Identification of Most Sensitive Node of Radial Distribution Networks

Thesis submitted in partial fulfillment of requirement for the award of degree of

Master of Engineering
in
Power System and Electric Drives

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CERTIFICATE

I hereby certify that the Thesis entitled **"Identification of Most Sensitive Node of Radial Distribution Networks"** in the partial fulfillment of the requirement for the degree of MASTER OF ENGINEERING in POWER SYSTEM and ELECTRIC DRIVES, submitted in Electrical and Instrumentation Department, Thapar University, Patiala is an authentic work carried out under the guidance of Dr. SMARAJIT GHOSH, Professor and Head EIED, Thapar University and refers other researcher's works, which are duly listed in the reference section.

To the best of my knowledge the matter embodied in the thesis has not been submitted to any other university/institute for the award of any degree or diploma.

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This is certifying that the above statement made by the student is correct and true to the best of my knowledge and belief.

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Dedicated to My Family and Friends

ABSTRACT

In this thesis work an attempt has been made to propose a suitable voltage stability index (VSI) for distribution network. An exhaustive literature survey in voltage stability index has been carried out. The VSIs available in literature are based on reduction of network, which gave the inexact result. The available VSIs did not include the voltage angle. The proposed VSI has been derived without reduction of the network and without neglecting the voltage angle.

The proposed method has been tested on a 85-node radial distribution network for constant power, constant current, constant impedance, composite and exponential load modeling for substation voltage 1.000 p.u., 1.025 p.u. and 1.050 p.u. respectively.

The proposed result obtained by the proposed method has been compared with the other method proposed by Chakravorty and Das [**19**]. To derive the results, the load-flow proposed by Ghosh [**34**] has been used in this thesis work.

The line and load data are available in [**35**].

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LIST OF SYMBOLS

jj : Branch number

N(i) : Total number of nodes of feeder, lateral(s) or sub lateral(s)

NB(i) : Total number of branches of feeder, lateral(s) or sub lateral(s)

NN(i) : Array for storing the nodes

PR(jj) : Active power at the branch–jj entering the node NN(jj+1)

QR(jj) : Reactive power at the branch–jj entering the node NN(jj+1)

PS(jj) : Active power at the branch–jj coming out of the node NN(jj)

QS(jj) : Reactive power at the branch–jj coming out of the node NN(jj)

I(jj) : Current through the branch–jj

LP(jj) : Real power loss of the branch–jj

LQ(jj) : Reactive power loss of the branch–jj

PL(m2): Real power load at the node m2

QL(m2): Reactive power load at the node m2

V(m2) : Complex value of the voltage at the node m2

Z(jj) : Impedance of the branch–jj

R(jj): Resistance of branch–jj

X(jj): Resistance of branch–jj

CHAPTER 1

INTRODUCTION

1.1 OVERVIEW

Distribution system is a part of power system. The transmission networks are loop in nature while the distribution systems are radial (practically weakly meshed) in nature. Distribution systems are generally unbalanced. Balanced distribution system consists of three-phase laterals with the loads being equally distributed among the three phases. Unbalanced distribution system, on the other hand, is a mixture of three-phase, two-phase and single-phase laterals. Voltage instability is due to inability of power system to maintain steady-state voltage at all buses following a disturbance, increased in load demand or change in operating condition given initial operating condition. The main factor that causes voltage instability is inability of distribution system to meet the reactive power demand. Load stability also known as voltage stability as they are main driving force into voltage instability. Hence, by analyzing the load can determine the voltage stability of a distribution system.

Power systems operation becomes more important as the load demand increases all over the world. This rapid increase in load demand forces power systems to operate near critical limits due to economical and environmental constraints. The objective in power systems operation is to serve energy with acceptable voltage and frequency to consumers at minimum cost.

Reliability and stability are also important parameters for power systems and should be satisfied. By reliability, it is meant that the system has adequate reserves in the face of changing energy demand. By stability, it is meant that upon occurrence of a contingency, the system could recover to its original state and supply the same quality service as before. All these objectives can be achieved by proper planning, operation and control of power generation, transmission and distribution systems.

Voltage stability problems may occur in power system when there is an increase in load demand. Voltage instability is one of the main problems in power systems. In voltage stability problem some or all bus voltages decrease due to insufficient power delivered to loads. In case of voltage stability problems, serious blackouts may occur in a considerable part of a system. This can cause severe social and economic problems. In fact, more than 55 cases of voltage instability or voltage collapse were reported all over the world between 1965 and 2012. For example, a voltage collapse in North India in the year 2012 resulted in service interruptions to millions of people. When the necessity of electricity to industry and community in all fields of the life is considered, the importance of a blackout can be understood more easily. Therefore, special analysis should be performed in order to examine the voltage stability in power systems.

As power systems are operated under increasingly stressed conditions, the ability to maintain voltage stability becomes a growing concern. Voltage stability refers to ‘‘the ability of a power system to maintain steady voltages at all buses in the system after being subjected to a disturbance from a given initial operating condition’’. If voltage stability exists, the voltage and power of the system will be controllable at all times. In general, the inability of the system to supply the required demand leads to voltage instability (voltage collapse). The nature of voltage instability phenomena can be either fast (short-term, with voltage collapse in the order of fractions of a second to a few seconds) or slow (long-term, with voltage collapse in minutes to hours). Short-term voltage stability problems are usually associated with the rapid response of voltage controllers (e.g., generators’ Automatic Voltage Regulator [AVR]) and power electronic converters, such as those encountered in flexible AC transmission system or FACTS controllers and high voltage DC (HVDC) links. The analysis of voltage stability for a given system state involves the examination of two aspects:

- Proximity: how close is the system to voltage instability?
- Mechanism: when voltage instability occurs. What are the key contributing factors? What are the voltage-weak points, and what areas are involved?

Proximity gives a measure of voltage security whereas mechanism provides information useful in determining system modifications or operating strategies, which could be used to prevent voltage instability. Voltage stability is indeed a dynamic phenomenon and can be studied using extended transient/midterm stability simulations. However, such simulations do not readily provide sensitivity information or the degree of stability. They are also time consuming in terms of CPU and engineering required for analysis of results. Therefore, the application of dynamic simulations is limited to investigation of specific voltage collapse situations including fast or transient voltage collapse and for coordination of protection and controls. Voltage stability analysis often requires examination of a wide range of system conditions and a large number of contingency scenarios. For such applications, the approach based on steady state analysis is more attractive and if used properly, can provide much insight into the voltage/reactive power problem.

1.2 INTRODUCTION TO DISTRIBUTION SYSTEM

System that comprises those parts of an electric power system between the subtransmission system and the consumers' service switches is known as distribution system. It includes distribution substations; primary distribution feeders; distribution transformers; secondary circuits, including the services to the consumer; and appropriate protective and control devices. Sometimes, the subtransmission system is also included in the definition. Single line diagram of a typical low tension distribution system is shown in Fig. 1.1.

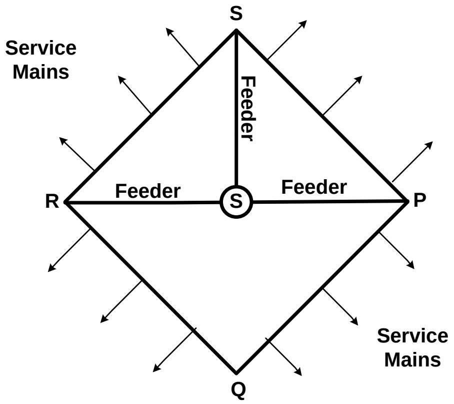

Figure 1.1 Single Line Diagram of a Typical Low Tension Distribution System

The distribution of power over a utility company network is a complex process involving a number of power-generating plants, transmission lines, and substations. The physical size of a metropolitan power distribution and control system is immense. Substations use massive transformers, oil-filled circuit breakers, huge strings of insulators, and high-tension conductors in distributing power to customers. Power distribution and transmission networks interconnect generating plants into an area grid, to which area loads are attached. Most utility systems are interconnected to one extent or another. In this way, power-generating resources can be shared as needed. The potential for single-point failure also is reduced in a distributed system. A typical power-distribution network is shown in Fig.1.2.

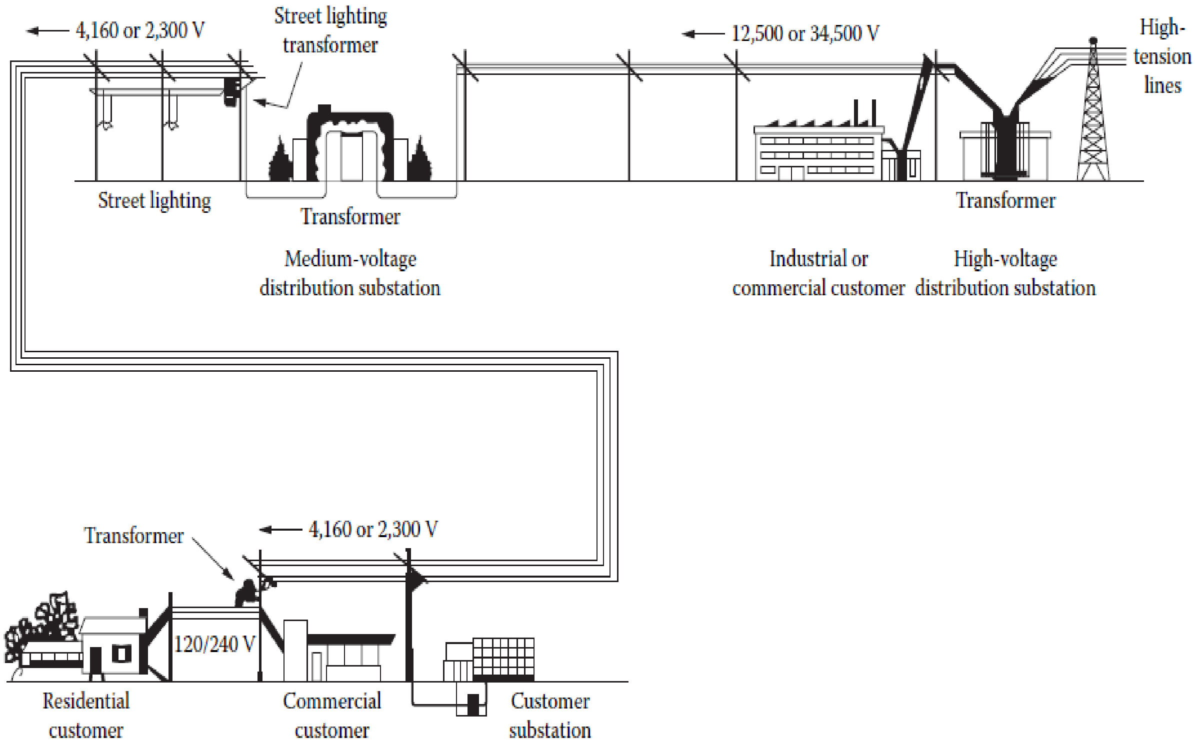

Figure 1.2 Simplified Power-Distribution Architecture **[1]**

Electric power distribution is the portion of the power delivery infrastructure that takes the electricity from the highly meshed, high-voltage transmission circuits and delivers it to customers. Primary distribution lines are “medium-voltage” circuits, normally thought of as 600V to 35kV. At a distribution substation, a substation transformer takes the incoming transmission-level voltage (35kV to 230kV) and steps it down to several distribution primary circuits, which fan out from the substation. Close to each end user, a distribution transformer takes the primary-distribution voltage and steps it down to a low-voltage secondary circuit (commonly 120/240V; other utilization voltages are used as well). From the distribution transformer, the secondary distribution circuits connect to the end user where the connection is made at the service entrance. Power transmission lines operate at voltage levels from 2.3 kV for local distribution to 500 kV or more for distribution between cities or generating plants. Long-distance, direct current transmission lines also are used, with potentials of 500 kV and higher.

Underground power lines are limited to short runs in urban areas. Increased installation costs and cable heat-management considerations limit the use of high-voltage underground lines. Wide variations in standard voltage levels can be found within any given system. Each link in the network is designed to transfer energy with the least I^2R loss, thereby increasing overall system efficiency. The following general classifications of power-distribution systems can be found in common use:

1.2.1 Radial System

The simplest of all distribution networks, a single substation supplies power to all loads in the system as shown in Fig. 1.3.

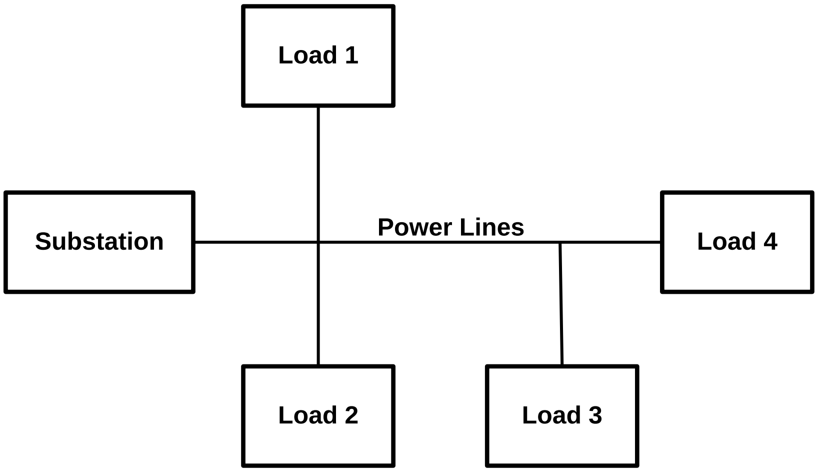

Figure 1.3 Radial Power-Distribution System

1.2.2 Ring System

Distribution lines encircle the service area, with power being delivered from one or more sources into substations near the service area. Power is then distributed from the substations through the radial transmission lines as shown in Fig. 1.4.

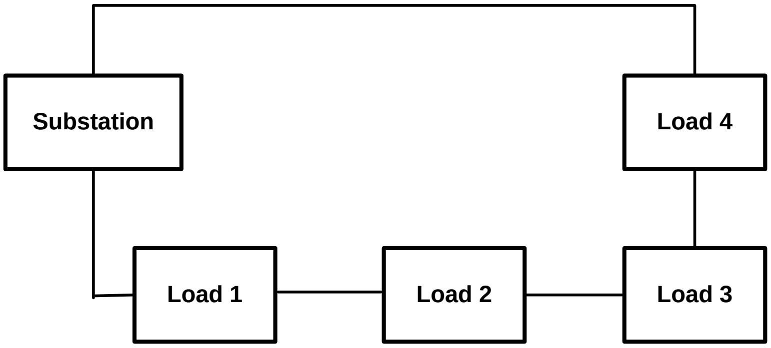

Figure 1.4 Ring Power-Distribution System

1.2.3 Network System

A combination of the radial and ring distribution systems. Although such a system is more complex than either of the previous configurations, reliability is improved significantly. The network system, illustrated in Fig. 1.5, is one of the most common power-distribution configurations.

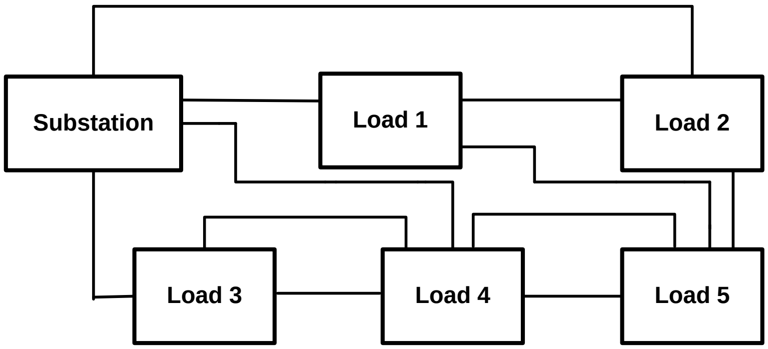

Figure 1.5 Network Power-Distribution System

Some substations are entirely enclosed in buildings, whereas others are built entirely in the open with all equipment enclosed in one or more metal-clad units. The final design of the type of substation depends on economic factors; future load growth; and environmental, legal, and social issues.

1.2.4 Distribution Substations

Distribution substations serve as the source for primary distribution feeders. They receive bulk electric power at high voltages and reduce the voltage to distribution primary values. Also associated with a substation are provisions for protection from faults, for voltage regulation, and for data acquisition and monitoring. The equipment generally installed in a distribution substation includes:

- Power transformers
- Oil or air circuit breakers
- Voltage regulators
- Protective relays
- Air break and disconnect switches
- Surge arresters
- Measuring instruments
- Storage batteries and capacitors (in some installations)

1.3 REQUIREMENT OF A DISTRIBUTION SYSTEM

A considerable amount of effort is necessary to maintain an electric power supply within the requirements of various types of consumers. Some of the requirements of a good distribution system are: proper voltage, availability of power on demand, and reliability

1.3.1 Proper Voltage

One important requirement of a distribution system is that voltage variations at consumer’s terminals should be as low as possible. The changes in voltage are generally caused due to the variation of load on the system. Low voltage causes loss of revenue, inefficient lighting and possible burning out of motors. High voltage causes lamps to burn out permanently and may cause failure of other appliances. Therefore, a good distribution system should ensure that the voltage variations at consumer’s terminals are within permissible limits. The statutory limit of voltage variations is ±10% of the rated value at the consumer’s terminals. Thus, if the declared voltage is 230 V, then the highest voltage of the consumer should not exceed 244 V while the lowest voltage of the consumer should not be less than 216 V.

1.3.2 Availability of Power Demand

Power must be available to the consumers in any amount that they may require from time to time. For example, motors may be started or shut down, lights may be turned on or off, without advance warning to the electric supply company. As electrical energy cannot be stored, therefore, the distribution system must be capable of supplying load demands of the consumers. This necessitates that operating staff must continuously study load patterns to predict in advance those major load changes that follow the known schedules.

1.3.3 Reliability

Modern industry is almost dependent on electric power for its operation. Homes and office buildings are lighted, heated, cooled and ventilated by electric power. This calls for reliable service. Unfortunately electric power, like everything else that is man-made, can never be absolutely reliable. However, the reliability can be improved to a considerable extent by

a) Inter-connected system,
b) Reliable automatic control system and
c) Providing additional reserve facilities.

1.4 DESIGN CONSIDERATIONS IN DISTRIBUTION SYSTEM

Good voltage regulation of a distribution network is probably the most important factor responsible for delivering good service to the consumers. For this purpose, design of feeders and distributors requires careful consideration.

1.4.1 Feeders

A feeder is designed from the point of view of its current carrying capacity while the voltage drop consideration is relatively not important. It is because voltage drop in a feeder can be compensated by means of voltage regulating equipment at the sub-station.

1.4.2 Distributors

A distributor is designed from the point of view of the voltage drop in it. It is because a distributor supplies power to the consumers and there is a statutory limit of voltage variations at the consumer terminal (± 10% of rated value). The size and length of the distributor should be such that voltage at the consumer’s terminals is within the permissible limits.

1.5 IMPORTANCE OF LOAD-FLOW STUDY

Analysis of distribution system using Load-flow is important in the field of power systems. Distribution systems are predominantly characterized by their high R/X ratio and radial topology. Matrix based iterative methods do not lend themselves for radial distribution systems owing to these characteristics. So load-flow studies are adopted, which proved to be the best tool to analyze a power system condition. Load-flow studies are important for planning and future expansion of distribution systems as well as in determining the best operation of existing systems. The principal information obtained from the load-flow study is the magnitude and phase angle of the voltage at each bus, and the real and reactive power flowing in each line. Some points related to the importance of load-flow study are given below:

- Load flow analysis is very important in planning stages of new networks or addition to existing ones like adding new generator sites, meeting increase load demand.
- The load-flow solution gives the nodal voltages and phase angles and hence the power injection at all the buses and power flows through interconnecting power channels.
- It is helpful in determining the best location as well as optimal capacity of proposed substation.
- It determines the voltage of the buses. The voltage level at the certain buses must be kept within the closed tolerances.

1.6 VOLTAGE ANALYSIS

Distribution systems are designed to maintain service voltages within specified limits during normal and emergency conditions. Typical voltage limits are:

- For service to residential customers, the voltage at the point of delivery shall not exceed ±5% of the nominal voltage.
- For service to commercial or industrial customers, the voltage at the point of delivery shall not exceed ±7.5% of the nominal voltage.
- The maximum allowable voltage imbalance for a three-phase service shall be ±2.5%.

The goal of voltage analysis is to determine whether the voltages on different line sections remain within the specified limits under varying load conditions. Thus, voltage analysis facilitates the effective placement of capacitors, voltage regulators, and other voltage regulation devices on the distribution system. Load-flow analysis is a computer-aided tool that is typically used in this planning task. Voltage analysis begins with an accurate representation, or map, of the feeder circuits, starting at the substation. The map generally consists of details and electrical characteristics (such as kVA ratings, impedances, and other parameters) of the conductors and cables on the system, substation and distribution transformers, series and shunt capacitors, voltage regulators, and related devices. Before the analysis can begin, feeder loading must be known. Several different methods can be used for this task. If the utility maintains a database on each customer connected to a distribution transformer, it can use the billing data to determine the kilowatt-hours supplied by each transformer for a given month. Methods can then be used to convert the kilowatt-hours to a non-coincident peak kilo-volt-ampere demand for all distribution transformers connected on the feeder. If this information is not available, the kilo-volt-ampere rating of the transformer and a representative power factor can be used as the load. With the metered demand at the substation, the transformer loads can be allocated, for each phase, such that the allocated loads plus losses will equal the metered substation demand. Accurately representing the load types or models is an important issue in voltage analysis. Several load models are available, including:

- Spot and distributed loads
- Star and delta connected loads
- Constant power, constant current, constant impedance, or a combination of these methods

1.7 DEFINITION AND CLASSIFICATION OF VOLTAGE STABILITY

Power system stability is defined as characteristics for a power system to remain in state of equilibrium after a disturbance. Traditionally, the stability problem has been the rotor angle stability, i.e. maintaining synchronous operation. Instability may also occur without loss of synchronism, in which case the concern is the control and stability voltage. Fig. 1.6 shows the classification of voltage stability.

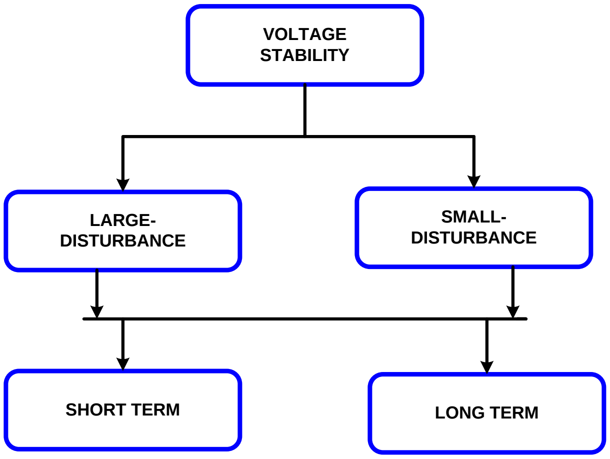

Figure 1.6 Classification of Voltage Stability

1.7.1 Voltage Stability

The voltage stability is the ability of a power system to maintain steady acceptable voltage at all buses in the system at normal operating condition and after being subjected to a disturbance. Power system is voltage stable if voltage after a disturbance is close to voltage at normal operating condition. A power system becomes unstable when voltages uncontrollably decrease due to outage of equipment (generator, line, transformer, bus bar, etc.), increment of load, decrement of production and/or weakening of voltage control.

1.7.2 Voltage Instability

Voltage instability stems from the load dynamics to restore power consumption beyond the capability of the combined transmission and generation system.

1.7.3 Voltage Collapse

Voltage collapse is the process by which voltage instability leads to the loss of voltage in a significant part of the system. This condition results from reactive losses significantly exceeding the reactive resources available to supply them. Circuits loaded above surge impedance loadings and reduced output of shunt capacitors as voltages decline can lead to accelerating voltage drops. Voltage collapse can look like both a steady-state problem with time to react and a problem where no effective operator intervention is possible. It is very hard to predict the area that will be affected or electrically isolated from the grid.

1.8 CLASSIFICATION OF POWER SYSTEM STABILITY

Power system stability is classified as rotor angle and voltage stability. A classification of power system stability based on time scale and driving force criteria is shown in Table 1.1. The driving forces for an instability mechanism are named generator-driven and load driven.

The rotor angle stability is divided into small-signal and transient stability. The small-signal stability is present for small disturbances in the form of undamped electromechanical oscillations. The transient stability is due to lack of synchronizing torque and is initiated by large disturbances. The time frame is called short-term timescale, because the dynamics typically last for a few seconds. The voltage problem is load-driven as described above. The voltage stability may be divided into short and long-term voltage stability according to time scale of load components dynamics as shown in Table 1.1.

Table 1.1 Classification of Power System Stability

Time Scale	Generator-driven		Load-driven	
Short-term	Rotor angle stability		Short term voltage stability	
	Small-signal	Transient		
Long-term	Frequency stability		Long-term voltage stability	
			Small disturbance	Large Disturbance

Short-term voltage stability is characterized by components such as induction motor, excitation of synchronous generators and electronically controlled devices such as HVDC and static VAr compensator. The timescale of short-term voltage stability is same as rotor angle stability. The modeling and the analysis of these problems are similar. The distinction between rotor angle and short-term voltage instability is sometimes difficult, because most practical voltage collapse includes some elements of both voltage and angle instability. When short-term dynamics have died out sometime after the disturbance, the system enters a slower time frame.

The dynamics of the long-term time scale lasts for several minutes. Two types of stability problems emerge in the long term time scale: frequency and voltage problems. Frequency problems may appear after a major disturbance resulting in power system islanding. Frequency instability is related to the active power imbalance between generators and loads. An island may be either under or over-generated when the system frequency either declines or rises.

The analysis of long-term voltage stability requires detailed modeling of long-term dynamics. The long-term voltage stability is characterized by scenarios such as load recovery by the action of on-load tap changer or through load self-restoration, delayed corrective control actions such as shunt compensation switching or load shedding. The long-term dynamics such as response of power plant controls, boiler dynamics and automatic generation control also effect long-term voltage stability. The modeling of long-term voltage stability requires consideration of transformer on-load tap changers, characteristics of static loads, manual control actions of operators, and automatic generation control.

For purpose of analysis, it is sometimes useful to classify voltage stability into small and large disturbances. Small disturbance voltage stability considers the power systems ability to control voltages after small disturbances, e.g. change in load. The analysis of small disturbance voltage stability is done in steady state. In that case power system can be liberalized around an operating point and the analysis is typically based on eigen value and eigen vector technique. Large disturbances voltage stability can be studied by using non-linear time domain simulations in the short term time frame and load-flow analysis in the long term time frame. The voltage stability is, however, a single problem on which a combination of both linear and non-linear tools can be used.

1.9 CAUSES OF VOLTAGE INSTABILITY

The following are the causes of voltage instability:

1. The load on distribution line is too high.
2. The voltage sources are too far from the load centers.
3. The source voltages are too low.
4. There is insufficient load reactive compensation.

1.10 SCOPE OF THE THESIS WORK

The literature survey shows that the voltage stability analysis or detection of most sensitive node of the distribution network had been done using the reduced network and failed to incorporate to use the voltage angle. They had simplified the equation neglecting the voltage angle.

1.11 OBJECTIVES OF THE THESIS WORK

The thesis work endeavors to derive a new expression of voltage stability index (VSI) and its applications in planning of power distribution system. The objectives are divided into the following:

- To derive a new expression of VSI to be computed for all nodes of the distribution networks.
- To identify the most sensitive nodes of distribution networks.

The detailed literature survey of voltage stability analysis of distribution networks has been presented in chapter-2.

1.12 ORGANIZATION OF THE THESIS WORK

Chapter 1 introduces the introduction of distribution system, voltage stability of the distribution systems, scope and objectives of the thesis.

Chapter 2 presents the literature survey of distribution system.

Chapter 3 presents the assumption, formulation of voltage stability index, examples and results and conclusions.

Chapter 4 presents overall conclusion and future scope of further research work.

References

Appendix-A Line Data and Load Data of 85-node Radial Distribution Network

CHAPTER 2

LITERATURE SURVEY ON VOLTAGE STABILITY ANALYSIS OF DISTRIBUTION SYSTEMS

Voltage stability analysis is currently one of the most significant fields of research in the power systems area. In the last few years, many contributions to a better knowledge of the various aspects of voltage problems have been reported in the literature, where the problem has been explored from many different points of view. Voltage collapse in addition, has been an active subject of research for years.

In the following section, a detailed literature review on the voltage stability and voltage stability index are introduced. Literature survey shows that a lot of work still needs to be done on voltage stability analysis of power distribution system.

O'Grady and Pai [2] extended the singular value decomposition method for detecting voltage collapse to include voltage dependent loads, Q-limited generation sources and also proposed a new indicator called the condition number which was more sensitive than the minimum singular value.

Kessel and Glavitsch [3] proposed a method for online testing of power system aimed at the detection of voltage instabilities. An indicator L was defined which varied in the range between 0 (no load of system) and 1 (voltage collapse). Based on the basic concept of such an indicator various models were derived which allowed to predict voltage instability or the proximity of a collapse.

Kurita and Sakuraj [4] studied the reason of the power system failure occurred in the year 1987 in the service area of The Tokyo Electric Power Company, Inc. (TEPCO). Some of the major reasons found out were voltage instability, high power demand and the rate of increase of power demand. They also gave the importance of voltage stability studies.

Merritt *et al.* [5] compared two approaches for steady-state bulk transmission system voltage analysis. The first one was a conventional approach using both a power flow program and the other one was "Optimal Power Flow program" and a "Security Constrained Optimization Program". The later was used in solving difficult capacitor planning studies.

Brownell *et al.* [6] presented the recordings of increased load demand of a system and showed the voltage collapse of the system. They also proposed urgent compensation of reactive power.

Jasmon and Lee [7] proposed a voltage stability analysis of radial distribution networks. They reduced the whole network by a single line diagram that was valid only at the derived operating point. They had put voltages of all nodes equal to 1.0 pu for simplifying the derivation of voltage stability index. This method was unable to handle changing load pattern.

Gao *et al.* [8] provided a voltage stability assessment technique for large power systems using modal analysis. The method computes a specified number of the smallest Eigen values of a reduced Jacobian matrix and the associated bus, branch, and generator participations.

Ajjarapu *et al.* [9] proposed both the capability and usefulness of the continuation power flow in voltage stability analysis of transmission system. Their method could be used incorporating nonlinear load models into the process so that a more accurate assessment of voltage stability performance could be made.

Chebbo *et al.* [10] suggested a method about studying the voltage collapse using Thevenin's theorem. The voltage collapse proximity indicator was incorporated in a reactive power dispatch algorithm for minimizing the risk of a voltage collapse in the system.

Morison *et al.* [11] presented static and dynamic techniques for voltage stability analysis in power system. Using a small test system, results of time domain simulations were presented to clarify the phenomenon of voltage instability and to better understand modeling requirements. The same system was then analyzed using a static approach in which modal analysis was performed wing system conditions, or snapshots, which approximate different stages along the time domain trajectory. The results obtained using the dynamic and static method were compared and shown to be consistent.

Rahman *et al.* [12] proposed a method to study the voltage collapse using Thevenin's theorem. They suggested a voltage stability index.

Lind and Karlsson [13] presented a method for voltage stability studies including radial distribution system, with cascaded tap-changers. In their model of the network, all lines and transformers explicitly represented and compared with a simple model consisting of impedance in series with a modified tap-changer.

Moghavvemi [14] proposed a method for determining the voltage stability factor based on the concept of power flow thorough a single line equivalent. Adopting the technique of reducing the power system network into the equivalent single line system, a voltage stability factor was derived and used to examine the overall system stability.

Gubina *et al.* [15] proposed a method to study voltage stability analysis of radial distribution networks reducing the system model in the form of single line equivalent. The approach was based on exact two-bus equivalent of a radial network. The equivalent was built on assumptions, which enabled uniform transform of selected phasor variables of the radial network.

Chen and Wang [16] described the DistFlow method to find the load flow solutions for radial power networks from which an equivalent 2-bus network could be obtained during the solving process where only one feasible voltage solution existed for a radial power network which could be judged directly from the sign of the Jacobian determinant of the equivalent 2-bus network obtained.

Kashem *et al.* [17] used only local measurements to find a proximity index based on the voltage to power sensitivities for predicting voltage collapse on a load bus which was a measure of closeness of the current operating point to the stability limit point. The index could also identify the marginally stable operating point which might extend beyond the maximum power transfer point by estimating the network equivalent as viewed from a load bus.

Kashem *et al.* [18] presented an algorithm for enhancement of voltage stability by network reconfiguration. They performed the network reconfiguration by altering the topological structure of distribution feeders by which voltage stability could be maximized for a particular set of loads in distribution system.

Chakravorty and Das [19] proposed a voltage stability index for identifying the most sensitive node of the network. They handled the composite load using power convergence and used the load-flow technique. They had shown that the critical loading for constant current load was maximum.

Musirin *et al.* [20] presented a Fast Voltage Stability Index (FVSI) simplified from a pre-developed voltage stability index referred to a line initiated from the voltage quadratic equation at the sending end of a representation of a 2-bus system. The line index in the inter-connected system in which the value that was closed to 1.00 indicated that the line reached its instability limit.

Haque [21] proposed a simple equivalent model of the power system to generate the voltage stability boundary involving very little computations without any repetitive load flow simulations and thus various voltage stability margins of the critical bus or area in a large power system could be directly determined from the generated stability boundary for different initial operating conditions with a high potential for on-line application.

Ranjan *et al.* [22] suggested a new voltage stability index to identify the most sensitive node of the network. They assumed the equality of magnitude of voltage for sending end node and receiving end node of each branch while deriving voltage stability index. They had shown that critical loading for constant impedance load was maximum. In their proposed method they compared the result for critical loading with the result obtained by [12]. They were silent regarding the result of critical loading for composite load modeling.

Wang *et al.* [23] developed a two-bus equivalent methodology based on tracking the weakest power transmission path by defining the weak power draining buses incorporating the electrical distance information and reactive generation reserves so that the on line voltage stability of the power system could be assessed correctly instead of using Thevenin equivalent parameters.

Smon *et al.* [24] simplified the determination of the Thevenin's parameters and enabled derivation of the new local voltage stability index using Tellegen's theorem and adjoint networks which required the voltage and current phasors measurements only, to evaluate the system's voltage stability at a bus and therefore it was very appealing for PMU-based online monitoring and protection schemes as it involved one-step calculation procedure.

Jabr and Pal [25] formulated the radial distribution load capability computation as a conic quadratic optimization problem. Static voltage stability index was used as by product of the conic solution which gave a measure of the distance to collapse. The proposed formulation could be extended to unbalanced three-phase networks.

Eminoglu and Hocaoglu [26] presented a network topology-based voltage stability index for identifying the most sensitive bus to the voltage collapse in the radial distribution networks. The developed index was based on the transferred active and reactive power equation of the distribution line. Network topology algorithm based on the bus-injection to branch-current (BIBC) matrix was used for implementation of developed stability index to the radial distribution networks.

Arun and Aravindhababu [27] presented a reconfiguration algorithm for enhancing voltage stability and to improve the voltage profile besides minimizing losses without incurring any additional cost for installation of capacitors, tap changing transformers and related switching equipment in the distribution system.

Eidiani [28] presented a method for assessing static voltage stability in transmission and distribution networks. The expanded Newton–Raphson–Seydel (NRS) and Down-Hill (DH) algorithms were employed. Standard CPF and expanded NRS methods were compared to his proposed method. These algorithms were tested on 420 bus transmission and 4438 bus distribution networks.

Chanda and Das [29] proposed a voltage stability indicator derived from voltage equation of radial distribution system. The voltage stability indicator (VSI) could identify the condition of load buses in the voltage collapse point of view. The developed VSI was tested on a standard 32-bus radial distribution system for reliability test. A relationship between the proposed indicator and active and reactive losses of different branches of the system was also established, from which the weak branch could be determined easily without detail loss calculation.

Ali *et al.* [30] developed voltage stability index based on the transferred active and reactive power of a distribution line. The developed index utilized the continuous power flow solution calculated using real time power flow software in distribution management systems.

Mahmoud [31] proposed catastrophe theory for finding voltage stability index (VSI) for identifying the most sensitive bus in radial distribution network and the voltage stability boundary of a distribution system.

Guiping *et al.* [32] proposed an equivalent system considering distribution network with high/medium voltage and an approach of online tracking of voltage stability margin of the equivalent system were proposed. By applying the theorem of voltage stability, it was derived that because of power loss caused by the distribution line in the equivalent system, voltage collapse occurred at the distribution bus before at the transmission bus.

Gunalan *et al.* [33] investigated the impact of each load characteristic individually and also as combination on voltage stability of an IEEE 34 Bus Distribution System by increasing the load demand, and transient analysis. The most suitable load to study the voltage stability was also proposed. It was found that, with constant power loads, the reactive power demand increases significantly. Constant power load is found to be the suitable load to study the voltage stability.

Ghosh [34] presented a load-flow technique based on nodes beyond branches for solving radial distribution network, which was very efficient computationally.

CHAPTER 3

PROBLEM FORMULATION AND RESULTS

3.1 INTRODUCTION

Voltage stability is a property of power distribution system which enables it to stay in a state of equilibrium under normal operating condition and the system also comes back to an acceptable state of equilibrium after a disturbance. If the power consumption from the system goes beyond its capability, the voltage instability occurs immediately and a sequence of events accompanying voltage instability results in a low acceptable voltage profile of the distribution networks. The transmission system is loop in nature. On the other hand, the distribution system is radial in nature. The distribution networks have high R/X ratio compared to the transmission networks and hence the distribution networks are ill–conditioned in nature.

The modern power distribution networks are constantly being faced with an ever–growing load demand. Distribution networks experience distinct change of load from a lower to higher level every day. The distribution system experiences voltage collapse above certain critical loading conditions. Power system voltage stability is system’s capability to keep acceptable voltages in all buses in normal conditions after disturbances.

Voltage stability is a major concern in planning and operations of power systems. It is well known that voltage instability and collapse have led to major system failures. With the development of power markets, more and more electric utilities are facing voltage stability–imposed limits.

Literature survey has already discussed in chapter-2. The problem formulation and the results are presented in this Chapter. The load-flow proposed by Ghosh [34] is used this thesis work.

3.2 ASSUMPTION

The three-phase radial distribution networks are taken as balanced and hence can be represented by their single-line diagram.

3.3 SOLUTION METHODOLOGY

A single-line diagram of a radial distribution network is shown in Fig. 3.1 and Table 3.1 shows the branch number, sending end node and receiving end node of Fig. 3.1.

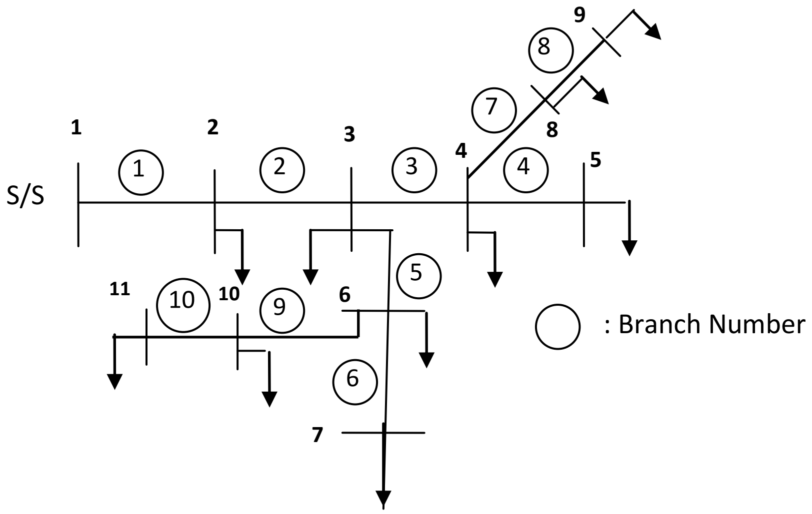

Figure 3.1 Single-line Diagram of a Radial Distribution Network [34]

Table 3.1: Branch number (jj), Sending end node (m1 = IS(jj)), Receiving end node (m2 = IR(jj)) and nodes beyond branches 1, 2, 3, …, 10 of Fig. 3.1 [34]

Branch Number (jj)	**Sending end m1 = IS(jj)**	**Receiving end m2 = IR (jj)**
1	1	2
2	2	3
3	3	4
4	4	5
5	3	6

6	6	7
7	6	8
8	7	9
9	4	10
10	10	11

The following symbols are used in this thesis work:

jj : Branch number

N(i) : Total number of nodes of feeder, lateral(s) or sub lateral(s)

NB(i) : Total number of branches of feeder, lateral(s) or sub lateral(s)

NN(i) : Array for storing the nodes

PR(jj) : Active power at the branch–jj entering the node NN(jj+1)

QR(jj) : Reactive power at the branch–jj entering the node NN(jj+1)

PS(jj) : Active power at the branch–jj coming out of the node NN(jj)

QS(jj) : Reactive power at the branch–jj coming out of the node NN(jj)

I(jj) : Current through the branch–jj

LP(jj) : Real power loss of the branch–jj

LQ(jj) : Reactive power loss of the branch–jj

PL(m2): Real power load at the node m2

QL(m2): Reactive power load at the node m2

V(m2) : Complex value of the voltage at the node m2

Z(jj) : Impedance of the branch–jj

R(jj): Resistance of branch–jj

X(jj): Resistance of branch–jj

$$\bar{V}(m1) = \bar{V}(m2) + \bar{I}(jj)\bar{Z}(jj) \tag{3.1}$$

$$V(m1)\angle\delta_1 = V(m2)\angle\delta_2 + [I_{re}(jj) + jI_{im}(jj)][\mathrm{R(jj)} + \mathrm{jX(jj)}]$$

$$\begin{aligned} V(m1)\cos\delta_1 + jV(m1)\sin\delta_1 = V(m2)\cos\delta_2 + V(m2)\sin\delta_2 + [I_{re}(jj)\,\mathrm{R(jj)} - \mathrm{I}_{im}(jj)X(jj)] \\ - j[I_{re}(jj)X(\mathrm{jj}) + \mathrm{I}_{im}(jj)R(jj)] \end{aligned} \tag{3.2}$$

From Eq. (3.2), we have

$$V(m1)\cos\delta_1 = V(m2)\cos\delta_2 + E \tag{3.3}$$

$$V(m1)\sin\delta_1 = V(m2)\sin\delta_2 + F \tag{3.4}$$

where

$$E = I_{re}(jj)\,\mathrm{R(jj)} - \mathrm{I}_{im}(jj)X(jj) \tag{3.5}$$

$$F = I_{re}(jj)X(\mathrm{jj}) + \mathrm{I}_{im}(jj)R(jj) \tag{3.6}$$

From Eq. (3.3) and Eq. (3.4), we have

$$V(m2)^2 + 2[E\cos\delta_2 + F\sin\delta_2]V(m2) + (E^2 + F^2) = V(m1)^2$$

i.e., $\{V(m2)\}^2 + 2[\mathrm{E}\cos\delta_2 + F\sin\delta_2]V(m2) + [(E^2 + F^2) - \{V(m1)^2\}] = 0$ (3.7)

i.e., $\{V(m2)\}^2 + 2CV(m2) + D = 0$ (3.8)

where $C = E\cos\delta_2 + F\sin\delta_2$ (3.9)

and $D = (E^2 + F^2) - \{V(m1)\}^2$ (3.10)

From Eq. (3.10) we have

$$V(m2) = -C \pm \sqrt{C^2 - 4D} \tag{3.11}$$

From Eq. (3.11), the feasible solution is

$$V(m2) = -C + \sqrt{C^2 - 4D} \tag{3.12}$$

Since V(m2) is always > 0, the following condition must be satisfied.

$$\sqrt{C^2 - 4D} > C$$

i.e., $C^2 - 4D > C^2$

i.e., $D < 0$

i.e., $(E^2 + F^2) - \{V(m1)\}^2 < 0$

i.e., $\{V(m1)\}^2 - (E^2 + F^2) > 0$ (3.13)

Let $L(m2) = \{V(m1)\}^2 - (E^2 + F^2)$ (3.14)

Here $L(m2) = \{V(m1)\}^2 - (E^2 + F^2)$ is taken as voltage sensitivity index (VSI).

The real power loss of branch-jj is given by

$LP(jj) = |I(jj)|^2 R(jj)$ (3.15)

The reactive power loss of branch-jj is given by

$LQ(jj) = |I(jj)|^2 X(jj)$ (3.16)

The voltage stability index proposed by **Chakravorty and Das [19]** is shown below

$$VSI(m2) = \{|V(m1)|^4 - 4.0\{P(m2)x(jj) - Q(m2)r(jj)\}^2 - 4.0\{P(m2)r(jj) + Q(m2)x(jj)\}|V(m1)|^2 \quad (3.17)$$

The algorithm for load-flow proposed in [34] used here to compute the stability index, which is shown below.

The algorithm for computation of PR(jj) and QR(jj) and also PS(jj) and QS(jj) is shown below.

Step 1 **:** Read the number of feeder (A), lateral(s) (B) and sub lateral(s) (C).

Step 2 **:** Compute the number of branches of feeder, lateral(s) or sub lateral(s) respectively.

Step 3 **:** Compute the branch current

Step 4 **:** Compute the real and imaginary part of voltage. Compute the voltage of each node using Eq. (3.1).

Step 5 **:** Compute angle of voltage for each node.

Step 6 : Compute the stability index of each node using Eq. (3.14) and Eq. (3.17).

Step 7 : Compute the real and reactive power losses of each branch using Eq. (3.15) and (3.16) respectively.

Step 8 : Print the results.

3.4 LOAD MODELING

A balanced load can be represented either as constant power, constant current, constant impedance or as an exponential load and the general expression of load is shown below.

$$P(m2) = P_n [a_0 + a_1 V(m2) + a_2 V^2(m2) + a_3 V^{e1}(m2)] \quad (3.18)$$

$$Q(m2)= Q_n [b_0 + b_1 V(m2) + b_2 V^2(m2) + b_3 V^{e1}(m2)] \quad (3.19)$$

where, P_n and Q_n are nominal real and reactive power respectively and V(m2) is the voltage at node m2.

For all the loads, Eq. (3.18) and Eq. (3.19) are modeled as:

$$a_0 + a_1 + a_2 + a_3 = 1.0 \quad (3.20)$$

$$b_0 + b_1 + b_2 + b_3 = 1.0 \quad (3.21)$$

For constant power (CP) load $a_0 = b_0 = 1$ and $a_i = b_i = 0$ for i = 1, 2, 3. For constant current (CI) load $a_1 = b_1 = 1$ and $a_i = b_i = 0$ for i = 0, 2, 3. For constant impedance (CZ) load $a_2 = b_2 = 1$ and $a_i = b_i = 0$ for i = 0, 1, 3. Composite load modeling is combination of CP, CI and CZ. For exponential load $a_3 = b_3 = 1$ and $a_i = b_i = 0$ for i = 0, 1, 2 and e1 and e2 are 1.38 and 3.22 respectively [34].

3.5 EXAMPLE

To demonstrate the proposed method, 85-node radial distribution is taken. Figure 3.2 shows the diagram. The line data and load data are shown in Appendix-A. The base values are 11 kV and 100 MVA. The different load modelings are used. The total loads on the system for CP load modeling are 2028.00 kW and 1521.00 kVAr respectively.

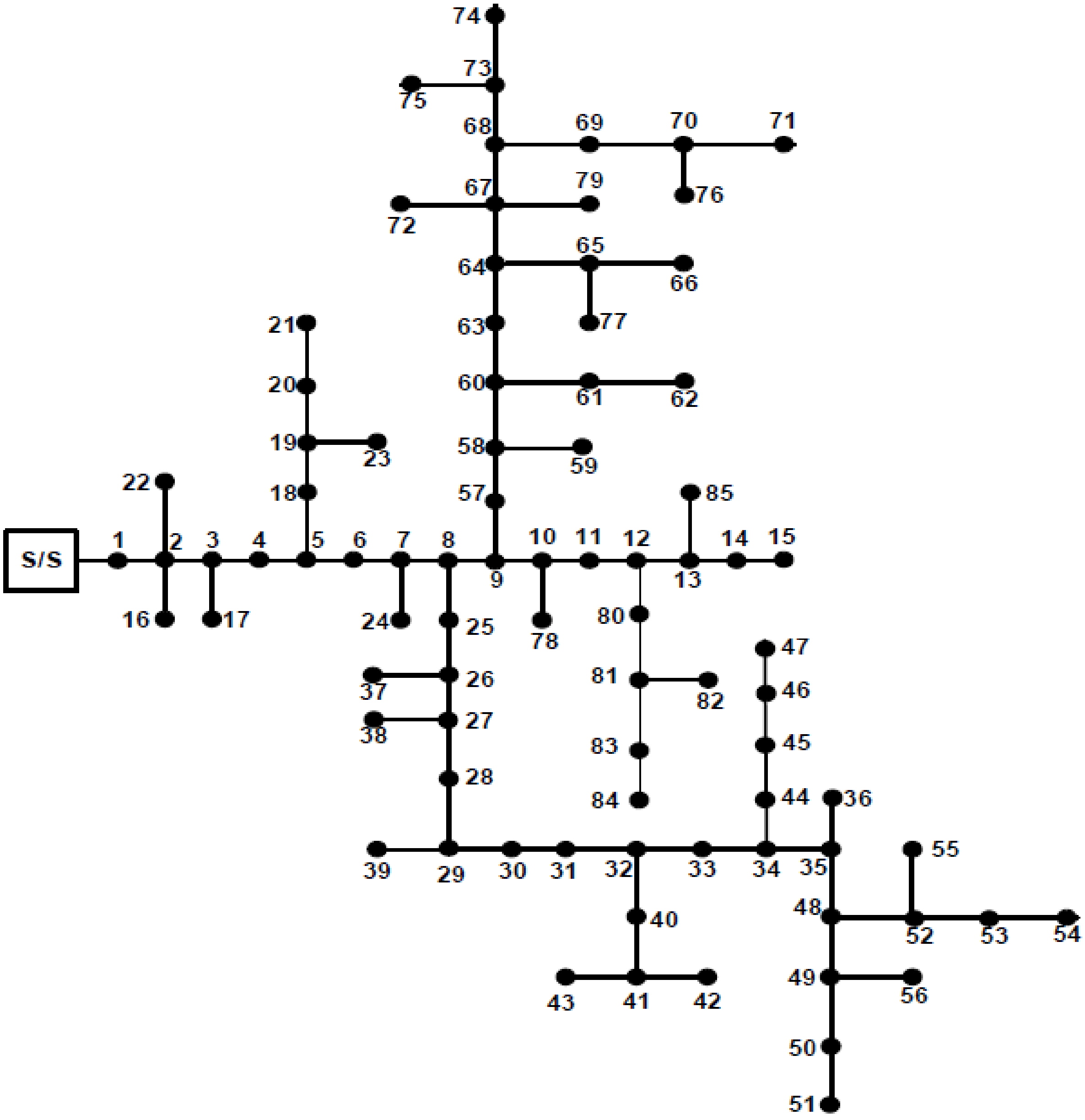

Figure 3.2 85 Node Radial Distribution Network

3.6 RESULTS

Table 3.2 shows the R/X ratio of each branch of Fig. 3.2.

Table 3.3 shows the magnitude of voltage in p.u. of each node and angle of voltage of each node for constant power load modeling. Table 3.4 show the voltage stability index of each node of

Fig. 3.2 for constant power load modeling. Table 3.5 and Table 3.6, Table 3.7 and Table 3.8, Table 3.9 and Table 3.10, Table 3.11 and Table 3.12 show the above results for constant current(CI) constant impedance (CZ) , composite (40% CP + 30% CI + 30% CZ) and exponential load modeling respectively. The above cases are for substation voltage of 1.00 p.u.

Table 3.13 shows the magnitude of voltage in p.u. of each node and angle of voltage of each node for constant power load. Table 3.14 shows the voltage stability index of each node of Fig. 3.2 for constant power load modeling. Table 3.15 and Table 3.16, Table 3.17 and Table 3.18, Table 3.19 and Table 3.20, Table 3.21 and Table 3.22 show the above results for constant current(CI) constant impedance (CZ) , composite (40% CP + 30% CI + 30% CZ) and exponential load modeling respectively. The above cases are for substation voltage of 1.025 p.u.

Table 3.23 shows the magnitude of voltage in p.u. of each node and angle of voltage of each node for constant power load modeling. Table 3.24 shows the voltage stability index of each node of Fig. 3.2 for constant power load modeling. Table 3.25 and Table 3.26, Table 3.27 and Table 3.28, Table 3.29 and Table 3.30, Table 3.31 and Table 3.32 show the above results for constant current(CI) constant impedance (CZ) , composite (40% CP + 30% CI + 30% CZ) and exponential load modeling respectively. The above cases are for substation voltage of 1.050 p.u.

Table 3.33, Table 3.34 and Table 3.35 show real power and reactive power loss, total kW and kVAr load, minimum voltage, VSI of most sensitive node for different load modeling for substation voltage 1.000 p.u., 1.025 p.u., 1.050 p.u. respectively.

Table 3.2 R/X Ratio of Each Branch

Branch Number	R/X Ratio of Each Branch
2	1.440000
3	1.455357
4	1.456376
5	1.459459
6	1.459732
7	1.462366
8	1.459756
9	1.459459
10	1.458537
11	1.458445
12	1.992674
13	1.458537
14	1.462366
15	1.461883
16	2.410596
17	2.407408
18	2.411765
19	2.412879
20	2.407408
21	2.408823
22	2.411215
23	2.426666
24	2.407408
25	2.407408
26	2.410596
27	2.415929
28	2.415929
29	2.415929
30	2.415929
31	2.415929
32	2.426666
33	2.426666
34	2.408823
35	2.412879
36	2.426666
37	2.410596
38	2.408654
39	2.415929
40	2.407408

41	2.408654
42	2.415929
43	2.407408
44	2.408654
45	2.410053
46	2.410053
47	2.415929
48	2.412879
49	2.426666
50	2.410596
51	2.407408
52	2.409171
53	2.407408
54	2.415929
55	2.415929
56	2.415929
57	2.415929
58	2.408823
59	2.426666
60	2.415929
61	2.410596
62	2.414458
63	2.426666
64	2.410596
65	2.426666
66	2.426666
67	2.407408
68	2.407408
69	2.410596
70	2.407408
71	2.415929
72	2.426666
73	2.411405
74	2.415929
75	2.408654
76	2.415929
77	2.459459
78	2.412879
79	2.415929
80	2.410596
81	2.410596

82	2.459459
83	2.410596
84	2.947059
85	2.408823

Table 3.3 Magnitude of Voltage (p.u.) for Each Node and its Angle (radian) for CP Load Modeling for Substation Voltage 1.000 p.u.

Node Number	Voltage magnitude (p.u.)	Voltage Angle (rad.)
1	1.000000	0.000000
2	0.997066	0.000092
3	0.992769	0.000250
4	0.987295	0.000452
5	0.984634	0.000554
6	0.974974	0.000924
7	0.968938	0.001165
8	0.942795	0.002216
9	0.941549	0.002269
10	0.939123	0.002378
11	0.937241	0.002462
12	0.935826	0.002734
13	0.935148	0.002764
14	0.934943	0.002774
15	0.934820	0.002780
16	0.996842	0.000150
17	0.992324	0.000364
18	0.982907	0.001002
19	0.981880	0.001270
20	0.981597	0.001343
21	0.981341	0.001410
22	0.996591	0.000214
23	0.981790	0.001293
24	0.968651	0.001240
25	0.938876	0.003256
26	0.935861	0.004062
27	0.931908	0.005130
28	0.930074	0.005627
29	0.926691	0.006549
30	0.923595	0.007398
31	0.922138	0.007800
32	0.921228	0.008053
33	0.920499	0.008256
34	0.917321	0.009133
35	0.915574	0.009619
36	0.915513	0.009636
37	0.935672	0.004114

38	0.931385	0.005273
39	0.926405	0.006628
40	0.920774	0.008179
41	0.920108	0.008363
42	0.920017	0.008389
43	0.919957	0.008405
44	0.916184	0.009450
45	0.915454	0.009653
46	0.915029	0.009772
47	0.914957	0.009792
48	0.914040	0.010047
49	0.913856	0.010099
50	0.913538	0.010188
51	0.913296	0.010256
52	0.912126	0.010582
53	0.911731	0.010692
54	**0.911441**	**0.010774**
55	0.911836	0.010663
56	0.913784	0.010119
57	0.939780	0.002745
58	0.934895	0.004061
59	0.934801	0.004087
60	0.931925	0.004871
61	0.931165	0.005080
62	0.930643	0.005223
63	0.931220	0.005065
64	0.928491	0.005811
65	0.928372	0.005844
66	0.928277	0.005871
67	0.927083	0.006197
68	0.925044	0.006757
69	0.923534	0.007174
70	0.923144	0.007282
71	0.922963	0.007332
72	0.926988	0.006223
73	0.924030	0.007037
74	0.923887	0.007077
75	0.923698	0.007129
76	0.922857	0.007362
77	0.928361	0.005848
78	0.938794	0.002467

79	0.926903	0.006247
80	0.934737	0.003031
81	0.934381	0.003128
82	0.934334	0.003141
83	0.933882	0.003265
84	0.933757	0.003309
85	0.934880	0.002837

Table 3.4 Voltage Stability Index of Each Node by Proposed Method and by Method of Chakravorty and Das *et al.* [19] for CP Load Modeling for Substation Voltage 1.000 p.u.

Node Number	**VSI**	
	by Chakravorty and Das *et al.*[19]	**Proposed Method**
2	0.999888	0.999888
3	0.988315	0.994140
4	0.970908	0.985590
5	0.950140	0.974751
6	0.939938	0.969504
7	0.903593	0.950575
8	0.879835	0.938840
9	0.790076	0.888862
10	0.785908	0.886515
11	0.776764	0.881952
12	0.771623	0.878421
13	0.766974	0.875770
14	0.764419	0.874502
15	0.763681	0.874118
16	0.987429	0.994140
17	0.969645	0.985590
18	0.938394	0.969504
19	0.932166	0.966106
20	0.928931	0.964089
21	0.927428	0.963532
22	0.986432	0.994140
23	0.929127	0.964089
24	0.880377	0.938841
25	0.789581	0.888861
26	0.776397	0.881487
27	0.767087	0.875834
28	0.753749	0.868452
29	0.748289	0.865036
30	0.736889	0.858756
31	0.727373	0.853028
32	0.723076	0.850339
33	0.720150	0.848661
34	0.717948	0.847318

35	0.708086	0.841478
36	0.702520	0.838276
37	0.766468	0.875835
38	0.752519	0.868452
39	0.736553	0.858757
40	0.719753	0.848661
41	0.718807	0.847825
42	0.716446	0.846598
43	0.716258	0.846598
44	0.707054	0.841478
45	0.703646	0.839393
46	0.701405	0.838057
47	0.700813	0.837278
48	0.702706	0.838276
49	0.698008	0.835469
50	0.697066	0.835133
51	0.695738	0.834551
52	0.698008	0.835468
53	0.691718	0.831974
54	**0.690102**	**0.831254**
55	0.691300	0.831974
56	0.697227	0.835133
57	0.785439	0.886515
58	0.780020	0.883186
59	0.763617	0.874029
60	0.763000	0.874028
61	0.753036	0.868483
62	0.750122	0.867069
63	0.754187	0.868483
64	0.751984	0.867170
65	0.743209	0.862096
66	0.742525	0.861875
67	0.743209	0.862096
68	0.738711	0.859483
69	0.730419	0.855707
70	0.727463	0.852915
71	0.725666	0.852194
72	0.738408	0.859483
73	0.732234	0.855707
74	0.728576	0.853832
75	0.727982	0.853832

76	0.725332	0.852194
77	0.742791	0.861875
78	0.776749	0.881952
79	0.738137	0.859483
80	0.765736	0.875770
81	0.763409	0.873733
82	0.762094	0.873068
83	0.761081	0.873068
84	0.760213	0.872135
85	0.763878	0.874502

Table 3.5 Magnitude of Voltage (p.u.) for Each Node and its Angle (radian) for CI Load Modeling for Substation Voltage 1.000 p.u.

Node Number	Voltage magnitude (p.u.)	Voltage Angle (rad.)
1	1.000000	0.000000
2	0.997247	0.000087
3	0.993221	0.000234
4	0.988107	0.000424
5	0.985624	0.000519
6	0.976663	0.000862
7	0.971063	0.001085
8	0.946828	0.002055
9	0.945668	0.002104
10	0.943398	0.002204
11	0.941638	0.002282
12	0.940315	0.002534
13	0.939682	0.002563
14	0.939490	0.002572
15	0.939375	0.002577
16	0.997024	0.000144
17	0.992779	0.000348
18	0.983929	0.000959
19	0.982921	0.001221
20	0.982642	0.001293
21	0.982392	0.001358
22	0.996773	0.000208
23	0.982832	0.001244
24	0.970785	0.001158
25	0.943215	0.003009
26	0.940438	0.003748
27	0.936806	0.004723
28	0.935122	0.005177
29	0.932020	0.006017
30	0.929184	0.006789
31	0.927850	0.007154
32	0.927017	0.007384
33	0.926351	0.007568
34	0.923447	0.008363
35	0.921852	0.008802
36	0.921797	0.008818
37	0.940262	0.003796

38	0.936319	0.004855
39	0.931755	0.006090
40	0.926599	0.007498
41	0.925986	0.007667
42	0.925903	0.007690
43	0.925847	0.007705
44	0.922406	0.008650
45	0.921738	0.008834
46	0.921349	0.008942
47	0.921282	0.008960
48	0.920452	0.009190
49	0.920284	0.009236
50	0.919993	0.009317
51	0.919772	0.009378
52	0.918707	0.009672
53	0.918347	0.009772
54	**0.918082**	**0.009846**
55	0.918442	0.009746
56	0.920218	0.009255
57	0.944025	0.002544
58	0.939493	0.003758
59	0.939404	0.003782
60	0.936739	0.004504
61	0.936032	0.004697
62	0.935546	0.004830
63	0.936087	0.004683
64	0.933562	0.005368
65	0.933452	0.005399
66	0.933364	0.005423
67	0.932260	0.005722
68	0.930377	0.006236
69	0.928983	0.006618
70	0.928623	0.006717
71	0.928456	0.006763
72	0.932172	0.005747
73	0.929440	0.006493
74	0.929308	0.006529
75	0.929134	0.006577
76	0.928358	0.006790
77	0.933441	0.005402
78	0.943089	0.002288

79	0.932094	0.005768
80	0.939297	0.002811
81	0.938965	0.002901
82	0.938921	0.002913
83	0.938499	0.003028
84	0.938382	0.003068
85	0.939431	0.002631

Table 3.6 Voltage Stability Index of Each Node by Proposed Method and by Method of Chakravorty and Das *et al.* [19] for CI Load Modeling for Substation Voltage 1.000 p.u.

Node Number	VSI	
	by Chakravorty and Das *et al.*[19]	Proposed Method
2	0.999800	0.999800
3	0.989032	0.994501
4	0.972685	0.986489
5	0.953271	0.976356
6	0.943725	0.971455
7	0.909869	0.953870
8	0.887670	0.942962
9	0.803681	0.896482
10	0.799750	0.894287
11	0.791077	0.889999
12	0.786205	0.886682
13	0.781797	0.884193
14	0.779374	0.883002
15	0.778674	0.882641
16	0.988149	0.994501
17	0.971429	0.986489
18	0.942203	0.971455
19	0.936070	0.968115
20	0.932884	0.966133
21	0.931405	0.965586
22	0.987155	0.994501
23	0.933077	0.966133
24	0.888161	0.942963
25	0.803210	0.896482
26	0.790895	0.889655
27	0.782206	0.884423
28	0.769755	0.877604
29	0.764668	0.874453
30	0.754035	0.868661
31	0.745163	0.863383
32	0.741158	0.860905
33	0.738430	0.859360
34	0.736379	0.858125

35	0.727191	0.852755
36	0.722005	0.849811
37	0.781618	0.884424
38	0.768591	0.877605
39	0.753716	0.868662
40	0.738057	0.859360
41	0.737170	0.858586
42	0.734956	0.857450
43	0.734779	0.857450
44	0.726227	0.852755
45	0.723042	0.850833
46	0.720949	0.849601
47	0.720396	0.848883
48	0.722179	0.849811
49	0.717801	0.847232
50	0.716923	0.846923
51	0.715684	0.846388
52	0.717801	0.847231
53	0.711943	0.844023
54	**0.710438**	**0.843361**
55	0.711553	0.844023
56	0.717073	0.846923
57	0.799302	0.894287
58	0.794208	0.891182
59	0.778772	0.882646
60	0.778188	0.882646
61	0.768809	0.877479
62	0.766054	0.876156
63	0.769898	0.877479
64	0.767828	0.876258
65	0.759579	0.871538
66	0.758933	0.871332
67	0.759579	0.871538
68	0.755351	0.869109
69	0.747562	0.865602
70	0.744785	0.863009
71	0.743096	0.862340
72	0.755065	0.869109
73	0.749267	0.865602
74	0.745828	0.863860
75	0.745269	0.863860

76	0.742782	0.862340
77	0.759184	0.871332
78	0.791061	0.889999
79	0.754810	0.869109
80	0.780623	0.884193
81	0.778417	0.882280
82	0.777171	0.881656
83	0.776211	0.881656
84	0.775388	0.880780
85	0.778861	0.883002

Table 3.7 Magnitude of Voltage (p.u.) for Each Node and its Angle (radian) for CZ Load Modeling for Substation Voltage 1.000 p.u.

Node Number	Voltage magnitude (p.u.)	Voltage Angle (rad.)
1	1.000000	0.000000
2	0.997403	0.000082
3	0.993611	0.000221
4	0.988807	0.000400
5	0.986478	0.000489
6	0.978116	0.000808
7	0.972891	0.001015
8	0.950295	0.001915
9	0.949209	0.001961
10	0.947075	0.002054
11	0.945422	0.002127
12	0.944180	0.002363
13	0.943584	0.002389
14	0.943404	0.002398
15	0.943296	0.002402
16	0.997181	0.000139
17	0.993173	0.000334
18	0.984811	0.000092
19	0.983820	0.001178
20	0.983546	0.001248
21	0.983300	0.001312
22	0.996931	0.000203
23	0.983733	0.001200
24	0.972621	0.001086
25	0.946944	0.002796
26	0.944370	0.003478
27	0.941009	0.004375
28	0.939453	0.004792
29	0.936590	0.005562
30	0.933974	0.006270
31	0.932743	0.006604
32	0.931976	0.006814
33	0.931362	0.006982
34	0.928691	0.007707
35	0.927224	0.008108
36	0.927173	0.008123
37	0.944203	0.003523

38	0.940554	0.004498
39	0.936343	0.005630
40	0.931589	0.006919
41	0.931021	0.007074
42	0.930944	0.007095
43	0.930892	0.007109
44	0.927731	0.007970
45	0.927115	0.008138
46	0.926756	0.008237
47	0.926695	0.008254
48	0.925937	0.008461
49	0.925783	0.008504
50	0.925515	0.008577
51	0.925311	0.008633
52	**0.924334**	**0.008900**
53	0.924003	0.008991
54	0.923760	0.009058
55	0.924091	0.008968
56	0.925722	0.008521
57	0.947675	0.002370
58	0.943447	0.003497
59	0.943364	0.003520
60	0.940879	0.004189
61	0.940218	0.004368
62	0.939763	0.004492
63	0.940272	0.004354
64	0.937923	0.004988
65	0.937821	0.005016
66	0.937738	0.005039
67	0.936713	0.005315
68	0.934964	0.005789
69	0.933668	0.006142
70	0.933334	0.006233
71	0.933179	0.006275
72	0.936631	0.005338
73	0.934093	0.006026
74	0.933970	0.006060
75	0.933808	0.006104
76	0.933088	0.006300
77	0.937810	0.005019
78	0.946784	0.002133

79	0.936557	0.005358
80	0.943224	0.002621
81	0.942912	0.002705
82	0.942871	0.002717
83	0.942474	0.002824
84	0.942365	0.002861
85	0.943349	0.002453

Table 3.8 Voltage Stability Index of Each Node by Proposed Method and by Method of Chakravorty and Das *et al.* [19] for CZ Load Modeling for Substation Voltage 1.000 p.u.

Node Number	**VSI**	
	by Chakravorty and Das *et al.*[19]	**Proposed Method**
2	0.999888	0.999888
3	0.989651	0.994812
4	0.974220	0.987264
5	0.955974	0.977739
6	0.946998	0.973138
7	0.915297	0.956712
8	0.894461	0.946517
9	0.815518	0.903060
10	0.811797	0.900998
11	0.803552	0.896951
12	0.798918	0.893822
13	0.794728	0.891475
14	0.792424	0.890351
15	0.791758	0.890011
16	0.988770	0.994812
17	0.972969	0.987264
18	0.945498	0.973138
19	0.939454	0.969852
20	0.936313	0.967902
21	0.934857	0.967364
22	0.987779	0.994812
23	0.936504	0.967902
24	0.894902	0.946518
25	0.815070	0.903059
26	0.803515	0.896702
27	0.795368	0.891833
28	0.783697	0.885498
29	0.778934	0.882572
30	0.768975	0.877200
31	0.760668	0.872307
32	0.756918	0.870010
33	0.754363	0.868579
34	0.752445	0.867436

35	0.743848	0.862466
36	0.738997	0.859744
37	0.794810	0.891834
38	0.782596	0.885498
39	0.768673	0.877200
40	0.754013	0.868579
41	0.753177	0.867858
42	0.751094	0.866800
43	0.750928	0.866800
44	0.742948	0.862466
45	0.739962	0.860684
46	0.737999	0.859542
47	0.737476	0.858877
48	0.739160	0.859744
49	0.735064	0.857359
50	0.734242	0.857073
51	0.733084	0.856577
52	0.735064	0.857358
53	0.729587	0.854393
54	**0.728181**	**0.853782**
55	0.729224	0.854393
56	0.734382	0.857073
57	0.811372	0.900998
58	0.806563	0.898087
59	0.791987	0.890092
60	0.791436	0.890092
61	0.782578	0.885254
62	0.779968	0.884010
63	0.783607	0.885254
64	0.781655	0.884112
65	0.773873	0.879700
66	0.773263	0.879507
67	0.773873	0.879700
68	0.769885	0.877431
69	0.762551	0.874157
70	0.759925	0.871737
71	0.758336	0.871112
72	0.769616	0.877431
73	0.764151	0.874157
74	0.760908	0.872530
75	0.760384	0.872530

76	0.758041	0.871112
77	0.773500	0.879507
78	0.803535	0.896951
79	0.769376	0.877431
80	0.793616	0.891475
81	0.791515	0.889671
82	0.790330	0.889083
83	0.789422	0.889083
84	0.788637	0.888257
85	0.791938	0.890351

Table 3.9 Magnitude of Voltage (p.u.) for Each Node and its Angle (radian) for CC Load Modeling for Substation Voltage 1.000 p.u.

Node Number	Voltage magnitude (p.u.)	Voltage Angle (rad.)
1	1.000000	0.000000
2	0.997226	0.000088
3	0.993171	0.000238
4	0.988016	0.000432
5	0.985513	0.000528
6	0.976474	0.000878
7	0.970825	0.001105
8	0.946375	0.002095
9	0.945206	0.002145
10	0.942919	0.002247
11	0.941145	0.002326
12	0.939813	0.002581
13	0.939174	0.002610
14	0.938981	0.002619
15	0.938865	0.002624
16	0.997004	0.000146
17	0.992728	0.000352
18	0.983815	0.000969
19	0.982805	0.001231
20	0.982526	0.001304
21	0.982275	0.001369
22	0.996753	0.000210
23	0.982717	0.001255
24	0.970546	0.001178
25	0.942728	0.003060
26	0.939925	0.003809
27	0.936256	0.004796
28	0.934555	0.005256
29	0.931422	0.006107
30	0.928556	0.006890
31	0.927208	0.007259
32	0.926366	0.007492
33	0.925693	0.007679
34	0.922758	0.008485
35	0.921146	0.008931
36	0.921090	0.008947
37	0.939746	0.003857

38	0.935765	0.004930
39	0.931155	0.006181
40	0.925945	0.007608
41	0.925325	0.007779
42	0.925241	0.007802
43	0.925185	0.007818
44	0.921706	0.008776
45	0.921031	0.008963
46	0.920638	0.009072
47	0.920571	0.009091
48	0.919730	0.009324
49	0.919561	0.009371
50	0.919266	0.009453
51	0.919043	0.009515
52	0.917966	0.009813
53	0.917602	0.009915
54	**0.917334**	**0.009990**
55	0.917698	0.009888
56	0.919494	0.00939
57	0.943549	0.002589
58	0.938977	0.003817
59	0.938888	0.003841
60	0.936199	0.004571
61	0.935487	0.004766
62	0.934997	0.004900
63	0.935541	0.004752
64	0.932994	0.005445
65	0.932883	0.005476
66	0.932794	0.005501
67	0.931681	0.005804
68	0.929780	0.006323
69	0.928373	0.006710
70	0.928009	0.006810
71	0.927841	0.006856
72	0.931592	0.005828
73	0.928835	0.006583
74	0.928701	0.006620
75	0.928525	0.006668
76	0.927742	0.006884
77	0.932872	0.005479
78	0.942607	0.002331

79	0.931512	0.005850
80	0.938787	0.002860
81	0.938452	0.002951
82	0.938408	0.002963
83	0.937982	0.003079
84	0.937865	0.003120
85	0.938922	0.002678

Table 3.10 Voltage Stability Index of Each Node by Proposed Method and by Method of Chakravorty and Das *et al.* [19] for CC Load Modeling for Substation Voltage 1.000 p.u.

Node Number	VSI	
	by Chakravorty and Das *et al.*[19]	Proposed Method
2	0.999888	0.999888
3	0.988952	0.994461
4	0.972486	0.986388
5	0.952920	0.976176
6	0.943301	0.971237
7	0.909164	0.953501
8	0.886790	0.942500
9	0.802146	0.895626
10	0.798188	0.893414
11	0.789462	0.889095
12	0.784561	0.885754
13	0.780127	0.883248
14	0.777688	0.882048
15	0.776984	0.881685
16	0.988068	0.994461
17	0.971229	0.986388
18	0.941777	0.971237
19	0.935635	0.967891
20	0.932444	0.965905
21	0.930963	0.965357
22	0.987074	0.994461
23	0.932638	0.965905
24	0.887286	0.942501
25	0.801672	0.895625
26	0.789258	0.888736
27	0.780498	0.883457
28	0.767946	0.876575
29	0.762816	0.873393
30	0.752094	0.867546
31	0.743149	0.862217
32	0.739109	0.859715
33	0.736358	0.858154
34	0.734290	0.856907

35	0.725023	0.851483
36	0.719793	0.848509
37	0.779907	0.883458
38	0.766774	0.876575
39	0.751773	0.867547
40	0.735982	0.858154
41	0.735089	0.857373
42	0.732858	0.856227
43	0.732680	0.856227
44	0.724051	0.851483
45	0.720840	0.849542
46	0.718730	0.848298
47	0.718172	0.847574
48	0.719968	0.848509
49	0.715553	0.845904
50	0.714666	0.845592
51	0.713417	0.845051
52	0.715553	0.845903
53	0.709644	0.842662
54	**0.708125**	**0.841994**
55	0.709251	0.842662
56	0.714818	0.845592
57	0.797738	0.893413
58	0.792607	0.890283
59	0.777063	0.881679
60	0.776475	0.881678
61	0.767030	0.876469
62	0.764257	0.875136
63	0.768126	0.876469
64	0.766041	0.875237
65	0.757733	0.870478
66	0.757082	0.870271
67	0.757733	0.870478
68	0.753474	0.868029
69	0.745627	0.864491
70	0.742830	0.861876
71	0.741129	0.861201
72	0.753186	0.868029
73	0.747345	0.864491
74	0.743881	0.862734
75	0.743318	0.862734

76	0.740812	0.861201
77	0.757335	0.870271
78	0.789447	0.889095
79	0.752929	0.868029
80	0.778945	0.883248
81	0.776726	0.881321
82	0.775472	0.880692
83	0.774507	0.880692
84	0.773679	0.879810
85	0.777172	0.882048

Table 3.11 Magnitude of Voltage (p.u.) for Each Node and its Angle (radian) for Exp. Load Modeling for Substation Voltage 1.000 p.u.

Node Number	Voltage magnitude (p.u.)	Voltage Angle (rad.)
1	1.000000	0.000000
2	0.997401	-0.000049
3	0.993607	-0.000107
4	0.988798	-0.000193
5	0.986466	-0.000235
6	0.978095	-0.000434
7	0.972864	-0.000556
8	0.950240	-0.001135
9	0.949153	-0.001158
10	0.947017	-0.001192
11	0.945362	-0.001220
12	0.944108	-0.001061
13	0.943513	-0.001071
14	0.943332	-0.001073
15	0.943224	-0.001075
16	0.997179	0.000008
17	0.993167	0.000002
18	0.984793	0.000169
19	0.983799	0.000410
20	0.983525	0.000477
21	0.983278	0.000536
22	0.996929	0.000071
23	0.983712	0.000432
24	0.972592	-0.000493
25	0.946836	-0.000495
26	0.944221	-0.000002
27	0.940804	0.000642
28	0.939222	0.000939
29	0.936310	0.001486
30	0.933648	0.001986
31	0.932396	0.002221
32	0.931614	0.002369
33	0.930990	0.002487
34	0.928270	0.002993
35	0.926776	0.003271
36	0.926724	0.003281
37	0.944052	0.000033

38	0.940342	0.000735
39	0.936059	0.001536
40	0.931221	0.002445
41	0.930644	0.002557
42	0.930565	0.002572
43	0.930513	0.002582
44	0.927293	0.003177
45	0.926666	0.003296
46	0.926301	0.003365
47	0.926239	0.003377
48	0.925465	0.003516
49	0.925308	0.003546
50	0.925035	0.003597
51	0.924828	0.003636
52	0.923833	0.003819
53	0.923496	0.003881
54	0.923248	0.003928
55	0.923585	0.003865
56	0.925246	0.003557
57	0.947597	-0.000851
58	0.943306	-0.000009
59	0.943222	0.000009
60	0.940700	0.000507
61	0.940029	0.000643
62	0.939568	0.000737
63	0.940084	0.000629
64	0.937699	0.001097
65	0.937594	0.001118
66	0.937511	0.001135
67	0.936469	0.001338
68	0.934692	0.001684
69	0.933376	0.001942
70	0.933036	0.002008
71	0.932879	0.002039
72	0.936386	0.001355
73	0.933807	0.001858
74	0.933682	0.001883
75	0.933518	0.001915
76	0.932786	0.002058
77	0.937584	0.001121
78	0.946722	-0.001130

79	0.936311	0.001369
80	0.94314	-0.000862
81	0.942824	-0.000797
82	0.942782	-0.000788
83	0.94238	-0.000706
84	0.942269	-0.000675
85	0.943274	-0.001021

Table 3.12 Voltage Stability Index of Each Node by Proposed Method and by Method of Chakravorty and Das *et al.* [19] for Exp. Load Modeling for Substation Voltage 1.000 p.u.

Node Number	**VSI**	
	by Chakravorty and Das *et al.*[19]	**Proposed Method**
2	0.999800	0.999800
3	0.989645	0.994809
4	0.974201	0.987254
5	0.955938	0.977721
6	0.946952	0.973114
7	0.915217	0.956670
8	0.894358	0.946463
9	0.815330	0.902956
10	0.811606	0.900892
11	0.803353	0.896841
12	0.798715	0.893709
13	0.794488	0.891341
14	0.792182	0.890216
15	0.791516	0.889875
16	0.988764	0.994809
17	0.972948	0.987254
18	0.945447	0.973114
19	0.939383	0.969817
20	0.936232	0.967861
21	0.934770	0.967321
22	0.987772	0.994809
23	0.936423	0.967861
24	0.894794	0.946464
25	0.814877	0.902956
26	0.803143	0.896499
27	0.794867	0.891553
28	0.783009	0.885113
29	0.778168	0.882138
30	0.768047	0.876675
31	0.759603	0.871698
32	0.755790	0.869362
33	0.753193	0.867905
34	0.751242	0.866742

35	0.742501	0.861685
36	0.737568	0.858914
37	0.794302	0.891553
38	0.781895	0.885113
39	0.767742	0.876676
40	0.752838	0.867905
41	0.751988	0.867173
42	0.749873	0.866098
43	0.749705	0.866098
44	0.741587	0.861685
45	0.738551	0.859872
46	0.736556	0.858710
47	0.736025	0.858033
48	0.737733	0.858914
49	0.733567	0.856486
50	0.732732	0.856195
51	0.731555	0.855690
52	0.733567	0.856485
53	0.727997	0.853467
54	0.726567	0.852844
55	0.727628	0.853467
56	0.732875	0.856195
57	0.811175	0.900891
58	0.806296	0.897939
59	0.791512	0.889827
60	0.790954	0.889827
61	0.781968	0.884917
62	0.779322	0.883655
63	0.783009	0.884917
64	0.781028	0.883758
65	0.773131	0.879279
66	0.772512	0.879083
67	0.773131	0.879279
68	0.769084	0.876974
69	0.761640	0.873649
70	0.758974	0.871191
71	0.757361	0.870557
72	0.768811	0.876974
73	0.763263	0.873649
74	0.759972	0.871996
75	0.759440	0.871996

76	0.757062	0.870557
77	0.772752	0.879083
78	0.803326	0.896841
79	0.768568	0.876974
80	0.793363	0.891341
81	0.791233	0.889513
82	0.790033	0.888917
83	0.789113	0.888917
84	0.788316	0.888080
85	0.791687	0.890216

Table 3.13 Magnitude of Voltage (p.u.) for Each Node and its Angle (radian) for CP Load Modeling for Substation Voltage 1.025 p.u.

Node Number	Voltage magnitude (p.u.)	Voltage Angle (rad.)
1	1.025000	0.000000
2	1.022147	0.000088
3	1.017970	0.000239
4	1.012649	0.000432
5	1.010063	0.000529
6	1.000677	0.000883
7	0.994811	0.001112
8	0.969410	0.002112
9	0.968199	0.002163
10	0.965841	0.002265
11	0.964011	0.002345
12	0.962636	0.002602
13	0.961977	0.002631
14	0.961778	0.002640
15	0.961658	0.002645
16	1.021929	0.000143
17	1.017535	0.000347
18	1.008379	0.000955
19	1.007379	0.001209
20	1.007102	0.001279
21	1.006853	0.001342
22	1.021684	0.000204
23	1.007291	0.001232
24	0.994532	0.001183
25	0.965604	0.003095
26	0.962676	0.003858
27	0.958837	0.004866
28	0.957056	0.005336
29	0.953772	0.006207
30	0.950766	0.007008
31	0.949351	0.007387
32	0.948467	0.007626
33	0.94776	0.007817
34	0.944675	0.008645
35	0.942978	0.009103
36	0.942919	0.009119
37	0.962492	0.003907

38	0.958329	0.005001
39	0.953494	0.006282
40	0.948027	0.007744
41	0.947379	0.007919
42	0.947291	0.007943
43	0.947232	0.007958
44	0.943570	0.008943
45	0.942862	0.009135
46	0.942449	0.009247
47	0.942379	0.009266
48	0.941489	0.009507
49	0.941311	0.009555
50	0.941001	0.009639
51	0.940767	0.009703
52	0.939632	0.010010
53	0.939248	0.010114
54	**0.938966**	**0.010191**
55	0.939350	0.010087
56	0.941241	0.009575
57	0.966481	0.002612
58	0.961734	0.003857
59	0.961642	0.003881
60	0.958848	0.004622
61	0.958110	0.004819
62	0.957602	0.004955
63	0.958163	0.004805
64	0.955512	0.005510
65	0.955397	0.005541
66	0.955305	0.005566
67	0.954144	0.005874
68	0.952164	0.006403
69	0.950696	0.006796
70	0.950317	0.006898
71	0.950142	0.006945
72	0.954052	0.005899
73	0.951179	0.006667
74	0.951039	0.006705
75	0.950856	0.006754
76	0.950039	0.006973
77	0.955385	0.005544
78	0.965521	0.002350

79	0.953969	0.005921
80	0.961577	0.002883
81	0.961231	0.002975
82	0.961186	0.002987
83	0.960746	0.003104
84	0.960625	0.003145
85	0.961717	0.002700

Table 3.14 Voltage Stability Index of Each Node by Proposed Method and by Method of Chakravorty and Das *et al.* [19] for CP Load Modeling for Substation Voltage 1.025 p.u.

Node Number	VSI	
	by Chakravorty and Das *et al.*[19]	Proposed Method
2	1.103813	1.050625
3	1.091575	1.044785
4	1.073335	1.036262
5	1.051564	1.025458
6	1.040862	1.020226
7	1.002710	1.001354
8	0.977734	0.989649
9	0.883142	0.939756
10	0.878738	0.937410
11	0.869069	0.932849
12	0.863632	0.929318
13	0.858714	0.926668
14	0.856010	0.925400
15	0.855229	0.925016
16	1.090645	1.044785
17	1.072008	1.036262
18	1.039238	1.020226
19	1.032684	1.016829
20	1.029278	1.014812
21	1.027697	1.014255
22	1.089597	1.044785
23	1.029485	1.014812
24	0.978305	0.989650
25	0.882618	0.939755
26	0.868693	0.932390
27	0.858855	0.926743
28	0.844750	0.919368
29	0.838975	0.915955
30	0.826911	0.909680
31	0.816834	0.903956
32	0.812283	0.901267
33	0.809183	0.899590
34	0.806850	0.898248

35	0.796396	0.892410
36	0.790494	0.889208
37	0.858200	0.926744
38	0.843448	0.919368
39	0.826554	0.909680
40	0.808762	0.899590
41	0.807759	0.898754
42	0.805256	0.897528
43	0.805056	0.897528
44	0.795302	0.892410
45	0.791687	0.890325
46	0.789310	0.888989
47	0.788682	0.888210
48	0.790691	0.889208
49	0.785707	0.886401
50	0.784708	0.886066
51	0.783299	0.885484
52	0.785707	0.886401
53	0.779034	0.882908
54	0.777319	0.882187
55	0.778590	0.882908
56	0.784879	0.886066
57	0.878241	0.937410
58	0.872515	0.934084
59	0.855173	0.924932
60	0.854520	0.924932
61	0.843977	0.919389
62	0.840892	0.917975
63	0.845195	0.919389
64	0.842864	0.918076
65	0.833576	0.913004
66	0.832851	0.912783
67	0.833576	0.913004
68	0.828812	0.910391
69	0.820029	0.906615
70	0.816897	0.903824
71	0.814992	0.903103
72	0.828491	0.910391
73	0.821952	0.906616
74	0.818076	0.904741
75	0.817446	0.904741

76	0.814638	0.903103
77	0.833133	0.912783
78	0.869054	0.932849
79	0.828205	0.910391
80	0.857404	0.926668
81	0.854941	0.924630
82	0.853550	0.923966
83	0.852478	0.923966
84	0.851559	0.923033
85	0.855438	0.925400

Table 3.15 Magnitude of Voltage (p.u.) for Each Node and its Angle (radian) for CI Load Modeling for Substation Voltage 1.025 p.u.

Node Number	Voltage magnitude (p.u.)	Voltage Angle (rad.)
1	1.025000	0.000000
2	1.022247	0.000085
3	1.018221	0.000230
4	1.013107	0.000415
5	1.010624	0.000508
6	1.001663	0.000844
7	0.996063	0.001062
8	0.971828	0.002010
9	0.970668	0.002058
10	0.968398	0.002156
11	0.966638	0.002232
12	0.965316	0.002478
13	0.964682	0.002505
14	0.964490	0.002514
15	0.964375	0.002519
16	1.022024	0.000141
17	1.017779	0.000340
18	1.008929	0.000937
19	1.007921	0.001193
20	1.007642	0.001263
21	1.007392	0.001326
22	1.021773	0.000204
23	1.007832	0.001215
24	0.995785	0.001133
25	0.968216	0.002940
26	0.965439	0.003661
27	0.961806	0.004611
28	0.960123	0.005053
29	0.957021	0.005872
30	0.954185	0.006624
31	0.952851	0.006979
32	0.952018	0.007203
33	0.951352	0.007382
34	0.948448	0.008157
35	0.946853	0.008585
36	0.946797	0.008600
37	0.965262	0.003708

38	0.961320	0.004740
39	0.956756	0.005943
40	0.951600	0.007315
41	0.950987	0.007479
42	0.950904	0.007501
43	0.950848	0.007516
44	0.947407	0.008436
45	0.946739	0.008616
46	0.946350	0.008721
47	0.946283	0.008738
48	0.945453	0.008962
49	0.945285	0.009007
50	0.944994	0.009086
51	0.944773	0.009145
52	0.943708	0.009432
53	0.943348	0.009529
54	0.943083	0.009601
55	0.943443	0.009504
56	0.945219	0.009025
57	0.969025	0.002487
58	0.964493	0.003670
59	0.964405	0.003694
60	0.961739	0.004397
61	0.961032	0.004585
62	0.960546	0.004714
63	0.961087	0.004571
64	0.958562	0.005239
65	0.958452	0.005269
66	0.958364	0.005292
67	0.957261	0.005584
68	0.955378	0.006084
69	0.953983	0.006456
70	0.953623	0.006552
71	0.953456	0.006597
72	0.957172	0.005608
73	0.954441	0.006334
74	0.954308	0.006370
75	0.954134	0.006416
76	0.953358	0.006623
77	0.958441	0.005272
78	0.968089	0.002238

79	0.957094	0.005628
80	0.964298	0.002747
81	0.963965	0.002835
82	0.963921	0.002847
83	0.963499	0.002958
84	0.963382	0.002998
85	0.964431	0.002572

Table 3.16 Voltage Stability Index of Each Node by Proposed Method and by Method of Chakravorty and Das *et al.* [19] for CI Load Modeling for Substation Voltage 1.025 p.u.

Node Number	VSI	
	by Chakravorty and Das *et al.*[19]	Proposed Method
2	1.103813	1.050625
3	1.092000	1.044988
4	1.074390	1.036775
5	1.053469	1.026386
6	1.043179	1.021361
7	1.006668	1.003329
8	0.982715	0.992141
9	0.891984	0.944449
10	0.887733	0.942196
11	0.878353	0.937795
12	0.873083	0.934389
13	0.868315	0.931834
14	0.865693	0.930611
15	0.864935	0.930241
16	1.091049	1.044988
17	1.073036	1.036775
18	1.041539	1.021361
19	1.034927	1.017937
20	1.031491	1.015904
21	1.029896	1.015343
22	1.089978	1.044988
23	1.031700	1.015904
24	0.983245	0.992142
25	0.891475	0.944448
26	0.878156	0.937441
27	0.868758	0.932071
28	0.855286	0.925071
29	0.849781	0.921835
30	0.838270	0.915889
31	0.828665	0.910469
32	0.824327	0.907925
33	0.821372	0.906338
34	0.819151	0.905069

35	0.809197	0.899554
36	0.803578	0.896531
37	0.868122	0.932072
38	0.854026	0.925071
39	0.837925	0.915889
40	0.820968	0.906338
41	0.820008	0.905543
42	0.817609	0.904376
43	0.817418	0.904376
44	0.808152	0.899554
45	0.804702	0.897580
46	0.802435	0.896315
47	0.801835	0.895578
48	0.803767	0.896530
49	0.799023	0.893881
50	0.798070	0.893564
51	0.796728	0.893014
52	0.799023	0.893881
53	0.792674	0.890585
54	0.791042	0.889905
55	0.792251	0.890585
56	0.798233	0.893564
57	0.887249	0.942196
58	0.881739	0.939008
59	0.865041	0.930246
60	0.864410	0.930246
61	0.854261	0.924942
62	0.851279	0.923582
63	0.855439	0.924942
64	0.853199	0.923687
65	0.844270	0.918842
66	0.843571	0.918630
67	0.844270	0.918842
68	0.839693	0.916348
69	0.831259	0.912746
70	0.828253	0.910084
71	0.826424	0.909397
72	0.839384	0.916348
73	0.833106	0.912746
74	0.829382	0.910957
75	0.828777	0.910957

76	0.826084	0.909397
77	0.843843	0.918630
78	0.878336	0.937795
79	0.839108	0.916348
80	0.867044	0.931834
81	0.864658	0.929870
82	0.863309	0.929229
83	0.862271	0.929229
84	0.861380	0.928330
85	0.865138	0.930611

Table 3.17 Magnitude of Voltage (p.u.) for Each Node and its Angle (radian) for CZ Load Modeling for Substation Voltage 1.025 p.u.

Node Number	Voltage magnitude (p.u.)	Voltage Angle (rad.)
1	1.025000	0.000000
2	1.022338	0.000082
3	1.018452	0.000221
4	1.013527	0.000400
5	1.011140	0.000489
6	1.002569	0.000808
7	0.997214	0.001015
8	0.974052	0.001915
9	0.972939	0.001961
10	0.970752	0.002054
11	0.969058	0.002127
12	0.967784	0.002363
13	0.967174	0.002389
14	0.966989	0.002398
15	0.966879	0.002402
16	1.022110	0.000139
17	1.018002	0.000334
18	1.009431	0.000920
19	1.008416	0.001178
20	1.008135	0.001248
21	1.007883	0.001312
22	1.021854	0.000203
23	1.008327	0.001200
24	0.996937	0.001086
25	0.970618	0.002796
26	0.967979	0.003478
27	0.964535	0.004375
28	0.962940	0.004792
29	0.960005	0.005562
30	0.957323	0.006270
31	0.956062	0.006604
32	0.955275	0.006814
33	0.954647	0.006982
34	0.951908	0.007707
35	0.950405	0.008108
36	0.950352	0.008123
37	0.967809	0.003523

38	0.964068	0.004498
39	0.959752	0.005630
40	0.954879	0.006919
41	0.954297	0.007074
42	0.954218	0.007095
43	0.954165	0.007109
44	0.950924	0.007970
45	0.950293	0.008138
46	0.949925	0.008237
47	0.949863	0.008254
48	0.949085	0.008461
49	0.948927	0.008504
50	0.948653	0.008577
51	0.948444	0.008633
52	0.947443	0.008900
53	0.947104	0.008991
54	0.946854	0.009058
55	0.947193	0.008968
56	0.948865	0.008521
57	0.971367	0.002370
58	0.967033	0.003497
59	0.966948	0.003520
60	0.964402	0.004189
61	0.963724	0.004368
62	0.963257	0.004492
63	0.963780	0.004354
64	0.961372	0.004988
65	0.961266	0.005016
66	0.961182	0.005039
67	0.960131	0.005315
68	0.958338	0.005789
69	0.957011	0.006142
70	0.956668	0.006233
71	0.956509	0.006275
72	0.960047	0.005338
73	0.957445	0.006026
74	0.957319	0.006060
75	0.957153	0.006104
76	0.956416	0.006300
77	0.961256	0.005019
78	0.970453	0.002133

79	0.959972	0.005358
80	0.966805	0.002621
81	0.966485	0.002705
82	0.966443	0.002717
83	0.966036	0.002824
84	0.965924	0.002861
85	0.966933	0.002453

Table 3.18 Voltage Stability Index of Each Node by Proposed Method and by Method of Chakravorty and Das *et al.* [19] for CZ Load Modeling for Substation Voltage 1.025 p.u.

Node Number	**VSI**	
	by Chakravorty and Das *et al.*[19]	**Proposed Method**
2	1.103813	1.050625
3	1.092389	1.045174
4	1.075357	1.037244
5	1.055218	1.027238
6	1.045309	1.022403
7	1.010317	1.005145
8	0.987319	0.994435
9	0.900180	0.948778
10	0.896073	0.946611
11	0.886971	0.942360
12	0.881857	0.939072
13	0.877232	0.936607
14	0.874689	0.935426
15	0.873954	0.935068
16	1.091417	1.045174
17	1.073976	1.037244
18	1.043653	1.022403
19	1.036982	1.018951
20	1.033516	1.016902
21	1.031908	1.016337
22	1.090323	1.045174
23	1.033726	1.016902
24	0.987806	0.994436
25	0.899686	0.948777
26	0.886932	0.942099
27	0.877938	0.936983
28	0.865056	0.930327
29	0.859799	0.927253
30	0.848805	0.921609
31	0.839637	0.916468
32	0.835497	0.914055
33	0.832677	0.912551
34	0.830560	0.911350

35	0.821070	0.906129
36	0.815716	0.903269
37	0.877323	0.936984
38	0.863841	0.930327
39	0.848473	0.921609
40	0.832291	0.912551
41	0.831367	0.911794
42	0.829069	0.910682
43	0.828886	0.910682
44	0.820077	0.906129
45	0.816780	0.904257
46	0.814614	0.903057
47	0.814037	0.902358
48	0.815895	0.903269
49	0.811374	0.900763
50	0.810467	0.900463
51	0.809188	0.899942
52	0.811374	0.900763
53	0.805328	0.897648
54	0.803776	0.897005
55	0.804928	0.897648
56	0.810622	0.900463
57	0.895603	0.946611
58	0.890295	0.943553
59	0.874206	0.935153
60	0.873598	0.935153
61	0.863821	0.930071
62	0.860940	0.928763
63	0.864956	0.930071
64	0.862801	0.928871
65	0.854212	0.924236
66	0.853539	0.924033
67	0.854212	0.924236
68	0.849810	0.921851
69	0.841714	0.918412
70	0.838816	0.915869
71	0.837062	0.915213
72	0.849513	0.921852
73	0.843480	0.918412
74	0.839901	0.916702
75	0.839323	0.916702

76	0.836737	0.915213
77	0.853800	0.924033
78	0.886953	0.94236
79	0.849249	0.921852
80	0.876004	0.936606
81	0.873685	0.934711
82	0.872378	0.934093
83	0.871375	0.934093
84	0.870509	0.933226
85	0.874152	0.935426

Table 3.19 Magnitude of Voltage (p.u.) for Each Node and its Angle (radian) for CC Load Modeling for Substation Voltage 1.025 p.u.

Node Number	Voltage magnitude (p.u.)	Voltage Angle (rad.)
1	1.025000	0.000000
2	1.022236	0.000086
3	1.018195	0.000233
4	1.013059	0.000421
5	1.010565	0.000515
6	1.001559	0.000856
7	0.995931	0.001078
8	0.971572	0.002041
9	0.970407	0.002090
10	0.968128	0.002189
11	0.966361	0.002266
12	0.965033	0.002513
13	0.964397	0.002541
14	0.964204	0.002550
15	0.964089	0.002555
16	1.022014	0.000142
17	1.017753	0.000343
18	1.008870	0.000944
19	1.007863	0.001199
20	1.007585	0.001270
21	1.007335	0.001333
22	1.021764	0.000205
23	1.007775	0.001222
24	0.995652	0.001149
25	0.967940	0.002979
26	0.965147	0.003705
27	0.961492	0.004663
28	0.959798	0.005110
29	0.956677	0.005935
30	0.953823	0.006694
31	0.952480	0.007053
32	0.951642	0.007279
33	0.950971	0.007459
34	0.948049	0.008241
35	0.946443	0.008674
36	0.946387	0.008689
37	0.964969	0.003752

38	0.961004	0.004793
39	0.956411	0.006006
40	0.951222	0.007391
41	0.950605	0.007557
42	0.950521	0.007580
43	0.950465	0.007594
44	0.947001	0.008523
45	0.946328	0.008705
46	0.945937	0.008811
47	0.945870	0.008829
48	0.945033	0.009054
49	0.944864	0.009100
50	0.944571	0.009180
51	0.944349	0.009240
52	0.943276	0.009529
53	0.942913	0.009627
54	0.942647	0.009700
55	0.943009	0.009602
56	0.944798	0.009119
57	0.968757	0.002521
58	0.964202	0.003713
59	0.964113	0.003737
60	0.961434	0.004445
61	0.960724	0.004634
62	0.960236	0.004764
63	0.960779	0.004620
64	0.958241	0.005293
65	0.958130	0.005323
66	0.958042	0.005347
67	0.956932	0.005640
68	0.955039	0.006144
69	0.953637	0.006520
70	0.953275	0.006616
71	0.953107	0.006662
72	0.956844	0.005664
73	0.954097	0.006397
74	0.953964	0.006432
75	0.953789	0.006479
76	0.953008	0.006688
77	0.958119	0.005326
78	0.967818	0.002271

79	0.956765	0.005685
80	0.964011	0.002784
81	0.963677	0.002872
82	0.963633	0.002884
83	0.963209	0.002997
84	0.963092	0.003036
85	0.964145	0.002608

Table 3.20 Voltage Stability Index of Each Node by Proposed Method and by Method of Chakravorty and Das *et al.* [19] for CC Load Modeling for Substation Voltage 1.025 p.u.

Node Number	VSI	
	by Chakravorty and Das *et al.*[34]	Proposed Method
2	1.103813	1.050625
3	1.091955	1.044967
4	1.074278	1.036720
5	1.053267	1.026288
6	1.042934	1.021241
7	1.006250	1.003120
8	0.982188	0.991877
9	0.891046	0.943953
10	0.886779	0.941690
11	0.877369	0.937272
12	0.872082	0.933854
13	0.867299	0.931289
14	0.864668	0.930061
15	0.863908	0.929689
16	1.091006	1.044967
17	1.072927	1.036720
18	1.041295	1.021241
19	1.034689	1.017820
20	1.031257	1.015789
21	1.029664	1.015228
22	1.089937	1.044967
23	1.031466	1.015789
24	0.982722	0.991878
25	0.890536	0.943952
26	0.877153	0.936907
27	0.867707	0.931507
28	0.854168	0.924467
29	0.848634	0.921212
30	0.837063	0.915231
31	0.827407	0.909779
32	0.823047	0.907219
33	0.820076	0.905623
34	0.817843	0.904346

35	0.807835	0.898796
36	0.802185	0.895754
37	0.867069	0.931508
38	0.852903	0.924467
39	0.836717	0.915232
40	0.819670	0.905623
41	0.818705	0.904823
42	0.816296	0.903651
43	0.816103	0.903651
44	0.806785	0.898796
45	0.803316	0.896810
46	0.801037	0.895537
47	0.800434	0.894796
48	0.802375	0.895754
49	0.797605	0.893087
50	0.796647	0.892768
51	0.795297	0.892214
52	0.797605	0.893087
53	0.791220	0.889770
54	0.789579	0.889085
55	0.790795	0.889770
56	0.796810	0.892768
57	0.886294	0.941689
58	0.880762	0.938488
59	0.863997	0.929685
60	0.863363	0.929685
61	0.853173	0.924356
62	0.850181	0.922991
63	0.854355	0.924356
64	0.852105	0.923095
65	0.843139	0.918226
66	0.842437	0.918014
67	0.843139	0.918226
68	0.838542	0.915719
69	0.830071	0.912100
70	0.827051	0.909424
71	0.825214	0.908733
72	0.838232	0.915720
73	0.831926	0.912100
74	0.828186	0.910301
75	0.827578	0.910301

76	0.824872	0.908733
77	0.842710	0.918014
78	0.877353	0.937272
79	0.837954	0.915720
80	0.866024	0.931289
81	0.863630	0.929317
82	0.862277	0.928674
83	0.861235	0.928674
84	0.860342	0.927771
85	0.864111	0.930061

Table 3.21 Magnitude of Voltage (p.u.) for Each Node and its Angle (radian) for Exp Load Modeling for Substation Voltage 1.025 p.u.

Node Number	Voltage magnitude (p.u.)	Voltage Angle (rad.)
1	1.025000	0.000000
2	1.022337	0.000003
3	1.018450	0.000021
4	1.013524	0.000032
5	1.011136	0.000037
6	1.002562	0.000007
7	0.997204	-0.000007
8	0.974032	-0.000108
9	0.972919	-0.000107
10	0.970731	-0.000095
11	0.969036	-0.000086
12	0.967756	0.000101
13	0.967145	0.000104
14	0.966960	0.000105
15	0.966850	0.000106
16	1.022111	0.000065
17	1.018002	0.000140
18	1.009429	0.000479
19	1.008415	0.000742
20	1.008135	0.000814
21	1.007883	0.000879
22	1.021855	0.000133
23	1.008326	0.000765
24	0.996927	0.000062
25	0.970561	0.000604
26	0.967894	0.001153
27	0.964410	0.001869
28	0.962797	0.002201
29	0.959827	0.002810
30	0.957112	0.003367
31	0.955836	0.003630
32	0.955039	0.003795
33	0.954402	0.003926
34	0.951629	0.004491
35	0.950105	0.004802
36	0.950052	0.004813
37	0.967722	0.001192

38	0.963939	0.001973
39	0.959571	0.002866
40	0.954638	0.003879
41	0.954049	0.004004
42	0.953969	0.004021
43	0.953916	0.004032
44	0.950632	0.004697
45	0.949993	0.004829
46	0.949621	0.004906
47	0.949558	0.004919
48	0.948769	0.005075
49	0.948609	0.005108
50	0.948331	0.005165
51	0.948120	0.005209
52	0.947104	0.005414
53	0.946761	0.005484
54	0.946508	0.005535
55	0.946851	0.005465
56	0.948546	0.005121
57	0.971332	0.000233
58	0.966957	0.001167
59	0.966871	0.001187
60	0.964299	0.001740
61	0.963615	0.001891
62	0.963144	0.001994
63	0.963671	0.001876
64	0.961238	0.002395
65	0.961132	0.002419
66	0.961046	0.002438
67	0.959985	0.002663
68	0.958172	0.003048
69	0.956831	0.003334
70	0.956484	0.003408
71	0.956324	0.003443
72	0.959899	0.002681
73	0.957270	0.003241
74	0.957143	0.003269
75	0.956975	0.003305
76	0.956229	0.003463
77	0.961121	0.002421
78	0.970430	-0.000027

79	0.959823	0.002698
80	0.966768	0.000321
81	0.966446	0.000393
82	0.966403	0.000403
83	0.965993	0.000494
84	0.965880	0.000527
85	0.966902	0.000158

Table 3.22 Voltage Stability Index of Each Node by Proposed Method and by Method of Chakravorty and Das *et al.* [19] for Exp. Load Modeling for Substation Voltage 1.025 p.u.

Node Number	VSI	
	by Chakravorty and Das *et al.*[19]	Proposed Method
2	1.103813	1.050625
3	1.092388	1.045174
4	1.075350	1.037241
5	1.055204	1.027231
6	1.045292	1.022395
7	1.010287	1.005130
8	0.987279	0.994416
9	0.900106	0.948739
10	0.895998	0.946572
11	0.886894	0.942319
12	0.881778	0.939030
13	0.877128	0.936551
14	0.874584	0.935370
15	0.873849	0.935013
16	1.091419	1.045174
17	1.073976	1.037241
18	1.043639	1.022395
19	1.036975	1.018947
20	1.033512	1.016900
21	1.031906	1.016335
22	1.090330	1.045174
23	1.033722	1.016900
24	0.987765	0.994416
25	0.899609	0.948739
26	0.886721	0.941989
27	0.877629	0.936818
28	0.864605	0.930087
29	0.859288	0.926977
30	0.848170	0.921267
31	0.838894	0.916064
32	0.834705	0.913622
33	0.831852	0.912099
34	0.829709	0.910883

35	0.820106	0.905597
36	0.814687	0.902700
37	0.877010	0.936819
38	0.863381	0.930087
39	0.847834	0.921267
40	0.831462	0.912099
41	0.830528	0.911333
42	0.828205	0.910210
43	0.828021	0.910210
44	0.819102	0.905597
45	0.815767	0.903702
46	0.813575	0.902487
47	0.812991	0.901780
48	0.814868	0.902700
49	0.810292	0.900162
50	0.809375	0.899859
51	0.808081	0.899331
52	0.810292	0.900162
53	0.804173	0.897007
54	0.802602	0.896356
55	0.803768	0.897007
56	0.809531	0.899859
57	0.895525	0.946572
58	0.890166	0.943485
59	0.873927	0.935005
60	0.873314	0.935005
61	0.863444	0.929873
62	0.860538	0.928554
63	0.864588	0.929873
64	0.862412	0.928661
65	0.853737	0.923979
66	0.853058	0.923774
67	0.853737	0.923979
68	0.849292	0.921570
69	0.841115	0.918094
70	0.838186	0.915525
71	0.836413	0.914862
72	0.848992	0.921570
73	0.842897	0.918094
74	0.839283	0.916367
75	0.838699	0.916367

76	0.836085	0.914862
77	0.853321	0.923774
78	0.886870	0.942319
79	0.848725	0.921570
80	0.875892	0.936551
81	0.873553	0.934640
82	0.872234	0.934017
83	0.871224	0.934017
84	0.870349	0.933142
85	0.874042	0.935370

Table 3.23 Magnitude of Voltage (p.u.) for Each Node and its Angle (radian) for CP Load Modeling for Substation Voltage 1.050 p.u.

Node Number	Voltage magnitude (p.u.)	Voltage Angle (rad.)
1	1.050000	0.000000
2	1.047224	0.000085
3	1.043159	0.000228
4	1.037983	0.000413
5	1.035467	0.000506
6	1.026338	0.000843
7	1.020634	0.001062
8	0.995931	0.002015
9	0.994753	0.002063
10	0.992458	0.002160
11	0.990678	0.002236
12	0.989340	0.002479
13	0.988699	0.002507
14	0.988505	0.002515
15	0.988388	0.002520
16	1.047011	0.000137
17	1.042735	0.000332
18	1.033825	0.000911
19	1.032849	0.001153
20	1.032579	0.001219
21	1.032337	0.001279
22	1.046772	0.000195
23	1.032764	0.001174
24	1.020361	0.001130
25	0.992230	0.002946
26	0.989383	0.003669
27	0.985652	0.004623
28	0.983921	0.005068
29	0.980729	0.005891
30	0.977808	0.006649
31	0.976433	0.007008
32	0.975574	0.007233
33	0.974886	0.007414
34	0.971888	0.008196
35	0.970240	0.008629
36	0.970183	0.008644
37	0.989205	0.003715

38	0.985158	0.004751
39	0.980459	0.005962
40	0.975145	0.007345
41	0.974516	0.007510
42	0.974431	0.007532
43	0.974373	0.007547
44	0.970815	0.008478
45	0.970127	0.008659
46	0.969725	0.008765
47	0.969657	0.008783
48	0.968793	0.009010
49	0.968620	0.009056
50	0.968319	0.009135
51	0.968091	0.009196
52	0.966988	0.009486
53	0.966615	0.009584
54	0.966341	0.009657
55	0.966714	0.009558
56	0.968551	0.009074
57	0.993081	0.002489
58	0.988465	0.003667
59	0.988376	0.003691
60	0.985658	0.004392
61	0.984940	0.004578
62	0.984446	0.004707
63	0.984992	0.004565
64	0.982414	0.005232
65	0.982302	0.005261
66	0.982212	0.005285
67	0.981084	0.005576
68	0.979158	0.006076
69	0.977732	0.006448
70	0.977363	0.006545
71	0.977192	0.006589
72	0.980994	0.005600
73	0.978200	0.006326
74	0.978065	0.006362
75	0.977887	0.006408
76	0.977092	0.006616
77	0.982291	0.005264
78	0.992147	0.002241

79	0.980914	0.005621
80	0.988310	0.002745
81	0.987973	0.002832
82	0.987929	0.002844
83	0.987501	0.002954
84	0.987383	0.002993
85	0.988445	0.002572

Table 3.24 Voltage Stability Index of Each Node by Proposed Method and by Method of Chakravorty and Das *et al.* [19] for CP Load Modeling for Substation Voltage 1.050 p.u.

Node Number	VSI	
	by Chakravorty and Das *et al.*[19]	Proposed Method
2	1.215506	1.102500
3	1.202702	1.096678
4	1.183608	1.088181
5	1.160808	1.077408
6	1.149593	1.072191
7	1.109589	1.053370
8	1.083364	1.041693
9	0.983822	0.991878
10	0.979176	0.989533
11	0.968971	0.984973
12	0.963231	0.981443
13	0.958036	0.978793
14	0.955180	0.977525
15	0.954355	0.977141
16	1.201725	1.096678
17	1.182214	1.088181
18	1.147887	1.072191
19	1.140999	1.068794
20	1.137419	1.066777
21	1.135757	1.066220
22	1.200625	1.096678
23	1.137637	1.066777
24	1.083966	1.041694
25	0.983270	0.991877
26	0.968585	0.984520
27	0.958205	0.978879
28	0.943317	0.971510
29	0.937219	0.968100
30	0.924474	0.961829
31	0.913823	0.956108
32	0.909011	0.953420
33	0.905733	0.951744
34	0.903266	0.950402

35	0.892207	0.944567
36	0.885960	0.941366
37	0.957513	0.978880
38	0.941941	0.971510
39	0.924096	0.961829
40	0.905287	0.951744
41	0.904226	0.950908
42	0.901578	0.949682
43	0.901366	0.949682
44	0.891049	0.944567
45	0.887222	0.942482
46	0.884706	0.941146
47	0.884041	0.940367
48	0.886170	0.941366
49	0.880894	0.938559
50	0.879835	0.938224
51	0.878343	0.937642
52	0.880894	0.938559
53	0.873828	0.935066
54	0.872011	0.934345
55	0.873357	0.935066
56	0.880016	0.938224
57	0.978652	0.989533
58	0.972612	0.986209
59	0.954307	0.977063
60	0.953616	0.977062
61	0.942481	0.971522
62	0.939221	0.970107
63	0.943769	0.971522
64	0.941306	0.970209
65	0.931492	0.965138
66	0.930725	0.964917
67	0.931492	0.965138
68	0.926457	0.962526
69	0.917171	0.958751
70	0.913858	0.955959
71	0.911843	0.955239
72	0.926117	0.962526
73	0.919204	0.958751
74	0.915105	0.956876
75	0.914439	0.956876

76	0.911469	0.955239
77	0.931023	0.964917
78	0.968954	0.984973
79	0.925814	0.962526
80	0.956653	0.978793
81	0.954052	0.976756
82	0.952582	0.976091
83	0.951450	0.976091
84	0.950479	0.975158
85	0.954576	0.977525

Table 3.25 Magnitude of Voltage (p.u.) for Each Node and its Angle (radian) for CI Load Modeling for Substation Voltage 1.050 p.u.

Node Number	Voltage magnitude (p.u.)	Voltage Angle (rad.)
1	1.050000	0.000000
2	1.047247	0.000083
3	1.043221	0.000225
4	1.038107	0.000407
5	1.035624	0.000498
6	1.026663	0.000827
7	1.021063	0.001040
8	0.996828	0.001968
9	0.995668	0.002015
10	0.993398	0.002110
11	0.991638	0.002184
12	0.990316	0.002424
13	0.989682	0.002451
14	0.989490	0.002459
15	0.989375	0.002464
16	1.047024	0.000138
17	1.042779	0.000333
18	1.033929	0.000916
19	1.032921	0.001166
20	1.032642	0.001234
21	1.032392	0.001296
22	1.046773	0.000199
23	1.032832	0.001188
24	1.020785	0.001109
25	0.993216	0.002875
26	0.990439	0.003578
27	0.986807	0.004504
28	0.985123	0.004935
29	0.982022	0.005733
30	0.979186	0.006467
31	0.977851	0.006813
32	0.977018	0.007031
33	0.976352	0.007206
34	0.973449	0.007960
35	0.971854	0.008378
36	0.971798	0.008392
37	0.990262	0.003623

38	0.986320	0.004630
39	0.981757	0.005802
40	0.976601	0.007140
41	0.975988	0.007300
42	0.975904	0.007322
43	0.975848	0.007336
44	0.972408	0.008233
45	0.971740	0.008408
46	0.971350	0.008510
47	0.971284	0.008527
48	0.970454	0.008745
49	0.970286	0.008789
50	0.969995	0.008866
51	0.969774	0.008924
52	0.968709	0.009203
53	0.968349	0.009298
54	0.968084	0.009368
55	0.968444	0.009273
56	0.970220	0.008807
57	0.994025	0.002432
58	0.989493	0.003586
59	0.989405	0.003609
60	0.986739	0.004295
61	0.986032	0.004478
62	0.985546	0.004604
63	0.986087	0.004465
64	0.983563	0.005116
65	0.983453	0.005145
66	0.983364	0.005168
67	0.982261	0.005452
68	0.980378	0.005939
69	0.978984	0.006302
70	0.978624	0.006396
71	0.978457	0.006439
72	0.982173	0.005475
73	0.979441	0.006183
74	0.979309	0.006218
75	0.979135	0.006263
76	0.978359	0.006465
77	0.983442	0.005147
78	0.993089	0.002189

79	0.982094	0.005495
80	0.989298	0.002686
81	0.988966	0.002772
82	0.988922	0.002783
83	0.988499	0.002892
84	0.988383	0.002930
85	0.989432	0.002515

Table 3.26 Voltage Stability Index of Each Node by Proposed Method and by Method of Chakravorty and Das *et al.* [19] for CI Load Modeling for Substation Voltage 1.050 p.u.

Node Number	VSI	
	by Chakravorty and Das *et al.*[19]	**Proposed Method**
2	1.215506	1.102500
3	1.202807	1.096725
4	1.183870	1.088311
5	1.161365	1.077667
6	1.150294	1.072518
7	1.110994	1.054037
8	1.085195	1.042569
9	0.987372	0.993666
10	0.982784	0.991355
11	0.972659	0.986840
12	0.966971	0.983347
13	0.961823	0.980725
14	0.958991	0.979471
15	0.958173	0.979091
16	1.201784	1.096725
17	1.182414	1.088311
18	1.148529	1.072517
19	1.141413	1.069008
20	1.137715	1.066925
21	1.135999	1.066350
22	1.200633	1.096725
23	1.137940	1.066925
24	1.085767	1.042570
25	0.986823	0.993665
26	0.972448	0.986477
27	0.962301	0.980969
28	0.947753	0.973787
29	0.941807	0.970467
30	0.929373	0.964366
31	0.918994	0.958805
32	0.914306	0.956193
33	0.911112	0.954565
34	0.908712	0.953263

35	0.897951	0.947603
36	0.891876	0.944500
37	0.961615	0.980970
38	0.946393	0.973787
39	0.929000	0.964367
40	0.910675	0.954565
41	0.909638	0.953749
42	0.907045	0.952552
43	0.906838	0.952552
44	0.896821	0.947603
45	0.893091	0.945576
46	0.890638	0.944278
47	0.889990	0.943521
48	0.892080	0.944500
49	0.886950	0.941780
50	0.885919	0.941455
51	0.884468	0.940890
52	0.886950	0.941780
53	0.880082	0.938397
54	0.878317	0.937699
55	0.879625	0.938397
56	0.886096	0.941455
57	0.982262	0.991355
58	0.976315	0.988085
59	0.958289	0.979097
60	0.957607	0.979096
61	0.946646	0.973655
62	0.943426	0.972260
63	0.947919	0.973655
64	0.945499	0.972368
65	0.935855	0.967396
66	0.935100	0.967179
67	0.935855	0.967396
68	0.930910	0.964837
69	0.921798	0.961141
70	0.918548	0.958409
71	0.916571	0.957704
72	0.930576	0.964837
73	0.923793	0.961141
74	0.919769	0.959305
75	0.919115	0.959305

76	0.916204	0.957704
77	0.935394	0.967179
78	0.972641	0.986840
79	0.930278	0.964837
80	0.960451	0.980725
81	0.957874	0.978710
82	0.956418	0.978053
83	0.955296	0.978053
84	0.954335	0.977131
85	0.958392	0.979471

Table 3.27 Magnitude of Voltage (p.u.) for Each Node and its Angle (radian) for CZ Load Modeling for Substation Voltage 1.050 p.u.

Node Number	Voltage magnitude (p.u.)	Voltage Angle (rad.)
1	1.050000	0.000000
2	1.047273	0.000082
3	1.043292	0.000221
4	1.038247	0.000400
5	1.035801	0.000489
6	1.027022	0.000808
7	1.021536	0.001015
8	0.997809	0.001915
9	0.996669	0.001961
10	0.994429	0.002054
11	0.992693	0.002127
12	0.991388	0.002363
13	0.990763	0.002389
14	0.990574	0.002398
15	0.990461	0.002402
16	1.047040	0.000139
17	1.042831	0.000334
18	1.034051	0.000920
19	1.033011	0.001178
20	1.032724	0.001248
21	1.032465	0.001312
22	1.046777	0.000203
23	1.032920	0.001200
24	1.021252	0.001086
25	0.994291	0.002796
26	0.991588	0.003478
27	0.988060	0.004375
28	0.986426	0.004792
29	0.983419	0.005562
30	0.980672	0.006270
31	0.979380	0.006604
32	0.978574	0.006814
33	0.977931	0.006982
34	0.975125	0.007707
35	0.973585	0.008108
36	0.973531	0.008123
37	0.991414	0.003523

38	0.987581	0.004498
39	0.983160	0.005630
40	0.978168	0.006919
41	0.977572	0.007074
42	0.977491	0.007095
43	0.977437	0.007109
44	0.974117	0.007970
45	0.973471	0.008138
46	0.973094	0.008237
47	0.973030	0.008254
48	0.972233	0.008461
49	0.972072	0.008504
50	0.971790	0.008577
51	0.971577	0.008633
52	0.970551	0.008900
53	0.970203	0.008991
54	0.969948	0.009058
55	0.970295	0.008968
56	0.972008	0.008521
57	0.995059	0.002370
58	0.990619	0.003497
59	0.990532	0.003520
60	0.987923	0.004189
61	0.987229	0.004368
62	0.986751	0.004492
63	0.987286	0.004354
64	0.984820	0.004988
65	0.984712	0.005016
66	0.984625	0.005039
67	0.983549	0.005315
68	0.981712	0.005789
69	0.980352	0.006142
70	0.980001	0.006233
71	0.979838	0.006275
72	0.983462	0.005338
73	0.980798	0.006026
74	0.980668	0.006060
75	0.980498	0.006104
76	0.979742	0.006300
77	0.984701	0.005019
78	0.994123	0.002133

79	0.983385	0.005358
80	0.990385	0.002621
81	0.990057	0.002705
82	0.990014	0.002717
83	0.989598	0.002824
84	0.989483	0.002862
85	0.990516	0.002453

Table 3.28 Voltage Stability Index of Each Node by Proposed Method and by Method of Chakravorty and Das *et al.* [19] for CZ Load Modeling for Substation Voltage 1.050 p.u.

Node Number	**VSI**	
	by Chakravorty and Das *et al.*[19]	**Proposed Method**
2	1.215506	1.102500
3	1.202927	1.096780
4	1.184170	1.088458
5	1.161993	1.077958
6	1.151081	1.072884
7	1.112549	1.054774
8	1.087223	1.043535
9	0.991267	0.995624
10	0.986744	0.993350
11	0.976722	0.988889
12	0.971090	0.985439
13	0.965996	0.982851
14	0.963196	0.981612
15	0.962387	0.981237
16	1.201856	1.096780
17	1.182650	1.088458
18	1.149258	1.072884
19	1.141912	1.069261
20	1.138095	1.067112
21	1.136324	1.066518
22	1.200652	1.096780
23	1.138326	1.067112
24	1.087759	1.043536
25	0.990723	0.995623
26	0.976678	0.988614
27	0.966774	0.983246
28	0.952589	0.976262
29	0.946800	0.973036
30	0.934694	0.967113
31	0.924597	0.961718
32	0.920038	0.959186
33	0.916933	0.957608
34	0.914602	0.956348

35	0.904152	0.950869
36	0.898255	0.947868
37	0.966097	0.983247
38	0.951251	0.976262
39	0.934328	0.967113
40	0.916508	0.957608
41	0.915491	0.956813
42	0.912960	0.955647
43	0.912758	0.955647
44	0.903058	0.950869
45	0.899428	0.948904
46	0.897042	0.947645
47	0.896407	0.946912
48	0.898453	0.947867
49	0.893475	0.945238
50	0.892476	0.944923
51	0.891068	0.944376
52	0.893475	0.945238
53	0.886817	0.941969
54	0.885108	0.941294
55	0.886377	0.941969
56	0.892646	0.944923
57	0.986227	0.993350
58	0.980382	0.990141
59	0.962665	0.981326
60	0.961996	0.981326
61	0.951229	0.975993
62	0.948056	0.974621
63	0.952479	0.975993
64	0.950106	0.974733
65	0.940648	0.969870
66	0.939906	0.969657
67	0.940648	0.969870
68	0.935801	0.967368
69	0.926885	0.963758
70	0.923694	0.961090
71	0.921762	0.960402
72	0.935474	0.967368
73	0.928830	0.963758
74	0.924889	0.961964
75	0.924252	0.961964

76	0.921404	0.960402
77	0.940194	0.969657
78	0.976702	0.988889
79	0.935182	0.967368
80	0.964645	0.982851
81	0.962091	0.980862
82	0.960651	0.980214
83	0.959547	0.980214
84	0.958593	0.979303
85	0.962605	0.981612

Table 3.29 Magnitude of Voltage (p.u.) for Each Node and its Angle (radian) for CC Load Modeling for Substation Voltage 1.050 p.u.

Node Number	Voltage magnitude (p.u.)	Voltage Angle (rad.)
1	1.050000	0.000000
2	1.047244	0.000084
3	1.043215	0.000227
4	1.038094	0.000411
5	1.035608	0.000503
6	1.026630	0.000836
7	1.021020	0.001052
8	0.996738	0.001991
9	0.995577	0.002039
10	0.993304	0.002135
11	0.991543	0.002209
12	0.990219	0.002450
13	0.989584	0.002477
14	0.989392	0.002486
15	0.989277	0.002491
16	1.047022	0.000139
17	1.042774	0.000335
18	1.033917	0.000920
19	1.032912	0.001169
20	1.032635	0.001238
21	1.032385	0.001300
22	1.046773	0.000200
23	1.032824	0.001191
24	1.020742	0.001121
25	0.993118	0.002902
26	0.990334	0.003608
27	0.986692	0.004539
28	0.985004	0.004973
29	0.981893	0.005775
30	0.979049	0.006512
31	0.977710	0.006861
32	0.976875	0.007080
33	0.976207	0.007255
34	0.973294	0.008014
35	0.971693	0.008434
36	0.971637	0.008449
37	0.990157	0.003654

38	0.986204	0.004665
39	0.981628	0.005844
40	0.976456	0.007189
41	0.975841	0.007350
42	0.975758	0.007372
43	0.975702	0.007386
44	0.972249	0.008288
45	0.971579	0.008464
46	0.971189	0.008567
47	0.971122	0.008585
48	0.970288	0.008804
49	0.970120	0.008848
50	0.969828	0.008925
51	0.969606	0.008984
52	0.968538	0.009264
53	0.968176	0.009360
54	0.967910	0.009430
55	0.968272	0.009335
56	0.970054	0.008866
57	0.993931	0.002458
58	0.989391	0.003616
59	0.989303	0.003639
60	0.986632	0.004327
61	0.985924	0.004511
62	0.985437	0.004637
63	0.985979	0.004497
64	0.983449	0.005151
65	0.983339	0.005180
66	0.983250	0.005203
67	0.982145	0.005488
68	0.980258	0.005978
69	0.978860	0.006342
70	0.978499	0.006436
71	0.978332	0.006480
72	0.982056	0.005512
73	0.979319	0.006223
74	0.979186	0.006258
75	0.979012	0.006303
76	0.978234	0.006506
77	0.983328	0.005183
78	0.992995	0.002214

79	0.981978	0.005532
80	0.989200	0.002713
81	0.988867	0.002799
82	0.988823	0.002810
83	0.988400	0.002920
84	0.988284	0.002958
85	0.989334	0.002542

Table 3.30 Voltage Stability Index of Each Node by Proposed Method and by Method of Chakravorty and Das *et al.* [19] for CC Load Modeling for Substation Voltage 1.050 p.u.

Node Number	VSI	
	by Chakravorty and Das *et al.*[19]	Proposed Method
2	1.215506	1.102500
3	1.202796	1.096720
4	1.183842	1.088297
5	1.161307	1.077640
6	1.150222	1.072484
7	1.110852	1.053970
8	1.085011	1.042481
9	0.987016	0.993487
10	0.982423	0.991173
11	0.972291	0.986654
12	0.966597	0.983157
13	0.961445	0.980533
14	0.958612	0.979277
15	0.957793	0.978897
16	1.201777	1.096720
17	1.182391	1.088297
18	1.148462	1.072484
19	1.141367	1.068985
20	1.137680	1.066908
21	1.135969	1.066334
22	1.200630	1.096720
23	1.137904	1.066908
24	1.085585	1.042482
25	0.986467	0.993486
26	0.972062	0.986282
27	0.961892	0.980760
28	0.947311	0.973560
29	0.941350	0.970231
30	0.928885	0.964114
31	0.918478	0.958536
32	0.913778	0.955917
33	0.910576	0.954284
34	0.908169	0.952979

35	0.897379	0.947301
36	0.891286	0.944188
37	0.961205	0.980761
38	0.945949	0.973560
39	0.928511	0.964114
40	0.910139	0.954284
41	0.909098	0.953467
42	0.906500	0.952266
43	0.906293	0.952266
44	0.896246	0.947301
45	0.892506	0.945269
46	0.890047	0.943966
47	0.889397	0.943208
48	0.891491	0.944188
49	0.886346	0.941459
50	0.885313	0.941134
51	0.883857	0.940567
52	0.886346	0.941459
53	0.879459	0.938065
54	0.877688	0.937365
55	0.879000	0.938065
56	0.885490	0.941134
57	0.981901	0.991172
58	0.975945	0.987898
59	0.957893	0.978895
60	0.957210	0.978894
61	0.946233	0.973443
62	0.943009	0.972047
63	0.947507	0.973443
64	0.945083	0.972154
65	0.935423	0.967173
66	0.934666	0.966955
67	0.935423	0.967173
68	0.930469	0.964608
69	0.921340	0.960905
70	0.918084	0.958167
71	0.916104	0.957461
72	0.930134	0.964608
73	0.923339	0.960905
74	0.919307	0.959065
75	0.918652	0.959065

76	0.915736	0.957461
77	0.934961	0.966955
78	0.972273	0.986654
79	0.929836	0.964608
80	0.960072	0.980533
81	0.957494	0.978516
82	0.956036	0.977858
83	0.954914	0.977858
84	0.953951	0.976935
85	0.958012	0.979277

Table 3.31 Magnitude of Voltage (p.u.) for Each Node and its Angle (radian) for Exp. Load Modeling for Substation Voltage 1.050 p.u.

Node Number	Voltage magnitude (p.u.)	Voltage Angle (rad.)
1	1.050000	0.000000
2	1.047272	0.000055
3	1.043291	0.000148
4	1.038245	0.000255
5	1.035799	0.000306
6	1.027019	0.000444
7	1.021532	0.000536
8	0.997802	0.000908
9	0.996662	0.000931
10	0.994421	0.000990
11	0.992685	0.001035
12	0.991378	0.001249
13	0.990753	0.001266
14	0.990563	0.001271
15	0.990450	0.001274
16	1.047041	0.000121
17	1.042833	0.000277
18	1.034058	0.000785
19	1.033023	0.001070
20	1.032737	0.001148
21	1.032480	0.001219
22	1.046781	0.000195
23	1.032933	0.001095
24	1.021249	0.000611
25	0.994263	0.001691
26	0.991543	0.002296
27	0.987991	0.003084
28	0.986346	0.003449
29	0.983317	0.004121
30	0.980550	0.004735
31	0.979248	0.005024
32	0.978436	0.005206
33	0.977787	0.005351
34	0.974959	0.005974
35	0.973406	0.006318
36	0.973352	0.006330
37	0.991368	0.002338

38	0.987510	0.003198
39	0.983056	0.004182
40	0.978027	0.005299
41	0.977427	0.005436
42	0.977345	0.005455
43	0.977290	0.005467
44	0.973943	0.006201
45	0.973292	0.006347
46	0.972912	0.006432
47	0.972847	0.006447
48	0.972043	0.006619
49	0.971880	0.006656
50	0.971596	0.006719
51	0.971381	0.006767
52	0.970346	0.006993
53	0.969996	0.007070
54	0.969738	0.007128
55	0.970089	0.007050
56	0.971816	0.006670
57	0.995044	0.001305
58	0.990582	0.002331
59	0.990495	0.002353
60	0.987873	0.002960
61	0.987175	0.003125
62	0.986695	0.003239
63	0.987232	0.003109
64	0.984752	0.003680
65	0.984643	0.003706
66	0.984556	0.003727
67	0.983473	0.003974
68	0.981626	0.004398
69	0.980258	0.004713
70	0.979905	0.004794
71	0.979741	0.004832
72	0.983387	0.003995
73	0.980706	0.004611
74	0.980576	0.004641
75	0.980405	0.004680
76	0.979645	0.004855
77	0.984632	0.003709
78	0.994114	0.001065

79	0.983309	0.004013
80	0.990371	0.001491
81	0.990042	0.001569
82	0.989998	0.001580
83	0.989580	0.001680
84	0.989465	0.001716
85	0.990505	0.001325

Table 3.32 Voltage Stability Index of Each Node by Proposed Method and by Method of Chakravorty and Das *et al.* [19] for Exp Load Modeling for Substation Voltage 1.050 p.u.

Node Number	**VSI**	
	by Chakravorty and Das *et al.*[19]	**Proposed Method**
2	1.215506	1.102500
3	1.202924	1.096779
4	1.184165	1.088456
5	1.161984	1.077954
6	1.151070	1.072879
7	1.112534	1.054767
8	1.087205	1.043527
9	0.991238	0.995609
10	0.986715	0.993335
11	0.976692	0.988874
12	0.971060	0.985424
13	0.965955	0.982830
14	0.963154	0.981591
15	0.962345	0.981216
16	1.201862	1.096779
17	1.182660	1.088456
18	1.149257	1.072879
19	1.141949	1.069275
20	1.138152	1.067137
21	1.136390	1.066546
22	1.200667	1.096779
23	1.138382	1.067137
24	1.087745	1.043527
25	0.990693	0.995609
26	0.976564	0.988558
27	0.966598	0.983156
28	0.952321	0.976125
29	0.946491	0.972877
30	0.934302	0.966912
31	0.924134	0.961478
32	0.919541	0.958927
33	0.916413	0.957337
34	0.914064	0.956066

35	0.903535	0.950545
36	0.897594	0.947519
37	0.965919	0.983157
38	0.950979	0.976125
39	0.933935	0.966913
40	0.915986	0.957337
41	0.914962	0.956537
42	0.912415	0.955363
43	0.912213	0.955363
44	0.902435	0.950545
45	0.898778	0.948565
46	0.896375	0.947296
47	0.895735	0.946558
48	0.897793	0.947519
49	0.892776	0.944868
50	0.891770	0.944551
51	0.890352	0.944000
52	0.892776	0.944868
53	0.886066	0.941572
54	0.884344	0.940892
55	0.885623	0.941572
56	0.891941	0.944551
57	0.986197	0.993335
58	0.980322	0.990111
59	0.962520	0.981253
60	0.961849	0.981253
61	0.951029	0.975892
62	0.947843	0.974514
63	0.952283	0.975892
64	0.949897	0.974626
65	0.940388	0.969736
66	0.939643	0.969522
67	0.940388	0.969736
68	0.935514	0.967220
69	0.926550	0.963589
70	0.923339	0.960905
71	0.921396	0.960213
72	0.935186	0.967220
73	0.928504	0.963589
74	0.924542	0.961785
75	0.923902	0.961785

76	0.921037	0.960213
77	0.939932	0.969522
78	0.976670	0.988874
79	0.934893	0.967220
80	0.964600	0.982830
81	0.962035	0.980834
82	0.960591	0.980183
83	0.959484	0.980183
84	0.958524	0.979269
85	0.962562	0.981591

Table 3.33 Real Power and Reactive Power Loss, Total kW and kVAr Load, Minimum Voltage, VSI of Most Sensitive Node for Different Load Modeling for Substation Voltage 1.000 p.u.

Load Model	**Real Power Loss (kW)**	**Reactive Power Loss (kVAr)**	**Total Real Power Load (kW)**	**Total Reactive Power Load (kVAr)**	**Node Number**	**Minimum Voltage**	**VSI of Most Sensitivity**	
							By Chakravorty and Das *et al.* [19]	**By Proposed Method**
CP	141.57	88.85	2028	1520.99	54	0.911441	0.690102	0.83124
CI	122.000977	76.656166	1913.156372	1434.867554	54	0.918082	0.710438	0.843361
CZ	106.370613	66.905907	1805.90564	1354.429199	54	0.923144	0.728181	0.853782
CC	124.132339	77.983101	1926.01416	1444.510986	54	0.917334	0.708125	0.841994
Exp	106.526123	66.998886	1871.536011	1263.438232	54	0.919494	0.726567	0.852844

Table 3.34 Real Power and Reactive Power Loss, Total kW and kVAr Load, Minimum Voltage, VSI of Most Sensitive Node for Different Load Modeling for Substation Voltage 1.025 p.u.

Load Model	**Real Power Loss (kW)**	**Reactive Power Loss (kVAr)**	**Total Real Power Load (kw)**	**Total Reactive Power Load (kVAr)**	**Node Number**	**Minimum Voltage**	**VSI of Most Sensitivity**	
							By Chakravorty and Das *et al.* [19]	**By Proposed Method**
CP	133.671967	83.896141	2028.000366	1520.999268	54	0.926301	0.777319	0.882187
CI	122.001022	76.656189	1963.854736	1472.891113	54	0.943083	0.791042	0.889905
CZ	111.755646	70.292984	1897.328979	1422.997070	54	0.942979	0.803776	0.897005
CC	123.216965	77.409889	1970.657959	1477.993652	54	0.942647	0.789579	0.889085
Exp	111.697136	70.253395	1936.517822	1368.168945	54	0.944798	0.802602	0.896356

Table 3.35 Real Power and Reactive Power Loss, Total kW and kVAr Load, Minimum Voltage, VSI of Most Sensitive Node for Different Load Modeling for Substation Voltage 1.050 p.u.

Load Model	**Real Power Loss (kW)**	**Reactive Power Loss (kVAr)**	**Total Real Power Load (kw)**	**Total Reactive Power Load (kVAr)**	**Node Number**	**Minimum Voltage**	**VSI of Most Sensitivity**	
							By Chakravorty and Das *et al.* [19]	**By Proposed Method**
CP	126.445702	79.365555	2028.000366	1520.999268	54	0.966341	0.872011	0.934345
CI	122.001083	76.656227	2014.554077	1510.915527	54	0.968084	0.878317	0.937699
CZ	117.273628	73.763710	1991.010742	1493.257812	54	0.969948	0.885108	0.941294
CC	122.447800	76.928383	2016.016846	1512.012573	54	0.967910	0.877688	0.937365
Exp	117.183662	73.706612	2002.041382	1478.627075	54	0.944798	0.884344	0.940892

3.7 Conclusions

A voltage stability index (VSI) is proposed in this thesis work without reduction of the network. The voltage angle has also been taken care of during derivation of the voltage stability index. The VSI identifies the most sensitive node of the network. To demonstrate the effectiveness of the proposed method, one example (85 node radial distribution network) has been considered. The proposed method has been compared with the method Chakravorty and Das [19]. The results by the proposed method for 1.0 , 1.025 and 1.050 p.u. substation voltage for different types load modeling i.e., constant power, constant current, constant impedance, composite and exponential load modeling are better than that of obtained by the method das and Chakravorty and Das [19].

CHAPTER 4

CONCLUSION AND FUTURE SCOPE

4.1 Overall Conclusions

A voltage stability index (VSI) is proposed in this thesis work without reduction of the network. The voltage angle has also been taken care of during derivation of the voltage stability index. The VSI identifies the most sensitive node of the network. Since the most sensitive node is prone to voltage collapse, the increase of load to this node should be taken care of. Otherwise, voltage collapse will occur. The aim of the thesis work is limited to identify this node of the network so that the future planner can take care it during planning. To demonstrate the effectiveness of the proposed method, one example (85 node radial distribution network) has been considered. The proposed method has been compared with the method Das and Chakraborty [19]. The results by the proposed method for 1.0 , 1.025 and 1.050 p.u. substation voltage for different types load modeling i.e., constant power, constant current, constant impedance, composite and exponential load modeling are better than that of obtained by the method Das and Chakraborty [19].

4.2 Future Scope of Further Research Work

The following works can be carried out in future:

(i) Use of Fuzzy logic to take care of uncertainty of load in the analysis of VSI

(ii) Use of Fuzzy logic to take care of uncertainty of branch parameters in the analysis of VSI

(iii) Derivation of VSI for unbalanced system

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APPENDIX-A

Table A1 Line Data and Load Data of 85-node Radial Distribution Network [35]

Branch Number (jj)	Sending-end	Receiving-end	Resistance of each branch (jj)	Reactance of each branch (jj)	Real power load of each receiving-end node (m2)	Reactive power load of each receiving-end node (m2)
	Node (m1)	Node	(W)	(W)	(kW)	(kVAr)
		(m2)				
1	1	2	0.108	0.075	0	0
2	2	3	0.163	0.112	0	0
3	3	4	0.217	0.149	44.80	33.60
4	4	5	0.108	0.074	0	0
5	5	6	0.435	0.298	0	0
6	6	7	0.272	0.186	0	0
7	7	8	1.197	0.820	28.22	21.16
8	8	9	0.108	0.074	0	0
9	9	10	0.598	0.410	0	0
10	10	11	0.544	0.373	44. 80	33.60
11	11	12	0.544	0.273	0	0
12	12	13	0.598	0.410	0	0
13	13	14	0.272	0.186	28.22	21.16
14	14	15	0.326	0.223	28.22	21.16
15	2	16	0.728	0.302	28.22	21.16
16	3	17	0.455	0.189	89.60	67.20
17	5	18	0.820	0.340	44. 80	33.60
18	18	19	0.637	0.264	44. 80	33.60
19	19	20	0.455	0.189	28.22	21.16
20	20	21	0.819	0.340	28.22	21.16
21	2	22	1.548	0.642	28.22	21.16
22	19	23	0.182	0.075	44. 80	33.60
23	7	24	0.910	0.378	28.22	21.16
24	8	25	0.455	0.189	28.22	21.16
25	25	26	0.364	0.151	44. 80	33.60
26	26	27	0.546	0.226	0	0
27	27	28	0.273	0.113	44. 80	33.60
28	28	29	0.546	0.226	0	0
29	29	30	0.546	0.226	28.22	21.16
30	30	31	0.273	0.113	28.22	21.16

31	31	32	0.182	0.075	0	0
32	32	33	0.182	0.075	11.20	8.40
33	33	34	0.819	0.340	0	0
34	34	35	0.637	0.264	0	0
35	35	36	0.182	0.075	28.22	21.16
36	26	37	0.364	0.151	44. 80	33.60
37	27	38	1.002	0.416	44. 80	33.60
38	29	39	0.546	0.226	44. 80	33.60
39	32	40	0.455	0.189	28.22	21.16
40	40	41	1.002	0.416	0	0
41	41	42	0.273	0.113	28.22	21.16
42	41	43	0.455	0.189	28.22	21.16
43	34	44	1.002	0.416	28.22	21.16
44	44	45	0.911	0.378	28.22	21.16
45	45	46	0.911	0.378	28.22	21.16
46	46	47	0.546	0.226	11.20	8.40
47	35	48	0.637	0.264	0	0
48	48	49	0.182	0.075	0	0
49	49	50	0.364	0.151	29.02	21.76
50	50	51	0.455	0.189	44.80	33.60
51	48	52	1.366	0.567	0	0
52	52	53	0.455	0.189	28.22	21.16
53	53	54	0.546	0.226	44.80	33.60
54	52	55	0.546	0.226	44.80	33.60
55	49	56	0.546	0.226	11.2	8.40
56	9	57	0.273	0.113	44.80	33.60
57	57	58	0.819	0.340	0	0
58	58	59	0.182	0.075	44.80	33.60
59	58	60	0.546	0.226	44.80	33.60
60	60	61	0.728	0.302	44.80	33.60
61	61	62	1.002	0.415	44.80	33.60
62	60	63	0.182	0.075	11.2	8.40
63	63	64	0.728	0.302	0	0
64	64	65	0.182	0.075	0	0
65	65	66	0.182	0.075	44.80	33.60
66	64	67	0.455	0.189	0	0
67	67	68	0.910	0.378	0	0
68	68	69	1.092	0.453	44.80	33.60
69	69	70	0.455	0.189	0	0

70	70	71	0.546	0.226	28.22	21.16
71	67	72	0.182	0.075	44.80	33.60
72	68	73	1.184	0.491	0	0
73	73	74	0.273	0.113	44.80	33.60
74	73	75	1.002	0.416	28.22	21.16
75	70	76	0.546	0.226	44.80	33.60
76	65	77	0.091	0.037	11.20	8.40
77	10	78	0.637	0.264	44.80	33.60
78	67	79	0.546	0.226	28.22	21.16
79	12	80	0.728	0.302	44.80	33.60
80	80	81	0.364	0.151	0	0
81	81	82	0.091	0.037	44.80	33.60
82	81	83	1.092	0.453	28.22	21.16
83	83	84	1.002	0.340	11.20	8.40
84	13	85	0.819	0.340	28.22	21.16