# A COMPARATIVE ANALYSIS OF THRESHOLDING TECHNIQUES USED IN IMAGE DENOISING THROUGH WAVELETS

***Thesis submitted in partial fulfillment of the requirements for the award of degree of***

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# CERTIFICATE

I hereby certify that the work which is being presented in the thesis entitled, **"A Comparative Analysis Of Thresholding Techniques Used In Image Denoising Through Wavelets"**, in partial fulfillment of the requirements for the award of degree of **Master of Engineering** in **Electronics and Instrumentation Control** submitted in Electrical & Instrumentation Engineering Department of Thapar University, Patiala, is an authentic record of my own work carried out under the supervision of **Mr. M.D.SINGH**, **Sr. Lecturer,** EIED, Thapar University, Patiala.

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

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This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

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# ABSTRACT

Historically, the field of image processing grew from electrical engineering as an extension of the signal processing branch. The massive amount of data required for images is a primary reason for the development of many sub areas within the field of computer imaging such as image segmentation and compression.

Whatever may be the way of transmission, the data tends to get noisy and thereby the further processing does not lead to good results. Hence, it is very essential to keep the data close to originality.

The prime focus of this thesis is related to the pre processing of an image. The pre processing being worked upon is the de noising of images. In order to achieve this in terms of the concerned work, wavelet transforms have been applied: Discrete wavelet transform and Un decimated  Discrete wavelet transform.

In this thesis, a new thresholding technique has been presented alongwith the standard thresholding techniques like soft and hard thresholding. And a comparative analysis of different combinations of the suggested threshold values and thresholding techniques has been carried out very efficiently. A new constraint, of either thresholding the low pass components or keeping them as such before applying the inverse DWT and UDWT, has also been added. This has been done in order to find more possible combinations that can lead to the best denoising technique.

MATLAB codes have been developed for all the possible combinations, separately.

# TABLE OF CONTENTS

# LIST OF ABBREVIATIONS

| | |
|---|---|
| DWT | Discrete Wavelet Transform |
| UDWT | Undecimated Discrete Wavelet Transform |
| PSNR | Peak Signal to Noise Ratio |
| MAD | Median Absolute Deviation |
| STD | Standard Deviation |
| UNIV | Universal Threshold |
| MS | MAD Soft |
| SS | STD Soft |
| US | UNIV Soft |
| MH | MAD Hard |
| SH | STD Hard |
| UH | UNIV Hard |
| MC | MAD Custom |
| SC | STD Custom |
| UC | UNIV Custom |

# Chapter 1

# INTRODUCTION

Image processing is a field that continues to grow, with new applications being developed at an ever increasing pace. It is a fascinating and exciting area to be involved in today with application areas ranging from the entertainment industry to the space program. One of the most interesting aspect of this information revolution is the ability to send and receive complex data that transcends ordinary written text. Visual information, transmitted in the form of digital images, has become a major method of communication for the 21$^{st}$ century.

Image processing is any form of signal processing for which the input is an image, such as photographs or frames of video and the output of image processing can be either an image or a set of characteristics or parameters related to the image. Most image-processing techniques involve treating the image as a two-dimensional signal and applying standard signal-processing techniques to it.

There are applications in image processing that require the analysis to be localized in the spatial domain. The classical way of doing this is through what is called Windowed Fourier Transform. Central idea of windowing is reflected in Short Time Fourier Transform (STFT). The STFT conveys the localized frequency component present in the signal during the short window of time.

The same concept can be extended to a two-dimensional spatial image where the localized frequency components may be determined from the windowed transform. This is one of the basis of the conceptual understanding of wavelet transforms. Hence, wavelet transforms have been kept as the main consideration in this thesis.

It is well known that while receiving the input image some abberations get introduced alongwith it and hence a noisy image is what we are left with for future processing.

The image de-noising naturally corrupted by noise is a classical problem in the field of signal or image processing. Additive random noise can easily be removed using simple threshold methods.

De-noising of natural images corrupted by noise using wavelet techniques is very effective because of its ability to capture the energy of a signal in few energy transform values. The wavelet de-noising scheme thresholds the wavelet coefficients arising from the wavelet transform. The wavelet transform yields a large number of small coefficients and a small number of large coefficients.

Simple de-noising algorithms that use the wavelet transform consist of three steps.

- Calculate the wavelet transform of the noisy signal.
- Modify the noisy wavelet coefficients according to some rule.
- Compute the inverse transform using the modified coefficients.

The problem of Image de-noising can be summarized as follows,

Let $A(i, j)$ be the noise-free image and $B(i, j)$ the image corrupted with noise $Z(i, j)$,

$$B(i, j) = A(i, j) + \sigma Z(i, j) \qquad ...(1.1)$$

The problem is to estimate the desired signal as accurately as possible according to some criteria. In the wavelet domain, the problem can be formulated as

$$Y(i, j) = W(i, j) + N(i, j) \qquad ...(1.2)$$

where $Y(i, j)$ is noisy wavelet coefficient; $W(i, j)$ is true coefficient and $N(i, j)$ noise.

In this thesis work, the algorithm has been carried out by using variety of inputs. Like two type of wavelet transforms have been used- Discrete wavelet transform and Undecimated Discrete wavelet transform. Five types of test images have been used for analysis. Furthermore, four type of noise has been added to the test images. The modification of the noisy wavelet coefficients has been done by using three types of thresholding techniques, each being applied for three different threshold values. A further add on has been made to the existing algorithms by not thresholding the low pass components for the case of DWT, whereas thresholding them for the case of UDWT.

The performance of the applied image de-noising algorithms has been investigated in terms of two parameters PSNR (peak signal to noise ratio) and MSSIM (mean structural similarity index value between two images).

The coding for the presented image de noising algorithms has been done using MATLAB 7.0 for all the combinations separately.

## Organization of thesis is as follows:

This thesis is organized as per the following format.

Chapter 2 involves the description of the entire literature survey that has been done during the present study.

Chapter 3 gives a complete idea of the basic fundamentals of Wavelet Transforms. It also contains a full description of the procedural transforms that have been used in the concerned work.

Chapter 4 gives a brief overview about image denoising. It also explains how the Wavelet Transforms are used for image denoising.

Chapter 5 provides with the entire proposed algorithm, in an explicit form, that is followed during denoising images while using Wavelet Transforms.

Chapter 6 consists of the results obtained (both in the form of tables and graphs) for the images denoised using Wavelet Transforms. PSNR and MSSIM are the two parameters that have been taken as the basis for the discussion of results.

Chapter 7 discusses the experimental results obtained during the study.

Chapter 8 concludes this thesis work. This chapter also gives a brief outline of the future scope of the proposed work.

APPENDIX A consists of all the original standard test images that have been used in this thesis.

APPENDIX B contains the samples of noisy images that are to be denoised using the proposed algorithm.

APPENDIX C has been dedicated to the denoised image samples that have been obtained using the proposed algorithm.

# Chapter 2

# LITERATURE SURVEY

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This chapter deals with the survey of various research papers that have contributed in the denoising of images using wavelet transforms, in one way or other.

There is a growing demand of image processing in diverse application areas, such as multimedia computing, secured image data communication, biomedical imaging, biometrics, remote sensing, texture understanding, pattern recognition, content-based image retrieval, compression, and so on. And wavelet transform has been providing a major contribution in all the above mentioned areas since long time. But the quest of betterment never ends.

It is very essential to keep the useful data in the exact original form for further processing and wavelet denoising being the latest technique that has proved its command over this issue. The following literature review discusses denoising using wavelet transforms in a wide scenario, i.e. using a number of thresholding techniques for a wide variety of test images.

John C. Wood, Kevin Johnson [1] did denoising of synthetic, phantom, and volunteer cardiac images either in the complex or magnitude domains. For superior edge resolution of real and imaginary images, they suggested denoising prior to rectification. Magnitude and complex denoising significantly improved SNR, SBR, and CNR.

Wilfred L. Rosenbaum, M. Stella Atkinsa, Gordon E. Sarty [2] applied wavelet shrinkage denoising algorithms and Nowak's algorithm for denoising the magnitude images. The wavelet shrinkage denoising methods were performed using both hard and soft thresholding. It was suggested that changes in mean relative SNR are statistically associated with type of threshold and type of wavelet. Nowak's data-adaptive wavelet filtering was found to provide the best overall performance as compared to direct wavelet shrinkage.

Fabrizio Argenti, Gionatan Torricelli [3] assumed Wiener-like filtering, for noise reduction, performed in a shift-invariant wavelet domain by means of an adaptive rescaling of the coefficients of undecimated octave decomposition calculated from the parameters of the noise model, and the wavelet filters. The proposed method resulted in excellent background smoothing as well as preservation of edge sharpness and fine details. LLMMSE estimation in an undecimated wavelet domain tested on both synthetically speckled images and ultrasonic images demonstrated an efficient rejection of the distortion due to speckle.

Jiecheng Xie [4] mentioned the denoising method based on a doubly stochastic process model of wavelet coefficients that gave a new spatially varying threshold using the MDL principle. This method outperformed the traditional thresholding method in both MSE error and compression gain.

Alle Meije Wink and Jos B.T.M.Roerdink [5] evaluated two denoising methods for the simulation of an fMRI series with a time signal in an active spot – by the average temporal SNR inside the original activated spot and by the shape of the spot detected by thresholding the temporal SNR maps. These methods were found to be better suited for low SNRs but for reasonable quality images they were not preferred as they introduced heavy decompositions. Hence, wavelet based denoising methods were used as they preserved sharpness of the images, retained the original shapes of active regions as well and produced a smaller total number of errors than Gaussian smoothing. But both Gaussian and wavelet based smoothing methods introduced severe deformations and blurred the edges of the active spot. For low SNR both methods are found to be on par. For high SNR –Wavelet performed better than Gaussian giving a maximum output of above 10 db.

Hyeokho Choi, Richard G.Baranuik [6] defined Besov Balls, a convex set of images whose Besov norms are bounded from above by their radii, in multiple wavelet domains. And projected them onto their intersection using the projection onto convex sets (POCS) algorithm. It corresponded to a type of wavelet shrinkage for image denoising. This algorithm provided significant improvement over conventional wavelet shrinkage algorithm, based on a single wavelet domain such as hard thresholding in a single wavelet domain.

Byung-Jun Yoon and P. P. Vaidyanathan [7] proposed the custom thresholding scheme and demonstrated that it outperformed the traditional soft and hard-thresholding schemes, since the custom thresholding function adapted well to the characteristics of the given signal, resulting in a smaller estimation error. This research work has been taken as the main basis of our thesis work.

Tai-Chiu Hsung, Daniel Pak-Kong Lun and K.C.Ho [8] improved the traditional wavelet method by applying Multivariate Shrinkage on multiwavelet transform coefficients. $1^{st}$ly a simple $2^{nd}$ order orthogonal pre filter design method was used for applying multi wavelet of higher multiplicities (preserving orthogonal pre filter for any multiplicity). Then threshold selections were studied using Stein's unbiased risk estimator (SURE) for each resolution level, provided the noise structure is known. Numerical experiments showed that – (a) multivariate shrinkage of higher multiplicity usually gave better performance and (b) the proposed LSURE substantially outperformed the traditional SURE in multivariate shrinkage denoising, particularly at high multiplicity.

Aleksandra Piˇzurica, Alle Meije Wink, Ewout Vansteenkiste, Wilfried Philips and Jos B.T.M. Roerdink [9] analyzed two types of filters for magnetic resonance images (MRI): noise suppression in magnitude MRI images and denoising blood oxygen level-dependent (BOLD) response in functional MRI images (fMRI). The noise distribution in magnitude MRI images was Rician, while the noise distribution in BOLD images was shown to follow a Gaussian model. And hence different methods were evaluated based on signal to noise ratio improvement and based on the preservation of the shape of the activated regions in fMRI. In the case of fMRI, wavelet-based denosing methods were shown to be effective in terms of improving SNR as well as preserving the shape of the activated region. However, the results on real fMRI data, where denoising was combined with statistical parametric mapping, were somewhat disappointing compared to the purely simulated cases and hence proved to be a tradeoff between sensitivity (the ability to detect the target region) and specificity (the ability to not detect non-target regions) in fMRI analysis.

Yong Sun Kim and Jong Beom Ra [10] proposed, for 2-dimensional B-mode ultrasound images, an image enhancement algorithm based on a multi-resolution approach. Directional filtering and noise reducing procedures had been performed from the coarse to fine resolution images that were obtained from the wavelet-transformed data. For directional filtering, the structural feature at each pixel was examined through the eigen-analysis. If the pixel belonged to the edge region, then two-step directional filtering was done, namely, directional smoothing along the tangential direction of the edge to improve its continuity, and directional sharpening along the normal direction to enhance the contrast. The proposed speckle reduction scheme was based on the structural information rather than the statistics of the magnitude of wavelet coefficients as in the existing methods, for in ultrasound images. The speckle energy was found to be comparable to the signal energy in a wide range of frequency bands and it was not easy to discriminate speckle from the signal only by using magnitude statistics of wavelet coefficients in the decomposed image. To discriminate speckle from the signal, the structural information from the wavelet decomposed image was obtained by performing the eigen-analysis at each resolution scale. Then, based on the structural information, the directional filtering and speckle reduction procedures were applied adaptively to the multi-resolution image. The proposed algorithm considerably improved the subjective image quality without generating any noticeable artifact, and provided better performance compared with the existing enhancement schemes.

Kai-qi Huang, Zhen-ye Wu, George S.K.Fung, Francis H.Y.Chan [11] presented a new approach to color image denoising taking into consideration Human Visual System. A contrast sensitivity function (CSF) implementation was employed in the subband of wavelet domain, based on an invariant single factor weighting and noise making was adopted in succession. The method adopted holded more details while smoothing and reduced most noises. Improvements were said to be possible by using a more complex HVS model.

Nai-Xiang Lian, Vitali Zagorodnov, Member, IEEE, and Yap-Peng Tan [12] proposed a new denoising method, based on the minimum cut algorithm, in order to exploit both the interscale and intrascale correlations of wavelet coefficients. (Since Wavelet thresholding is efficient in edge-preserving for grayscale images especially when it exploits the interscale correlations of wavelet coefficients. Intrascale correlations can further improve the denoising performance, but the gain for gray scale images is generally small.) This proposed method achieved up to 5-db gain in peak SNR for color-difference images and led to fewer visual color artifacts. The interscale correlations were exploited by updating local state probabilities using the states of the neighbouring coefficients. Experimental results suggested substantial PSNR improvement in the denoised Cb and Cr components. This improvement translated into fewer color artifacts in the resultant images.

C.O.S Sorzano, E.Ortiz, M.Lopez, J.Rodrigo [13] proposed an improvement of the image denoising method based on wavelet, done by replacing the parameter estimation step by a constrained nonlinear optimization. Algorithm was tested with images simulating electron microscopy (EM) conditions as well as EM images. The Bayesian approach proposed by Bijaoui was proven favorable to other wavelet-thresholding denoising techniques. Performance was improved by solving simultaneously for all the involved signal and noise parameters. Two methods had been introduced for this purpose. Method 1- based on the analysis of the PDF of the measured image, while Method 2- based on its power decomposition. The parameter search was optionally constrained by a priori knowledge about the SNR or about the nature of the images (here, energy tended to concentrate in low frequency components). A methodology to estimate the algorithm parameters had been devised (like noise power and signal power), for different noise levels at each wavelet scale. It was found feasible to apply the proposed algorithm in case of low-pass filtered images that resembled negative-staining conditions in electron microscopy, since the proposed methodology did not require the knowledge of the explicit form of the filter.

Brij N.Singh, Arvind K.Tiwari [14] used ECG signal to access the cardiovascular condition of humans. A mother wavelet basis function was selected and applied for denoising while signal peaks close to full amplitude were retained. The results revealed suitability of Daubechies mother wavelet of order 8 as the most appropriate wavelet basis function for the denoising application and the selected basis function was found to be optimal in RMSE and preserved the peaks of the ECG signal as well.

Cajo J.F. ter Braak [15] extended the normal Bayesian linear model by assigning a flat prior to the $\delta$th power of the variance components of the regression coefficients ($0 < \delta \leq 1/2$) in order to improve prediction accuracy. The expected shrinkage was a sigmoid function of the squared value of the least-squares estimate divided by its standard error. This gave a small amount of shrinkage for large values and, provided $\delta$ is small, heavy shrinkage for small values. The limit behavior for both small and large values approached that of the ideal coordinate wise shrinker in terms of the expected squared error of prediction, when $\delta$ was kept close to 0. The proposed Bayesian shrinkage model yielded a lower mean squared error than soft thresholding (lasso) and was found competitive with wavelet shrinkage methods based on mixture prior distributions. In the BSS proposed here, a single hyper parameter $\delta$ governed the shrinkage, whereby coefficients that were small in statistical sense were shrunken to near zero and large coefficients were effectively not shrunken. In the context of wavelets the hyper parameter was made level-dependent via the rule $\delta_j = 1/m$ with $m$ being the number of coefficients of the level. BSS performed better than the lasso in terms of mean squared error. BSS proved to be the only parameterized shrinking method that approached the ideal shrinker.

O. Tumšys, R. Raišutis, Prof. K.Baršauskas [16] in their study presented de-noising results of the ultrasonic echo-signals using the discrete wavelet transform with level-dependent thresholds and applied it for simulated and measured ultrasonic signals, obtained from austenitic steel sample and glass fiber reinforced plastic composite pipe. "Symlet-10" mother wavelet using "Minimax" threshold selection rule provided significant improvement for ultrasonic grain noise reduction in the case of detection of intergranular stress corrosion cracks in austenitic steel samples. The signals reflected by regular discontinuities, like interfaces between the adjacent layers, covered the useful signals from the internal defects (like SDH). But the proposed wavelet de-noising procedure could not eliminate these reflections from the interfaces.

Zhao Jian, Cao Zhengwen and Zhou Mingquan [17] explained the main concept of wavelet-fractal based SAR (synthetic aperture radar) image processing. They demonstrated that after wavelet transformation, multifractal spectrum of the signal was different from that of noise. This difference was used to alleviate the noise produced by SAR image. The proposed method focused on adjusting the Hölder exponent $\alpha$ of multifractal spectrum. And it was suggested that after simulation, $\alpha$ should be adjusted to 1.72-1.73. The more the value of $\alpha$ exceeded 1.73, the less distinctive the edges of SAR image became. It was found that denoising was optional at $\alpha$ = 1.72-1.73 or could be said that a smooth and denoised SAR image was produced.

D.Giaouris, J.W.Finch [18] showed that the denoising scheme based on the WT did not distort the signal and the noise component after the process was found to be small. But this process imposed a certain delay on the signal and was relatively complicated. In fixed frequency case, no improvement had been noted. But WT was employed where the useful components existed at widely spread and varying frequencies and the bandwidths were uncertain.

S.Poornachandra [19] used the wavelet-based denoising for the recovery of signal contaminated by white additive Gaussian noise and investigated the noise free reconstruction property of universal threshold. The procedure was known as Subband adaptive. Parameters were chosen by difference in mean method. The S-median-DM and S-median thresholds were found to have higher SNR and lower MSE than the universal threshold. This proposed technique found its application in denoising of biological and communication signals.

Li Zhen, He Zhengjia, Zi Yanyang, Wang Yanxue [20] proposed a wavelet denoising method by incorporating neighboring coefficients (NeighCoeff), and it gave better results than the traditional term-by-term approaches. This method exploited only the intra-scale dependency of wavelet coefficients. Here, by designing the prediction operator and update operator, a customized wavelet based on the lifting scheme was constructed directly to match the transient properties of a given signal. The NeighCoeff denoising algorithm was improved by taking into consideration the intra- and inter-scale dependency of wavelet coefficients. The optimal prediction operator and update operator were designed by the rules of the Kurtosis maximization and reconstruction error minimization, respectively. A customized wavelet was constructed, which adapted to the transient characteristics of the inspected signal. The proposed method was tested for simulated signal and bearing vibration signals. It performed better than non-adaptive lifting scheme and traditional Daubechies 8 wavelet.

Yinpeng Jin, Elsa D. Angelini, Peter D. Esser, Andrew F. Laine [21] introduced a cross-scale regularization scheme, which took into account cross-scale coherence of structured signals. Wavelet thresholding was also compared to denoising with a brushlet expansion. The proposed regularization scheme eliminated the need for threshold parameter settings, making the denoising process less tedious and suitable for clinical practice. For SPECT and PET data, the directional texture of the noise components led to the failure of most of the standard 2D denoising methods since directional patterns appeared as significant spatial features in 2D slices. The denoising quality was significantly improved when compared to the clinical reconstruction method as it provided better edge definition and improved the visibility of structure. A parallel approach used a brushlet multi-scale expansion and spatially adaptive hard thresholding to remove texture noise components in 2D slices. A very good performance of brushlet denoising was observed at removing background noise that enhanced physiological data as well (with higher details of brightness levels within different structures) as compared to wavelet denoising but less spatial delineation of the anatomical contours was also noticed.

From the above review of research papers, it is quite clear that wavelet has provided a very handsome amount of contribution in image denoising. A good number of aforesaid methods have been applied to different type of images. But among these papers, we found that one of the techniques, custom thresholding using wavelets, was developed only for signals (one dimensional) and has not been applied to two dimensional problems like for example images. Hence, we modified and proposed the same technique for images, in this thesis, taking its application for signals (in one dimension) as the main basis.

# Chapter 3
# WAVELET TRANSFORMS

This chapter discusses the fundamental basics of wavelet. It also contains a brief overview of the types of wavelet transforms used in this work.

## 3.1 Introduction to Wavelet

The concept of wavelet was hidden in the works of mathematicians even more than a century ago. In 1873, Karl Weirstrass mathematically described how a family of functions can be constructed by superimposing scaled versions of a given basis function. The term wavelet was originally used in the field of seismology to describe the disturbances that emanate and proceed outward from a sharp seismic impulse [22].

Wavelet means a "small wave". The smallness refers to the condition that the window function is of finite length (compactly supported) [23].

A wave is an oscillating function of time or space and is periodic. In contrast, wavelets are localized waves. They have their energy concentrated in time and are suited to analysis of transient signals. While Fourier Transform and STFT use waves to analyze signals, the Wavelet Transform uses wavelets of finite energy [22].

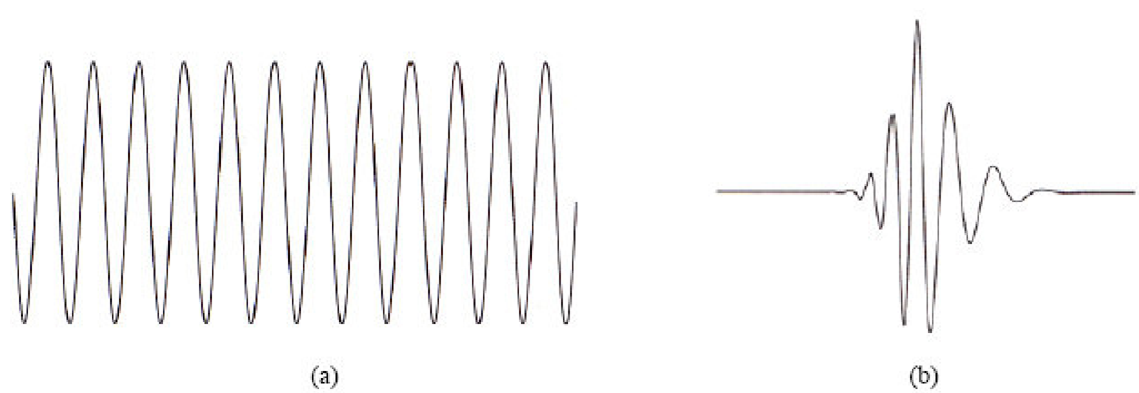



**Figure 3.1 Difference between Wave and Wavelet (a) wave (b) wavelet.**

In wavelet analysis the signal to be analyzed is multiplied with a wavelet function and then the transform is computed for each segment generated. The Wavelet Transform, at high frequencies, gives good time resolution and poor frequency resolution, while at low frequencies, the Wavelet Transform gives good frequency resolution and poor time resolution.

An arbitrary signal can be analyzed in terms of scaling and translation of a single mother wavelet function (basis). Wavelets allow both time and frequency analysis of signals simultaneously because of the fact that the energy of wavelets is concentrated in time and still possesses the wave-like (periodic) characteristics. As a result, wavelet representation provides a versatile mathematical tool to analyze transient, time-variant (non stationary) signals that are not statistically predictable especially at the region of discontinuities-a feature that is typical of images having discontinuities at the edges [24].

## 3.2 Mathematical Representation of Wavelet

Wavelets are functions generated from one single function (basis function) called the prototype or mother wavelet by dilations (scalings) and translations (shifts) in time (frequency) domain.

If the mother wavelet is denoted by $\psi$ (t), the other wavelets $\psi_{a,b}$ (t) can be represented as

$$\psi_{a,b}(t) = (1 * \psi((t-b)/a))/\sqrt{|a|} \qquad \text{...( 3.1)}$$

where a and b are two arbitrary real numbers. The variables ‘a’ and ‘b’ represent the parameters for dilations and translations respectively in the time axis.

The mother wavelet can be essentially represented as

$$\psi(t) = \psi_{1,0}(t) \qquad \text{...(3.2)}$$

For any arbitrary a $\neq$ 1 and b = 0, we can derive that

$$\psi_{a,0}(t) = (1 * \psi(t/a))\sqrt{|a|} \qquad \text{...(3.3)}$$

As shown above, $\psi_{a,0}(t)$ is nothing but a time-scaled (by a) and amplitude-scaled (by $\sqrt{|a|}$ ) version of the mother wavelet function $\psi$ (t). The parameter ‘a’ causes contraction of $\psi$ (t) in the time axis when a < 1 and expansion or stretching when a > 1. That's why the parameter ‘a’ is called the dilation (scaling) parameter. For a < 0, the function $\psi_{a,b}(t)$ results in time reversal with dilation. Mathematically, we can substitute't' in equation by (t – b) to cause a translation or shift in the time axis resulting in the wavelet function $\psi_{a,b}(t)$ as shown in equation 3.1.

The function $\psi_{a,b}(t)$ is a shift of $\psi_{a,0}(t)$ in right along the time axis by an amount b when b > 0 whereas it is a shift in left along the time axis by an amount b when b < 0. That's why the variable b represents the translation in time (shift in frequency) domain [22].

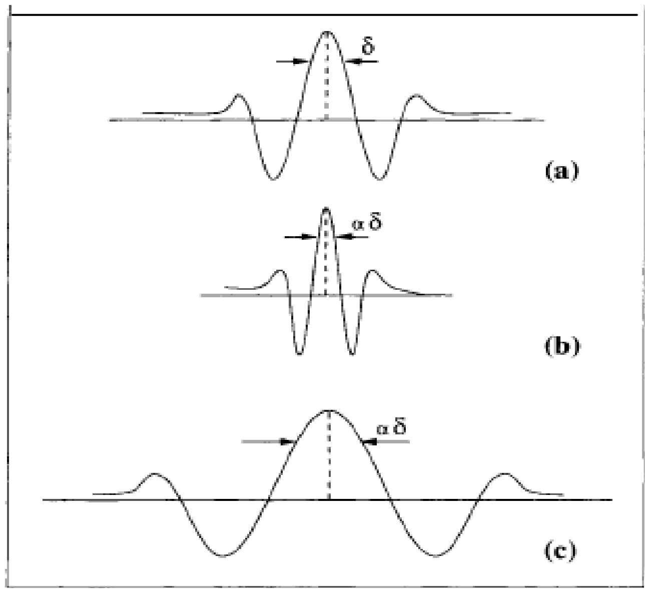



**Figure 3.2 a) Mother wavelet, ψ (t);b) ψ (t/α): o<α<1;c)ψ (t/α):α >1**

## 3.3 Translation and Scale in WT

**TRANSLATION** is related to the location of the window, as the window is shifted through the signal. It corresponds to time information in the transform domain [23]. It simply means delaying (or hastening) its onset.

Mathematically, delaying a function $\psi(t)$ by $k$ is represented by $\psi(t-k)$ [25].

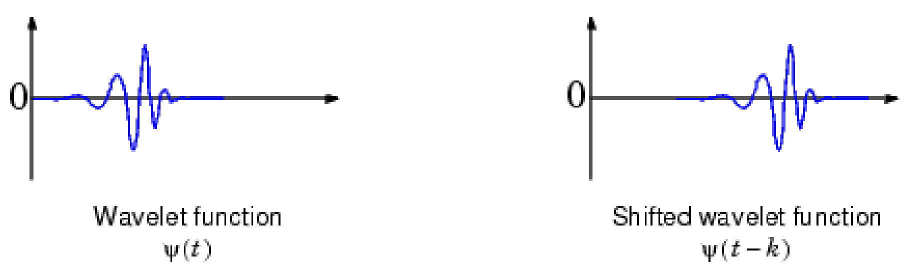



**Figure 3.3 Translation**

However, we do not have a frequency parameter, (as in STFT). Instead, we have scale parameter which is defined as inverse of frequency.

**SCALE** is a parameter in the WAVELET analysis that is quite similar to the scale used in maps. In case of maps, high scales correspond to a non-detailed global view and low scales correspond to a detailed view.

Similarly in case of frequency, low frequencies (high scales) correspond to a global information of a signal (that usually spans the entire signal), whereas high frequencies (low scales) correspond to a detailed information of a hidden pattern in the signal (that usually lasts for a relatively short time).Scaling, as a mathematical operation, either dilates or compresses a signal. Larger scales correspond to dilated (or stretched out) signals and small scales correspond to compressed signals.

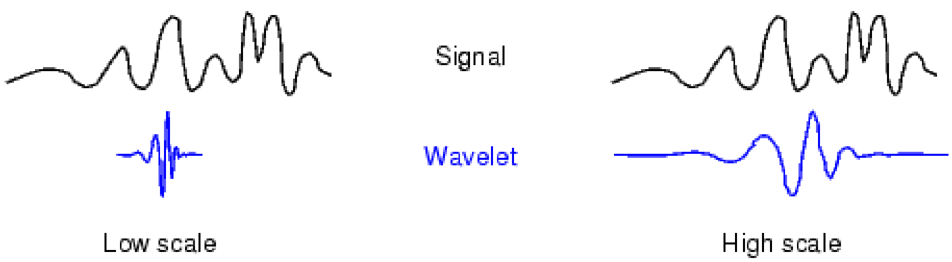



**Figure 3.4 Scaling**

If f (t) is a given function, then f (st) corresponds to a contracted (compressed) version of f (t) if s>1 and to an expanded (dilated) version of f (t) if s<1.However, in WT, the scaling term is used in the denominator and hence s>1 dilates the signal and s<1 compresses the signal [23].

## 3.4 Multi-Resolution Analysis in WT

MULTI-RESOLUTION ANALYSIS, as the name itself suggests, analyzes the signal at different frequencies with different resolutions. Here, every spectral component is not resolved equally as was the case in the STFT.

MRA provides an alternative approach to analyze any signal, although the TIME and FREQUENCY resolution problems are results of a phenomenon (the Heisenberg's Uncertainty Principle) and exist regardless of the transform used.

MRA is designed to give good time resolution and poor frequency resolution at high frequencies and good frequency resolution and poor time resolution at low frequencies.

This approach makes sense especially when the signal at hand has high frequency components for short durations and low frequency components for long durations. In practical applications as well, we face such problems [23, 28].

## 3.5 Properties of Wavelet

- 'Regularity' defined as: if r is an integer and a function is r-time continuously differentiable at $x_0$, then the regularity is r. If r is not an integer, let $n$ be the integer such that n <r<n+1, then function has a regularity of r in $x_0$ if its derivative of order $n$ resembles $(x-x_0)^{r-n}$ locally around $x_0$. This property is useful for getting nice features, such as smoothness, of the reconstructed signals.
- The support of a function is the smallest space-set (or time-set) outside of which function is identically zero.
- The number of vanishing moments of wavelets determines the order of the polynomial that can be approximated and is useful for compression purposes.
- The wavelet symmetry relates to the symmetry of the filters and helps to avoid dephasing in image processing. Among the orthogonal families, the Haar wavelet is the only symmetric wavelet. For biorthogonal wavelets it is possible to synthesize wavelet functions and scaling functions that are symmetric or antisymmetric [22].

## 3.6 Types of Wavelet Transforms

There are mainly two types of Wavelet Transforms-

- Continuous Wavelet Transformation (CWT)
- Discrete Wavelet Transformation (DWT)

Since our algorithm is to be based on discrete wavelet transform, so we will discuss only the concepts of DWT (leaving CWT as such) in the following paragraphs.

Two commonly used abbreviations are DWT and IDWT

**DWT** stands for Discrete Wavelet Transformation. It is the Transformation of sampled data, e.g. transformation of values in an array, into wavelet coefficients.

**IDWT** is Inverse Discrete Wavelet Transformation: procedure converts wavelet coefficients into the original sampled data.

Here the case of square images has been considered. Let us take an N by N image.

### 3.6.1 Decomposition Process

To start with, the image is high and low-pass filtered along the rows and the results of each filter are down- sampled by two. Those two sub-signals correspond to the high and low frequency components along the rows and are each of size N by N/2. Then each of these sub-signals is again high and low-pass filtered, along the column data. The results are again down-sampled by two.

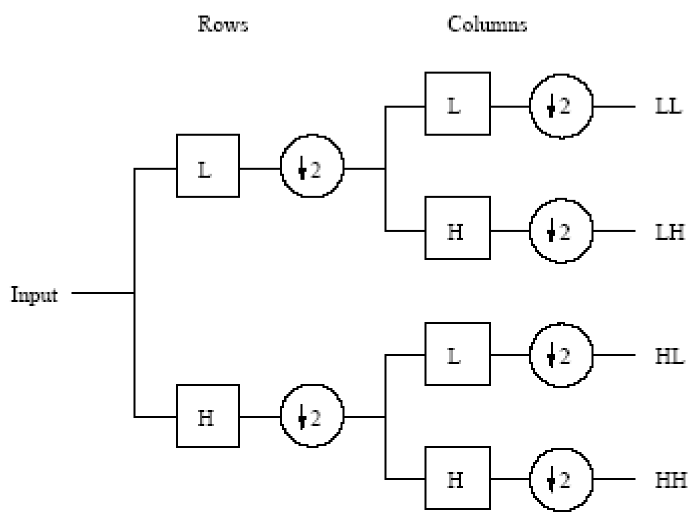



**Figure 3.5 One decomposition step of the two dimensional image**

As a result the original data is split into four sub-images each of size N/2 by N/2 containing information from different frequency components. Figure 3.5 shows the level one decomposition step of the two dimensional grayscale image. Figure 3.6 shows the four sub bands in the typical arrangement.

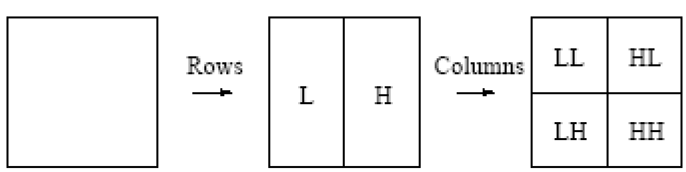



**Figure 3.6 One DWT decomposition step**

The LL subband is the result of low-pass filtering both the rows and columns and it contains a rough description of the image as such. Hence, the LL subband is also called the approximation subband. The HH subband is high-pass filtered in both directions and contains the high-frequency components along the diagonals as well. The HL and LH images are the result of low-pass filtering in one direction and high-pass filtering in another direction. LH contains mostly the vertical detail information that corresponds to horizontal edges. HL represents the horizontal detail information from the vertical edges. All three subbands HL, LH and HH are called the detail subbands, because they add the high-frequency detail to the approximation image.

### 3.6.2 Composition Process

The inverse process is shown in Figure 3.7. The information from the four sub-images is up-sampled and then filtered with the corresponding inverse filters along the columns. The two results that belong together are added and then again up-sampled and filtered with the corresponding inverse filters. The result of the last step is added together in order to get the original image again. Note that there is no loss of information when the image is decomposed and then composed again at full precision.

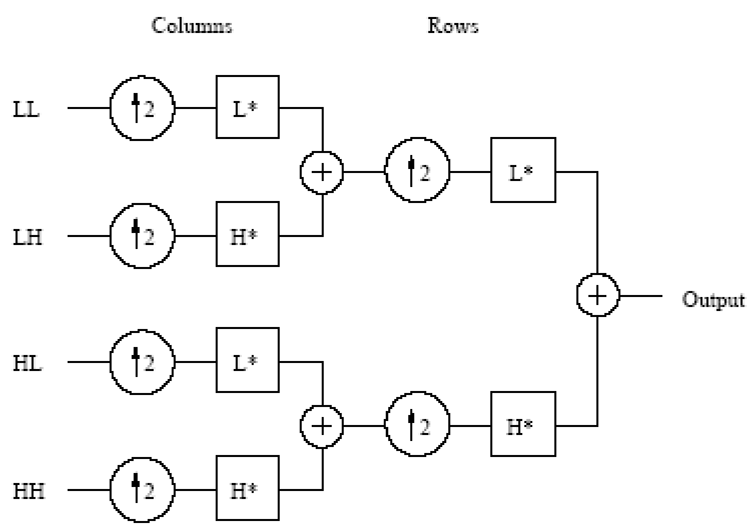



**Figure 3.7 One composition step of the four sub images**

With DWT we can decompose an image more than once. Decomposition can be continued until the signal has been entirely decomposed or can be stopped before by the application at hand.

Mostly two ways of decomposition are used. They are:

i.) Pyramidal decomposition

ii.) Packet decomposition

### 3.6.3 Pyramidal Decomposition

Pyramidal decomposition is the simplest and most common form of decomposition used. For the pyramidal decomposition we only apply further decompositions to the LL subband. Figure 3.8 shows a systematic diagram of three decomposition steps. At each level the detail subbands are the final results and only the approximation subband is further decomposed [25].

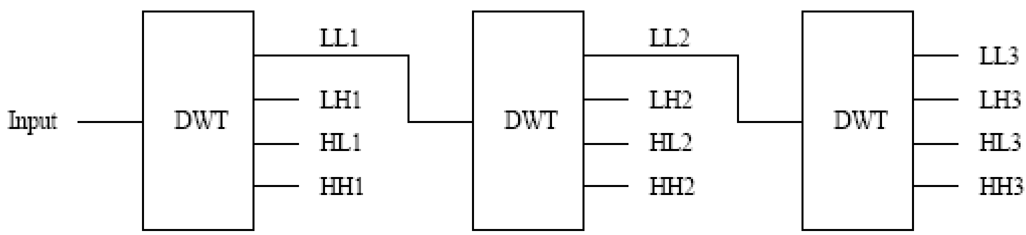



**Figure 3.8 Three decomposition steps of an image using Pyramidal Decomposition**

Figure 3.9 shows the pyramidal structure that result from this decomposition. At the lowest level there is one approximation subband and there are a total of nine detail subbands at the different levels. After L decompositions, a total of D (L) = 3 * L + 1 subbands are obtained.

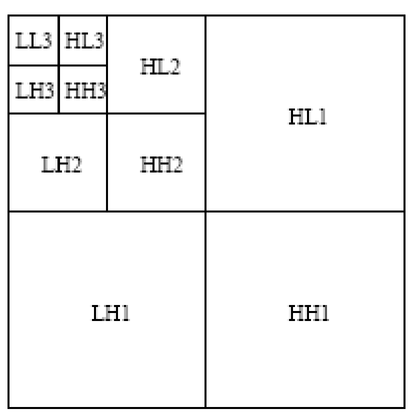



**Figure 3.9 Pyramid after three decomposition steps**

Figure 3.10 is an example of this decomposition process. It shows the “Lena" image after one, two and three pyramidal decomposition steps [27].

**Figure 3.10 Pyramidal decomposition of Lena image (1, 2 and 3 times)**

### 3.6.4 Wavelet Packet Decomposition

For the wavelet packet decomposition, the decomposition is not limited to the approximation subband only but a further wavelet decomposition of all subbands on all levels is considered. In figure 3.11, the system diagram for a complete two level wavelet packet decomposition has been shown.

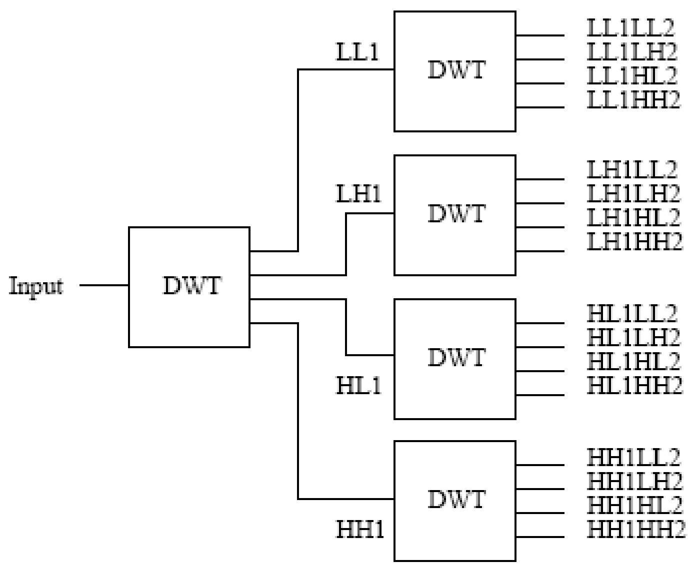



**Figure 3.11 Two complete decomposition steps using wavelet packet decomposition**

In figure 3.12, the resulting subband structure is on display. Again the simple decomposition step from 3.5 is used as a basic building block. The composition step is equivalent to the pyramidal case. All four subbands on one level are used as input for the inverse transformation and a resultant in the subband on the higher level is obtained. This process is repeated again and again until the original image is reproduced.

| | | | |
|---|---|---|---|
| LL1LL2 | LL1HL2 | HL1LL2 | HL1HL2 |
| LL1LH2 | LL1HH2 | HL1LH2 | HL1HH2 |
| LH1LL2 | LH1HL2 | HH1LL2 | HH1HL2 |
| LH1LH2 | LH1HH2 | HH1LH2 | HH1HH2 |

**Figure 3.12 Subband structure after two level packet decomposition.**

The discrete wavelet transform is very efficient from the computational point of view. Its only drawback is that it is not translation invariant. Translations of the original signal lead to different wavelet coefficients. In order to overcome this and to get more complete characteristic of the analyzed signal the undecimated wavelet transform was proposed. The general idea behind it is that it doesn't decimate the signal. Thus it produces more precise information for the frequency localization. From the computational point of view the undecimated wavelet transform has larger storage space requirements and involves more computations [25].

## 3.7 Undecimated Wavelet Transform

UDWT is based on the idea of no decimation. It applies the wavelet transform and omits both down-sampling in the forward and up-sampling in the inverse transform. More precisely, it applies the transform at each point of the image and saves the detail coefficients and uses the low-frequency coefficients for the next level. The size of the coefficients array does not diminish from level to level. By using all coefficients at each level, we get very well allocated high-frequency information. From level to level there is very small step in the width of the scaling filter - instead of 8 pixels at the third level of DWT; here its width is 5 pixels. Generally, the step is not a power of 2 but a sum with 2. This property is good for noise removal because the noise is usually spread over small number of neighboring pixels. With this transform the number of pixels involved in computing a given coefficient grows slower and so the relation between the frequency and spatial information is more precise. In the ideal case, this means removal of the noise only at the places that it really exists, without affecting the neighboring pixels. It gives the best results in terms of visual quality (less blurring for larger noise removal) [26].

## 3.8 Wavelet Families

There are a number of basis functions that can be used as the mother wavelet for Wavelet Transformation. Since the mother wavelet produces all wavelet functions used in the transformation through translation and scaling, it determines the characteristics of the resulting Wavelet Transform. Therefore, the details of the particular application should be taken into account and the appropriate mother wavelet should be chosen in order to use the Wavelet Transform effectively.

Figure 3.13 illustrates some of the commonly used wavelet functions. Haar wavelet is one of the oldest and simplest wavelet. Therefore, any discussion of wavelets starts with the Haar wavelet. Daubechies wavelets are the most popular wavelets. They represent the foundations of wavelet signal processing and are used in numerous applications. These are also called Maxflat wavelets as their frequency responses have maximum flatness at frequencies 0 and R. This is a very desirable property in some applications. The Haar, Daubechies, Symlets and Coiflets are compactly supported orthogonal wavelets. These wavelets along with Meyer wavelets are capable of perfect reconstruction. The Meyer, Morlet and Mexican Hat wavelets are symmetric in shape. The wavelets are chosen based on their shape and their ability to analyze the signal in a particular application [25].

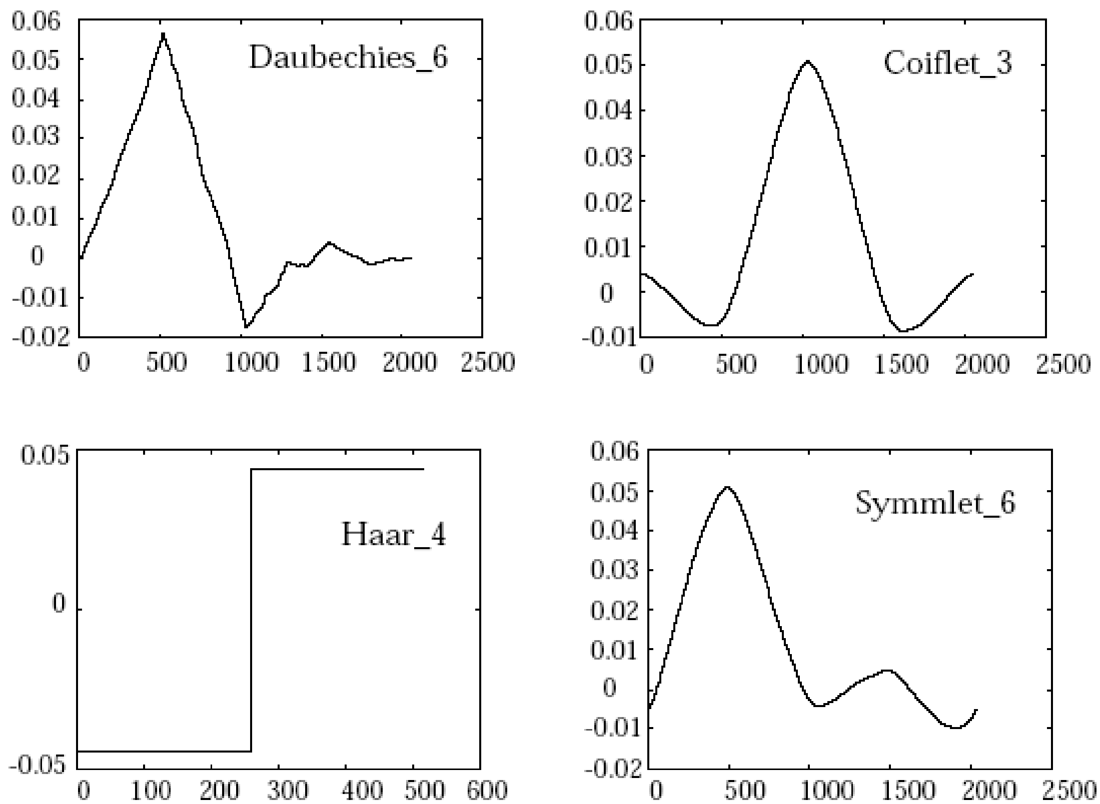



**Figure 3.13 Several different families of wavelets**

## 3.9 Wavelet Domain Advantages

Why we prefer denoising in wavelet domain, because it has many advantages. Like:

- Wavelet-based denoising provides multi-resolution hierarchical characteristics. Hence an image can be denoised at different levels of resolution and can be sequentially processed from low resolution to high resolution.
- High robustness as compared to common signal processing.

# Chapter 4

# IMAGE DENOISING

This chapter gives a brief explanation of image denoising, both in general and using wavelet. It also contains a small description of various image types and their file formats used in our work. In the end, the proposed thresholding technique for image denoising using wavelet has been mentioned.

## 4.1 Introduction

Image Denoising is a delicate and difficult task. A tradeoff between noise reduction and the preservation of actual image features occurs, in order to enhance the relevant image content. Reducing noise has always been one of the standard problems of image processing. A multitude of methods have been proposed to remove noise as it is well known that every source of noise creates a different type of noise.

The purpose of denoising is to suppress the noise from the observed signal, and help the recovery of functions of that signal. In statistical terms, this corresponds to a non parametric regression, where an orthogonal basis expansion is used to estimate the unknown function using a time regression setting.

The majority of methods are optimally applied under standard assumptions (normally distributed independent data, dyadic sample size, equal observational spacing, fixed sample points, stationarity in covariance). Often such conditions do not apply to the data at hand, and hence modified wavelet expansions have been suggested and implemented in response to this problem.

## 4.2 Wavelet Denoising

Many popular techniques of image denoising using wavelets are based on wavelet shrinkage and wavelet thresholding.

The computational advantage of using such estimation approaches is provided by algorithms of fast implementation. The main idea behind it is that, if the wavelet coefficients' estimates are bigger in absolute value than a certain specified threshold then the same value is either retained as such or is diminished by the amount corresponding to the threshold. The smaller coefficients are instead eliminated, hence sparsifying the wavelet expansion.

The simplest way to describe this is by considering a signal corrupted by noise:

$$y(t_i) = f(t_i) + \sigma \in (t_i) \qquad ...(4.1)$$

The goal is recovering the underlying and unknown function f measured at time $t_i$ from the noisy data $y_i$ , where the disturbance is delivered by the $\in_i$.

Wavelets provide a good expansion basis for the unknown function, because it can satisfy both parsimony and sparsity properties simultaneously; while the latter comes from the inherent distribution of the "energy" of the function over the coefficient vector, hence naturally selecting a subset of them as the most significant. It further strengthens the concept by assuming that only a few coefficients $d_{j,k}$, detail coefficients, carry information about the unknown function, while the rest are same as noise.

Denoising steps can be sketched as DWT $\Rightarrow d \Rightarrow \hat{d} \Rightarrow$ IDWT, where $\hat{d}$ is transformed according to various possible linear or nonlinear rules. In this thesis, UDWT and IUDWT have also been used at the same place where DWT and IDWT have been used in the above statement. Three thresholding techniques have been applied in this work -hard (discontinuous function), soft (continuous function) and custom (combination of hard and soft thresholding rules)

Reconstruction from each level shows that the pattern resulting from each reconstructed level does not seem considerably different from each other, even after a relevant part of noise is effectively removed. There could be a number of possible reasons for the lack of power of levels, in discriminating across levels on their own: one is that the features used are probably structured so that after thresholding and denoising still something common is left [30].

This thesis work involves the denoising of intensity images with formats like JPG, BMP, PNG and TIF.

## 4.3 Image types

Four types of images are there:

- **Intensity images** – An intensity image is a data matrix whose values represent intensities within some range. For the elements of class uint8 or class uint16 of an intensity image, the integer values lie between [0, 255] and [0, 65535], respectively. And if the image is of class double, then the associated values are floating- point numbers. Conventionally, the intensity images with scaled, class double data type have a range of [0, 1]. In MATLAB, an intensity image is stored as a single matrix, with each element of the matrix corresponding to one image pixel.
- **Binary images –** A binary image is a logical array of 0s and 1s. Pixels with the value 0 are displayed as black; pixels with the value 1 are displayed as white. In MATLAB, a binary image must be of class logical that is why the intensity images that happen to contain only 0's and 1's are not taken as binary images.
- **Indexed images** – An indexed image consists of a data matrix, X, and a colormap matrix termed as "map". The "map" is an m-by-3 array of class double containing floating-point values in the range [0, 1]. Its every row specifies the red, green, and blue components of a single color. For these images pixel values are directly mapped to their corresponding colormap values. The color of each image pixel is determined by using the corresponding value of X as an index into map. The value 1 points to the first row in map, the value 2 points to the second row, and so on.
- **RGB images –** An RGB image is also referred as a true-color image. In MATLAB these images are stored in the form of an m-by-n-by-3 data array that defines red, green, and blue color components for each individual pixel. The color of each pixel is determined by the combination of the red, green, and blue intensities stored in each color plane at the pixel's location. Graphics file formats store RGB images as 24-bit images, where the red, green, and blue components are 8 bits each. An RGB array can be of class double, uint8, or uint16. In an RGB array of class double, each color component is a value between 0 and 1. A pixel whose color components are (0, 0, 0) is displayed as black, and a pixel whose

color components are (1, 1, 1) is displayed as white. The three color components for each pixel are stored along the third dimension of the data array [25].

## 4.4 Image file formats

A variety of image file formats are available at present. Like TIFF, JPEG, GIF, BMP, PNG, XWD, etc. These are explained as follows:

- TIFF – stands for Tagged Image File Format. Its extension is recognized both as '.tif', '.tiff'. It is a file format used for storing images, including photographs and line art. It grew to accommodate grayscale images, then color images. Today, it is a popular format for high-color-depth images, alogwith JPEG and PNG.
- JPEG – stands for Joint Photographic Experts Group. It has '.jpg', '.jpeg' as the allowed extensions. It is the most common format for storing and transmitting photographic images on the World Wide Web and is a commonly used method of compression for photographic images.
- GIF – stands for Graphics Interchange Format. Its extension is recognized as '.gif'. It is an 8-bit-per-pixel bitmap image format and uses a palette of up to 256 distinct colors from the 24-bit RGB color space. It also supports animations and allows a separate palette of 256 colors for each frame. But its color limitation makes it unsuitable for reproducing color photographs and other continuous color images.
- BMP – stands for Windows Bitmap. Its extension is '.bmp'. It is an image file format that is used to store bitmap digital images, especially on Microsoft Windows and OS/2 operating systems.
- PNG – stands for Portable Network Graphics. It has the extension '.png'. It is a bitmapped image format and employs lossless data compression. PNG was created to improve and replace the GIF format.
- XWD – stands for X Window Dump. Its extension is '.xwd'. In actual computing world, XWD is a program that captures the screen or window contents and hence the same name is also referred to the image format it uses to save the dump [33].

## 4.5 Wavelet image denoising using Custom Thresholding

Till date custom thresholding technique has only been applied to estimate a signal that is corrupted by an additive noise, using wavelet transform. In this thesis, we have tried to propose the same custom thresholding technique for denoising images using wavelet transforms.

Custom thresholding refers to a combination of both soft and hard thresholding techniques. It is continuous around the threshold value and is adaptive to the characteristics of the input signal (in this work it has been tried and tested on the standard test images instead of signals). It is similar to the hard thresholding function but follows a smooth transition around the threshold value, thld.

Custom thresholding function can be defined as follows:

$$f_c(x) = \{x - sign(x)(1-\alpha)\lambda........if|x| \geq \lambda \qquad ...(4.2)$$

$$\{\ 0.........if|x| \leq \gamma$$

$$\{\ \alpha\lambda(\frac{|x|-\gamma}{\lambda-\gamma})^2\{(\alpha-3)(\frac{|x|-\gamma}{\lambda-\gamma})+4-\alpha\}.....otherwise$$

where $0 < \gamma < \lambda; 0 \leq \alpha \leq 1$

$\lambda = thld$, $\gamma$ is the cut-off value value, below which the wavelet coefficients are set to be zero, and $\alpha$ is the parameter that decides the shape of the thresholding function.

Generally, it is known that soft thresholding outperforms hard thresholding but there are some cases where hard thresholding yields a much superior result and in those cases it has been said that the quality of the estimate can be improved by using custom thresholding function [7].

# Chapter 5

# THE EXPLICIT ALGORITHM

This chapter contains the stepwise, detailed methodology that is followed while denoising images using wavelet transforms. For better and easy understanding, a complete flowchart of the discussed methodology has been shown at the end of this chapter.

The main algorithm, followed in order to fulfill the aim of this thesis, is as follows:

**Step 1:**

Read the original standard image (LENA.bmp, MANDRILL.png, BARBARA.jpg, BOAT.png, MRI.jpg, BRAIN.tif).

**Step 2:**

Resize the loaded image to a standard size of 256 × 256. The images taken for testification have a lot of variation in their sizes and hence cannot be compared on the same basis. For large sized images, such as $512 \times 512$, the computation time for denoising is found to be more. And if the image size is taken smaller than $256 \times 256$ , then the useful data is liable to get lost.

All the "resized" standard images are shown in the Appendix A.

**Step 3:**

Noise is added to the standard test images using the following type of available noise:

- **Gaussian noise** - This type of noise adds normal distributed noise to the original image. The noise is independent of the image it is applied to. The value of the pixel is altered by the additive Gaussian noise as

  $$J(k,l) = x(k,l) + n \qquad ...(5.1)$$

  where $n$ is the noise, $n \sim N(0,v)$ , being distributed normally with variance $v$ .

The noisy pixels which are generated are anywhere between black and white, distributed according to the Gaussian curve. The width of the curve is adjusted with the variance parameter. In our case, variance is taken 0.09 which is quite within the permissible limits of [0 1].

- **Poisson noise** - Poisson noise is generated from the data instead of adding artificial noise to the data. . If I, the original image, is double precision, then input pixel values are interpreted as means of Poisson distributions scaled up by 1e12. If I is uint8 or uint16, then input pixel values are used directly without scaling. Poisson noise generates a noise sequence of integer numbers having a Poisson probability distribution
- $p(x) = \frac{\mu^x}{x!} \cdot e^{-\mu}$ ...(5.2)
- **Speckle noise** - Speckle adds multiplicative noise to the image according to the following formula:

  $$J = I + n * I \quad ...(5.3)$$

  where n is an array with the size of the original image, filled with random values resulting from a normal distribution (Gaussian distribution) with mean 0 and are controlled by the variance. With this type of noise, noise generation is dependent on the original image, hence the product in the formula. In dark areas (where values are 0 or close to 0) no noise is generated. Variance is taken to be 0.09.
- **Salt & Pepper noise -** This is the simplest type of noise among all. According to a given density D more or less pixels are flipped randomly to black (0) or white (1). D is just a measure of the amount of noise to be added, not a value! This type of noise is also independent of the image it is applied to. D has been taken 0.09 [29].

Some samples of noisy images are shown in the Appendix B.

**Step 4:**

Make the noisy image to undergo wavelet transform, both DWT and UDWT.

- In general, linear approximation systems are often sub-optimal, due mainly to the functional complexity involved in any cases. Thus, instead of following the rule of selecting N approximating terms, it is preferable to adhere to adaptive criteria and nonlinear schemes like wavelet transform.

- A well-known orthogonal basis expansion is obtained by discrete wavelet transform $WT^d$, by which a map $f \to w$ is implemented via a bank of quadrature mirror filters , by $w = W^d f$, and coefficients at high/low scales (with high and low frequency content, respectively) are obtained. If an orthonormal wavelet basis is used, such as daublets, symmlets or coiflets, then

  $$y = f + \xi \qquad \text{...(5.4)}$$

  becomes transformed in:

  $$W^d y = W^d f + W^d \xi \equiv g + \eta \qquad \text{...(5.5)}$$

  This transformation preserves Gaussianity (as from the noise $\xi$) and produces decorrelation for autocorrelated systems.

- An extension of the above is non-orthogonal non-decimated wavelet transform $WT^u$, i.e. a conservative transform for which the expansion coefficients are not eliminated while obtaining them resolution-wise, unlike with transforms where the decimation occurs when changing scale. It is characterized by a matrix $W^u$ of size $\overline{N} \times N$, for, $\overline{N} \geq N$ and a redundant system is found, together with a pseudo-inverse transform $W^{u-}$, such that

  $$W^{u-} W = I \qquad \text{...(5.6)}$$

  Now for $y = f + z$, $WT^u$ decomposes as:

  $$W^u y = W^u f + W^u z \equiv h + \in \qquad \text{...(5.7)}$$

  while the Gaussian property is still preserved.

- There are various wavelet families that can be used to approximate many types of functions that when transformed assume a sparser or simplified structure. Wavelets refer to a set of functions generated by dilation and translation of a compactly supported scaling function (or father wavelet) and a mother wavelet, $\phi$ and $\psi$, respectively associated with a multi-resolutive analysis of $L_2(R)$. Multi-resolution techniques represent both adaptive and time-frequency localized solutions, deal with non-linear complex dynamics and non-stationary systems, and have strong computational and theoretical motivation.

Generally speaking, with a $WT^d$, a sequence of smoothed signals and of details , giving information at finer resolution levels, is found and may be used to represent a signal expansion:

$$f(x) = \sum_k c_{j_0k}\phi_{j_0k}(x) + \sum_{j>j_0}\sum_k d_{jk}\psi_{jk}(x) \qquad ...(5.8)$$

where $\phi_{j_0k}$ is associated with the corresponding coarse resolution coefficients $c_{j_0k}$ and $d_{jk}$ are the detail coefficients, given as

$$c_{jk} = \int f(x)\phi_{jk}(x)dx \qquad ...(5.9)$$

$$d_{jk} = \int f(x)\psi_{jk}(x)dx \qquad ...(5.10)$$

In short, the first term of the right hand side of the above equation is the projection of f onto the coarse approximating space $V_{j_0}$ while the second term represents the cumulated details [30].

**Step 5:**

After the noisy image is decomposed into approximation and detail coefficients using wavelet transform, it is made to undergo the following thresholding rules having various threshold values. In addition, two cases have been considered- one where the low pass components are not thresholded and the other being the one where the low pass components have been thresholded. The thresholding techniques applied are as follows,

- **Soft Thresholding** – refers to the procedure where firstly the input elements with absolute value lower than the set threshold value, are set to zero and are then scaled to the non-zero coefficients toward zero. It eliminates discontinuity and gives more visually pleasant images.

$$x = abs(y) \qquad ...(5.11)$$

$$x = sign(y).*(x \geq thld).*(x - thld) \qquad ...(5.12)$$

where y is the input, thld is the threshold value and x is the thresholded output.

- **Hard Thresholding** – refers to the procedure where the input elements with absolute value lower than the set threshold value, are set to zero. It is discontinuous at the point where $|x| = thld$ and yields abrupt artifacts in the recovered images especially when the noise energy is significant.

$$x = (abs(y) > thld).*\, y) \qquad ...(5.13)$$

- **Custom Thresholding** – refers to a combination of both soft and hard thresholding techniques. It is continuous around the threshold and is adaptive to the characteristics of the input signal (in this work it has been tried and tested on the standard test images instead of signals). It is similar to the hard thresholding function but follows a smooth transition around the threshold value, thld.

  Custom thresholding function can be defined as follows:

$$f_c(x) = \{x - sign(x)(1-\alpha)\lambda ........ if\, |x| \geq \lambda \qquad ...(5.14)$$

$$\{\, 0 .......... if\, |x| \leq \gamma$$

$$\{\, \alpha\lambda(\frac{|x|-\gamma}{\lambda-\gamma})^2 \{(\alpha-3)(\frac{|x|-\gamma}{\lambda-\gamma}) + 4 - \alpha\} ..... otherwise$$

  where $0 < \gamma < \lambda; 0 \leq \alpha \leq 1$

  $\lambda = thld$, $\gamma$ is the cut-off value value, below which the wavelet coefficients are set to be zero, and $\alpha$ is the parameter that decides the shape of the thresholding function.

  Generally, it is known that soft thresholding outperforms hard thresholding but there are some cases where hard thresholding yields a much superior result and in those cases it has been said that the quality of the estimate can be improved by using custom thresholding function [7].

Each thresholding technique has been applied for three set threshold values. The threshold values are:

- **STD** – STD stands for standard deviation. It is the classical numerical standard deviation estimation method.

  $$thld = c * std(tmp(:)) \qquad ...(5.15)$$

  $$tmp = xd(floor(mx/2)+1:mx, floor(nx/2)+1:nx); DWT$$

  $$tmp = xh(:, c\_offset : c\_offset + nx - 1); UDWT$$

  where $c = 3.0$ for DWT and $c = 3.6$ for UDWT ; $mx$ and $nx$ give the size of the noisy image and $xd$ is the decomposed image in DWT and $xh$ is the high frequency component of the decomposed image in case of UDWT .

- **MAD** – MAD stands for median absolute deviation. The MAD is less efficient than the standard deviation as an estimate of the spread when all the data is from the normal distribution. The default version of MAD, based on means, is also commonly referred to as the average absolute deviation (AAD).

  $$thld = c * median(abs(tmp(:)))/.67 \qquad ...(5.16)$$

  $$tmp = xd(floor(mx/2)+1:mx, floor(nx/2)+1:nx); DWT$$

  $$tmp = xh(:, c\_offset : c\_offset + nx - 1); UDWT$$

  where $c = 3.0$ for DWT and $c = 3.6$ for UDWT ; $mx$ and $nx$ give the size of the noisy image and $xd$ is the decomposed image in DWT and $xh$ is the high frequency component of the decomposed image in case of UDWT .

- **UNIV** – UNIV stands for universal threshold value. It estimates the standard deviation using median.

  $$thld = c * median(median(abs(xd)))/0{,}6745; forDWT \qquad ...(5.17)$$

  $$thld = c * median(median(abs(xh)))/0.6745; forUDWT \qquad ...(5.18)$$

  where $c = 3.0$ for DWT and $c = 3.6$ for UDWT ; $mx$ and $nx$ give the size of the noisy image and $xd$ is the decomposed image in DWT and $xh$ is the high frequency component of the decomposed image in case of UDWT [25].

**Step 6**:

After the decomposed image coefficients are thresholded using the above mentioned three threshold values with each of the thresholding technique, the denoised image is reconstructed using inverse wavelet transforms- IDWT and IUDWT.

A few denoised image samples obtained by the use of both type of wavelet transforms are shown in the Appendix C.

**Step 7:**

Then two parameters, PSNR (peak signal to noise ratio) and MSSIM (mean structural similarity index) are calculated for all the standard images with their noisy and denoised counterparts, respectively. Hence, we get a good amount of comparison between the noisy and denoised images keeping the set standard image intact.

- **PSNR –** PSNR stands for the peak signal to noise ratio. It is an engineering term used to calculate the ratio between the maximum possible power of a signal and the power of corrupting noise that affects the fidelity of its representation. Because many signals have a very wide dynamic range, PSNR is usually expressed in terms of the logarithmic decibel scale. It is most commonly used as a measure of quality of reconstruction in image compression etc. It is calculated as the following:

$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}\left\| I(i,j) - K(i,j)\right\|^2 \qquad ...(5.19)$$

$$PSNR = 10.*\log_{10}\left(\frac{MAX_I^2}{MSE}\right) \qquad ...(5.20)$$

where I and K are the original and noisy/ denoised image, respectively. $MAX_I$ is the maximum possible pixel value of the image. When the pixels are represented using 8 bits per sample, this is equivalent to 255, and in this work as well it is 255.

At one time, we calculate PSNR for original with noisy image and refer it as PSNR (O/N). After the image is denoised, it is calculated for original with denoised image and is then referred as PSNR (O/D). Hence, it shows the improvement in the noisy image after denoising, if any. An identical image to the original will yield an undefined PSNR as the MSE will become equal to zero due to no error. In this case the PSNR value can be thought of as approaching infinity as the MSE approaches zero; this shows that a higher PSNR value provides a higher image quality [31].

- **MSSIM –** MSSIM stands for mean structural similarity index. Natural image signals are highly structured. Their pixels exhibit strong dependencies, especially when they are spatially proximate, and these dependencies carry important information about the structure of the objects in the visual scene. The structural information in an image is defined as those attributes that represent the structure of objects in the scene, independent of the average luminance and contrast.

  The system separates the task of similarity measurement into three comparisons: luminance, contrast and structure. For discrete signals, luminance is estimated as the mean intensity and the standard deviation (the square root of variance) is taken as an estimate of the signal contrast. Then, the signal is normalized (divided) by its own standard deviation, so that the two signals being compared have unit standard deviation.

  Suppose x and y are two nonnegative image signals, which have been aligned with each other. First, the luminance of each signal is compared. The luminance comparison function $l(x, y)$ is given as a function of $\mu_x$ and $\mu_y$, where

  $$\mu_x = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad ...(5.21)$$

  Then the mean intensity from the signal is removed. In discrete form, the resulting signal $x - \mu_x$ corresponds to the projection of vector x onto the hyperplane defined by $\sum_{i=1}^{N} x_i = 0$ .The standard deviation which gives an estimate of the contrast is given by

  $$\sigma_x = \left(\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu_x)^2\right)^{1/2} \qquad ...(5.22)$$

The contrast comparison $c(x, y)$ is the comparison of $\sigma_x$ and $\sigma_y$. After this the signal is normalized (divided) by its own standard deviation, so that the two signals being compared have unit standard deviation. The structure comparison $s(x, y)$ is conducted on these normalized signals $(x - \mu_x)/\sigma_x$ and $(y - \mu_y)/\sigma_y$ .

Finally, the three components are combined to yield an overall similarity measure:

$$S(x, y) = f(l(x, y), c(x, y), s(x, y)) \quad ...(5.23)$$

The three components are relatively independent. The similarity measure should also satisfy the following conditions:

1. Symmetry: $S(x, y) = S(y, x)$;
2. Bounded ness: $S(x, y) \leq 1$;
3. Unique maximum: $S(x, y) = 1$ if and only if x = y (in discrete representations, $x_i = y_i$ for all $i = 1,2.........N$ )

For luminance comparison, we define

$$l(x, y) = \frac{2\mu_x\mu_y + C_1}{\mu_x^2 + \mu_y^2 + C_1} \quad ...(5.24)$$

where the constant $C1$ is included to avoid instability when $\mu_x^2 + \mu_y^2$ is very close to zero. Specifically, we choose $C_1 = (K_1L)^2$ where $L$ is the dynamic range of the pixel values (255 for 8-bit grayscale images), and $K_1 << 1$ is a small constant.

Similar considerations also apply to contrast comparison and structure comparison.

The contrast comparison function takes a similar form:

$$c(x, y) = \frac{2\sigma_x\sigma_y + C_2}{\sigma_x^2 + \sigma_y^2 + C_2} \quad ...(5.25)$$

where $C_2 = (K_2L)^2$ and $K_2 << 1$.

Structure comparison is conducted after luminance subtraction and variance normalization. The structure comparison function is defined as follows:

$$s(x, y) = \frac{\sigma_{xy} + C_3}{\sigma_x \sigma_y + C_3} \quad \text{...(5.26)}$$

where

$$\sigma_{xy} = \frac{1}{N-1}\left(\sum_{i=1}^{N}(x_i - \mu_x)(y_i - \mu_y)\right) \quad \text{...(5.27)}$$

Hence a specific form of the SSIM index is obtained, given as

$$SSIM(x, y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^{\,2} + \mu_y^{\,2} + C_1)(\sigma_x^{\,2} + \sigma_y^{\,2} + C_2)} \quad \text{...(5.28)}$$

For image quality assessment, it is useful to apply the SSIM index locally rather than globally. First, image statistical features are usually highly spatially non-stationary. Second, image distortions, which may or may not depend on the local image statistics, may also be space-variant. Third, at typical viewing distances, only a local area in the image can be perceived with high resolution by the human observer at one time instance.

For this thesis, the local statistics $\mu_x, \sigma_x$ and $\sigma_{xy}$ are computed within a local $8 \times 8$ square window, which moves pixel-by-pixel over the entire image. At each step, the local statistics and SSIM index are calculated within this local window. Value of $K_1$ and $K_2$ has been taken 0.01 and 0.03, respectively.

In practice, one usually requires a single overall quality measure of the entire image. Hence, a mean SSIM (MSSIM) index value is calculated to evaluate the overall image quality:

$$MSSIM(X, Y) = \frac{1}{M}\sum_{j=1}^{M} SSIM(x_j, y_j) \quad \text{...(5.29)}$$

where X and Y are the reference and the distorted images, respectively; $x_j$ and $y_j$ are the image contents at the *j*-th local window; and *M* is the number of local windows in the image [32].

The MSSIM value is referred as MSSIM (O/N) when calculated for original image with noisy image and is similarly referred as MSSIM (O/D) when calculated for original image with denoised image.

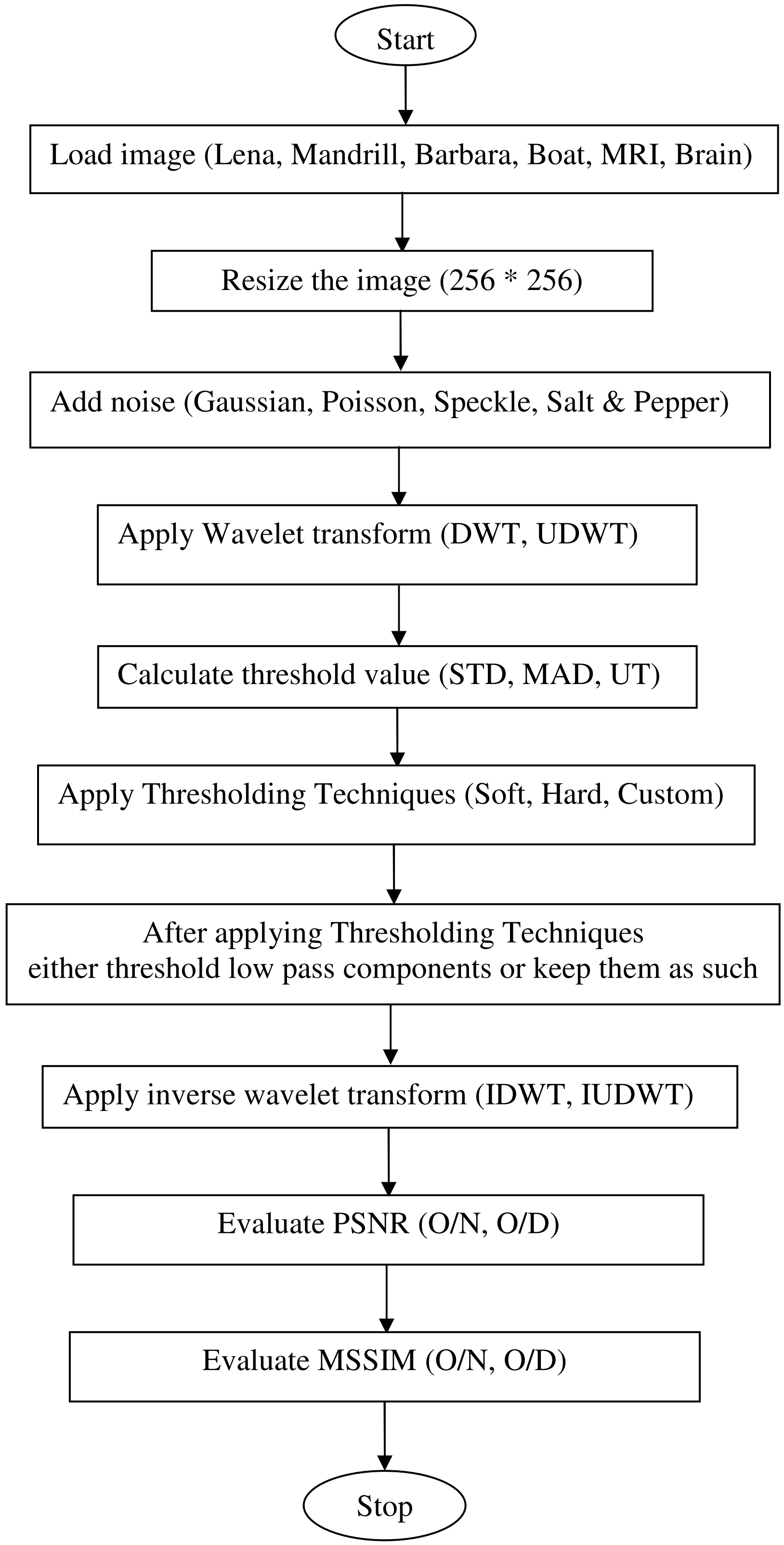



**Figure 5.1 Flowchart for Image Denoising Algorithm Using Wavelet Transform**

# Chapter 6

# EXPERIMENTAL RESULTS

The present chapter contains the results, obtained after following the wavelet denoising algorithm as discussed in the previous chapter. The results have been demonstrated in the form of comparison tables. At the chapter end, a graphical representation has also been done for a quick analysis of results. All the techniques have been tested for all the assumed standard test images.

## Image1. LENA

### Not Thresholding Low Pass Components

**a. GAUSSIAN (0, 0.09)**

**Table 6.1 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 11.6806 | 20.9731 | 0.1150 | 0.4263 |
| STD | Soft | 11.6366 | 20.8682 | 0.1121 | 0.4218 |
| UNIV | Soft | 11.6978 | 20.9860 | 0.1140 | 0.4249 |
| MAD | Hard | 11.7021 | 20.3862 | 0.1141 | 0.4036 |
| STD | Hard | 11.6757 | 20.2582 | 0.1135 | 0.3968 |
| UNIV | Hard | 11.6526 | 20.5144 | 0.1122 | 0.4078 |
| MAD | Custom | 11.6723 | 20.2806 | 0.1131 | 0.3821 |
| STD | Custom | 11.6173 | 20.2900 | 0.1122 | 0.3825 |
| UNIV | Custom | 11.6748 | 20.2725 | 0.1143 | 0.3792 |

**Table 6.2 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 11.6903 | 21.5381 | 0.1139 | 0.4987 |
| STD | Soft | 11.6609 | 21.4971 | 0.1134 | 0.5021 |
| UNIV | Soft | 11.6358 | 21.5237 | 0.1120 | 0.5021 |
| MAD | Hard | 11.6847 | 21.5544 | 0.1158 | 0.5027 |
| STD | Hard | 11.6451 | 21.4983 | 0.1137 | 0.5040 |
| UNIV | Hard | 11.6493 | 21.4672 | 0.1113 | 0.4939 |
| MAD | Custom | 11.6525 | 21.7645 | 0.1161 | 0.5013 |
| STD | Custom | 11.6572 | 21.5028 | 0.1121 | 0.4939 |
| UNIV | Custom | 11.6392 | 21.4404 | 0.1131 | 0.4946 |

**b. POISSON**

**Table 6.3 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 27.1959 | 27.6951 | 0.6659 | 0.7895 |
| STD | Soft | 27.2228 | 27.3216 | 0.6672 | 0.7781 |
| UNIV | Soft | 27.1794 | 27.1046 | 0.6653 | 0.7715 |
| MAD | Hard | 27.1911 | 29.1167 | 0.6665 | 0.8106 |
| STD | Hard | 27.2235 | 28.9063 | 0.6667 | 0.8111 |
| UNIV | Hard | 27.2266 | 28.7761 | 0.6669 | 0.8098 |
| MAD | Custom | 27.2280 | 26.5822 | 0.6673 | 0.7713 |
| STD | Custom | 27.1529 | 26.5464 | 0.6655 | 0.7720 |
| UNIV | Custom | 27.1702 | 26.5192 | 0.6651 | 0.7710 |

**Table 6.4 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 27.1960 | 27.6247 | 0.6664 | 0.8130 |
| STD | Soft | 27.1616 | 27.3435 | 0.6653 | 0.8044 |
| UNIV | Soft | 27.2455 | 27.0206 | 0.6673 | 0.7979 |
| MAD | Hard | 27.1676 | 30.4224 | 0.6646 | 0.8619 |
| STD | Hard | 27.1922 | 30.0409 | 0.6661 | 0.8549 |
| UNIV | Hard | 27.2065 | 29.7740 | 0.6665 | 0.8499 |
| MAD | Custom | 27.1379 | 28.1916 | 0.6633 | 0.8341 |
| STD | Custom | 27.2039 | 28.0250 | 0.6663 | 0.8294 |
| UNIV | Custom | 27.1887 | 27.9512 | 0.6668 | 0.8280 |

**c. SPECKLE (v=0.09)**

**Table 6.5 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 16.4085 | 23.5782 | 0.2940 | 0.5728 |
| STD | Soft | 16.4225 | 23.4101 | 0.2927 | 0.5668 |
| UNIV | Soft | 16.4561 | 23.5052 | 0.2926 | 0.5741 |
| MAD | Hard | 16.4376 | 21.9102 | 0.2924 | 0.4915 |
| STD | Hard | 16.4434 | 22.6509 | 0.2968 | 0.5354 |
| UNIV | Hard | 16.4239 | 22.6955 | 0.2946 | 0.5364 |
| MAD | Custom | 16.4147 | 22.5164 | 0.2942 | 0.5065 |
| STD | Custom | 16.4167 | 22.8263 | 0.2922 | 0.5209 |
| UNIV | Custom | 16.4218 | 22.8247 | 0.2931 | 0.5230 |

**Table 6.6 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 16.4147 | 24.2224 | 0.2940 | 0.6465 |
| STD | Soft | 16.4134 | 23.9567 | 0.2937 | 0.6445 |
| UNIV | Soft | 16.4278 | 24.0203 | 0.2927 | 0.6425 |
| MAD | Hard | 16.4402 | 24.6551 | 0.2942 | 0.6325 |
| STD | Hard | 16.4349 | 24.7047 | 0.2948 | 0.6494 |
| UNIV | Hard | 16.3988 | 24.3393 | 0.2939 | 0.6460 |
| MAD | Custom | 16.4457 | 24.2153 | 0.2953 | 0.6259 |
| STD | Custom | 16.4385 | 24.6139 | 0.2941 | 0.6397 |
| UNIV | Custom | 16.4154 | 24.4789 | 0.2938 | 0.6390 |

**d. SALT & PEPPER (d=0.09)**

**Table 6.7 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 15.9726 | 22.3437 | 0.2756 | 0.4871 |
| STD | Soft | 15.9335 | 22.9595 | 0.2746 | 0.5584 |
| UNIV | Soft | 15.9553 | 22.8721 | 0.2756 | 0.5248 |
| MAD | Hard | 15.9366 | 17.8906 | 0.2742 | 0.2993 |
| STD | Hard | 15.9465 | 20.4522 | 0.2742 | 0.4367 |
| UNIV | Hard | 15.9700 | 18.8584 | 0.2766 | 0.3438 |
| MAD | Custom | 15.8764 | 19.9567 | 0.2726 | 0.3813 |
| STD | Custom | 15.9930 | 21.3980 | 0.2731 | 0.4740 |
| UNIV | Custom | 15.8233 | 20.5173 | 0.2677 | 0.4116 |

**Table 6.8 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 15.7775 | 23.3928 | 0.2632 | 0.6011 |
| STD | Soft | 15.8691 | 23.5281 | 0.2647 | 0.6311 |
| UNIV | Soft | 15.8749 | 23.5503 | 0.2726 | 0.6301 |
| MAD | Hard | 15.9301 | 20.8222 | 0.2669 | 0.4319 |
| STD | Hard | 15.9252 | 23.5629 | 0.2758 | 0.5930 |
| UNIV | Hard | 15.9032 | 22.2768 | 0.2705 | 0.5127 |
| MAD | Custom | 16.0557 | 22.2345 | 0.2805 | 0.5141 |
| STD | Custom | 16.0030 | 23.2269 | 0.2801 | 0.5986 |
| UNIV | Custom | 15.8889 | 22.6254 | 0.2730 | 0.5600 |

**Thresholding Low Pass Components**

**a. GAUSSIAN (0, 0.09)**

**Table 6.9 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 11.6841 | 15.5153 | 0.1130 | 0.3818 |
| STD | Soft | 11.6408 | 13.0917 | 0.1146 | 0.3646 |
| UNIV | Soft | 11.6427 | 12.5990 | 0.1140 | 0.3465 |
| MAD | Hard | 11.6625 | 16.6448 | 0.1131 | 0.2186 |
| STD | Hard | 11.6534 | 20.0022 | 0.1111 | 0.3826 |
| UNIV | Hard | 11.6513 | 20.1488 | 0.1113 | 0.3985 |
| MAD | Custom | 11.6700 | 17.9586 | 0.1123 | 0.2668 |
| STD | Custom | 11.6415 | 19.6722 | 0.1129 | 0.3673 |
| UNIV | Custom | 11.6459 | 19.7773 | 0.1141 | 0.3825 |

**Table 6.10 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 11.6969 | 11.7562 | 0.1136 | 0.3809 |
| STD | Soft | 11.6827 | 11.8619 | 0.1130 | 0.3783 |
| UNIV | Soft | 11.6825 | 11.5306 | 0.1144 | 0.3681 |
| MAD | Hard | 11.6804 | 20.7012 | 0.1130 | 0.4788 |
| STD | Hard | 11.6458 | 21.1944 | 0.1128 | 0.4794 |
| UNIV | Hard | 11.6604 | 20.5930 | 0.1123 | 0.4684 |
| MAD | Custom | 11.6377 | 20.4641 | 0.1144 | 0.4865 |
| STD | Custom | 11.7054 | 20.4713 | 0.1158 | 0.4823 |
| UNIV | Custom | 11.6645 | 20.2548 | 0.1134 | 0.4758 |

**b. POISSON**

**Table 6.11 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 27.1768 | 27.1323 | 0.6653 | 0.8184 |
| STD | Soft | 27.2417 | 24.6842 | 0.6683 | 0.7754 |
| UNIV | Soft | 27.1617 | 23.9698 | 0.6638 | 0.7688 |
| MAD | Hard | 27.2077 | 28.9890 | 0.6672 | 0.7626 |
| STD | Hard | 27.1919 | 28.8813 | 0.6666 | 0.8094 |
| UNIV | Hard | 27.1970 | 28.7105 | 0.6667 | 0.8095 |
| MAD | Custom | 27.1262 | 26.4001 | 0.6633 | 0.7365 |
| STD | Custom | 27.2000 | 26.4691 | 0.6659 | 0.7685 |
| UNIV | Custom | 27.1933 | 26.4341 | 0.6652 | 0.7704 |

**Table 6.12 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 27.1966 | 24.1714 | 0.6666 | 0.8057 |
| STD | Soft | 27.2057 | 23.6423 | 0.6676 | 0.7984 |
| UNIV | Soft | 27.1505 | 23.0069 | 0.6649 | 0.7894 |
| MAD | Hard | 27.2045 | 30.4810 | 0.6672 | 0.8596 |
| STD | Hard | 27.2211 | 30.4525 | 0.6671 | 0.8567 |
| UNIV | Hard | 27.2076 | 29.7721 | 0.6658 | 0.8502 |
| MAD | Custom | 27.1922 | 28.0598 | 0.6655 | 0.8349 |
| STD | Custom | 27.2196 | 27.8982 | 0.6667 | 0.8316 |
| UNIV | Custom | 27.1911 | 27.7629 | 0.6654 | 0.8280 |

**c. SPECKLE (v=0.09)**

**Table 6.13 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 16.4080 | 20.0773 | 0.2958 | 0.53818 |
| STD | Soft | 16.4153 | 17.3730 | 0.2950 | 0.5286 |
| UNIV | Soft | 16.4162 | 16.7371 | 0.2949 | 0.5328 |
| MAD | Hard | 16.4198 | 19.2495 | 0.2925 | 0.3816 |
| STD | Hard | 16.4147 | 22.5833 | 0.2954 | 0.5332 |
| UNIV | Hard | 16.4265 | 22.6752 | 0.2943 | 0.5328 |
| MAD | Custom | 16.4232 | 20.8646 | 0.2934 | 0.4289 |
| STD | Custom | 16.4032 | 22.4039 | 0.2932 | 0.5241 |
| UNIV | Custom | 16.3985 | 22.3787 | 0.2921 | 0.5232 |

**Table 6.14 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 16.4435 | 16.6270 | 0.2940 | 0.5949 |
| STD | Soft | 16.4367 | 15.8244 | 0.2937 | 0.5758 |
| UNIV | Soft | 16.4268 | 15.7241 | 0.2949 | 0.5737 |
| MAD | Hard | 16.4626 | 24.6712 | 0.2934 | 0.6377 |
| STD | Hard | 16.4461 | 24.3845 | 0.2931 | 0.6443 |
| UNIV | Hard | 16.4165 | 24.3065 | 0.2953 | 0.6450 |
| MAD | Custom | 16.4278 | 23.5518 | 0.2950 | 0.6255 |
| STD | Custom | 16.4309 | 23.5056 | 0.2948 | 0.6369 |
| UNIV | Custom | 16.4063 | 23.5310 | 0.2935 | 0.6323 |

**d. SALT & PEPPER (d=0.09)**

**Table 6.15 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 15.9200 | 19.7447 | 0.2713 | 0.4193 |
| STD | Soft | 15.9307 | 16.7041 | 0.2744 | 0.5210 |
| UNIV | Soft | 15.9424 | 18.3367 | 0.2735 | 0.5175 |
| MAD | Hard | 15.9725 | 16.8220 | 0.2743 | 0.2672 |
| STD | Hard | 15.9791 | 20.3510 | 0.2759 | 0.4401 |
| UNIV | Hard | 16.0146 | 18.8361 | 0.2743 | 0.3388 |
| MAD | Custom | 15.9704 | 19.2606 | 0.2769 | 0.3371 |
| STD | Custom | 15.8786 | 21.2028 | 0.2675 | 0.4681 |
| UNIV | Custom | 15.9188 | 20.4324 | 0.2711 | 0.4126 |

**Table 6.16 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 16.0128 | 18.7687 | 0.2755 | 0.5825 |
| STD | Soft | 15.8974 | 15.5502 | 0.2747 | 0.5720 |
| UNIV | Soft | 15.8434 | 16.5841 | 0.2671 | 0.5933 |
| MAD | Hard | 16.0651 | 20.5755 | 0.2811 | 0.4261 |
| STD | Hard | 15.8776 | 23.4393 | 0.2709 | 0.5980 |
| UNIV | Hard | 15.8848 | 22.4347 | 0.2716 | 0.5278 |
| MAD | Custom | 15.9359 | 21.9507 | 0.2728 | 0.5161 |
| STD | Custom | 15.8087 | 22.6964 | 0.2683 | 0.5963 |
| UNIV | Custom | 15.9137 | 22.5284 | 0.2745 | 0.5631 |

# Image2. MANDRILL

**Not Thresholding Low Pass Components**

**a. GAUSSIAN (0, 0.09)**

**Table 6.17 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 11.5655 | 18.0752 | 0.2036 | 0.2557 |
| STD | Soft | 11.5712 | 18.3631 | 0.2014 | 0.2627 |
| UNIV | Soft | 11.5959 | 18.3884 | 0.2059 | 0.2674 |
| MAD | Hard | 11.5447 | 18.2396 | 0.2010 | 0.2686 |
| STD | Hard | 11.5847 | 18.0638 | 0.2043 | 0.2704 |
| UNIV | Hard | 11.6041 | 18.2391 | 0.2072 | 0.2631 |
| MAD | Custom | 11.5662 | 18.2363 | 0.2003 | 0.2818 |
| STD | Custom | 11.5725 | 18.0721 | 0.2048 | 0.2845 |
| UNIV | Custom | 11.6016 | 18.2700 | 0.2032 | 0.2842 |

**Table 6.18 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 11.5420 | 17.5915 | 0.2021 | 0.2983 |
| STD | Soft | 11.5825 | 17.6195 | 0.2032 | 0.3071 |
| UNIV | Soft | 11.5722 | 17.6099 | 0.2039 | 0.2999 |
| MAD | Hard | 11.5631 | 17.9760 | 0.2015 | 0.3066 |
| STD | Hard | 11.5735 | 17.7237 | 0.2010 | 0.2993 |
| UNIV | Hard | 11.5988 | 17.6376 | 0.2036 | 0.3099 |
| MAD | Custom | 11.5597 | 17.9102 | 0.2017 | 0.3195 |
| STD | Custom | 11.5609 | 17.8145 | 0.2026 | 0.3210 |
| UNIV | Custom | 11.5376 | 17.7127 | 0.2003 | 0.3171 |

**b. POISSON**

**Table 6.19 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 26.8206 | 19.9888 | 0.8371 | 0.4862 |
| STD | Soft | 27.0061 | 19.4332 | 0.8374 | 0.4465 |
| UNIV | Soft | 27.0234 | 19.4647 | 0.8370 | 0.4492 |
| MAD | Hard | 26.9996 | 22.4592 | 0.8372 | 0.6252 |
| STD | Hard | 26.8422 | 21.7122 | 0.8381 | 0.5590 |
| UNIV | Hard | 27.0393 | 21.4900 | 0.8378 | 0.5548 |
| MAD | Custom | 27.0199 | 21.0658 | 0.8382 | 0.5814 |
| STD | Custom | 26.9573 | 20.6366 | 0.8374 | 0.5450 |
| UNIV | Custom | 26.9770 | 21.0641 | 0.8353 | 0.5427 |

**Table 6.20 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 26.7940 | 19.7871 | 0.8372 | 0.4685 |
| STD | Soft | 26.8117 | 19.4305 | 0.8363 | 0.4403 |
| UNIV | Soft | 26.8950 | 19.3254 | 0.8394 | 0.4318 |
| MAD | Hard | 27.0088 | 21.8878 | 0.8368 | 0.6167 |
| STD | Hard | 27.0070 | 20.8450 | 0.8370 | 0.5417 |
| UNIV | Hard | 27.0145 | 20.9950 | 0.8368 | 0.5362 |
| MAD | Custom | 26.9030 | 20.9834 | 0.8370 | 0.5912 |
| STD | Custom | 26.8943 | 20.4322 | 0.8367 | 0.5409 |
| UNIV | Custom | 26.9218 | 20.3925 | 0.8383 | 0.5367 |

**c. SPECKLE (v=0.09)**

**Table 6.21 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 16.0680 | 19.3778 | 0.4306 | 0.3407 |
| STD | Soft | 16.0782 | 19.0546 | 0.4314 | 0.3429 |
| UNIV | Soft | 16.0971 | 19.2837 | 0.4324 | 0.3401 |
| MAD | Hard | 16.0847 | 19.3447 | 0.4340 | 0.3740 |
| STD | Hard | 16.1087 | 19.3585 | 0.4320 | 0.3611 |
| UNIV | Hard | 16.0882 | 19.4089 | 0.4307 | 0.3596 |
| MAD | Custom | 16.1346 | 19.7551 | 0.4323 | 0.3980 |
| STD | Custom | 16.0957 | 19.7465 | 0.4302 | 0.3969 |
| UNIV | Custom | 16.0890 | 19.7457 | 0.4272 | 0.3865 |

**Table 6.22 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 16.1258 | 18.5735 | 0.4348 | 0.3700 |
| STD | Soft | 16.1135 | 18.6339 | 0.4340 | 0.3676 |
| UNIV | Soft | 16.1106 | 18.8305 | 0.4305 | 0.3687 |
| MAD | Hard | 16.0989 | 19.4973 | 0.4296 | 0.3893 |
| STD | Hard | 16.1238 | 19.2875 | 0.4321 | 0.3873 |
| UNIV | Hard | 16.1061 | 18.7751 | 0.4320 | 0.3800 |
| MAD | Custom | 16.0953 | 19.6400 | 0.4315 | 0.4278 |
| STD | Custom | 16.1057 | 19.1123 | 0.4321 | 0.4205 |
| UNIV | Custom | 16.1149 | 19.5262 | 0.4333 | 0.4180 |

**d. SALT & PEPPER (d=0.09)**

**Table 6.23 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 16.0657 | 18.5220 | 0.4471 | 0.3536 |
| STD | Soft | 16.0358 | 18.6925 | 0.4443 | 0.3368 |
| UNIV | Soft | 16.1293 | 19.0553 | 0.4490 | 0.3451 |
| MAD | Hard | 15.9905 | 17.6526 | 0.4397 | 0.3256 |
| STD | Hard | 16.0396 | 18.6029 | 0.4441 | 0.3235 |
| UNIV | Hard | 15.9745 | 18.1140 | 0.4381 | 0.3280 |
| MAD | Custom | 16.0948 | 18.9379 | 0.4478 | 0.3691 |
| STD | Custom | 15.9881 | 19.1080 | 0.4418 | 0.3496 |
| UNIV | Custom | 16.0519 | 19.1053 | 0.4455 | 0.3643 |

**Table 6.24 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 15.9856 | 18.5440 | 0.4402 | 0.3725 |
| STD | Soft | 16.1506 | 18.4734 | 0.4497 | 0.3654 |
| UNIV | Soft | 16.0240 | 18.4516 | 0.4433 | 0.328 |
| MAD | Hard | 16.0313 | 19.7068 | 0.4445 | 0.3912 |
| STD | Hard | 16.1109 | 19.2321 | 0.4462 | 0.3772 |
| UNIV | Hard | 15.9574 | 19.1858 | 0.4392 | 0.3729 |
| MAD | Custom | 16.0603 | 18.7953 | 0.4425 | 0.4236 |
| STD | Custom | 15.9974 | 18.6944 | 0.4411 | 0.3958 |
| UNIV | Custom | 16.0320 | 18.6656 | 0.4414 | 0.4003 |

**Thresholding Low Pass Components**

**a. GAUSSIAN (0, 0.09)**

**Table 6.25 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 11.5968 | 13.5511 | 0.2027 | 0.2671 |
| STD | Soft | 11.5775 | 11.3491 | 0.2016 | 0.2329 |
| UNIV | Soft | 11.5661 | 10.8434 | 0.2047 | 0.2211 |
| MAD | Hard | 11.5888 | 15.9456 | 0.2039 | 0.2128 |
| STD | Hard | 11.5626 | 17.9639 | 0.2055 | 0.2642 |
| UNIV | Hard | 11.5703 | 17.9984 | 0.2023 | 0.2696 |
| MAD | Custom | 11.5853 | 16.8063 | 0.2055 | 0.2638 |
| STD | Custom | 11.6004 | 17.8495 | 0.2041 | 0.2825 |
| UNIV | Custom | 11.5589 | 17.8520 | 0.2027 | 0.2797 |

**Table 6.26 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 11.5574 | 9.9843 | 0.2032 | 0.2336 |
| STD | Soft | 11.5663 | 10.1845 | 0.2040 | 0.2350 |
| UNIV | Soft | 11.5577 | 9.7403 | 0.2006 | 0.2259 |
| MAD | Hard | 11.5594 | 17.3795 | 0.2059 | 0.3103 |
| STD | Hard | 11.5779 | 17.5719 | 0.2050 | 0.3093 |
| UNIV | Hard | 11.5949 | 17.4891 | 0.2037 | 0.3038 |
| MAD | Custom | 11.6102 | 17.1228 | 0.2072 | 0.3235 |
| STD | Custom | 11.5854 | 17.1282 | 0.2039 | 0.3262 |
| UNIV | Custom | 11.5561 | 17.0074 | 0.2027 | 0.3218 |

**b. POISSON**

**Table 6.27 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 26.8803 | 20.1619 | 0.8370 | 0.5945 |
| STD | Soft | 26.9611 | 17.4803 | 0.8360 | 0.4394 |
| UNIV | Soft | 27.0340 | 17.5436 | 0.8372 | 0.4435 |
| MAD | Hard | 26.8198 | 24.4246 | 0.8384 | 0.7409 |
| STD | Hard | 27.0493 | 21.7399 | 0.8375 | 0.5606 |
| UNIV | Hard | 26.6450 | 21.3244 | 0.8369 | 0.587 |
| MAD | Custom | 26.9130 | 21.8576 | 0.8361 | 0.6459 |
| STD | Custom | 26.8042 | 20.2927 | 0.8381 | 0.5451 |
| UNIV | Custom | 26.6801 | 20.3981 | 0.8376 | 0.5432 |

**Table 6.28 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 26.9974 | 17.7254 | 0.8372 | 0.4619 |
| STD | Soft | 26.9040 | 16.8712 | 0.8369 | 0.4317 |
| UNIV | Soft | 26.8250 | 16.5180 | 0.8372 | 0.4219 |
| MAD | Hard | 27.0102 | 21.7971 | 0.8373 | 0.6170 |
| STD | Hard | 27.0431 | 20.8083 | 0.8381 | 0.5424 |
| UNIV | Hard | 26.8276 | 20.6860 | 0.8370 | 0.5358 |
| MAD | Custom | 26.9435 | 20.9064 | 0.8377 | 0.5918 |
| STD | Custom | 26.7454 | 20.3288 | 0.8393 | 0.5403 |
| UNIV | Custom | 27.0131 | 20.2844 | 0.8371 | 0.5378 |

**c. SPECKLE (v=0.09)**

**Table 6.29 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 16.1135 | 16.5705 | 0.4300 | 0.3843 |
| STD | Soft | 16.1074 | 14.1385 | 0.4307 | 0.3225 |
| UNIV | Soft | 16.1004 | 13.9267 | 0.4293 | 0.3151 |
| MAD | Hard | 16.1188 | 18.2614 | 0.4310 | 0.3812 |
| STD | Hard | 16.0972 | 19.3204 | 0.4281 | 0.3546 |
| UNIV | Hard | 16.0822 | 19.3459 | 0.4320 | 0.3533 |
| MAD | Custom | 16.1044 | 19.1250 | 0.4305 | 0.4138 |
| STD | Custom | 16.1406 | 19.5750 | 0.4316 | 0.3958 |
| UNIV | Custom | 16.1065 | 19.5249 | 0.4331 | 0.3928 |

**Table 6.30 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 16.0963 | 13.3908 | 0.4321 | 0.3383 |
| STD | Soft | 16.1106 | 13.2263 | 0.4298 | 0.3353 |
| UNIV | Soft | 16.1089 | 12.7847 | 0.4315 | 0.3300 |
| MAD | Hard | 16.1075 | 19.4835 | 0.4272 | 0.3901 |
| STD | Hard | 16.1084 | 19.2808 | 0.4321 | 0.3872 |
| UNIV | Hard | 16.1214 | 19.3458 | 0.4314 | 0.3844 |
| MAD | Custom | 16.1086 | 18.7549 | 0.4331 | 0.4260 |
| STD | Custom | 16.1419 | 18.6748 | 0.4321 | 0.4188 |
| UNIV | Custom | 16.1308 | 18.8225 | 0.4334 | 0.4160 |

**d. SALT & PEPPER (d= 0.09)**

**Table 6.31 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 16.1155 | 16.9498 | 0.4497 | 0.3877 |
| STD | Soft | 16.0673 | 14.1232 | 0.4450 | 0.3166 |
| UNIV | Soft | 15.9696 | 14.7019 | 0.4392 | 0.3268 |
| MAD | Hard | 15.9950 | 16.7783 | 0.4379 | 0.3395 |
| STD | Hard | 16.0165 | 18.6226 | 0.4406 | 0.3293 |
| UNIV | Hard | 16.0814 | 18.2297 | 0.4434 | 0.3193 |
| MAD | Custom | 15.9560 | 18.3319 | 0.4392 | 0.3738 |
| STD | Custom | 16.1543 | 19.1048 | 0.4474 | 0.3641 |
| UNIV | Custom | 16.0913 | 18.9853 | 0.4472 | 0.3611 |

**Table 6.32 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 16.1599 | 14.8648 | 0.4506 | 0.3609 |
| STD | Soft | 16.0818 | 13.3146 | 0.4438 | 0.3348 |
| UNIV | Soft | 16.0281 | 13.4912 | 0.4454 | 0.3404 |
| MAD | Hard | 15.8962 | 19.6999 | 0.4330 | 0.3904 |
| STD | Hard | 16.0440 | 19.4639 | 0.4443 | 0.3702 |
| UNIV | Hard | 15.8814 | 19.2673 | 0.4341 | 0.3781 |
| MAD | Custom | 16.0096 | 18.5696 | 0.4400 | 0.4206 |
| STD | Custom | 16.0948 | 18.4011 | 0.4499 | 0.3951 |
| UNIV | Custom | 16.0753 | 18.5047 | 0.4464 | 0.4045 |

# Image3. BARBARA

**Not Thresholding Low Pass Components**

**a. GAUSSIAN (0, 0.09)**

**Table 6.33 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 11.7589 | 19.3194 | 0.1708 | 0.3857 |
| STD | Soft | 11.7900 | 19.4244 | 0.1708 | 0.3973 |
| UNIV | Soft | 11.7878 | 19.4546 | 0.1723 | 0.3995 |
| MAD | Hard | 11.7453 | 19.1951 | 0.1694 | 0.3947 |
| STD | Hard | 11.7891 | 19.0627 | 0.1721 | 0.3920 |
| UNIV | Hard | 11.8107 | 19.2185 | 0.1693 | 0.3955 |
| MAD | Custom | 11.8064 | 19.1537 | 0.1734 | 0.3893 |
| STD | Custom | 11.7731 | 18.9984 | 0.1701 | 0.3773 |
| UNIV | Custom | 11.8054 | 19.2422 | 0.1725 | 0.3919 |

**Table 6.34 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 11.7474 | 20.0395 | 0.1697 | 0.4507 |
| STD | Soft | 11.7928 | 20.0990 | 0.1690 | 0.4599 |
| UNIV | Soft | 11.8127 | 20.1097 | 0.1725 | 0.4551 |
| MAD | Hard | 11.7790 | 20.1244 | 0.1703 | 0.4610 |
| STD | Hard | 11.8044 | 20.1246 | 0.1730 | 0.4622 |
| UNIV | Hard | 11.7948 | 20.0893 | 0.1719 | 0.4587 |
| MAD | Custom | 11.7863 | 20.1636 | 0.1709 | 0.4629 |
| STD | Custom | 11.7712 | 20.2155 | 0.1692 | 0.4673 |
| UNIV | Custom | 11.8043 | 20.1598 | 0.1694 | 0.4622 |

**b. POISSON**

**Table 6.35 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 27.4336 | 24.5114 | 0.7842 | 0.7326 |
| STD | Soft | 27.4164 | 22.8354 | 0.7841 | 0.6471 |
| UNIV | Soft | 27.4575 | 23.8558 | 0.7862 | 0.7019 |
| MAD | Hard | 27.4545 | 26.5378 | 0.7865 | 0.8123 |
| STD | Hard | 27.4551 | 24.2546 | 0.7861 | 0.7243 |
| UNIV | Hard | 27.4588 | 25.7286 | 0.7848 | 0.7875 |
| MAD | Custom | 27.4383 | 24.0809 | 0.7854 | 0.7429 |
| STD | Custom | 27.4235 | 23.5048 | 0.7851 | 0.6998 |
| UNIV | Custom | 27.4223 | 23.9341 | 0.7855 | 0.7314 |

**Table 6.36 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 27.4202 | 24.4576 | 0.7835 | 0.7366 |
| STD | Soft | 27.4536 | 22.9934 | 0.7856 | 0.6670 |
| UNIV | Soft | 27.4550 | 23.9375 | 0.7851 | 0.7123 |
| MAD | Hard | 27.4505 | 27.8180 | 0.7863 | 0.8453 |
| STD | Hard | 27.4654 | 24.6633 | 0.7863 | 0.7399 |
| UNIV | Hard | 27.4707 | 27.1294 | 0.7858 | 0.8264 |
| MAD | Custom | 27.4245 | 25.5071 | 0.7851 | 0.7913 |
| STD | Custom | 27.4382 | 24.2080 | 0.7851 | 0.7290 |
| UNIV | Custom | 27.4239 | 25.2729 | 0.7853 | 0.7824 |

**c. SPECKLE (v=0.09)**

**Table 6.37 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 16.7459 | 21.4374 | 0.4212 | 0.5434 |
| STD | Soft | 16.7424 | 21.2495 | 0.4215 | 0.5319 |
| UNIV | Soft | 16.7664 | 21.3502 | 0.4244 | 0.5356 |
| MAD | Hard | 16.7597 | 20.3531 | 0.4230 | 0.5296 |
| STD | Hard | 16.7553 | 20.8894 | 0.4206 | 0.5362 |
| UNIV | Hard | 16.7585 | 20.7895 | 0.4220 | 0.5347 |
| MAD | Custom | 16.7799 | 21.0279 | 0.4242 | 0.5375 |
| STD | Custom | 16.7638 | 21.3155 | 0.4226 | 0.5450 |
| UNIV | Custom | 16.7756 | 21.1963 | 0.4202 | 0.5390 |

**Table 6.38 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 16.7880 | 21.9428 | 0.4233 | 0.5889 |
| STD | Soft | 16.7694 | 21.8089 | 0.4227 | 0.5775 |
| UNIV | Soft | 16.7859 | 21.8751 | 0.4197 | 0.5850 |
| MAD | Hard | 16.7619 | 22.4625 | 0.4207 | 0.6188 |
| STD | Hard | 16.7884 | 22.1005 | 0.4234 | 0.5975 |
| UNIV | Hard | 16.7779 | 22.2885 | 0.4231 | 0.6106 |
| MAD | Custom | 16.7455 | 22.5240 | 0.4266 | 0.6184 |
| STD | Custom | 16.7611 | 22.3258 | 0.4209 | 0.6107 |
| UNIV | Custom | 16.7804 | 22.4280 | 0.4222 | 0.6169 |

**d. SALT & PEPPER (d=0.09)**

**Table 6.39 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 15.6658 | 20.7707 | 0.3720 | 0.4756 |
| STD | Soft | 15.7006 | 20.8167 | 0.3661 | 0.4884 |
| UNIV | Soft | 15.6998 | 20.8435 | 0.3717 | 0.4794 |
| MAD | Hard | 15.7396 | 17.7747 | 0.3682 | 0.3569 |
| STD | Hard | 15.6745 | 19.3276 | 0.3690 | 0.4246 |
| UNIV | Hard | 15.5593 | 18.2755 | 0.3620 | 0.3678 |
| MAD | Custom | 15.7709 | 19.2619 | 0.3757 | 0.4136 |
| STD | Custom | 15.7014 | 19.8908 | 0.3728 | 0.4408 |
| UNIV | Custom | 15.7723 | 19.4904 | 0.3735 | 0.463 |

**Table 6.40 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 15.7755 | 21.5134 | 0.3752 | 0.5453 |
| STD | Soft | 15.7345 | 21.4526 | 0.3712 | 0.5466 |
| UNIV | Soft | 15.7382 | 21.4682 | 0.3727 | 0.5468 |
| MAD | Hard | 15.7649 | 20.2905 | 0.3779 | 0.4749 |
| STD | Hard | 15.6206 | 21.2524 | 0.3663 | 0.5296 |
| UNIV | Hard | 15.8040 | 20.7541 | 0.3776 | 0.4939 |
| MAD | Custom | 15.7183 | 21.1008 | 0.3696 | 0.5113 |
| STD | Custom | 15.6591 | 21.3919 | 0.3671 | 0.5328 |
| UNIV | Custom | 15.5999 | 21.2147 | 0.3653 | 0.5199 |

**Thresholding low pass components**

**a. GAUSSIAN (0, 0.09)**

**Table 6.41 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 11.7901 | 15.5910 | 0.1685 | 0.3707 |
| STD | Soft | 11.8114 | 13.2270 | 0.1710 | 0.3152 |
| UNIV | Soft | 11.7900 | 12.9673 | 0.1708 | 0.3099 |
| MAD | Hard | 11.8432 | 16.4894 | 0.1744 | 0.2740 |
| STD | Hard | 11.7820 | 18.6925 | 0.1707 | 0.3618 |
| UNIV | Hard | 11.8354 | 18.8872 | 0.1708 | 0.3675 |
| MAD | Custom | 11.7894 | 17.3635 | 0.1724 | 0.3065 |
| STD | Custom | 11.7793 | 18.7546 | 0.1705 | 0.3719 |
| UNIV | Custom | 11.7589 | 18.7309 | 0.1708 | 0.3659 |

**Table 6.42 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 11.7453 | 12.0948 | 0.1694 | 0.3163 |
| STD | Soft | 11.7891 | 12.1324 | 0.1721 | 0.3171 |
| UNIV | Soft | 11.7928 | 11.9726 | 0.1690 | 0.3102 |
| MAD | Hard | 11.8064 | 19.6633 | 0.1734 | 0.4334 |
| STD | Hard | 11.7731 | 19.5868 | 0.1701 | 0.4242 |
| UNIV | Hard | 11.7474 | 19.5294 | 0.1697 | 0.4209 |
| MAD | Custom | 11.8107 | 19.5452 | 0.1693 | 0.4509 |
| STD | Custom | 11.8054 | 19.5316 | 0.1725 | 0.4541 |
| UNIV | Custom | 11.7878 | 19.6207 | 0.1723 | 0.4511 |

**b. POISSON**

**Table 6.43 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 27.4176 | 24.9532 | 0.7846 | 0.7957 |
| STD | Soft | 27.4290 | 20.0099 | 0.7858 | 0.6212 |
| UNIV | Soft | 27.4905 | 21.7901 | 0.7870 | 0.6865 |
| MAD | Hard | 27.4227 | 27.6312 | 0.7852 | 0.8320 |
| STD | Hard | 27.4515 | 24.2463 | 0.7854 | 0.7233 |
| UNIV | Hard | 27.4614 | 25.7262 | 0.7857 | 0.7852 |
| MAD | Custom | 27.4332 | 24.2287 | 0.7851 | 0.7468 |
| STD | Custom | 27.4313 | 23.4357 | 0.7848 | 0.6995 |
| UNIV | Custom | 27.4825 | 23.8922 | 0.7868 | 0.7345 |

**Table 6.44 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 27.4056 | 22.2316 | 0.7854 | 0.7258 |
| STD | Soft | 27.4661 | 19.3151 | 0.7860 | 0.6251 |
| UNIV | Soft | 27.4349 | 21.2895 | 0.7851 | 0.6931 |
| MAD | Hard | 27.4372 | 27.7465 | 0.7841 | 0.8438 |
| STD | Hard | 27.4486 | 24.6569 | 0.7855 | 0.7379 |
| UNIV | Hard | 27.4184 | 27.1641 | 0.7855 | 0.8263 |
| MAD | Custom | 27.4630 | 25.4249 | 0.7860 | 0.7949 |
| STD | Custom | 27.4525 | 24.0183 | 0.7862 | 0.7279 |
| UNIV | Custom | 27.4102 | 25.1463 | 0.7848 | 0.7829 |

**c. SPECKLE (v=0.09)**

**Table 6.45 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 16.7459 | 19.2158 | 0.4212 | 0.5394 |
| STD | Soft | 16.7424 | 16.2875 | 0.4215 | 0.4526 |
| UNIV | Soft | 16.7664 | 16.8665 | 0.4244 | 0.4682 |
| MAD | Hard | 16.7597 | 18.6672 | 0.4230 | 0.4857 |
| STD | Hard | 16.7553 | 20.7152 | 0.4206 | 0.5154 |
| UNIV | Hard | 16.7585 | 20.6823 | 0.4220 | 0.5200 |
| MAD | Custom | 16.7799 | 20.0643 | 0.4242 | 0.5083 |
| STD | Custom | 16.7638 | 20.9432 | 0.4226 | 0.5361 |
| UNIV | Custom | 16.7756 | 20.8754 | 0.4202 | 0.5328 |

**Table 6.46 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 16.7880 | 16.5819 | 0.4233 | 0.5004 |
| STD | Soft | 16.7459 | 15.3851 | 0.4212 | 0.4599 |
| UNIV | Soft | 16.7756 | 15.8254 | 0.4202 | 0.4723 |
| MAD | Hard | 16.7424 | 22.2160 | 0.4215 | 0.5934 |
| STD | Hard | 16.7597 | 21.7040 | 0.4230 | 0.5657 |
| UNIV | Hard | 16.7585 | 22.0592 | 0.4220 | 0.5863 |
| MAD | Custom | 16.7553 | 22.0079 | 0.4206 | 0.6105 |
| STD | Custom | 16.7799 | 21.6353 | 0.4242 | 0.5948 |
| UNIV | Custom | 16.7638 | 21.9770 | 0.4226 | 0.6084 |

**d. SALT & PEPPER (d=0.09)**

**Table 6.47 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 15.7845 | 19.0987 | 0.3780 | 0.4524 |
| STD | Soft | 15.6837 | 16.0639 | 0.3712 | 0.4554 |
| UNIV | Soft | 15.7057 | 17.2777 | 0.3720 | 0.4631 |
| MAD | Hard | 15.8220 | 16.7371 | 0.3784 | 0.3524 |
| STD | Hard | 15.7412 | 19.3007 | 0.3709 | 0.4149 |
| UNIV | Hard | 15.6917 | 18.2732 | 0.3694 | 0.3706 |
| MAD | Custom | 15.7644 | 18.6495 | 0.3739 | 0.3979 |
| STD | Custom | 15.8000 | 19.9318 | 0.3735 | 0.4484 |
| UNIV | Custom | 15.8107 | 19.4944 | 0.3798 | 0.4297 |

**Table 6.48 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 15.7286 | 17.5286 | 0.3700 | 0.5084 |
| STD | Soft | 15.6658 | 14.9968 | 0.3720 | 0.4574 |
| UNIV | Soft | 15.5593 | 15.9057 | 0.3620 | 0.4804 |
| MAD | Hard | 15.7006 | 20.2359 | 0.3661 | 0.4628 |
| STD | Hard | 15.7396 | 21.1350 | 0.3682 | 0.5200 |
| UNIV | Hard | 15.7014 | 20.7307 | 0.3728 | 0.4952 |
| MAD | Custom | 15.6745 | 21.0437 | 0.3690 | 0.5162 |
| STD | Custom | 15.6629 | 20.9789 | 0.3699 | 0.5309 |
| UNIV | Custom | 15.7709 | 21.0978 | 0.3757 | 0.5190 |

# Image4. BOAT

### Not Thresholding Low Pass Components

**a. GAUSSIAN (0, 0.09)**

**Table 6.49 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 11.7547 | 20.0252 | 0.1406 | 0.3745 |
| STD | Soft | 11.7786 | 20.1197 | 0.1418 | 0.3834 |
| UNIV | Soft | 11.7544 | 20.0170 | 0.1400 | 0.3709 |
| MAD | Hard | 11.7405 | 19.5872 | 0.1379 | 0.3608 |
| STD | Hard | 11.7651 | 19.4679 | 0.1375 | 0.3535 |
| UNIV | Hard | 11.7264 | 19.7543 | 0.1370 | 0.3637 |
| MAD | Custom | 11.7562 | 19.5456 | 0.1407 | 0.3454 |
| STD | Custom | 11.7409 | 19.5065 | 0.1419 | 0.3446 |
| UNIV | Custom | 11.7339 | 19.6911 | 0.1407 | 0.3536 |

**Table 6.50 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 11.7512 | 20.3847 | .1383 | 0.4396 |
| STD | Soft | 11.7744 | 20.3899 | 0.1416 | 0.4469 |
| UNIV | Soft | 11.7633 | 20.3571 | 0.1403 | 0.4463 |
| MAD | Hard | 11.7368 | 20.3978 | 0.1400 | 0.4506 |
| STD | Hard | 11.7565 | 20.3458 | 0.1405 | 0.4415 |
| UNIV | Hard | 11.7675 | 20.4456 | 0.1397 | 0.4456 |
| MAD | Custom | 11.7289 | 20.3453 | 0.1400 | 0.4429 |
| STD | Custom | 11.7792 | 20.4480 | 0.1426 | 0.4472 |
| UNIV | Custom | 11.7458 | 20.3493 | 0.1403 | 0.4392 |

**b. POISSON**

**Table 6.51 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 26.7708 | 25.7681 | 0.6840 | 0.7556 |
| STD | Soft | 26.7888 | 25.2816 | 0.6841 | 0.7412 |
| UNIV | Soft | 26.7919 | 24.9912 | 0.6849 | 0.7307 |
| MAD | Hard | 26.7571 | 27.8334 | 0.6835 | 0.8026 |
| STD | Hard | 26.8046 | 27.5752 | 0.6849 | 0.7989 |
| UNIV | Hard | 26.8382 | 27.2749 | 0.6856 | 0.7899 |
| MAD | Custom | 26.8182 | 24.6321 | 0.6852 | 0.7252 |
| STD | Custom | 26.7997 | 24.4906 | 0.6850 | 0.7245 |
| UNIV | Custom | 26.8074 | 24.7632 | 0.6852 | 0.7239 |

**Table 6.52 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 26.7649 | 25.7476 | 0.6834 | 0.7661 |
| STD | Soft | 26.7986 | 25.3977 | 0.6854 | 0.7532 |
| UNIV | Soft | 26.7539 | 24.8544 | 0.6817 | 0.7320 |
| MAD | Hard | 26.7893 | 29.1952 | 0.6838 | 0.8484 |
| STD | Hard | 26.7938 | 28.4366 | 0.6843 | 0.8372 |
| UNIV | Hard | 26.8218 | 27.9733 | 0.6843 | 0.8184 |
| MAD | Custom | 26.7949 | 26.2639 | 0.6863 | 0.7968 |
| STD | Custom | 26.7964 | 26.0366 | 0.6836 | 0.7863 |
| UNIV | Custom | 25.7423 | 25.7553 | 0.6829 | 0.7743 |

**c. SPECKLE (v=0.09)**

**Table 6.53 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 15.4922 | 22.1033 | 0.3230 | 0.5030 |
| STD | Soft | 15.4758 | 22.0097 | 0.3215 | 0.5028 |
| UNIV | Soft | 15.4855 | 21.9752 | 0.3222 | 0.4904 |
| MAD | Hard | 15.4817 | 20.5651 | 0.3209 | 0.4368 |
| STD | Hard | 15.5069 | 21.4104 | 0.3244 | 0.4749 |
| UNIV | Hard | 15.4734 | 21.3755 | 0.3241 | 0.4750 |
| MAD | Custom | 15.5141 | 21.1627 | 0.3232 | 0.4409 |
| STD | Custom | 15.4906 | 21.5703 | 0.3239 | 0.4653 |
| UNIV | Custom | 15.5064 | 21.5124 | 0.3240 | 0.4602 |

**Table 6.54 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 15.5262 | 22.2863 | 0.3227 | 0.5638 |
| STD | Soft | 15.5036 | 22.2418 | 0.3242 | 0.5570 |
| UNIV | Soft | 15.5044 | 22.2276 | 0.3239 | 0.5562 |
| MAD | Hard | 15.4928 | 23.1521 | 0.3222 | 0.5692 |
| STD | Hard | 15.5001 | 22.5260 | 0.3237 | 0.5707 |
| UNIV | Hard | 15.5038 | 22.5571 | 0.3229 | 0.5731 |
| MAD | Custom | 15.4853 | 22.5287 | 0.3222 | 0.5545 |
| STD | Custom | 15.4974 | 22.5419 | 0.3233 | 0.5647 |
| UNIV | Custom | 15.5059 | 22.5768 | 0.3222 | 0.5711 |

**d. SALT & PEPPER (d=0.09)**

**Table 6.55 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 15.7269 | 21.3106 | 0.3064 | 0.4421 |
| STD | Soft | 15.7529 | 21.4190 | 0.3907 | 0.4804 |
| UNIV | Soft | 15.7983 | 21.4609 | 0.3128 | 0.4612 |
| MAD | Hard | 15.8429 | 17.5614 | 0.3134 | 0.3126 |
| STD | Hard | 15.7219 | 19.6902 | 0.3082 | 0.3972 |
| UNIV | Hard | 15.8980 | 18.4438 | 0.3177 | 0.3246 |
| MAD | Custom | 15.8144 | 19.6941 | 0.3154 | 0.3781 |
| STD | Custom | 15.7506 | 20.6948 | 0.3097 | 0.4311 |
| UNIV | Custom | 15.6337 | 20.0455 | 0.3029 | 0.3961 |

**Table 6.56 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 15.8534 | 22.1106 | 0.3120 | 0.5389 |
| STD | Soft | 15.8755 | 22.0086 | 0.3166 | 0.5527 |
| UNIV | Soft | 15.9152 | 22.0210 | 0.3164 | 0.5532 |
| MAD | Hard | 15.7394 | 19.9999 | 0.3089 | 0.4063 |
| STD | Hard | 15.6641 | 21.8753 | 0.3078 | 0.5202 |
| UNIV | Hard | 15.6375 | 21.5992 | 0.3011 | 0.4806 |
| MAD | Custom | 15.7246 | 21.3924 | 0.3071 | 0.4976 |
| STD | Custom | 15.8256 | 21.8397 | 0.3125 | 0.5358 |
| UNIV | Custom | 15.7787 | 21.7702 | 0.3079 | 0.5232 |

**Thresholding low pass components**

**a. GAUSSIAN (0, 0.09)**

**Table 6.57 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 11.7603 | 15.0643 | 0.1386 | 0.3511 |
| STD | Soft | 11.7708 | 12.7851 | 0.1390 | 0.3251 |
| UNIV | Soft | 11.7705 | 12.3982 | 0.1401 | 0.3071 |
| MAD | Hard | 11.7452 | 16.5066 | 0.1385 | 0.2324 |
| STD | Hard | 11.7493 | 19.3188 | 0.1386 | 0.3408 |
| UNIV | Hard | 11.7194 | 19.5501 | 0.1394 | 0.3503 |
| MAD | Custom | 11.7392 | 17.5915 | 0.1399 | 0.2612 |
| STD | Custom | 11.7447 | 19.1057 | 0.1375 | 0.3424 |
| UNIV | Custom | 11.7670 | 19.1808 | 0.1428 | 0.3484 |

**Table 6.58 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 11.7671 | 11.5490 | 0.1396 | 0.3340 |
| STD | Soft | 11.7447 | 11.5745 | 0.1414 | 0.3381 |
| UNIV | Soft | 11.7844 | 11.2850 | 0.1385 | 0.3290 |
| MAD | Hard | 11.7615 | 20.1210 | 0.1419 | 0.4223 |
| STD | Hard | 11.7003 | 20.1328 | 0.1396 | 0.4199 |
| UNIV | Hard | 11.7574 | 20.0812 | 0.1392 | 0.4177 |
| MAD | Custom | 11.7458 | 19.5930 | 0.1374 | 0.4361 |
| STD | Custom | 11.7471 | 19.5621 | 0.1399 | 0.4345 |
| UNIV | Custom | 11.7580 | 19.4916 | 0.1415 | 0.4359 |

**b. POISSON**

**Table 6.59 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 26.8326 | 25.7963 | 0.6850 | 0.8011 |
| STD | Soft | 26.7801 | 23.2067 | 0.6831 | 0.7336 |
| UNIV | Soft | 26.7856 | 22.7117 | 0.6843 | 0.7207 |
| MAD | Hard | 26.8431 | 28.0408 | 0.6858 | 0.7663 |
| STD | Hard | 26.7932 | 27.5571 | 0.6851 | 0.7971 |
| UNIV | Hard | 26.7607 | 27.2600 | 0.6836 | 0.7909 |
| MAD | Custom | 26.8583 | 24.9368 | 0.6855 | 0.6978 |
| STD | Custom | 26.7984 | 24.5180 | 0.6846 | 0.7272 |
| UNIV | Custom | 26.7962 | 24.5493 | 0.6853 | 0.7227 |

**Table 6.60 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 26.8214 | 23.0790 | 0.6847 | 0.7525 |
| STD | Soft | 26.7875 | 22.5324 | 0.6836 | 0.7378 |
| UNIV | Soft | 26.8061 | 21.5519 | 0.6841 | 0.7120 |
| MAD | Hard | 26.8411 | 29.2474 | 0.6854 | 0.8483 |
| STD | Hard | 26.8136 | 28.4783 | 0.6849 | 0.8350 |
| UNIV | Hard | 26.8298 | 28.2452 | 0.6864 | 0.8208 |
| MAD | Custom | 26.7835 | 26.1183 | 0.6841 | 0.7937 |
| STD | Custom | 26.7692 | 25.9203 | 0.6835 | 0.7857 |
| UNIV | Custom | 26.7784 | 25.6813 | 0.6832 | 0.7769 |

**c. SPECKLE (v=0.09)**

**Table 6.61 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 15.4935 | 18.9200 | 0.3218 | 0.4539 |
| STD | Soft | 15.4909 | 16.0084 | 0.3223 | 0.4177 |
| UNIV | Soft | 15.5112 | 16.0419 | 0.3237 | 0.4187 |
| MAD | Hard | 15.4971 | 18.1527 | 0.3237 | 0.3627 |
| STD | Hard | 15.5032 | 21.2268 | 0.3219 | 0.4493 |
| UNIV | Hard | 15.5039 | 21.2275 | 0.3221 | 0.4461 |
| MAD | Custom | 15.5013 | 19.6264 | 0.3217 | 0.3900 |
| STD | Custom | 15.4682 | 21.1512 | 0.3217 | 0.4453 |
| UNIV | Custom | 15.5148 | 21.1538 | 0.3256 | 0.4472 |

**Table 6.62 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 15.4785 | 15.6519 | 0.3221 | 0.4639 |
| STD | Soft | 15.4715 | 14.8238 | 0.3221 | 0.439 |
| UNIV | Soft | 15.4649 | 14.7745 | 0.3235 | 0.4419 |
| MAD | Hard | 15.4710 | 22.3445 | 0.3218 | 0.5249 |
| STD | Hard | 15.4903 | 21.9048 | 0.3227 | 0.5241 |
| UNIV | Hard | 15.5240 | 22.5613 | 0.3228 | 0.5256 |
| MAD | Custom | 15.5063 | 21.8557 | 0.3224 | 0.5361 |
| STD | Custom | 15.4593 | 21.6763 | 0.3210 | 0.5352 |
| UNIV | Custom | 15.5108 | 21.7326 | 0.3225 | 0.5355 |

### d. SALT & PEPPER (d=0.09)

**Table 6.63 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 15.8495 | 18.9159 | 0.3138 | 0.4054 |
| STD | Soft | 15.8272 | 15.9648 | 0.3154 | 0.4459 |
| UNIV | Soft | 15.7624 | 17.2226 | 0.3093 | 0.4477 |
| MAD | Hard | 15.8587 | 16.6168 | 0.3151 | 0.2994 |
| STD | Hard | 15.8653 | 19.6898 | 0.3186 | 0.3904 |
| UNIV | Hard | 15.7360 | 18.3485 | 0.3093 | 0.3219 |
| MAD | Custom | 15.6983 | 18.8097 | 0.3060 | 0.3338 |
| STD | Custom | 15.8914 | 20.5325 | 0.3140 | 0.4350 |
| UNIV | Custom | 15.8975 | 20.1239 | 0.3165 | 0.4099 |

**Table 6.64 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 15.6993 | 17.8222 | 0.3031 | 0.5125 |
| STD | Soft | 15.8816 | 15.0765 | 0.3184 | 0.4904 |
| UNIV | Soft | 15.7704 | 15.6587 | 0.3086 | 0.5006 |
| MAD | Hard | 15.8166 | 19.8974 | 0.3164 | 0.4123 |
| STD | Hard | 15.7808 | 21.6275 | 0.3191 | 0.5196 |
| UNIV | Hard | 15.8136 | 21.4925 | 0.3126 | 0.4751 |
| MAD | Custom | 15.7080 | 21.1792 | 0.3078 | 0.4933 |
| STD | Custom | 15.6520 | 21.2493 | 0.3038 | 0.5265 |
| UNIV | Custom | 15.8758 | 21.3895 | 0.3183 | 0.5235 |

# Image5. MRI

### Not Thresholding Low Pass Components

**a. GAUSSIAN (0, 0.09)**

**Table 6.65 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 12.2992 | 19.3145 | 0.1034 | 0.3515 |
| STD | Soft | 12.3381 | 19.4033 | 0.1062 | 0.3512 |
| UNIV | Soft | 12.3108 | 19.4295 | 0.1029 | 0.3532 |
| MAD | Hard | 12.2592 | 18.3110 | 0.1012 | 0.2905 |
| STD | Hard | 12.2927 | 18.6408 | 0.1011 | 0.3060 |
| UNIV | Hard | 12.3293 | 18.7555 | 0.1039 | 0.3159 |
| MAD | Custom | 12.3106 | 18.4034 | 0.1042 | 0.2780 |
| STD | Custom | 12.2891 | 18.6317 | 0.1012 | 0.2884 |
| UNIV | Custom | 12.3007 | 18.7026 | 0.1030 | 0.2931 |

**Table 6.66 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 12.4021 | 20.0706 | 0.1061 | 0.4112 |
| STD | Soft | 12.2596 | 19.7761 | 0.1016 | 0.4012 |
| UNIV | Soft | 12.3035 | 19.9023 | 0.1055 | 0.4026 |
| MAD | Hard | 12.2973 | 19.8815 | 0.1043 | 0.3905 |
| STD | Hard | 12.2767 | 19.9061 | 0.1057 | 0.4036 |
| UNIV | Hard | 12.3386 | 19.9986 | 0.1028 | 0.4020 |
| MAD | Custom | 12.3690 | 19.8284 | 0.1050 | 0.3831 |
| STD | Custom | 12.2834 | 19.7716 | 0.1037 | 0.3819 |
| UNIV | Custom | 12.3365 | 19.9294 | 0.1047 | 0.3911 |

**b. POISSON**

**Table 6.67 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 30.4446 | 32.5587 | 0.8377 | 0.8961 |
| STD | Soft | 30.3510 | 30.3897 | 0.8357 | 0.8645 |
| UNIV | Soft | 30.4827 | 30.9829 | 0.8383 | 0.8759 |
| MAD | Hard | 30.3853 | 31.1250 | 0.8361 | 0.8507 |
| STD | Hard | 30.4250 | 31.7680 | 0.8372 | 0.8797 |
| UNIV | Hard | 30.4345 | 31.7744 | 0.8372 | 0.8740 |
| MAD | Custom | 30.3878 | 28.3895 | 0.8375 | 0.8267 |
| STD | Custom | 30.3976 | 28.5647 | 0.8376 | 0.8425 |
| UNIV | Custom | 30.3846 | 28.5277 | 0.8365 | 0.8389 |

**Table 6.68 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 30.3983 | 33.3131 | 0.8362 | 0.9145 |
| STD | Soft | 30.4933 | 31.1543 | 0.8391 | 0.8848 |
| UNIV | Soft | 30.4304 | 31.1430 | 0.8364 | 0.8841 |
| MAD | Hard | 30.3861 | 33.0285 | 0.8372 | 0.8953 |
| STD | Hard | 30.3823 | 33.9370 | 0.8364 | 0.9180 |
| UNIV | Hard | 30.4011 | 34.0637 | 0.8358 | 0.9184 |
| MAD | Custom | 30.4631 | 30.8867 | 0.8375 | 0.8912 |
| STD | Custom | 30.3484 | 30.5289 | 0.8363 | 0.8905 |
| UNIV | Custom | 30.3835 | 30.6082 | 0.8362 | 0.8908 |

**c. SPECKLE (v=0.09)**

**Table 6.69 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 20.5175 | 24.9996 | 0.6133 | 0.7035 |
| STD | Soft | 20.5271 | 26.3582 | 0.6119 | 0.7613 |
| UNIV | Soft | 20.4925 | 26.2602 | 0.6136 | 0.7500 |
| MAD | Hard | 20.5127 | 20.9771 | 0.6127 | 0.6099 |
| STD | Hard | 20.4456 | 24.1853 | 0.6113 | 0.6977 |
| UNIV | Hard | 20.5011 | 22.7182 | 0.6130 | 0.6529 |
| MAD | Custom | 20.4627 | 23.0491 | 0.6111 | 0.6374 |
| STD | Custom | 20.4606 | 24.8259 | 0.6130 | 0.7039 |
| UNIV | Custom | 20.4848 | 24.1097 | 0.6116 | 0.6760 |

**Table 6.70 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 20.4412 | 26.2400 | 0.6126 | 0.7527 |
| STD | Soft | 20.5175 | 27.2691 | 0.6127 | 0.8040 |
| UNIV | Soft | 20.4865 | 27.4708 | 0.6125 | 0.8089 |
| MAD | Hard | 20.4970 | 21.9085 | 0.6128 | 0.6497 |
| STD | Hard | 20.4355 | 27.2070 | 0.6098 | 0.7943 |
| UNIV | Hard | 20.5246 | 25.8236 | 0.6117 | 0.7527 |
| MAD | Custom | 20.4369 | 25.3996 | 0.6125 | 0.7180 |
| STD | Custom | 20.5047 | 27.0183 | 0.6124 | 0.7882 |
| UNIV | Custom | 20.5144 | 26.6182 | 0.6133 | 0.7664 |

**d. SALT & PEPPER (d=0.09)**

**Table 6.71 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 14.7680 | 20.4032 | 0.2484 | 0.3972 |
| STD | Soft | 14.8010 | 22.5806 | 0.2528 | 0.4909 |
| UNIV | Soft | 14.8512 | 21.5338 | 0.2550 | 0.4450 |
| MAD | Hard | 14.6940 | 16.0725 | 0.2433 | 0.2500 |
| STD | Hard | 14.7895 | 19.1845 | 0.2545 | 0.3973 |
| UNIV | Hard | 14.8807 | 16.8895 | 0.2546 | 0.2805 |
| MAD | Custom | 14.8972 | 18.0170 | 0.2571 | 0.3035 |
| STD | Custom | 14.7143 | 19.7360 | 0.2499 | 0.3840 |
| UNIV | Custom | 14.6337 | 18.3461 | 0.2469 | 0.3217 |

**Table 6.72 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 14.7211 | 22.3620 | 0.2417 | 0.4932 |
| STD | Soft | 14.9203 | 23.7242 | 0.2581 | 0.5554 |
| UNIV | Soft | 14.8610 | 23.6040 | 0.2586 | 0.5505 |
| MAD | Hard | 14.7021 | 17.5369 | 0.2528 | 0.3438 |
| STD | Hard | 14.7939 | 22.1989 | 0.2545 | 0.5149 |
| UNIV | Hard | 14.7274 | 20.0572 | 0.2492 | 0.4354 |
| MAD | Custom | 14.9144 | 20.1116 | 0.2591 | 0.4020 |
| STD | Custom | 14.7836 | 22.0758 | 0.2533 | 0.4919 |
| UNIV | Custom | 14.7966 | 21.0869 | 0.2560 | 0.4455 |

**Thresholding Low Pass Components**

**a. GAUSSIAN (0, 0.09)**

**Table 6.73 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 12.2897 | 19.2397 | 0.1032 | 0.4206 |
| STD | Soft | 12.3478 | 16.9943 | 0.1040 | 0.4752 |
| UNIV | Soft | 12.3193 | 16.9073 | 0.1040 | 0.4705 |
| MAD | Hard | 12.2989 | 15.6699 | 0.1017 | 0.1628 |
| STD | Hard | 12.3388 | 19.9916 | 0.1038 | 0.4037 |
| UNIV | Hard | 12.3046 | 20.3005 | 0.1026 | 0.4216 |
| MAD | Custom | 12.2902 | 17.3514 | 0.1044 | 0.2135 |
| STD | Custom | 12.2775 | 19.9931 | 0.1023 | 0.3376 |
| UNIV | Custom | 12.3220 | 20.3620 | 0.1029 | 0.3514 |

**Table 6.74 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 12.2879 | 16.3906 | 0.1039 | 0.4806 |
| STD | Soft | 12.2977 | 15.8326 | 0.1034 | 0.4619 |
| UNIV | Soft | 12.2896 | 15.7285 | 0.1007 | 0.4495 |
| MAD | Hard | 12.3344 | 22.7841 | 0.1054 | 0.5341 |
| STD | Hard | 12.2696 | 23.0225 | 0.1026 | 0.5572 |
| UNIV | Hard | 12.2717 | 22.9934 | 0.1033 | 0.5490 |
| MAD | Custom | 12.2854 | 22.3127 | 0.1025 | 0.4756 |
| STD | Custom | 12.3765 | 22.5944 | 0.1063 | 0.5024 |
| UNIV | Custom | 12.2862 | 22.4622 | 0.1032 | 0.4989 |

**b. POISSON**

**Table 6.75 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 30.3641 | 32.2081 | 0.8362 | 0.8234 |
| STD | Soft | 30.4076 | 28.4307 | 0.8379 | 0.7682 |
| UNIV | Soft | 30.4208 | 29.3359 | 0.8377 | 0.7802 |
| MAD | Hard | 30.4105 | 30.6700 | 0.8367 | 0.8352 |
| STD | Hard | 30.4483 | 31.5694 | 0.8380 | 0.7863 |
| UNIV | Hard | 30.4124 | 31.6118 | 0.8368 | 0.7831 |
| MAD | Custom | 30.4762 | 28.1863 | 0.8382 | 0.8114 |
| STD | Custom | 30.3858 | 28.4111 | 0.8360 | 0.7606 |
| UNIV | Custom | 30.4217 | 28.4225 | 0.8374 | 0.7671 |

**Table 6.76 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 30.4491 | 31.6000 | 0.8373 | 0.8123 |
| STD | Soft | 30.4628 | 28.3638 | 0.8374 | 0.7750 |
| UNIV | Soft | 30.3943 | 28.1960 | 0.8361 | 0.7735 |
| MAD | Hard | 30.4383 | 32.9060 | 0.8372 | 0.8113 |
| STD | Hard | 30.4042 | 33.6106 | 0.8363 | 0.8160 |
| UNIV | Hard | 30.4305 | 33.7936 | 0.8378 | 0.8194 |
| MAD | Custom | 30.3866 | 30.6931 | 0.8362 | 0.8415 |
| STD | Custom | 30.3920 | 30.2338 | 0.8368 | 0.7940 |
| UNIV | Custom | 30.3692 | 30.3622 | 0.8367 | 0.8001 |

**c. SPECKLE (v=0.09)**

**Table 6.77 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 20.4713 | 23.3397 | 0.6104 | 0.5773 |
| STD | Soft | 20.5087 | 22.3303 | 0.6108 | 0.6331 |
| UNIV | Soft | 20.4737 | 23.6553 | 0.6114 | 0.6371 |
| MAD | Hard | 20.4632 | 20.6172 | 0.6124 | 0.5928 |
| STD | Hard | 20.4950 | 24.1464 | 0.6109 | 0.6001 |
| UNIV | Hard | 20.5001 | 22.6233 | 0.6134 | 0.5571 |
| MAD | Custom | 20.4690 | 22.7028 | 0.6130 | 0.5753 |
| STD | Custom | 20.5030 | 24.6511 | 0.6123 | 0.6120 |
| UNIV | Custom | 20.4987 | 23.9319 | 0.6118 | 0.5778 |

**Table 6.78 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 20.4992 | 25.0905 | 0.6124 | 0.6422 |
| STD | Soft | 20.5699 | 21.8171 | 0.6138 | 0.6519 |
| UNIV | Soft | 20.4778 | 22.9916 | 0.6120 | 0.6664 |
| MAD | Hard | 20.5185 | 21.9324 | 0.6117 | 0.5458 |
| STD | Hard | 20.5102 | 27.2662 | 0.6143 | 0.6933 |
| UNIV | Hard | 20.4809 | 25.6577 | 0.6119 | 0.6463 |
| MAD | Custom | 20.5064 | 25.3503 | 0.6127 | 0.6200 |
| STD | Custom | 20.4826 | 26.6827 | 0.6126 | 0.6893 |
| UNIV | Custom | 20.4864 | 26.3258 | 0.6132 | 0.6661 |

### d. SALT & PEPPER (d=0.09)

**Table 6.79 DWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 14.7945 | 19.2822 | 0.2492 | 0.3590 |
| STD | Soft | 14.8216 | 18.8543 | 0.2543 | 0.5496 |
| UNIV | Soft | 14.7526 | 20.6614 | 0.2534 | 0.4703 |
| MAD | Hard | 14.6893 | 15.2538 | 0.2502 | 0.2359 |
| STD | Hard | 14.6782 | 19.9129 | 0.2458 | 0.4212 |
| UNIV | Hard | 14.7663 | 17.2816 | 0.2468 | 0.2951 |
| MAD | Custom | 14.7703 | 17.7221 | 0.2455 | 0.2884 |
| STD | Custom | 14.7769 | 20.3927 | 0.2517 | 0.4103 |
| UNIV | Custom | 14.8032 | 18.7983 | 0.2513 | 0.3309 |

**Table 6.80 UDWT**

| Threshold value | Thresholding technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 14.7709 | 21.4755 | 0.2524 | 0.5180 |
| STD | Soft | 14.7946 | 17.8224 | 0.2569 | 0.5653 |
| UNIV | Soft | 14.8480 | 19.4481 | 0.2552 | 0.5896 |
| MAD | Hard | 14.7820 | 17.6676 | 0.2462 | 0.3371 |
| STD | Hard | 14.7322 | 23.8633 | 0.2469 | 0.5829 |
| UNIV | Hard | 14.7436 | 20.9999 | 0.2437 | 0.4624 |
| MAD | Custom | 14.8275 | 20.3668 | 0.2530 | 0.4020 |
| STD | Custom | 14.7783 | 23.0870 | 0.2481 | 0.5294 |
| UNIV | Custom | 14.7706 | 21.6586 | 0.2508 | 0.4561 |

## Image6. BRAIN

### Not Thresholding Low Pass Components

**a. GAUSSIAN (0, 0.09)**

**Table 6.81 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 12.2770 | 18.4388 | 0.1682 | 0.3964 |
| STD | Soft | 12.3020 | 18.4957 | 0.1698 | 0.4041 |
| UNIV | Soft | 12.2990 | 18.4103 | 0.1725 | 0.3989 |
| MAD | Hard | 12.3199 | 17.9212 | 0.1700 | 0.3535 |
| STD | Hard | 12.2740 | 18.1113 | 0.1691 | 0.3746 |
| UNIV | Hard | 12.2819 | 18.2958 | 0.1682 | 0.3834 |
| MAD | Custom | 12.2445 | 17.8032 | 0.1694 | 0.3395 |
| STD | Custom | 12.2872 | 18.0466 | 0.1680 | 0.3518 |
| UNIV | Custom | 12.3006 | 18.0964 | 0.1709 | 0.3592 |

**Table 6.82 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 12.2798 | 18.9410 | 0.1696 | 0.4587 |
| STD | Soft | 12.3240 | 18.9723 | 0.1701 | 0.4562 |
| UNIV | Soft | 12.2565 | 18.8984 | 0.1714 | 0.4570 |
| MAD | Hard | 12.2755 | 19.1816 | 0.1685 | 0.4565 |
| STD | Hard | 12.2754 | 19.1610 | 0.1674 | 0.4615 |
| UNIV | Hard | 12.2893 | 19.1505 | 0.1697 | 0.4601 |
| MAD | Custom | 12.2920 | 19.1560 | 0.1710 | 0.4507 |
| STD | Custom | 12.2484 | 19.0736 | 0.1704 | 0.4500 |
| UNIV | Custom | 12.2559 | 19.1197 | 0.1700 | 0.4484 |

**b. POISSON**

**Table 6.83 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 30.0472 | 30.0227 | 0.8632 | 0.8992 |
| STD | Soft | 30.0538 | 27.8252 | 0.8634 | 0.8590 |
| UNIV | Soft | 30.1672 | 28.2525 | 0.8654 | 0.8670 |
| MAD | Hard | 30.1082 | 30.7325 | 0.8639 | 0.8927 |
| STD | Hard | 30.1025 | 30.1453 | 0.8629 | 0.8891 |
| UNIV | Hard | 30.0974 | 30.4479 | 0.8636 | 0.8945 |
| MAD | Custom | 30.0434 | 24.8783 | 0.8620 | 0.8223 |
| STD | Custom | 30.1176 | 24.8459 | 0.8640 | 0.8257 |
| UNIV | Custom | 30.0676 | 24.8612 | 0.8639 | 0.8251 |

**Table 6.84 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 30.0789 | 30.5527 | 0.8640 | 0.9136 |
| STD | Soft | 30.1647 | 28.3440 | 0.8659 | 0.8791 |
| UNIV | Soft | 30.1049 | 27.7793 | 0.8646 | 0.8699 |
| MAD | Hard | 30.0994 | 32.6779 | 0.8648 | 0.9336 |
| STD | Hard | 30.0890 | 32.0929 | 0.8642 | 0.9233 |
| UNIV | Hard | 30.0647 | 32.0831 | 0.8631 | 0.9231 |
| MAD | Custom | 30.0924 | 27.5566 | 0.8635 | 0.8955 |
| STD | Custom | 30.1785 | 27.1663 | 0.8650 | 0.8828 |
| UNIV | Custom | 30.0471 | 27.1298 | 0.8624 | 0.8826 |

**c. SPECKLE (v=0.09)**

**Table 6.85 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 20.2425 | 24.2882 | 0.6101 | 0.7652 |
| STD | Soft | 20.2013 | 23.5617 | 0.6085 | 0.7508 |
| UNIV | Soft | 20.2116 | 23.9014 | 0.6098 | 0.7604 |
| MAD | Hard | 20.2200 | 21.2096 | 0.6087 | 0.6565 |
| STD | Hard | 20.2476 | 23.1042 | 0.6112 | 0.7382 |
| UNIV | Hard | 20.1811 | 22.4295 | 0.6087 | 0.7124 |
| MAD | Custom | 20.2179 | 21.7630 | 0.6107 | 0.6672 |
| STD | Custom | 20.2099 | 22.4764 | 0.6087 | 0.7219 |
| UNIV | Custom | 20.2338 | 22.3261 | 0.6111 | 0.7084 |

**Table 6.86 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 20.2236 | 25.3413 | 0.6099 | 0.8186 |
| STD | Soft | 20.1698 | 24.0762 | 0.6074 | 0.7874 |
| UNIV | Soft | 20.1969 | 24.2772 | 0.6095 | 0.7939 |
| MAD | Hard | 20.1958 | 22.7920 | 0.6086 | 0.7460 |
| STD | Hard | 20.2082 | 25.4468 | 0.6096 | 0.8228 |
| UNIV | Hard | 20.2793 | 25.1564 | 0.6085 | 0.8147 |
| MAD | Custom | 20.2215 | 24.1082 | 0.6110 | 0.7788 |
| STD | Custom | 20.2074 | 24.3411 | 0.6104 | 0.8037 |
| UNIV | Custom | 20.1662 | 24.2883 | 0.6052 | 0.7966 |

**d. SALT & PEPPER (d=0.09)**

**Table 6.87 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 14.9790 | 20.0079 | 0.3349 | 0.4750 |
| STD | Soft | 14.8905 | 20.7880 | 0.3310 | 0.5259 |
| UNIV | Soft | 14.8476 | 20.6020 | 0.3325 | 0.5011 |
| MAD | Hard | 14.8525 | 15.9881 | 0.3325 | 0.3359 |
| STD | Hard | 14.8092 | 18.5338 | 0.3299 | 0.4475 |
| UNIV | Hard | 14.9352 | 16.9636 | 0.3403 | 0.3640 |
| MAD | Custom | 14.9715 | 17.7467 | 0.3363 | 0.3759 |
| STD | Custom | 14.8342 | 18.7708 | 0.3325 | 0.4256 |
| UNIV | Custom | 14.8682 | 18.1017 | 0.3315 | 0.3853 |

**Table 6.88 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 14.9496 | 21.4022 | 0.3345 | 0.5532 |
| STD | Soft | 14.8508 | 21.4951 | 0.3341 | 0.5810 |
| UNIV | Soft | 14.8204 | 21.6645 | 0.3297 | 0.5913 |
| MAD | Hard | 14.9405 | 17.7492 | 0.3366 | 0.4278 |
| STD | Hard | 14.8932 | 21.3422 | 0.3308 | 0.5699 |
| UNIV | Hard | 15.0391 | 20.2699 | 0.3385 | 0.5125 |
| MAD | Custom | 14.9066 | 19.7117 | 0.3353 | 0.4703 |
| STD | Custom | 15.0020 | 21.0577 | 0.3414 | 0.5440 |
| UNIV | Custom | 14.8610 | 20.5237 | 0.3310 | 0.5005 |

**Thresholding low pass components**

**a. GAUSSIAN (0, 0.09)**

**Table 6.89 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 12.3020 | 18.2179 | 0.1696 | 0.4628 |
| STD | Soft | 12.2846 | 15.9824 | 0.1697 | 0.4391 |
| UNIV | Soft | 12.3028 | 15.7184 | 0.1685 | 0.4196 |
| MAD | Hard | 12.2920 | 15.6053 | 0.1718 | 0.2423 |
| STD | Hard | 12.2825 | 18.8453 | 0.1695 | 0.4112 |
| UNIV | Hard | 12.3109 | 18.9421 | 0.1700 | 0.4169 |
| MAD | Custom | 12.2885 | 16.9603 | 0.1685 | 0.2756 |
| STD | Custom | 12.2734 | 18.9726 | 0.1693 | 0.3937 |
| UNIV | Custom | 12.2849 | 19.2316 | 0.1717 | 0.4117 |

**Table 6.90 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 12.3831 | 15.3021 | 0.1728 | 0.4311 |
| STD | Soft | 12.2966 | 14.9313 | 0.1707 | 0.4033 |
| UNIV | Soft | 12.2902 | 14.6415 | 0.1710 | 0.3918 |
| MAD | Hard | 12.2653 | 20.9500 | 0.1715 | 0.5260 |
| STD | Hard | 12.2829 | 21.0020 | 0.1685 | 0.5313 |
| UNIV | Hard | 12.3283 | 21.0324 | 0.1731 | 0.5291 |
| MAD | Custom | 12.2740 | 20.7543 | 0.1697 | 0.5213 |
| STD | Custom | 12.2497 | 20.7515 | 0.1690 | 0.5287 |
| UNIV | Custom | 12.3286 | 20.8683 | 0.1698 | 0.5336 |

**b. POISSON**

**Table 6.91 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 30.1235 | 30.5770 | 0.8643 | 0.8566 |
| STD | Soft | 30.0441 | 26.2826 | 0.8634 | 0.7597 |
| UNIV | Soft | 30.1011 | 26.9205 | 0.8646 | 0.7696 |
| MAD | Hard | 30.0453 | 30.4085 | 0.8623 | 0.8799 |
| STD | Hard | 30.1468 | 29.9863 | 0.8650 | 0.7961 |
| UNIV | Hard | 30.1464 | 30.2782 | 0.8639 | 0.8013 |
| MAD | Custom | 30.1566 | 24.6025 | 0.8647 | 0.8072 |
| STD | Custom | 30.0516 | 24.5691 | 0.8627 | 0.7558 |
| UNIV | Custom | 30.1038 | 24.6056 | 0.8638 | 0.7687 |

**Table 6.92 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 30.1444 | 29.0727 | .8659 | 0.8053 |
| STD | Soft | 30.1207 | 26.1463 | 0.8643 | 0.7656 |
| UNIV | Soft | 30.0757 | 25.4204 | 0.8636 | 0.7552 |
| MAD | Hard | 30.0815 | 32.3284 | 0.8639 | 0.8362 |
| STD | Hard | 30.1022 | 31.7084 | 0.8631 | 0.8162 |
| UNIV | Hard | 30.0664 | 31.6854 | 0.8639 | 0.8156 |
| MAD | Custom | 30.0835 | 27.2519 | 0.8640 | 0.8599 |
| STD | Custom | 30.0480 | 26.7313 | 0.8635 | 0.7847 |
| UNIV | Custom | 30.1542 | 26.7692 | 0.8650 | 0.7853 |

**c. SPECKLE (v=0.09)**

**Table 6.93 DWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 20.2221 | 23.1634 | 0.6098 | 0.6452 |
| STD | Soft | 20.2363 | 20.7073 | 0.6107 | 0.6345 |
| UNIV | Soft | 20.2008 | 21.4914 | 0.6095 | 0.6489 |
| MAD | Hard | 20.2778 | 20.6468 | 0.6117 | 0.6175 |
| STD | Hard | 20.2176 | 22.9375 | 0.6106 | 0.6414 |
| UNIV | Hard | 20.1970 | 22.3915 | 0.6094 | 0.6180 |
| MAD | Custom | 20.2118 | 21.3239 | 0.6093 | 0.5869 |
| STD | Custom | 20.2113 | 22.2571 | 0.6109 | 0.6281 |
| UNIV | Custom | 20.1676 | 22.0226 | 0.6090 | 0.6079 |

**Table 6.94 UDWT**

| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 20.1545 | 23.2515 | 0.6075 | 0.6989 |
| STD | Soft | 20.1907 | 20.0362 | 0.6079 | 0.6411 |
| UNIV | Soft | 20.1913 | 20.5883 | 0.6106 | 0.6555 |
| MAD | Hard | 20.2317 | 22.8024 | 0.6081 | 0.6368 |
| STD | Hard | 20.1775 | 25.2919 | 0.6091 | 0.7162 |
| UNIV | Hard | 20.1590 | 24.9534 | 0.6088 | 0.7091 |
| MAD | Custom | 20.2254 | 23.8824 | 0.6084 | 0.6732 |
| STD | Custom | 20.1978 | 23.9610 | 0.6078 | 0.7001 |
| UNIV | Custom | 20.2231 | 24.0273 | 0.6098 | 0.6990 |

### d. SALT & PEPPER (d=0.09)

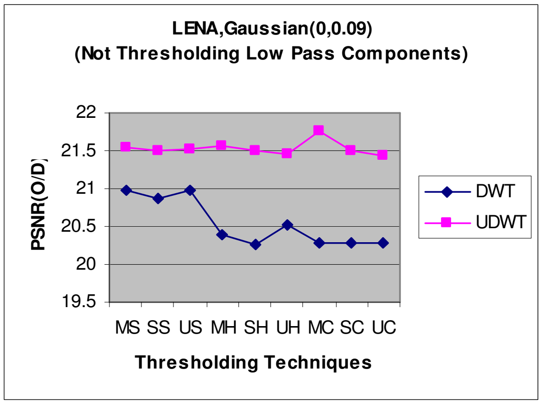
**Table 6.95 DWT**

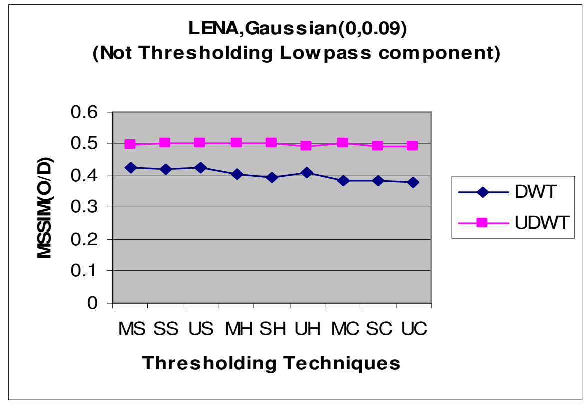
| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 14.8259 | 19.1541 | 0.3320 | 0.4393 |
| STD | Soft | 14.8470 | 17.7891 | 0.3351 | 0.5353 |
| UNIV | Soft | 14.8566 | 19.2224 | 0.3323 | 0.5265 |
| MAD | Hard | 15.0676 | 15.5587 | 0.3419 | 0.3277 |
| STD | Hard | 14.8681 | 19.1430 | 0.3346 | 0.4635 |
| UNIV | Hard | 14.9055 | 17.0791 | 0.3382 | 0.3670 |
| MAD | Custom | 14.8355 | 17.3048 | 0.3305 | 0.3594 |
| STD | Custom | 14.9278 | 19.3473 | 0.3349 | 0.4458 |
| UNIV | Custom | 14.7692 | 18.1894 | 0.3293 | 0.3934 |

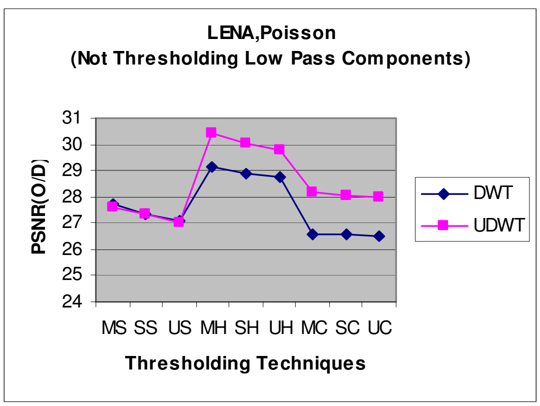
**Table 6.96 UDWT**

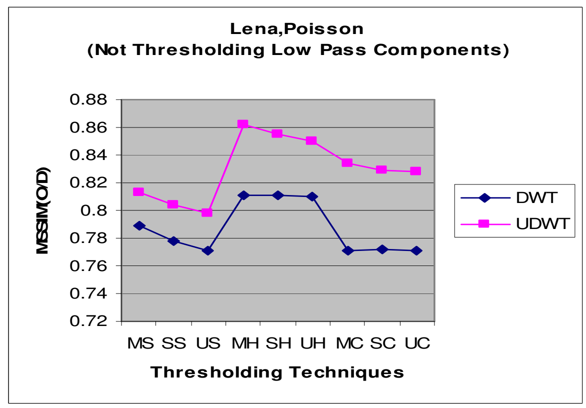
| Threshold value | Thresholding Technique | PSNR(O/N) | PSNR(O/D) | MSSIM(O/N) | MSSIM(O/D) |
|---|---|---|---|---|---|
| MAD | Soft | 14.8049 | 20.2796 | 0.3313 | 0.5579 |
| STD | Soft | 14.8573 | 16.7923 | 0.3308 | 0.5240 |
| UNIV | Soft | 14.9219 | 17.8373 | 0.3349 | 0.5632 |
| MAD | Hard | 14.8955 | 17.9678 | 0.3289 | 0.4253 |
| STD | Hard | 14.8979 | 22.1952 | 0.3326 | 0.5986 |
| UNIV | Hard | 14.8508 | 20.5286 | 0.3266 | 0.5168 |
| MAD | Custom | 14.8591 | 19.9898 | 0.3301 | 0.4709 |
| STD | Custom | 14.8758 | 21.5151 | 0.3300 | 0.5527 |
| UNIV | Custom | 14.8488 | 20.8269 | 0.3360 | 0.5119 |

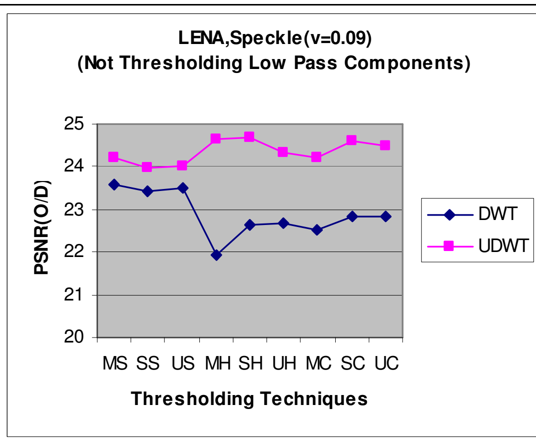
Following graphs give a simultaneous comparison of PSNR and MSSIM obtained after applying all the combinations proposed for denoising images by wavelet transforms, both DWT and UDWT.

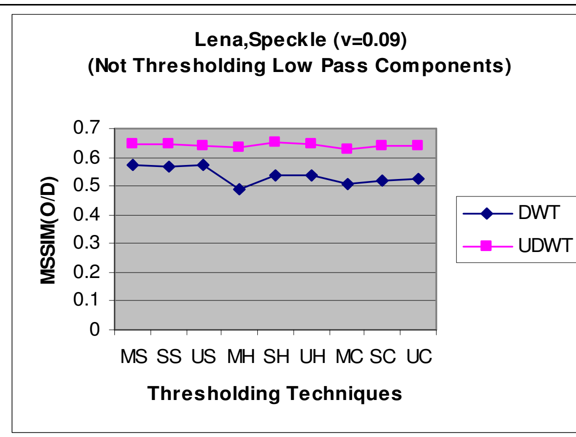


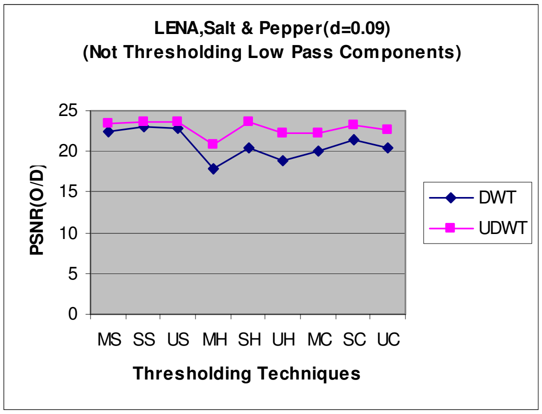



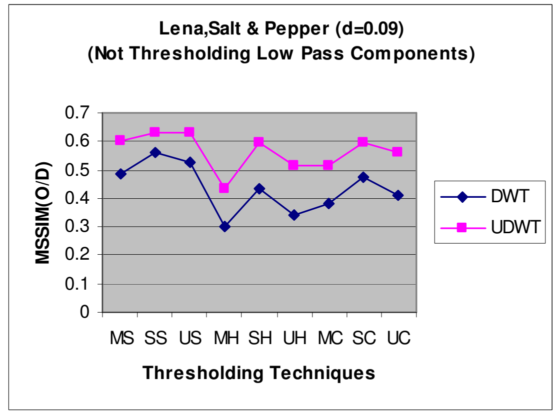



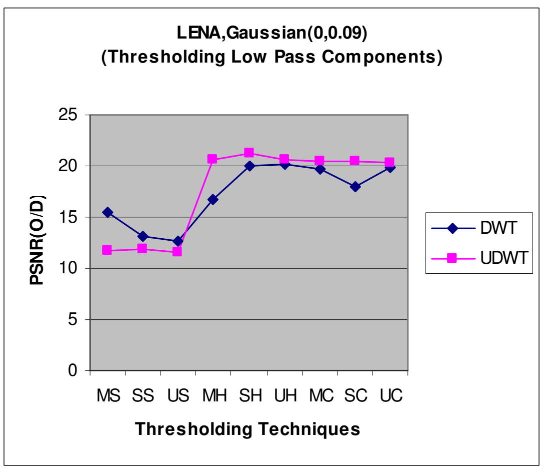



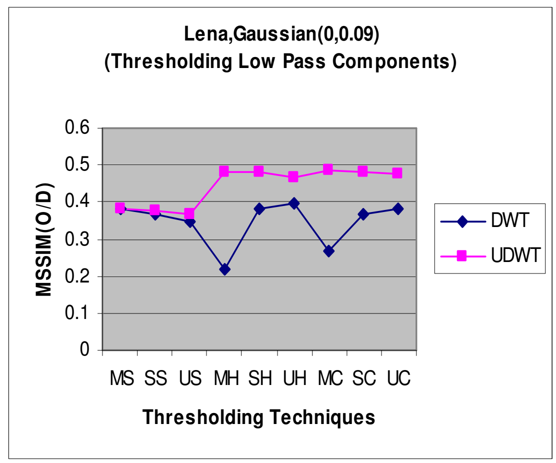



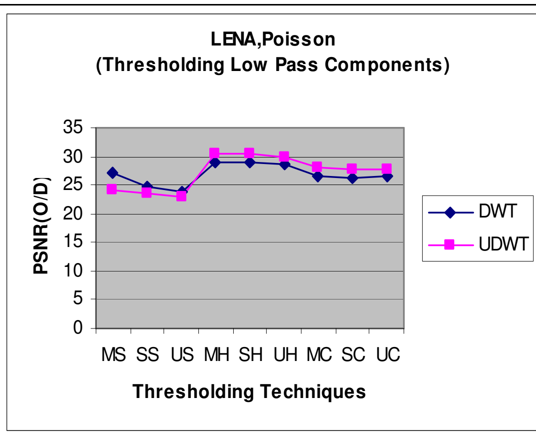



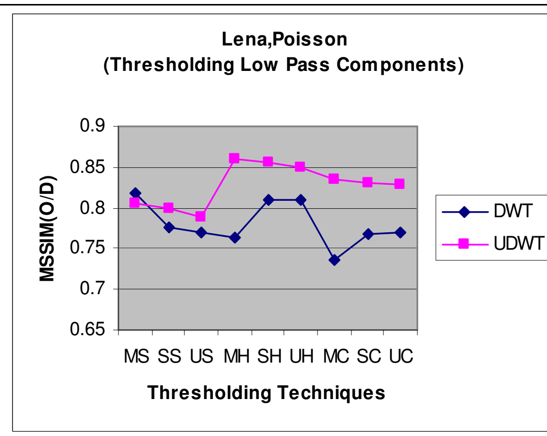

LENA,Salt & Pepper(d=0.09)
(Not Thresholding Low Pass Components)
PSNR(O/D)
0
5
10
15
20
25
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

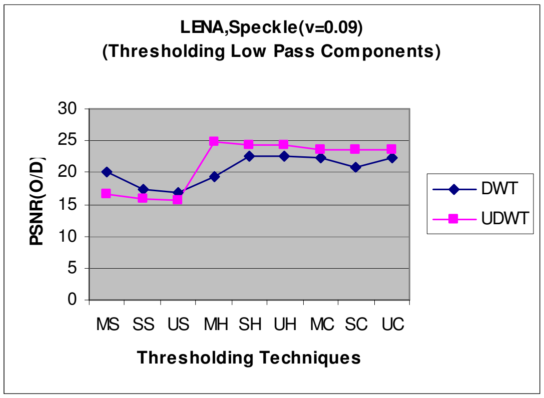

Lena,Salt & Pepper (d=0.09)
(Not Thresholding Low Pass Components)
MSSIM(O/D)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

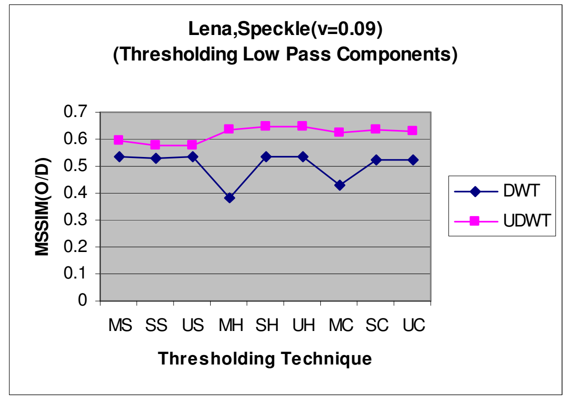

LENA,Gaussian(0,0.09)
(Thresholding Low Pass Components)
PSNR(O/D)
0
5
10
15
20
25
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

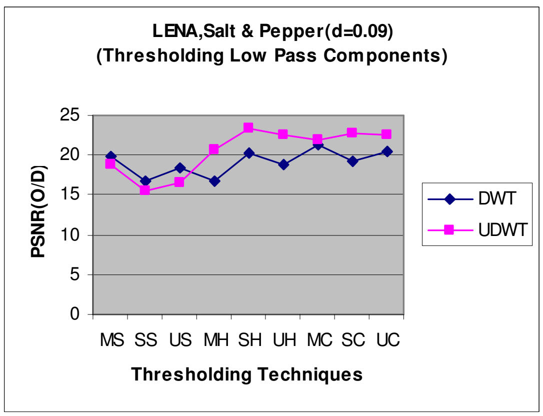

Lena,Gaussian(0,0.09)
(Thresholding Low Pass Components)
MSSIM(O/D)
0
0.1
0.2
0.3
0.4
0.5
0.6
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

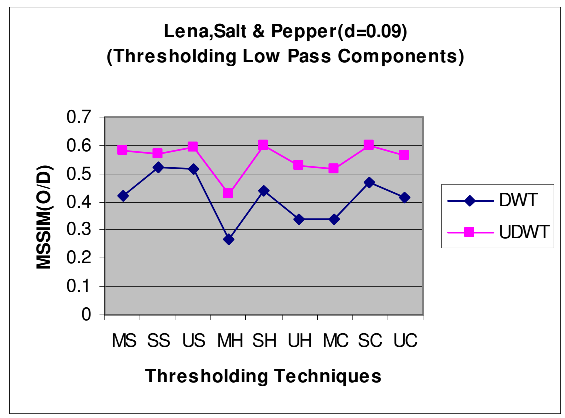

LENA,Poisson
(Thresholding Low Pass Components)
PSNR(O/D)
0
5
10
15
20
25
30
35
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

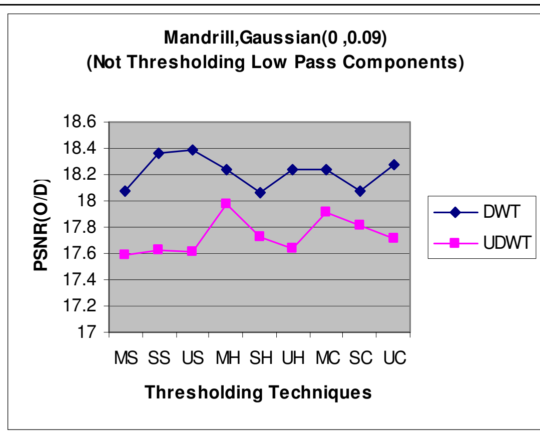

Lena,Poisson
(Thresholding Low Pass Components)
MSSIM(O/D)
0.65
0.7
0.75
0.8
0.85
0.9
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

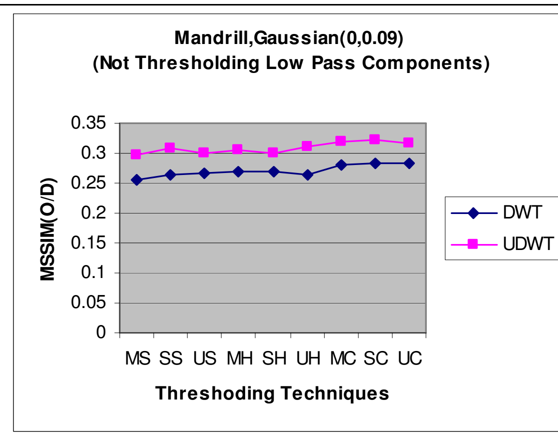

LENA,Speckle(v=0.09)
(Thresholding Low Pass Components)
PSNR(O/D)
30
25
20
15
10
5
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

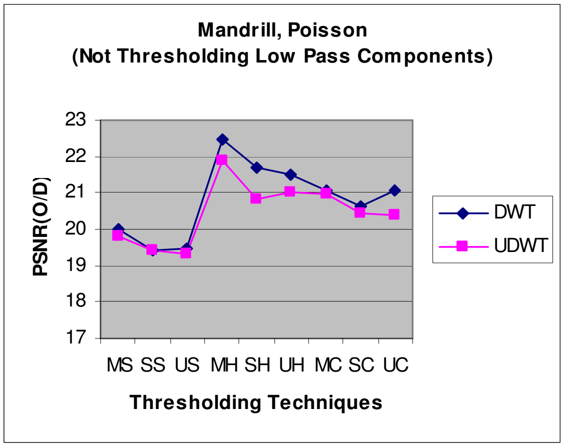

Lena,Speckle(v=0.09)
(Thresholding Low Pass Components)
MSSIM(O/D)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
MS SS US MH SH UH MC SC UC
Thresholding Technique
DWT
UDWT

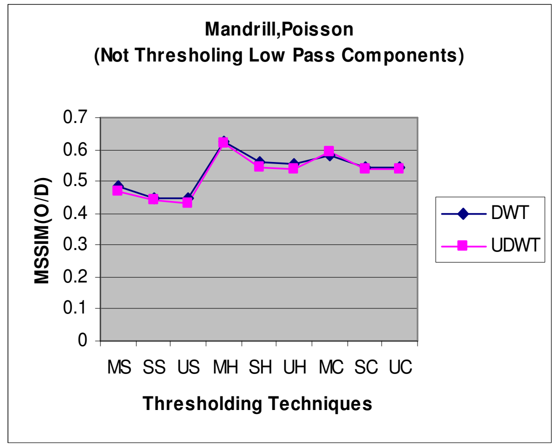

LENA,Salt & Pepper(d=0.09)
(Thresholding Low Pass Components)
PSNR(O/D)
25
20
15
10
5
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

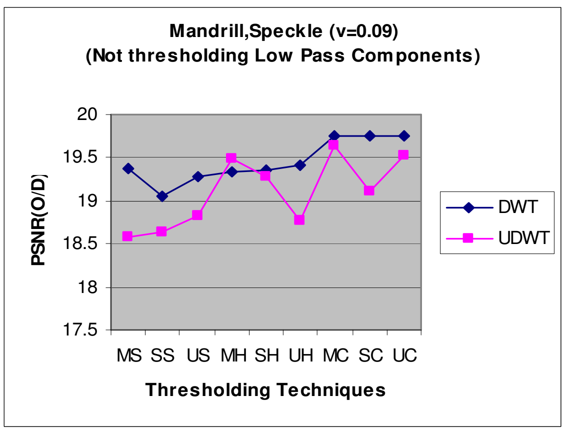

Lena,Salt & Pepper(d=0.09)
(Thresholding Low Pass Components)
MSSIM(O/D)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

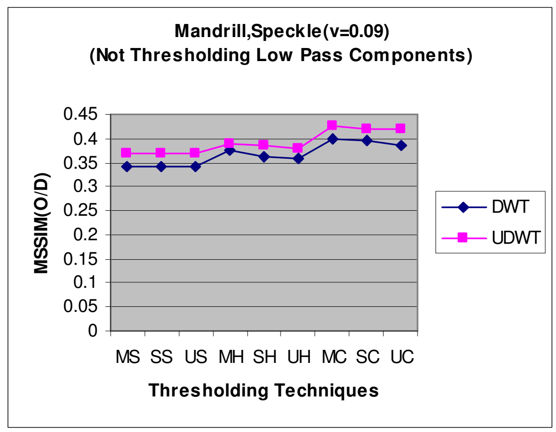

Mandrill,Gaussian(0 ,0.09)
(Not Thresholding Low Pass Components)
PSNR(O/D)
18.6
18.4
18.2
18
17.8
17.6
17.4
17.2
17
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

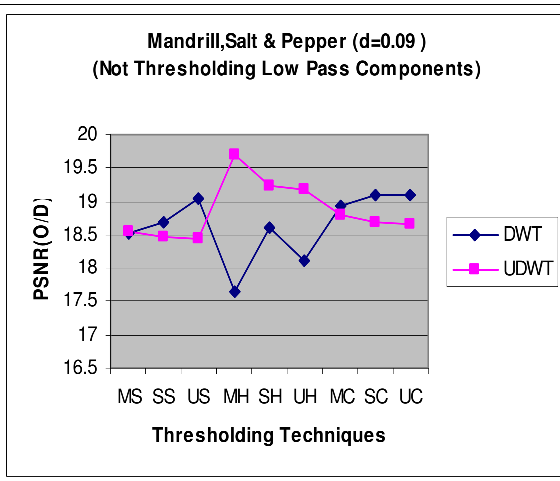

Mandrill,Gaussian(0,0.09)
(Not Thresholding Low Pass Components)
MSSIM(O/D)
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
MS SS US MH SH UH MC SC UC
Threshoding Techniques
DWT
UDWT

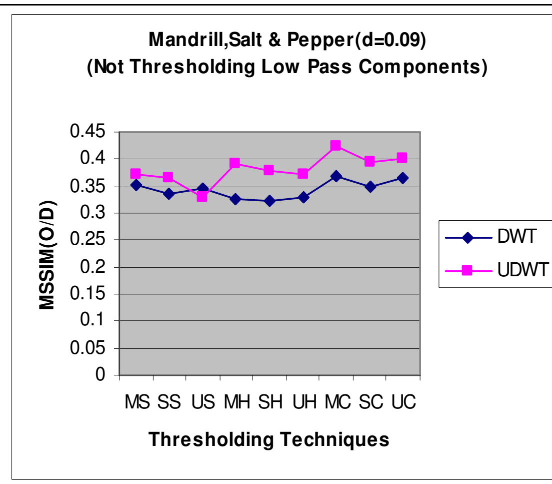

Mandrill, Poisson
(Not Thresholding Low Pass Components)
PSNR(O/D)
23
22
21
20
19
18
17
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

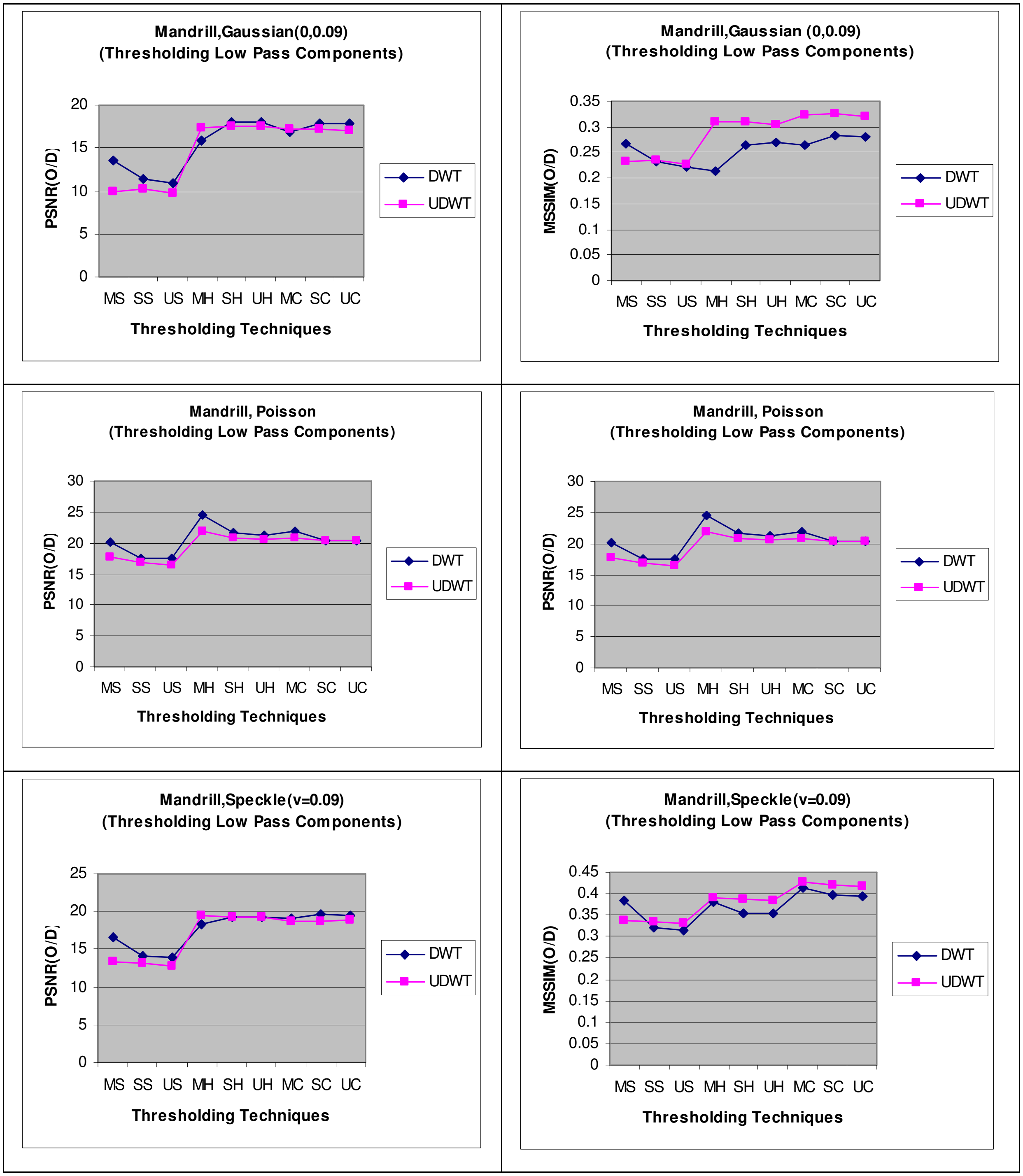

Mandrill,Poisson
(Not Thresholing Low Pass Components)
MSSIM(O/D)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

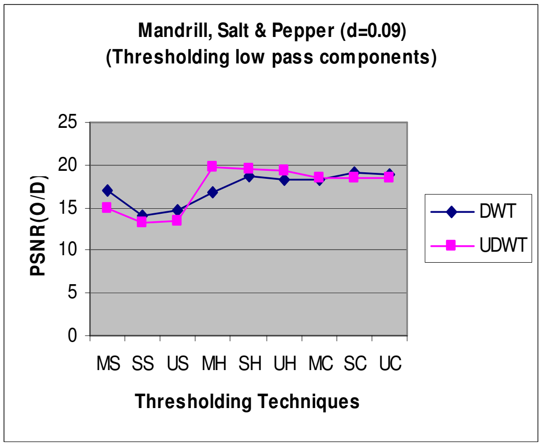

Mandrill,Speckle (v=0.09)
(Not thresholding Low Pass Components)
PSNR(O/D)
20
19.5
19
18.5
18
17.5
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

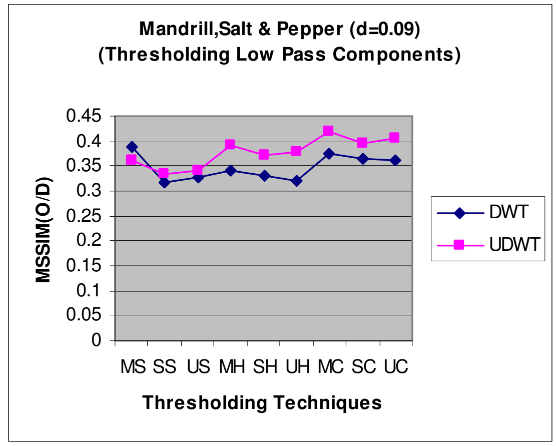

Mandrill,Speckle(v=0.09)
(Not Thresholding Low Pass Components)
MSSIM(O/D)
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

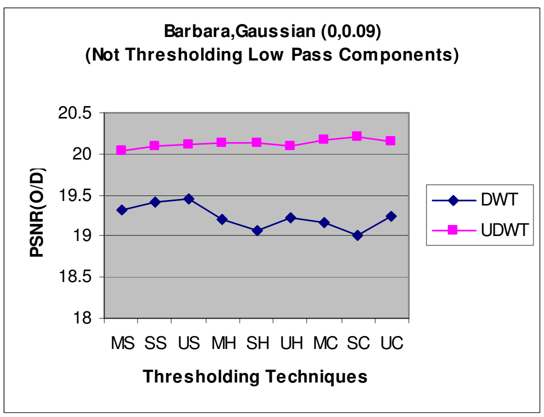

Mandrill,Salt & Pepper (d=0.09 )
(Not Thresholding Low Pass Components)
PSNR(O/D)
20
19.5
19
18.5
18
17.5
17
16.5
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

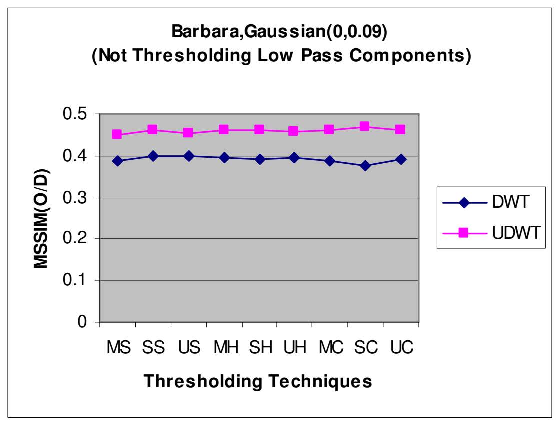

Mandrill,Salt & Pepper(d=0.09)
(Not Thresholding Low Pass Components)
MSSIM(O/D)
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

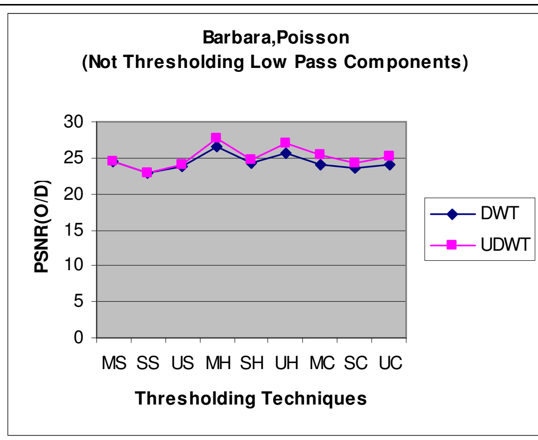

Mandrill,Gaussian(0,0.09)
(Thresholding Low Pass Components)
PSNR(O/D)
0 5 10 15 20
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT
Mandrill,Gaussian (0,0.09)
(Thresholding Low Pass Components)
MSSIM(O/D)
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT
Mandrill, Poisson
(Thresholding Low Pass Components)
PSNR(O/D)
0 5 10 15 20 25 30
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT
Mandrill, Poisson
(Thresholding Low Pass Components)
PSNR(O/D)
0 5 10 15 20 25 30
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT
Mandrill,Speckle(v=0.09)
(Thresholding Low Pass Components)
PSNR(O/D)
0 5 10 15 20 25
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT
Mandrill,Speckle(v=0.09)
(Thresholding Low Pass Components)
MSSIM(O/D)
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

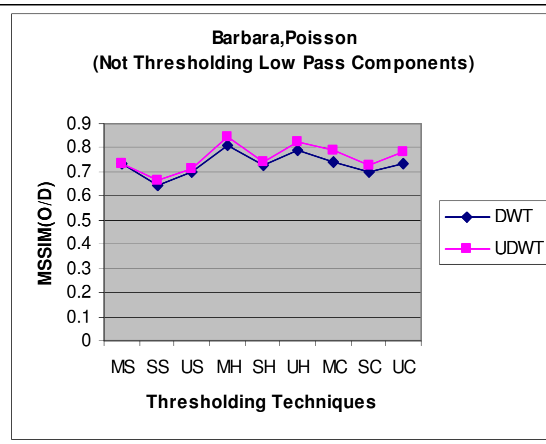

Mandrill, Salt & Pepper (d=0.09)
(Thresholding low pass components)
PSNR(O/D)
25
20
15
10
5
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

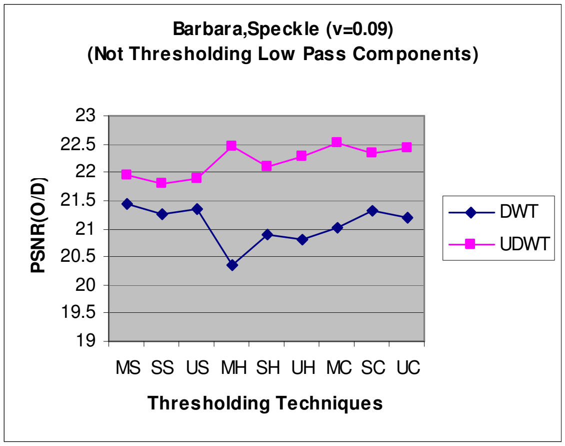

Mandrill,Salt & Pepper (d=0.09)
(Thresholding Low Pass Components)
MSSIM(O/D)
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

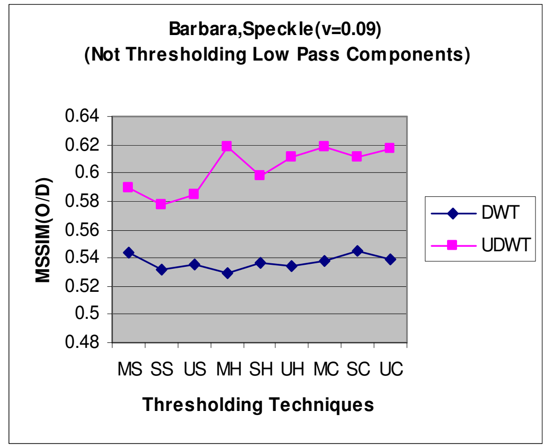

Barbara,Gaussian (0,0.09)
(Not Thresholding Low Pass Components)
PSNR(O/D)
20.5
20
19.5
19
18.5
18
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

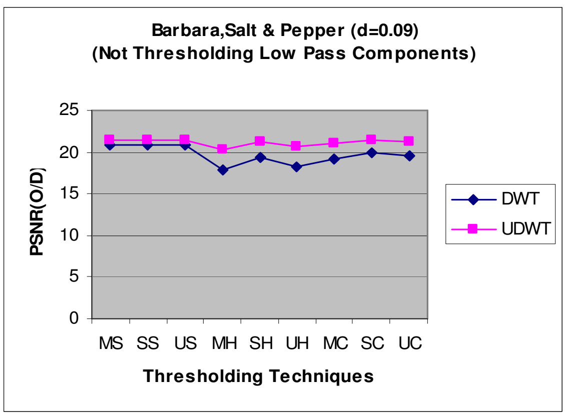

Barbara,Gaussian(0,0.09)
(Not Thresholding Low Pass Components)
MSSIM(O/D)
0.5
0.4
0.3
0.2
0.1
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

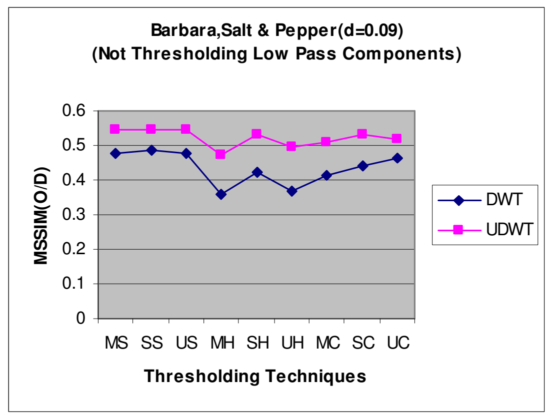

Barbara,Poisson
(Not Thresholding Low Pass Components)
PSNR(O/D)
30
25
20
15
10
5
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

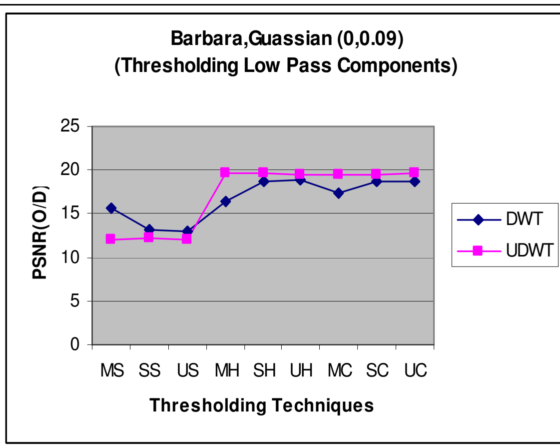

Barbara,Poisson
(Not Thresholding Low Pass Components)
MSSIM(O/D)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

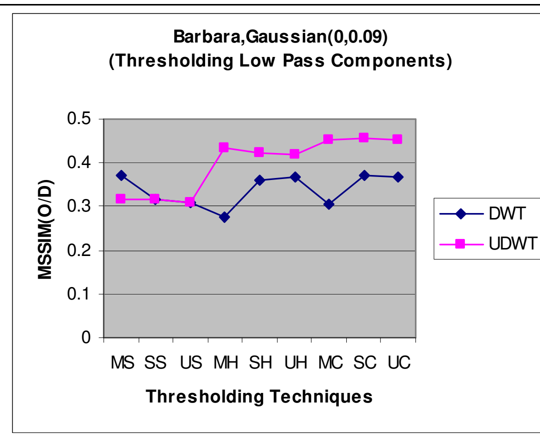

Barbara,Speckle (v=0.09)
(Not Thresholding Low Pass Components)
PSNR(O/D)
23
22.5
22
21.5
21
20.5
20
19.5
19
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

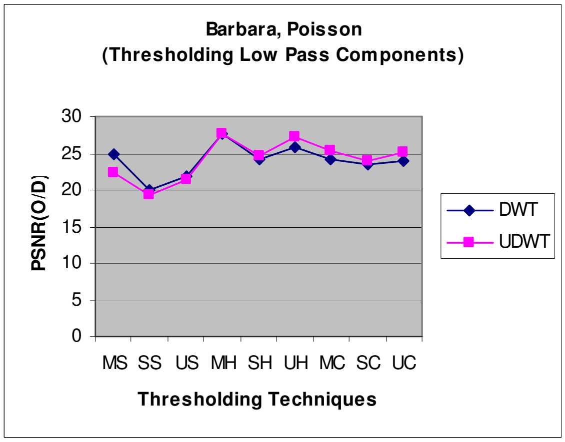

Barbara,Speckle(v=0.09)
(Not Thresholding Low Pass Components)
MSSIM(O/D)
0.64
0.62
0.6
0.58
0.56
0.54
0.52
0.5
0.48
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

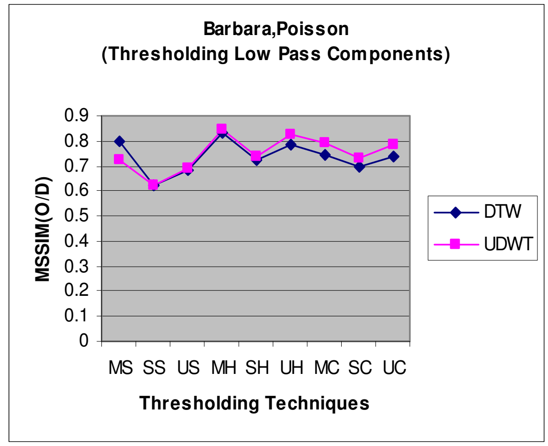

Barbara,Salt & Pepper (d=0.09)
(Not Thresholding Low Pass Components)
PSNR(O/D)
25
20
15
10
5
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

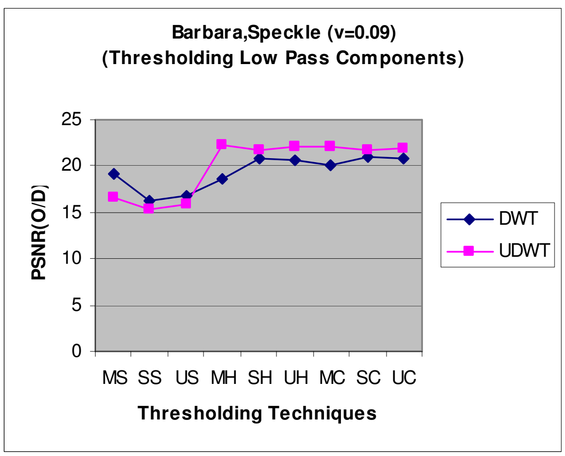

Barbara,Salt & Pepper(d=0.09)
(Not Thresholding Low Pass Components)
MSSIM(O/D)
0.6
0.5
0.4
0.3
0.2
0.1
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

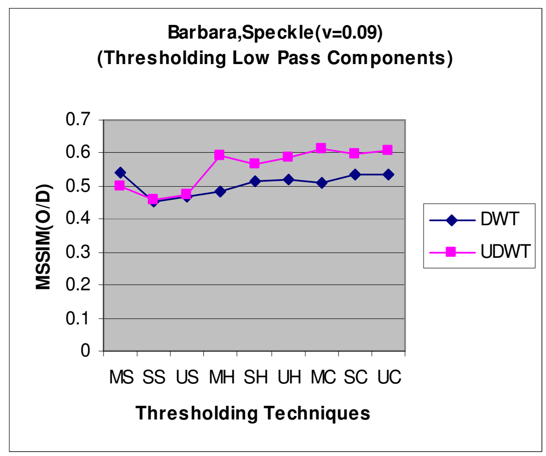

Barbara,Guassian (0,0.09)
(Thresholding Low Pass Components)
PSNR(O/D)
25
20
15
10
5
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

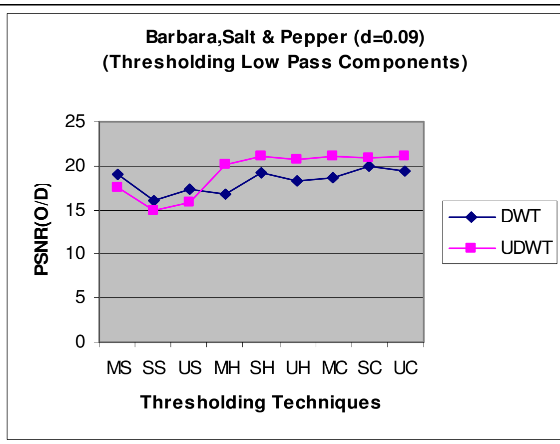

Barbara,Gaussian(0,0.09)
(Thresholding Low Pass Components)
MSSIM(O/D)
0.5
0.4
0.3
0.2
0.1
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

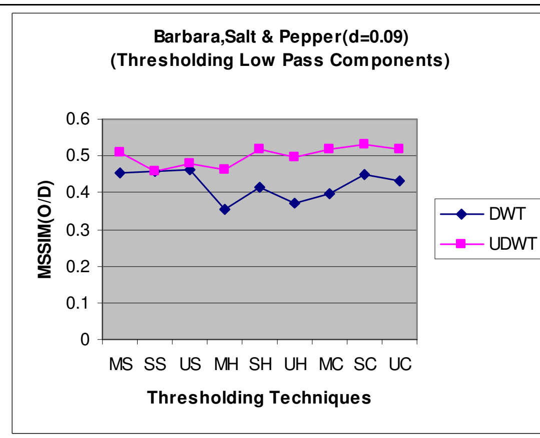

Barbara, Poisson
(Thresholding Low Pass Components)
PSNR(O/D)
30 25 20 15 10 5 0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

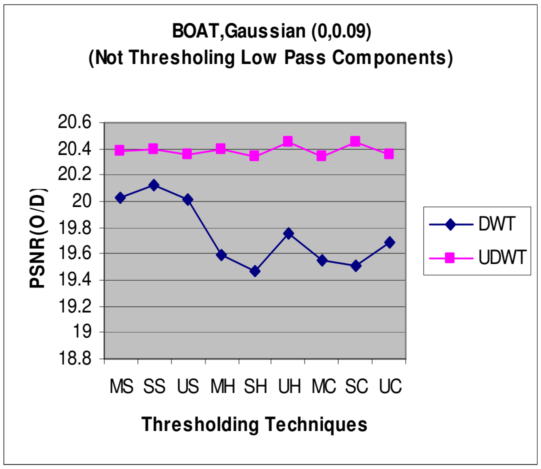

Barbara,Poisson
(Thresholding Low Pass Components)
MSSIM(O/D)
0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DTW
UDWT

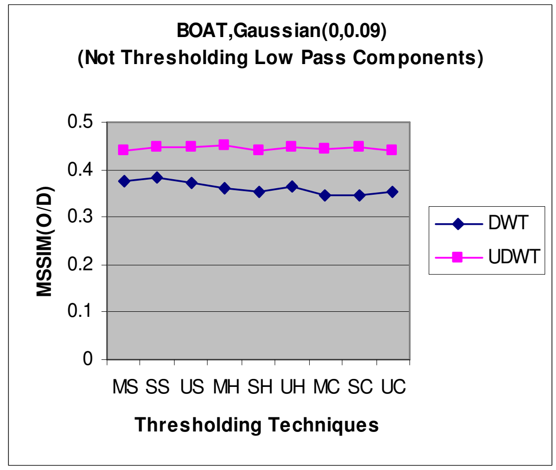

Barbara,Speckle (v=0.09)
(Thresholding Low Pass Components)
PSNR(O/D)
25 20 15 10 5 0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

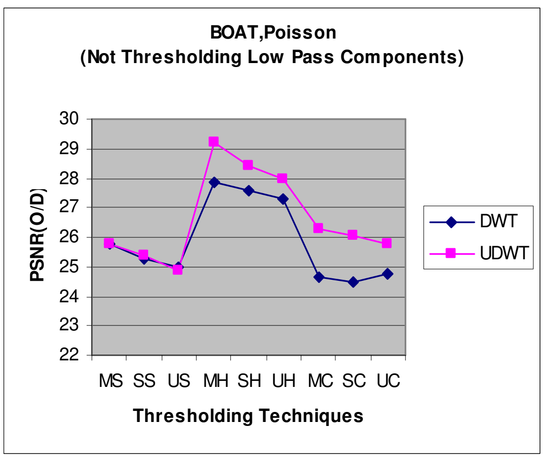

Barbara,Speckle(v=0.09)
(Thresholding Low Pass Components)
MSSIM(O/D)
0.7 0.6 0.5 0.4 0.3 0.2 0.1 0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

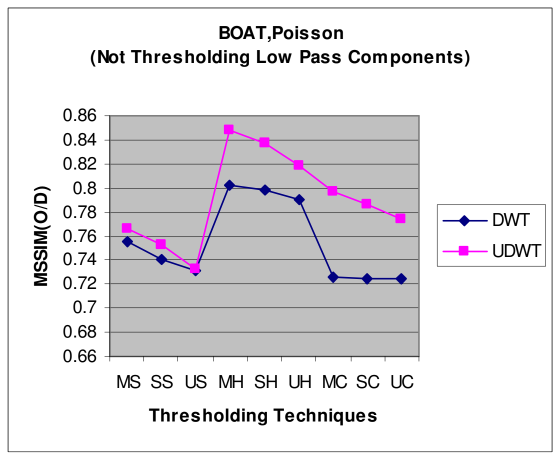

Barbara,Salt & Pepper (d=0.09)
(Thresholding Low Pass Components)
PSNR(O/D)
25 20 15 10 5 0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

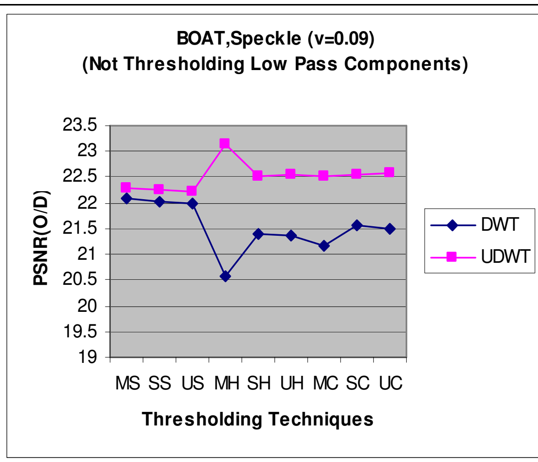

Barbara,Salt & Pepper(d=0.09)
(Thresholding Low Pass Components)
MSSIM(O/D)
0.6 0.5 0.4 0.3 0.2 0.1 0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

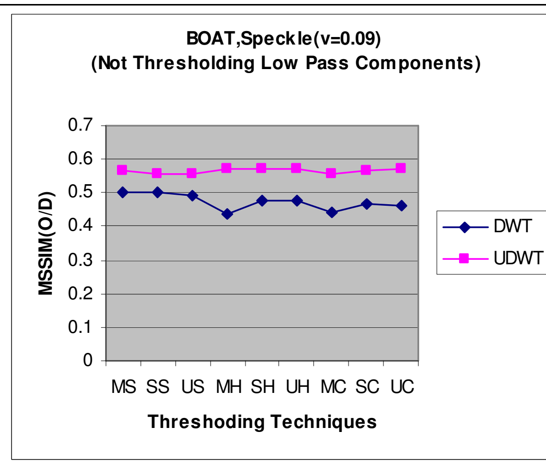

BOAT,Gaussian (0,0.09)
(Not Thresholing Low Pass Components)
PSNR(O/D)
20.6
20.4
20.2
20
19.8
19.6
19.4
19.2
19
18.8
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

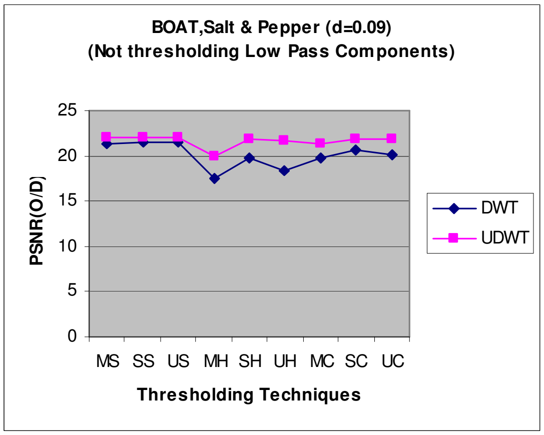

BOAT,Gaussian(0,0.09)
(Not Thresholding Low Pass Components)
MSSIM(O/D)
0.5
0.4
0.3
0.2
0.1
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

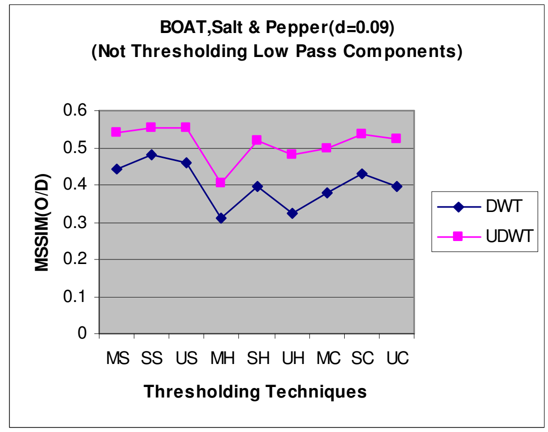

BOAT,Poisson
(Not Thresholding Low Pass Components)
PSNR(O/D)
30
29
28
27
26
25
24
23
22
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

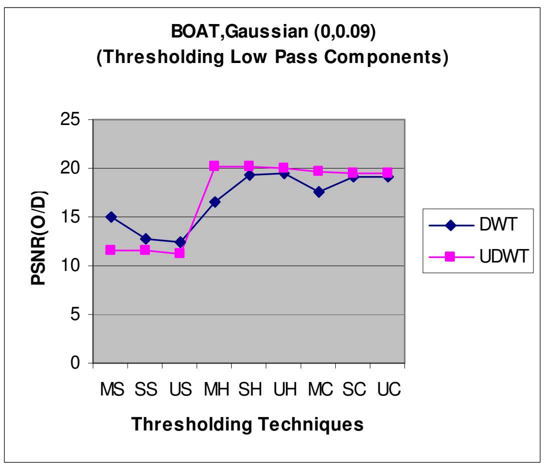

BOAT,Poisson
(Not Thresholding Low Pass Components)
MSSIM(O/D)
0.86
0.84
0.82
0.8
0.78
0.76
0.74
0.72
0.7
0.68
0.66
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

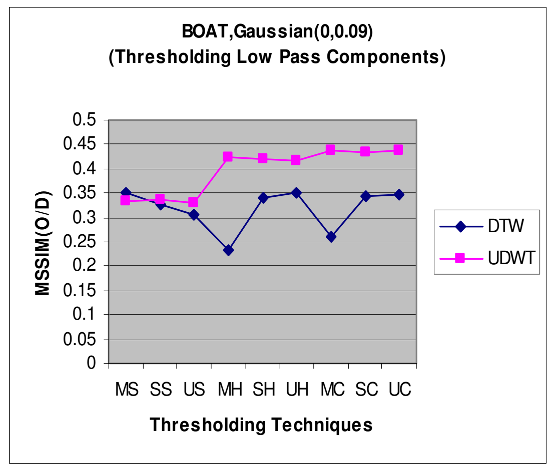

BOAT,Speckle (v=0.09)
(Not Thresholding Low Pass Components)
PSNR(O/D)
23.5
23
22.5
22
21.5
21
20.5
20
19.5
19
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

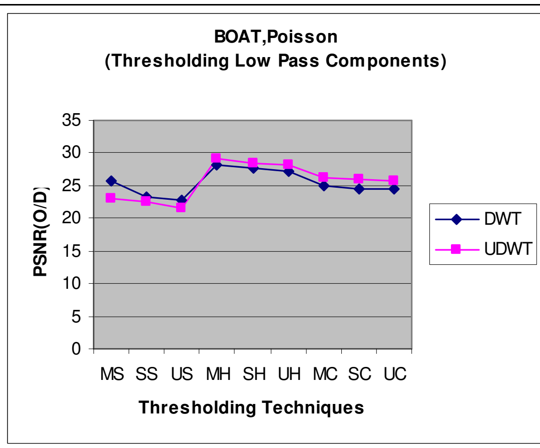

BOAT,Speckle(v=0.09)
(Not Thresholding Low Pass Components)
MSSIM(O/D)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
MS SS US MH SH UH MC SC UC
Threshoding Techniques
DWT
UDWT

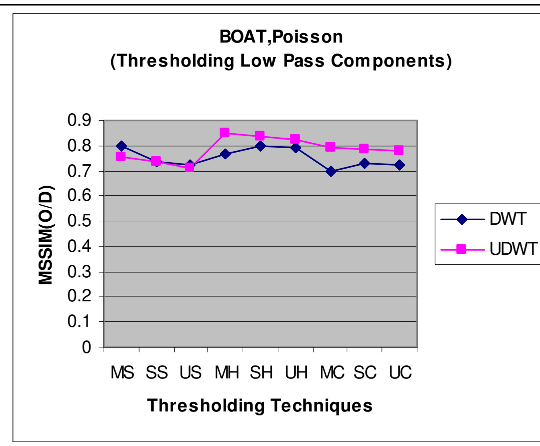

BOAT,Salt & Pepper (d=0.09)
(Not thresholding Low Pass Components)
PSNR(O/D)
0
5
10
15
20
25
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

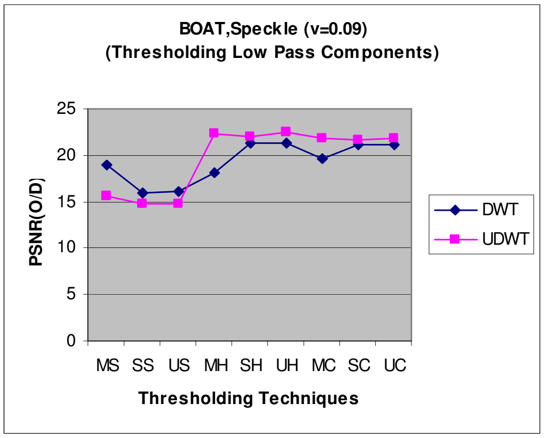

BOAT,Salt & Pepper(d=0.09)
(Not Thresholding Low Pass Components)
MSSIM(O/D)
0
0.1
0.2
0.3
0.4
0.5
0.6
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

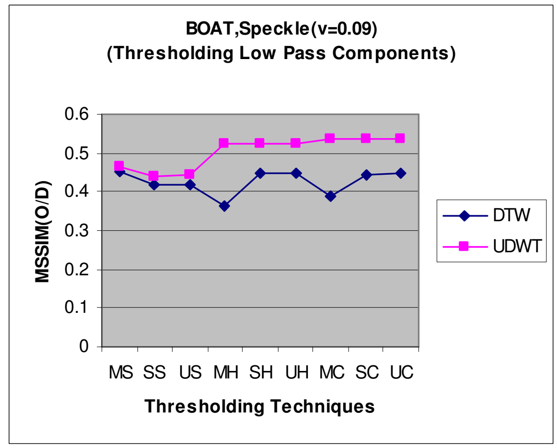

BOAT,Gaussian (0,0.09)
(Thresholding Low Pass Components)
PSNR(O/D)
0
5
10
15
20
25
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

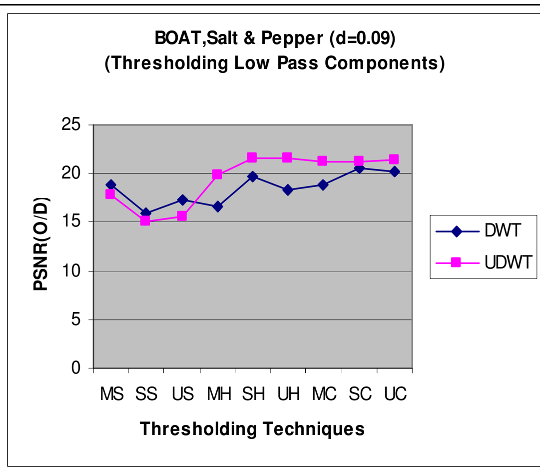

BOAT,Gaussian(0,0.09)
(Thresholding Low Pass Components)
MSSIM(O/D)
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DTW
UDWT

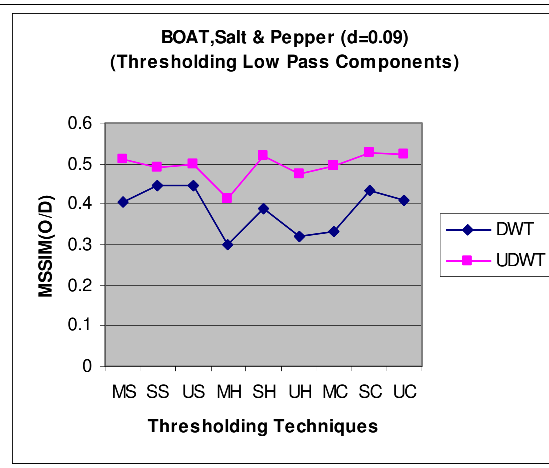

BOAT,Poisson
(Thresholding Low Pass Components)
PSNR(O/D)
0
5
10
15
20
25
30
35
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

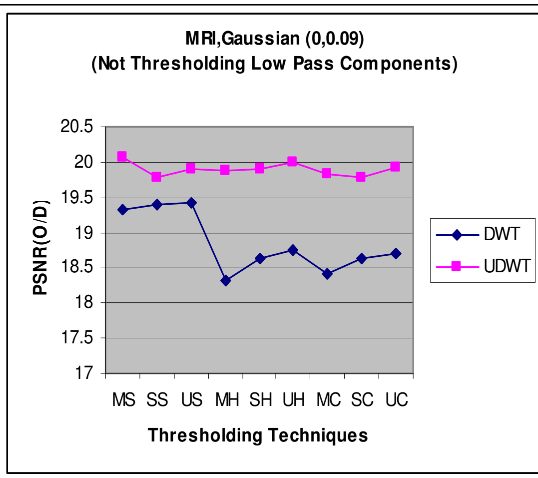

BOAT,Poisson
(Thresholding Low Pass Components)
MSSIM(O/D)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

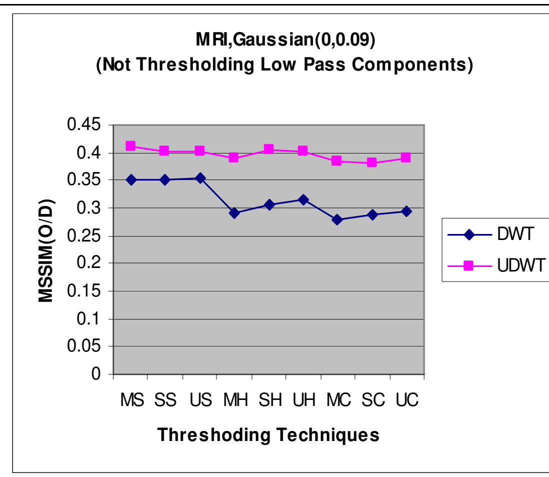

BOAT,Speckle (v=0.09)
(Thresholding Low Pass Components)
PSNR(O/D)
25
20
15
10
5
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT


BOAT,Speckle(v=0.09)
(Thresholding Low Pass Components)
MSSIM(O/D)
0.6
0.5
0.4
0.3
0.2
0.1
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DTW
UDWT

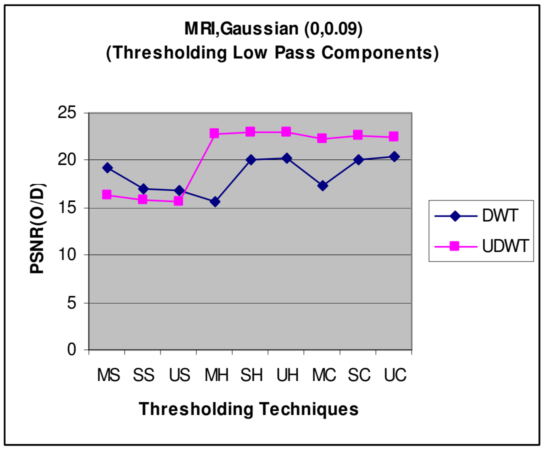

BOAT,Salt & Pepper (d=0.09)
(Thresholding Low Pass Components)
PSNR(O/D)
25
20
15
10
5
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

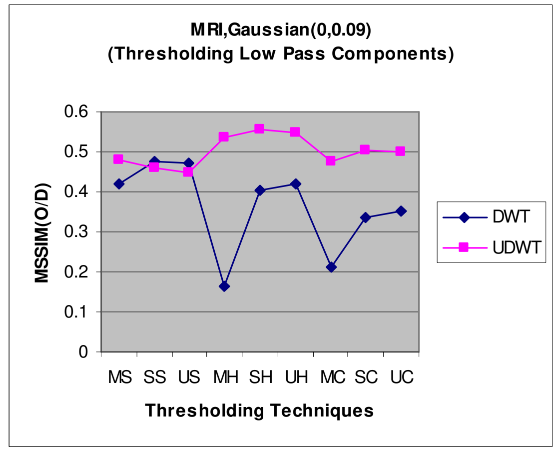

BOAT,Salt & Pepper (d=0.09)
(Thresholding Low Pass Components)
MSSIM(O/D)
0.6
0.5
0.4
0.3
0.2
0.1
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

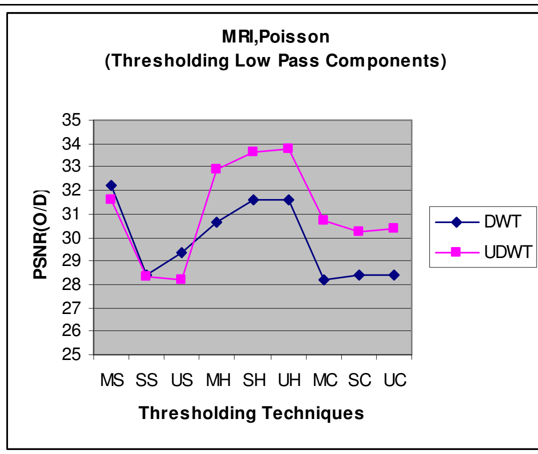

MRI,Gaussian (0,0.09)
(Not Thresholding Low Pass Components)
PSNR(O/D)
20.5
20
19.5
19
18.5
18
17.5
17
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

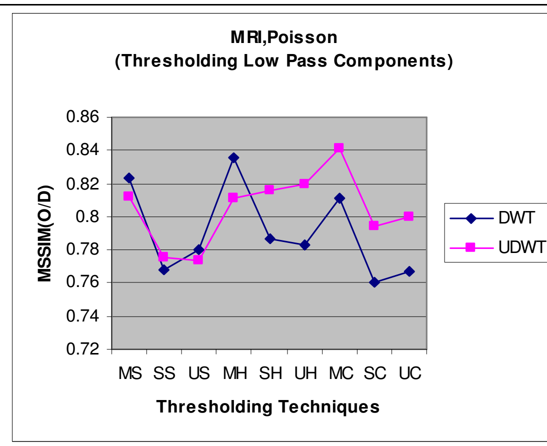

MRI,Gaussian(0,0.09)
(Not Thresholding Low Pass Components)
MSSIM(O/D)
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
MS SS US MH SH UH MC SC UC
Threshoding Techniques
DWT
UDWT

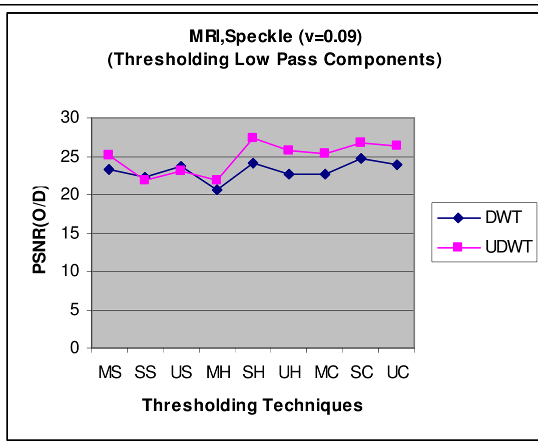

MRI,Poisson
(Not Thresholding Low Pass Components)
PSNR(O/D)
35
34
33
32
31
30
29
28
27
26
25
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT
MRI,Poisson
(Not Thresholding Low Pass Components)
MSSIM(O/D)
0.94
0.92
0.9
0.88
0.86
0.84
0.82
0.8
0.78
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT
MRI,Speckle (v=0.09)
(Not Thresholding Low Pass Components)
PSNR(O/D)
30
25
20
15
10
5
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT
MRI,Speckle(v=0.09)
(Not Thresholding Low Pass Components)
MSSIM(O/D)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT
MRI,Salt & Pepper (d=0.09)
(Not Thresholding Low Pass Components)
PSNR(O/D)
25
20
15
10
5
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT
MRI,Salt & Pepper (d=0.09)
(Not Thresholding Low Pass Components)
MSSIM(O/D)
0.6
0.5
0.4
0.3
0.2
0.1
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

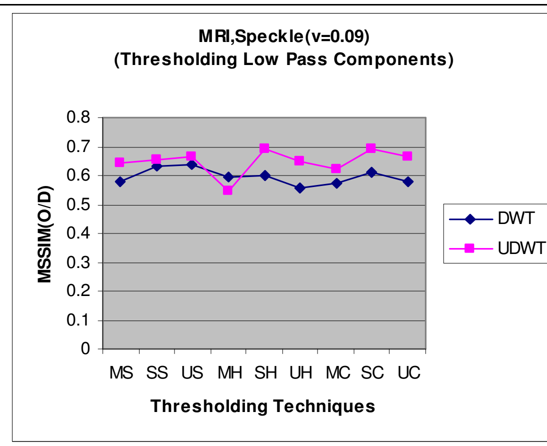

MRI,Gaussian (0,0.09)
(Thresholding Low Pass Components)
PSNR(O/D)
25
20
15
10
5
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

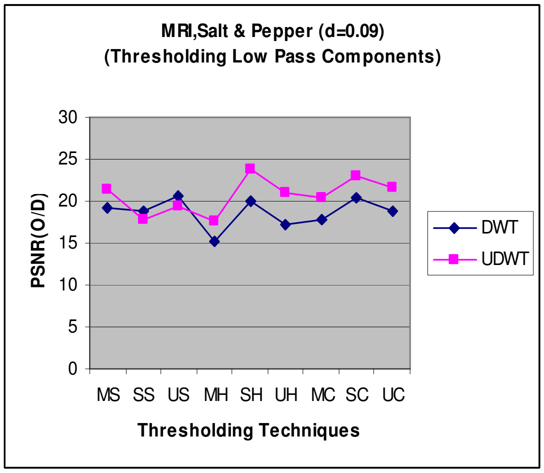

MRI,Gaussian(0,0.09)
(Thresholding Low Pass Components)
MSSIM(O/D)
0.6
0.5
0.4
0.3
0.2
0.1
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

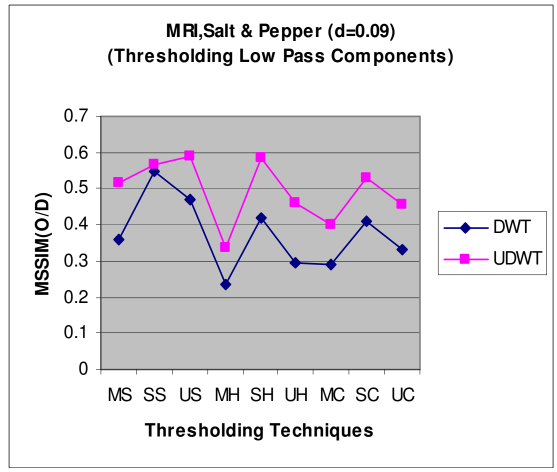

MRI,Poisson
(Thresholding Low Pass Components)
PSNR(O/D)
35
34
33
32
31
30
29
28
27
26
25
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

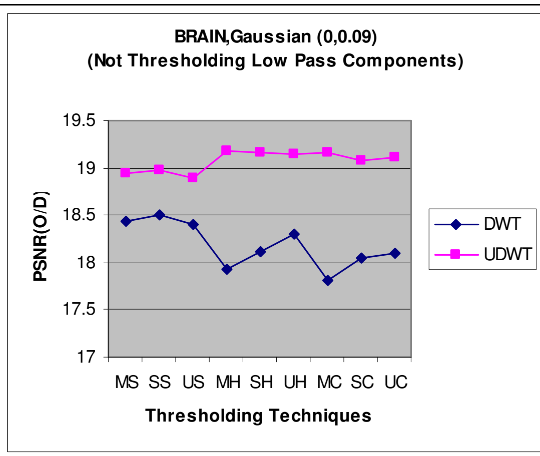

MRI,Poisson
(Thresholding Low Pass Components)
MSSIM(O/D)
0.86
0.84
0.82
0.8
0.78
0.76
0.74
0.72
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

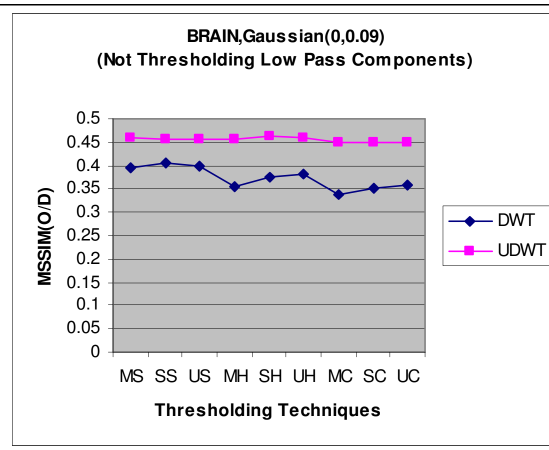

MRI,Speckle (v=0.09)
(Thresholding Low Pass Components)
PSNR(O/D)
30
25
20
15
10
5
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

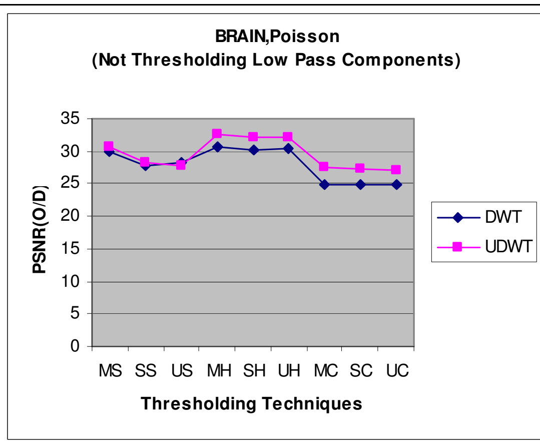

MRI,Speckle(v=0.09)
(Thresholding Low Pass Components)
MSSIM(O/D)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

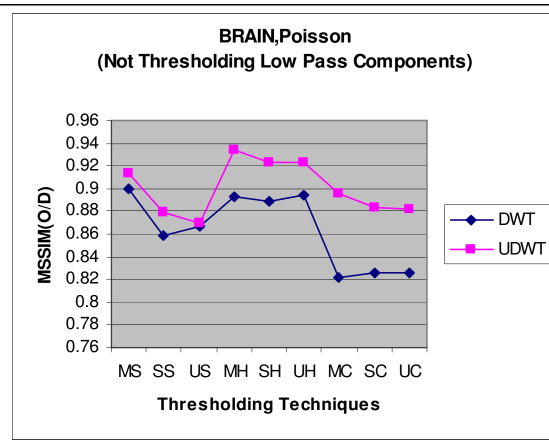

MRI,Salt & Pepper (d=0.09)
(Thresholding Low Pass Components)
PSNR(O/D)
30
25
20
15
10
5
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

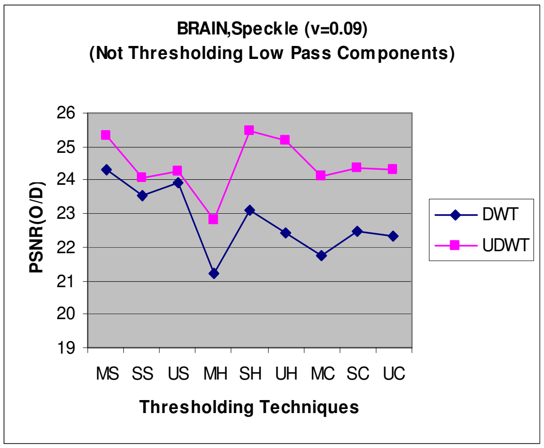

MRI,Salt & Pepper (d=0.09)
(Thresholding Low Pass Components)
MSSIM(O/D)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

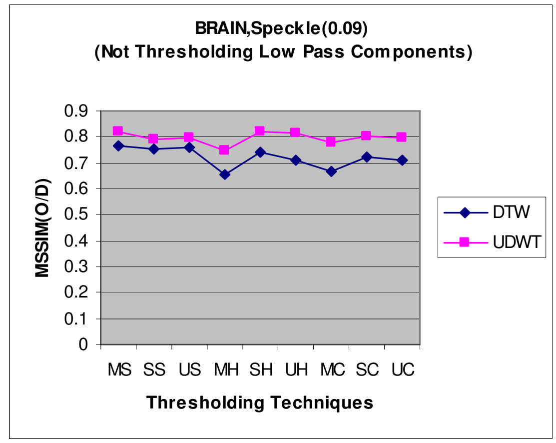

BRAIN,Gaussian (0,0.09)
(Not Thresholding Low Pass Components)
PSNR(O/D)
19.5
19
18.5
18
17.5
17
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

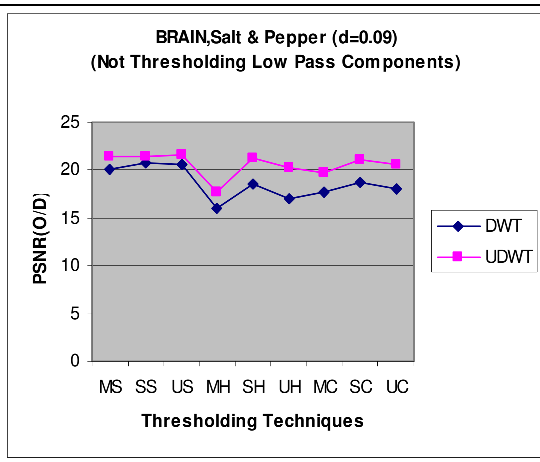

BRAIN,Gaussian(0,0.09)
(Not Thresholding Low Pass Components)
MSSIM(O/D)
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

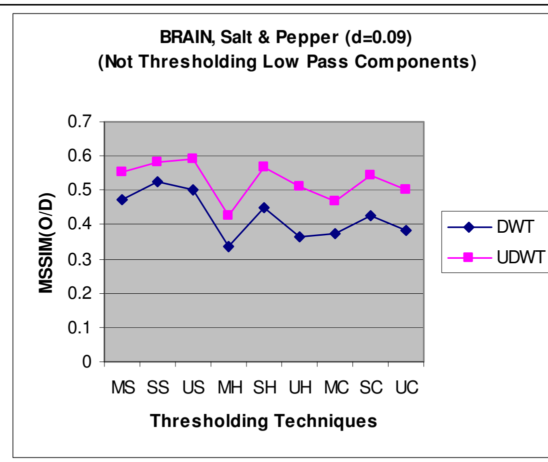

BRAIN,Poisson
(Not Thresholding Low Pass Components)
PSNR(O/D)
35
30
25
20
15
10
5
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

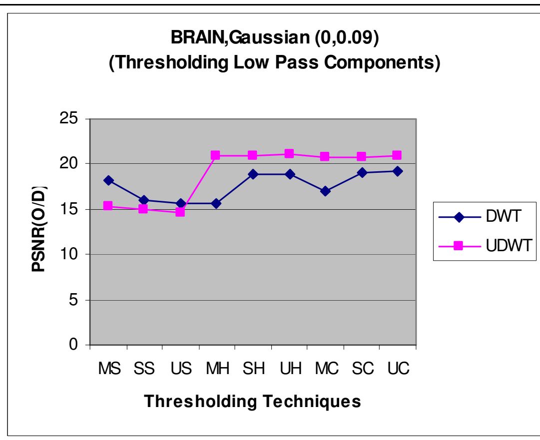

BRAIN,Poisson
(Not Thresholding Low Pass Components)
MSSIM(O/D)
0.96
0.94
0.92
0.9
0.88
0.86
0.84
0.82
0.8
0.78
0.76
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

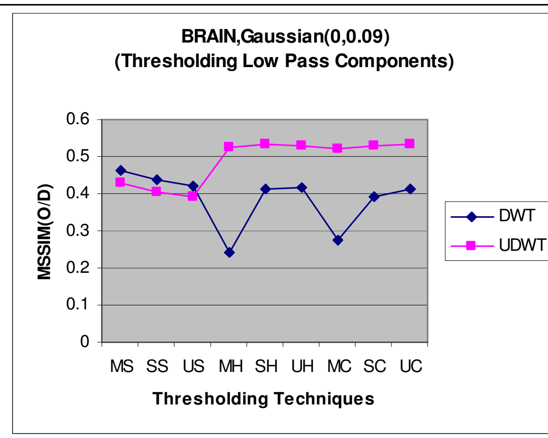

BRAIN,Speckle (v=0.09)
(Not Thresholding Low Pass Components)
PSNR(O/D)
26
25
24
23
22
21
20
19
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

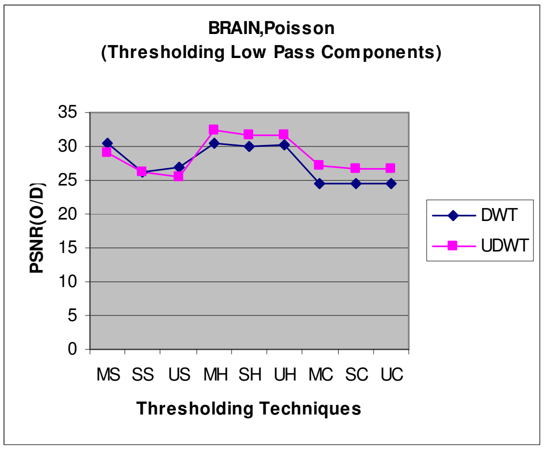

BRAIN,Speckle(0.09)
(Not Thresholding Low Pass Components)
MSSIM(O/D)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DTW
UDWT

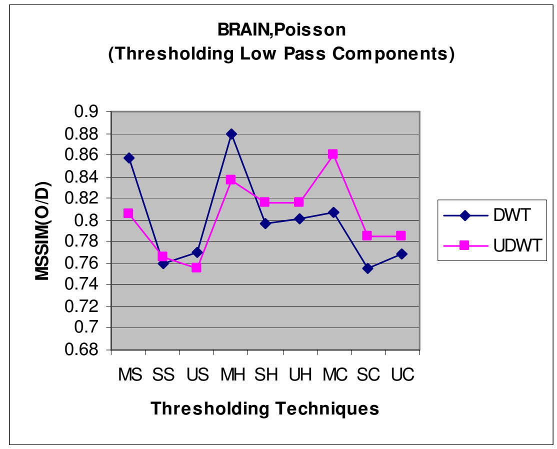

BRAIN,Salt & Pepper (d=0.09)
(Not Thresholding Low Pass Components)
PSNR(O/D)
25
20
15
10
5
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

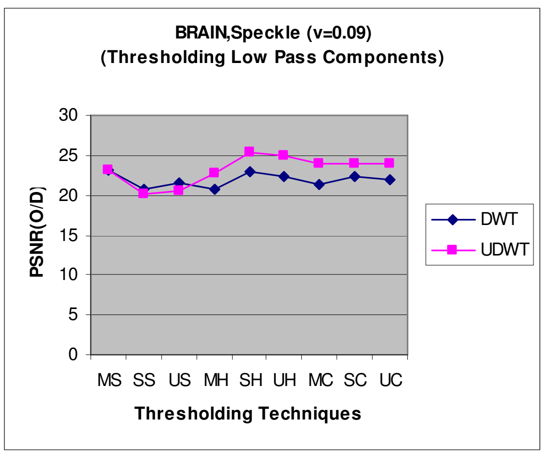

BRAIN, Salt & Pepper (d=0.09)
(Not Thresholding Low Pass Components)
MSSIM(O/D)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

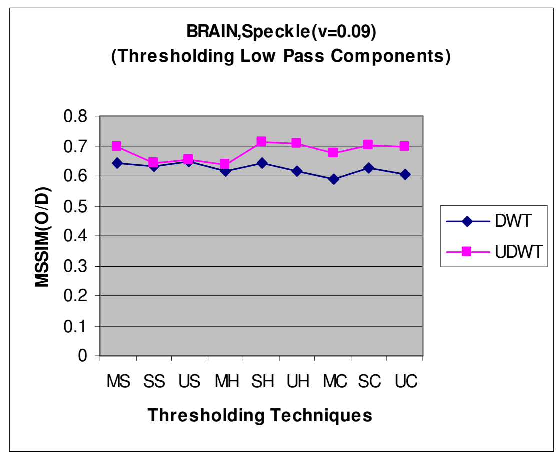

BRAIN,Gaussian (0,0.09)
(Thresholding Low Pass Components)
PSNR(O/D)
25
20
15
10
5
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

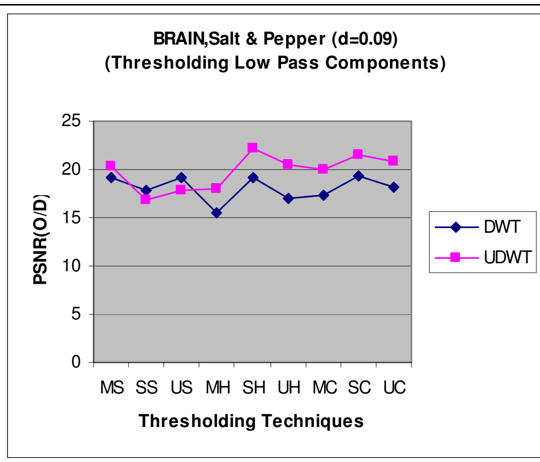

BRAIN,Gaussian(0,0.09)
(Thresholding Low Pass Components)
MSSIM(O/D)
0.6
0.5
0.4
0.3
0.2
0.1
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

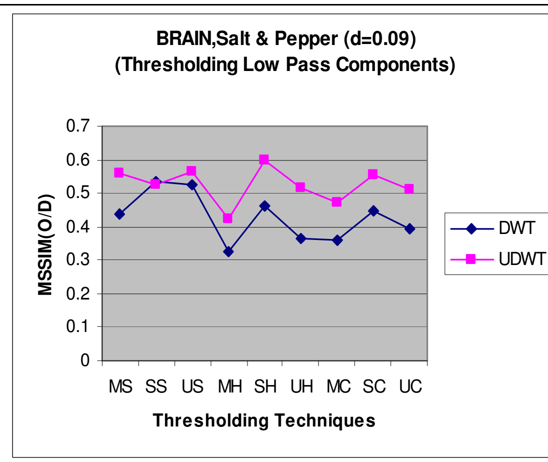

BRAIN,Poisson
(Thresholding Low Pass Components)
PSNR(O/D)
35
30
25
20
15
10
5
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT


BRAIN,Poisson
(Thresholding Low Pass Components)
MSSIM(O/D)
0.9
0.88
0.86
0.84
0.82
0.8
0.78
0.76
0.74
0.72
0.7
0.68
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT


BRAIN,Speckle (v=0.09)
(Thresholding Low Pass Components)
PSNR(O/D)
30
25
20
15
10
5
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT


BRAIN,Speckle(v=0.09)
(Thresholding Low Pass Components)
MSSIM(O/D)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT


BRAIN,Salt & Pepper (d=0.09)
(Thresholding Low Pass Components)
PSNR(O/D)
25
20
15
10
5
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

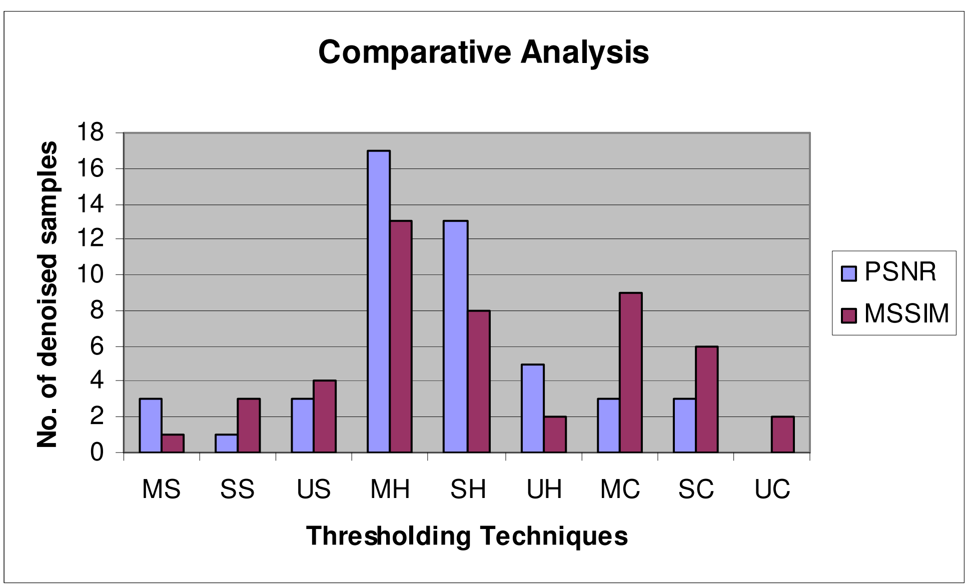

BRAIN,Salt & Pepper (d=0.09)
(Thresholding Low Pass Components)
MSSIM(O/D)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
MS SS US MH SH UH MC SC UC
Thresholding Techniques
DWT
UDWT

# Chapter 7

# DISCUSSION OF RESULTS

This chapter contains discusses the results obtained by following the proposed algorithm in chapter 5. Here, we have graphically demonstrated the contribution of each thresholding technique for denoising images using wavelet transforms. The samples have been studied only on the basis of the thresholding techniques applied. The concept of denoising through DWT and UDWT (along with the cases of not thresholding low pass components or thresholding them) have been sidelined for the time being because results clearly showed that UDWT outperformed DWT.

- The following graph shows a comparative analysis of all the thresholding techniques used for denoising images in terms of the two parameters, PSNR and MSSIM.

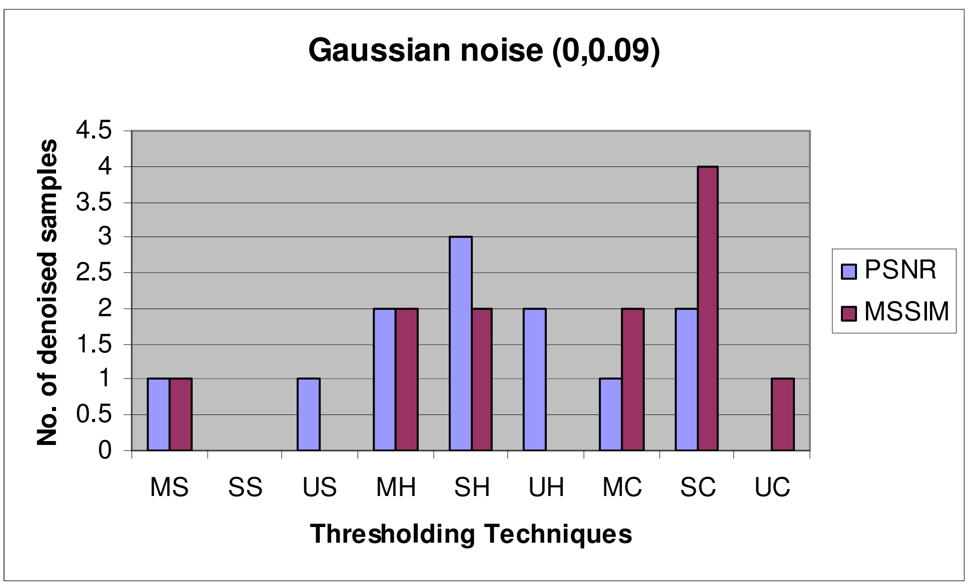


- The following graphs demonstrate the best thresholding technique out of all for each additive noise again in terms of PSNR and MSSIM, using UDWT. The samples (only few are there) that gave improved results for DWT have not been included because even their presence does not make a much difference to the results obtained in general using UDWT.

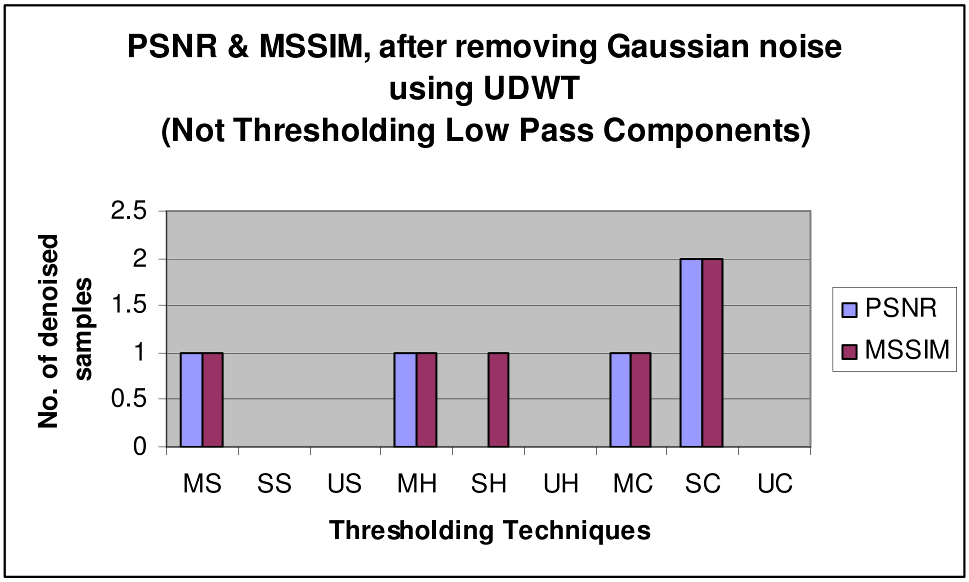
  It is quite visible from graphical representation, of the obtained results, shown in chapter 6, that UDWT has outperformed DWT, in all the tried combinations for denoising images.

1. Gaussian noise

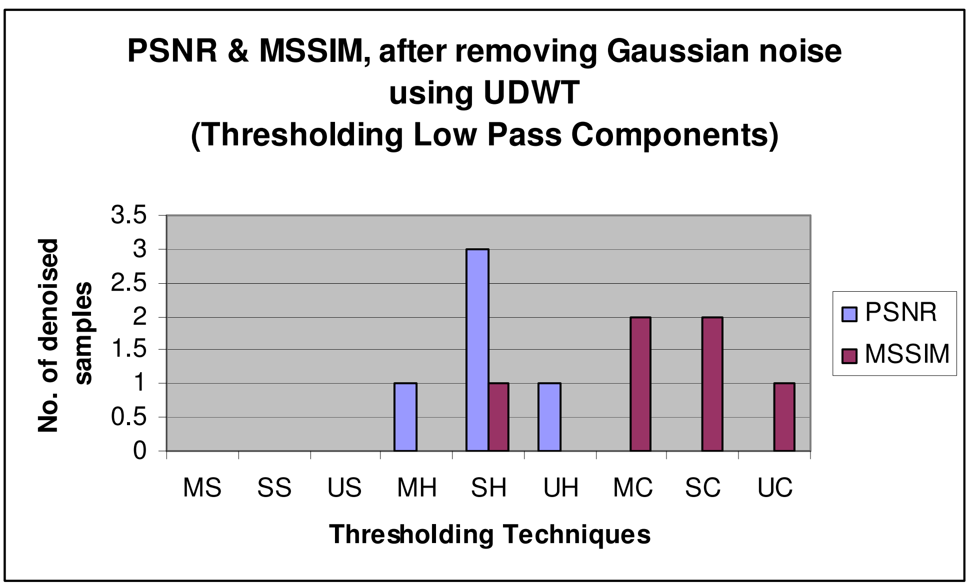
   ➢ following results are shown for both 'not thresholding and thresholding low pass components' together, using UDWT.



➢ results are shown only for 'not thresholding low pass components'.

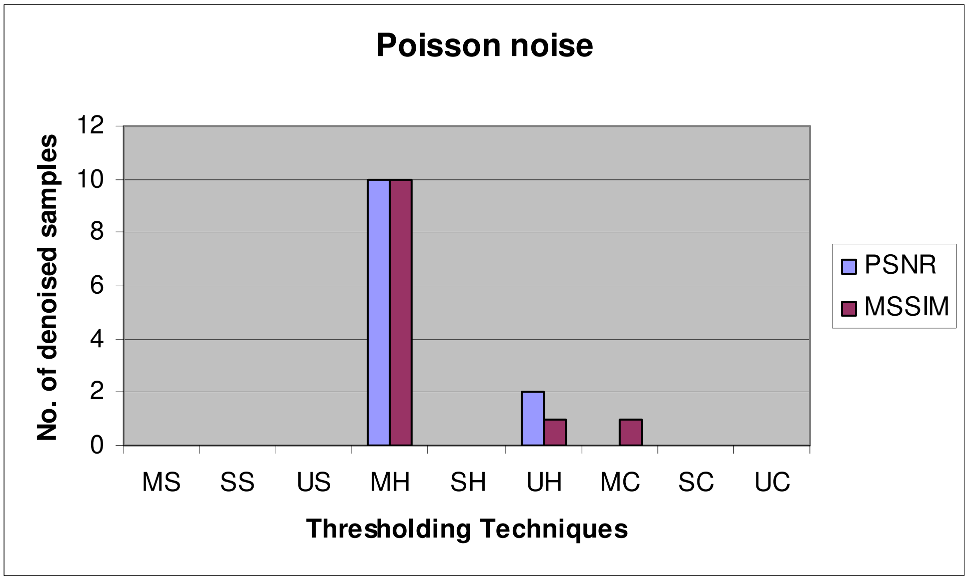



➢ results are shown only for 'thresholding low pass components'.

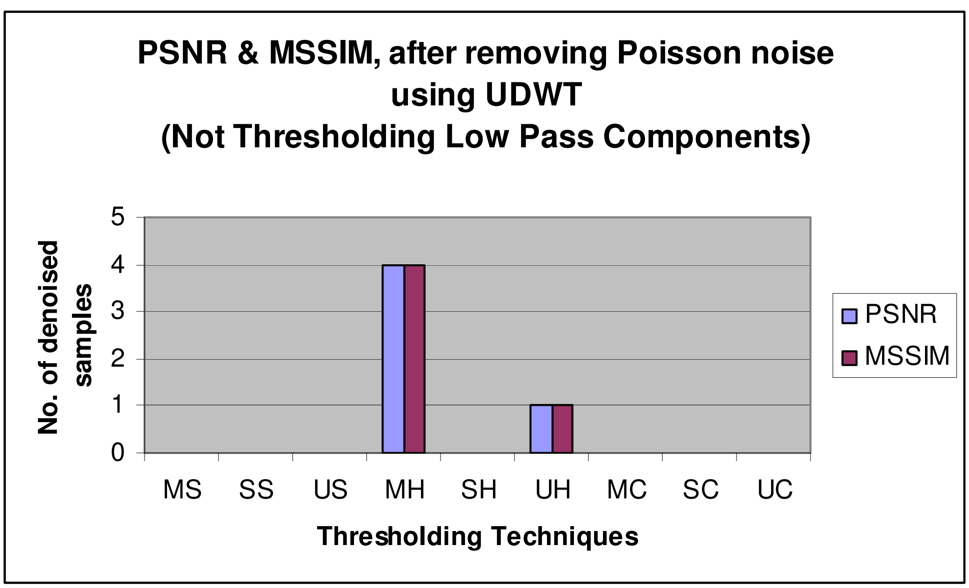


2. Poisson noise

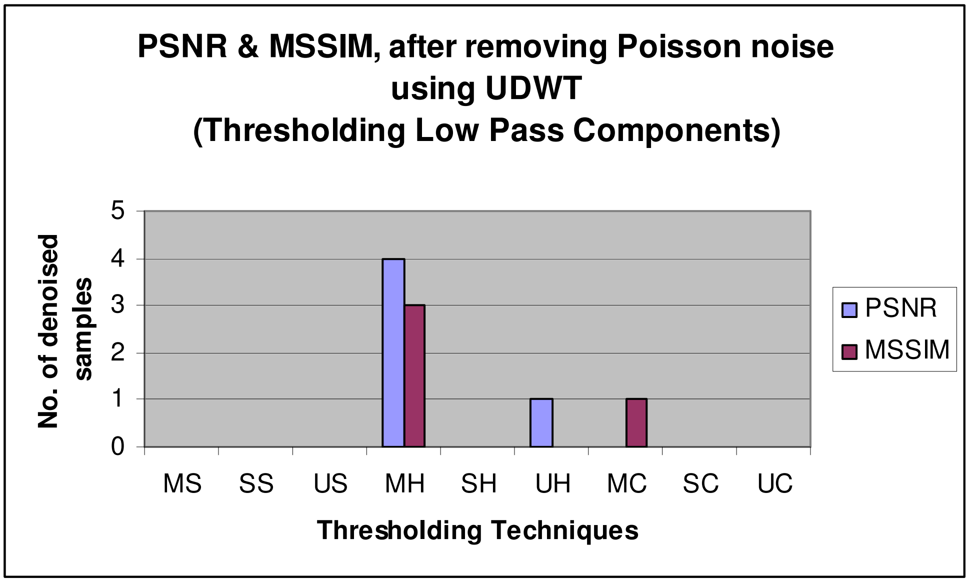
- following results are shown for both 'not thresholding and thresholding low pass components' together, using UDWT.




- results are shown only for 'not thresholding low pass components'.

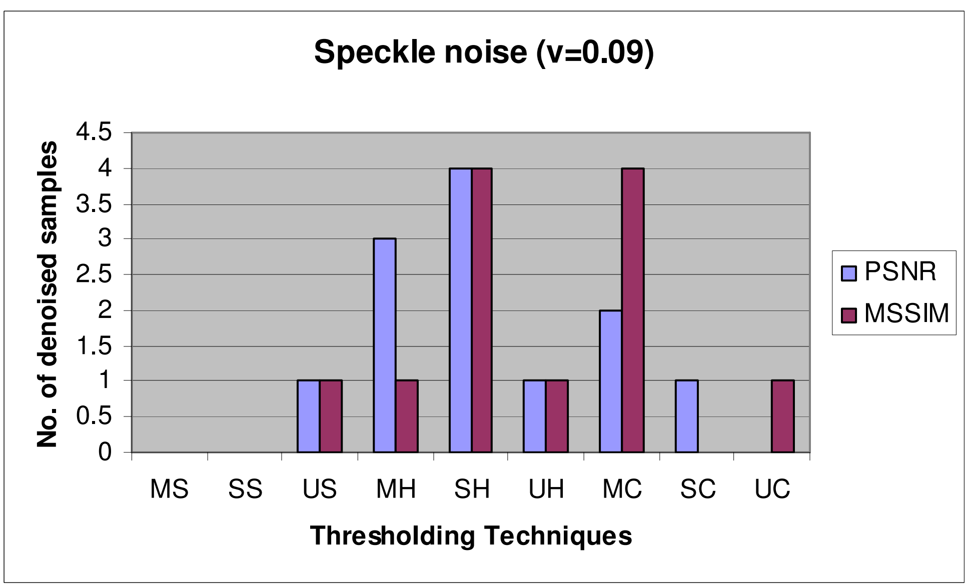


➢ results are shown only for 'thresholding low pass components'.

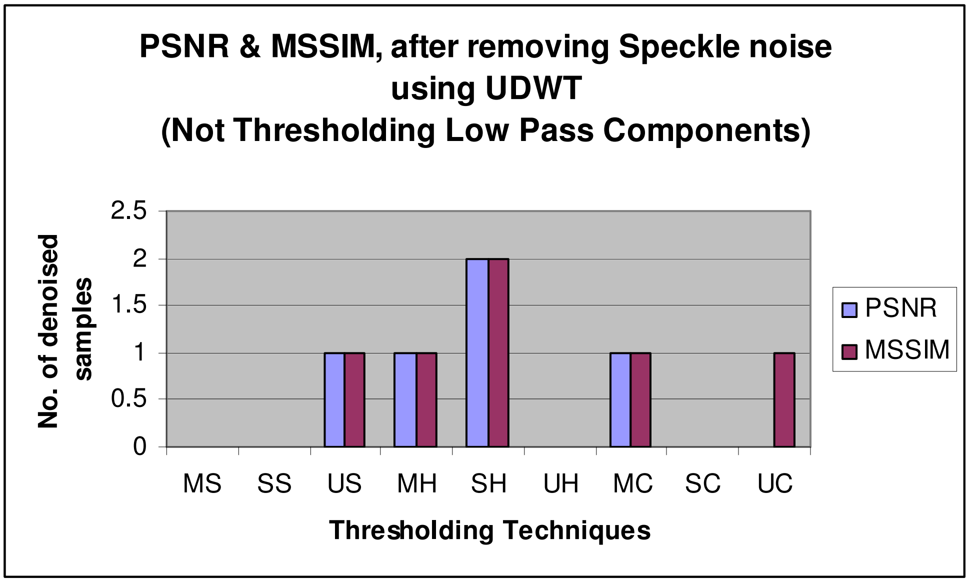



3. Speckle noise

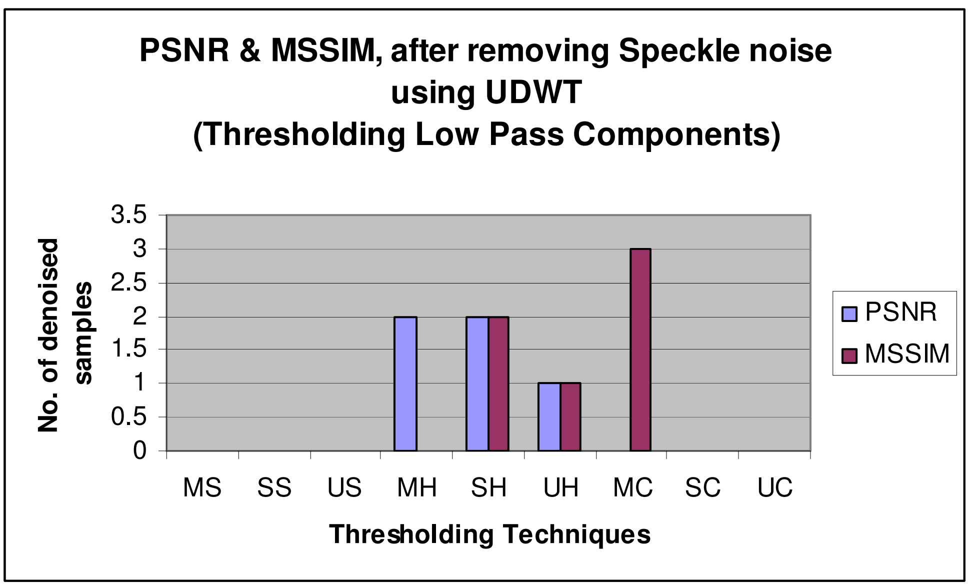
➢ following results are shown for both 'not thresholding and thresholding low pass components' together, using UDWT.



➢ results are shown only for 'not thresholding low pass components'.

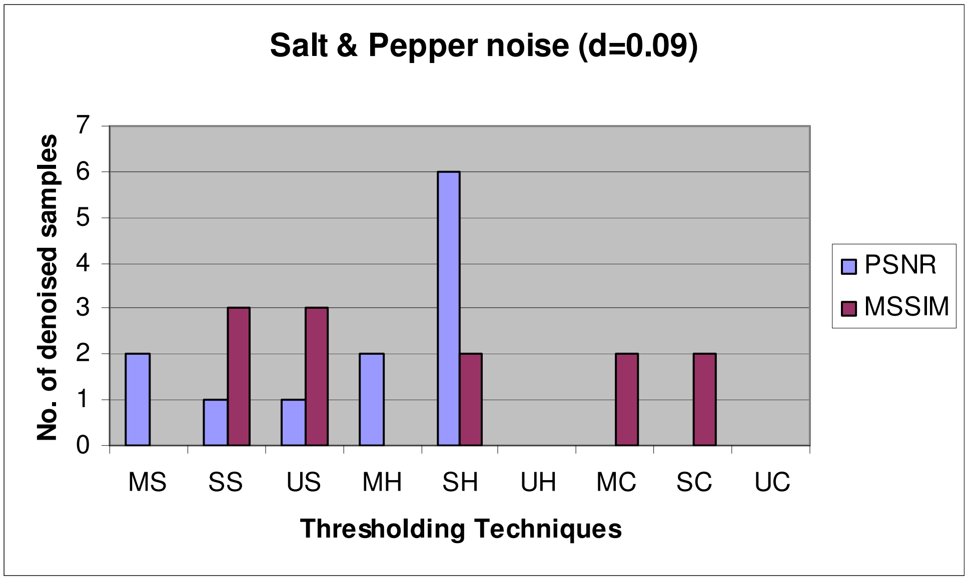



➢ results are shown only for 'thresholding low pass components'.

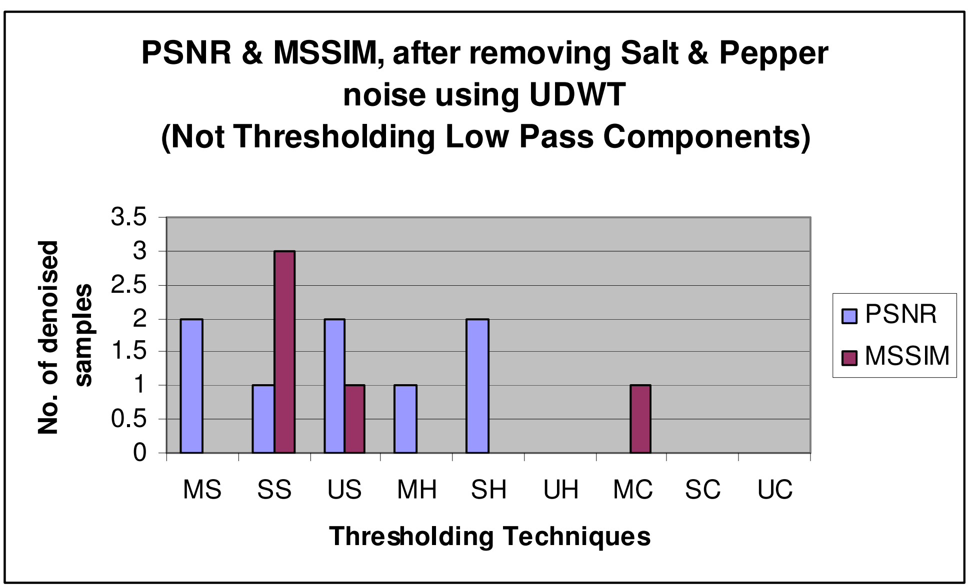


4. Salt & Pepper

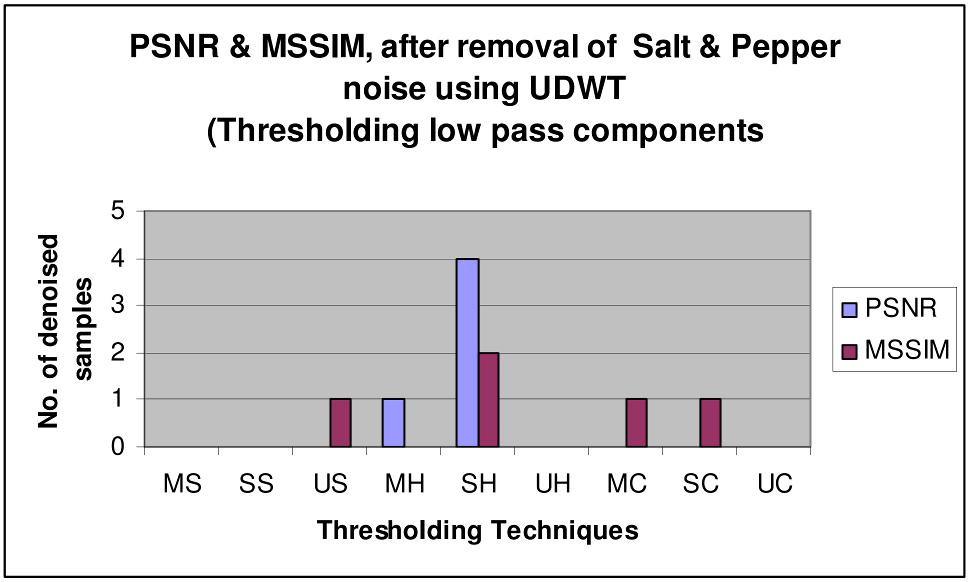
- following results are shown for both 'not thresholding and thresholding low pass components' together, using UDWT.




- results are shown only for 'not thresholding low pass components'.



➢ results are shown only for ‘thresholding low pass components’.



# Chapter 8

# CONCLUSION AND FUTURE SCOPE

This chapter concludes the work in this thesis in terms of the various input and output parameters that have been considered while denoising images using wavelet transforms. It also provides with a look up in the future scope of our work area.

## 8.1 CONCLUSION

This thesis presents a comparative analysis of various image denoising techniques using wavelet transforms. A lot of combinations have been applied in order to find the best method that can be followed for denoising intensity images. The image formats that have been used in this work are JPG, BMP, TIF and PNG.

The analysis, of all the obtained experimental results, demonstrates that UDWT outperforms DWT for denoising all of the above mentioned images (whether the low pass components are thresholded or are kept as such).

- UDWT denoises the images with more precision as compared to DWT because of its inborn quality of keeping the data intact to a greater extent.
- In UDWT, the step of down sampling the image in the forward run (decomposition process) and up sampling it in the reverse run (composition process) has been omitted. This way the useful data is not lost and a better denoised image is obtained.
- Both PSNR and MSSIM show an apprehensive improvement, if the noisy images are denoised using UDWT.

CASE 1: For not thresholding low pass components, while denoising images using UDWT, the following pattern of results has been obtained for the used noise types,

1. For GAUSSIAN noise (0, 0.09) – both Hard and Custom Thresholding techniques give improved results.

2. For POISSON noise – only Hard Thresholding technique is found to give good denoised images.
3. For SPECKLE noise (v=0.09) – both Hard and Custom Thresholding techniques give good results, rather it can be said that the combination of these two, results in better denoising. It has been observed that for some images Hard Thresholding improves the PSNR but the best MSSIM value is obtained for Custom Thresholding.
4. For SALT & PEPPER (d=0.09) – only Soft Thresholding contributes in the improvement of results.

CASE 2: For thresholding low pass components, while denoising images using UDWT, the following pattern of results has been obtained for the used noise types,

1. For GAUSSIAN noise (0, 0.09) – PSNR improves by the use of Hard Thresholding technique while better MSSIM is obtained by the use of Custom Thresholding technique.
2. For POISSON noise – only Hard Thresholding technique is found to give good denoised images.
3. For SPECKLE noise (v=0.09) – Hard Thresholding technique appears to be the most effective. Custom Thresholding technique also performs well.
4. For SALT & PEPPER (d=0.09) – a combination of Hard and Custom Thresholding plays a major role in the improvement of results. Hard, alone, is also found to be good for denoising images having additional speckle noise.

In all, it can be concluded that the proposed thresholding technique i.e. Custom Thresholding, leads to fairly good results as far as denoising of intensity images is concerned.

It can be noted that, for images where Hard Thresholding technique yields much superior results, as compared to Soft Thresholding, in those cases the quality of the estimate can be improved by using the proposed Custom Thresholding technique.

Like it is well said that nothing is perfect and there always exists an exception. On the similar grounds an exception has been found in the form of the PNG image

“MANDRILL”. Here, DWT still gives better results as compared to UDWT because the histogram of this image has been observed to be more wide spread as compared to other test images.

## 8.2 FUTURE SCOPE

The field of image processing has been growing at a very fast pace. The day to day emerging technology requires more and more revolution and evolution in the image processing field.

The well known saying “A picture says a thousand words” can be taken as the main motive behind the need of image processing.

The work proposed in this thesis also portrays a small contribution in this regard. The proposed denoising technique can provide a good platform for further research work in this respect.

This work can be further enhanced to denoise the other type of images, as well, like RGB, Indexed and Binary images. It will provide a good add on to the already existing denoising techniques used for denoising these images.

Moreover, for future work we can train our algorithm using various AI techniques like fuzzy logic or neural network, in order to attain the best output without performing calculations for each and every combination. Use of AI techniques will lead to the optimal solution directly, with more efficiency and less tedious work.

# APPENDIX A

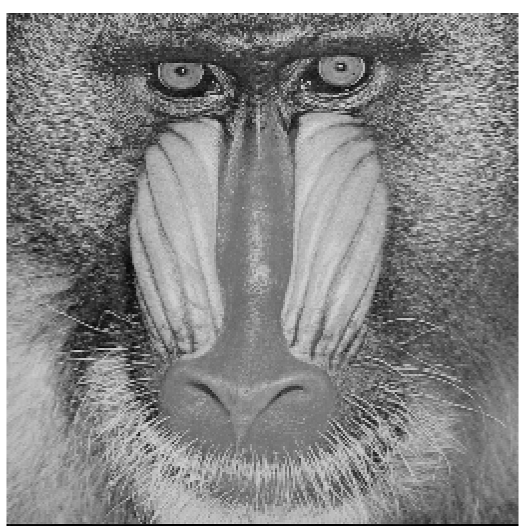
The following standard test images [34], resized from their original sizes to a standard size of $256 \times 256$, have been used for adding the assumed noise types.

1. LENA.bmp

Original Image

2. MANDRILL.png

Original Image



3. BARBARA.jpg

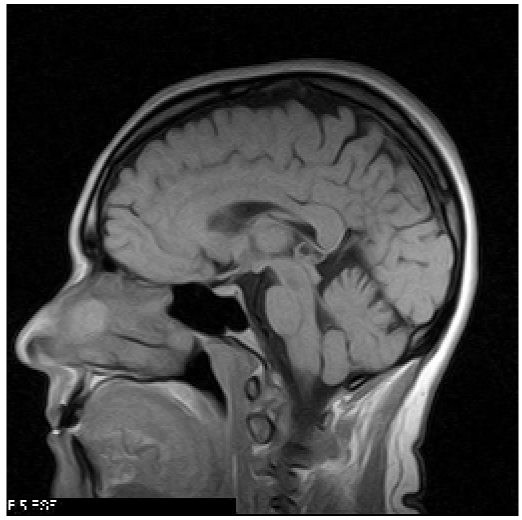
Original Image

4. BOAT.png

Original Image

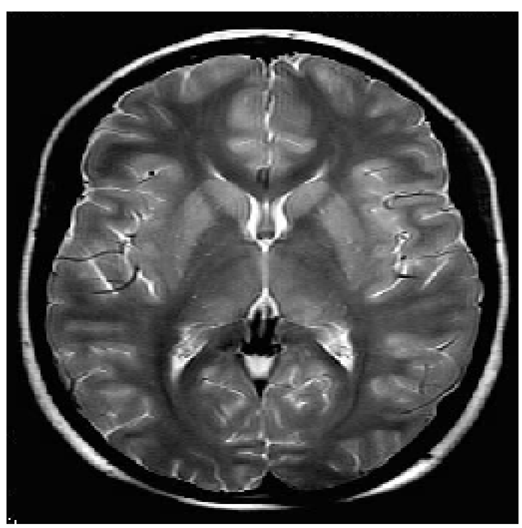
5. MRI.jpg

Original Image



6. BRAIN.tif

Original Image



# APPENDIX B

The following are the noisy images obtained after the deliberate addition of the specified noise to the original image LENA.bmp, as given in APPENDIX A.

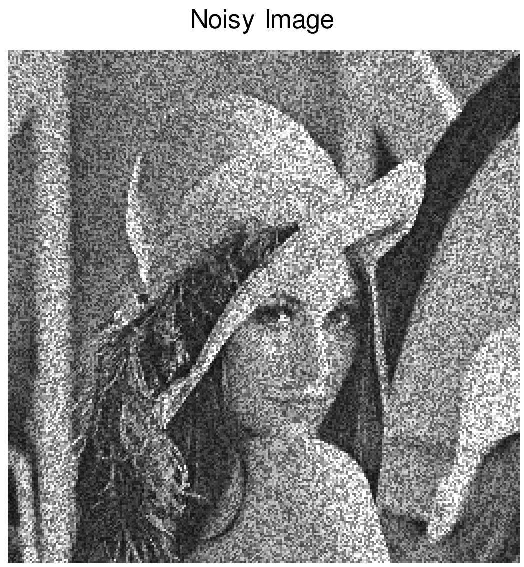
- After addition of Gaussian noise (0,0.09)

Noisy Image

- After addition of Poisson noise

Noisy Image

- After addition of Speckle noise (v=0.09)




- After addition of Salt & Pepper noise (d=0.09)


The same above mentioned noises are added in the similar manner to all other test images as well.

# APPENDIX C

Here denoised image samples for the noisy image LENA.bmp (noised by Gaussian noise) have been shown.

- SM has been used for DWT while not thresholding low pass components.

Denoised Image

- SM has been used for UDWT while not thresholding low pass components.

Denoised Image

- HS has been used for UDWT while thresholding low pass components.

Denoised Image

- CS has been used for UDWT while thresholding low pass components.

Denoised Image

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