COST-TIME TRADE OFF PAIRS FOR FIXED CHARGE BI-CRITERION INDEFINITE QUADRATIC TRANSPORTATION PROBLEM WITH ENHANCED FLOW

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DEDICATED

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GOD, MY PARENTS

AND

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CERTIFICATE

*I hereby certify that the work which is being presented in the thesis entitled "**Cost-time trade off pairs for fixed charge bi-criterion indefinite quadratic transportation problem with enhanced flow**" in partial fulfillment of the requirements for the award of degree of Master of Science, School of Mathematics and Computer Applications, Thapar University, Patiala is an authentic record of my own work carried out under the supervision of Dr. Mahesh Kumar Sharma.*

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

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ABSTRACT

The fixed charge transportation problem (FCTP) is an extension of the classical transportation problem in which a fixed cost (for example set up cost) arises in which each origin has a fixed cost coefficient independent of the amount transported in addition to the usual cost coefficients

The Linear functions are the most useful and widely used in operational research. Also quadratic functions and quadratic problems are the least difficult to handle out of all non-linear programming problems. A fair number of functional relationships occurring in the real world are truly quadratic. For example kinetic energy carried by a rocket or an atomic particle is proportional to the square of its velocity .in statistics, the variance of a given sample of observations is a quadratic function of the values that constitute the sample. So there are countless other non-linear relationships occurring in nature, capable of being approximated by quadratic functions.

The fixed charge bi-criteria indefinite quadratic transportation problem with enhanced flow which is an extension of the fixed charge bi-criteria transportation problem has been studied in the present work. Thesis consists of three chapters. Chapter 1 is introductory in nature which includes the some variants of transportation problem and literature related to the topic. In chapter 2 a fixed charge bi-criteria indefinite quadratic transportation problem with enhanced flow (Khurana and Arora, 2011) has been reviewed. In chapter 3, the problem considered in chapter 2 has been modified in which the fixed cost called set up cost is incurred at every origin where the quantity is transported only and new trade off pairs has been obtained.

CONTENTS

CHAPTER 1

"INTRODUCTION"

1 INTRODUCTION

The transportation problem is one of many well-structured in operation research that has been extensively studied in literature. The transportation problem is the subclass of the linear programming problems for which simple and practical computational procedures have been developed that take the advantage of the special structure of the problem.

In the term linear programming refers to the mathematical programming. In the context, it refers to a planning process that allocates resources labour, material, machines, capital in the best possible (optimal) way so that costs are minimized or profits are maximized. In linear programming these resources are known as decision variables. The criterion for selecting the best values of decision variables (e.g., minimizing costs) is known as objective function. Limitations on resource availability from what is known as constraint set.

1.1 COST MINIMIZING TRANSPORTATION PROBLEM

Classical transportation problem is a particular class of linear programming, which is associated with day-to-day activities in our real life and mainly deals with logistics. It helps in solving problems on distribution and transportation of resources from one place to another. The goods are transported from a set of *m* origins (e.g., factories) to a set of *n* destinations (e.g., shops) to meet the specific requirements.

In other words, transportation problems deal with the transportation of a single product manufactured at different plants (supply origins) to a number of different warehouses (demand destinations). The objective is to satisfy the demand at destinations from the supply constraints at the minimum transportation cost possible. To achieve this objective, we must know the quantity of available supplies and the quantities demanded. A cost minimization transportation problem is formulated as

Minimize $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}$

Subject to

$$\sum_{j=1}^{n} x_{ij} \le a_{i}, \qquad a_i > 0, i = 1, 2, ..., m$$

$$\sum_{i=1}^{m} x_{ij} \ge b_{j}, \qquad b_j > 0, \ j = 1, 2, ..., n$$

$$x_{ij} \geq 0 \quad \forall\ i, j$$

where

i=1, 2,…,m is the set of origins.

j=1, 2,…,n is the set of destinations.

x_{ij} =the quantity transported from the i^{th} origin to the j^{th} destination.

c_{ij} =per unit cost in transporting goods from i^{th} origin to the j^{th} destination.

a_i =the amount available at the i^{th} origin.

b_j =the demand of the j^{th} destination.

It is assumed that the total availability is equal to total demand, i.e.

$(\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j)$

Then the problem is a balanced transportation problem, otherwise it is unbalanced.

But in certain life situation, the total availability may not be equals to the total requirement, i.e., $(\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j)$ For example, if supply is greater than demand $(\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j)$ then a fictitious destination may be used to create the desired equality. If the demand exceeds supply $(\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j)$, then a fictitious source may be introduced. The aim is to minimize the objective function satisfying the above constraints. In classical transportation problem in linear programming the traditional objective is to minimize the total cost.

1.2 TIME MINIMIZING TRANSPORTATION PROBLEM

In a time minimizing transportation problem, the time of transporting goods from *m* origins to *n* destinations is minimized, satisfying certain conditions in respect of availabilities and source requirement at the destination.

Time minimization transportation problem is formulated as

Minimize $\max \sum_{i=1}^{m} \sum_{j=1}^{n} [t_{ij} / x_{ij} > 0]$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i$$

$$\sum_{i=1}^{m} x_{ij} = b_j$$

$$x_{ij} \geq 0, \text{for}(\text{i}=1,2.....m;\text{j}=1,2,....n)$$

a_i =the amount available at the i^{th} origin.

b_j =the demand of the j^{th} destination.

The time of transportation remains independent of the amount of commodity sent as long as $x_{ij} > 0$. It is assumed that

(1) The carriers have sufficient capacity to carry goods from an origin to a destination in a single trip.
(2) They start simultaneously from the respective origin.

The time minimizing-transportation problems are of importance when it is required to transport perishable goods. Sometimes there may exists emergency situations such as those requiring police services, fire services, ambulance services, etc.,when the time of transportation is of greater importance than cost of transportation.

Some methods for minimizing the time of transportation have been established. In such situations rather than minimizing the cost ,the objective is to minimize the maximum time to transport all supply to destinations satisfying certain conditions in respect of availabilities at sources and requirements at the destinations.

For any given feasible solution, $X=\left[x_{ij}\right]$ satisfying the above constraints, the time of transportation is the maximum of t_{ij} 's among the cells in which there are positive allocations, corresponding to the solution $X=\left[x_{ij}\right]$, the time of transportation is

$$Z = \max \sum_{i=1}^{m} \sum_{j=1}^{n} [t_{ij} / x_{ij} > 0]$$

The aim is to minimize this time of transportation.

1.3 FIXED CHARGE TRANSPORTATION PROBLEM

The fixed charge transportation problem (FCTP) is an extension of classical transportation problem in which fixed cost is incurred for every origin. The fixed charge transportation problem was originally formulated by Hirsch and Danzig. Many distribution problems in practice can only be modelled as FCTPs. For example, rails, roads and trucks have invariable used freight rates which consist of a fixed cost and a variable cost. The fixed cost may represent the cost of renting a vehicle, landing fees at airport, set up costs for machines in manufacturing environment etc.

The problem can be formulated mathematically as:

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} + \sum_{i=1}^{m} F_i \quad ,$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_{i,} \qquad a_i > 0, i = 1, 2........, m$$

$$\sum_{i=1}^{m} x_{ij} = b_{j,} \qquad b_j > 0, j = 1, 2, ..., n$$

$$x_{ij} \geq 0 \quad \forall \ i, j$$

i=1, 2,…,m is the set of origins.

j=1, 2,…,n is the set of destinations.

x_{ij} =the quantity transported from the i^{th} origin to the j^{th} destination.

c_{ij} =per unit cost in transporting goods from i^{th} origin to the j^{th} destination.

a_i =the amount available at the i^{th} origin.

b_j =the demand of the j^{th} destination.

F_i =the fixed cost associated with origin *i*.

1.4 FIXED CHARGE INDEFINITE QUADRATIC TRANSPORTATION PROBLEM

Formulation of fixed charge indefinite quadratic transportation problem is given as follow

$$\text{Minimize} \quad \left(\sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij}\right)\left(\sum_{i\in I}\sum_{j\in J} d_{ij}x_{ij}\right) + \sum_{i\in I} \mathrm{F}_i,$$

$$\text{Subject to} \quad \sum_{j\in J} x_{ij} = a_i \qquad i\in I$$

$$\sum_{i\in I} x_{ij} = b_j \qquad j\in J$$

$$x_{ij} \geq 0 \qquad i \in I,\ j \in J$$

I=1, 2,…,m is the set of origins.

J=1, 2,…,n is the set of destinations.

x_{ij} =the quantity transported from the i^{th} origin to the j^{th} destination.

c_{ij} =per unit cost in transporting goods from i^{th} origin to the j^{th} destination.

d_{ij} =per unit depreciation cost in transporting goods from i^{th} origin to the j^{th} destination

t_{ij} =the time of transporting the product from i^{th} origin to the j^{th} destination which is independent of the amount of commodity transported, so long x_{ij} >0

F_i =the fixed cost associated with the i^{th} origin which is independent of the amount of commodity transported, for x_{ij} >0

= 0 for x_{ij} =0.

a_i =the amount available at the i^{th} origin.

b_j =the demand of the j^{th} destination.

1.5 MULTI-CRITERIA OPTIMIZATION

Multi-criteria optimization (or multi-objective programming), also known as multi-attribute optimization, is the process of simultaneously optimizing two or more conflicting objectives subject to certain constraints.

Normally, existing multi-objective transportation models use a minimization of the total cost objective as one of their objectives. The other objectives may concern about quantity of goods delivered, energy consumption, total delivery time, etc. Consider m origins and n destinations and also the quantities available at each origin and the quantities to be transported to each destination. The total quantities required at the destinations may differ from the total quantities available at the origins .For such situations; the problem is balanced by introducing fictitious origin or destination; whichever is needed in order to get precisely the same quantities at the origins and the destinations. Specifically, a balanced transportation problem is considered as it amounts to no loss of generality.

Multi –criteria optimization problems can be found in various fields: product and process design, the oil and gas industry or wherever optimal decisions need to be taken in the presence of trade-offs between two or more conflicting objectives.

For nontrivial multi-criteria optimization problems, one cannot identify a single solution that simultaneously minimizes each objective to its fullest. While searching for solutions, one reaches points such that, when attempting to improve an objective further, other objectives suffer as a result. A solution is called non-dominated if it cannot be eliminated from consideration because there is at least another solution which improves an objective without worsening another one.

This multi-criteria problem can be formulated as:

$$\begin{aligned} &\text{Optimize } f(X) = (f_1(X), f_2(X), ..., f_k(X)) \\ &\text{subject to } g_j(X) \leq = \geq b_j,\ j = 1, 2, ..., m \\ &\qquad X \geq 0 \\ &\qquad X = (x_1, x_2, ..., x_n)^T \end{aligned}$$

Where, $f(X)$ is the objective function to optimize. $(f_1(X), f_2(X),..., f_k(X))$ are k number of distinct objective functions subject to m constraints. X is a vector consists of decision making variables $x_1, x_2,..., x_n$.

. 1.6 FIXED CHARGE BI-CRITERION TRANSPORTATION PROBLEM

Most of the practical transportation problems have two objectives: minimizing of cost and minimizing of time. The cost minimizing transportation problem and time minimizing transportation problem cannot be viewed as two independent problems. Most of the methods develop so far have given importance to minimize cost then time or to minimize time then cost.

If one is interested in obtaining a solution which minimizes cost and time simultaneously is called bi-criteria transportation problem. The fixed charge bi-criteria transportation problem is an extension of the bi-criteria transportation problem which is more complex to solve. In the classical transportation problem the cost of transportation is directly proportional to the number of units transported, but on account of quantity discounts, price breaks etc. the transportation may not be linear. Thus in real world situations, when a commodity is transported, a fixed cost is incurred in the objective function. The fixed cost may represent the cost of renting a vehicle, landing fees in a manufacturing environment, a new facility costs money to be constructed etc.

A fixed charge bi-criteria transportation problem formulated as

Minimizing $$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{i=1}^{m} F_i \quad , \quad \max[t_{ij} / x_{ij} > 0]$$

Subject to
$$\sum_{j=1}^{n} x_{ij} = a_i$$
$$\sum_{i=1}^{m} x_{ij} = b_j$$
$$x_{ij} \geq 0, \qquad for \ (i = 1, 2........m; \ j = 1, 2,......n).$$

Where i=1, 2,…,m is the set of origins.

j=1, 2,…,n is the set of destinations.

x_{ij} =the quantity transported from the i^{th} origin to the j^{th} destination.

c_{ij} =per unit cost in transporting goods from i^{th} origin to the j^{th} destination.

a_i =the amount available at the i^{th} origin.

b_j =the demand of the j^{th} destination.

F_i =the fixed cost associated with origin *i*.

t_{ij} = the time of transportation of product from i^{th} origins to the j^{th} destinations.

1.7 FIXED CHARGE BI-CRITERION INDEFINITE QUADRATIC TRANSPORTATION PROBLEM

Linear functions are the most useful and widely used in modelling of mathematical optimization problems. Also quadratic functions and quadratic problems are the least difficult to handle out of all non-linear programming problems. A fair number of functional relationships occurring in the real world are truly quadratic. For example kinetic energy carried by a rocket or an atomic particle is proportional to the square of its velocity .in statistics, the variance of a given sample of observations is a quadratic function of the values that constitute the sample. So there are countless other non-linear relationships occurring in nature, capable of being approximated by quadratic functions.

$$\text{Minimize} \quad \left(\sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij}\right)\left(\sum_{i\in I}\sum_{j\in J} d_{ij}x_{ij}\right)+\sum_{i\in I}\mathrm{F}_i, \quad \max\left[t_{ij} / x_{ij} > 0\right]$$

$$\text{Subject to} \quad \sum_{j\in J} x_{ij} = a_i \qquad i\in I$$

$$\sum_{i\in I} x_{ij} = b_j \qquad j\in J$$

$$x_{ij} \geq 0 \qquad i \in I,\ j \in J$$

I=1, 2,…,m is the set of origins.

J=1, 2,…,n is the set of destinations.

x_{ij} =the quantity transported from the i^{th} origin to the j^{th} destination.

c_{ij} =per unit cost in transporting goods from i^{th} origin to the j^{th} destination.

d_{ij} =per unit depreciation cost in transporting goods from i^{th} origin to the j^{th} destination.

t_{ij} =the time of transporting the product from i^{th} origin to the j^{th} destination which is independent of the amount of commodity transported, so long x_{ij} >0

F_i =the fixed cost associated with the i^{th} origin which is independent of the amount of commodity transported, for x_{ij} >0

= 0 for x_{ij} =0.

a_i =the amount available at the i^{th} origin.

b_j =the demand of the j^{th} destination.

1.8 LITERATURE SURVEY

There are different types of transportation problems and the simplest of them is now standard in the literature was first presented by Hitchcock (1941). It usually aims to minimize the total transportation cost. Other objectives that can be set are a minimization of the total delivery time, a maximization of profits, etc. from the investigation; the entire existing objectives in single objective transportation model are represented by quantitative information. This may cause the negligence of some crucial points which cannot be described by quantitative data.

Later independently, by Koopman(1947) .Koopman began to spearhead research on the potentialities of linear programs for the study of the problems in economics. His historic paper "optimum utilization of the transportation potations systems" was based on his war time experience. Because of this and the work done earlier by Hitchcock, the classical case is often referred as the Hitchcock-Koopman's transportation problem. Kantorovich (1942) publishes the paper on a continuous version of the problem and later with Gavurin, an applied study of the capacitated transportation problem (Kantorovich and Gavurin 1949).

The time minimizing transportation problem has been studied by Hammer (1969),Garfinkel and Rao(1971) and Szwarc (1971).Hammer (1969) and Szwarc(1971) used labeling techniques to solve the problem. Garfunkel and Rao(1971) solved the problem by introducing a sufficiently large cost M on certain routes. Sometimes there may exist emergency situations such as those requiring police services, fire services, ambulance services, etc., when the time of transportation is of greater importance than cost of transportation. Some methods for minimizing the time of transportation have been established. Several methods for minimizing the time of transportation are also developed. Then Bhatia *et.al*.(1975) developed a technique for minimizing time in a transportation problem. The procedure involved finite number of iterations and is based on moving from one basic feasible solution to another till the last solution is arrived at. The algorithm given by them consists of determination of an initial basic feasible solution which can be found by the methods applicable in the case of the common cost minimizing transportation problem and finding an adjacent better basic feasible solution, the procedure is repeated until no better adjacent basic feasible solution can be found. This repeated procedure deals with the determination of a cell not in the basis which, when introduced ,will either reduce the time of transportation or reduce the allocation in at least one of the cells belongs to Q, where Q is the set of cell with positive allocations and corresponding time equal to the time of transportation.

The transportation problem with two-objectives known as the bi-criterion problem has been studied by many research workers. In this type of problem there are two objectives-one primary and the other secondary. The primary objective is to minimize the total cost of transportation problem and the secondary objective is to minimize the duration of transportation.

In a classical transportation the cost of transportation is directly proportional to th3e number of units of the commodity transported. But in real world situations when a commodity is transported, a fixed cost is incurred in objective function. Fixed charge transportation problem has been investigated by many research workers in which a fixed charge is associated with each route that can be opened, in addition to the variable transportation cost proportional to the amount of goods shipped. The fixed charge problem was first formulated by Hirch and Dantzig in (1954). In (1961) Balinski showed the fixed charge problem to the special case of the fixed charge problem and presented as approximate solution. In (1968) Murty devised an exact solution based on searching among the adjacent extreme points of the transportation problem; however, he presented only one sample problem, solved by hand. He pointed out that the method was the most useful for the case in which the fixed charge quite small compared to transportation cost. Paul Gray (1968) uses an alternate approach to Murty ‘s namely a search among the extreme points according to their associated fixed charges. To improve computability, the special structure of the transportation problem is exploited extensively. David R.Denzler described an applied to the set of test problems to explore the margin of error. The result indicates that the proposed fixed charge simplex algorithm is capable of finding optimal or near optimal solutions to moderates the fixed charge problems. In the absence of an exact method, this heuristic should prove useful in solving this fundamental non linear programming problem. Also Basu*et.al.(*1994) developed an algorithm for the optimum time-cost trade-off in a fixed charge linear transportation problem giving same priority to cost and time. Puri and Swarup (1974) established a method of solving the fixed charge problem by breaking it into two problems, one a simple linear programming problem and other a zero-one problem with no constraints of usual type. Bhatia *et.al.*(1976) provided the time-cost trade off pair in a transportation problem. Barr *et.al.(*1981) presented a branch and bound algorithm for solving large scale fixed charge transportation problem where all cells do not exist. The algorithm exploits the absence of full problem density in several ways, thus yielding a procedure which is especially applicable to solving real world problems which are normally quite sparse. Coopper and Drebes(1967) provided two heuristic methods produce optimal solutions in well over 90 percent of the several hundred problems investigated and very close to optimal (a few percent) in the remaining cases. Sandrock (1988) presented a low technology algorithm for the solution of small, fixed charge problems. Prasad *et.al.*(1993) considered that the unit costs and the bottleneck time for the time-cost trade off solutions of the generalized trade-off problem. Also , a direct method is outlined for the case involving a finite set of discrete alternatives of unit cost-time pairs for each pair of supply-demand points. Balinski (1961) has given an approximate method of solving the fixed charge problem. Gottileb and Paulmann (1998) presented two genetic algorithms for FCTP. Both algorithms incorporate knowledge about the properties of optimal solutions. The algorithms mainly differ in the technique used to deal with the inherent constraints of FCTP they compared both genetic algorithms on randomly generated instances.

Basu*et.al.*(1994) developed a technique for solving the fixed charge bi-criterion transportation problem. Arora*et.al.*(1996) developed a fixed charge bi-criterion transportation problem wherein there is a restriction on the total flow is studied. An algorithm to find the efficient cost-time trade off pairs in a fixed charge bi-criteria transportation problem with restriction flow is presented. Gottileb and Paulmann (1998) presented two genetic algorithms for FCTP. Both algorithms incorporate knowledge about the properties of optimal solutions. The algorithms mainly differ in the technique used to deal with the inherent constraints of FCTP; they compared both genetic algorithms on randomly generated instances.

Recently, Khurana and Arora (2011) purposed an algorithm to find an efficient cost-time trade off pairs in a fixed charge bi-criteria quadratic transportation problem is presented. A related fixed charge bi-criteria quadratic transportation problem is formulated and the efficient cost-time trade off pairs to the given problem is shown to be derivable from this related problem.

1.9 PRESENT WORK

In the present thesis a fixed charge bi-criteria indefinite quadratic transportation problem has been studied by Khurana and Arora (2011) has been reviewed in chapter 2. In chapter 3 the problem considered in chapter 2 has been modified in which the fixed cost called set up cost is incurred at every origin where the quantity is transported only and new trade - off pairs has been obtained.

CHAPTER-2

"FIXED CHARGE BI-CRITERION INDEFINITE QUADRATIC TRANSPORTATION PROBLEM WITH ENHANCED FLOW"

2.1 INTRODUCTION

Bi-criteria transportation problem is an extension of single objective transportation problem. In this type of problem there are two objectives: one of minimizing the total cost and the second is minimize the total time of transportation.

The fixed charge bi-criterion indefinite quadratic transportation problem is an extension of the fixed cost called set up cost is incurred for every origin. In the classical transportation problem the cost of transportation is directly proportional to the number of units transported, but on account of quantity, price breaks etc. the transportation may not be linear. Thus in real world situations, when a commodity is transported, a fixed cost is incurred in the objective function. The fixed cost may represent the cost of renting a vehicle, landing fees in an airport, set up costs for machines in a manufacturing environment.

Sometimes, situations arise when because of extra demand in the market; the total flow needs to be enhanced, compelling some of the factories to increase their productions in order to be able to meet the extra demand. The total flow from the factories in the market is now increased by the amount of extra demand. This gives an idea to develop an algorithm to find the cost-time trade off pairs for the case of enhanced flow for a quadratic transportation problem.

In this chapter, we have reviewed the work done by Khurana and Arora (2011). They considered the problem of fixed charge bi-criterion indefinite quadratic transportation problem with enhanced flow. They discussed the case when the flow gets enhanced due to the extra demand in the market for a fixed charge indefinite transportation problem and shall also find the cost-time trade off pairs for the problem.

2.2 FORMULATION OF FIXED CHARGE BI-CRITERION QUADRATIC TRANSPORTATION PROBLEM

Linear functions are the most useful and widely used in modelling of mathematical optimization problems. Also quadratic functions and quadratic problems are the least difficult to handle out of all non-linear programming problems. A fair number of functional relationships occurring in the real world are truly quadratic. For example kinetic energy carried by a rocket or an atomic particle is proportional to the square of its velocity .in statistics, the variance of a given sample of observations is a quadratic function of the values that constitute the sample. So there are countless other non-linear relationships occurring in nature, capable of being approximated by quadratic functions. Consider the fixed charge bi-criteria quadratic transportation problem (FCBQTP) given by Khurana and Arora (2011) formulated the problem.

$$\text{Minimize} \quad \left(\sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij}\right)\left(\sum_{i\in I}\sum_{j\in J} d_{ij}x_{ij}\right) + \sum_{i\in I} \mathrm{F}_i \ , \quad \max_{\substack{i\in I\\ j\in J}} \left[t_{ij} / x_{ij} > 0\right]$$

$$\text{Subject to} \quad \sum_{j\in J} x_{ij} = a_i \qquad i \in I$$

$$\sum_{i\in I} x_{ij} = b_j \qquad j \in J$$

$$x_{ij} \geq 0 \qquad i \in I,\ j \in J$$

$I = 1, 2, ..., m$ is the set of origins.

$J = 1, 2, ..., n$ is the set of destinations.

x_{ij} =the quantity transported from the i^{th} origin to the j^{th} destination.

c_{ij} =per unit cost in transporting goods from i^{th} origin to the j^{th} destination.

d_{ij} =per unit depreciation cost in transporting goods from i^{th} origin to the j^{th} destination.

t_{ij} =the time of transporting the product from i^{th} origin to the j^{th} destination which is independent of the amount of commodity transported, so long x_{ij} >0

F_i =the fixed cost associated with the i^{th} origin which is independent of the amount of commodity transported, for x_{ij} >0

= 0 for x_{ij} =0.

a_i =the amount available at the i^{th} origin.
b_j =the demand of the j^{th} destination.

The total flow in the problem is $\sum_{i \in I} a_i = \sum_{j \in J} b_j$

In the above problem the cost of transporting one unit from the i^{th} origin to the j^{th} destination is $\left(\sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} \right)$, but while transporting goods from one origin to the other destination, some fraction of goods get damaged so the total cost of damaged goods is $\left(\sum_{i \in I} \sum_{j \in J} d_{ij} x_{ij} \right)$. The aim is to minimize the two costs simultaneously; therefore the product of two costs $\left(\sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} \right) \left(\sum_{i \in I} \sum_{j \in J} d_{ij} x_{ij} \right)$ is considered.

Also it is to minimize the fixed cost associated with i^{th} origin and the time of transportation from i^{th} origin to j^{th} destination. If in the problem the total availability is not equal to the total demand, then some of the source and destination constraints are satisfied as inequalities. Sometimes because of extra demand in the market, the total flow from the factories in the market is increased.

Let P(>max($\sum_{i \in I} a_i$, $\sum_{j \in J} b_j$)) be the enhanced flow. This flow constraint changes the structure of the transportation problem .The resulting fixed charge bi-criterion quadratic transportation with

Enhanced flow is (P_1):

$$\text{Minimize} \quad \left(\sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} \right) \left(\sum_{i \in I} \sum_{j \in J} d_{ij} x_{ij} \right) + \sum_{i \in I} \mathrm{F}_i \ , \quad \max_{\substack{i \in I \\ j \in J}} \left[t_{ij} / x_{ij} > 0 \right]$$

$$\text{Subject to} \quad \left.\begin{aligned} & \sum_{j \in J} x_{ij} \geq a_i && i \in I \\ & \sum_{i \in I} x_{ij} \geq b_j && j \in J \\ & \sum_{i \in I} \sum_{j \in J} x_{ij} = P(> Max(\sum_{i \in I} a_i , \sum_{j \in J} b_j)) && \\ & x_{ij} \geq 0 && i \in I, j \in J \\ & c_{ij}, d_{ij} \geq 0, && i \in I, j \in J \end{aligned}\right\} \quad \ldots\ldots\ldots (2.1)$$

2.3 SOLUTION PROCEDURE

For the solution of (FCBQTP with enhanced flow), we separate it into the two problems $\left(P_1^{'}\right)$ and $\left(P_1^{''}\right)$ where

$\left(P_1^{'}\right)$: Minimize the cost function

$$\left(\sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij}\right)\left(\sum_{i\in I}\sum_{j\in J} d_{ij}x_{ij}\right)+\sum_{i\in I}\mathrm{F}_i \quad \text{....subject to constraints (2.1)}$$

$\left(P_1^{''}\right)$: Minimize the time function $\max_{\substack{i\in I\\ j\in J}}\{t_{ij} / x_{ij} > 0\}$ ....subject to constraints (2.1)

For formulation of $F_i = (i=1,2,...,m)$ it is assumed that $F_i = (i=1,2,...,m)$ has p number of steps so that

$$\begin{aligned}
F_i &= \sum_{r=1}^{p}\delta_{ir}F_{ir},\ i=1,2,...,m\\
\text{where } \delta_{i1} &= 1 \quad \text{if } \sum_{j=1}^{n} x_{ij} > K_{i1},\ i=1,2,.....,m\\
&= 0 \quad \text{otherwise;}\\
\delta_{i2} &= 1 \quad \text{if } \sum_{j=1}^{n} x_{ij} > K_{i2},\ i=1,2,.....,m\\
&= 0 \quad \text{otherwise;}\\
\text{and so on...}\\
\delta_{ip} &= 1 \quad \text{if } \sum_{j=1}^{n} x_{ij} > K_{ip},\ i=1,2,.....,m\\
&= 0 \quad \text{otherwise;}
\end{aligned}$$

Here $0 = K_{i1} < K_{i2} <,..............., < K_{ip} < \max\{a_i\}$.

Also $K_{i1}, K_{i2},..............., K_{ip} (i=1,2,.....m)$ are constants and are chosen randomly as defined above $F_{ir}(i=1,2,........,m; r=1,2,.....p)$ are fixed costs.

In order to deal with the flow constraints $\sum_{i\in I}\sum_{j\in J} x_{ij} = P$, a related fixed charge bi –criterion transportation problem is formulated by adding a fictitious factory with availability equal to $(P-\sum_{j\in J} b_j)$ and a fictitious destination with demand equal to $\left(P-\sum_{j\in J} a_i\right)$. Hence the related fixed charge bi-criterion quadratic transportation problem (P_2) is

$$\text{Minimize} \quad \left(\sum_{i\in I}\sum_{j\in J} c'_{ij}x_{ij}\right)\left(\sum_{i\in I}\sum_{j\in J} d'_{ij}x_{ij}\right)+\sum_{i\in I} F_i, \quad \max_{\substack{i\in I\\ j\in J}}\{t'_{ij} / x_{ij} > 0\}$$

$$\text{Subject to} \quad \left.\begin{array}{ll} \sum_{j\in J} x_{ij} = a'_i & i \in I' \\ \sum_{i\in I} x_{ij} = b'_j & j \in J' \\ x_{ij} \geq 0,\ i \in I', j \in J & \end{array}\right\} \text{.........................(2.2)}$$

where

$$I' = \{1, 2, ..., m+1\} = I \cup \{m+1\}$$
$$J' = \{1, 2, ..., m+1\} = J \cup \{n+1\}$$
$$a'_i = a_i \quad i \in I\ , \quad a'_{m+1} = (P - \sum_{j\in J} b_j)$$
$$b'_j = b_j \quad j \in J, \quad b'_{n+1} = (P - \sum_{i\in I} a_i)$$
$$\text{where} \quad P(> \max(\sum_{i\in I} a_i, \sum_{j\in J} b_j))$$

$$c'_{\mathrm{ij}} = c_{ij}\ , \quad (i, j) \in I \times J\ ;\ d'_{\mathrm{ij}} = d_{ij}\ , \quad (i, j) \in I \times J$$
$$t'_{\mathrm{ij}} = t_{ij}\ , \quad (i, j) \in I \times J$$

$$\text{Let} \quad c'_{m+1,j} = c_{lj} \text{ and } d'_{m+1,j} = d_{lj} \text{ such that } c_{lj}d_{lj} = \min_{i\in I}(c_{ij}d_{ij})$$
$$c'_{i,n+1} = c_{ik} \text{ and } d'_{i,n+1} = d_{ik} \text{ such that } c_{ik}d_{ik} = \min_{j\in J}(c_{ij}d_{ij})$$
$$t'_{i,n+1} = t'_{m+1,j} = 0\ ,\ j \in J\ ;\ t'_{m+1,n+1} > \max_{\substack{i\in I\\ j\in J}}\{t'_{ij} / x_{ij} > 0\}$$

$$c'_{m+1,n+1} = \text{M} = d'_{m+1,n+1}$$

$$F_{m+1} = 0 \text{ where M is a large positive number.}$$

(RFCBQTP) is separated into two problems (P_2') and (P_2'').

$$(\text{P}_2'): \quad \text{Minimize} \left(\sum_{i\in I}\sum_{j\in J} c'_{ij}x_{ij}\right)\left(\sum_{i\in I}\sum_{j\in J} d'_{ij}x_{ij}\right) + \sum_{i\in I} \text{F}_i, \text{ ...subject to (2.2)}$$

$$(\text{P}_2''): \quad \underset{\substack{i\in I \\ J\in J}}{\text{Max}}\left[t'_{ij} / x_{ij} > 0\right] \text{ ....subject to constraints (2.2)}$$

2.4 THEORETICAL DEVELOPMENTS

Khurana and Arora (2011) proposed the following theoretical developments related to the problem considered.

THEOREM 1: let $X=\{x_{ij}\}$ be a basic feasible solution of (QTP) with basis matrix B. Then it will be an optimal basic feasible solution if

$$R_{ij} \geq 0 \qquad \forall \text{ cells } (i, j) \notin B$$
$$= 0 \qquad \forall \text{ cells } (i, j) \in B$$

where $R_{ij} = \theta_{ij}(z'_{ij} - d_{ij})(z_{ij} - c_{ij}) - Z_1(z'_{ij} - d_{ij}) - Z_2(z_{ij} - c_{ij})$

$$u_i + v_j = z_{ij} \qquad \forall\ (i, j) \notin B$$
$$u'_i + v'_j = z'_{ij} \qquad \forall\ (i, j) \notin B$$

also
$$\left.\begin{aligned} u_i + v_j &= c_{ij} && \forall\ (i, j) \in B \\ u'_i + v'_j &= d_{ij} && \forall (i, j) \in B \end{aligned}\right\} \text{....................(2.3)}$$

Z_1 and Z_2 respectively equals the value of $\left(\sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij}\right)$ and $\left(\sum_{i\in I}\sum_{j\in J} d_{ij}x_{ij}\right)$ at the current basic feasible solution corresponding to the basis B and θ_{ij} is the level at which a non basic cell (i, j) enters the basis replacing some basic cell of *B*.

Proof: Let Z^0 be the objective value of the problem (*QTP*).

Let $Z^0 = Z_1 Z_2$

Let $\hat{Z}$ be the value of the objective function at the current basic feasible $\hat{X} = [x_{ij}]$ corresponding to the basis B obtained on entering the cell (i, j) into the basis .

$$\text{Then} \quad \hat{Z} = [Z_1 + \theta_{ij}(c_{ij} - z_{ij})][Z_2 + \theta_{ij}(d_{ij} - z'_{ij})]$$
$$\hat{Z} - Z^0 = Z_1 Z_2 + Z_1\theta_{ij}(d_{ij} - z'_{ij}) + Z_2\theta_{ij}(c_{ij} - z_{ij}) + \theta_{ij}^2(c_{ij} - z_{ij})(d_{ij} - z'_{ij}) - Z_1 Z_2$$

$$= Z_1\theta_{ij}(d_{ij} - z'_{ij}) + Z_2\theta_{ij}(c_{ij} - z_{ij}) + \theta_{ij}^2(c_{ij} - z_{ij})(d_{ij} - z'_{ij})$$
$$= \theta_{ij}[Z_1(d_{ij} - z'_{ij}) + Z_2(c_{ij} - z_{ij}) + \theta_{ij}(c_{ij} - z_{ij})(d_{ij} - z'_{ij})]$$

This basic feasible solution will give an improved value of Z if $\hat{Z} < Z^0$,i.e., if

$$\theta_{ij}\ [Z_1(d_{ij} - z'_{ij}) + Z_2(c_{ij} - z_{ij}) + \theta_{ij}(c_{ij} - z_{ij})(d_{ij} - z'_{ij})] < 0 \ .$$

Since $\theta_{ij} \geq 0$

$$\left\{Z_1(d_{ij} - z'_{ij}) + Z_2(c_{ij} - z_{ij}) + \theta_{ij}(c_{ij} - z_{ij})(d_{ij} - z'_{ij}) < 0\right\}, \ldots\ldots., (2.4)$$

Therefore one can move from one basic feasible solution to another basic feasible solution on entering the cell (i, j) into the basis for which condition (2.4) is satisfied.

It will be an optimal basic feasible solution if

$$R_{ij} = \theta_{ij}(z'_{ij} - d_{ij})(z_{ij} - c_{ij}) - Z_1(z'_{ij} - d_{ij}) - Z_2(z_{ij} - c_{ij}) \geq 0$$

Also it can be easily seen that $R_{ij} = 0\ \forall\ cells\ (i, j) \in B$.

Corner feasible solution

When $y_{m+1,n+1}=0$ in the problem (P_2) provides a basic feasible solution $[y_{ij}]\ i \in I^{'}, j \in J^{'}$ to (P_2) is called corner feasible solution (cfs) which implies that bottom right corner cell of the transformed transportation matrix in the problem (P_2) must take the value zero .

Theorem 2: Every corner feasible solution of (P_2) provides a basic feasible solution to (P_1) and conversely.

Proof: let $[y_{ij}]\ , i \in I^{'}\ , j \in J^{'}$ be a cfs to (P_2). Define $y_{ij}=x_{ij}\ ,(i.j) \in I \times J\ , [x_{ij}]$ so defined can be established to be a basic feasible to (P_1).

Conversely, given $[x_{ij}]$ to be a basic feasible solution to (P_1) then $y_{ij}\ , (i.j) \in I^{'} \times J^{'}$

Where $I^{'}=\{1,2,....,m+1\}$

$J^{'}=\{1,2,.....,n+1\}$

defined by the transformation

$$y_{ij}=x_{ij}\ ,(i,j) \in I \times J$$

$$y_{i,n+1}=a_i-\sum_{j \in J} x_{ij}, \quad i \in I$$

$$y_{m+1,j}=b_j-\sum_{i \in I} x_{ij}, \quad j \in J$$

$$y_{m+1,n+1}=0 \ \ (as\ c_{m+1,n+1}=\mathrm{M}=d_{m+1,n+1})$$

can be defined to be a cfs to (P_2) .

Remark 1: A cfs of (P_2) is also a cfs of (P_2').

Remark 2: The value of the objective function of $(P_2^{'})$ at a corner feasible solution is equal to the value of the objective function of $(P_1^{'})$ at its corresponding basic feasible solution.

Remark 3: A non-corner feasible solution to $(P_2^{'})$ can't provide a feasible solution to $(P_1^{'})$.

Remark 4: An optimal solution to $(P_2^{'})$ has to be a corner feasible solution.

Remark 5: Optimal corner feasible solution to $(P_2^{'})$ provides an optimal solution to $(P_1^{'})$.

2.5 ALGORITHM

STEP 1: Given the fixed charge bi-criterion transportation problem .Separate it into two problems $(P_1^{'})$ and $(P_1^{''})$ .let the flow be enhanced to P(>max($\sum_{i\in I} a_i , \sum_{j\in J} b_j$)) . Introduce an additional row with availability=P- $\sum_{j\in J} b_j$ and an additional column with demand=P- $\sum_{i\in I} a_i$.

STEP 2: Form the problem $(P_2^{'})$. Find its initial basic feasible solution $\{y_{ij}^{'}\}$ .Let B be its corresponding basis.

STEP 3: Calculate the fixed cost of the current basic feasible solution and denote it by F^1 (current), where $F^1(\text{current}) = \sum_{i\in I} F_i$.

STEP 4: Find R_{ij} $\forall$ $(i, j) \notin B$

Where $R_{ij} = \theta_{ij}(z_{ij}^{'} - d_{ij})(z_{ij} - c_{ij}) - Z_1(z_{ij}^{'} - d_{ij}) - Z_2(z_{ij} - c_{ij})$

and denote it by $(R_{ij})_1$.

STEP 5: Find $A_{ij}^{'} = (R_{ij})_1 \times (E_{ij})_1$

Where $A_{ij}^{'}$ is the change in the variable cost obtained on introducing a non basic cell ((i, j) with value $(E_{ij})_1$ (for all $(i, j) \notin B$) into the basis .

STEP 6: Find $F_{ij}^{'}$ (difference) =change in fixed cost = $F_{ij}^{'}$ (NB) - F^{1}(current) where $F_{ij}^{'}$ (NB) is the total fixed cost obtained on introducing the variable x_{ij} with value $(E_{ij})_1$ (for all $(i, j) \notin B$) into the current basis to form a new basis.

STEP 7: Find $\Delta_{ij}^{'} = F_{ij}^{1}$ **(**difference)+ $A_{ij}^{'}$ (for all $(i, j) \notin B$)

If all $\Delta_{ij}^{1} \geq 0$, then it is not possible to decrease the total cost i.e.,(variable cost +fixed cost).Go to step 8. But at least one $\Delta_{ij}^{1} < 0$ find min { $\Delta_{ij}^{1} / \Delta_{ij}^{1} < 0$, $(i, j) \notin B$)= Δpq (say) .Then the cell (p,q) enters the basis .

STEP 8: Let Z^1 be the optimal cost of $(P_2^{'})$ yielded by the basic feasible solution $\{y_{ij}^{'}\}$.

STEP 9: Find $\mathrm{T}^1 = \max\limits_{\substack{i \in I \\ j \in J}} \{t_{ij} / y_{ij}^{1} > 0\}$ from the problem $(P_2^{''})$.

Then the corresponding pair (Z^1, T^1) is the first cost –time trade off pair for the problem (P_2) and subsequently for the problem (P_1) . To find the next best cost time trade off pair, go to step 10.

STEP 10: Define $c_{ij}^{1} = \begin{cases} \mathrm{M} & if\ \ t_{ij} \geq \mathrm{T}^1 \\ c_{ij} & if\ \ t_{ij} < \mathrm{T}^1 \end{cases}$

Where M is a sufficiently large positive number, form the corresponding fixed charge quadratic transportation problem with variable cost c_{ij}^{1}. Repeat the above process till we get the problem to be infeasible.

The complete set of cost –time trade off pairs of (P_1) at the end of the qth iteration is given by $(\mathrm{Z}^1,\mathrm{T}^1),(\mathrm{Z}^2,\mathrm{T}^2),,,,,,,,,,(\mathrm{Z}^q,\mathrm{T}^q)$

Where $\mathrm{Z}^1 < \mathrm{Z}_2 < .... < \mathrm{Z}^q$ and $\mathrm{T}^1 > \mathrm{T}^2 > .... > \mathrm{T}^q$.

2.6 NUMERICAL EXAMPLE

The fixed charge bi-criterion quadratic transportation problem with enhanced flow is:

$$\text{Minimize} \quad \left(\sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij}\right)\left(\sum_{i\in I}\sum_{j\in J} d_{ij}x_{ij}\right)+\sum_{i\in I}\mathrm{F}_i \;, \quad \max_{\substack{i\in I\\ j\in J}}\left[t_{ij} / x_{ij} > 0\right]$$

$$\left.\begin{array}{lll}\text{Subject to} & \sum_{j\in J} x_{ij} \geq a_i & i \in I \\ & \sum_{i\in I} x_{ij} \geq b_j & j \in J \\ & \sum_{i\in I}\sum_{j\in J} x_{ij} = P(> Max(\sum_{i\in I} a_i, \sum_{j\in J} b_j)) & \\ & x_{ij} \geq 0 & i \in I, j \in J \\ & c_{ij}, d_{ij} \geq 0, & i \in I, j \in J \end{array}\right\} \ldots\ldots\ldots\ldots (2.1)$$

Table 1 gives the values of variable costs. $c_{ij}'s$ and $d_{ij}'s\,(i=1,2,3;\, j=1,2,3)$ and upper rows shows the $c_{ij}'s$ and lower rows shows the $d_{ij}'s$ and table – 2 gives the values of time $t_{ij}\,(i=1,2,3;\, j=1,2,3)$.

Introducing a dummy source and a dummy destination in table 1 with

$c_{i(n+1)} = c_{iq}$ and $d_{i(n+1)} = d_{iq}$ such that $c_{iq}d_{iq} = \min_{j\in J} c_{ij}d_{ij} \quad \forall i \in I$

$c_{(m+1)j} = c_{pj}$ and $d_{(m+1)j} = d_{pj}$ such that $c_{pj}d_{pj} = \min_{i\in I} c_{ij}d_{ij} \quad \forall j \in J$

$c_{44} = \mathrm{M} = d_{44}$ whereM is a positive large number.

and $\quad b_4 = P - \sum_{i=1}^{3} a_i = 50 - 40 = 10$

$\quad a_4 = P - \sum_{j=1}^{3} b_j = 50 - 40 = 10$

They form the corresponding problem (P_2') . Similarly on introducing a dummy source and dummy destination in table 2 with

$t_{i4} = \min_{j\in J} t_{ij}\ , i = 1,2,3\ ; t_{4j} = \min_{i\in I} t_{ij}, \quad j = 1,2,3; \ t_{44} > \underset{\substack{i\in I\\ j\in J}}{Max}\ t_{ij} = 16$

And taking $t_{44} = 18$ with $a_4 = 10, b_4 = 10$ .They form the corresponding problem $(P_2^{''})$.

Table 1

Destination $j\rightarrow$ Sources $i\downarrow$	1	2	3	a_i
1	5 1	9 2	9 4	19
2	4 3	6 7	2 4	10
3	4 2	1 9	2 5	11
b_j	15	10	15	

Table 2

Destination $j\rightarrow$ Sources $i\downarrow$	1	2	3	a_i
1	8	14	2	19
2	7	5	11	10
3	12	9	16	11
b_j	15	10	15	

The fixed costs are

$F_{11} = 100, F_{12} = 50, F_{13} = 50$
$F_{21} = 150, F_{22} = 50, F_{23} = 50$
$F_{31} = 200, F_{32} = 100, F_{33} = 50$

the total cost which is to be minimized is given by$(\sum_{i=1}^{3}\sum_{j=1}^{3} c_{ij}x_{ij} \sum_{i=1}^{3}\sum_{j=1}^{3} d_{ij}x_{ij}) + \sum_{i=1}^{3} F_i$

where $F_i = \sum_{l=1}^{p} \delta_{il} F_{il}, \quad i = 1,2,.....m$

where $\delta_{i1} = 1$ if $\sum_{j=1}^{3} x_{ij} > 0, \quad i = 1,2,....., m$

$= 0$ otherwise;

$\delta_{i2} = 1$ if $\sum_{j=1}^{3} x_{ij} > 7, \quad i = 1,2,....., m$

$= 0$ otherwise;

$\delta_{i3} = 1$ if $\sum_{j=1}^{3} x_{ij} > 10, \quad i = 1,2,....., m$

$= 0$ otherwise;

Let the flow be enhanced to P=50

Where P=50>max ($\sum_{i=1}^{3} a_i = 40,\ \sum_{j=1}^{3} b_j = 40$)

Introducing a dummy source and a dummy destination in table 1with:

$c_{i(n+1)} = c_{iq}$ and $d_{i(n+1)} = d_{iq}$ such that $c_{iq}d_{iq} = \min_{j\in J} c_{ij}d_{ij} \quad \forall i \in I$

$c_{(m+1)j} = c_{pj}$ and $d_{(m+1)j} = d_{pj}$ such that $c_{pj}d_{pj} = \min_{i\in I} c_{ij}d_{ij} \quad \forall j \in J$

$c_{44} = \mathrm{M} = d_{44}$ where M is a positive large number.

And $b_4 = P - \sum_{i=1}^{3} a_i = 50 - 40 = 10$

$a_4 = P - \sum_{j=1}^{3} b_j = 50 - 40 = 10$

Similarly on introducing a dummy source and dummy destination in table 2 with

$t_{i4} = \min_{j\in J} t_{ij}\ , i = 1,2,3\ ; t_{4j} = \min_{i\in I} t_{ij}, \quad j = 1,2,3;\ t_{44} > \underset{\substack{i\in I\\ j\in J}}{Max}\ t_{ij} = 16$

By taking $t_{44} = 18$ with $a_4 = 10, b_4 = 10$ .they got table 3 and table 4

Table 3

Destination $j \rightarrow$ Sources $i \downarrow$	1	2	3	4
1	5 1	9 2	9 4	5 1
2	4 3	6 7	2 4	2 4
3	4 2	1 9	2 5	4 2
	5 1	1 9	2 4	M M

Table 4

Destination $j \rightarrow$ Sources $i \downarrow$	1	2	3	4
1	8	14	2	2
2	7	5	11	5
3	12	9	16	9
4	7	5	2	18

A basic feasible solution of table 3 is given in table 5.

The right hand side value of table 5 gives the total fixed cost of the current solution

Table 5

Destination $j \rightarrow$ Sources $i \downarrow$	1	2	3	4	
1	5 (9) 1	9 (10) 2	9 4	5 1	200
2	4 3	6 7	2 (10) 4	2 4	200
3	4 (1) 2	1 9	2 5	4 (10) 2	200
4	5 (5) 1	1 9	2 (5) 4	M M	0
					600

$$Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} = 234 \quad , Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{m} d_{ij}x_{ij} = 116$$

Apply step 4 the values of $R_{ij} \ \forall \ (i, j) \notin B$ are obtained which are given in table 6.

Table6

(i, j)	(1,3)	(1,4)	(2,1)	(2,2)	(2,4)	(3,2)	(3,3)	(4,2)
R_{ij}	812	0	342	747	309	550	116	430

Apply step 5 of algorithm the values of A_{ij}^{1} are obtained which are displayed in table 7.

Table 7

(i, j)	(1,3)	(1,4)	(2,1)	(2,2)	(2,4)	(3,2)	(3,3)	(4,2)
A_{ij}^1	4060	0	1710	3735	1545	550	116	2150

Applying step 6 the following results are obtained which are tabulated in table 8.

Table 8

(i, j)	(1,3)	(1,4)	(2,1)	(2,2)	(2,4)	(3,2)	(3,3)	(4,2)
1	200	150	200	200	200	200	200	200
2	200	200	200	200	150	200	200	200
3	200	300	200	200	200	200	200	200
F_{ij} (NB)	600	650	600	600	550	600	600	600
F_{ij} (difference)	0	50	0	0	-50	0	0	0

Apply step 7 of the algorithm the values of Δ_{ij}^1 which are displayed in table 9

Table 9

(i, j)	(1,3)	(1,4)	(2,1)	(2,2)	(2,4)	(3,2)	(3,3)	(4,2)
Δ_{ij}^1	4060	50	1710	3735	1495	550	116	2150

Here all $\Delta_{ij}^1 > 0$ (for all $(i, j) \notin B$)

It is now not possible to decrease the total cost (variable cost +fixed cost) .

Minimum cost Z^1 =(234$\times$116)+600=27144+600=27744 and corresponding time= $T^1 = 14$.

Hence the first cost –time trade off pair is (27744,14).

Define $c_{ij}^1 = \begin{cases} M & \text{if } t_{ij} \geq T^1 = 14 \\ c_{ij} & \text{if } t_{ij} < T^1 = 14 \end{cases}$

On solving the problem, the optimal solution is given in table 10.

Table10

Destination $j \rightarrow$ Sources $i \downarrow$	1	2	3	4	
1	5 (15) 1	M 2	9 4	5 (4) 1	200
2	4 3	6 7	2 (5) 4	2 (5) 4	150
3	4 2	1 (10) 9	M 5	4 (1) 2	300
4	5 1	1 9	2 (10) 4	M M	0
					650

$$Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} = 149 \quad , Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{m} d_{ij}x_{ij} = 191$$

Apply step 4 values of R_{ij} $\forall\ (i, j) \notin B$ are obtained which are given in table 11.

Table11

(i, j)	(1,3)	(2,1)	(2,2)	(3,1)	(4,1)	(4,2)
R_{ij}	1259	223	601	0	81	64

Apply step 5 of algorithm the values of A_{ij}^{1} are obtained which are displayed in table 12.

Table 12

(i, j)	(1,3)	(2,1)	(2,2)	(3,1)	(4,1)	(4,2)
A_{ij}^{1}	5036	1115	3005	0	405	320

Applying step 6 the following results are obtained which are tabulated in table 13.

Table 13

(i, j)	(1,3)	(2,1)	(2,2)	(3,1)	(4,1)	(4,2)
1	200	150	200	200	150	200
2	150	200	200	150	200	200
3	300	300	200	350	300	200
F_{ij} (NB)	650	650	600	700	650	600
F_{ij} (difference)	0	0	-50	50	0	-50

Apply step 7 of the algorithm the values of are Δ_{ij}^{1} obtained which are displayed in table 14

Table 14

(i, j)	(1,3)	(2,1)	(2,2)	(3,1)	(4,1)	(4,2)
Δ_{ij}^{1}	5036	1115	2955	50	405	270

Here all $\Delta_{ij}^{1} > 0$ (for all $(i, j) \notin B$)

It is now not possible to decrease the total cost (variable cost +fixed cost) .

Minimum cost Z^1 =(149$\times$191)+650=28459+600=29109 and corresponding time= T^1 =11.

Hence the second cost –time trade off pair is (29109,11).

Define $c_{ij}^{1} = \begin{cases} \text{M} & \text{if } t_{ij} \geq T^1 = 11 \\ c_{ij} & \text{if } t_{ij} < T^1 = 11 \end{cases}$

On solving the problem, the optimal solution is obtained in table 15.

Table15

Destination j→ Sources i↓	1	2	3	4	
1	5 (14) 1	M 2	9 (5) 4	5 1	200
2	4 (1) 3	6 7	M 4	2 (9) 4	150
3	M 2	1 (10) 9	M 5	4 (1) 2	300
4	5 1	1 9	2 (10) 4	M M	0
					650

$$Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} = 171 \;, Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{m} d_{ij}x_{ij} = 205$$

Apply step 4 the values of $R_{ij}\ \forall\ (i,j) \notin B$ are obtained which are given in table 16.

Table16

(i, j)	(1,4)	(2,2)	(4,1)	(4,2)
R_{ij}	221	499	1435	1640

Apply step 5 of algorithm we get the values of A_{ij}^{1} which are displayed in table 17.

Table 17

(i, j)	(1,4)	(2,2)	(4,1)	(4,2)
A_{ij}^{1}	1989	4491	14350	14760

.

Applying step 6 the following results are obtained which are tabulated in table 18.

Table 18

(i, j)	(1,4)	(2,2)	(4,1)	(4,2)
1	150	200	200	200
2	200	200	150	200
3	300	200	300	200
F_{ij} (NB)	650	600	650	600
F_{ij} (difference)	0	-50	0	-50

Apply step 7 of the algorithm the values of are Δ_{ij}^{1} obtained which are displayed in table 19

Table 19

(i, j)	(1,4)	(2,2)	(4,1)	(4,2)
Δ_{ij}^{1}	1989	4441	14350	14710

Here all $\Delta_{ij}^{1} > 0$ (for all $(i,j) \notin B$)

It is now not possible to decrease the total cost (variable cost +fixed cost).

Minimum cost Z^{1} =(171$\times$205)+650=35055+650=35705 and corresponding time= T^{1} =9.

Hence the third cost –time trade off pair is (35705, 9).

After this iteration, solution is infeasible. Then three cost-time trades off pairs as

{(27744, 14), (29109, 11), (35705,9)}

CHAPTER-3

COST-TIME TRADE-OFF PAIRS FOR FIXED CHARGE BI-CRITERION INDEFINITE QUADRATIC TRANSPORTATION PROBLEM WITH ENHANCED FLOW

INTRODUCTION

In chapter 2 (FCQTP with enhanced flow) in which a fixed cost called set up cost is incurred for every origin is considered. However in real life problems it is possible that the fixed cost may incurred at each origin. In this chapter it is considered that the set up cost is incurred for every cell where the quantity is transported only and the solution procedure is modified. The algorithm used in chapter 2 is applied to the same example to find the cost-time trade off pairs for FCQTP with enhanced flow.

3.1 SOLUTION PROCEDURE

In order to solve the (FCQTP) problem (P_1) we separate it into two problems $(P_1^{'})$ and $(P_1^{''})$

$(P_1^{'})$: Minimize $\left(\sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij}\right)\left(\sum_{i\in I}\sum_{j\in J} d_{ij}x_{ij}\right)+\sum_{i\in I}\mathrm{F}_i$ .subject to constraints(2.1)

$(P_1^{''})$: Minimize the time function $\max_{\substack{i\in I\\ j\in J}}\{t_{ij} / x_{ij} > 0\}$ ….subject to constraints (2.1)

For formulation of $F_i = (i = 1, 2, ..., m)$ it is assumed that $F_i = (i = 1, 2, ..., m)$ has p number of steps so that

$$
\begin{aligned}
F_i \; &= \sum_{r=1}^{p} \delta_{ir} F_{ir}, \\
\text{where } \delta_{i1} &= 1 \quad \text{if } \sum_{j=1}^{n} x_{ij} > K_{i1},\; i = 1, 2, ..., m \\
&= 0 \quad \text{otherwise;} \\
\delta_{i2} &= 1 \quad \text{if } \sum_{j=1}^{n} x_{ij} > K_{i2},\; i = 1, 2, ..., m \\
&= 0 \quad \text{otherwise;} \\
&\text{and so on...} \\
\delta_{ip} &= 1 \quad \text{if } \sum_{j=1}^{n} x_{ij} > K_{ip},\; i = 1, 2, ..., m \\
&= 0 \quad \text{therwise;}
\end{aligned}
$$

Here $0 = K_{i1} < K_{i2} <, ..., < K_{ip} < \max\{a_i\}$.

Also $K_{i1}, K_{i2}, ..., K_{ip} (i = 1, 2, ...m)$ are constants and are chosen randomly as defined above $F_{ir} (i = 1, 2, ..., m; r = 1, 2, ...p)$ are fixed costs.

In order to deal with the flow constraints $\sum_{i\in I}\sum_{j\in J} x_{ij} = P$, a related fixed charge bi –criterion transportation problem is formulated by adding a fictitious factory with availability

equal to $(P - \sum_{j\in J} b_j)$ and a fictitious destination with demand equal to $(P - \sum_{j\in J} a_i)$. Hence the related fixed charge bi-criterion quadratic transportation problem (P_2) is

$$\text{Minimize} \quad \left(\sum_{i\in I}\sum_{j\in J} c'_{ij} x_{ij}\right)\left(\sum_{i\in I}\sum_{j\in J} d'_{ij} x_{ij}\right) + \sum_{i\in I} F_i, \quad \max_{\substack{i\in I\\ j\in J}}\{t'_{ij} / x_{ij} > 0\}$$

$$\text{Subject to} \quad \left.\begin{array}{ll} \sum_{j\in J} x_{ij} = a'_i & i \in I' \\ \sum_{i\in I} x_{ij} = b'_j & j \in J' \\ x_{ij} \ge 0, \ i \in I', j \in J' & \end{array}\right\} \text{..................(3.2)}$$

$$I' = \{1, 2, \ldots\ldots, m+1\} = I \cup \{m+1\}$$
$$J' = \{1, 2, \ldots\ldots, m+1\} = J \cup \{n+1\}$$
$$a'_i = a_i \quad i \in I, \quad a'_{m+1} = (P - \sum_{j\in J} b_j)$$
$$b'_j = b_j \quad j \in J, \quad b'_{n+1} = (P - \sum_{i\in I} a_i)$$
$$\text{where} \quad P(> \max(\sum_{i\in I} a_i, \sum_{j\in J} b_j))$$

$$c'_{ij} = c_{ij}, \quad (i, j) \in I \times J \ ; d'_{ij} = d_{ij}, \quad (i, j) \in I \times J$$
$$t'_{ij} = t_{ij}, \quad (i, j) \in I \times J$$
$$Let\ c'_{m+1,j} = c_{lj} \text{ and } d'_{m+1,j} = d_{lj} \text{ such that } c_{lj} d_{lj} = \min_{i\in I}(c_{ij} d_{ij})$$
$$c'_{i,n+1} = c_{ik} \text{ and } d'_{i,n+1} = d_{ik} \text{ such that } c_{ik} d_{ik} = \min_{j\in J}(c_{ij} d_{ij})$$
$$t'_{i,n+1} = t'_{m+1,j} = 0, \ j \in J \ ; t'_{m+1,n+1} > \operatorname*{Max}_{\substack{i\in I\\ j\in J}}\{t'_{ij} / x_{ij} > 0\}$$
$$c'_{m+1,n+1} = \text{M} = d'_{m+1,n+1}$$
$$F_{m+1} = 0 \text{ where M is a large positive number.}$$

(RFCBQTP) is separated into two problems $(P_2^{'})$ and $(P_2^{''})$.

$(P_2^{'})$: Minimize $\left(\sum_{i\in I}\sum_{j\in J} c^{'}_{ij} x_{ij}\right)\left(\sum_{i\in I}\sum_{j\in J} d^{'}_{ij} x_{ij}\right) + \sum_{i\in I} F_i$, ...subject to (3.2)

$(P_2^{''})$: $\text{Max}_{\substack{i\in I \\ j\in J}} \left[t^{'}_{ij} / x_{ij} > 0\right]$ ....subject to constraints (3.2)

Khurana and Arora (2011) developed some theory related to this problem and is given in previous chapter

(3.2) NUMERICAL EXAMPLE

Table 1 gives the values of variable costs. c_{ij}'s and d_{ij}'s $(i = 1,2,3; j = 1,2,3)$ and upper rows shows the c_{ij}'s and lower rows shows the d_{ij}'s and table – 2 gives the values of time t_{ij} $(i = 1,2,3; j = 1,2,3)$.

The fixed costs for every cell are:

$F_{11} = 100, F_{12} = 50, F_{13} = 50$
$F_{21} = 150, F_{22} = 50, F_{23} = 50$
$F_{31} = 200, F_{32} = 100, F_{33} = 50$

Table 1

Destination $j \rightarrow$ Sources $i \downarrow$	1	2	3	a_i
1	5 1	9 2	9 4	19
2	4 3	6 7	2 4	10
3	4 2	1 9	2 5	11
b_j	15	10	15	

Table 2

Destination $j\rightarrow$ Sources $i\downarrow$	1	2	3	a_i
1	8	14	2	19
2	7	5	11	10
3	12	9	16	11
b_j	15	10	15	

The total cost which is to be minimized is given by $(\sum_{i=1}^{3}\sum_{j=1}^{3} c_{ij}x_{ij}\sum_{i=1}^{3}\sum_{j=1}^{3} d_{ij}x_{ij})+\sum_{i=1}^{3} F_i$

where $F_i = \sum_{l=1}^{p}\delta_{il}F_{il}, \quad i=1,2,.....m$

where $\delta_{i1} = 1$ if $\sum_{j=1}^{3} x_{ij} > 0, \quad i=1,2,.....,m$

$= 0$ otherwise;

$\delta_{i2} = 1$ if $\sum_{j=1}^{3} x_{ij} > 0, \quad i=1,2,.....,m$

$= 0$ otherwise;

$\delta_{i3} = 1$ if $\sum_{j=1}^{3} x_{ij} > 0, \quad i=1,2,.....,m$

$= 0$ otherwise;

Let the flow be enhanced to P=50

Where P=50>max $(\sum_{i=1}^{3} a_i = 40.\sum_{j=1}^{3} b_j = 40)$

Introducing a dummy source and a dummy destination in table 1with:

$c_{i(n+1)} = c_{iq}$ and $d_{i(n+1)} = d_{iq}$ such that $c_{iq}d_{iq} = \min_{j\in J} c_{ij}d_{ij} \quad \forall i \in I$

$c_{(m+1)j} = c_{pj}$ and $d_{(m+1)j} = d_{pj}$ such that $c_{pj}d_{pj} = \min_{i\in I} c_{ij}d_{ij} \quad \forall j \in J$

$c_{44} = \mathrm{M} = d_{44}$ where M is a positive large number.

And $b_4 = P - \sum_{i=1}^{3} a_i = 50 - 40 = 10$

$$a_4 = P - \sum_{j=1}^{3} b_j = 50 - 40 = 10$$

Similarly on introducing a dummy source and dummy destination in table 2 with

$$t_{i4} = \min t_{ij} \ , i = 1,2,3 \ ; t_{4j} = \min t_{ij}, \quad j = 1,2,3; \ t_{44} > \underset{\substack{i \in I \\ j \in J}}{Max} \ t_{ij} = 16$$

and taking $t_{44} = 18$ with $a_4 = 10, b_4 = 10$.

Table 3

Destination $j \rightarrow$ Sources $i \downarrow$	1	2	3	4
1	5 1	9 2	9 4	5 1
2	4 3	6 7	2 4	2 4
3	4 2	1 9	2 5	4 2
4	5 1	1 9	2 4	M M

Table 4

Destination $j \rightarrow$ Sources $i \downarrow$	1	2	3	4
1	8	14	2	2
2	7	5	11	5
3	12	9	16	9
4	7	5	2	18

A basic feasible solution of table 3 is given in table 5.

The right hand side value of table 5 gives the total cost of the current solution.

Table 5

Destination $j \rightarrow$ Sources $i \downarrow$	1	2	3	4	
1	5 (9) 1	9 (10) 2	9 4	5 1	150
2	4 3	6 7	2 (10) 4	2 4	50
3	4 (1) 2	1 9	2 5	4 (10) 2	200
4	5 (5) 1	1 9	2 (5) 4	M M	0
					400

$$Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} = 234 \quad , Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{m} d_{ij}x_{ij} = 116$$

Apply step 4 the values of R_{ij} $\forall \; (i, j) \notin B$ are obtained which are given in table 6.

Table 6

(i, j)	(1,3)	(1,4)	(2,1)	(2,2)	(2,4)	(3,2)	(3,3)	(4,2)
R_{ij}	812	0	342	747	309	550	116	430

Apply step 5 of algorithm the values of A_{ij}^{1} are obtained which are displayed in table 7.

Table 7

(i, j)	(1,3)	(1,4)	(2,1)	(2,2)	(2,4)	(3,2)	(3,3)	(4,2)
A_{ij}^{1}	4060	0	1710	3735	1545	550	116	2150

Applying step 6 the following results are obtained which are tabulated in table 8.

Table 8

(i, j)	(1,3)	(1,4)	(2,1)	(2,2)	(2,4)	(3,2)	(3,3)	(4,2)
1	200	50	150	150	150	150	150	150
2	50	50	200	100	50	50	50	50
3	200	200	200	200	200	100	50	200
F_{ij} (NB)	450	300	550	450	400	300	250	400
F_{ij} (Difference)	50	-100	150	50	0	-100	-150	0

Apply step 7 of the algorithm the values of Δ_{ij}^{1} are obtained which are displayed in table 9

Table 9

(i, j)	(1,3)	(1,4)	(2,1)	(2,2)	(2,4)	(3,2)	(3,3)	(4,2)
Δ_{ij}^{1}	4110	-100	1860	3785	1545	450	-34	2150

In table 9 min { $\Delta_{ij}^{1} / \Delta_{ij}^{1} < 0$, $(i, j) \notin B$) = -100 at (1, 4) cell.

Therefore, the variable to enter the basis is x_{14} and the new solution is given in table 10.

Table 10

Destination $j \rightarrow$ Sources $i \downarrow$	1	2	3	4	
1	5 (5) 1	9 (10) 2	9 4	5 (9) 1	50
2	4 3	6 7	2 (10) 4	2 4	50
3	4 (10) 2	1 9	2 5	4 (1) 2	200
4	5 (5) 1	1 9	2 (5) 4	M M	0
					300

$$Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} = 234 \quad , Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{m} d_{ij}x_{ij} = 116$$

Apply step 4 the values of $R_{ij} \ \forall \ (i, j) \notin B$ are obtained which are given in table 11.

Table 11

(i, j)	(1,1)	(1,3)	(2,1)	(2,2)	(2,4)	(3,2)	(3,3)	(4,2)
R_{ij}	0	812	342	807	345	550	116	654

Apply step 5 of algorithm the values of A_{ij}^1 are obtained which are displayed in table 12.

Table 12

(i, j)	(1,1)	(1,3)	(2,1)	(2,2)	(2,4)	(3,2)	(3,3)	(4,2)
A_{ij}^1	0	4060	1710	807	345	550	580	654

Applying step 6 the following results are obtained which are tabulated in table 13.

Table 13

(i, j)	(1,1)	(1,3)	(2,1)	(2,2)	(2,4)	(3,2)	(3,3)	(4,2)
1	150	100	50	50	50	50	50	50
2	50	50	200	100	50	50	50	50
3	200	200	200	200	200	300	250	200
F_{ij} (NB)	400	350	450	350	300	400	350	300
F_{ij} (difference)	100	50	150	50	0	100	50	0

Apply step 7 of the algorithm the values of Δ_{ij}^1 are obtained which are displayed in table 14.

Table 14

(i, j)	(1,1)	(1,3)	(2,1)	(2,2)	(2,4)	(3,2)	(3,3)	(4,2)
Δ_{ij}^1	100	4110	1860	857	345	650	630	654

Here all $\Delta_{ij}^1 > 0$ (for all $(i, j) \notin B$)

It is now not possible to decrease the total cost (variable cost +fixed cost) .

Minimum cost $Z^1 = (234 \times 116) + 300 = 27144 + 300 = 27444$ and corresponding time= $T^1 = 14$.

Hence the first cost –time trade off pair is (27444,14).

Define $c_{ij}^{1} = \begin{cases} \text{M} & \text{if } t_{ij} \geq T^{1} = 14 \\ c_{ij} & \text{if } t_{ij} < T^{1} = 14 \end{cases}$

On solving the problem, the optimal solution is obtained in table 15.

Table 15

Destination $j \rightarrow$ Sources $i \downarrow$	1	2	3	4	
1	5 (15) 1	M 2	9 4	5 (4) 1	100
2	4 3	6 7	2 (5) 4	2 (5) 4	50
3	4 2	1 (10) 9	M 5	4 (1) 2	100
4	5 1	1 9	2 (10) 4	M M	0
					250

$$Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} = 149 \quad , Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{m} d_{ij}x_{ij} = 191$$

Apply step 4 the values of $R_{ij} \ \forall \ (i, j) \notin B$ are obtained which are given in table 16.

Table 16

(i, j)	(1,3)	(2,1)	(2,2)	(3,1)	(4,1)	(4,2)
R_{ij}	1259	223	601	0	81	64

Apply step 5 of algorithm the values of A_{ij}^{1} are obtained which are displayed in table 17.

Table 17

(i, j)	(1,3)	(2,1)	(2,2)	(3,1)	(4,1)	(4,2)
A_{ij}^1	5036	1115	3005	0	405	320

Applying step 6 the following results are obtained which are tabulated in table 18

Table 18

(i, j)	(1,3)	(2,1)	(2,2)	(3,1)	(4,1)	(4,2)
1	150	100	100	100	100	100
2	50	200	100	50	50	50
3	100	100	100	300	100	100
F_{ij} (NB)	300	400	300	450	250	250
F_{ij} (difference)	50	150	50	200	0	0

Apply step 7 of the algorithm the values of are Δ_{ij}^1 obtained which are displayed in table 19.

Table 19

(i, j)	(1,3)	(2,1)	(2,2)	(3,1)	(4,1)	(4,2)
Δ_{ij}^1	5086	1265	3055	200	405	320

Here all $\Delta_{ij}^1 > 0$ (for all $(i, j) \notin B$)

It is now not possible to decrease the total cost (variable cost +fixed cost) .

Minimum cost $Z^1 = (149 \times 191) + 250 = 28459 + 250 = 28709$ and corresponding time= $T^1 = 11$.

Hence the second cost –time trade off pair is (28709,11).

Define $c_{ij}^1 = \begin{cases} M & \text{if } t_{ij} \geq T^1 = 11 \\ c_{ij} & \text{if } t_{ij} < T^1 = 11 \end{cases}$

On solving the problem, the optimal solution is obtained in table 20.

Table20

Destination $j \rightarrow$ Sources $i \downarrow$	1	2	3	4	
1	5 (14) 1	M 2	9 (5) 4	5 1	150
2	4 (1) 3	6 7	M 4	2 (9) 4	150
3	M 2	1 (10) 9	M 5	4 (1) 2	100
4	5 1	1 9	2 (10) 4	M M	0
					400

$$Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} = 171 \;, Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{m} d_{ij}x_{ij} = 205$$

Apply step 4 the values of $R_{ij} \;\forall\; (i, j) \notin B$ are obtained which are given in table 21.

Table21

(i, j)	(1,4)	(2,2)	(4,1)	(4,2)
R_{ij}	221	499	1435	1640

Apply step 5 of algorithm the values of A_{ij}^{1} are obtained which are displayed in table 22.

Table 22

(i, j)	(1,4)	(2,2)	(4,1)	(4,2)
A_{ij}^{1}	1989	4541	14350	14760

Applying step 6 the following results are obtained which are tabulated in table 23.

Table 23

(i, j)	(1,4)	(2,2)	(4,1)	(4,2)
1	150	150	150	150
2	150	200	150	150
3	100	100	100	100
F_{ij} (NB)	400	450	400	400
F_{ij} (*difference*)	0	50	0	0

Apply step 7 of the algorithm the values of Δ_{ij}^{1} are obtained which are displayed in table 24.

Table 24

(i, j)	(1,4)	(2,2)	(4,1)	(4,2)
Δ_{ij}^{1}	1989	4541	14350	14760

Here all $\Delta_{ij}^{1} > 0$ (for all $(i, j) \notin B$)

It is now not possible to decrease the total cost (variable cost +fixed cost) .

Minimum cost Z^1=(171$\times$205)+400=35055+400=35455 and corresponding time= T^1=9.

Hence the third cost –time trade off pair is (35455, 9).

After this iteration, solution is infeasible. Then three cost-time trades off pairs have been obtained.

{(27444, 14), (28709, 11), (35455, 9)}

(3.3) CONCLUSION

The fixed charge bi-criterion transportation problem with enhanced flow given by Khurana and Arora (2011) is modified. And the three cost-time trade off pairs has been obtained.

REFERENCES

(1) Adlakha , V .and Kowalski ,K.,(2004) , a simple algorithm for the source induced transportation problem , *journal of operational research society* ,55(12), 1275-1280.

(2) Adlakha, V.,Krzysztof K.,Vemuganti, R. and Lev ,B.(2007): More-for-less algorithm for fixed charge transportation problems, Omega, 35,1-20.

(3) Adlakha,V., Kowalski, K. and Lev, B.(2010): A branching method for the fixed charge transportation problem, Omega, 38,393-397.

(4) Aneja ,Y.P. and Nair ,K.P.K (1979),Bicriteria Transportation Problem ,*Management Science, 25*,73-78 .

(5) Arnold, P. Jones and Richard M .Sonald ,(1969) ,*A branch and bound algorithm for multi-level fixed charge transportation problem, Management Science* ,16(1),67-76,.

(6) Arora S. R., Khurana A. (2011). "Fixed charge bi-criteria indefinite quadratic transportation problem with enhanced flow", *REVISTA INVESTIGACION OPERATIONAL*, Vol. 32, pp. 133-145,2011.

(7) ArchanaKhurana, DeepaThirwani and S.R. Arora ,(1997), An Algorithm For Solving Fixed Charge Bi-criterion Indefinite Quadratic Transportation Problem With Restricted Flow .,*International Journal of Optimization :Theory, Methods and* Applications,1(4),367-380.

(8) Balinnski,M.L.,(1961), Fixed Cost Transportation Problem, *Naval Research Logistics Quarterly* ,8,41-54.

(9) Barr, R.S., Glover ,F., and Klingman,D.,(1981),A new optimization method for large-scale fixed charge transportation problem ,*operations research*,29,448-463.

(10) Basu, M., Pal, B.B and Kundu ,A.,(1994), An Algorithm for the optimum time-cost trade-off in a fixed -charge bi-criterion transportation problem,*Optimization*,40,53-68.

(11) Basu, M., Pal, B.B and Kundu ,A.,(1994),An algorithm for finding the optimum solution of solid fixed –charge transportation problem, *Optimization*,31,283-91.

(12) Bhatia ,H.L., Swarup,K. and Puri M.C. (1975), A procedure for time minimization transportation problem, *Indian J.Pure Appl. Math.*,8,920-29.

(13) Cooper, L. and Drebes,C.(1967), An approximate solution method for the fixed charge transportation problem,*Nav.Res.Log.Quart*,14,101-113.

(14) Deepika Gupta,(2009), "Multi-Index Fixed Charge Bi-Criteria Transportation Problem", Msc Thesis Thapar University Patiala.

(15) Garfinkel, R.S and Rao,M.R.(1971),The bottleneck transportation problem *Nav. Res.Log.Quart*.,18,465-472.

(16) Gottileb, J and Paulmann, L., (1998), Genetic algorithm for fixed charge transportation problem, *Evolutionary Computation Proceeding, IEEE World Congress on Computational Intelligence.*, 330-33

(17) Gray, P.(1971), Exact solution of the fixed charge transportation problem, *Operation Research*,19,1529-38.*Nav. Res. Log*

(18) Haley,K.B. (1962),The solid Transportation Problem, *Operations Research*,10,448-63.

(19) Haley,K.B. (1963),The Multi-Index Problem,*Operations Research*,11,368-79.

(20) Hammer,P.L.(1969), Time minimizing transportation problems *Nav. Res. Log. Quart.*,36,345-357.

(21) Hitchcock, F. 1. (1941), the distribution of the product from several sources to numerous localities. *J. Math. Phys*.,20,224-30.

(22) Hirch, W.M. and Dantzig, G.B., (1954), Notes on linear programming: Part-xix, The fixed charge problem, The RAND Corporation ,RM-1383, Santa Monica, California 1954

(23) KantiSwarup, "Operation Research", Publish By Sultan Chand and Sons,p.p.171-175, 205-206.

(24) Kantorovich, L.V., and Gavarin, M.K.,(1949). The Application of Mathematical Methods to Problems of Freight Flow Analysis (Translation).

(25) Kasana, H.S. and Kumar, K.D(2003). *Introductory operationsResearch. Theory and Applications.Springer-Verlag, Berlin.*

(26) Koopman, T.C.(1947). Optimum utilization of the transportation system. *Proc.Intern.Statis..,* Washington, D.C.,5.

(27) Murty, K.A.(1968) Solving the fixed-charge problem by ranking the extreme points,*Operations Research* ,16 ,268-79.

(28) Prakash, S.(1981) Transportation problem with objectives to minimize cost and duration of transportation,*Opsearch* ,18,235-38.

(29) Prasad, V.R., Nair, K.P.K. and Aneja, Y.P.,(1993), A Generalised Time-Cost Trade-Off Transportation Problem, *The Journal of the Operational Research Society*, 44, 1243-1248.

(30) Puri, M.C. and Swaroop,k., (1974) ,Simple enumerative method for fixed charge problem ,*Indian Journal of Puri and Applied Mathematics*,5 (10),894-901.

(31) Ramakrishnan, C.S., (1977), A note on the time minimizing transportation problem, *opsearch*,14, 207-209.

(32) Sandrock, K. (1988), A simple algorithm for solving small fixed-charge transportation problems, *Journal of the Operational Research Society*,39,465-75.

(33) Steinberg, D.I., (1970), The Fixed Charge, *Naval ResearchLogistics Quarterly*,17,217-236.