DYNAMIC ANALYSIS INCLUDING STABILITY OF NARROW FLEXIBLY SUPPORTED JOURNAL BEARINGS WITH MICROPOLAR LUBRICANT

A

Thesis Report

Submitted in partial fulfillment of the requirement for the award of degree

MASTER OF ENGINEERING

IN

CAD/CAM & ROBOTICS

Submitted By

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CERTIFICATE

This is to certify that the work which is being presented in this thesis report entitled ***"Dynamic analysis including stability of narrow flexibly supported journal bearings with micropolar lubricant"*** in partial fulfilment of requirement for the award of the Master Degree in CAD/CAM and Robotics, submitted in the Mechanical Engineering Department, Thapar University, Patiala, is an authentic record of the initial work carried out by me under the guidance of **Dr. A.K.CHATTOPADHYAY,** Professor & Head of Department, Mechanical Engineering Department, Mody Institute of Technology and Science, Lakshmangarh, Rajasthan.

The matter embodied in this report has not been submitted in part or full to any other university or institute for the award of any degree.

DATED: 9th July 2010

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This is to certify that above declaration made by the student concerned is correct to the best of my knowledge & belief.

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ABSTRACT

The aim of the present thesis is to investigate and predict theoretically the static and dynamic characteristics including the stability of flexibly supported narrow hydrodynamic journal bearings lubricated under the micropolar lubricants.

The theoretical study has been presented using the linear analysis based on the perturbation method.

Parametric studies have been conducted and stability characteristics have been obtained and comparison of the results of Newtonian and Micropolar fluid using linear analysis for different parameters have been compared and presented. A modified Reynolds equation has been obtained using micropolar lubrication theory and the solution of pressure obtained from this equation is used in the governing differential equations of motion to arrive at a stability threshold.

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NOTATIONS

C	Radial clearance (m)
C_B	Body couple per unit mass of micropolar fluid (m^2/s^2)
D	Journal diameter (m)
D_{ij}	Damping co-efficients of micropolar fliud film, for i=R,φ and j=R,φ (Ns/m)
$\overline{D}_{ij}$	Dimensionless damping co-efficient of micropolar fluid film
F_B	Body force per unit mass of micropolar fluid (m/s^2)
F_i	Force component along the R and φ direction
h	Local film thickness (m)
$\overline{h}$	Non-dimensional film thickness =h/C
l_m	Non-dimensional characteristic length of the micropolar fluid=Λ/C
M	Mass parameter (kg)
$\overline{m}$	Non-dimensional parameter $=MC\omega^2/W_0$
$\overline{M}_C$	Critical value of non-dimensional parameter
d	Damping co-efficient of the support
$\overline{d}$	Non-Dimensional damping co-efficient of the support
N	Coupling number= $[\chi/(2\mu+\chi)]^{1/2}$
P	Micropolar film pressure in the film region in the non-linear analysis (Pa)
$\overline{p}$	Non-dimensional micropolar film pressure in the film region in the non-linear analysis.
p_i	Local micropolar film pressure in the film region , i=0,1 and 2 for the steady State & first order perturbed film pressure along the ε_0 and φ directions (Pa)
$\overline{p}_i$	Non-dimensional local micropolar film pressure in the film region , i=0,1 and 2 for the steady state and first Order perturbed film pressure along the ε_0 and φ directions (Pa)
S_{ij}	Stiffness of the micropolar fluid film for i=R,φ and j=R, φ (N/m)
$\overline{S}_{ij}$	Dimensionless damping co-efficient of the micropolar fluid film = $2C^3S_{ij}/(\mu\omega R^3 L)$ For i=R , φ and j=R , φ
t	Time (s)
U	Velocity of journal=ωR (m/s)

$\vec{v}$	Microrotational velocity vector
$\vec{V}$	Velocity vector (m/s)
W_0	Steady state load in the bearing (N)
$\overline{W_0}$	Non-dimensional steady state load in the bearing $= 2C^2W_0/(\mu\omega R^3 L)$
x	Cartesian coordinate axis along the circumferential direction= $R\theta$ (m)
z	Cartesian coordinate axis along the bearing axis (m)
$\bar{z}$	Non-dimensional Cartesian coordinate axis along the bearing axis $= 2z/L$
α, β, γ	Viscosity co-efficient of the micropolar fluid (kg m/s)
ε_i	Perturbed eccentricity for i=R and φ
ε_0	Steady state eccentricity ratio
K	Stiffness co-efficient of the support
$\overline{K}$	Non dimensional stiffness co-efficient of the support
θ	Circumferential coordinate (rad)
λ_R	Whirl ratio
Λ	Characteristic length of the micropolar fluid= $[\gamma/(4\mu)]^{1/2}$ (m)
μ, λ	Newtonian viscocity co-efficient (Pa s)
μ_V	Effective viscosity coefficient of the micropolar fluid $= (2\mu + \chi)/2$ (Pa s)
π^*	Thermodynamic pressure
ρ	Mass density (kg/m^3)
τ	Non-dimensional time $= \omega t$
Φ	Attitude angle (rad)
Φ_0	Steady state attitude angle (rad)
χ	Spin viscosity coefficients of the micropolar fluid (Pa s)
ω	Angular velocity of the journal (rad/s)
ω_p	Angular velocity of the orbital motion of the journal centre (rad/s)

ABBREVIATION USED

SRR — Stiffness in r-direction due to disturbance in r-direction — S_{rr}

SPP — Stiffness in φ-direction due to disturbance in φ-direction — $S_{\varphi\varphi}$

SRP — Stiffness in φ direction due to disturbance in r-direction — $S_{r\varphi}$

SPR — Stiffness in r-direction due to disturbance in φ-direction — $S_{\varphi r}$

DRR — Damping co-efficient in r-direction due to disturbance in r-direction — D_{rr}

DPP — Damping co-efficient in φ-direction due to disturbance in φ-direction — $D_{\varphi\varphi}$

DRP — Damping co-efficient in φ-direction due to disturbance in r-direction — $D_{r\varphi}$

DPR — Damping co-efficient in r-direction due to disturbance in φ-direction — $D_{\varphi r}$

KBAR--- $\overline{K}$

DBAR--- $\overline{d}$

INTRODUCTION

*A **bearing** is a mechanical element which locates two machine parts relative to each other and permits a relative motion between them.*

The use of bearings is in vogue for centuries, but the rapid development of science and technology has reflected on the innovation design and development of bearings with high precisions and optimum performance ever in adverse working environment. Many of the challenging applications of bearings now-a-days can be found in space technology, electronics, cryogenics, nuclear engineering, computer science and technology. Most of the bearings in use are lubricated by liquid and gases though solid lubricants viz. graphite, molybdenum disulphide etc. are also used in some cases e.g. in extrusion process in hot and cold working.

However, all bearings used today are categorized into two main classes depending on their functions:

1. Rolling contact bearings and
2. Sliding contact bearings.

The rolling contact bearings use rolling elements at the points of contact of the surfaces in relative motion and thus reduce the friction. They consist of different ball bearings and roller bearings.

The sliding friction bearings are based upon two concepts of lubrication:

1. Sliding friction thin film lubrication or boundary film lubrication and
2. Sliding friction thick film lubrication.

The sliding friction thick film lubrication is further categorized into two different groups, namely:

1. Self-acting or hydrodynamic lubrication, and
2. Externally pressurized or hydrostatic lubrication.

In hydrodynamic journal bearing, the load supporting high pressure fluid film is created due to the shape and relative motion between the two surfaces. The moving surface pulls the lubricant into a wedge-shaped zone, at a velocity sufficiently high to create the high pressure film necessary to separate the two surfaces against the load. Principle of hydrodynamic journal bearing is shown in fig. 1.

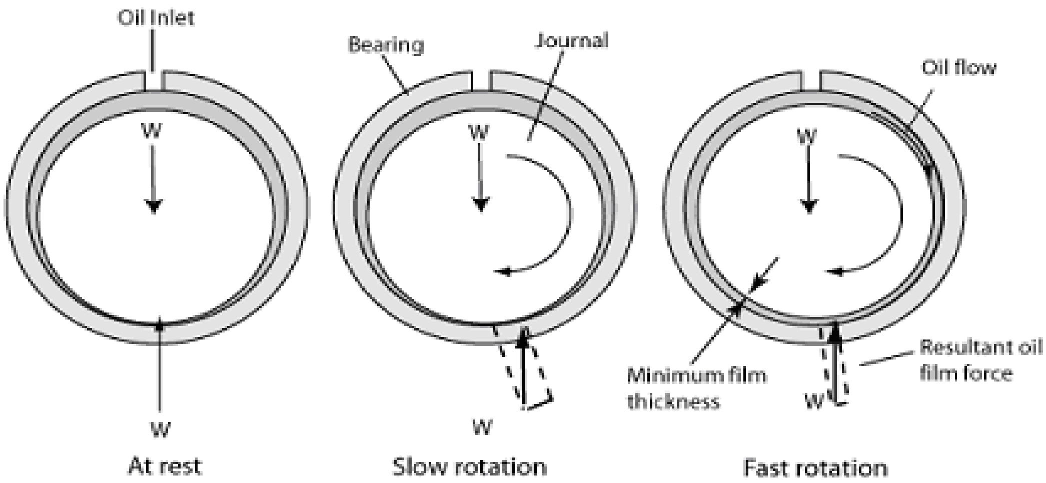

Fig.1 Principle of hydrodynamic journal bearing

Hydrodynamic bearings are generally used in cases when the relative velocity are high enough as a result of continuous increase in the sizes and speeds of the rotating machinery or due to use of fluids having kinematic viscosity, the oil film close in the bearings frequently becomes turbulent.

Again keeping in mind the extremely small radial clearance to facilitate case of alignment flexibly supported bearings have come into vogue. Therefore, it is important to do a study on stability of such flexibly supported journal bearing. Flexibly supported bearing is shown in fig. 2.

As the magnitude of hydrodynamic pressure depends upon the relative tangential or normal velocity, sometimes the developed pressure may not be sufficient for obtaining the desired load carrying capacity, particularly at the low relative velocity and thereby demands external supply of pressurized lubricants. This type of lubrication is called externally pressurized or hydrostatic lubrication. These bearings are also known as hybrid bearings as their operational principle includes both self-acting and external pressurization.

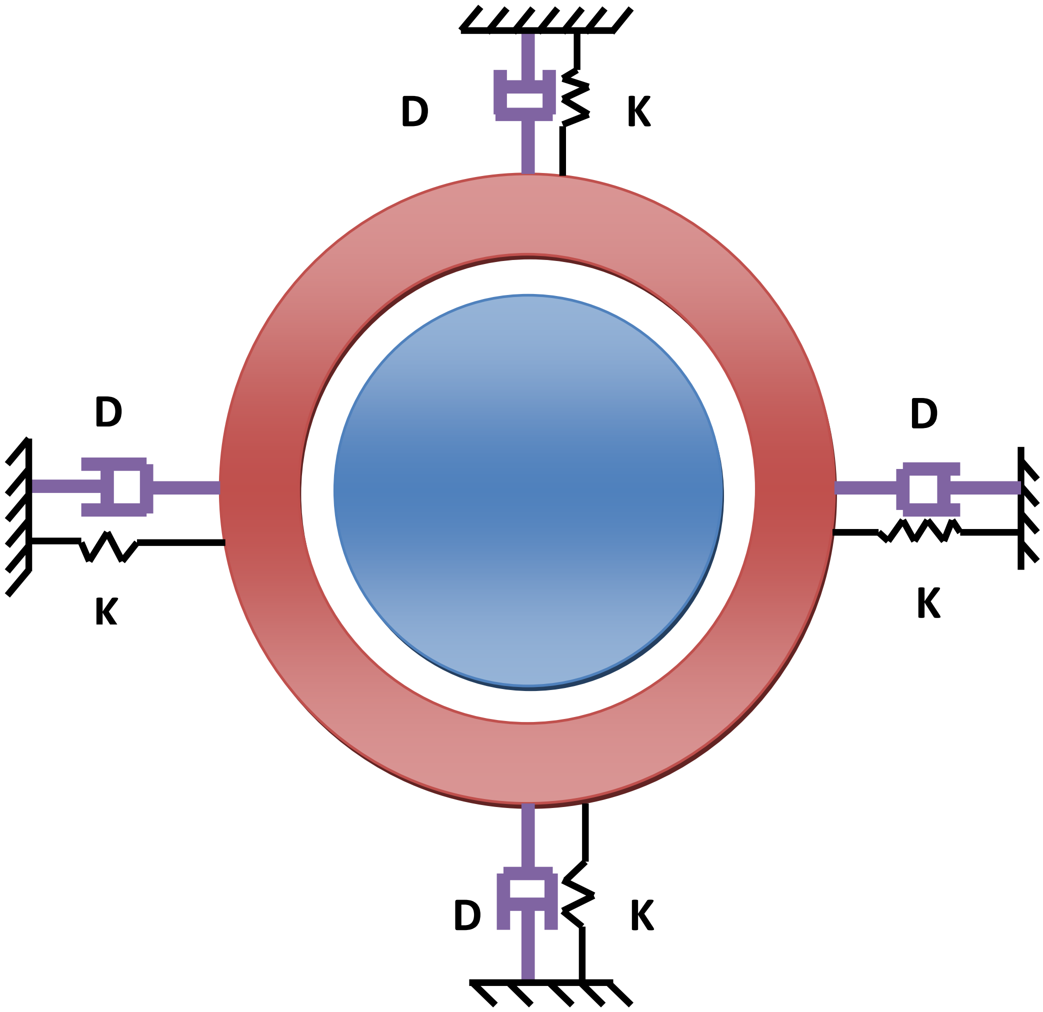

Fig.2 Schematic diagram of journal bearing on flexible support

However, it has been observed that the hydrodynamic journal bearings can show excellent performances in withstanding static load, but fail easily under dynamic conditions if not critically designed for the specific purpose.

Though dry lubrication like lubrication with graphite, molybdenum disulphide etc. are used in some cases, *e.g.* in extrusion processes in cold and hot working, most of the above types of bearings are lubricated with fluids *i.e.* with liquids and gases. Under the general industrial operating conditions such Newtonian lubricating fluids become contaminated with microstructures like dust and worn out metal particles. As the classical continuum theory becomes erroneous in the analysis of such lubrication, micro continuum approach seems to be more appropriate.

Based upon the microcontinuum approach, the first consistent theory of fluid microcontinua was presented by **Eringen [40]**. The microfluid was considered to contain microstructures having individual mass and velocity, in as much as the macrovolume element contains the microvolume elements and those microstructures can translate, rotate and deform independently of the motion of the macrovolume.

The theory included the mechanism to support the local stress moments and body moments and to consider the influence of the spin inertia of each microelement. But this theory of microfluid is too complicated and the mathematical model is not so acquiescent to the solution of the non-trivial problems. **Eringen**, therefore, again simplified the theory **[45]**, postulating a subclass of the microfluids, called micropolar fluids, considering the microrotational effects and ignoring the microdeformation effects of the substructures and permitting the surface and the body couples.

The theory included two independent kinematic vector fields representing the translation velocities and the angular or spin velocities of the microstructures *i.e.* translation velocity vector and microrotational velocity vector. Although contemporary theories of fluid microcontinua were also presented by various authors, the basic field equations, however, remained similar to those of **Eringen [40].** Physically the fluid consisting of bar like elements may be represented as micropolar fluid.

The theory of micropolar fluid, thus, may be a satisfactory model for analyzing the flow behaviours of real fluid suspensions, polymeric fluids and fluid consisting of polymeric additives as is mixed with the engine oil for enhancing lubricating effectiveness, certain anisotropic fluids, *e.g.* liquid crystals consisting of dumbbell shaped molecules and possibly animal blood.

Hence the theory of micropolar fluid has found its application as a refined model for studying the behaviour of such fluid flows as in the field of lubrication, where the clearance in a bearing is comparable to the average grain size or molecular size of a non-Newtonian fluid *viz.* a polymeric fluid or in the practical usage where the lubricants under general operating conditions become fluid suspensions after being contaminated with dirt and worn out metal particles. For an important practical example, in the area of nuclear power plant, where sodium, which is used in a dual role of a heat transfer agent and a lubricant in the journal bearing supporting the mixing screw in the batch mixers exhibits the Non-Newtonian micropolar characteristics.

ABOUT THE THESIS

The aim of the present thesis is to investigate and predict theoretically the static and dynamic characteristics including the stability of flexibly supported narrow hydrodynamic journal bearings lubricated under the micropolar lubricants.

From the basic equations *i.e.* the equation of continuity of mass, the equations for the balance of linear momentum and the balance of angular momentum, which are coupled together due to the existence of two kinematic vector fields of the microstructures and characterizing the micropolar fluid, the derivation of Reynolds equation has been modified.

The theoretical prediction of hydrodynamic pressures in the bearing is obtained by the solution of modified Reynolds equation satisfying the appropriate boundary conditions. The steady state pressure profile is obtained easily by an analytical method. The steady state parameters like load carrying capacity and attitude angle can be easily obtained, once the pressure profile over the entire bearing surface has been found.

For dynamic pressure, using linearized analysis, first order perturbations of eccentricity ratio and attitude angle are used, and the resulting equations are solved by analytical method. Knowledge of the perturbed pressures enables one to obtain the dynamic characteristics in terms of the components of stiffness and damping coefficients. Stiffness and damping coefficients aside, the dynamic characteristics also include the threshold stability and the whirl ratio.

In addition to the linearized stability analysis have also been considered in the bearing system to get actual threshold of stability and the values obtained in parametric study in which we will varies different parameters and check the stability.

The dissertation has been divided into five chapters.

- A brief review of the relevant literature is included in Chapter 1.
- The basic governing equations have been derived in generalized form and are presented in Chapter 2 for ready reference.

A parametric study has been made for various characteristics of journal bearings lubricated with micropolar lubricants and are reported in the following chapters:

- Chapter 3: Steady state characteristic of hydrodynamic journal bearings under micropolar lubrication is analyzed in this chapter.
- Chapter 4: This chapter presents the dynamic characteristics in terms of the components of stiffness and damping coefficients and also the stability threshold of hydrodynamic journal bearings lubricated with micropolar lubricants.

Finally, important conclusions based on the work presented in the earlier chapters of this dissertation are summarized and are reported in Chapter 5.

A list of references and appendices are provided at the end of the dissertation.

CHAPTER 1: LITERATURE REVIEW

1.1 INTRODUCTION

Human interest in the constituent parts of tribology probably started with the aim of reducing their physical labour in moving heavy loads. Today's science of tribology took its first rolling with the invention of wheel.

Bearing and bearing lubrication play an important role in the design of machine components and has direct effect on machine performance characteristics like vibration, reliability etc.

The emphasis on this field started century back, with **NEWTON [1]** in 1668. He was derived the first formulation of a hypothesis regarding the force necessary to overcome the viscous resistance of a fluid. **PETROFF [2]** being the first person to consider viscous friction in journal bearing as a source of power loss in 1883.

TOWER, B. [3] [FIRST REPORT ON FRICTION EXPERIMENTS IN 1883**]**

Tower's experiments in 1883 and 1885, gave the new light to the development of the science of lubrication. BERUCHAMP TOWER'S experiments for the determination of suitable methods of lubricating railway axle bearings, which were partial bearings lubricated from an oil bath, were the first significant experimental studies and the developed pressure distribution in the bearing clearance under the loaded condition as measured through a number of pressure gauges at the midplane in the circumferential direction was found to exhibit a peak, which was several times higher than the mean pressure calculated on the basis of the projected area.

REYNOLDS, O. [4] [THEORY OF LUBRICATION AND ITS APPLICATION IN 1886**]**

Just after three years of **Tower**'s first experiments, **Reynolds** presented a theoretical study to justify the experimental results of **Tower.** The experiments of **Tower** and the analysis of **Reynolds** may be considered as the entrance to the concept of hydrodynamic lubrication and the field of **Tribology**.

Since the development of the concept of the hydrodynamic lubrication about 120 years ago with the experiments of **Tower** and theoretical study of **Reynolds** lot of theoretical, experimental and analytical studies were presented by a number of authors. A few of them considered as benchmark work are only briefly reported.

KINGSBURY, A. [5] [EXPERIMENTS WITH AN AIR LUBRICATED JOURNAL IN 1897]

Kingsbury built an experimental set up to study the behaviour of gas journal bearing. This concept of an electrical analogy gave a new dimension to the study in this field

SOMMERFELD, A. [6] [1904]

The first exact Solution of Reynolds equation was given by **Sommerfeld** for an infinitely long journal bearing using full film boundary condition along with the load and friction characteristics in terms of eccentricity ratio in 1904.

HARRISON, W. J. [7] [THE HYDRODYNAMICAL THEORY OF LUBRICATION WITH SPECIAL REFERENCE TO AIR AS A LUBRICANT IN 1913]

Harrison enriched the hydrodynamic lubrication through his experiments with special reference to air as lubricant. **Rayleigh [8]** first investigated the effect of lubricated film shape on the operating characteristics of a bearing using the calculus of vibrations. **Christopherson [9]** showed that southwell's relaxation method was very suitable for lubrication problems. By the use of this method **Cameron** and **Wood [10]** obtained the solution of finite bearings and presented bearing characteristics for two slenderness ratios using realistic boundary conditions. During the same time **Vogelpohl [11]** developed an approximate solution based on infinite series. **Ocvirk [12]** *presented the solution of the Reynolds equation using narrow bearing approximation in 1955.*

Theoretical research on the dynamic characteristics of impervious self-acting oil bearing was initiated by **Harrison [13]**, who showed that hydrodynamic equations for a light rigid rotor, supported by a pair of infinitely long bearings with full film, are satisfied by an orbital motion of the journal axis under a constant load.

However, the derivation of Reynolds equation is based upon the assumptions relating to cavitation, fluid inertia forces and non-linear flow effects. **Coles** and **Hughes [14]** initiated investigation regarding cavitation in the lubricating film. Their experiments showed that the end of pressure curve was coinciding with the portion of zero pressure derivation. However **Jacobson** and **Floberg [15]**, indicated that the pressure cavitated somewhere beyond that point. The recent studies of film rupture and reformation for lightly loaded bearing contacts have led to modification in Reynolds boundary condition **Kahlert [16],** and **Osterle** and **Saibel [17]** pioneered research in the area of consecutive inertia forces.

G.M.RENTZEPIS AND **B.STERNLICHT [18]** [ON THE STABILITY OF ROTORS IN CYLINDRICAL JOURNAL BEARINGS IN 1961]

As high speed is essential to generate fluid film by self action, hydrodynamic bearing have found applications mostly in high speed machinery such as turbines, turbo generations, turbo pumps etc. **Rentzepis** and **Sternlicht** studied the stability of rotors in cylindrical journal bearings. They analytically determined the regions of stability for plain cylindrical journal bearings. The linear “variational” equation of motion was employed to obtain the stability regions bounded by families of load carrying capacity and operating eccentricity curves. These results were applied to the quassi static equilibrium case for gas lubricated cylindrical journal bearings of L/D=2.0. They showed that there exists a minimum in the stability curves. There production was supported by experimental guidance. The results of this work seemed to bridge together observation on stability at very small clearances and large ones.

C.H.T.PAN AND **STERNLICHT [19]** [ON THE TRANSLATORY WHIRL MOTION OF A VERTICAL ROTOR IN PLAIN CYLINDRICAL GAS-DYNAMIC JOURNAL BEARINGS IN 1962]

C.H.T. Pan and **Sternlicht [19]** worked on the translatory motion of a vertical rotor in plain cylindrical gas dynamic journal bearings. They indicate that for the theoretical prediction of the dynamic characteristics of a rotor system, it is necessary to have an accurate knowledge of the bearing fluid film forces under dynamical conditions.

Sternlicht and **L.W.Winn [20]** considered the phenomenon of half frequency whirl instability. There analysis and experimental results showed good correlation. **J.S.Ausman [21]** used the linearised approximation to the time dependent Reynolds equation and solved simultaneously with the equation of motion of rigidly mounted rotor. He found the dimensionless bearing compressibility parameter, the bearing stability parameter, and a critical operating eccentricity ratio below which the bearing is unstable. Whirl frequency at the threshold point was computed to be equal to or slightly less than one half the rotor speed.

R.HOLMES [22] [THE ROLE OF OIL-FILM BEARINGS IN PROMOTING SHAFT INSTABILITY AND THE REMEDIAL EFFECT OF DAMPING IN 1980]

R. Holmes examined the oil whirl instability in the context of rigid and flexible rotor systems and described the use of external damping to militate against such instability. He established that for a flexible rotor bearing system the small amount of external damping at a position in the shaft span increased the instability onset speed and this was most pronounced when the eccentricity ratio for the bearings was high. He suggested the external point damping as a useful method for achieving stability control. In this sphere **Allaire, P. E. [23]** plotted graphs for the stability threshold and whirl ratio against sommerfield number and indicated the stable and unstable regions.

Extensive theoretical and experimental work has been carried out on the investigation of the stability of self acting gas journal bearings in which bearings are rigidly supported.

But due to high speed of present day machines, and keeping in mind the small radial clearance, it is important to make a study on the stability of flexible supported self acting oil journal bearings.

Flexible supports facilitate case of alignment. An application of such a system in the textile field was demonstrated at the National Engineering Laboratory. The work of **MARSH [24] KERR [25]** has shown that the stability characteristic of this system are affected by both the stiffness and damping of the bearing support. Marsh in his theoretical treatment finally used a numerical method to obtain the dynamic pressure field of the bearing in solving the equations of motion for the system.

D.A.Boefey[26] examine the stability characteristics of a bearing on a flexible damped support using a purely analytical approach. His analysis was based on the linearized solution to the time dependent Reynolds equation.

However, the experimental evidences since the early years of last century put forward the realization of the existence of a separate lubrication theory for fluid suspension and the effort for establishing a theory was intensified since mid of the last century. The early studies on such theories of fluid micro-continua have no concern with the research works of bearings. The theory was later simplified to an acceptable fluid film lubrication theory, called micropolar lubrication theory and in subsequent years this simplified micropolar lubrication theory was introduced in the analyses of bearings.

The present work has, therefore, been aimed at studying the lubrication effectiveness of hydrodynamic journal bearings using micropolar lubrication under the above mentioned operational situations. Hence, in what follow, an attempt has been made to present a brief review of the available literature in the following sequence:

(a) A review of the theory of micropolar fluid.

(b) A review of bearings with micropolar lubrication.

1.2 THEORY OF MICROPOLAR FLUID

The development of the theory of micropolar fluid was initiated with the development of the theory on micro-continuum approach. For the application of the classical continuum theory in the field of film lubrication, the most practical importance is whether the deep molecular orientation results in any rheological abnormalities in the vicinity of the solid surfaces and the significant differences in the behaviour of the molecules of the liquid in intimate contact with and adhering to the surfaces or to absorbed layers on the surface from the behaviour of the adjacent molecules of the fluid in the bulk are found obvious and the experimental evidences may be worth mentioning herein.

KINGSBURY, A. [27] [A NEW OIL TESTING MACHINE AND SOME OF ITS RESULTS IN 1903]

Kingsbury observed such rheological abnormalities that varied from a small enhancement of viscosity to rigidity in the adhering layer ranging from 10^{-3} mm to 10^{-7} mm and attributed this difference in the behaviour of the molecules to a possible intensified viscosity in that part of the fluid within the range of the attraction of the surface molecules of the metal.

In 1909 **Cosserat** and **Cosserat [28]** developed the theories of oriented media in which there existed three directors at each point. The work of **Jeffery [29]** was the first to study the effect of the behavior of the Newtonian fluid of infinite extent in presence of a single ellipsoidal particle and the result indicated the enhancement of the viscosity of such fluid suspension than its base fluid. **Hardy** and **Nottage [30]** concluded that to be beyond the range of surface influence, the liquid film must be between 0.00762 to 0.01015 mm. in thickness. **Needs [31]**, through a series of experiments of the boundary surface influences on the viscosity of thin lubricating films squeezed in between two optically plane parallel circular disks approaching each other with no tangential velocity and film thickness down to 0.000635 mm, found an increasing time lag for the plates in their time of approach than that predicted by classical Newtonian theory for the film thickness below 0.00127 mm, indicating the increase in the effective viscosity of the film in that zone. He found the evidences where the effective viscosity in the boundary film increased nearly five times than that in the bulk. **Buckley**'s **[32]** experiment showed the total loss of rigidity in clean liquid at a distance of 0.3 x 10-4 mm from a solid boundary. The unexpected behaviour of the fluid containing oblong molecules in a shear field was recognized by **Anzelius [33]** and this dependence of the stress and moment at each point of the fluid was expressed by him as a function of the usual rate of strain tensor and the orientation of the substructure. In an experiment of the Couette type flow performed on leuben oil containing additives of aluminum napthenate up to 2%, **Henniker [34]** found an extensive tenfold increase in the viscosity within 50000A of the surface. Later by squeeze film experiments of various fluids (*e.g.* solutions of pure fatty acids in pure hydrocarbon solvents) between two plates submerged in the fluid and by measuring the gap between the plates as a function of time **Fuks [35, 36]** found the existence of a residual film between the plates even after squeezing for a very long time, similar to those obtained by **Needs[37]** and also observed that the thickness of such residual film increased with the increase in fatty acid concentration, the chain length of the hydrocarbon in both solvent and fatty acid and the surface energy of the solid substrate**.** A few more experiments by **Hoyt** and **Fabula[38]** and by **Vogel** and **Patterson[39]** showed that the mixing of some extremely small amount of polymeric additives in the lubricating fluid reduced the skin friction near a rigid body than in such a fluid to an extent of 30-35%.

ERINGEN, A. C. [40] [SIMPLE MICROFLUIDS IN 1964]

Subsequent to the introduction of micro-elastic solids, **Eringen** in 1964 proposed a similar theory— the basic equations, jump conditions and constitutive equations of ***'simple microfluent'*** media, exhibiting similar micro-effects. These fluids, named as ***'simple microfluids'*** are considered to be a fluent medium and a generalization of the Stokesian fluids whose properties and behavior are affected by the local motions of the material particles contained in each of its volume element. Non-linear Stokesian fluids are considered to be a special class of simple micro-fluids. As these fluids possess local inertia, Eringen extended continuum theory to consider:

(a) Conservation of micro-inertia moments and

(b) Balance of first stress moments, and developed and discussed the concepts of inertial spin, body moments, gyration tensors, stress moments, wherein the stresses and stress moments were expressed as the functions of deformation rate tensor and various microdeformation rate tensors. A simple microfluid in its simplest form has as many as twenty two viscosity coefficients and the fluids in which gyrational effects are important, such as anistropic fluids, vortex fluids and fluid having surface tensions are guessed to fall in this group.

ERINGEN, A. C. [41, 42] [MECHANICS OF MICROMORPHIC MATERIALS AND THEORY OF MICROPOLAR CONTINUA IN 1965]

In 1965 **Eringen** presented the theory of micromorphic material. As even the linear theory was too complicated for engineering application, **Eringen [43]** simplified it with the consideration of a subclass of micromorphic materials, named as micropolar media. The study however presented the discussions on deformation and motion, laws of motion, constitutive equations, theory of micropolar media, micropolar viscoelasticity, micropolar materials with attenuating neighbourhood and the problem of micropolar channel flow, with the assumptions of linear constitutive equations, microisotropic medium and negligible heat conduction.

As an extension of the above study, the linear theory with basic equations, theory of micropolar elasticity, couple stress theory, non-negative internal energy and the Uniqueness theorem of micropolar elasticity were presented by **Eringen [44]** in the same year. Finally, the totality of the microcontinuum theory was achieved by **Eringen[45]**, when he presented the laws of motion, constitutive equations of microfluids, the definition of micropolar fluids— the theories of its thermodynamics and field equations and the study of such a fluid flow in a circular pipe.

1.3 BEARINGS WITH MICROPOLAR LUBRICATION

ERINGEN, A. C. [46] [THEORY OF MICROPOLAR FLUIDS IN 1966]

Based on the theory of micropolar lubrication the first application of the theory was presented by **Eringen** himself for the steady motion of micropolar fluids in a circular channel in which the profiles for the velocity, microrotational velocity, shear stress difference and the couple stress on the fluid surface adjacent to the wall were presented graphically. The velocity profile was found to lose its parabolic nature and was smaller than that of classical Navier-Stokes fluid. Though the shearing stress remained the same as that determined by classical theory, the surface shear was found to be reduced by an amount equivalent to the effect of the distributed couples aroused on the fluid surface in a thin layer adjacent to the surface thus, indicating the development of a boundary layer phenomenon not present in the Navier-Stokes theory.

BALARAM, M. [47] [MICROPOLAR SQUEEZE FILMS IN 1975]

Balaram presented an analysis of micropolar squeeze films between two rectangular plates of infinite length along with the expressions for the pressure, the load carrying capacity of the squeeze film and the relationship of film thickness with time. The results showed an improvement in load carrying capacity with the increase in the density of the macromolecular volume but a decrease in load capacity with an increase in substructure particle size.

PRAKASH, J. AND **CHRISTENSEN, H. [48]** [RHEOLOGICAL ANOMALIES IN THIN HYDRODYNAMIC FILMS — A MICROCONTINUUM VIEW IN 1976]

Prakash and **Christensen** developed the hydrodynamic theory for the two-dimensional micropolar lubrication of a rigid cylinder on a plane surface in order to make critical analysis of micropolar effects revealed in the results of **Fuks [35, 36].**

PRAKASH, J. AND **SINHA, P. [49]** [A STUDY OF SQUEEZING FLOW IN MICROPOLAR FLUID LUBRICATED JOURNAL BEARINGS IN 1976]

Prakash and **Sinha** also presented a theoretical analysis of squeeze films in both full and half journal bearings with a steady, laminar, incompressible flow of micropolar fluid between two eccentric cylinders in relative normal motion along with the expressions for

Reynolds equation, nondimensional pressure distribution, load capacity and response time. The results supported the experimental results **[31, 34]** of increase in the effective viscosity in narrow passages. In 1976, in another study, **Prakash** and **Sinha [50]** presented the Reynolds equation and the expressions for various bearing characteristics of squeeze films for the general case of dynamically loaded infinitely long journal bearing for steady incompressible two-dimensional micropolar fluid flow under a sinusoidal load with no journal rotation. The graphical representation of results exhibited decreasing velocity of approach with increasing micropolar effect.

ZAHEERUDDIN, KH. AND **ISA, MD. [51] [**MICROPOLAR FLUID LUBRICATION OF ONE-DIMENSIONAL JOURNAL BEARINGS IN 1978]

An analysis providing the expressions for different hydrodynamic characteristics such as the load carrying capacity, the volume flow flux and the frictional force, of one dimensional journal bearings, both infinitely long and infinitely short, with micropolar lubrication, was presented by **Zaheeruddin** and **Isa.** The results showed that an increase in the equivalent viscosity of the micropolar fluid increased the load carrying capacity and the time of approach, but decreases the coefficient of friction.

ZAHEERUDDIN, KH. [52] [THE DYNAMIC BEHAVIOUR OF SQUEEZE FILMS IN ONE-DIMENSIONAL POROUS JOURNAL BEARINGS LUBRICATED BY A MICROPOLAR FLUID IN 1981]

Zaheeruddin is presented the another paper that the generalized Reynolds equation in dynamically loaded one-dimensional, both infinitely long and infinitely short types of porous journal bearings operating under a cyclic load and lubricated with micropolar fluids. The analysis was to show the effects of microstructures present in the lubricants, the permeability of the bearing material and the bearing wall thickness on the operating eccentricity ratio. The three-dimensional Reynolds equation was first presented by **Singh** and **Sinha [53]**, which the authors solved to yield the expressions for the velocity distribution, microrotational velocities flow flux and frictional torques.

HUANG, T-W. AND **WENG, C-I. [54]** [DYNAMIC CHARACTERISTICS OF FINITE-WIDTH JOURNAL BEARINGS WITH MICROPOLAR FLUIDS IN 1990]

Huang and **Weng** studied the dynamic characteristics of finite width journal bearing with micropolar lubrication using linear stability theory and graphically presented the stiffness, damping coefficients and the critical stability parameter of the journal bearings for width to diameter 30 ratio 1.0 and 10 under both Newtonian and micropolar lubrication. The micropolar fluids exhibited larger normal stiffness coefficient but smaller normal damping coefficient, and comparatively narrower stable region of the journal. The features of increasing effective viscosity and load capacity under micropolar lubrication resulted in lower limit values of the Sommerfeld number for completely stable operation. A long journal was found to exhibit significant difference in the values of critical mass parameter for the Newtonian and micropolar fluids, whereas the short bearing was found to be almost equally stable in both types of lubrication. The steady flow of a micropolar fluid between two rotating disks of finite radius rotating at different same speeds was investigated by **Chaturani** and **Narasimman [55]** and the equations of motion were reduced to a set of ordinary non-linear coupled differential equations, which were then linearized by quasilinearisation technique. The numerical solutions were obtained using fourth order Runga-Kutta method via orthonormalisation.

CHATTOPADHYAY. A.K., KARMAKAR, S., [56] [NON LINEAR ANALYSIS OF FLEXIBLY SUPPORTED FINITE TURBOLENT FLOW OIL JOURNAL BEARINGS IN 1997]

Chattopadhyay. A.K., Karmakar, S., a theoretical analysis is presented investigating the stability of a symmetrical, rigid rotor on a flexibly supported journal bearing. Oil-film flow in the bearing is assumed to be turbulent and the dynamics of the system is studied by calculating the components of the fluid film force by solving the generalized Reynolds equation modified to include the effect of turbulence and using these in the non-linear equation of motion of the journal and the bearing. The modified Reynolds equation is solved for pressure distribution by using finite difference method with S.O.R. scheme.

DAS, S., GUHA, S.K., AND CHATTOPADHYAY, A.K. [57] [THEORETICAL ANALYSIS OF STABILITY CHARACTERISTIC OF HYDRODYNAMIC JOURNAL BEARINGS LUBRICATED WITH MICROPOLAR FLUID IN 2004]

The aim of the present paper is to study theoretically, using finite difference techniques, the stability characteristics of hydrodynamic journal bearings lubricated with micropolar fluids. The theoretical study has been presented using both the linear analysis based on the perturbation method and non-linear analysis using Runge-Kutta method.

Parametric studies have been conducted and stability characteristics have been obtained and a comparison of the results of linear and non-linear analyses for different parameters have been compared and presented. A modified Reynolds equation has been obtained using micropolar lubrication theory and the solution of pressure obtained from this equation is used in the governing differential equations of motion to arrive at a stability threshold.

There are some recent researches related with hydrodynamic journal bearing under micropolar lubricant are done which is given below:

SURESH VERMA, VIJAY KUMAR, K. D. GUPTA [62] [ANALYSIS OF MULTIRECESS HYDROSTATIC JOURNAL BEARING OPERATING WITH MICROPOLAR LUBRICANT IN 2009 **]**

This paper presents a theoretical study of the performance characteristics of a constant flow valve compensated multirecess hydrostatic journal bearings operating with micropolar lubricant. The finite element method and iterative procedure have been used to solve the modified Reynolds equation governing the micropolar lubricant flow in the bearing. The performance characteristics are presented for a wide range of nondimensional load, lubricant flow, and micropolar parameters. It has been observed that the micropolar parameters significantly influence the performance characteristics of the bearing.

K. PRABHAKARAN NAIR, MOHAMMED SHABBIR AHMED, SAED THAMER AL-QAHTANI [63] [STATIC AND DYNAMIC ANALYSIS OF HYDRODYNAMIC JOURNAL BEARING OPERATING UNDER NANO LUBRICANTS IN 2009]

In this paper, the static and dynamic analysis of hydrodynamic journal bearing operating under nano lubricants is presented. The load carrying capacity of journal bearing mainly depends upon the viscosity of the lubricant. The addition of nano particles on commercially available lubricant may enhance the viscosity of lubricant and in turn changes the performance characteristics. In the proposed work, to obtain pressure distribution in the clearance space of the journal bearing, modified Raynolds equation is used. An iterative procedure is used to establish the film extent of the bearing. The modified Raynolds equation is solved by using the powerful numerical technique, finite element method. The static characteristics in terms of load capacity, attitude angle and end leakage and dynamic characteristics in terms of stiffness coefficients, damping coefficients, threshold speed and damped frequency of whirl are evaluated for different eccentricity ratios.

RAVINDRA R. NAVTHAR, DR. N.V. HALEGOWDA [64] [STABILITY ANALYSIS OF HYDRODYNAMIC JOURNAL BEARING USING STIFFNESS COEFFICIENTS IN 2010**]**

Journal bearings are widely applied in different rotating machineries. These bearings allow for transmission of large loads at mean speed of rotation. These bearings are susceptible to large amplitude lateral vibration due to self exited instability which is known as oil whirl or Synchronous whirl. This paper presents a method to determine the Synchronous whirl i.e. Stability of hydrodynamic Journal bearings by using dynamic characteristics such as stiffness coefficients. Analysis shows that Bearing operating at a speed of 800 rpm and 150N load remains stable up to a speed of 1666rpm. This is verified experimentally on journal bearing test rig by operating the bearing up to 1666 rpm and observing the pressure distribution plot .Different journal speeds and loads are considered for the analysis.

1.4 CONCLUDING REMARKS

From the above survey it seems that lot of study can still be done on flexibly supported journal /bearing performance with micropolar fluid as lubricant. Especially the dynamic performance of flexibly support bearings including stability analysis deserves special attention, as there is scant literature available on this.

It is the objective of this work to investigate both the steady state and dynamic characteristics including the stability of a flexibly support self acting journal bearing lubricated with micropolar fluid.

CHAPTER 2: BASIC GOVERNING EQUATION

2.1 INTRODUCTION

In this chapter, starting from the basic field equations and using certain assumptions, modified Reynolds equation has been derived and a brief discussion of the various non-dimensional parameters related to micropolar fluid has been made. The basic field equations of the micropolar fluid were developed by **Eringen [45]**, and later adopted by **Prakash** and **Sinha [49]** followed by the model of **Singh** and **Sinha [53]**.

2.2 FIELD EQUATIONS

The field equations for micropolar fluids in vectorial form are **[49]:**

Principle of conservation of mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \vec{V}\right) = 0 \qquad \text{... (2.1)}$$

Conservation of linear momentum:

$$(\lambda + 2\mu)\nabla\left(\nabla \cdot \vec{V}\right) - \frac{(2\mu + \chi)}{2}\nabla \times \nabla \times \vec{V} + \chi\nabla \times \vec{v} - \nabla \pi^* + \rho F_B = \rho \frac{D\vec{V}}{Dt} \qquad \text{… (2.2)}$$

Conservation of angular momentum:

$$(\alpha + \beta + \gamma)\nabla(\nabla \cdot \vec{v}) - \gamma\nabla \times \nabla \times \vec{v} + \chi\nabla \times \vec{V} - 2\chi\vec{v} + \rho C_B = \rho j \frac{D\nu}{Dt} \qquad \text{… (2.3)}$$

Where **ρ** is the mass density, $\vec{V}$ is the velocity vector and $\vec{v}$ is the microrotational velocity vector. π^* is the thermodynamic pressure and is to be replaced by the hydrodynamic film pressure***, p,*** since, $\pi^* = -\left[\frac{\partial E}{\partial \rho^{-1}}\right] = p$ where E is the internal energy **[45]** and p is to be determined by the boundary conditions. μ and λ are the familiar viscosity coefficients of the classical fluid mechanics, while $\boldsymbol{\alpha}$,$\boldsymbol{\beta}$ and$\boldsymbol{\gamma}$ are the new viscosity coefficients derived as the combinational effects of the gyroviscosities for micropolar fluid as defined by **Eringen[45]**. $\boldsymbol{\chi}$ is also a new viscosity coefficient for micropolar fluid, termed as *spin viscosity*, which establishes the link between the velocity vector$\vec{V}$, and the microrotational velocity vector$\vec{v}$. $\boldsymbol{F}_B$ is the body force per unit mass, $\boldsymbol{C}_B$ is the body couple (moment) per unit mass and j is the microinertia constant. λ, μ and $\boldsymbol{\chi}$ are all of dimension$\left[ML^{-1}T^{-1}\right]$, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are all of dimension $\left[ML\,T^{-1}\right]$and j is of dimension [L^2] **[49]**. If microrotation velocity and local fluid vorticity are considered to be identically equal and is imposed as a possible solution of the balance equations, which seems to be a natural solution and is desirable in reducing the number of constitutive coefficients, it can be shown that $j = \Lambda^2$**[49]**. $\frac{D}{Dt}$ indicates the material differentiation.

Constitutive equations for the stress tensor t_{kl} and the couple stress tensor m_{kl} are **[49]:**

$$t_{kl} = \left(-\pi^* + \lambda \vec{V}_{r,r}\right)\delta_{k1} + \left(\mu - \frac{1}{2}\chi\right)\left(\vec{V}_{k,1} + \vec{V}_{1,k}\right) + \chi\left(\vec{V}_{1,k} - \eta_{k1r}\vec{v}_r\right) \qquad \dots (2.4)$$

$$m_{k1} = \alpha \vec{v}_{r,r}\delta_{k1} + \beta v_{k,1} + \gamma \vec{v}_{1,k}$$

Where, η_{k1r} is an alternating tensor. An index followed by a comma represents partial differentiation with respect to the space variable x_k.

2.3 DERIVATION OF MODIFIED REYNOLDS EQUATION

The above micropolar effects are then incorporated into the lubrication problem for a journal bearing to derive the modified Reynolds equation with certain simplifications.

2.3.1 BASIC ASSUMPTIONS

The basic assumptions in micropolar lubrication to a journal bearing include the usual lubrication assumptions in deriving Reynolds equation and the assumptions to generalize the micropolar effects **[53]**:

1. The flow is incompressible and steady, *i.e.* $\boldsymbol{\rho}$ = constant and $\frac{\partial \rho}{\partial t} = 0$
2. The flow is laminar *i.e.* free of vortices and turbulences.
3. Body forces and body couples are negligible, *i.e.* $\boldsymbol{F}_B = 0$ and $\boldsymbol{C}_B = 0$.
4. The film is very thin in comparison to the length and the span of the bearing. Thus, the curvature effect of the fluid film may be ignored and the rotational velocities may be replaced by the translatory velocities.
5. No slip occurs at the bearing surfaces.
6. Bearing surfaces are smooth, non-porous and rigid *i.e.* no effects of surface roughness or porosity and the surface can withstand infinite pressure and stress theoretically without having any deformation.
7. No fluid flow exists across the fluid film, *i.e.* the lubrication characteristics are independent of *y*-direction.

8. The micropolar properties are also independent of *y*-direction. The velocity vector, the microrotational velocity vector and the fluid film pressure are given as:

$$\left.\begin{aligned} \vec{V} &= \left[\vec{V}_X(x,y,z), \vec{V}_y(x,y,z), \vec{V}_Z(x,y,z)\right] \\ \vec{v} &= \left[\vec{v}_1(x,y,z), \vec{v}_2(x,y,z), \vec{v}_3(x,y,z)\right] \\ p &= p(x,y,z) \end{aligned}\right\} \quad \text{... (2.5)}$$

Note that for $\boldsymbol{\alpha} = \boldsymbol{\beta} = \boldsymbol{\gamma} = \boldsymbol{\chi} = 0$ and for negligible body couple per unit mass equation (2.3) yields $\vec{v} = 0$ and so, equation (2.2) reduces to the classical Navier-Stokes equation. For $\boldsymbol{\chi} = 0$ the velocity vector and the microrotational velocity vector are uncoupled and the global motion of the fluid becomes free of the microrotation and their effects.

2.3.2 DIRECTIONAL FLOW EQUATIONS

With the above assumptions (2.1) reduces to

$$\left.\begin{aligned} &\nabla \cdot \vec{V} = 0 \\ \text{i.e.} \quad &\frac{\partial \vec{V}_X}{\partial x} + \frac{\partial \vec{V}_y}{\partial y} + \frac{\partial \vec{V}_Z}{\partial z} = 0 \end{aligned}\right\} \quad \text{… (2.6)}$$

It can also be considered that,

$$\nabla \vec{v} = 0 \quad \text{... (2.7)}$$

The explanation for equation (2.7) can be given following **Tipei [58]** as:

In case of a flow in thin layers bounded by solid surfaces, as in lubrication problem, the component of $\vec{v}$ perpendicular to the boundaries becomes negligible with respect to the other two components. Since the derivatives with respect to the normal *y*-direction are much larger than the other derivatives, *i.e.* $\frac{\partial}{\partial y} >> \frac{\partial}{\partial x} \, or \, \frac{\partial}{\partial z}$, hence $\nabla(\nabla.\vec{v}) = \frac{\partial^2 \vec{v}_1}{\partial x \partial y} + \frac{\partial^2 \vec{v}_3}{\partial z \partial y}$, while the other components are much smaller. The component of $(\nabla \times \vec{v})$ upon *y*-coordinate *i.e.* & $\left[\frac{\partial \vec{v}_1}{\partial z} - \frac{\partial \vec{v}_3}{\partial x}\right]$ is an order of magnitude smaller and may be neglected and hence equation (2.3) as projected upon *y*-axis yields $\nabla(\nabla.\vec{v}) \approx 0$. This result can be obtained directly under usual approximations in lubrication theory **[59]** by considering $\nabla^2 \vec{v} >> \nabla(\nabla, \vec{v})$.

Now from the vector calculus $\left(\nabla\times\nabla\times\vec{V}\right)$ and $\left(\nabla\times\nabla\times\vec{v}\right)$ can be written as:

$$\nabla\times\nabla\times\vec{V}=\nabla\left(\nabla.\vec{V}\right)-\nabla^2\vec{V} \qquad \text{and} \qquad \nabla\times\nabla\times\vec{v}=\nabla\left(\nabla.\vec{v}\right)-\nabla^2\vec{v} \qquad \text{... (2.8)}$$

With the basic assumptions and equation (2.8), equation (2.2) yields the following three equations along *x*-, *y*- and *z*-directions respectively:

$$\frac{(2\mu+\chi)}{2}\left(\frac{\partial^2\vec{V}_X}{\partial x^2}+\frac{\partial^2\vec{V}_X}{\partial y^2}+\frac{\partial^2\vec{V}_X}{\partial z^2}\right)+\chi\left(\frac{\partial\vec{v}_3}{\partial y}-\frac{\partial\vec{v}_2}{\partial z}\right)-\frac{\partial p}{\partial x}=\rho\left(\vec{V}_X\frac{\partial\vec{V}_X}{\partial x}+\vec{V}_Y\frac{\partial\vec{V}_X}{\partial y}+\vec{V}_Z\frac{\partial\vec{V}_X}{\partial z}\right)$$

… (2.9)

$$\frac{(2\mu+\chi)}{2}\left(\frac{\partial^2\vec{V}_Y}{\partial x^2}+\frac{\partial^2\vec{V}_Y}{\partial y^2}+\frac{\partial^2\vec{V}_Y}{\partial z^2}\right)+\chi\left(\frac{\partial\vec{v}_1}{\partial y}-\frac{\partial\vec{v}_3}{\partial x}\right)-\frac{\partial p}{\partial y}=\rho\left(\vec{V}_X\frac{\partial\vec{V}_Y}{\partial x}+\vec{V}_Y\frac{\partial\vec{V}_Y}{\partial y}+\vec{V}_Z\frac{\partial\vec{V}_Y}{\partial z}\right)$$

… (2.10)

$$\frac{(2\mu+\chi)}{2}\left(\frac{\partial^2\vec{V}_Z}{\partial x^2}+\frac{\partial^2\vec{V}_Z}{\partial y^2}+\frac{\partial^2\vec{V}_Z}{\partial z^2}\right)+\chi\left(\frac{\partial\vec{v}_2}{\partial x}-\frac{\partial\vec{v}_1}{\partial y}\right)-\frac{\partial p}{\partial z}=\rho\left(\vec{V}_X\frac{\partial\vec{V}_Z}{\partial x}+\vec{V}_Y\frac{\partial\vec{V}_Z}{\partial y}+\vec{V}_Z\frac{\partial\vec{V}_Z}{\partial z}\right)$$

… (2.11)

Similarly, with the basic assumptions and equation (2.8), equation (2.3) yields the following three equations along *x*-, *y*- and *z*-directions respectively:

$$\gamma\left(\frac{\partial^2\vec{v}_1}{\partial x^2}+\frac{\partial^2\vec{v}_1}{\partial y^2}+\frac{\partial^2\vec{v}_1}{\partial z^2}\right)+\chi\left(\frac{\partial\vec{V}_Z}{\partial y}-\frac{\partial\vec{V}_Y}{\partial z}\right)-2\chi\vec{v}_1=\rho j\left(\vec{V}_X\frac{\partial\vec{v}_1}{\partial x}+\vec{V}_Y\frac{\partial\vec{v}_1}{\partial y}+\vec{V}_Z\frac{\partial\vec{v}_1}{\partial z}\right)$$

… (2.12)

$$\gamma\left(\frac{\partial^2\vec{v}_2}{\partial x^2}+\frac{\partial^2\vec{v}_2}{\partial y^2}+\frac{\partial^2\vec{v}_2}{\partial z^2}\right)+\chi\left(\frac{\partial\vec{V}_X}{\partial z}-\frac{\partial\vec{V}_Z}{\partial x}\right)-2\chi\vec{v}_2=\rho j\left(\vec{V}_X\frac{\partial\vec{v}_2}{\partial x}+\vec{V}_Y\frac{\partial\vec{v}_2}{\partial y}+\vec{V}_Z\frac{\partial\vec{v}_2}{\partial z}\right)$$

… (2.13)

$$\gamma\left(\frac{\partial^2\vec{v}_3}{\partial x^2}+\frac{\partial^2\vec{v}_3}{\partial y^2}+\frac{\partial^2\vec{v}_3}{\partial z^2}\right)+\chi\left(\frac{\partial\vec{V}_Y}{\partial x}-\frac{\partial\vec{V}_X}{\partial y}\right)-2\chi\vec{v}_3=\rho j\left(\vec{V}_X\frac{\partial\vec{v}_3}{\partial x}+\vec{V}_Y\frac{\partial\vec{v}_3}{\partial y}+\vec{V}_Z\frac{\partial\vec{v}_3}{\partial z}\right)$$

… (2.14)

2.3.3 NON-DIMENSIONAL SCHEME

Equations (2.9) to (2.14) when non-dimensionalised with the following substitutions

$$\left.\begin{array}{l}
\bar{x} = \dfrac{x}{R};\;\; \bar{y} = \dfrac{y}{R};\; \bar{z} = \dfrac{z}{L/2};\;\; \delta_1 = \dfrac{C}{R};\;\; \delta_2 = \dfrac{C}{L/2};\;\; \delta_3 = \dfrac{h_{\min}}{R};\; \delta_4 = \dfrac{L/2}{R}; \\[2ex]
\bar{V}_i = \dfrac{V_i}{U}, \text{ where i = x, y, z;} \qquad \bar{v}_i = v_i \dfrac{C}{U}, \text{ where i = 1, 2, 3;} \qquad \xi = \dfrac{h_{\min}}{C}; \\[2ex]
\bar{p} = \dfrac{ph^2{}_{\min}}{\left(\mu + \dfrac{\chi}{2}\right)UR};\quad R_e = \dfrac{2\rho CU}{(2\mu+\chi)};\quad R_R = \dfrac{\rho jUh}{4\mu\Lambda};\quad N = \left(\dfrac{\chi}{2\mu+\chi}\right)^{1/2}; \\[2ex]
l_m = \dfrac{C}{\Lambda};\; L_R = \dfrac{h_{\min}}{\Lambda};\text{ where } \Lambda = \left(\dfrac{\gamma}{4\mu}\right)^{1/2} \text{ and is of dimension of length.}
\end{array}\right\} \quad \text{... (2.15)}$$

The following equations will result:

$$\left(\delta_1^2 \frac{\partial^2 \bar{V}_X}{\partial \bar{x}^2} + \frac{\partial^2 \bar{V}_X}{\partial \bar{y}^2} + \delta_2^2 \frac{\partial^2 \bar{V}_X}{\partial \bar{z}^2}\right) + 2N^2\left(\frac{\partial \bar{v}_3}{\partial \bar{y}} - \delta_2 \frac{\partial \bar{v}_2}{\partial \bar{z}}\right) - \frac{1}{\xi^2}\frac{\partial \bar{p}}{\partial \bar{x}}$$
$$= R_e\left(\delta_1 \bar{V}_X \frac{\partial \bar{V}_X}{\partial \bar{x}} + \bar{V}_Y \frac{\partial \bar{V}_X}{\partial \bar{y}} + \delta_2 \bar{V}_Z \frac{\partial \bar{V}_X}{\partial \bar{z}}\right) \quad \text{... (2.16)}$$

$$\xi\delta_3\left(\delta_1^2 \frac{\partial^2 \bar{V}_Y}{\partial \bar{x}^2} + \frac{\partial^2 \bar{V}_Y}{\partial \bar{y}^2} + \delta_2^2 \frac{\partial^2 \bar{V}_Y}{\partial \bar{z}^2}\right) + 2\xi\delta_3 N^2\left(\delta_2 \frac{\partial \bar{v}_1}{\partial \bar{z}} - \delta_1 \frac{\partial \bar{v}_3}{\partial \bar{x}}\right) - \frac{\partial \bar{p}}{\partial \bar{y}}$$
$$= \xi\delta_3 R_e\left(\delta_1 \bar{V}_X \frac{\partial \bar{V}_Y}{\partial \bar{x}} + \bar{V}_Y \frac{\partial \bar{V}_Y}{\partial \bar{y}} + \delta_2 \bar{V}_Z \frac{\partial \bar{V}_Y}{\partial \bar{z}}\right) \quad \text{... (2.17)}$$

$$\xi^2\delta_4\left(\delta_1^2 \frac{\partial^2 \bar{V}_Z}{\partial \bar{x}^2} + \frac{\partial^2 \bar{V}_Z}{\partial \bar{y}^2} + \delta_2^2 \frac{\partial^2 \bar{V}_Z}{\partial \bar{z}^2}\right) + 2\xi^2\delta_4 N^2\left(\delta_1 \frac{\partial \bar{v}_2}{\partial \bar{x}} - \frac{\partial \bar{v}_1}{\partial \bar{y}}\right) - \frac{\partial \bar{p}}{\partial \bar{z}}$$
$$= \xi^2\delta_4 R_e\left(\delta_1 \bar{V}_X \frac{\partial \bar{V}_Z}{\partial \bar{x}} + \bar{V}_Y \frac{\partial \bar{V}_Z}{\partial \bar{y}} + \delta_2 \bar{V}_Z \frac{\partial \bar{V}_Z}{\partial \bar{z}}\right) \quad \text{... (2.18)}$$

$$\left(\delta_1^2 \frac{\partial^2 \bar{v}_1}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}_1}{\partial \bar{y}^2} + \delta_2^2 \frac{\partial^2 \bar{v}_1}{\partial \bar{z}^2}\right) + \frac{N^2 L_R^2}{2(1-N^2)\xi^2}\left(\frac{\partial \bar{V}_Z}{\partial \bar{y}} - \delta_2 \frac{\partial \bar{V}_Z}{\partial \bar{z}}\right) - \frac{N^2 L_R^2}{(1-N^2)\xi^2}\bar{v}_1$$
$$= R_R\left(\delta_1 \bar{V}_X \frac{\partial \bar{v}_1}{\partial \bar{x}} + \bar{V}_Y \frac{\partial \bar{v}_1}{\partial \bar{y}} + \delta_2 \bar{V}_Z \frac{\partial \bar{v}_1}{\partial \bar{z}}\right) \quad \text{... (2.19)}$$

$$\left(\delta_1^2 \frac{\partial^2 \bar{v}_2}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}_2}{\partial \bar{y}^2} + \delta_2^2 \frac{\partial^2 \bar{v}_2}{\partial \bar{z}^2}\right) + \frac{N^2 L_R^2}{2(1-N^2)\xi^2}\left(\delta_2 \frac{\partial \bar{V}_X}{\partial \bar{z}} - \delta_1 \frac{\partial \bar{V}_Z}{\partial \bar{x}}\right) - \frac{N^2 L_R^2}{(1-N^2)\xi^2}\bar{v}_2$$
$$= R_R\left(\delta_1 \bar{V}_X \frac{\partial \bar{v}_2}{\partial \bar{x}} + \bar{V}_Y \frac{\partial \bar{v}_2}{\partial \bar{y}} + \delta_2 \bar{V}_Z \frac{\partial \bar{v}_2}{\partial \bar{z}}\right) \quad \text{... (2.20)}$$

$$\left(\delta_1^2 \frac{\partial^2 \bar{v}_3}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}_3}{\partial \bar{y}^2} + \delta_2^2 \frac{\partial^2 \bar{v}_3}{\partial \bar{z}^2}\right) + \frac{N^2 L_R^2}{2(1-N^2)\xi^2}\left(\delta_1 \frac{\partial \bar{V}_Y}{\partial \bar{x}} - \frac{\partial \bar{V}_X}{\partial \bar{y}}\right) - \frac{N^2 L_R^2}{(1-N^2)\xi^2}\bar{v}_3$$
$$= R_R\left(\delta_1 \bar{V}_X \frac{\partial \bar{v}_3}{\partial \bar{x}} + \bar{V}_Y \frac{\partial \bar{v}_3}{\partial \bar{y}} + \delta_2 \bar{V}_Z \frac{\partial \bar{v}_3}{\partial \bar{z}}\right) \quad \text{... (2.21)}$$

2.3.4 STUDY OF NON-DIMENSIONAL PARAMETERS

In this section the interpretations of the non-dimensional parameters are given briefly. As the current analysis is aimed at showing the micropolar effects on the hydrodynamic journal bearings, a separate importance is given in discussion of the micropolar properties at the end of this section.

❖ **MICROPOLAR PARAMETERS**

N and Λ are two parameters distinguishing a micropolar fluid from a Newtonian fluid. *N* is a non-dimensional parameter called the *coupling number,* which couples the linear and angular momentum equations arising out of the microrotational effect of the suspended particles in the fluid. Λ, a dimensional parameter represents the interaction between the micropolar fluid and the film gap. Λ is termed as the *characteristic length* of the micropolar fluid and has the dimension of the length. l_m is a non-dimensional quantity and is termed as the *non-dimensional characteristic length* of the micropolar fluid. For $\chi = 0$, $N = 0$ the equations representing linear and angular momentums are uncoupled and equation (2.2) reduces to that of classical Navier-Stokes equation. On the other hand the strong micropolar effect is exhibited with the increase in the value of Λ or decrease in clearance in between the journal and bearing *i.e.* when non-dimensional characteristic length l_m decreases. The second case is most likely here as *C* is usually very small in lubrication theory. Hence for $N \longrightarrow 0$ or for $l_m \longrightarrow \infty$, (*i.e.* characteristic length of the microstructure is small), the micropolar characteristic is lost and the lubrication problem reduces to that of the classical hydrodynamic lubrication. Again, as the gradient of the microrotational velocity across the film thickness is very small, when $l_m \longrightarrow 0$, the velocity and other flow characteristics will reduce to their equivalents in the Newtonian theory with μ everywhere replaced by $\left(\mu + \frac{1}{2}\chi\right)$Hence$\left(\mu + \frac{1}{2}\chi\right)$ togetherly may be considered as the *effective viscosity,* μ_e, which arises due to the microrotational effect. The thermodynamic restrictions require that $0 \le N \le 1$. *j*, the microinertia constant is evolved due to the microrotational effects.

❖ **OTHER NON-DIMENSIONAL PARAMETERS AND ORDER ANALYSIS**

L_R, a non-dimensional quantity, may be termed as *length ratio*. During the initial stage of the application of the micropolar theory to the lubrication flow problems, researchers**[45, 58]** used either NL_R or Nl_m as the parameter in discussing the micropolar fluid; but as these products characterize the combinational effects, later N and l_m were considered separately to study their effects as can be found in **Prakash** and **Sinha[49]**. Re may be considered as the *modified Reynolds number*, where μ has been replaced by effective viscosity, μ_e, *i.e.* $\left(\mu + \frac{1}{2}\chi\right)$ and this number is always less than the classical Reynolds number. Generally for lubrication problem, Re << 1 and is usually of the order of 10^{-3}. As j is the square of a length typical of the microstructure as discussed earlier, it is reasonable to consider R_R << 1. Also with the assumptions that $\delta_1,\delta_2,\delta_3$ are of the order of 10^{-3} and δ_4, $\boldsymbol{\xi}$, L_R are of the order of 1, equations (2.16) to (2.21) will reduce to

$$\frac{\partial^2 \overline{V}_X}{\partial \overline{y}^2} + 2N^2 \frac{\partial \overline{v}_3}{\partial \overline{y}} - \frac{1}{\xi^2}\frac{\partial \overline{p}}{\partial \overline{x}} = 0 \qquad \text{... (2.22)}$$

$$\frac{\partial \overline{p}}{\partial \overline{y}} = 0 \qquad \text{... (2.23)}$$

$$\frac{\partial^2 \overline{V}_Z}{\partial \overline{y}^2} - 2N^2 \frac{\partial \overline{v}_1}{\partial \overline{y}} - \frac{1}{\xi^2 \delta_4}\frac{\partial \overline{p}}{\partial \overline{z}} = 0 \qquad \text{... (2.24)}$$

$$\frac{\partial^2 \overline{v}_1}{\partial \overline{y}^2} + \frac{N^2 L_R^2}{2(1-N^2)\xi^2}\frac{\partial \overline{V}_Z}{\partial \overline{y}} - \frac{N^2 L_R^2}{(1-N^2)\xi^2}\overline{v}_1 = 0 \qquad \text{... (2.25)}$$

$$\frac{\partial^2 \overline{v}_2}{\partial \overline{y}^2} - \frac{N^2 L_R^2}{(1-N^2)\xi^2}\overline{v}_2 = 0 \qquad \text{... (2.26)}$$

$$\frac{\partial^2 \overline{v}_3}{\partial \overline{y}^2} - \frac{N^2 L_R^2}{2(1-N^2)\xi^2}\frac{\partial \overline{V}_X}{\partial \overline{y}} - \frac{N^2 L_R^2}{(1-N^2)\xi^2}\overline{v}_3 = 0 \qquad \text{... (2.27)}$$

4.3.5 VELOCITY AND MICROROTATIONAL VELOCITY VECTORS

Now to solve the velocity components and the microrotational velocity components equations (2.22) to (2.27) are returned to their dimensional forms as follows:

$$\frac{(2\mu+\chi)}{2}\frac{\partial^2\vec{V}_X}{\partial y^2}+\chi\frac{\partial v_3}{\partial y}-\frac{\partial p}{\partial x}=0 \qquad ... (2.28)$$

$$\frac{\partial p}{\partial y}=0 \qquad ... (2.29)$$

$$\frac{(2\mu+\chi)}{2}\frac{\partial^2\vec{V}_Z}{\partial y^2}-\chi\frac{\partial v_1}{\partial y}-\frac{\partial p}{\partial z}=0 \qquad ... (2.30)$$

$$\gamma\frac{\partial v_1}{\partial y^2}+\chi\frac{\partial\vec{V}_Z}{\partial y}-2\chi v_1=0 \qquad ... (2.31)$$

$$\gamma\frac{\partial^2 v_2}{\partial y^2}-2\chi v_2=0 \qquad ... (2.32)$$

$$\gamma\frac{\partial^2 v_3}{\partial y^2}-\chi\frac{\partial\vec{V}_X}{\partial y}-2\chi v_3=0 \qquad ... (2.33)$$

As $x=R\theta$, equation (2.28) reduces to

$$\frac{(2\mu+\chi)}{2}\frac{\partial^2\vec{V}_X}{\partial y^2}+\chi\frac{\partial v_3}{\partial y}-\frac{1}{R}\frac{\partial p}{\partial\theta}=0 \qquad ... (2.34)$$

Solving equations (2.30) and (2.31) the expressions for $\vec{v}_1$ and $\vec{V}_Z$ are evaluated as follows

$$\vec{v}_1=A_1+A_2e^{ay}+A_3e^{-ay}+\left(\frac{1}{2\mu}\frac{\partial p}{\partial z}\right)y \qquad ... (2.35)$$

$$\vec{V}_Z=2A_1y+2A_2\frac{N^2}{a}e^{ay}-2A_3\frac{N^2}{a}e^{-ay}+A_4+\left(\frac{1}{2\mu}\frac{\partial p}{\partial z}\right)y^2 \qquad ... (2.36)$$

Where, A_1, A_2, A_3 and A_4 are four constants to be evaluated from the boundary conditions and a is a non-dimensional quantity given by, $a=\frac{N}{\Lambda}=\sqrt{\frac{4\mu N^2}{\gamma}}$

Boundary conditions for $\vec{v}_1$ and $\vec{V}_z$ are:

(i) At y = 0 $\vec{v}_1=0$ and $\vec{V}_z=0$ ... (2.37)

(ii) At y = h $\vec{v}_1=0$ and $\vec{V}_z=0$

Using the above boundary conditions in equations (2.35) and (2.36) A_1, A_2, A_3 and A_4 are evolved as:

$$\left.\begin{aligned} A_1&=-\frac{h}{4\mu}\frac{\partial p}{\partial z}; & A_2&=-\frac{h}{4\mu}\frac{\partial p}{\partial z}\frac{1}{(e^{ah}-1)};\\ A_3&=\frac{h}{4\mu}\frac{\partial p}{\partial z}\frac{1}{(1-e^{-ah})}; & A_4&=\frac{hN^2}{2\mu a}\frac{\partial p}{\partial z}\frac{(1+e^{-ah})}{(1-e^{-ah})};\end{aligned}\right\} \qquad ... (2.38)$$

With the substitutions of A_1, A_2, A_3 and A_4 the expressions for $\vec{v}_1$ and $\vec{V}_Z$ become

$$\vec{v}_1 = \frac{h}{4\mu}(\sinh ay)\left\{\frac{(\cosh ay - 1)}{\sinh ay} - \frac{(\cosh ah + 1)}{\sinh ah}\right\}\frac{\partial p}{\partial z} + \frac{y}{2\mu}\frac{\partial p}{\partial z} \qquad \text{... (2.39)}$$

$$\vec{V}_Z = \frac{y(y-h)}{2\mu}\frac{\partial p}{\partial z} + \frac{h}{2\mu}\frac{N^2}{a}\left\{\sinh ay - \frac{(\cosh ay - 1)(\cosh ah + 1)}{\sinh ah}\right\}\frac{\partial p}{\partial z} \qquad \text{... (2.40)}$$

Similarly, solving equations (2.33) and (2.34) with the following boundary conditions

(i) At y = 0 $\vec{v}_3 = 0$ and $\vec{V}_x = 0$

(ii) At y = h $\vec{v}_3 = 0$ and $\vec{V}_x = U$ ... (2.41)

The following equations will result:

$$\vec{v}_3 = -\frac{h}{4\mu R}(\sinh ay)\left\{\frac{(\cosh ay - 1)}{\sinh ay} - \frac{(\cosh ah + 1)}{\sinh ah}\right\}\frac{\partial p}{\partial \theta} - \frac{y}{2\mu R}\frac{\partial p}{\partial \theta}$$
$$+ \frac{U}{2\left[\frac{2N^2}{a} - h\frac{(\cosh ah + 1)}{\sinh ah}\right]}\left\{\sinh ay - \frac{(\cosh ay - 1)(\cosh ah + 1)}{\sinh ah}\right\} \qquad \text{... (2.42)}$$

$$\vec{V}_X = \frac{y(y-h)}{2\mu R}\frac{\partial p}{\partial \theta} + \frac{h}{2\mu R}\frac{N^2}{a}\left\{\sinh ay - \frac{(\cosh ay - 1)(\cosh ah + 1)}{\sinh ah}\right\}\frac{\partial p}{\partial \theta} -$$
$$\frac{U}{2\left[\frac{2N^2}{a} - h\frac{(\cosh ah + 1)}{\sinh ah}\right]}\left[\frac{2y(\cosh ah + 1)}{\sinh ah} + \frac{2N^2}{a}(\sinh ay)\left\{\frac{(\cosh ay - 1)}{\sinh ay} - \frac{(\cosh ah + 1)}{\sinh ah}\right\}\right]$$

... (2.43)

Now, from the principle of conservation of mass as stated in equation (2.1), it follows that

$$\frac{\partial(\rho\vec{V}_Y)}{\partial y} = -\frac{\partial \rho}{\partial t} - \frac{\partial(\rho\vec{V}_X)}{\partial x} - \frac{\partial(\rho\vec{V}_Z)}{\partial z} \qquad \text{... (2.44)}$$

Integrating both sides with respect to *y* within the limit 0 and *h*, equation (2.43) becomes:

$$\rho(\vec{V}_h - \vec{V}_0) = -\int_0^h \frac{\partial \rho}{\partial t}dy - \int_0^h \frac{\partial(\rho\vec{V}_X)}{\partial x}dy - \int_0^h \frac{\partial(\rho\vec{V}_Z)}{\partial z}dy \qquad \text{... (2.45)}$$

Where, $(\vec{V}_Y)_{Y=h} = \vec{V}_h$ and $(\vec{V}_Y)_{Y=0} = \vec{V}_0$

As the upper limit *h* is the function of the coordinates *x*, *z* the integration before differentiation can be performed by using Leibnitz's rule and using the boundary conditions that

(i) At y = 0 $\vec{V}_x = 0$ and $\vec{V}_z = 0$

(ii) At y = h $\vec{V}_x = U$ and $\vec{V}_z = 0$ ... (2.46)

And noting that $V_h = U\frac{\partial h}{\partial x} + \frac{\partial h}{\partial t}$ and V_0=0, the following equation will result:

$$\frac{\partial}{\partial x}\left[\frac{\rho h}{12\mu}\left\{h^2 + 12\Lambda^2 - 6N\Lambda h \coth\frac{Nh}{2\Lambda}\right\}\frac{\partial p}{\partial x}\right] + \frac{\partial}{\partial z}\left[\frac{\rho h}{12\mu}\left\{h^2 + 12\Lambda^2 - 6N\Lambda h \coth\frac{Nh}{2\Lambda}\right\}\frac{\partial p}{\partial z}\right]$$
$$= \frac{\partial}{\partial x}\left(\frac{\rho U h}{2}\right) + \frac{\partial(\rho h)}{\partial t} \quad \text{... (2.47)}$$

For incompressible fluid flow equation (2.47) reduces to give the ***Reynolds equation for incompressible fluid flow in micropolar lubrication:***

$$\frac{\partial}{\partial x}\left[\frac{h^3}{\mu}\Phi(\Lambda, N, h)\frac{\partial p}{\partial x}\right] + \frac{\partial}{\partial z}\left[\frac{h^3}{\mu}\Phi(\Lambda, N, h)\frac{\partial p}{\partial z}\right] = 6\frac{\partial(Uh)}{\partial x} + 12\frac{\partial h}{\partial t} \quad \text{... (2.48)}$$

Where $\Phi(\Lambda, N, h) = \left\{1 + 12\frac{\Lambda^2}{h^2} - 6\frac{N\Lambda}{h}\coth\frac{Nh}{2\Lambda}\right\}$ ... (2.49)

Introducing the rotating coordinate system and considering that the journal centre is rotating an angular velocity $\boldsymbol{\omega}_p$, equation (2.48) becomes:

$$\frac{\partial}{\partial x}\left[\frac{h^3}{\mu}\Phi(\Lambda, N, h)\frac{\partial p}{\partial x}\right] + \frac{\partial}{\partial z}\left[\frac{h^3}{\mu}\Phi(\Lambda, N, h)\frac{\partial p}{\partial z}\right] = 6\frac{\partial(Uh)}{\partial x} + 12\frac{\partial h}{\partial t} - 12\omega_p\frac{\partial h}{\partial \theta} \quad \text{... (2.50)}$$

Equation (2.50) represents the ***modified Reynolds equation in rotating coordinate system in micropolar lubrication.***

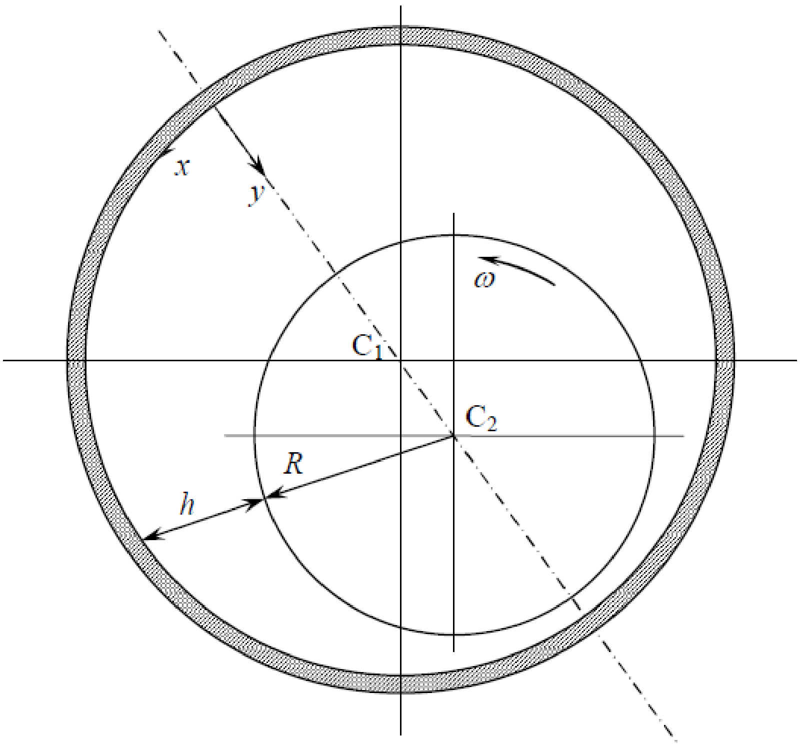

Fig.2.1 Bearing Configuration for the Derivation of Reynolds Equation.

CHAPTER 3: STEADY STATE CHARACTERISTICS OF HYDRODYNAMIC JOURNAL BEARINGS LUBRICATED WITH MICROPOLAR FLUIDS

3.1 INTRODUCTION

This chapter deals with the theoretical analysis of the steady state characteristics of finite journal bearings lubricated with micropolar fluids. The steady state analysis is important for the subsequent linear analysis for determining the threshold of instability of the journal bearings.

A good amount of work has been reported on the steady state characteristics of rotor/bearing systems lubricated with micropolar fluids. **Prakash** and **Sinha[49]** studied the steady state characteristics of an infinitely long journal bearing considering a two dimensional flow field. **Huang *et al.* [60]**, **Khonsari** and **Brewe[61]** made same study on the steady state characteristics of journal bearing system lubricated with incompressible micropolar lubricant. In the present chapter a comprehensive parametric study has been made of the steady state characteristics *viz.* load parameter and attitude angle, of the journal/bearing systems lubricated with micropolar fluids. In the analysis the governing modified Reynolds equation in the non-dimensional form satisfying the boundary conditions has been solved by analytically. The steady state non-dimensional pressure profiles thus obtained is used to determine the steady state characteristics mentioned above.

3.2 THEORETICAL ANALYSIS

3.2.1 GOVERNING EQUATIONS

A schematic diagram of a journal bearing in micropolar lubrication with the appropriate coordinate system used for the purpose of analysis is shown in *Fig.3.1*. The journal is considered to rotate 45^0 with a steady angular velocity about its axis. The governing modified Reynolds equation (2.47) for two-dimensional flow of micropolar lubricant under steady state condition is written as follows:

$$\frac{\partial}{\partial x}\left[\frac{{h_0}^3}{\mu}\Phi(\Lambda, N, h_0)\frac{\partial p_0}{\partial x}\right]+\frac{\partial}{\partial z}\left[\frac{{h_0}^3}{\mu}\Phi(\Lambda, N, h_0)\frac{\partial p_0}{\partial z}\right]=6U\frac{\partial h_0}{\partial x} \qquad \text{... (3.1)}$$

With the following substitutions

$$\theta = \frac{x}{R}, \qquad \bar{z} = \frac{2z}{L}, \qquad \bar{h}_0 = \frac{h_0}{C}, \qquad \bar{p}_0 = \frac{p_0 C^2}{\mu \omega R^2}, \qquad l_m = \frac{C}{\Lambda}$$

Equation (3.1) will reduce to its non-dimensional form as:

$$\frac{\partial}{\partial\theta}\left[\bar{g}\left(l_m, N, \bar{h}_0\right)\frac{\partial \bar{p}_0}{\partial\theta}\right] + \left(\frac{D}{L}\right)^2 \frac{\partial}{\partial\bar{z}}\left[\bar{g}\left(l_m, N, \bar{h}_0\right)\frac{\partial \bar{p}_0}{\partial\bar{z}}\right] = \frac{1}{2}\frac{\partial \bar{h}_0}{\partial\theta} \qquad \text{... (3.2)}$$

Where

$$\overline{\Phi}\left(l_m, N, \bar{h}_0\right) = \left\{1 + \frac{12}{\bar{h}_0^2 l_m^2} - 6\frac{N}{\bar{h}_0 l_m}\coth\left(\frac{N l_m \bar{h}_0}{2}\right)\right\}$$

And

$$\bar{g}\left(l_m, N, \bar{h}_0\right) = \frac{\bar{h}_0^3}{12}\overline{\Phi}\left(l_m, N, \bar{h}_0\right) = \frac{\bar{h}_0^3}{12} + \frac{\bar{h}_0}{l_m^2} - \frac{N\bar{h}_0^2}{2l_m}\coth\left(\frac{N l_m \bar{h}_0}{2}\right)$$

As $\bar{h}_0 = 1 + \varepsilon_0 \cos\theta = \bar{h}_0(\theta)$ is a function of θ only, $\bar{g}\left(l_m, N, \bar{h}_0\right) = \bar{g}(\theta)$ is also a function of θ only.

Equation (3.2) is further simplified to the following non-dimensional form under steady state condition

$$C_A \frac{\partial \bar{h}_0}{\partial\theta}\frac{\partial \bar{p}_0}{\partial\theta} + C_B\left\{\frac{\partial^2 \bar{p}_0}{\partial\theta^2} + \left(\frac{D}{L}\right)^2 \frac{\partial^2 \bar{p}_0}{\partial\bar{z}^2}\right\} = \frac{1}{2}\frac{\partial \bar{h}_0}{\partial\theta} \qquad \text{... (3.3)}$$

Where,

$$\left.\begin{aligned} C_A &= \frac{\bar{h}_0^2}{4} + \frac{1}{l_m^2} - \frac{N\bar{h}_0}{l_m}\coth\left(\frac{N l_m \bar{h}_0}{2}\right) + \frac{N^2\bar{h}_0^2}{4}\operatorname{cosech}^2\left(\frac{N l_m \bar{h}_0}{2}\right) \\ C_B &= \frac{\bar{h}_0^3}{12} + \frac{\bar{h}_0}{l_m^2} - \frac{N\bar{h}_0^2}{2l_m}\coth\left(\frac{N l_m \bar{h}_0}{2}\right) \end{aligned}\right\} \qquad \text{... (3.4)}$$

Boundary conditions for equation (3.3) are as follows:

1. The pressures at the ends of the bearing are zero

   $\bar{p}_0(\theta, \pm 1) = 0$

2. The pressures distribution is symmetrical about the midplane of the bearing

   $\dfrac{\partial \bar{p}_0(\theta, 0)}{\partial \bar{z}} = 0$

3. Cavitation boundary condition

   $\dfrac{\partial \bar{p}_0(\theta_2, \bar{z})}{\partial \theta} = 0$, $\quad \bar{p}_0(\theta, \bar{z}) = 0 \quad$ for $\theta \geq \theta_2$

… (3.5)

Where, θ_2 represents the angular coordinate at which the film cavitates.

3.3 NUMERICAL SOLUTION FOR PRESSURES

3.3.1 ANALYTICAL METHOD

Equation 3.3 is solved using the analytical method to obtain the steady state pressure distribution, $\bar{p}_0$, satisfying the boundary conditions as given in equation (3.5).

From equation (3.3)

$$C_A \frac{\partial \bar{h}_0}{\partial \theta} \frac{\partial \bar{p}_0}{\partial \theta} + C_B \left\{ \frac{\partial^2 \bar{p}_0}{\partial \theta^2} + \left(\frac{D}{L} \right)^2 \frac{\partial^2 \bar{p}_0}{\partial \bar{z}^2} \right\} = \frac{1}{2} \frac{\partial \bar{h}_0}{\partial \theta}$$

$$C_B \left(\frac{D}{L} \right)^2 \frac{\partial^2 \bar{p}_0}{\partial \bar{z}^2} = \frac{1}{2} \frac{\partial \bar{h}_0}{\partial \theta}$$

$$C_B \left(\frac{D}{L} \right)^2 \frac{\partial^2 \bar{p}_0}{\partial \bar{z}^2} = \frac{1}{2} \frac{\partial (1 + \varepsilon_0 \cos \theta)}{\partial \theta}$$

$$C_B \left(\frac{D}{L} \right)^2 \frac{\partial^2 \bar{p}_0}{\partial \bar{z}^2} = -\frac{\varepsilon_0 \sin \theta}{2}$$

Integrating both sides of the above equation

$$C_B \left(\frac{D}{L} \right)^2 \frac{\partial \bar{p}_0}{\partial \bar{z}} = -\frac{\varepsilon_0 \sin \theta}{2} \bar{z} + C_1$$

Again integrating

$$C_B \left(\frac{D}{L} \right)^2 \bar{p}_0 = -\frac{\varepsilon_0 \sin \theta}{2} \frac{\bar{z}^2}{2} + C_1 \bar{z} + C_2$$

3.3.2 PRESSURE PROFILES

We are calculating the coefficients C_1 and C_2 using the boundary condition which is given in equation (3.5)

So the value of steady state perturbed pressure is given as

$$\boxed{\overline{p}_0 = \left(\frac{L}{D}\right)^2 \frac{\varepsilon_0 \sin\theta}{4C_B}\left(1 - \overline{z}^2\right)}$$

3.4 RESULTS AND DISCUSSION

The schematic diagram of the journal is shown in *Fig 3.1.* Since the steady state characteristics *viz.*, load carrying capacity and attitude angle of the journal are dependent on the steady state film pressure $\overline{p}_0$, which, in turn, depends upon the micropolar parameters l_m, N, and eccentricity ratio ε_o, detailed parametric studies are done to show their effects in the respective non-dimensional form of steady state load, attitude angle and the results have been compared with the respective values for Newtonian fluid, thereby validating the methods and computer codes.

3.4.1 LOAD CARRYING CAPACITY

3.4.1.1 EFFECT OF COUPLING NUMBER (*N*)

when coupling number N is taken as a parameter then the variation of the dimensionless load capacity of the journal bearings as a function of l_m for L/D = 0.1 and ε_o = 0.5 is shown in *Fig 3.2.* In *Fig.3.2* we can see that for any coupling number N, the load capacity reduces with l_m and approaches asymptotically to the Newtonian value as $l_m \to \infty$ at a finite value of l_m the load parameter increases as coupling number is increased. Moreover, as $l_m \to 0$ the load parameter increases rapidly as N increases. This is expected since the micropolar effect will be significant either when the characteristic material length is large or the clearance is small. The second case is very important here as C is usually very small in hydrodynamic journal bearing. As $l_m \to 0$ the velocity and the other flow characteristics will reduce to their equivalents in the Newtonian theory with μ everywhere replaced by $\left(\mu+\frac{1}{2}\chi\right)$, as gradient of microrotational velocity across film thickness is very small. Hence effectively the viscosity has been enhanced. So, when the non-dimensional load has been referred to the Newtonian viscosity, it is increased by a factor $\left(\mu+\frac{1}{2}\chi\right)/\mu$ at $l_m \to 0$. The decrease in the load capacity to the asymptotic value at $l_m \to \infty$ can be attributed to the same reason as given in the explanation of steady state pressure. $\left(\mu+\frac{1}{2}\chi\right)/\mu$ is equal to $\{1/(1-N^2)\}$ by virtue of definition of *N*. So at $l_m \to 0$ at higher value of *N*, $\{1/(1-N^2)\}$ is more and so $\overline{W_o}$ is more. In between $l_m \to 0$ and $l_m \to \infty$, at $N \neq 0$, the effect of angular momentum equation will modify Reynolds' equation through coupling number. Hence following the same explanation as that given for the steady state pressure, the enhanced pressure will increase the non-dimensional load carrying capacity.

3.4.1.2 EFFECT OF ECCENTRICITY RATIO (ε_o)

Eccentricity ratio is one of the important parameter which is considered for the variation of load parameter with l_m for *L/D* = 0.1 and N^2 = 0.5 is shown in Fig 3.3. It can be seen that for a particular value of l_m, the non-dimensional load carrying capacity though increases in both types of lubrication as ε_o is increased, the rate of increase in $\overline{W_o}$ in micropolar lubrication is found to be more rapid than that in the Newtonian lubrication. It is also found that the load parameter at any eccentricity ratio is considerably higher than that for Newtonian value at lower values of l_m and converges to that for Newtonian fluid as $l_m \to \infty$.

3.4.2 ATTITUDE ANGLE

3.4.2.1 EFFECT OF COUPLING NUMBER (*N*)

Fig.3.4 shows the variation of the attitude angle with micropolar parameter l_m for *L/D* = 0.1 and ε_o = 0.5 when coupling number *N* as treated a parameter. It can be seen from the figure that for a particular value of l_m attitude angle decreases as *N* is increased. Furthermore, as l_m increases the values of the attitude angle converge asymptotically to that for the Newtonian fluid. For any coupling number, attitude angle initially decreases with increase in l_m reaching a minimum and then reversing the trend as l_m is further increased. It is found that the optimum value of l_m at which ϕ_0 becomes a minimum increases with a decrease in *N*. It can be verified form the figure that the optimum value of l_m for minimum value of ϕ_0 shifts from around l_m = 6.0 at N^2 = 0.9 to around 16.0 at N^2 = 0.3. It can be demonstrated that to the left of the optimum l_m, micropolar effect becomes significant and to the right of this micropolar effect diminishes. Further at l_m = 0, the attitude angle remains the same as that for the Newtonian fluid. The reason is that components of non-dimensional forces along the line of centres and perpendicular to it will be increased by the same factor $\left(\mu + \frac{1}{2}\chi\right)/\mu$ and the attitude angle being the function of the ratio of the two will remain unaffected.

3.4.2.2 EFFECT OF ECCENTRICITY RATIO (ε_o)

Effect of the eccentricity ratio on attitude angle has been shown in *Fig.3.5*, for *L/D* = 0.1 and N^2 = 0.5. It is found from the figure that at any l_m, attitude angle decreases with increase in ε_o. Furthermore, the effect of micropolar parameter l_m becomes more pronounced at higher ε_o. The optimum micropolarity is found to be almost independent of ε_o and occurs at l_m around 10.0.

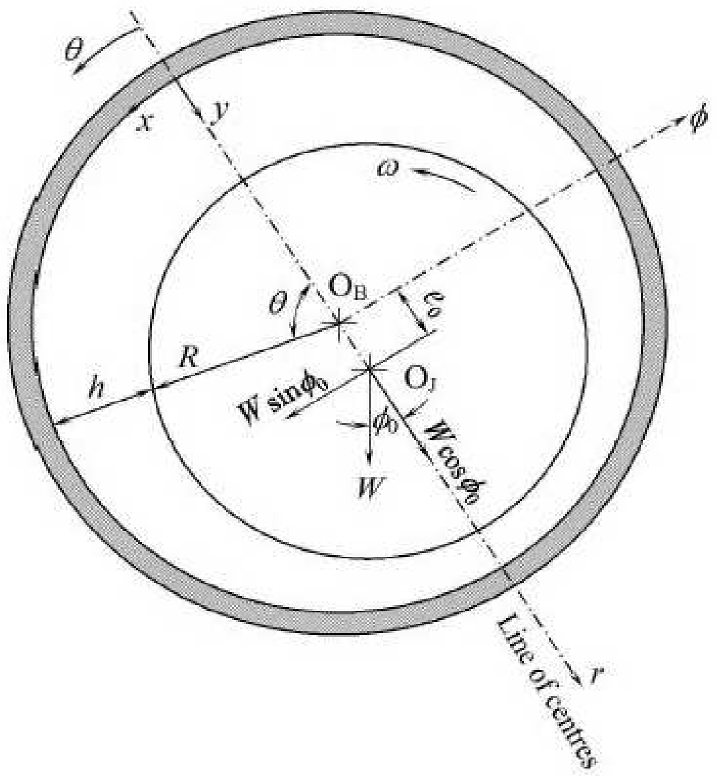

Fig 3.1 Schematic diagram of hydrodynamic journal bearing under steady state condition

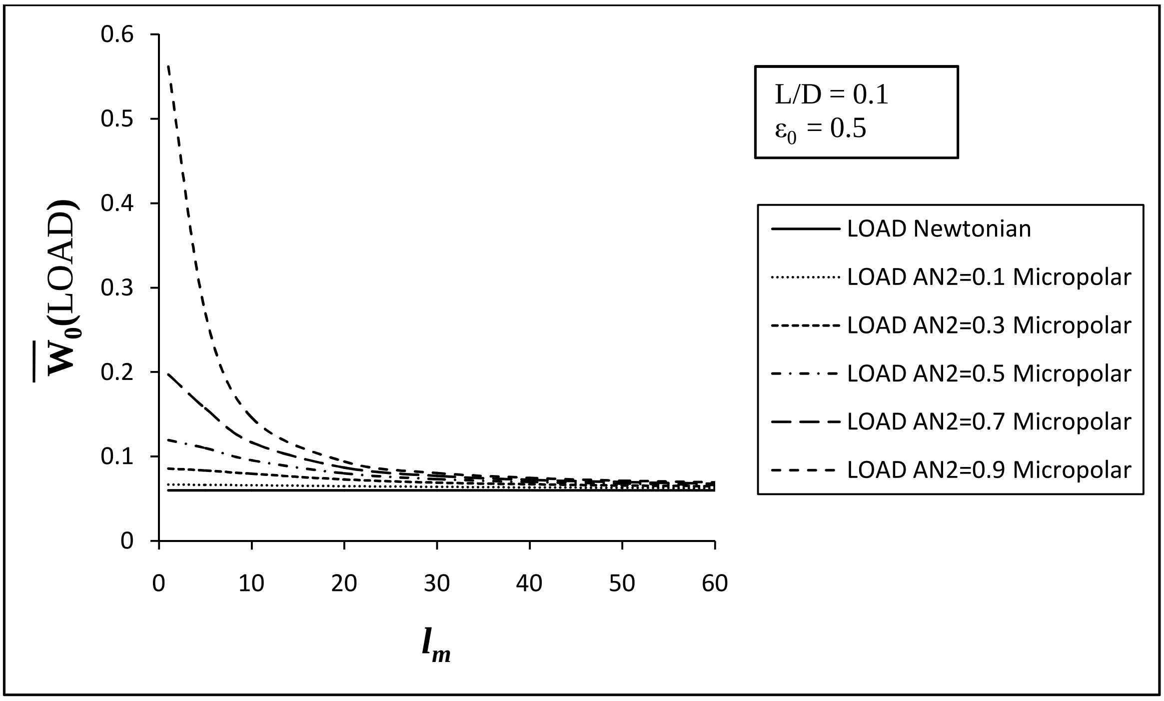

Fig.3.2: Variation of load parameter with l_m for various values of N (AN).

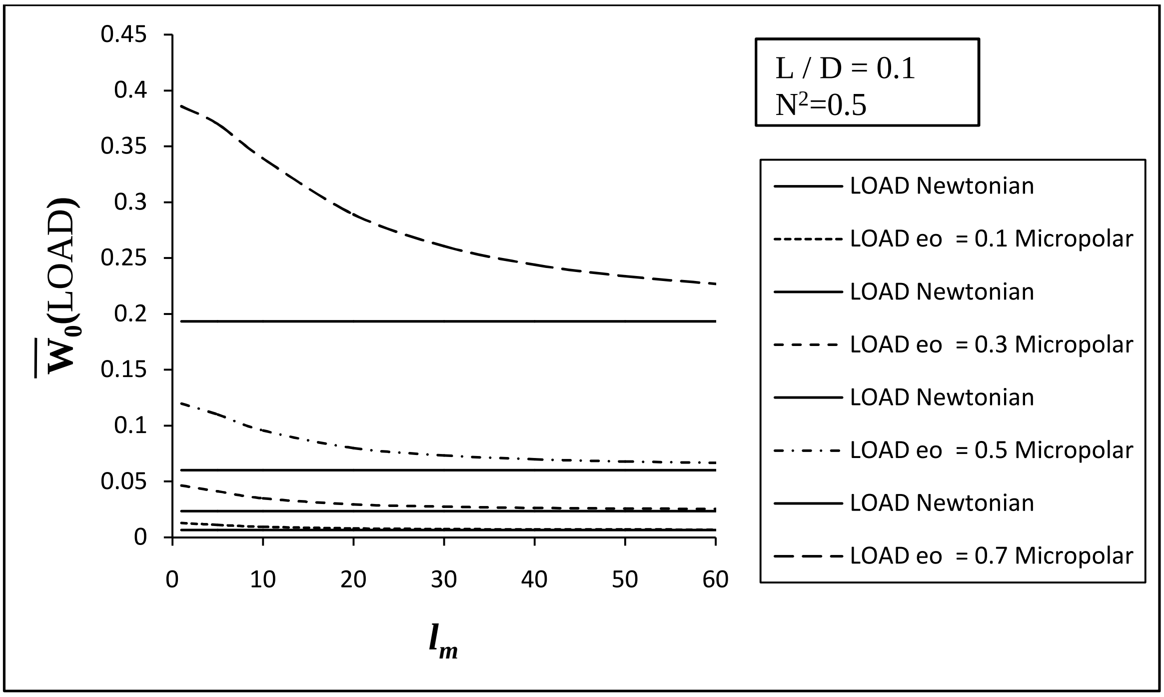

Fig.3.3: Variation of load parameter with l_m for various values of ε_0.

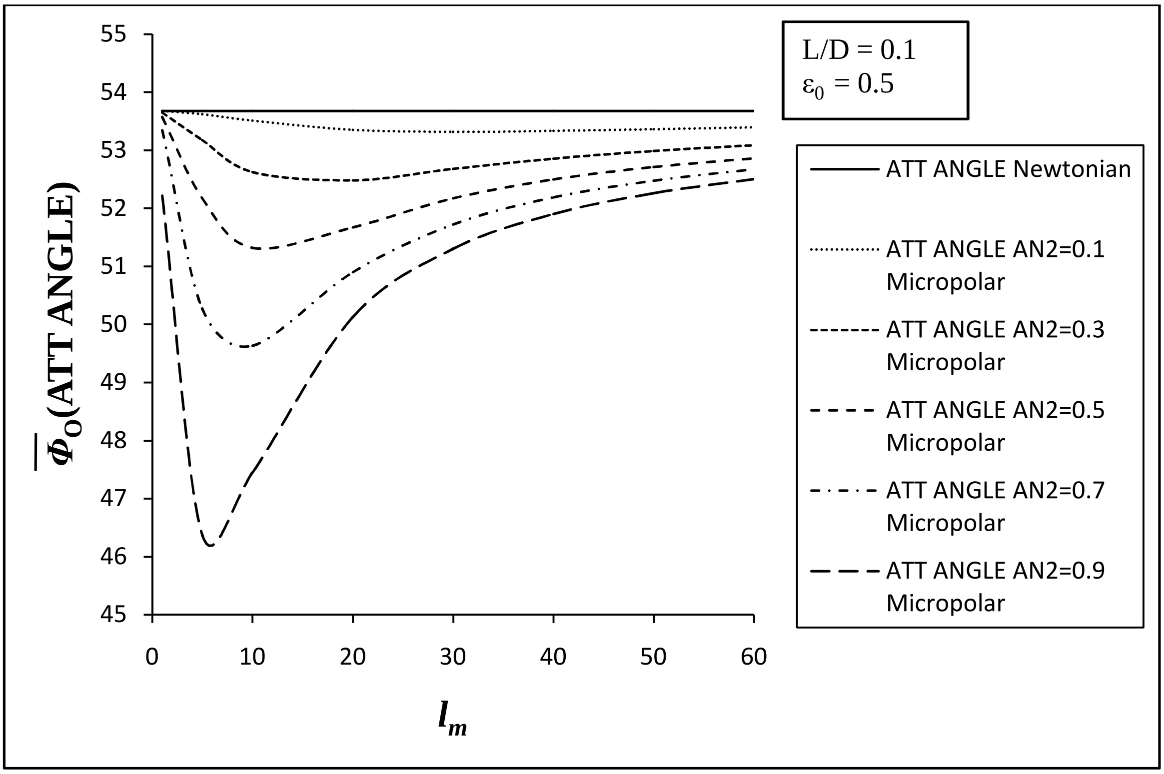

Fig.3.4: Variation of attitude angle with l_m for various values of N (AN).

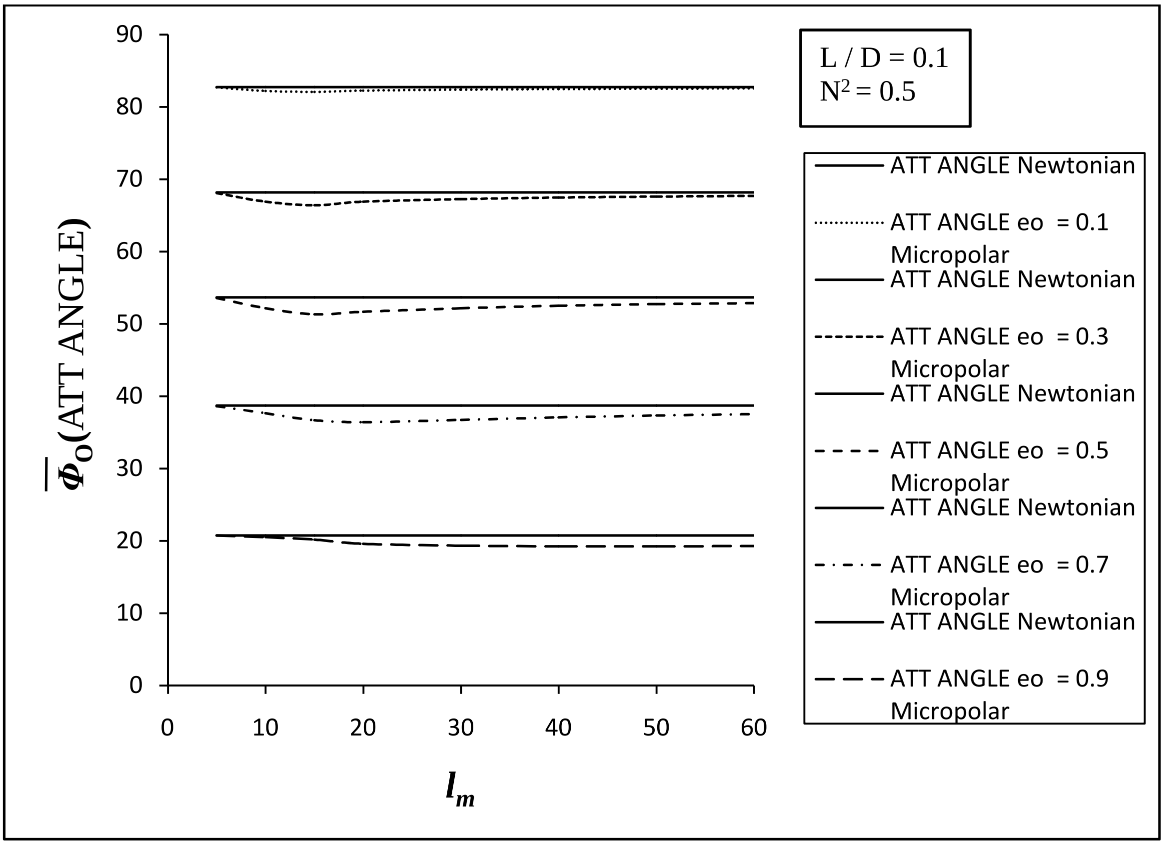

Fig.3.5: Variation of attitude angle with l_m for various values of ε_0.

CHAPTER 4: DYNAMIC CHARACTERISTICS OF HYDRODYNAMIC JOURNAL BEARINGS LUBRICATED WITH MICROPOLAR FLUIDS.

4.1 INTRODUCTION

This chapter deals with the theoretical analysis of the dynamic characteristics of flexibly supported hydrodynamic journal bearings (narrow) under micropolar fluid. A lot of work has been done on the performance of hydrodynamic journal bearings. The method used in obtaining close approximation to the load carrying capacity of a bearing with any geometry, has the basic drawback that the actual shape of the pressure profile is far from satisfactory. This argument must be considered seriously when the data of steady state performance have to be used in the calculation of dynamic characteristics. Then derivatives of the pressure are necessary, and the accuracy of the calculation depends considerably upon the smoothness of the data and on the close similarity of the approximate and exact profiles. Therefore the direct numerical methods of solution of the equations governing the problem have been chosen has a better method. Dynamic co-efficient i.e. the oil film stiffness and damping co-efficient have been determined by perturbation technique followed by numerical method. These dynamic co-efficient were used to write down the equation of motion which were then linearized to obtain the stability parameter

Apart from the stiffness and damping coefficients another dynamic quantity is the threshold of oil whirl stability, an important phenomenon exhibited by rotor/bearing system. Because of the oil whirl, the shaft center shifts away from the static equilibrium position, whirls in an orbit and may ultimately come in contact with the bearing surface, and the bearing may fail due to seizure. Therefore, a study of oil whirl instability is important in bearing design.

Keeping this in view, the threshold for oil whirl for a rigid rotor in self acting cylindrical journal bearings of finite length lubricated with micropolar fluid has been analyzed theoretically. In this analysis, the journal is also assumed to execute small harmonic oscillation about its static equilibrium position, such that first order perturbation can be applied without serious error. The equations of motion of the journal have been written using the oil stiffness and damping coefficients, and the resulting equations have been investigated for oil whirl stability. The results of the analysis are shown in the form of graphs and the effects of various parameters like micropolar parameters, eccentricity ratio, and slenderness ratio are studied and discussed.

4.1.1 STABILITY:

Rotating machines are normally supported in fluid film bearings. Hydrodynamic bearings have relatively low coefficient of fluid friction and these provide viscous damping which reduce lateral vibrations. On the other hand, rolling element bearings have adequate stiffness and virtually no damping. The fluid film hydrodynamic bearings are quite useful in damping some vibrations which are present in the rotating machinery.

There are number of causes for large vibrations in rotating machinery. They may be unbalance, hydrodynamic bearing themselves, seals, friction rubs and internal friction due to stress reversal in shrunk on parts. The journal motion which gives rise to large vibrations is called instability. The instability occurring at a frequency less than that of rotating speed is called subsynchronous whirl.

4.2 THEORETICAL ANALYSIS

4.2.1 GOVERNING EQUATIONS

A schematic diagram of a journal bearing in micropolar lubrication with the appropriate coordinate system used for the purpose of analysis is shown in fig 4.1. The journal is considered to rotate with a steady angular velocity about its axis and it is further assumed that the journal undergoes a steady whirl in an elliptic orbit with a frequency ω_p about its mean steady state position. The governing modified Reynolds equation (2.47) for two-dimensional flow of micropolar lubricant with the following substitutions

$$\theta = \frac{x}{R}, \qquad \bar{z} = \frac{2z}{L}, \qquad \bar{h} = \frac{h}{C}, \qquad \bar{p} = \frac{pC^2}{\mu\omega R^2}, \qquad l_m = \frac{C}{\Lambda}, \qquad \tau = \omega t \quad \ldots (4.1)$$

will reduce to its non-dimensional form as

$$\frac{\partial}{\partial\theta}\left[\bar{g}\left(l_m, N, \bar{h}\right)\frac{\partial\bar{p}}{\partial\theta}\right] + \left(\frac{D}{L}\right)^2 \frac{\partial}{\partial\bar{z}}\left[\bar{g}\left(l_m, N, \bar{h}\right)\frac{\partial\bar{p}}{\partial\bar{z}}\right] = \frac{1}{2}\left(1-2\phi'\right)\frac{\partial\bar{h}}{\partial\theta} + \frac{\partial\bar{h}}{\partial\tau} \quad \ldots (4.2)$$

Where,

$$\overline{\Phi}\left(l_m, N, \bar{h}\right) = \left\{1 + \frac{12}{\bar{h}^2 l_m^2} - 6\frac{N}{\bar{h}\, l_m}\coth\left(\frac{N l_m \bar{h}}{2}\right)\right\}$$

$$\bar{g}\left(l_m, N, \bar{h}\right) = \frac{\bar{h}^3}{12}\bar{\Phi}\left(l_m, N, \bar{h}\right) = \frac{\bar{h}^3}{12} + \frac{\bar{h}}{l_m^2} - \frac{N\bar{h}^2}{2l_m}\coth\left(\frac{Nl_m\bar{h}}{2}\right)$$

And

$$\phi' = \frac{\partial \phi}{\partial \tau}$$

As $\bar{h} = \bar{h}(\theta)$ is a function of θ only. $\bar{g}\left(l_m, N, h = \bar{g}(\theta)\right)$ is also function of θ only.

4.2.2 PERTURBATION TECHNIQUE

The concept of the perturbation technique is to provide small order perturbation to the eccentricity ratio and attitude angle assuming the journal to execute small harmonic oscillation about its static equilibrium position, to eliminate the time dependent terms without serious error. First order perturbation method is utilized for non-dimensional pressure and local film thickness with assumption that the journal whirls about its mean steady state position given by the eccentricity ratio ε_0 with amplitudes $\mathrm{Re}\left(\varepsilon_1 e^{i\lambda_r t}\right)$ and $\mathrm{Re}\left(\varepsilon_0 e^{i\lambda_r t}\right)$ along the line of centres and perpendicular to the line of centres respectively for which the non-dimensional pressure and local film thickness will be as follows:

$$\left.\begin{aligned} \bar{p} &= \bar{p}_0 + \bar{p}_1\varepsilon_1 e^{i\lambda R\tau} + \bar{p}_2\varepsilon_0\phi_1 e^{i\lambda R\tau} \\ \bar{h} &= \bar{h}_0 + \varepsilon_1 e^{i\lambda R\tau}\cos\theta + \varepsilon_0\phi_1 e^{i\lambda R\tau}\sin\theta \end{aligned}\right\} \quad \text{... (4.3)}$$

Where,

$$\bar{h}_0 = 1 + \varepsilon_0\cos\theta \quad ; \quad \varepsilon = \varepsilon_0 + \varepsilon_1 e^{i\lambda R\tau} \quad ; \quad \phi = \phi_0 + \phi_1 e^{i\lambda R\tau} \quad \text{and} \quad \lambda_R = \frac{\omega_p}{\omega}$$

Parameters with suffix '0' correspond to steady-state condition.

Substituting equation (4.3) into equation

$$\left(\frac{D}{L}\right)^2 \frac{\partial}{\partial \bar{z}}\left[\bar{g}\left(l_m, N, \bar{h}\right)\frac{\partial \bar{p}}{\partial \bar{z}}\right] = \frac{1}{2}(1 - 2\phi)\frac{\partial \bar{h}}{\partial \theta} + \frac{\partial \bar{h}}{\partial \tau} \text{(For narrow bearing)}$$

and collecting the first order terms of ε_1 and $\varepsilon_0\phi_1$ the following two equations are obtained.

$$C_B\left(\frac{D}{L}\right)^2 \frac{\partial^2 \overline{p_1}}{\partial \overline{z}^2} + (C_A\cos\theta)\left(\frac{D}{L}\right)^2 \frac{\partial^2 \overline{p_0}}{\partial \overline{z}^2} = -\frac{1}{2}\sin\theta + i\lambda_r\cos\theta \quad \text{... (4.4)}$$

$$C_B(\frac{D}{L})^2\frac{\partial^2\overline{p_2}}{\partial\overline{z}^2}+(C_A\sin\theta)(\frac{D}{L})^2\frac{\partial^2\overline{p_0}}{\partial\overline{z}^2}=\frac{1}{2}\cos\theta+i\lambda_r(\sin\theta-\frac{1}{\varepsilon_0}\frac{\partial\overline{h_0}}{\partial\theta}) \quad \text{... (4.5)}$$

Where, C_A and C_B are obtained from equation (3.4).

Equations (4.4) and (4.5) represent the governing differential equations for perturbed pressures.

The boundary conditions for the perturbed pressures are as follows:

1. The perturbed pressures at the ends of the bearing are zero.

$$\overline{p}_1(\theta,\pm1)=\overline{p}_2(\theta,\pm1)=0$$

2. The perturbed pressure distributions are symmetrical about the midplane of the bearing.

$$\frac{\partial\overline{p}_1(\theta,0)}{\partial\overline{z}}=\frac{\partial\overline{p}_2(\theta,0)}{\partial\overline{z}}=0 \quad \text{… (4.6)}$$

3. All perturbed pressure in the cavitated zone are zero.

$$\overline{p}_1(\theta_1,\overline{z})=\overline{p}_2(\theta_1,\overline{z})=0,$$

$$\overline{p}_1(\theta,\overline{z})=\overline{p}_2(\theta,\overline{z})=0, \qquad \theta\geq\theta_2$$

Where, θ_2 represents the angular coordinate where the film cavitates.

4.2.3 NUMERICAL SOLUTION OF PRESSURES

4.2.3.1 ANALYTICAL METHOD

Equation (4.4) and (4.5) are solved using analytical method to obtain the perturbed pressure distribution $\overline{p}_1\,\&\,\overline{p}_2$, satisfying the boundary conditions as given in equation (4.6)

$$C_B(\frac{D}{L})^2\frac{\partial^2\overline{p_1}}{\partial\overline{z}^2}+(C_A\cos\theta)(\frac{D}{L})^2\frac{\partial^2\overline{p_0}}{\partial\overline{z}^2}=-\frac{1}{2}\sin\theta+i\lambda_r\cos\theta$$

We have already calculate the value of perturbed pressure p_0

$$\overline{p}_0=\left(\frac{L}{D}\right)^2\frac{\varepsilon_0\sin\theta}{4C_B}\left(1-\overline{z}^2\right)$$

Put the value of p_0 in the above equation and we will get

$$C_B(\frac{D}{L})^2\frac{\partial^2\overline{p_1}}{\partial\overline{z}^2}+(C_A\cos\theta)\frac{(-\varepsilon_0\sin\theta)}{2C_B}=-\frac{1}{2}\sin\theta+i\lambda_r\cos\theta$$

$$C_B\left(\frac{D}{L}\right)^2 \frac{\partial^2 \overline{p_1}}{\partial \overline{z}^2} = (C_A \cos\theta)\frac{\varepsilon_0 \sin\theta}{2C_B} - \frac{1}{2}\sin\theta + i\lambda_r \cos\theta$$

Integrate the above equation on both sides

$$C_B\left(\frac{D}{L}\right)^2 \frac{\partial \overline{p_1}}{\partial \overline{z}} = \frac{1}{2}\left(\frac{C_A\varepsilon_0 \sin\theta\cos\theta}{2C_B} - \sin\theta\right)\overline{z} + i\lambda_r \cos\theta\overline{z} + C_1$$

Again integrating the above equation

$$C_B\left(\frac{D}{L}\right)^2 \overline{p}_1 = \frac{1}{2}\left(\frac{C_A\varepsilon_0 \sin\theta\cos\theta}{2C_B} - \sin\theta\right)\frac{\overline{z}^2}{2} + i\lambda_r \cos\theta\frac{\overline{z}}{2} + C_1\overline{z} + C_2$$

4.2.3.2 PRESSURE PROFILES

We have calculate the coefficients of C_1 and C_2 using the boundary condition which is given in equation (4.6). After calculating the coefficients of C_1 and C_2 we will get the value of perturbed pressure p_1

$$\boxed{\overline{p}_1 = C_B\left(\frac{L}{D}\right)^2\left[\frac{1}{4}\left(\frac{C_A\varepsilon_0 \sin\theta\cos\theta}{C_B} - \sin\theta\right) + i\lambda_r \cos\theta\right]\left[\overline{z}^2 - 1\right]}$$

Similarly we are calculating the perturbed pressure p_2

So the value of perturbed pressure p_2 is

$$\boxed{\overline{p}_2 = C_B\left(\frac{L}{D}\right)^2\left[\frac{1}{4}\left(\frac{C_A\varepsilon_0 \sin^2\theta}{C_B} + \cos\theta\right) + i\lambda_r \sin\theta\right]\left[\overline{z}^2 - 1\right]}$$

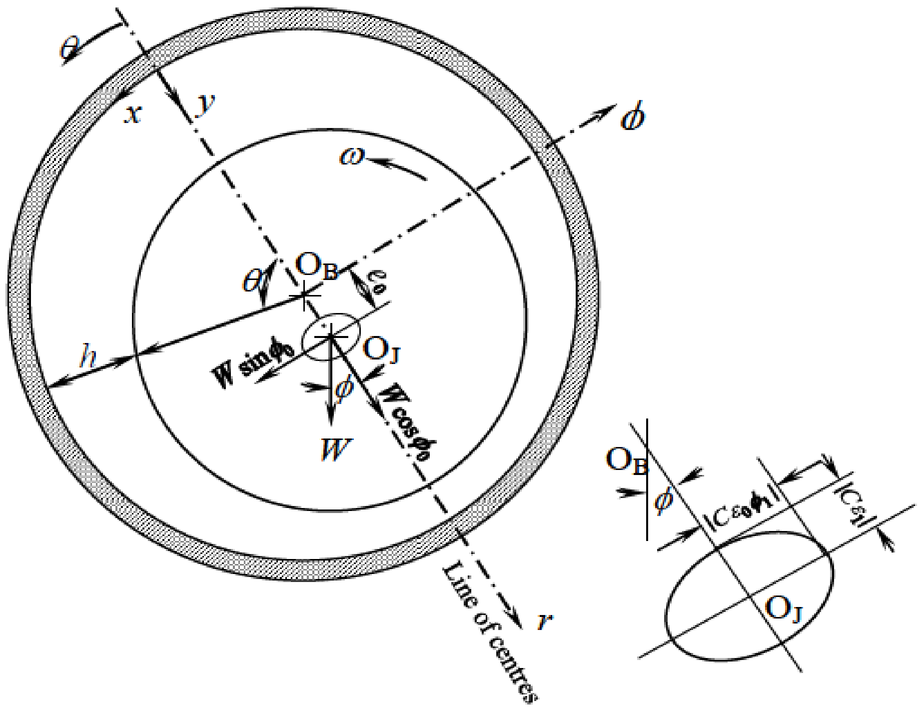

Fig 4.1. Schematic diagram of hydrodynamic journal bearing under dynamic operating condition.

4.2.4 STIFFNESS AND DAMPING COEFFICIENTS

The components of the dynamic load along the line of centres and perpendicular to the line of centres corresponding to perturbed pressure ($p_1\varepsilon_1 e^{i\lambda_R t}$) in *R*-direction can be written as under

$$(W_d)_R e^{i\lambda R\tau} = -2\int_0^{L/2}\int_{\theta_1}^{\theta_2} p_1 \cos\theta\varepsilon_1 e^{i\lambda R\tau} R d\theta dz \qquad \dots (4.7)$$

And

$$(W_d)_\phi e^{i\lambda R\tau} = -2\int_0^{L/2}\int_{\theta_1}^{\theta_2} p_1 \sin\theta\varepsilon_1 e^{i\lambda R\tau} R d\theta dz \qquad \dots (4.8)$$

As the journal executes small harmonic oscillation about its steady state position in an elliptic orbit, the components of dynamic load can be expressed as a spring load and viscous damping load as

$$(W_d)_R e^{i\lambda R\tau} = S_{RR} C\varepsilon_R + D_{RR}\frac{d}{dt}(C\varepsilon_R) \qquad \dots (4.9)$$

$$(W_d)_\phi e^{i\lambda R\tau} = S_{\phi R} C\varepsilon_R + D_{\phi R}\frac{d}{dt}(C\varepsilon_R) \qquad \dots (4.10)$$

Combining equations (4.7) through (4.10), non-dimensionalising and noting that $\varepsilon_R = \varepsilon_1 e^{i\lambda R t}$, the following non-dimensional components of stiffness and damping coefficient result

$$\left.\begin{aligned}
\bar{S}_{RR} &= -\,\mathrm{Re}\left[2\int_0^1 \int_{\theta_1}^{\theta_2} \bar{p}_1 \cos\theta d\theta d\bar{z}\right] \\
\bar{S}_{\phi R} &= -\,\mathrm{Re}\left[2\int_0^1 \int_{\theta_1}^{\theta_2} \bar{p}_1 \sin\theta d\theta d\bar{z}\right] \\
\bar{D}_{RR} &= -\,\mathrm{Im}\left[2\int_0^1 \int_{\theta_1}^{\theta_2} \bar{p}_1 \cos\theta d\theta d\bar{z}\right] \Big/ \lambda_R \\
\bar{D}_{\phi R} &= -\,\mathrm{Im}\left[2\int_0^1 \int_{\theta_1}^{\theta_2} \bar{p}_1 \sin\theta d\theta d\bar{z}\right] \Big/ \lambda_R
\end{aligned}\right\} \qquad \text{... (4.11)}$$

Similarly, considering dynamic displacement of the journal along Φ direction, it can be shown that:

$$\left.\begin{aligned}
\bar{S}_{R\phi} &= -\,\mathrm{Re}\left[2\int_0^1 \int_{\theta_1}^{\theta_2} \bar{p}_2 \cos\theta d\theta d\bar{z}\right] \\
\bar{S}_{\phi\phi} &= -\,\mathrm{Re}\left[2\int_0^1 \int_{\theta_1}^{\theta_2} \bar{p}_2 \sin\theta d\theta d\bar{z}\right] \\
\bar{D}_{R\phi} &= -\,\mathrm{Im}\left[2\int_0^1 \int_{\theta_1}^{\theta_2} \bar{p}_2 \cos\theta d\theta d\bar{z}\right] \Big/ \lambda_R \\
\bar{D}_{\phi\phi} &= -\,\mathrm{Im}\left[2\int_0^1 \int_{\theta_1}^{\theta_2} \bar{p}_2 \sin\theta d\theta d\bar{z}\right] \Big/ \lambda_R
\end{aligned}\right\} \qquad \text{... (4.12)}$$

Where,

$$\bar{S}_{ij} = \frac{2C^3 S_{ij}}{\mu\omega R^3 L} \text{ And } \bar{D}_{ij} = \frac{2C^3 D_{ij}}{\mu R^3 L},\ i = R,\phi \text{ and } j = R,\phi$$

As the dynamic pressure distributions $\bar{p}_1$ and $\bar{p}_2$ have been obtained by analytical method, the above components of stiffness and damping coefficients can be easily obtained by numerical integration using Simpson's 1/3-rd rule.

4.2.5. EQUATION OF MOTION

The dynamic pressure distribution $\bar{p}_1$ & $\bar{p}_2$ have been obtained by analytical method and stiffness and damping co-efficient also have been calculated by numerical integration using Simpson's 1/3 rule. Stiffness and damping co-efficient thus obtained can now be employed to study the stability of the flexibility supported hydrodynamic oil journal bearing can be written as:

Equation of motion of rotor in ‘r’ direction is:

$$m_R\left[A_r + \frac{d^2 e_r}{dt^2} - e_r\left(\frac{d\phi}{dt}\right)^2\right] + S_{rr}e_r + D_{rr}\frac{de_r}{dt} + S_{r\phi}e_\phi + D_{r\phi}\frac{de_\phi}{dt} = w_0\cos\phi \qquad \ldots (4.13)$$

Equation of motion of rotor in ‘Φ’ direction is:

$$m_R\left[A_\phi + \frac{d^2 e_\phi}{dt^2} + 2e_r e_\phi\right] + S_{\phi\phi}e_\phi + D_{\phi\phi}\frac{de_\phi}{dt} + S_{\phi r}e_r + D_{\phi r}\frac{de_r}{dt} = w_0\sin\phi \qquad \ldots (4.14)$$

Equation of motion of the bearing (neglecting the weight of bearing) in ‘r’ direction:

$$m_b[A_r] - S_{rr}e_r - D_{rr}\frac{de_r}{dt} - S_{r\phi}e_\phi - D_{r\phi}\frac{de_\phi}{dt} = -\left(Kx_b + d\frac{dx_b}{dt}\right)\sin\phi - \left(Ky_b + d\frac{dy_b}{dt}\right)\cos\phi$$

… (4.15)

And equation of motion of bearing in ‘Φ’ direction is:

$$m_b[A_\phi] - S_{\phi\phi}e_\phi - D_{\phi\phi}\frac{de_\phi}{dt} - S_{\phi r}e_r - D_{\phi r}\frac{de_r}{dt} = -\left(Kx_b + d\frac{dy_b}{dt}\right)\cos\phi + \left(Ky_b + d\frac{dy_b}{dt}\right)\sin\phi$$

… (4.16)

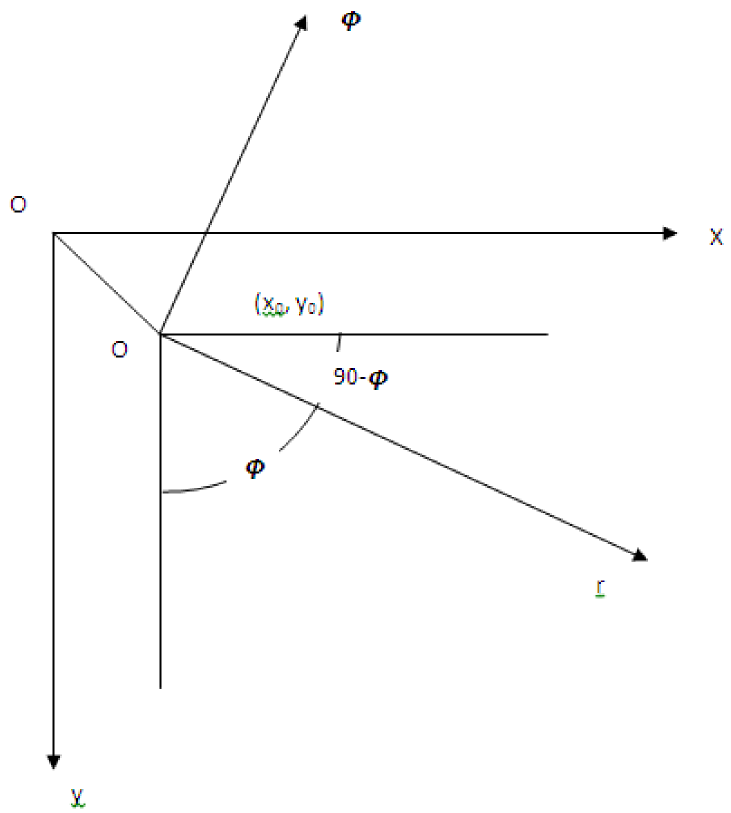

Fig 4.2 Schematic diagram of co-ordinate system

Where K and d are the external stiffness and damping co-efficient respectively.

Here for the system

$A_r = \ddot{x}_b \sin\phi + \ddot{y}_b \cos\phi$ And $A_\phi = \ddot{x}_b \cos\phi - \ddot{y}_b \sin\phi$

Now the following substitution are made

$$x_b = c.\overline{x}_b = c\left(\overline{x}_0 + \overline{x}_1 e^{i\lambda_R t}\right)$$

$$y_b = c.\overline{y}_b = c\left(\overline{y}_0 + \overline{y}_1 e^{i\lambda_R t}\right)$$

$$\phi = \phi_0 + \phi_1 e^{i\lambda_R t}$$

$$e_r = \overline{e}_r.c = c\left(\varepsilon_0 + \varepsilon_1 e^{i\lambda_R t}\right)$$

$$e_\phi = \overline{e}_\phi.c = c\left(\varepsilon_0\phi_0 + \varepsilon_0\phi_1 e^{i\lambda_R t}\right)$$

$$\cos\phi = \cos\phi_0 - \phi_1 e^{i\lambda_R t}\sin\phi_0$$

$$\sin\phi = \sin\phi_0 - \phi_1 e^{i\lambda_R t}\cos\phi_0$$

$$S_{ij} = \frac{\eta U R^2 L}{2C^3}.\overline{S}_{ij}$$

$$D_{ij} = \frac{\eta R^3 L}{2C^3}.\overline{D}_{ij}$$

$$w_0 = \frac{\eta U R^2 L}{2C^3}.\overline{w}_0$$

$$T = \omega_p.t \quad : \quad \omega_p/\omega = \lambda \quad : \quad m_b/m_R = Z$$

$\overline{m} = \dfrac{m_R C w^2}{w_0}$ (Bearing stability parameter)

Here $\overline{x}_0$, $\overline{y}_0$ are steady state non-dimensional co-ordinates of centre of bearing.

ε_0 =Steady state eccentricity ratio

ϕ_0 =steady state altitude angle

$\overline{x}_1, \overline{y}_1, \varepsilon_1, \varepsilon_0$ & ϕ_1 are 1st order perturbation

Now using the above substitution and equation co-efficient on both sides, we have the equation of motion in the following non-dimensional form

$$-\lambda^2\overline{m}\overline{w}_0\left[\overline{x}_1\sin\phi_0 + \overline{y}_1\cos\phi_0\right] + \left(-\lambda^2\overline{m}\overline{w}_0 + \overline{S}_{rr} + i\lambda\overline{D}_{rr}\right)\varepsilon_1 + \left(\overline{S}_{r\phi}\varepsilon_0 + i\lambda\overline{D}_{r\phi}\varepsilon_0 + \overline{w}_0\sin\phi_0\right)\phi_1 = 0 \quad \ldots (4.17)$$

$$-\lambda^2\overline{m}\overline{w}_0[\overline{x}_1\cos\phi_0-\overline{y}_1\sin\phi_0]+\left(\overline{S}_{\phi r}+i\lambda\overline{D}_{\phi r}\right)\varepsilon_1+\left(\overline{S}_{\phi\phi}\varepsilon_0-\lambda^2\overline{m}\overline{w}_0+\overline{S}_{rr}+i\lambda\overline{D}_{\phi\phi}\varepsilon_0+\overline{w}_0\cos\phi_0\right)\phi_1=0 \quad \text{... (4.18)}$$

$$\begin{aligned}&\left(-\lambda^2 z\overline{m}\overline{w}_0+\overline{k}+i\lambda\overline{d}\right)\sin\phi_0\overline{x}_1+\left(-\lambda^2 z\overline{m}\overline{w}_0+\overline{k}+i\lambda\overline{d}\right)\cos\phi_0\overline{y}_1+\left(\overline{S}_{rr}+i\lambda\overline{D}_{rr}\right)\varepsilon_1\\&+\left(-\overline{S}_{r\phi}\varepsilon_0-i\lambda\overline{D}_{r\phi}\varepsilon_0+\overline{k}\overline{x}_0\cos\phi_0-\overline{k}\overline{y}_0\sin\phi_0\right)\phi_1=0\end{aligned} \quad \text{... (4.19)}$$

$$\begin{aligned}&\left(-\lambda^2 z\overline{m}\overline{w}_0+\overline{k}+i\lambda\overline{d}\right)\cos\phi_0\overline{x}_1+\left(-\lambda^2 z\overline{m}\overline{w}_0-\overline{k}-i\lambda\overline{d}\right)\sin\phi_0\overline{y}_1+\left(-\overline{S}_{\phi r}-i\lambda\overline{D}_{\phi r}\right)\varepsilon_1\\&+\left(-\overline{S}_{\phi\phi}\varepsilon_0-i\lambda\overline{D}_{\phi\phi}\varepsilon_0-\overline{k}\overline{x}_0\sin\phi_0-\overline{k}\overline{y}_0\cos\phi_0\right)\phi_1=0\end{aligned} \quad \text{... (4.20)}$$

For steady state condition

$$\overline{k}\overline{x}_0\sin\phi_0+\overline{k}\overline{y}_0\cos\phi_0=\overline{S}_{rr}\varepsilon_0+\overline{S}_{r\phi}\varepsilon_0\phi_0$$

And

$$\overline{k}\overline{x}_0\cos\phi_0-\overline{k}\overline{y}_0\sin\phi_0=\overline{S}_{\phi r}\varepsilon_0+\overline{S}_{\phi\phi}\varepsilon_0\phi_0$$

From which $\overline{x}_0$ & $\overline{y}_0$ are calculated as

$$\overline{x}_0=\left(1/\overline{k}\right)\left[\left(\overline{S}_{rr}\varepsilon_0+\overline{S}_{r\phi}\varepsilon_0\phi_0\right)\sin\phi_0+\left(\overline{S}_{\phi r}\varepsilon_0+\overline{S}_{\phi\phi}\varepsilon_0\phi_0\right)\cos\phi_0\right]$$

$$\overline{y}_0=\left(1/\overline{k}\right)\left[\left(\overline{S}_{rr}\varepsilon_0+\overline{S}_{r\phi}\varepsilon_0\phi_0\right)\cos\phi_0-\left(\overline{S}_{\phi r}\varepsilon_0+\overline{S}_{\phi\phi}\varepsilon_0\phi_0\right)\sin\phi_0\right]$$

Now the following substitution are imposed

$$\overline{P}_1=\left(\overline{x}_1\sin\phi_0+\overline{y}_1\cos\phi_0\right)$$

$$\overline{q}_1=\left(\overline{y}_1\sin\phi_0-\overline{x}_1\cos\phi_0\right)$$

$$\overline{b}_1=\left(\overline{x}_0\sin\phi_0+\overline{y}_0\cos\phi_0\right)$$

$$\overline{b}_2=\left(\overline{y}_0\sin\phi_0-\overline{x}_0\cos\phi_0\right)$$

Therefore the equation of motion takes the configuration as

$$-\lambda^2\overline{m}\overline{w}_0\overline{P}_1+\left(-\lambda^2\overline{m}\overline{w}_0+\overline{S}_{rr}+i\lambda\overline{D}_{rr}\right)\varepsilon_1+\left(\overline{S}_{r\phi}\varepsilon_0-i\lambda\overline{D}_{r\phi}\varepsilon_0+\overline{w}_0\sin\phi_0\right)\phi_1=0 \quad \text{... (4.21)}$$

$$\lambda^2\overline{m}\overline{w}_0\overline{q}_1+\left(\overline{S}_{\phi r}+i\lambda\overline{D}_{\phi r}\right)\varepsilon_1+\left(-\lambda^2\overline{m}\overline{w}_0\varepsilon_0+\overline{S}_{\phi\phi}\varepsilon_0+i\lambda\overline{D}_{\phi\phi}\varepsilon_0+\overline{w}_0\cos\phi_0\right)\phi_1=0 \quad \text{... (4.22)}$$

$$\left(-z\lambda^2\overline{m}\overline{w}_0-\overline{k}-i\lambda\overline{d}\right)\overline{P}_1+\left(-\overline{S}_{rr}-i\lambda\overline{D}_{rr}\right)\varepsilon_1+\left(-\overline{S}_{r\phi}\varepsilon_0-i\lambda\overline{D}_{r\phi}\varepsilon_0-\overline{k}\overline{b}_2\right)\phi_1=0 \quad \text{... (4.23)}$$

And

$$\left(z\lambda^2 \overline{m}\overline{w}_0 - \overline{k} - i\lambda\overline{d}\right)\overline{q}_1 + \left(-\overline{S}_{\phi r} - i\lambda\overline{D}_{\phi r}\right)\varepsilon_1 + \left(-\overline{S}_{\phi\phi}\varepsilon_0 - i\lambda\overline{D}_{\phi\phi}\varepsilon_0 - \overline{k}\overline{b}_1\right)\phi_1 = 0 \qquad \text{... (4.24)}$$

Now for non-trivial solution, the determinant of the above equations (4.21), (4.22), (4.23), (4.24) must vanish. Equating real and imaginary parts of equation (obtained by expanding the determinant of the equations for non-trivial solution) separately to zero, the following two equations result:

$$A_{11}\overline{m}^4 + A_{12}\overline{m}^3 + A_{13}\overline{m}^2 + A_{14}\overline{m} + A_{15} = 0 \qquad \text{... (4.25)}$$

$$A_{12}\overline{m}^3 + A_{22}\overline{m}^2 + A_{23}\overline{m} + A_{24} = 0 \qquad \text{... (4.26)}$$

Where

$$A_{11} = C_{21}.\lambda^8\overline{w}_0^{\ 4} \qquad \text{... (4.27)}$$

$$A_{12} = C_{22}.\lambda^6\overline{w}_0^{\ 3} \qquad \text{... (4.28)}$$

$$A_{13} = C_{23}.\lambda^6\overline{w}_0^{\ 2} + C_{24}.\lambda^4\overline{w}_0^{\ 2} \qquad \text{... (4.29)}$$

$$A_{14} = C_{25}.\lambda^4\overline{w}_0 + C_{27}.\lambda^2\overline{w}_0 \qquad \text{... (4.30)}$$

$$A_{15} = C_{26}.\lambda^4 + C_{28}.\lambda^2 + C_{29} \qquad \text{... (4.31)}$$

$$A_{21} = C_{31}.\lambda^6\overline{w}_0^{\ 3} \qquad \text{... (4.32)}$$

$$A_{22} = C_{32}.\lambda^4\overline{w}_0^{\ 2} \qquad \text{... (4.33)}$$

$$A_{23} = C_{33}.\lambda^4\overline{w}_0 + C_{34}.\lambda^2\overline{w}_0 \qquad \text{... (4.34)}$$

$$A_{24} = C_{35}.\lambda^2 + C_{36} \qquad \text{... (4.35)}$$

$$C_{21} = -Z^2\varepsilon_0 \qquad \text{... (4.36)}$$

$$C_{22} = Z\overline{S}_{rr}\varepsilon_0 + Z\overline{S}_{\phi\phi}\varepsilon_0 + 2Z\overline{k}\varepsilon_0 + Z^2\overline{S}_{\phi\phi}\varepsilon_0 + Z^2\overline{w}_0\cos\phi_0 + Z\overline{k}\overline{b}_1 \qquad \text{... (4.37)}$$

$$\begin{aligned}C_{23} = {} & \overline{D}_{rr}\overline{D}_{\phi\phi}\varepsilon_0 - \overline{D}_{r\phi}\overline{D}_{\phi r}\varepsilon_0 - 2Z\overline{D}_{\phi r}\overline{D}_{r\phi}\varepsilon_0 + \overline{d}\overline{D}_{rr}\varepsilon_0 + 2Z\overline{D}_{rr}\overline{D}_{\phi\phi}\varepsilon_0 + \overline{d}\overline{D}_{\phi\phi}\varepsilon_0 + D^2\overline{D}_{rr}\overline{D}_{\phi\phi}\varepsilon_0 + \\ & 2Z\overline{d}\overline{D}_{rr}\varepsilon_0 + 2Z\overline{d}\overline{D}_{\phi\phi}\varepsilon_0 - Z^2\overline{D}_{\phi r}\overline{D}_{r\phi}\varepsilon_0 + \overline{d}^2\varepsilon_0\end{aligned} \qquad \text{... (4.38)}$$

$$
\begin{aligned}
&C_{24} = -\bar{S}_{rr}\bar{S}_{\phi\phi}\varepsilon_0 - \bar{S}_{rr}\bar{k}\bar{b}_1 + \bar{S}_{r\phi}\bar{S}_{\phi r}\varepsilon_0 + \bar{S}_{\phi r}\bar{k}\bar{b}_1 + 2Z\bar{S}_{r\phi}\bar{S}_{\phi r}\varepsilon_0 + Z\bar{S}_{\phi r}\bar{k}\bar{b}_2 \\
&- Z\bar{S}_{rr}\bar{w}_0\cos\phi_0 - \bar{k}\bar{S}_{rr}\varepsilon_0 - 2Z\bar{S}_{rr}\bar{S}_{\phi\phi}\varepsilon_0 - Z\bar{k}\bar{S}_{rr}\bar{b}_1 - \bar{k}\bar{S}_{\phi\phi}\varepsilon_0 - \bar{k}^2\bar{b}_1 + \\
&Z\bar{S}_{\phi r}\bar{w}_0\sin\phi_0 - \bar{k}^2\varepsilon_0 - 2Z\bar{k}\bar{S}_{rr}\varepsilon_0 - 2Z\bar{k}\bar{S}_{\phi\phi}\varepsilon_0 - Z^2\bar{S}_{rr}\bar{S}_{\phi\phi}\varepsilon_0 - 2Z\bar{k}\bar{w}_0\cos\phi_0 \\
&- Z^2\bar{S}_{rr}\bar{w}_0\cos\phi_0 + Z^2\bar{S}_{r\phi}\bar{S}_{\phi r}\varepsilon_0 + Z^2\bar{S}_{\phi r}\bar{w}_0\sin\phi_0
\end{aligned}
$$

... (4.39)

$$
\begin{aligned}
&C_{25} = 2\bar{k}\bar{D}_{\phi r}\varepsilon_0 + 2\bar{d}\bar{D}_{\phi r}\bar{S}_{r\phi}\varepsilon_0 + \bar{d}\bar{k}\bar{b}_2\bar{D}_{\phi r} + 2\bar{d}\bar{S}_{\phi r}\bar{D}_{r\phi}\varepsilon_0 - 2\bar{k}\bar{D}_{\phi\phi}\bar{D}_{rr}\varepsilon_0 - 2\bar{d}\bar{S}_{rr}\bar{D}_{\phi\phi}\varepsilon_0 \\
&- 2\bar{d}\bar{S}_{\phi\phi}\bar{D}_{rr}\varepsilon_0 - \bar{d}\bar{D}_{rr}\bar{w}_0\cos\phi_0 - \bar{d}\bar{D}_{rr}\bar{k}\bar{b}_1 + \bar{d}\bar{D}_{\phi r}\bar{w}_0\sin\phi_0 - \bar{d}^2\bar{S}_{rr}\varepsilon_0 - \bar{d}^2\bar{S}_{\phi\phi}\varepsilon_0 - \\
&\bar{d}^2\bar{w}_0\cos\phi_0 - 2Z\bar{k}\bar{D}_{\phi\phi}\bar{D}_{rr}\varepsilon_0 - 2\bar{d}\bar{k}\bar{D}_{rr}\varepsilon_0 - 2\bar{d}\bar{k}\bar{D}_{\phi\phi}\varepsilon_0 - Z\bar{d}\bar{D}_{rr}\bar{S}_{\phi\phi}\varepsilon_0 - 2Z\bar{d}\bar{D}_{rr}\bar{w}_0\cos\phi_0 \\
&- 2Z\bar{d}\bar{S}_{rr}\bar{D}_{\phi\phi}\varepsilon_0 + 2Z\bar{k}\bar{D}_{\phi r}\bar{D}_{r\phi}\varepsilon_0 + 2Z\bar{d}\bar{S}_{r\phi}\bar{D}_{\phi r}\varepsilon_0 + 2Z\bar{d}\bar{D}_{\phi r}\bar{w}_0\sin\varepsilon_0 + 2Z\bar{d}\bar{S}_{\phi r}\bar{D}_{r\phi}\varepsilon_0
\end{aligned}
$$

... (4.40)

$$
\begin{aligned}
&C_{27} = -2\bar{k}\bar{S}_{r\phi}\bar{S}_{\phi r}\varepsilon_0 - \bar{k}^2\bar{S}_{\phi r}\bar{b}_2 + 2\bar{k}\bar{S}_{rr}\bar{S}_{\phi\phi}\varepsilon_0 + \bar{k}\bar{S}_{rr}\bar{w}_0\cos\phi_0 + \\
&\bar{k}^2\bar{S}_{rr}\bar{b}_1 - \bar{k}\bar{S}_{\phi r}\bar{w}_0\sin\phi_0 + \bar{k}^2\bar{S}_{rr}\varepsilon_0 + \bar{k}^2\bar{S}_{\phi\phi}\varepsilon_0 + 2Z\bar{k}\bar{S}_{rr}\bar{S}_{\phi\phi}\varepsilon_0 + \bar{k}^2\bar{w}_0\cos\phi_0 + \\
&2Z\bar{k}\bar{S}_{rr}\bar{w}_0\cos\varepsilon_0 - 2Z\bar{k}\bar{S}_{r\phi}\bar{S}_{\phi r}\varepsilon_0 - 2Z\bar{k}\bar{S}_{\phi r}\bar{w}_0\sin\phi_0
\end{aligned}
$$

... (4.41)

$$C_{26} = -\bar{d}^2\bar{D}_{rr}\bar{D}_{\phi\phi}\varepsilon_0 + \bar{d}^2\bar{D}_{r\phi}\bar{D}_{\phi r}\varepsilon_0$$

... (4.42)

$$
\begin{aligned}
&C_{28} = -\bar{d}^2\bar{S}_{rr}\bar{S}_{\phi\phi}\varepsilon_0 + \bar{d}^2\bar{S}_{rr}\bar{w}_0\cos\phi_0 + \bar{k}^2\bar{D}_{rr}\bar{D}_{\phi\phi}\varepsilon_0 + 2\bar{d}\bar{k}\bar{D}_{rr}\bar{S}_{\phi\phi}\varepsilon_0 + \\
&2\bar{d}\bar{k}\bar{D}_{rr}\bar{w}_0\cos\phi_0 + 2\bar{d}\bar{k}\bar{D}_{\phi\phi}\bar{S}_{rr}\varepsilon_0 - \bar{k}^2\bar{D}_{\phi r}\bar{D}_{r\phi}\varepsilon_0 - \bar{d}^2\bar{S}_{r\phi}\bar{S}_{\phi r}\varepsilon_0 - \bar{d}^2\bar{S}_{\phi r}\bar{w}_0\sin\phi_0 - \\
&2\bar{d}\bar{k}\bar{S}_{r\phi}\bar{D}_{\phi r}\varepsilon_0 - 2\bar{d}\bar{k}\bar{D}_{\phi r}\bar{w}_0\sin\phi_0 - 2\bar{d}\bar{k}\bar{S}_{\phi r}\bar{D}_{r\phi}\varepsilon_0
\end{aligned}
$$

... (4.43)

$$C_{29} = -\bar{k}^2\bar{S}_{rr}\bar{S}_{\phi\phi}\varepsilon_0 - \bar{k}^2\bar{S}_{rr}\bar{w}_0\cos\phi_0 + \bar{k}^2\bar{S}_{r\phi}\bar{S}_{\phi r}\varepsilon_0 + \bar{k}^2\bar{S}_{\phi r}\bar{w}_0\sin\phi_0$$

... (4.44)

$$C_{31} = Z\bar{D}_{rr}\varepsilon_0 + Z\bar{D}_{\phi\phi}\varepsilon_0 + 2Z\bar{d}\varepsilon_0 + Z^2\bar{D}_{rr}\varepsilon_0 + Z^2\bar{D}_{\phi\phi}\varepsilon_0 = 0$$

... (4.45)

$$
\begin{aligned}
C_{32} ={}& -\bar{S}_{rr}\bar{D}_{\phi\phi}\varepsilon_0 - \bar{D}_{rr}\bar{S}_{\phi\phi}\varepsilon_0 - \bar{D}_{rr}\bar{k}\bar{b}_1 + \bar{S}_{r\phi}\bar{D}_{\phi r}\varepsilon_0 + \bar{D}_{\phi r}\bar{k}\bar{b}_2 + \bar{S}_{\phi r}\bar{D}_{r\phi}\varepsilon_0 + 2Z\bar{D}_{\phi r}\bar{S}_{r\phi}\varepsilon_0 + \\
& Z\bar{D}_{r\phi}\bar{k}\bar{b}_2 + 2Z\bar{S}_{\phi r}\bar{D}_{r\phi}\varepsilon_0 - Z\bar{D}_{rr}w_0\cos\phi_0 - \bar{k}\bar{D}_{rr}\varepsilon_0 + Z\bar{D}_{\phi r}w_0\sin\phi_0 - Z\bar{d}\bar{k}\varepsilon_0 - 2Z\bar{d}\bar{S}_{\phi\phi}\varepsilon_0 \\
& -2Z\bar{d}w_0\cos\phi_0 - Z^2\bar{S}_{\phi\phi}\bar{D}_{rr}\varepsilon_0 - Z^2\bar{D}_{rr}w_0\cos\phi_0 - Z^2\bar{S}_{rr}\bar{D}_{\phi\phi}\varepsilon_0 + Z^2\bar{D}_{\phi r}\bar{S}_{r\phi}\varepsilon_0 + \\
& Z^2\bar{D}_{\phi r}w_0\sin\phi_0 + Z^2\bar{S}_{\phi r}\bar{D}_{r\phi}\varepsilon_0 - 2Z\bar{k}\bar{D}_{rr}\varepsilon_0 - 2Z\bar{k}\bar{D}_{\phi\phi}\varepsilon_0
\end{aligned}
\qquad \ldots (4.46)
$$

$$
C_{33} = 2\bar{d}\bar{D}_{\phi r}\bar{D}_{r\phi}\varepsilon_0 - 2\bar{d}\bar{D}_{rr}\bar{D}_{\phi\phi}\varepsilon_0 - 2Z\bar{d}\bar{D}_{rr}\bar{D}_{\phi\phi}\varepsilon_0 - \bar{d}^2\bar{D}_{rr}\varepsilon_0 - \bar{d}^2\bar{D}_{\phi\phi}\varepsilon_0 \qquad \ldots (4.47)
$$

$$
\begin{aligned}
C_{34} ={}& -2\bar{D}_{\phi r}\bar{S}_{r\phi}\varepsilon_0 - k\bar{D}_{r\phi}\bar{b}_2 - 2\bar{k}\bar{S}_{\phi r}\bar{D}_{r\phi}\varepsilon_0 - 2\bar{d}\bar{S}_{\phi r}k\bar{b}_2 + 2\bar{k}\bar{S}_{rr}\bar{D}_{\phi\phi}\varepsilon_0 + 2\bar{k}\bar{S}_{\phi\phi}\bar{D}_{rr}w_0\cos\phi_0 + \\
& 2\bar{d}\bar{S}_{\phi\phi}\bar{S}_{rr}\varepsilon_0 + \bar{d}\bar{S}_{rr}w_0\cos\phi_0 + \bar{d}\bar{k}\bar{b}_1\bar{S}_{rr} + \bar{k}^2\bar{b}_1\bar{D}_{rr} - \bar{d}\bar{S}_{\phi r}w_0\sin\phi_0 - \bar{k}\bar{D}_{\phi r}w_0\sin\phi_0 + \\
& 2\bar{d}\bar{k}\bar{S}_{rr}\varepsilon_0 + 2\bar{d}\bar{k}\bar{S}_{\phi\phi}\varepsilon_0 + 2\bar{d}\bar{k}w_0\cos\phi_0 + 2Z\bar{d}\bar{S}_{rr}\bar{S}_{\phi\phi}\varepsilon_0 + 2Z\bar{d}\bar{S}_{rr}w_0\cos\phi_0 + \bar{k}^2\bar{D}_{\phi\phi}\varepsilon_0 + \\
& 2Z\bar{k}\bar{S}_{\phi\phi}\bar{D}_{rr}\varepsilon_0 + 2Z\bar{k}\bar{D}_{rr}w_0\cos\phi_0 + 2Z\bar{k}\bar{S}_{rr}\bar{D}_{\phi\phi}\varepsilon_0 - 2Z\bar{k}\bar{D}_{\phi r}w_0\sin\phi_0 - 2Z\bar{k}\bar{S}_{\phi r}\bar{D}_{r\phi}\varepsilon_0 - \\
& 2Z\bar{d}\bar{S}_{r\phi}\bar{S}_{\phi r}\varepsilon_0 - 2Z\bar{d}\bar{S}_{\phi r}w_0\sin\phi_0
\end{aligned}
\qquad \ldots (4.48)
$$

$$
\begin{aligned}
C_{35} ={}& 2\bar{d}\bar{k}\bar{D}_{rr}\bar{D}_{\phi\phi}\varepsilon_0 + \bar{d}^2\bar{S}_{\phi\phi}\bar{D}_{rr}\varepsilon_0 + \bar{d}^2\bar{D}_{rr}\bar{w}_0\cos\phi_0 + \bar{d}^2\bar{S}_{rr}\bar{D}_{\phi\phi}\varepsilon_0 - \bar{d}^2\bar{S}_{r\phi}\bar{D}_{\phi r}\varepsilon_0 - \\
& \bar{d}^2\bar{D}_{\phi r}\bar{w}_0\sin\phi_0 - \bar{d}^2\bar{S}_{\phi r}\bar{D}_{r\phi}\varepsilon_0
\end{aligned}
\qquad \ldots (4.49)
$$

$$
\begin{aligned}
C_{36} ={}& 2\bar{d}\bar{k}\bar{S}_{rr}\bar{S}_{\phi\phi}\varepsilon_0 - 2\bar{d}\bar{k}\bar{S}_{rr}\bar{w}_0\cos\phi_0 - \bar{k}^2\bar{S}_{\phi\phi}\bar{D}_{rr}\varepsilon_0 - \bar{k}^2\bar{D}_{rr}\bar{w}_0\cos\phi_0 - \bar{k}^2\bar{S}_{rr}\bar{D}_{\phi\phi}\varepsilon_0 + \\
& \bar{k}^2\bar{S}_{r\phi}\bar{D}_{\phi r}\varepsilon_0 + \bar{k}^2\bar{D}_{\phi r}\bar{w}_0\sin\phi_0 + \bar{k}^2\bar{S}_{\phi r}\bar{D}_{r\phi}\varepsilon_0 + 2\bar{d}\bar{k}\bar{S}_{r\phi}\bar{S}_{\phi r}\varepsilon_0 + 2\bar{d}\bar{k}\bar{S}_{\phi r}\bar{w}_0\sin\phi_0
\end{aligned}
\qquad \ldots (4.50)
$$

The critical mass parameter & whirl ratio will be determined by satisfying the equations (4.25) & (4.26) employing Newton Raphson method.

4.3 RESULTS AND DISCUSSION

Since the dynamic characteristics expressed in terms of non-dimensional components of stiffness and damping coefficients and the critical mass parameter along with the corresponding whirl ratio, representing the stability of the journal, are dependent on the steady state and perturbed film pressures $\overline{P_0}$, $\overline{P_1}$ and $\overline{P_2}$ and thus, in turn, depend upon the micropolar parameters (l_m, N), eccentricity ratio ε_o, detailed parametric studies are done to show their effects on the non-dimensional components of stiffness and damping coefficient and finally on critical mass parameter and whirl ratio.

4.3.1 NON-DIMENSIONAL COMPONENTS OF STIFFNESS AND DAMPING COEFFICIENTS

4.3.1.1 EFFECT OF COUPLING NUMBER (N)

Figs.4.3 to 4.10 shows the variation of the non-dimensional components of stiffness and damping coefficient as function of l_m, when coupling number (N) is considered as a parameter, keeping ε_o and L/D fixed at 0.5 and 0.1 respectively.

It can be observed from the figures that at any value l_m, the direct components of stiffness $\overline{S}_{RR}$ and $\overline{S}_{\phi\phi}$ increase with an increase in coupling number. For any of value of N, $\overline{S}_{RR}$ and $\overline{S}_{\phi\phi}$ decrease with l_m and all the family of curves converge to the asymptotic value of these for Newtonian fluid as l_m becomes infinitely large. However, at $l_m \to 0$, $\overline{S}_{RR}$ and $\overline{S}_{\phi\phi}$ are much larger compared to those of Newtonian fluids. The reason for such enhanced components of stiffness at $l_m \to 0$ is the same as that for load parameter. Cross components of stiffness $\overline{S}_{R\phi}$ and $\overline{S}_{\phi R}$ follow the same trend as the direct components.

Direct and cross components of damping coefficient increase with coupling number for any value of l_m, but at large l_m $(l_m \to \infty)$, they approach the corresponding values for Newtonian fluid.

4.3.1.2 EFFECT OF STEADY STATE ECCENTRICITY RATIO (ε_o)

Figs.4.11 to 4.18 show the variations of non-dimensional components of stiffness and damping coefficients for different values of ε_o and l_m, when other factors *viz., L/D* and N are kept fixed. It can be observed from the referred figures that an increase in ε_o increases the magnitudes of the dynamic coefficients. It is further seen that all the curves in the micropolar fluid lubrication regime converge to those for Newtonian fluid lubrication regime at $l_m \rightarrow \infty$ at $l_m \rightarrow 0$ the values of all the coefficients are more than those for Newtonian fluid because of the reason discussed in Chapter 2.

4.3.2 CRITICAL MASS PARAMETER $(\overline{M}_C)$

4.3.2.1 EFFECT OF COUPLING NUMBER (*N*)

Fig.4.19 shows the variation of critical mass parameter representing the threshold of stability as function of l_m, when ε_o = 0.3, $\overline{K}$=.001, $\overline{d}$ =.001and *L/D* = 0.1 and *N* is considered as a parameter. It is found from the figure that the critical mass parameter increases as *N* is increased and it is also shown in the figure that the critical mass parameter is suddenly up and down in the range of 0.17 to 0.3 at eccentricity ratio 0.1 and 0.3 although critical mass parameter is increased when the value of l_m is increased at eccentricity ratio 0.5 to 0.9. Moreover, at particular value of *N (0.5, 0.7 & 0.9),* stability improves initially as l_m is increased, reaches a maximum to that for Newtonian fluid at large value of l_m.

The increase in the stability parameter in micropolar lubricant regime can be explained from the steady state analysis in which it has been shown in steady state analysis that load parameter of a journal bearing increases in the micropolar fluid than in the Newtonian fluid. So, with the same load in a Newtonian fluid the journal will run at a higher eccentricity ratio than that in micropolar lubricant and consequently the stability parameter in micropolar lubricant regime will increase.

4.3.2.2 EFFECT OF STEADY STATE ECCENTRICITY RATIO (ε_o)

The effect of variation of ε_o on $\overline{M}_C$ for N^2 = 0.5, $\overline{K}$=.001, $\overline{d}$ =.001 and *L/D* = 0.1 is shown in *Fig.4.20* It is found from the figure that the critical mass parameter increases with increase in l_m at ε_o =0.1 and ε_o =0.3 but at ε_o =0.5 critical mass parameter decrease with increase in l_m. For $\varepsilon_o \leq$ 0.3, the stability parameter is less in micropolar lubrication compared to that in Newtonian fluid. However, at higher ε_o (ε_o = 0.5) it is found that stability parameter is more in micropolar lubrication compared to that in Newtonian fluid. All the curves, however, show the tendency of converging to the value corresponding to the Newtonian fluid at large value of l_m.

Fig. 4.24 shows the variation of critical mass parameter ($\overline{M}_C$) with external stiffness ($\overline{K}$) at various values of ε_o at N^2 = 0.1, l_m =5, $\bar{d}$ =.001, $\overline{Z}$ =.1 and L/D= 0.1.It is found from the figure that critical mass parameter ($\overline{M}_C$) initially decrease at very low value of ($\overline{K}$) and then increase with increasing the value of ($\overline{K}$) at different eccentricity ratio (ε_o). As we can shown in the figure that the critical mass parameter increase with decrease in eccentricity ratio. Threshold stability decreases with eccentricity ratio.

4.3.2.3 EFFECT OF EXTERNAL DAMPING COEFFICIENTS ($\bar{d}$)

Fig. 4.23 shows the variation of critical mass parameter ($\overline{M}_C$) with external stiffness ($\overline{K}$) at various values of $\bar{d}$ at N^2 = 0.1, l_m =5, $\bar{d}$ =.001, $\overline{Z}$ =.1 and L/D= 0.1. It is found from the fig. that critical mass parameter ($\overline{M}_C$) decrease with increase in external stiffness ($\overline{K}$) at $\bar{d}$ =0.01 but at $\bar{d}$ =0.001 critical mass parameter ($\overline{M}_C$) approximately zero with increase in external stiffness ($\overline{K}$). Threshold stability decreases with damping.

4.3.3 WHIRL RATIO (λ_R)

4.3.3.1 EFFECT OF COUPLING NUMBER (*N*)

Fig.4.21 shows the variation of whirl ratio with l_m, when *N* is taken as a parameter for ε_o = 0.5 and *L/D* = 0.1. It can be seen from the figure that as coupling number increases, whirl ratio decreases. At a particular value of *N*, the whirl ratio increase with increase in l_m at eccentricity ratio 0.1 to 0.7 but at eccentricity ratio 0.9 whirl ratio initially decrease then increase with l_m , and ultimately converges to that for Newtonian fluid when l_m is indefinitely large. However, the variation of whirl ratio is very small.

4.3.3.2 EFFECT OF STEADY STATE ECCENTRICITY RATIO (ε_o)

The effect of ε_o on whirl ratio can be found from *Fig.4.22* as a function of l_m for N^2 = 0.3 and *L/D* = 0.1. It can be seen from the figure that as eccentricity ratio increases, whirl ration increases but in the range of 0.1 to 0.3. After that whirl ratio decreases. Whirl ratio increases with increase in l_m, and ultimately converges to that for Newtonian fluid when l_m is indefinitely large.

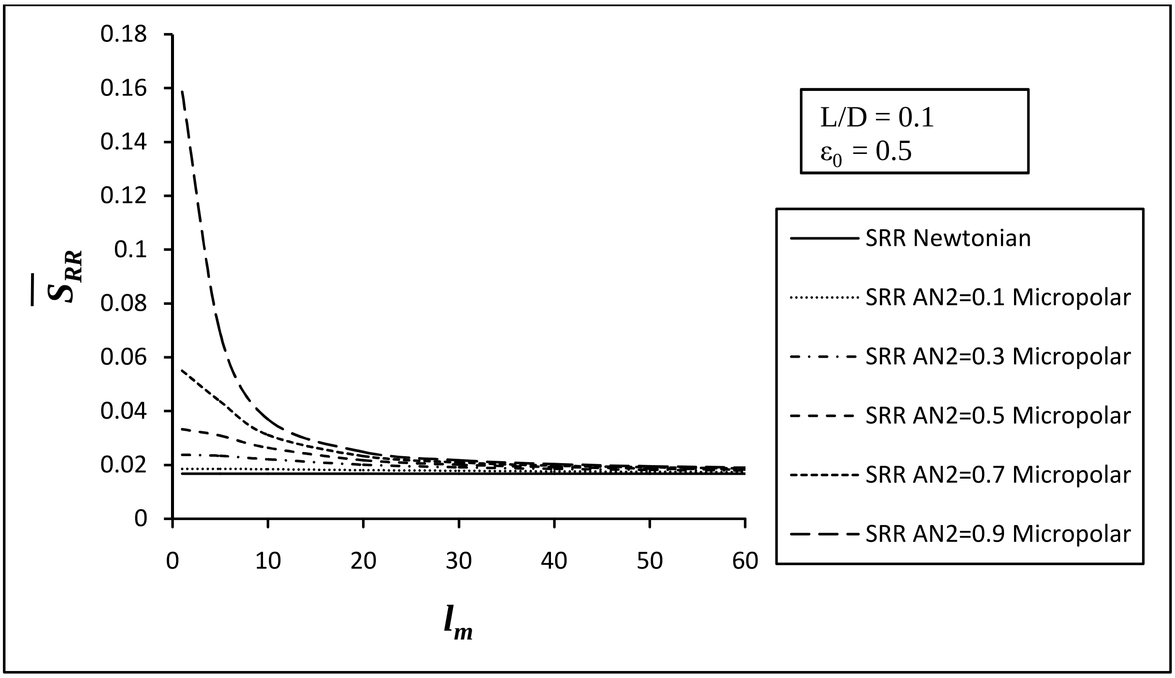

Fig.4.3: Variation of $\overline{S}_{RR}$ with l_m for various values of N (AN).

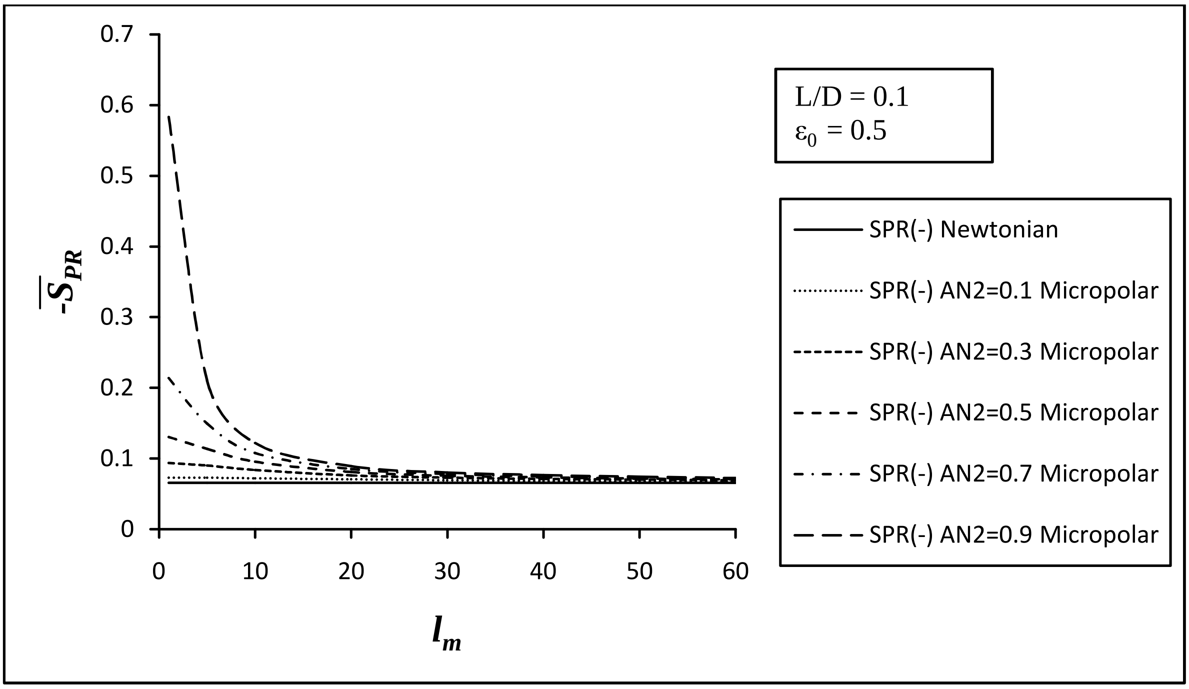

Fig.4.4: Variation of $\overline{S}_{PR}$ with l_m for various values of N (AN).

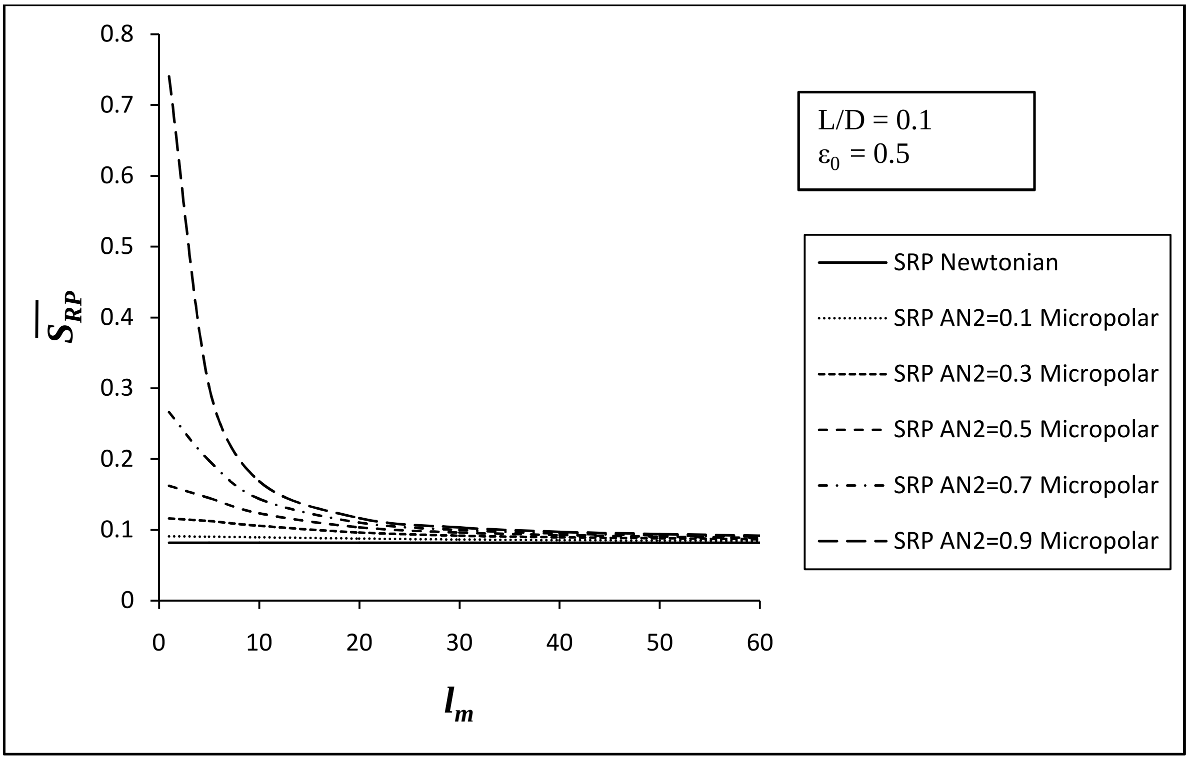

Fig.4.5: Variation of $\overline{S}_{RP}$ with l_m for various values of N (AN).

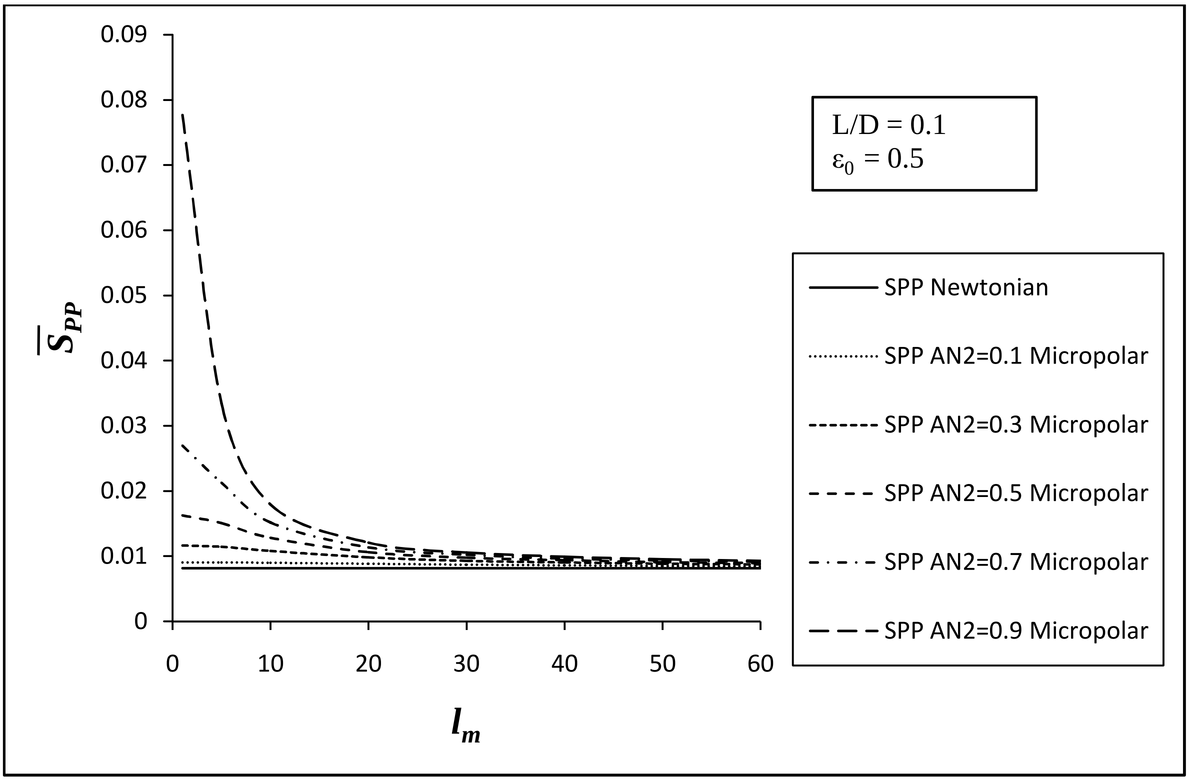

Fig.4.6: Variation of $\overline{S}_{PP}$ with l_m for various values of N (AN).

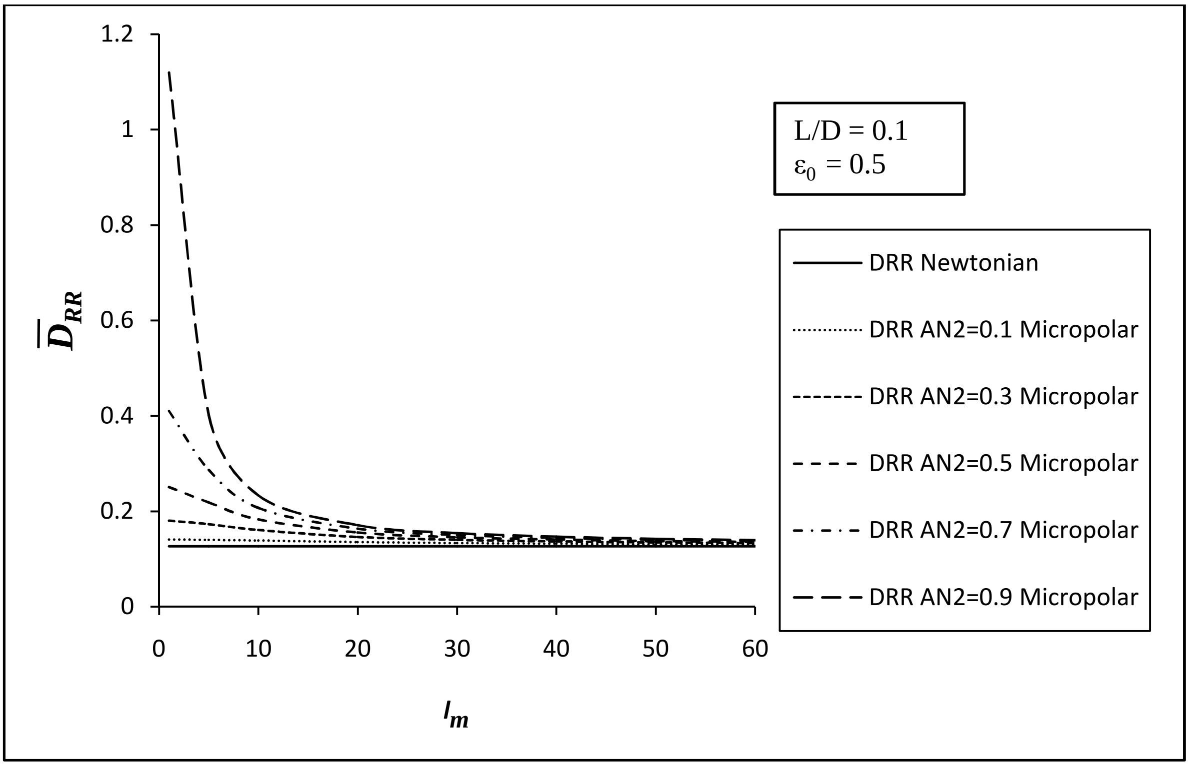

Fig.4.7: Variation of $\overline{D}_{RR}$ with l_m for various values of N (AN).

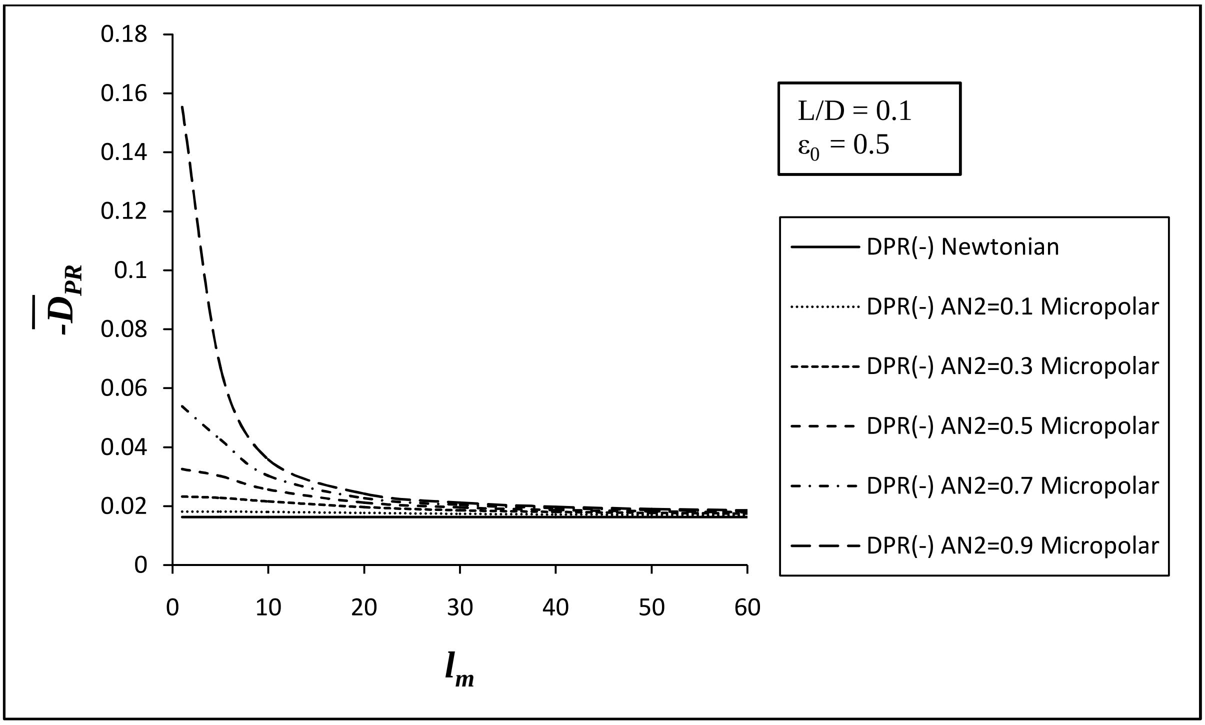

Fig.4.8: Variation of $\overline{D}_{PR}$ with l_m for various values of N (AN).

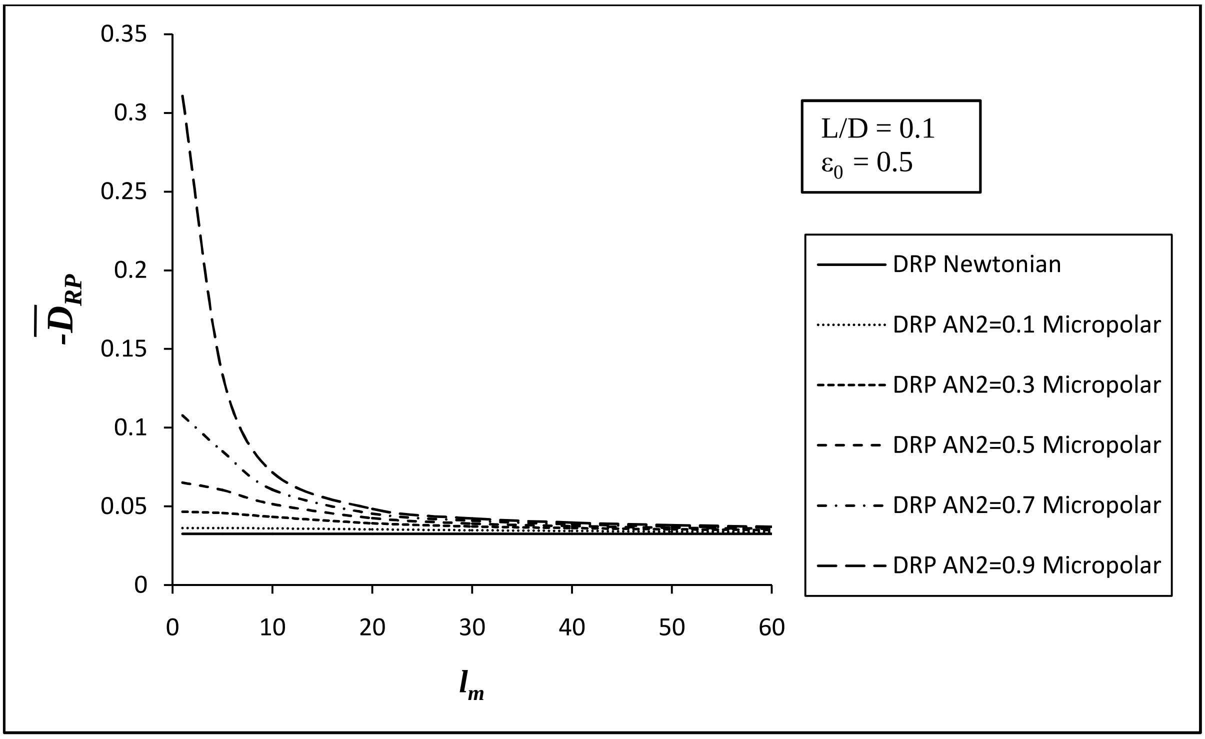

Fig.4.9: Variation of $\overline{D}_{RP}$ with l_m for various values of N (AN).

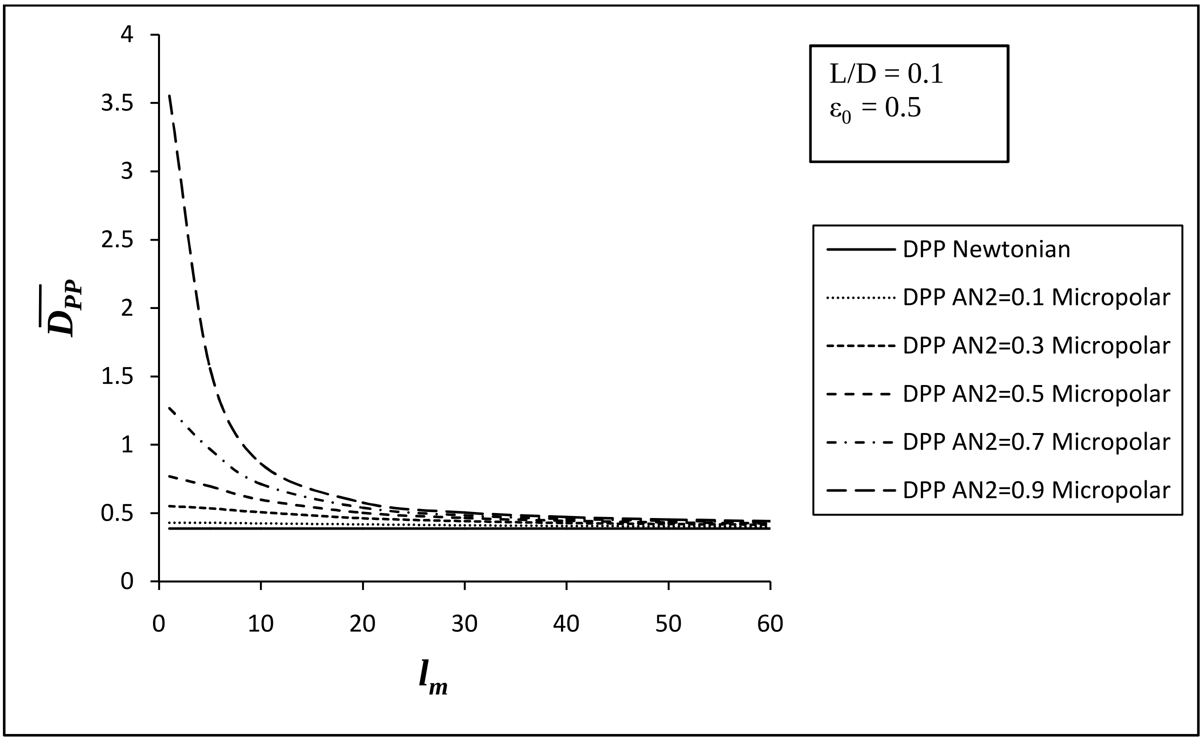

Fig.4.10: Variation of $\overline{D}_{PP}$ with l_m for various values of N (AN).

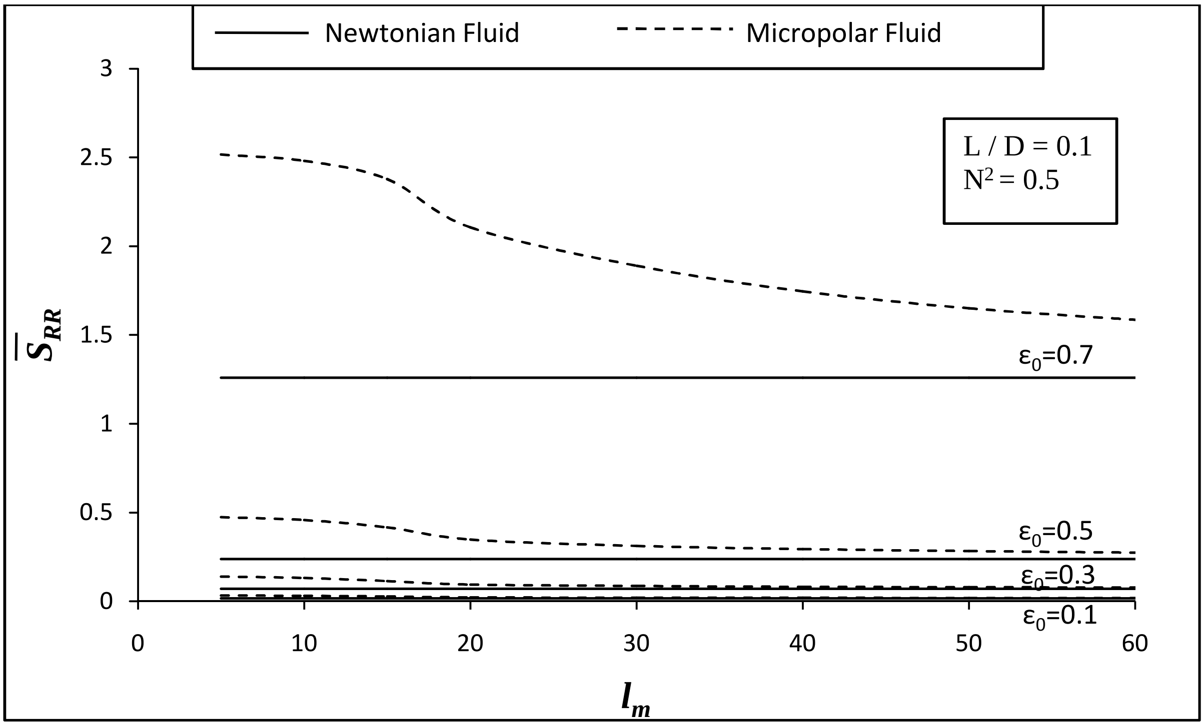

Fig.4.11: Variation of $\overline{S}_{RR}$ with l_m for various values of ε_0

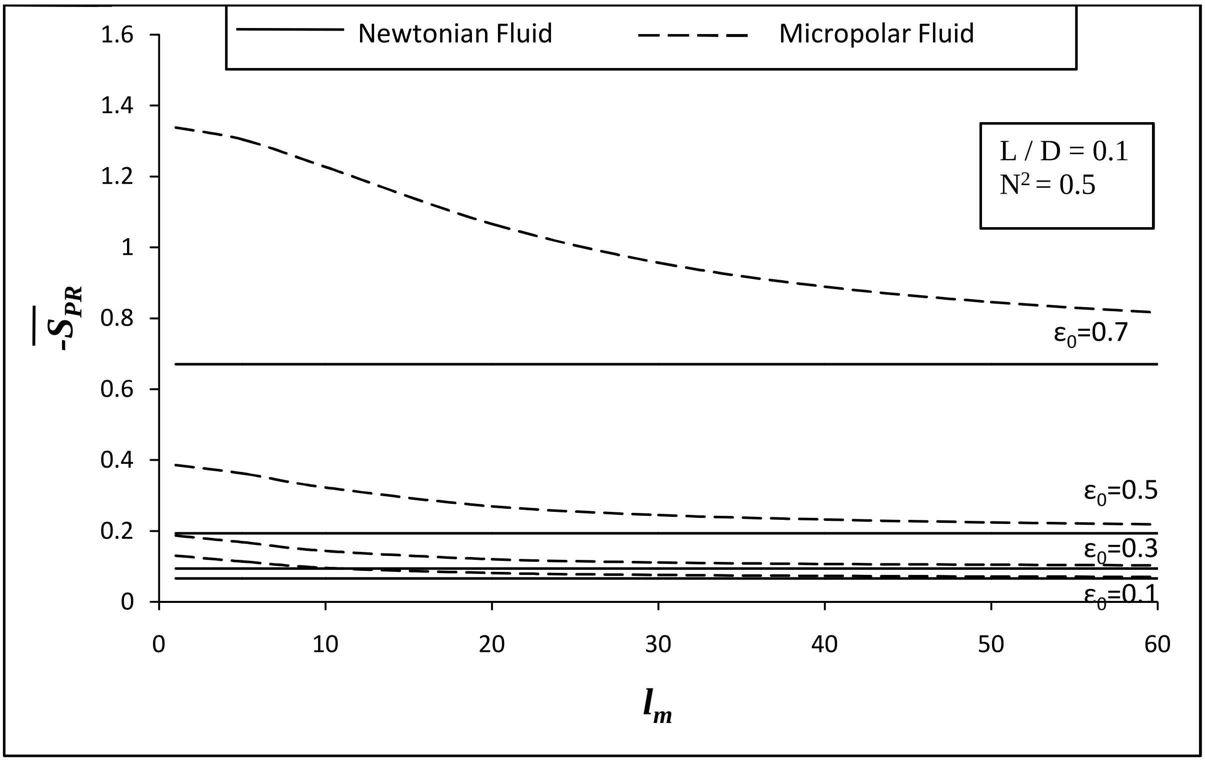

Fig.4.12: Variation of $\overline{S}_{PR}$ with l_m for various values of ε_0.

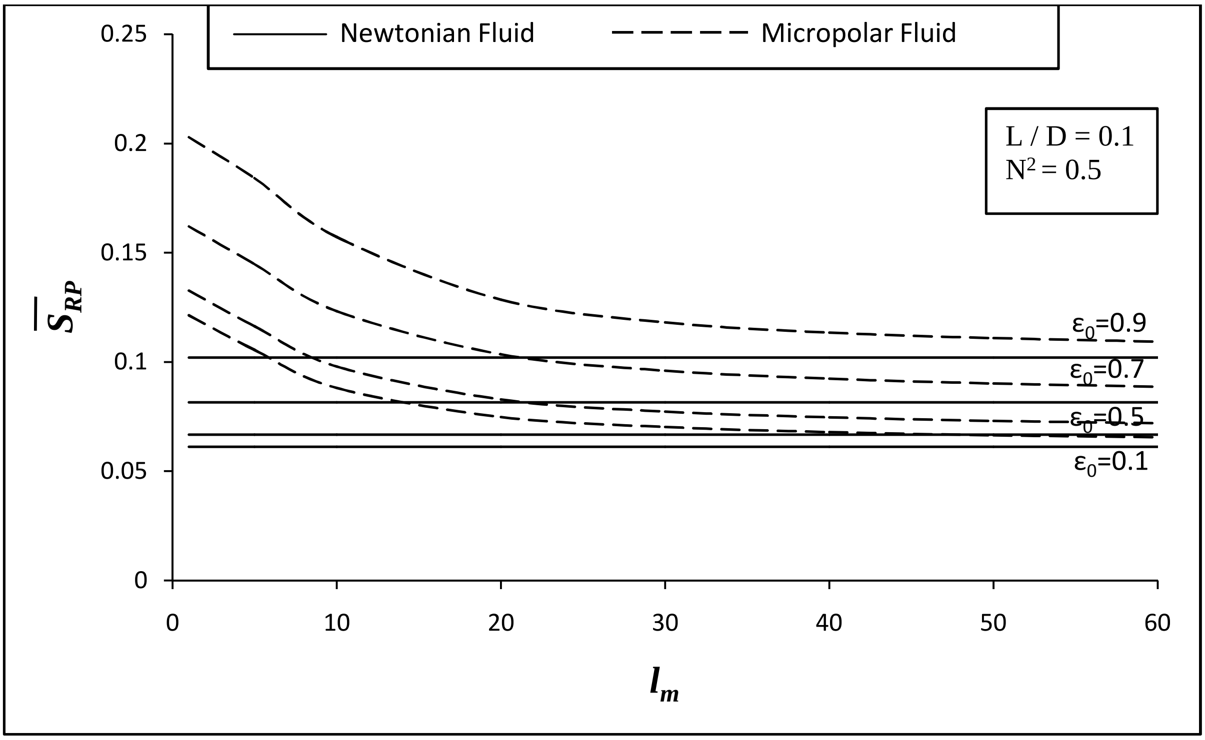

Fig.4.13: Variation of $\overline{S}_{RP}$ with l_m for various values of ε_0.

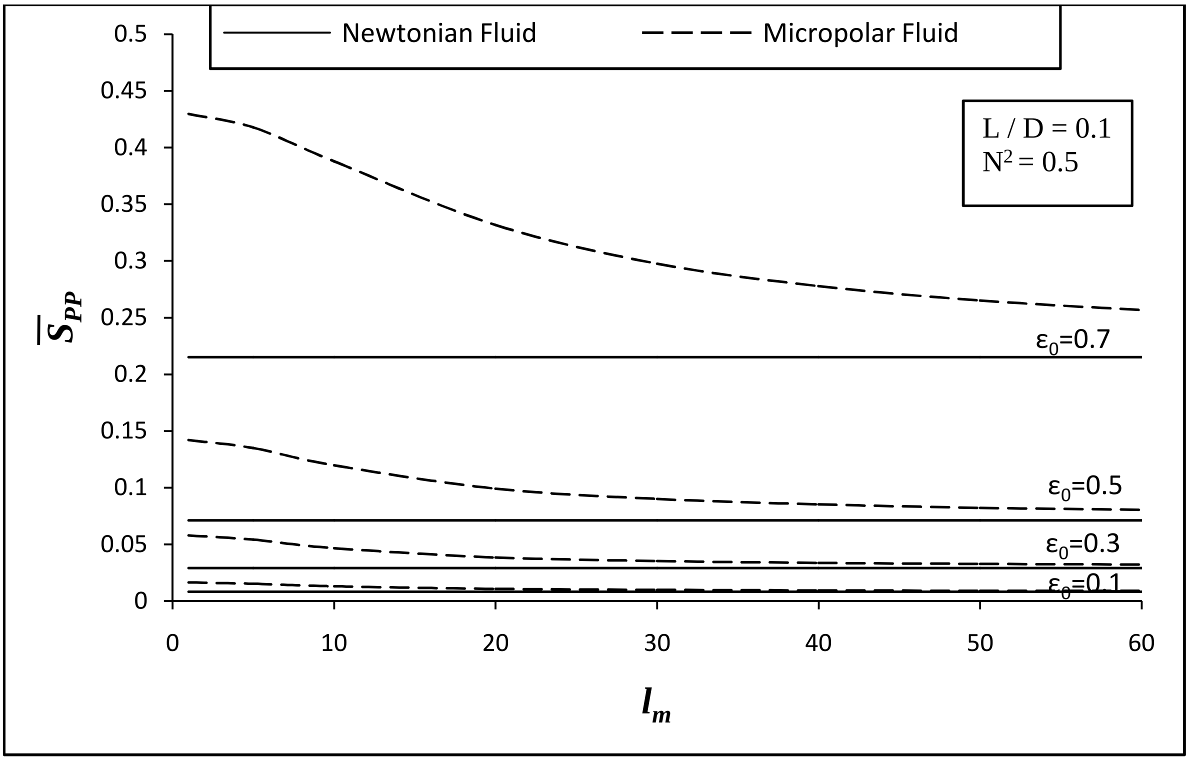

Fig.4.14: Variation of $\overline{S}_{PP}$ with l_m for various values of ε_0.

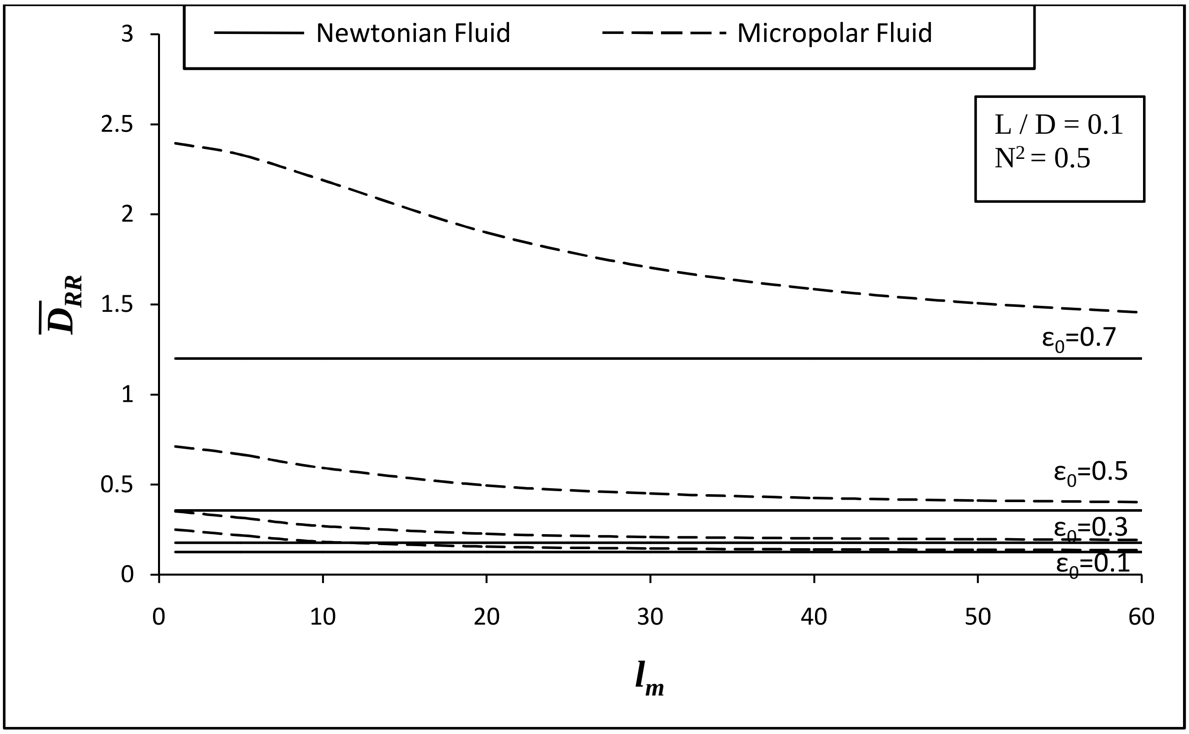

Fig.4.15: Variation of $\overline{\mathrm{D}}_{\mathrm{RR}}$ with l_m for various values of ε_0.

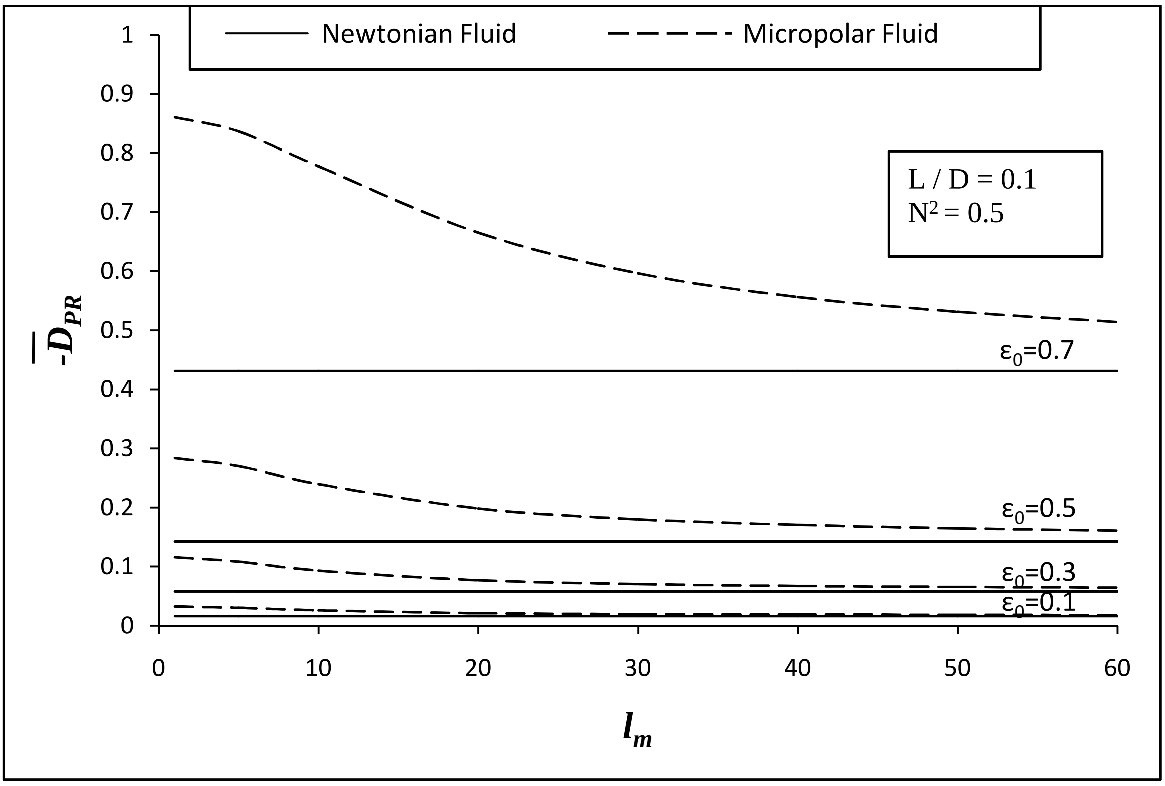

Fig.4.16: Variation of $\overline{\mathrm{D}}_{\mathrm{PR}}$ with l_m for various values of ε_0.

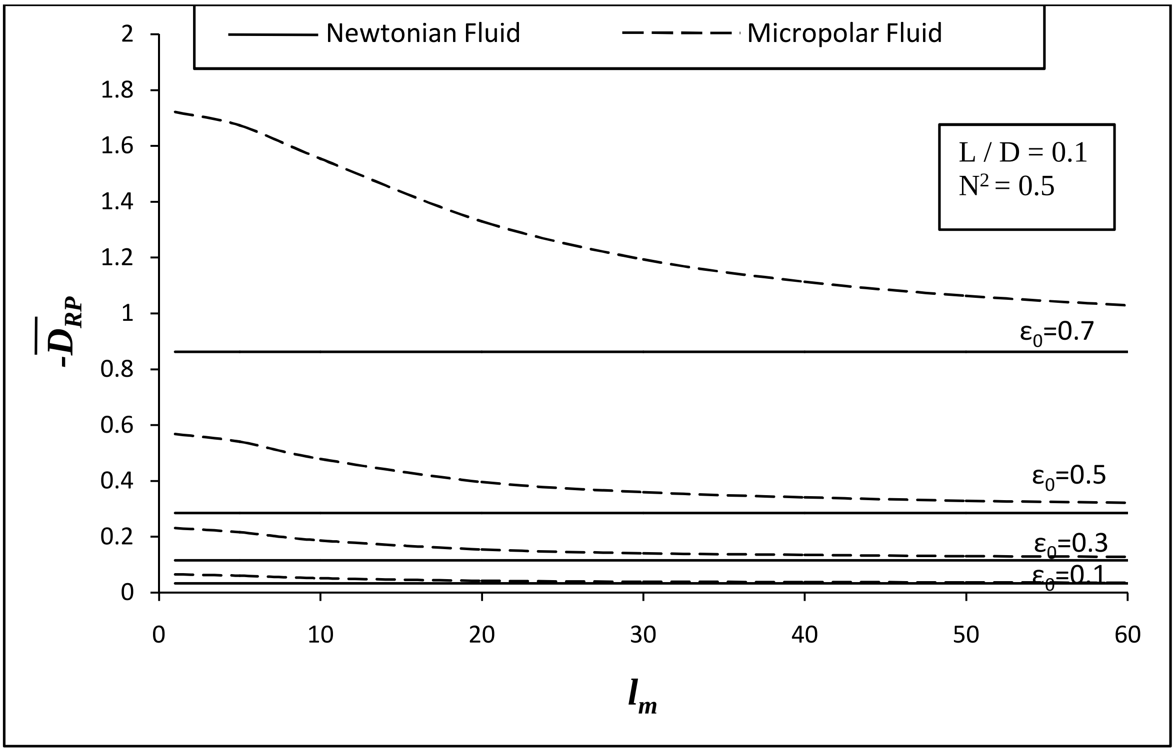

Fig.4.17: Variation of $\overline{D}_{RP}$ with l_m for various values of ε_0.

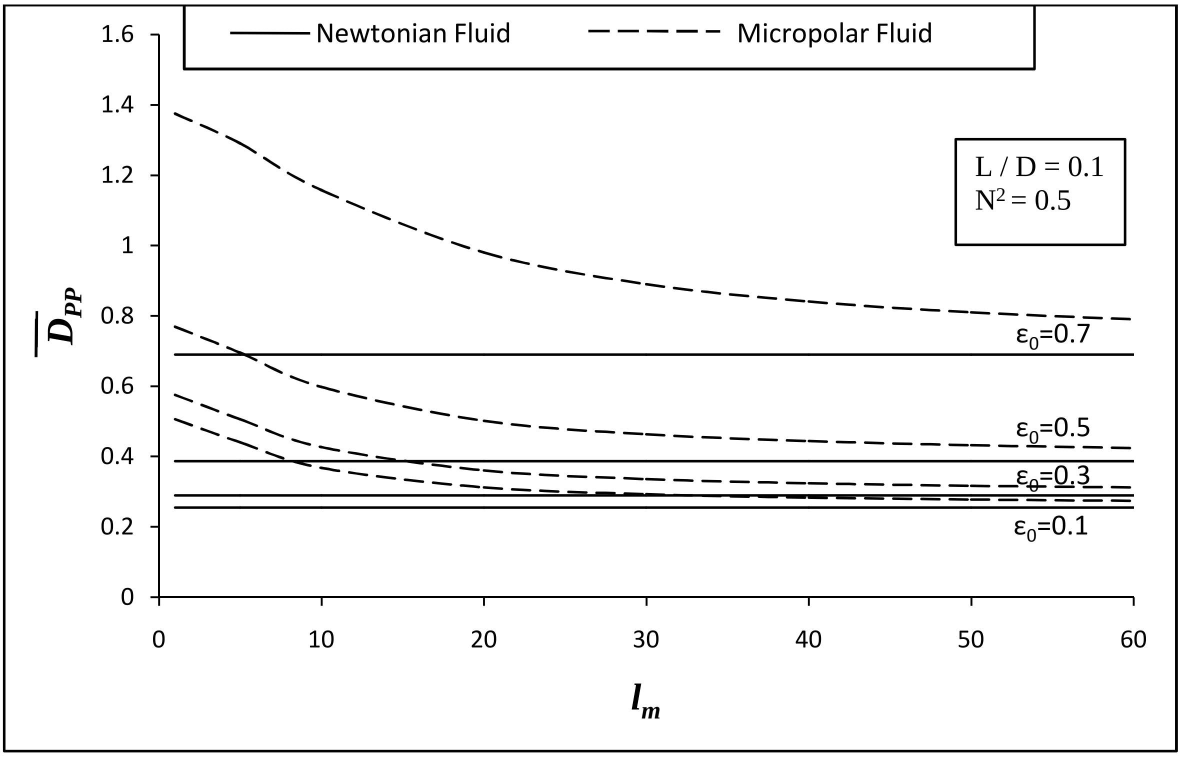

Fig.4.18: Variation of $\overline{D}_{PP}$ with l_m for various values of ε_0.

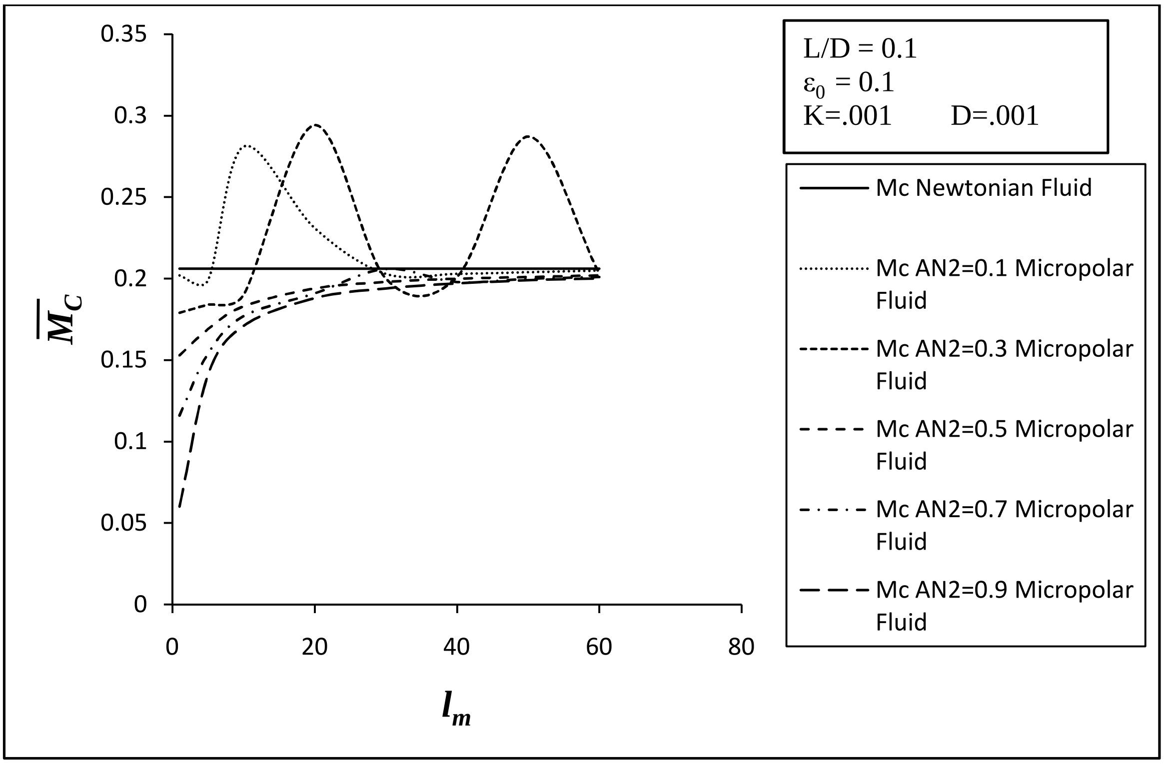

Fig.4.19: Variation of $\overline{M}_C$ with l_m for various values of N (AN).

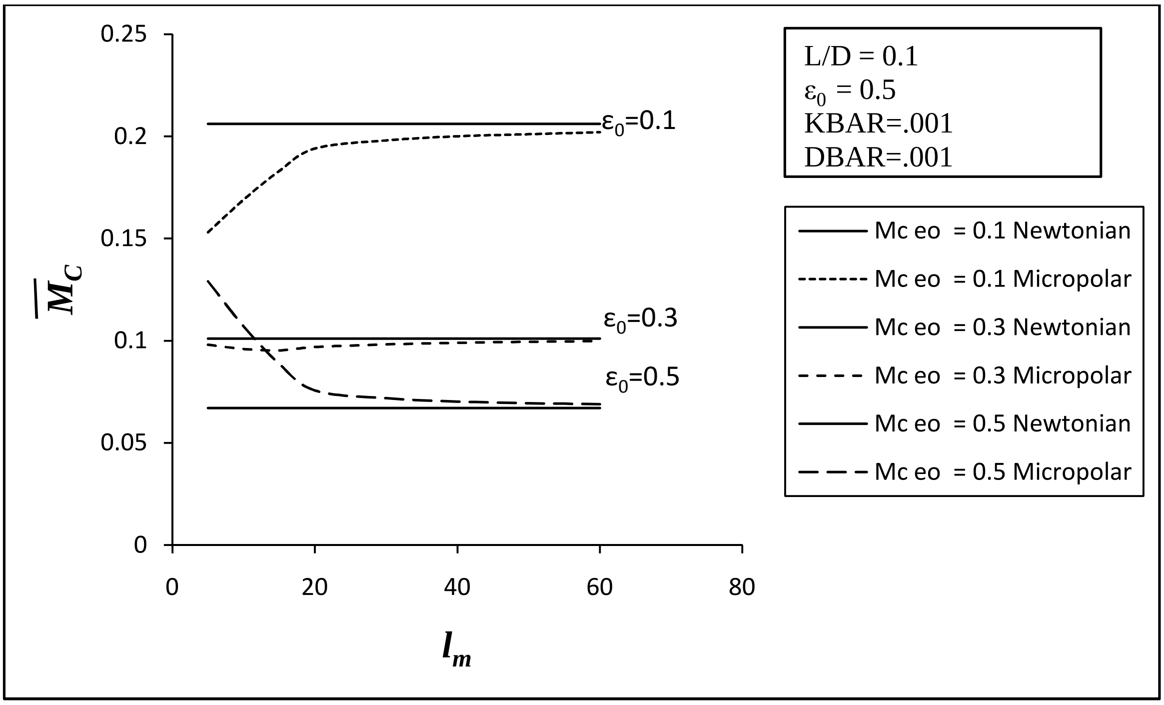

Fig.4.20: Variation of $\overline{M}_C$ with l_m for various values of ε_0.

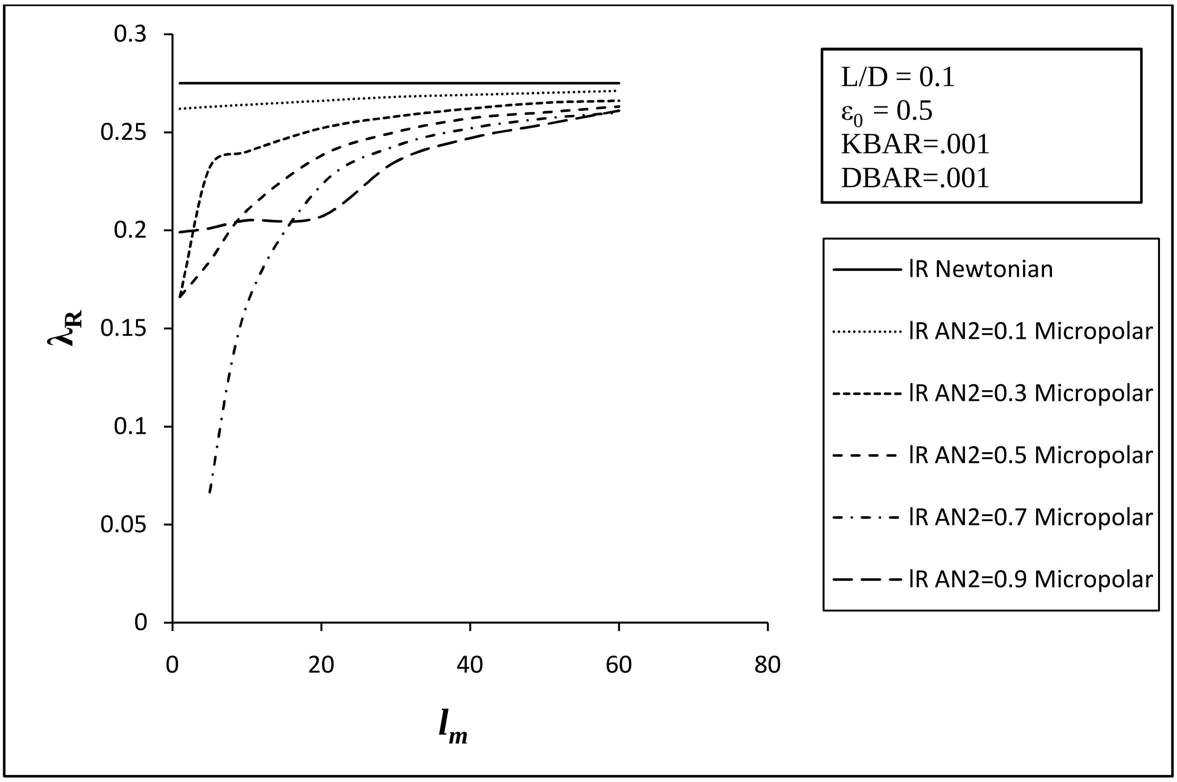

Fig.4.21: Variation of λ_R with l_m for various values of N (AN).

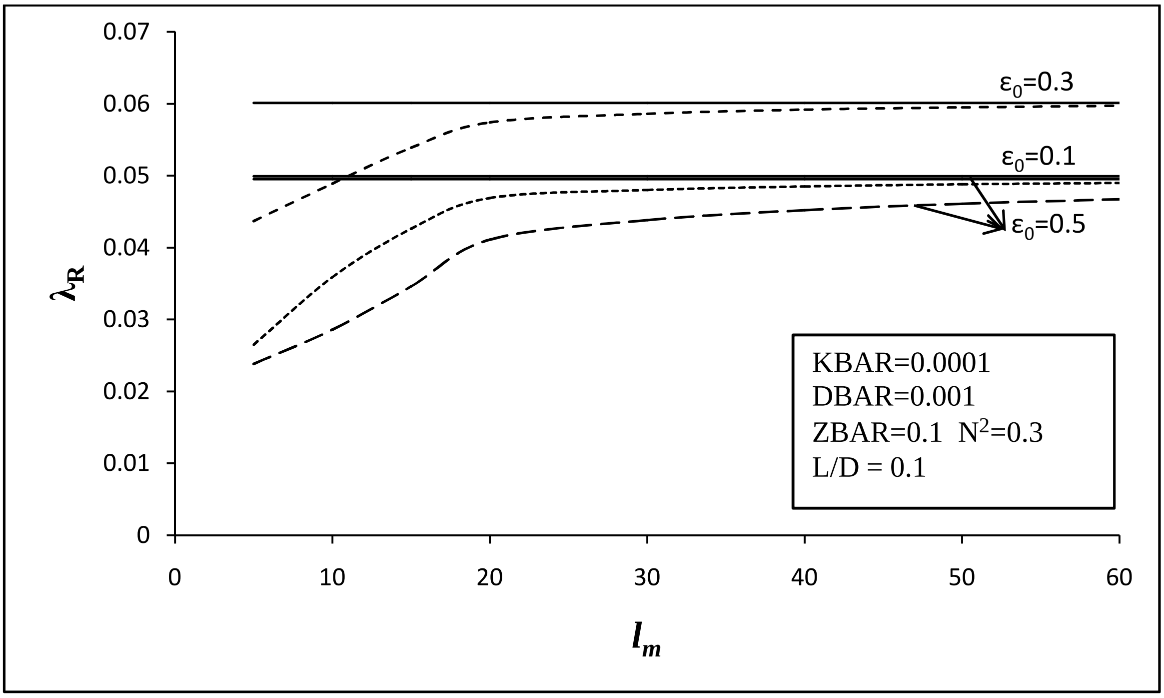

Fig.4.22: Variation of λ_R with l_m for various values of ε_0.

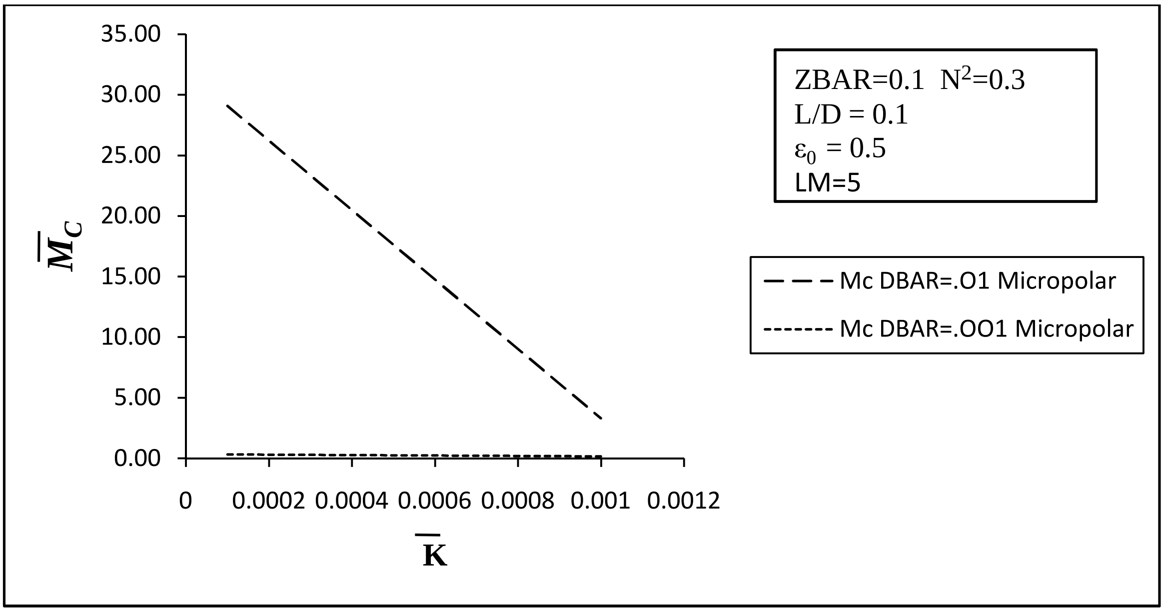

Fig.4.23: Variation of $\overline{M}_C$ with $\overline{K}$ for various values of DBAR ($\bar{d}$)

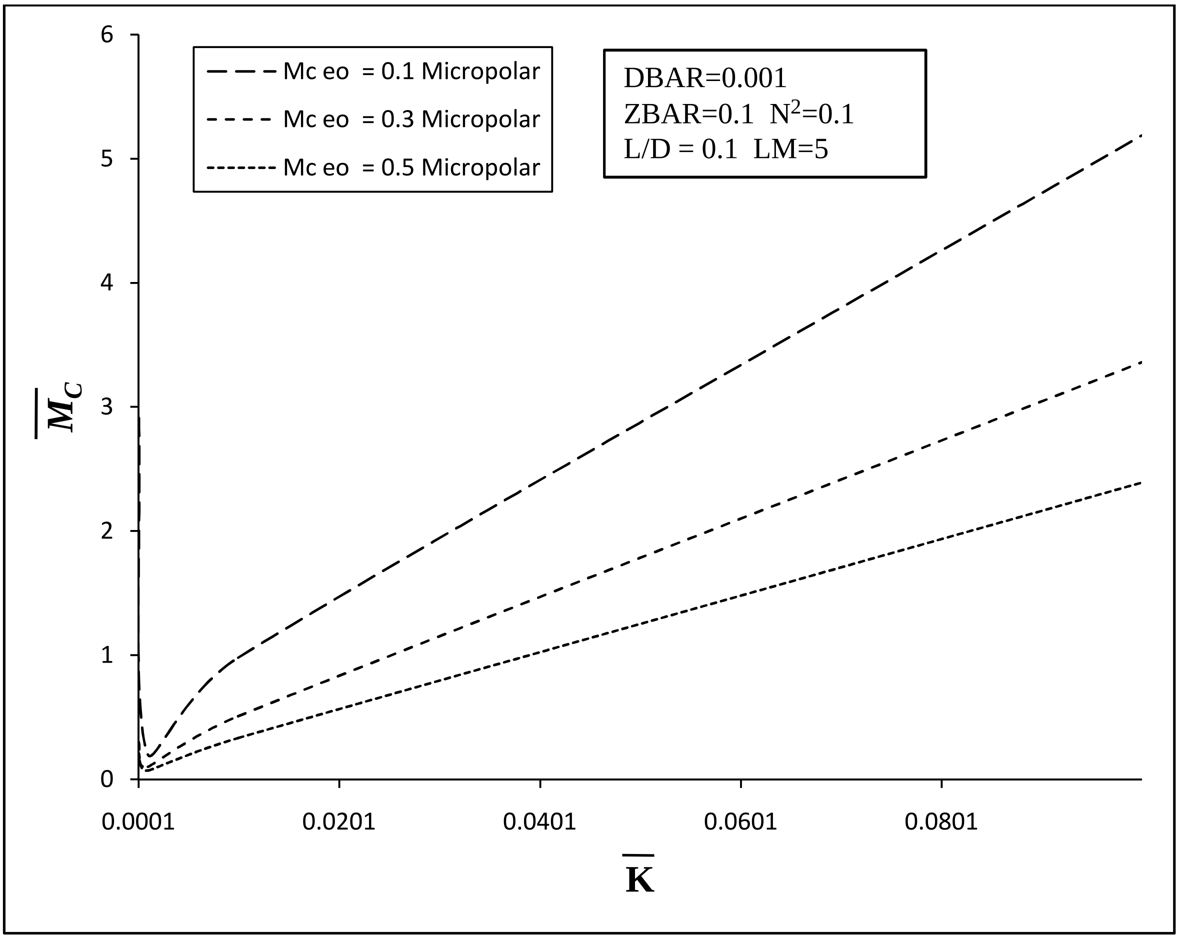

Fig.4.24: Variation of $\overline{M}_C$ with $\overline{K}$ for various values of $\varepsilon_{0.}$

CHAPTER 5: CONCLUSION AND SCOPE OF FUTURE WORK

5.1 CONCLUSIONS

In the present dissertation, the steady state and dynamic analysis including stability of flexibly supported infinitely short (Narrow) hydrodynamic journal bearings with micropolar lubrication are studied theoretically. Characteristics of bearing have been studied and presented for a wide range of micropolar lubrication parameters as well as bearing parameters, so as to provide a guideline for the design of such bearings.

From the studies and the results reported in the previous chapters, the following conclusions may be drawn.

5.1.1 Conclusions for Steady State Characteristics of Hydrodynamic Journal Bearings lubricated with Micropolar Fluids.

1. The non-dimensional load carrying capacity increases as the coupling number increases and non-dimensional characteristic length decreases. The load parameter converges to the Newtonian value as $N \rightarrow 0$ and $l_m \rightarrow \infty$.
2. Other parameters remaining the same, the load parameter increases in both types of lubrication with the increase in eccentricity ratio. The load parameter remains always higher in the micropolar fluid than that in the Newtonian fluid.

5.1.2 Conclusions for Dynamic Characteristics of Hydrodynamic Journal Bearings Lubricated with Micropolar Fluids.

1. All non-dimensional components of direct and cross stiffness coefficients exhibit the similar relationship with the micropolar parameters as that of dimensionless steady state load.
2. At a certain value of l_m, all dimensionless response coefficients increase as the eccentricity ratio increases when other parameters remain the same.
3. More micropolarity represents more stability in the bearing. However, for a specific micropolar fluid an optimum micropolar effect can be found by the suitable choice of the lubricant's grain size.
4. Higher eccentricity ratio favours the bearing towards higher stability.

5.1.3 Conclusions for Dynamic Characteristics of Flexibly Supported Hydrodynamic Journal Bearings Lubricated with Micropolar Fluids.

1. The magnitude of components of direct film stiffness co-efficient increase with ε_0
2. The magnitude of cross component of film stiffness co-efficient increase with ε_0
3. The magnitude of direct components of film damping coefficients increase with ε_0.
4. Threshold stability increase with damping.
5. Threshold stability decrease with eccentricity ratio (ε_0).

5.2 SCOPE OF FUTURE WORK

Following are some suggested areas, which are related to the present work and deserve to be explored:

1. A suitable experimentation to validate the theoretical results of the present analysis.
2. Theoretical and experimental study of journal bearings lubricated with micropolar fluid and considering the elastic deformation of the bearing liners.
3. Thermodynamic analysis of journal bearing systems lubricated with micropolar fluids.
4. Only a selected set of variables have considered in analyzing different parameters for time shortage, so there is ample scope for similar analysis of the bearing considering wide range of variables.
5. The present analysis is carried out only for hydrodynamic (also called self-acting) bearings. Although a hydrodynamic bearing has a few advantages like reliability, little attention and low cost, it can be operated if the design speed is low or load is high. Moreover in a hydrodynamic bearing the film breaks down during starting and stopping of the machine. Externally pressurized (Hydrostatic) bearing where fluid is supplied from an external source, has very low frictional characteristics, high positional accuracy and can carry heavy loads at low speeds. For these advantages the type of bearings are used in many fields (i.e. in gyroscopes, precision instruments, telescopes and machine tools). Therefore similar analysis could be carried out for this type of bearings and the results could be compared with the present results.

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APPENDICES

APPENDIX-I **Computer Listing for Steady State Analysis of Hydrodynamic Journal Bearings Lubricated with Micropolar Fluids.**

```
      INTEGER PRECAV
      REAL LM, LM2
      DIMENSION P(47), THETA(100), HH0(55)
      DIMENSION CC1(55),CC2(55)
      OPEN(5, FILE='PRESS.DAT')
      OPEN(3, FILE='NEWTON.DAT')
C
410   PI=4.0*ATAN(1.0)
      WRITE(*,*)'IPT=?'
      READ(*,*)IPT
      WRITE(*,*)'MICROPOLAR CHARACTERISTICS LM, AN2 (=N2)'
      READ(*,*)LM,AN2
      WRITE(*,*)'GIVE VALUES FOR: R (=SLENDERNESS RATIO)'
      READ(*,*)R
      WRITE(*,*)'GIVE VALUES FOR:EPH0'
      READ(*,*)EPH0
      WRITE(*,*)'GIVE VALUE FOR: ORF'
      READ(*,*)ORF
C
      WRITE(3,1510)LM,AN2
1510  FORMAT(//32X,'MICROPOLAR FLUID CHARACTERISTICS:',3X,'LM=',F10.3,
     #3X,'N^2=',F7.4)
C
      RLAMDA=1.0
      WRITE(3,1520)RLAMDA,ORF
1520  FORMAT(/5X,'RLAMDA=',F4.1,5X,'ORF='F5.2/)
      WRITE(3,1530)
1530  FORMAT(3X,'L/D',4X,'EPH0',2X,'ITER',2X,'SS LOAD',2X,
     #'ATT ANGLE',2X,'SOM.NO.',2X,'ANG_STRT',2X,'H_STRT',2X,'PMAX_ANG',
     #2X,'H_PMAX',3X,'CAV_ANG',3X,'H_CAV',2X,'END FLOW',2X,
     #'FRIC.DRAG',2X,'FRIC.PARA'/)
C
      WRITE(*,1220)KNT,RLAMDA,LM,AN2,R,EPH0
1220  FORMAT(/4X,'KNT=',I8,4X,'RLAMDA=',F4.2,3X,'LM=',F7.2,3X,
     #'AN2=',F6.3,3X,'R=',F3.1,3X,'EPH0=',F5.2)
C
      I5=IPT+2
      I6=I5+1
      I4=I5-1
      I3=I4-1
      I2=I3-1
      AI3=I3
C
      DELTH=2.0*PI/AI3
      DELTH2=DELTH*DELTH
C     DELZ2=DELZ*DELZ
C     DELR2=DELTH2/DELZ2
C
      RR=1.0/R
      RR2=RR*RR
      LM2=LM*LM
      AN=SQRT(AN2)
```

```
      AN3=AN2*AN
C     DENO=2.*(1.0+RR2*DELR2)
C
      ITER=1
      SPO=0.
========================================================================
C     CALCULATION OF FILM THICKNESS AND THE FUNCTION F1=COTH(N*LM*H0/2)
C     AND F2={COSECH(N*LM*H0/2)^2
========================================================================
C
      DO 10 I=2,I5
      AI=I-2
      THETAI=AI*DELTH
      THETA(I)=THETAI
      H0=1.0+EPH0*COS(THETAI)
      HH0(I)=H0
      A=0.5*AN*LM*H0
      TANHA=TANH(A)
      COTHA=1.0/TANHA
      SINHA=SINH(A)
      CSCHA=1.0/SINHA
      CSCHA2=CSCHA*CSCHA
      H02=H0*H0
      H03=H02*H0
      CC1(I)=H02/4.+1./LM2-AN*H0*COTHA/LM+AN2*H02*CSCHA2/4.
      CC2(I)=H03/12.+H0/LM2-0.5*AN*H02*COTHA/LM
      P(I)=EPH0*SIN(THETAI)/(4*CC2(I))
      IF(P(I).LT.0)  P(I)=0.
      IF(I.EQ.2) P(I5)=P(2)
      IF(I.EQ.I4) P(1)=P(I4)
10    CONTINUE
========================================================================
C     TEST FOR CAVITATION
========================================================================
C
90    I=5
130   IF(P(I))100,100,110
110   IF(I.EQ.I5) GOTO 120
      I=I+1
      GOTO 130
120   WRITE(*,*)'NO CAVITATION IN THE BEARING'
100   NCAVI=I
      PRECAV=NCAVI-1
      THCAV=(NCAVI-2)*DELTH
      THETACAV=THCAV*180.0/PI
      H0CAV=1.0+EPH0*COS(THCAV)
C
      I=NCAVI
 708  IF(P(I).GT.0.) GO TO 707
      I=I+1
      IF(I.EQ.I5) I=2
      GO TO 708
 707  NFILMS=I-1

========================================================================
C     EVALUATION OF LOAD, ATTITUDE ANGLE AND SOMMERFELD NUMBER
========================================================================
C
      WX=0.0
      WY=0.0
      DO 210 I=2,PRECAV,2
```

```
      IF(I.EQ.PRECAV) GO TO 1501
C     IF(I.EQ.I5) THEN
C     I=2
C     P(1)=P(I4)
C     ENDIF
      AI=I-2
      TH1=AI*DELTH
      TH2=TH1+DELTH
      TH3=TH2+DELTH
      TERM1=P(I)*COS(TH1)+4.*P(I+1)*COS(TH2)+P(I+2)*COS(TH3)
      TERM2=P(I)*SIN(TH1)+4.*P(I+1)*SIN(TH2)+P(I+2)*SIN(TH3)
      WX=WX+TERM1*DELTH/3.
      WY=WY+TERM2*DELTH/3.
      GOTO 210
1501  M=PRECAV-1
      AI=M-2
      TH1=AI*DELTH
      TH2=TH1+DELTH
      TH3=TH2+DELTH
      TERM1=-P(M)*COS(TH1)+8.*P(M+1)*COS(TH2)+5.*P(M+2)*COS(TH3)
      TERM2=-P(M)*SIN(TH1)+8.*P(M+1)*SIN(TH2)+5.*P(M+2)*SIN(TH3)
      WX=WX+TERM1*DELTH/12.
      WY=WY+TERM2*DELTH/12.
210   CONTINUE
C
      WX=(-4/3)*WX*R**2
      WY=(-4/3)*WY*R**2
      W=2*SQRT(WX*WX+WY*WY)
      ATTRAD=ATAN(-WY/WX)
      ATT=ATTRAD*180./PI
      SN=1.0/(PI*W)
      WRITE(*,*)'W=',W,'ATTITUDE=',ATT,'SOMMERFELD NO=',SN
      WRITE(5,*)'W=',W,'ATTITUDE=',ATT,'SOMMERFELD NO=',SN
C
      WRITE(*,*)'PROGRAM TERMINATED'
      STOP
      END
```

APPENDIX-II: **Computer Listing for Dynamic Analysis using Perturbation Technique of Hydrodynamic Journal Bearings Lubricated with Micropolar Fluids.**

```
      COMPLEX FTH1(100), FTH2(100)
      COMPLEX AM11,AM12,AM21,AM22
      COMPLEX WDX1,WDY1,WDX2,WDY2
      INTEGER PRECAV
      REAL LM,LM2
      DIMENSION P(100),THETA(100),HH0(100)
      DIMENSION RLM(10),CC1(100),CC2(100)
      OPEN(3,FILE='RESULT2N.DAT')
      OPEN(1,FILE='VALLM2.DAT')
      PI=4.0*ATAN(1.0)
      WRITE(*,*)'IPT=?(=44)'
      READ(*,*)IPT
      WRITE(*,*)'GIVE VALUES FOR:EPH0'
      READ(*,*)EPH0
      WRITE(*,*)'GIVE VALUES FOR: R (=SLENDERNESS RATIO)'
      READ(*,*)R
      WRITE(*,*)'GIVE VALUE FOR: ORF (1.05)'
      READ(*,*)ORF
      KNT=1
      AN2=0.1
      READ(1,*)(RLM(ILM),ILM=1,8)
470   IF(AN2.GT.0.9) GOTO 180
      ILM=1
450   IF(ILM.GT.8) GOTO 460
      LM=RLM(ILM)
      WRITE(3,1510)LM,AN2
1510  FORMAT(32X,'MICROPOLAR FLUID CHARATERISTICS:',3X,'LM=',F10.3,
     #3X,'N^2=',F7.4//)
C
      RLAMDA=1.0
      R=0.1
      EPH0=0.1
      WRITE(3,1520)RLAMDA
1520  FORMAT(5X,'RLAMDA=',F4.1/)
      WRITE(3,1010)
1010  FORMAT(3X,'L/D',2X,'EPH0',1X,'ITER',2X,'SS LOAD',2X,'ATT ANGLE',
     #1X,'IP1',1X,'IP2',6X,'SRR',9X,'SPR',9X,'DRR',9X,'DPR',9X,'SRP',9X,
     #'SPP',9X,'DRP',9X,'DPP',4X,'STABILITY',2X,'WH RATIO',2X,'MASS
     #PARA',2X,'SN'/)
410   IF (EPH0.GE.1.0) GOTO 420
C
      WRITE(*,1220)KNT,RLAMDA,LM,AN2,R,EPH0
1220  FORMAT(/4X,'KNT=',I8,4X,'RLAMDA=',F4.2,3X,'LM=',F7.2,3X,
     #'AN2=',F6.3,3X,'R=',F3.1,3X,'EPH0=',F5.2)
C
      I5=IPT+2
      I6=I5+1
      I4=I5-1
      I3=I4-1
      I2=I3-1
      AI3=I3
C
      DELTH=2.0*PI/AI3
      DELTH2=DELTH*DELTH
C
      LM2=LM*LM
```

```
      AN=SQRT(AN2)
      AN3=AN2*AN
=========================================================================
C     CALCULATION OF FILM THICKNESS AND THE FUNTIONS F1=COTH(N*LM*H0/2)
C     AND F2={COSECH(N*LM*H0/2)}^2
=========================================================================
      DO 10 I=2,I5
      AI=I-2
      THETAI=AI*DELTH
      THETA(I)=THETAI
      H0=1.0+EPH0*COS(THETAI)
      HH0(I)=H0
      A=0.5*AN*LM*H0
      TANHA=TANH(A)
      COTHA=1.0/TANHA
      SINHA=SINH(A)
      CSCHA=1.0/SINHA
      CSCHA2=CSCHA*CSCHA
      H02=H0*H0
      H03=H02*H0
      CC1(I)=H02/4.+1./LM2-AN*H0*COTHA/LM+AN2*H02*CSCHA2/4.
      CC2(I)=H03/12.+H0/LM2-0.5*AN*H02*COTHA/LM
      P(I)=EPH0*SIN(THETAI)/(4*CC2(I))
      IF(P(I).LT.0)  P(I)=0
      IF(I.EQ.2) P(I5)=P(2)
      IF(I.EQ.I4) P(1)=P(I4)
10    CONTINUE
=========================================================================
C     TEST FOR CAVIATION
=========================================================================
90    I=5
130   IF(P(I))100,100,110
110   IF(I.EQ.I5) GOTO 120
      I=I+1
      GOTO 130
120   WRITE(*,*)'NO CAVITATION IN THE BEARING'
100   NCAVI=I
      PRECAV=NCAVI-1
      THCAV=(NCAVI-2)*360.0/AI3
      THETACAV=THCAV*180.0/PI
      H0CAV=1.0+EPH0*COS(THCAV)
=========================================================================
C     EVALUATION OF LOAD AND ATTITUDE ANGLE
=========================================================================
      WX=0.0
      WY=0.0
      DO 140 I=2,PRECAV,2
      IF(I.EQ.PRECAV) GOTO 150
      AI=I-2
      TH1=AI*DELTH
      TH2=TH1+DELTH
      TH3=TH2+DELTH
      TERM1=P(I)*COS(TH1)+4.*P(I+1)*COS(TH2)+P(I+2)*COS(TH3)
      TERM2=P(I)*SIN(TH1)+4.*P(I+1)*SIN(TH2)+P(I+2)*SIN(TH3)
      WX=WX+TERM1*DELTH/3.
      WY=WY+TERM2*DELTH/3.
      GOTO 140
150   M=PRECAV-1
      AI=M-2
      TH1=AI*DELTH
      TH2=TH1+DELTH
```

```
      TH3=TH2+DELTH
      TERM1=-P(M)*COS(TH1)+8.*P(M+1)*COS(TH2)+5.*P(M+2)*COS(TH3)
      TERM2=-P(M)*SIN(TH1)+8.*P(M+1)*SIN(TH2)+5.*P(M+2)*SIN(TH3)
      WX=WX+TERM1*DELTH/12.
      WY=WY+TERM2*DELTH/12.
      GOTO 71
140   CONTINUE
C
71    W=2*SQRT(WX*WX+WY*WY)*(2/3.)*R**2
      ATTRAD=ATAN(-WY/WX)
      ATT=ATTRAD*180./PI
      SN=1./(PI*W)
C

========================================================================
C     CALCULATION OF PERTURBED FILM PRESSURE, P1 & P2
========================================================================
      DO 9126 I=2,I5
      FTH1(I)=CMPLX(0.,0.)
9126  FTH2(I)=CMPLX(0.,0.)
      DO 210 I=2,PRECAV
      THETAI=THETA(I)
      CC1I=CC1(I)
      CC2I=CC2(I)
      CBE=(0.5*(CC1I*EPH0*SIN(THETAI)*COS(THETAI)/CC2I-SIN(THETAI)))
     #/CC2I
      CBF=(RLAMDA*COS(THETAI))/CC2I
      CBA=(0.5*(CC1I*EPH0*SIN(THETAI)*SIN(THETAI)/CC2I+COS(THETAI)))/CC2
I
      CBB=(2*RLAMDA*SIN(THETAI))/CC2I
      FTH1(I)=CMPLX(CBE,CBF)
      FTH2(I)=CMPLX(CBA,CBB)
210   CONTINUE
========================================================================
C     EVALUATION OF STIFFNESS
========================================================================
300   WDX1=CMPLX(0.,0.)
      WDY1=CMPLX(0.,0.)
      WDX2=CMPLX(0.,0.)
      WDY2=CMPLX(0.,0.)
      IF(NCAVI.EQ.I5) THEN
      NMOD=I3
            ELSE
            NMOD=PRECAV
      ENDIF
      DO 310 I=2,NMOD,2
      IF(I.EQ.PRECAV) GOTO 320
      AI=I
      TH1=(AI-2.0)*DELTH
      TH2=TH1+DELTH
      TH3=TH2+DELTH
C
      AM11=FTH1(I)*COS(TH1)+4.*FTH1(I+1)*COS(TH2)+FTH1(I+2)*COS(TH3)
      AM12=FTH1(I)*SIN(TH1)+4.*FTH1(I+1)*SIN(TH2)+FTH1(I+2)*SIN(TH3)
      WDX1=WDX1+AM11*DELTH/3.
      WDY1=WDY1+AM12*DELTH/3.
C
      AM21=FTH2(I)*COS(TH1)+4.*FTH2(I+1)*COS(TH2)+FTH2(I+2)*COS(TH3)
      AM22=FTH2(I)*SIN(TH1)+4.*FTH2(I+1)*SIN(TH2)+FTH2(I+2)*SIN(TH3)
      WDX2=WDX2+AM21*DELTH/3.
      WDY2=WDY2+AM22*DELTH/3.
```

```
      GOTO 310
C
320   M=PRECAV-1
      AI=M-2
      TH1=AI*DELTH
      TH2=TH1+DELTH
      TH3=TH2+DELTH
C
      AM11=-FTH1(M)*COS(TH1)+8.*FTH1(M+1)*COS(TH2)+5.*FTH1(M+2)*COS(TH3)
      AM12=-FTH1(M)*SIN(TH1)+8.*FTH1(M+1)*SIN(TH2)+5.*FTH1(M+2)*SIN(TH3)
      WDX1=WDX1+AM11*DELTH/12.
      WDY1=WDY1+AM12*DELTH/12.
C
      AM21=-FTH2(M)*COS(TH1)+8.*FTH2(M+1)*COS(TH2)+5.*FTH2(M+2)*COS(TH3)
      AM22=-FTH2(M)*SIN(TH1)+8.*FTH2(M+1)*SIN(TH2)+5.*FTH2(M+2)*SIN(TH3)
      WDX2=WDX2+AM21*DELTH/12.
      WDY2=WDY2+AM22*DELTH/12.
      GOTO 311
310   CONTINUE
311   WDX1=(-2/3.)*R**2*WDX1
      WDX2=(-2/3.)*R**2*WDX2
      WDY1=(-2/3.)*R**2*WDY1
      WDY2=(-2/3.)*R**2*WDY2
========================================================================
C     STIFFNESS AND DAMPING COEFFICIENTS FOR THE PERTURBED FILM
C     PRESSURE, P1
========================================================================
      SRR=-REAL(WDX1)
      SPR=-REAL(WDY1)
      WDX1A=AIMAG(WDX1)
      WDY1A=AIMAG(WDY1)
      DRR=-WDX1A/RLAMDA
      DPR=-WDY1A/RLAMDA
========================================================================
C     STIFFNESS AND DAMPING COEFFICINTS FOR THE PERTURBED FILM
C     PRESSURE. P2
========================================================================
      SRP=-REAL(WDX2)
      SPP=-REAL(WDY2)
      WDX2A=AIMAG(WDX2)
      WDY2A=AIMAG(WDY2)
      DRP=-WDX2A/RLAMDA
      DPP=-WDY2A/RLAMDA
      WRITE(*,*)'WDX1=',WDX1,'WDY1=',WDY1,'WDX2=',WDX2,'WDY2=',WDY2
      WRITE(*,*) 'SRR=',SRR,'SPP=',SPP
      WRITE(3,1020)R,EPH0,W,ATT,SRR,SPR,DRR,DPR,SRP,
     #SPP,DRP,DPP
1020  FORMAT(F7.5,1X,F4.2,1X,10E16.5)
C
      EPH0=EPH0+0.1
      KNT=KNT+1
      GOTO 410
420   ILM=ILM+1
      GOTO 450
460   AN2=AN2+0.2
      GOTO 470
180   WRITE(*,*)'PROGRAM TERMINATED'
      STOP
      END
```

APPENDIX-III: **Computer Listing for Bearing stability parameter of Hydrodynamic Journal Bearings lubricated with Micropolar Fluids.**

```
      IMPLICIT REAL*8(A-H,O-Z)
C     DIMENSION A(5),XR(3),F(3)
      PARAMETER (PI=3.1415927)
C     OPEN(5,FILE='STD1.DAT')
C     OPEN(7,FILE='STO.DAT')
C     WRITE(*,2)
C2    FORMAT(1X,3HSK,2HD,1HZ)
14    WRITE(*,*) 'KBAR,D,Z'
      READ(*,*)SK,D,Z
      WRITE(*,*)'EPH0=','W=','ATT=','SRR=','SPR=','DRR=','DPR='
     #,'SRP=','SPP=','DRP=','DPP='
      READ(*,*)EPH0,W,ATT,SRR,SPR,DRR,DPR,SRP,SPP,DRP,DPP
      PHI=ATT*PI/180.
      X=((SRR+SRP*PHI)*DSIN(PHI)+(SPR+SPP*PHI)*DCOS(PHI))*EPH0/SK
      Y=((SRR+SRP*PHI)*DCOS(PHI)-(SPR+SPP*PHI)*DSIN(PHI))*EPH0/SK
      B1=(X*DSIN(PHI)+Y*DCOS(PHI))
      B2=(Y*DSIN(PHI)-X*DCOS(PHI))

      A1=-(Z**2)*EPH0*(W**4)

      A2=(Z*SRR*EPH0+Z*SPP*EPH0+Z*(SK*B1+2.*SK*EPH0)+(Z**2)*(SRR*EPH0+
     #SPP*EPH0+W*DCOS(PHI)))*(W**3)

      A31=((DRR*DPP-DPR*DRP)*EPH0+2.*Z*(DRR*DPP-DRP*DPR)*EPH0+D*EPH0*
     #(DRR+DPP)+2.*Z*D*EPH0*(DRR+DPP)+(Z**2)*(DRR*DPP-DRP*DPR)*EPH0+
     #(D**2)*EPH0)*(W**2)

      A32=((SRP*SPR-SRR*SPP)*EPH0+2.*Z*(SRP*SPR-SRR*SPP-SK*SRR-SK*SPP)*
     #EPH0-SK*(SRR+SPP)*EPH0-Z*W*SRR*DCOS(PHI)*(1.+Z)+Z*W*SPR
     #*DSIN(PHI)*(1.+Z)+(Z**2)*(SRP*SPR-SRR*SPP)*EPH0-2.*Z*SK*
     #W*DCOS(PHI)-(SK**2)*EPH0-SK*B1*(SRR+Z*SRR+SK)+SK*B2*SPR*(Z+1))
     #*(W**2)

      A42=(2.*SK*(DPR*DRP-DPP*DRR)*EPH0+2.*D*(DPR*SRP-SRR*DPP)*EPH0
     #+D*SK*B2*DPR+2.*D*(SPR*DRP-SPP*DRR)*EPH0-D*W*DRR*(1.+2.*Z)
     #*DCOS(PHI)-(D**2)*W*DCOS(PHI)-D*SK*B1*DRR+D*W*DPR*DSIN(PHI)*
     #(1.+2.*Z)-(D**2)*EPH0*(SRR+SPP)+2.*Z*SK*(DPR*DRP-DPP*DRR)*EPH0-
     #2.*D*SK*(DRR+DPP)*EPH0-2.*Z*D*(DRR*SPP+SRR*DPP-SRP*DPR-SPR*DRP)
     #*EPH0)*W

      A51=(D**2)*(DRP*DPR-DRR*DPP)*EPH0

      A41=(2.*SK*(SRR*SPP-SRP*SPR)*EPH0+(SK**2)*(SRR+SPP)*EPH0+SK*W*SRR*
     #DCOS(PHI)*(1.+2.*Z)-SK*W*SPR*DSIN(PHI)*(1.+2.*Z)+(SK**2)*W*
     #DCOS(PHI)+2.*Z*SK*(SRR*SPP-SPR*SRP)*EPH0-(SK**2)*B2*SPR+
     #(SK**2)*B1*SRR)*W

      A52=(D**2)*(SRR*SPP-SPR*SRP)*EPH0+(SK**2)*(DRR*DPP-DRP*DPR)*EPH0
     #+2.*D*SK*(DRR*SPP+SRR*DPP-SRP*DPR-SPR*DRP)*EPH0+W*D*COS(PHI)*
     #(D*SRR+2.*SK*DRR)-D*W*DSIN(PHI)*(D*SPR+2.*SK*DPR)

      A53=(SK**2)*(SRP*SPR-SRR*SPP)*EPH0+(SK**2)*W*(SPR*DSIN(PHI)-SRR*
     #DCOS(PHI))

      BB1=(Z*(DRR+DPP+2.*D)*EPH0+(Z**2)*(DRR+DPP)*EPH0)*(W**3)
```

```
      BB2=((SRP*DPR+SPR*DRP-SRR*DPP-DRR*SPP)*EPH0+2.*Z*(DPR*SRP+SPR*DRP-
     #SRR*DPP-SPP*DRR)*EPH0-D*(SRR+SPP)*EPH0-Z*W*DRR*DCOS(PHI)*(1.+Z)+
     #Z*W*DPR*DSIN(PHI)*(1.+Z)-2.*Z*D*W*DCOS(PHI)-(Z**2)*(SPP*DRR+SRR
     #*DPP-DPR*SRP-SPR*DRP)*EPH0-2.*Z*SK*(DRR+DPP)*EPH0-2.*Z*D*(SPP+SRR
     #)*EPH0-SK*(DPP+DRR)*EPH0-D*B1*SK-2*D*SK*EPH0-B1*DRR*SK*(1.+Z)+
     #SK*B2*(DPR+Z*DPR))*(W**2)

      BB32=(2.*D*DPR*DRP*EPH0-2.*D*DRR*DPP*EPH0-2.*Z*D*DRR*DPP*EPH0-
     #(D**2)*(DRR+DPP)*EPH0+DPR*DRP*D*EPH0)*W

      BB31=(2.*SK*(SRR*DPP+SPP*DRR-DPR*SRP-SPR*DRP)*EPH0+2.*D*(SPP*SRR-
     #SRP*SPR)*EPH0+SK*W*DRR*(1.+2.*Z)-D*W*SPR*DSIN(PHI)*
     #(1.+2.*Z)+D*SRR*DCOS(PHI)*(1.+2.*Z)-SK*W*DPR*DSIN(PHI)*
     #(1.+2.*Z)+2.*D*SK*DCOS(PHI)+2.*D*SK*(SRR+SPP)*EPH0+
     #(SK**2)*(DRR+DPP)*EPH0+2.*SK*Z*(SPP*DRR+SRR*DPP-SRP*DPR-SPR*DRP)*
     #EPH0+2.*Z*D*(SRR*SPP-SRP*SPR)*EPH0-SK*B2*(SK*DPR-D*SPR)+
     #SK*B1*(D*SRR+SK*DRR))*W

      BB41=2.*D*SK*DRR*DPP*EPH0+(D**2)*(SPP*DRR+SRR*DPP-SRP*DPR-SPR*DRP)
     #*EPH0+(D**2)*W*(DRR*DCOS(PHI)-DPR*DSIN(PHI))-2*D*SK*DPR*DRP

      BB42=2.*SK*D*(SRP*SPR-SRR*SPP)*EPH0+(SK**2)*(SRP*DPR+SPR*DRP-
     #SPP*DRR-SRR*DPP)*EPH0-SK*W*DCOS(PHI)*(2.*D*SRR+SK*DRR)+
     #SK*W*DSIN(PHI)*(SK*DPR+2.*D*SPR)
========================================================================
C     SIMULTANEOUS SOLUTION BY NEWTON RAPHSON-METHOD
========================================================================
401   ITER=1
      WRITE(*,*)'LAMDA,SM,'
      READ(*,*)SL,SM
      S=SL**2*SM
      V=SL**2
200   F1=A1*S**4+A2*S**3+(A31*V+A32)*S**2+(A42*V+A41)*S+
     #(A51*V**2+A52*V+A53)
      F2=BB1*S**3+BB2*S**2+(BB32*V+BB31)*S+(BB41*V+BB42)
      IF(ITER.NE.1) GO TO 20
      WRITE(*,*) 'F1=',F1,'F2=',F2
      WRITE(*,*)'FURTHER INITIALIZE? IF YESS PRESS 1'
      READ(*,*) LOGIC
      IF(LOGIC.EQ.1) GO TO 401
20    DF1S=4.*A1*S**3+3.*A2*S**2+2.*(A31*V+A32)*
     #S+(A42*V+A41)
      DF2S=3.*BB1*S**2+2.*BB2*S+(BB32*V+BB31)
      DF1V=A31*S**2+A42*S+2*A51*V+A52
      DF2V=BB32*S+BB41
      DEN=DF1S*DF2V-DF2S*DF1V
      H=(F2*DF1V-F1*DF2V)/DEN
      AK=-(F2*DF1S-F1*DF2S)/DEN
      WRITE(*,*)'K=',AK,'H=',H,'ERR1=',ERR1,'ERR2=',ERR2
      WRITE(*,*)'V=',V,'M=',S/V
      AS=S+H
      AV=V+AK
C     ERR1=1.-S/AS
C     ERR2=1.-V/AV
      ERR1=DABS(S-AS)
      ERR2=DABS(V-AV)
      IF((ERR1).GT.1.0E-08) GO TO 100
      IF((ERR2).GT.1.0E-08) GO TO 100
      WRITE(*,*)S,V
      IF(V) 400,400,2000
400   WRITE(*,*) 'INFINTELY STABLE'
```

```
      GO TO 2001
2000  SL=SQRT(V)
      SM=S/V
      WRITE(*,*) 'WHIRL RATIO=',SL,'CRITICAL MASS=',SM
      WRITE(*,*)SRR,SPR,DRR,DPR
2001  WRITE(*,*)'INDEX=?,INDEX=1 IMPLIES GOING FOR NEXT CALCULATION'
      READ(*,*) INDEX
      IF (INDEX.EQ.1) GO TO 310
      STOP
100   IF(ITER.GT.100) GO TO 300
      S=AS
      V=AV
      ITER=ITER+1
      GO TO 200
300   WRITE(*,*)'MODIFY THE INITIAL VALUES AS ITER IS >100'
      GO TO 401
310   GO TO 14
C     WRITE(7,26)EPH0,SM,SL
C26   FORMAT(1X,'R=',F7.1,2X,'EPH0=',F3.1,2X,'SM='F12.9,2X,'SL=',F12.9)
C     WRITE(*,27)EPH0,SM,SL
C27   FORMAT(1X,'R=',F7.1,2X,'EPH0=',F3.1,2X,'SM='F12.9,2X,'SL=',F12.9)
C14   CONTINUE
      END
```

APPENDIX-IV: **computer listing for calculating load carrying capacity, stiffness & damping of hydrodynamic journal bearings lubricated with newtonian fluids.**

```
      COMPLEX FTH1(100), FTH2(100)
      COMPLEX AM11,AM12,AM21,AM22
      COMPLEX WDX1,WDY1,WDX2,WDY2
      INTEGER PRECAV
      REAL LM,LM2
      DIMENSION P(100),THETA(100),HH0(100)
      DIMENSION RLM(10),CC1(100),CC2(100)
      OPEN(3,FILE='RESULT2.DAT')
      WRITE(*,*)'LM=?','AN2=?'
        READ(*,*)LM,AN2
C
      RLAMDA=1.0
C     IR=1
C430  IF(IR.GT.5) GOTO 440
C     R=RLD(IR)
      R=0.1
      EPH0=0.1
      WRITE(3,1520)RLAMDA
1520  FORMAT(5X,'RLAMDA=',F4.1/)
      WRITE(3,1010)
1010  FORMAT(3X,'L/D',2X,'EPH0',1X,'ITER',2X,'SS LOAD',2X,'ATT ANGLE',
     #1X,'IP1',1X,'IP2',6X,'SRR',9X,'SPR',9X,'DRR',9X,'DPR',9X,'SRP',9X,
     #'SPP',9X,'DRP',9X,'DPP',4X,'STABILITY',2X,'WH RATIO',2X,'MASS
     #PARA',2X,'SN'/)
410   IF (EPH0.GE.1.0) GOTO 420
C
      WRITE(*,1220)KNT,RLAMDA,LM,AN2,R,EPH0
1220  FORMAT(/4X,'KNT=',I8,4X,'RLAMDA=',F4.2,3X,'LM=',F7.2,3X,
     #'AN2=',F6.3,3X,'R=',F3.1,3X,'EPH0=',F5.2)
C
      IPT=44
        PI=4.*ATAN(1.0)
      I5=IPT+2
      I6=I5+1
      I4=I5-1
      I3=I4-1
      I2=I3-1
      AI3=I3
C
      DELTH=2.0*PI/AI3
      DELTH2=DELTH*DELTH
C
      LM2=LM*LM
      AN=SQRT(AN2)
      AN3=AN2*AN
========================================================================
C     CALCULATION OF FILM THICKNESS AND THE FUNTIONS F1=COTH(N*LM*H0/2)
C     AND F2={COSECH(N*LM*H0/2)}^2
========================================================================
      DO 10 I=2,I5
      AI=I-2
      THETAI=AI*DELTH
      THETA(I)=THETAI
      H0=1.0+EPH0*COS(THETAI)
      HH0(I)=H0
      A=0.5*AN*LM*H0
```

```
      TANHA=TANH(A)
      COTHA=1.0/TANHA
      SINHA=SINH(A)
      CSCHA=1.0/SINHA
      CSCHA2=CSCHA*CSCHA
      H02=H0*H0
      H03=H02*H0
      CC1(I)=H02/4.+1./LM2-AN*H0*COTHA/LM+AN2*H02*CSCHA2/4.
      CC2(I)=H03/12.+H0/LM2-0.5*AN*H02*COTHA/LM
      P(I)=EPH0*SIN(THETAI)/(4*CC2(I))
      IF(P(I).LT.0)  P(I)=0
      IF(I.EQ.2) P(I5)=P(2)
      IF(I.EQ.I4) P(1)=P(I4)
10    CONTINUE
========================================================================
C     TEST FOR CAVIATION
========================================================================
90    I=5
130   IF(P(I))100,100,110
110   IF(I.EQ.I5) GOTO 120
      I=I+1
      GOTO 130
120   WRITE(*,*)'NO CAVITATION IN THE BEARING'
100   NCAVI=I
      PRECAV=NCAVI-1
      THCAV=(NCAVI-2)*360.0/AI3
      THETACAV=THCAV*180.0/PI
      H0CAV=1.0+EPH0*COS(THCAV)
========================================================================
C     EVALUATION OF LOAD AND ATTITUDE ANGLE
========================================================================
      WX=0.0
      WY=0.0
      DO 140 I=2,PRECAV,2
      IF(I.EQ.PRECAV) GOTO 150
      AI=I-2
      TH1=AI*DELTH
      TH2=TH1+DELTH
      TH3=TH2+DELTH
      TERM1=P(I)*COS(TH1)+4.*P(I+1)*COS(TH2)+P(I+2)*COS(TH3)
      TERM2=P(I)*SIN(TH1)+4.*P(I+1)*SIN(TH2)+P(I+2)*SIN(TH3)
      WX=WX+TERM1*DELTH/3.
      WY=WY+TERM2*DELTH/3.
      GOTO 140
150   M=PRECAV-1
      AI=M-2
      TH1=AI*DELTH
      TH2=TH1+DELTH
      TH3=TH2+DELTH
      TERM1=-P(M)*COS(TH1)+8.*P(M+1)*COS(TH2)+5.*P(M+2)*COS(TH3)
      TERM2=-P(M)*SIN(TH1)+8.*P(M+1)*SIN(TH2)+5.*P(M+2)*SIN(TH3)
      WX=WX+TERM1*DELTH/12.
      WY=WY+TERM2*DELTH/12.
140   CONTINUE
C
      W=2*SQRT(WX*WX+WY*WY)
      ATTRAD=ATAN(-WY/WX)
      ATT=ATTRAD*180./PI
      SN=1./(PI*W)
C
```

```
==========================================================================
C     CALCULATION OF PERTURBED FILM PRESSURE, P1 & P2
==========================================================================
      DO 9126 I=2,I5
      FTH1(I)=CMPLX(0.,0.)
9126  FTH2(I)=CMPLX(0.,0.)
      DO 210 I=2,PRECAV
      THETAI=THETA(I)
      CC1I=CC1(I)
      CC2I=CC2(I)
      CBE=0.5*(CC1I*EPH0*SIN(THETAI)*COS(THETAI)/CC2I-SIN(THETAI))
      CBF=RLAMDA*COS(THETAI)
      CBA=0.5*(CC1I*EPH0*SIN(THETAI)/CC2I+COS(THETAI))
      CBB=2*RLAMDA*SIN(THETAI)
      FTH1(I)=CMPLX(CBE,CBF)
      FTH2(I)=CMPLX(CBA,CBB)
210   CONTINUE
==========================================================================
C     EVALUATION OF STIFFNESS
==========================================================================
300   WDX1=CMPLX(0.,0.)
      WDY1=CMPLX(0.,0.)
      WDX2=CMPLX(0.,0.)
      WDY2=CMPLX(0.,0.)
      IF(NCAVI.EQ.I5) THEN
      NMOD=I3
            ELSE
            NMOD=PRECAV
      ENDIF
      DO 310 I=2,NMOD,2
      IF(I.EQ.PRECAV) GOTO 320
      AI=I
      TH1=(AI-2.0)*DELTH
      TH2=TH1+DELTH
      TH3=TH2+DELTH
C
      AM11=FTH1(I)*COS(TH1)+4.*FTH1(I+1)*COS(TH2)+FTH1(I+2)*COS(TH3)
      AM12=FTH1(I)*SIN(TH1)+4.*FTH1(I+1)*SIN(TH2)+FTH1(I+2)*SIN(TH3)
      WDX1=WDX1+AM11*DELTH/3.
      WDY1=WDY1+AM12*DELTH/3.
C
      AM21=FTH2(I)*COS(TH1)+4.*FTH2(I+1)*COS(TH2)+FTH2(I+2)*COS(TH3)
      AM22=FTH2(I)*SIN(TH1)+4.*FTH2(I+1)*SIN(TH2)+FTH2(I+2)*SIN(TH3)
      WDX2=WDX2+AM21*DELTH/3.
      WDY2=WDY2+AM22*DELTH/3.
      GOTO 310
C
320   M=PRECAV-1
      AI=M-2
      TH1=AI*DELTH
      TH2=TH1+DELTH
      TH3=TH2+DELTH
C
      AM11=-FTH1(M)*COS(TH1)+8.*FTH1(M+1)*COS(TH2)+5.*FTH1(M+2)*COS(TH3)
      AM12=-FTH1(M)*SIN(TH1)+8.*FTH1(M+1)*SIN(TH2)+5.*FTH1(M+2)*SIN(TH3)
      WDX1=WDX1+AM11*DELTH/12.
      WDY1=WDY1+AM12*DELTH/12.
C
      AM21=-FTH2(M)*COS(TH1)+8.*FTH2(M+1)*COS(TH2)+5.*FTH2(M+2)*COS(TH3)
      AM22=-FTH2(M)*SIN(TH1)+8.*FTH2(M+1)*SIN(TH2)+5.*FTH2(M+2)*SIN(TH3)
      WDX2=WDX2+AM21*DELTH/12.
```

```
      WDY2=WDY2+AM22*DELTH/12.
      GOTO 311
310   CONTINUE
311   WDX1=(-2/3.)*R**2*WDX1
      WDX2=(-2/3.)*R**2*WDX2
      WDY1=(-2/3.)*R**2*WDY1
      WDY2=(-2/3.)*R**2*WDY2
========================================================================
C     STIFFNESS AND DAMPING COEFFICIENTS FOR THE PERTURBED FILM
C     PRESSURE, P1
========================================================================
      SRR=-REAL(WDX1)
      SPR=-REAL(WDY1)
      WDX1A=AIMAG(WDX1)
      WDY1A=AIMAG(WDY1)
      DRR=-WDX1A/RLAMDA
      DPR=-WDY1A/RLAMDA
========================================================================
C     STIFFNESS AND DAMPING COEFFICINTS FOR THE PERTURBED FILM
C     PRESSURE. P2
========================================================================
      SRP=-REAL(WDX2)
      SPP=-REAL(WDY2)
      WDX2A=AIMAG(WDX2)
      WDY2A=AIMAG(WDY2)
      DRP=-WDX2A/RLAMDA
      DPP=-WDY2A/RLAMDA
      WRITE(*,*)'WDX1=',WDX1,'WDY1=',WDY1,'WDX2=',WDX2,'WDY2=',WDY2
      WRITE(3,1020)R,EPH0,W,ATT,SRR,SPR,DRR,DPR,SRP
     #SPP,DRP,DPP
1020  FORMAT(F7.5,1X,F4.2,1X,10E16.5)
C
      EPH0=EPH0+0.1
      KNT=KNT+1
      GOTO 410
      END
```