been submitted for the award of any other degree.

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3

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TABLE OF CONTENTS

NOMENCLATURE

u, v, w = Displacement in x, y, z direction

ψ_x, ψ_y = Rotations about x and y directions

Q_{ij} = Material constants

$\overline{Q}_{ij}$ = Transformed material constants

N_x, N_y = Number of grid points in x and y direction

M_x, M_y = Plane moment resultant in x and y direction

Q_x , Q_y = Shear force in x and y direction respectively

p_x, p_y, p_x = Force in x , y and z direction respectively

M_x, M_y = Moments in x and y directions respectively

k_{ij}^2 = Shear correction factor = 5/6

N = Number of layers

K_w = Elastic restraint parameter in z- direction

S_ϕ = Elastic restraint parameter in rotational direction

$C_{ij}^{(n)}$ = Weighting coefficients for n_{th} order

L = Linear differential operator $\frac{d}{dr}$

ν, E , G = poisson's ratio, young's modulus, shear modulus

D = Plate flexural rigidity = $Eh^3/12(1-\nu^2)$

z = Thickness coordinate

A_{ij} = Extensional stiffness

D_{ij} = Bending stiffness

B_{ij} = Bending-extensional coupling stiffness

a = Length of plate

b = Width of plate

t = Thickness of plate

LIST OF FIGURES

ABSTRACT

In the present scenario composite plate are playing a vital role in different applications. So it becomes important to study the vibration behavior of the composite plate.

The present work aims free vibration analysis of composite plate with elastic restrained edges using a numerical solution technique called Differential Quardrature Method. First order shear deformation theory is used for mathematical formulation. Symmetric and Anti-symmetric laminated composite plates have been studied with respect to different parameters. The differential quardrature method has been applied for different sets of grid points. Further the work was extended with the convergence study for symmetric laminated composite plate.

CHAPTER 1

INTRODUCTION TO COMPOSITES

1.1 INTRODUCTION

Composite materials are used in almost all aspects of the industrial and commercial fields in aircraft, ships, common vehicles etc. Their most attractive properties are the high strength to weight ratio and high stiffness to weight ratio. However, these new materials also have some problems such as fiber breakage, matrix cracking and delamination.

In the continuing quest for improved performance, which may be specified by various criteria including weight reduction, more strength and lower cost, currently used engineering materials have reached the limit of their usefulness. Thus materials scientists are always striving to produce either improved traditional materials or completely new materials [1].

1.1.1 DEFINITION

A composite material is combination of two or more distinct constituents or phases but this definition is not sufficient and three other criteria have to be satisfied before the material can be a composite. First both the constituents have to be present in the reasonable proportions, say greater than 5%, secondly constituent phases having different properties and hence the composite properties are noticeably different from constituent's properties. Thirdly there are distinct recognizable interfaces between the constituent phases. The composite materials possess characteristic properties such as high stiffness and strength, low weight, high temperature performance, good corrosion resistance, high hardness and conductivity that are not possible in any of its constituents alone. Analysis of these properties reveals that they depend on the following:

1. Properties of the individual constituents.
2. Relative amounts of the constituents.
3. Size and shape of the constituents (i.e. Morphology).

4. Degree of bonding between constituents.
5. Orientation of the various constituents.

1.1.2 HISTORY OF COMPOSITES

One of the earliest known composite materials is adobe brick in which straw (a fibrous material) is mixed with mud or clay (an adhesive with strong compressive strength). The straw allows the water in the clay to evaporate and distribute cracks in the clay uniformly, greatly improving the strength of this early building material. Another form of a composite material is the ubiquitous construction material called plywood. Plywood (Fig.1.1) uses natural materials (thin slabs of wood) held together by a strong adhesive, making the structure stronger than just the wood itself. In nature bamboo is often cited as an example of a wood composite structure, combining a cellulose fiber and lignin, with the lignin providing the adhesive to hold the fibers together.

FIG 1.1 THIN SLABS IN PLYWOOD

Reinforced concrete is a combination of two remarkable materials, concrete (a composite by itself) and steel that takes advantage of the strengths of each material to overcome their individual limitations in each. Steel has very high tensile strength, while concrete has very high compressive strength. In combination, they make a superior material for road and bridge construction.

Today, when we speak of composite materials or just ‘composites’, we are referring to the highly engineered combinations of polymer resins and reinforcing materials such as glass fibers. A fiberglass composite structure is a combination of glass fibers of various lengths and resins such as vinyl ester or polyester. The term FRP is often used, meaning Fiber Reinforced Plastic. FRP is a very general term for many different combinations of reinforcement materials and bonding resins. Thus, the term “composites” is used extremely broadly to describe many materials with many different properties targeted at an even larger number of applications. [1].

1.2 PARAMETERS OF COMPOSITE MATERIALS

Following two parameters are commonly used to measure the relative advantage derived from composite materials.

(a) Specific modulus

(b) Specific strength

SPECIFIC MODULUS: it is defined as the ratio of Young's Modulus (E) and density (ρ) of the materials.

SPECIFIC STRENGTH: it is defined as ratio between strength and density of the materials.

These two ratios are high in composite materials. As an example the strength of a graphite/epoxy unidirectional composite material is same as steel but the specific strength of composite material is three times that of steel. This means that for the same application of a rod, we required 1/3rd diameter than that of steel, thereby saving material and energy cost.

1.2.1 MATERIAL SELECTION PARAMETERS

For selecting a composite material for a particular application, the following parameters are to be commonly considered:

1. Strength
2. Toughness
3. Join ability
4. Corrosion
5. Wear Resistance
6. Affordability
7. Thermal behavior

1.3 CLASSIFICATION OF COMPOSITE MATERIALS

Composites are classified on the following two bases [1]:

1.3.1 CLASSIFICATION BASED ON GEOMETRY OF REINFORCEMENT

Based on this, the composites are of the following types [1]:

(a) Particulate Composites

(b) Flake Composites

(c) Fiber Composites

1.3.2 CLASSIFICATION BASED ON MATRIX

Based on the type of matrix, Composites are of the following types [1]:

(a) Polymer Matrix Composite (PMC)

(b) Metal Matrix Composite (MMC)

(c) Ceramic Matrix Composite (CMC)

1.3.1 CLASSIFICATION BASED ON GEOMETRY OF REINFORCEMENT

PARTICULATE COMPOSITES

This class of composite consists of particles reinforced in a matrix of metal, alloy or ceramics. They usually exhibit isotropic properties. Particulate composites have advantages such as improved strength, increased operating temperature and oxidation resistance.

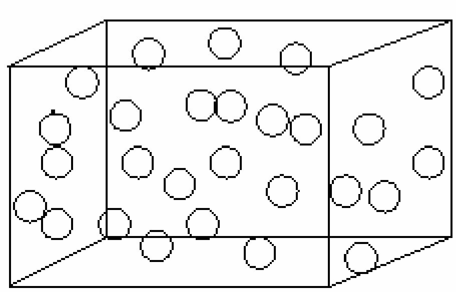

FIGURE 1.2 PARTICLE REINFORCED COMPOSITES

FLAKE COMPOSITES

It consists of flat reinforcement in the matrix. Typical flake materials are glass, mica, aluminum and silver. Flake composites provide advantages such as high out of plane flexural modulus, higher strength and low cost. However, flakes can not be oriented easily therefore a limited number of materials are available for use.

FIBER COMPOSITES

These composites consist of matrix reinforced by short (discontinuous) or long (continuous) fiber. Fibers are generally anisotropic and examples include carbon aramids.

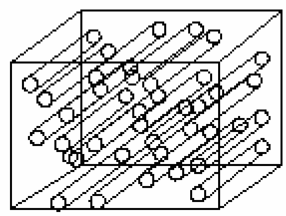

FIGURE 1.3 FIBER-REINFORCED COMPOSITES

1.3.2 CLASSIFICATION BASED ON MATRIX

POLYMER MATRIX COMPOSITE (PMC)

Polymer matrix composites are the most advanced composites. These composites consist of a polymer (e.g. epoxy, polyester, urethane etc) reinforced by thin-diameter fibers(e.g. graphite , aramids, boron etc).These are commonly employed due to their low cost, high strength, and simple manufacturing principle. As an example, graphite/ epoxy composites are approximately five times stronger than steel on a weight for weight basis. Main drawback of Polymer Matrix Composites (PMCs) include low operating temperature, high coefficient of thermal and moisture expansion and low elastic properties in certain directions. However, the advantages include its strength, low cost, high chemical resistance and good insulating property.

METAL MATRIX COMPOSITE (MMC)

This class of composite materials consists of metallic matrix which is usually ductile. The ductile matrix can be aluminum, copper, magnesium, titanium, nickel, super alloy or even an intermetallic compound. The reinforcing fibers may be graphite, boron carbide, alumina or silicon carbide. Fine whiskers of sapphire, silicon carbide, silicon nitride, wires of titanium, tungsten, molybdenum, beryllium and stainless steel etc have also been used as reinforcement.

Compared to conventional engineering materials, these composites offer higher stiffness and strength, especially at elevated temperatures, low coefficient of thermal expansion and enhanced resistance to fatigue, abrasion and wear. Compared to organic matrix materials, they offer high heat resistance and improved electrical and thermal conductivity.

Graphite-reinforced aluminum can be designed to have near zero thermal expansion in the fiber direction. Aluminum oxide-reinforced aluminum matrix composites have been extensively used in automotive connecting rods to provide high stiffness with low weight. Aluminum reinforced with silicon carbide whiskers has been fabricated into aircraft wing panels providing 20-40% weight saving. Fiber reinforced super alloy has potential future for applications such as turbine blades.

CERAMIC MATRIX COMPOSITE (CMC)

Ceramic matrix composites possess properties like high melting points, good corrosion resistance, stability at elevated temperatures and high compressive strength. These materials can even be used at very high temperatures (i.e. above 1500°C).

1.4 LAMINATED COMPOSITES

WHAT IS A LAMINATE?

High stiffness and strength usually require a high proportion of fibers in the composite. This is achieved by aligning a set of long fibers in a thin sheet (a lamina or ply). However, such material is highly anisotropic, generally being weak and compliant (having a low stiffness) in the transverse direction. Commonly high strength and stiffness are required in various directions within a plane. The solution is to stack and weld together a number of sheets, each having the fibers oriented in different directions. Such a stack is termed a laminate [3].

1.5 LIMITATIONS OF COMPOSITES

Limitations of composite materials are given below:

a) High cost of fabrication of composite materials. For example, a part made up of graphite/epoxy composite may cost up to 10 to 15 times of materials cost.

b) Mechanical characterization of composite structure is more complex than that of a metal structure. Composite materials are not isotropic (i.e. do not possess same properties in all the direction). For example, a single layer of graphite/epoxy composite requires nine stiffness and strength constants for conducting mechanical analysis while in the case of steel only four stiffness and strength constant are required.

c) Repair of composite is not a simple process as compared to metals.

d) Composites do not have good combination of strength and fracture toughness as compared to metals.

e) Composites do not necessarily give higher performance in the all the property used for material selection.

CHAPTER 2

THEORIES OF LAMINATED COMPOSITE PLATES

2.1 EQUATION OF ELASTICITY

The equations governing the motion of a solid body can be classified into following basic categories [2]:

1. Kinematics (strain-displacement relation)
2. Kinetics (conservation of momenta)
3. Constitutive equation (stress-strain relations)

2.1.1 KINEMATICS (STRAIN-DISPLACEMENT RELATION)

Kinematics is the study of geometric changes or deformation in a body, without consideration of forces causing the deformation.

2.1.2 KINETICS (CONSERVATION OF MOMENTA)

Kinetics is the study of static or dynamic equilibrium of forces acting on a body.

2.1.3 CONSTITUTIVE EQUATION (STRESS-STRAIN RELATIONS)

The constitutive equations describe the constitutive behavior of the body and relate the dependent variables introduced in the kinetic description to those in the kinematics and thermodynamic descriptions.

2.2 THE CLASSICAL LAMINATED PLATE THEORY

It is the simplest equivalent single layer laminated theory. It is based on the Kirchoff’s hypothesis [2]:

1. Straight line perpendicular to the mid surface. (i.e. transverse normal) before deformation remains straight after deformation (they do not bend).
2. The transverse normal remains unstretched. (they keep the same length)
3. The transverse normal rotates such that they remain perpendicular to the mid surface after deformation. (they always make a right angle to the neutral plane)

Thus it neglects the effect of both transverse shear and transverse normal. The displacement components u, v and w along the x, y and z coordinate directions, respectively are given by [2]:

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_0}{\partial x} \quad (2.1)$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_0}{\partial y} \quad (2.2)$$

$$w(x, y, z, t) = w_0 (x, y, t) \quad (2.3)$$

Where t denotes time and u_0, v_0, w_0 are the displacement component along the x, y and z coordinates directions respectively, of a point on the mid plane (i.e. z=0).

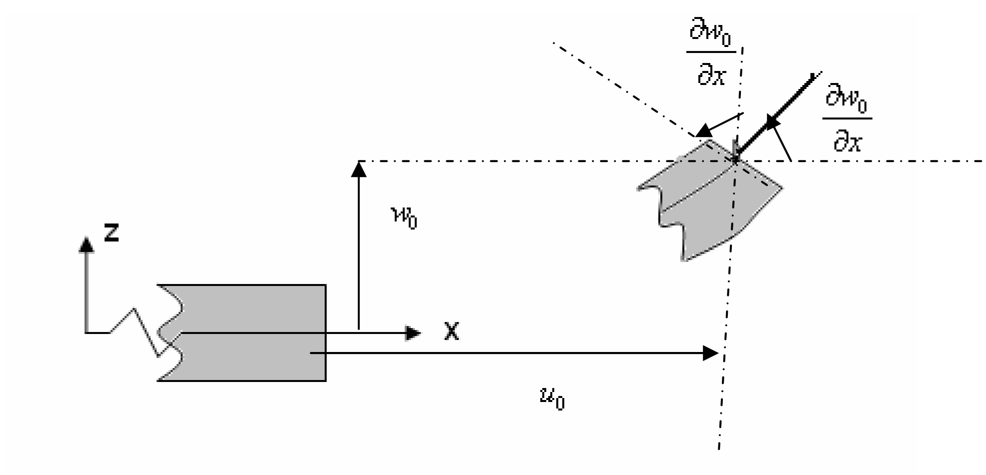

FIGURE 2.1 UNDEFORMED AND DEFORMED GEOMETRIES OF AN EDGE OF A PLATE UNDER THE KIRCHHOFF HYPOTHESIS

The nonlinear strains associated with the above displacement field are given by following equations:

$$\varepsilon_{xx}= \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 - z\frac{\partial^2 w_0}{\partial x^2} \quad (2.4)$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right) - 2z\frac{\partial^2 w_0}{\partial x \partial y} \tag{2.5}$$

$$\varepsilon_{yy} = \frac{\partial u_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 - z\frac{\partial^2 u_0}{\partial y^2} \tag{2.6}$$

$$\varepsilon_{xz} = 0 \tag{2.7}$$

$$\varepsilon_{yz} = 0 \tag{2.8}$$

$$\varepsilon_{zz} = 0 \tag{2.9}$$

These strains are called von-karman strains and the transverse strains (ε_{xz}, ε_{yz}, ε_{zz}) are identically zero in the classical plate theory.

2.3 THE FIRST ORDER SHEAR DEFORMATION THEORY

It is also one of the equivalent single layer laminate theory. This theory assumes that transverse normal do not remain perpendicular to the mid surface after deformation. This amounts to include the transverse shear strains because of transverse shear.

The displacement field of first order theory is given by [2]:

$$u(x,y,z,t) = u_0(x,y,t) + z\phi_x(x,y,t) \tag{2.10}$$

$$v(x,y,z,t) = v_0(x,y,t) + z\phi_y(x,y,t) \tag{2.11}$$

$$w(x, y,z,t) = w_0(x,y,t) \tag{2.12}$$

Where u, v and w are the displacement components along x, y and z axes respectively, and u_0, v_0 and w_0 are the displacement components along the x, y and z coordinate direction respectively ,of a point on the mid plane (i.e. z = 0).

$\phi_x = \frac{\partial u}{\partial z}$ and $\phi_y = \frac{\partial v}{\partial z}$ are the rotations of a transverse normal about y and x axes respectively. The quantities (u_0, v_0, w_0, ϕ_x, ϕ_y) are called the generalized displacement.

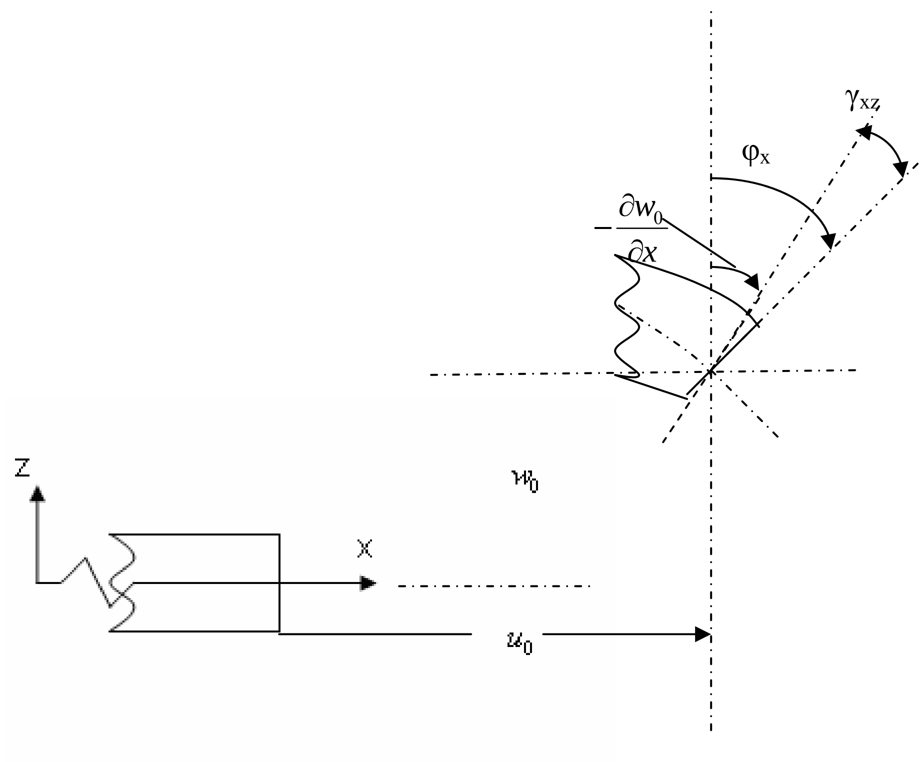

FIGURE 2.2 DEFORMED GEOMETRIES OF AN EDGE OF A PLATE UNDER THE ASSUMPTIONS OF FIRST ORDER SHEAR DEFORMATION THEORY

The nonlinear strains associated with the above displacement field are given by following equations:

$$\varepsilon_{xx} = \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 + z\frac{\partial \phi_x}{\partial x} \tag{2.13}$$

$$\varepsilon_{yy} = \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 + z\frac{\partial \phi_y}{\partial y} \tag{2.14}$$

$$\gamma_{xy} = \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right) + z\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right) \tag{2.15}$$

$$\gamma_{xz} = \frac{\partial w_0}{\partial x} + \phi_x, \quad \gamma_{yz} = \frac{\partial w_0}{\partial y} + \phi_y, \quad \varepsilon_{zz} = 0 \tag{2.16}$$

Here the strains ($\varepsilon_{xx,}$ $\varepsilon_{yy,}$ γ_{xy}**)** are linear through the laminate thickness, while the transverse shear strains ($\gamma_{xz,}$ γ_{yz}) are constant in first order laminated plate theory.

CHAPTER 3

DIFFERENTIAL QUADRATURE METHOD

3.1 INTRODUCTION

The essence of the method relies on the idea that a derivative of a function with respect to a variable at any discreet point can be approximated as a weighted linear sum of the function values at all the discreet points chosen in the overall domain of that variable. The differential quadrature method is first employed to convert the differential equations into a set of linear algebraic equations. Then by solving the algebraic equations, solutions to the problem are obtained.

3.2 METHODOLOGY

Supposing that there are N_x, grid points in the x-direction and N_y in the y-direction with x_1, x_2,…… x_{Nx} and y_1, y_2, y_3……., y_{Ny} as the coordinates, the n^{th}-order partial derivative with respect to x, the m^{th}-order partial derivative of f(x, y) with respect to y and the $(n + m)^{th}$-order partial derivative of f(x, y) with respect to both x and y can be expressed discretely at the point (x_i, y_i) as[4] :

$$f_x^{(n)}(x_i, y_j) = \sum_{k=1}^{N_x} C_{ik}^{(n)} f(x_k, y_j); \qquad n=1,2,\ldots\ldots N_x-1$$

$$f_y^{(m)}(x_i, y_j) = \sum_{k=1}^{N_y} C_{jk}^{(m)} f(x_j, y_k); \qquad m=1,2,\ldots\ldots N_x-1$$

$$f_{xy}^{(n+m)}(x_i, y_j) = \sum_{k=1}^{N_x} C_{ik}^{(n)} \sum_{l=1}^{N_y} \overline{C}_{jl}^{m} f(x_k, y_l);$$

for i=1,2,…….N_x-1 and j=1,2…..N_y

Where $C_{ij}^{(n)}$ and $\overline{C}_{ij}^{(m)}$ are weighting coefficients associated with n^{th}-order partial derivative of f(x, y) with respect to x at the discrete point x_i and m^{th}-order derivative with respect to y at y_i, respectively.

Using the generalised and simplified DQ method. the weighting coefficients in above equation, i.e $C_{ij}^{(n)}$ and $\overline{C}_{ij}^{(m)}$ can be determined as follows :

$$C_{ij}^{(1)} = \frac{M^{(1)}(x_i)}{(x_i - x_j)M^{(1)}(x_j)}; \qquad i,j=1,2\ldots\ldots N, \quad \text{but } j \neq i,$$

$$\overline{C}_{ij}^{(1)} = \frac{p^{(1)}(y_i)}{(y_i - y_j)p^{(1)}(y_j)} \qquad i,j=1,2,\ldots\ldots N, \quad \text{but } j \neq i,$$

where

$$M^{(1)}(x_i) = \prod_{j=1,\, j\neq i}^{N_x} (x_i - x_j)$$

$$p^{(1)}(y_i) = \prod_{j=1,\, j\neq i}^{N_y} (y_i - y_j)$$

and

$$C_{ij}^{(n)} = n\left(C_{ij}^{(n-1)} C_{ij}^{(1)} - \frac{C_{ij}^{(n-1)}}{x_i - x_j} \right)$$

for $i,j=1,2\ldots\ldots N$, but $j \neq i$,

$$\overline{C}_{ij}^{(m)} = m\left(\overline{C}_{ij}^{(m-1)} \overline{C}_{ij}^{(1)} - \frac{\overline{C}_{ij}^{(m-1)}}{y_i - y_j} \right)$$

$$C_{ij}^{(n)} = -\sum_{i=1,\, j\neq i}^{N_x} C_{ij}^{(n)}$$

$$\overline{C}_{ij}^{(m)} = -\sum_{j=1,\, j\neq i}^{N_y} \overline{C}_{ij}^{(m)}$$

Once the functional values at all grid points are calculated, the value at any point could be readily obtained in terms of the polynomial approximation:

$$f(x, y_j) = \sum_{i=1}^{N_x} f(x_i, y_j) r_i(x)$$

$$f(x_i, y\ ) = \sum_{j=1}^{N_y} f(x_i, y_j) s_i(y)$$

$$f(x, y\ ) = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} f(x_i, y_j) r_i(x) s_j$$

where $r_i(x)$ and $s_j,(y)$ are the Lagrange interpolation polynomials along the x and y directions, respectively, and given in the following forms :

$$r_i(x) = \frac{(x - x_1)(x - x_2)...............(x - x_{N_x})}{M^{(1)}(x_i)}$$

$$s_j(y) = \frac{(y - y_1)(y - y_2)...............(y - y_{N_y})}{P^{(1)}(y_i)}$$

3.3 CHOICE OF SAMPLING GRID POINTS

The accuracy of the quadrature method is dictated by the choice of location of the sampling grid points. It is natural and convenient to choose sampling grid points with equal spacing, i.e.

$$X_i = \frac{i}{n} \times \Delta x \qquad \text{for } i = 0,1,2,\ldots\ldots\ldots\ldots n\ ;$$

Where: Δx is the length of the domain interval under consideration. However non-uniform grid points have been demonstrated to enhance the accuracy of the quadrature solutions.

Non-uniform sampling grid points are given by:

$$X_i = \frac{1}{2}\left(1-\cos\left(\frac{i\pi}{n}\right)\right)\times \Delta x \qquad \text{for } i=0,1,2,\ldots\ldots\ldots\ldots n \;;$$

Where: Δx is the length of the domain interval under consideration, X_i=sampling grid point and n=total no. of grid points.

3.4. IMPOSITION OF THE BOUNDARY CONDITIONS

The differential quadrature method (DQM) has been successfully used to tackle various initial or boundary value problem of physical and engineering science efficiently and accurately. However, the imposition of the given initial/boundary conditions can be difficult when more then one boundary conditions are specified at boundary point. This situation is very commonly found in structural mechanics problem. There are two techniques of applying boundary conditions in DQM which are explained as follows:

3.4.1 The δ technique

Bert, sung [5] proposed this technique to impose the two given boundary conditions at each boundary point for structural mechanics problems. The δ technique consists of placing a series of two grid points separated from each other by a small distance δ near the boundary edge. One of the boundary conditions is applied at the grid points located on the boundary edge while the other is applied to the adjacent auxiliary δ-grid points. It can be seen that one boundary condition is exactly imposed while the other is approximately imposed only.

3.4.2 Modified Weighing Coefficients Technique

T.C. Fung [6] applied this procedure to the simply supported boundary condition only.

Consider a linear m^{th} order ordinary differential equation in the form as:

$$\alpha_0 \frac{d^m y}{dx^m} + \alpha_1 \frac{d^{m-1} y}{dx^{m-1}} + \ldots\ldots\ldots\ldots + \alpha_m y = f(x) \text{ for } 0<x<L \text{ and } \alpha_0 \neq 0 \qquad (3.1)$$

using differential quardrature method the analogous equations of the governing differential equations at the n sampling grid points $x_1, x_2 \ldots\ldots\ldots x_n$ can be written as:

$$\alpha_0[A^{(m)}] + \alpha_1[A^{(m-1)}] + \ldots\ldots\ldots\ldots\ldots\alpha_{m-1}[A^{(1)}] + \alpha_m[A^{(0)}] = \{f\} \qquad (3.2)$$

where

$$[A^{(0)}] = [I], \{Y\} = \begin{Bmatrix} y_1 \\ . \\ . \\ . \\ y_n \end{Bmatrix}, \qquad \{f\} = \begin{Bmatrix} f(x_1) \\ . \\ . \\ . \\ f(x_n) \end{Bmatrix} \qquad (3.3)$$

and $y_1,\ y_2,\ y_3 \ldots\ldots\ y_N$ are the approximate values of $y(x)$ at $x_1,\ x_2,\ x_3 \ldots\ldots\ x_N$, respectively. Equation (3.2) is solved by applying boundary conditions. For an m^{th} order equation, there should be m boundary conditions. The solution procedure can be implemented in following ways:

(i) Select (n-m) equation (3.2) and construct the m differential quadrature analogous equations of the boundary conditions at the boundary points. The n unknowns $y_1,\ y_2,\ y_3 \ldots\ldots\ y_N$ are then solved from the combined n equations.

(ii) Select (n-m) equation (3.2) and construct the m differential quadrature analogous equations of the boundary conditions at the boundary and adjacent points to solve for $y_1,\ y_2,\ y_3 \ldots\ldots\ y_N$. This is the δ technique [] and the boundary conditions are only satisfied approximately.

(iii) Modify the weighting coefficient matrices $[A^{(r)}]$ to incorporate the given boundary conditions. The boundary conditions are satisfied exactly by the interpolated solutions.

Approach to modify the weighting coefficient matrices:

To elaborate the method lets take the example of a simply supported beam, the boundary conditions at the two ends can be expressed as $y(x_1) = 0,\ y''(x_1) = 0,\ y(x_N) = 0,$ and $y''(x_N) = 0$. To impose the boundary conditions in the weighting coefficient matrices, all the elements in the columns corresponding to x_1 and x_N in the weighting coefficient matrix $[A^{(1)}]$ are set to zero, i.e. $[A^{(1)}]$ is modified to $[\tilde{A}^{(1)}]$ as

$$[\tilde{A}^{(1)}] = \begin{bmatrix} 0 & A_{12}^{(1)} & . & . & A_{1,n-1}^{(1)} & 0 \\ 0 & A_{12}^{(1)} & . & . & A_{2,n-1}^{(1)} & 0 \\ . & . & . & . & & . \\ . & . & . & . & & . \\ 0 & A_{n2}^{(1)} & . & . & A_{n,n-1}^{(1)} & 0 \end{bmatrix} \tag{3.4}$$

$$[\tilde{A}^{(2)}] = [A^{(1)}][\tilde{A}^{(1)}]$$

and

$$[\tilde{A}^{(4)}] = [\tilde{A}^{(2)}][\tilde{A}^{(2)}]$$

TC Fung [6] proposed that second procedure is more general and can be tackle mixed-type non-homogenous boundary conditions directly.

Let the m boundary conditions for the differential equation in Equation (3.1) be given in the following non-homogenous mixed form as:

$$\gamma_{i1} \frac{d^{m-1}y}{dx^{m-1}}\Big|_{x=\bar{x}_i} + \gamma_{i1} \frac{d^{m-2}y}{dx^{m-2}}\Big|_{x=\bar{x}_i} + \ldots\ldots\ldots\ldots \gamma_{im} y(x_i) = \beta_i \quad \text{For i = 1,2…………m} \tag{3.5}$$

where γ_{ii} and β_i are the constant coefficients, $\bar{x}_1, \ldots\ldots\ldots\ldots \bar{x}_m$ are the co-ordinates of the boundary points. For boundary values problems, $\bar{x}_i$ is generally normalized and would be 0 or 1.

If the n differential quadrature analogous equations of the governing differential equations in Equation (3.2) are to be used, additional auxiliary sampling grid points are required. Let the m additional auxiliary sampling grid points $x_{N+1}, x_{N+2} ....x_{N+M}$. Applying differential quadrature rule for n+m, equation i.e.

$$\begin{Bmatrix} y_1^{(r)} \\ . \\ . \\ y_n^{(r)} \\ y_{n+1}^{(r)} \\ . \\ . \\ y_{n+m}^{(r)} \end{Bmatrix} = \begin{bmatrix} A_{11}^{(r)} & . & . & . & A_{1n}^{(r)} & A_{1,n+1}^{(r)} & . & . & . & A_{1,n+m}^{(r)} \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ A_{n1}^{(r)} & . & . & . & A_{nn}^{(r)} & A_{n,n+1}^{(r)} & . & . & . & A_{n,n+m}^{(r)} \\ A_{n+1,1}^{(r)} & . & . & . & A_{n+1,n}^{(r)} & A_{n+1,n+1}^{(r)} & . & . & . & A_{n+1,n+m}^{(r)} \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ A_{n+m,1}^{(r)} & . & . & . & A_{n+m,n}^{(r)} & A_{n+m,n+1}^{(r)} & . & . & . & A_{n+m,n+m}^{(r)} \end{bmatrix} \begin{Bmatrix} y_1 \\ . \\ . \\ . \\ y_n \\ y_{n+1} \\ . \\ . \\ . \\ y_{n+m} \end{Bmatrix} \qquad (3.8)$$

$$\{Y_1^{(r)}\} = [A_1^{(r)} \quad A_2^{(r)}] \begin{Bmatrix} Y_1 \\ Y_2 \end{Bmatrix} \qquad \text{And} \qquad \{Y_2^{(r)}\} = [A_3^{(r)} \quad A_4^{(r)}] \begin{Bmatrix} Y_1 \\ Y_2 \end{Bmatrix}$$

the differential quadrature analogous equation of the boundary condition in Equation (3.5) can be written as:

$$[\gamma_{i1} \quad \gamma_{i2} \quad . \quad . \quad \gamma_{im}] \begin{bmatrix} A_{j1}^{(m-1)} & . & . & . & A_{jn}^{(m-1)} & A_{j,n+1}^{(m-1)} & . & . & . & A_{j,n+m}^{(m-1)} \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ A_{j1}^{(0)} & . & . & . & A_{jn}^{(0)} & A_{j,n+1}^{(0)} & . & . & . & A_{j,n+m}^{(m-1)} \end{bmatrix} \begin{Bmatrix} y_1 \\ . \\ . \\ y_n \\ y_{n+1} \\ . \\ . \\ y_{n+m} \end{Bmatrix} = \beta_i$$

As a result, the m boundary conditions can be collectively written as:

$$\begin{bmatrix} \Gamma_{11} & . & . & . & \Gamma_{1n} & \Gamma_{1,n+1} & . & . & . & \Gamma_{1,n+m} \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ \Gamma_{m1} & . & . & . & \Gamma_{m,n} & \Gamma_{m,n+1} & . & . & . & \Gamma_{m,n+m} \end{bmatrix} \begin{Bmatrix} y_1 \\ . \\ . \\ y_n \\ y_{n+1} \\ . \\ . \\ y_{n+m} \end{Bmatrix} = \begin{Bmatrix} \beta_1 \\ \\ \\ \beta_m \end{Bmatrix}$$

In short form $[\Gamma_1]\{Y_1\} + [\Gamma_2]\{Y_2\} = \beta$

Hence substituting the value of $\{Y_2\}$ we get:

$$\{Y_1^{(r)}\} = \left([A_1^{(r)}] - [A_2^{(r)}][\Gamma_2]^{-1}[\Gamma_1]\right)\{Y_1\} + [A_2^{(r)}][\Gamma_2]^{-1}\{\beta\} = [\ddot{A}_1^{(r)}]\{Y_1\} + [\ddot{B}_1^{(r)}]\{\beta\}$$

This is modified differential quadrature rule with non-homogenous mixed type boundary conditions in (3.5) imposed.

3.5 LITERATURE REVIEW

Following is a review of the application of differential quadrature method to free vibration analysis of composite plates with edge elastically restrained against translation and rotation.

A. R. Setoodeh, P. Malekzadeh [7], A differential quadrature (DQ) methodology was employed for the static and stability analysis of irregular quadrilateral straight-sided thin plates. A four-noded super element was used to map the irregular physical domain into a square computational domain. Second order transformation schemes with relative ease and low computational effort were employed to transform the fourth order governing equations of thin plates between the domains.

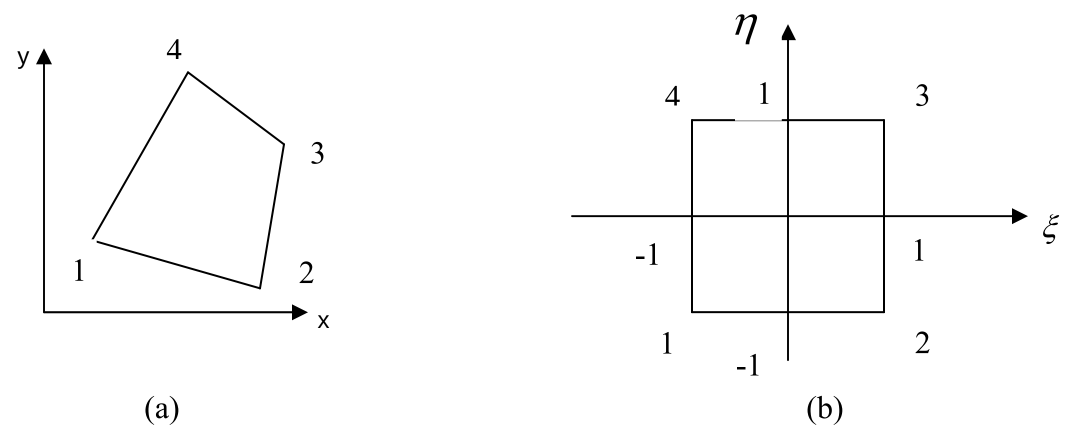

FIGURE 3.1. (a) AN ARBITRARY STRAIGHT-SIDED QUADRILATERAL PLATE (PHYSICAL DOMAIN), (B) COMPUTATIONAL DOMAIN.

The physical domain was mapped into the computational domain according to the following transformation law.

$$x = \sum_{i=1}^{N_s} x_i \psi_i(\xi,\eta), \qquad y = \sum_{i=1}^{N_s} y_i \psi_i(\xi,\eta),$$

Where x_i and y_i are the coordinates of node i in the physical domain, and Ns is the number of nodal points. $\psi_i(\xi,\eta)$ is the shape function associated with node i:

$$\psi_i(\xi,\eta) = \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i) \qquad \text{i=1….4}$$

P. Malekzadeh, G. Karami b [8], presented a differential quadrature (DQ) solution for free vibration analysis of thick plates of continuously varying thickness on two-parameter elastic foundations. The formulations were based on the first-order shear deformation theory taking into account the effects of rotary inertia. The thickness of the plate varied in one or two directions. The thickness variation might be assumed linear or non-linear. Different types of boundary conditions, including free edges and corners, loaded edges with in-plane forces were formulated.

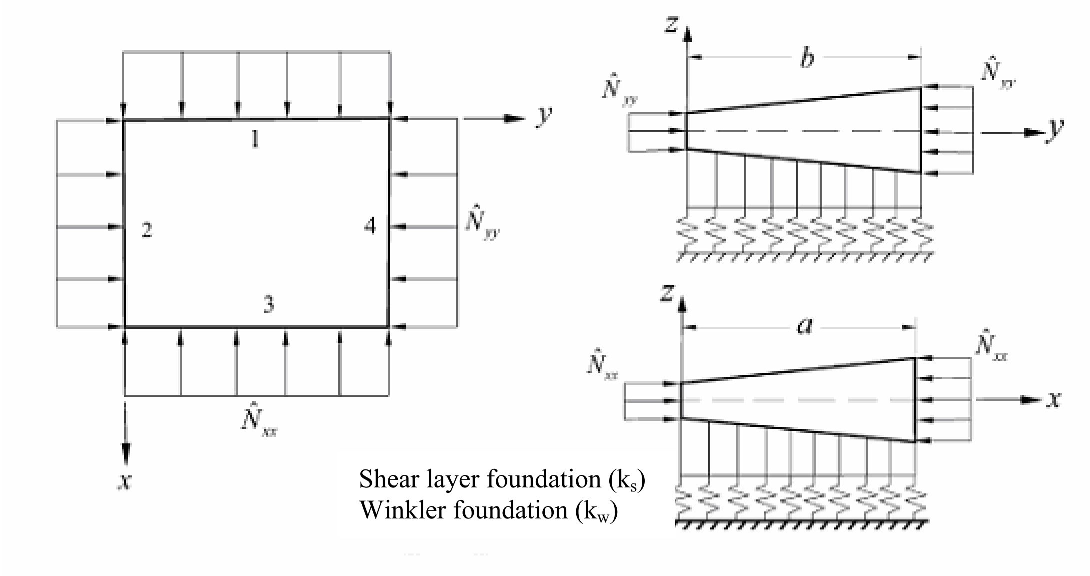

FIGURE 3.2 THE GEOMETRY OF A THICK PLATE

they choose these set of grid points in terms of natural coordinate directions x and y as

$$x_i = \frac{1}{2}\left[1-\cos\left[\frac{(i-1)\pi}{(N_x-1)}\right]\right];$$

$$y_j = \frac{1}{2}\left[1-\cos\left[\frac{(j-1)\pi}{(N_y-1)}\right]\right];$$

for $i = 1,2,.....N_x$ and $j = 1,2,.....N_y$

Accurated results were obtained for higher order mode of vibration with only few grid points, which shows the advaintageous point of low computation cost of method.

P. Malekzadeh *et. al.*[9]**,** introduced a semi-analytical differential quadrature element method (DQEM) based on first-order shear deformation theory for free vibration analysis of thick plates with two opposte edge simply supported and two other edges under general boundary condition.. By decomposing the plate into a series of sub domains or elements, any discontinuity in geometry, material properties, and elastic foundations in one direction can be handled conveniently. Classical, as well as non-classical boundary conditions can be imposed.Accurate solutions were obtained for higher order modes of vibrations with only few grid points. One benefits of this semi analytical DQEM was reducing the storage requirements and the computational efforts needed to assemble the global equations.

K.M. Liew *et. al.*[10] adopted the first-order shear deformation theory in the moving least squares differential quadrature (MLSDQ) procedure for predicting the free vibration behavior of moderately thick symmetrically laminated composite plates. The transverse deflection and two rotations of the laminate were independently approximated with the moving least squares (MLS) approximation. The weighting coefficients used in the MLSDQ approximation were obtained through the fast computation of the MLS shape functions and their partial derivatives.

To illustrate the present MLSDQ method, the plate was discretized firstly by a finite number of nodes (x_i, y_i) associated with three nodal parameters (W^i, ψ_x^i, ψ_y^i) for each node. Then, a circle of influence was formed for each discrete point and the MLS approximation of the general displacement components (W^i, ψ_x^i, ψ_y^i) was achieved in the domain of influence. They obtained numerical results of the netural frequencies for laminates with various boundary conditions, modulus ratio, stacking sequence and span to thickness ratio. They also founs that higher order basis assumptions provides more accurate results for regular node patterns in comparison with low order ones.

P. Malekzadeh, S.A. Shahpari [11],developed a differential quadrature (DQ) procedure for free vibration analysis of variable thickness moderately thick plates with edges elastically restrained against translation and rotation. The governing equations were based on the first order shear deformation theory and the rotary inertia effects were considered. Comparisons with known thin plate and uniform thickness Mindlin plate solutions were carried out to verify the applicability and accuracy of the analysis. Plates with linear or nonlinear varying thickness in one or two directions can be considered. It was demonstrated that using this DQ procedure, classical boundary conditions such as simply supported, clamped and free edges for variable thickness, thin as well as moderately thick plates can be simulated without any numerical difficulties. The effects of different combinations of constraints at edges, aspect ratio, thickness-to-length ratio, and stiffness parameter values on accuracy and convergence behaviors of the plates were presented. Some new results were presented for bi-linearly variable thickness plates with elastically restrained edges.

The numerical results show that the DQ method can yields convergent and accurate solutions for variable thickness thin and moderately thick plates with elastic edges supports or classical boundary conditions using few grid points. Thus DQM can be used as a computationally efficient numerical tool to obtain the natural frequencies of variable thickness plates with different types of boundary conditions.

Ahmed S. Ashour [12] analyzed the natural frequencies of symmetrically laminated plates of variable thickness using the finite strip transition matrix technique. They determined the natural frequencies of such plates for edges with being elastically restrained against both rotation and transition or both. A successive conjunction of the classical finite strip method and the transition matrix method were applied to develop a new modification of the finite strip method to reduce the complexity of the problem. The displacement function was expressed as the product of a basic trigonometric series function in the longitudinal direction and an unknown function that has to be determined in the other direction. Using the new transition matrix, after necessary simplification and the satisfaction of the boundary conditions, yields a set of simultaneous equations that leads to the characteristic matrix of vibration. The mode shapes and the frequency parameters for different combinations of elastic or transnational restraint coefficients had been presented and compared with those available from other methods in the literature. Also, the effect of the tapered ratio and the aspect ratio on the natural frequencies and the mode shapes of the plates were presented.

S. Hatami *et. al.*[13], studied the free vibration of axially moving symmetrically laminated plates subjected to in-plan forces by classical plate theory. Two-dimensional axially moving materials had a wide range of industrial applications such as papers, plastics and composites in producing lines, power transmission and conveyor belts, etc. In many of these instances, the moving material was not isotropic, but was a single-layer orthotropic material or consists of several orthotropic layers. This category includes symmetric cross-ply and angle-ply laminates and anisotropic plates. Firstly, an exact method is developed to analyze vibration of multi-span traveling cross-ply laminates, and then a semi-analytical finite strip method is extended for moving symmetric laminated plates in general, with arbitrary boundary conditions. By the finite strip method intermediate elastic or rigid supports can also be added to the model of the moving plate. The supports may be in the form of point, line or local distributed supports.

P. Malekzadeh, M. Farid [14], explored the applicability and accuracy of a recently developed differential quadrature (DQ) methodology, for nonlinear analysis of composite plates. For this purpose, the large deformation analysis of symmetric and antisymmetric cross ply, thin, elastic rectangular laminated plates rested on nonlinear elastic foundations were investigated. Thin plate theory in conjunction with Green's strain and Von Karman assumptions was used for modeling the nonlinear behavior of the plates. The nonlinear governing equations were discretized at whole domain grid points and boundary conditions were implemented exactly at boundary grid points. DQ solutions based on the first-order shear deformation theory (FSDT) were also obtained and comparative studies were made between two approaches for different cases. Convergence of the methodology was demonstrated and the results were compared with existing solutions of other methods. It was shown that accurate results were obtained even when using only small number of grid points.

A.S. Ashour [15] presented the semi-analytical solutions to determine the natural requencies and the mode shapes of angle-ply laminated plates with edges elastically restrained. The finite strip transition matrix technique was extended to apply to angle-ply laminated plates with edges elastically restrained against rotation and translation. The effect of the edge conditions on the natural frequencies and mode shapes were presented. The convergence and comparison of the results with those available in the literature indicate the accuracy and the validity of the proposed technique. The effects of the elastic restraint parameters on the mode shapes were illustrated in graphic forms.

K. M. Liew *et. al.*[16] used a global numerical technique (The differential quadrature method) for its suitability to solve the boundary value problem of symmetric cross-ply laminates using the first order shear deformation plate theory.

The bending behaviours of symmetric cross-ply laminates, subject to different boundary constraints, are investigated. In this study, the method is used to transform the sets of governing differential equations and boundary conditions of the laminated plates into sets of linear algebraic equations. Boundary conditions along the edges are implemented through the discrete grid points by constraining the displacements, bending moments and rotations.

G. Karami *et. al.*[17], applied a differential quadrature method for static, free vibration, and stability analysis of skewed and trapezoidal composite thin plates. To mechanize the modelling procedure, a general transformation scheme is employed to transfer the variation of the variables in the computational to the physical domain and vice versa. Examples are shown to show the accuracy and convergence of the solutions under different geometrical parameters and boundary conditions. The accuracy is demonstrated by comparing the results with those of other numerical methods.

CHAPTER 4

PROBLEM FORMULATION

4.1 GOVERNING EQUATION FOR LAMINATED COMPOSITE RECTANGULAR PLATE USING FSDT

Let's consider a laminated rectangular plate with length a, width b and total thickness t, having N orthotropic layer which are stacked with two of their principle material direction along the side of rectangle. The layer applied at one or both sides of laminated plate have their poling direction in z-direction, perpendicular to plate. The first layer has one face (bottom layer) at z= -t/2 and the N_{th} layer has one face at z=t/2. The axis is taken in the middle of the mid-plane of the plate.

The displacement field of first order theory is given by [2]:

$$u(x,y,z,t) = u_0(x,y,t) + z\phi_x(x,y,t) \quad (4.1)$$

$$v(x,y,z,t) = v_0(x,y,t) + z\phi_y(x,y,t) \quad (4.2)$$

$$w(x, y, z, t) = w_0(x, y, t) \quad (4.3)$$

Where u, v and w are the displacement components along x, y and z axes respectively and u_0, v_0 and w_0 are the displacement components along the x, y and z coordinate direction respectively of a point on the mid plane (i.e. z = 0).

$\phi_x = \dfrac{\partial u}{\partial z}$ and $\phi_y = \dfrac{\partial v}{\partial z}$ are the rotations of a transverse normal about y and x axis respectively. The quantities (u_0, v_0, w_0, ϕ_x, ϕ_y) are called the generalized displacement.

The linear strains associated with the above displacement field are given by following equations:

$$\varepsilon_{xx} = \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 + z\frac{\partial \phi_x}{\partial x} \quad (4.4)$$

$$\varepsilon_{yy} = \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 + z\frac{\partial \phi_y}{\partial y} \quad (4.5)$$

$$\gamma_{xy} = \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right) + z\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right) \quad (4.6)$$

$$\gamma_{xz} = \frac{\partial w_0}{\partial x} + \phi_x, \quad \gamma_{yz} = \frac{\partial w_0}{\partial y} + \phi_y, \quad \varepsilon_{zz} = 0 \quad (4.7)$$

Here the strains (ε_{xx}, ε_{yy}, γ_{xy}) are linear through the laminate thickness, while the transverse shear strains (γ_{xz}, γ_{yz}) are constant in first order laminated plate theory.

The governing equations of symmetric angle-ply laminated plate based on FSDT are given as [3]:

$$D_{11}\frac{\partial^2\phi^x}{\partial x^2} + D_{12}\frac{\partial^2\phi^y}{\partial x\,\partial y} + D_{16}\left(2\frac{\partial^2\phi^x}{\partial x\,\partial y} + \frac{\partial^2\phi^y}{\partial x^2}\right) + D_{26}\frac{\partial^2\phi^y}{\partial y^2} + D_{66}\left(\frac{\partial^2\phi^x}{\partial y^2} + \frac{\partial^2\phi^y}{\partial x\,\partial y}\right) -$$

$$k_{55}A_{55}\left(\phi^x + \frac{\partial w}{\partial x}\right) - k_{45}A_{45}\left(\phi^y + \frac{\partial w}{\partial y}\right) = I_2\frac{\partial^2\phi^x}{\partial t^2} \quad (4.8)$$

$$D_{16}\frac{\partial^2\phi^x}{\partial x^2} + + D_{66}\left(2\frac{\partial^2\phi^x}{\partial x\,\partial y} + \frac{\partial^2\phi^y}{\partial x^2}\right) + D_{12}\frac{\partial^2\phi^x}{\partial x\,\partial y} + D_{22}\frac{\partial^2\phi^y}{\partial y^2} + D_{26}\left(\frac{\partial^2\phi^x}{\partial y^2} + 2\frac{\partial^2\phi^y}{\partial x\,\partial y}\right) -$$

$$k_{45}A_{45}\left(\phi^x + \frac{\partial w}{\partial x}\right) - k_{44}A_{44}\left(\phi^y + \frac{\partial w}{\partial y}\right) = I_2\frac{\partial^2\phi^y}{\partial t^2} \quad (4.9)$$

$$k_{55}A_{55}\left(\frac{\partial\phi^x}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) + k_{45}A_{45}\left(\frac{\partial\phi^y}{\partial x} + \frac{\partial\phi^x}{\partial y} + 2\frac{\partial^2 w}{\partial x\,\partial y}\right) + k_{45}A_{44}\left(\frac{\partial\phi^y}{\partial y} + \frac{\partial^2 w}{\partial y^2}\right) = I_0\frac{\partial^2 w}{\partial t^2}$$

(4.10)

Where A_{ij} and D_{ij} are stretching and bending stiffness, k_{ij} (i, j= 4, 5) are the shear correction factor, w is the transverse displacement, ϕ^x and ϕ^y are the bending rotation about y-axis and x-axis, t is time and I_i (i=0, 2) are the mass inertia of the plate defined as [2]:

$$I_i = \int_{-h/2}^{h/2} \rho z^i dz \quad i = 0, 2 \tag{4.11}$$

Here ρ and t denote the density of each lamina and total thickness of plate respectively.

Considering a plate with all edges elastically restrained against translation and rotation, as shown in Fig. 4.1, the boundary conditions along each edge are as follows:

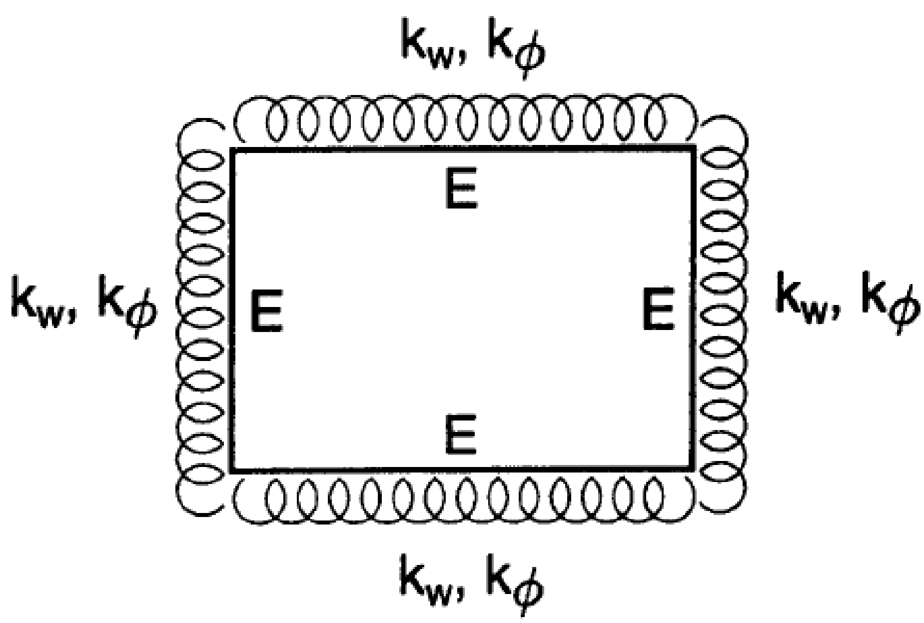

FIGURE 4.1 RECTANGULAR PLATE WITH ELASTIC RESTRAINTS ON FOUR EDGES

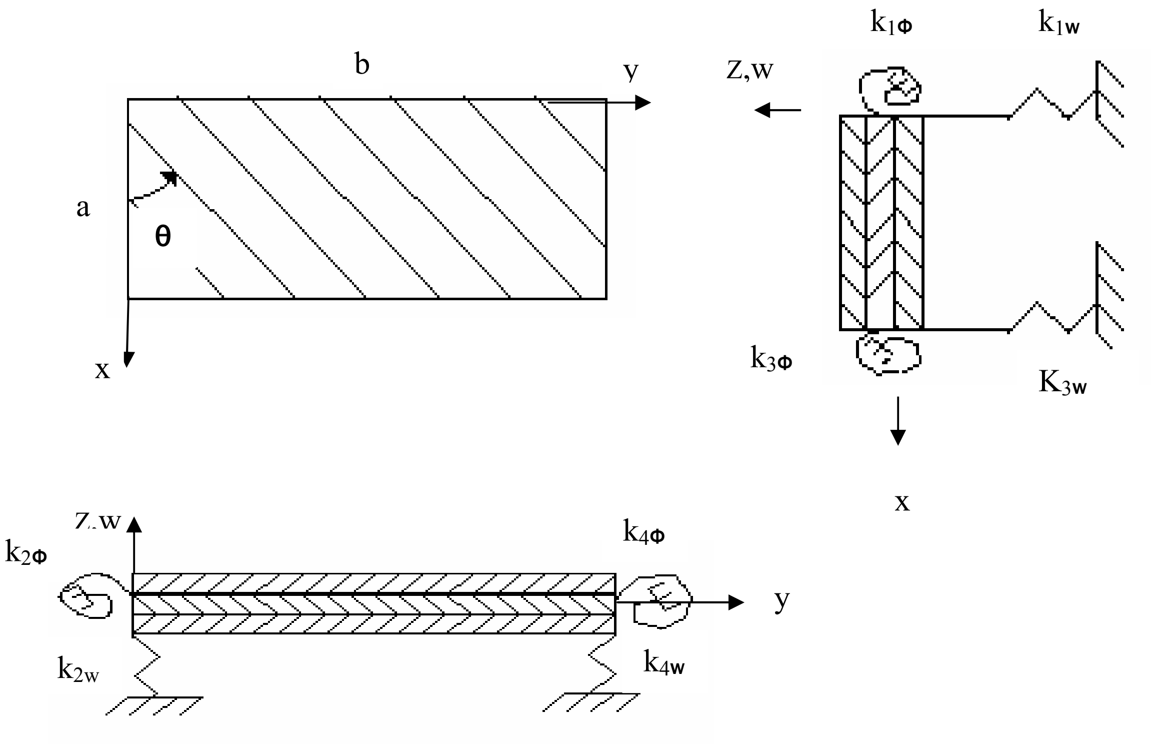

FIGURE 4.2 THE GEOMETRY AND BOUNDARY CONDITIONS OF A LAMINATED PLATE.

Boundary conditions along edges x = 0 and a:

$$Q_x + n_{ix} k_w w = 0, \qquad M_x + n_{ix} k_\phi \phi^x = 0, \qquad M_{xy} = 0, \tag{4.12}$$

where i = 1 for edge x = 0 and i = 3 for edge x = a (see Fig. 4.2). n_{ix}, k_w and k_ϕ (i = 1,3) are the x-component of unit normal along edges i, the coefficients of transverse and torsional support respectively.

Boundary conditions along edges y = 0 and b:

$$Q_y + n_{jy} k_w w = 0, \qquad M_y + n_{jy} k_\phi \phi^y = 0, \qquad M_{xy} = 0, \tag{4.13}$$

Where j = 2 for y = 0 and j = 4 for edge y = b (see Fig. 4.2). n_{jy} (j = 2, 4) are the y component of unit normal along edges j.

In the above equations, M_x and M_y are bending moments per unit length about y- and x-axis, M_{xy} is the twisting moment, Q_x and Q_y are the shear force per unit length along x and y edges, respectively. The bending moments and shear forces can be expressed in terms of the displacement gradients using the constitutive relations as [3]

$$\left.\begin{aligned}
M_x &= D_{11}.\frac{\partial \phi^x}{\partial x} + D_{12}.\frac{\partial \phi^y}{\partial y} + D_{16}.(\frac{\partial \phi^x}{\partial y} + \frac{\partial \phi^y}{\partial x}) \\
M_y &= D_{12}.\frac{\partial \phi^x}{\partial x} + D_{22}.\frac{\partial \phi^y}{\partial y} + D_{26}.(\frac{\partial \phi^x}{\partial y} + \frac{\partial \phi^y}{\partial x}) \\
M_{xy} &= D_{16}.\frac{\partial \phi^x}{\partial x} + D_{26}.\frac{\partial \phi^y}{\partial y} + D_{66}.(\frac{\partial \phi^x}{\partial y} + \frac{\partial \phi^y}{\partial x}) \\
Q_x &= k_{55}A_{55}.(\phi^x + \frac{\partial w}{\partial x}) + k_{45}A_{45}.(\phi^y + \frac{\partial w}{\partial y}) \\
Q_y &= k_{45}A_{45}.(\phi^x + \frac{\partial w}{\partial x}) + k_{44}A_{44}.(\phi^y + \frac{\partial w}{\partial y})
\end{aligned}\right\} \quad (4.14)$$

where A_{ij} and D_{ij} are the extensional and bending stiffness of the plate. Using (4.12) and (4.13) and without any additional formulations, a wide spectrum of boundary conditions can be developed by allowing the coefficients of elastic supports to take their natural limiting values of zeros and infinity.

A zero transverse displacement and rotation conditions along the boundaries can be obtained by approaching k_w and k_ϕ to infinity in (4.12)–(4.14). In this work, values of $k_w = 10^8(b^3/D_{22})$ and $k_w = 10^8(a^3/D_{22})$ are used to simulate zero transverse displacement along edges x = 0, a and along edges y = 0, b, respectively. Also, zero rotation along edges x = 0, a and along edges y = 0,b are simulated by using $k_\phi = 10^8(b/D_{22})$ and $k_\phi = 10^8(a/D_{22})$ along x = 0,a and y = 0,b edges, respectively. Different types of boundary conditions considered in the present work are as follows:

E_{RV}: Along edges x =0 and x = a:

$$M_x + n_{ix}k_\phi \phi^x = 0, \qquad Q_x + n_{ix}k_w w = 0, \qquad \phi^y = 0,$$

Along edges y =0 and y = b:

$$M_y + n_{jy}k_\phi \phi^y = 0, \qquad Q_y + n_{jy}k_w w = 0, \qquad \phi^x = 0, \qquad (4.15)$$

Since the free vibration of plates is a harmonic motion, one can assume the following periodic form for the field variables (w, ϕ^x , ϕ^y)

$$w(x,y,t) = W(x,y)e^{i\omega t},\ \phi^x(x,y,t) = \psi^x(x,y)e^{i\omega t},\ \phi^y(x,y,t) = \psi^y(x,y)e^{i\omega t}, \qquad (4.16)$$

4.2 DIFFERENTIAL QUARDRATURE ANALOGOUS EQUILIBRIUM EQUATIONS:

Differential quardrature method is basically used to convert the differential equation into the algebric form so that it can be easily solved and solution can be found. The derivatives of the field variables may be transformed into computational domain efficiently by modifying the weighting coefficients as

$$\left.\begin{aligned} A_{ij}^x &= \frac{\overline{A_{ij}^x}}{a}, \qquad B_{ij}^x = \frac{\overline{B_{ij}^x}}{a^2}, \\ A_{ij}^y &= \frac{\overline{A_{ij}^y}}{b}, \qquad B_{ij}^y = \frac{\overline{B_{ij}^y}}{b^2}, \end{aligned}\right\} \qquad (4.17)$$

Where, $\overline{A_{ij}^x}$, $\overline{A_{ij}^y}$ and $\overline{B_{ij}^x}$, $\overline{B_{ij}^y}$ are the weighting coefficients of first and second order derivatives in x- and y-directions, respectively. Details on calculations of weighting coefficients can be found in the work of Shu and Richards [16]. In this study, a cosine rule is used for generation of grid points in x- and y-directions [17]. The DQ analogs of the governing equations and boundary conditions take the following forms, respectively:

$$D_{11}(\sum_{m=1}^{N_x} B_{im}^x \psi_{mj}^x) + D_{12}(\sum_{m=1}^{N_x}\sum_{n=`}^{N_y} A_{im}^x A_{jn}^y \psi_{mn}^y) + D_{16}[(2\sum_{m=1}^{N_x}\sum_{n=`}^{N_y} A_{im}^x A_{jn}^y \psi_{mn}^x) + (\sum_{m=1}^{N_x} B_{im}^x \psi_{mj}^y)] + D_{26}(\sum_{n=1}^{N_y} B_{jn}^y \psi_{in}^y) + D_{66}[\sum_{m=1}^{N_x}\sum_{n=1`}^{N_y} A_{im}^x A_{jn}^y \psi_{mn}^y)$$
$$+\sum_{n=1}^{N_y} B_{jn}^y \psi_{in}^x] - \mathrm{k}_{55}\mathrm{A}_{55}(\psi_{ij}^x + \sum_{m=1}^{N_x} A_{im}^x W_{mj}) - \mathrm{k}_{45}\mathrm{A}_{45}(\psi_{ij}^y + \sum_{n=1}^{N_y} A_{jn}^y W_{in}) + I_2\omega^2\psi_{ij}^x = 0$$

(4.18)

$$D_{16}(\sum_{m=1}^{N_x} B_{im}^{x}\psi_{mj}^{x}) + D_{66}(\sum_{m=1}^{N_x}\sum_{n=`}^{N_y} A_{im}^{x}A_{jn}^{y}\psi_{mn}^{x} + \sum_{m=1}^{N_x} B_{im}^{x}\psi_{mj}^{y}) + D_{12}(\sum_{m=1}^{N_x}\sum_{n=`}^{N_y} A_{im}^{x}A_{jn}^{y}\psi_{mn}^{x}) + \;)] + D_{22}(\sum_{n=1}^{N_y} B_{jn}^{y}\psi_{in}^{y})$$

$$+ D_{26}(\sum_{n=1}^{N_y} B_{jn}^{y}\psi_{in}^{x} + 2\sum_{m=1}^{N_x}\sum_{n=1`}^{N_y} A_{im}^{x}A_{jn}^{y}\psi_{mn}^{y}) - k_{45}A_{45}(\psi_{ij}^{x} + \sum_{m=1}^{N_x} A_{im}^{x}W_{mj}) - k_{44}A_{44}(\psi_{ij}^{y} + \sum_{n=1}^{N_y} A_{jn}^{y}W_{in}) + I_2\omega^2\psi_{ij}^{y} = 0 \qquad (4.19)$$

$$k_{55}A_{55}(\sum_{m=1}^{N_x} A_{im}^{x}\psi_{mj}^{x} + \sum_{m=1}^{N_y} B_{im}^{x}W_{mj}) + k_{45}A_{45}(\sum_{m=1}^{N_x} A_{im}^{x}\psi_{mj}^{y} + \sum_{n=1}^{N_y} A_{jn}^{y}\psi_{in}^{x} + 2\sum_{m=1}^{N_x}\sum_{n=`}^{N_y} A_{im}^{x}A_{jn}^{y}W_{mn}) + k_{44}A_{44}(\sum_{n=1}^{N_y} A_{jn}^{y}\psi_{in}^{y} + \sum_{n=1}^{N_y} B_{jn}^{y}W_{in}) + I_0\omega^2 W_{ij} = 0 \qquad (4.20)$$

4.3 DESCRITIZATION OF BOUNDARY CONDITION:

The DQ analogous of boundary conditions are obtained as follows:

Bending moment along edges x=0 or a:

$$D_{11}(\sum_{m=1}^{N_x} A_{im}^{x}\psi_{mj}^{x}) + D_{12}(\sum_{m=1}^{N_y} A_{jm}^{y}\psi_{im}^{y}) + D_{16}(\sum_{m=1}^{N_y} A_{jm}^{y}\psi_{im}^{x} + \sum_{m=1}^{N_x} A_{im}^{x}\psi_{mj}^{y}) + n_{Ix}k_{I\phi}\psi_{ij}^{x} = 0,$$

For i=1, I=1 or i=N_x, I=2; j=2 …Ny-1 (4.21)

Bending moment along edges y=0 or b:

$$D_{21}(\sum_{m=1}^{N_x} A_{im}^{x}\psi_{mj}^{x}) + D_{22}(\sum_{m=1}^{N_y} A_{jm}^{y}\psi_{im}^{y}) + D_{26}(\sum_{m=1}^{N_y} A_{jm}^{y}\psi_{im}^{x} + \sum_{m=1}^{N_x} A_{im}^{x}\psi_{mj}^{y}) + n_{Jx}k_{J\phi}\psi_{ij}^{y} = 0$$

For j=1, J=3 or j=N_y, J=4; i=2 …Nx-1 (4.22)

Shear force along edges x=0 or a:

$$k_{55}A_{55}(\psi_{ij}^{x} + \sum_{m=1}^{N_x} A_{im}^{x}\psi_{mj}^{x}) + k_{45}A_{45}(\psi_{ij}^{y} + \sum_{n=1}^{N_y} A_{jn}^{y}\psi_{in}^{y}) + n_{Ix}k_{Iw}W_{ij} = 0, \omega$$

For i=1, I=1 or i=N_x, I=2; j=2 …Ny-1 (4.23)

Shear force along edges y=0 or b:

$$k_{45}A_{45}(\phi_{ij}^{x} + \sum_{m=1}^{N_x} A_{im}^{x}w_{mj}) + k_{44}A_{44}(\phi_{ij}^{y} + \sum_{n=1}^{N_y} A_{jn}^{y}w_{in}) + n_{jx}k_{Jw}W_{ij} = 0,$$

For j=1, J=3 or j=N_y, J=4; i=2 …Nx-1 (4.24)

At corner grid points, where the numbers of equations are greater than the number of unknowns, two of the boundary conditions can be removed. There is no sensitivity on which of the two that can be removed. In the analysis, the shear forces and twisting moments are applied at corner grid points. To establish the eigenvalue system of equations, the degrees of freedom are separated into the boundary and the domain degrees of freedom as

$$\{U\}_b = \begin{Bmatrix} W \\ \psi^x \\ \psi^y \end{Bmatrix}_b , \qquad \{U\}_d = \begin{Bmatrix} W \\ \psi^x \\ \psi^y \end{Bmatrix}_d$$

Where subscripts 'd' and 'b' stand for the domain and boundary degrees of freedom.
Rearranging the discretized governing equations, the assembled form of them in a matrix form becomes,

$$[S]\{U\}_d - \omega^2 [M]\{U\}_d = 0,$$

Where S= stiffness matrix.
By solving the above equations, natural frequencies can be obtained.

CHAPTER 5

RESULT AND DISCUSSION

A plate of laminated cross-ply laminates having four edges elastically restraint (EEEE) as shown in figure 4.1 is considered. The eigen value are determined in terms of Non-dimensional frequency parameter $(\omega b^2 / \pi^2)(\sqrt{\rho t / D_{22}})^{1/2}$ for the considered plate.

Variation of Non-dimensional frequency parameter with respect to aspect ratio a/b, thickness ratio t/b and elastic restraint parameter $K_w = k_w b^3 / D_{22}$ and $S_\phi = k_\phi b^3 / D_{22}$ has been studied and shown in figures.

The considered plate is made up of 9 layers, that have the same material and geometrical properties in all directions and lie in the order of (0°, 90°/………/0°). The considered elastic properties are:

Possion ratio ν_{12} = 0.25,

Shear moduli $G_{12} / E_2 = G_{13} / E_2 = 0.6$,

$G_{23} / E_2 = 0.5$,

Shear correction factor $k_5 = k_6 = 5/6$,

Aspect ratio a / b = 1,

Thickness to width ratio t / b = 0.2,

Numbers of layers $N_x = N_y = 9$,

K_w = 100,

S_φ = 100,

these are the default properties of the considered plate, which are fixed unless specified otherwise.

Convergence study has been done for the considered cross play laminated plate, which is presented in the following tables. Convergence study has been done on the basis of following data. Number of grid point has been varied from 6 to 14. No of layer considered are nine.

a / b = 1

$N_x=N_y$	$\overline{\omega_1}$	$\overline{\omega_2}$	$\overline{\omega_3}$
6	0.875058	1.10932	1.56037
7	0.874179	1.11087	1.4667
8	0.874189	1.10916	1.47815
9	0.874205	1.10922	1.47058
10	0.874206	1.10928	1.4711
11	0.874206	1.10928	1.47143
12	0.874206	1.10928	1.47142
13	0.874206	1.10928	1.4714
14	0.874206	1.10928	1.4714

TABLE: 1

a / b = 2

$N_x=N_y$	$\overline{\omega_1}$	$\overline{\omega_2}$	$\overline{\omega_3}$
6	0.875058	1.10932	1.56037
7	0.874179	1.11087	1.4667
8	0.874189	1.10916	1.47815
9	0.874205	1.10922	1.47058
10	0.874206	1.10928	1.4711
11	0.874206	1.10928	1.47143
12	0.874206	1.10928	1.47142
13	0.874206	1.10928	1.4714
14	0.874206	1.10928	1.4714

TABLE: 2

Convergence tables show that the values of frequency parameters, after increment of certain grid point become constant. Means there is no further change in frequency parameter. It shows the convergences of this method.

The method has been applied to compute the first three Non-dimensional frequency parameter for plate with two aspect ratio a / b = 1 and 2 respectively.

E_1/E_2 = 40, Number of layers ($N_x = N_y$) = 9.

The elastic restraint parameter K_w varies from 10^{-1} to 10^8, thickness ratio t / b ranges from 0.001 to 0.2 and value of S_ϕ is being taken 0.0, 10^2 and 10^8. The variotin of Non-dimensional frequency parameter $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})^{1/2}$ with respect to elastic lateral edge restraint parameter K_w has been shown in the following figures.

N = 9, a / b = 1, $S_\phi = 0$

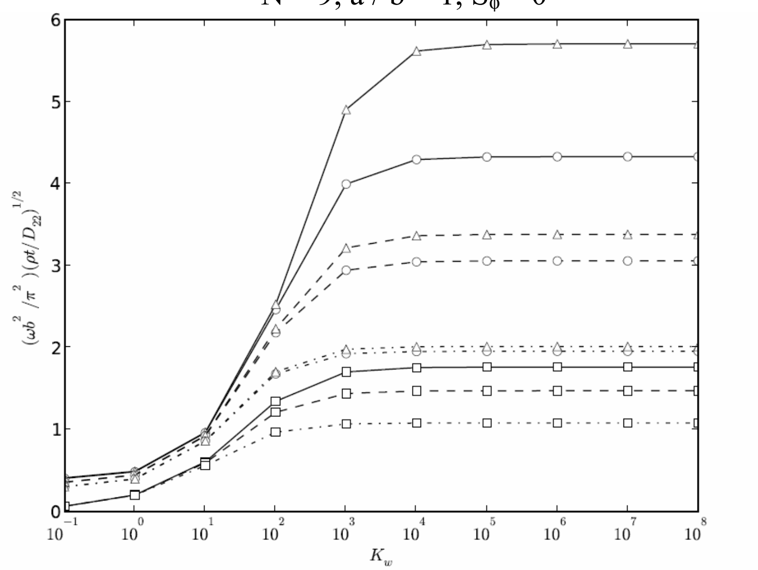

FIGURE 5.1. FREQUENCY PARAMETER, $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})^{1/2}$, VERSUS ELASTICAL LATERAL EDGE RESTRAINED PARAMETER, K_W, FOR SYMMETRIC CROSS-PLY LAMINATED PLATE (KEY:- t / b=0.001;--t / b=0.1;O-.-O t / b=0.2),□ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

In the above figure variation of Non-dimensional frequency parameter has been shown with respect to K_w . Aspect ratio (a / b) has been taken 1. Value of S_ϕ has been taken 0. Figure shows that value of Non-dimensional frequency parameter increases slowly with increase in value of K_w till 10. As soon as value of K_w increases 10 onwards , there is sudden rise in value of Non-dimensional frequency parameter till value of K_w reached 10^4 and after this value there is almost negligible change in value of Non-dimensional frequency parameter.

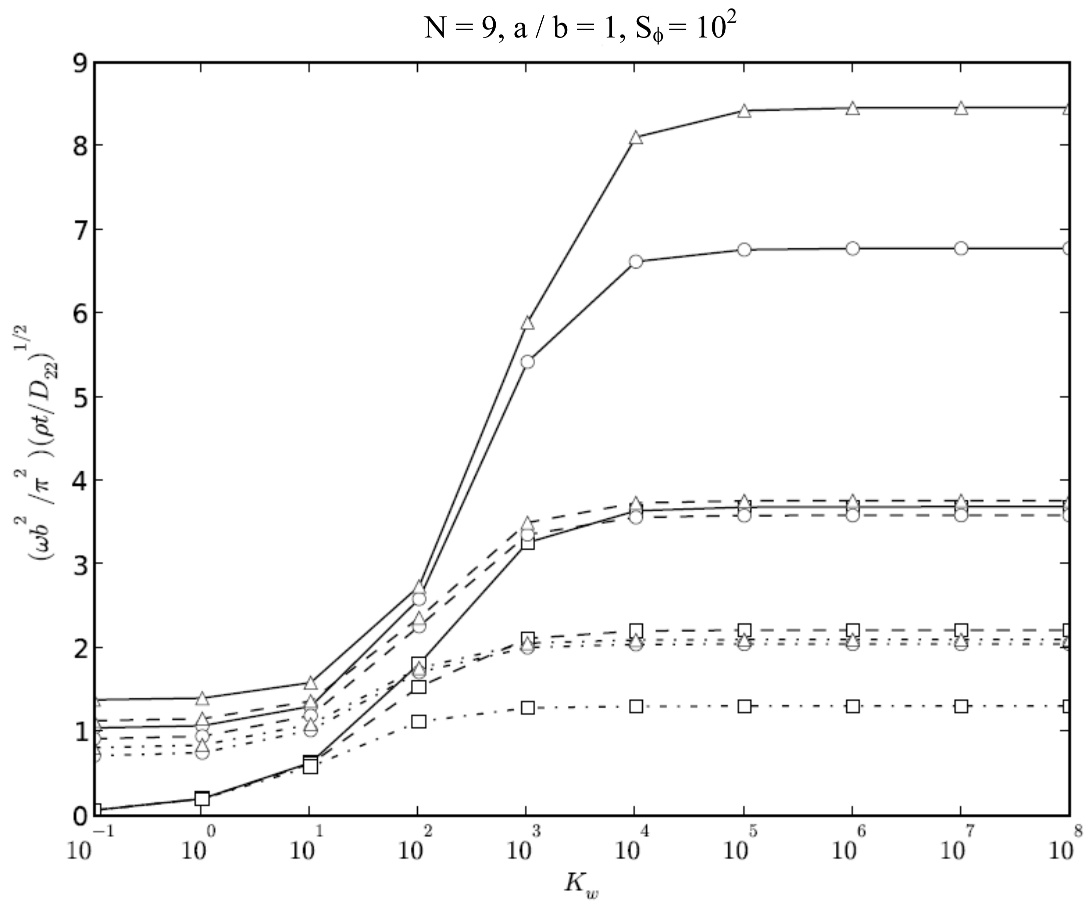

FIGURE 5.2. FREQUENCY PARAMETER, $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})^{1/2}$, VERSUS ELASTICAL LATERAL EDGE RESTRAINED PARAMETER, Kw, FOR SYMMETRIC CROSS-PLY LAMINATED PLATE (KEY:- T/B=0.001;--T/B=0.1;O-.-O T/B=0.2),□ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

In the above figure variation of Non-dimensional frequency parameter has been shown with respect to K_w . Aspect ratio (a / b) has been taken 1. Value of S_ϕ has been taken 10^2. Figure shows that value of Non-dimensional frequency parameter increases slowly with increase in value of K_w till 10. As soon as value of K_w increases 10 onwards , there is sudden rise in value of Non-dimensional frequency parameter till value of K_w reached 10^5 and after this value there is almost negligible change in value of Non-dimensional frequency parameter.

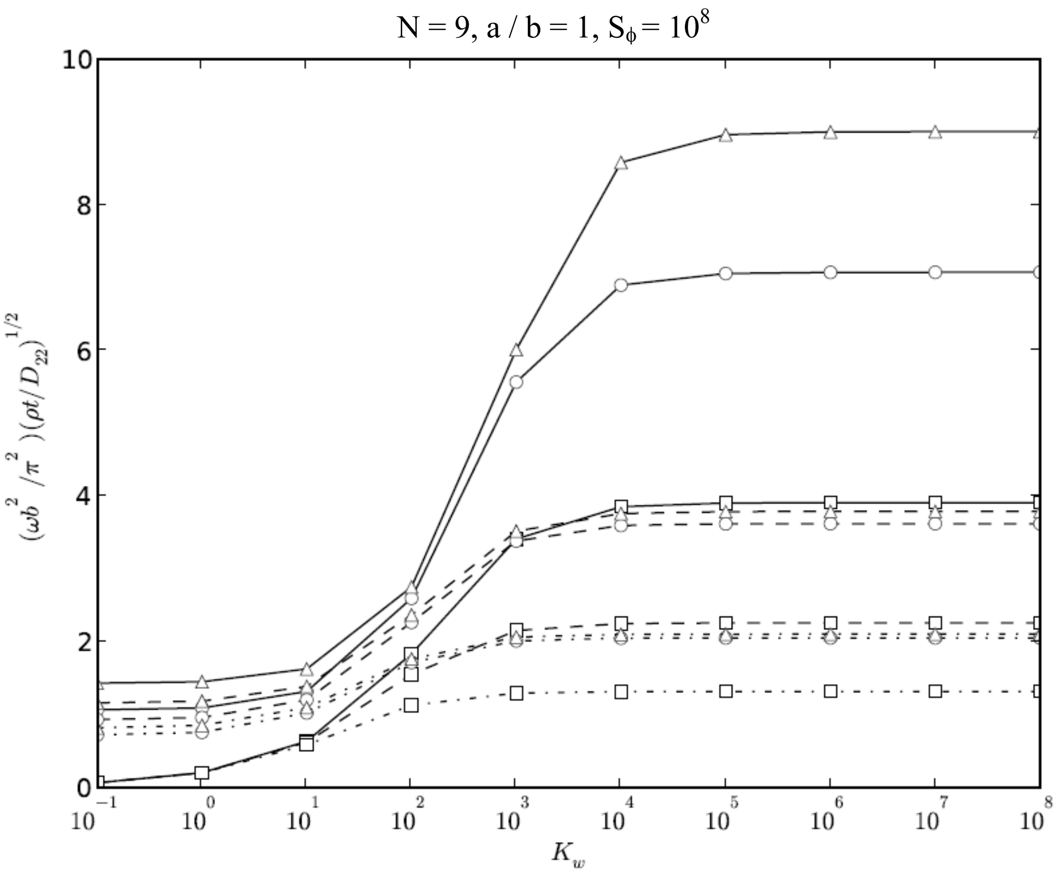

FIGURE 5.3. FREQUENCY PARAMETER, $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})^{1/2}$,VERSUS ELASTICAL LATERAL EDGE RESTRAINED PARAMETER, Kw, FOR SYMMETRIC CROSS-PLY LAMINATED PLATE (KEY:- T/B=0.001;--T/B=0.1;O-.-O T/B=0.2),□ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

In the above figure variation of Non-dimensional frequency parameter has been shown with respect to K_w . Aspect ratio (a / b) has been taken 1. Value of S_ϕ has been taken 10^8. Figure shows that value of Non-dimensional frequency parameter increases slowly with increase in value of K_w till 10. As soon as value of K_w increases 10 onwards , there is sudden rise in value of Non-dimensional frequency parameter till value of K_w reached 10^4 and after this value there is almost negligible change in value of Non-dimensional frequency parameter.

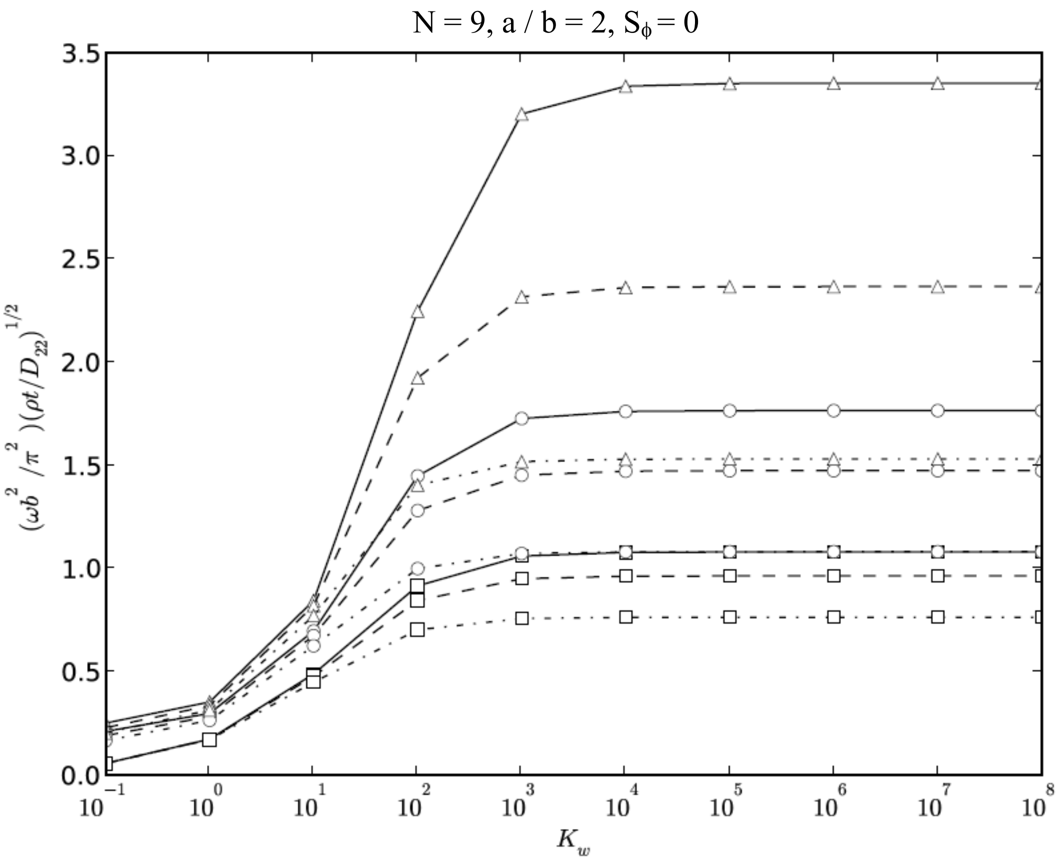

FIGURE 5.4 FREQUENCY PARAMETER, $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})^{1/2}$,VERSUS ELASTICAL LATERAL EDGE RESTRAINED PARAMETER, Kw, FOR SYMMETRIC CROSS-PLY LAMINATED PLATE (KEY:- T/B=0.001;--T/B=0.1;O-.-O T/B=0.2),□ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

In the above figure variation of Non-dimensional frequency parameter has been shown with respect to K_w . Aspect ratio (a / b) has been taken 2. Value of S_ϕ has been taken 0. Figure shows that value of Non-dimensional frequency parameter increases slowly with increase in value of K_w till 10^0. As soon as value of K_w increases 10^0 onwards , there is sudden rise in value of Non-dimensional frequency parameter till value of K_w reached 10^3 and after this value there is almost negligible change in value of Non-dimensional frequency parameter.

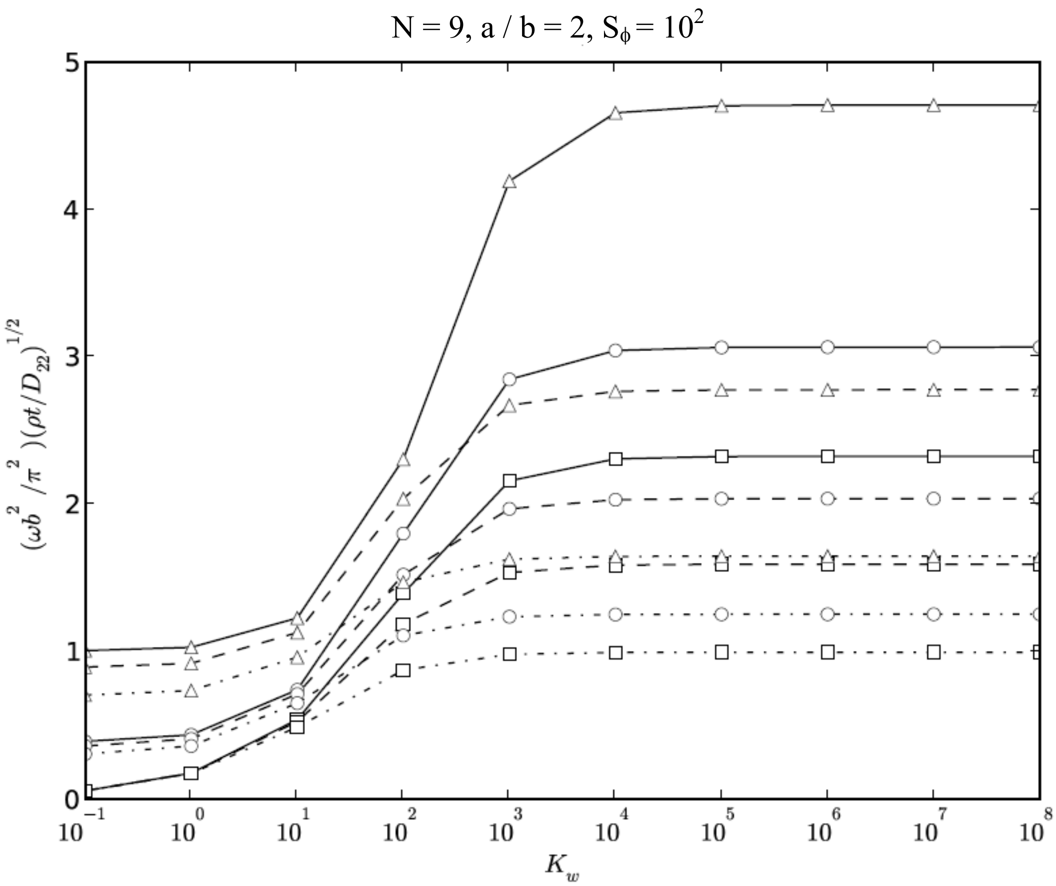

FIGURE 5.5 FREQUENCY PARAMETER, $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})^{1/2}$,VERSUS ELASTICAL LATERAL EDGE RESTRAINED PARAMETER, Kw, FOR SYMMETRIC CROSS-PLY LAMINATED PLATE (KEY:- T/B=0.001;--T/B=0.1;O-.-O T/B=0.2),□ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

In the above figure variation of Non-dimensional frequency parameter has been shown with respect to K_w . Aspect ratio (a / b) has been taken 2. Value of S_ϕ has been taken 10^2. Figure shows that value of Non-dimensional frequency parameter increases slowly with increase in value of K_w till 10^1. As soon as value of K_w increases 10^1 onwards , there is sudden rise in value of Non-dimensional frequency parameter till value of K_w reached 10^3 and after this value there is almost negligible change in value of Non-dimensional frequency parameter.

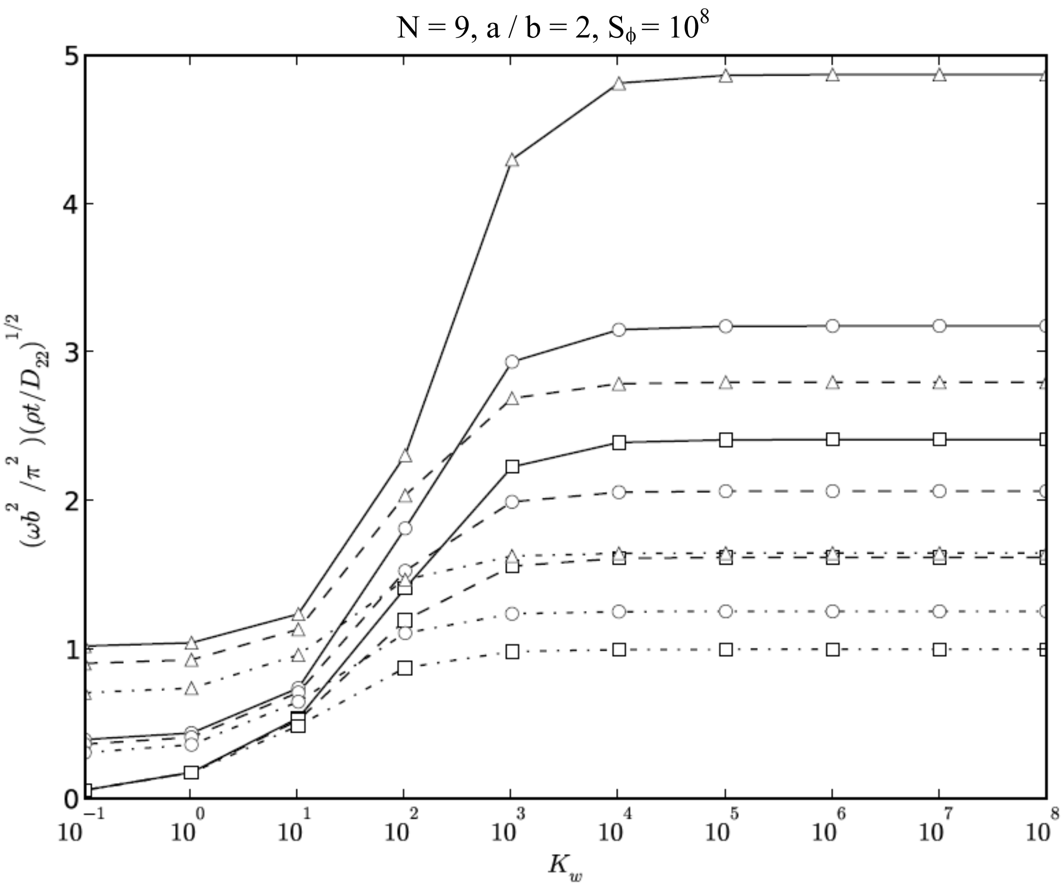

FIGURE 5.6. FREQUENCY PARAMETER, $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})^{1/2}$,VERSUS ELASTICAL LATERAL EDGE RESTRAINED PARAMETER, Kw, FOR SYMMETRIC CROSS-PLY LAMINATED PLATE (KEY:- T/B=0.001;--T/B=0.1;O-.-O T/B=0.2),□ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

In the above figure variation of Non-dimensional frequency parameter has been shown with respect to K_w . Aspect ratio (a / b) has been taken 2. Value of S_ϕ has been taken 10^8. Figure shows that value of Non-dimensional frequency parameter increases slowly with increase in value of K_w till 10^1. As soon as value of K_w increases 10^1 onwards , there is sudden rise in value of Non-dimensional frequency parameter till value of K_w reached 10^4 and after this value there is almost negligible change in value of Non-dimensional frequency parameter

A further study has been done for Lminated composite plate having 4 numbers of layers, thus making it asymmetric composite plate. Non-dimensional frequency parameter for the considered plate with two aspect ratio a / b = 1 and 2 has been determined.

$E_1/E_2 = 40$, Number of layers (N_x=N_y) = 4.

The elastic restraint parameter (K_w) varies from 10^{-1} to 10^{8}, whereas thickness ratio t / b ranges from 0.001 to 0.2 and value of S_ϕ is being taken 0.0, 10^2 and 10^8. The variotin of Non-dimensional frequency parameter $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})^{1/2}$ with respect to elasticlateral edge restraint parameter K_w has been shown in the following figures

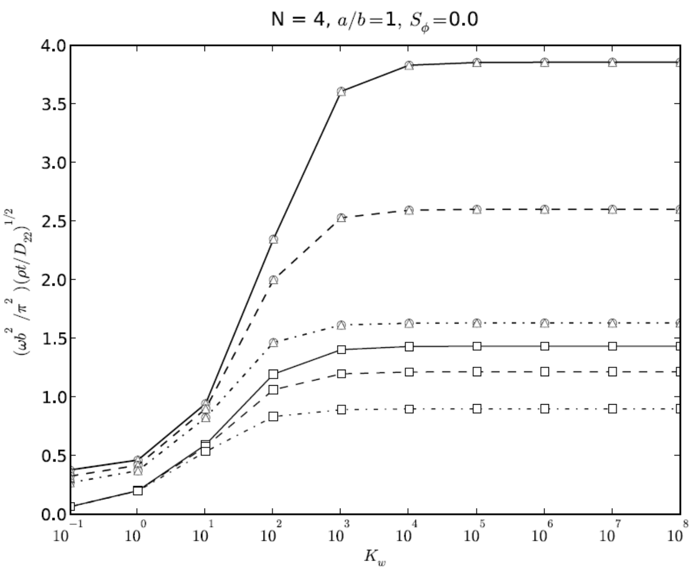

FIGURE 5.7 FREQUENCY PARAMETER, $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})^{1/2}$,VERSUS ELASTICAL LATERAL EDGE RESTRAINED PARAMETER, Kw, FOR SYMMETRIC CROSS-PLY LAMINATED PLATE (KEY:- T/B=0.001;--T/B=0.1;O-.-O T/B=0.2),□ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

In the above figure variation of Non-dimensional frequency parameter has been shown with respect to K_w . Aspect ratio (a / b) has been taken 1. Value of S_ϕ has been taken 0. Figure shows that value of Non-dimensional frequency parameter increases slowly with increase in value of K_w till 10^0. As soon as value of K_w increases 10^0 onwards , there is sudden rise in value of Non-dimensional frequency parameter till value of K_w reached 10^3 and after this value there is almost negligible change in value of Non-dimensional frequency parameter

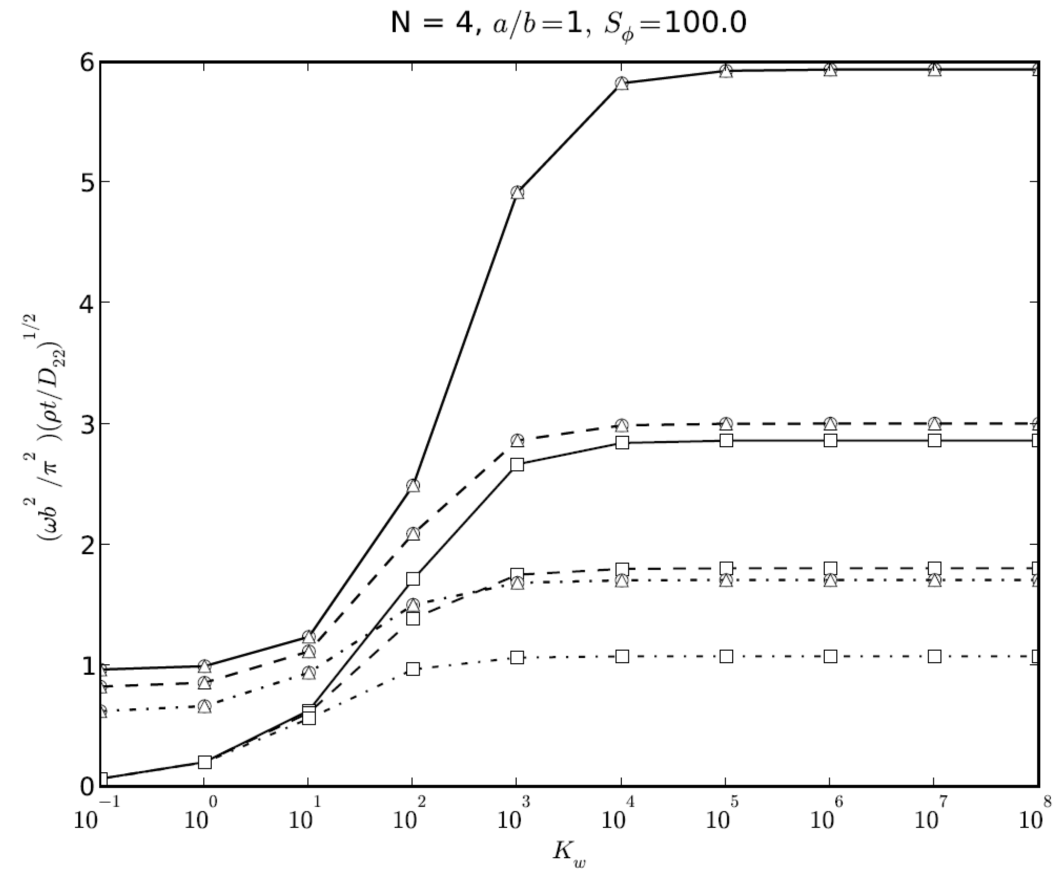

FIGURE 5.8 FREQUENCY PARAMETER, $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})^{1/2}$,VERSUS ELASTICAL LATERAL EDGE RESTRAINED PARAMETER, Kw, FOR SYMMETRIC CROSS-PLY LAMINATED PLATE (KEY:- T/B=0.001;--T/B=0.1;O-.-O T/B=0.2),□ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

In the above figure variation of Non-dimensional frequency parameter has been shown with respect to K_w . Aspect ratio (a / b) has been taken 1. Value of S_ϕ has been taken 10^2. Figure shows that value of Non-dimensional frequency parameter increases slowly with increase in value of K_w till 10^1. As soon as value of K_w increases 10^1 onwards , there is sudden rise in value of Non-dimensional frequency parameter till value of K_w reached 10^3 and after this value there is almost negligible change in value of Non-dimensional frequency parameter

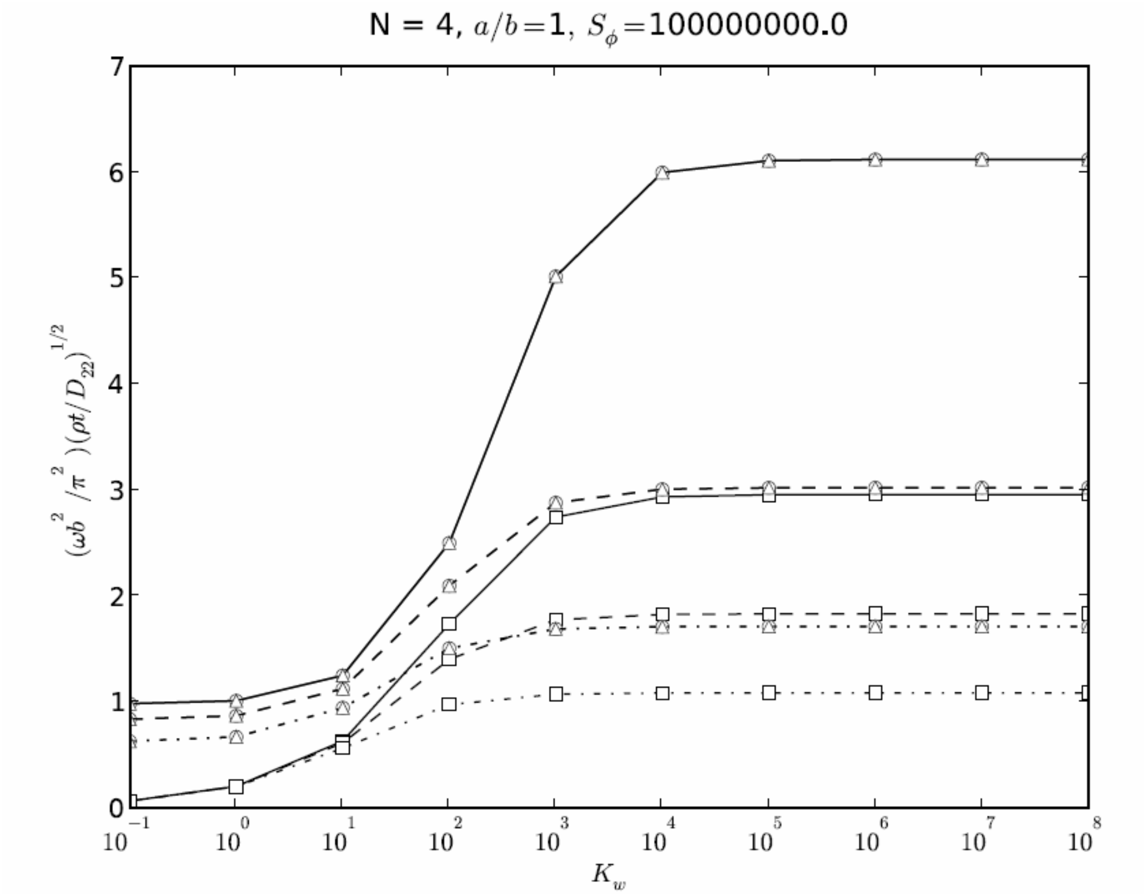

FIGURE 5.9 FREQUENCY PARAMETER, $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})^{1/2}$,VERSUS ELASTICAL LATERAL EDGE RESTRAINED PARAMETER, Kw, FOR SYMMETRIC CROSS-PLY LAMINATED PLATE (KEY:- T/B=0.001;--T/B=0.1;O-.-O T/B=0.2),□ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

In the above figure variation of Non-dimensional frequency parameter has been shown with respect to K_w . Aspect ratio (a / b) has been taken 1. Value of S_ϕ has been taken 10^8. Figure shows that value of Non-dimensional frequency parameter increases slowly with increase in value of K_w till 10^1. As soon as value of K_w increases 10^1 onwards , there is sudden rise in value of Non-dimensional frequency parameter till value of K_w reached 10^3 and after this value there is almost negligible change in value of Non-dimensional frequency parameter

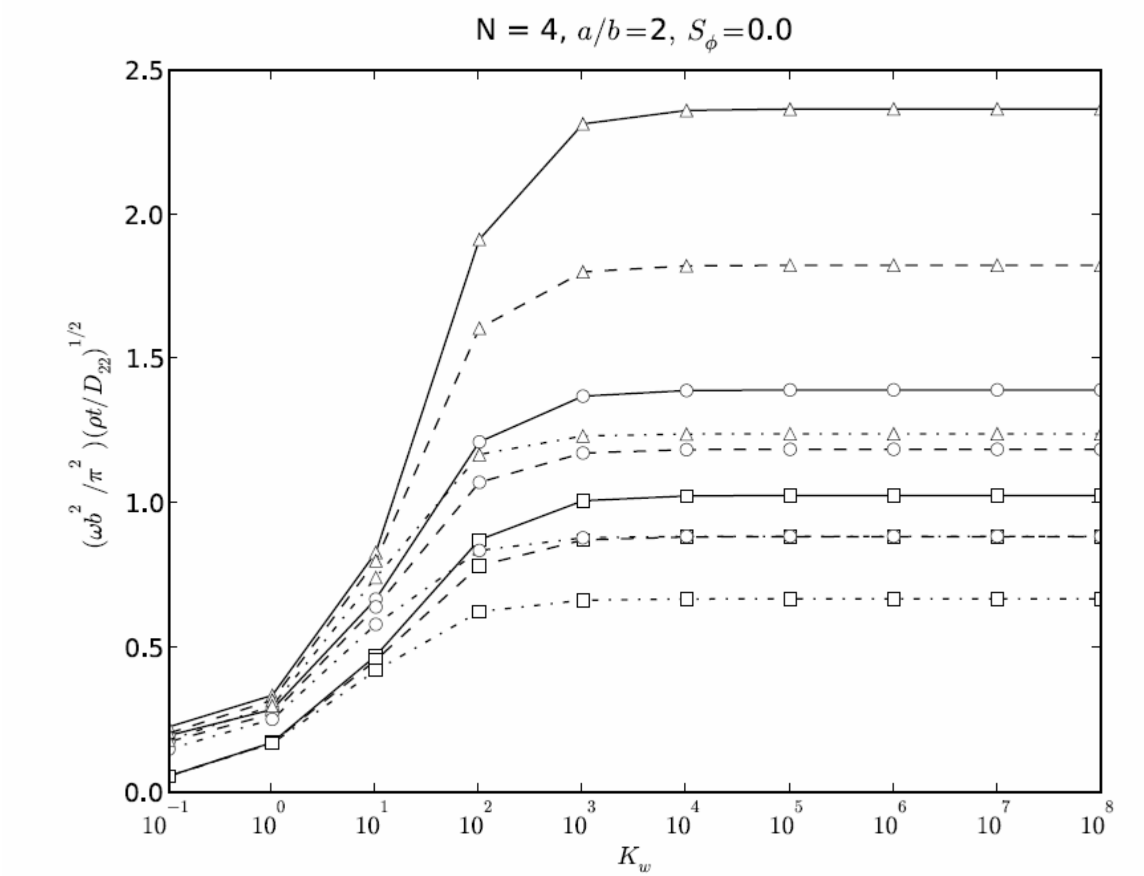

FIGURE 5.10 FREQUENCY PARAMETER, $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})^{1/2}$,VERSUS ELASTICAL LATERAL EDGE RESTRAINED PARAMETER, Kw, FOR SYMMETRIC CROSS-PLY LAMINATED PLATE (KEY:- T/B=0.001;--T/B=0.1;O-.-O T/B=0.2),□ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

In the above figure variation of Non-dimensional frequency parameter has been shown with respect to K_w . Aspect ratio (a / b) has been taken 2. Value of S_ϕ has been taken 0. Figure shows that value of Non-dimensional frequency parameter increases slowly with increase in value of K_w till 10^0. As soon as value of K_w increases 10^0 onwards , there is sudden rise in value of Non-dimensional frequency parameter till value of K_w reached 10^3 and after this value there is almost negligible change in value of Non-dimensional frequency parameter

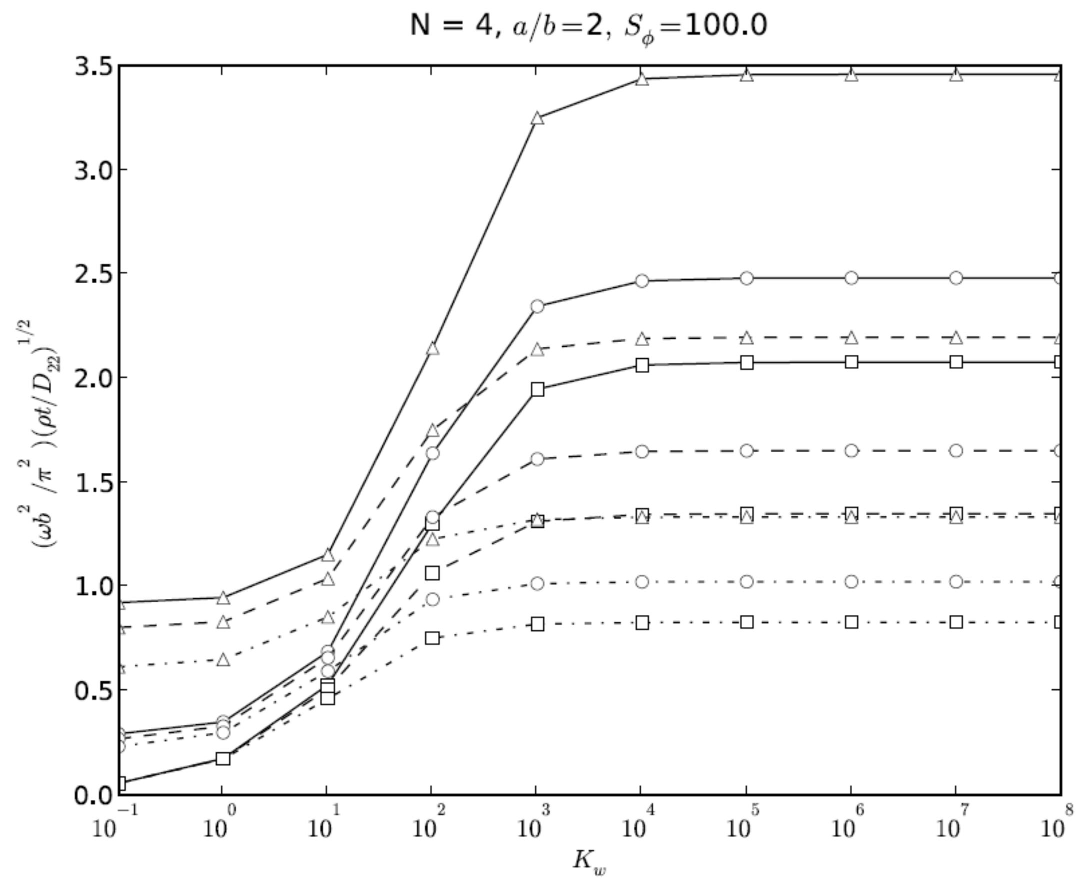

FIGURE 5.11 FREQUENCY PARAMETER, $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})^{1/2}$,VERSUS ELASTICAL LATERAL EDGE RESTRAINED PARAMETER, Kw, FOR SYMMETRIC CROSS-PLY LAMINATED PLATE (KEY:- T/B=0.001;--T/B=0.1;O-.-O T/B=0.2),□ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

In the above figure variation of Non-dimensional frequency parameter has been shown with respect to K_w. Aspect ratio (a / b) has been taken 2. Value of S_ϕ has been taken 10^2. Figure shows that value of Non-dimensional frequency parameter increases slowly with increase in value of K_w till 10^0. As soon as value of K_w increases 10^0 onwards , there is sudden rise in value of Non-dimensional frequency parameter till value of K_w reached 10^3 and after this value there is almost negligible change in value of Non-dimensional frequency parameter

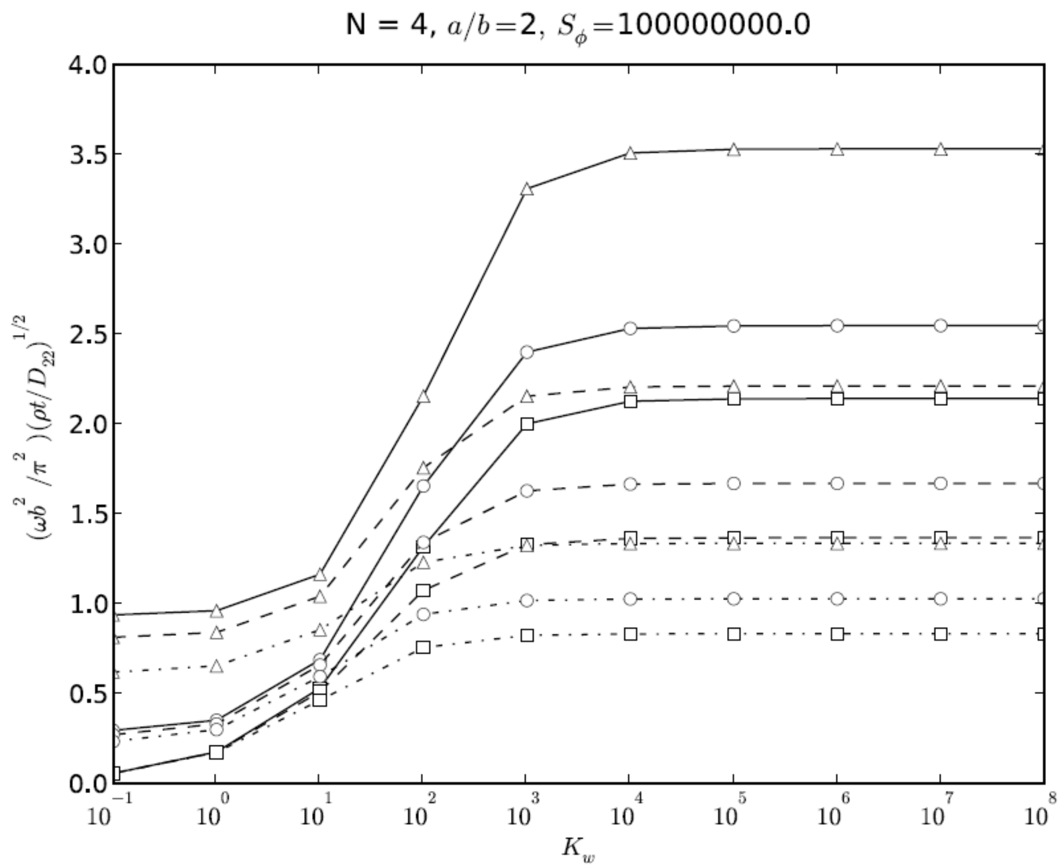

FIGURE 5.12 FREQUENCY PARAMETER, $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})^{1/2}$, VERSUS ELASTICAL LATERAL EDGE RESTRAINED PARAMETER, K_W, FOR SYMMETRIC CROSS-PLY LAMINATED PLATE (KEY:- T/B=0.001;--T/B=0.1;O-.-O T/B=0.2),□ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

In the above figure variation of Non-dimensional frequency parameter has been shown with respect to K_w . Aspect ratio (a / b) has been taken 2. Value of S_ϕ has been taken 10^8. Figure shows that value of Non-dimensional frequency parameter increases slowly with increase in value of K_w till 10^1. As soon as value of K_w increases 10^1 onwards , there is sudden rise in value of Non-dimensional frequency parameter till value of K_w reached 10^3 and after this value there is almost negligible change in value of Non-dimensional frequency parameter

CHAPTER 6

CONCLUSION AND FUTURE SCOPE

6.1 CONCLUTION

Free vibration analysis of angle-ply laminated composite plate with edges elastically restrained against translation and rotation, has been present here with differential quardrature method(DQM). It is observed that the Non-dimensional frequency parameter increases with increasing elastic restraint. The initial rise of frequency parameter with respect to increase of elastic restraint parameter is relatively rapid. However when the elastic restraint parameter is greater than a certain large value (say 10^4), the rate at which frequency parameter approaches the upper bound is relatively slow. This is almost certainly due to the nature of the stiff elastic restraint that are approaching the classical supporting edge conditions of the plate.

Further more it is also observed that the frequency parameter decreases as the plate thickness ratio increases. The frequency parameter of the thicker plate is lower that of a thinner plate due the effect of transverse shear deformation and rotary inertia.

6.2 FUTURE SCOPE

(i) Variation of Non-dimensional parameter can be studied with respect to $S\varphi$.

(ii) DQM method can be applied for free vibration analysis of composite plate with free corner.

(iii) DQM method can be applied for free vibration analysis of composite plate with different kind of support (simply supported, clamped etc.).

(iv) DQM method can be applied for free vibration analysis of thin composite plates with free corners.

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