

**GENERATION AND DYNAMICS OF CAVITY SOLITON IN VCSEL
USING INTERBAND AND INTRABAND GRAPHENE SATURABLE
ABSORBER.**

*A thesis submitted in partial fulfilment of the requirements for the award of
degree of*

Master of Science

in

Physics

Submitted by:

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2019**

CERTIFICATE

I hereby declare that the work which has been presented in this thesis entitled "**Generation and dynamics of cavity soliton in VCSEL using interband and intraband graphene saturable absorber**" is an authentic record of my own work carried out for the partial fulfillment of the requirement for the award of the degree of Masters of Science in Physics at Thapar Institute Of Engineering And Technology , Patiala (Punjab), under the guidance of **Dr. Soumendu Jana**, Associate Professor, School of Physics and Materials Science and refers other researcher's work which are duly listed in the reference section. The intellectual content of this thesis is the product of my own work and contains no material which to a substantial extent has been accepted for the award of any other degree at this or any other educational institution, except where due acknowledgment is made in the thesis.

Date: 15th july, 2019


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It is to certify that the above statement made by the candidate is correct and true to the best of my knowledge and belief.



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Komal Gupta

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I dedicate this work to My parents,

'Sh. Sanjeev Kumar and Smt. Sushma Gupta'

and My brothers

'Ayush Gupta and Anik Gupta'

and My friends who encouraged and supported

me.

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ABSTRACT

In this thesis we study theoretically formation of cavity soliton in Vertical Cavity Surface Emitting Laser (VCSEL) with graphene saturable absorber. Saturable absorber of graphene can be generated through interband and intraband mechanism. The role of both of these interband and intraband has been identified in generation and dynamics of cavity soliton in VCSEL. We show the existence and dynamics and interactions of two or more cavity solitons in infrared and far infrared regions. The effect of replacing a common semiconductor saturable absorber by graphene saturable absorber is highlighted. For greater strength of graphene saturable absorber reduces the required feedback strength. The interaction of cavity solitons for different phase difference and gaps are demonstrated. A comprehensive stability analysis of cavity soliton based on VCSEL embedded with saturable absorber is also done. Parametric zones for stable and unstable cavity solitons are identified. The results obtained by the investigation may lead to experimental realization of cavity soliton with graphene saturable absorber.

CHAPTER-1

INTRODUCTION

1.1 Introduction to Soliton

Soliton is an important topic of pure and experimental research in various fields related to Physics, Biology, chemistry, engineering and even behavioural science. Soliton is basically a confined structure like vibration, beam, wave or pulse that maintains its size and shape in spite of collisions with another such structure. The story of soliton started in the year 1834. During his working time in a shipyard at Union canal in Glasgow- Edinburgh, J. S. Russell monitored a lump of water running along the head of bow of a boat, which continued in the canal even after the boat had stopped. The water wave travelled vigorously without any change or deformity in its shape and after a long while it gets diminished gradually. After nearly a decade's investigation he named such kind of waves as 'translational waves' [1]. He gave the view that the water wave equations likely to have solitary wave solutions. These kind of waves were later named as solitary waves. The soliton concept grows under the conditions of conservative systems, i.e., a system having no loss. The stability of soliton originates from the balance of 'nonlinearity' and 'dispersion'. To understand the concept of soliton let's consider a light wave. What happens to a light wave after travelling some distance? The answer is that after travelling some distance it starts diminishing due to dispersion/ diffraction and/or loss. But what will happen if a wave propagates a long distance and yet does not show any change in its initial size and shape? Is that possible? It sounds to be impossible but it may be true. Generally, a pulse (a beam) broadens due to dispersion. The nonlinearity may create self-phase modulation, which eventually does the contraction of the pulse. When these two phenomena's perfectly counter balance each other one gets a conservative soliton. Now, if there is a loss in the system the soliton will not be stable at all.

To stabilize the soliton in a lossy or dissipative system the loss/(s) during the wave propagation is compensated by an external gain. [2].

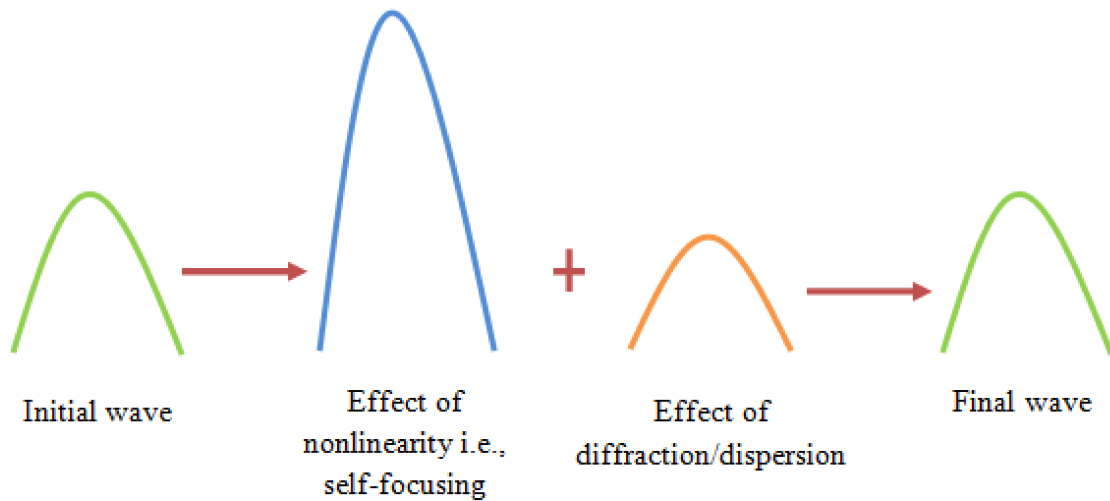


Fig. 1.1 The balance between nonlinearity and diffraction and soliton formation.

In 1847, Stokes gave the solution of the periodic nonlinear waves in deep water[3]. In 1965, Los Alamos et al. studied the energy the flow in the 1D lattice in which same masses were attached by nonlinear springs. They anticipated that the energy which is given in input to wavelength mode will split between all modes of system but they found that system showed the distribution of energy between the few number of modes [4]. In this line an improvement was noticed when Zabusky & Kruskal in 1965 gave the the mathematical conceptualization . However no experimental verification was reported. This was later carried out using KdV equation. They declared that stable wave exists in the system that retains their shape during traveling and retain their shape even after collision with another wave of similar nature. This was called as ‘Soliton’[5]. After this, Gardner et al. gave the solution to existing value problem for solving KdV equation to grow the mathematical mechanism [6]. Then in 1971, Zakharov et al. justified that this method can also work for other nonlinear equations [7]. To solve the ‘inverse scattering transform’ Ablowitz et al., in 1974, gave the technique. In 1985, Airy et al. reported a nonlinear wave equation[8]. Soliton concept has progressed so much in modern physics that it would not

be too dramatic to say that almost any wave can be called as soliton. Solitons can be formed in conservative as well as non-conservative systems. In the conservative system there exist the balance between nonlinearity and diffraction. Whereas in a non-conservative system, apart from above requirement, additional energy flow is provided to bring soliton into existence. The solitons in the non-conservative system are called dissipative solitons. Phenomena of cavity soliton and feedback solitons was given by T. Ackemann, W.J. Firth and prior to them by N. N. Rosanov provided the concept of dissipative solitons in laser cavity with SA.

1.2 Conservative and Dissipative Solitons

As discussed soliton is a localized structure which can maintain shape during its transit and also after the collision [9]. Solitons are the solutions of nonlinear Schrödinger equation in mathematical language, when the diffraction or dispersion effects are counterbalanced by nonlinear terms. Soliton lies in the positive frequency domain, while, linear waves exist under negative domain [10]. It means that both these cannot influence each other. It means that solitons are conserved in nature. So, for the existence of stationary soliton above mentioned condition is required in the conservative system [11]. Such type of conservative solitons system possess various conservative terms like total energy, momentum, etc. However, when some of the perturbations accompany the soliton then the system no longer can produce the conservative soliton [10]. As we know that no practical system under ideal conditions can exist; system always accompany some kind of loss whatever small loss is there. Such types of ‘lossy’ systems hold a different group of solitons known as dissipative solitons (DS) [12]. Dissipative soliton are observed in a range of physical [13, 14, 15], mathematical [16], chemical systems [17], also biological systems [18] and even some natural systems can host soliton. Continuous supply of external energy is the essential requirement of dissipative soliton that keeps the localised structures ‘alive’. This continuous energy flow is not requirement for the conservative soliton. In

this dissertation, the research is inclined to optical CSs, that is a special family of spatial dissipative soliton (SDS).

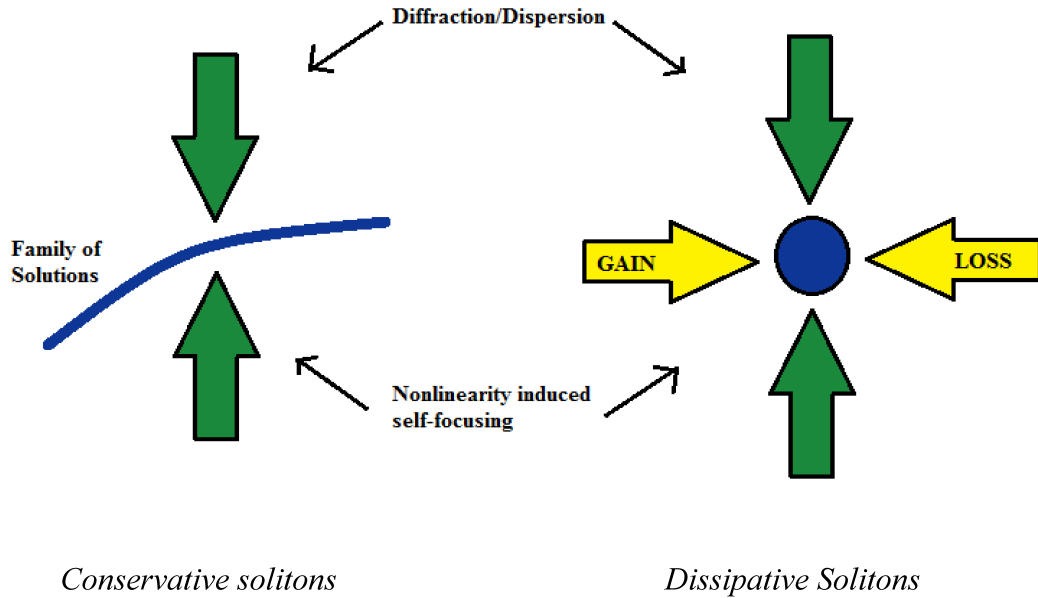


Fig. 1.2 Continuous and discrete solutions in conservative and dissipative systems.

1.3 Cavity Solitons

Cavity soliton(CS) comes under a special family of SDS that can be excited in broad area device particularly in, VCSEL (Vertical cavity surface emitting laser) [19,20]. Optical solitons are self-trapped beam or pulse that face no change in their shape and size neither during transit and nor after the collisions with other localised structure on solitons [21, 22, 23]. The spatial solitons are the result of two competing phenomena: broadening due to self-diffraction and contraction due to self-focusing [24,25].

They can be viewed as bright (dark) spots on the homogeneous dark (bright) background on the transverse plane of the cavity Fig. (1.4). One can excite one/many CS by hitting particular point with writing beam. They take place at a point which is away from equilibrium as the loss is there and therefore it requires the continuous supply of the energy. CS have some properties of SDS in common (Fig. 1.3).

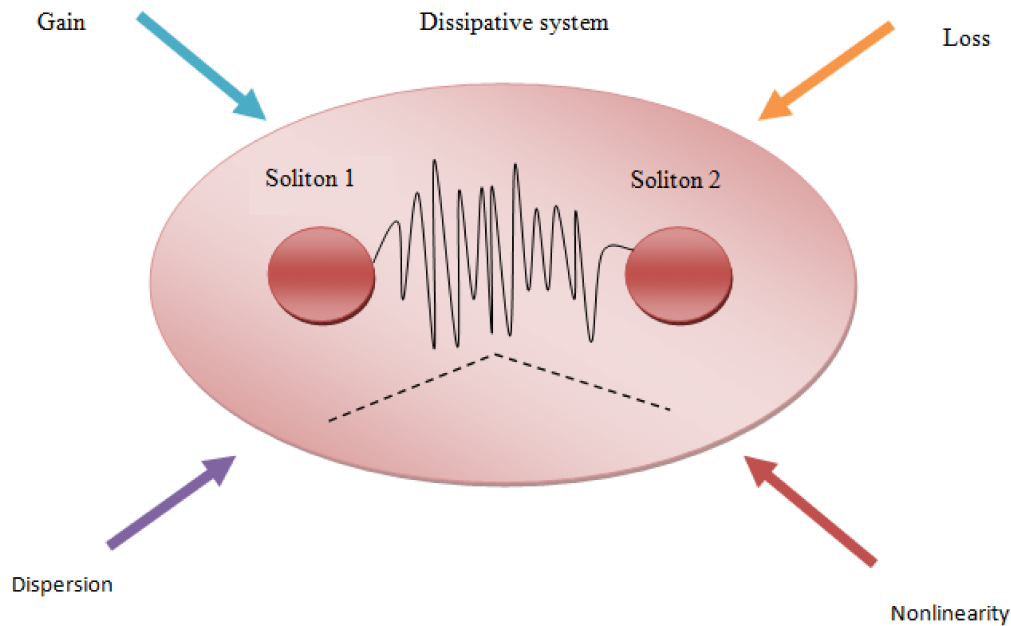


Fig. 1.3 Categorization of dissipative and cavity solitons.

A broad area device like VCSEL is ideal to investigate the generation of CS or cluster of CS moreover the width area is very suitable to trace the dynamics of CS. There exist various VCSEL models to obtain cavity solitons such as VCSEL with holding beam (HB), VCSEL with Bragg reflector, VCSEL with saturable absorber and many more. The cavity losses can be reimbursed by different mechanism of providing gain viz. holding beam [26], frequency selective feedback (FSF) [27, 28, 29, 30] and saturable absorber (SA) [1,31-33] . SA are widely used in VCSEL to excite CSs. Various SA used are semiconductor saturable absorber mirrors (SESAM)[34], lead sulfide suspended in glasses [35], carbon nanotubes (CNT) [36-38], Gallium arsenide. Out of these, SESAM is more pronounced.

CS can be written and erased by the writing beam itself , making them convenient for all-optical erasable memory [26,39- 40]. The cavity soliton can be drifted by introducing suitable gradients in the VCSEL cavity. That drift can be uniform or non uniform.

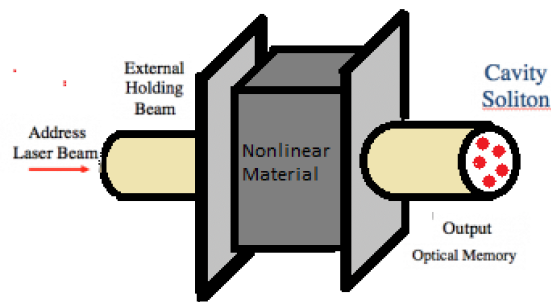


Fig. 1.4 A coherent, stationary, quasipplane-wave holding field drives an optical cavity containing a nonlinear medium.(cavity soliton).

1.4 Recent work done on Cavity soliton

The innovative work in the field of SDS and CS stated in the further section, antecede the path for revolutionary research work in different system contours. In upcoming section we outline the intriguing work on Cavity soliton.

Concept of the dissipative soliton is popular from last few decades. This concept came in 1960 by the development of the classical theory of soliton. Zabusky and Kruskal gave the mathematical concept in 1965 but that time no experimental verification was available. When they finally succeeded in giving experimental verification, they indicated that solitons be generated everywhere.

The soliton concept is divided into three parts, (a) standard soliton theory, which states that formation of soliton because of the balance between the nonlinearity and the diffraction, (b) nonlinear dynamics theory which gave idea about the soliton dynamics and burification, (c) the theory of pattern formation.

Prigogine's idea that gave idea that when there is continuous supply of energy self organised structure of soliton exist. Depending on the supply of energy DS can attain different shapes.

Moloney and his co-workers may be considered as the leading investigators of CS. They found a switching of wave when a Gaussian beam is made to pass through the ring cavity, which has kerr medium in z-direction. VCSEL is an ideal host of cavity soliton system. It was given by

Wilmsen in 1999. The dynamics of cavity soliton in VCSEL with frequency selective feedback and the behavioural changes in system stability was demonstrated by Paulav et al. [55]. The spontaneous dynamics of CS including the time delay feedback was reported in the same year by Tlidi et al. [56,57]. Then Scroggie et al. published the theoretical concept related to the formation of single frequency solitons [58]. After that Prati et al. declared the spontaneous motion in cavity soliton laser of CS with saturable absorber [59]. Firth et al. demonstrated the stability of fundamental and vertical soliton in 2D system in VCSEL with external feedback eliminating delay [60]. Ackeman et al. declared the disordering mapping in VCSEL in further year [61]. The spontaneous dynamics and the motion of the soliton was stated by O.A.Egorov et al. [62]. Generation, spontaneous dynamics and interaction of twin CS in VCSEL with saturable absorber with theoretical proof was reported [63]. Research on CS can be divided in different groups subjecting to the nature of cavity [64, 65], scheme [26] and the operational conditions [66, 67]. Now considering the schemes, there are three basic schemes that attracted the interest of many researchers i.e., (a) cavity soliton by using injected beam [66], (b) cavity soliton with FSF [58] and (c) cavity soliton using SA [68, 69]. Theoretical and experimental evidences of the cavity solitons in the VCSEL was claimed by the F. Prati et al. [67]. Cs can be generated as well as destroyed with same writing beam with opposite phase [69]. To retain the excitations in lossy cavity, holding beam is needed. But HB may create complexity. So HB was replaced with feedback. This self sustained mechanism in system gives rise to CSL [58, 66].

FSF was used to eliminate the need of HB and this makes the system simple. If one can control the feedback, a wide range of stable CS clusters can be found. It can also maintain excitation in lossy system. CSs are studied in a VCSEL with SA. SA in VCSEL is a widely used scheme for CS. SA can create bistability of CS.

1.5 Motivation

Although, SA is widely used, most of the cases a semiconductor saturable absorber mirror is

used. For last some years graphene has projected as a versatile SA. For investigating the generation and dynamics of CSs in the present study we use graphene as saturable absorber. Graphene absorbs over a wide spectral range. Using this property, we studied the existence, dynamics and interaction of CS in the infrared and far infrared range. Thus it creates a new path for research in understanding the peculiar properties of graphene that make it suitable to use it as a SA and also for understanding behaviour of soliton in different wavelength ranges. The SA mechanism of graphene comes from interband and intraband contribution. We aim to identify their contribution in cavity soliton generation and dynamics.

1.6 Objectives

Thus we propose to investigate,

- Generation and dynamics of cavity soliton in VCSEL using GSA in different wavelength ranges for interband and intraband SA domain.
- Stability analysis of cavity soliton in VCSEL with interband and intraband GSA.

CHAPTER-2

THE SYSTEM COMPONENTS AND MATHEMATICAL MODELLING

This section discusses the essential elements of soliton theory and the mathematical modelling.

2.1 The system components:

Cavity solitons are generally formed in VCSEL. The major components are FSF and SA. In our case we use graphene as saturable absorber.

2.1.1 VCSEL (Vertical Cavity Surface Emitting Laser)

Vertical cavity surface emitting laser are modern age of semiconductor laser which was invented in mid of 1980's. Typical VCSEL is shown in Fig.. (2.1). It became the best technology and displaced its rival edge emitting lasers due to its low manufacturing cost and reliability. VCSEL structure is shown in Fig.. (2.2). Using molecular beam epitaxy (MBE) Semiconductor materials layers called epi. are grown on the top of one another and finally results in VCSEL. Between the two Bragg reflected mirrors, active layer is sandwiched in the structure of VCSEL as shown in Fig.. (2.3). These mirrors are called as doped distributed Bragg reflectors (DBR) and the reflectivity of these mirrors is near about 99.5-99.7s%. By the action of those mirrors amplification of radiation takes place and eventually the radiation comes out from the top surface. VCSEL has many advantages. It has long operating lifetime and high output power per unit area, $\sim 1200\text{W}/\text{cm}^2$. VCSEL work efficiently at temperature upto 80°C therefore it is suitable for room temperature operation.

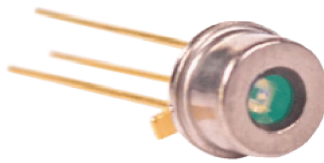


Fig. 2.1 Typical VCSEL

(Image courtesy: https://www.rpphotonics.com/vertical_cavity_surface_emitting_lasers.html)

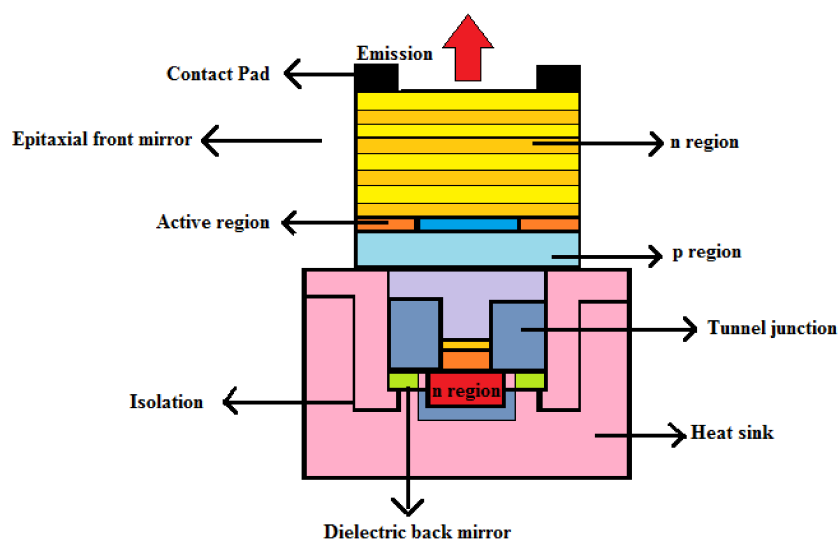


Fig. 2.2 Structure of typical VCSEL

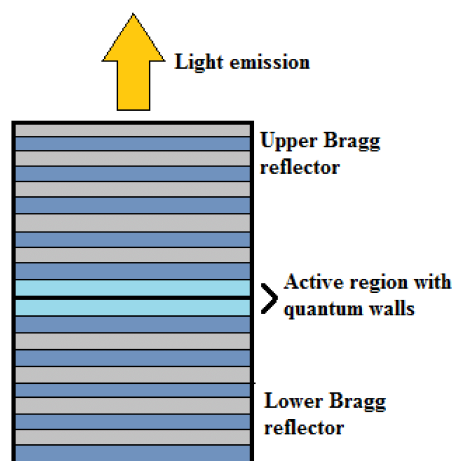


Fig. 2.3 VCSEL structure showing the active region sandwiched between the DBR's

2.1.2 Graphene : Saturable absorber

As discussed earlier CSs are predicted in a vertical-cavity surface-emitting laser with a saturable absorber [41]. A saturable absorber is optical component which have a certain optical loss and is reduced at high optical intensities. Similar effects can occur in semiconductors, where excitation of electrons from the valence band into the conduction band reduces the absorption for photon energies just above the band gap energy. The relation of absorption coefficient α on the intensity of incident laser I is given as $\alpha = \frac{\alpha_0}{(1 + \frac{I}{I_s})}$ [42]. By using SA it is possible to obtain cavity soliton without using HB. Till date various types of saturable absorbers like SESAM [43], PbS[44], CNT's [45, 46, 47] are available. Here we plan to use graphene in VCSEL, thus SA used in VCSEL must have fast recovery time.

Graphene is a 2D material. Graphene is the allotrope of carbon. Each vertex of hexagonal structure is occupied by the carbon atom [48]. Carbon atoms are sp^2 hybridised. It is the strongest of the materials even 200 times stronger than steel. Its electronics and optical properties are explained as the term of mass less Dirac fermions [49]. The most interesting fact about the Graphene is its zero band gap. It is flexible conductor so it can be used in the display of the electronic devices [50]. Graphene has absorption of 2.3%. Recovery time of graphene is very fast of order of few femtoseconds. Because of the gapless structure of graphene, it can be used as the broadband SA[51].

After understanding these things let's concern with our topic of interest i.e. how graphene can be used as SA in VCSEL for obtaining cavity soliton?. Thus saturable absorber used in VCSEL should have very fast recovery time and we found that graphene has fast recovery time of few femtoseconds, it should have high damage threshold, low saturable losses. Hence, properties of graphene fits for the use of SA in VCSEL.

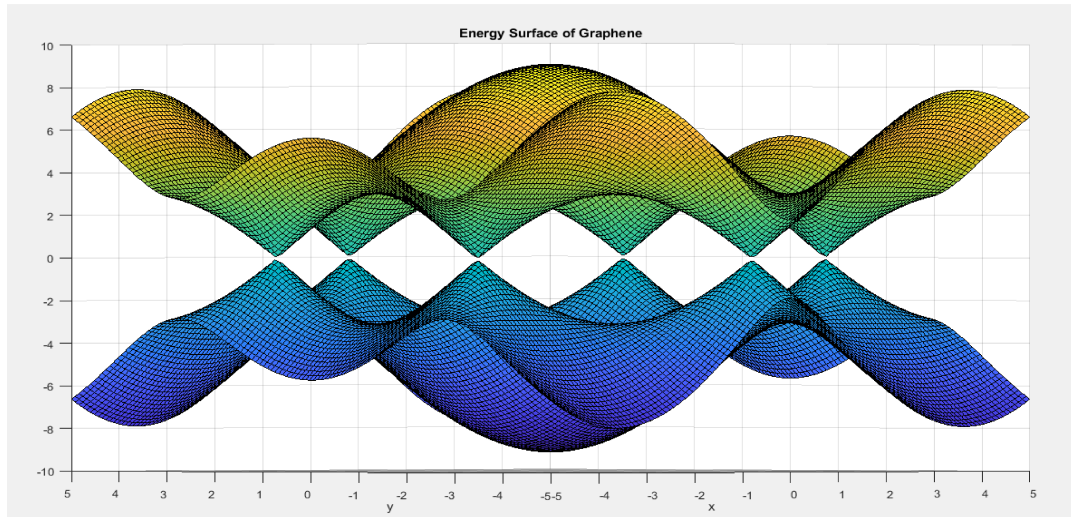


Fig. 2.4 Typical energy band for graphene

2.1.3 Interband and Intraband absorption

Graphene has attracted interests of many researchers in the past decades[52]. It's optical and electronic properties are demonstrated in manner of mass less Dirac fermions [53, 54]. It's exact 2D hexagonal crystal structure leads to absence of energy gap. This gapless band structure gives rise to optical properties in graphene as shown in Fig.. (2.5) which are remarkably different from the conventional semiconductors. In the case of conventional semiconductors, the interband transition occurs when photon energy is greater than bandgap. The case is different in Graphene. In graphene, according to Pauli's blocking the interband transition takes place when the photon energy is greater than twice of Fermi energy E_F as shown in Fig.. (2.5(a)) . If we take example of undoped Graphene, with zero E_F , the interband transition can ideally occurs at low photon energy. On the other hand intraband transition occurs at low energy photon excitation as shown in Fig.. (2.5(b)). The optical absorption of graphene in the far-infrared and infrared regions arises from two types of contributions, those from intraband and from interband optical transitions. Interband lies in range from Mid-IR to UV and Intraband lies in range of Far-IR. The basic comparison that can be made is that intraband is $1/10^{\text{th}}$ of the interband [76].

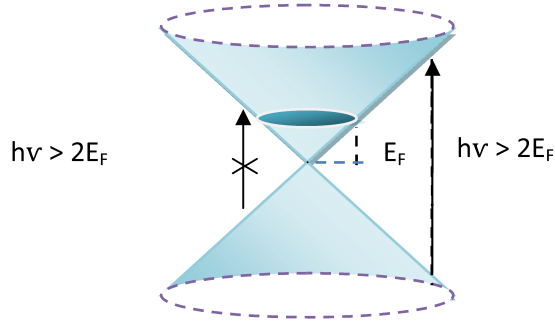


Fig. 2.5(a)

Interband transitions

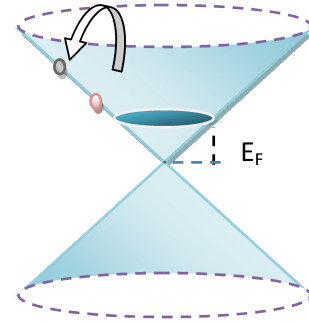


Fig. 2.5(b)

Intraband transitions

Fig. 2.5 Optical absorption of Graphene

2.2 The Mathematical modelling

2.2.1 COMPLEX GINZBURG-LANDAU'S EQUATION(CGLE)

One of the most-studied nonlinear equations in the physics are complex Ginzburg-Landau equation (CGLE). For developing the Ginzburg landau equation, Vitalii Ginzburg received Nobel Prize in Physics 2003 and Alex Abrikosov received Nobel Prize in Physics 2003 for stationary solution to the Ginzburg landau equation. The mathematical modelling of CS generally involves CGLE or we can say that the equation that governs non-linear dynamics of cavity soliton is CGLE. CGLE also describes dynamics in dissipative system. CGLE can be considered as non-linear Schrodinger equation (NLSE) [70].

Following examples in ref [72] of CGLE representing DS read as:

$$\frac{\partial A}{\partial z} = (1 + i\alpha) \frac{\partial^2 A}{\partial x^2} + A - (1 - i\beta)|A|^2 A \quad (2.2)$$

$$\frac{\partial A}{\partial t} = (1 + i\alpha) \frac{\partial^2 A}{\partial x^2} + A - (1 - i\beta)|A|^2 A \quad (2.3)$$

Where,

$A(x, y, z, t)$ is the complex amplitude,

$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the transverse laplace operator,

α is the linear dispersion and, β is the non – linear dispersion..

CGLE for our system is:

$$\frac{\partial E}{\partial t} = \left[-(1 - i\theta) + \frac{\mu(1-i\alpha)}{1+g|E|^2} - \frac{\gamma(1-i\beta)}{1+sg|E|^2} + \alpha_{ns} + i\Delta \right] E + (a - ib)E \quad (2.1)$$

Where,

E is the intracavity electric field of photon; μ, γ are pumping parameter of active and passive medium respectively; α, β are line width factor for active and passive medium respectively ; s is saturation parameter; $a = \frac{\sigma\lambda^2}{\lambda^2 + \Omega_0^2}$ and $b = \frac{\sigma\lambda\Omega_0}{\lambda^2 + \Omega_0^2}$, λ is band width of filter reflector; α_{ns} is non saturable absorption loss; g represent a coefficient of saturation of GSA; θ is detuning parameter; $(1 - i\theta)$ represents linear loss or gain, $\frac{\mu(1-i\alpha)}{1+g|E|^2}$ shows saturable absorption for active medium; $\frac{\gamma(1-i\beta)}{1+sg|E|^2}$ shows saturable absorption for passive medium; Δ is the transverse ∇ operator. ∇E is diffraction term . $(a-ib)E$ is the feedback contribution [71].

The theoretical work on dissipative cavity soliton mainly focus to solve the governing equations (2.1). Both theoretical and numerical techniques have been used in understanding the equations. Theoretical work overcomes the numerical investigations. It's mainly due to the complex or complicated nature of the system. However, for such investigations, split step Fourier method (SSFM), with some modifications is used[73]. In this work we used SSFM, which is a numerical method.

2.2.2 SPLIT STEP FOURIER METHOD (SSFM)

Generally the governing CGLE is non intergrable (through inverse scattering technique). However, some approximate analytical method (variational method) [74] analytical-numerical

method (separation of Eigen value method) [75] have been successfully used to find steady state soliton solutions. Preferably the governing equation is solved through numerical approaches. A pseudo-spectral numerical method i.e., SSFM is used to solve the NLSE. SSFM is used to solve the pulse-propagation problem in non-linear dissipative medium. The SSFM is the approach of choice for solving the NLSE due to its easy operation and faster speed compared to other methods, especially time-domain finite difference methods. The methodology is described as:

We consider a NLSE as:

$$\frac{\partial A}{\partial z} + \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial x^2} - i\gamma |A|^2 A = 0 \quad (2.4)$$

$$\frac{\partial A}{\partial z} = \left[-\frac{i\beta_2}{2} \frac{\partial^2}{\partial x^2} + i\gamma |A|^2 \right] A \quad (2.5)$$

Equation (2) can be written as:

$$\frac{\partial A}{\partial z} = (\hat{D} + \hat{N}) A, \quad (2.6)$$

$$\text{Where, } \hat{D} = -\frac{i\beta_2}{2} \frac{\partial^2}{\partial x^2}, \hat{N} = i\gamma |A|^2 \quad (2.7)$$

$\hat{D}$ is the differential operator that captures dispersion term. $\hat{N}$ is the nonlinear operator. Dispersion and nonlinearity act together. We assume that dispersion and nonlinearity act independently over a small distance h ; so propagation is done from z to $z + h$ in two steps, where h is step size.

Step 1: non-linearity acts alone i.e., $\hat{D} = 0$.

$$\text{Equation (2.6) becomes } \frac{\partial A}{\partial z} = (\hat{N}) A \quad (2.8)$$

By integration from limit $A(z, t)$ to $A(z + h, t)$

$$\int_{A(z,t)}^{A(z+h,t)} \frac{\partial A}{A} = \hat{N} \int_z^{z+h} \partial z \quad (2.9)$$

$$A(z+h, t) = A(z, t) \exp(h\hat{N}) \quad (2.10)$$

Step 2: Dispersion acts alone i.e. , $\hat{N} = 0$

$$\text{Equation (2.6) becomes } \frac{\partial A}{\partial z} = (\widehat{D}) A \quad (2.11)$$

By integration from limit $A(z, t)$ to $A(z+h, t)$

$$\int_{A(z,t)}^{A(z+h,t)} \frac{\partial A}{A} = \widehat{D} \int_z^{z+h} \partial z \quad (2.12)$$

$$A(z+h, t) = A(z, t) \exp(h\widehat{D}) \quad (2.13)$$

Combining step 1 and step 2, the solution becomes

$$A(z+h, t) = \exp(h\widehat{D}) \exp(h\hat{N}) A(z, t) \quad (2.14)$$

Operator $\exp(h\widehat{D})$ can be assessed using Fourier theorem and by changing $\frac{\partial}{\partial t}$ by $-i\omega$. It means that Fourier transformation can convert the time domain to frequency domain. So operator $\widehat{D}$ is expressed as,

$$\widehat{D} = \frac{i\beta_2}{2} \omega^2 \quad (2.15)$$

$$\exp(h\widehat{D}) B(z, t) = F_T^{-1} \exp(h\widehat{D}(-i\omega)) F_T B(z, t) \quad (2.16)$$

The exact solution of equation (2.4) is given as,

$$A(z+h, t) = \exp(h(\hat{N} + \widehat{D})) A(z, t) \quad (2.17)$$

Equation (2.13) can be expressed as:

$$A(z+h, t) \approx \exp\left(\frac{h}{2} \widehat{D}\right) \exp\left(\int_z^{z+h} \hat{N}(z') (dz')\right) \exp\left(\frac{h}{2} \widehat{D}\right) A(z, t) \quad (2.18)$$

This equation (2.18) is called as symmetrised SSFM because, exponential operator is symmetric. $\hat{N}$ operator depend on z . Equation (2.13) and (2.18) become similar if step size h is very small due to which middle value is approximately equal to $\exp(h\hat{N})$.

CHAPTER 3

GENERATION AND DYNAMICS OF CAVITY SOLITON

We solve the governing CGLE that corresponds to the microcavity of VCSEL coupled with frequency selective feedback (FSF). Importantly, the VCSEL has a GSA layer. We investigate CS formation emphasizing the interband and intraband contribution of GSA. The interband SA becomes dominant at lower wavelength zone, while intraband SA becomes prominent. The spontaneous and interaction dynamics of CS at different wavelengths is shown in preceding section. The optical absorption of Graphene in the far-infrared and infrared regions arises from two types of contributions intraband and interband optical transitions. Interband lies in range from Mid-IR to UV and Intraband lies in range of Far-IR. The spontaneous and interaction dynamics of the soliton at different wavelength are shown in preceding section.

Section 1: Generation and spontaneous drift of cavity soliton with intraband and interband GSA

The Fig. (3.1) above show the generation dynamics of CS at constant wavelength of 700 nm as interband lies in between Mid-IR to UV. The dynamics of CS is affected by changing the saturable nonlinearity term 'g' and corresponding feedback strength 'σ'. In Fig. 3.1(a), firstly solitons interact then moves slowly apart but when we increase the nonlinearity the velocity of the solitons increases as in Fig.. 3.1 (b).

Other parameters are $\theta = 1.1, \mu = 1.1, \alpha = 2.5, \beta = 0.05, \gamma = 0.3, \lambda = 0.7, s = 10$ which remains constant throughout.

.

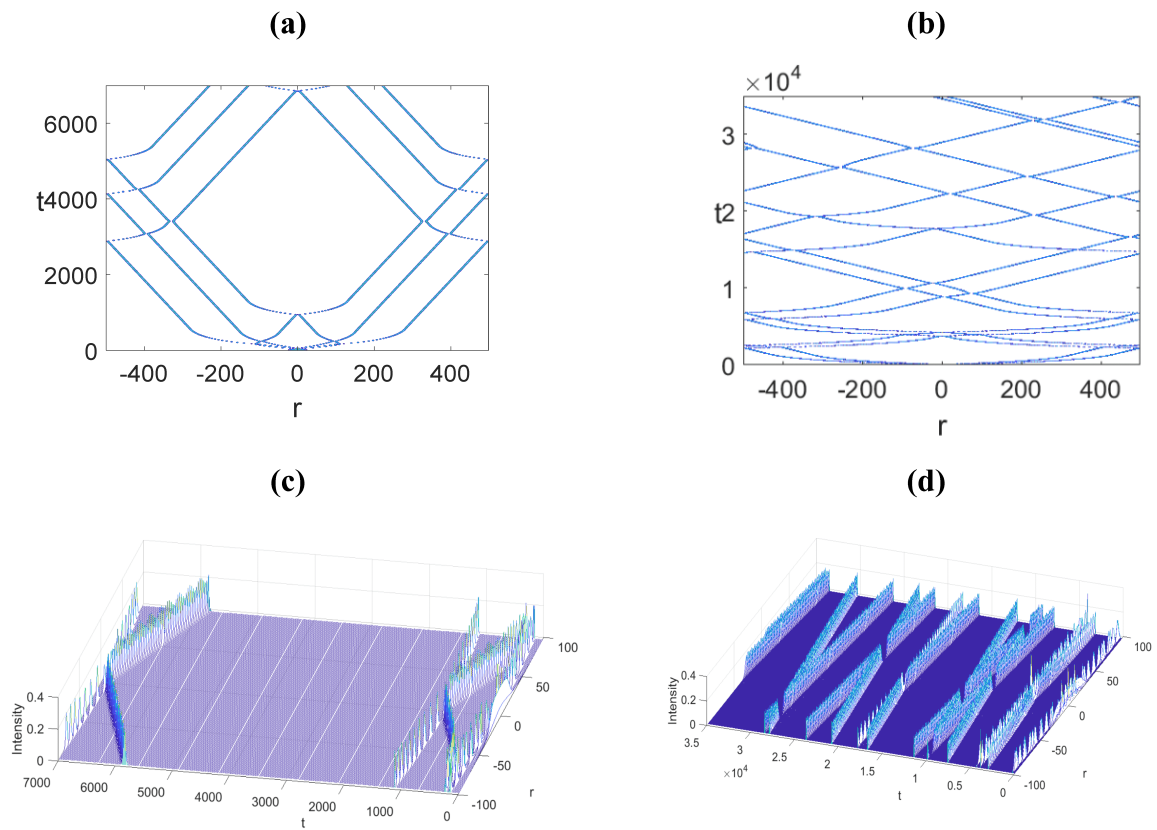


Fig. 3.1 Interband at constant wavelength of 700nm
 (a) The generation and dynamics of CS at $g = 4, \sigma = 0.41$
 (b) The generation and dynamics of CS at $g = 9, \sigma = 0.15$
 (c and d) are the corresponding 3D plots.

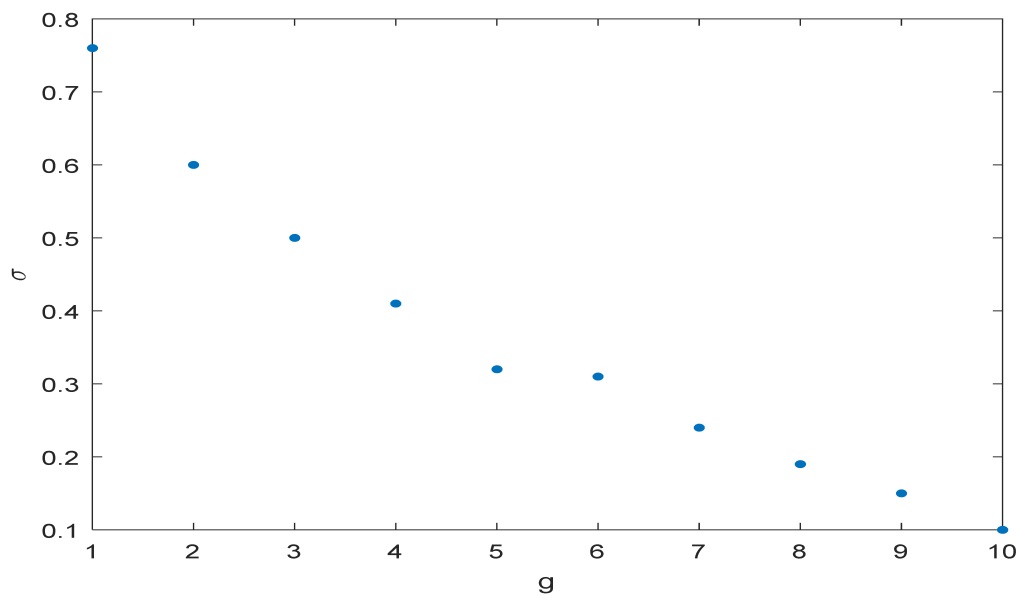


Fig. 3.2 Variation of g with respect to σ in case of Interband

The Fig. 3.2 shows the variation of g with respect to σ , as the nonlinearity increases, the feedback coefficient σ decreases. This variation of g with respect to σ is almost same in case of intraband.

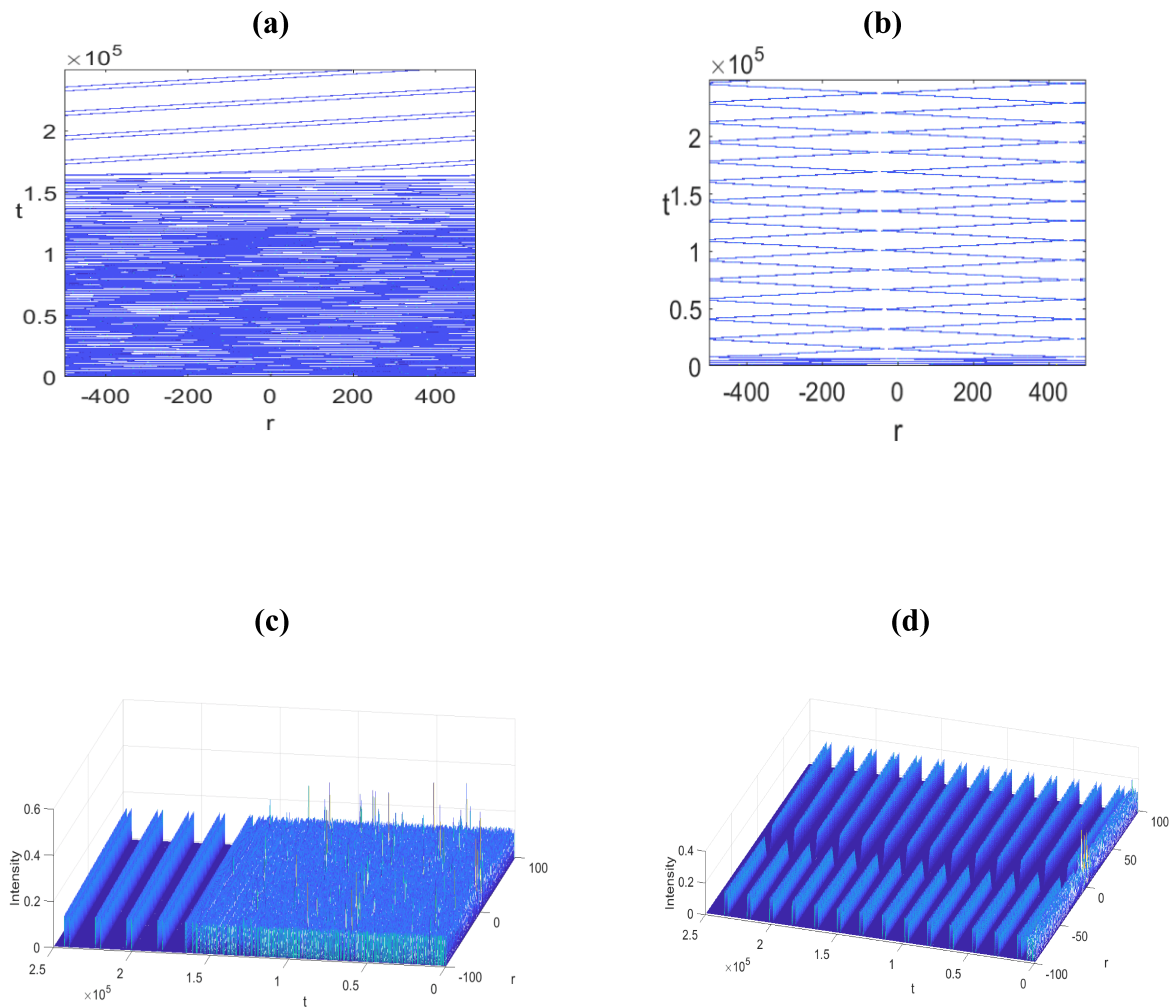
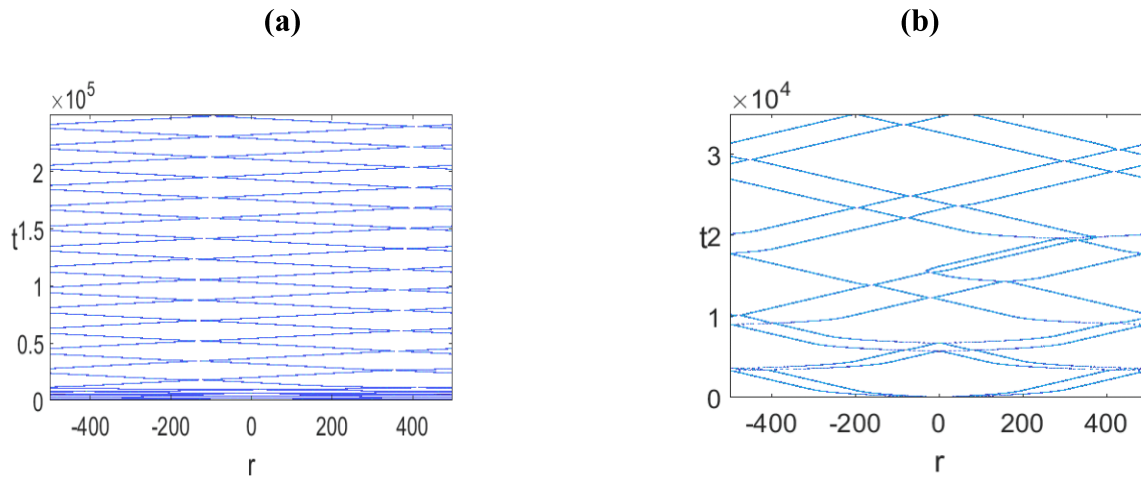


Fig. 3.3 Intraband at constant wavelength of 25000nm
 (a) The dynamics of CS at $g = 0.6, \sigma = 0.92$
 (b) The dynamics of CS at $g = 7, \sigma = 0.88$
 (c and d) are the corresponding 3D plots

The above Fig. 3.3 show the dynamics of CS at constant wavelength of 25000 nm as intraband lies in range of far IR. In Fig. 3.3 (a) cavity soliton at first are moving so fast that their dynamics are hard to understand (can be seen as clumsy part). After some time the velocity decreases and

cavity soliton moves uniformly in one direction. When the nonlinearity increases the solitons undergoes uniform oscillations as in Fig. 3.3(b).



*Fig. 3.4 Comparison of generation and dynamics of CS in interband and intraband saturable absorption
(a) Intraband at $g=0.8$, $\sigma = 0.83$ (b) Interband at $g=8$, $\sigma = 0.19$*

Fig. (3.4) shows the comparison of intraband and interband absorption at nonlinearity value with the ratio of 1/10 because the intraband is 1/10th of the interband absorption. Fig. 3.4(a) demonstrates the dynamics of cavity soliton in light of intraband absorption and on the other hand Fig. 3.4 (b) shows dynamics in the interband absorption. The comparison that can be made is that in intraband absorption (Fig. 3.4(a)), the soliton initially moves very fast which can be seen as clumsy part and after some time it oscillates with uniform velocity, which is less than the initial velocity. Considering the interband absorption (Fig. 3.4(b)), the soliton oscillates with velocity which is very less as compared to intraband.

It was concluded that the velocity of soliton in case of intraband absorption is much higher as compared to interband absorption.

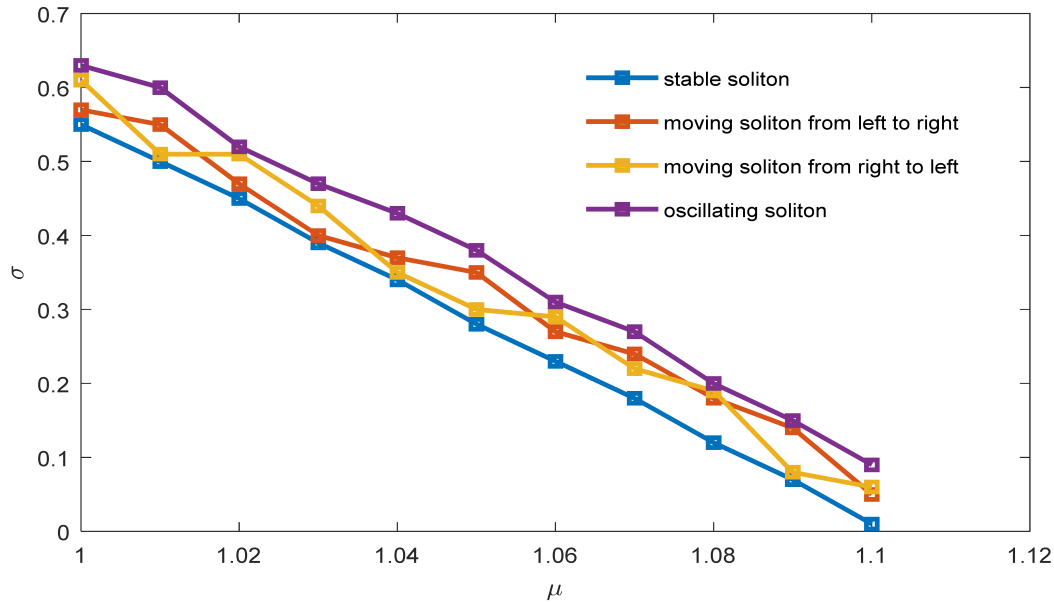


Fig. 3.5 Multi stability of CS in case of interband. Variation of μ creates four states of CS at four different sets of σ . The snaking behaviour between two moving states (left to right and right to left) are sandwiched between stable and oscillating soliton.

The variation of system parameters may lead to intriguing dynamical states of soliton. In order to explore that we vary pump parameter μ and find CS for different feedback strength σ at a fixed GSA strength. Interestingly we find four states of CS for a particular μ , however with different σ . Fig. 3.5 shows four lines that correspond to stable CS, left to right moving CS, right to left moving CS and oscillating CS. These four curves implies that CS can have multi stable states. One can see kind of snaking in between stable and oscillating soliton curves. This snaking is a signature feature of cavity soliton system. The Fig. 3.5 shows multi stability of CS which can be manipulated for all optical switching device applications.

Section 2: Interaction dynamics of cavity soliton with intraband and interband GSA.

A. INTERBAND

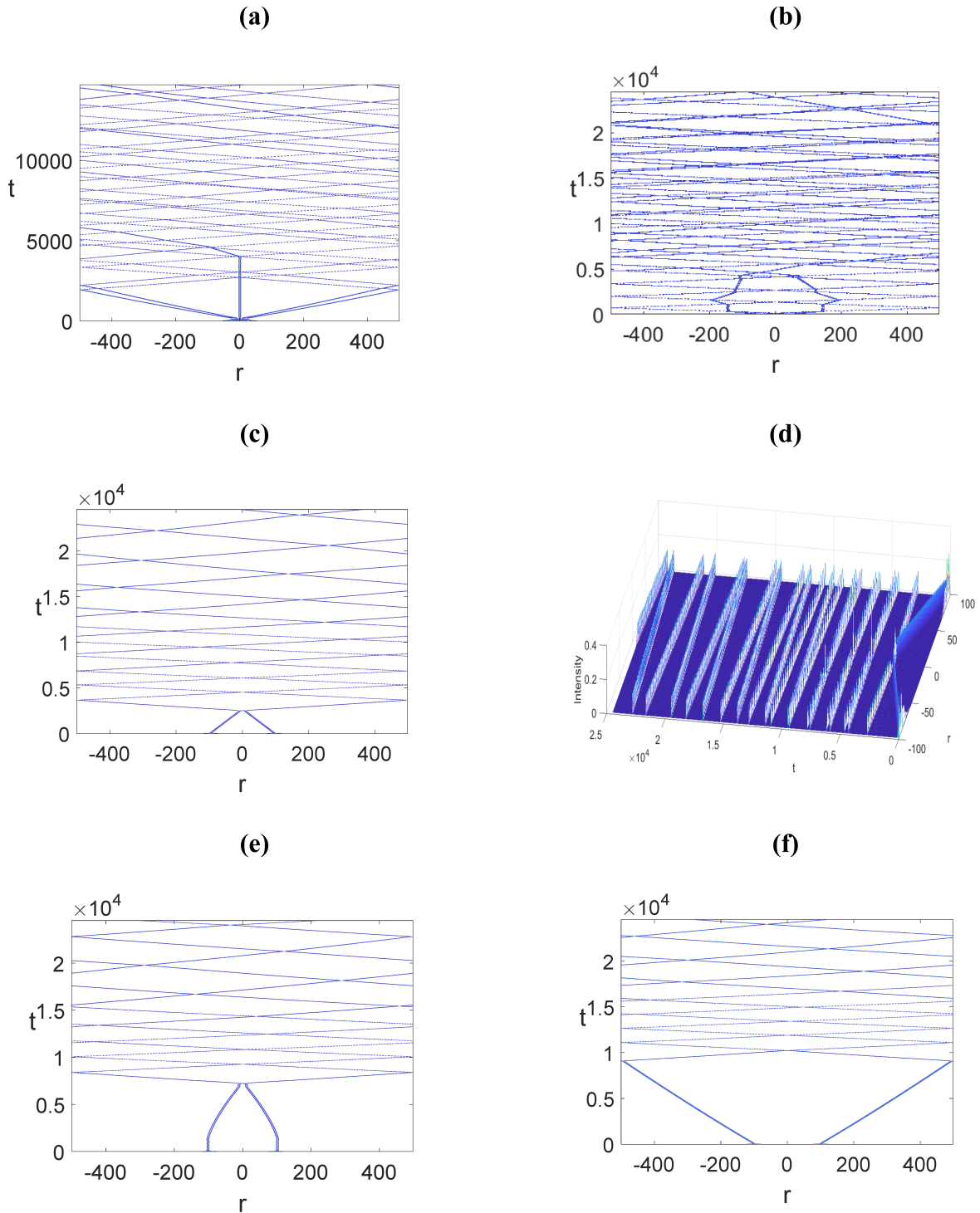
First we consider the interband GSA

1. Interaction of two solitons of same amplitude

Interaction of two solitons, keeping amplitude constant is further divided into two parts i.e.,

at different spatial separation and at different phase gap.

(i) Different spatial separation(X_0)



*Fig. 3.6 The dynamics of CS at different spatial separation with constant nonlinearity factor $g = 10$ and feedback strength constant $\sigma = 0.70$
(a) at $X_0 = 35$, (b) at $X_0 = 65$, (c) at $X_0 = 95.4$, (d) is the corresponding 3D plot of (c), (e) at $X_0 = 97$, (f) at $X_0 = 100$*

Interaction dynamics profile of solitons at same amplitude and different spatial separation is shown in Fig. (3.6). The initial field very shortly divided into two CSs and then oscillating and moves continuously. There is no common trend that solitons follows. On changing the separation, every time there appears new dynamics. As shown in Fig. 3.6(e), at first solitons attract slowly after that they repel each other and then they undergoes oscillations. A similar type of dynamics is observed in Fig.. 3.6(c). In Fig.. 3.6(a), solitons first repel each other and then oscillates with high velocity. Similar behaviour in Fig. 3.6(f) is noticed with the difference that the velocity of oscillation first case (Fig. 3.6(a)) is more. In Fig. 3.6(b), there is different dynamics and velocity with which soliton is moving is very high.

(ii) Different phase gap(φ)

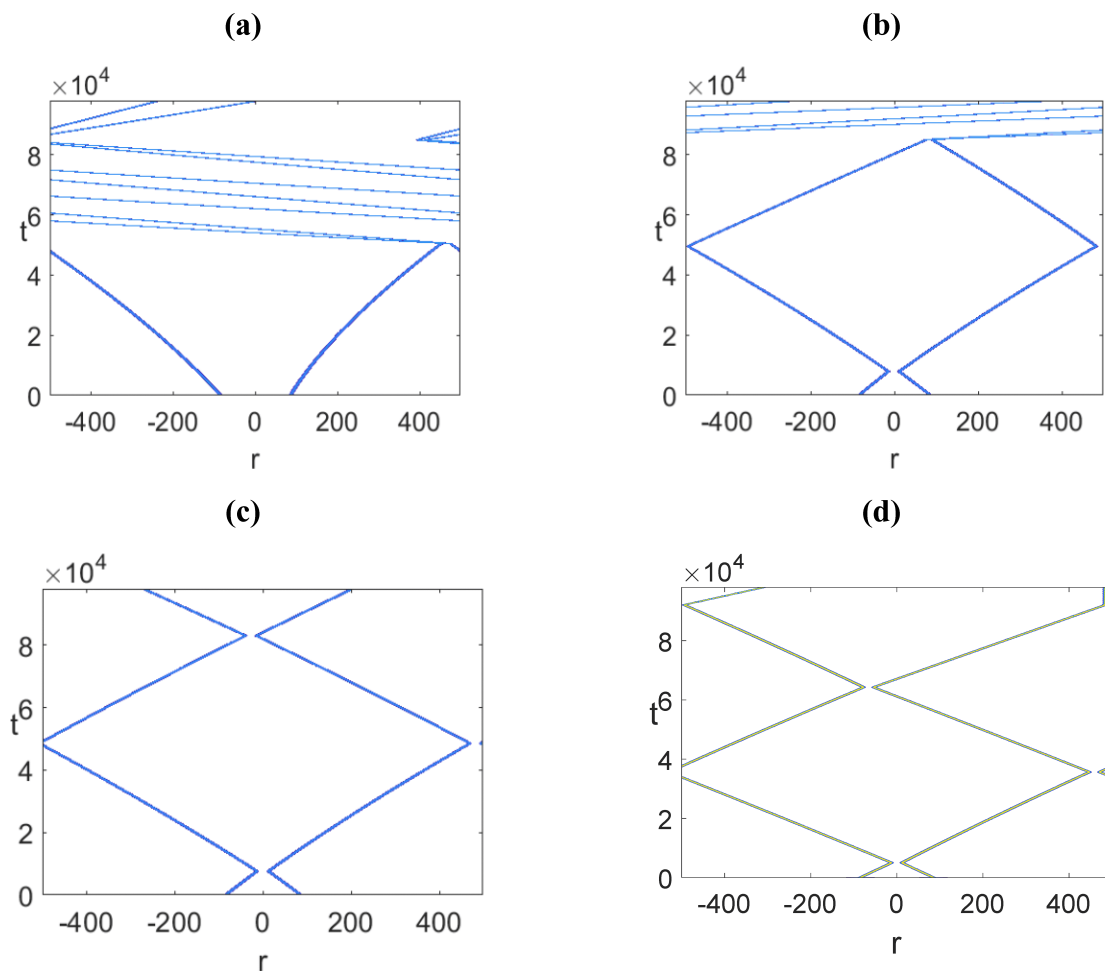


Fig. 3.7 The kinetics of CS at different phase with constant $g = 10$, other parameters are

$$\sigma = 0.65, X_0 = 85$$

(a) at $\varphi = \frac{\pi}{2}$, (b) at $\varphi = \frac{\pi}{12}$, (c) at $\varphi = \frac{\pi}{25}$, (d) at $\varphi = \frac{\pi}{80}$

The interacting profiles of cavity soliton at different phase gap is shown in Fig. (3.7). It was seen that at phase gap of $\varphi = \frac{\pi}{2}$ (Fig. 3.7(a)), the soliton repel each other in starting and after some time they started moving in one direction. The trend that was observed was that every soliton first attract each other and then repel after the phase gap of $\frac{\pi}{2}$. Like in Fig. 3.7(b), 3.7(c), 3.7(d), solitons first attract each other then repel and then again attract.

2. Interaction of two CSs at different amplitude

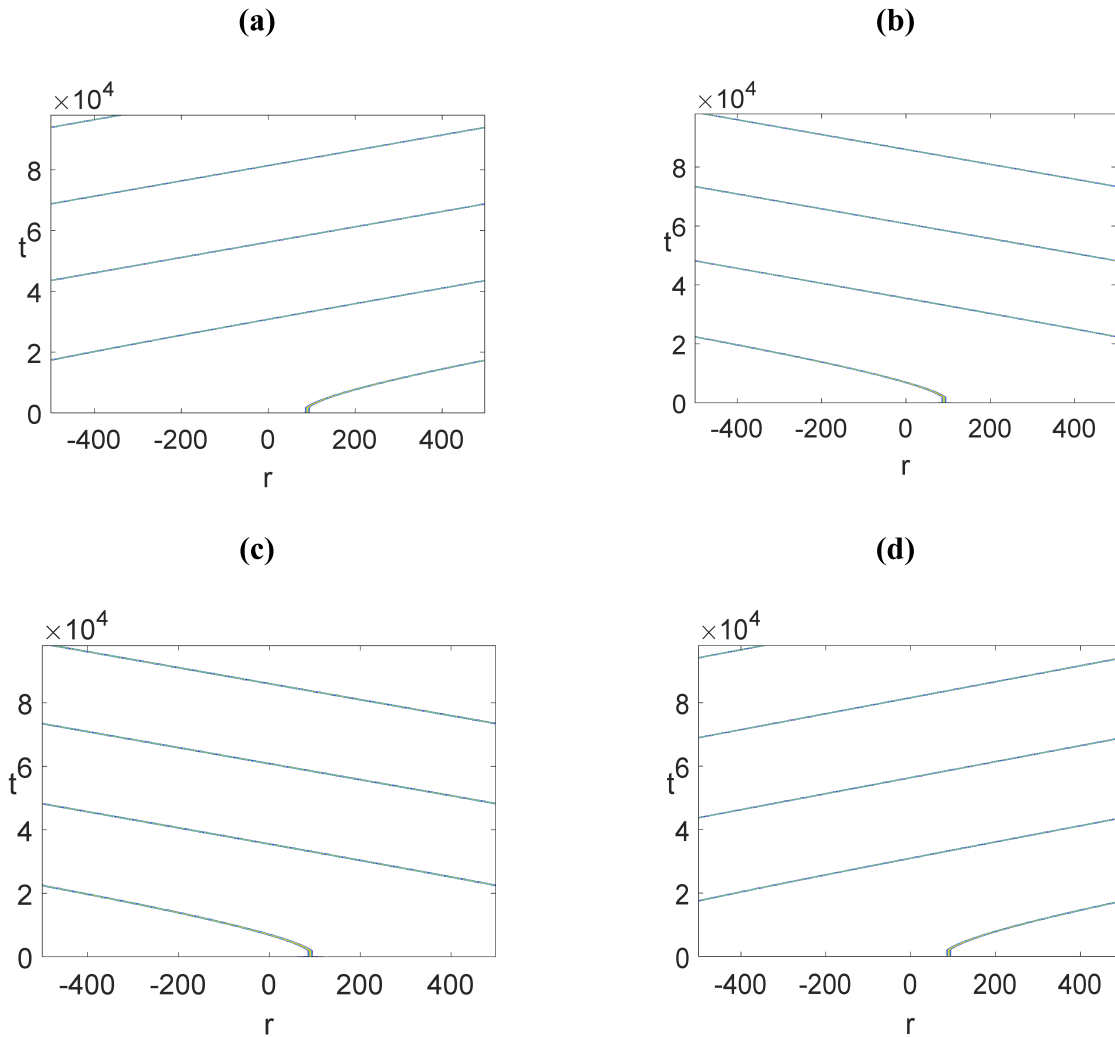


Fig. 3.8 Dynamics of CS at different amplitude with constant $g = 10$, $\sigma = 0.70$, and $X_0 = 90$ at amplitude (a) $A = 0.1$, (b) $A = 0.2$, (c) $A = 0.3$, (d) $A = 0.4$

The motion of solitons with different amplitude is shown in Fig. 3.8. The trend noticed was that the direction of soliton flip flop on increasing the amplitude (see in Fig. 3.8(a), 3.8(b)), same happens for amplitude $A=0.3$ and $A=0.4$. In each case the soliton moves uniformly with the only difference in the direction of soliton.

B. INTRABAND

1. Interaction of two solitons at same amplitude.

(i) Different spatial separation(X_0)

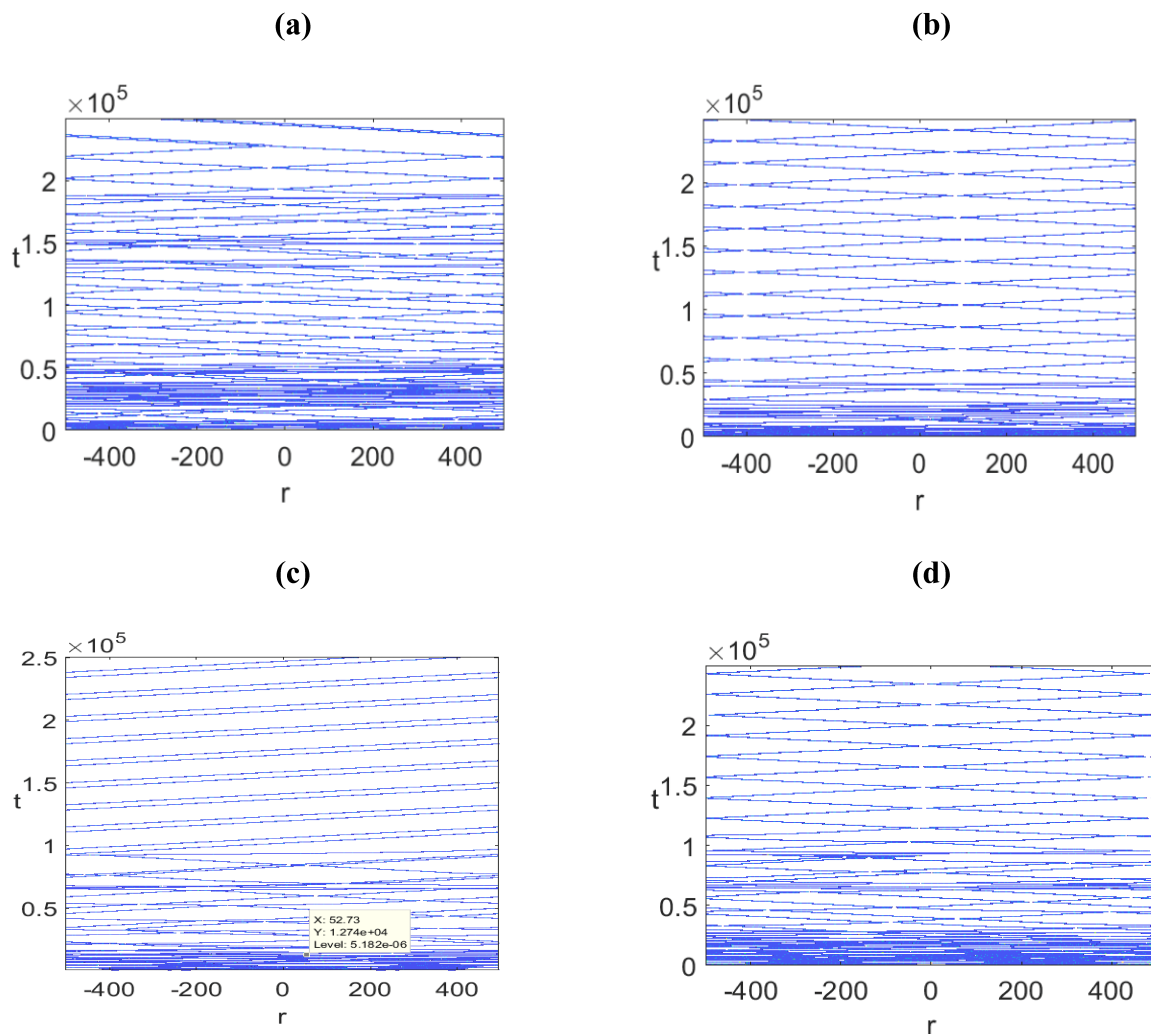


Fig. 3.9 The dynamics of CS at different spatial separation with constant nonlinearity factor $g = 0.7$ and feedback constant $\sigma = 0.88$

(a) at $X_0 = 30$, (b) at $X_0 = 50$, (c) at $X_0 = 70$, (d) at $X_0 = 120$

Fig. 3.9 shows the interacting profiles of two cavity solitons at varying spatial distance with constant ‘g’ and ‘ σ ’. In case of intraband also no common trend is followed. There is random movement of soliton in each case. If we see Fig. 3.9(a), the dynamics can’t be understood clearly as the soliton is moving very fast. One thing that can be seen in above Fig.s is that soliton in each case is moving very fast (clumsy part). At different separation, there is different dynamics sometimes it oscillates, sometimes it move uniformly in one direction and some times the direction flip flop with respect to first soliton.

(i) Different phase gap (φ)

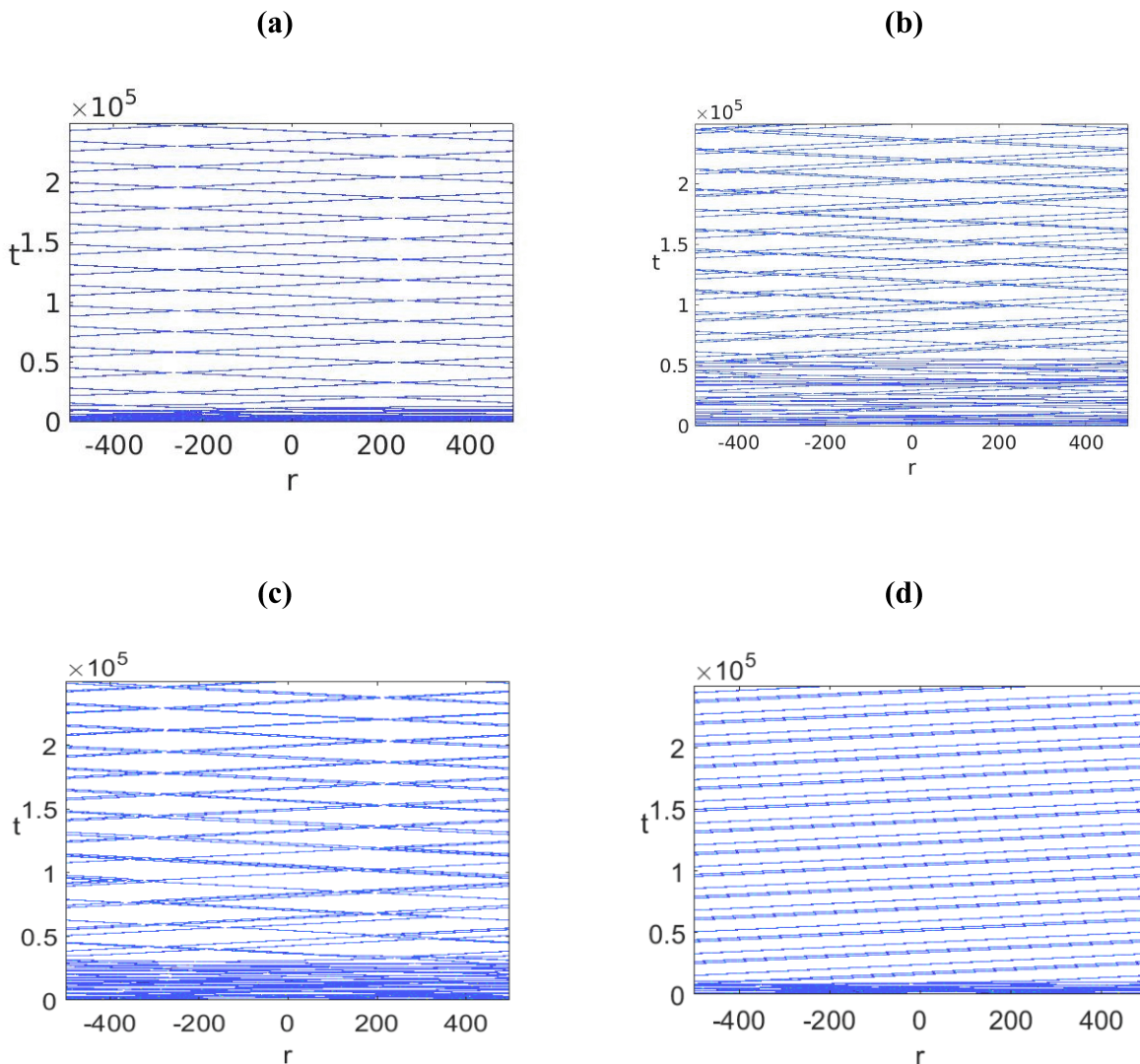


Fig.3.10 Interaction dynamics of CS at different phase gap
(a) at $\varphi = \pi/8$, (b) at $\varphi = \pi/15$, (c) at $\varphi = \pi/45$, (d) at $\varphi = \pi/100$

Fig. 3.10 shows dynamics of solitons at different phase gap. In all the interactions, velocity of the soliton is very high due to which dynamics is hard to predict and after some time they follow different random trend.

2. Interaction of two CSs at different amplitude.

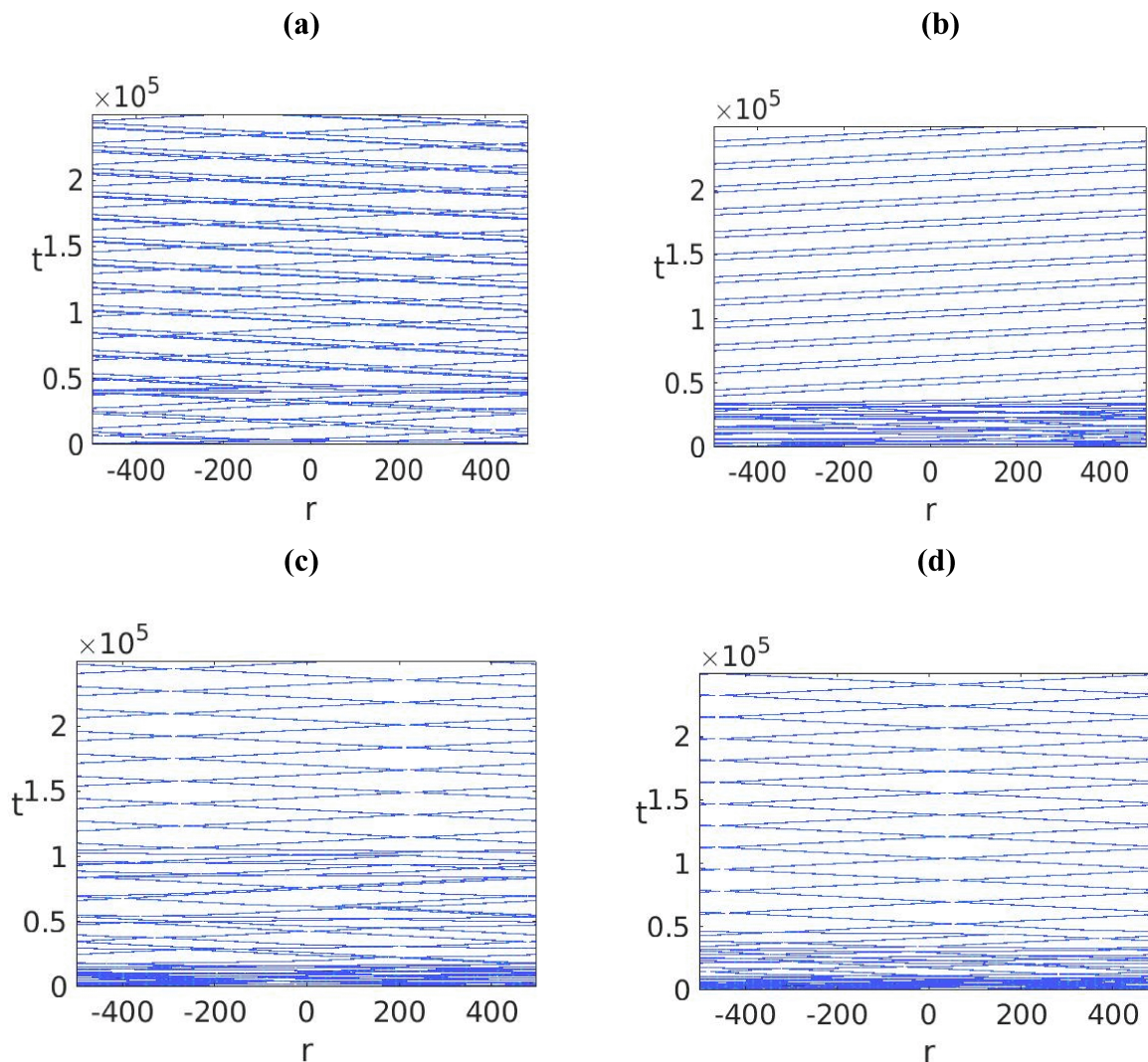


Fig. 3.11 Dynamics of CSs at different amplitude with constant $g = 0.7$, $\sigma = 0.88$, and $X_0=90$ at amplitude (a) $A=0.2$, (b) $A=0.5$, (c) $A=0.7$, (d) $A=0.9$.

At different amplitudes again the dynamics is random for every value. Some times it oscillates (Fig. 3.11(c) and 3.11(d)), some times it move uniformly in one direction with high velocity (Fig. 3.11(b) and some time it moves so fast that it's dynamics is hard to understand.

CHAPTER-4

STABILITY ANALYSIS

4.1 Stability analysis of cavity soliton based on VCSEL with saturable absorber.

In the previous sections we discussed the generation, dynamics and interactions of cavity solitons at different wavelengths. The important part that is needed to be discussed is the stability of the soliton. It is important to know that where the soliton is stable and where it is unstable. One must know the conditions, which will make soliton stable or unstable. In this section the conditions for the stable and unstable soliton are discussed.

We proceed considering the CGLE that describes dynamics in dissipative system.

For the stability analysis we take the following CGLE

$$\frac{\partial E}{\partial T} = [-(1-i\theta)E + i\nabla^2 E - \varepsilon^2(\theta + \nabla^2)^2 E + \left[\frac{(1-i\alpha)\mu}{1+g_1|E|^2} - \frac{(1-i\beta)\gamma}{1+sg_2|E|^2} \right] E + \alpha_{ns}E + (a - ib)E], \quad (4.A)$$

where,

E is electric field of photon; μ, γ are pumping parameter for active and passive medium respectively; α, β are line width factor for active and passive medium respectively; s is saturation parameter; $a = \sigma\lambda^2/(\lambda^2 + \Omega_0^2)$ and $b = \sigma\lambda\Omega_0/(\lambda^2 + \Omega_0^2)$, λ is band width of filter reflector; s feedback field; α_{ns} is non saturable absorption loss; g represent a coefficient of saturation of GSA; θ is detuning parameter, $(1 - i\theta)$ represents linear loss or gain, $\frac{\mu(1-i\alpha)}{1+g_1|E|^2}$ shows saturable absorption for active medium; $\frac{\gamma(1-i\beta)}{1+sg_2|E|^2}$ shows saturable absorption for passive medium; $i\Delta$ is diffraction term. $(a-ib)E$ is the feedback contribution.

Equation (4.A) can be rewritten as (by opening the third term in right hand side)

$$\frac{\partial E}{\partial T} = (1 - i\theta)E + i\nabla^2 E - \varepsilon^2(\theta^2 + \nabla^4 + 2\theta\nabla^2)E + \left[\frac{(1-i\alpha)\mu}{1+g_1|E|^2} - \frac{(1-i\beta)\gamma}{1+sg_2|E|^2} \right] E + \alpha_{ns}E + (a - ib)E \quad (4.1)$$

For steady state $\nabla^2 E = 0, \nabla^4 E = 0, \frac{\partial E}{\partial t} = 0 \therefore E_0 = 0$. This is the trivial solution. The non trivial solution can be found as $E_0 = \sqrt{I_0} e^{i\theta t}$

Substituting the above relation in equation (4.1), we get,

$$\frac{\partial E}{\partial t} = i\theta\sqrt{I_0} e^{i\theta t} = [-(1 - i\theta) - \varepsilon^2\theta^2 + \frac{(1-i\alpha)\mu}{1+g_1(I_0)} - \frac{(1-i\beta)\gamma}{1+g_2s(I_0)} + \alpha_{ns}E + (a - ib)E]\sqrt{I_0} e^{i\theta t} \quad (4.2)$$

Equating the real part of above equation(4.2), we get,

$$\theta^2 = \frac{1}{\varepsilon^2} \left[\frac{\mu}{1 + g_1 I_0} - \frac{\gamma}{1 + g_2 s I_0} + \alpha_{ns} + a - 1 \right] \quad (4.2a)$$

Equating the imaginary part, we get,

$$\frac{1}{\alpha} \left(\frac{\beta\gamma}{1+g_2sI_0} - b \right) = \frac{\mu}{1+g_1I_0} \quad (4.2b)$$

Substituting equation (4.2b) in (4.2a), we get,

$$\theta^2 = \frac{1}{\varepsilon^2} \left[\frac{\gamma}{1 + g_2 s I_0} \left(\frac{\beta}{\alpha} - 1 \right) + \alpha_{ns} + a - \frac{b}{\alpha} - 1 \right] \quad (4.3)$$

Now equation (4.2b) can be simplified as,

$$\beta\gamma - b - \mu\alpha = -g_1 I_0 \beta\gamma + b g_1 I_0 + s g_2 b I_0 + s b g_1 g_2 I_0^2 + \mu \alpha g_2 s I_0$$

This results in

$$I_0 = \frac{\frac{1}{b}(\beta\gamma g_1 - \alpha\mu s g_2) - g_1 - g_2 s \pm \sqrt{[(g_1 + g_2) + \frac{(\alpha\mu s g_2 - \beta\gamma g_1)1}{b}]^2 - 4g_1 g_2 s(\alpha\mu - \beta\gamma + b)}}{2g_1 g_2 s}$$

Now simplifying equation (4.2a), we get,

$$(1 + \varepsilon^2\theta^2 - \alpha_{ns} - a)(1 + g_1 I_0)(1 + s g_2 I_0) = \mu(1 + s g_2 I_0) - \gamma(1 + g_1 I_0)$$

In this case too we get I_0 as:

$$I_0 = \frac{\frac{\mu s g_2 - \gamma g_1}{1 + \varepsilon^2 \theta^2 - \alpha_{ns} - a} - s g_2 - 1 \pm \sqrt{\left(\frac{\mu s g_2 - \gamma g_1}{1 + \varepsilon^2 \theta^2 - \alpha_{ns} - a} + s g_2 + \gamma g_1\right)^2 - 4 s g_1 g_2 \left(1 + \frac{\gamma - \mu}{1 + \varepsilon^2 \theta^2 - \alpha_{ns} - a}\right)}}{2 s g_1 g_2}$$

We now proceed for linear stability analysis. We consider a slight perturbation in the initial electric field

$\square_\theta$. So the perturbed electric field can be written as $E = E_0 + \delta E e^{iqx}$, where $E_0 = \sqrt{I_0} e^{i\theta t}$

$$E = \sqrt{I_0} e^{i\theta t} + \delta E e^{iqx} \quad (4.4)$$

Taking complex conjugate of equation (4.4), we get,

$$E^* = \sqrt{I_0} e^{-i\theta t} + \delta E^* e^{-iqx}$$

Again equation (4.1) can be rearranged as,

$$\frac{\partial E}{\partial t} = \left[-1 + i\theta - \varepsilon^2 \theta^2 + \left[\frac{(1 - i\alpha)\mu}{1 + g_1 |E|^2} - \frac{(1 - i\beta)\gamma}{1 + g_2 s |E|^2} \right] + \alpha_{ns} + a - ib \right] E + [-2\theta \varepsilon^2 + i] \nabla^2 E - \varepsilon^2 \nabla^4 E \quad (4.5)$$

Putting equation (4.4) in (4.5), we get,

$$\begin{aligned} i\theta \sqrt{I_0} e^{i\theta t} + \frac{\partial}{\partial t} (\delta E) e^{iqx} \\ = \left[\frac{(1 - i\alpha)\mu}{1 + g_1 E E^*} - \frac{(1 - i\beta)\gamma}{1 + g_2 s E E^*} - 1 + i\theta - \varepsilon^2 \theta^2 + \alpha_{ns} + a - ib \right] (E_0 + \delta E e^{iqx}) \\ + (i - 2\theta \varepsilon^2) \left(\frac{\partial^2}{\partial x^2} e^{iqx} \right) \delta E - \varepsilon^2 \left(\frac{\partial^4}{\partial x^4} e^{iqx} \right) \delta E \end{aligned} \quad (4.6)$$

Now ,

$$E E^* = (\sqrt{I_0} e^{i\theta t} + \delta E e^{iqx}) (\sqrt{I_0} e^{-i\theta t} + \delta E^* e^{-iqx}) \quad (i)$$

$$= I_0 + E_0 \delta E^* e^{-iqx} + \delta E e^{iqx} E_0^*$$

$$\frac{1}{g_1 E E^*} \cong \frac{1}{1+g_1 I_0} \left[1 - \frac{g_1 E_0^* \delta E e^{iqx} + g_1 E_0 \delta E^* e^{-iqx}}{(1+g_1 I_0)} \right] \quad (ii)$$

$$\frac{1}{g_2 s E E^*} \cong \frac{1}{1+g_2 s I_0} \left[1 - \frac{g_2 s E_0^* \delta E e^{iqx} + g_2 s E_0 \delta E^* e^{-iqx}}{(1+g_2 s I_0)} \right] \quad (iii)$$

$$\frac{\partial^2}{\partial x^2} e^{iqx} = -q^2 e^{iqx} \quad (iv)$$

$$\frac{\partial^4}{\partial x^4} e^{iqx} = q^4 e^{iqx} \quad (v)$$

Putting all these values (i-v) in equation (4.6), we get,

$$\begin{aligned} i\theta E_0 + \frac{\partial}{\partial t} (\delta E) e^{iqx} &= \left[\frac{(1-i\alpha)\mu}{1+g_1 I_0} - \frac{(1-i\beta)\gamma}{1+g_2 s I_0} - 1 + i\theta - \varepsilon^2 \theta^2 + \alpha_{ns} + a - ib \right] (E_0 + \delta E e^{iqx}) \\ &- \frac{E_0 + \delta E e^{iqx}}{(1+g_1 I_0)^2} (1-i\alpha)\mu g_1 [E_0^* \delta E e^{iqx} + E_0 \delta E^* e^{-iqx}] + \frac{E_0 + \delta E e^{iqx}}{(1+g_2 s I_0)^2} (1-i\beta)\gamma s g_2 [E_0^* \delta E e^{iqx} + E_0 \delta E^* e^{-iqx}] \\ &+ (2\theta\varepsilon^2 - i)q^2 \delta E e^{iqx} \\ &- \varepsilon^2 q^4 \delta E e^{iqx} \end{aligned} \quad (4.7)$$

Neglecting small derivatives(i.e. $\delta E e^{iqx}$)and taking first term in right hand side of equation (4.7) we get,

$$= \left\{ \left[\frac{\mu}{1+g_1 I_0} - \frac{\gamma}{1+g_2 s I_0} - 1 + i\theta - \varepsilon^2 \theta^2 + \alpha_{ns} + a \right] - i \left[\frac{(i\alpha)}{1+g_1 I_0} - \frac{(i\beta)}{1+g_2 s I_0} + \theta + b \right] \right\} (E_0 + \delta E e^{iqx}) \quad (4.8)$$

Substituting values from equation (4.2a) and (4.2b)in the above equation (4.8), we get,

$$i\theta E_0 + \frac{\partial}{\partial t} (\delta E) e^{iqx} = i\theta E_0 + i\theta \delta E e^{iqx} + \text{others terms of equation (4.7)}$$

Therefore above relation implies,

$$\begin{aligned} \partial_t (\delta E) e^{iqx} &= \\ &\left[i\theta - \frac{\mu g_1 I_0 (1-i\alpha)}{(1+g_1 I_0)^2} + \frac{\gamma s g_2 I_0 (1-i\beta)}{(1+g_2 s I_0)^2} + (2\theta\varepsilon^2 - i)q^2 - \varepsilon^2 q^4 \right] \delta E e^{iqx} + \left[-\frac{\mu g_1 E_0^2 (1-i\alpha)}{(1+g_1 I_0)^2} + \frac{\gamma s g_2 E_0^2 (1-i\beta)}{(1+g_2 s I_0)^2} \right] \delta E^* e^{-iqx} \end{aligned} \quad 4.8(a)$$

Taking conjugate of equation 4.8(a), we get,

$$\begin{aligned}\partial_t(\delta E)e^{-iqx} = & \left[-i\theta - \frac{\mu g_1 I_0(1+i\alpha)}{(1+g_1 I_0)^2} + \frac{\gamma s g_2 I_0(1+i\beta)}{(1+g_2 s I_0)^2} + (2\theta\epsilon^2 - i)q^2 - \epsilon^2 q^4 \right] \delta E^* e^{-iqx} \\ & + \left[-\frac{\mu g_1 E_0^2(1+i\alpha)}{(1+g_1 I_0)^2} + \frac{\gamma s g_2 E_0^2(1+i\beta)}{(1+g_2 s I_0)^2} \right] \delta E e^{iqx}\end{aligned}\quad 4.8(b)$$

Case 1: $E_0 = I_0 = 0$ and $\theta = 0$, $Q = (\delta E)e^{iqx}$

Equation (4.7) becomes,

$$\begin{aligned}\partial_t(Q) = & [(1-i\alpha)\mu - (1-i\beta)\gamma - 1 + \alpha_{ns} + a - ib](Q) - iq^2 Q - \epsilon^2 q^4 Q \\ = & [(\mu - \gamma - 1 + \alpha_{ns} + a - \epsilon^2 q^4) + i(\beta\gamma - \alpha\mu - b - q^2)]Q\end{aligned}$$

If $\mu < 1 + \epsilon^2 q^4 - \alpha_{ns} - a$ system is stable

If $\mu > 1 + \epsilon^2 q^4 - \alpha_{ns} - a$ system is unstable

The above analysis show bifurcation. Actually, one may get

HOPF burification with frequency $\beta\gamma - \alpha\mu - b - q^2$

Case 2: If $E_0 \neq 0 = \sqrt{I_0}e^{i\theta t}$

The equation 4.8(a) becomes,

$$\begin{aligned}\partial_t(Q) = & \left[i\theta - \frac{\mu g_1 I_0(1-i\alpha)}{(1+g_1 I_0)^2} + \frac{\gamma s g_2 I_0(1-i\beta)}{(1+g_2 s I_0)^2} + (2\theta\epsilon^2 - i)q^2 - \epsilon^2 q^4 \right] Q \\ & + \left[-\frac{\mu g_1 E_0^2(1-i\alpha)}{(1+g_1 I_0)^2} + \frac{\gamma s g_2 E_0^2(1-i\beta)}{(1+g_2 s I_0)^2} \right] Q^*\end{aligned}$$

Or, $\partial_t(Q) = AQ + BQ^*$ where,

$$A = i\theta - \frac{\mu g_1 I_0(1-i\alpha)}{(1+g_1 I_0)^2} + \frac{\gamma s g_2 I_0(1-i\beta)}{(1+g_2 s I_0)^2} + (2\theta\epsilon^2 - i)q^2 - \epsilon^2 q^4,$$

$$B = -\frac{\mu g_1 E_0^2 (1 - i\alpha)}{(1 + g_1 I_0)^2} + \frac{\gamma s g_2 E_0^2 (1 - i\beta)}{(1 + g_2 s I_0)^2}$$

Complex conjugate of the above equation is $\partial_t(Q^*) = A^* Q^* + B^* Q$

These two can be written as

$$\partial_t \begin{bmatrix} Q \\ Q^* \end{bmatrix} = \begin{bmatrix} A & B \\ B^* & A^* \end{bmatrix} \begin{bmatrix} Q \\ Q^* \end{bmatrix},$$

Corresponding eigen values can be determined as,

$$\lambda = \frac{2\text{Re}(A) \pm \sqrt{(2\text{Re}(A))^2 - 4[|A^2 - B^2|]}}{2}$$

$$\text{Re}(A) \pm \sqrt{-\text{img}(A)^2 + |B^2|}$$

If, λ is real and is positive then it corresponds to stability of soliton and if λ is negative then it corresponds to unstable soliton

$$\lambda \rightarrow \text{real} \rightarrow -\text{ve}(\text{stability}) \text{ to } + \text{ve}(\text{unstability})$$

If, $|\lambda| \ll 1$ then it can be neglected

$$\text{If, } 2\text{Re}(A) = |A^2| - |B^2|$$

$$\lambda = \frac{|A^2| - |B^2|}{2\text{Re}(A)}, \text{ It means instability depends on } |A^2| - |B^2| \text{ and } \text{Re}(A)$$

$$\lambda = 0 \text{ for } |A^2| = |B^2|$$

$$\text{Now, } \text{Re}(A) = -\frac{\mu g_1 I_0}{(1 + g_1 I_0)^2} + \frac{\gamma s g_2 I_0}{(1 + g_2 s I_0)^2} + (2\theta \epsilon^2) q^2 - \epsilon^2 q^4$$

$$= B I_0 + (2\theta \epsilon^2) q^2 - \epsilon^2 q^4$$

$$\text{img}(A) = \theta - q^2 + \frac{\mu g_1(\alpha) I_0}{(1 + g_1 I_0)^2} - \frac{\gamma s g_2 I_0(\beta)}{(1 + g_2 s I_0)^2}$$

$$= CI_0 - q^2 + \theta$$

$$|B^2| = I_0^2 \left\{ \left[-\frac{\mu g_1}{(1 + g_1 I_0)^2} + \frac{\gamma s g_2}{(1 + g_2 s I_0)^2} \right]^2 + \left[\frac{\mu g_1(-\alpha)}{(1 + g_1 I_0)^2} + \frac{\gamma s g_2(\beta)}{(1 + g_2 s I_0)^2} \right]^2 \right\}$$

$$|A^2| - |B^2| = \text{Re}(A)^2 + \text{img}(A)^2 - |B^2|$$

$$= (BI_0 + (2\theta\epsilon^2)q^2 - \epsilon^2 q^4)^2 + (CI_0 - q^2 + \theta)^2 - I_0^2(B^2 + c^2)$$

After solving the above equation and neglecting ϵ^4 we get,

$$|B^2| = 4BI_0\epsilon^2\theta q^2 - 2\epsilon^2 q^4 BI_0 + \theta^2 + q^4 - 2\theta q^2 - 2q^2 CI_0 + 2\theta CI_0$$

$$0 = q^4 - 2q^2 \left[\frac{CI_0}{(1 - 2\epsilon^2 BI_0)} + \theta \right] + \left[\frac{2\theta CI_0 + \theta^2}{(1 - 2\epsilon^2 BI_0)} \right]$$

$$\text{Or, } q^2 = \theta + \frac{(CI_0)}{1 - 2\epsilon^2 BI_0} \pm \sqrt{\theta^2 + \frac{(CI_0)^2}{(1 - 2\epsilon^2 BI_0)^2} - \frac{\theta^2}{1 - 2\epsilon^2 BI_0}}$$

The factor $\frac{\theta^2}{1 - 2\epsilon^2 BI_0}$ in above equation is nearly equal to θ^2 as ϵ^2 is very small

$$\text{Therefore, } q^2 = \theta + \frac{(CI_0)}{1 - 2\epsilon^2 BI_0} \pm \sqrt{\frac{(CI_0)^2}{(1 - 2\epsilon^2 BI_0)^2}}$$

This gives two solutions

$$q^2 = \theta \quad \text{and} \quad q^2 = \theta + \frac{(2CI_0)}{1 - 2\epsilon^2 BI_0}$$

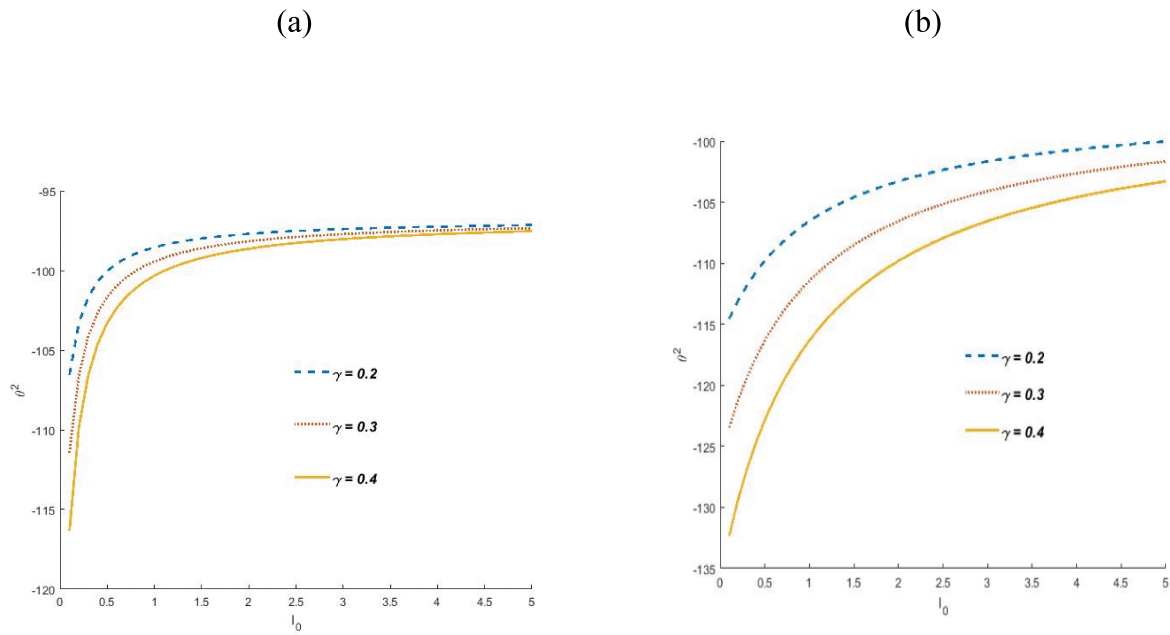


Fig.4.1 Represents the variation of θ^2 with respect to I_0
(a) for interband (b) for intraband

Fig. 4.1 shows the variation of θ^2 with respect to I_0 at different values of γ in both interband (Fig. (a)) and in intraband (Fig. (b)). On comparing both the Fig. 4.1(a) and Fig. 4.1 (b), it was noticed that the saturation in case of interband is quick as compared to intraband.

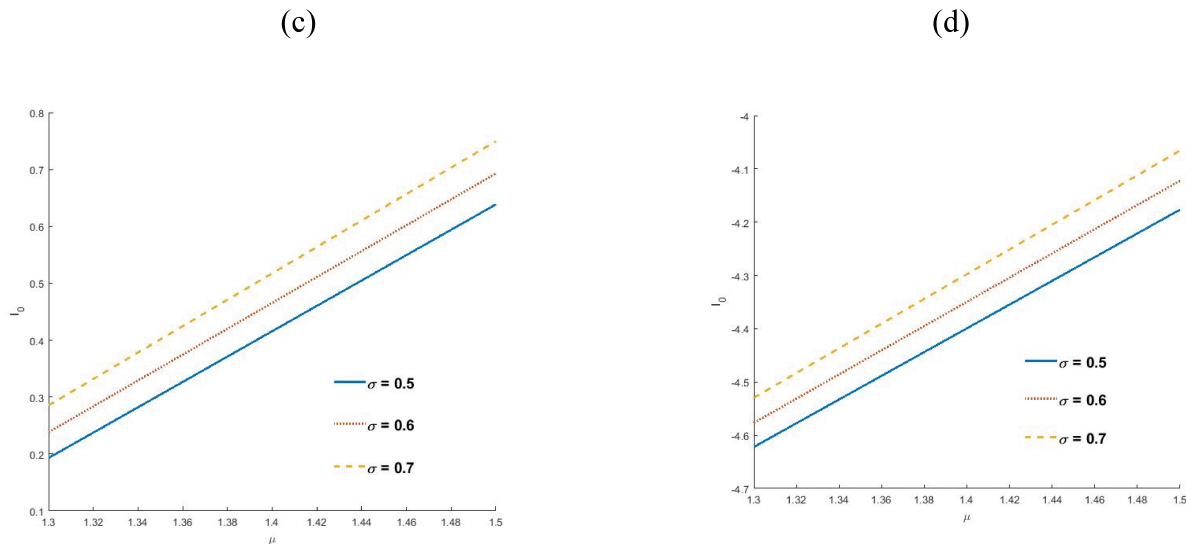


Fig 4.2. shows the variation of I_0 with respect to μ
(c) interband (d) intraband

Fig. 4.2 represents the variation of I_0 with respect to μ at varying σ for interband and intraband GSA. With increasing pump parameter μ , the intensity linearly increases. For a particular μ the intensity is higher for higher sigma. The above features are similar for both interband and intraband GSA. It can be noticed that I_0 is higher in intraband case with respect to that in interband case.

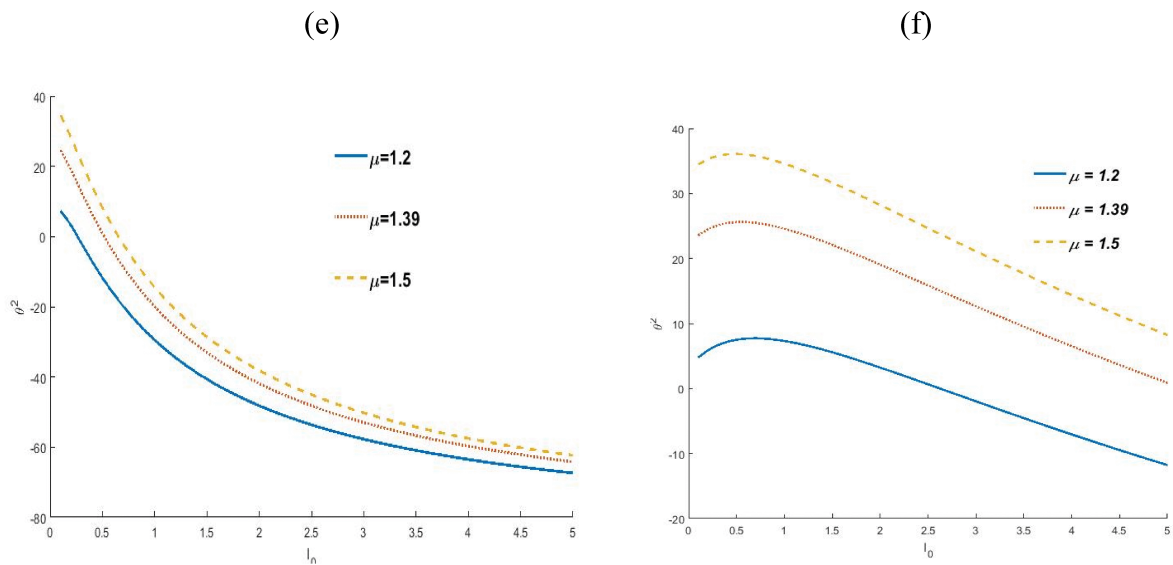


Fig4.3. Represents the variation of θ^2 with respect to I_0 (e) for interband (f) for intraband

Fig.4.3 shows the variation of θ^2 with respect to I_0 at different values of μ for both interband and intraband GSA. In case of interband θ^2 nonlinearly reduces whereas for intraband slightly increases at the beginning then decreases almost linearly with increase in I_0 .

4.2 Conclusion

We generate cavity soliton in VCSEL with feedback mechanism emphasizing interband and intraband saturable absorption mechanism of GSA. The generated CS exhibits different interesting spontaneous dynamics, which causes due to a slow of presenting the cavity system. The snaking and multistability curves of CS is shown. The velocity of the cavity soliton is much

higher in case of intraband than in interband absorption. The interaction dynamics of two CSs of same as well as different amplitudes are studied with different spatial gap . In both the cases CS dynamics can be tuned by the spatial gap. Similar tuning by phase gap is found for interaction of two CSs having equal amplitude as well as unequal amplitude. The stability analysis of the cavity soliton system has been extensively parametric zones for stable and unstable CSs has been identified. The results of this investigation gives an insight of the role of GSA interband and intraband mechanism in CS formation. This knowledge can be utilised for experimental verification of the role of interband and intraband GSA in CS formation. Also, it can be utilised to fabricate an all-optical tuning devices.

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In this thesis we study theoretically [formation of cavity soliton in Vertical Cavity Surface Emitting Laser \(VCSEL\)](#) with graphene saturable absorber. Saturable absorber of graphene can be generated through interband and intraband mechanism. The role of both of these interband and intraband has been identified in [generation and dynamics of cavity soliton in VCSEL](#). We show the existence and dynamics and interactions of two or more cavity solitons in infrared and far infrared regions. The effect of replacing a common semiconductor saturable absorber by graphene saturable absorber is highlighted. For greater strength of graphene saturable absorber reduces the required feedback strength. The interaction of cavity solitons for different phase difference and gaps are demonstrated. A comprehensive stability analysis of [cavity soliton based on VCSEL embedded with saturable absorber](#) is also done. Parametric zones for stable and unstable cavity solitons are identified. The results obtained by the investigation may lead to experimental realization of cavity soliton with graphene saturable absorber. Soliton is a important topic of pure and experimental research [in various fields](#) related to [Physics](#), Biology, [chemistry, engineering and even](#) behavioural [science](#). Soliton is basically confined [structure like vibration](#), beam, [wave or pulse that](#) maintains [its](#) size [and](#) shape in spite of collisions with another such structure. The story of soliton started in the year 1834. During his working time in a shipyard at Union canal in Glasgow- Edinburgh, J. S. Russell monitored a lump of water running along the head of bow of a boat, which continued in the canal even after the boat had stopped. The water wave travelled vigorously without any change or deformity in its shape and after a long while it gets diminished gradually. After nearly a decade's investigation he named such kind of waves as 'translational waves' [1]. He gave the view that the water wave equations likely to have solitary wave solutions. These kind of waves were later named as solitary waves. The soliton concept grow under the conditions of conservative systems, i.e., a system having no loss. The stability of soliton originates from the balance of 'nonlinearity' and 'dispersion'. To understand the concept of soliton let's consider a light wave. What happens to light wave after travelling some distance? The answer is that after travelling some distance it starts diminishing due to dispersion/ diffraction and/or loss. But what will happen if a wave propagate a long distance and yet do not show any change in its initial size and shape? Is that possible? It sounds to be impossible but it may be true. Generally, a pulse (a beam) broadens due to dispersion. The nonlinearity may create self-phase modulation, which eventually do the contraction of the pulse . When these two phenomena's perfectly counter balance each other one gets conservative soliton. Now, if there is a loss in the system the soliton will not be stable at all. To stabilize the soliton in a lossy or dissipative system the loss/(s) during the wave propagation is [compensated by an external gain](#). [2]. In 1847, Stokes gave [the solution of the periodic nonlinear waves in deep water](#)[3]. In 1965, Los Alamos et al. studied the energy the flow in the 1D [lattice in which](#) same [masses](#)