Performance Analysis of Periodic Precoding for Channel Estimation in MIMO- OFDM System

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DECLARATION

I hereby declare that the dissertation entitled as "**Performance analysis of Periodic Precoding for channel estimation in MIMO- OFDM systems**" is an authentic record of my study carried out as requirement for the award of degree of ME (Electronics and Communication Engineering) at Thapar University, Patiala, under the supervision of **Dr. Ashutosh Kumar Singh** during 4th semester, 2015-2016.

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ABSTRACT

In modern wireless communication systems, multiple antennas at transmitter and receiver are employed along with OFDM to achieve very high data rate with significant system reliability. To enhance this task of MIMO-OFDM system we explore blind channel estimation. It is a technique in which no information about transmitted data is used for channel estimation, but statistical properties are considered.

In contrast to previous methods of estimating channel, a precoder is used in this dissertation which is able to remove problems like channel-order overestimation, bandwidth expansion caused by oversampling at the receiver and limitations of subspace and training methods. Moreover precoders can be used in both time and frequency dispersive channels. A periodic precoder passes a periodic sequence through each subcarrier of OFDM symbol. A different periodic sequence coefficient is given to each subcarrier of the system to make the system time-invariant. A toeplitz channel matrix is taken and super diagonal elements of the expression are taken for calculations and then cyclostationarity of received signal is generated. Because of cyclostationarity induced, the only indentifiabilty condition imposed is channel impulse response is full column rank. Eigen values and Eigen vectors of channel product matrices are calculated and used to estimate channel. The proposed method is able to work in both conditions of more inputs and more outputs.

A technique is stated to choose an optimal periodic sequence in such a way so that it reduces noise at the receiver. BER for MQAM signal is also calculated using SNR of the system. It is observed that SNR depends on precoding sequence and estimated channel and so thus BER and variation of BER with channel estimation error is observed.

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GLOSSARY OF ACRONYMS

MIMO	Multiple input-multiple output
ISI	Inter symbol interference
OFDM	Orthogonal frequency division multiplexing
WLAN	Wireless local area network
WWAN	Wireless wide area network
WMAN	Wireless metropolitan area network
4G	Fourth generation
5G	Fifth generation
SISO	Single input single output
ISI	Inter symbol interference
ICI	Inter carrier interference
PAR	Peak to average ratio
QAM	Quadrature amplitude modulation
BPSK	Binary phase shift key
QPSK	Quad phase shift key
MCM	Multi carrier modulation
IFFT	Inverse fast fourier transform
BER	Bit error rate
SNR	Signal to noise ratio
CP	Cyclic prefix
CSIT	Channel state information at transmitter
SLNR	Signal to leakage plus noise ratio
SVD	Singular value decomposition

Information and data rate passed on communication system is growing these days. Major use of wireless is seen in mobile communication and video downloading which require real time connections [1].

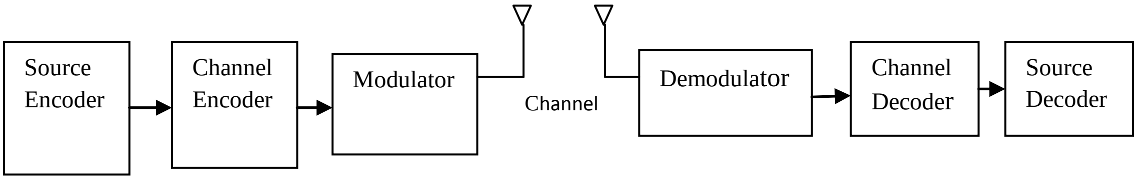


Fig 1.1: Block diagram of wireless communication system

Fig 1.1 illustrates basic elements of any digital wireless communication system [2]. Here, encoder receives data in analog form like audio, speech or any video which is converted to digital form. Then it is passed through channel encoder which processes the digital data so that it can be regenerated at the receiver. Before passing through physical channel i.e. the medium between transmitter and receiver, it is passed through a modulator. Physical channel includes noise due to different sources .At the receiver the process gets reversed to obtain the transmitted signal. This transmitted signal contains errors due to scattering, deflection and reflection of signal in the channel.

In wireless communication, high data rate and spectral efficiency is limited by interference and fading due to other users [3]. Therefore, a reliable system is needed to remove these errors to enhance the performance. Multiple input multiple output (MIMO) technology has reduced these limitations by providing link reliability and diversity gain [4]. Although, the complexity of system increases with MIMO, however, it is still considered as emerging field because it saves bandwidth, thus reducing the cost of system. It is also helpful in other research areas like information theory and coding, digital signal processing, channel estimation, antenna design, interference reduction [5]. There are many practical applications of this system e.g. wireless local area network (WLAN) such as pre IEEE 802.1 launched by Marvell Semiconductor, Airgo Networks, Metalink Technologies, Inc., Atheros Communications, Inc., Broadcom Corporation, Inc., and Inc. Nowadays it is used in Wi-Fi and LTE. Further research is going on for its usage in LTE advanced [6]. The performance of MIMO system alone degrades due to inter-symbol interference (ISI), however, it can be countered when MIMO is used with orthogonal frequency division multiplexing (OFDM). Researchers are fascinated by MIMO-OFDM as it provides wireless solution to WLAN, wireless wide area network (WWAN), wireless metropolitan area network (WMAN) and other 4G (fourth generation) and 5G (fifth generation) mobile solution [7].

1.1 MIMO SYSTEM

In any random wireless system, a link is formed between transmitter and receiver through multiple antennas as shown in fig 1.2 [6]. This arrangement is made between transmit antennas (TX) and receive antennas (RX) so that quality of service and bit error rate or the data rate can be improved [8]. This antenna technology is called MIMO system.

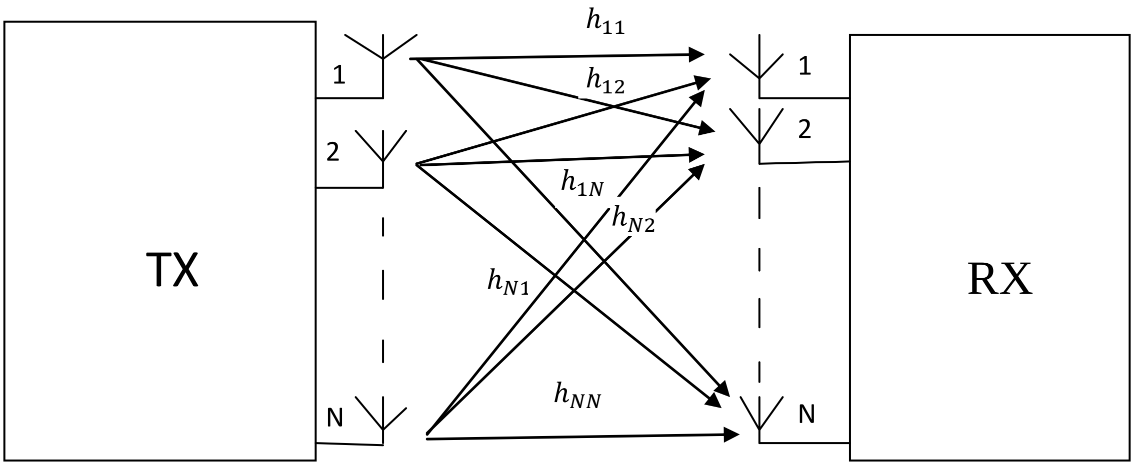


Fig 1.2: MIMO System

Here, system output y is given by:

$$Y = HX + N \tag{1.1}$$

where,

H is the channel matrix given as $H = \begin{bmatrix} h_{1,1} & \cdots & h_{1,M_{TX}} \\ \vdots & \ddots & \vdots \\ h_{M_{RX},1} & \cdots & h_{M_{RX},M_{TX}} \end{bmatrix}$,

$\mathrm{h}_{\mathrm{r,t}}$ is the channel gain for single input and single output between r^{th} receive and t^{th} transmit antenna. Any k^{th} column of H represents spatial signature of k^{th} transmit antennas with all receive antennas, X is input and N is the additive noise.

In MIMO system, capacity increases with min $(\mathrm{M}_{\mathrm{RX}}, \mathrm{M}_{\mathrm{TX}})$, diversity increases due to multipath propagation in turn reduces interference, thus, it can be used in single user and multi user case [4]. In this system, benefit of having channel knowledge at receiver improves bit error rate, capacity, suppress interference, thus improving performance of system. However, it can further be improved if characteristics of channel are also known at transmitter side. Thus it motivates researchers to estimate the channel at transmitter side in advance [9].

1.1.1 Need for MIMO system

MIMO system has replaced SISO (single input single output) due to its various applications which are listed below:

TO INCREASE SYSTEM CAPACITY: - MIMO system enhances the data rate by transmitting several independent data lines within the same bandwidth which is called spatial multiplexing [10]. These data streams can be separated by the receiver if appropriate channel conditions are met. The channel capacity of each data line is increased by multiplicative factor of total no. of lines [11]. Thus capacity of MIMO system is increased by spatial multiplexing. Hence for consistent increase in capacity the no. of data streams should be minimum of number of transmit and receive antennas i.e. min {MT, MR} [5].

TO INCREASE RANGE AND SNR**: -** Array gain is defined as increased SNR at the receiver due to consistent combining of input signals at a receiver [12]. The consistent combining is due to spatial processing at transmitting and receiving antennas. Array gain reduces addition of noise at the receiver and hence improves range of a wireless system [6].

TO REDUCE CHANNEL FADING: - The signal intensity is not fixed at the receiver of wireless system, it constantly fades. This fading can be diminished by spatial diversity which generates several self sufficient copies if transmitted signal. As number of self sufficient copies or diversity order increases, it is assumed that one of the copies is free from deep fading, hence received signal becomes more reliable and quality enhances. For a MIMO systems with MT transmit antennas and MR receiver antennas, the spatial diversity order is MT× MR [13].

NO NEED TO INCREASE BANDWIDTH AND TRANSMIT POWER: - Capacity, range and reliability get better in MIMO system through proper utilization of space dimension which in turn boosts throughput and link range within the same B.W. and transmitted power [4].

1.2 OFDM SYSTEM

OFDM is a frequency-division multiplexing (FDM) technique used as a digital multi-carrier modulation method. This system is able to overcome the limitations of MIMO system, like, reducing ISI (inter symbol interference) and increase in data rate without increase in bandwidth. All subcarriers are orthogonal to each other and are modulated with different modulation scheme like quadrature amplitude modulation (QAM) or phase-shift keying. Hence this system increases spectral efficiency with no adjacent channel interference [14]. The OFDM system is preferred to single-carrier system as its complex structure can easily solve for large delay spreads. Equalizers are designed for some particular delay spread value [15]. Secondly, in MIMO system as delay spread value exceeds this particular value, its performance start to degrade where as this type of problem does not occur in OFDM blocks [4]. It has many advantages like maximum throughput, deals with multipath, decreases narrowband interference, provides frequency diversity, creates single frequency network [16], likewise it has some disadvantages such as :

LARGE PEAK TO AVERAGE POWER RATIO - In OFDM system N random variables are modulated with different frequencies. At some frequencies these modulated signals add up where as in some they cancel each other. This increases peak to average ratio (PAR) of the system. This problem is seen more towards transmitter. This was solved earlier by using power amplifiers at the transmitter side so as to avoid clipping of transmitted signal, but it also had consequences of large power consumption. Hence new method is to be needed to solve these problems [16].

TIME OFFSET - In OFDM system, delay spread is caused due to the occurrence of time offset which in turn generates inter symbol interference (ISI) and results into loss of data [17].

FREQUENCY OFFSET - When there is difference in frequency occurs at the transmitter and receiver end it is called frequency offset. As a result of this inter-carrier interference (ICI) occur and orthogonality disappear [17].

In OFDM system first the input signal is divided into various subcarriers which are passed in parallel to modulation block as shown in fig.1.3 [17]. Different subcarriers can be modulated with any one of the schemes like, QAM, Binary phase shift key (BPSK) or Quadrature phase shift key (QPSK) .Then inverse fast Fourier transform (IFFT) is perform to obtain OFDM symbol which is passed through a unknown channel. In receiver side the process is repeated in reverse order. But the received signal contains some errors due to addition of noise at the channel [17].

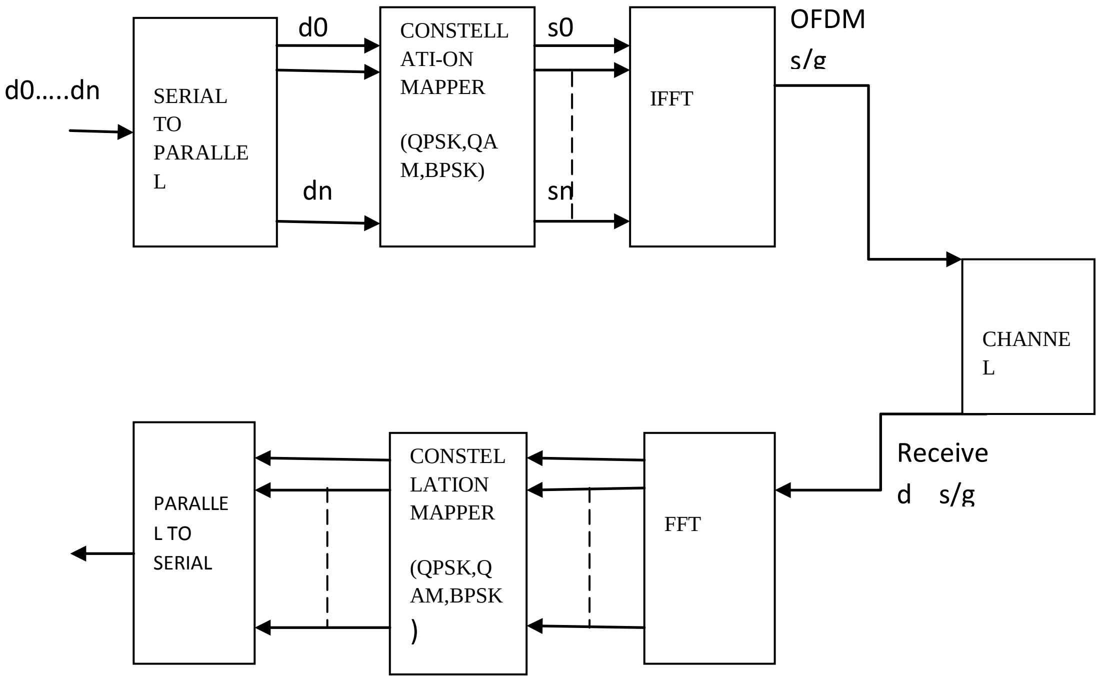


Fig 1.3: Block diagram of OFDM system

The simulation result shown in fig 1.4 [18] depict graph between bit error rate (BER) and signal to noise ratio (SNR) curve for OFDM system in which it can be seen that the accuracy of to OFDM generates its results [18].

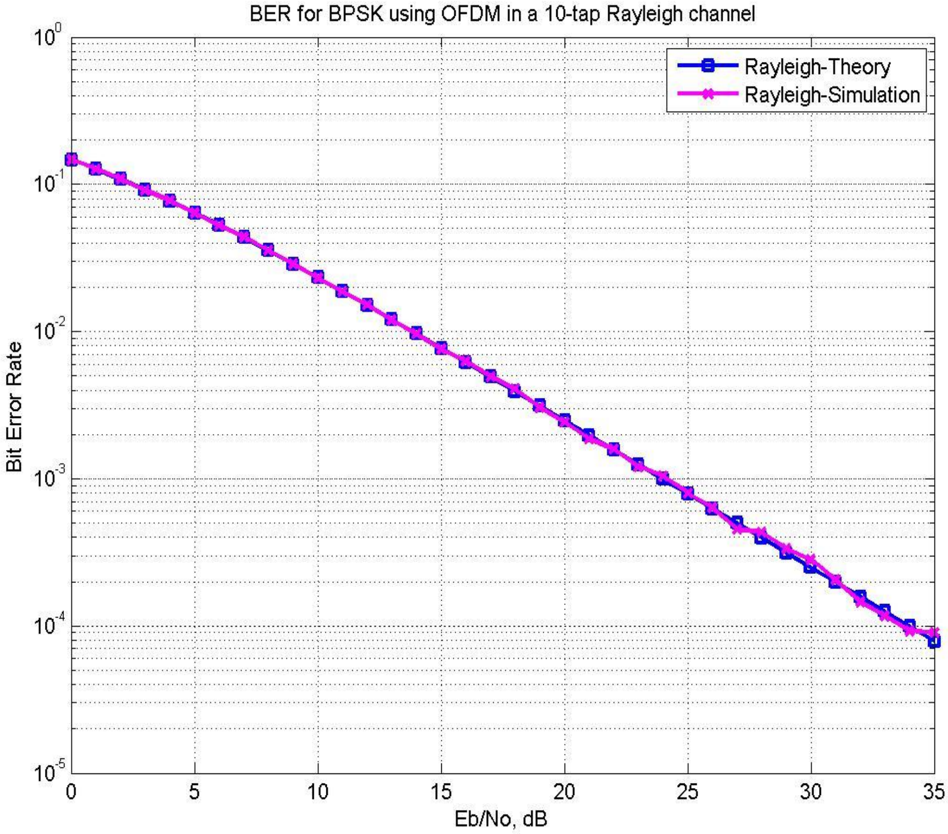


Fig 1.4: BER versus (vs.) SNR curve for OFDM system

The specification adopted for this simulation is included in table 1.1.

TABLE 1.1 Values of Parameter for BER vs. SNR for OFDM system

PARAMETER	**VALUES**
No. of symbols	10,000
No. of subcarriers	52
Range of SNR	0:35
FFT size	64
Modulation	BPSK
No. of multipath	10

The graph is plotted for Rayleigh channel and blue curve gives theoretical BER and pink curve shows simulation BER i.e. difference in transmitted and received bits in OFDM system. The curve almost overlaps the theoretical value.

1.2.1 Multi-Carrier Modulation

Multi-Carrier Modulation (MCM) is the method of transmitting data by dividing the stream into several bit streams, each of which has a much lower bit rate, and by using these sub streams to modulate several carriers. The advantages of MCM include relative immunity to fading caused by transmission over more than one path at a time (multipath fading), less susceptibility than single-carrier systems to interference caused by impulse noise, and enhanced immunity to inter-symbol interference. Limitations include, difficulty in synchronizing the carriers under marginal conditions and a relatively strict requirement that amplification be linear [19].

1.2.2 OFDM Symbol

OFDM symbol is generated by multiplexing several overlapping subcarriers and adding a cyclic prefix (CP). Prior to OFDM symbol, as shown in fig 1.5, the addition of CP actually ensures the elimination of ISI. At the receiver CP is removed and only information bearing part is further processed [15].

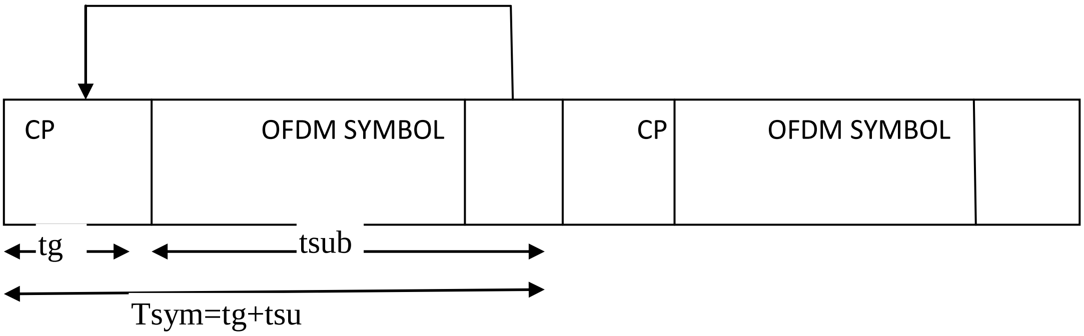


Fig 1.5: OFDM Symbol

$$Tsym=tg+tsub \tag{1.2}$$

where,

Tsym is symbol length, tsub is subcarrier length and tg is CP length.

CP is necessary in every OFDM system as multipath distortion is experienced by any radio communication system, hence, the received signal gets distorted. This distortion takes place due to differing delay in channel sinusoids. Therefore, CP and guard time have length longer than the maximum delay spread. This will remove interference between channel i.e. ICI and among consecutive blocks i.e. ISI. During IFFT interval, each sub-carrier has fixed no. of cycles which ensures orthogonality. At the receiver, multipath fading is observed in the input signal due to delayed and scaled sub carriers. So, till Guard Interval (GI) is longer than maximum channel delay then orthogonality principle sustains in subcarriers as cycles are in FFT interval range. Hence due to CP and GI wideband fading is converted to narrowband fading [17].

1.2.3 MIMO-OFDM Model

OFDM adds frequency multiplexing to the MIMO system which consists of spatial and temporal multiplexing. First the input data in binary form at the transmitter is encoded and then modulated with a modulation scheme. Thereafter, Serial to parallel conversion of data with different subcarriers is done is shown in fig 1.6 [14]. IFFT of subcarriers is done and the size of IFFT is taken larger than or equal to no. of subcarriers. Now CP is added to remove ISI which was seen in MIMO system. Then it is transformed to analog form and sent to receiver through channel. At the receiver all the process is repeated in reverse .An additional block of detector is placed at the receiving end. The output thus obtained at the receiver is not accurately same as transmitted signal but more accurate than MIMO system [14]. In MIMO system if sub-carriers are introduced and for each sub-carrier separate detection is done then the system will become slow. However, if OFDM is introduced then due to parallel transmission of subcarriers, the subcarriers can be detected faster than MIMO system.

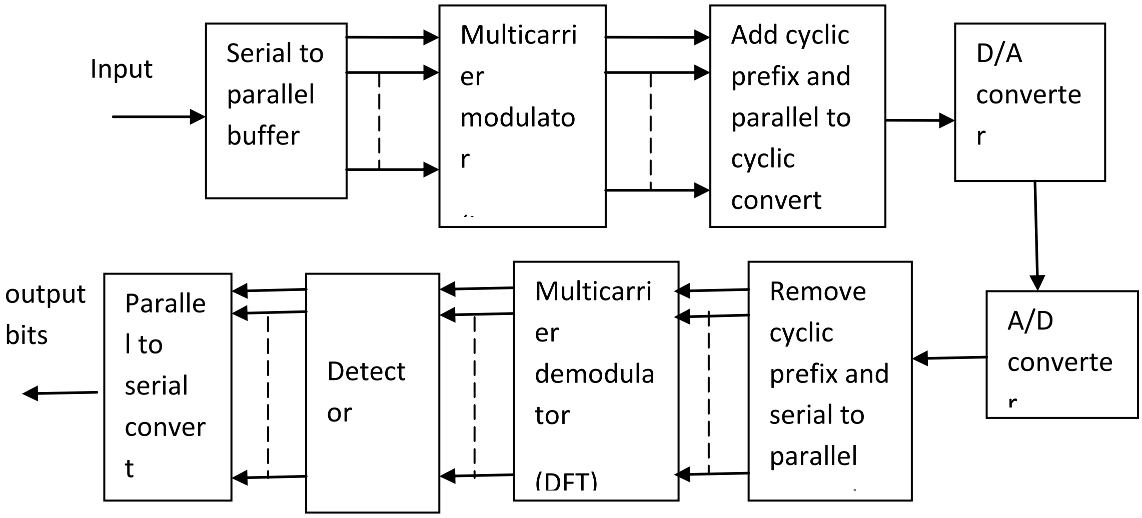


Fig1.6: Block diagram of transceiver of MIMO-OFDM system

1.3 PRECODING

Precoding is a method that utilizes the channel side information at the transmitter (CSIT). Precoder is the last block at the transmitter side. CSIT is beneficial in enhancing the transmission rate, boosting coverage and diminishing receiver complexity in MIMO system [20]. The block diagram of transmitter with precoding is shown below.

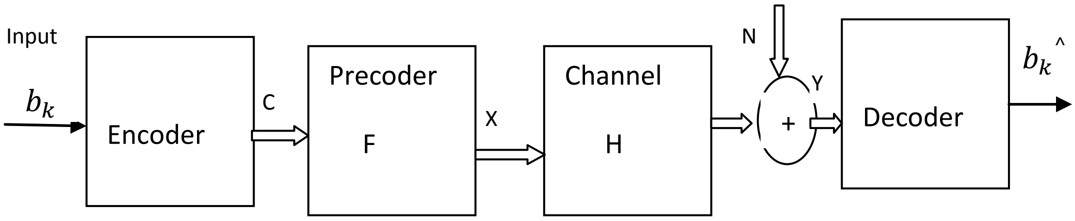


Fig 1.7: Block diagram of Transmitter with Precoder

Here first the encoder converts analog signal to digital format and then in precoder the CSIT is multiplied with this signal at the transmitter and then passed through the channel, where noise gets added to the signal output Y. Thus can be written:

$$\mathrm{Y} = \mathrm{HFC} + \mathrm{N} \tag{1.3}$$

where,

F is a precoder matrix, C is block of code, H is channel matrix and N is noise [21].

1.3.1 Types of Precoding

There are two types of precoding exists in the literature. They are linear precoding and non-linear precoding.

Linear precoding has a structure of beam former which directs its beam thus power in particular directions according to the requirement. It can have single or multiple beams [21]. It is used to increase diversity and throughput [22]. Linear precoding is computationally less complex. Hence, exploring this coding scheme is a key interest these days. It is a statistical method. At low SNRs, power distributed increases the capacity of the channel where as at higher SNR i.e. above 15 db, it decreases the capacity [23]. In order to increase the capacity at all SNRs, the antenna distribution should be defined accordingly. Mathematically, it is given by:

$$F = U_F D V_F^{*} \tag{1.4}$$

where,

F is precoder matrix, left singular vectors U_F are the orthogonal beam directions (patterns), squared singular values D^2 are the beam power loadings, right singular vectors V_F is termed as the input shaping matrix [24]. For transmitting a constant power from a system, condition below must be satisfied .

$$tr(FF^*) = 1 \tag{1.5}$$

Since, block code C is Gaussian distributed and has zero mean, its covariance is given as:

$$\mathrm{Q} = \frac{1}{\mathrm{T}} \mathrm{E}(\mathrm{CC}^*) \tag{1.6}$$

However, the covariance of output Y is given by :

$$E(YY^*) = FQF^* \tag{1.7}$$

Linear precoding can be further classified into two types i.e. redundant precoding and non-redundant precoding.

In Redundant precoding lowering of bandwidth efficiency occurs, hence, it is preferred to save performance of system. Precoder A has a tall structure of K × N with K>N where K and N are rows and columns respectively of a matrix [25].

where as in non-redundant precoding bandwidth can be saved by using non-redundant coding which is done by choosing matrix A with square structure, i.e. N × N where, N is row and column of a matrix [26]. It enhances multipath diversity of the OFDM system. Cross correlation of received signal is done and channel is estimated using diagonal matrix ambiguity. It can be operated without knowledge of channel length is less affected by stationary noise. It does not increase the power of the system rather distributes equal power between OFDM blocks [27].

Non-linear precoding uses *DIRTY PAPER TECHNIQUE* [28] to cancel interference in system. It was shown that the capacity of a channel where the transmitter knows the interfering, signal is the same as if there were no interference. In this technique full channel state information is required [24]. It is better than linear precoding but computationally complicated. The most famous non-linear precoding method is QR decomposition method where channel is given by the equation:

$$H=LQ \tag{1.8}$$

where

H is a unitary matrix, Q is precoded Hermitian matrix and L is lower triangular channel matrix. For first user, there is no interference according to this method, whereas, for the second user interference is experienced by the first user and so on [24].

1.3.3 Periodic Precoding

Periodic precoder is a linear precoder which consists of a sequence which repeats itself after a particular period e.g.

$$p\ (n+N) = p\ (n) \tag{1.9}$$

where

N is the period of the precoder. It is normally used in estimating channel in MIMO systems [29].These sequences are applicable in both conditions i.e. when number of transmitting antennas (M_{TX}) is more than receiving antennas and vice-versa [29]. Inducing cyclostationarity at the transmitter is one of the main characteristic of periodic precoder. The cyclostationarity is induced by the cyclic prefix at the transmitter [30]. Channel is estimated by using cyclic statistics i.e. covariance or correlation of received signal.

Periodic sequences are of two types:-

Constant modulus - Constant modulus periodic sequences are non-circular stationary sequences which are accompanied by conjugate cyclostationarity to remove higher order statistics which generally occurs during covariance computation. To induce conjugate cyclostationarity, periodic sequence is multiplied with complex exponential at some characteristic frequency [31].

Non-constant modulus - Non–constant modulus periodic sequences accompanied with cyclostationarity helps to alleviate restriction on channel zeros and channel order overestimation [32].

Periodic precoder has many advantages over various methods of estimating channel as discussed below:-

- Cyclostationarity [33] at the transmitter helps to reduce the indentifiabilty condition imposed on the channel. Now the only condition impose is that the channel impulse response matrix should be full column rank.
- The channel can have more outputs or more inputs: - It means number of transmit antennas can be more than receive antennas or number of receive antennas can be more than transmit antennas [34].
- The method is shown to be robust with respect to channel-order overestimation [34].
- By optimally choosing precoding sequence, the performance of MIMO system is increased as the effect of noise and channel estimation error are reduced [29].

1.4 CHANNEL ESTIMATION

Multipath propagation is the most important thing seen in the wideband wireless communication systems. To recover the transmitted signal at the receiver, it is essential to know some information about the channel. The complex wireless channel is time and frequency selective and in MIMO system space selectivity has a major role [35]. According to Jakes' model, in the multipath propagation parameters to each path is given by angle at the receiver or transmitter antennas, propagation delay and complex amplitude. These parameters are allocated by making statistical assumptions. In channel modelling channels are described through general model [36].

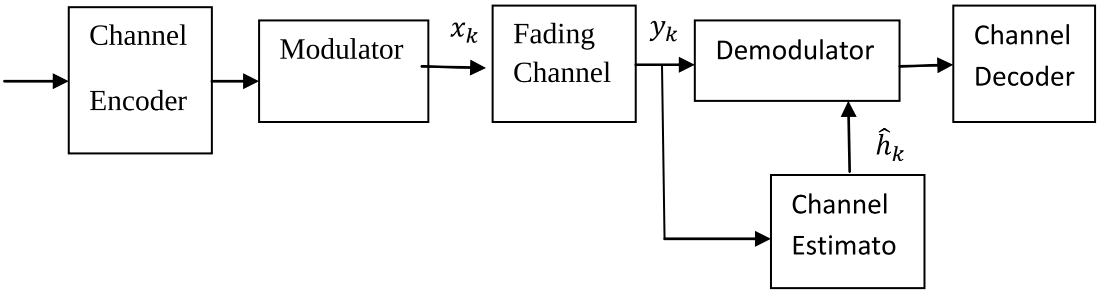


Fig 1.8: Block diag. for Channel Estimation

For channel estimation first channel encoder encodes the input from different atmosphere to binary form and then modulated as shown in fig. 1.8 [37]. Then, this modulated scheme is passed through channel where fading takes place or interference occurs. The output received signal is then demodulated and decoded. The channel is estimated using output and comparing it with input signal. Thereafter the estimated channel is sent to the demodulator to get more accurate results. In OFDM system channel estimation is easier as sub carriers have less spacing, therefore enables its use As the channel estimation is used in high speed systems where fast channel estimation is possible with lesser delay.

1.4.1 Techniques for Channel Estimation

There are various techniques for estimating channels such as subspace method, training method, pilot symbol method, DFT method, recoding method. This can be categorised in semi-blind and blind channel estimation. Their comparison is included later to find out the best one.

1.4.1.1 Training Method

In practical systems, channels are estimated using periodic cluster of known training symbols [38], therefore focus is mostly on these techniques. Earlier methods involved usage of known training symbols. More accurate results can be obtained by using convolution of unknown data symbols which is called advanced training method. In semi-blind methods training symbols are used along with blind techniques. In training method, maximum likelihood (ML) method is used to estimate channel. It can be done in two ways, namely, deterministic ML (DML) and Gaussian ML (GML) methods [39]. In DML, unknown parameters are determined where as in GML unknown parameters are calculated with Gaussian distribution. Both of these methods do not use finite alphabet property but uses discrete distribution of random variables [40] to calculate unknown parameters.

1.4.1.2 *Pilot Method*

Next method can be pilot symbol insertion [41] in which pilot symbols which are known to both transmitter and receiver can either be of block type or comb type which is employed on each subcarriers of OFDM symbol. Many interpolation techniques such as second order interpolation, low pass interpolation, linear and spline are used for channel estimation in pilot method. After this, LS (least square) error [42] and MMSE (minimum mean square error) are calculated. Simulation results in fig 1.9 shows channel estimation error for LS and MMSE methods [43]. Results show, that MMSE method is better than LS as mean square error (MSE) of channel in MMSE is lesser than LS method.

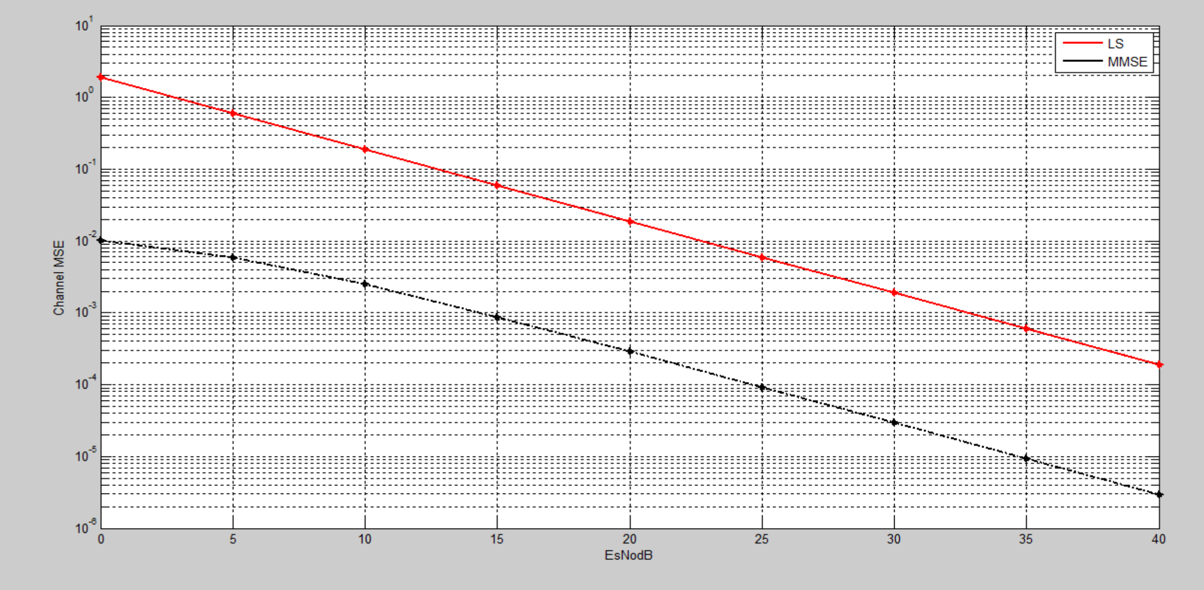


Fig 1.9: Channel mean square error vs. SNR for LS and MMSE method

To perform this simulation, the values of various parameter included is shown in table 1.2

TABLE 1.2 Values of Parameter for Channel MSE vs. SNR

PARAMETER	**VALUES**
Subcarriers	72
Cyclic prefix length	8
FFT size	64
Symbols	1500
SNR (db)	0:5:40
Channel order	10

1.5 BLIND CHANNEL ESTIMATION

Earlier semi blind channel estimation techniques were used as training method for estimating channel. In this technique, transmitter sends a signal which is already known by the receiver. But this method had serious drawbacks, such as, it would increase operating time of wireless devices and most wireless devices are battery operated so they don’t survive for longer time [44]. Moreover training increases the length of transmitted signal and hence transmission rate of data decreases. To avoid this channel effects equalizer can be used. This equalizer can be made by either knowing channel information or without channel estimation. Blind equalization can also be used as it does not require any transmitted data for channel estimation; instead it uses statistical and structural properties like finite alphabet, constant modulus, sub-spaces orthogonality of the communication signals [44]. Thus, if blind channel estimation methods can be sensibly used, then amount of training required can be reduced significantly. Typically, some special property of the transmitted signal is exploited for blind channel estimation.

It is of two types statistical and deterministic

In statistical methods, second order statistics of received signal is calculated which are induced by the cyclic prefix at the transmission side. The channel is estimated using these cyclic properties introduced by cyclic prefix or by subspace method imposed on pre-DFT received blocks where as in deterministic methods, post DFT received blocks are used to extract finite alphabet property of input signals, e.g. maximum likelihood and Bayesian method.

1.5.1 Techniques for Blind Channel Estimation

Various methods of blind channel estimation are reported in literature.

1.5.1.1 *Subspace-based Channel Estimation*

The subspace-based channel estimation technique is the most commonly used technique in many OFDM systems. It involves usage of second order characteristics of received signal .It can be used in various ways like singular value decomposition (SVD) can also be used or precoder can be used at transmitter side. The basic approach followed by subspace is that separating noise signal and data signal. It uses the property that noise signal is orthogonal to data signal and thus both can be separated. This method, however, increases the computational complexity as correlation of received signal is done and then Eigen values are calculated [45]. For accurately calculating the channel large numbers of equations are needed to be evaluated, required which can be done by oversampling. Another way of using subspace method which has been used so far is using redundancy in CP insertion in SISO-OFDM systems. Now a subspace method can be used in OFDM systems without CP [41].

1.5.1.2 DFT-based channel estimation

The DFT-based channel estimation technique is also very much used, in which the channel estimate by LS or MMSE is first passed through IDFT and then DFT is done for that channel to eliminate the effect of noise and improve the performance [46]. A comparison of true and estimated channel using DFT method is shown in fig 1.10 [46].

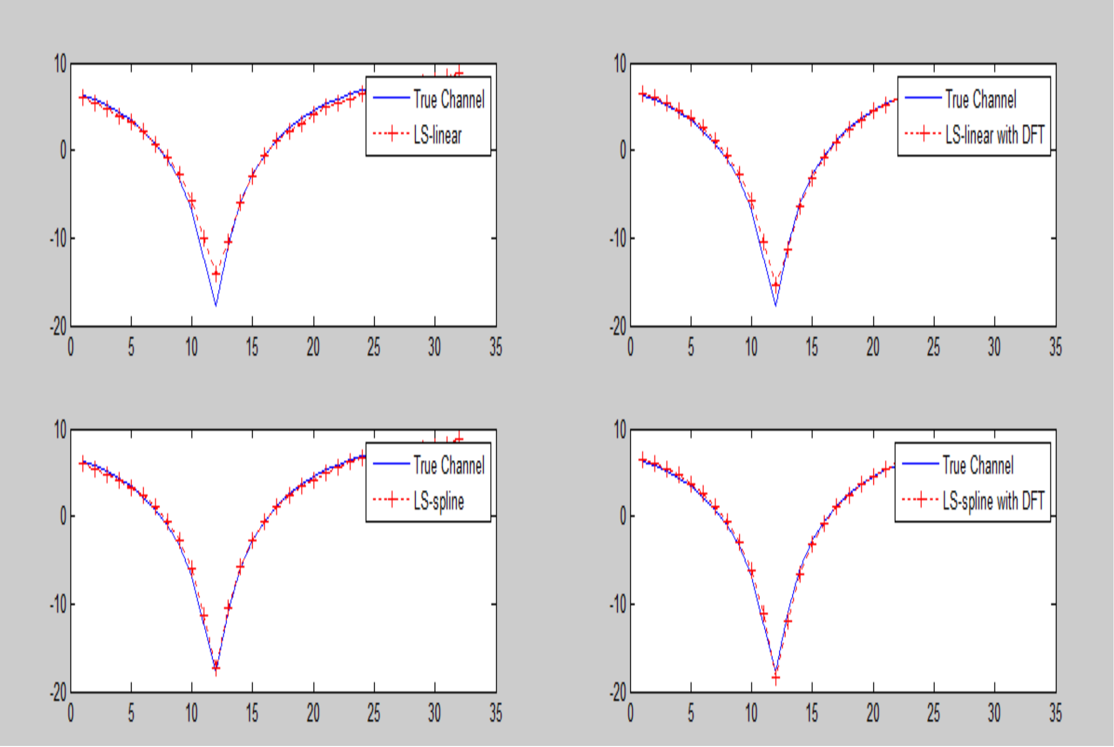


Fig 1.10: Comparison of true and estimated channel using DFT method

Here, are plots of channel estimation (H) are done in MIMO-OFDM system using DFT method and without DFT method. LS (least square) method is used in without DFT graph.

As it can be depicted from the above plots that linear interpolation with DFT gives better output than linear interpolation without DFT as it is capable of removing noise from the LS system and therefore provides more accurate results. Also spline interpolation gives better results as compared to linear interpolation because interpolant is smoother in spline interpolation. However, computation in spline interpolation is more complex than linear due to use of low degree of polynomials. The LS channel estimate of a system is given below:

$$H^{\wedge}{}_{LS}[K] = \frac{Y[K]}{X[K]} \tag{1.10}$$

where,

$Y[K]$ is the output signal and $X[K]$ is the input signal.

Taking the IDFT of the channel estimate $\{H^{\wedge}(K)\}_{K=0}^{N-1}$

$$\text{IDFT}\{H^{\wedge}(K)\} = h[n] + z[n] \triangleq h^{\wedge}[n],\ \text{n=0, 1.........N-1} \tag{1.11}$$

The parametric values used for above simulation is given in table 1.3

TABLE 1.3 Values of parameter for DFT method

PARAMETER	VALUES
Nfft	32
Ng	Nfft/8
Nofdm	Nfft+Ng
Nsym	100
Nps	4
Nbps	Nfft/Nps
M	2^Nbps
SNR	30
Es	1

where,

z[n] is the noise in time domain. Coefficients $\{h^{\wedge}[n]\}$ that contain only noise are removed, the coefficients with maximum delay L can be defined as:

$$h^{\wedge}[n] = \begin{cases} h[n] + z[n], & n = 0,1 \dots\dots\dots N-1 \\ 0 & otherwise \end{cases} \tag{1.12}$$

and remaining L elements are changed to frequency domain as follows:

$$H^{\wedge}{}_{DFT}[K] = DFT\{h^{\wedge}[n]\} \tag{1.13}$$

1.5.2 Precoding Based Blind Channel Estimation

Since, training method cannot be applied to certain communication system precoding based blind channel estimation is used here. Moreover MIMO channels require large amount of training sequences [47]. Although SS method has simple structure but it requires some specifications. Firstly, the receive antennae must be strictly more than the transmit antennae which may not be satisfied by many existing standards, e.g. the IEEE 802.11standard uses only 2 × 2 transmit and receive antennae pairs. Besides, in the case of SISO systems, the equal number of transmit and receive antennae is obviously used, which does not satisfy the requirement. Secondly, the accurate knowledge of the channel order is difficult to obtain [27]. The channel order over-estimation may cause significant performance degradation. Thus, blind channel estimation for MIMO-OFDM systems has been studied extensively in recent years. Subspace (SS) based method for blind channel estimation of MIMO-OFDM systems, which exploits the redundancy, introduce by the CP. In general, most of the existing SS algorithms for MIMO-OFDM systems are processed in the time domain before removing the guard interval. Hence, they are either designed for CP based OFDM or ZP based OFDM, therefore not suitable for both simultaneously. Besides, some strict requirements must be satisfied for the SS methods [48].

Precoding based algorithm is a way to solve these problems. Frequency domain linear precoding is an effective method which can balance the lack of the multipath diversity for OFDM systems to reduce the negative effects of channel zeros at certain subcarriers. It also motivates an alternative method for blind channel estimation [49].

Results in fig 1.11[50] show the magnitude and phase of actual channel and estimated channel for a precoder based channel estimation technique. It is observed that the magnitude shows small variation in actual and estimated channel. Although there is more difference in phases of actual and estimated channel.

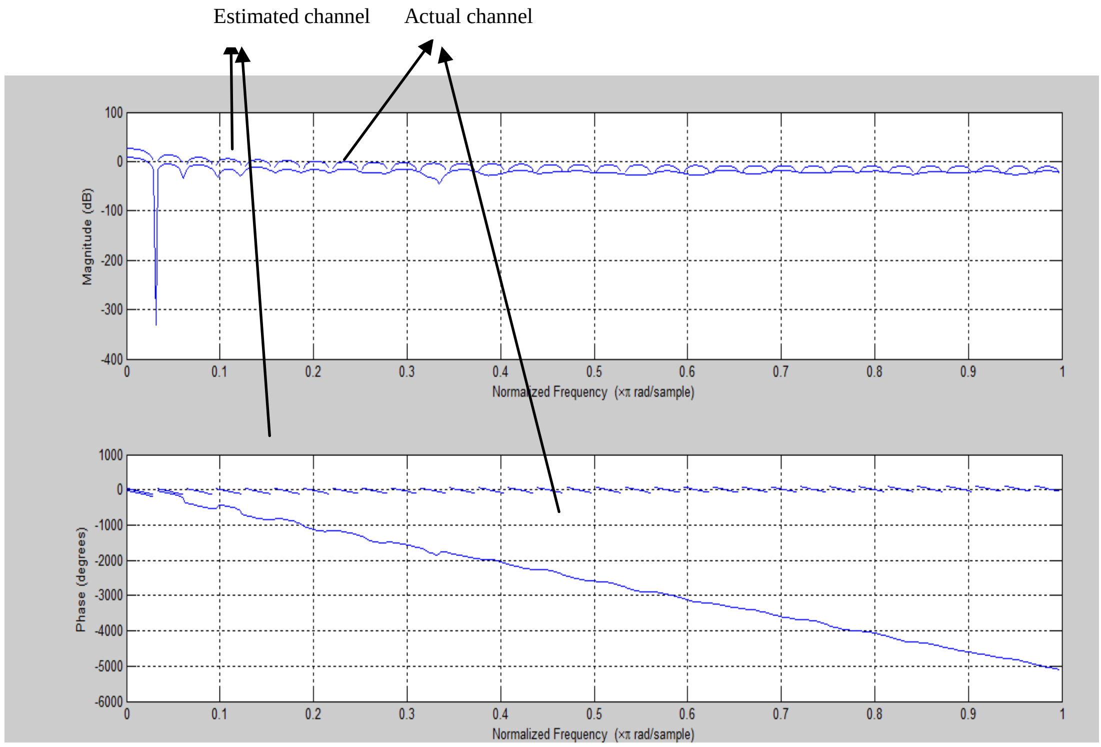


Fig 1.11 Magnitude and Phase of Actual and Estimated channel using Precoding method

The various parameter included in obtaining the above graph is shown in table 1.4

TABLE 1.4 Values of Parameter for Precoding method

PARAMETER	VALUES
No. of subcarriers	64
SNR	50
CP length	8
H11	[0.2710 + 0.1843i]
H12	[-0.0009 + 0.0718i]
H21	[0.2710 + 0.1843i]
H22	[0.2356 - 0.0318i]

1.5 THESIS OBJECTIVE

The objective of this thesis is to estimate channel with minimum errors in MIMO-OFDM system. Hence, periodic precoders is used which not only reduces complexity but also reduces noise and channel errors at receiver.

1.6 ORGANISATION OF THESIS

This dissertation has been divided into four chapters. First chapter is the introduction chapter to the MIMO-OFDM system, channel estimation techniques and precoding techniques. Second chapter includes the literature survey. The analysis of periodic precoder based channel estimation and BER calculation are considered in third chapter, whereas the conclusion and future scope of the thesis are included in fourth chapter. References and list of publication are also given in this chapter.

For this work reported in the thesis, a total of twenty three reputed research papers are investigated deeply. Their comparative study actually formulated the problem tackled and reported in this thesis. The details about all these papers are included below:

H. Bolcskei, E. Zurich "**MIMO-OFDM wireless systems: basics, perspectives, and challenges" [4]:** MIMO-OFDM is the new solution to WLAN, WMAN and next generation like 4G and 5G.In this paper basics of MIMO and OFDM are defined. It directs towards space-frequency signalling. It defines how MIMO-OFDM can be used in multiuser system and single user system**.** Complex multi-user system is discussed and hardware is implemented for the same.

Spencer, Q. H Peel, C.B. Swindlehurst, A.L. Haardt "An introduction to the multiuser MIMO downlink" [24]: MIMO is the new focus of interest in wireless communication. Since MIMO can be used for higher capacity, higher throughput, reduction in interference and enhanced multipath diversity. MIMO is now used for WLAN, WMAN, mobile applications for next generation system, it is necessary for further up gradation of the system like in multi user system. For this advantage of higher capacity of the system is taken where multiple users can operate with one system with the help of space division multiple access. In this paper two types of techniques are described like linear and non-linear techniques. Linear techniques are defined by beam forming and power distribution. Non-linear is defined by dirt paper technique where interference is considered negligible for first user and second user experiences some interference from first user and like this approach goes for further users. Hence the method can be employed for multiple users.

H. Liu, X.G. Xia "Precoding Techniques for under sampled Multi-receiver Communication Systems" [21]: In this paper, blind signal is recovered from under sampled data at the multi-receiver. It is exploited that by using a precoder at transmitter with increase in bandwidth under sampled data can be identified. The main focus of this paper is the formation of an algorithm for under sampled system and deriving equations for a resistant precoder acting as filters. It is also analysed that blind channel estimation for linear time invariant channel can also be done by using non-redundant precoding.

F. Li and H. Jafarkhani "Multiple-Antenna Interference Cancellation and Detection for Two Users Using Precoders" [48]: This paper is about removal of interference due to two users by a precoder assuming both channels are known to each other. The main focus of this system is cancellation of interference with the use of multiple antennas without loss of diversity and increase in complexity. In previous papers it is shown that the interference due to two users can be cancelled with two antennas at transmitting and receiving end. The author has reduced complexity in this paper with the introduction of diversity of 4. Then he further exploited the results for multiple antennas. His main idea is to design precoders, using the channel information, to make it possible for different users to transmit over orthogonal spaces. This property of orthogonality is used by the receiver to detect the transmitted signal. This detection of signal is mathematically derived and gives proof for full diversity for both users.
This analysis has been done for two users only but for any number of users this cannot be implemented, a new system is needed to provide full diversity with less complexity.

M. Sadek and S. Aissa "Leakage Based Precoding for Multi-User MIMO-OFDM Systems" [25]: Till now different precoders have been proposed which is used to reduce interference in multi-user MIMO system. Out of these precoders, the precoder that uses signal to leakage plus noise ratio (SLNR) to optimise its value is chosen. This precoder is used in this paper in MIMO-OFDM system so that not only noise is reduced but also ISI is reduced. Moreover in this paper channel identification is also determined by feedback of channel given by receiver and thus transmitter can regenerate the channel.

R. Zhang "Blind OFDM channel estimation through linear precoding: a subspace approach" [27]: Linear precoder is used for blind channel estimation in OFDM system in this paper. This precoder generates a low rank structure in the received signal. It uses properties of subspace method to estimate channel up to scalar ambiguity.
Precoder is placed before IFFT which generates a code which is designed such that errors between actual channel and estimated channel are reduced.

F. Khalid and J. Speidel "Robust Hybrid Precoding for Multiuser MIMO Wireless Communication Systems" [49]: This paper proposes hybrid precoding in which multiple precoders work for multiple optimisation. This precoder is designed for multiuser MIMO system and two iterative algorithms are proposed for robustness against channel estimation errors namely hybrid diversity maximisation robust (HDMR) and enhanced HDM (EHDM). HDMR optimises the process in two steps with lower complexity among various multi-users. EHDM optimises the process in three stages and shows results better than HDMR with respect to spatial multiplexing situations.

Y.S. Chen and C.A. Lin "Blind-Channel Identification for MIMO Single-Carrier Zero-Padding Block-Transmission Systems" [51]: In this paper periodic precoder has been used for MIMO single carrier system. Data is transmitted through different blocks and then zero padding is done for the system. Channel is estimated in two steps, first is evaluating lower triangular linear system and second is calculating Eigen values and Eigen vectors of channel product matrices.

Periodic precoder uses cyclostationarity of transmitted signal to produce cyclic properties of received signal. Periodic sequence of period N is used to estimate channel with reduction of noise. Correlation of noise with periodic sequence should be minimum. Optimal periodic sequence is generated by normalising power of elements of periodic sequence.

A. Liu and V. Lau "Two-Stage Subspace Constrained Precoding in Massive MIMO Cellular System" [52]: Precoder with subspace properties has been introduced in this paper that uses spatial channel correlation to find gain given by massive antenna with good channel state information (CSI). Precoder is divided into two matrices at each base station- inner precoder and transmit subspace control matrix. Inner precoder depends on CSI for spatial multiplexing gain where as transmit subspace control reduces inter cell interference by using channel statistics and optimises QOS problem in which SINR probability for each user should not exceed outage probability. A bi-convex approximation technique is used which has three components-semi definite relaxation, random matrix theory and chance constrained optimisation. Then to tackle bi-convex approximation problem a proper solution is proposed. Simulation results give considerable gain for various base stations.

Channel is estimated using downlink channel estimation and pilot symbols used depends on channel coherence time which further decreases as M increases. Channel reciprocity and channel feedback are impossible in channel estimation.

Real time CSI is required in massive MIMO system to reduce inter-cell interference (ICI) which is difficult to obtain due to backhaul signal latency.

N.M. Tran, D.H. Pham, H. DTuan and H.H. Nguyen "Orthogonal Affine Precoding and Decoding for Channel Estimation and Source Detection in MIMO Frequency-Selective Fading Channels" [50]: In this paper a linear precoder with training sequence induced is used which is called affine precoder in MIMO-OFDM system. For channel estimation data is neglected and for data detection training sequence is dropped at the receiver. The proposed method performance is better than existing methods and complexity is also reduced. Power is also optimally distributed between data signals and training signal.

S. Noh, Y. Sung and M.D. Zoltowski "A New Precoder Design for Blind Channel Estimation in MIMO-OFDM Systems" [53]: Blind channel estimation is done with a new precoder proposed in which a linear precoder is used in MIMO-OFDM system. Some data symbols of proportionate length of channel are linearly precoded are transmitted as subcarriers with equal spacing. Hence some subcarriers are sent in conventional way and some with linearly precoded spacing. Therefore maximum likelihood detection (MLD) is done for each subcarrier which are not precoded. This was not possible in earlier precoding methods. Hence MLD can be implemented in this precoding system. The precoder is proposed such that channel estimation error and signal error are minimised. This is called optimisation of precoder.

X. Dong and Z. Ding "Downlink Wireless Channel Estimation for Linear MIMO Transmission Precoding" [54]: In this paper linear precoding is exploited for channel estimation in MIMO system. As precoder gives CSIT the precoder used in this paper is called transmission precoder. The channel is estimated by adopting proper algorithm with a feedback using precoder which also improves capacity and diversity of the system. Receiver sends back some of its data received to the mobile station for quadratic channel product estimation. This quadratic channel product is used to propose downlink precoders that know channel information. This method can be implemented for both frequency selective and flat fading channels.

M. Tran, A. Nix, and A. Doufexi "Mobile WiMAX: Impact of channel estimation error on the Performance of Limited Feedback Linear Precoding" [55]. In this paper mobile WiMAX standard (802.16e) is used with limited feedback precoding. This system is compared to open loop precoding system and limited feedback shows better results for channel estimation in MIMO system. Actual channel and estimated channel shows errors in practical MIMO-OFDM system. Hence perfect channel estimation is required. Researchers are studying to reduce channel estimation error with limited feedback precoding. Channel estimation error is determined in three steps. Firstly MMSE channel estimation is done that uses correlation of received signal. Secondly low rank (LR) estimator does not require correlation used in MMSE system. Thirdly in ZF estimator no information about correlation is used. Now three estimators are compared and MMSE shows 0.2db loss in performance, 0.5db is observed in LR case and 4-5db in ZF case.

J. Wang and D. P. Palomar "Robust MMSE Precoding in MIMO Channels with Pre-Fixed Receivers" [56]: Robust precoding is discussed in this paper which uses MMSE technique for channel estimation in MIMO system. Pre fixed receivers are used which are less complex and channel according to change in channel but do not make use of CSI given by precoders. Receivers adapt with following types of CSI- perfect CSI, statistical CSI and deterministic-imperfect CSI in which actual channel is near to the estimated channel which generates worst results in robust precoding which is mainly discussed in this paper. Under optimal conditions left singular vector of robust precoding is equal to right singular vector, statistical CSI, channel mean, deterministic imperfect CSI and perfect CSI. Power is allocated for closed system using the proposed method. Deterministic imperfect CSI is used so that robustness of the MIMO system can be improved. Uncertainty in a region is considered for determining imperfect CSI.

J.Y. Wu “**How Much Coherent Interval Should be Dedicated to Non-Redundant Diagonal Precoding for Blind Channel Estimation in Single-Carrier Block Transmission?” [57]:** Earlier for blind channel estimation transmit precoding has been used but channel capacity is not discussed in these papers. Here in this paper diagonal linear precoder is used in single carrier MIMO system with addition of cyclic prefix for channel estimation using covariance of received signal. When perfect CSI is seen at the receiver then capacity deteriorates in optimal precoder scheme. Due to finite samples in covariance matrix channel estimation occurs as coherent time is finite. So the coherent time interval is to be detected for more accurate results and to maximise capacity. Matrix perturbation theory is used to determine capacity by taking uncertainty in channel estimation. The capacity is dependent on non-redundant diagonal precoding. An approximate formula is formulated for maximum capacity in the closed form channel estimation method with precoding interval to be calculated. Simulation results are given for the proposed method.

Y.S. Chen “**A Simple Precoding-Based Blind Channel Estimation for Space-Time Block-Coded Single-Carrier with Frequency-Domain Equalization Systems” [58]:** In this paper non-redundant precoding is used in MIMO space time single carrier frequency domain equalisation (SC-FDE) for blind channel estimation which uses covariance matrix for finding coefficients of channel**.** By finding channel coefficients with the use of only division and subtraction, Eigen values and Eigen vectors are generated of Hermitian channel product matrix. An algorithm is defined for channel estimation and reduction of noise using proposed precoders. Simulations are defined to give evidence to this method and numerical results.

17. P.M. Castrot, M. Joham, L. Castedot and W. Utschick “**Robust MMSE linear precoding for Multiuser MISO systems with limited feedback and channel prediction” [59]:** Here multi input single output (MISO) system is used for multi users and a robust a MMSE precoder with a feedback is used. Closed form structure is formed that is used to provide CSI with a no. of feedbacks. A method is proposed to determine past channel which can be used to predict future and present channel estimate with the use of rank reduction. Numerical results are verified with simulations of the proposed method.

J. Jose, A. Ashikhmin, P. Whiting, and S. Vishwanath "Channel Estimation and Linear Precoding in Multiuser Multiple-Antenna TDD Systems" [60]: Traditional approaches in the analysis of downlink systems decouple the precoding and the channel estimation problems. However, in cellular systems with mobile users, these two problems are in fact tightly coupled. In this paper, this coupling is explicitly studied by accounting for channel training overhead and estimation error while determining the overall system throughput. The paper studies the problem of utilizing imperfect channel estimates for efficient linear precoding and user selection. It presents precoding methods that take into account the degree of channel estimation error. Information-theoretic lower and upper bounds are derived to evaluate the performance of these precoding methods. In typical scenarios, these bounds are close.

A. Papazafeiropoulos and T. Ratnarajah "Linear Precoding for Downlink Massive MIMO with Delayed CSIT and Channel Prediction" [61]:

In this paper linear precoding is proposed for massive multi user MIMO in downlink time division duplex (TDD) system. The base stations consist of large no. of antennas which increases as no. of users increases. To remove practical hindrances like pilot contamination and delayed CSI a regularized zero forcing (RZF) precoder is used which increases capacity also. The deterministic sum rate predicts channel although channel determination decreases because of delayed CSIT. Hence complexity also decreases with RZF precoding and better results are obtained for practical systems.

B.S. Thian, H.D. Nguyen, and S. Sun "Statistical Precoding for MIMO Systems with Channel Estimation Errors" [62]: In rapidly changing channel it is difficult to obtain CSI in MIMO system. Hence statistical CSI is used as statistics are not altered for longer time. The precoder which gives statistical CSI is proposed in this paper which is able to remove errors considerably as compared to other precoding schemes used so far.

It is assumed in this paper that transmitter knows only statistical properties of channel and receiver knows both statistical and instantaneous estimate. A feedback at transmitter is also not required as transmitter will estimate from statistical properties of channel.

Kan Zheng, Jian Su, Wenbo Wang "Iterative DFT-based Channel Estimation for MIMO-OFDM Systems"[46]: Since earlier comb type pilot method with interpolation shows mean square errors (MSE) in MIMO-OFDM system and subcarriers should be close to follow sampling theorem. A DFT method is introduced in this paper for channel estimation with phase shifted pilot symbols. Channel impulse response (CIR) and channel frequency response (CFR) can be easily calculated in time domain and frequency domain respectively. Simulation results shows comparison between Linear Minimum Mean Square Error (LMMSE) of comb type pilot and DFT method.

Mohammad Torabi, Sonia A¨ ıssa, Senior Member, IEEE, and M. Reza Soleymani, Senior Member, IEEE "On the BER Performance of Space-Frequency Block Coded OFDM Systems in Fading MIMO Channels" [63]: In this paper BER expressions for space-frequency block coded OFDM (SFBC-OFDM) system are derived. Modulation schemes MPSK (M-ary phase shift keying) and M-ary QAM are used to explore performance of BER with change in channel estimation errors.BER expressions are compared with different SFBC-OFDM forms.

Ching-An Lin and Yi-Sheng Chen "Blind Identification of MIMO Channels Using Optimal Periodic Precoding" [64]: Interest in blind identification of channel in wireless system is growing these days. Precoding is an effective technique for exploring channel state information. A new precoder has been introduced called periodic precoder. Periodic precoder uses cyclostationarity of transmitted signal to produce cyclic properties of received signal .Periodic sequence of period N is used to estimate channel with reduction of noise. Correlation of noise with periodic sequence should be minimum. Optimal periodic sequence is generated by normalising power of elements of periodic sequence. This has not been used for OFDM system so far.

From the above papers it can be concluded that MIMO-OFDM has replaced MIMO system due to its vast advantages. It is also stated that to improve performance of MIMO-OFDM system CSI is necessary. Hence a new term called precoding is introduced. Precoding has many applications in MIMO-OFDM systems. It is used for cancellation of interference, for increase in capacity and one of the common applications discussed in these papers is blind channel estimation with different precoding designs. Precoding is used in both closed loop (limited feedback) and open loop design. Closed loop shows better results than open loop design. Non-redundant linear precoding is commonly used in many papers which is primarily used for blind channel estimation. After this a new precoder called periodic precoder was introduced. This precoder has reduced further complexity of MIMO system compared to other precoding schemes. But it has not been implemented in MIMO-OFDM system. So my work is to use periodic precoder in MIMO-OFDM system and calculate BER for the same.

CHAPTER - 3 ANALYSIS OF PERIODIC PREODER BASED CHANNEL ESTIMATION

Periodic precoder is used in the proposed method which is able to remove problems like channel-order overestimation, bandwidth expansion caused by oversampling at the receiver and limitations of subspace and training methods [64]. Moreover it can be used in both time and frequency dispersive channels. A periodic precoder passes a periodic sequence through each subcarrier of OFDM symbol. A different periodic sequence is given to each block of the system to make the system time-invariant. A toeplitz channel matrix is taken and super diagonal elements of the expression are taken for calculations and then cyclostationarity of received signal is generated. Because of cyclostationarity induced, the only indentifiabilty condition imposed is channel impulse response is full column rank. Eigen values and Eigen vectors of channel product matrices are calculated and used to estimate channel. The proposed method is able to work in both conditions of more inputs and more outputs. A technique is stated to choose a optimal periodic sequence in such a way so that it reduces noise at the receiver. BER for MQAM signal is also calculated using SNR of the system .We will observe that SNR depends on precoding sequence and estimated channel and so thus BER and variation of BER with channel estimation error is observed.

NOTATIONS

The notations used here are given as: bold upper-case is used for matrices and bold lower-case is used for vectors. C^{T} represents transpose of the matrix C and C^{H} indicates transpose and conjugate of matrix C. $\mathrm{C} \otimes \mathrm{D}$ is the kronecker product of C and D. C^{+}denotes the pseudo inverse of C. $\|\mathrm{C}\|_{\mathrm{F}}$ and $\|\mathrm{C}\|_{2}$ denotes forbenius norm and L_2 norm of C respectively. $[\mathrm{C}]_{\mathrm{i,j}}$ denotes element of C at i^{th} row and j^{th} column and $[\mathrm{b}_{\mathrm{i,j}}]$ represents matrix composed of element $\mathrm{b}_{\mathrm{i,j}}$ at the i^{th} row and j^{th} column. diag (d1, d2......dn) is the diagonal matrix with elements of d1,d2.......dn. E[y] denotes expectation of y. $\kappa(\mathrm{C})$ denotes condition number of C. I_{N} is the identity matrix of the order $\mathrm{N} \times \mathrm{N}$.

$0_{\mathrm{M} \times \mathrm{N}}$ is the zero matrix of dimension $\mathrm{M} \times \mathrm{N}$.The symbols $\mathbb{R}$ and $\mathbb{C}$ represents real and complex numbers, respectively.

3.1 SYSTEM MODEL

The model of MIMO-OFDM system has been described with M_{TX} transmit antennas and M_{RX} receive antennas is shown in fig 3.1.

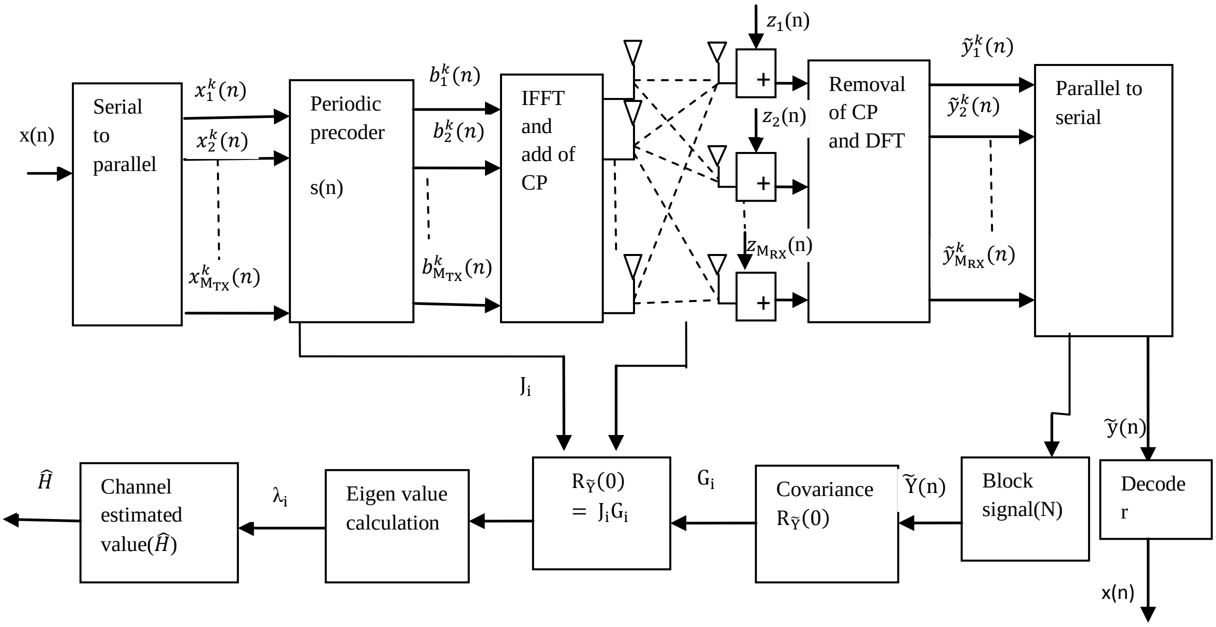


Fig 3.1: Block diagram of Channel Estimation using Periodic Precoder

The output of periodic precoder is given as:

$$b_i^k(n) = s(n)x_i^k(n) \tag{3.1}$$

where,

i=1,2............ M_{TX} and $x_i^k(n)$ is the input kth OFDM symbol given to transmit antenna i which is equivalent to $x_i^k(n)$ $=[x_i^k(n,0), x_i^k(n,1) \ldots \ldots \ldots \ldots \ldots .. x_i^k(n, M-1)]^T \in \mathbb{C}^M$, M is the number of OFDM subcarriers and s(n) is a periodic precoder with period N i.e. s(n)=s(N+n),∀ n.

Now the resulting signal is transformed by IDFT and then cyclic prefix is added having length L_{cp}which is stated as L_{cp}=M/4. Hence symbol length is given by M'=M+L_{cp}.The CP reduces intersymbol interference (ISI) if CP has length equal to or longer than the channel length. This is called OFDM modulation. Now this signal is transmitted through a finite impulse response (FIR) channel.

The impulse response between j^{th} receive antenna and i^{th} transmit antenna [53] is defined as:

$$h_{j,i} = [h_{j,i}(0), h_{j,i}(1) \dots\dots\dots\dots\dots . h_{j,i}(L-1)]^T \ ,1 \le i \le M_{TX}, 1 \le j \le M_{RX} \qquad (3.2)$$

where,

L is the order of the channel.

When the signal passes through FIR channel, it gets affected by noise. However, at the receiver side cyclic prefix is removed before further precoding. The received signal is given by:-

$$y_j^k(n) = \sum_{i=0}^{M_{TX}} \sum_{l=0}^{L-1} h_{j,i}(l) W^H \, b_i^k(n-l) + z_j\,(n) \qquad (3.3)$$

where,

W^H is the normalized IDFT, $z_j(n) \sim \mathcal{CN}(0, \sigma_z^2 I)$ is a noise vector.

This equation can be written in more concise form as b(n)=s(n)x(n),then

$$y(n)=\sum_{l=0}^{L-1} H(l) W^H \, b(n-l) + z(n) \qquad (3.4)$$

where,

b(n),x(n)$\epsilon \mathbb{C}^{M_{TX}M}$ and y(n),z(n) $\epsilon\ \mathbb{C}^{M_{RX}M}$ are vector signals formed by combining the respective scalar signals together.

E.g. $x(n) = [\, x_1^k(n) x_2^k(n) \dots\dots\dots\dots .. x_{M_{TX}}^k(n)]$ and H(l) $\epsilon\ \mathbb{C}^{M_{RX}M \times M_{TX}M}$ are matrix with ji^{th} element given by $h_{j,i}(l)$ .Now since the periodic sequence s(n) is periodically varying with time, the received signal y(n) and input signal x(n) are time varying. In order to make it time invariant [65], block input and block output is taken. Block signal can be described as:-

X(n) =$[x(Nn)^T, x(Nn+1)^T \dots\dots\dots\dots\dots\dots .. x(Nn+N-1)^T]^T \ \epsilon\ \mathbb{C}^{M_{TX}MN}$ and similarly Y(n), Z(n) and B(n) are defined.

$$B\,(n) = SX\,(n)\,, \forall n \qquad (3.5)$$

where,

$S=\mathrm{diag}(s(0)(I_M \otimes I_{M_{TX}}),s(1)(I_M \otimes I_{M_{TX}}),.................s(N-1)(I_M \otimes I_{M_{TX}})]\in \mathbb{R}^{M_{TX}MN\times M_{TX}MN}$ is a diagonal matrix. Now writing (3.4) in terms of block signal :-

$$Y(n) = H_0W^HB(n) + H_1W^HB(n-1) + z(n) \qquad (3.6)$$

where,

H_0 is an $M_{RX}NM \times M_{TX}NM$ block lower triangular toeplitz matrix with $[H(0)^T\, H(1)^T \ldots\ldots\ldots\ldots . H(L-1)^T\, 0^T_{M_{RX}\times M_{TX}} \ldots\ldots\ldots . . 0^T_{M_{RX}\times M_{TX}}]^T \in \mathbb{C}^{M_{RX}NM\times M_{TX}M}$ as its first block column (i.e., the first M_{TX} columns) and H_1 is an $M_{RX}NM \times M_{TX}NM$ block upper triangular toeplitz matrix with $[0_{M_{RX}\times M_{TX}} \ldots\ldots\ldots\ldots . 0_{M_{RX}\times M_{TX}}\, H(L-1) H(L-2) \ldots\ldots\ldots\ldots H(1)] \in \mathbb{C}^{M_{RX}M\times M_{TX}NM}$ as its first block row(i.e. the first M_{RX} rows).

Finally, the received signal is obtained by taking DFT of Y (n).

$$\widetilde{Y(n)}=\widetilde{H_0}B(n) + \widetilde{H_1}B(n-1) + \widetilde{Z(n)} \qquad (3.7)$$

where,

$\widetilde{H_0} = WH_0W^H, \widetilde{H_1} = WH_1W^H$ and $\widetilde{Z(n)} = WZ(n)$ when W is the normalized $MNM_{RX} \times MNM_{RX}$ DFT matrix.

The following properties of matrices are used in the derivation of channel estimation [66].

- Firstly, for any $m \times m$ matrix $A=[a_{i,j}]_{0\le i,j\le m-1}$, define $\mathcal{L}_l(A) = [a_{0,l}\, a_{1,l+1} \ldots\ldots\ldots\ldots . . a_{m-1-l,m-1}]^T$ for $0\le l \le m-1$ ie $\mathcal{L}_l(A)$ is the vector formed from the l^{th} superdiagnol of A.
- Secondly, for any $M_{RX}n \times M_{RX}n$ matrix $B=[B_{i,j}]_{0\le i,j\le n-1}$ where $B_{i,j}$ is a block matrix of dimension $M_{RX} \times M_{RX}$ define $\Lambda_l(B)=[B^T_{0,l}\, B^T_{1,l+1} \ldots\ldots\ldots\ldots\ldots\ldots B^T_{n-1-l,n-1}]^T$ for $0\le l \le n-1$ ie $\Lambda_l(B)$ is the matrix formed from the l^{th} block super diagonal of B.

After this following assumption has been considered throughout the thesis:-

Assumption (A1). x (n) and z(n) are zero-mean white vector sequences ,and x(n) and z(n) are uncorrelated. Mathematically, $E[x(k)x(l)^H] = \delta(k-l)I_{M_{TX}} \in \mathbb{R}^{M_{TX}\times M_{TX}}$, $E[z(k)z(l)^H] = \delta(k-l)\sigma_z^2 I_{M_{RX}} \in \mathbb{R}^{M_{RX}\times M_{RX}}$, $E[x(k)z(l)'] = 0_{M_{TX}\times M_{RX}}\ \forall\, k,l$ where, $\delta(.)$ is the kronecker delta function.

Rank $([H(0)^T\ H(1)^T \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots H(L)^T]^T) = M_{TX}$.

An upper bound $L^{\wedge}$ of the channel order L is known and the period $N > L^{\wedge} + 1$

3.2. BLIND CHANNEL ESTIMATION

In this section, estimation of channel using periodic sequences with or without noise has been studied. In 3.2.1, channel is estimated without considering noise. In section 3.2.2, the affect of variation in precoding sequence of noise is analysed. And lastly, an algorithm has been defined based on study of estimation of channel using periodic precoder is included in section 3.2.3.

3.2.1 Estimation Method without Noise

In the absence of noise (3.7) will become

$$\widetilde{Y(n)} = \widetilde{H_0} SX(n) + \widetilde{H_1} SX(n-1) \tag{3.8}$$

Due to absence of noise, channel order L can be easily determined .Now for channel estimation co-variance of output is taken and given based on assumption A1.

$$R_{\tilde{Y}}(0) = E[\widetilde{Y(n)}\widetilde{Y(n)}^H\ ] = \widetilde{H_0} S^2 \widetilde{H_0}^H + \widetilde{H_1} S^2 \widetilde{H_1}^H \tag{3.9}$$

Let $Q \in \mathbb{R}^{N\times N}$ is a matrix whose first super diagonal are all one i.e. $\mathcal{L}_1(A) = [11 \ldots\ldots\ldots\ldots 1]^T \in \mathbb{R}^{(N-1)}$ and all remaining entries are zero. As $\widetilde{H_0}$ and $\widetilde{H_1}$ have block toeplitz structures, $\widetilde{H_0}$ and $\widetilde{H_1}$ can be written as $\widetilde{H_0} = \sum_{l=0}^{L-1} Q^l \otimes \widetilde{H(l)}$ and $\widetilde{H_1} = \sum_{l=0}^{L-1} (Q^T)^{N-l} \otimes \widetilde{H(l)}$, respectively.

Although, S_s can be defined as $S_s = \mathrm{diag}[s(0), s(1) \ldots\ldots\ldots\ldots s(N-1)] \in \mathbb{R}^{N\times N}$. Therefore $\widetilde{H_0} S^2 \widetilde{H_0}^H$ can be written as:

$$\widetilde{H_0} S^2 \widetilde{H_0}^H = \sum_{l=0}^{L-1} Q^l \otimes \widetilde{H(l)} (S_s{}^2 \otimes I_{M_{TX}M}) \sum_{k=0}^{L-1} (Q^k \otimes \widetilde{H(k)}\ )^H$$

$$= \sum_{l=0}^{L-1} \sum_{k=0}^{L-1} Q^l \otimes \widetilde{H(l)} (S_s{}^2 \otimes I_{M_{TX}M}) \left((Q^T)^k \otimes \widetilde{H(k)}^{\ H}\right)$$

$$= \sum_{l=0}^{L-1}\sum_{k=0}^{L-1} (Q^{l} {S_s}^2 (Q^T)^k) \otimes \left(\widetilde{H(l)}\ \widetilde{H(k)}^{\ H} \right) \tag{3.10}$$

where,

The identities used are as $(C \otimes D)^H = C^H \otimes D^H$ and $(C \otimes D)(E \otimes F) = (CE) \otimes (DF)$. Similarly $\widetilde{H_1} S^2 \widetilde{H_1}^H$ can be written as:-

$$\widetilde{H_1} S^2 \widetilde{H_1}^H = \sum_{l=0}^{L-1}\sum_{k=0}^{L-1} ((Q^T)^{N-l} {S_s}^2 Q^{N-k}) \otimes \left(\widetilde{H(l)}\ \widetilde{H(k)}^{\ H} \right) \tag{3.11}$$

Let $0 \leq l, k \leq L-1$ be two non-negative integers. Then the following is true [64]:-

- For k=l+i, where $0 \leq i \leq L-1-l$, both $Q^l {S_s}^2 (Q^T)^k$ and $(Q^T)^{N-l} {S_s}^2 Q^{N-k}$ are upper triangular matrices with only the respective ith upper diagonals nonzero
- For $k < l$ both $\mathcal{L}_i(Q^l {S_s}^2 (Q^T)^k)$ and $\mathcal{L}_i((Q^T)^{N-l} {S_s}^2 Q^{N-k})$ are lower triangular with zero diagonal matrices.

$$\mathcal{L}_i(Q^l {S_s}^2 (Q^T)^k) = [0 \ldots\ldots\ldots\ldots\ldots 0\ s(0)^2\ s(1)^2 \ldots\ldots\ldots\ldots . s(N-1-l-i)^2]^T \tag{3.12}$$

$$\mathcal{L}_i\left((Q^T)^{N-l} {S_s}^2 Q^{N-k}\right) = [s(N-l)^2 . \ldots\ldots\ldots\ldots . s(N-1)^2 0 \ldots\ldots\ldots\ldots\ldots 0]^T \tag{3.13}$$

$$\mathcal{L}_i(Q^l {S_s}^2 (Q^T)^k) + \mathcal{L}_i((Q^T)^{N-l} {S_s}^2 Q^{N-k}) = \begin{cases} [s(N-l)^2 . \ldots\ldots\ldots\ldots . s(N-1)^2 s(0)^2\ s(1)^2 \ldots\ldots\ldots\ldots . s(N-1-l-i)^2]^T & \text{if } i = k-l \geq 0 \\ 0_{(N-i)\times 1} & \text{if } i \neq k-l \end{cases} \tag{3.14}$$

The above equations are true, since

$$\Lambda_i((Q^l {S_s}^2 (Q^T)^l) \otimes (\widetilde{H(l)}\ \widetilde{H(k)}^{\ H}) = \mathcal{L}_i(Q^l {S_s}^2 (Q^T)^l)\ \otimes \left(\widetilde{H(l)}\ \widetilde{H(k)}^{\ H}\right) \tag{3.15}$$

$$\Lambda_i((Q^T)^{N-l} {S_s}^2 Q^{N-k}) \otimes (\widetilde{H(l)}\ \widetilde{H(k)}^{\ H}) = \mathcal{L}_i((Q^T)^{N-l} {S_s}^2 Q^{N-k}) \otimes (\widetilde{H(l)}\ \widetilde{H(k)}^{\ H}) \tag{3.16}$$

Hence $\Lambda_i\left(R_{\tilde{Y}}(0)\right)$ can be derived from (3.9)-(3.11) and (3.14)-(3.16) as shown in (3.17)

$$\Lambda_i\left(R_{\tilde{Y}}(0)\right) = \Lambda_i\left(\widetilde{H_0} S^2 \widetilde{H_0}^H + \widetilde{H_1} S^2 \widetilde{H_1}^H\right) = \sum_{l=0}^{L-1}\sum_{k=0}^{L-1} \Lambda_i((Q^l {S_s}^2 (Q^T)^l) \otimes (\widetilde{H(l)}\ \widetilde{H(k)}^{\ H}) + \Lambda_i((Q^T)^{N-l} {S_s}^2 Q^{N-k}) \otimes (\widetilde{H(l)}\ \widetilde{H(k)}^{\ H}))$$

$$= \sum_{l=0}^{L-1-i} [\mathrm{s}(\mathrm{N}-l)^2 . \dots\dots\dots\dots . \mathrm{s}(\mathrm{N}-1)^2 \;\; \mathrm{s}(0)^2\, \mathrm{s}(1)^2 \dots\dots\dots\dots . \mathrm{s}(\mathrm{N}-1-l-\mathrm{i})^2]^{\mathrm{T}}$$

$$\otimes (\widetilde{\mathrm{H}(l)}\; \widetilde{\mathrm{H}(l+\imath)}^{\;H})$$

$$=\sum_{l=0}^{L-1-i}[\mathrm{s}(\mathrm{N}-l)^2\mathrm{I}_{\mathrm{RX}} . \dots\dots\dots\dots . \mathrm{s}(\mathrm{N}-1)^2\mathrm{I}_{\mathrm{RX}} \;\; \mathrm{s}(0)^2\mathrm{I}_{\mathrm{RX}}\, \mathrm{s}(1)^2\mathrm{I}_{\mathrm{RX}} \dots\dots\dots\dots . \mathrm{s}(\mathrm{N}-1-l-\mathrm{i})^2\mathrm{I}_{\mathrm{RX}}]^{\mathrm{T}}\, (\widetilde{\mathrm{H}(l)}\; \widetilde{\mathrm{H}(l+\iota)}^{\;H}) \qquad (3.17)$$

The right hand side of (3.17) is formed by combining channel product matrices $\widetilde{\mathrm{H}(l)}\; \widetilde{\mathrm{H}(l+\iota)}^{\;H}$ as coefficients with block columns. The (3.17) can be written as:

$$\Lambda_{\mathrm{i}}\big(\mathrm{R}_{\tilde{\mathrm{Y}}}(0)\big) = \mathrm{J}_{\mathrm{i}}\mathrm{G}_{\mathrm{i}} \quad ,\forall 0 \le \mathrm{i} \le \mathrm{L}-1 \qquad (3.18)$$

where ,

$\mathrm{J}_i \in \mathbb{R}^{\mathrm{M_{RX}}(\mathrm{N}-\mathrm{i})\times \mathrm{M_{RX}}(\mathrm{L}-i)}$ as defined below in (3.19). Here it is noted that J_i is obtained from J_0 by deleting its last i $\mathrm{M_{RX}}$ rows and last i $\mathrm{M_{RX}}$ columns.

$$\mathrm{J}_i = \begin{bmatrix} \mathrm{s}(0)^2 & \mathrm{s}(\mathrm{N}-1)^2 & \cdots & \mathrm{s}(\mathrm{N}-\mathrm{L}-1+\mathrm{i})^2 \\ \mathrm{s}(1)^2 & \mathrm{s}(0)^2 & \cdots & \mathrm{s}(\mathrm{N}-\mathrm{L}+\mathrm{i})^2 \\ \vdots & \vdots & \vdots & \vdots \\ \mathrm{s}(\mathrm{N}-2-\mathrm{i})^2 & \mathrm{s}(\mathrm{N}-3-\mathrm{i})^2 & & \mathrm{s}(\mathrm{N}-\mathrm{L}-3)^2 \\ \mathrm{s}(\mathrm{N}-1-\mathrm{i})^2 & \mathrm{s}(\mathrm{N}-2-\mathrm{i})^2 & & \mathrm{s}(\mathrm{N}-\mathrm{L}-2)^2 \end{bmatrix} \otimes \mathrm{I}_{\mathrm{M_{RX}}} \qquad (3.19)$$

and G_{i} can be written as in (3.20) ,where $0 \le \mathrm{i} \le \mathrm{L}-1$

$$\mathrm{G}_{\mathrm{i}} = [\big(\widetilde{\mathrm{H}(0)}\; \widetilde{\mathrm{H}(\imath)}^{\;H}\big)^{\mathrm{T}} \;\big(\widetilde{\mathrm{H}(1)}\; \widetilde{\mathrm{H}(\imath+1)}^{\;H}\big)^{\mathrm{T}} \dots (\widetilde{\mathrm{H}(\mathrm{L}-\imath)}\; \widetilde{\mathrm{H}(\mathrm{L}-1)}^{\;H})^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{C}^{\mathrm{M_{RX}}(\mathrm{L}-\mathrm{i})\times \mathrm{M_{RX}}} \qquad (3.20)$$

Since $N > L$, the L equations in (3.18) are over determined and for the noise free case, these equations are consistent. If J_i is full column rank, then the solution can be obtained as

$$\mathrm{G}_{\mathrm{i}} = (\mathrm{J}_i{}^{\mathrm{T}}\mathrm{J}_i)^{-1}\mathrm{J}_i{}^{\mathrm{T}}\Lambda_{\mathrm{i}}\big(\mathrm{R}_{\tilde{\mathrm{Y}}}(0)\big) \qquad (3.21)$$

Now channel product matrices $\widetilde{\mathrm{H}(l)}\; \widetilde{\mathrm{H}(\mathrm{k})}^{\;H}$ can be obtained from G_{i} ,$0 \le \mathrm{i} \le \mathrm{L}-1$ as computed from (3.21). Channel impulse response matrices $\mathrm{H}(0), \mathrm{H}(1), \dots\dots\dots . \mathrm{H}(L-1)$ are calculated using G_{i}.

Let R be the Hermitian matrix defined by $\Lambda_i(R) = G_i$ for i=0, 1...........L-1, from this R can be simplified as

$$R=\widetilde{H}\widetilde{H}^{H} \tag{3.22}$$

where,

$\widetilde{H}$=[$\widetilde{H(0)}^{T}\ \widetilde{H(1)}^{T} \ldots\ldots\ldots\ldots.H\widetilde{(L-1)}^{T}]^{T}$, Since $\widetilde{H} = WHW^{H}$, (3.22) can be written as

R= $(WHW^{H})(WH^{H}W^{H})$

W^{+}R= $(W^{+}WHW^{H}WH^{H}W^{H})$

W^{+}R= $HW^{H}WH^{H}W^{H}$

W^{+}RW= $HW^{H}WH^{H}W^{H}W$, since $W^{H}W = I_N$

W^{+}RW=HH^{H}, Now replace W^{+}RW with P

$$P=HH^{H} \tag{3.23}$$

$$P = \begin{bmatrix} H(0)^{T}H(0)^{H} & H(0)^{T}H(1)^{H} & \cdots & H(0)^{T}H(L-1)^{H} \\ H(1)^{T}H(0)^{H} & H(1)^{T}H(1)^{H} & \cdots & H(1)^{T}H(L-1)^{H} \\ \vdots & \vdots & \ddots & \vdots \\ H(L-1)^{T}H(0)^{H} & H(L-1)^{T}H(1)^{H} & \cdots & H(L-1)^{T}H(L-1)^{H} \end{bmatrix}$$

Since rank(H)= M_{TX} from assumption (A1). Therefore rank of P is also M_{TX} .So P has positive Eigen values, say $\lambda_1, \ldots\ldots..\lambda_{M_{TX}}$, since P is Hermitian and positive definite. P can be written in form of λ_i as

$$P = \sum_{i=1}^{M_{TX}}(\sqrt{\lambda_i}\, t_i)(\sqrt{\lambda_i}t_i)^{H} \tag{3.24}$$

where t_i is a unit norm Eigen vector of P associated with $\lambda_i > 0$

$$H^{\wedge} = [\sqrt{\lambda_1}t_1\ \sqrt{\lambda_2}t_2 \ldots\ldots\ldots\ldots.\sqrt{\lambda_{M_{TX}}}t_{M_{TX}}] \ \ \epsilon\ \mathbb{C}^{M_{RX}(L)\times M_{TX}} \tag{3.25}$$

It is assumed that H can only be identified up to a unitary matrix ambiguity $U\ \epsilon\ \mathbb{C}^{M_{TX}\times M_{TX}}$

i.e.$H^{\wedge} = HU$, since

$\mathrm{H}^{\wedge}\mathrm{H}^{\wedge \mathrm{H}} = \mathrm{HH}^{H} = \mathrm{P}$. Therefore it can be said that ambiguity matrix U is important to methods for blind identification of multiple input systems using only second-order statistics. It is observed that J_{i}, i=0,1........L-1 is completely calculated by precoding sequence.

3.2.2 More Inputs, More Outputs

Thus by optimally selecting the precoding sequence, J_{i} can be made full column rank. From assumption (A1), the channel matrix H can be assumed to be full column rank. Hence, channel cannot be reduced column wise. If $\mathrm{M}_{\mathrm{RX}} > \mathrm{M}_{\mathrm{TX}}$ (more outputs), then (A1) is generally satisfied but if $\mathrm{M}_{\mathrm{TX}} > \mathrm{M}_{\mathrm{RX}}$(more inputs), then (A1) is satisfied if $\mathrm{LM}_{\mathrm{RX}} \geq \mathrm{M}_{\mathrm{TX}}$. It is observed that if there are more inputs as compared to output, even if accurate channel estimate is calculated, channel equalisation and source separation is difficult.

3.2.3 Channel Order Overestimation

Till now, it is assumed that L-1 the channel order is known, only if $L^{\wedge} \geq L-1$ as well as $N > L^{\wedge}+1$, then calculations will be same as given in section 3.2.1, thus matrix R $(\mathrm{M}_{\mathrm{RX}}(L^{\wedge}+1) \times \mathrm{M}_{\mathrm{RX}}(L^{\wedge}+1))$ can be calculated in the same way as in (3.22). The last $L^{\wedge}-(L-1)$ block rows (i.e., $(L^{\wedge}-(L-1))\mathrm{M}_{\mathrm{RX}}$ rows) of R are zero and same is for $L^{\wedge}-(L-1)$ block columns. Therefore R, is of rank M_{TX}, so it has M_{TX} Eigen values and Eigen vectors of the form $t^{\wedge} = [t^{T}\ 0 \cdots 0]^{T} \epsilon \mathbb{C}^{\mathrm{M}_{\mathrm{RX}}(L^{\wedge}+1)}$, where, $t \epsilon \mathbb{C}^{\mathrm{M}_{\mathrm{RX}}(L)}$.Hence with these M_{TX} Eigen vectors and Eigen values of R, channel impulse response with unitary ambiguity can be estimated.

3.3. OPTIMAL SELECTION OF PRECODING SEQUENCE

In this section, effect of noise in calculation of channel matrix is considered. In previous section from (3.21) it is observed that G_{i} is dependent on J_{i}. Therefore by changing precoding sequences J_{i} can be made full column rank and accurate estimate of G_{i} can be made. Thus we could say that noise causes channel estimation error. This error can be mitigated by optimally selecting the precoding sequence and accurate channel can be estimated using $R_{\tilde{Y}}(0)$. Then an algorithm for channel estimation has been defined using periodic precoder in MIMO-OFDM system.

3.3.1. Condition for Optimality

Covariance of (3.7) is taken and considering (A1), variance of noise gets added in (3.8).Now same procedure is followed as in section 3.2, except effect of noise is considered on channel estimation error. Therefore, precoding sequence is selected in such a way that channel estimation error can be minimised

$$R_{\tilde{Y}}(0) = E[\widetilde{Y(n)}\widetilde{Y(n)}^{H}] \quad = \widetilde{H_0}S^2\widetilde{H_0}^{H} + \widetilde{H_1}S^2\widetilde{H_1}^{H} + \sigma_z^2 I_{M_{RX}} \otimes I_N \tag{3.26}$$

As it can be seen that noise variance occurs at only diagonal elements of $R_{\tilde{Y}}(0)$, thus change occurs only at diagonal elements and other elements of the matrix remains same as in (3.8) of section 3.2.Thus using i=0 ,so that only those elements are considered where noise is added.

$$\Lambda_0\left(R_{\tilde{Y}}(0)\right) = \Lambda_0\left(\widetilde{H_0}S^2\widetilde{H_0}^{H} + \widetilde{H_1}S^2\widetilde{H_1}^{H}\right) + \sigma_z^2\Lambda_0(I_{M_{RX}} \otimes I_N) \tag{3.27}$$

From (3.18) and (3.8), (3.27) can be written as

$$\Lambda_0\left(R_{\tilde{Y}}(0)\right) = J_0G_0 + A \tag{3.28}$$

where,

$A=\sigma_z^2[I_{M_{RX}}\ I_{M_{RX}} \ldots\ldots\ldots\ldots\ldots I_{M_{RX}}]^T$, $G_0{}^{\wedge}$ is the least square approximation of G_0, defined as

$$G_0{}^{\wedge} = (J_0{}^{T}J_0)^{-1}J_0{}^{T}(J_0G_0 + A) \tag{3.29}$$

$$= G_0 + (J_0{}^{T}J_0)^{-1}J_0{}^{T}A = G_0 + V \tag{3.30}$$

where,

G_0 is added to a perturbation term due to noise. Since $J_0V = A$, V is defined as least square solution of this equation. Now if J_0 is orthogonal to every column of A then V=0 which is not possible as A has nonnegative terms and J_0 is positive. Therefore precoding sequence is chosen in such a way that J_0 is almost orthogonal to A and V is almost 0. Hence we define s_{ki} and a_i as the columns of J_0 and A respectively in (3.31) and (3.32).

$$J_0 = [p_{01}\ p_{02} \cdots p_{0M_{RX}} \cdots p_{L1}\ p_{L2} \cdots p_{LM_{RX}}] \tag{3.31}$$

$$A=\sigma_z^2[I_{M_{RX}}\ I_{M_{RX}} \cdots I_{M_{RX}}]^T = [a_1\ a_2 \cdots a_{M_{RX}}] \tag{3.32}$$

Now $\Lambda_0(P^\wedge) = G_0{}^\wedge$,from eq (30) and (40), where, P' is product of channel matrices with changed diagonal elements. Since diagonal elements of matrix P are now changed, therefore new Eigen values $(\lambda_1{}^\wedge, \ldots \ldots .. \lambda_{M_{TX}}{}^\wedge)$ will be generated now as compared to (3.23) and correspondingly new Eigen vectors are generated $(t_1{}^\wedge \ldots \ldots \ldots . t_{M_{TX}}{}^\wedge)$. $P^\wedge =$

$$\begin{bmatrix} 0 & H(0)^T H(1)^H & \cdots & H(0)^T H(L-1)^H \\ H(1)^T H(0)^H & 0 & \cdots & H(1)^T H(L)^H \\ \vdots & \vdots & \ddots & \vdots \\ H(L-1)^T H(0)^H & H(L-1)^T H(1)^H & \cdots & 0 \end{bmatrix} + G_0{}^\wedge \tag{3.33}$$

Hence $P^\wedge$ can be written as $P^\wedge = \sum_{i=1}^{M_{TX}} (\sqrt{\lambda_i{}^\wedge}\, t_i{}^\wedge)(\sqrt{\lambda_i{}^\wedge}\, t_i{}^\wedge)^\wedge$. Hence estimated channel with noise can be further defined as

$$H^\wedge = [\sqrt{\lambda_1{}^\wedge}\, t_1{}^\wedge\ \sqrt{\lambda_2{}^\wedge}\, t_2{}^\wedge \ldots \ldots \ldots \ldots . \sqrt{\lambda_{M_{TX}}{}^\wedge}\, t_{M_{TX}}{}^\wedge] \quad \epsilon\ \mathbb{C}^{M_{RX}(L) \times M_{TX}} \tag{3.34}$$

As observed from the block matrix of J_0 and A, p_{ki} is orthogonal to a_j i.e. $p_{ki}{}^T a_j = 0$ if $j \neq$ i.eg $p_{01}{}^T a_2 = 0$ as shown below:

$$p_{01}{}^T a_2 = [s(0)^2\ 0 \cdots\ 0 \cdots s(N-1)^2\ 0 \cdots\ 0][0\ \sigma_z^2\ 0 \cdots\ 0 \cdots\ 0\ \sigma_z^2\ 0\ \cdots 0]^T = 0 \tag{3.35}$$

and every $p_{ki}{}^T a_i$ gives the same value $\sigma_z^2 \sum_{n=0}^{N-1} s(n)^2$ for k=0,1....L-1 and i=1,2.......... M_{RX} ,

$$p_{01}{}^T a_1 = [s(0)^2\ 0 \cdots\ 0 \cdots s(N-1)^2\ 0 \cdots\ 0][0\ \sigma_z^2\ 0 \cdots\ 0 \cdots\ \sigma_z^2\ 0\ \cdots 0]^T = \sigma_z^2 \sum_{n=0}^{N-1} s(n)^2 \tag{3.36}$$

Hence we will seek only relation between columns of p_{01} and a_1(k=0,i=1),for other values of k and i result will remain the same. Therefore correlation coefficient is given by

$$\theta = \frac{p_{01}{}^{T} a_1}{\|p_{01}\|_2 \|y_1\|_2} \tag{3.37}$$

By Cauchy-Schwarz inequality value of θ lies between 0 and $1(0 \leq \theta \leq 1)$.We have to make θ as small as possible by appropriately choosing p_{01}, hence the perturbation term V could be decreased. So we choose precoding sequence in such a way that:-

$$\frac{1}{N}\sum_{n=0}^{N-1} |s(n)|^2 = 1 \tag{3.38}$$

$$|s(n)|^2 \geq \tau > 0 \qquad \forall 0 \leq n \leq N-1 \tag{3.39}$$

Thus optimal periodic sequence should be selected such that normalised power gain of periodic sequence is equal to1 as given in (3.38). From (3.39) it is shown that the power gain of sequence at any time is greater than τ.Therefore only one peak value occurs in the sequence in a single period.

$$s(n) = \begin{cases} \sqrt{N(1-\tau)+\tau} \ldots\ldots\ldots\ldots\ldots n = m \\ \sqrt{\tau} \ , \quad \ldots\ldots\ldots\ldots\ldots.. n \neq m \end{cases} \quad 0 \leq n \leq N-1 \tag{3.40}$$

where, $0 \leq m \leq N-1$, Here θ can be calculated using τ as $\theta = \frac{1}{\sqrt{N(1-\tau)^2+\tau(2-\tau)}}$. When τ decreases θ also decreases and thus noise also decreases and we can say that estimated channel is near to actual channel.

3.3.2 Optimal Selection of m

From above equation it can be seen that m plays crucial role in selection of periodic sequences. In this sub -section the dependence of m in detail is studied which will further help in our calculations. J_i will take different values according to values of m and thus which would effect the calculation of G_i as given in (3.8) and $G_0{}^{\wedge}$ in (3.18).

$$\varphi = \max_{0 \leq i \leq L} \kappa(J_i{}^{T} J_i) \tag{3.41}$$

where,

$\kappa(C)$ is the condition no.of C

As value of $J_i^T J_i$ will change condition number will also change. If the value of condition number is large, wrong values of $J_i^T J_i$ are generated and error occurs in calculation of G_i. Our main purpose is to select m such that the largest condition number of the matrix $J_i^T J_i$, i=0, 1...........L is minimised. As maximum value occur at any one of the N points of the periodic sequence. For different precoding sequences different φ is calculated. The following proposition states that value of m should be selected carefully because some values result in J_i as rank deficient and $\varphi = \infty$.

3.3.3 Algorithm to Estimate a Channel

At last an algorithm on the basis of all the study of channel estimation using periodic precoder is written. All explanation described so far has been defined in concise form in few steps. This is a quick review of how channel estimation can be done with the conditions applied.

- First calculate the covariance of received signal by taking time average which gives estimated value of $R_{\tilde{Y}}(0)$,

$$R_{\tilde{Y}}(0)^{\wedge} = \frac{1}{k}\sum_{i=1}^{k} \widetilde{Y(\imath)}\widetilde{Y(\imath)}' \tag{3.42}$$

  where,
  K is number of data blocks (ie.KM is the number of samples for each transmitter).
- Select the precoding sequence s(n) such that J_i is full column rank. Now obtain J_0 such that J_0 and A are almost orthogonal to each other so that noise effect is minimised.
- Now calculate G_i using J_i from (3.8),$1 \leq i \leq L$. Similarly obtain $G_0{}^{\wedge}$ from J_0 using (3.18) keeping in mind the condition $\frac{1}{N}\sum_{n=0}^{N-1}|s(n)|^2 = 1$.
- 4. FromG_i, $1 \leq i \leq L$ and $G_0{}^{\wedge}$channel product matrices are obtained as given in (3.34).
- 5. Form the matrix $P^{\wedge}$ as in (3.34) and solve Eigen values and Eigen vectors for$P^{\wedge}$, compute channel impulse response using these Eigen values. By assumption (A1) M_{TX} Eigen values and vectors are possible.

3.4 BER ANALYSIS

In this section signal to noise ratio (SNR) and bit error rate (BER) have been calculated using input MQAM signal. First of all SNR is calculated which is used in the BER expression. SNR is calculated to see the effect of precoder on the MIMO-OFDM system. In this section, it is analysed that addition of precoder will increase SNR and decrease BER. In this section, expression of SNR is mathematically calculated which is dependent on precoder and similarly for BER.

3.4.1. Calculation of SNR

The SNR for MIMO-OFDM system with periodic precoder is being calculated in this sub-section. The MQAM input signal is first decoded from the received signal as given in below equation. The error generated is treated as noise. First we consider a case without precoder is considered.

$$\widetilde{y(n)}=\widetilde{H}x(n)+\widetilde{z(n)} \tag{3.43}$$

This is general output of MIMO-OFDM system. Now the signal is decoded using zero forcing decoding to get input signal.

$$x(n)^{\wedge}=x(n)+\widetilde{H}^{+}\widetilde{z(n)} \tag{3.44}$$

$$e(n)=\widetilde{H}^{+}\widetilde{z(n)} \tag{3.45}$$

$$x(n)^{\wedge}=x(n)+e(n) \tag{3.46}$$

After this the case when periodic precoder has been added to the MIMO-OFDM system, the output of MIMO-OFDM system will be:

$$\widetilde{y(n)}=\widetilde{H}Sx(n)+\widetilde{z(n)} \tag{3.47}$$

Now again signal will be decoded and input signal will be taken from the equation below:

$$\overline{x(n)^{\wedge}}=x(n)+S^{-1}e(n) \tag{3.48}$$

Now SNR is calculated by taking ratio of power of input signal to power of error being generated while calculating input signal. Now after simplifying this expression.

$$\mathrm{SNR}=\gamma_s=\frac{\sum_{i=0}^{N-1}E[x(Nn+i)^2]}{\sum_{i=0}^{N-1}\overline{s(\imath)}^2E[e(Nn+i)^2]},\ 0\le i\le N-1 \tag{3.49}$$

where,

$\overline{s(\imath)}$ is the i^{th} element of S^{-1}.

Now using (3.45), (3.49) can be written as:

$$\gamma_s=\frac{\sum_{i=0}^{N-1}E[x(Nn+i)^2]}{\sum_{i=0}^{N-1}\overline{s(\imath)}^2(\widetilde{H}^{+})^2E[\tilde{z}(Nn+i)^2]} \tag{3.50}$$

Since from assumption A1 $E[\tilde{z}(n)^2]=\sigma_z^2 I_{M_{RX}N}$, therefore above (3.50) can be reduced to

$$\gamma_s = \frac{\sum_{i=0}^{N-1} E[x(Nn+i)^2]}{\sigma_z^2 I_{M_{RX}\,N} (\widetilde{H}^{+})^2 \sum_{i=0}^{N-1} \overline{s(\imath)^2}} \tag{3.51}$$

3.4.2. BER Performance of MQAM-Precoded OFDM

For OFDM system BER expression can be written as:

$$BER = \frac{1}{MN} \sum_{i=0}^{M-1} \sum_{k=0}^{N-1} BER[k,i] \tag{3.52}$$

where instantaneous BER[k,i] is the bit error rate of k-th sub carrier of the i-th block of OFDM system.

In this sub section we have used the expression for SNR from the above (3.51) and used in the calculation of BER for MQAM signal .Here input signal is assumed to be square MQAM and gray bit mapping is being done with α bits/symbol. These α bits are allotted to each sub carrier k where M=2^ α .Now instantaneous BER mathematical expression for MIMO-OFDM system is written as in [4][5].

$$\mathrm{BER}_{\mathrm{MQAM}}[\mathrm{k,i}] = \frac{2\left(1-\frac{1}{\sqrt{2\beta}}\right)}{\beta} erfc\left(\sqrt{\frac{1.5\gamma_s |H[k]|^2}{2^{\beta}-1}}\right) \tag{3.53}$$

erfc(x) is the complementary error function: $erfc(x) = \int_x^{\infty} \exp\,(-t^2)dt$

(3.53) can be reduced to exponential form using approximation:

$$\mathrm{BER}_{\mathrm{MQAM}}[\mathrm{k,i}] = 0.2\exp\left(-\frac{1.6\gamma_s |H[k]|^2}{2^{\beta}-1}\right) \tag{3.54}$$

Now using (3.53) BER expression in (3.52) can be more simplified for a OFDM symbol as :

$$\mathrm{BER}_{\mathrm{MQAM}} = \frac{2\left(1-\frac{1}{\sqrt{2\beta}}\right)}{\mathrm{MN}\beta} \sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\gamma_s |H[k]|^2}{2^{\beta}-1}}\right) \tag{3.55}$$

Similarly using (3.52) , (3.55) can be written as stated in next equation :-

$$\mathrm{BER}_{\mathrm{MQAM}} = \frac{0.2}{\mathrm{NM}} \sum_{k=0}^{N-1} \exp\left(-\frac{1.6\gamma_s |H[k]|^2}{2^{\beta}-1}\right) \tag{3.56}$$

Now taking the mean of BER expression:

$$\overline{BER}_{MQAM} = \int_0^{\infty} \mathrm{BER}_{\mathrm{MQAM}}\, p(\gamma)d\gamma. \tag{3.57}$$

Now using assumption mean of BER can be calculated for (3.57) as given in [3].

$$\overline{BER}_{MQAM} = 0.2\left(1 + \frac{1.6\gamma_s}{2^{\beta}-1}\right)^{-1} \tag{3.58}$$

3.4.3 BER Performance of MQAM-Precoded OFDM with Channel Estimation

Now BER will be calculated using channel estimation which has been calculated earlier in (3.58) and channel estimation error (σ_e) will also be used to calculate BER as given in [3].

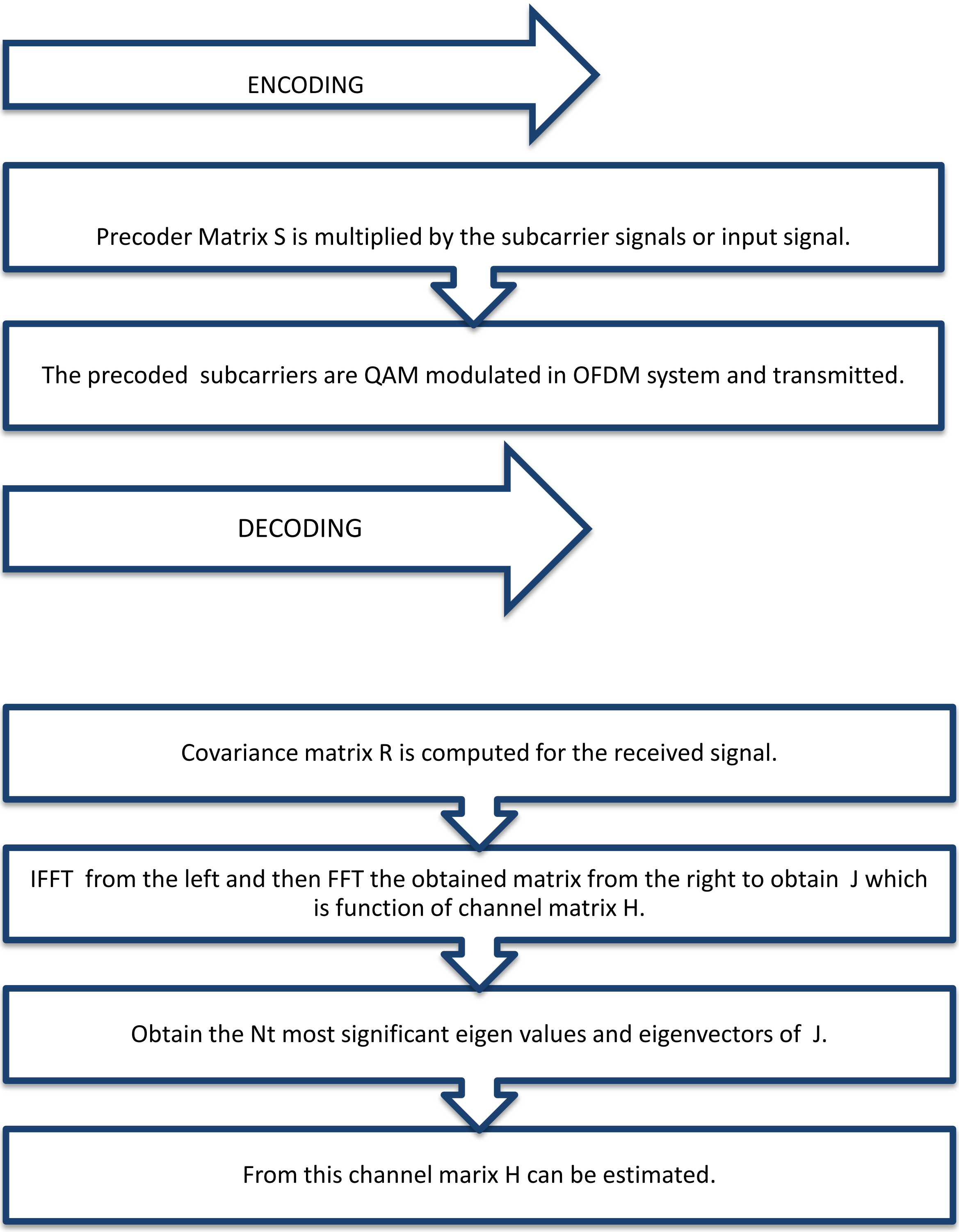
$$E[(H - H^{\wedge})(H - H^{\wedge})'|H^{\wedge}] = \sigma_e{}^2 \mathrm{I}_{\mathrm{M_{RX}\,N}}$$

$$\mathrm{BER}_{\mathrm{MQAM}} = \frac{2\left(1-\frac{1}{\sqrt{2^{\beta}}}\right)}{\mathrm{N}\beta} \sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\gamma_s \sum_{j=1}^{M_{RX}} \sum_{i=1}^{M_{TX}} \left|H^{\wedge}{}_{j,i}[k]\right|^2}{(2^{\beta}-1)(1+\sigma_e{}^2\gamma_s)}}\right) \tag{3.59}$$

$$\mathrm{BER}_{\mathrm{MQAM}} = \frac{0.2}{\mathrm{N}} \sum_{k=0}^{N-1} \exp\left(-\frac{1.6\gamma_s \sum_{j=1}^{M_{RX}} \sum_{i=1}^{M_{TX}} \left|H^{\wedge}{}_{j,i}[k]\right|^2}{(2^{\beta}-1)(1+\sigma_e{}^2\gamma_s)}\right) \tag{3.60}$$

$$\overline{BER}_{MQAM} = 0.2\left(1 + \frac{1.6\gamma_s}{(2^{\beta}-1)(1+\sigma_e{}^2\gamma_s)}\right)^{-1} \tag{3.61}$$

3.5 PROPOSED METHODOLOGY



ENCODING

Precoder Matrix S is multiplied by the subcarrier signals or input signal.

The precoded subcarriers are QAM modulated in OFDM system and transmitted.

DECODING

Covariance matrix R is computed for the received signal.

IFFT from the left and then FFT the obtained matrix from the right to obtain J which is function of channel matrix H.

Obtain the Nt most significant eigen values and eigenvectors of J.

From this channel marix H can be estimated.

3.6 RESULTS AND DISCUSSION

Various parameters of the proposed method is listed below :

TABLE 3.1 Parameters and values for proposed method

PARAMETER	VALUES
Symbols	100
SNR (db)	0:20
Blocks	3
FFT	64

In this section,we have generated 100 random signals over two transmitters and receivers with channel order of 2 to show the performance of the described method. Every element of channel impulse response is a toeplitz gaussian random variable with zero mean and unit variance. The normalised mean square error is defined as NRMSE=$\frac{1}{\|H\|_F}\sqrt{\frac{1}{I}\sum_{i=1}^{I}\left\|\hat{H}^{(i)} - H^{(i)}\right\|_F^2}$,where F is the forbenius norm and I is the no. of random signals or monte carlo runs generated. $\hat{H}^{(i)} = [\hat{H}^{(i)}(0)^T\ \hat{H}^{(i)}(1)^T \ldots\ldots\ldots\hat{H}^{(i)}(L)^T]^T$ is the matrix in which unitary matrix ambiguity is removed by least square method.The noise is taken as zero mean white gaussian noise as stated above.

3.6.1 Subcarriers are given same periodic sequence cofficients

In this case each subcarrier is given same precoding sequence coefficient, precoding sequence coefficients are only changed in different blocks. Therefore period of periodic sequence is same as no. of blocks.

3.6.1.1 Optimal selection of precoding sequence

The NRMSE is plotted in db for different sequences with change in number of symbol blocks. Three blocks are taken with $\tau = 0.6$ and 4 periodic sequences are taken with period three for comparison which satisfies (3.38) and (3.39). S_0 the first periodic sequence is selected as $\{\sqrt{1.8}, \sqrt{0.6}, \sqrt{0.6}\}$ satisfies (3.40), S_1 as $\{\sqrt{0.6}, \sqrt{1.8}, \sqrt{0.6}\}$, S_2 as $\{\sqrt{0.6}, \sqrt{0.6}, \sqrt{1.8}\}$ and S_3 as $\{1,1,1\}$ which is the case of no precoding.

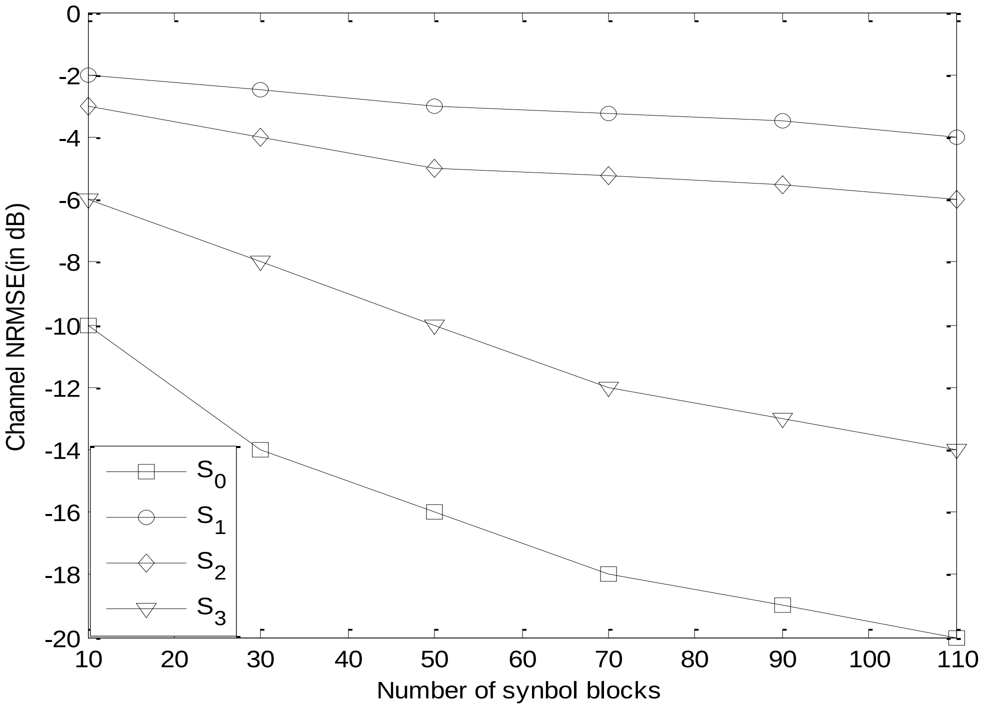


Fig 3.2: Channel NRMSE vs different Symbol blocks

Fig(3.2) shows NRMSE decreases with increase in no. of symbol blocks. S_0 is the optimal precoding sequence as NRMSE is least in this case. After this comes S_3, S_2 and then S_1. S_0 shows better results than other precoding sequence as it can be cosidered as without precoding in the system. Hence, it shows that periodic precoder gives good results only if optimality condition is applied not for all the cases.

3.6.1.2 *BER vs SNR for different τ*

In this simulation BER is plotted for different values of τ with respect to SNR keeping in range of 0 to 20db.

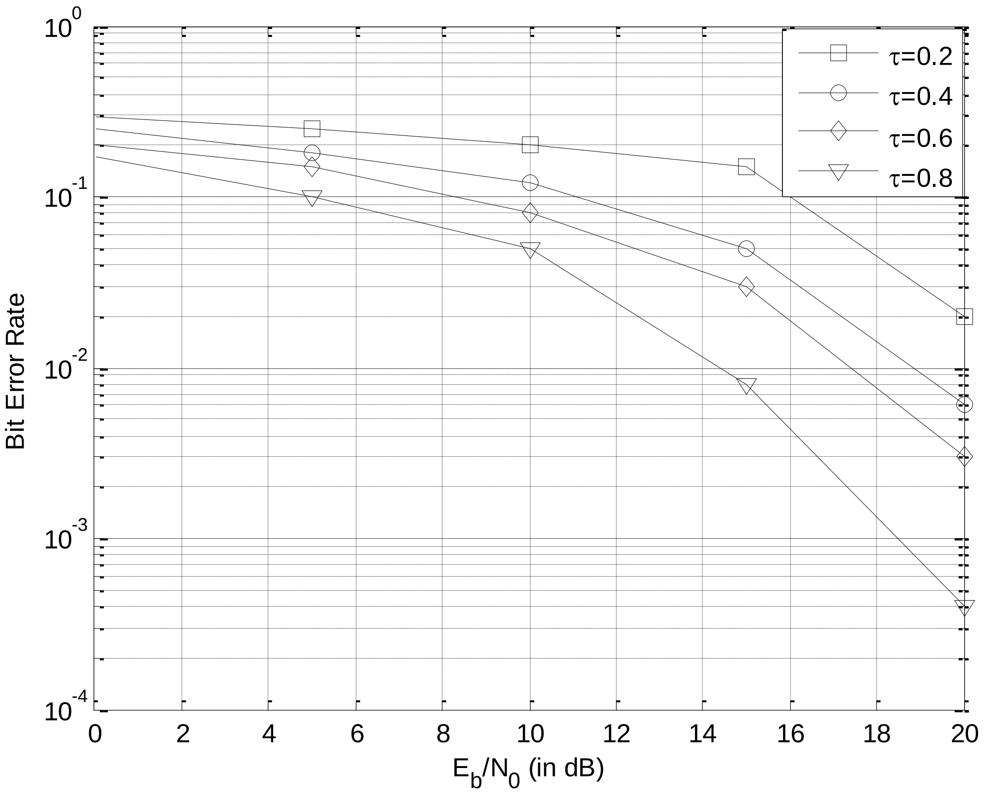


Fig 3.3: BER vs SNR for different values of τ

In this simulation BER shows better performance with increasing value of τ. BER results show reverse of NRMSE results which decreases with decreasing value of τ. Hence τ = 0.8 shows best results for high as well as low SNR.

3.6.1.3 *Channel NRMSE vs. SNR for Different values of* τ

According to numerical analysis with decreasing value of τ correlation of periodic sequence with noise should decrease as given in (3.41). This is shown by the simulation results shown below:

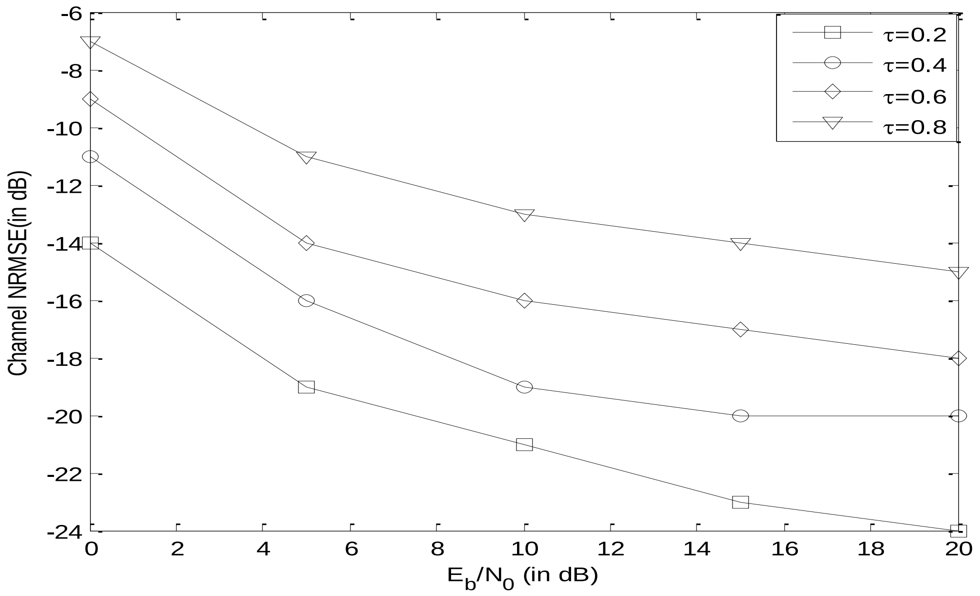


Fig 3.4: Channel NRMSE vs SNR

In this simulation, we use variation of optimal precoding sequences with values of τ.It is observed that channel NRMSE decreases with increasing SNR.As τ decreases NRMSE also decreases, hence best results are shown by $\tau = 0.2$.This can be explained on the basis of (3.41) where variation of τ changes correlation coefficient. Lesser the value of τ lesser will be effect of noise on the system. Here 100 symbol blocks are taken, for smoother results 1000 blocks can be taken. Moreover for all values of τ NRMSE curve gets constant for snr greater than 10 db.

3.6.1.4: Channel NRMSE vs. overestimated channel for different τ

The simulation result shown below is change in channel NRMSE with change in value of channel order which is given in terms of overestimated channel order.

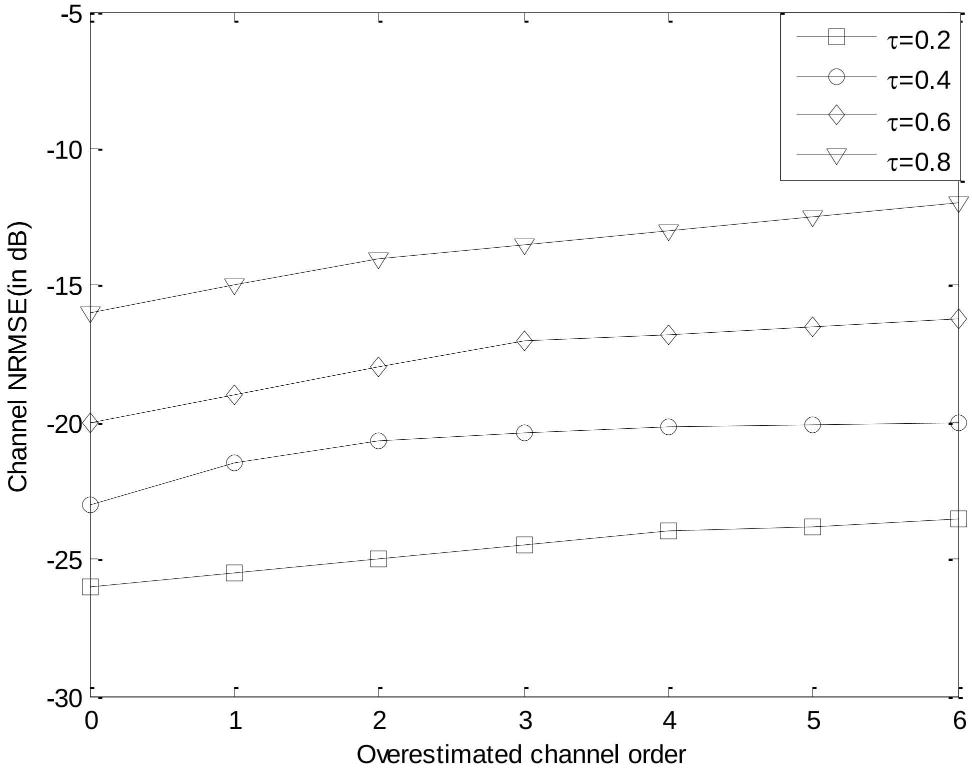


Fig 3.5: Channel NRMSE v/s Overestimated channel order$\widehat{(L}-L)$

Simulation above shows increase of NRMSE with increase in overestimated channel order i.e.$\hat{L}-L$ for 100 blocks. Results for different values of τ is shown which gives optimal precoding sequence according to (3.43) .N= $\hat{L}+2$ for each $\hat{L}$ and $0 \leq (\hat{L}-L) \leq 6$,SNR =10db and m=0 case is taken. Least value shows best results and NRMSE curve is almost constant for each value of τ with respect to overestimated channel order. For $\tau = 0.6$ NRMSE changes from -20db to -18db as $0 \leq (\hat{L}-L) \leq 3$.

3.6.2 Subcarriers are given Different Precoding Sequence Coefficients

Each subcarrier is given different precoding sequence coefficients in this case. Precoding sequence coefficient changes with subcarriers as well as blocks. Now the precoding sequence has period equal to product of subcarriers and number of blocks.

TABLE 3.2 Parameters and values of proposed method (2)

PARAMETER	VALUES
Symbols	256
SNR (db)	0:20
Blocks	4
FFT	64

3.6.2.1 Channel NRMSE vs different Number of Samples for different sequences

The NRMSE is plotted in db for different sequences with change in number of symbol blocks. Four blocks are taken with $\tau = 0.6$ and 4 periodic sequences are taken with period three for comparison which satisfies (3.38) and (3.39). S_0 the first periodic sequence is selected as $\sqrt{103},\underline{\sqrt{0.6}.............\sqrt{0.6}}_{\text{(upto 255 terms)}}$ satisfies (3.40), S_1 as $\sqrt{0.6},\underline{\sqrt{103}.............\sqrt{0.6}}_{\text{(upto 255 terms)}}$, $S_2 as$ $\sqrt{0.6},\underline{\sqrt{0.6}.............\sqrt{103}}_{\text{(upto 255 terms)}}$ and S_3 as 1,1,1......1(upto 256 terms) which is the case of no precoding.

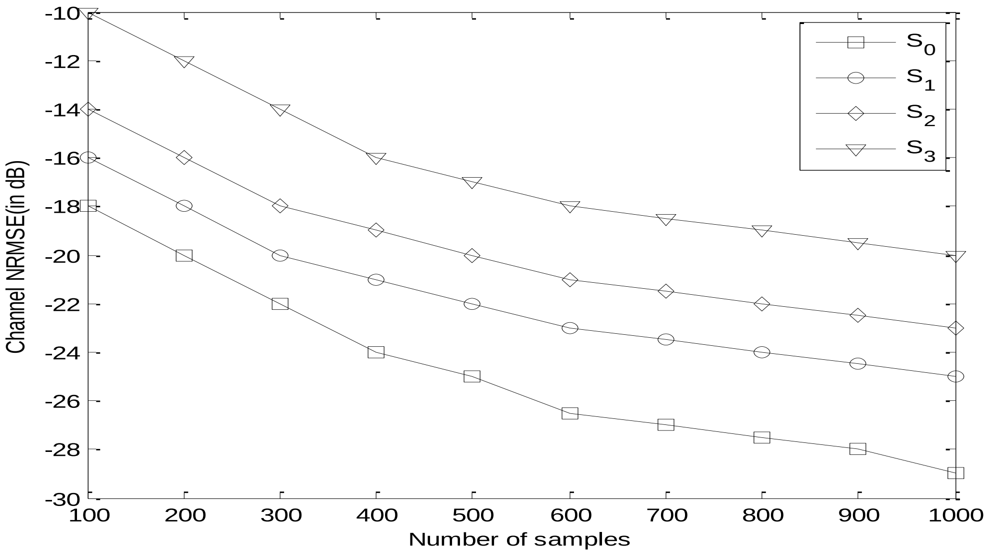


Fig 3.7: Channel NRMSE vs different Number of Samples

Channel NRMSE is plotted with different number of samples (at each transmitter). It is observed that optimal sequence shows best results as compared to other results.

3.6.2.2 BER vs SNR for different τ

In this simulation BER is plotted for different values of τ with respect to SNR keeping in range of 0 to 20db.

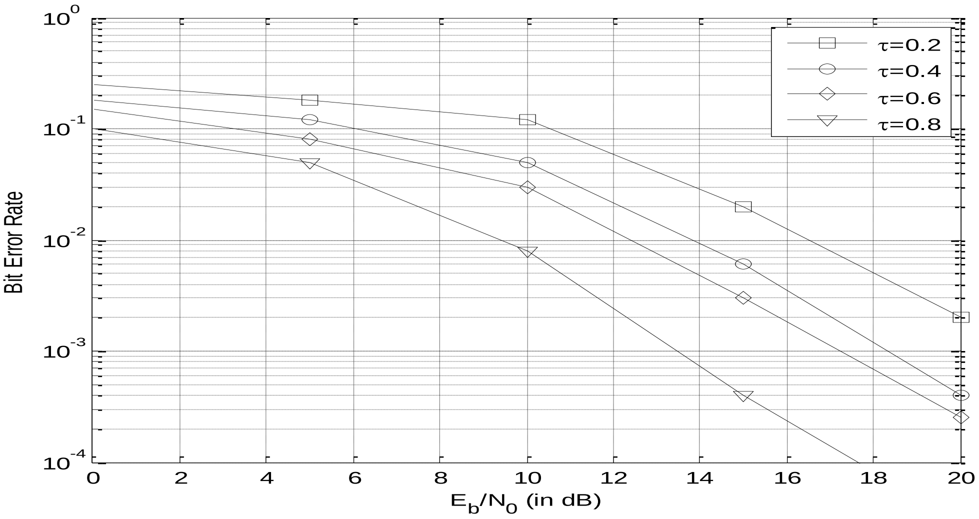


Fig 3.5 BER vs. SNR for different values of τ (2)

Simulation results show that when each subcarrier is given different periodic sequence coefficients then performance of the system improves as compared to system where each subcarrier is given same periodic sequence coefficients. Although results appear same as in both BER decreases with increasing value of τ.

3.6.2.3 Channel NRMSE vs SNR for different τ

According to numerical analysis with decreasing value of τ correlation of periodic sequence with noise should decrease as given in (3.41). This is shown by the simulation results shown below:

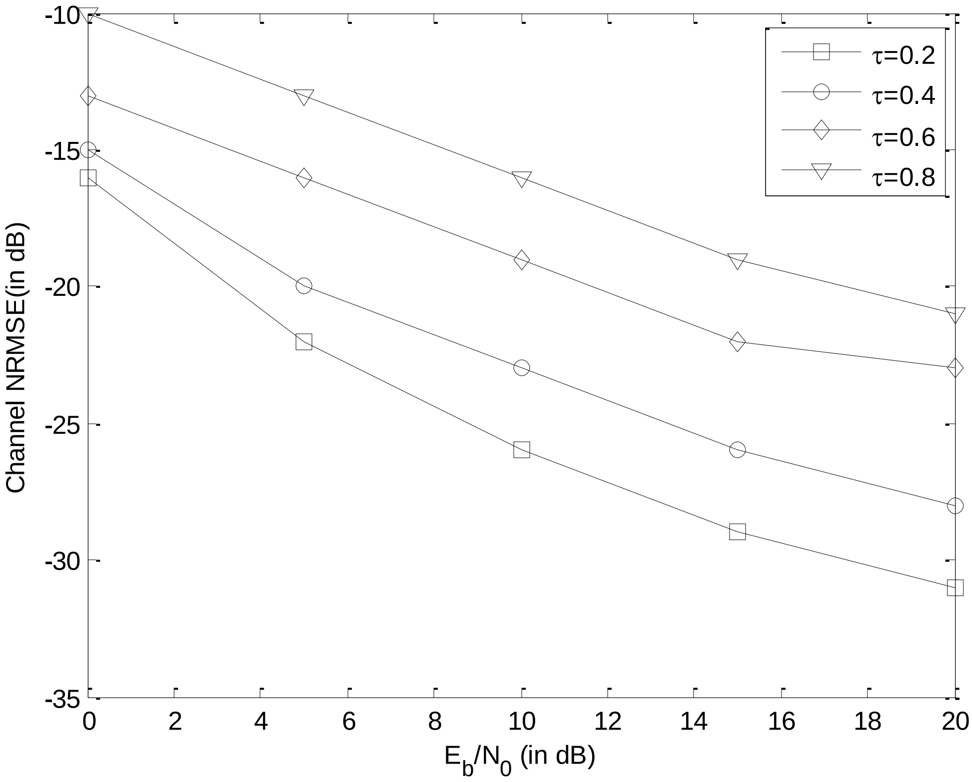


Fig 3.6: Channel NRMSE vs SNR

Channel NRMSE decreases with decreasing value of τ. Although for all τ NRMSE is decreasing for all values of SNR. Compared to earlier case of same precoding sequence coefficient to each subcarrier these results show better performance. Since each subcarrier is given different precoding coefficient it is easy to identify them at the receiver, although complexity increases in this case but more accurate results are obtained for a given system.

3.6.2.4 Channel NRMSE vs. Overestimated channel order$(\widehat{L} - L)$

The simulation result shown below is change in channel NRMSE with change in value of channel order which is given in terms of overestimated channel order.

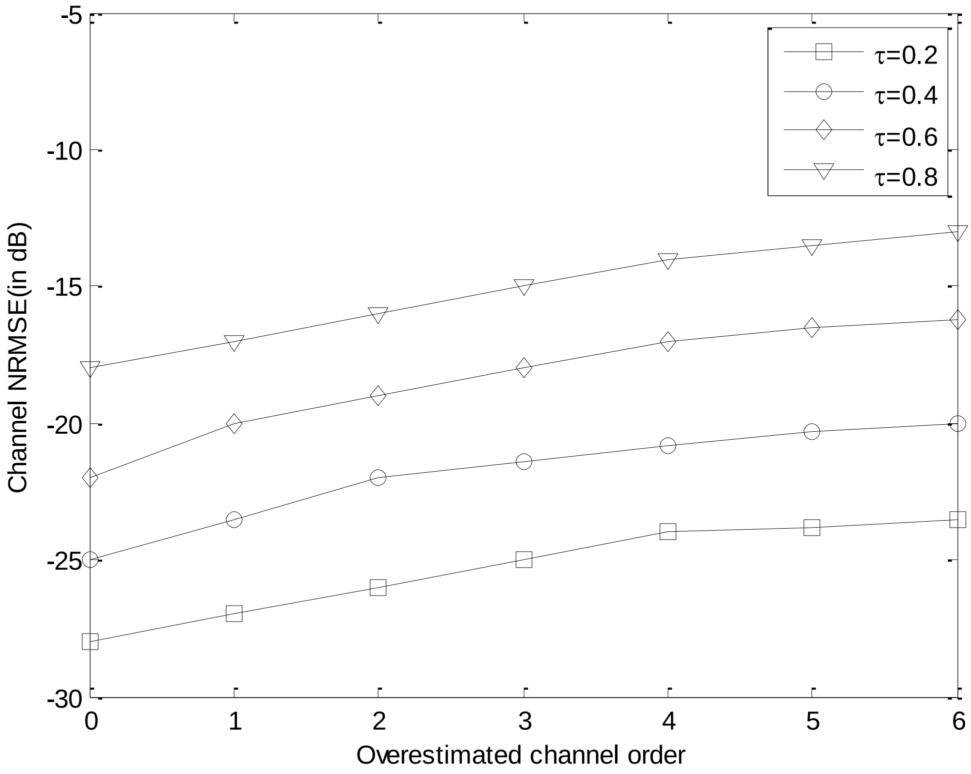


Fig 3.8: Channel NRMSE vs. Overestimated channel order$(\widehat{L} - L)$

Since it is discussed in the paper that periodic precoder does not require overestimated channel order. Hence the results above also show that with increase in overestimated channel order the channel NRMSE increases, therefore it works better for lesser channel order. Moreover compared to previous result in 3.5.1 it shows better results as performance is improved by assigning different precoding sequence coefficients to different subcarriers.

CHAPTER - 4

CONCLUSION AND FUTURE SCOPE

CONCLUSION

From the above study and results, it can be concluded that blind channel estimation should be preferred for channel estimation as data transmission rate increases and less bandwidth is required as compared to semi blind channel estimation. Blind channel estimation uses no information about transmitted data, but statistical properties are considered. Periodic precoding method is applied to MIMO-OFDM system and results are observed for two cases. First when each subcarrier is given same precoding sequence coefficient and second when each subcarrier is given different precoding sequence coefficient. NRMSE and BER plots are compared for both cases. It is concluded that second case gives better results than first case as each subcarrier is given different precoding sequence coefficient, therefore it is easier for receiver to decode it with less errors. Different sequences are also compared and optimal precoding sequence gives best results. Reduction of BER shows this precoder is able to remove noise from the MIMO-OFDM system. In this method, correlation of sequence with noise has been calculated. Thereafter, by optimally choosing periodic sequence, the effect of noise can be further reduced.

FUTURE SCOPE

For future Hybrid precoders can also be designed using periodic precoders and other precoders to enhance the performance of BER as well as channel estimation error in MIMO-OFDM system. Moreover periodic precoder can also be used for other applications like reducing interference and increasing capacity which has not been used so far.

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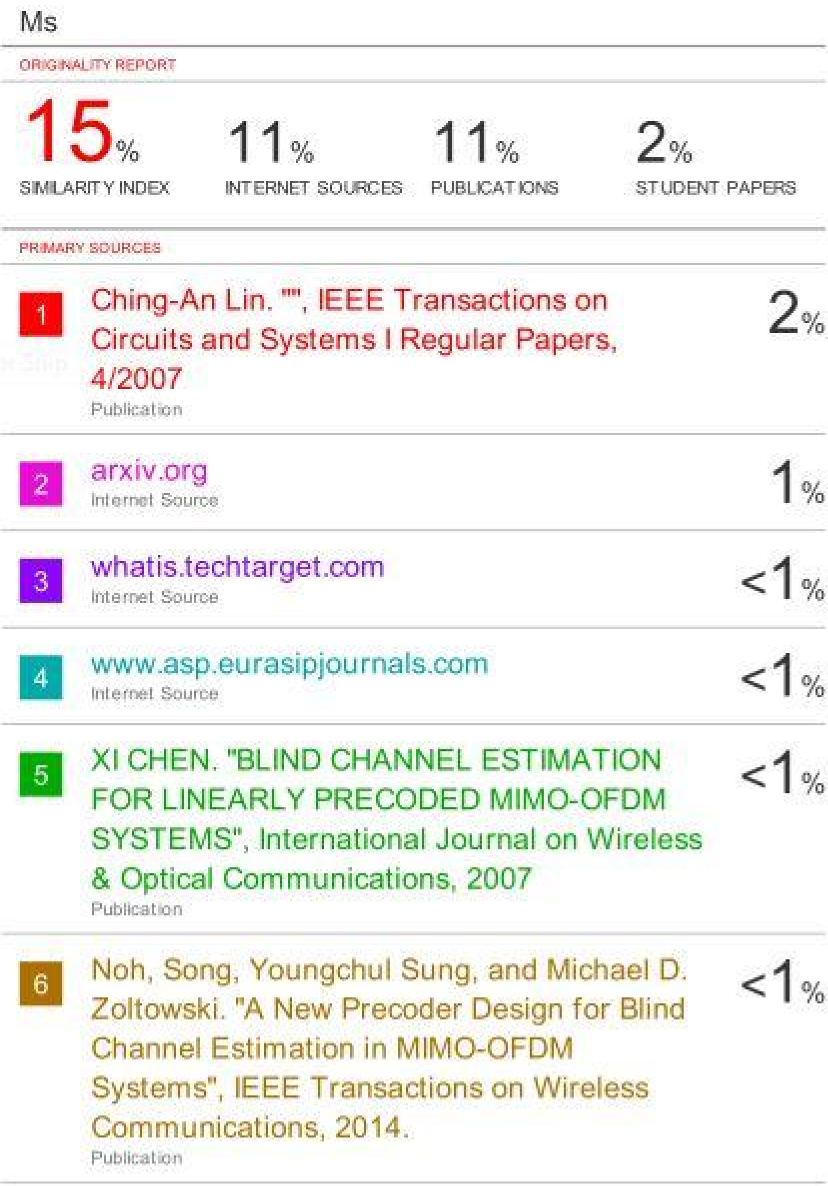
LIST OF PUBLICATIONS

Shivani Gupta, Bhasker Gupta, Ashutosh Kumar Singh "Optimized Periodic Precoder Based Blind Channel Estimation for MIMO-OFDM Systems" submitted to Taylor and Francis journal, 2016.



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