SIMULATION STUDIES OF CHANNEL CODED MSK IN WIRELESS ENVIRONMENT

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CERTIFICATE

I, Simranjit Singh hereby declare that this report entitled "**Simulation studies of channel coded MSK in wireless environment**" is an authentic record of my own work carried out towards the partial fulfillment for the award of degree of M.E. (Master of Engineering) in Electronics and Communication Engineering Department, Thapar University, Patiala, under the guidance of Mrs. Surbhi Sharma, Lecturer, ECED, during January to June 2009.

The matter presented in this report has not been submitted in any other University or Institute for the award of any degree.

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This is certified that the above statement made by the student is correct to the best of my knowledge and belief.

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ABSTRACT

The Low-Density Parity-Check codes are among the most powerful forward error-correcting codes and are poised to become a standard in the next digital transmission system generations. In addition to their error correction capacity, the LDPC codes have a relatively simple decoding algorithm, so they are likely to be the coding choice in future.

In 2003, LDPC codes beat all other competing codes to become the error correcting code in the new DVB-S2 standard for the satellite transmission of digital television.

In this thesis an attempt has been made to further improve the error performance of LDPC codes by combining them with two of the most powerful modulation techniques, MSK and GMSK. The performance of LDPC coded MSK/GMSK modulated signals is tested over fading conditions ranging from very severe to moderate. The Nakagami fading conditions have been modeled by simulating the fading envelope. This adaptive fading model has been used to generate various fading conditions for testing the BER performance of LDPC codes.

In this thesis, a comparative study of LDPC coding using MSK and GMSK modulation has also been done. The proposed system when compared with the uncoded system provides a coding gain of 3.3 dB for m=1 and it is 4dB for m=2 for MSK and 3.9dB for m=1 and 5dB for m=2 for GMSK. It is also shown that for a given BER, LDPC coded signal requires less SNR when compared to uncoded signal.

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LIST OF ABBREVIATIONS

ASK	Amplitude Shift Keying
AWGN	Additive White Gaussian Noise
BCH	Bose Chaudhuri Hocquenghem
BER	Bit Error Rate
CPM	Continuous Phase Modulation
DPSK	Differential Phase Shift Keying
DVB	Digital Video Broadcasting
EXIT	Extrinsic Information Transfer
FAX	Facsimile
FSK	Frequency Shift Keying
GMSK	Gaussian Minimum Shift Keying
GSM	Global System for Mobile Communications
LDPC	Low Density Parity Check
MATLAB	Matrix Laboratory
MIMO	Multiple Input Multiple Output
MSK	Minimum Shift Keying
PAM	Pulse Amplitude Modulation
PSK	Phase Shift Keying
QAM	Quadrature Amplitude Modulation
QPSK	Quadrature Phase Shift Keying
RAM	Random Access Memory
SISO	Serial In Serial Out
SNR	Signal-to-Noise Ratio

TCM	Trellis Coded Modulation
VHDL	Very High Speed Integrated Circuit Hardware Description Language
VLSI	Very Large Scale Integration

CHAPTER 1
Introduction

The first chapter introduces the system model and gives a brief description of the components used in it. The background of telecommunications and coding theory has also been discussed in this chapter.

1.1 Introduction

Digital transmission of information has great advantages and it increasingly dominates communication systems. Digital communications are increasingly becoming popular because of greater data processing options and flexibilities as compared to analog communications. In this thesis the fundamental problem of digital communications has been addressed, i.e. to reproduce the originally transmitted signal at the receiver with lesser errors and to mitigate the effects of fading. In communications, our basic aim is to transmit some information reliably over a channel which may be anything from a pair of wires to an open environment. It is not mandatory to use digital communication if we can find a better method than it. But for now, that is the best we can do, because until now the most efficient method is to transmit a data in the form of discrete pulses, i.e. digitally. As is known, in digital communication, the signal is transmitted in the form short discrete pulses, which can have one of the two levels, i.e. 1 or 0, fully on or fully off. During transmission, for a distortion to affect the information, it should be large enough to displace the current digital level to the other level. The levels of distortion which are smaller than that can be dealt with by using amplifiers and repeaters.

1.1.1 System Model

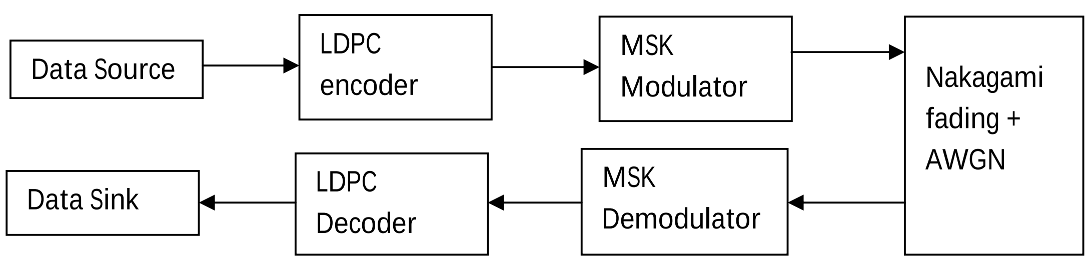

Figure 1.1: System Model

The data source generates an information signal that is to be transmitted to the receiver. After the message is produced by the *data source* it processed at the transmitter end before being transmitted on a noisy channel. The processing is done to compensate the effects of various signal impairments caused by the channel and to enable the waveforms to be detected easily at the receiver end.

The two main blocks at the transmitter end are encoder and modulator. The message from data source is fed into the *channel encoder (LDPC)*. Typically, a channel encoder adds redundant bits to the incoming data to make the signal more immune to various channel distortions. These redundant bits are removed at the receiver at recover the original message. The encoded signal is then modulated using a *MSK/GMSK modulator* to transform the signal waveforms to better withstand the channel impairments. In the case of GMSK, the rectangular input pulses are converted to Gaussian shaped pulses. Modulation is the method by which the message symbols are converted to waveforms that are compatible with the requirements imposed by the channel.

The *channel* is the transmission medium, through which this waveform is transmitted. It is responsible for distortion the message signal and degrading the performance of the digital communication system. For wireless channels, the distortions of the channel are much more severe and compared to a wired medium. The distortions in wireless channel are often due to noise (*AWGN*) and fading. A suitable model for modeling the fading conditions accurately is *Nakagami fading*. In this fading model the adaptive parameter m defines the severity of fading. As the value of m increases the fading becomes less severe. The fading is severest when m=0.5. All the work being done at the transmitter is just to mitigate the degradation effects of these two phenomena (Noise and Fading).

After the degraded signal is received at the receiver, it is processed by the demodulator and decoder which try to correct some of the errors in the transmission. The process at the receiver is the reverse of that at the transmitter. The received waveform is demodulated by the *MSK/GMSK demodulator* block. The iterative *LDPC decoder* then decodes the demodulated waveform and removes some of the errors. If good quality decoders with good error detection and correction are used, the effects of this degradation can be mitigated significantly.

The term *Data Sink* is used to describe a computer or any other medium capable of receiving data. All this trouble is undertaken just to ensure that the correct information which is intelligible is imparted to the data sink.

1.2 History and Background

The history of communications dates back to 3500 B.C when the Sumerians developed cuneiform writing and the Egyptians developed hieroglyphic writing. It was then that the human race started developing different ways of communicating their messages. The major breakthrough in telecommunications came when Samuel Morse invented the Morse code.

1.2.1 Timeline

The timeline of telecommunications is given below:

- 1793 - The Chappe brothers established the first commercial semaphore system between two locations near Paris.
- 1837 - Cooke and Wheatstone obtain a patent on telegraph. Morse publicly demonstrates his telegraph.
- 1843 - FAX invented by the Alexander Bain.
- 1844 - Electric telegraph was demonstrated by Samuel Morse. This was a major breakthrough in communications and marked the beginning of a new era.
- 1865 - James Clark Maxwell invents four simple electromagnetic equations that describe all known electromagnetic phenomena of the time.
- 1864 - James Clerk Maxwell proves that wireless telegraphy is possible.
- 1866 - The first Transatlantic telegraph cable is laid.
- 1876 - Alexander Graham Bell invents the telephone.
- 1895 - Guglielmo Marconi invented the radio.
- 1901 Marconi completes the first transatlantic radio transmission from Newfoundland.
- 1907 - The world's first transatlantic commercial wireless services is established by Marconi with stations at Clifden, Ireland and Glace Bay, Nova Scotia

- 1909 - Marconi gets a joint Nobel Prize in Physics, with Karl Ferdinand Braun because of their work in the development of wireless telegraphy.
- 1910 - Thomas Edison demonstrated the first talking motion picture.
- 1925 - John Logie Baird transmits the first experimental television signal.
- 1934 - Joseph Begun invents the first tape recorder for broadcasting first magnetic recording.
- 1940’s - Spread Spectrum

Before 1948, communication was strictly an engineering discipline, with little scientific theory to back it up. Although a number of communication systems were in use at that time, communication engineering wasn’t considered to be a hard science because the engineers at that time didn’t have the reasons to back up their claims. Till 1948, there was no concept of ‘information’ and at that time, the transmission of audio signals and moving pictures was considered to be separate. In 1948, Claude E. Shannon known as, “The Father of Information Theory” published his ground breaking work [1], entitled, “A Mathematical Theory of Communication” that altered our basic think about communication. In his paper two major concepts were given:

1. The concept of information and the modeling of information sources
2. The concept of a channel and on the limits of reliable communication on unreliable channels

Given a communication channel, Shannon proved that there exists a number, called the *capacity* of the channel, such that reliable transmission is possible for rates arbitrarily close to the capacity, and reliable transmission is not possible for rates above capacity. He demonstrated that it is possible to achieve error free transmission on a noisy communication channel through coding. His work focuses on the problem of how best to encode the information a sender wants to transmit. This paper was the beginning of a new era in the field of error control coding. It was also proved in his paper that there exist codes which approach capacity, for which the probability of error of the maximum likelihood decoder goes to zero as the block length tends to infinity. Although capacity approaching codes are very good but the theorems given in the paper are non-constructive and do not offer any advice on how to find there codes. Soon after the publication of this paper, the researchers found out that random codes are capacity achieving. This spurred a lot of research activity in the field of error control coding and also gave the communication engineers the concepts of ‘Information’ and ‘Channel’.

More than a decade ago in 1993, C. Berrou and A. Glavieux [3] presented a new scheme for channel codes decoding: the turbo codes, and their associated turbo decoding algorithm. Turbo codes made possible to get within a few tenth of dB away from the Shannon limit, for a bit error rate of 10^{-5}. This made other researchers realize the importance of iterative decoding and also made them aware that other capacity approaching codes existed. Recently, LDPC codes have attracted much attention in the world of coding. They were originally invented in the 1960’s and forgotten. They were again brought to life in 1995 by Mackay and Neal [5]. Unlike many other classes of codes LDPC codes are already equipped with very fast (probabilistic) encoding and decoding algorithms. This makes LDPC codes not only attractive from a theoretical point of view, but also perfect for practical applications. They are already being implemented for satellite communication standards and 4G standards.

1.3 Channel coding

The encoder can be subdivided into two parts, source encoder and channel encoder. In this thesis, we are not concerned with source coding, so the term encoder simply means channel encoder. The channel encoder makes the message signal more immune to noise and distortion by adding redundant information in the discrete domain. This technique can thought of as a medium for accommodating desirable system trade-offs such as error performance vs. bandwidth or power vs. bandwidth. It is possible to get as much as 10dB performance improvement by the use of channel coding. At the receiver end the decoder uses this redundancy to efficiently extract the originally transmitted message with as less errors as possible. Basically channel coding is used to maximize the information rate. Channel codes are of three types

1. Block codes
2. Convolutional codes
3. Turbo codes

A number of channel codes have been used over time. A few of them are hamming codes, BCH codes, Reed Solomon codes, Turbo codes, LDPC codes and Trellis codes. A few important properties of these codes are discussed below.

1.3.1 Hamming codes

Named after their inventor, Richard Hamming, Hamming codes are one of the earliest channel codes that were actually effective. They are especially suitable for correcting all single errors and detecting two or fewer errors in a block. These codes have a minimum distance of 3 and are very effective when used with syndrome decoding.

- Block length: $n=2^m-1$
- Information Bits: $k=2^m-m-1$
- Parity Check Bits: $n-k=m$
- Correctable Errors: $t=1$

These codes became very popular when they were invented but are not used much nowadays because of the development of more efficient coding schemes. Due to the simplicity of Hamming codes they are used in computer memory RAM.

1.3.2 BCH codes

The BCH codes are a generalization of the Hamming codes and allow multiple error correction. The abbreviation BCH comprises of the names of the inventors of these codes, Bose, Chaudhuri and Hocquenghem. These codes belong to a special class of codes called cyclic codes which enable them to be decoded very easily by the use of an elegant algebraic method called syndrome decoding.

BCH codes for $m\geq3$ and $t<2^{m-1}$:

- Block Length: $n=2^m-1$
- Parity Check Bits: $n-k\leq mt$
- Minimum Distance: $d\geq2t+1$

The very famous R-S codes are a subclass of BCH codes.

1.3.3 Reed-Solomon codes

In 1960, Irving Reed and Gus Solomon introduced a new class of error-correcting codes that are now called Reed-Solomon (R-S) codes. Reed–Solomon is an error-correcting code that works by oversampling a polynomial constructed from the input data. These codes are amongst one of the most popular channel codes and are used in a number of commercial applications and data transmission technologies. Most prominent applications of RS codes include CDs, DVDs, Blu-Ray Discs, DSL and WiMAX. The main idea behind RS codes is that the data which is to be encoded is first seen as a polynomial. The code is based on a theorem of algebra that states that any k distinct points uniquely determine a polynomial of degree, at most, k − 1.

- Block Length: $n=2^m-1$
- Minimum Hamming Distance: n-k+1

1.3.4 Low density parity check (LDPC) codes

LDPC codes were invented by Robert Gallager [2] in his Ph.D thesis at M.I.T in 1962. Unlike many other classes of codes, LDPC codes are equipped with very fast encoding and decoding algorithms which makes them very useful for practical applications. At the time when these codes were invented, the hardware was not available for their implementation, so they were not given much attention. Now with the advancement of VLSI and DSP it is finally possible to implement these codes practically. The main attraction of LDPC codes is that they approach the Shannon limit of a channel. There have been simulations that perform within 0.04dB of the Shannon limit. The codes that approached the Shannon limit are not the ones originally proposed by Gallager, but rather a modified version called irregular LDPC codes proposed by Mackay *et. al.* [9]. In 1981 R.M Tanner [7] introduced a recursive approach to the construction of LDPC codes described in the language of graphical theory. The LDPC codes are characterized by a sparse parity check matrix in which the number of 1’s is very less as compared to the number of 0’s [30]. LDPC codes are regarded as the best codes as their performance is the best when compared to other good codes such as turbo codes [28]. They are set to be used as next generation satellite digital video broadcasting standard, DVB-S2 and also strong contenders for coding in 4G wireless systems and hard disk drives.

1.4 Modulation schemes

After the signal has been encoded, it is modulated using a carrier to transform the signal into waveforms that can better withstand the channel impairments. By modulating, we vary some parameter of the carrier according to the message signal and send it over the channel. The basic bandpass modulation/demodulation techniques are divided into two parts:

1. Coherent – ASK, PSK, FSK, CPM (Continuous Phase Modulation) and Hybrids
2. Noncoherent – Differential PSK, ASK, FSK, CPM and Hybrids

In coherent modulation schemes, the information about the phase and frequency of the carrier is needed at the receiver for detection. It is more efficient in terms of performance but the receiver design becomes more complex. In non-coherent modulation schemes, no information about the phase and frequency of the carrier is required for detection. In this scheme the receiver design is less complex but at the cost of degraded performance. A number of modulation schemes have been proposed in the past. The ones that became popular are:

- QPSK (Quadrature Phase Shift Keying)
- DPSK (Differential Phase Shift Keying)
- QAM (Quadrature Amplitude Modulation)
- MSK (Minimum Shift Keying)
- GMSK (Gaussian Minimum Shift Keying)

All these modulation schemes have a single objective of transforming the signal into waveforms to make it more immune to the distortions offered by the channel. Out of the modulation schemes given above, the best performance has been obtained from GMSK (Gaussian Minimum Shift Keying). GMSK is a continuous-phase frequency-shift keying modulation scheme. GMSK is just a modified form of MSK (Minimum Shift Keying) in which a pre-modulation Gaussian filter is added before the modulator. GMSK increases spectral efficiency with sharper cutoff and has excellent power efficiency due to constant envelope. Because of its performance the GMSK technique has been used extensively in second generation digital cellular and cordless telephone applications.

1.5 Fading

Radio waves propagate from a transmitting antenna, and travel through free space undergoing absorption, reflection, refraction, diffraction, and scattering. They are greatly affected by the ground, buildings, trees and other objects present in their path. All the things are responsible for the characteristics of the received signal. The two main factors that affect the reliable communication of a message over the channel are noise and fading. Noise in communication systems is generally modeled as AWGN (Additive White Gaussian Noise) because it is convenient to deal with noise of additive nature rather than multiplicative nature. Noise affects transmission over a channel by degrading the signal quality. Over large distances, signal quality is shown to degrade even without the presence of large quantities of AWGN. This degradation is known as **fading** [29] and the errors introduced by fading are much difficult to deal with as compared to the errors introduced by noise. Fading appears as unpredictable changes in the measured signal, thus amplifying or attenuating the received power. Fading is of two types:

1. Large Scale Fading
2. Small Scale Fading

Large scale fading: This type of fading is seen over large distances and is caused by shadowing introduced by obstacles where a large obstruction such as a hill or large building obscures the main signal path between the transmitter and the receiver. The amplitude change caused by shadowing is often modeled using a log-normal distribution. Large scale fading is also called Shadow fading.

Small scale fading: Small scale fading is observed over smaller distances as compared to large scale fading. In small scale fading, different replicas of the same signal are created by reflection, diffraction and scattering. Small scale fading is caused by constructive and destructive addition of different multipath components. The small scale fading can be further categorized as:

- Fast or Slow Fading
- Frequency Selective or Flat Fading

Small scale fading models that have been widely used for modeling the fading environment are:

- Rayleigh fading
- Rician fading
- Nakagami fading

1.5.1 Rayleigh fading

Rayleigh fading is used for modeling severe fading conditions as in urban areas where there is a lot of congestion. Rayleigh fading models assume that the signal that has passed through the Rayleigh channel will fading according to the Rayleigh distribution. This means that the envelope of the channel response of Rayleigh channel will be Rayleigh distributed. The pdf of Rayleigh distribution is given by:

$$P_r(R) = \frac{2r}{\Omega} e^{-\frac{r^2}{\Omega}}, \qquad r \geq 0 \tag{1.1}$$

This model is the most appropriate when there is no dominant signal along the line of sight between the transmitter and receiver and is used mostly for heavily built urban environment.

1.5.2 Rician fading

Rician fading is used to model fading conditions which are less severe as compared to Rayleigh fading and where dominant line of sight path propagation is present between the transmitter and receiver. In Rician fading the dominant component is much stronger than other multipath components. Rician fading model assumes that the signal that has passed through the Rician channel will fade according to the Rician distribution. This means that the envelope of the channel response of Rician channel will be Rician distributed. The pdf of Rician distribution is given by:

$$P_r(R) = \frac{r}{\sigma^2} exp^{-\frac{(r^2+A^2)}{2\sigma^2}} I_0(\frac{Ar}{\sigma^2}) \tag{1.2}$$

where, I_0 (.) is the modified Bessel function of the first kind and zero order. K parameter is given by: $K=A^2/2\sigma^2$

1.5.3 Nakagami fading

The Rayleigh and Rician fading models described above fall short of describing long distance fading effects with sufficient accuracy. M. Nakagami observed this fact and then formulated a parametric gamma function which was inspired by his experiments in high frequency long distance propagation. The model proposed by Nakagami uses an adaptive m parameter to describe the fading conditions. It is shown that fading conditions less and more severe than Rayleigh and Rician fading can also be accurately modeled by Nakagami fading. Nakagami fading model assumes that the signal that has passed through the channel will fade according to the Nakagami distribution. This means that the envelope of the channel response of Nakagami channel will be Nakagami distributed. The pdf of can be given by:

$$P_R(r) = \frac{m^m r^{2m-1}}{\Omega^m \Gamma(m)} \exp\{-\frac{mr^2}{\Omega}\} \quad (1.3)$$

Γ (m) is the gamma function and m is the shape factor with the constraint ($m \geq 1/2$)
Experimental and theoretical [19, 20] works have shown that the Nakagami distribution is the best-fit distribution for data obtained from many urban multipath radio channels. Therefore, we have used Nakagami fading to model the fading conditions in this thesis.

1.6 Thesis Objective

As discussed above, the main motive of any communication system is to reliably transmit the information to the receiver and reduce the number of errors induced by the channel. In this thesis, we have worked on reducing the transmission errors and improving the BER. We try to combine the best encoding (LDPC) and modulation (GMSK/MSK) schemes and test them over different fading conditions. Error performance of LDPC coded GMSK/MSK modulated signals was analyzed by comparing their BER with different SNR values.

- Simulation of PDF’s for Nakagami fading for different values of m
- Accurate simulation of Nakagami fading envelope

- Error performance of uncoded MSK and GMSK over Nakagami fading channel for different values of m
- Error performance of LDPC coded MSK and GMSK over Nakagami fading channel for different values of m
- Comparison of uncoded and LDPC coded MSK and GMSK

1.7 Thesis Organization

This thesis includes five chapters. An outline of each chapter is given below:

The 1st chapter gives an introduction to the digital communications and coding theory. Some of the problems of wireless communications such as noise and fading are also addressed in this chapter.

Chapter 2 is dedicated to the literature survey. The research papers which are relevant to this thesis are discussed here.

Chapter 3 presents a study of the structure, encoding and decoding of LDPC codes. In this chapter MSK, GMSK and Nakagami fading have also been discussed in detail. Some of the simulation results of Nakagami fading have been included in this chapter.

Chapter 4 includes meaningful results which are supported by theoretical arguments and computer simulations. In this chapter we have presented the analysis and results of error performance of coded and uncoded MSK and GMSK in Nakagami fading environment. These simulations have been done on MATLAB R2007b (version 7.5). Various tables of SNR for fixed BER values are also included in this chapter.

Chapter 5 concludes this thesis, summarizing the major results and offering suggestions for further work on this topic.

In the end, a list of important references to this thesis is given, without which this work could not have been possible.

CHAPTER 2
Literature Survey

This chapter surveys some important studies in the field of our research and outlines the main ideas contained in them.

Low density parity check codes were invented by Robert Gallager [2] in his Ph.D thesis at M.I.T in 1963. The codes invented by Robert Gallager are called low-density-parity-check (LDPC) codes because the parity check matrix used is low density (sparse) which contains only a few 1's in comparison to 0's. He discovered an iterative decoding algorithm which he applied to a new class of codes and for these codes to perform well he needed the parity check matrices to be sparse. The codes which he developed give a good performance and are called regular LDPC codes because they use parity check matrices in which the number of 1's and 0's in each row and column are equal. Still, the LDPC codes were ignored for a long time because of the requirement of high complexity computation and lack of hardware support at that time.

Turbo codes were first introduced in 1993 by Berrou, Glavieux, and Thitimajshima, and reported in [3, 4]. In these papers it is shown that the turbo coding scheme achieves a bit-error probability of 10^{-5} using a rate 1/2 code over an additive white Gaussian noise (AWGN) channel and BPSK modulation at an E_b/N_0 of 0.7 dB. Turbo codes are a type of concatenated codes in which two or more simple codes combine to give a new code with much better error correction properties. Although these codes were complex to work with they attracted a lot of attention towards iterative codes.

In the year 1995, LDPC codes were rediscovered by Mackay and Neal [5] and they set up a link between their iterative algorithms to the Pearl's belief algorithm [6], from the artificial intelligence community (Bayesian networks). The work of Mackay and Neal gave rise to a lot of research activity in the field of LDPC codes. Most of the main articles related to the LDPC codes are gathered in a special issue of the IEEE's Transactions on Information Theory (IEEE 2001).

Gallager's LDPC codes were not originally described in the language of graphical theory. R.M Tanner [7], in 1981 introduced a recursive approach to the construction of codes which generalized the LDPC code construction and suggested that the design of algorithms for encoding and decoding is amenable to the basic techniques of graph theory. In this paper A method is described for constructing long error-correcting codes from one or more shorter error-correcting codes, referred to as subcodes, and a bipartite graph. Also, a lot of work was done by Wiberg [8] in the field of graphical theory of LDPC codes. Hence, LDPC codes are at the confluence of two major revolutions in the channel coding community: the graph-based code-description, and the iterative decoding techniques.

As is said earlier, LDPC codes get the closest to approaching the Shannon capacity. The LDPC codes given by Gallager used regular parity check matrices, but these codes are not the ones that approach capacity. The codes which approach capacity were given by D.J.C Mackay [9] *et. al.* and are called Irregular LDPC codes. In this paper, irregular parity check matrix is used instead of a regular parity check matrix. An irregular matrix is one in which the number of 1's and 0's are not equal in every row and column. Also, in this paper an alternative construction of the graph is given, the advantages of which are twofold:

- It is seen that a "super-Poisson" construction which gives a small improvement in empirical performance over a random construction.
- The new construction proposed enables LDPC codes to be encoded at a better rate than the original LDPC codes.

Therefore, most of the work being done in the field of LDPC codes today is using irregular LDPC codes.

Another major development in the field of LDPC codes was when T. J. Richardson, A. Shokrollahi, and R. Urbanke [10] proposed a density evolution scheme for the optimization of irregular LDPC codes. Density evolution is an analytical technique which has been used to understand limits of performance of LDPC decoders. In this paper, they designed capacity approaching irregular LDPC codes using density evolution algorithm. This algorithm tracks the probability density function (pdf) of the messages through the graph nodes under the assumption that the cycle free (there should be no cycles in a tanner graph) hypothesis is verified. It is a kind of belief propagation algorithm with pdf messages instead of log likelihood ratios messages. In this paper, it is shown that LDPC codes approach the Shannon capacity. They used highly irregular bipartite graphs for the construction of irregular LDPC codes.

Chung et al. [11] designed rate−1/2 irregular LDPC codes of block length 10^7 for binary-input AWGN channels that approach the Shannon limit very closely. In this paper improved algorithms have been used to construct low density parity check codes. Simulation results with a somewhat simpler code show that we can achieve within 0.04 dB of the Shannon limit at a bit error rate of 10^{-6} using a block length of 10^7. This is the paper in which the results have shown that LDPC codes come closest to approach the limit.

S. ten Brink [12] also contributed towards the optimization of iterative decoding schemes. This paper proposes a new scheme for analyzing iteratively decoded codes. This paper proposes a new scheme for visualizing the convergence behavior of iterative decoding schemes. Each constituent decoder is represented by a mutual information transfer characteristic which describes the flow of extrinsic information through the soft in soft out decoder. The scheme proposed in this paper is more commonly known as EXIT. The exchange of extrinsic information between constituent decoders is plotted in an extrinsic information transfer chart called EIT chart by the author. This method is both faster and more insightful than simulating a code.

T. Richardson and R. Urbanke [13] demonstrated an efficient and effective encoding method of parity check matrix H in 2001. In this paper, the authors have shown the method of exploiting the sparseness of the parity check matrix to obtain efficient encoders. As it is known that the complexity of encoding is essentially quadratic in the block length. In this method, the associated coefficients are made very small so that the encoding of codes of length 100,000 is practically possible. It is also shown in this paper that optimized codes actually admit linear time coding.

For decoding LDPC codes, Gallager himself gave an iterative algorithm [2]. The decoding of LDPC codes is usually processed by iterative algorithms generally called message passing algorithms. This algorithm is present in Gallager's work and is also used in the Artificial Intelligence community [6]. S. Chung *et. al.* have presented a Gaussian approximation for message densities under density evolution to simplify the analysis of the decoding algorithm [14]. The infinite-dimensional problem of iteratively calculating message densities has been converted to a one dimensional problem of updating by means of Gaussian densities. The advantages of this simplification are twofold:

- It helps to calculate the threshold quickly
- It helps to understand the decoder better

It is shown that by using Gaussian approximation, the sum-product decoding algorithm can be visualized.

MSK (Minimum Shift Keying) was developed in the late 1960's and is one of the first techniques that were actually spectrally efficient. A lot of work was done on this modulation technique by Subbarayan Pasupathy [15]. The MSK scheme was shown to be a special case of continuous phase FSK signaling with frequency deviation equal to the bit rate. MSK can also be viewed as a form of offset QPSK signaling in which the symbol pulse is a half-cycle sinusoid rather than the usual rectangular form. It combines many attractive attributes such as constant envelope, compact spectrum, the error rate performance of BPSK, and simple demodulation and synchronization circuits in one modulation format. From the various results given in this paper, MSK is spectrally efficient and has a good error performance compared to the other techniques.

GAUSSIAN minimum-shift keying (GMSK) was first proposed in 1981 by Kazuaki Murota and Kenkichi Hirade [16] and has become very popular due to its compact power spectral density and excellent error performance. It is a bandwidth-efficient constant envelope digital modulation technique, and therefore, it is suitable for mobile communications in fading channels. It has been used in cellular systems such as GSM and DCS1800. In this technique the author has used a pre-modulation Gaussian filter before MSK modulator to obtain Gaussian shaped pulses. In the paper the results support the fact that GMSK is the most bandwidth efficient modulation technique. This modulation technique has become very popular and is currently being used in GSM mobile communication as their modulation standard with BT product of 0.3.

One of the most frustrating problems that the researchers face while trying to communicate a signal reliably over an unreliable channel is fading. When a signal is transmitted over large distances, its level is known to degrade even without the presence of noise; this is due to a phenomenon called fading. There are a number of different fading models that have been used to model the fading conditions, such as Gaussian, Rayleigh [17, 18], Rician and Nakagami. In this thesis we are primarily concerned about the Nakagami fading model. It was developed by M. Nakagami [21] in 1960 because it matched empirical results for short ionospheric propagation. It is an adaptive fading model used to model fading conditions ranging from severe to no fading. There is an adaptive m-parameter in Nakagami fading, the values of which are varied to model the fading conditions.

- It is a good model for outdoor in urban areas where line-of-sight (LOS) does not exists (Rayleigh, m=l).
- Nakagami distribution with large m (m>2) is a good model for indoor propagation where LOS exists and in mobile satellite systems.

Gayatri S. Prabhu and P. Mohana Shankar in [19] have simulated various flat fading channels, their envelopes, pdfs and also their outage probabilities. The basic concepts of fading have been reinforced using MATLAB simulations. The concept of outage is also demonstrated using MATLAB.

Li Tang and Zhu Hongbo [20] have presented a simple and accurate model of Nakagami fading. In this paper they have simulated and analyzed the statistical performances of the Nakagami fading channel in wireless communication with MATLAB. In this paper the second order characteristics such as average fade duration and level crossing rate have also been analyzed and discussed in detail.

LiDuan MA *et. al.* [22] have analyzed the performance GMSK transmission system using R-S coding over AWGN and Rayleigh channel. The transmitted signal is kept constant and the performance of GMSK and R-S code combination is compared. Better BER was obtained for the coded GMSK system as compared to uncoded system. Also, the optimal code rate was found to be a function of the total system bandwidth.

M.K. Caldera and K.S Chung [23] studied the performance of TCM (trellis coded modulation) scheme in which the modulation scheme used was GMSK. In this paper Convolutional coding has been combined with GMSK. A comparative analysis of coded vs. uncoded waveform was done over frequency selective fading channels and the results showed that there was a significant improvement in the bit error rate of the TCM scheme. The TCM scheme also shows less sensitivity to carrier phase errors than the uncoded scheme.

S. Elnoubi *et. al.* [24] did a comparative study of MSK and GMSK over Nakagami fading channel with and without diversity. In this paper the Nakagami fading variables have been simulated and the results are presented for the BER performance of both coherent and non coherent MSK and GMSK. The results show that the error performance of MSK is better than GMSK. Also, it is shown that GMSK is more bandwidth efficient than MSK. The best results are obtained after applying diversity technique when the value of m is the greatest.

Gee L. Lui *et. al.* did a comparative study of LDPC coded quaternary GMSK [25] and ldpc coded QPSK for different adjacent channel interference levels. The simulation results are presented over a FDMA communication system for ldpc coded quaternary GMSK and LDPC coded QPSK for various adjacent channel interference levels and it is shown that the error performance of coded GMSK is much better than coded QPSK with severe adjacent channel interference.

In [26] Taki M. *et. al.* have proposed a new scheme for detection of LDPC coded GMSK modulated signals. This new technique is based on Laurent decomposition of CPM signal into PAM pulses. The received signal was passed through a filter matched with highest energy pulse and the output was equalized to eliminate linear and non-linear distortions due to other pulses. This detection system employed a filter that was matched with the highest energy pulse through which the signal was passed. The output was equalized to eliminate linear and non-linear distortions due to other pulses. The filters employed were Wiener linear equalizer and Turbo like equalizer. These filters combined with the LDPC decoder were used for detection. This system gives a performance close to the Maximum-Likelihood receiver but the complexity is much smaller.

CHAPTER 3
Low Density Parity Check Codes

In the 3rd chapter, the structure, encoding and decoding of LDPC codes has been explained briefly. Short notes on MSK and Nakagami fading have also been included in the last three sections of this chapter.

3.1 An Introduction to LDPC codes

LDPC codes are a class of Linear Block Codes. The name comes from the characteristic of their parity-check matrix which contains only a few 1's in comparison to the amount of 0's. Their main advantage is that they provide a performance which is very close to the channel capacity. Impractical to implement when invented by Robert Gallager in his Ph.D thesis, they are the best channel codes available today. After the invention, these codes were largely forgotten and there was no activity till mid-late 90s when there was a renewed interest in LDPC codes. Turbo codes, discovered in 1993, became the coding scheme of choice in the late 1990s, used for applications such as deep-space satellite communications. However, in the last few years, the advances in low density parity check codes have developed them past turbo codes, and LDPC is the most efficient scheme discovered as of 2009. There are two different methods of representing LDPC codes:

- Matrix Representation
- Graphical Representation

3.1.1 Matrix Representation

LDPC codes are defined by a sparse parity check matrix. Below given is an example for a low density matrix of dimension $n \times m$ for a (6, 3) code. Now we can define two parameters, w_r is the number of ones in each row and w_c is the number of ones in each column. For a matrix to be called low density, there are two conditions: $w_r << m$ and $w_c << n$.

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \tag{3.1}$$

In order to meet these conditions, the parity check matrix should be very large, so the example matrix cannot be really called low density. The matrix shown in the figure below can be actually called low density. In this figure the blue boxes are ones and the rest are zeros.

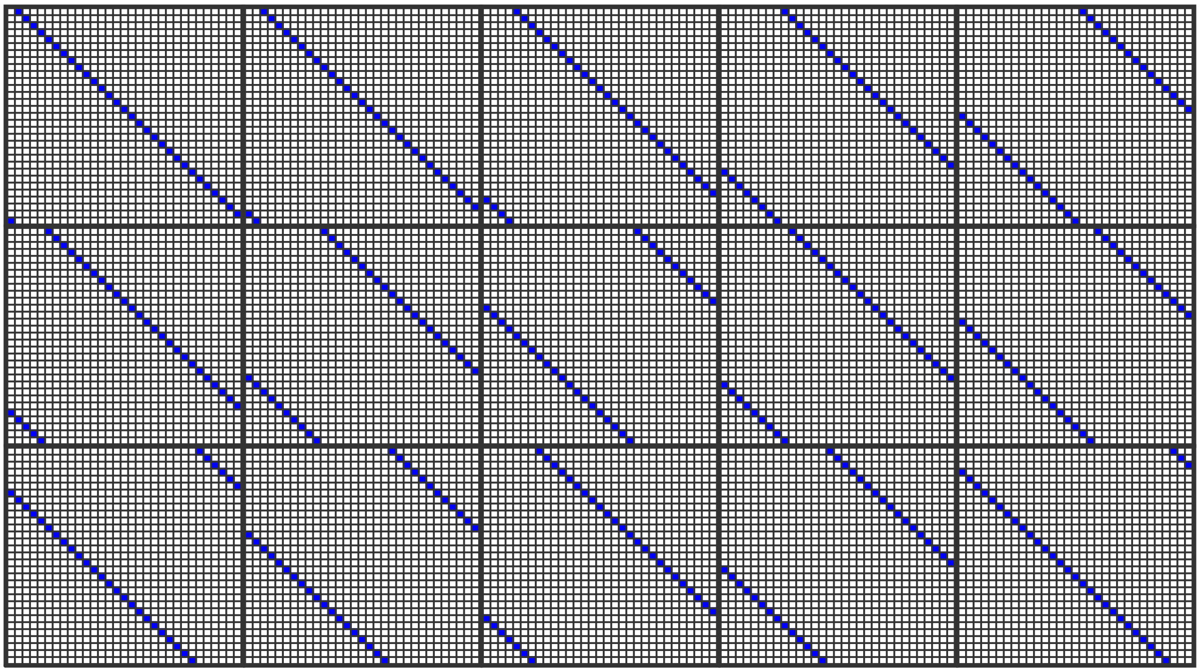

Figure 3.1: Sparse parity check matrix for (155, 64) code

3.1.2 Graphical Representation

Tanner [7] introduced an effective graphical representation for LDPC codes. Not only provide these graphs a complete representation of the code, they also help to describe the decoding algorithm. These graphs called Tanner graphs are bipartite graphs. It means that the nodes of the graph are separated into two distinctive sets and edges are only connecting nodes of two different types. The two types of nodes in a Tanner graph are called variable nodes (v-nodes) and check nodes -nodes). The graph gives rise to a linear code of block length n and dimension at least n- r in the following way: The n coordinates of the codewords are associated with the n message nodes. The codewords are those vectors $(c_1 \ldots . c_n)$such that for all check nodes the sum of the neighboring positions among the message nodes is zero. Figure (3.2) is an example for such a Tanner graph and represents the same code as the matrix in equation (3.1). The graphical representation is analogous to a matrix representation by looking at the adjacency matrix of the graph: let H be a binary $r \times n$ matrix in which the entry (i, j) is 1 if and only if the i^th^ check node is connected to the j^th^ message node in the graph. Then the LDPC code defined by the graph is the set of vectors $c = (c_1 \ldots c_n)$ such that $H \cdot c^T = 0$. The matrix H is called a parity check matrix for the code. The sparsity of the graph structure is key property that allows for the algorithmic efficiency of LDPC codes. For a good decoding performance the Tanner graph should be cycle free as in figure 3.2. In a Tanner graph, short cycles shown in figure 3.3 should be avoided as they are bad for decoding performance.

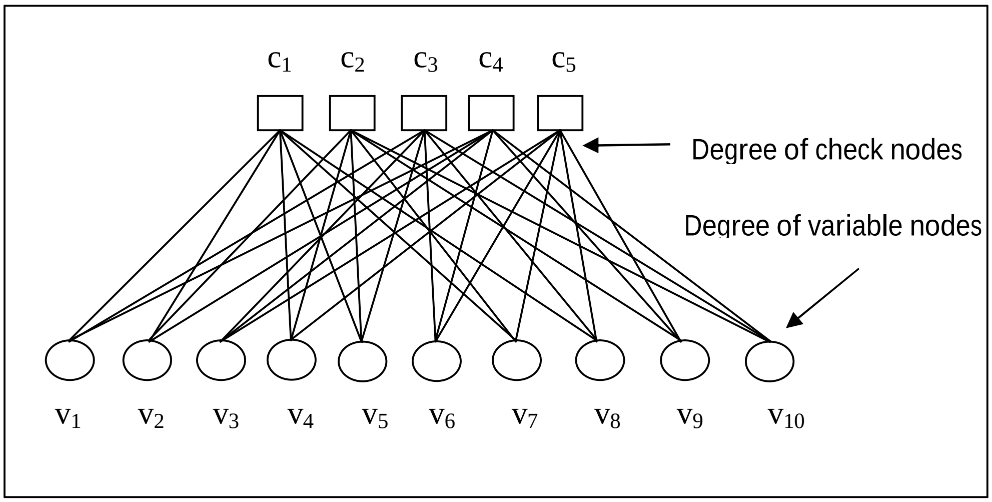

Figure 3.2: Tanner graph of H matrix in equation (3.1)

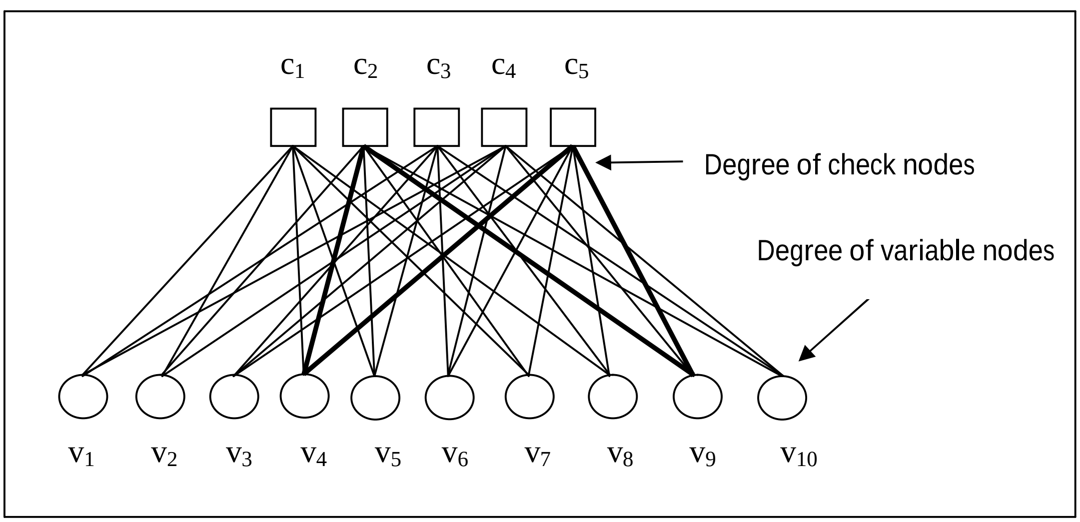

Figure 3.3: Tanner graph with short cycle of length 4.

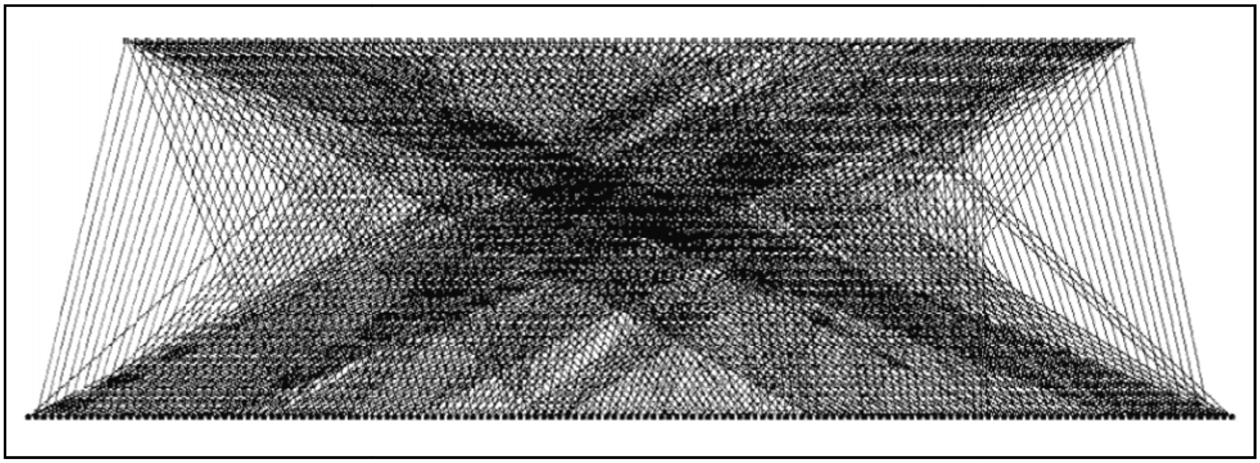

Figure 3.4: Tanner Graph for (155, 64) code shown in Fig 3.1

3.1.3 Regular and Irregular LDPC codes

A LDPC code is called regular if w_c is constant for every column and $w_r = w_c \cdot (n/m)$ is also constant for every row. The example matrix is regular with $w_c = 3$ and $w_r = 6$. It's also possible to see the regularity of this code while looking at the graphical representation. There is the same number of incoming edges for every v-node and also for all the c-nodes. If the number of edges emanating from a variable nodes is called variable node degree d_v and the number of edges emanating from a check node is called check node degree d_c, then the rate of (d_v, d_c) regular LDPC code is $R=1-d_v/d_c$. The number of 1's in the parity check matrix H is $N.d_v$, while the total number of elements in H is $N^2.R_c$, where N is the length of the code. Obviously, when N increases linearly and the number of elements increase quadratically. Hence, the parity check matrix is sparse with large N. The sparse characteristic of the parity check matrix is essential because the number of 1's presents the relations between a variable node and a check node. As the number of 1's increase, the tanner graph becomes more complex and in turn the decoder becomes more complex. So the sparsity of the parity check matrix directly controls the decoder complexity.

If H is low density but the numbers of 1's in each row or column aren't constant the code is called an irregular LDPC code. It means that irregular LDPC codes have nodes with different degrees. The degrees of the variable nodes and check nodes are chosen according to some distribution. For convenient description, the degree distribution is often presented in polynomial form. The variable node distribution is denoted by $\lambda(x)$ and it can be expressed as

$$\lambda(x) = \sum_{i=2}^{d_v} \lambda_i x^{i-1} \tag{3.2}$$

where λ_i is the fraction of edges emanating from variable nodes of degree i and d_v is the maximum variable degree or the irregular LDPC code. Note that the coefficient λ_i is associated with x^{i-1}, rather than x^i. Similarly, the check node degree distribution is denoted by $\rho(x)$ and can be expressed as

$$\rho(x) = \sum_{i=2}^{d_c} \rho_i x^{i-1} \tag{3.3}$$

where ρ_i is the fraction of edges emanating from check nodes of degree i and d_c is the maximum check degree. The number of variable nodes of degree u if the (λ, ρ) irregular LDPC code of length N is

$$N \frac{\frac{\lambda_i}{i}}{\sum_{j\geq 2}\frac{\lambda}{j}} = N \frac{\frac{\lambda_i}{i}}{\int_0^1 \lambda(x)dx} \tag{3.4}$$

The total number of edges emanating from the variable nodes is

$$E = N \sum_{i\geq 2} \frac{\lambda_i}{\int_0^1 \lambda(x)dx} = N \frac{1}{\int_0^1 \lambda(x)dx} \tag{3.5}$$

The quantity E can also be expressed in terms of the total number of check nodes M as

$$E = M \frac{1}{\int_0^1 \rho(x)dx} \tag{3.6}$$

From equations (3.5) and (3.6), the relation between M and N is

$$M = N \frac{\int_0^1 \rho(x)dx}{\int_0^1 \lambda(x)dx} \tag{3.7}$$

Assuming that all these check equations are linear independent, the design rate is equal to

$$r(\lambda, \rho) = \frac{N - M}{N} = 1 - \frac{\int_0^1 \rho(x)dx}{\int_0^1 \lambda(x)dx} \tag{3.8}$$

For a given length and a given degree distribution, there is a set of codes called ensemble of codes.

3.2 Decoding LDPC codes: Belief Propagation

The algorithms used for decoding LDPC codes are iterative algorithms called message passing algorithms. The reason for their name is that at each round of the algorithms messages are passed from message nodes to check nodes, and from check nodes back to message nodes. The messages from message nodes to check nodes are computed based on the observed value of the message node and some of the messages passed from the neighboring check nodes to that message node. An important aspect is that the message that is sent from a message node v to a check node c must not take into account the message sent in the previous round from c to v.

One important subclass of message passing algorithms is the belief propagation algorithm. This algorithm is present in Gallager's work [2], and it is also used in the Artificial Intelligence community [6]. The name belief propagation was given by the artificial intelligence community. The principle of belief propagation is to transmit “belief” along the edges. Here, the artificial intelligence community prefers the term “belief” to the term “probability” because they define probability as the “degree of belief” [6].The message that is passed from node v to node c is the probability that v has a certain value given the observed value of that message node and all the values communicated to v in the prior round from check nodes incident to v other than c. On the other hand, the message passed from c to v is the probability that v has a certain value given all the messages passed to c in the previous round from message nodes other than v.

Using the assumption of code bit independence and Baye's rule the APP $p\left(x_j = b \middle| S_j, r\right)$ can be rewritten as

$$p\left(x_j = b \middle| S_j, r\right) = Kp\left(r_j \middle| x_j = b\right)p(S_j|x_j = b, r) \tag{3.9}$$

where K is a constant for both $b = 0,1$ and can be ignored. The first term is easy to compute, which for Gaussian noise

$$p(r_j|x_j = b) = \frac{1}{(2\pi\sigma^2)^{1/2}} \exp \frac{\left(r_j + (-1)^b\right)^2}{2\sigma^2} \tag{3.10}$$

And for a BSC (with crossover probability p)

$$p(r_j|x_j = 0) = p^{r_j}(1-p)^{1-r_j} \tag{3.11}$$

$$p(r_j|x_j = 1) = p^{1-r_j}(1-p)^{r_j} \tag{3.12}$$

The second term in (3.10) is the probability that all the parity checks connected to x_j are satisfied given r and x_j = b. The term $S_j = \{S_{0j}, \dots, S_{kj}\}$ is a collection of events, where S_{mj} is the event that m'th parity node connected to x_j is satisfied. Again by independence of the code bits $(c_0, \dots, c_{N-1})$ this can be written as

$$p(S_j|x_j = b, r) = p(S_{0j}, \dots, S_{kj}|x_j = b, r) = \prod_{m\in M(j)} p(S_{mj}|x_j = b, r) \tag{3.13}$$

where $p(S_{mj}|x_j = b, r)$ is the probability that m'th parity check connected to the bit x_j is satisfied given x_j = b and r. If b=0, this is the probability that the code bits other than x_j connected to the m'th parity check have an even number of 1's. If b=1, the other code bits must have odd parity. Using this fact we next show that $p(S_{mj}|x_j = b, r)$ has a relatively simple form.

As a preliminary calculation, suppose two bits satisfy a parity check constraint $x1 \square x2 = 0$, and it is known that $p_1 = P(x_1 = 1)$ and $p_2 = P(x_2 = 1)$. Let $q_1 = 1 - p_1$ and $q_2 = 1 - p_2$. Then the probability that the check is satisfied is

$$P(x1 \square x2 = 0) = (1-p_1)(1-p_2) + p_1p_2$$

$$= 2p_1p_2 - p_1 - p_2 + 1$$

which can be rewritten as,

$$2P(x1 \oplus x2 = 0) - 1 = (1 - 2p_1)(1 - 2p_2) = (q_0 - p_0)(q_1 - p_1) \qquad (3.14)$$

Now suppose that L + 1 bits satisfy an even parity-check constraint $x_0 \oplus x_1 \oplus \ldots . \oplus x_L = 0$, then for known probabilities $\{p_1, p_2, \ldots., p_L\}$ corresponding to the bits $\{x_1, x_2, \ldots., x_L\}$, it is possible to generalize (3.15) to find the probability distribution on the binary sum $Z_L = x_0 \oplus x_1 \oplus \ldots . x_L$, where $Z_L = Z_{L-1} \oplus x_L$

$$2P(Z_L = 0) - 1 = \big(1 - 2P(Z_{L-1} = 1)\big)(1 - 2p_L)$$

$$= (2P(z_{L-1} = 0) - 1)(1 - 2p_L)$$

where $p_L = P(x_L = 1)$. Applying this recursively yields

$$2P(z_L = 0) - 1 = \prod_{i=1}^{L}(1 - 2p_i) \qquad (3.15)$$

$$P(z_L = 0) = \frac{1}{2}(1 + \prod_{i=1}^{L}(1 - 2p_i) = \frac{1}{2}(1 + \prod_{i=1}^{L}(q_i - p_i) \qquad (3.16)$$

Similarly it is possible to show

$$P(z_L = 1) = \frac{1}{2}(1 - \prod_{i=1}^{L}(q_i - p_i) \qquad (3.17)$$

Returning to our calculation of $p\big(S_{mj} | x_j = b, r\big)$, if $x_j=0$ then we use $P(z_L=0)$ and if $x_j=1$, we use $P(z_L=1)$.

$$p\big(S_{mj} | x_j = 0, r\big) = \frac{1}{2}(1 + \prod_{n'=N(m)\backslash j}(q^0_{mn'} - q^1_{mn'}) \qquad (3.18)$$

$$p\big(S_{mj} | x_j = 1, r\big) = \frac{1}{2}(1 - \prod_{n'=N(m)\backslash j}(q^0_{mn'} - q^1_{mn'}) \qquad (3.19)$$

where $q^0_{mn'}$ is the probability that code bit $x_{n'}$ is zero, given r and excluding any information about $x_{n'}$ from parity check m. The exclusion is needed because we desire extrinsic knowledge about $x_{n'}$ from its parity checks to get extrinsic knowledge about x_j.

Note that the rightmost product in (3.20) is over all code bits connected to the m'th parity node except for x_j , since we are interested in the even or odd parity of the bits other than x_j. Combining these results with (3.10) and (3.14) we get the final expressions for the APPs.

$$p(x_j = 0|S_j, r) = Kp(r_j|x_j = 0)p(S_j|x_j = 0, r) \qquad (3.20)$$

$$Kp(r_j|x_j = 0) \prod_{m\in M(j)} \frac{1}{2}\left(1 + \prod_{n'=N(m)\backslash j} (q^0_{mn'} - q^1_{mn'} \right. \qquad (3.21)$$

$$p(x_j = 1|S_j, r) = Kp(r_j|x_j = 1)p(S_j|x_j = 1, r) \qquad (3.22)$$

$$p(r_j|x_j = 1) \prod_{m\in M(j)} \frac{1}{2}\left(1 - \prod_{n'=N(m)\backslash j} (q^0_{mn'} - q^1_{mn'} \right. \qquad (3.23)$$

Careful inspection of the APPs in (3.27) and (3.28) one sees that many of the calculations can be done in parallel. Furthermore, some can be viewed as "parity node" computations and others as "bit node" computations, for example, for b = 1, $p(r_j|x_j = 1)$ is

$$\underbrace{\overbrace{p(r_j|x_j = 1)}^{\text{prior}} \prod_{m\in M(j)} \overbrace{\tfrac{1}{2}\left(1 - \prod_{n'=N(m)\backslash j}(q^0_{mn'} - q^1_{mn'}\right)}^{\text{Parity node}}}_{\text{Bit node}} \qquad (3.24)$$

The notation can be simplified by letting $\delta q_{mj} = q^0_{mj} - q^1_{mj}$ and defining parity check equations as

$$r_{mj}^0 = \frac{1}{2}(1 + \prod_{n'=\bar{N}(\bar{m})\setminus j} \delta q_{mn'}) \quad (3.25)$$

$$r_{mj}^1 = \frac{1}{2}(1 - \prod_{n'=\bar{N}(\bar{m})\setminus j} \delta q_{mn'}) \quad (3.26)$$

For the BSC, the expressions in (3.27) and (3.28) can be simplified a great deal. We begin by fleshing out what happens in the first iteration of the decoder. The probabilities q_{mj}^0 and q_{mj}^1 get initialized as

$$q_{mj}^0 = p(r_j|x_j = 0) = p^{r_j}(1-p)^{1-r_j} \quad (3.27)$$

$$q_{mj}^1 = p(r_j|x_j = 1) = p^{1-r_j}(1-p)^{r_j} \quad (3.28)$$

Depending on the value of the received bit r_j the initialized q_{mj}^i's take on only one of the two values. The expressions in (3.25) and (3.26) can be simplified as follows. First recognize that if $r_j = 0$

$$q_{mj}^0 - q_{mj}^1 = p^{r_j}(1-p)^{1-r_j} - p^{1-r_j}(1-p)^{r_j}$$

$$= 1 - 2p$$

And if r_j=1, $q_{mj}^0 - q_{mj}^1 = -(1-2p)$. This implies that (3.25) and (3.26) can be rewritten

$$r_{mj}^0 = \frac{1}{2}(1 + \prod_{n'=\bar{N}(\bar{m})\setminus j} (1-2p)(-1)^{r_{n'}}) \quad (3.29)$$

$$= \frac{1}{2}(1 + (1-2p)^{|N(m)|-1} \prod_{n'=\bar{N}(\bar{m})\setminus j} (-1)^{r_{n'}} \quad (3.30)$$

$$r_{mj}^1 = \frac{1}{2}(1 - (1-2p)^{|N(m)|-1} \prod_{n'=\bar{N}(\bar{m})\setminus j} (-1)^{r_{n'}} \quad (3.31)$$

Note that the product on the RHS of both of these terms evaluates to either +1 or -1 depending on the number of 1's in the set N (m)\j and is very easy to compute given the value of the parity check computed for node m. Finally, r_{mj}^0 and r_{mj}^1take only one of two values, making this easy to implement in hardware, with no real calculations required. Namely

$r_{mj}^0 = \frac{1}{2}(1 - (1 - 2p)^{|N(m)|-1}$ if the parity of bits with indices from N(m)\j is odd

$= \frac{1}{2}(1 + (1 - 2p)^{|N(m)|-1}$ if the parity of bits with indices from N(m)\j is even

$r_{mj}^0 = \frac{1}{2}(1 - (1 - 2p)^{|N(m)|-1}$ if the parity of bits with indices from N(m)\j is even

$= \frac{1}{2}(1 + (1 - 2p)^{|N(m)|-1}$ if the parity of bits with indices from N (m)\j is odd (3.32)

Proceeding further the APPs can be simplified as:

$$p(x_j = 0|S_j, r) = p(r_j|x_j = 0) \prod_{m \in M(j)} \frac{1}{2}(1 \pm (1 - 2p)^{|N(m)|-1} \quad (3.33)$$

$$p(x_j = 1|S_j, r) = p(r_j|x_j = 1) \prod_{m \in M(j)} \frac{1}{2}(1 \pm (1 - 2p)^{|N(m)|-1} \quad (3.34)$$

which reduces to,

$$p(x_j = 0|S_j, r) = f_j^0 \prod_{m \in M^{odd}(j)} \frac{1}{2}(1 - (1 - 2p)^{|N(m)|-1} \prod_{m \in M^{even}(j)} \frac{1}{2}(1 + (1 - p)^{|N(m)|-1} \quad (3.35)$$

$$p(x_j = 1|S_j, r) = f_j^1 \prod_{m \in M^{odd}(j)} \frac{1}{2}(1 + (1 - 2p)^{|N(m)|-1} \prod_{m \in M^{even}(j)} \frac{1}{2}(1 - (1 - p)^{|N(m)|-1} \quad (3.36)$$

Where $M^{odd}(j)$ are the set of nodes connected to x_j with odd parity, and $M^{even}(j)$ are those with even parity. Finally, the first iteration calculations, which are extremely easy to calculate given the hard decision results of the parity checks, are:

$$p(x_j = 0|S_j, r) = f_j^0 \frac{1}{2}((1 - (1 - 2p)^{|N(m)|-1})^{|M^{odd}(j)|} \frac{1}{2}((1 + (1 - 2p)^{|N(m)|-1})^{|M^{even}(j)|}$$

$$p(x_j = 1|S_j, r) = f_j^1 \frac{1}{2}((1 + (1 - 2p)^{|N(m)|-1})^{|M^{odd}(j)|} \frac{1}{2}((1 - (1 - 2p)^{|N(m)|-1})^{|M^{even}(j)|} \quad (3.37)$$

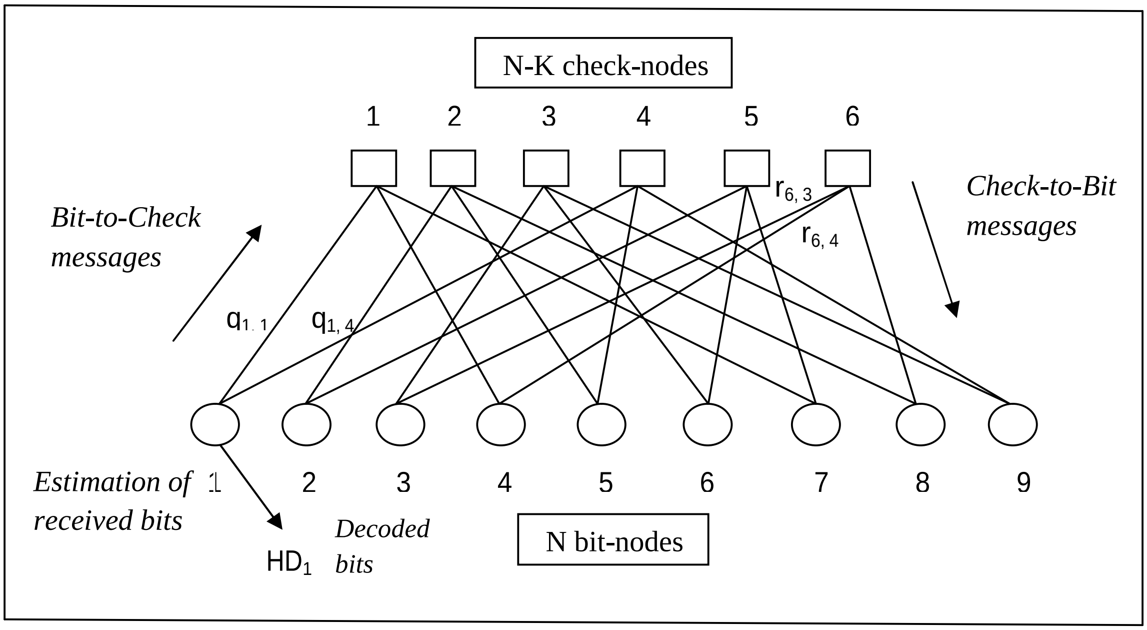

Figure 3.5: Message Passing Algorithm

3.3 MSK (Minimum Shift Keying)

In digital modulation, minimum shift keying (MSK) is a type of continuous-phase frequency shift keying that was developed in the late 1960s. MSK has a modulation index (h) of 0.5. A modulation index of 0.5 corresponds to the minimum frequency spacing that allows two FSK signals to be coherently orthogonal, and the name minimum shift keying implies the minimum frequency separation (i.e. bandwidth) that allows orthogonal detection. Minimum Shift Keying (MSK) is derived from Offset QPSK [15] by replacing the rectangular pulse in amplitude with a half-cycle sinusoidal pulse.

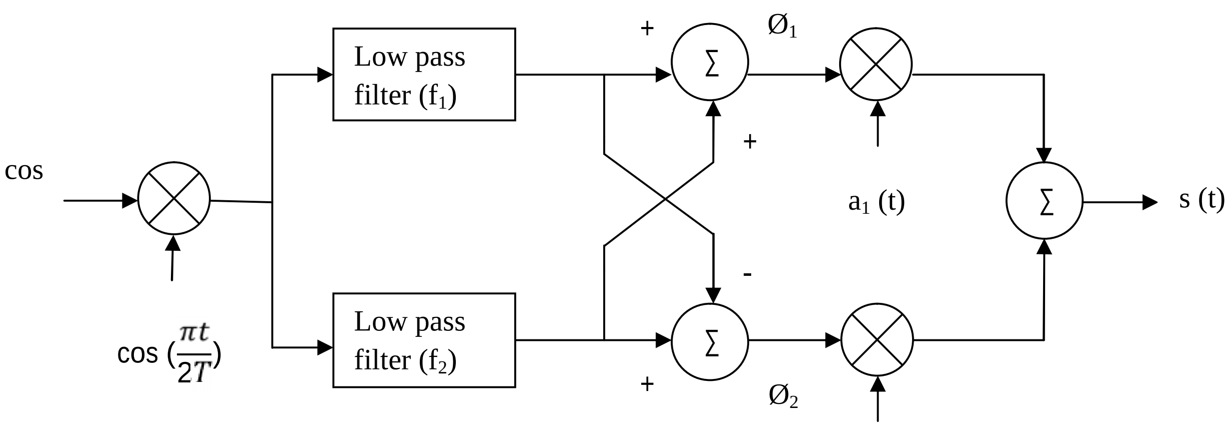

Figure 3.6: MSK Modulator [27]

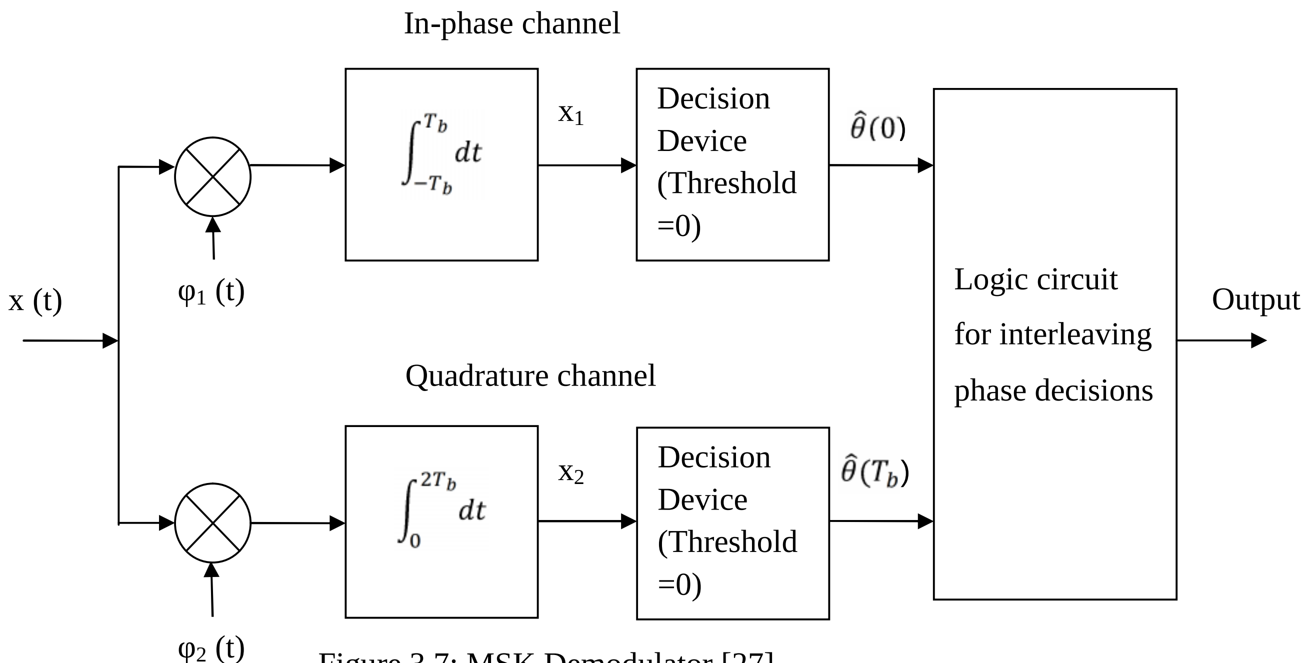

Figure 3.7: MSK Demodulator [27]

$$S(t) = \sqrt{\frac{2E_b}{T_b}} \cos\,[2\pi f_c t + \theta(t)] \tag{3.38}$$

Where θ (t) is the phase of the signal and it is a continuous function of time.

The phase θ (t) of the signal increases or decreases linearly with time during bit duration of T_b seconds as shown by the following equation:

$$\theta(t) = \theta(0) \pm \left(\frac{\pi h}{T_b}\right) * t\,,\ 0 \le t \le T_b \tag{3.39}$$

The plus sign indicates to sending bit 1 and the minus sign corresponds to sending a 0 bit.

$$f_1 = f_c + \frac{h}{2T_b} \tag{3.40}$$

$$f_2 = f_c - \frac{h}{2T_b} \tag{3.41}$$

Combining the two equations given above, we get:

$$f_c = \frac{1}{2}(f_1 + f_2) \tag{3.42}$$

$$h = T(f_1 - f_2) \qquad (3.43)$$

where f_c is the nominal carrier frequency which is nothing but the arithmetic mean of the frequencies f_1 and f_2. The difference between the frequencies f_1 and f_2, normalized with respect to the bit rate ($1/T_b$), defined the parameter h, which is referred to as the deviation ratio.

3.3.1 Signal Space Diagram

We know that

$$S(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left[2\pi f_c t + \theta(t)\right]$$

Using trigonometry we can express the above equation as

$$S(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \cos[\theta(t)] - \sqrt{\frac{2E_b}{T_b}} \sin(2\pi f_c t) \sin[\theta(t)] \qquad (3.44)$$

Consider the in phase components; it consists of a half cycle cosine pulse defined as

$$S_I(t) = \sqrt{\frac{2E_b}{T_b}} \cos[\theta(t)] \qquad (3.45)$$

$$= \pm\sqrt{\frac{2E_b}{T_b}} \cos\left(\frac{\pi}{2T_b}\right) t, \qquad -T_b \le t \le T_b \qquad (3.46)$$

Where plus sign corresponds to $\theta(0) = 0$ and minus sign corresponds to $\theta(0) = \pi$.
Similarly we see that the in the interval , $0 \le t \le +2Tb$ quadrature component $S_Q(t)$ consists of a half cycle sine pulse, whose polarity depends on $\theta(Tb)$ as shown by

$$S_Q(t) = \sqrt{\frac{2E_b}{T_b}} \sin[\theta(t)] \qquad (3.47)$$

$$= \pm\sqrt{\frac{2E_b}{T_b}} \sin\left(\frac{\pi}{2T_b}\right) t, \qquad 0 \le t \le 2T_b \qquad (3.48)$$

Where plus sign corresponds to θ (Tb) = π/2 and minus sign corresponds to θ (Tb) = -π/2.

MSK can signal may assume one of the four possible states depending on the values of θ(0) and θ(T_b).

From the expression

$$S(t) = \sqrt{\frac{2E_b}{T_b}} \square \cos(2\pi f_c t)\cos[\theta(t)] - \sqrt{\frac{2E_b}{T_b}} \square \sin(2\pi f_c t)\sin[\theta(t)]$$

We deduce the orthonormal basis functions $ø_1$ (t) and $ø_2$ (t) defined by

$$ø1(t) = \sqrt{\frac{2}{T_b}} * \cos\left(\frac{\pi}{2T_b}\right) t * \cos(2\pi f_c t), \quad 0 \le t \le T_b \qquad (3.49)$$

$$ø1(t) = \sqrt{\frac{2}{T_b}} * \sin\left(\frac{\pi}{2T_b}\right) t * \sin(2\pi f_c t), \quad 0 \le t \le T_b \qquad (3.50)$$

We may assume MSK expression as

$$S(t) = S_1 ø_1(t) + S_2 ø_2(t), \quad 0 \le t \le T_b \qquad (3.51)$$

Where coefficients S_1 and S_2 are related to the phase states θ (0) and θ (T_b)

$$S_1 = \int S(t) \square \square_1(t)dt, \quad -T_b \le t \le T_b \qquad (3.52)$$

$$S_2 = \int S(t) \square \square_2(t)dt, \quad -T_b \le t \le T_b \qquad (3.53)$$

This gives

$$S_1 = \sqrt{E_b} * \cos[\theta(0)], \quad -T_b \le t \le +T_b \qquad (3.54)$$

$$S_2 = \sqrt{E_b} * \sin[\theta(T_b)], \quad 0 \le t \le +2T_b \qquad (3.55)$$

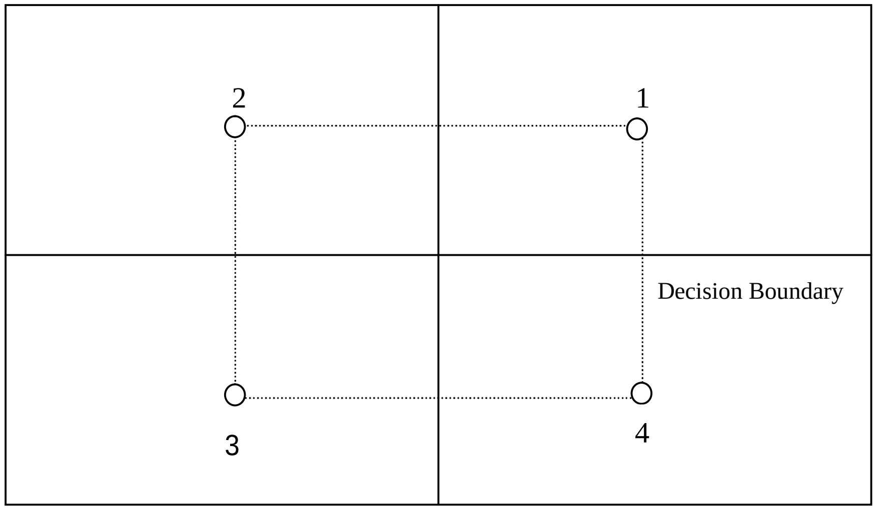

Figure 3.8: Signal Space Diagram

1. Message point m1 : symbol 0; [θ(0) = 0, θ(Tb) = -π/2; Coordinates (√ Eb , √ Eb)
2. Message point m2 : symbol 1; [θ(0) = π, θ(Tb) = -π/2; Coordinates (√ Eb , -√ Eb)
3. Message point m3 : symbol 0; [θ(0) = π, θ(Tb) = π/2; Coordinates (-√ Eb , -√ Eb)
4. Message point m4 : symbol 1; [θ(0) = 0, θ(Tb) = π/2; Coordinates (√ Eb , -√ Eb)

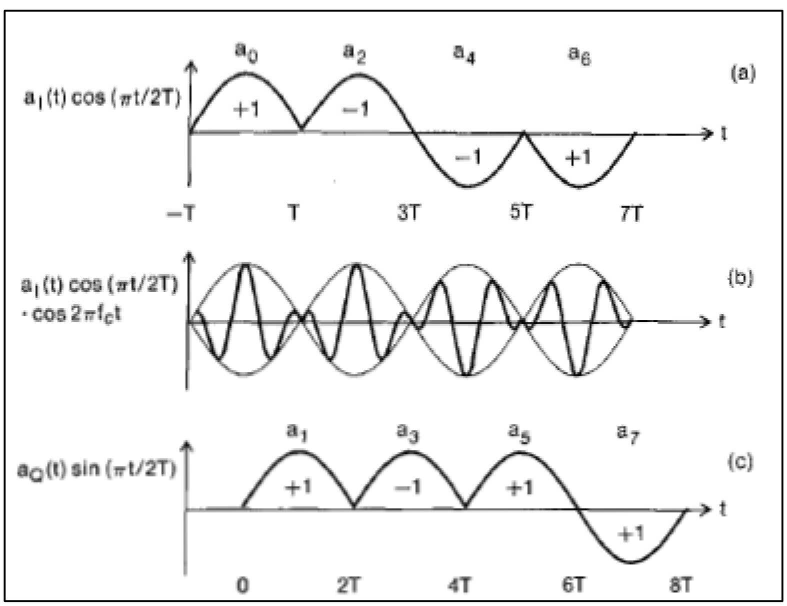

Figure 3.9: MSK waveforms [15]

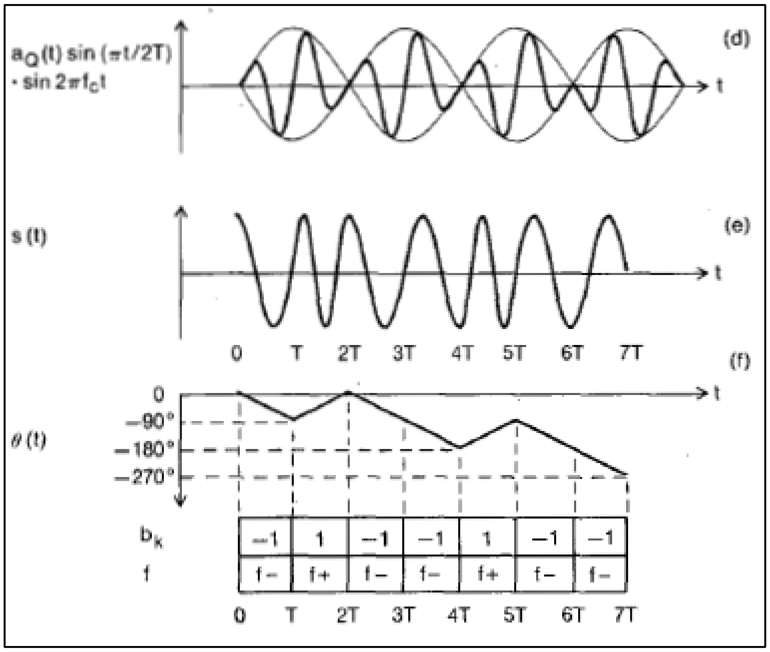

Figure 3.10: MSK waveforms continued [15]

3.4 GMSK (Gaussian Minimum Shift Keying)

In MSK we replace the rectangular pulse with a sinusoidal pulse. Obviously, other pulse shapes are also possible. A Gaussian-shaped impulse response filter generates a signal with low side lobes and narrower main lobe than the rectangular pulse. So in GMSK, we replace the rectangular pulse with a Gaussian pulse by using a Gaussian filter. A filter used to reduce the bandwidth of a baseband pulse train prior to modulation is called a pre-modulation filter. The Gaussian pre-modulation filter smoothens the phase trajectory of the MSK signal thus limiting the instantaneous frequency variations. The rest of the circuit is same as MSK except that a premodulation Gaussian filter is placed at the start. The BT product (where B is the bandwidth and T is the bit duration) in the premodulation filter defines the bandwidth of the system. The bandwidth reduction comes at a cost of increased ISI (Inter Symbol Interference). BER for GMSK is:

$$P_e = Q\left(\sqrt{2\alpha\left(\frac{E_b}{N_0}\right)}\right) \tag{3.56}$$

Where α is a constant related to BT as:

Table 3.1: Values of α for different values of BT

Values of BT	Values of α
0.25	0.68
∞	0.85

The case where BT → ∞ corresponds to MSK as the BER of MSK is given by:

$$P_e \approx Q(\sqrt{2\left(\frac{E_b}{N_0}\right)})$$

So we see that $P_e^{GMSK} > P_e^{MSK}$. This arises from the trade-off between power and bandwidth efficiency: GMSK achieves better bandwidth efficiency than MSK at the expense of power efficiency. The impulse response of the Gaussian filter is:

$$g(t) = \frac{1}{2T}[Q\left(2\pi B_b \frac{t-\frac{T}{2}}{\sqrt{ln2}}\right) - Q\left(2\pi B_b \frac{t+\frac{T}{2}}{\sqrt{ln2}}\right)] \tag{3.57}$$

For $0 \leq B_bT \leq \infty$

Where Q (t) is the Q-function

$$Q(t) = \int_t \frac{1}{\sqrt{2}} \exp\left(-\frac{x^2}{2}\right) dx \tag{3.58}$$

B_b is the bandwidth of the low pass filter having a Gaussian shaped spectrum, T is the bit period and BN=B_bT is the normalized bandwidth. In figure (3.10) a premodulation Gaussian filter is used before the MSK modulator. The purpose of this pre modulation filter is to turn the rectangular waveforms to Gaussian shaped pulses.

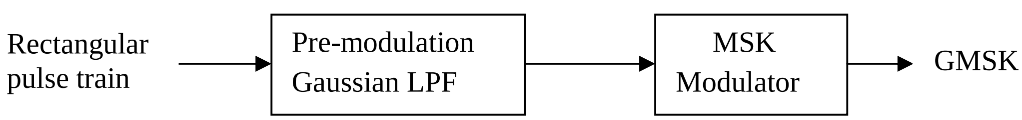

Figure 3.11: GMSK modulator [27]

Given below is the diagram of modulator which was initially used to modulate GMSK [15]. In this circuit a low pass filter and voltage controlled oscillator are used in series. This is a simpler implementation of the GMSK modulator, but its performance in not as good as the one shown in Figure (3.10).

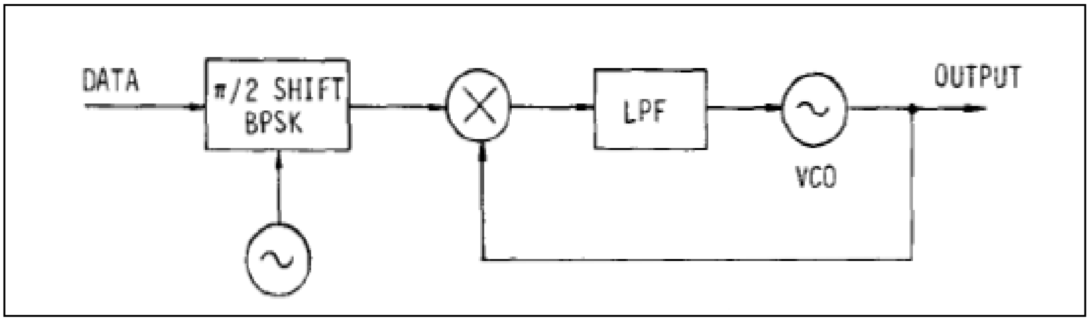

Figure 3.12: GMSK modulator model proposed initially [16]

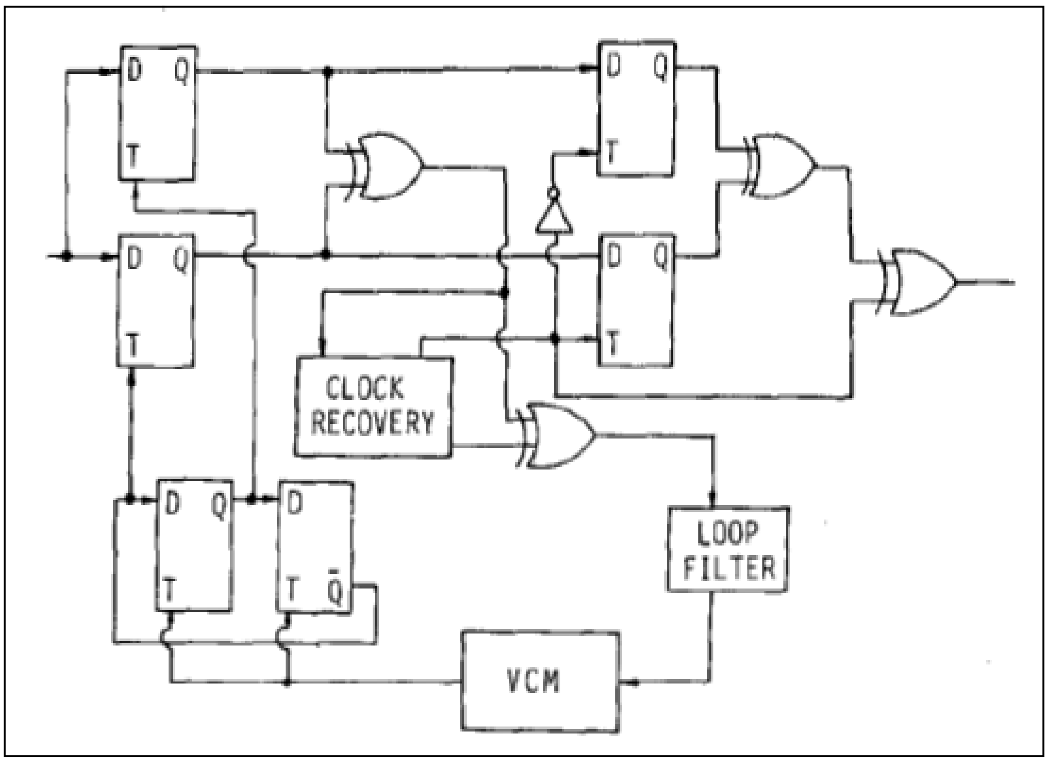

Figure 3.13: GMSK Demodulator model that was initially proposed [16]

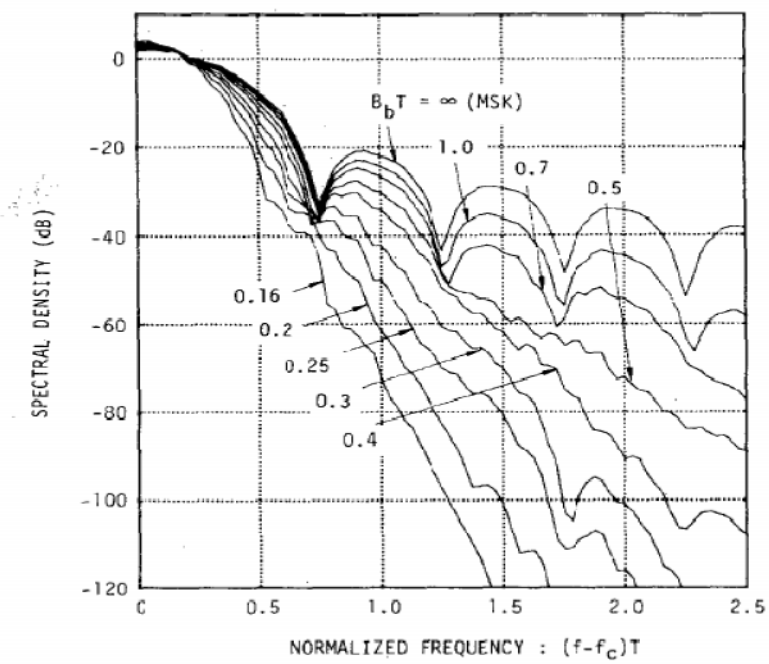

Figure 3.14: Power spectra of GMSK [16]

In Figure 3.14, the power spectra of GMSK signals for different B_bT products is compared with the power spectra of MSK with $B_bT=\infty$. It is clearly visible from the graph that GMSK is more spectrally efficient than MSK.

3.5 Nakagami Fading

Nakagami fading channels have received considerable attention in the study of various aspects of wireless systems. It is also called m-distribution and fits experimental data collected in a variety of fading better than Rayleigh, Rician, or Lognormal distributions. It can be used to model fading conditions which are more severe or less severe than Rayleigh fading. In this type of fading, we have a m parameter by which we can model the signal fading conditions from severe to no fading at all. For m=1/2, the distribution is a one sided Gaussian distribution, for m=1, the distribution is corresponds to Rayleigh fading, for fading more severe than Rayleigh fading, $1/2 \leq m \leq 1$ and for no fading at all $m=\infty$. When m=2 the distribution approximates Rician distribution.

The probability density function of Nakagami distribution is given as:

$$f(r) = \frac{2m^m r^{2m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{mr^2}{\Omega}\right) \tag{3.59}$$

$m \geq 1/2$; $r \geq 0$

m is Nakagami parameter or shape factor, describing the fading degree due to scattering and multipath interference

$$m = \frac{E^2\{r^2\}}{E\{[r^2 - E(r^2)]^2\}} \tag{3.60}$$

Ω is the average power of multipath scatter field
$\Omega = E\{r^2\}$

And, $\Gamma(m)$ is the gamma function.

Experimental and theoretical works have shown that the Nakagami distribution is the best-fit distribution for data obtained from many urban multipath radio channels. Due to its mathematical tractability, Nakagami distribution is also widely used in modeling wireless channels with diversity combining.

CHAPTER 4
RESULTS AND DISCUSSION

In this chapter, the simulation results are described. The simulation was done on MATLAB R2007b (7.5) software. We have done a comparative study of LDPC-coded and uncoded MSK and GMSK in Nakagami fading conditions.

4.1 Simulation Details

A bit stream (input) is sent through a vector encoder (LDPC) and modulated by using GMSK/MSK modulation technique. This encoder adds redundancy to the input bit stream by multiplying it with a sparse parity check matrix that is generated by using a MATLAB program. Thus we have a LDPC coded bit stream at the output of the encoder. The modulator converts the signal waveforms into Gaussian shaped pulses to better withstand the channel impairment effects. The modulator divides the input encoded signal into in-phase and quadrature components and Nakagami fading and AWGN is applied independently to both the components. The Nakagami fading coefficients are again generated by using a MATLAB program defined as a function. The output of the channel is a degraded signal which has been distorted by Nakagami fading and AWGN. The degraded signal is then demodulated by using GMSK/MSK demodulator. This demodulated signal is passed through an iterative LDPC decoder, which decodes the input signal and tries to detect and correct most of the errors. As the number of iterations increase, the error performance increases as well. The error performance is measured by computing the BER at different SNR values. Unless otherwise stated, we assume in the following that

1. The Nakagami fading model that we use in our simulations is flat
2. The B_bT product of the premodulation Gaussian filter in GMSK is always taken to be 0.25.
3. The receiver has complete knowledge about the channel

Details of parameters, schemes and values used

- Software used: MATLAB R2007b (7.5)
- Encoding scheme used: LDPC
- Dimensions of parity check matrix: 150× 300
- Modulation technique: GMSK and MSK
- Channel: Nakagami *plus* AWGN
- Number of iterations: 1000
- SNR values: 1-10dB
- No of coded bits: 300

4.2 Simulation Results of BER Performance of LDPC Codes

4.2.1 Comparison of uncoded GMSK signal for m=1 and m=2

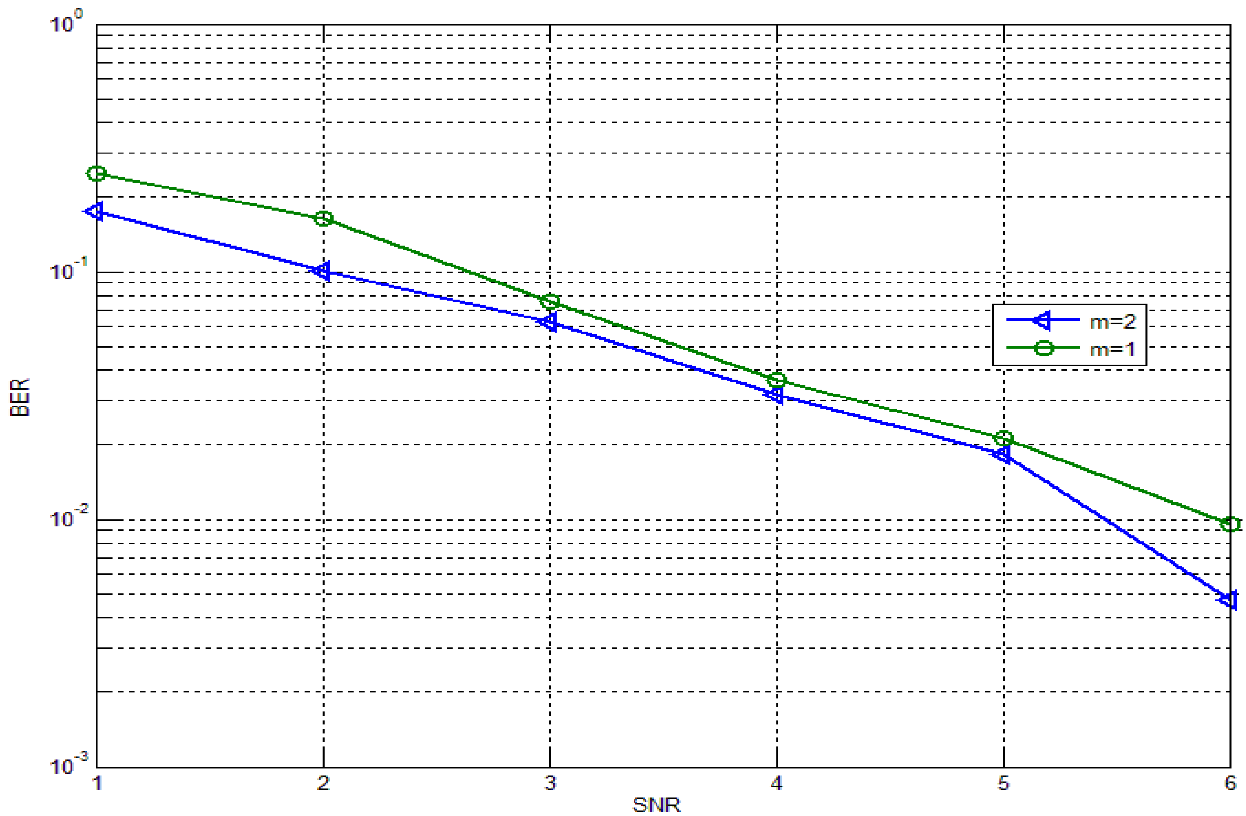

Fig 4.1 BER performance of uncoded GMSK modulated signal on a Nakagami fading channel for m=1 and m=2

4.2.2 Comparison of LDPC coded GMSK signal for m=1 and m=2

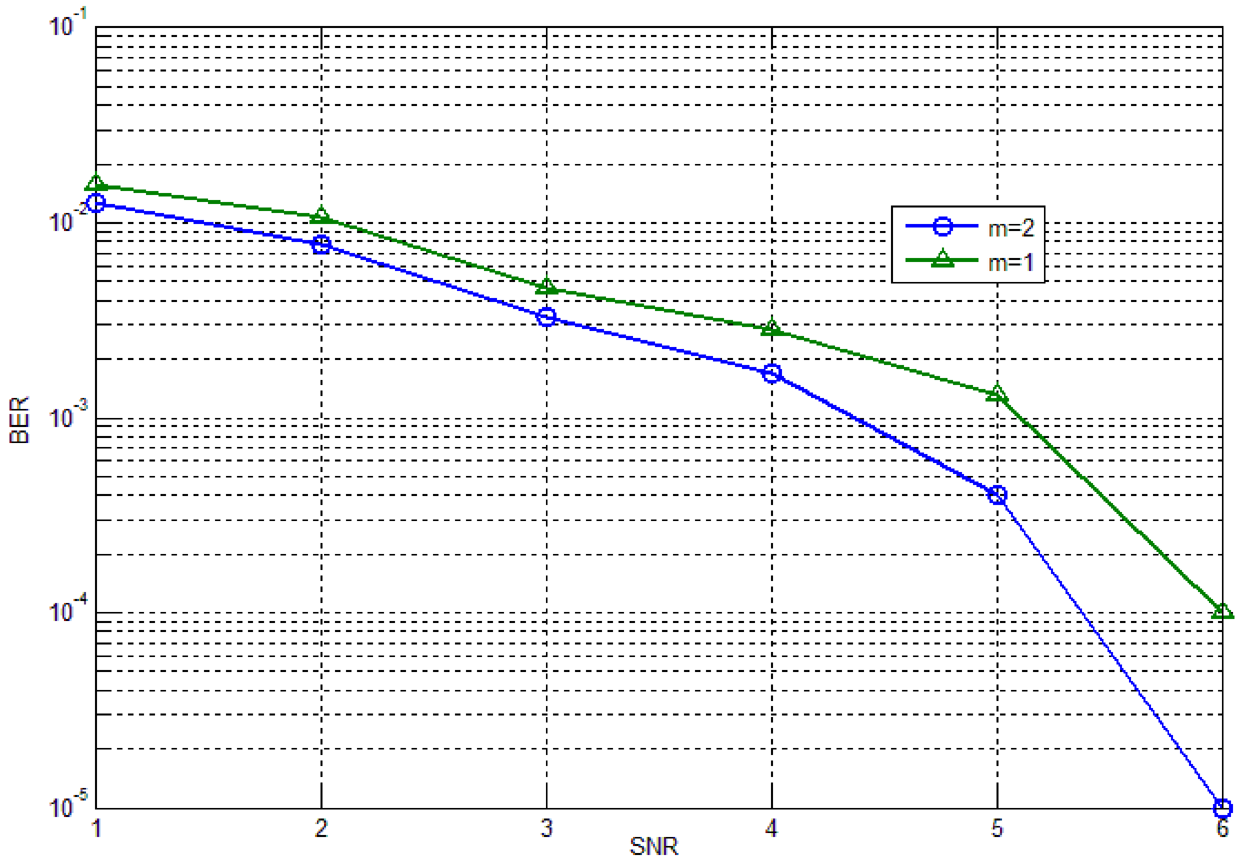

Fig 4.2 BER performance of LDPC coded GMSK modulated signal on a Nakagami fading channel for m=1 and m=2

4.2.3 Comparison of Uncoded and LDPC-coded GMSK signals for m=1 and m=2

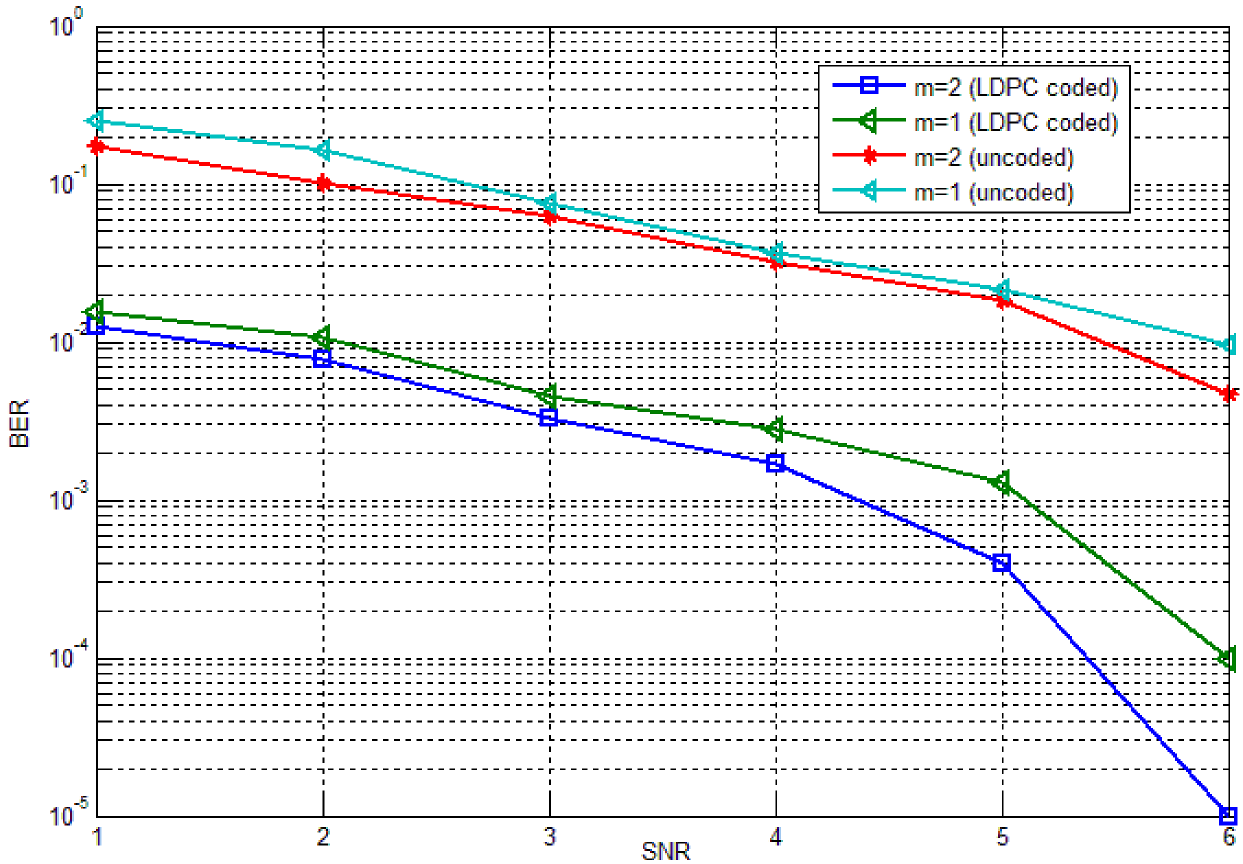

Fig. 4.3: BER performance of uncoded and LDPC-coded GMSK modulated signals on a Nakagami fading channel for m=1 and m=2

Table 4.1: Combined SNR performance for Nakagami channel for m=1 at BER of 10^{-2}

At BER of 10^{-2}	
Modulation Technique	**Signal To Noise Ratio (dB)**
Uncoded GMSK m=1	5.9
Coded GMSK m=1	2

Table 4.1 shows that for a BER of 10^{-2} for m=1 LDPC coded GMSK modulated signal gives about 3.9 dB coding gain over the use of uncoded GMSK. So the performance of coded GMSK is much better than the performance of uncoded GMSK.

Table 4.2: Combined SNR performance for Nakagami channel for m=1 at BER of 10^{-2}

At BER of 10^{-2}	
Modulation Technique	**Signal To Noise Ratio (dB)**
Uncoded GMSK m=2	5.4
Coded GMSK m=2	0.4

Table 4.2 shows that for a BER of 10^{-2} for m=2 LDPC coded GMSK modulated signal gives about 5 dB coding gain over the use of uncoded GMSK. So the performance of coded GMSK is much better than the performance of uncoded GMSK.

4.2.4 Comparison of Uncoded and LDPC-coded GMSK signals (Normalized) for m=1 and m=2

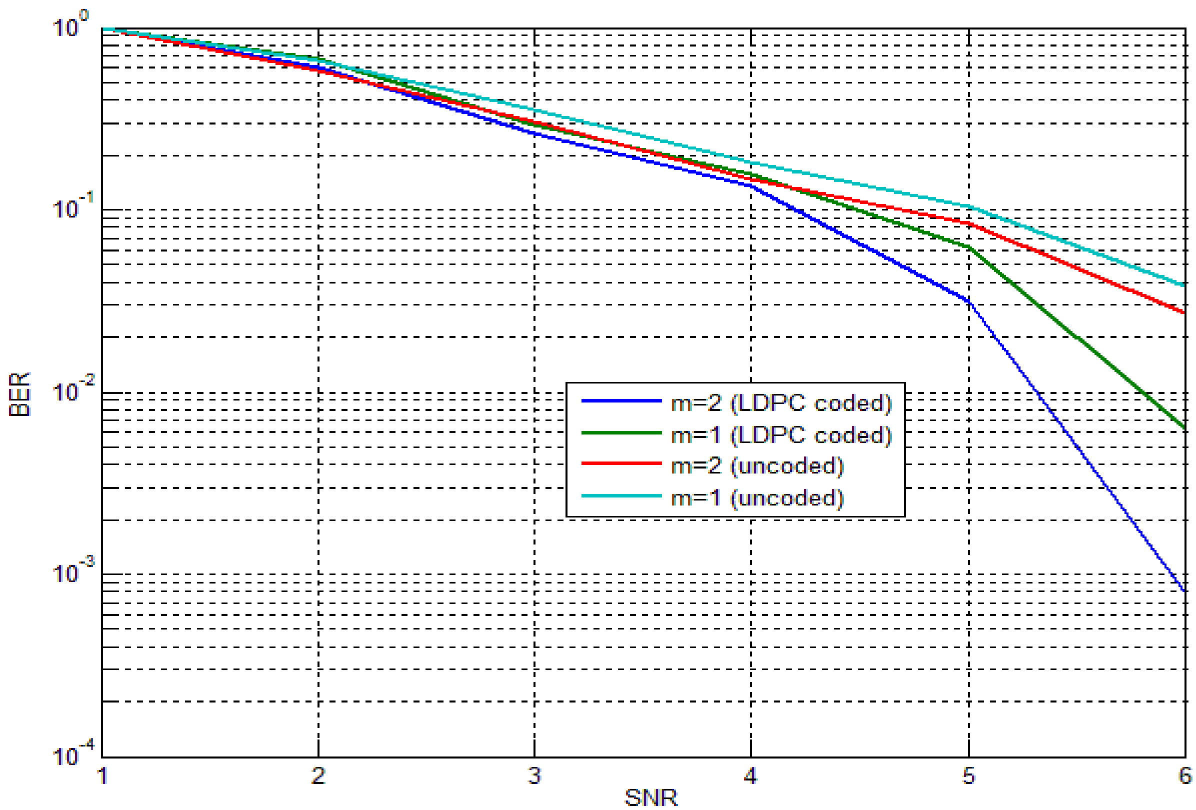

Fig. 4.4: BER performance of uncoded and LDPC coded GMSK modulated signals (Normalized) on a Nakagami fading channel for m=1 and m=2

4.2.5 Comparison of LDPC coded MSK and GMSK signal on Nakagami fading for m=1

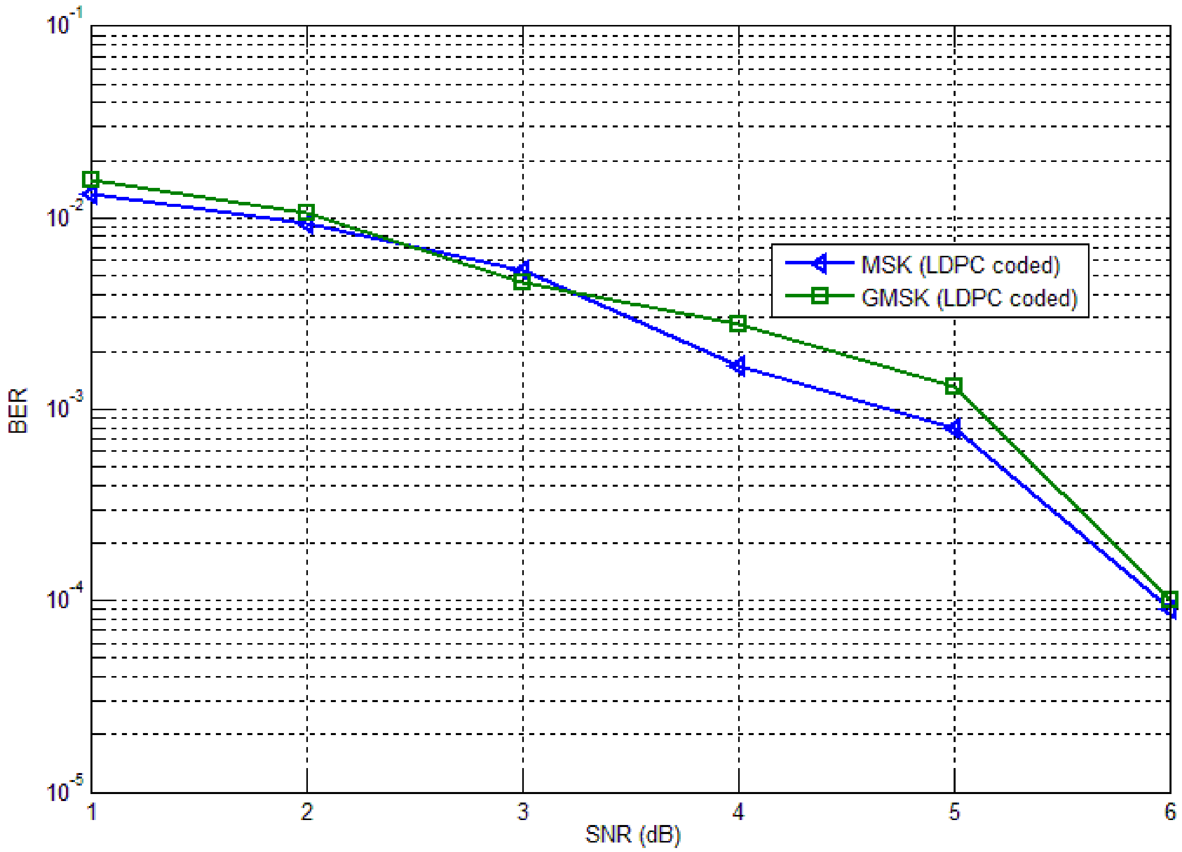

Fig. 4.5: BER performance of LDPC coded MSK modulated signal and LDPC coded GMSK modulated signal as compared on a Nakagami fading channel for m=1

4.2.6 Comparison of LDPC coded MSK and GMSK signal on Nakagami fading for m=2

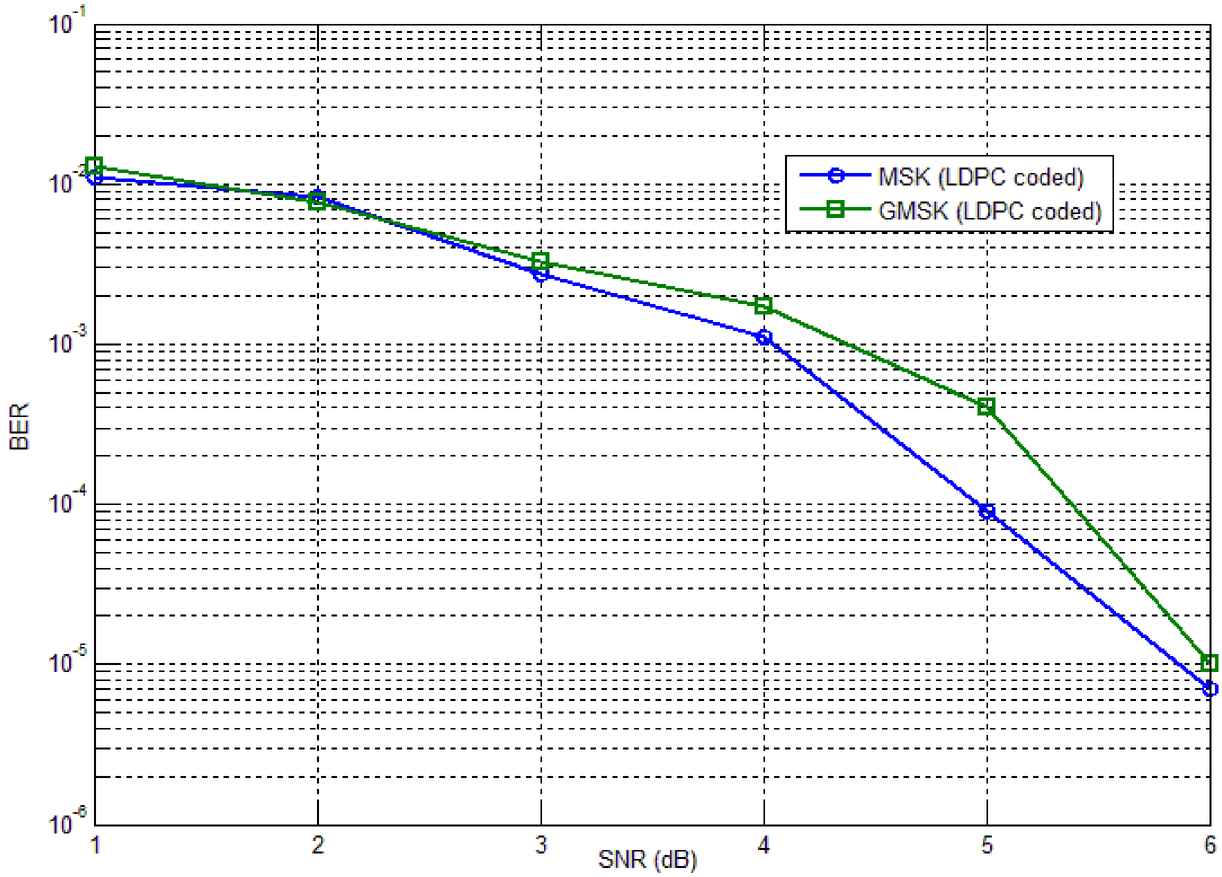

Fig 4.6: BER performance of LDPC coded MSK modulated signal and LDPC coded GMSK modulated signal on a Nakagami fading channel for m=2

4.2.7 Comparison of uncoded MSK and GMSK signal on Nakagami fading for m=1

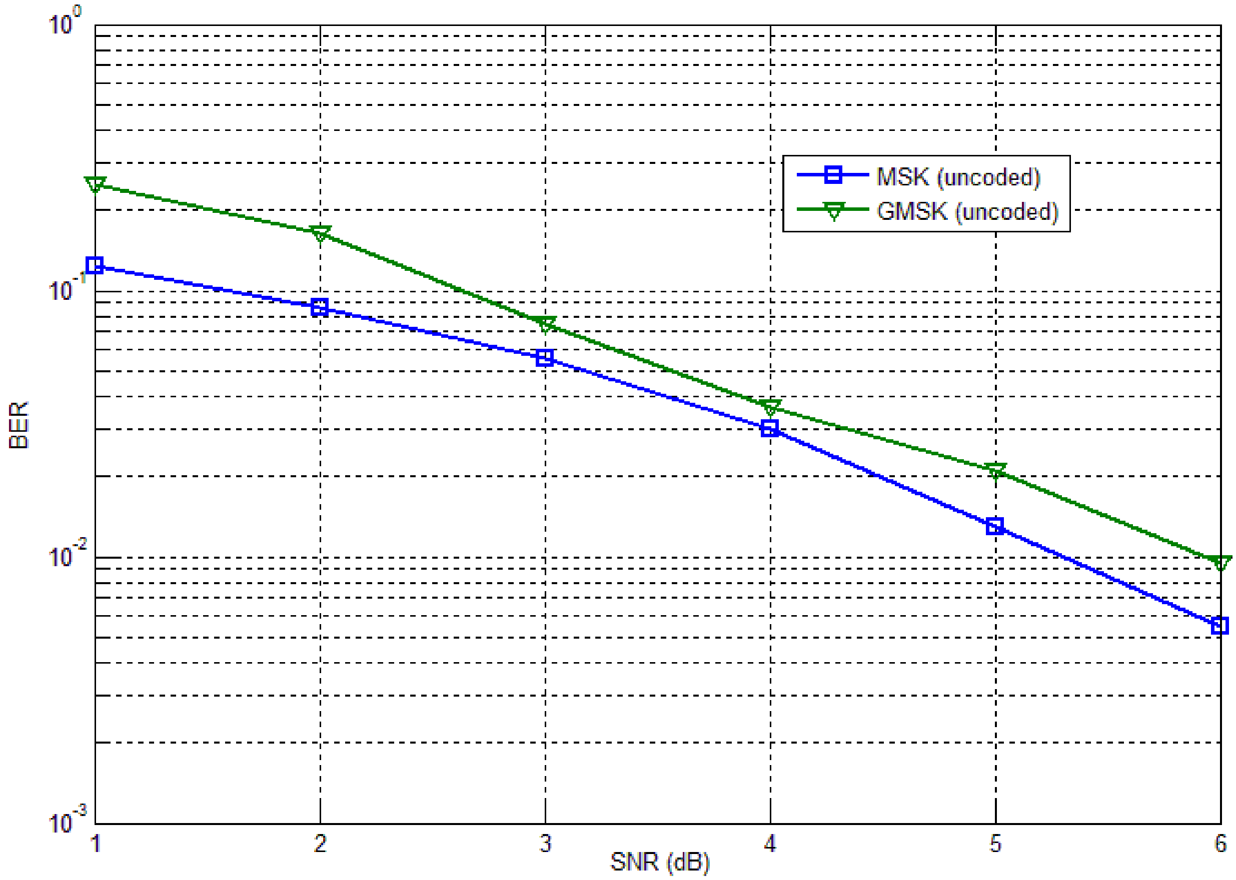

Fig. 4.7: BER performance of uncoded MSK modulated signal and uncoded GMSK modulated signal as compared on a Nakagami fading channel for m=1

4.2.8 Comparison of uncoded MSK and GMSK signal on Nakagami fading for m=2

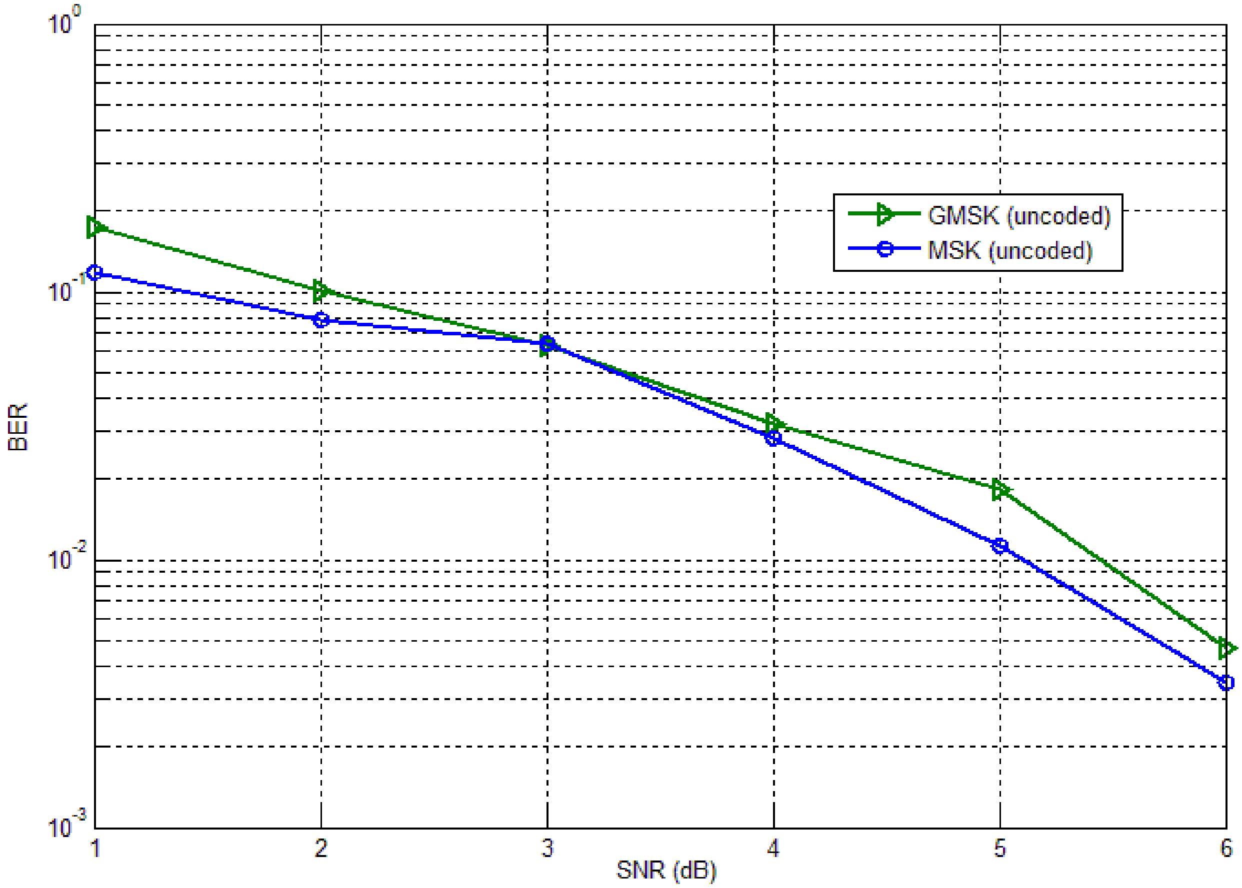

Fig. 4.8: BER performance of uncoded MSK modulated signal and uncoded GMSK modulated signal as compared on a Nakagami fading channel for m=2

4.2.9 Comparison of uncoded MSK and LDPC-coded MSK signal on Nakagami fading for m=1

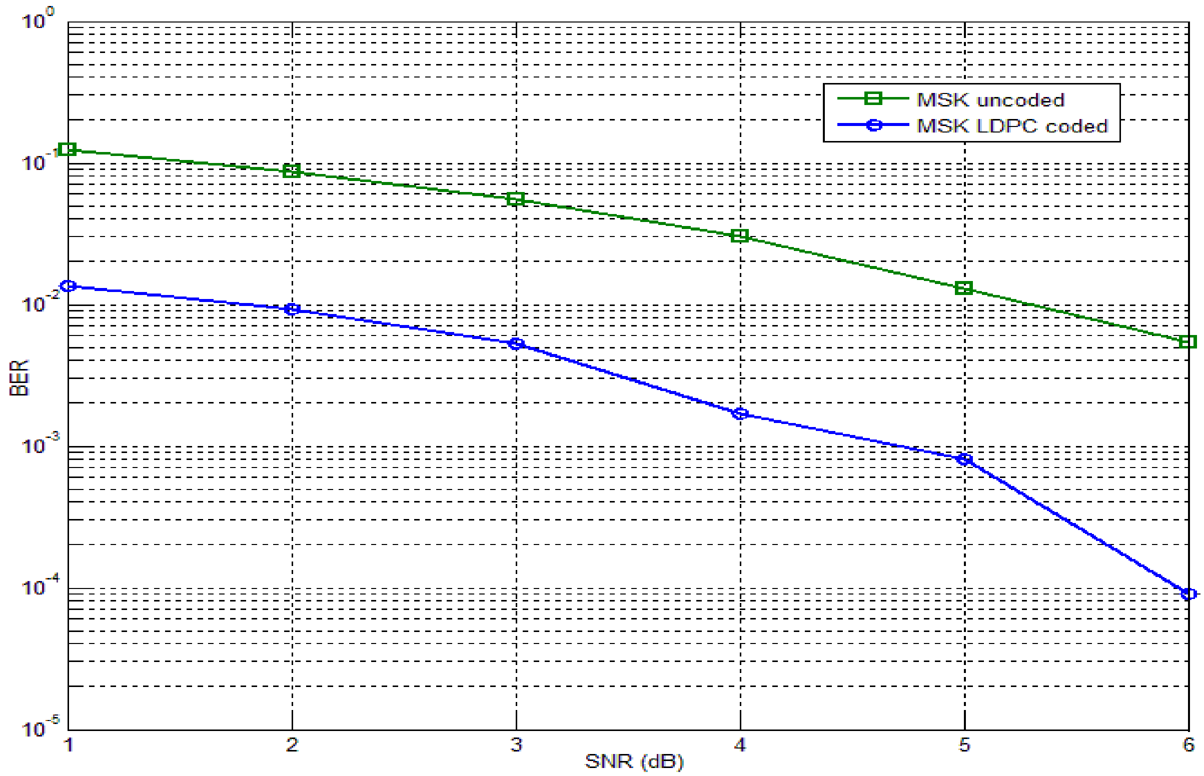

Fig. 4.9: BER performance of uncoded and LDPC-coded MSK modulated signal compared on a Nakagami fading channel for m=1

Table 4.3: Combined SNR performance for Nakagami channel for m=1 at BER of 10^{-2}

At BER of 10^{-2}	
Modulation Technique	**Signal To Noise Ratio (dB)**
Uncoded MSK m=1	5.3
Coded MSK m=1	2

Table 4.3 shows that for a BER of 10^{-2} for m=1 LDPC coded MSK modulated signal gives about 3.3 dB coding gain over the use of uncoded MSK. So the performance of coded MSK is much better than the performance of uncoded MSK.

4.2.10 Comparison of uncoded MSK and LDPC codec MSK signal on Nakagami fading for m=2

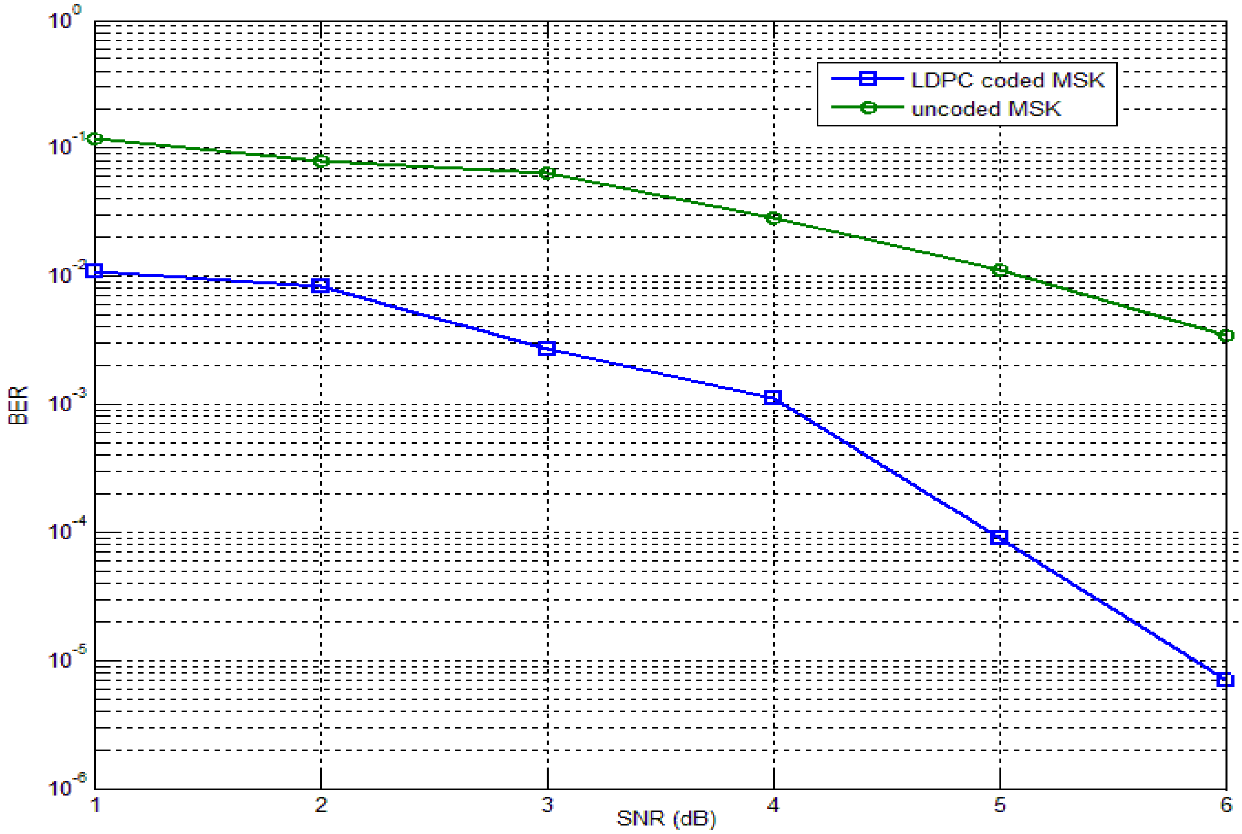

Fig. 4.10: BER performance of uncoded MSK modulated signal and coded MSK modulated signal compared on a Nakagami fading channel for m=2

Table 4.4: Combined SNR performance for Nakagami channel for m=2 at BER of 10^{-2}

At BER of 10^{-2}	
Modulation Technique	**Signal To Noise Ratio (dB)**
Uncoded MSK m=2	5
Coded MSK m=2	0.5

Table 4.4 shows that for a BER of 10^{-2} for m=1 LDPC coded MSK modulated signal gives about 4.5 dB coding gain over the use of uncoded MSK. So the performance of coded MSK is much better than the performance of uncoded MSK.

4.3 Simulation results of Nakagami fading

4.3.1 Nakagami distribution pdf

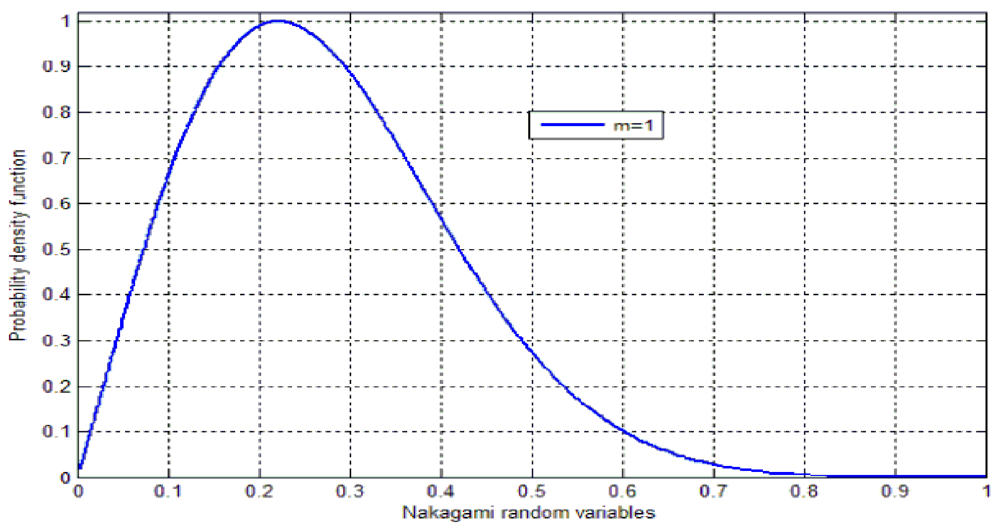

Figure 4.11: Nakagami distribution PDF for m=1

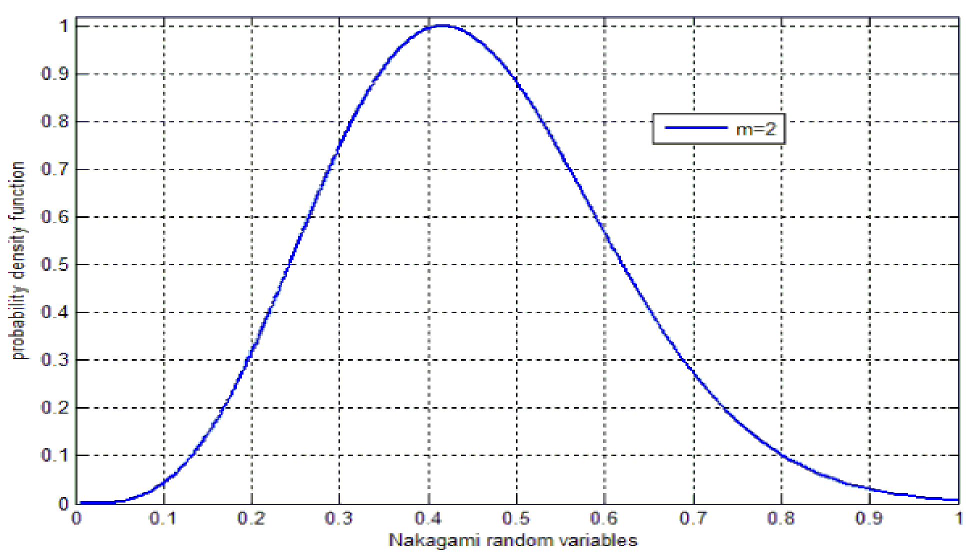

Figure 4.12: Nakagami distribution PDF for m=2

4.3.2 Nakagami fading envelope for m=1

For m=1, the Nakagami fading envelope approximates Rayleigh fading. The fading is severest in Nakagami fading when m=0.5 and there is no fading when m=∞. So, for m=1, the fading is still very severe and signal is known to fade rapidly for this fading envelope.

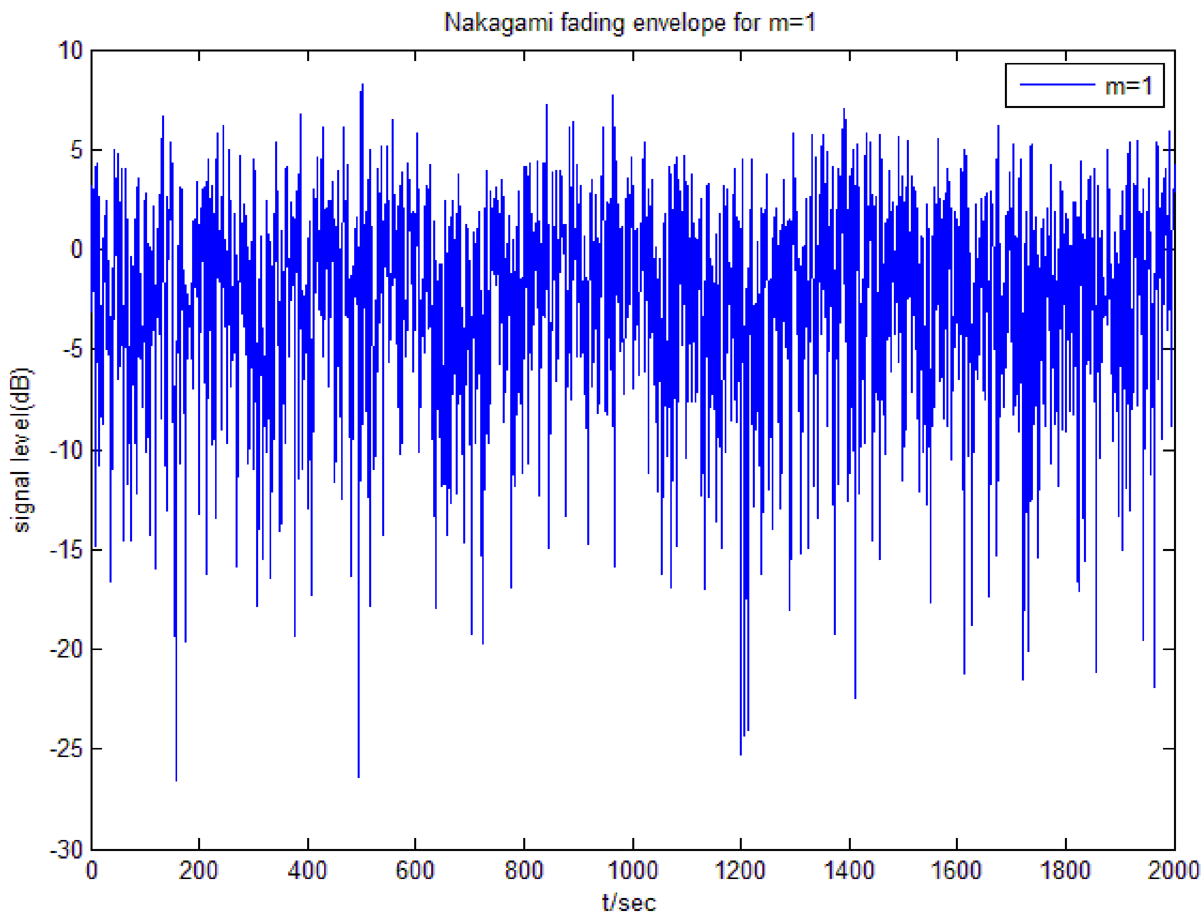

Fig. 4.13: Nakagami fading envelope for m=1 (Rayleigh fading)

4.3.3 Nakagami fading envelope for m=2

For m=2, the Nakagami fading approximates Rician distribution. The fading conditions are less severe as compared to the fading conditions for m=1. As the value of m goes on increasing, the severity of fading goes on decreasing.

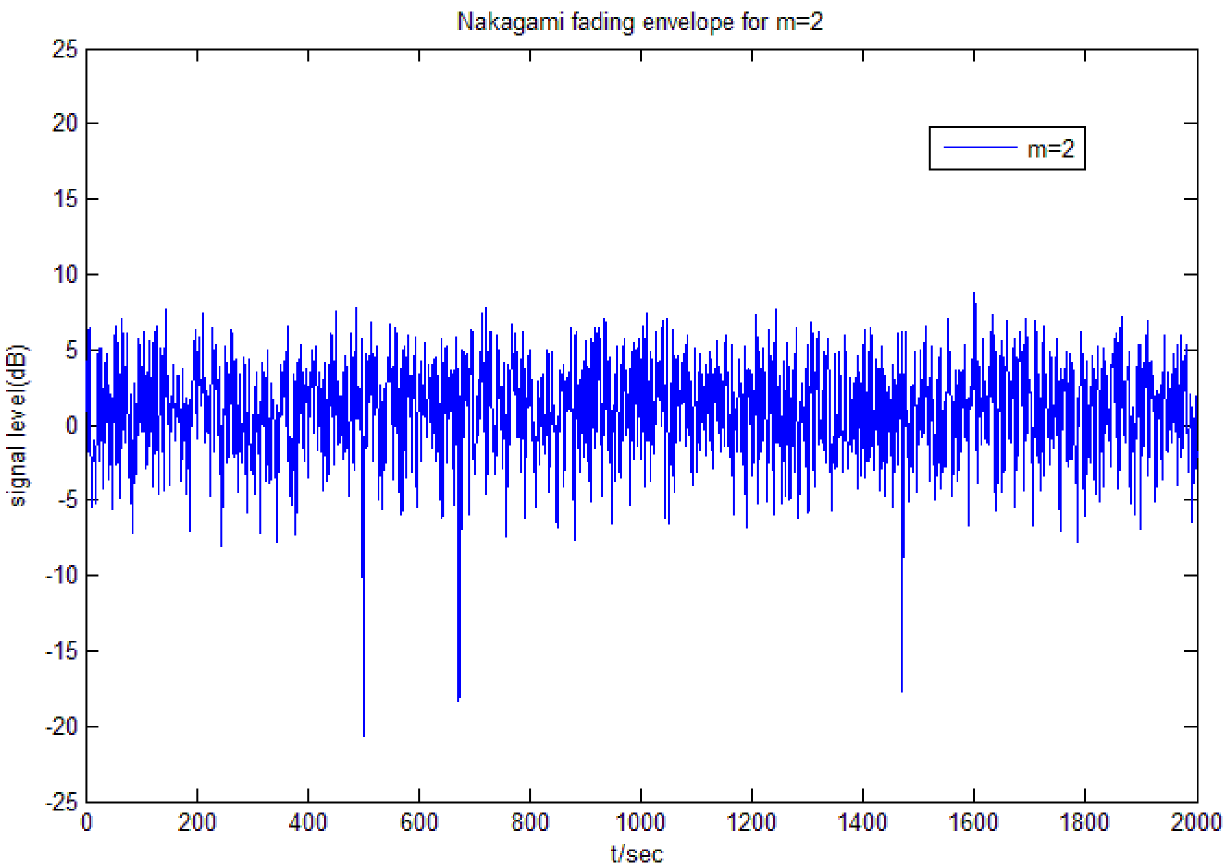

Fig. 4.14: Nakagami fading envelope for m=2 (≈ Rician fading)

4.3.4 Nakagami fading envelope for m=2

For m=10, the fading becomes very moderate and the signal level never fades substantially, so we have not taken the results for the envelope after m=10.

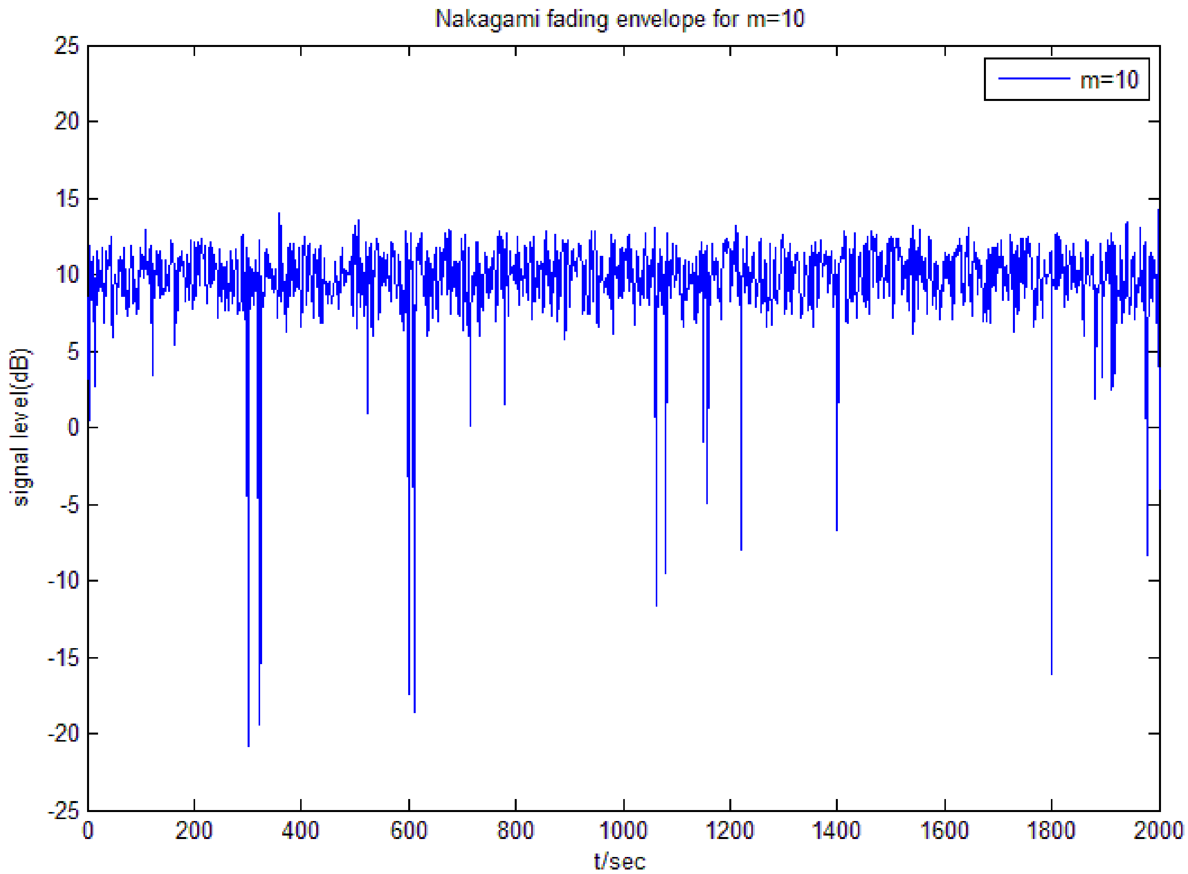

Fig. 4.15: Nakagami fading envelope for m=10

CHAPTER 5
Conclusions and Future Scope of Research

5.1 Conclusions

Channel coding is a very important part of the digital communication system. It helps to improve the performance of a system without much effort on the part of the designer. Channel codes have become very popular and are being used with almost all wireless communication systems. LDPC codes are amongst the most powerful forward error-correcting codes, as they have proved their error performance to be the closest to the Shannon limit (within 0.0045 dB).

In this thesis, a comparative study of LDPC coded and uncoded MSK and GMSK has been done in Nakagami fading channel for different values of m. It is seen that the error performance of uncoded and coded MSK is slightly better than GMSK. The BER when using MSK tends to cross 10^{-5} at SNR of 6dB while the BER of GMSK is slightly below 10^{-5}. Although, MSK has slightly better error performance, GMSK is more spectrally efficient as is evident by the result shown in chapter 3. The LDPC coded signals were compared to uncoded signals for the same modulation scheme and coding gain was determined. It has been shown that the coding gain for coded GMSK over uncoded GMSK is 3.9dB for m=1 and 5dB for m=2. It can be seen that the coding gain obtained by using LDPC codes is excellent. Also, the coding gain for LDPC coded and uncoded MSK waveforms have been computed. The coding gain for coded MSK is 3.3 dB for m=1 and it is 4dB for m=2. We can see from the results that by using LDPC coding with these modulation techniques, a very good coding gain is obtained.

In conclusion, it can be said that the BER performance of LDPC codes combined with MSK and GMSK is excellent even in severe fading conditions. When LDPC codes are used with GMSK, the transmission is more spectrally and power efficient. Both GMSK/MSK have great advantages over other modulation techniques and they are likely to be the modulation schemes of choice for the coming generation. Also, irregular LDPC codes are the best contenders for coding in the next generation wireless systems.

5.2 Future Scope

By using LDPC codes with MSK and GMSK modulation schemes, the BER performance of a wireless communication system is improved. The system which we have implemented is in SISO (single input single output). It can be improved further by implementing this setup in MIMO (multiple input multiple output) systems.

The hardware implementation of bandwidth efficient LDPC coded communication systems is still wide open for research. So naturally, it is a topic for further studies. The hardware implementation can be done using VHDL (or Verilog) for FPGA (Field Programmable Gate Arrays) or by Application specific integrated circuits (ASICs).

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