

Baseline Wander Estimation for ECG Characterization

*Thesis Submitted in partial fulfillment of the requirement for the award of
degree of*

**Master of Engineering
in
Electronics Instrumentation and Control**



By:

Vineet Kumar Mukamia
(800851019)

Under the supervision of:

Dr. Mandeep Singh

Assistant Professor

EIED.

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ELECTRICAL AND INSTRUMENTATION ENGINEERING
DEPARTMENT
THAPAR UNIVERSITY
PATIALA – 147004

Declaration

I hereby declare that the report entitled "Baseline Wander Elimination for ECG Characterization" is an authentic record of my own work carried out as requirements for the award of degree of M.E. (Electronic Instrumentation & Control) at Thapar University, Patiala, under the guidance of Dr. Mandeep Singh (AP, EIED) during January to July 2010.

Date: 13/7/10



Vineet Kumar Mukamia

Roll. No. 800851019

It is certified that the above statement made by the student is correct to the best of my knowledge and belief.



13/07/10

(-----)


Dr. Mandeep Singh

Assistant Professor, EIED

Thapar University, Patiala.


13/7/10

(-----)
Dr. Smarajit Ghosh
Professor & Head, EIED
Thapar University, Patiala.


14.7.10

(-----)
Dr. R. K. Sharma
Dean of Academic Affairs
Thapar University, Patiala.

Acknowledgment

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Place: Thapar University, Patiala

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Vineet Kumar Mukamia

Abstract

Baseline wander makes ECG characterization difficult since it makes manual and automatic analysis of ECG difficult, especially the measuring of ST-segment deviation. The removing of baseline wander from ECG can cause distortion of important clinical information since the spectrum of baseline wander and low frequency component of ECG signal usually overlaps. The aim of this study is to use wavelet packet analysis for estimation of ECG baseline wander. The main reasons for using wavelet transform are the properties of good representation non-stationary signal such as ECG signal and the possibility of dividing the signal into different bands of frequency. To test the propose method, ECG signals taken from MIT-BIH arrhythmia database are used. The method had been compared with traditional methods such as moving average, median etc. on the artificially constructed ECG with baseline wander. For the performance analysis we have use Percent Root Mean Square Difference (PRD). Obtained results show that the wavelet transform approach performs better than traditional methods.

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List of abbreviations

ECG	Electrocardiogram
HRECG	High-Resolution Electrocardiogram
IMF	Intrinsic Mode Functions
FIR	Finite Impulse Response
BW	Baseline Wander
TU	Thapar University
IEEE	Institute for Electrical and Electronic Engineers
ICA	Independent Component Analysis
VLP	Ventricular Late Potentials
WAF	Wavelet Adaptive Filter
WT	Discrete Wavelet Transform
BWT	Bionic Wavelet Transform
STFT	Short-Time Fourier Transform
CWT	Continuous Wavelet Transforms
DWT	Discrete Wavelet Transform
MIT/BIH	Massachusetts Institute of Technology / Beth Israel Hospital

ELECTROCARDIOGRAM AND ARTIFACTS

1.1 Introduction

Heart diseases, which are one of the death reasons of men/women, are among the important problems on this century. Early diagnosis and medical treatment of heart diseases can prevent sudden death of the patient. One of the ways to diagnose heart diseases is to use electrocardiogram (ECG) signals. ECG signals are formed of P wave, QRS complex, and T wave. They are designated by capital letters P, Q, R, S, and T. In the normal beat phase of a heart, the main parameters, inspected include the shape, the duration, and the relationship with each other of P wave, QRS complex, and T wave components and R-R interval. The changes in these parameters indicate an illness of the heart that may occur by any reason. All of the irregular beat phases are generally called arrhythmia and some arrhythmias are very dangerous for patient [1]. Some automatic ECG interpreting systems is available. Moreover, the computer-based interpreter systems are currently being developed to diagnose arrhythmia in time, and various methods are applied to these systems.

The ECG is a bioelectric signal, which records the heart's electrical activity versus time, therefore it is an important diagnostic tool for assessing heart function. The electrical current due to the depolarisation of the Sinus Atria (SA) node stimulates the surrounding myocardium and spreads into the heart tissues. A small proportion of the electrical current flow to the body surface. By applying electrodes on the skin at the selected points, the electrical potential generated by this current can be recorded as an ECG signal.

The interpretation of the ECG signal is an application of pattern recognition. The purpose of pattern recognition is to automatically categorise a system into one of a number of different classes [2]. An experienced cardiologist can easily diagnose various heart diseases just by looking at the ECG waveforms printout. In some specific cases, sophisticated ECG analysers achieve a higher degree of accuracy than

that of cardiologist, but at present there remains a group of ECG waveforms that are too difficult to identify by computers. However, the use of computerised analysis of easily obtainable ECG waveforms can considerably reduce the doctor's workload. Some analysers assist the doctor by producing a diagnosis; others provide a limited number of parameters by which the doctor can make his diagnosis [3].

During measurement of ECG noise (Anything other than muscular activity of heart) is superimposed on them, due to AC interference, loose electrode connection, malfunctioning of machine, patient movement like respiration etc all of them collectively called artifacts. Baseline wandering is one of the artifacts. The primary aim of this work is to remove this baseline wander from ECG so that it can be use by the computer for classification.

Generally, the ECG is one of the oldest and the most popular instrument-bound measurements in medical applications. It has followed the progress of instrumentation technology. Its most recent evolutionary step, to the computer-based system, has allowed patients to wear their computer monitor or has provided an enhanced, high resolution ECG that has opened new scene of ECG analysis and interpretations [4].

1.2 The heart function and ECG

The heart contains four chambers and several one-way valves as shown in Figure 1.1. A wall or septum divides the heart into left and right sides, in a double pump configuration. Each side is then further divided into an upper chamber, the atrium, and a lower chamber, the ventricle. The right side of the heart receives de-oxygenated blood from the venous systems, which is then pumped to the lungs via the pulmonary loop, where the carbon dioxide in the blood is exchanged for oxygen. The left side of the heart receives the oxygenated blood from the lungs and pumps it into the systemic loop for distribution throughout the body. The contraction of the various muscles of the heart enables the blood to be pumped. While the myocardial muscle cells can contract spontaneously, under normal conditions these contractions are triggered by action potentials originating from pacemaker cells situated in two areas of the heart the Sino-Atrial (SA) and Atrio-Ventricular (AV) nodes. The SA

pacemaker cells can spontaneously generate action potentials at 60-80 times per minute, but are themselves under the control of the sympathetic and parasympathetic nervous system. The SA node is generally the site to trigger the action potential for a heartbeat, but the AV node can take over this role if for some reason the SA node fails.

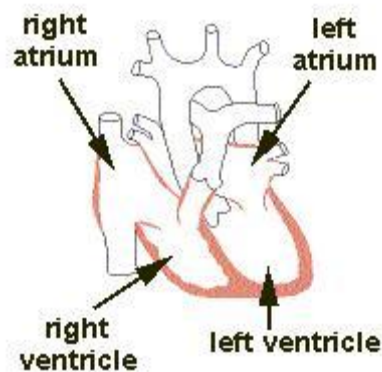


Figure1.1 The structure of the heart

The normal cycle of a heartbeat has the following sequence of events:

- a) The SA node generates an action potential, which spreads across both atria.
- b) This spreading action potential results in the simultaneous contraction of the left and right atria.
- c) This action potential is also passed to the AV node via the inter-nodal conducting fibres, taking about 40 msec.
- d) During the contraction of the atria, blood from the atria is pushed to the respective ventricle.
- e) The AV node's own action potential is triggered by the action potential arriving from the SA node. The AV action potential is spread to the ventricles via further conducting fibres, resulting in a delay of about 110 msec, which is sufficient time to ensure that the atrial contraction has finished.
- f) The AV action potential triggers both ventricles to contract and push blood into the arterial system. The left ventricle supplies the systemic arterial system while the right ventricle supplies the pulmonary system where the blood is oxygenated by the lungs.
- g) All muscles of the heart then relax and blood continues to flow due to the elastic recoil of the arterial walls. During this period both atria and ventricles

fill with blood as it returns from the body via the venous system. A series of one way valves at the input and outputs of the atria and ventricles determine the direction of blood flow.

1.3 Electrocardiography

The various propagating action potentials within the heart produce a current flow, which generates an electrical field that can be detected, in significantly attenuated form, at the body surface, via a differential voltage measurement system. The resulting measurement, when taken with electrodes in standardised locations, is known as the electrocardiogram (ECG). The ECG signal is typically in the range of $\pm 2\text{mv}$ and requires a recording bandwidth of 0.05 to 150Hz.

The ECG is a graphic representation of the electrical activity of the heart's conduction system recorded over a period of time. Under normal conditions, ECG tracings have a very predictable direction, duration, and amplitude. Because of this, the various components of the ECG tracing can be identified, assessed, and interpreted as to normal or abnormal function. The ECG is also used to monitor the heart's response to the therapeutic interventions. Because the ECG is such a useful tool in the clinical setting, the respiratory care practitioner must have a basic and appropriate understanding of ECG analysis. The essential knowledge components required for a systematic 12-ECG interpretation are discussed in the following chapter [5].

1.3.1 The standard 12 ECG system

The standard 12 ECG systems consist of four limb electrodes and six chest electrodes. Collectively, the electrodes (or leads) view the electrical activity of the heart from 12 different positions, 6 standard limb-leads and 6 pericardial chest-leads showed in Table1.1 [5]. Each lead:

- 1) Views the electrical activity of the heart from a different angle,
- 2) has a positive and negative component, and
- 3) monitors specific portions of the heart from the point of view of the positive

electrode in that lead.

Table 1.1 ECG lead system. [5]

Standard Leads	Limb Leads	Chest Leads
Biopolar Leads	Unipolar Leads	Unipolar Leads
Lead I	AVR	V1
Lead II	AVL	V2
Lead III	AVF	V3
		V4
		V5
		V6

The explanation of this three lead system will be in the development history of the ECG diagnostic system. The ECG, over a single cardiac cycle, has a characteristic morphology as shown in Figure 1.2 comprising a P wave, a QRS complex and a T wave. The normal ECG configurations are composed of waves, complexes, segments, and intervals recorded as voltage (on a vertical axis) against time (on a horizontal axis). A single waveform begins and ends at the baseline. When the waveform continues past the baseline, it changes into another waveform. Two or more waveforms together are called a complex. A flat, straight, or isoelectric line is called a segment. A waveform, or complex, connected to a segment is called an interval. All ECG tracings above the baseline are described as positive deflections. Waveforms below the baseline are negative deflections.

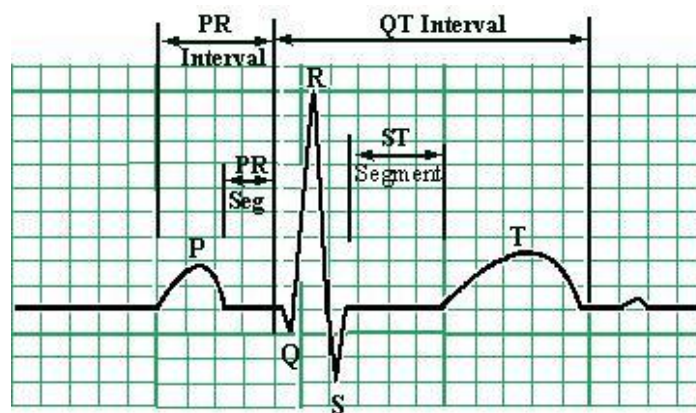


Figure 1.2 The human ECG signal over one cardiac cycle

1.3.2 The P wave

The propagation of the SA action potential through the atria results in contraction of the atria (depolarisation), producing the P wave. The magnitude of the P wave is normally low (50-100 μ V) and 100 msec in duration.

1.3.3 The PR interval

The PR interval begins with the onset of the P wave (P_i) and ends at the onset of the Q wave (Q_i). It represents the duration of the conduction through the atria to the ventricles. Normal measurement for PR interval is 120ms-200ms. It is shown in Figure 1.3.

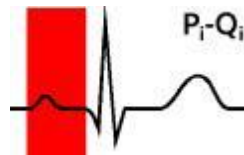


Figure 1.3 PR interval

The PR Segment begins with the endpoint of the P wave (P_t) and ends at the onset of the Q wave (Q_i). It represents the duration of the conduction from the atrioventricular node, down the bundle of its end through the bundle branches to the muscle. It is shown in Figure 1.4.

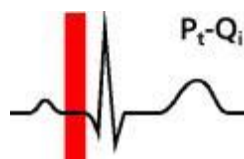


Figure 1.4 PR segment

1.3.4 The QRS complex

The QRS complex corresponds to the period of ventricular contraction or depolarisation. The atrial repolarisation signal is swamped by the much larger ventricular signal. It is the result of ventricular depolarisation through the Bundle Branches and Purkinje fibre. The QRS complex is much larger signal than the P wave

due to the volume of ventricular tissue involved, although some signal cancellation occurs as the waves of depolarisation in the left and right sides of the heart move in opposite directions. If either side of the heart is not functioning properly, the size of the QRS complex may increase. As shown in Figure 1.5. QRS can be measured from the beginning of the first wave in the QRS to where the last wave in the QRS returns to the baseline. Normal measurement for QRS is 60ms-100ms.

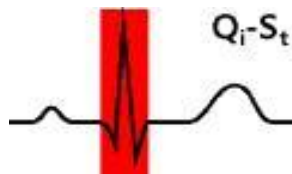


Figure 1.5 QRS duration

1.3.5 The ST segment

The ST segment represents the time between the ventricular depolarisation and the repolarisation. The ST segment begins at the end of the QRS complex (called J point) and ends at the beginning of the T wave. Normally, the ST segment measures 0.12 second or less. The precise end of depolarisation (S) is difficult to determine as some of the ventricular cells are beginning to repolarise. It is shown in Figure 1.6.

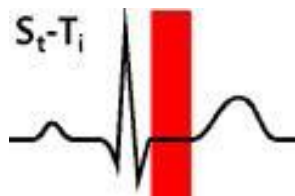


Figure1.6 ST segment

1.3.6 The T wave

The T wave results from the repolarisation of the ventricles and is of a longer duration than the QRS complex because the ventricular repolarisation happens more

slowly than depolarisation. Normally, the T wave has a positive deflection of about 0.5mv, although it may have a negative deflection. It may, however, be of such low amplitude that it is difficult to read. The duration of the T wave normally measures 0.20 second or less.

1.3.7 The QT interval

The QT interval begins at the onset of the Q wave (Q_i) and ends at the endpoint of the T wave (T_t), representing the duration of the ventricular depolarisation/repolarisation cycle. The normal QT interval measures about 0.38 second, and varies in males and females and with age. As a general rule, the QT interval should be about 40 percent of the measured R-R interval. The QT interval is shown in Figure 1.7.

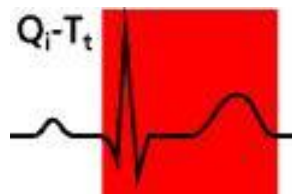


Figure 1.7 QT interval

1.3.8 Summary

Table 1.2 below provides approximate values for the duration of various waves and intervals in the normal adult ECG.

In the normal rhythm, the PR interval should not exceed 0.20 second. The QRS duration should not exceed 0.10 second. The P wave duration should not exceed 0.10 second. The T wave should be at least 0.20 second wide. A heartbeat rate between 60 and 100 is considered "normal," so the R-R interval should be between 0.6 and 1 second [6].

Table 1.2 Duration of waves and intervals in a normal adult human heart

Parameter	Duration (Sec)
Intervals	
P-R interval	0.12-0.20
Q-T interval	0.30-0.40
Waves	
P wave duration	0.08-0.10
QRS duration	0.06-0.10

1.4 Heart problems

There are a lot of heart problems like paced beats, right bundle branch block, atria premature beat and many different types of arrhythmia or conduction problems. And any changes from the normal morphology of the electrocardiogram can be used to diagnose these problems. The physician normally uses the ECG and other factors to determine the gross condition of the heart. And for this it needs an ECG which is free from the noise.

1.5 Development of the ECG diagnostic system

1.5.1 The development history

Kolliker and Mueller [7] using frogs discovered electric activity related to the heartbeat. Donder recorded the frog's heart muscle twitches, producing the first electrocardiographic signal. Waller originally observed the ECG in 1889 [8] using his

pet bulldog as the signal source and the capillary electrometer as the recording device. In 1903, Einthoven [7] enhanced the technology by employing the string galvanometer as the recording device and using a human subject with a variety of cardiac abnormalities.

Traditionally, the differential recording from a pair of electrodes in the body surface is referred to as a lead. Einthoven [7] defined three leads numbered with the Roman numerals II, III, and I. They are defined as:

$$\text{Lead I} = \text{VLA} - \text{VRA} \quad \text{.....(1.1)}$$

$$\text{Lead II} = \text{VLL} - \text{VRA} \quad \text{.....(1.2)}$$

$$\text{Lead III} = \text{VLL} - \text{VLA} \quad \text{.....(1.3)}$$

where the subscript RA=right arm, LA=left arm, and LL=left leg. Because the body is assumed purely resistive at ECG frequencies, the four limbs can be thought of as wires attached to the torso. Lead I could be recorded from the respective shoulders without a loss of cardiac information. The relationship of them is: I=I+III. The lead system presented in this research is largely focused on processing a modified limb II (MLII) obtained by placing the electrodes on the patient chest [9].

Not long after Einthoven described his string galvanometer, efforts were begun in the United States to create an electrocardiograph that used vacuum tubes. Between introduction of the string galvanometer and the hot stylus recorder for ECG, attempts were made to create direct inking ECG recorders. Despite the instant availability of inked recording of the ECG, those produced by the string galvanometer were superior, and it took some time for a competitor to appear. Such an instrument did appear in the form of the hot stylus recorder.

In 1933 Wilson added the concept of a “unipolar” recording, where tying the three limbs together creates a reference point and averaging their potentials so that individual recording sites on the limbs or chest surface would be differentially recorded with the same reference point.

However, from the mid-1930s until today, a standard 12-lead ECG system comprises 3 limb leads, 3 leads in which the limb potentials are referenced to a modified Wilson terminal, and 6 leads placed across the front of the chest and

referenced to the Wilson terminal. Figure 1.8 shows the development procedure of the 12-lead ECG and the 3-lead systems as were indicated in Table 1.1

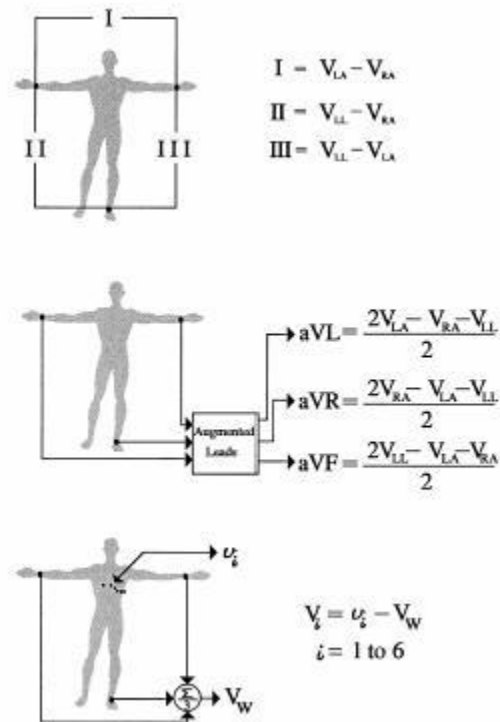


Figure 1.8 Development procedure of the 12-lead ECG

The final step toward modern electrocardiography was the introduction of the hot stylus recorder by Haynes [10] of the Bell Telephone Laboratories in New York. Following the end of World War II, vacuum tube electrocardiographs with heated stylus recorders became very popular and are still in use today. The vectorcardiogram uses a weighted set of recording sites to form an orthogonal xyz lead set, providing as much information as the 12-lead system, but with fewer electrodes. Cardiac surface mapping uses many recording sites (>64 electrodes) arranged on the body so that the cardiac surface potential can be computed and analysed over time. Other subsets of the 12 lead ECG are used in limited mode recording situations such as the tape recorded ambulatory ECG (usually 2 leads) or intensive care monitoring at the bedside (usually 1 or 2 leads) or telemeter within regions of the hospital from patients who are not confined to bed (1 lead).

Automated ECG interpretation was one of the earliest uses of computers in medical applications. This was initially achieved by linking the ECG diagnostic machine to a centralised computer via phone lines or computer network. The modern ECG machine is completely integrated with an analogue front end, a high-resolution analogue to digital converter and a microcomputer [7].

1.5.2 Computerised ECG interpretation

Application of the computer to the ECG for machine interpretation was one of the earliest uses of computers in medicine [11]. Of primary interest in the computer-based systems was replacement of the human reader and elucidation of the standard waves and intervals. Originally this was performed by linking the ECG machine to a centralised computer via phone lines or computer network. The modern ECG diagnostic machine is completely integrated with an analogy front and end, a 12-to 16-bit analogy to digital (A/D) converter, a central computational microprocessor, and dedicated input and output (I/O) processor.

The above-mentioned systems can compute a measurement matrix derived from the 12 lead signals and analyse this matrix with a set of rules (such as neural network) to obtain the final set of interpretive statements [12].

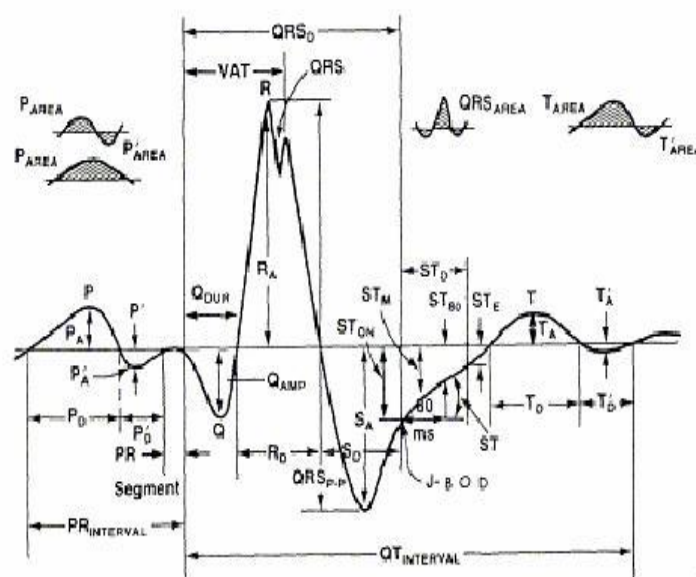


Figure 1.9 The ECG measurements that can be made with computer-based algorithm

Figure 1.9 [13] shows the ECG of a heartbeat and the types of measurement that might be made on each of the component waves of the ECG and used for classifying each beat type and subsequent cardiac rhythm. The depiction of the 12 analogy signals and this set of interpretive statement from the final output, are shown in Figure 1.10 [7].

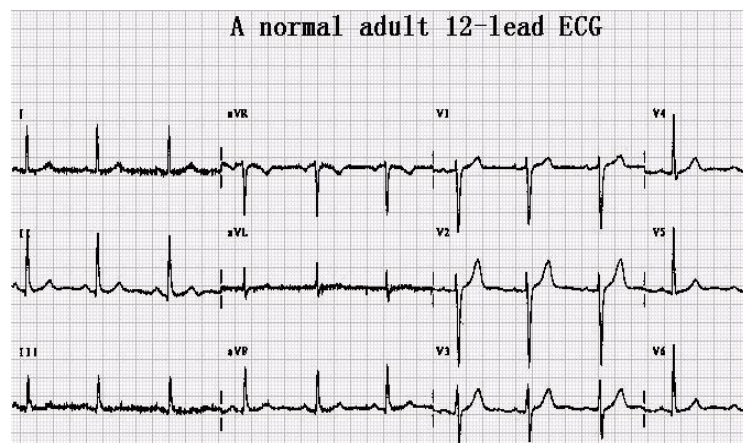


Figure 1.10 An example of an interpreted 12-lead ECG

The physician will over-read each ECG and modify or correct those statements, which are deemed inappropriate. The larger hospital based system will record this correction and maintain a large database of all ECGs accessible by any combination of parameters. There are hundreds of interpretive statements from which a specific diagnosis is made for each ECG, but there are only about five or six major classification groups for which the ECG is used. The first step is analysing an ECG requirement determination of the rate and rhythm for the atria and ventricles. Included here would be any conduction disturbances either in the relationship between the various chambers or within the chambers themselves. Then one can proceed to identify features that relate to the events that would occur with ischemia or an evolving myocardial infarction [7].

1.6 Artifacts

Electrocardiograph (ECG) artifacts are disturbances on ECG which is a measurement of cardiac potentials on the body surface. As a result of artifacts, normal components of the ECG can be distorted.

The word artifact is similar to artificial in the sense that it is often used to indicate something that is not natural (i.e. man-made). In electrocardiography, an ECG artifact is used to indicate something that is not "heart-made." These include (but are not limited to) electrical interference by outside sources, electrical noise from elsewhere in the body, poor contact, and machine malfunction. Artifacts are extremely common, and knowledge of them is necessary to prevent misinterpretation of a heart's rhythm [14].

1.6.1 Causes

Artifacts can be generated by patient's motion or other electrical devices attached to or implanted (e.g. deep brain stimulator) in the body. Tremors and shivering are good examples of motion induced artifacts. Simple movements such as brushing and combing the hair can cause ECG disturbances during ambulatory ECG monitoring. External sources of ECG artifacts mainly include power line electrical disturbances and radiofrequency based commercial (e.g. mobile phones) or medical devices. In operation theatres and intensive care unit various equipments can affect ECG measurement system (e.g. electrodes, leads, amplifier, and filters). Examples of equipment which can cause ECG artifacts are - electrocautery, transcutaneous nerve stimulator, hemo-filtration machines etc. [15].

1.6.1.1 Noise originating from sources outside the patient

a) Electrostatic sources

Patient acts as one plate of a capacitor. When a charged body is brought up close to an uncharged one, an equal & opposite charge develops on the uncharged body. e.g., if an unearthed body is close to any cable or lamp

element that is connected to mains, he will develop a surface charge of equal & opposite potential even though no current is flowing between the two bodies. As the mains potential has a frequency of 50 Hz, the induced potential will also have this frequency. Other sources of electrostatic charge include the operating table, other persons, electronic equipment.

b) Electromagnetic induction

An interference that occurs in the vicinity of wires carrying AC currents Results in 50 Hz interference. Due to the generation of a magnetic field by the flow of a current: all conductors carrying mains currents are surrounded by electromagnetic fields. The 50 Hz interference is a difference in potential, relative to the ground, that is impressed upon any subject in proximity to the wire carrying alternating current; the subject takes on a potential that is neither that of ground, nor of the power line, but somewhere in between. Since the utility current is fluctuating, the voltage of the subject is also fluctuating. Effect is minimised by the fact that the electromagnetic field generated by the live wire is to a greater degree cancelled out by the neutral cable flowing adjacent to the live cable but flowing in the opposite direction. However, if leakage of current occurs, the two currents are no longer equal & self-cancelling thereby generating an e.m.f. The effect is multiplied if the wires are coiled Leads connecting patient electrodes to sensitive amplifiers are most frequently affected.

c) Radiofrequency Interference [> 100 kHz]

May enter via Mains distribution system mixed up with 50 Hz current; sources include diathermy, electric motors. Radio-propagation whereby activated diathermy probe held in air acts as radio-transmitter aerial while the patient ECG lead acts as a receiving aerial.

d) Remedial action

- i. Differential analyser
- ii. High CMRR
- iii. Shielded patient circuitry with copper or aluminium enclosure
- iv. 'Floating' RL ECG

- v. Surrounding each lead with a braided copper screen to minimise electrostatic induction
- vi. Keep leads as short as possible
- vii. Mitigate effects of electromagnetic fields by ensuring all of patient's leads are of the same length, are closely bound together or even twisted together until close to the electrodes thereby ensuring that the induced signals are identical & therefore susceptible to CMRR
- viii. Eliminate source of unwanted electromagnetic radiation (only real cure)

1.6.1.2 Noise originating from patient-electrode contact

a) Unfortunately, recording electrodes do not act as a passive conductor. The placement of a metal next to an electrolyte solution as seen on the surface of skin produces an electrochemical half-cell resulting in the generation of an electromagnetic force. If a differential amplifier is connected to a pair of such electrodes, their output potentials are compared.

- If the cells are identical, the outputs will be self cancelling yielding zero output
- If the cells are not identical, the difference in potential between the two cells will be amplified

Additionally, the small current produced by the offset potential may result in polarisation. A polarised electrode will distort any signal

- b) Mechanical movement of recording electrodes results in changes in potential. Due to alteration in the physical dimensions of the electrode-skin half cell thus modifying cell potential and skin-electrode impedance
- c) Remedial action
- i. Abrade skin, remove hair, alcohol to ensure adhesion of electrode
 - ii. Use of Ag:AgCl electrode which does not polarise

1.6.1.3 Noise originating from the patient

- a) EMG [electromyogram]
 - i. Frequency (Hz) overlap those of ECG
 - ii. Signal can be much larger (increased mV) than ECG
 - iii. Muscular activity (especially shivering) can lead to gross interference
 - iv. 'Muscle artifact'
- b) Remedial action
 - i. Minimise patient movement i.e. relax
 - ii. Minimise shivering
 - iii. Avoid muscle area to place electrodes; use bony prominences

1.6.2 Characteristics

Artifacts on the ECG can distort individual or all components (P, QRS, T waves and PR and ST segments). Most of the time they are easily identifiable and neglected as they do not resemble any specific pattern. On occasions, changes may mimic specific arrhythmia like ventricular tachycardia and atrial flutter or fibrillation. It is important to differentiate these artifactual changes from genuine changes to prevent misdiagnosis. If ST segments are affected by artifacts, either ST segment depression or elevation can occur on the ECG. These changes can be misinterpreted as myocardial ischemia or infarction.

1.6.3 Consequences

Apart from the poor quality of ECG, artifacts can cause serious consequences particularly when they mimic like genuine changes. If ECG artifacts are not recognized by physician, anesthesiologist or intensivist unnecessary diagnostic and therapeutic measures could be taken. Such actions may subject patients to invasive investigations or they may receive unnecessary medications like antiarrhythmics.

1.6.4 Correction

Attention to basic principles such as proper electrodes placement and lead connections is required during ECG measurement. Well designed and maintained ECG measurement devices can withstand routine internal or external electrical and motion-related disturbances. However, it is not always possible to eliminate artifacts completely. It is essential that physicians keep high vigilance and interpret ECG keeping ECG artifacts in differential diagnosis list.

1.6.5 AC interference

Alternating current (AC) describes the type of electricity that we get from the wall. In the India, the electricity "changes direction" 50 times per second (i.e. 50 hertz, Many places in Europe use 50 Hz AC electricity. And USA use 60 Hz AC electricity). When an ECG machine is poorly grounded or not equipped to filter out this interference, you can get a thick looking ECG line (as shown in figure 1.11). If one were to look at this ECG line closely, he would see 50 up-and-down wave pattern in a given second (25 squares).

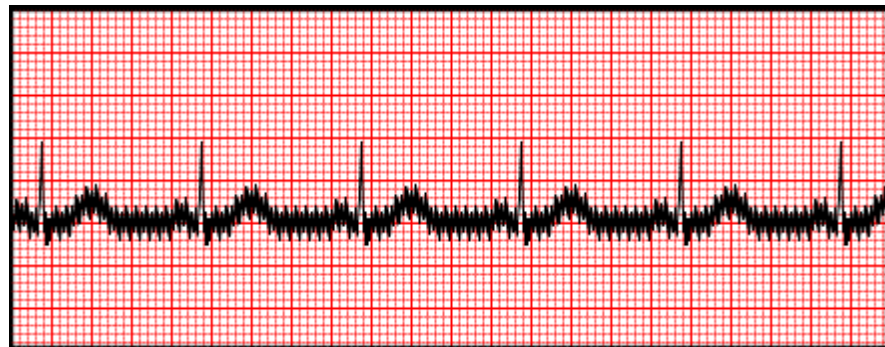


Figure 1.11 50Hz AC interference

1.6.6 Muscle tremor / noise

The heart is not the only thing in the body that produces measurable electricity. When your skeletal muscles undergo tremors, the ECG is bombarded with

seemingly random activity. The term noise does not refer to sound but rather to electrical interference.

Low amplitude muscle tremor noise can mimic the baseline seen in atrial fibrillation. Muscle tremors are often a lot more subtle than that shown in figure 1.12.

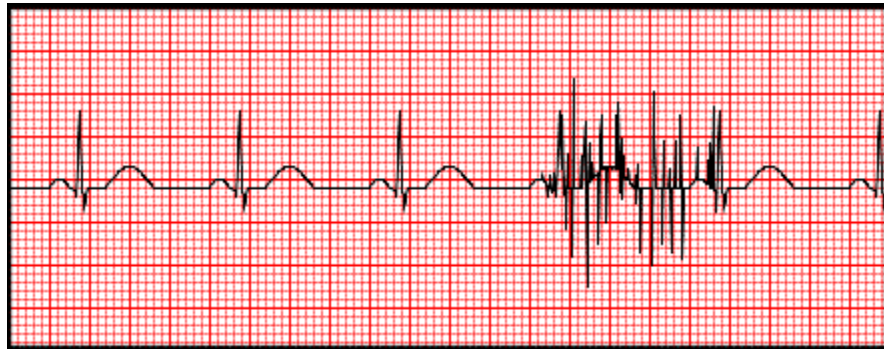


Figure 1.12 Muscle Tremors

1.6.7 Wandering baseline

In wandering baseline, the isoelectric line changes position. One possible cause is the cables moving during the reading. Patient movement, dirty lead wires/electrodes, loose electrodes, and a variety of other things can cause this as well.

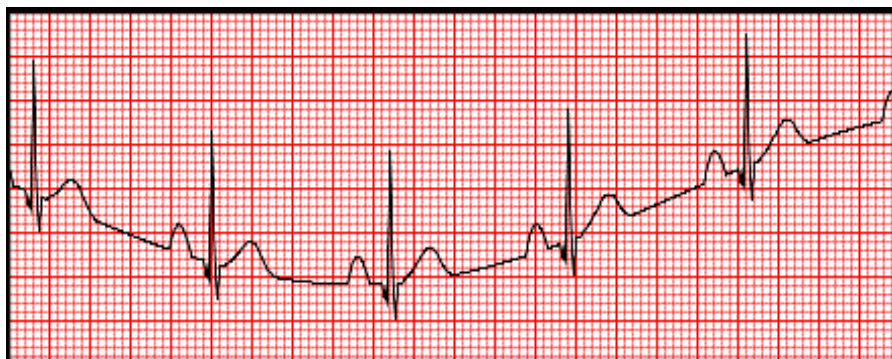


Figure 1.13 Wandering Baseline Artifact

1.7 Massachusetts Institute Of Technology / Beth Israel Hospital (MIT/BIH) database

This is a rich database of several hundred ECG recordings, extending over 200 hours [Moody, 1992]. Each recording contains one to three signals and ranges from 20 seconds to 24 hours in duration. Most of the signals have been annotated on beat-to-beat basis. In August 1989 a CD-ROM was produced containing the original MIT-BIH Arrhythmia Database (developed between 1975 and 1979, and first released in 1980), as well as a large number of supplementary recordings assembled for various research projects between 1981 and 1989. The CD-ROM contains approximately 600 megabytes of digitized ECG recordings, most with beat-by-beat annotations, having a total duration in excess of 200 hours. In September 1991, this data was made available on Internet (<http://www.physionet.org>) on PhysioNet, which is a web-based resource supplying well characterized physiologic signals and related open-source software to the biomedical research community. From September 2000, the data archive named PhysioBank, containing roughly 35 gigabytes of recorded signals and annotations was made available via PhysioNet [16].

The MIT-BIH Arrhythmia database contains 48 ECG signals that were recorded between 1975 and 1979 at the Beth Israel Hospital Arrhythmia Laboratory. The recordings were digitized at 360 samples per second per channel with 11-bit resolution over a 10 mv range. Each record was independently annotated by two or more cardiologists; disagreements were resolved to obtain the computer-readable reference annotations for each beat (approximately 110,000 annotations in all) included with the database. This directory contains the entire MIT-BIH Arrhythmia Database. About half (25 of 48 complete records, and reference annotation files for all 48 records) of this database has been freely available here since PhysioNet's inception in September 1999 [17].

1.8 The MATLAB environment

MATLAB is a powerful, comprehensive, and easy to use environment for technical computations. It provides engineers, scientist, and other technical

professionals with a single interactive system that integrates numeric computation, visualisation, and programming. MATLAB includes a family of application specific solutions called toolboxes.

One of its greatest strengths is that MATLAB allows building its own reusable tools. Customised special functions and programs can be easily created in MATLAB code. Biomedical engineers use MATLAB in research, design, and manufacturing of medical devices and to develop embedded algorithms and systems for medical instrumentation. MATLAB has several advantages over other traditional means of numerical computing.

- It allows quick and easy coding in a very high level language.
- Data structures require minimal attention, in particular, arrays need not be declared before first use.
- An interactive interface allows rapid experimentation and easy debugging.
- High-quality graphic and visualisation facilities are available.
- MATLAB M-files are completely portable across a wide range of platforms.
- Toolboxes can be added to extend the system, giving, for example, specialised signal processing facilities.

1.8.1 Signal processing toolbox

The Signal Processing Toolbox is a collection of MATLAB functions that provides a rich, customisable framework for analog and digital signal processing (DSP). Graphical user interfaces (GUIs) support interactive designs and analyses, while command-line functions support advanced algorithm development. The Signal Processing Toolbox is the ideal environment for signal analysis and DSP algorithm development. It uses industry-tested signal processing algorithms that have been carefully chosen and implemented for maximum efficiency and numeric reliability. Functions are mostly implemented as M-file routines written in the MATLAB language, giving access to the source code and algorithms. The open-system philosophy of MATLAB and the toolboxes enables making changes to existing functions or adding own experiments.

The main features of the signal processing toolbox are as follows [18]:

- A comprehensive set of signal and linear system models.
- Tools for analog filter design.
- Tools for finite impulse response (FIR) and infinite impulse response (IIR) digital filter design, analysis, and implementation.
- The most widely used transforms, such as fast Fourier transform (FFT) and discrete cosine transform (DCT).
- Methods for spectrum estimation and statistical signal processing.

1.8.2 Wavelet toolbox

Wavelet Toolbox software is a great way to work with wavelets. The toolbox, together with the power of MATLAB® software, really allows one to write complex and powerful applications, in a very short amount of time. The Graphic User Interface is both user-friendly and intuitive. It provides an excellent interface to explore the various aspects and applications of wavelets; it takes away the tedium of typing and remembering the various function calls. Wavelet Toolbox™ software is a collection of functions built on the MATLAB® technical computing environment. It provides tools for the analysis and synthesis of signals and images, and tools for statistical applications, using wavelets and wavelet packets within the framework of MATLAB. Wavelet Toolbox software provides two categories of tools [19]:

- Command-line functions
- Graphical interactive tools

The main features of MATLAB Wavelet Toolbox contains

- Continuous wavelet transforms (CWT),
- 1D and 2D discrete wavelet transforms (DWT),
- Wavelet packets, implemented as MATLAB objects
- Multiresolution decomposition and analysis of signals and images,
- User-extensible selection of wavelet basis functions,
- 1D and 2D wavelet packet transforms,
- Entropy-based wavelet packet tree pruning for "best-tree" and "best-level" analysis, and soft and hard thresholding De-noising.

1.9 Performance analysis

For the performance analysis, we have used the Percent Root Mean Square Difference (PRD) [47]. PRD has taken different definitions in different research reports. The most acceptable definition of PRD is given by the formula:

$$\text{PRD} = \sqrt{\frac{\sum_{i=1}^{i=N} (x_i - \hat{x}_i)^2}{\sum_{i=1}^{i=N} (x_i - \bar{x}_i)^2}} \times 100 \dots 1.4$$

Where x_i is the original standard signal and \hat{x}_i is the reconstructed signal \bar{x}_i is the mean signal, usually defined as the mean value of the original signal. However some researchers do not take into account an estimate of mean signal (\bar{x}_i) in the definition, therefore, PRD reported by them is obviously much less. Thus is it important to use the same definition of PRD, preferably taking an estimate of mean signal (\bar{x}_i) while comparing the performance of any algorithm.

1.10 Organization of the thesis

The work presented in this thesis has been covered in six chapters. Chapter one starts with a brief introduction to the concept of ECG particularly in the area of artifacts. The topics like cardiac signals, Electrocardiogram and its standard database, signal processing toolbox have then been discussed. The relevant research work carried out by different researchers has been presented in chapter two. The third chapter deals with introduction to wavelet transform, its advantages over Fourier transform, different types of wavelets, continuous, discrete wavelet transforms and wavelet packet analysis and different applications of wavelets in the area of bio-medical engineering. In chapter four, problem definition is given. A new baseline wander elimination method is proposed in this fifth chapter. The conclusion of the overall work, significant findings and the further scope of the work are given in the last chapter.

CHAPTER 2

LITERATURE REVIEW

McManus et al. has developed estimation procedures for baseline drift using cubic spline, polynomial, and rational functions. In a test set of 50 electrocardiograms (ECGs), each of 2.5-sec duration, baseline stability was significantly improved by application of any of these methods, except rational function approximation. Amplitude histograms of clinical ECGs after subtraction of estimated baseline distortions showed only small baseline variations over the recording period. For a quantitative validation of the estimation procedures, 10 ECGs with artificial baseline drift were constructed and analyzed by correlation and mean square error calculations. [20]

Alste proposed the linear phase filtering for the removal of baseline wander and power-line frequency components in electrocardiograms. In order to reduce the large number of computations involved in the digital filtering that were necessary, the desired filter spectrum was defined periodically. Making use of the property that the spectrum period was 50 Hz, the spectrum can be realized with a considerably reduced number of impulse response coefficients. This, in combination with the necessary impulse response symmetry, leads to a reduction in the number of multiplications per output sample by a factor of 10. A suitable impulse response is designed with a pass-band ripple of less than 0.5 dB and high stop-band attenuation. The applicability was demonstrated by applying the filtering to exercise electrocardiograms. [21]

Jake et al. study was to compare the cubic spline method with a multi-pole, null-phased digital filter in their ability to correct for baseline wander on 69 ECG segments with both normal and abnormal rhythms. A signal – pole 0.05Hz filter as recommended by the 1975 AHA report was also included in their study for comparison. A null-phase, 6-pole filters with a cut – off between 0.75 and 1.0 Hz can attenuate low frequency noise (i.e. correct baseline) as well as the cubic spline. However, such a filter will introduce minor changes in the original signal, especially in the presence of a long RR interval. The cubic spline was very dependent upon an

accurate determination of QRS on set. The single-pole, 0.05 Hz filter does very little to attenuate low frequency noise. [22]

Baseline wandering was a classical problem in ECG recording that generally produces artifactual data when measuring ECG parameters. Raimon et al. in their work presented and analysed a cascade adaptive filter for removing the baseline wander preserving the low frequency components of the ECG. This cascade adaptive filter work in two stages. The first stage was an adaptive notch filter at zero frequency. The second stage was an adaptive impulse correlated filter that, using a QRS detector, estimates the ECG signal correlated with the QRS occurrence. In this way, they preserved all the signal components correlated with the QRS complex. They analysed the frequency response of the filter, showing that the filter can be seen as a comb filter without the dc lobe. Finally, they have applied the method on ECG signals from the MIT-BIH database and compared its performance with the cubic spline approach. [23]

Sornmo applied the time-varying filtering techniques to the problem of baseline correction by letting the cut-off frequency of a linear filter be controlled by the low-frequency properties of the ECG signal. The time-varying filter was implemented as a bank of linear low-pass filters, in which each filter had a slightly differing cut-off frequency. Sampling rate decimation and interpolation are employed because the design of a filter for baseline reduction can be treated as a narrowband filtering problem. All filters have a linear phase response to reduce, for example, ST-segment distortion. The performance of the technique presented was studied on ECG signals with different types of simulated baseline wander. The results were compared with the performance of time-invariant linear filtering and cubic spline interpolation. [24]

A method of removing low frequency interference from an ECG signal was presented by Allen et al. as a simple alternative to some of the more computationally intensive techniques. The performance of the method was evaluated by examining changes in body surface isopotential map feature locations, due to baseline wander. The results show that although baseline wander can seriously interfere with isopotential map features, integrity can be restored by relatively simple methods. [25]

Sun et al. described a morphological approach to remove baseline wander from electrocardiogram (ECG) signals, with particular emphasis on preserving the ST segment of the original signal. The algorithm consists of two stages of morphological processing. First, the QRS complex and impulsive noise component due to skeletal muscle contractions etc., were detected and removed from the input signal. Second, the corrected QT interval and RR interval were used to determine a structuring element. With this structuring element, the same morphological operation as in the first stage is then applied to the QRS-removed signal to obtain and remove the baseline wander. The performance of the algorithm was evaluated with simulated and real ECGs. [26]

Norbert describes a linear phase finite impulse response (FIR) filters method. This new approach, based on the insensitive loss function, allows the design process to take into account not only constraints specified in the frequency domain, but also constraints on the output, time domain, signal. The performances of this method illustrated with a design of a high-pass filter used for ECG baseline wander reduction. [27]

The method used by Zhao to remove baseline wander and power line interference in ECG signal was based on Empirical Mode Decomposition and notch filter. Principles and characteristics of Empirical Mode Decomposition are presented; ECG signal was decomposed into a series of Intrinsic Mode Functions (IMFs). Then 50Hz notch filter was designed, by which the IMF of ECG signal containing 50Hz power line inference was filtered. The “clean” ECG signal was reconstructed by properly selecting IMFs. To evaluate the performance of the filter, Clinic ECG signals were used. [28]

Zeinab et al. show the ability of Independent Component Analysis (ICA) technique in removing baseline wandering from ECG by utilizing Single-Channel data. For applying ICA to single channel data, multi-channel signals were constructed by adding some delay to original data. For validation the effectiveness of the method, they applied ICA to constructed channels derived from each Frank lead in HRECG (High-Resolution Electrocardiogram) data as a pre-processing step in order to detect Ventricular Late Potentials (VLPs) by Simson's method. Results derived by this

approach were compared with those obtained from traditional high-pass filtering for removing baseline wandering. [29]

The removal of baseline wander (BW) was a very important step in the pre-processing stage of electrocardiogram (ECG). In Pan et al. proposed method Empirical Mode Decomposition (EMD) was used for accurate removal of the baseline wander (BW) in ECG. They briefly described the principles and characteristics of the EMD. To validate the proposed method, the recording from MIT/BIH database was used. They also applied the traditional median filter to remove BW in ECG for comparison with their EMD method. [30]

Markovsky et al. used Band-pass, Kalman, and adaptive filters for removal of resuscitation artifacts from human ECG signals. A database of separately recorded human ECG was used for evaluation of this method. The considered performance criterion is the signal to- noise ratio (SNR) improvement, defined as the ratio of the SNRs of the filtered signal and the given ECG signal. The empirical results show that for low SNR of the given signal, a band-pass filter yields the good performance, while for high SNR; an adaptive filter yields the good performance. [31]

Hargittai presented a multirate architecture with linear phase low-pass filter working at low sampling rate for removal of the baseline wander. Design trade off between transition band width and filter delay was considered. They determined the optimal decimation factor with respect to complexity and filter delay. For testing and assessment of behaviour of baseline filter they used test signals, normal and wide QRS complexes with different heart rate. [32]

The traditional method which was based on moving average filter can remove the baseline wander in electrocardiogram signals, but also causes the loss of motive ECG signals, which makes distortions of filtered ECG signals. Min Dai et al. proposed a modified moving average filter to selectively capture the low-frequency baseline wander noise and remove it from the detected signals in order to recover true ECG. The interval sampling data was taken into consideration when calculate the moving average in order to reduce the loss of useful ECG signals and distortions. The algorithm was developed for computer implementation using MATLAB. To validate the proposed methods, the recordings from MIT/BIH database were used. One of the

drawback of this filter approach is that it does not accommodate for quick baseline changes. [33]

According to Zahoor Baseline noise removal from electrocardiogram (ECG) signal was a blind source separation problem. Baseline noise distorts the low frequency segment of ECG signal. The low frequency segment is s-t segment. This segment was very important and has the information related to heart attack. They have applied projection pursuit gradient ascent algorithm to remove this noise from the measured ECG signal. This algorithm separates the independent signals from a mixture of signals. Efficient removal of baseline noise might give us certain information that are hidden from the doctors until now which may save the life of a person. Results for different baseline noise signals were analyzed. Different signal from MIT-BIH database were also analyzed for error in term of standard deviation and mean of error signal. Finally they did a comparative study of the results of different algorithms like kalman filter, cubic spline and moving average algorithms. [34]

Shivaram et al. presented a real-time algorithm for estimation and removal of baseline wander (BW) noise .This algorithm utilizes a real-time “T-P knot” baseline wander removal technique which was based on the repetitive backward subtraction of the estimated baseline from the ECG signal. The estimated baseline was interpolated from the ECG signal at midpoints between each detected R-wave. As each segment of the estimated baseline signal was subtracted from the ECG, a “flattened” ECG signal was produced for which the amplitude of each R-wave was analyzed. Testing of the algorithm was conducted in a pseudo real-time environment using MATLABTM, and test results are presented for simultaneously recorded ECG and respiration recordings from the PhysioNet/PhysioBank Fantasia database. [35]

Coifman et al. uses a library of orthonormal bases and an efficiency functional to match a basis to a given signal or family of signals. It permits de-noising of a variety of signals such as sound and images. The predefined libraries of modulated waveforms include orthogonal wavelet-packets, and localized trigonometric functions, have reasonably well controlled time-frequency localization properties. The idea is to build out of the library functions an orthonormal basis relative to which the given signal or collection of signals has the lowest information cost. The method

relies heavily on the remarkable orthogonality properties of the new libraries, all expansions in a given library conserve energy, hence are comparable. Several cost functional are useful, one of the most attractive is Shannon entropy, which has a geometric interpretation in this context. [36]

The use of an adaptive tree structure using wavelet packets as generalized wavelet decomposition for signal denoising was introduced by Coifman. The idea is to decompose a discrete signal using all possible wavelet packet bases of a given wavelet kernel, and then to find the “best” wavelet packet basis. Ramchandran et al. employ a framework that includes both rate and distortion. A fast algorithm is formulated to “prune” the complete tree, signifying the entire library of admissible wavelet packet bases, into that best basis subtree which minimizes the global distortion for a given coding bit budget or conversely which minimizes the total coding bit rate for a target quality. Arbitrary finite quantize sets are assumed to each hierarchical level of the basis-family tree. [37]

Amara Graps in the paper ‘An Introduction to Wavelet’ described wavelet as, Wavelets were mathematical functions that cut up data into different frequency components, and then study each component with a resolution matched to its scale. They have advantages over traditional Fourier methods in analyzing physical situations where the signal contains discontinuities and sharp spikes. Wavelets were developed independently in the fields of mathematics, quantum physics, electrical engineering, and seismic geology. Interchanges between these fields during the last ten years have led to many new wavelet applications such as image compression, turbulence, human vision, radar, and earthquake prediction. They introduce wavelets to the interested technical person outside of the digital signal processing field. They described the history of wavelets beginning with Fourier, compare wavelet transforms with Fourier transforms, state properties and other special aspects of wavelets, and finish with some interesting applications such as image compression, musical tones, and de-noising noisy data. [38]

A wavelet adaptive filter (WAF) for the removal of baseline wandering in ECG signals is described by Park et al. According to them, the WAF consists of two parts the first part is a wavelet transform that decomposes the ECG signal into seven frequency bands using Vaidyanathan Hoang wavelet. The second part is an adaptive

filter that uses the signal of the seventh lowest frequency band among the wavelet transformed signals as primary input and constant as reference input. To evaluate the performance of the WAF, two baseline wandering elimination filters are used, a commercial standard filter with a cut-off frequency of 0.5 hz and a general adaptive filter. The MIT/BIH database and the European ST-T database are used for the evaluation. [39]

Markus et al. had presented an approach to the adaptation of a wavelet filterbank based on perceptual and rate-distortion criteria. The system makes use of a wavelet-packet transform where each subband can have an individual time-segmentation. Boundary effects can be avoided by using overlapping blocks of samples and therefore switching bases is possible at every tree-level without affecting other subbands. A modified psychoacoustic model using perceptual entropy can control the switching of the wavelet filterbank and the individual time-segmentation of every subband allows taking advantage of temporal masking. [40]

In their article Francesco et al. extend the definition of wavelet variance to wavelet packets. They also adapt to wavelet packets an iterated cumulative sum of squares algorithm for the location of variance change points. Wavelet packets have greater de-correlation properties than standard wavelets in that they induce a finer partitioning of the frequency domain of the process generating the data. This allows their procedure to be applied to a wide class of processes [41]

Daqrouq had used discrete wavelet transform (DWT) for ECG signal processing, specifically for reduction of ECG baseline wandering. The main reasons for using discrete wavelet transform are the properties of good representation non-stationary signal such as ECG signal and the possibility of dividing the signal into different bands of frequency. This makes possible the detection and the reduction of ECG baseline wandering in low frequency subsignals. For testing presented method, ECG signals taken from MIT-BIH arrhythmia database are used. The method had been compared with traditional methods such FIR and on line averaging method and more advanced method such as wavelet adaptive filter (WAF). [42]

Zhang approached for BW correction and de-noising based on discrete wavelet transformation (DWT). They estimate the BW via coarse approximation in DWT with recommendations for how to select wavelets and the maximum depth for decomposition level. They reduce the high-frequency noise via Empirical Bayes posterior median wavelet shrinkage method with level dependent and position dependent thresholding values. The methods are applied to a real example. [43]

Marwa et al. propose a best basis selection method to choose a set of packets from a wavelet packet tree. Their goal is to obtain packets that show changes in both energy and frequency. The criterion adapted to choose the best basis is the Kullback-Leibler Distance (KLD). When there is no event to be detected, the estimated KLD follows roughly an exponential distribution depending on only one parameter the length of the windows partitioning the signal. When events are detected in a packet, the distribution of the estimated KLD deviates from the exponential distribution. The statistics Kolmogorov-Smirnov are used to measure the separation between experimental and theoretical cumulative distributions in order to highlight the presence of ruptures, then to select the most relevant packets. [44]

Sayadi et al. presented a method for ECG baseline correction using the adaptive bionic wavelet transform (BWT). In fact by the means of BWT, the resolution in the time–frequency domain can be adaptively adjusted not only by the signal frequency but also by the signal instantaneous amplitude and its first-order differential. Besides by optimizing the BWT parameters parallel to modifying their previous thresholding rule, one can handle ECG baseline correction. First an estimation of the baseline wandering frequency is obtained and then the adaptation can be used only in three successive scales in which the mid-scale has the closest centre frequency to the estimated frequency. Thus the implementation is possibly time consuming. Preliminary tests of BWT application to various ECG signals were constructed on the signals of MIT-BIH database. [45]

Electrocardiogram (ECG) signal compression is playing a vital role in biomedical applications. The signal compression is meant for detection and removing the redundant information from the ECG signal. Rizwan et al. deals with the

comparative study of ECG signal compression using pre-processing and without pre-processing approach on the ECG data. The performance and efficiency results are presented in terms of percent root mean square difference (PRD). Finally, the new PRD technique has been proposed for performance measurement and compared with the existing PRD technique; which has shown that proposed new PRD technique achieved minimum value of PRD with improved results. [46]

3.1 Introduction

A wavelet is a wave-like oscillation with amplitude that starts out at zero, increases, and then decreases back to zero. It can typically be visualized as a "brief oscillation" like one might see recorded by a seismograph or heart monitor. Generally, wavelets are purposefully crafted to have specific properties that make them useful for signal processing. Wavelets can be combined, using a "shift, multiply and sum" technique called convolution, with portions of an unknown signal to extract information from the unknown signal.

Let take an example, a wavelet could be created to have a frequency of Middle A and a short duration of roughly a 32nd note. If this wavelet were to be convolved at periodic intervals with a signal created from the recording of a song, then the results of these convolutions would be useful for determining when the Middle A note was being played in the song. Mathematically, the wavelet will resonate if the unknown signal contains information of similar frequency - just as a tuning fork physically resonates with sound waves of its specific tuning frequency. This concept of resonance is at the core of many practical applications of wavelet theory. [38]

As wavelets are a mathematical tool they can be used to extract information from many different kinds of data, including - but certainly not limited to - audio signals and images. Sets of wavelets are generally needed to analyze data fully. A set of "complementary" wavelets will deconstruct data without gaps or overlap so that the deconstruction process is mathematically reversible. Thus, sets of complementary wavelets are useful in wavelet based compression/decompression algorithms where it is desirable to recover the original information with minimal loss.

More technically, a wavelet is a mathematical function used to divide a given function or continuous-time signal into different scale components. Usually one can assign a frequency range to each scale component. Each scale component can then be

studied with a resolution that matches its scale. A wavelet transform is the representation of a function by wavelets. The wavelets are scaled and translated copies (known as "daughter wavelets") of a finite-length or fast-decaying oscillating waveform (known as the "mother wavelet"). Wavelet transforms have advantages over traditional Fourier transforms for representing functions that have discontinuities and sharp peaks, and for accurately deconstructing and reconstructing finite, non-periodic and/or non-stationary signals.

Wavelet transforms are classified into discrete wavelet transforms (DWTs) and continuous wavelet transforms (CWTs). Note that both DWT and CWT are continuous-time (analog) transforms. They can be used to represent continuous-time (analog) signals. CWTs operate over every possible scale and translation whereas DWTs use a specific subset of scale and translation values or representation grid.

A signal which we collected from the different sources like electrode, transducer or from antenna cannot be analyzed directly to draw useful conclusions. It needs to be processed in order to extract useful information from it. The signal is often transformed to different domains in order to better interpret the physical characteristics inherent in the original signal. The original signal can be reconstructed by performing inverse operation on the transformed signal without any loss of data. Some of the popular methods in signal processing include Fourier analysis, Wavelet Analysis, etc. All of these methods can be distinguished from each other by a way in which it maps the signal and have advantages over one another in terms of applicability for analyzing specific data type. A brief introduction of each method is given below.

3.2 Fourier analysis

Signal analysts already have at their disposal an impressive arsenal of tools. Perhaps the most well known of these is Fourier analysis, which breaks down a signal into constituent sinusoids of different frequencies. Another way to think of Fourier analysis is as a mathematical technique for transforming our view of the signal from time-based to frequency-based.

$$H(\omega) = \int_{-\infty}^{\infty} f(t)e^{-\omega t} dt \quad \dots (3.1)$$

Where ‘H(ω)’ is the Fourier transform of a signal ‘f(t)’.

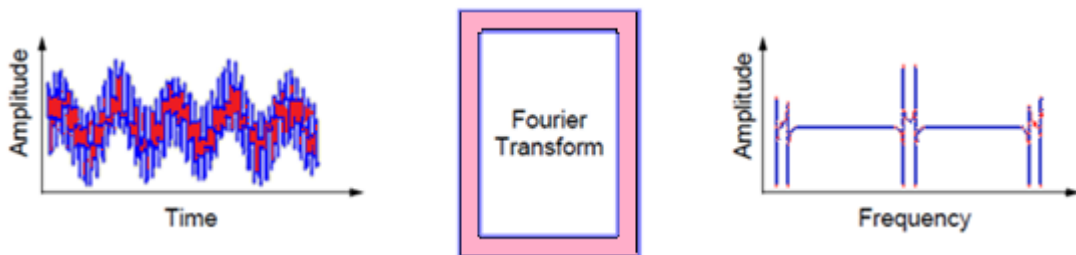


Figure 3.1 Fourier transform

For many signals, Fourier analysis is extremely useful because the signal’s frequency content is of great importance. So why do we need other techniques, like wavelet analysis? Fourier analysis has a serious drawback. In transforming to the frequency domain, time information is lost. When looking at a Fourier transform of a signal, it is impossible to tell when a particular event took place. If the signal properties do not change much over time — that is, if it is what is called a stationary signal — this drawback isn’t very important. However, most interesting signals contain numerous non-stationary or transitory characteristics: drift, trends, abrupt changes, and beginnings and ends of events. These characteristics are often the most important part of the signal, and Fourier analysis is not suited to detecting them.

3.3 Short-time fourier analysis

In an effort to correct this deficiency, Dennis Gabor (1946) adapted the Fourier transform to analyze only a small section of the signal at a time — a technique called windowing the signal. Gabor’s adaptation, called the Short-Time Fourier Transform (STFT), maps a signal into a two-dimensional function of time and

frequency. STFT employs a time-frequency representation ‘ $H(\omega, \tau)$ ’ of the signal ‘ $f(t)$ ’ as in the following equation.

$$H(\omega, \tau) = \int f(t)g * (t - \tau)e^{-\omega t} dt \quad \dots (3.2)$$

where $g(t-\tau)$ is a window function.

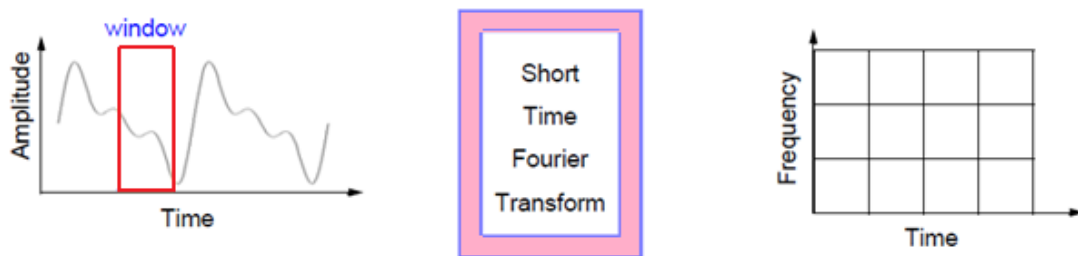


Figure 3.2 Short time fourier transform

The STFT represents a sort of compromise between the time- and frequency-based views of a signal. It provides some information about both when and at what frequencies a signal event occurs. However, you can only obtain this information with limited precision, and that precision is determined by the size of the window. While the STFT compromise between time and frequency information can be useful, the drawback is that once you choose a particular size for the time window, that window is the same for all frequencies. Many signals require a more flexible approach — one where we can vary the window size to determine more accurately either time or frequency.

3.4 Wavelet analysis

Wavelet analysis represents the next logical step: a windowing technique with variable-sized regions. Wavelet analysis allows the use of long time intervals where we want more precise low-frequency information, and shorter regions where we want high-frequency information.

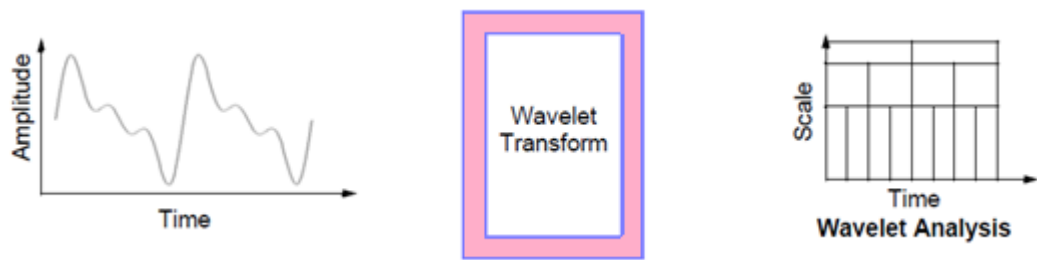


Figure 3.3 Wavelet transform

Here's what this looks like in contrast with the time-based, frequency-based, and STFT views of a signal:

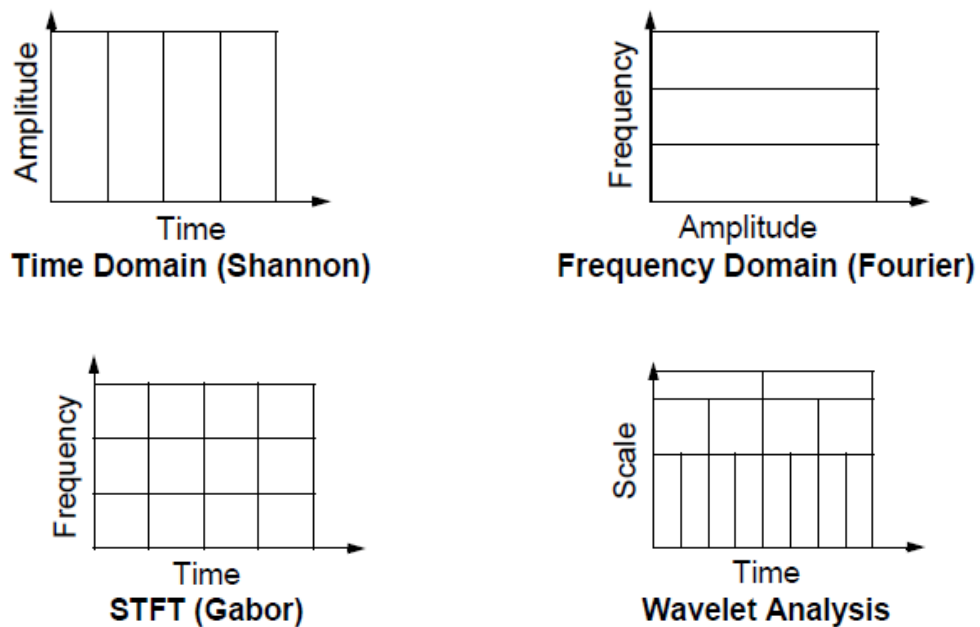


Figure 3.4 Time, frequency domain, STFT vs wavelet analysis

Here we can notice that wavelet analysis does not use a time-frequency region, but rather a time-scale region. Wavelet analysis is capable of revealing aspects of data that other signal analysis techniques miss aspects like trends, breakdown points, discontinuities in higher derivatives, and self-similarity. Furthermore, because it affords a different view of data than those presented by traditional techniques, wavelet analysis can often compress or de-noise a signal without appreciable degradation.

“What is a wavelet?”

A wavelet is a waveform of effectively limited duration that has an average value of zero. Compare wavelets with sine waves, which are the basis of Fourier analysis. Sinusoids do not have limited duration — they extend from minus to plus infinity. And where sinusoids are smooth and predictable, wavelets tend to be irregular and asymmetric.

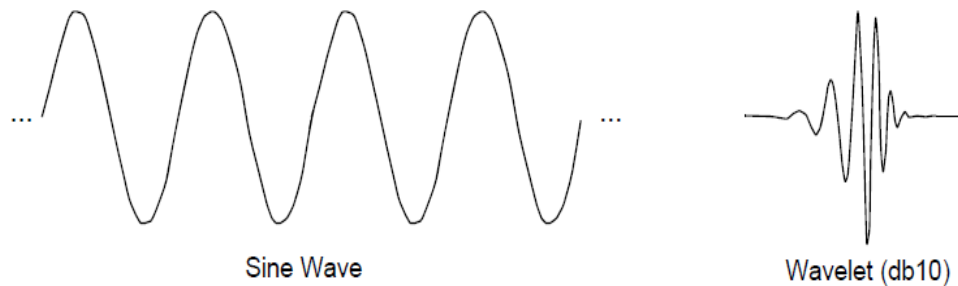


Figure 3.5 Sine wave and wavelet

Fourier analysis consists of breaking up a signal into sine waves of various frequencies. Similarly, wavelet analysis is the breaking up of a signal into shifted and scaled versions of the original (or mother) wavelet. Just looking at pictures of wavelets and sine waves, we can see intuitively that signals with sharp changes might be better analyzed with an irregular wavelet than with a smooth sinusoid, just as some foods are better handled with a fork than a spoon. It also makes sense that local features can be described better with wavelets that have local extent.

3.5 Continuous wavelet transform

Mathematically, the process of Fourier analysis is represented by the Fourier transform:

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt \quad \dots(3.3)$$

which is the sum over all time of the signal $f(t)$ multiplied by a complex exponential. The results of the transform are the Fourier coefficients, which when multiplied by a sinusoid of frequency yield the constituent sinusoidal components of the original signal. Graphically, the process looks like

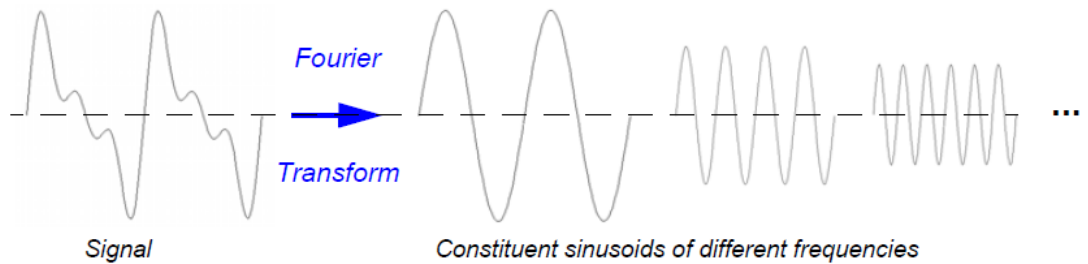


Figure 3.6 A signal and its constituent sinusoids of different frequencies

Similarly, the continuous wavelet transform (CWT) is defined as the sum over all time of the signal multiplied by scaled, shifted versions of the wavelet function:

$$C(\text{scale}, \text{position}) = \int_{-\infty}^{\infty} f(t)\psi(\text{scale}, \text{position}, t)dt \quad \dots(3.4)$$

The results of the CWT are many wavelet coefficients C , which are a function of scale and position. Multiplying each coefficient by the appropriately scaled and shifted wavelet yields the constituent wavelets of the original signal.

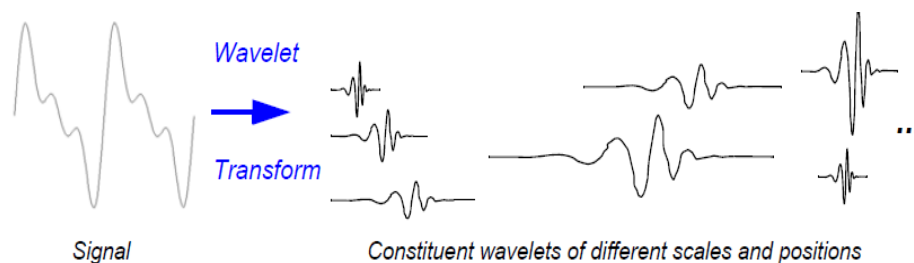


Figure 3.7 A signal and its constituent wavelets of different scales and positions

3.5.1 Mathematical description

Wavelets are generated from one single function (basis function) called the mother wavelet. Mother Wavelet is a prototype for generating the other window functions. The mother wavelet is scaled (or dilated) by a factor of 'a' and translated (or shifted) by a factor of b to give (under Morlet's original formulation):

$$\psi_{a,b}(t) = \left(\frac{1}{\sqrt{|a|}} \right) \times \psi\left(\frac{t-b}{a} \right) \quad \dots(3.5)$$

where a and b are two arbitrary real numbers. 'a' and 'b' represent the dilations and translations parameters respectively in the time axis. The parameter 'a' contracts (t) in the time axis when $a < 1$ and expands or stretches when $a > 1$. Hence 'a' is called the dilation (scaling) parameter. For $a < 0$, the function results in time reversal with dilation. Mathematically, when 't' is replaced in equation by (t - b) it causes a translation or shift in the time axis resulting in the wavelet function.

3.5.2 Scaling

We've already alluded to the fact that wavelet analysis produces a time-scale view of a signal, and now we're talking about scaling and shifting wavelets.

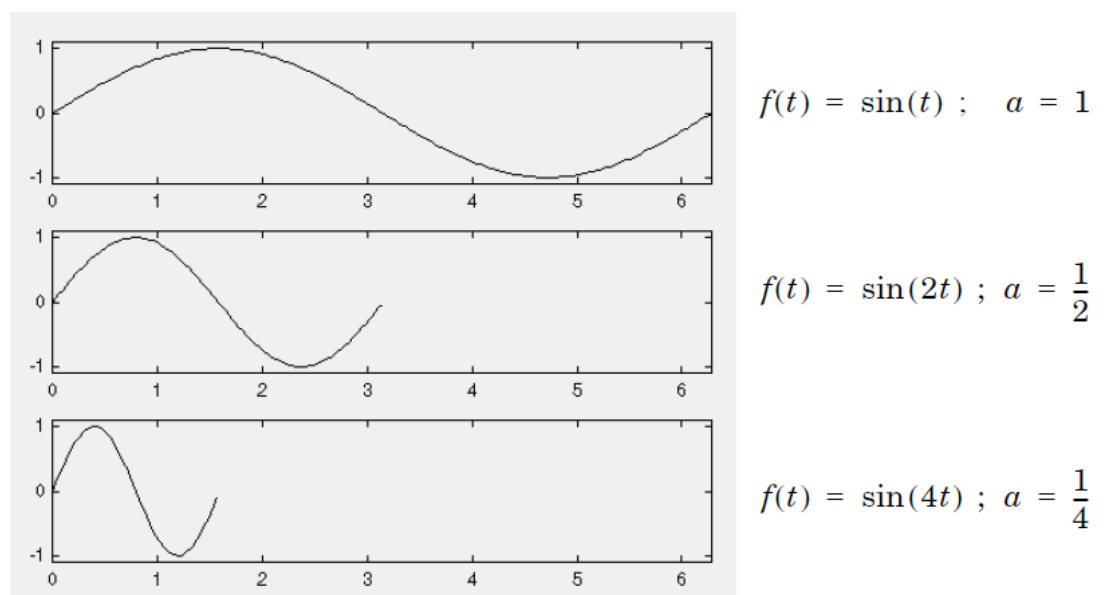


Figure 3.8 Scaling of a signal

Now question arise, what is scaling? Scaling a wavelet simply means stretching (or compressing) it. To go beyond colloquial descriptions such as “stretching,” we introduce the scale factor, often denoted by the letter ‘a’. If we’re talking about sinusoids, for example, the effect of the scale factor is very easy to see. Signal Constituent wavelets of different scales and positions. The scale factor works exactly the same with wavelets. The smaller the scale ‘a’ factor, the more “compressed” the wavelet.

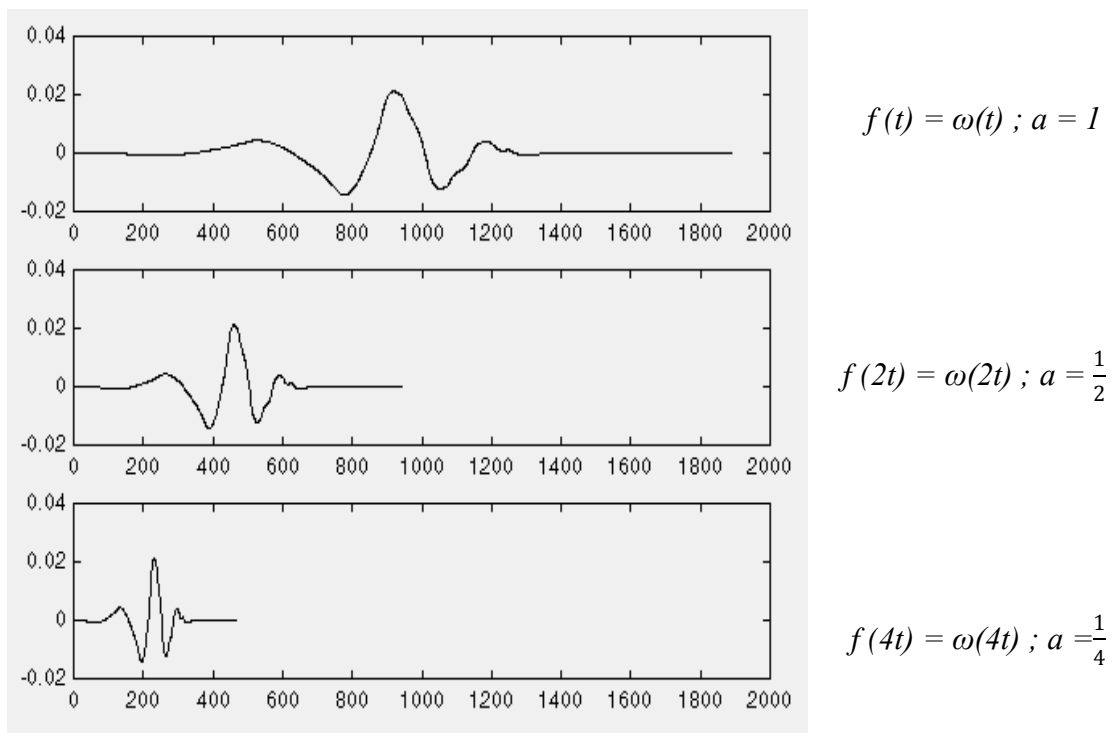


Figure 3.9 Scale is related to the frequency of the signal

It is clear from the diagrams that, for a sinusoid $\sin(\omega t)$, the scale factor is related (inversely) to the radian frequency . Similarly, with wavelet analysis, the scale is related to the frequency of the signal.

3.5.3 Shifting

Shifting a wavelet simply means delaying (or hastening) its onset. Mathematically, delaying a function $f(t)$ by k is represented by $f(t - k)$.

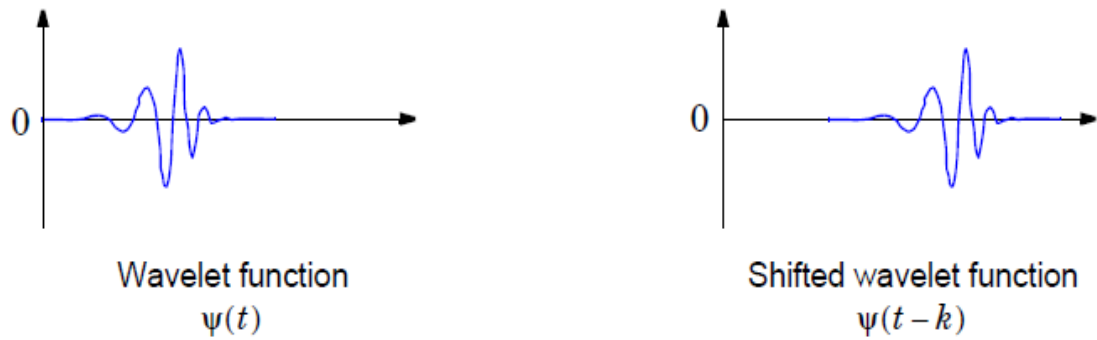


Figure 3.10 A wavelet and its shifted wavelet function

3.5.4 Scale and frequency

Notice that the scales in the coefficients plot (shown as y-axis labels) run from 1 to 31. Recall that the higher scales correspond to the most “stretched” wavelets. The more stretched the wavelet, the longer the portion of the signal with which it is being compared, and thus the coarser the signal features being measured by the wavelet coefficients.

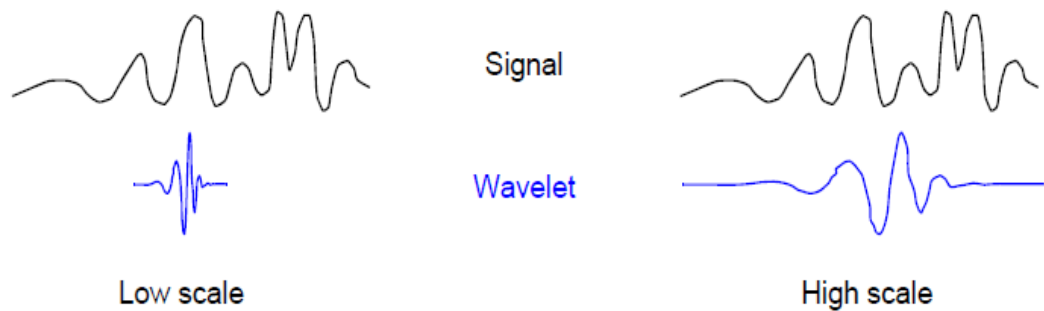


Figure 3.11 A signal is compared with a wavelet of low scale and high scale.

Thus, there is a correspondence between wavelet scales and frequency as revealed by wavelet analysis:

- Low scale $a \Rightarrow$ Compressed wavelet \Rightarrow Rapidly changing details \Rightarrow High frequency ω .
- High scale $a \Rightarrow$ Stretched wavelet \Rightarrow Slowly changing, coarse features \Rightarrow Low frequency ω .

3.5.5 Scale of nature

It's important to understand that the fact that wavelet analysis does not produce a time-frequency view of a signal is not a weakness, but strength of the technique. Not only is time-scale a different way to view data, it is a very natural way to view data deriving from a great number of natural phenomena. Consider a lunar landscape, whose ragged surface is a result of centuries of bombardment by meteorites whose sizes range from gigantic boulders to dust specks. If we think of this surface in cross section as a one-dimensional signal, then it is reasonable to think of the signal as having components of different scales, large features carved by the impacts of large meteorites, and finer features abraded by small meteorites.

3.6 Discrete wavelet transform

Calculating wavelet coefficients at every possible scale is a fair amount of work, and it generates an awful lot of data. It turns out, rather remarkably, that if we choose scales and positions based on powers of two — so-called dyadic scales and positions — then our analysis will be much more efficient and just as accurate. We obtain such an analysis from the discrete wavelet transform (DWT). An efficient way to implement this scheme using filters was developed in 1988 by Mallat [47]. The Mallat algorithm is in fact a classical scheme known in the signal processing community as a two-channel subband coder [48]. This very practical filtering algorithm yields a fast wavelet transform — a box into which a signal passes, and out of which wavelet coefficients quickly emerge. Let's examine this in more depth.

3.6.1 One-stage filtering: approximations and details

For many signals, the low-frequency content is the most important part. It is what gives the signal its identity. The high-frequency content, on the other hand, imparts flavour or nuance. Consider the human voice. If you remove the high-frequency components, the voice sounds different, but you can still tell what's being said. However, if you remove enough of the low frequency components, you hear gibberish. In wavelet analysis, we often speak of approximations and details. The

approximations are the high-scale, low-frequency components of the signal. The details are the low-scale, high-frequency components. The filtering process, at its most basic level, looks like this.

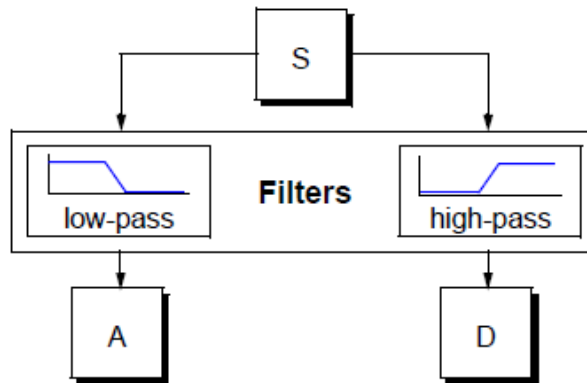


Figure 3.12 Approximations and details after signal filtering

The original signal, S , passes through two complementary filters and emerges as two signals. Unfortunately, if we actually perform this operation on a real digital signal, we wind up with twice as much data as we started with. Suppose, for instance, that the original signal S consists of 1000 samples of data. Then the resulting signals will each have 1000 samples, for a total of 2000. These signals A and D are interesting, but we get 2000 values instead of the 1000 we had. There exists a more subtle way to perform the decomposition using wavelets. By looking carefully at the computation, we may keep only one point out of two in each of the two 2000-length samples to get the complete information. This is the notion of down-sampling. We produce two sequences called cA and cD .

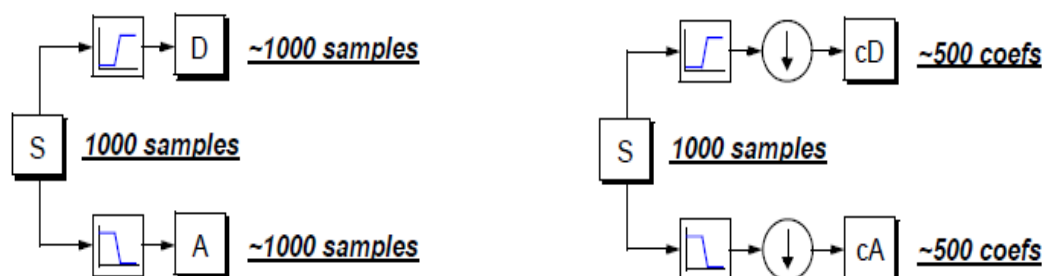


Figure 3.13 Effect of down-sampling

The process on the right, which includes down-sampling, produces DWT coefficients. To gain a better appreciation of this process, let's perform a one-stage discrete wavelet transform of a signal. Our signal will be a pure sinusoid with high-frequency noise added to it.

Here is a schematic diagram with real signals inserted into it.

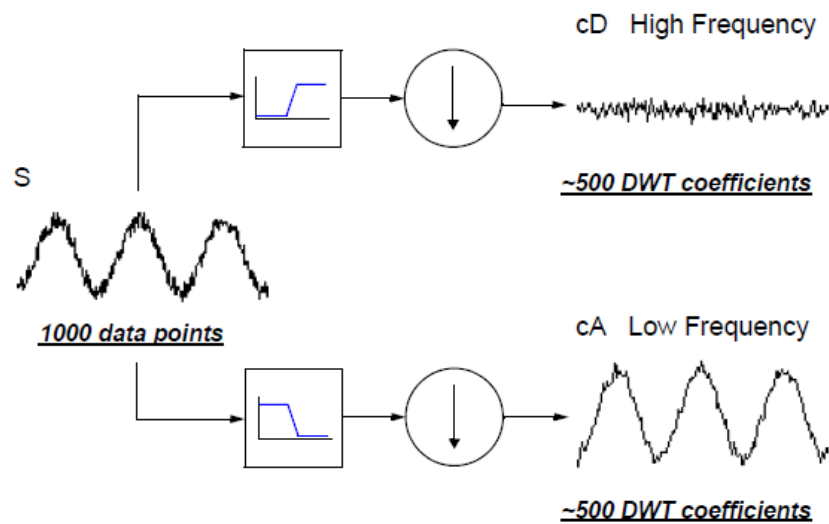


Figure 3.14 Schematic diagram with real signals

3.6.2 Multiple-level decomposition

The decomposition process can be iterated, with successive approximations being decomposed in turn, so that one signal is broken down into many lower resolution components. This is called the wavelet decomposition tree.

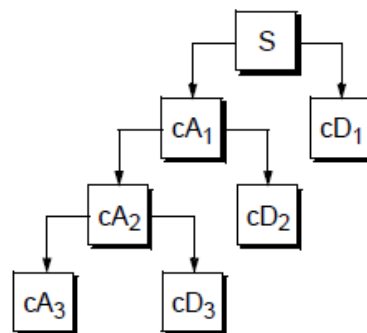


Figure 3.15 Multiple-level decomposition

Looking at a signal's wavelet decomposition tree can yield valuable information.

3.6.3 Number of levels

Since the analysis process is iterative, in theory it can be continued indefinitely. In reality, the decomposition can proceed only until the individual details consist of a single sample or pixel. In practice, you'll select a suitable number of levels based on the nature of the signal, or on a suitable criterion such as entropy.

3.7 Wavelet reconstruction

We've learned how the discrete wavelet transform can be used to analyze, or decompose, signals and images. This process is called decomposition or analysis. The other half of the story is how those components can be assembled back into the original signal without loss of information. This process is called reconstruction, or synthesis. The mathematical manipulation that effects synthesis is called the inverse discrete wavelet transforms (IDWT).

To synthesize a signal using Wavelet Toolbox™ software, we reconstruct it from the wavelet coefficients.

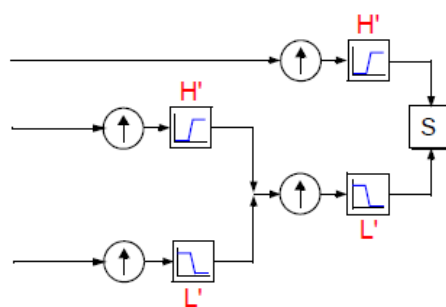


Figure 3.16 Wavelet reconstruction

Where wavelet analysis involves filtering and down-sampling, the wavelet reconstruction process consists of up-sampling and filtering. Up-sampling is the process of lengthening a signal component by inserting zeros between samples.

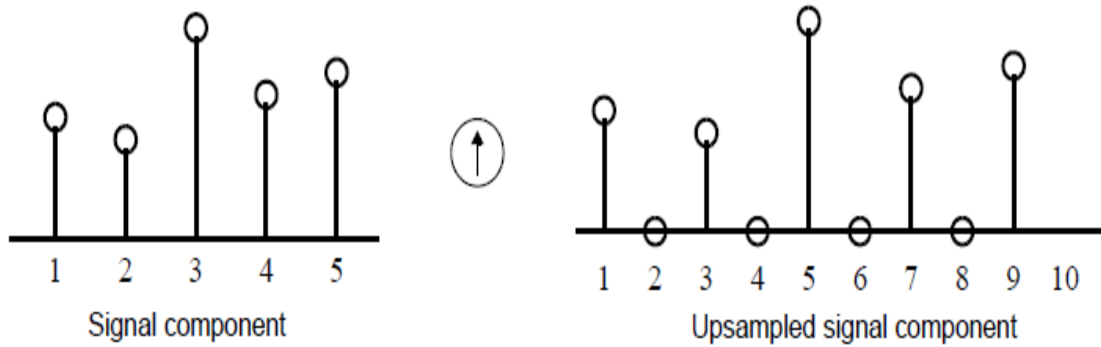


Figure 3.17 Up-sampling of signal

3.7.1 Reconstruction filters

The down-sampling of the signal components performed during the decomposition phase introduces a distortion called aliasing. It turns out that by carefully choosing filters for the decomposition and reconstruction phases that are closely related (but not identical), we can “cancel out” the effects of aliasing.

The low- and high-pass decomposition filters (L and H), together with their associated reconstruction filters (L' and H'), form a system of what is called quadrature mirror filters.

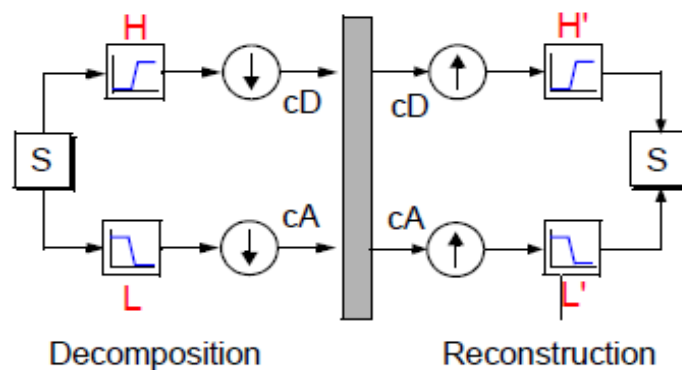


Figure 3.18 Quadrature mirror filters

3.7.2 Reconstructing approximations and details

We have seen that it is possible to reconstruct our original signal from the coefficients of the approximations and details.

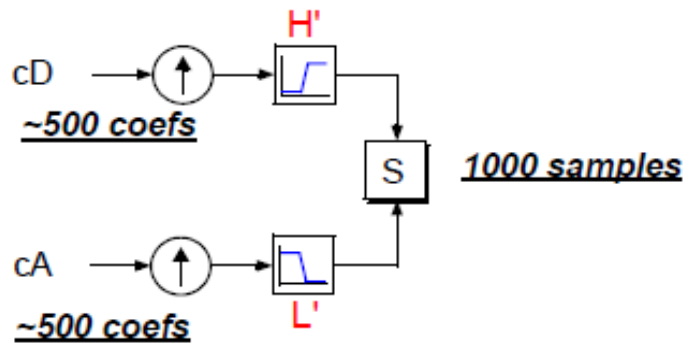


Figure 3.19 Reconstructing approximations and details

It is also possible to reconstruct the approximations and details themselves from their coefficient vectors. As an example, let's consider how we would reconstruct the first-level approximation A1 from the coefficient vector cA1. We pass the coefficient vector cA1 through the same process we used to reconstruct the original signal. However, instead of combining it with the level-one detail cD1, we feed in a vector of zeros in place of the detail coefficients vector:

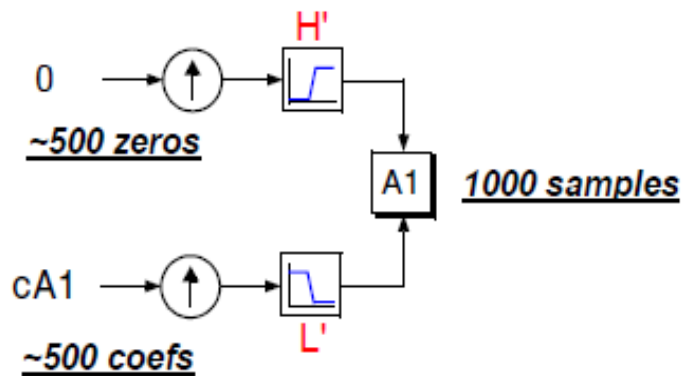


Figure 3.20 Reconstructing approximations and details with details being zero's

The process yields a reconstructed approximation A1, which has the same length as the original signal S and which is a real approximation of it. Similarly, we can reconstruct the first-level detail D1, using the analogous process.

The reconstructed details and approximations are true constituents of the original signal. In fact, we find when we combine them that

$$A_1 + D_1 = S \quad \text{.....(3.6)}$$

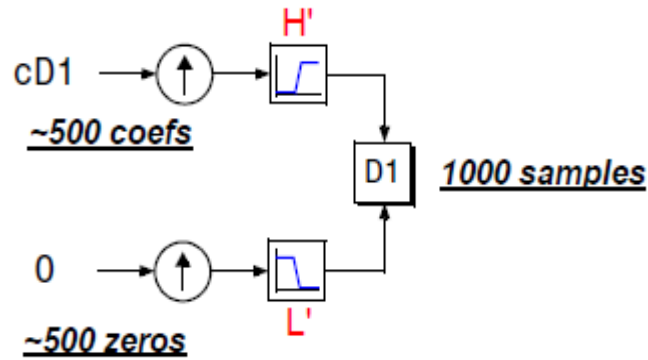


Figure 3.21 Reconstructing approximations and details with approximations being zero's

Note that the coefficient vectors $cA1$ and $cD1$ — because they were produced by down-sampling and are only half the length of the original signal — cannot directly be combined to reproduce the signal. It is necessary to reconstruct the approximations and details before combining them.

Extending this technique to the components of a multilevel analysis, we find that similar relationships hold for all the reconstructed signal constituents. That is, there are several ways to reassemble the original signal.

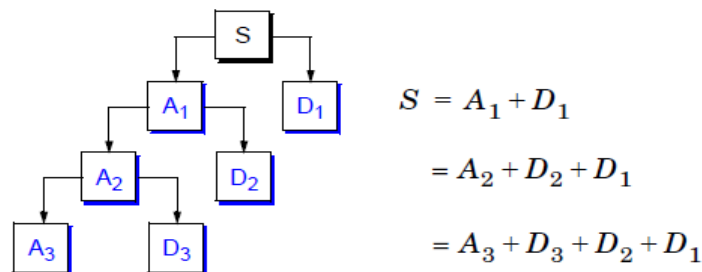


Figure 3.22 Reconstructed signal components

3.7.3 Multistep decomposition and reconstruction

A multistep analysis-synthesis process is represented in the figure 3.32. This process is divided in to two parts. They are as follow, breaking up a signal to obtain the wavelet coefficients, and reassembling the signal from the coefficients.

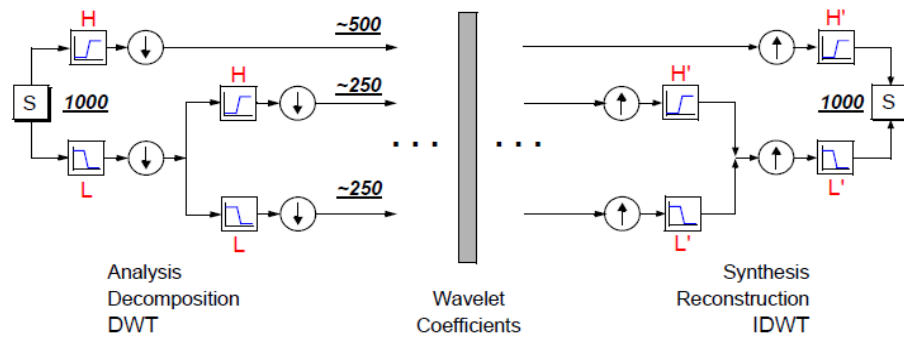


Figure 3.23 Multistep analysis-synthesis processes

Of course, there is no point breaking up a signal merely to have the satisfaction of immediately reconstructing it. We may modify the wavelet coefficients before performing the reconstruction step. We perform wavelet analysis because the coefficients thus obtained have many known uses, de-noising and compression being foremost among them. But wavelet analysis is still a new and emerging field. No doubt, many uncharted uses of the wavelet coefficients lie in wait. The toolbox can be a means of exploring possible uses and hitherto unknown applications of wavelet analysis. Explore the toolbox functions and see what you discover.

3.8 Wavelet packet analysis

The wavelet packet method is a generalization of wavelet decomposition that offers a richer range of possibilities for signal analysis. In wavelet analysis, a signal is split into an approximation and a detail. The approximation is then itself split into a second-level approximation and detail, and the process is repeated. For n -level decomposition, there are $n+1$ possible ways to decompose or encode the signal.

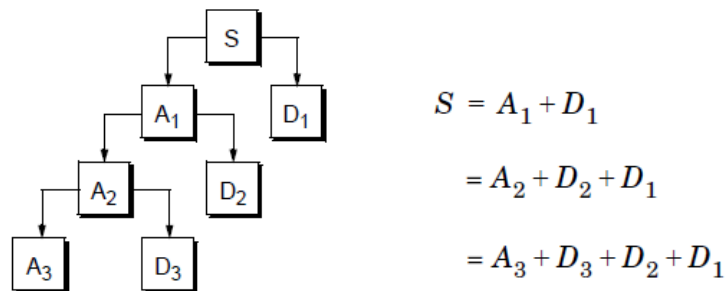


Figure 3.24 Wavelet decomposition and reconstruction

3.8.1 Wavelet packet decomposition

In wavelet packet analysis, the details as well as the approximations can be split. This yields more than 2^{2^n-1} different ways to encode the signal. This is the wavelet packet decomposition tree.

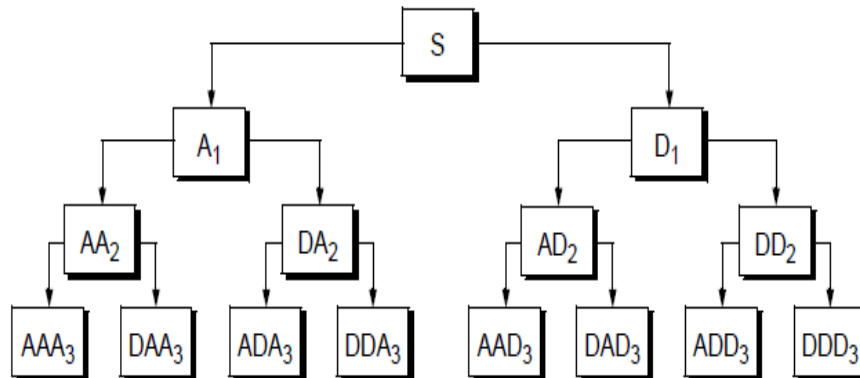


Figure 3.25 Wavelet packet decomposition tree.

The wavelet decomposition tree is a part of this complete binary tree. For instance, wavelet packet analysis allows the signal S to be represented as $A_1 + AAD_3 + DAD_3 + DD_2$. This is an example of a representation that is not possible with ordinary wavelet analysis. Choosing one out of all these possible encodings presents an interesting problem. In this toolbox, we use an entropy-based criterion to select the most suitable decomposition of a given signal. This means we look at each node of the decomposition tree and quantify the information to be gained by performing each split.

The wavelet packet decomposition (WPD) of a signal can be viewed as a step by step transformation of the signal from the time domain to the frequency domain. The top level of the WPD is the time representation of the signal. As each level of the decomposition is calculated there is an increase in the trade-off between time and frequency resolution. The bottom level of a fully decomposed signal is a. Frequency representation.

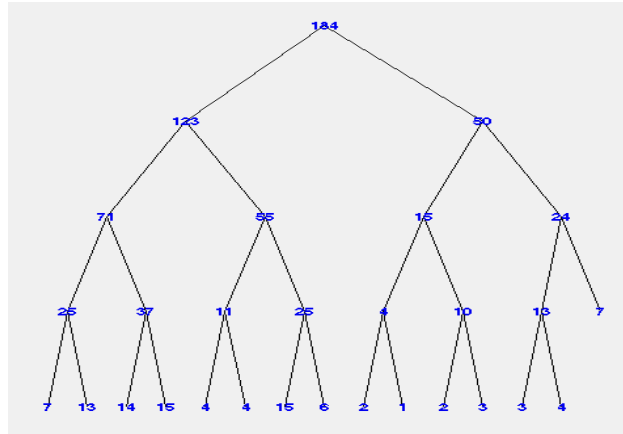


Figure 3.26 Wavelet packet decomposition

3.9 Wavelet properties

Various properties of wavelet transforms is described below:

- 1) Regularity
- 2) The window for a function is the smallest space-set (or time-set) outside which function is identically zero.
- 3) The order of the polynomial that can be approximated is determined by number of vanishing moments of wavelets.
- 4) The symmetry of the filters is given by wavelet symmetry. The Haar wavelet is the only symmetric wavelet among orthogonals. For biorthogonal wavelets both wavelet functions and scaling functions that are either symmetric or antisymmetric can be synthesized.
- 5) Orthogonality: Orthogonal filters lead to orthogonal wavelet basis functions; hence, the resulting wavelet transform is energy preserving. This implies that the mean square error (MSE) introduced during the quantization of the DWT coefficients is equal to the MSE in the reconstructed signal. This is desirable since it implies that the quantizer can be designed in the transform domain to take advantage of the wavelet decomposition structure. For orthogonal filter banks, the synthesis filters are transposes of analysis filters.

3.10 History of wavelets

From an historical point of view, wavelet analysis is a new method, though its mathematical underpinnings date back to the work of Joseph Fourier in the nineteenth century. Fourier laid the foundations with his theories of frequency analysis, which proved to be enormously important and influential.

The attention of researchers gradually turned from frequency-based analysis to scale-based analysis when it started to become clear that an approach measuring average fluctuations at different scales might prove less sensitive to noise.

The first recorded mention of what we now call a “wavelet” seems to be in 1909, in a thesis by Alfred Haar.

The concept of wavelets in its present theoretical form was first proposed by Jean Morlet and the team at the Marseille Theoretical Physics Center working under Alex Grossmann in France.

The methods of wavelet analysis have been developed mainly by Y. Meyer and his colleagues, who have ensured the methods’ dissemination. The main algorithm dates back to the work of Stephane Mallat in 1988. Since then, research on wavelets has become international.

3.11 Some application

The most common use of wavelets is in signal processing applications. For example:

- Compression applications. If we can create a suitable representation of a signal, we can discard the least significant" pieces of that representation and thus keep the original signal largely intact. This requires a transformation which separates the important" parts of the signal from less important parts. In the simplest case, we can decompose a signal into two parts: a low frequency part, which is some sort of average of the original signal, and a high frequency part, which is what remains after the low frequency part is subtracted from the original signal. If we are interested in the low frequency part and hence discard the high frequency part, what remains is a smoother representation of

the original signal with its low frequency components intact. Alternatively, if we are most interested in the high frequency part, we may be able to discard the low frequency part instead. This approach, which of decomposing a signal into two parts is common for all wavelets. Also fundamental to wavelet analysis is a hierarchical decomposition, in which we may apply further transforms to an already decomposed signal.

- Edge detection. With this application it is most important to identify the areas in which the input image changes quickly. We can discard the smooth (low frequency) parts. The simplest wavelet basis, the Haar basis (to be discussed later) is suitable for this application. Along this vein, the book by Strang and Nguyen describes a widely used application of wavelets, fingerprint compression, in which edge detection done prominently.[49]
- Fingerprint image compression The Federal Bureau of Investigation (FBI) has adopted a wavelet-based standard for digital fingerprint image compression. At compression ratios of about 20:1, the new standard will facilitate the rapid transmission of information that is crucial for effective police work. The FBI's fingerprint standard is just one way that wavelets are making an impact [50].
- Graphics. Two prominent uses of wavelets in graphics include Curve and surface representations; and Wavelet radiosity. These two reflect two quite different uses of wavelets.
- Numerical analysis. Wavelets are used in the solution of partial differential equations and integral equations.
- Signal-noise decomposition of time series. The continuous wavelet transform can provide a unique decomposition of a time-series into 'signal-like' and 'noise-like' components. From the overall wavelet spectrum, two mutually independent skeleton spectra can be extracted thus allowing the separate detection and monitoring in even non-stationary time-series. The idea of the method is to keep, from the overall wavelet expansion of the time-series, only the wavelet components of locally maximal amplitude at any given time or scale, thus obtaining the instantly maximal and scale maximal wavelet skeleton spectrum respectively [51].
- Classification of mammographic micro-calcifications. Breast cancer is the leading cause of death among women. Breast cancer can be detected earlier by

mammography than any other non-invasive examination. About 30% to 50% of breast cancers demonstrate tiny granule like deposits of calcium called micro-calcifications. It is difficult to distinguish between benign and malignant cases based on an examination of calcification regions, especially in hard-to-diagnose cases. Chitre et al., investigated the potential of using energy and entropy features computed from wavelet packets for their correlation with malignancy. Two types of Daubechies discrete filters were used as prototype wavelets. The classification results indicate the potential of using features derived from wavelet packets in discriminating micro-calcification regions into benign and malignant categories [52].

- Bio-signal compression for tele-medicine. A bio-signal is acquired by use of transducers or electrodes in the form of an array of elements at regular sampling intervals. Wavelet transform of the signal is taken on dyadic scale. A threshold is decided and zeroes replace all the samples with the value less than threshold. The remaining samples are replaced by a discrete positive or negative value. If all the descendents of a zero down the scale are also less than a threshold, a tree is formed. This gives tremendous compression, with reasonably good reconstructed signal upon taking the inverse wavelet transform. This compression technique helps in reducing the storage space as well as the bandwidth requirement for tele-medicine applications [53].

CHAPTER 4

PROBLEM DEFINITION

Many times when ECG signal is recorded from surface electrode that are connected to the chest of patient, the surface electrode are not tightly in contact with the skin as the patient breath the chest expand and contract producing a relative motion between skin and electrode. This results in shift in the baseline which is also known as low frequency baseline wander. Obviously the fundamental frequency of baseline wander is same as that of respiration frequency. It is required that this baseline wander is to removed from the ECG before extraction of any meaningful feature. Baseline wander makes manual and automatic analysis of ECG records difficult, especially in the detection of ST-segment deviations. This segment is very important and has the information related to heart attack. Since the spectrum of baseline wander and low frequency component of ECG signal usually overlaps, removing of baseline wander may cause distortion of important clinical information. This thesis work aims at removing this baseline wander signal while preserving the low frequency ECG clinical information.

PROPOSED SOLUTION AND ITS IMPLEMENTATION

5.1 Objective

Most of the signals in real life are accompanied with noise which may be random, Gaussian white noise etc. Till this noise is present with the signal, the received signal may be of very little use. Noise makes the process of information extraction from the signal a difficult task and results to incorrect output. Signal filtering is a process which can be thought as a “pre-processing step” for information extraction from the signal.

Let $S(n)$ be the original ECG signal with no noise(baseline wander), $V(n)$ be the baseline wander added to the signal before it is received for analysis or information extraction. The signal received is represented by $VS(n)$ as follows:

$$VS(n) = S(n) + V(n) \quad \dots 5.1$$

The purpose of the procedure of de-noising is to extract $S(n)$ from $VS(n)$ so that it can be used for intended purposes. Noise added to the signal is lower in frequency as compared to the original signal.

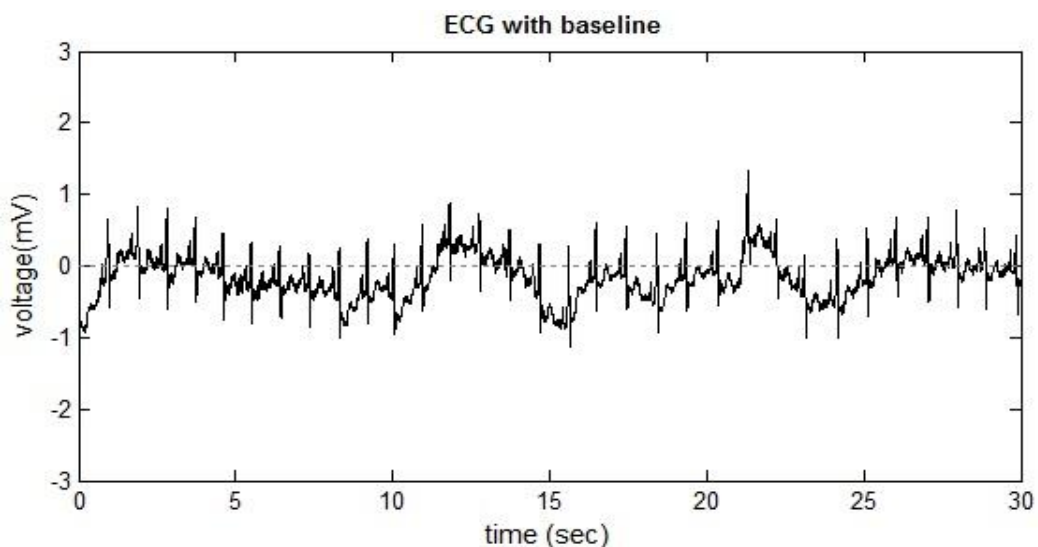


Figure 5.1 ECG Signal with baseline wander (MITBIH rec.108.23:00-23:30)

The ECG data is chosen from the MITBIH ECG database. All segments are of 360 Hz sampling rate and all present quite important baseline deviation. In addition, the segments include both normal and abnormal heartbeats. The used records are presented in the following fashion: MITBIH rec.xxx.yy:zz-tt:uu, where xxx is the record number in the database, yy:zz (resp.tt:uu) beginning (resp. ending) record times in min:sec.

5.2 Simulated baseline wander

In order to assess a baseline correction algorithm, one would need to acquire an ECG segment possessing an isoelectric level baseline, add a known signal simulating one or a combination of the above cited baseline deviation types, compute the baseline deviation from the (artificially) contaminated ECG and finally compute the error associated to the baseline extraction method.

This evaluation scheme had been adopted by previous authors for baseline deviation extraction [30] and other ECG related issues [54]. For the scheme to be effective, the used ECG segment must be as “clean” as possible. We ourselves adopted this technique, except that because of lack of recording facilities, we took as our “gold standard” chosen from the Massachusetts Institute of Technology—Beth Israel Hospital (MIT/BIH) public ECG database, possessing an almost (perceptually) isoelectric baseline. The obtained “standard” segment is shown in Fig. 5.2.

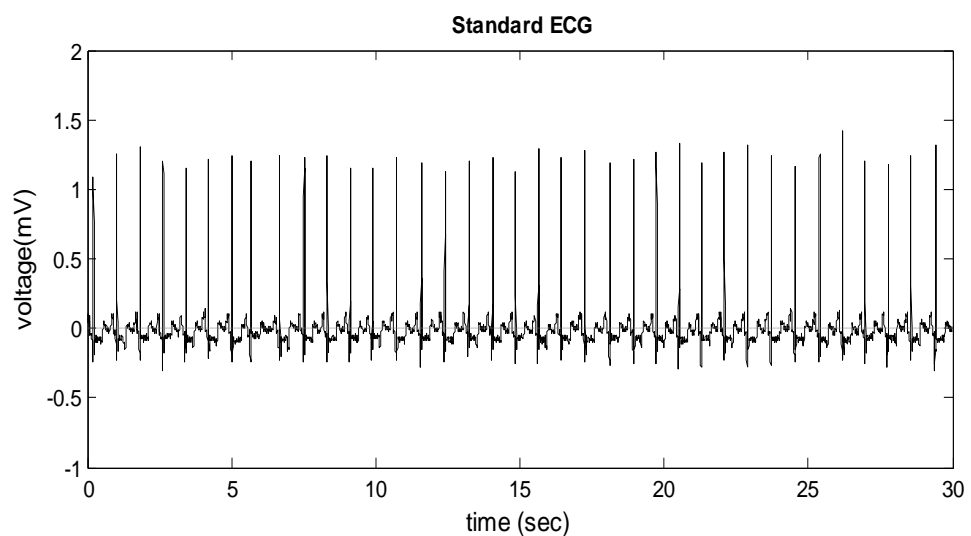


Figure 5.2 Standard ECG

We should emit the following precisions on the methodology and the used standard segment for the conducted experiments. First, as can be seen on Fig. 5.2, the standard segment is not fully noise free. This means that extraction of any added (simulated) noise may be affected by this pre-existing noise. Second, no prior information on the standard ECG has been used. We have constructed different artificial baseline, they are as follow:

1. B0, is a 0.1Hz sin wave
2. B1, is a 0.05Hz triangular wave
3. B2, is a 0.05Hz sin wave
4. B3 is a combination of B0 and B1
5. B4 is a combination of B0 and B2
6. B5 is a combination of B1 and B2
7. B6 is a combination of B0, b1 and B2
8. B7 is a combination of 0.2 Hz sin wave and B5
9. B8 is a combination of 0.3 Hz sin wave and B5

This artificial baseline are added to the standard ECG(fig 5.2) and we got the ECG with different baseline wandering. This is shown in the figure below.

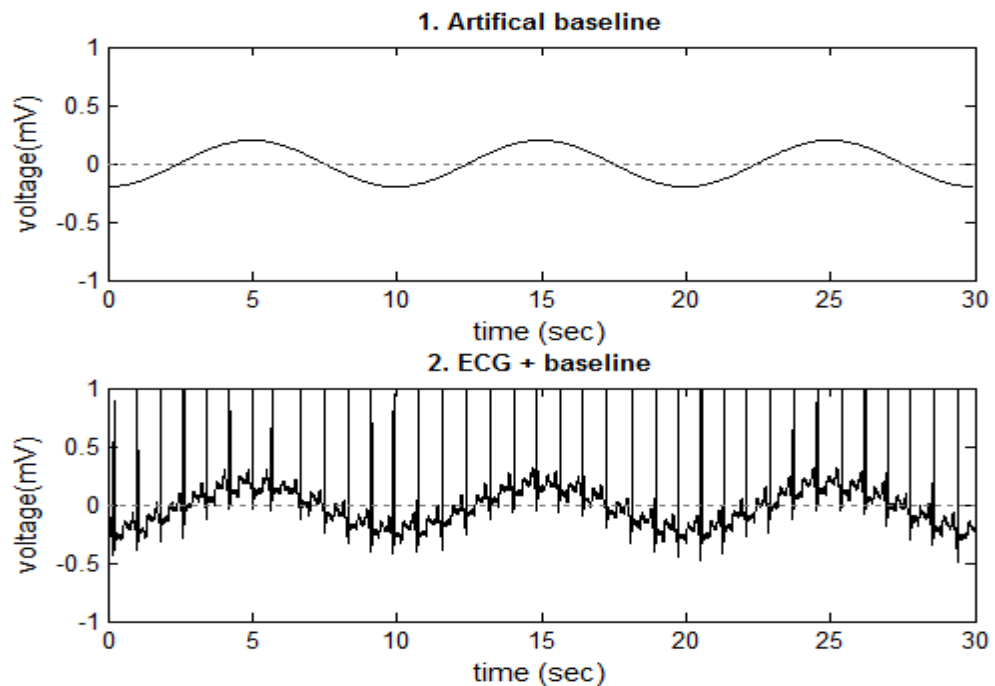


Figure 5.3 Standard ECG plus B0

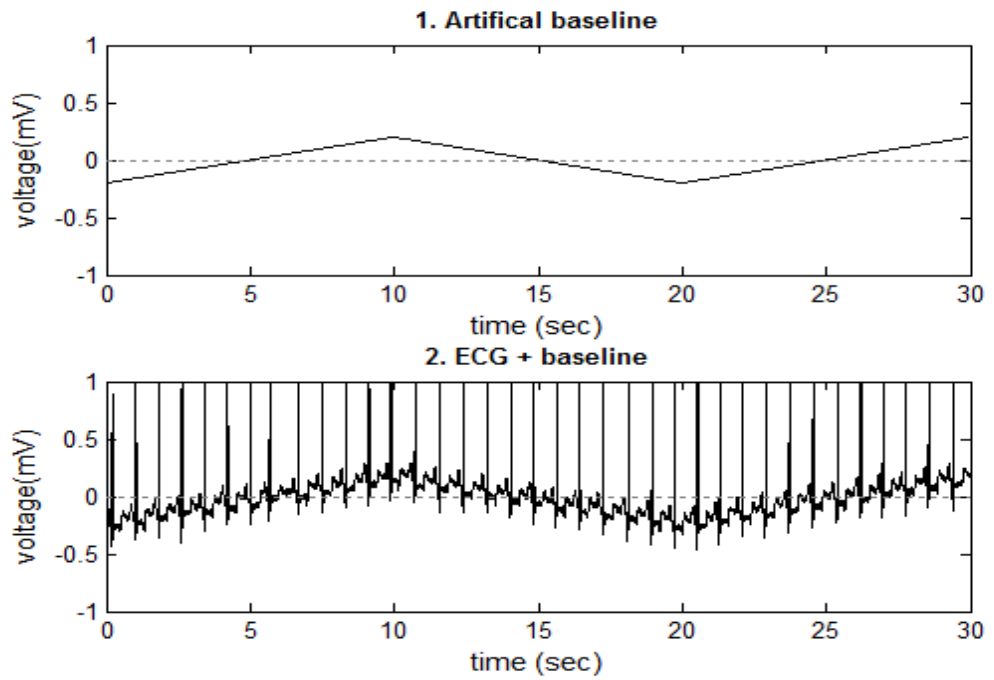


Figure 5.4 Standard ECG plus B1

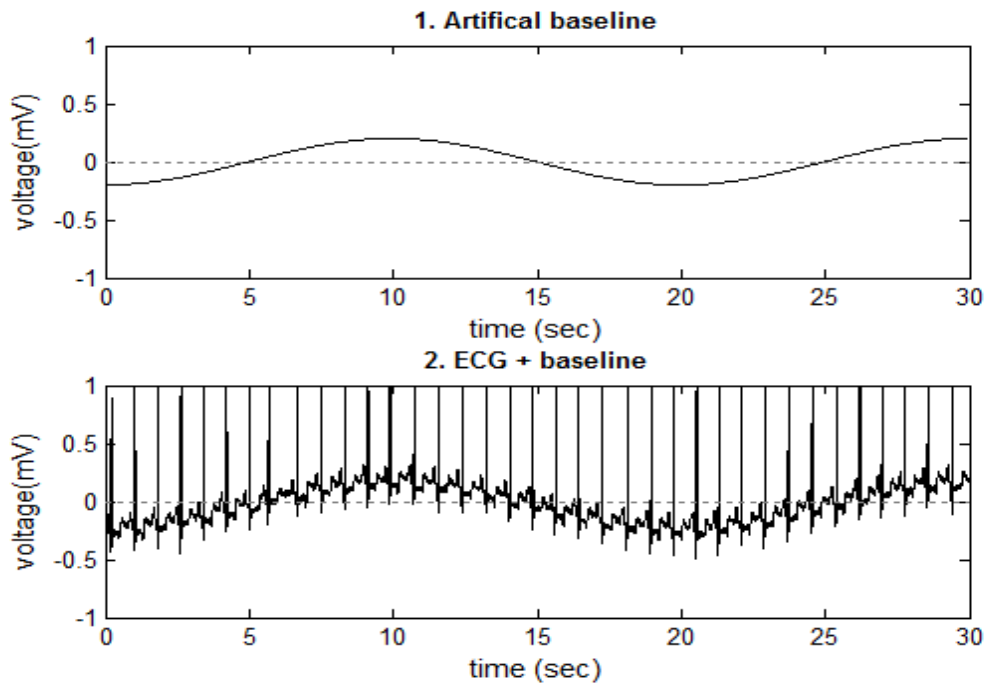


Figure 5.5 Standard ECG plus B2

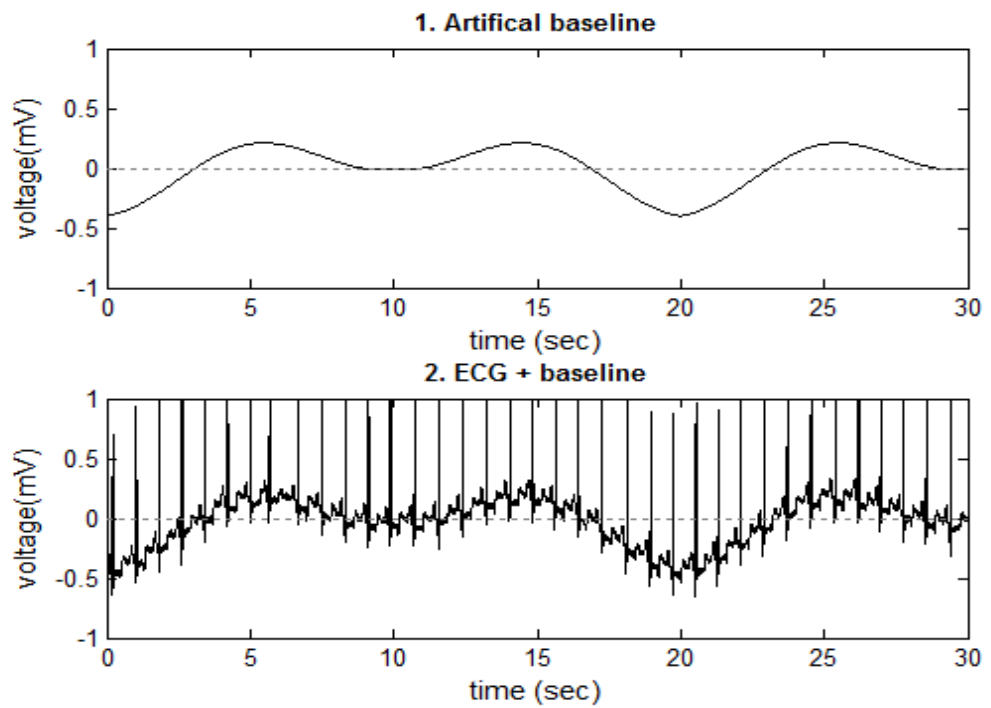


Figure 5.6 Standard ECG plus B3

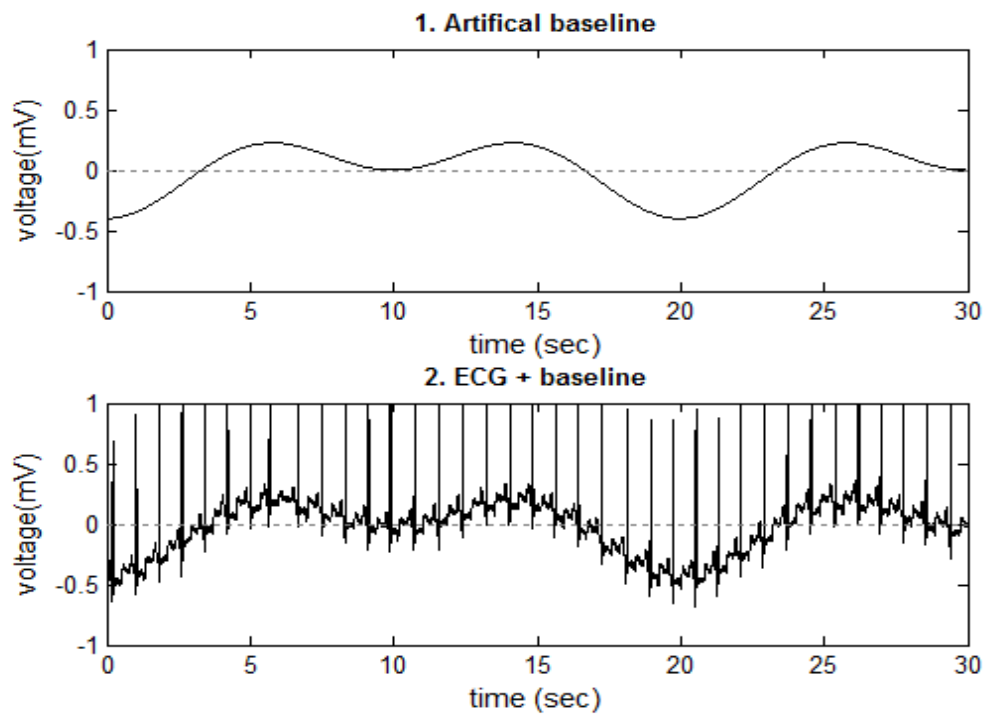


Figure 5.7 Standard ECG plus B4

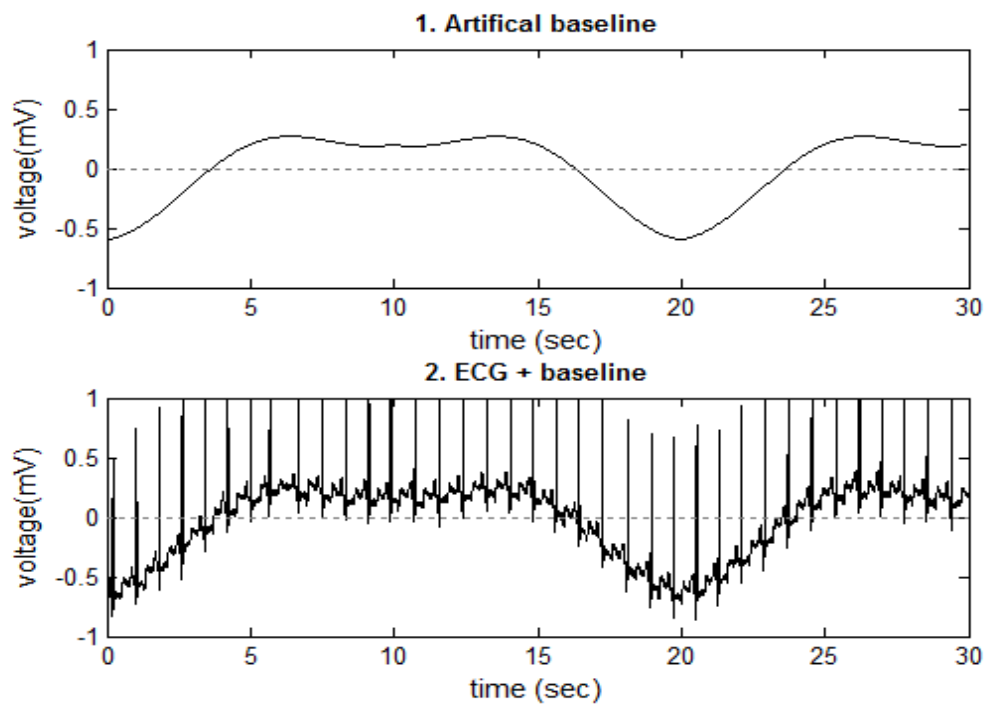


Figure 5.8 Standard ECG plus B5

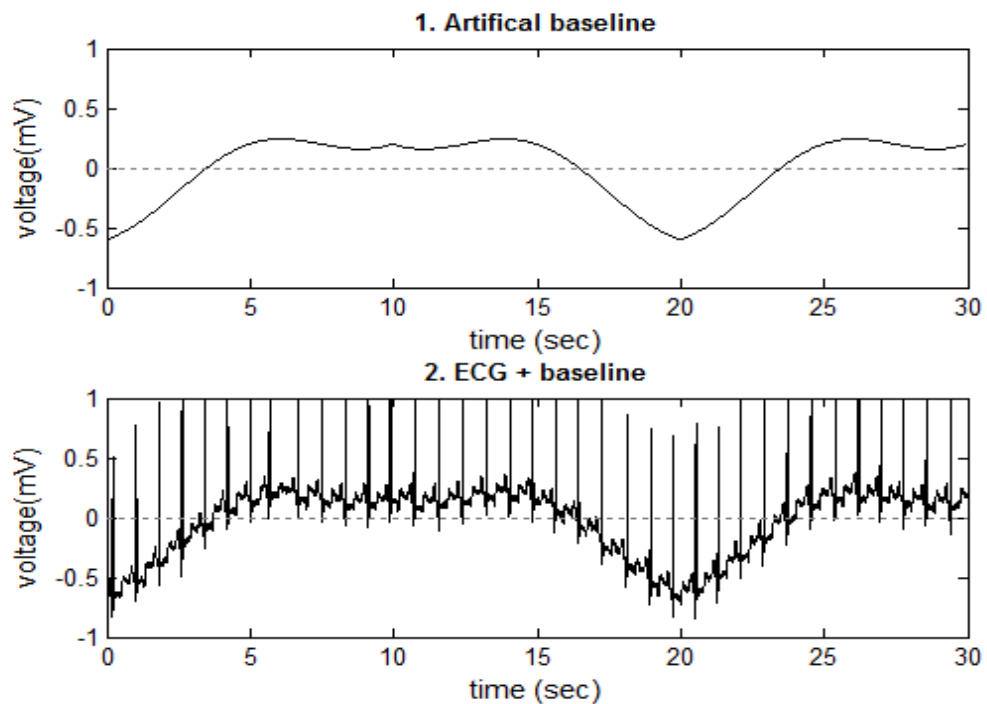


Figure 5.9 Standard ECG plus B6

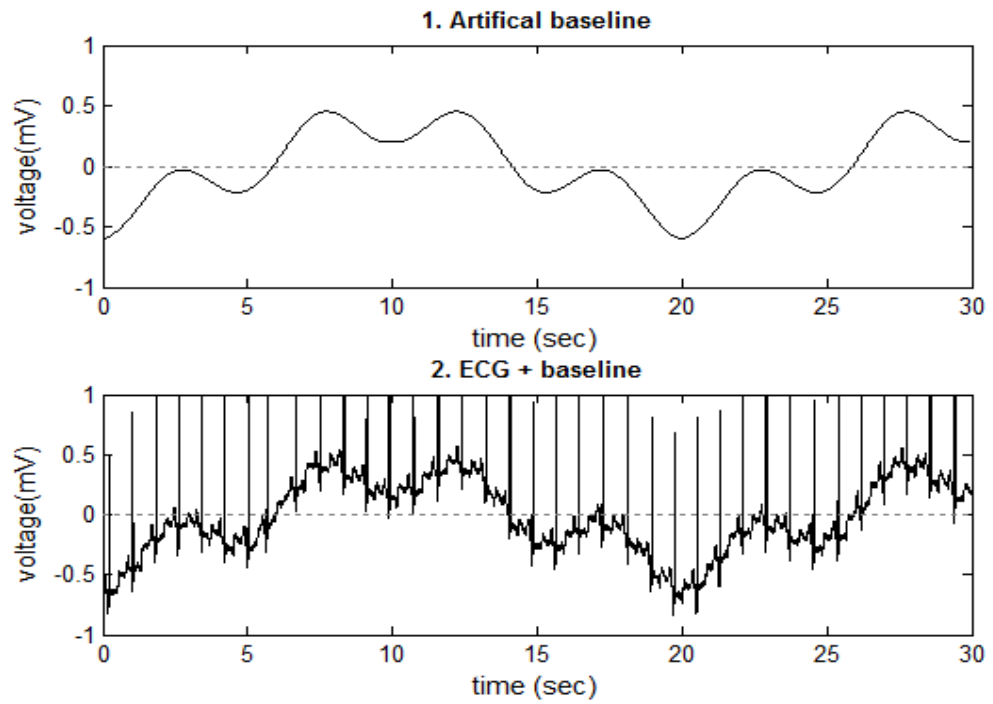


Figure 5.10 Standard ECG plus B7

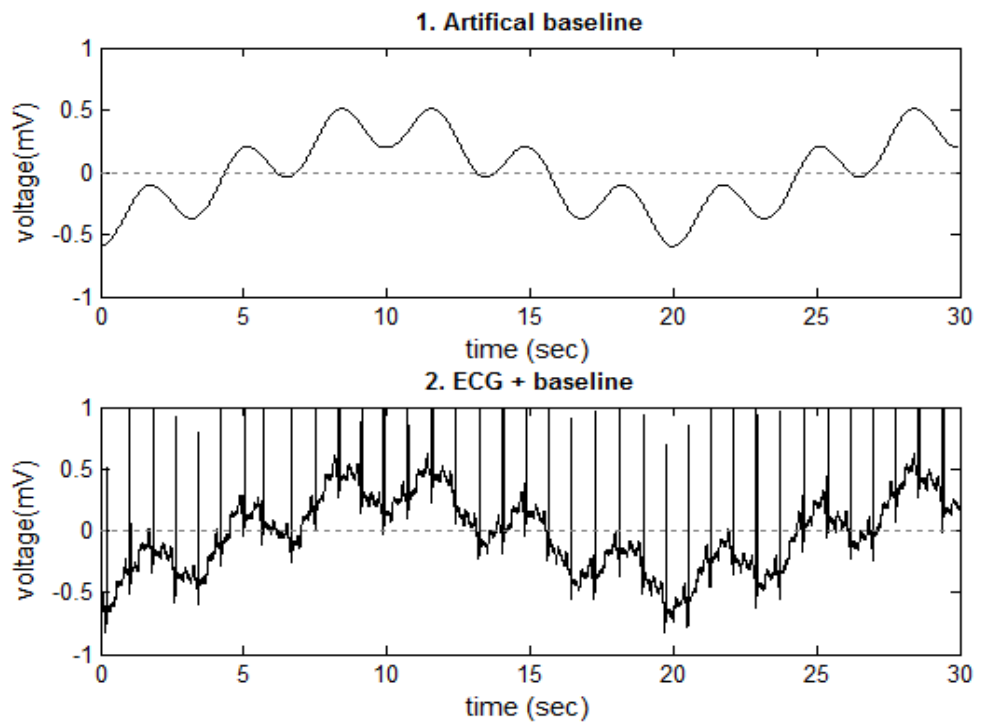


Figure 5.11 Standard ECG plus B8

5.3 Baseline estimation using wavelet transform.

Wavelet transformation is a linear operation that decomposes a signal into components that appear at different scales (or resolutions). De-noising is an interesting application of wavelet transform. In our research work, the properties of wavelet transforms have been employed to recover a signal from the signal with noise (baseline wander). The process of filtering can be broken into further steps which are:

1. Analysis
2. Decomposition
3. Threshold and
4. Reconstruction

To explain the procedure we have taken an ECG form MIT/BIH database which is shown below and all the above steps are apply on this ECG for explanation.

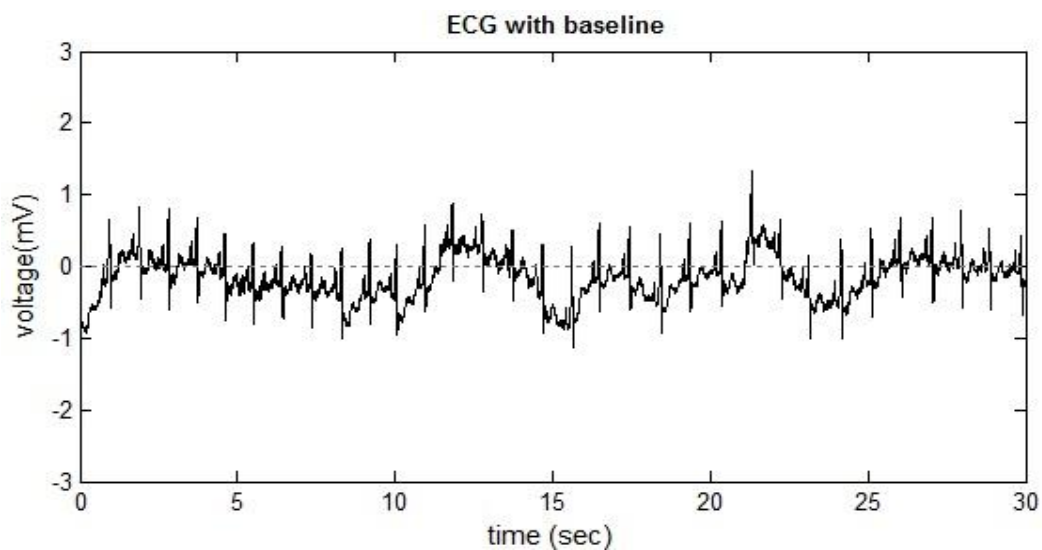


Figure 5.12 Input ECG signal with baseline wander (MITBIH rec.108.23:00-23:30)

5.3.1 Analysis step

Selecting an appropriate wavelet is a very important task in this step. The wavelet chosen should be similar to the signal that has to be filtered to give the best possible results.

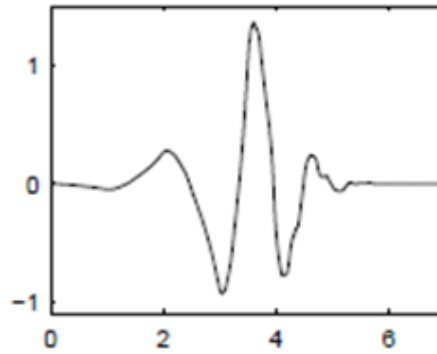


Figure 5.13 Daubechies family of wavelets (D4)

This “similarity” can be decided on the basis of the cross- correlation between the two functions. We have preferred the Daubechies family of wavelets because of their high number of vanishing moments making them capable of representing complex high degree polynomials. The result of our simulations showed that D4 wavelet provided sufficiently good signal output.

5.2.2 Decomposition

For a given wavelet, compute the wavelet decomposition of signal x at level N . One of the vectors has the “approximate” or smooth coefficients and the other has detail coefficients. The approximations are the high-scale, low-frequency components of the signal. The details are the low-scale, high-frequency components. Then approximations coefficients served as input for the next iteration of the same procedure. This procedure was carried out till the desire level is reached.

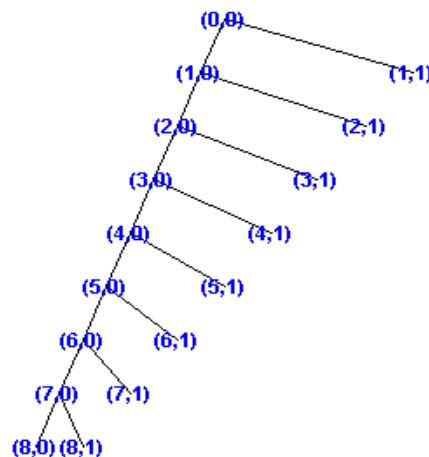


Figure 5.14 Wavelet tree (decompose to level 8)

To find the best level for decomposition of given ECG signal (standard ECG plus artificial baseline) we try the trial and error method. Here we taken the standard ECG mixed with the artificial baseline as mention above. Here we, first decompose the signal to the level 5 by D4 wavelet. It is done by MATLAB and for thersholding we chose the default value which we are geting form the toolbox. After this we did the performance analysis by PDR method. The above step is repeated for level 6,7,8, and 9. The results are shown in the table. We find that the average is decrease monolithically till the level 8 and then start increasing with further increases of level. On observing the table we concluded that we get the best result for D4 wavelet when signal is decompose to level eight.

Table 5.1 PRD value (%) for different decompose level 7, 8 and 9

Artificial baseline	Level 7	Level 8	Level 9
B0	18.83128	0.570889	1.538617
B1	18.83081	0.875334	1.86133
B2	18.83103	0.583233	0.857965
B3	18.83103	0.845154	2.130882
B4	18.83127	0.562538	1.549335
B5	18.83192	1.261423	3.386867
B6	18.83101	0.838872	2.145033
B7	18.83207	1.418767	13.66872
B8	18.83796	5.196422	42.3144
Average	18.83204	1.350292	7.717016

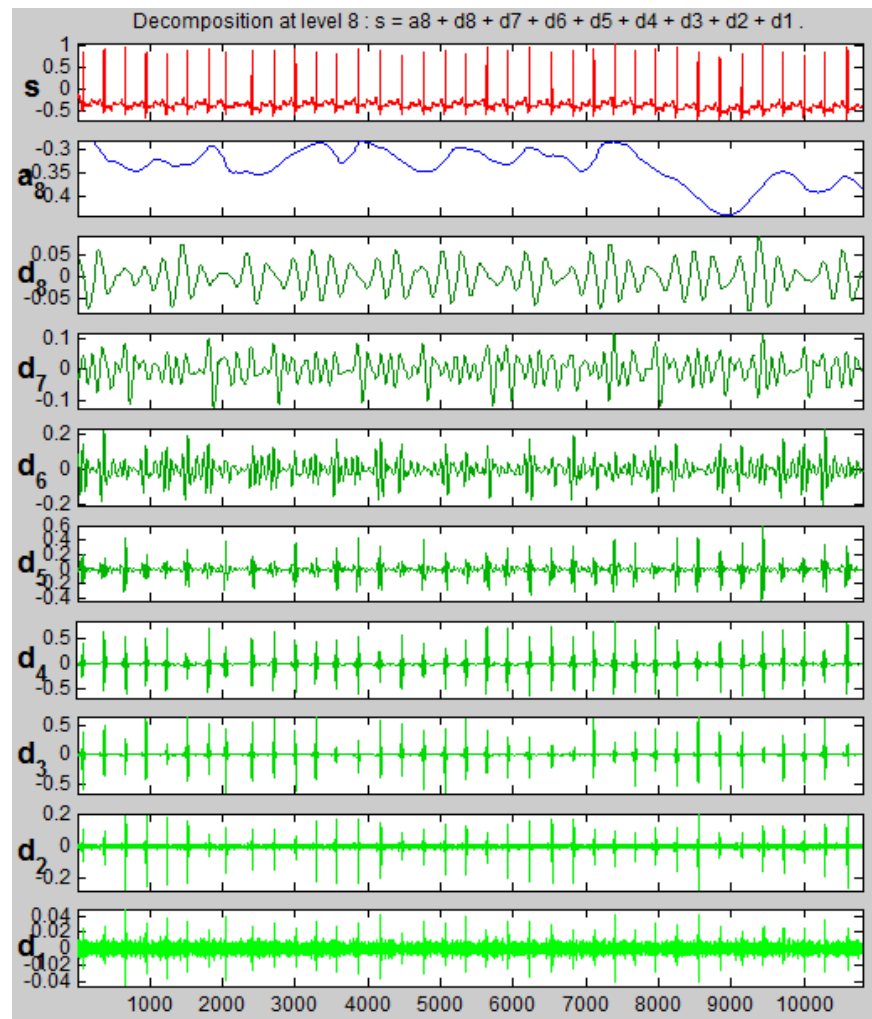


Figure 5.15 Decomposition

5.2.4 Thresholding step

After applying the selected wavelet transform to the input vector we obtained a numerically transformed vector which had the detail coefficients that are carried from one level to the next as it is and the finally we left with approximated value. To denoise the signal, the detail coefficients were made '0' after applying the transform. Then, the size of the input vector, which had sampled values of the signal, was known and it was also known that each time the size of the resultant vector had been reduced to half the original size.

5.2.5 Reconstruction

After applying the wavelet transform and threshold procedures, the inverse wavelet transform was applied. The output of this step, as seen in the figure, was the baseline wander. All the detail coefficients up to level 8 had been set to zero in the threshold step to obtain this output.

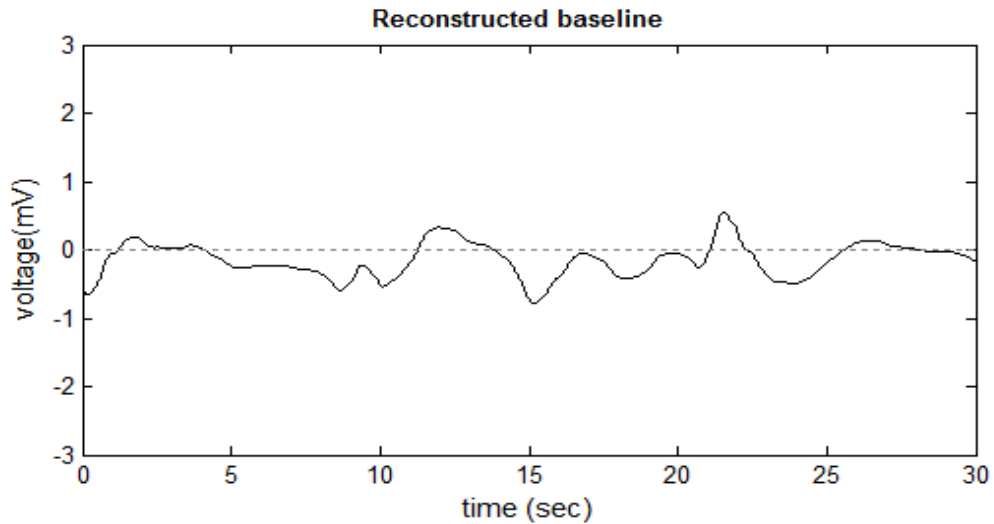


Figure 5.16 Reconstructed baseline

5.2.6 Baseline Wander Free ECG

After above five steps be getting the desire baseline. As we mention earlier in equation 5.1 we can get the baseline wander free ECG by simply subtracting baseline from the given ECG signal (figure 5.7).

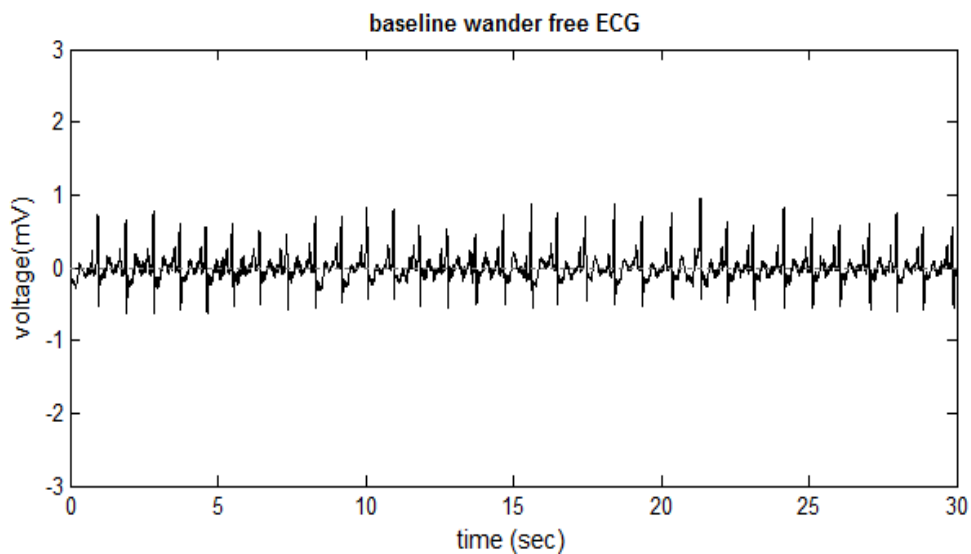


Figure 5.17 ECG (free from baseline wander)

Similarly baseline wander estimated in different ECG segment. All figures are presented in a unified fashion: (a) input ECG original, (b) extracted baseline wander, and (c) ECG out (baseline wander free).

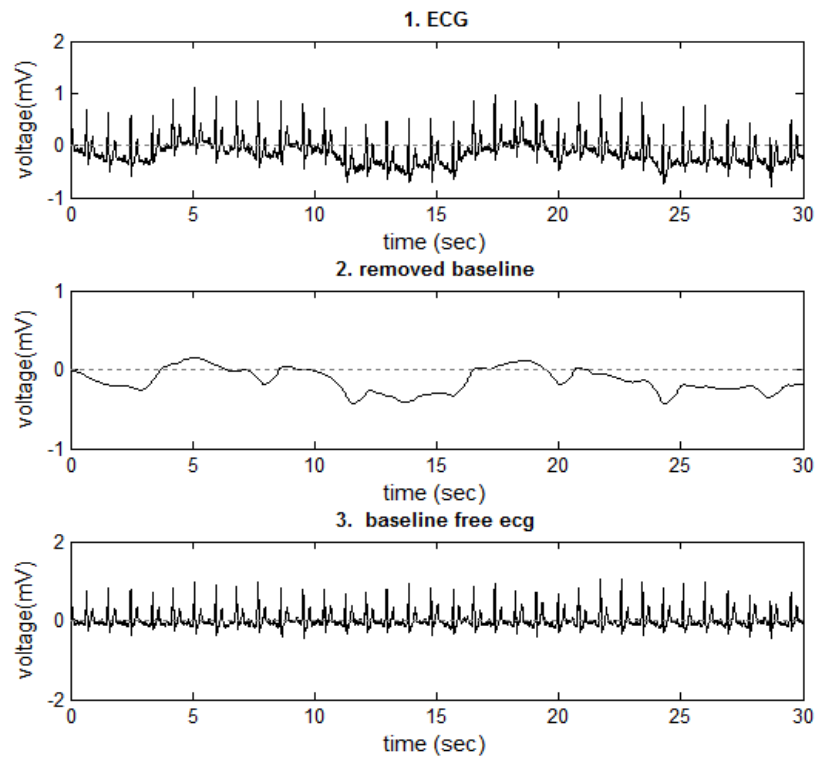


Figure 5.18 ECG_1 MITBIH rec.111.2:00-2:30

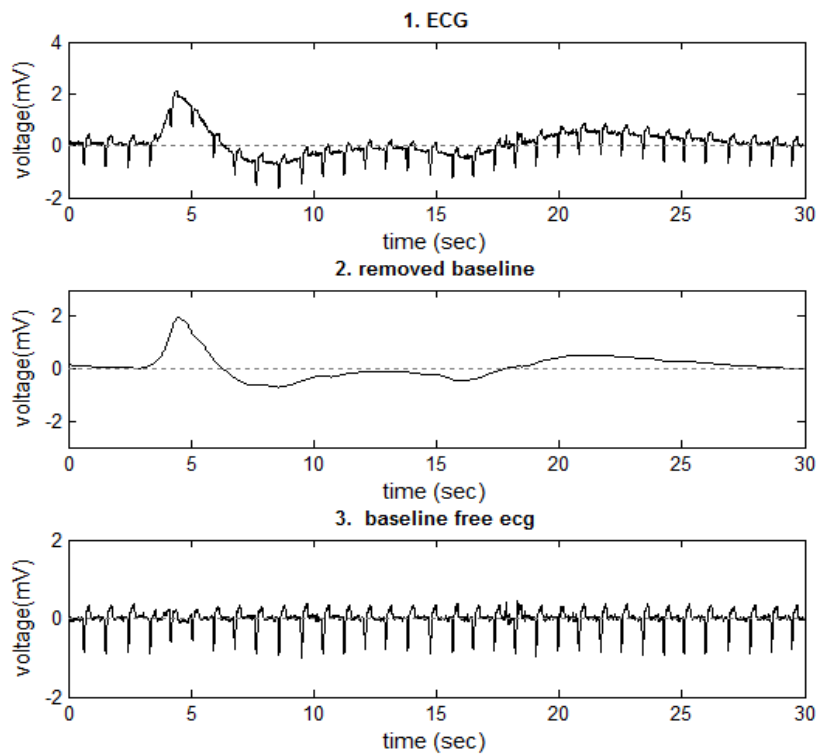


Figure 5.19 ECG_2 MITBIH rec.111.2:00-2:30

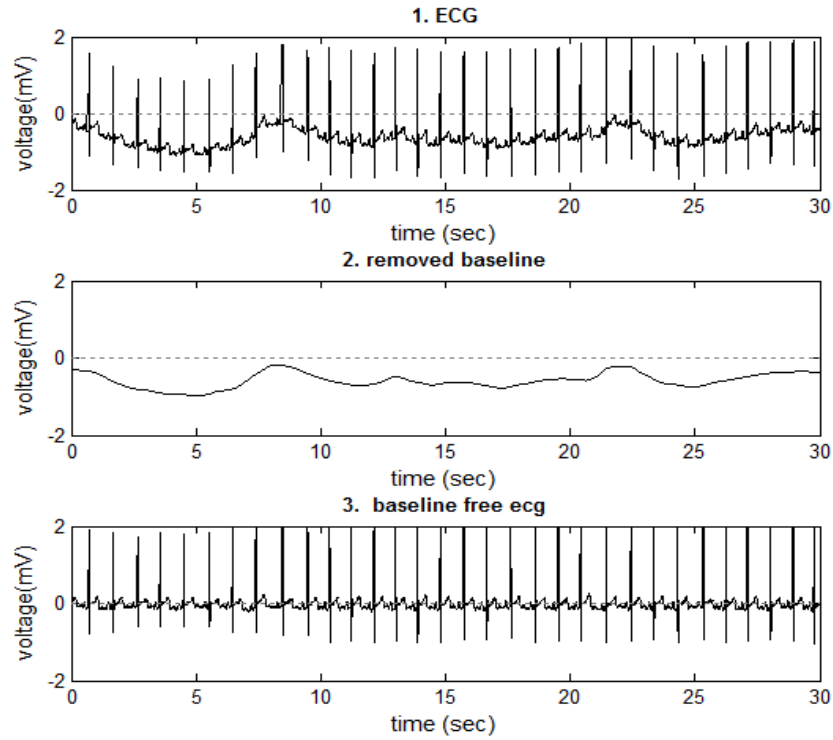


Figure 5.20 ECG_1 MITBIH rec.115.2:00-2:30

5.4 Comparison with other methods

To find effectiveness of our proposed method we compare it with different methods. They are chosen on the ground that they are very common one for this kind of problem. The chosen methods are:

- moving average filter (smooth 1),
- moving average filter 2 (smooth 2),
- median method
- low pass filter

5.4.1 Moving average filter (Smooth 1)

A moving average filter is a very simple FIR filter. The filter coefficients are found via the following equation:

$$b_i = \frac{1}{N+1} \quad i = 0, 1, \dots, N$$

To provide a more specific example, we select the filter order:

$$N = 2$$

The impulse response of the resulting filter is:

$$h[n] = \frac{1}{3}\delta[n] + \frac{1}{3}\delta[n - 1] + \frac{1}{3}\delta[n - 2]$$

The following figure shows the block diagram of such a 2nd-order moving-average filter.

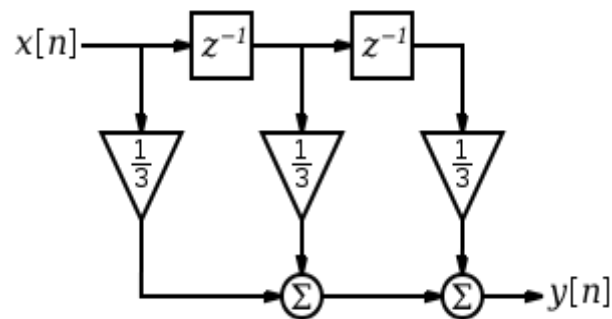


Figure 5.21 FIR Filter (Moving Average)

5.4.2 Moving average filter 2 (Smooth 2)

$Y[n]$ obtain form the above method is again passes though this moving average filter give the desire function smooth2.

5.4.3 Median method

The median of a finite list of numbers can be found by arranging all the observations from lowest value to highest value and picking the middle one. If there is an even number of observations, then there is no single middle value; the median is then defined to be the mean of the two middle values.

5.4.4 Low pass filter

A low-pass filter is a filter that passes low-frequency signals but attenuates (reduces the amplitude of) signals with frequencies higher than the cutoff frequency. The actual amount of attenuation for each frequency varies from filter to filter. It is sometimes called a high-cut filter, or treble cut filter when used in audio applications. A low-pass filter is the opposite of a high-pass filter.

5.5 Baseline estimation with different methods

For the baseline estimation with different methods we took ECG which is deliberately contaminated with simulated baseline (as mention is 5.3) and implement the different method of baseline estimation (Mention above) in MATLAB. The results are shown in the form of the figure. They are arrange as follow

1. The fist figure is the results of the estimation of baseline B_0 with different method second one is for B_1 and then B_2 and so on.
2. In the entire figures first we show the artificial baseline, and then result from smooth1, smooth2, low pass filter, median method, and last one is our proposed wavelet packet method.

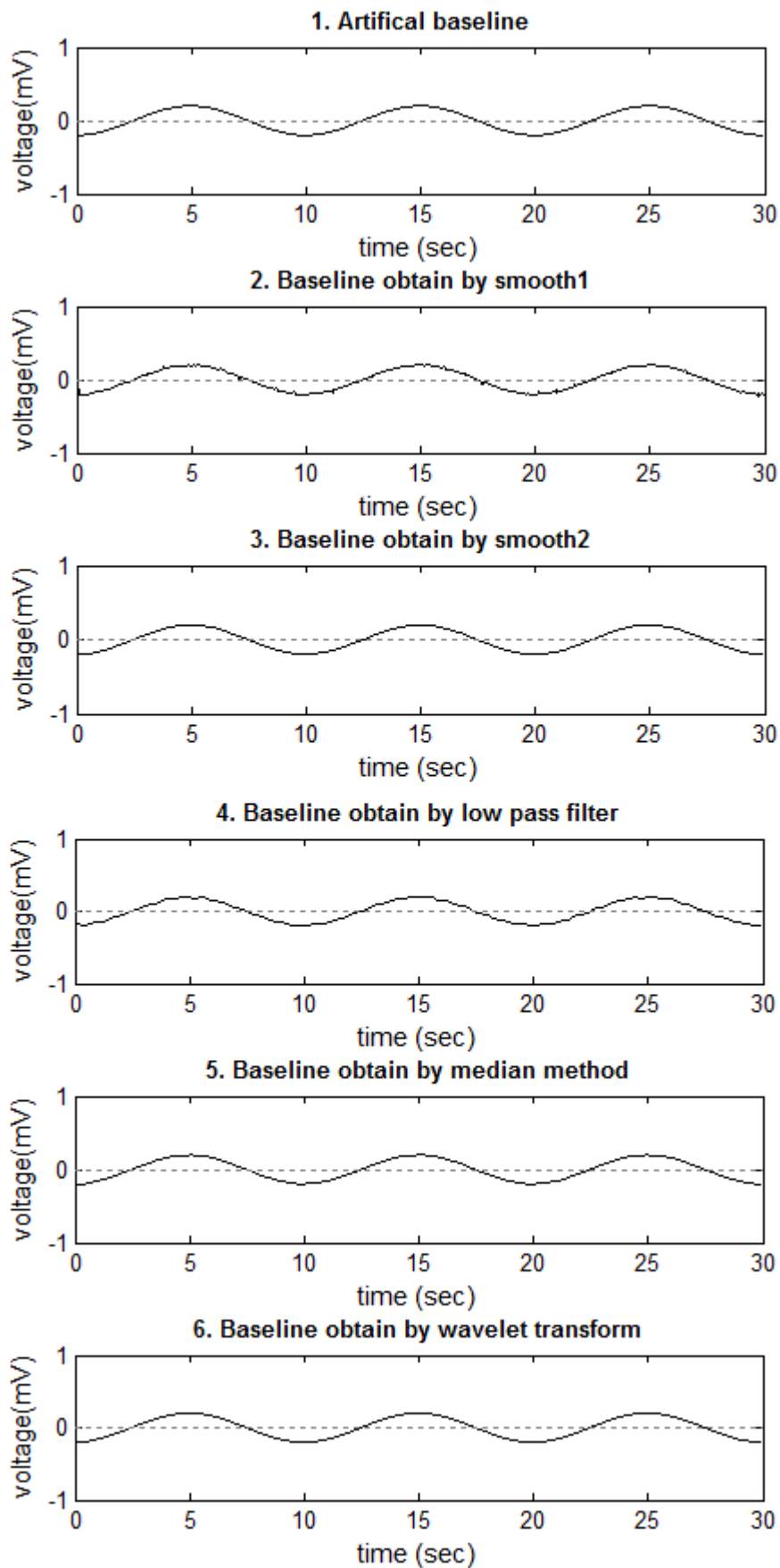


Figure 5.22 Baseline wander (B0) estimated by different methods

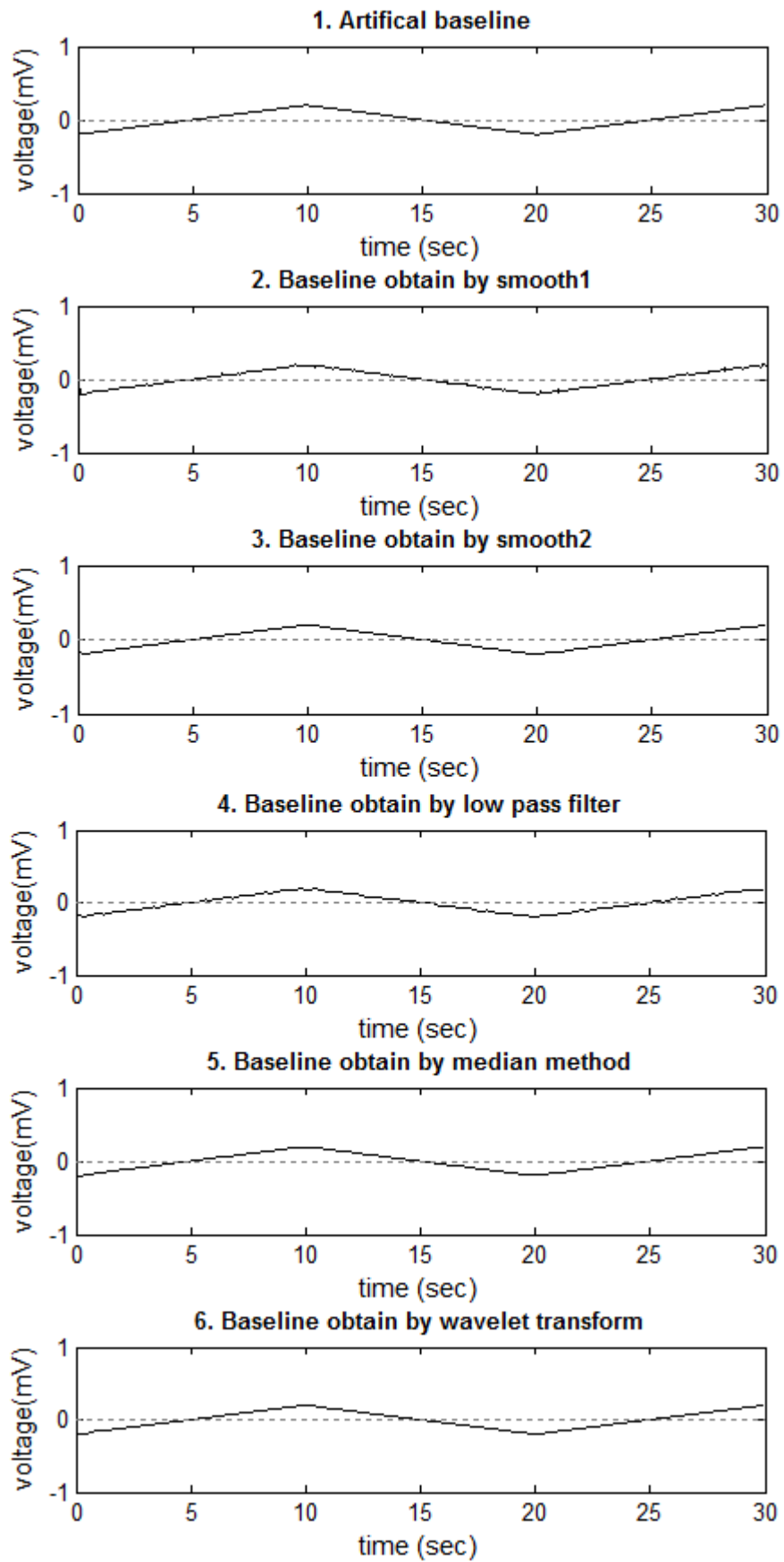


Figure 5.23 Baseline wander (B1) estimated by different Methods

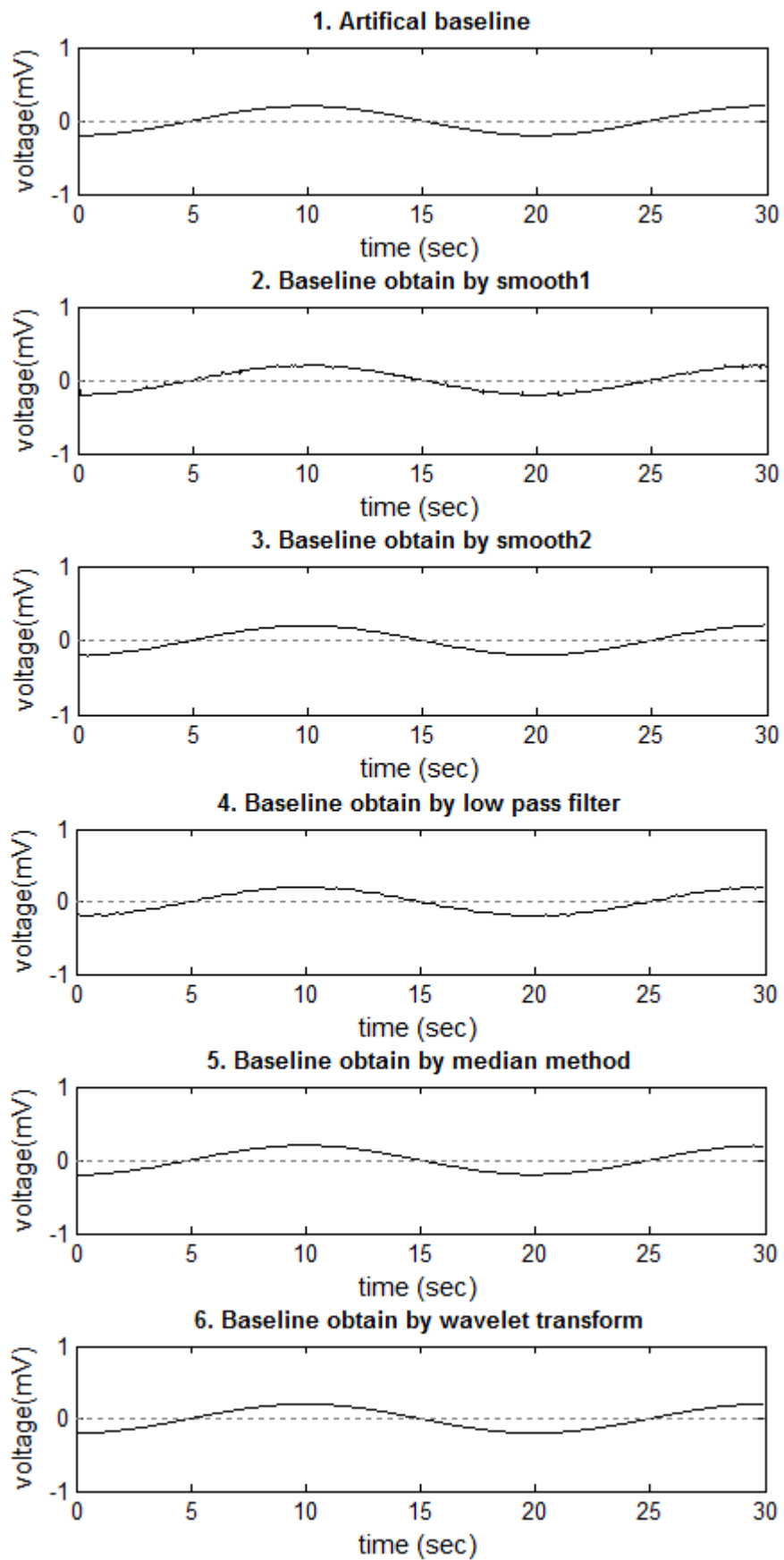


Figure 5.24 Baseline wander (B2) estimated by different methods

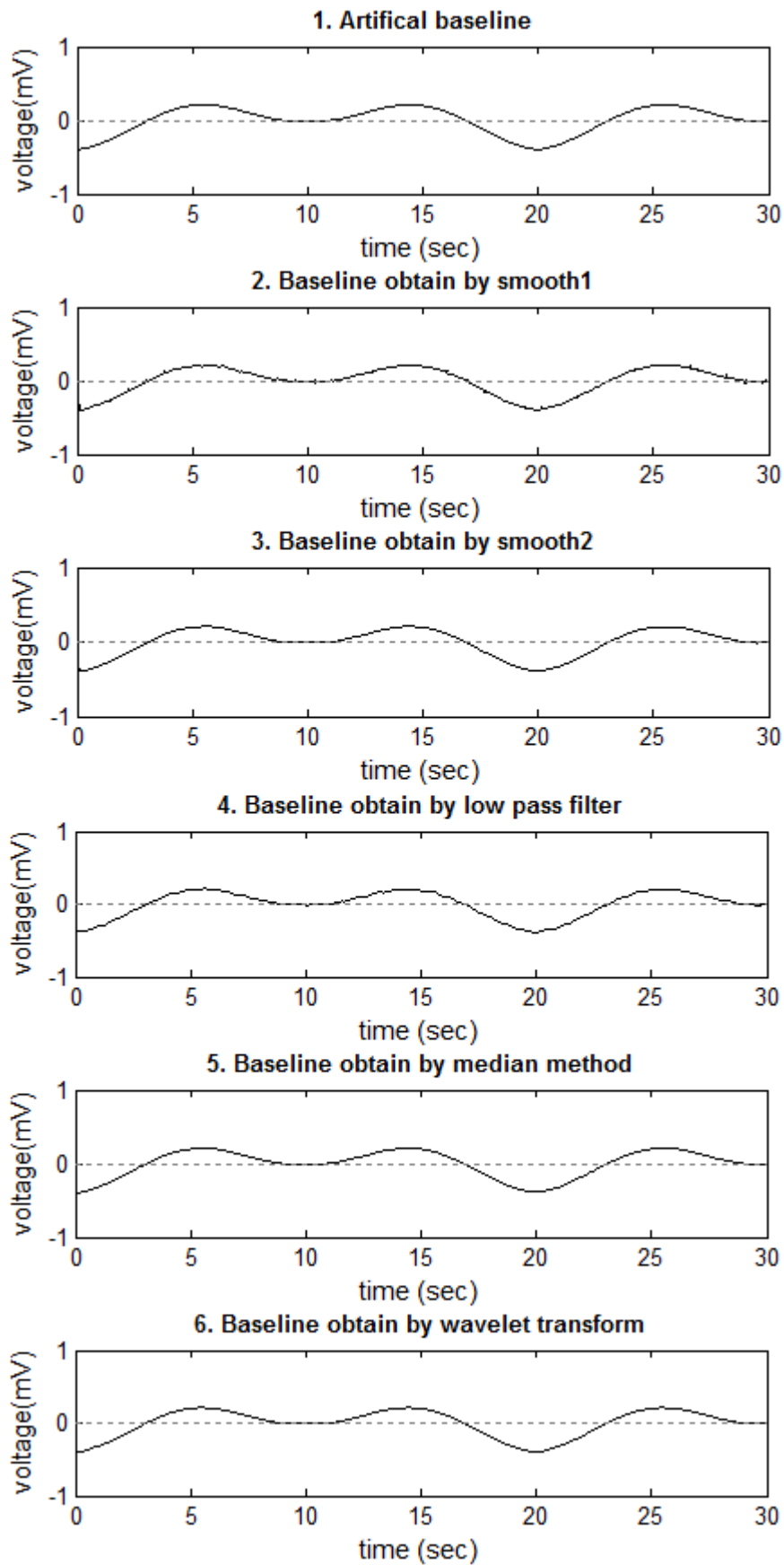


Figure 5.25 Baseline wander (B3) estimated by different methods

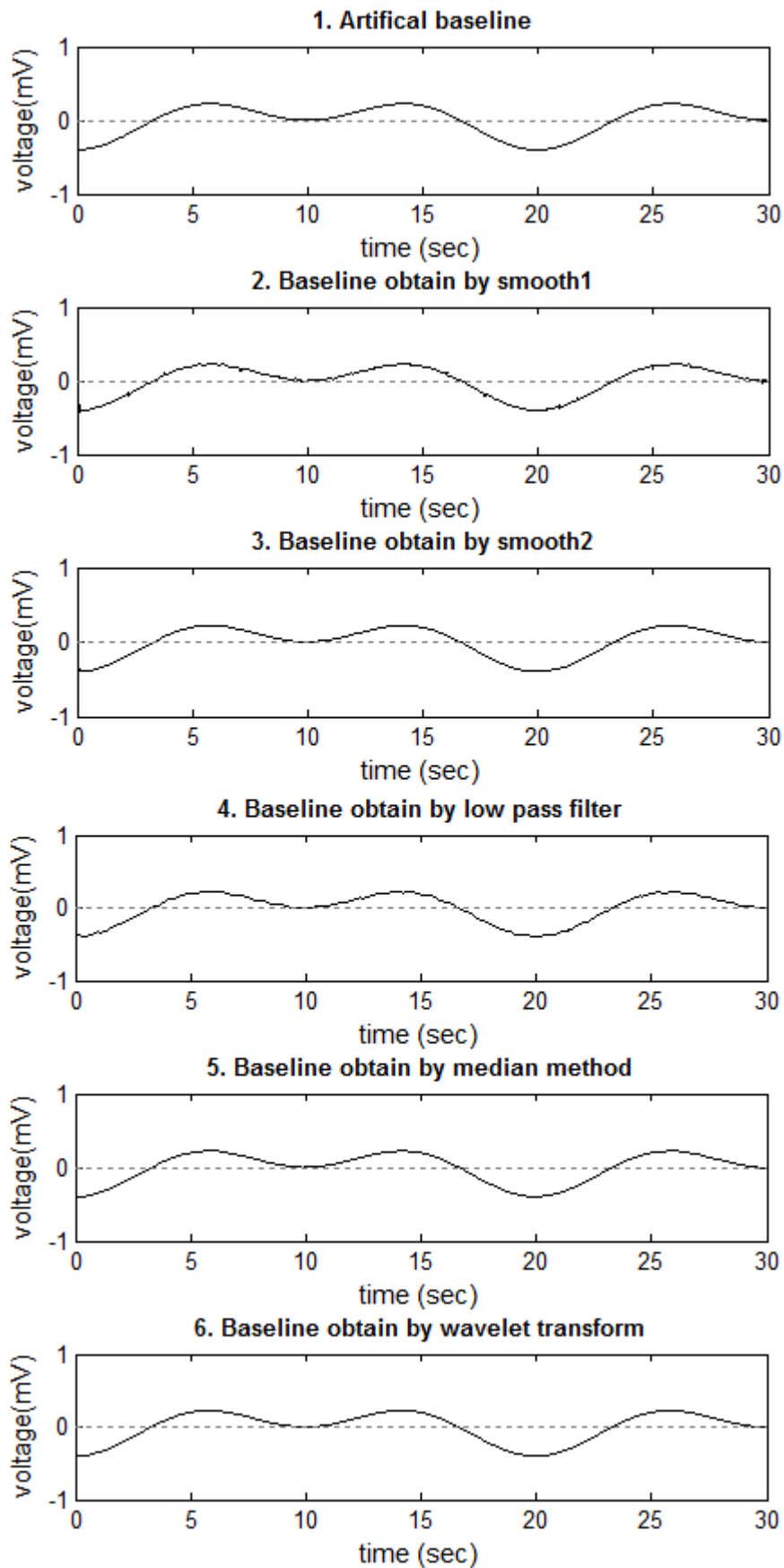


Figure 5.26 Baseline Wander (B4) estimated by different methods

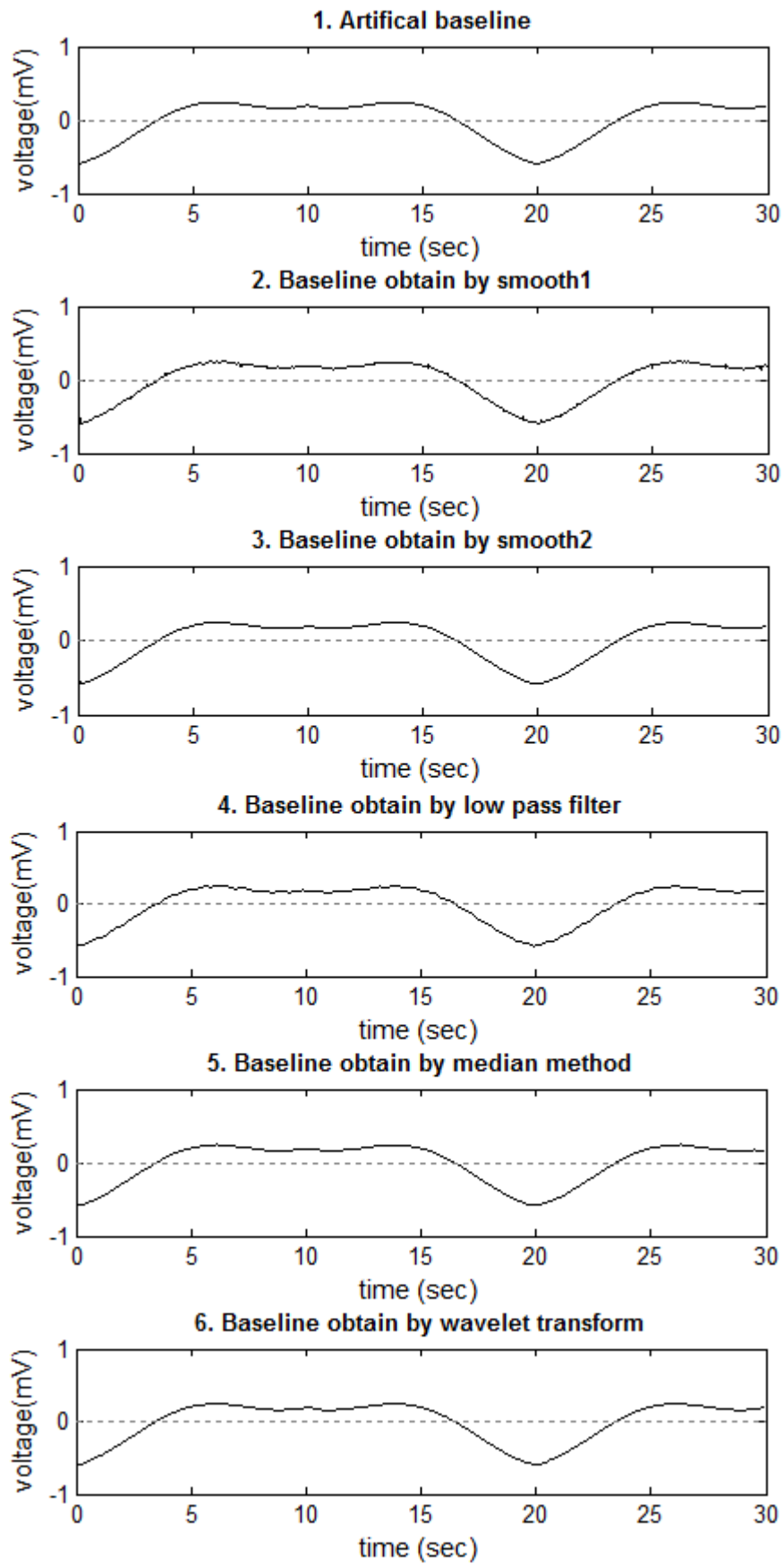


Figure 5.27 Baseline wander (B5) estimated by different methods

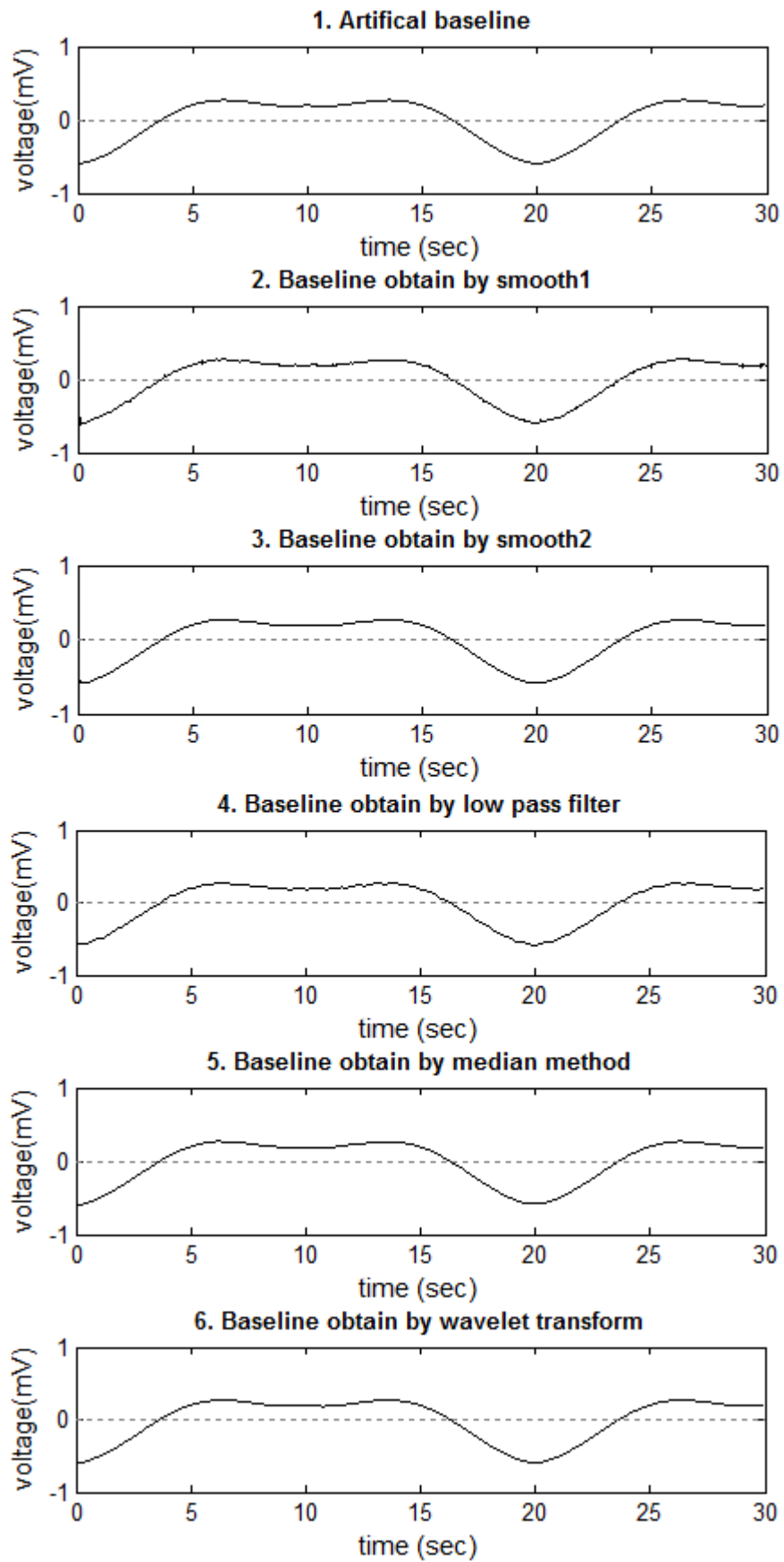


Figure 5.28 Baseline wander (B6) estimated by different methods

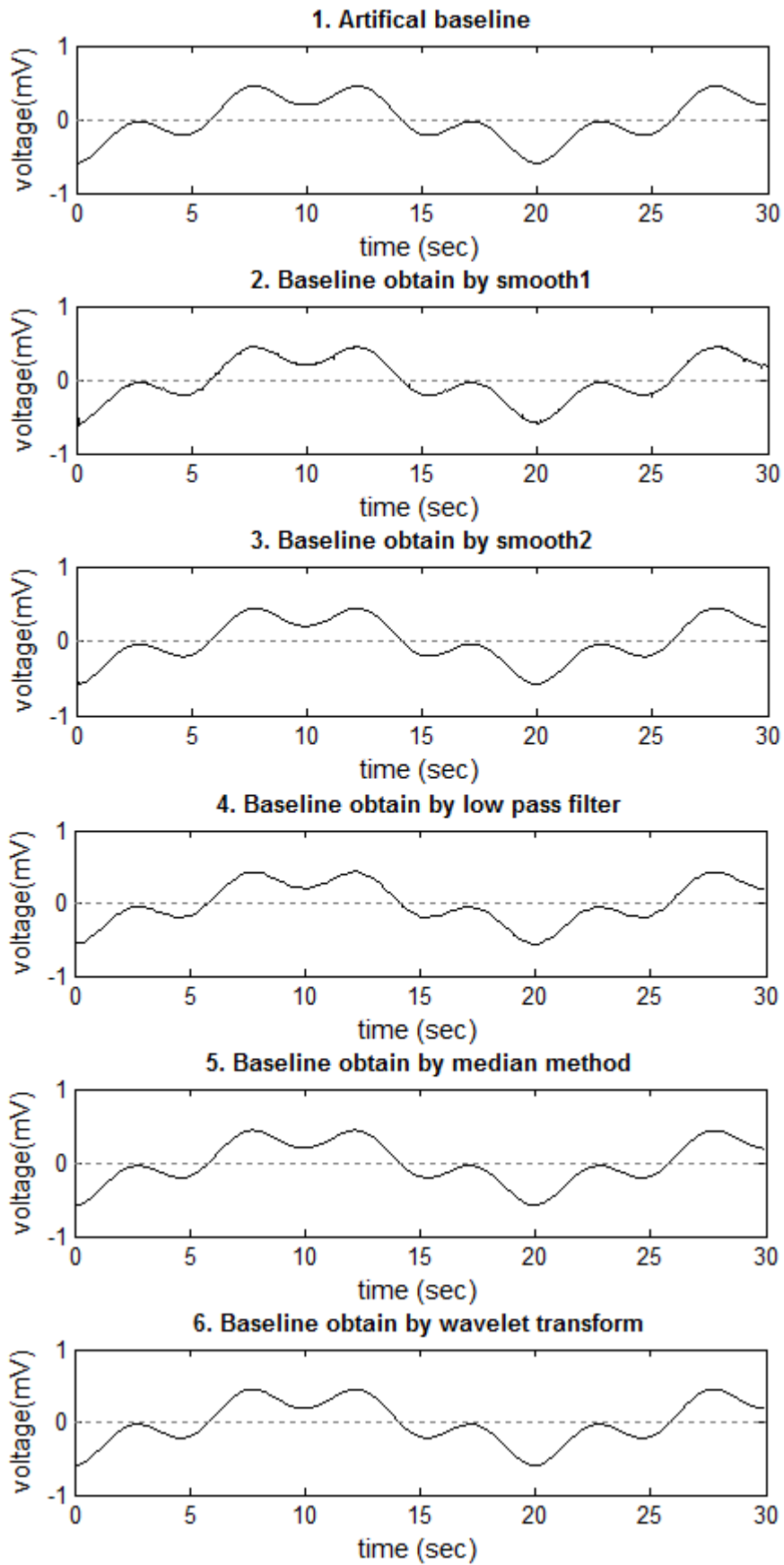


Figure 5.29 Baseline wander (B7) estimated by different methods

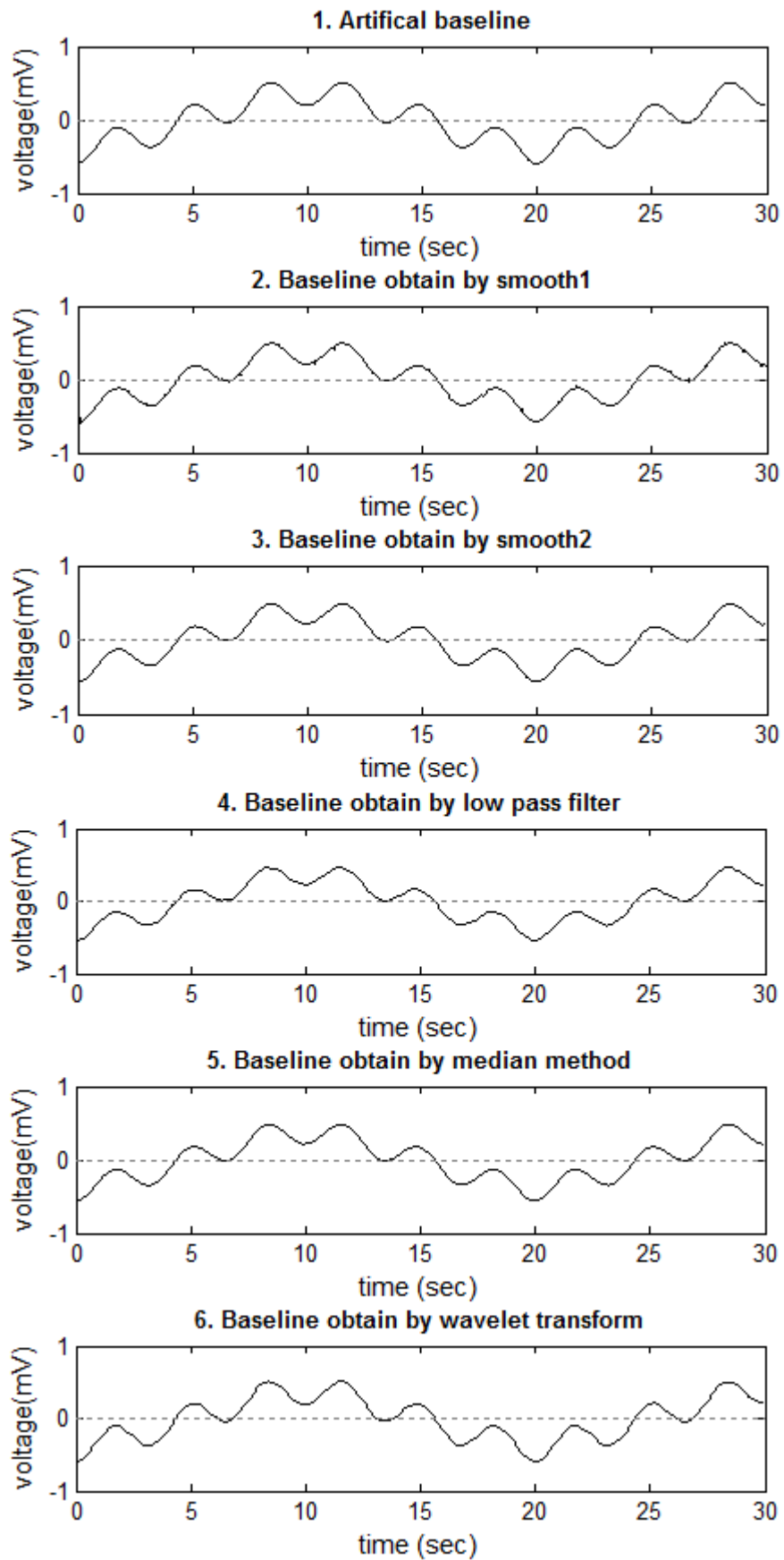


Figure 5.30 Baseline wander (B8) estimated by different methods

CONCLUSION AND SCOPE FOR FUTURE WORK

6.1 Conclusion

Wavelets with their “variable time frequency resolution” and properties such as MRA and high number of vanishing moments provide an effective way to analyze a signal. The process of signal filtering can be performed in quick time following this approach. This work has explored the feasibility of applying the wavelet transform to the estimation of the baseline wander in the ECG. We got good result when tested for the data from the MIT-BIH arrhythmia database. All ECG data used here are sampled at 360 Hz, and the resolution of each sample is 11 bits/sample. For performance analysis we use different baseline estimation method for the purpose of comparison. For this we construct artificial baseline which are added to a standard ECG which is free from the baseline wander, then implemented them on MATLAB and perform the baseline estimation. For the performance analysis, we have used the Percent Root Mean Square Difference (PRD) [47]. PRD calculated as

$$PRD = \sqrt{\sum_{i=1}^{i=N} (x_i - \hat{x}_i)^2 / \sum_{i=1}^{i=N} (x_i - \bar{x}_i)^2} \times 100 \quad \dots(6.1)$$

where x_i is the original standard signal and \hat{x}_i is the reconstructed signal \bar{x}_i is the mean signal.

The results are presented in the tabulated form in table 6.1. From the table we can conclude that it outperform the other methods. For low frequency its result are awesome. But as the frequency of baseline wander increase the effectiveness of the proposed method decreased but it is still much better than the other methods.

Table 6.1 performance analysis of different method by PRD(%)

Artificial baseline	low pass filter	smooth1	smooth2	median	wavelet transform
B0	4.9555	3.9191	2.6254	1.9856	0.5709
B1	4.3283	3.8730	2.3953	1.9219	0.8753
B2	4.1769	3.8452	2.3184	1.70237	0.5832
B3	5.1251	3.9478	2.7109	2.1913	0.8452
B4	5.0236	3.9206	2.6432	2.0020	0.5625
B5	5.5242	4.0388	2.9409	2.7306	1.2614
B6	5.2631	3.9564	2.7512	2.2708	0.8388
B7	11.0222	5.1374	5.691	5.8562	1.4188
B8	20.248	8.5746	11.5992	12.6239	5.1964
Avg.	7.2963	4.5792	3.9639	3.6983	1.3502

6.2 Future scope

The present work is good for the baseline wander with low frequency; lots of work can be done for to make wavelet transform method more effective for low as well as for high frequency noise (BW). The present wavelet used in our work is a D4 wavelet, other wavelet can be try to get better result. This process, as shown in our research, can be used to filter out noise from signals like ECG signal etc. and the algorithm can easily be implemented in C language resulting in direct practical implementations. This process can also be used for ECG feature extraction with suitable modification.

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