THEORETICAL ANALYSIS OF TORSIONAL SURFACE WAVE IN REGULAR AND IRREGULAR EARTHS'S SURFACE

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Submitted by

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CERTIFICATE

This is to certify that the thesis entitled "**THEORETICAL ANALYSIS OF TORSIONAL SURFACE WAVE IN REGULAR AND IRREGULAR EARTHS'S SURFACE**", being presented in partial fulfillment of the requirements for the award of the degree of Master of Science in the School of Mathematics, Thapar Institute of Engineering and Technology, Patiala, is a bonafide work carried out under the supervision of **Dr. Pramod Kumar Vaishnav.**

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

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This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

05/08/2019

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Nomenclature

F	Body force per unit volume
H,h	Thickness of the layer
r	Radial coordinate
θ	Circumferential coordinate
ω	Angular frequency
λ,μ	Lame's constant
Ω	Cubical dilation
ρ	Density of the medium
ω'	Rotational vector
ζ	Tensile initial stress
S_{ij},σ_{ij}	Stress components
$u_i=(u_i,v_i,w_i)$	Displacement components of solid
$U_i=(U_i,V_i,W_i)$	Displacement components of liquid
$v=v(r,z,t)$	Displacement component along θ direction
e_{ij}	Strain components
x_i	Cartesian co-ordinates
t	Time parameter

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Abstract

Theoretical analysis of Torsional surface wave in regular and irregular Earth's surface has been discussed in present thesis. For propagation of torsional wave, we utilized some specific mediums of the Earth with the impact of few parameters on phase velocity of torsional wave. Solution of governing equation of motion has been obtained by using separation of parameters method. The dispersion relations in anisotropic medium and porous medium have been obtained analytically. The standard dispersion relation of torsional surface wave have been gotten specifically in both discussed problems. The parametric impact on phase velocity of torsional wave is shown graphically by utilizing MATLAB software. This thesis contains three noteworthy chapters with addition of references towards the end. **Chapter 1** exhibits the introduction for the appropriate problems. It gives the clarification of concentrates in the field of theoretical seismology. In **Chapter 2**, propagation of Torsional surface wave in anisotropic channel with regular surface has been talked about. We assumed the initial stress in anisotropic layer lying over semi-limitless channel. The presence of assumed parameters in obtained dispersion relation approved the scientific effect of these parameters on phase velocity of Torsional surface wave. The standard dispersion relation of Torsional wave have been gotten in specific cases which approves the problem. In **Chapter 3**, the propagation of Torsional surface wave in porous medium with irregular interface is explained. The parabolic irregularity at the interface of porous layer and homogeneous half-space has been taken in assumed geometry. The dispersion relation for Torsional surface wave propagation have been obtained in presence of porosity and irregularity parameters. The effect of porosity and irregular interface hase been noticed in the graphical section. The present study is useful in earthquake engineering and seismology.

Chapter 1

Introduction

Seismology is the branch of science concerned with seismic waves and earthquakes that move through and around the Earth. It deals with artificially produced vibrations of the Earth and provides information about structure of inside of the Earth. Seismic waves travel through the Earth layer's and result in earthquakes, volcanic eruptions. Seismic waves are studied by Seismologists and is recorded by a seismometer.

An earthquake is a seismic event which generates seismic waves and causes the ground to vibrate. It is sudden release of energy in the Earth's crust. High intensity earthquakes results in disintegration of life and property. The Earth's crust is part into inflexible plates, which are all moving in respect to one another. The development delivers increasingly more weight on the ground until something in the end breaks along what's known as a land blame. This is the reason, in the event that you overlay a worldwide structural plate map and a worldwide earthquake map, you'll see a practically ideal cover between structural edges and earthquakes. Man-made blast can likewise deliver earthquake type highlights which produce seismic waves and can be distinguished.

1.1 Earth's structure

The Earth's inside is partitioned into three principle layers: crust, mantle and core. These layers are different both chemically and physically. The center of the Earth is around 1800 miles beneath the Earth's surface. The core of the earth is further partitioned into two layers, the inner core and outward core. The internal centre - the focal point of Earth, is strong and around 780 miles (1250 km) thick. The external center is hot to the point that the metal is constantly liquid, yet the internal center cannot melt despite the fact that temperatures their achieve is 3700°C. The outer core is around 1370 miles thick. Since, the Earth turns, the outward core twists around the inner core and that causes the Earth's attraction.

The mantle is regularly isolated into a few layers dependent on spiral discontinuities in seismic speed structure. These incorporate lithosphere, lower mantle, transition zone and asthenosphere. A negative seismic speed gradient in the upper mantle is normally recognized by seismologists as the lithosphere-asthenosphere limit. The outside layer ranges from 6 - 71 km top to bottom. It is presently all around acknowledged that the Earth is anisotropic, however from the seismological and minerlogical perspectives, this anisotropy is now and then hard to measure. The reasons are numerous causes of anisotropy and the different scales (from minute scale to worldwide size of Earth) that must be considered. In an anisotropic medium, physical properties change as a function of direction.

Seismological perceptions uncover exceedingly anisotropic patches at the base of the Earth's lower mantle, while main part of mantle has been seen to be largely isotropic. Seismology helps us to work out the elements of the internal and external core of the Earth. These wave consist of essential data about the interior structure of the Earth. As seismic waves go through the Earth, they are refracted or twisted, as beams of light twist when they pass however from glass prism.

1.2 Seismic waves and their classifications:

Seismic waves are waves of energy that move through Earth's layer. These are the vibrations produced by earthquakes. They travel through the Earth like a tidal wave goes through the sea, or the sound goes through the air. The time it takes for seismic waves to touch base at seismic observatories enable researchers to find the exact area of the earthquake that produced them. When accomplishing seismic observatories, their diverse travel times help scientists to discover wellspring of hypocenter.

There are two distinct sorts of seismic waves: body waves and surface waves. Body waves are waves which move inside Earth's body and surface waves go along Earth's surface. As these waves have diverse physical attributes, their landing times at seismic observatories are additionally used to describe inside of the Earth.

1) Body waves:

Body waves move through the within the Earth along ways constrained by material properties in regards to thickness and modulus. The thickness and modulus, along these lines, move according to temperature, piece, and material stage. Body waves are of two distinct types, P (primary) waves and S (secondary) waves. Each kind of wave shakes the ground in different ways.

i) P waves:

The main sort of body wave is known as the primary wave, and is generally alluded to as P-waves. This sort of seismic body wave goes at the best speed through the ground. As a longitudinal compressional waveform, P-waves move similarly as sound waves. As they spread out, they on the other hand push and pull the ground as they travel through it. P-waves can go through both strong shake and fluid material, for example, volcanic magma or seas. They travel at speeds extending from 1,600– 8,000 m/s, contingent upon the material they're traveling through. Due to their speed, they are the primary sort of wave to be felt and to enroll on a seismograph during an earthquake.

ii) S waves:

The second sort of body wave is known as the shear wave and is generally alluded to as S-waves. S-waves are a transverse waveform that shears ground sideways at right points to the course of movement. S-waves effectively affect ground surface depending upon their polarization and bearing of movement. Horizontally energized and vertically energized S-waves will move ground from side to side and all over with respect to the course they're moving, respectively. S-waves are ordinarily 40% slower than P-waves in some random material and have speeds extending from roughly 900– 4,500 m/s. These waves are second to enroll on a seismograph during a tremor. In spite of their slower speed, S-waves are regularly more disastrous than P-waves since they can have bigger amplitudes and can cause more noteworthy dimensions of ground shaking.

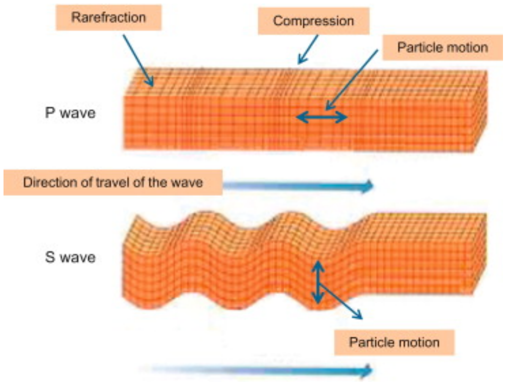

Figure 1.1: P and S wave propagation

2) Surface wave:

Seismic surface waves move along the Earth's surface. They can be named a sort of mechanical surface waves. They reduce as they get further from surface, which is the reason they are called surface waves. They move more step by step than seismic body waves (P and S). In large seismic tremors, surface waves can have a abundance of few centimeters.

There are a few sorts of surface waves, however the two most regular varities are Rayleigh waves and Love waves.

i) Love waves:

It was named in this way after Augustus Edward Hough Love, a Professor for Natural Philosophy at Oxford University who originally depicted the development of the waves named after him. Love waves have a transversal (opposite) development and are the most dangerous outside the immediate zone of the epicenter. Love waves can be destroying. They ordinarily move marginally quicker than Rayleigh waves, about 90% of S wave speed, and have largest amplitude.

ii) Rayleigh waves:

Rayleigh waves are the waves that move in a circular movement, delivering both a vertical and even segment of movement toward wave engendering. Molecule movement comprises of circular movements in the vertical plane and parallel to the direction of propagation. Material comes back to its unique shape after wave passes. These waves are used in non-destructive testing for detecting defects.

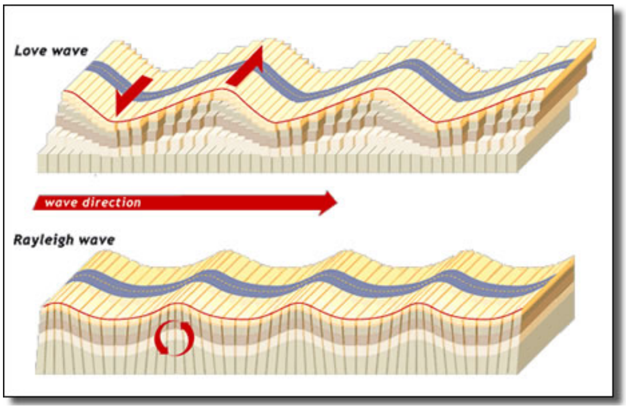

Figure 1.2: Love and Rayleigh wave propagation

iii) Torsional waves:

In Torsional waves, vibrations of medium are rotational movements around the bearing of propagation of wave. It implies the components of medium conveying the wave perform torsional motions about a hub parallel to the course of propagation. For instance, in event that you take an adaptable helical string and tenderly turn it toward one side, the bend will head out to opposite end as a wave pulse. Torsional waves can just exist in solid issue instead of gas, electromagnetic or fluid media which in total display just longitudnal or transverse waves. Spread of Torsional surface wave rely on medium where they engender as torsional surface waves proliferate fastly in nearness of flexible half-space in contrast with permeable half-space.

Chapter 2

Dispersion relation of Torsional wave in anisotropic medium with regular interface

2.1 Objective

This chapter deals with propagation of Torsional surface wave in anisotropic medium with regular interface. The solution of governing equation of motion has been gotten by utilizing separation of variables technique. The arrangement of wave equation represent the deportation in corresponding layer and half-space. For the Torsional wave propagation, we considered anisotropic medium as superficial layer lying over inhomogeneous semi-infinite medium under the impact of beginning stress. The dispersion curve is obtained in this medium which is converted into standard dispersion relation of Torsional wave. Impact of beginning stress parameters has been noticed in graphical section. It is noticed that, the presence of initial stress in dispersion curve affected phase velocity of Torsional surface wave. This investigation assume a significant job in field of theoretical seismology.

2.2 Mathematical structure of the problem

A heterogeneity model has been assumed to propagation of torsional surface wave. This model is all around furnished with an inhomogeneous anisotropic layer of limited thickness H lying over an inhomogeneous anisotropic half-space. Half-space is considered under initial stress with quadratic and hyperbolic variety in unbending nature and density, respectively. The source of coordinate system is situated at surface of half-space with z-axis vertically downwards and x-axis is taken along the head of wave propagation. The upper surface of anisotropic layer is stress free.

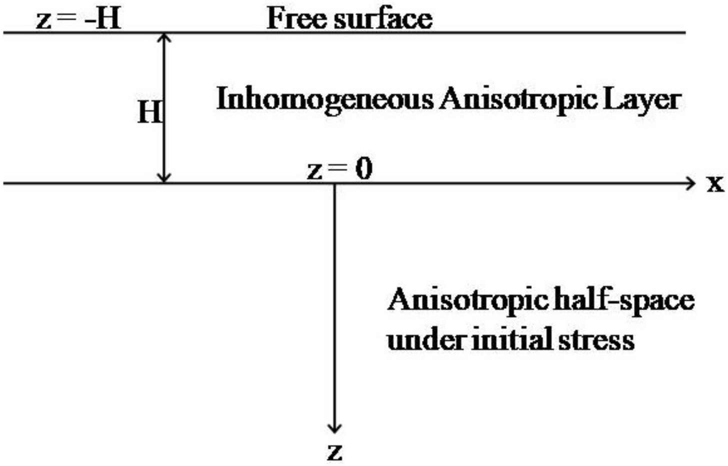

Figure 2.1: Geometry of the problem.

2.3 Dynamics and solution for an inhomogeneous anisotropic layer

The equation of movement in an anisotropic layer can be composed as:

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{\partial \sigma_{z\theta}}{\partial z} + \frac{2}{r}\sigma_{r\theta} = \rho \frac{\partial^2 v_1}{\partial t^2} \tag{2.3.1}$$

where θ and r be circumferential and radial coordinates, respectively, $v_1(r,z,t)$ is displacement component across θ direction and ρ is density . Stress-strain relation of an inhomegeneous anisotropic layer is given by:

$$\sigma_{r\theta} = 2Ne_{r\theta}, \sigma_{z\theta} = 2Le_{z\theta} \tag{2.3.2}$$

where strain components are expressed as $e_{r\theta} = \frac{1}{2}(\frac{\partial v_1}{\partial r} - \frac{v_1}{r})$, $e_{z\theta} = \frac{1}{2}\frac{\partial v_1}{\partial z}$. L and N is regarded to inflexibility of medium along z and r directions, respectively. Eq. (2.3.1) holds the below mentioned form by utilizing relation (2.3.2)

$$N\left(\frac{\partial^2 v_1}{\partial r^2} - \frac{v_1}{r^2} + \frac{1}{r}\frac{\partial v_1}{\partial r}\right) + \frac{\partial}{\partial z}\left(L\frac{\partial v_1}{\partial z}\right) = \rho\frac{\partial^2 v_1}{\partial t^2} \tag{2.3.3}$$

At the point, when wave propagates along radial direction with magnitude of displacement as a funcion of depth then the arrangement of condition (2.3.3) is written as:

$$v_1 = V_1^*(z)J_1(kr)e^{i\omega t} \tag{2.3.4}$$

ω denotes circular frequency of wave and J_1 is the Bessel's functions of order one of first kind. $V_1^*(z)$ is the solution of the following equation:

$$\frac{\mathrm{d}^2 V_1^*}{\mathrm{d}z^2} + \frac{1}{L}\frac{\mathrm{d}L}{\mathrm{d}z}\frac{\mathrm{d}V_1^*}{\mathrm{d}z} - \frac{k^2 N}{L}\left(1 - \frac{c^2\rho}{N}\right)V_1^* = 0 \tag{2.3.5}$$

where c=$\frac{\omega}{k}$ is velocity of propagation of torsional surface wave. Now, substituting $V_1^* = \frac{V_1}{\sqrt{L}}$ in equation (2.3.5), we get

$$\frac{\mathrm{d}^2 V_1}{\mathrm{d}z^2} - \frac{1}{2L}\left(\frac{\mathrm{d}^2 L}{\mathrm{d}z^2} - \frac{1}{2L}\left(\frac{\mathrm{d}L}{\mathrm{d}z}\right)^2\right)V_1 = \frac{k^2 N}{L}\left(1 - \frac{c^2\rho}{N}\right)V_1 \tag{2.3.6}$$

Contemplate the exponential variety in elastic moduli, density, rigidity in an inhomogeneous anisotropic layer i.e.

$$N = N_1 e^{\beta z}, L = L_1 e^{\beta z}, \rho = \rho_1 e^{\rho z} \tag{2.3.7}$$

where β is an inhomogeneity parameter and ρ_1, N_1, L_1 are density and rigidities as z → 0. Utilizing Eq. (2.3.7), Eq. (2.3.6) move toward becoming:

$$\frac{\mathrm{d}^2 V_1}{\mathrm{d}z^2} + m_1^2 V_1 = 0 \tag{2.3.8}$$

accordingly $m_1^2 = k^2\left(\frac{N_1}{L_1}\left(\frac{c^2}{c_1^2}-1\right)-\frac{\beta^2}{4k^2}\right)$ and $c_1 = \sqrt{\frac{N_1}{\rho_1}}$ is shear wave velocity in layer. The solution of Eq.(2.3.8) is given by:

$$V_1(z) = A_1 e^{-im_1 z} + A_2 e^{im_1 z} \tag{2.3.9}$$

where A_1 and A_2 are self-assertive constants and henceforth, displacement in an inhomogeneous anisotropic layer is stated as:

$$v_1 = \frac{1}{\sqrt{L_1}}\left(A_1 e^{-im_1 z} + A_2 e^{im_1 z}\right) J_1(kr) e^{i\omega t} \tag{2.3.10}$$

2.4 The displacement equation in an anisotropic half-space

If θ and r be circumferential and radial coordinates, individually, the equation of movement for the first focused anisotropic half-space is given by:

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{\partial \sigma_{z\theta}}{\partial z} + \frac{2}{r}\sigma_{r\theta} - \frac{\partial}{\partial z}\left(\frac{P}{2}\frac{\partial v_2}{\partial z}\right) = \rho \frac{\partial^2 v_2}{\partial t^2} \tag{2.4.1}$$

where $\sigma_{r\theta}$ and $\sigma_{z\theta}$ denotes incremental stress components of anisotropic half spaces, $v_2 = v_2(r,z,t)$ the dislodging along θ direction, ρ is thickness of the medium and P is the underlying compressive stress along radial coordinate r. Stress and strain connection for an anisotropic layer is given by:

$$\sigma_{r\theta} = N\left(\frac{\partial v_2}{\partial r} - \frac{v_2}{r}\right), \sigma_{z\theta} = L\frac{\partial v_2}{\partial z} \tag{2.4.2}$$

Now, Eq. (2.4.1) can be reduced to

$$\left(\frac{\partial^2 v_2}{\partial r^2} - \frac{v_2}{r^2} + \frac{1}{r}\frac{\partial v_2}{\partial z}\right) - \frac{1}{N}\frac{\partial}{\partial z}\left(G\frac{\partial v_2}{\partial z}\right) = \frac{\rho}{N}\frac{\partial^2 v_2}{\partial t^2} \tag{2.4.3}$$

where $G_n = L_n - \frac{P_n}{2}$, n= 1 and 2. Since the center layer does not contain any underlying pressure i.e. $P_1 = 0$. Eq. (2.4.2) takes the form:

$$\frac{\mathrm{d}^2 V_2^*}{\mathrm{d}z^2} + \frac{1}{G_n}\frac{\mathrm{d}G_n}{\mathrm{d}z}\frac{\mathrm{d}V_2^*}{\mathrm{d}z} + \frac{N}{G_n}\left(k^2 - \frac{\omega^2 \rho}{N}\right) V_2^*(z) = 0 \tag{2.4.4}$$

Assuming $V_2^*(z) = \frac{V_2(z)}{\sqrt{G_n}}$ Eq.(2.4.3) can be written as:

$$\frac{\mathrm{d}^2V_2}{\mathrm{d}z^2} - \left(\frac{1}{2G_n}\frac{\mathrm{d}^2G_n}{\mathrm{d}z^2} - \left(\frac{1}{2G_n}\frac{\mathrm{d}G_n}{\mathrm{d}z}\right)^2 + \frac{N}{G_n}\left(k^2 - \frac{\omega^2\rho}{N}\right)\right)V_2 = 0 \tag{2.4.5}$$

where $\omega = kc$, c is phase velocity of torsional surface waves in an initially stressed half space.

Solution of lower half space

Think about the hyperbolic variation in elastic moduli, initial stress and thickness with profundity z as:

$$N = N_2\cosh^2(\gamma z), L = L_2\cosh^2(\gamma z), \rho = \rho_2\cosh^2(\gamma z), P = P_2\cosh^2(\gamma z) \tag{2.4.6}$$

where γ is inhomogeneity parameter. Using relation (2.4.5), Eq.(2.4.4) takes the form

$$\frac{\mathrm{d}^2V_2}{\mathrm{d}z^2} - m_2^2V_2(z) = 0 \tag{2.4.7}$$

where $m_2^2 = k^2\left(\frac{\gamma^2}{k^2} + \frac{N_2}{G_2}\left(1 - \frac{c^2}{C_2^2}\right)\right)$. The solution of Eq.(2.4.6) is given by:

$$V_2(z) = A_3e^{-m_2z}$$

Therefore the solution of Eq.(2.4.1) is

$$v_2 = \frac{A_3e^{-m_2z}J_1(kr)e^{i\omega t}}{\sqrt{G_0}\cosh(\gamma z)} \tag{2.4.8}$$

2.5 Boundary conditions

1. The upper part of anisotropic layer is stress free hence density is zero at upper half-space i.e.

$$L_1\frac{\partial v_1}{\partial z} = 0 \; at \; z = -H$$

2. The displacement components are ceaseless at intersection of layer and the half-spaces i.e. z = 0

$$v_1 = v_2 \; , L_1\frac{\partial v_1}{\partial z} = L_2\frac{\partial v_2}{\partial z}$$

2.6 Dispersion relation

Phase velocity conditions will be gotten by utilizing of above mentioned boundary conditions in Eq.(2.3.10) and (2.4.8)

$$\sqrt{L_1}\left(-m_1A_1e^{im_1H}+m_1A_2e^{-im_1H}\right)=0 \tag{2.6.1}$$

$$\frac{1}{\sqrt{L_1}}\left(A_1+A_2\right)=\frac{A_3}{\sqrt{G_0}} \tag{2.6.2}$$

$$\sqrt{L_1}\left(-im_1A_1+im_1A_2\right)=\frac{-L_2m_2A_3}{\sqrt{G_0}} \tag{2.6.3}$$

Now, eliminating arbitrary constants from phase velocity

$$\begin{vmatrix} -\sqrt{L_1}m_1e^{im_1H} & \sqrt{L_1}m_1e^{-im_1H} & 0 \\ \frac{1}{\sqrt{L_1}} & \frac{1}{\sqrt{L_1}} & \frac{-1}{G_0} \\ -im_1\sqrt{L_1} & im_1\sqrt{L_1} & \frac{L_2m_2}{\sqrt{G_0}} \end{vmatrix}=0$$

which reduced to

$$\frac{-m_1m_2L_2}{\sqrt{G_0}}\left(e^{im_1H}+e^{-im_1H}\right)-\frac{im_1^2L_1}{\sqrt{G_0}}\left(e^{im_1H}-e^{-im_1H}\right)=0 \tag{2.6.4}$$

The generalized dispersion relation of torsional wave is obtained as follows:

$$\tan\left[kH\sqrt{\frac{N_1}{L_1}\left(\frac{c^2}{c_1^2}-1\right)-\frac{\beta^2}{4k^2}}\right]=\frac{L_2}{L_1}\frac{\sqrt{\frac{N_2}{G_2}\left(1-\frac{c^2}{c_2^2}\right)+\frac{\gamma^2}{k^2}}}{\sqrt{\frac{N_1}{L_1}\left(\frac{c^2}{c_2^2}-1\right)-\frac{\beta^2}{4k^2}}} \tag{2.6.5}$$

2.7 Validation of the problem

1. **Case-I:** If $N_1=L_1=\mu_1$,$N_2=L_2=\mu_2$ the lower half space has consistent thickness and inflexibility, the Eq. (2.6.5) accepts the structure as:

$$\tan\left[kH\sqrt{\left(\frac{c^2}{c_1^2}-1\right)-\frac{\beta^2}{4k^2}}\right]=\frac{\mu_2}{\mu_1}\frac{\sqrt{\frac{\mu_2}{G_2}\left(1-\frac{c^2}{c_2^2}\right)+\frac{\gamma^2}{k^2}}}{\sqrt{\left(\frac{c^2}{c_1^2}-1\right)-\frac{\beta^2}{4k^2}}} \tag{2.7.1}$$

2. **Case-II:** If $\frac{\beta}{k} = 0$, $\frac{\gamma}{k} = 0$ then Eq. (2.7.1) reduces to

$$\tan\left[kH\sqrt{\left(\frac{c^2}{c_1^2}-1\right)}\right] = \frac{\mu_2}{\mu_1}\frac{\sqrt{\frac{\mu_2}{G_2}\left(1-\frac{c^2}{c_2^2}\right)}}{\sqrt{\left(\frac{c^2}{c_1^2}-1\right)}} \tag{2.7.2}$$

3. **Case-III:** Setting $P_2 = 0$ where $G_2 = L_2 - \frac{P_2}{2}$, then Eq. (2.7.2) reduces to

$$\tan\left[kH\sqrt{\left(\frac{c^2}{c_1^2}-1\right)}\right] = \frac{\mu_2}{\mu_1}\frac{\sqrt{\left(1-\frac{c^2}{c_2^2}\right)}}{\sqrt{\left(\frac{c^2}{c_1^2}-1\right)}} \tag{2.7.3}$$

   Eq. (2.7.3) denotes the standard scattering relation of torsional wave in the anisotropic medium.

2.8 Numerical Results

The phase velocity of torsional wave in an inhomogeneous anisotropic layer has been determined numerically from the Eq. (2.6.5). The graphical portrayal demonstrates the phase velocity connection for the diverse estimations of β/k, γ/k, c/c_2 and kH with the fixed value of $N_1/L_1 = 0.7$, $\mu_2/\mu_1 = 0.2$, $N_2/G_2 = 0.2$ and $P_2 = 0.4$ Fig (2.2) to fig. (2.4) and fig.(2.6) to fig.(2.7) is plotted for wave number kH versus phase velocity c^2/c_1^2 which are dimensionless. Fig. (2.5) is plotted for undimensioned phase velocity c^2/c_1^2 versus undimensioned shear wave velocity c^2/c_2^2. Fig. (2.2) and fig. (2.4) speaks with effect of inhomogeneity parameter β/k and γ/k respectively on phase speed of torsional wave. Fig.(2.3) represents the impact of shear wave velocity on phase velocity of torsional surface wave. Fig.(2.5) shows the impact of dimensionless wave number kH on phase velocity of torsional surface wave. Fig. (2.6) represents effect of γ/k with $\beta/k = 0$ and fig. (2.7) shows impact of β/k with $\gamma/k = 0$. For graphical portrayal, MATLAB programming has been utilized to sum up the outcomes.

Fig. (2.2) speaks with the effect of inhomogeneity parameter β/k on phase speed of torsional surface wave. In this figure, every one of the bends have been plotted by taking β/K = 0.2, 0.4, 0.6, 0.8 and steady estimation of different parameters. It's been seen that phase speed of torsional surface wave increments at all points as estimation of β/K increments and it is more clearly observed at higher frequency.

Fig. (2.3) speaks with the impact of c/c_2 on phase velocity of torsional surface wave. The value of c/c_2 for bend 1, bend 2, bend 3 and bend 4 have been choosen as 0.2, 0.4, 0.6 and 0.8 separately and other parameters remains constant. It's been seen that phase speed of torsional surface wave diminishes as c/c_2 increments and impact of parameter is visible for low frequency.

Fig. (2.4) represents the impact of inhomogeneity parameter γ/k. The estimation of γ/k for bend 1, bend 2, bend 3 and bend 4 is supposed as 0.2, 0.4, 0.6 an 0.8 separately and other parameters remains constant. It's been noticed that phase velocity of torsional surface wave increments as value of γ/k increases.

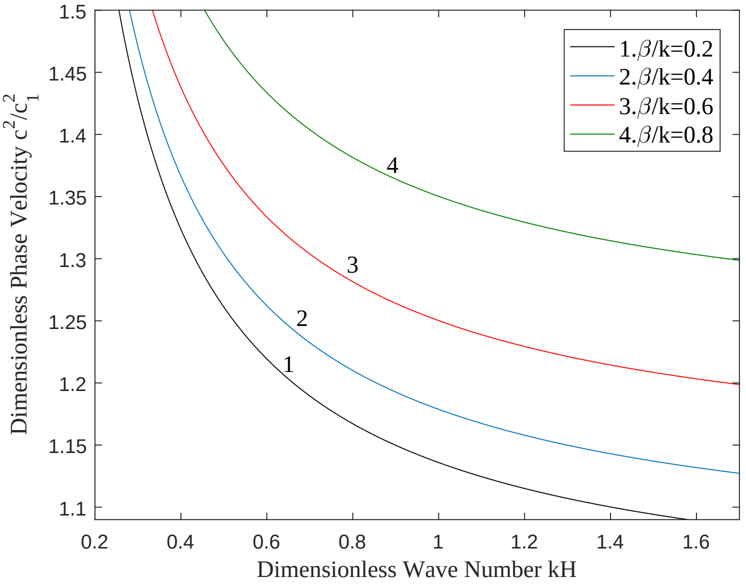

Figure 2.2: Dimensionless phase velocity c^2/c_1^2 versus dimensionless wave number kH for the different values of $\beta/K = 0.2, 0.4, 0.6, 0.8$

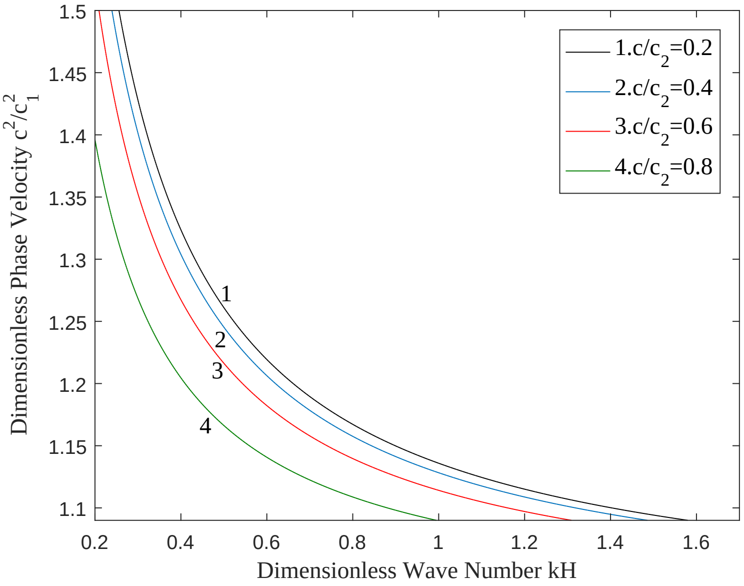

Figure 2.3: Dimensionless phase velocity c^2/c_1^2 versus dimensionless wave number kH for the different values of $c/c_2 = 0.2, 0.4, 0.6, 0.8$

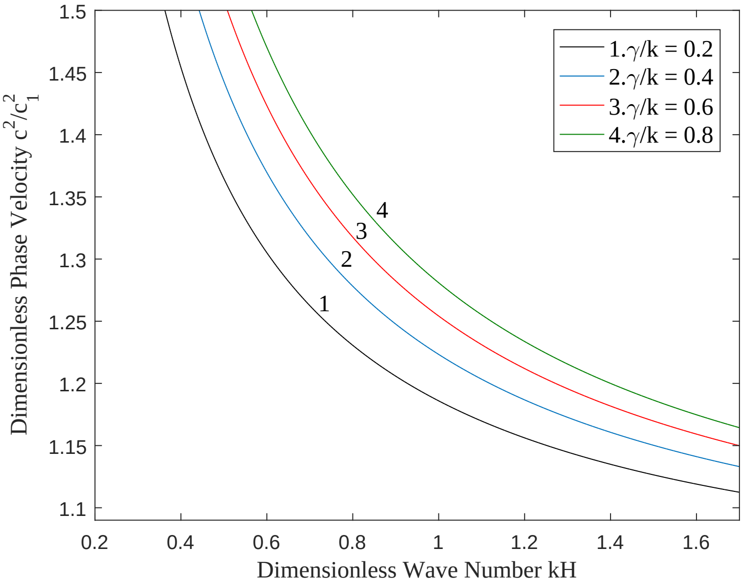

Figure 2.4: Dimensionless phase velocity c^2/c_1^2 versus dimensionless wave number kH for the different values of $\gamma/k = 0.2, 0.4, 0.6, 0.8$

Fig. (2.5) represents the effect of undimensioned wave number kH. The value of kH for all curves has been taken as 0.2, 0.3, 0.4 an 0.5, respectively and other parameters remains constant. It's been noticed that phase speed of torsional surface wave diminishes as value of kH increments and are shifted far at lower frequency.

Fig. (2.6) speaks with the effect of inhomogeneity parameter γ/k when β/k is negligible i.e. it is taken as zero. In this figure, the estimation of γ/k for bends 1, 2, 3 and 4 is taken as 0.2, 0.4, 0.6 and 0.8, separately. It's been seen that phase velocity of torsional surface wave increases with increase of inhomogeneity parameter and in absence of β/k.

Fig. (2.7) shows the effect of inhomogeneity parameter β/k when γ/k is taken as negligible i.e. it tends to zero. In this figure, all bends are plotted using value $\beta/k = 0.2$, 0.4, 0.6, 0.8 and constant value of other parameters. It's seen that phase speed of torsional surface wave increases with increment in inhomogeneity parameter and without γ/k.

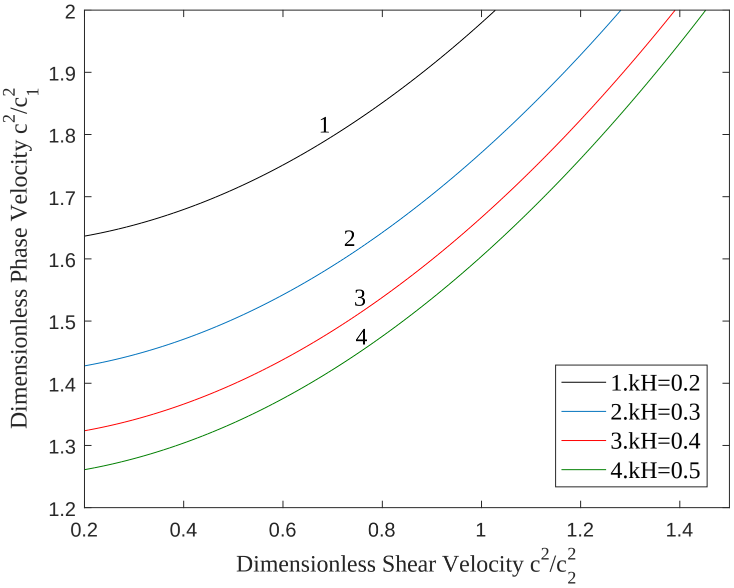

Figure 2.5: Dimensionless phase velocity c^2/c_1^2 versus dimensionless shear velocity c^2/c_2^2 for the different values of kH = 0.2, 0.3, 0.4, 0.5

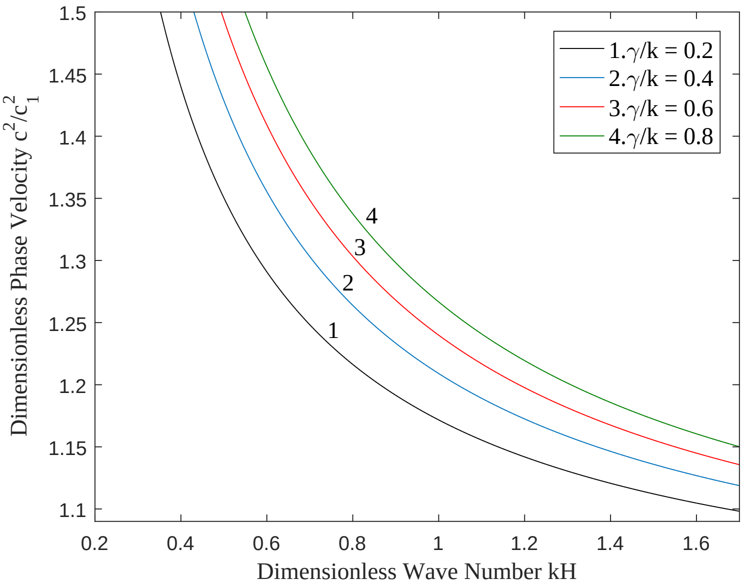

Figure 2.6: Dimensionless phase velocity c^2/c_1^2 versus dimensionless wave number kH for the different values of $\gamma/k = 0.2$, 0.4, 0.6, 0.8 and $\beta/k = 0$

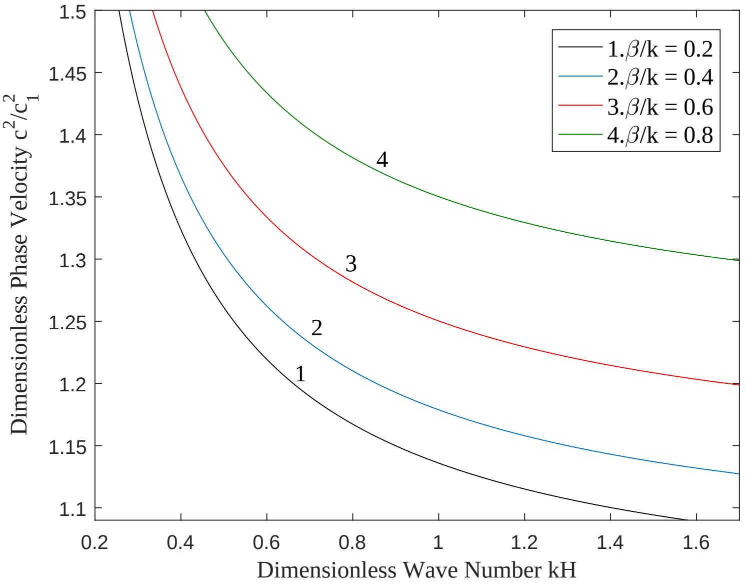

Figure 2.7: Dimensionless phase velocity c^2/c_1^2 versus dimensionless wave number kH for the different values of $\beta/K = 0.2$, 0.4, 0.6, 0.8 and $\gamma/k = 0$

2.9 Conclusions

The dispersion relation of Torsional surface wave in an inhomogeneous anisotropic medium that is lying over an anisotropic half-space have been inferred analytically. We infer that the geometry may permit the propagation of torsional surface waves in two unique mediums. Some exceptional cases have been obtained from generalized dispersion relation. Case-1 describes dispersion relation of torsional surface wave in inhomogeneous anisotropic layer and anisotropic half-space where anisotropy is changed to isotropy. Case-2 describes dispersion relation of torsional surface wave when layer is taken as homogeneous medium i.e. without inhomogeneity parameter. Case-3 represents the dispersion relation when the initial stress is negligible. From the above cases, we come to an end that phase speed of torsional surface wave increments as the value of β/k gets large, phase velocity of torsional surface wave diminishes as c/c_2 increases, phase velocity of torsional surface wave increments as value of γ/k increases. Phase velocity of torsional surface wave decreases as estimation of kH increments. Phase speed of torsional surface wave increases when inhomogeneity parameter γ/k is increased and β/k is kept as zero and vice-versa. We observe that graph of change in β/k for some value of γ/k is same as when γ/k is negligible. Also, graphs are similar when β/k is taken negligible and β/k has some value and γ/k varies. The obtained outcomes are helpful to discover the area of seismic tremors and may give important data about the choice of appropriate basic materials for common development.

Chapter 3

Effect of irregularity on torsional surface waves in an initially stressed anisotropic porous medium

3.1 Objective

This chapter deals with the propagation of Torsional surface wave in porous medium with irregular interface. The solution of governing equation of motion has been obtained by utilizing separation of variables techniques. The solution of wave equation represents the diplacement in corresponding layer and homogeneous half-space. For Torsional surface wave propagation, we considered porous medium as superficial layer lying over a homogeneous semi-infinite medium under impact of initial stress. The dispersion curve is obtained in this medium which is converted into standard dispersion relation of Torsional wave. The impact of beginning stress parameters has been noticed in graphical section. It has been seen that, the presence of initial stress in the dispersion curve affected the phase speed of Torsional surface wave.

3.2 Mathematical formulation of the problem

The propagation of torsional surface wave in an initially stressed anisotropic porous layer lying above a homogeneous half-space is discussed in this chapter. The parabolic irregularity is assumed at interface isolating the layer from half-space and expect the quadratic inhomogeneity in inflexibility and thickness. Depth of irregularity is taken as h and length is assumed as 2l. Along x-axis, the torsional wave is propagating and z-axis is taken vertically downward to the bearing of wave propagation. The upper part of anisotopic layer is stress free. The equation of irregularity at interface is taken as z = ε F(r), where

$$F(r) = \begin{cases} h\left(1-\frac{r^2}{l^2}\right); & |r| \leq l \\ 0 \quad ; & |r| > l \end{cases} \tag{3.2.1}$$

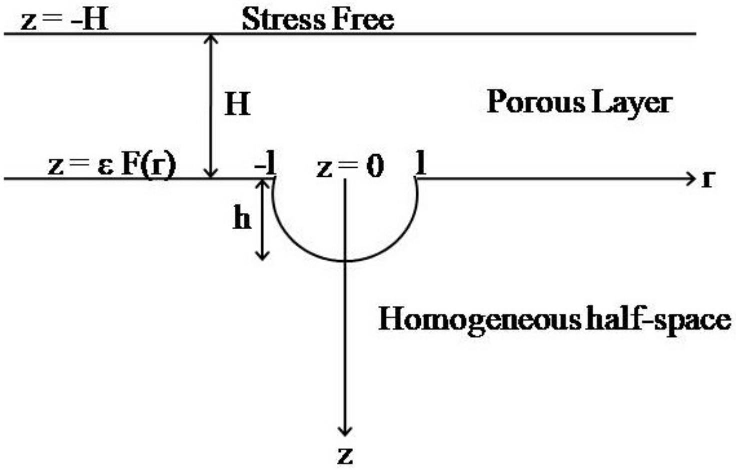

Figure 3.1: Geometry of the problem.

3.3 Dynamics and solution of layer

The dynamical conditions of movement in anisotropic permeable channel under the impact of initial stress P in absence of body force are:

$$\left.\begin{array}{l}\frac{\partial S_{rr}}{\partial r}+\frac{1}{r}\frac{\partial S_{r\theta}}{\partial \theta}+\frac{\partial S_{rz}}{\partial z}+\frac{S_{rr}-S_{\theta\theta}}{r}-P\frac{\partial w'_{\theta}}{\partial z}=\frac{\partial^2}{\partial t^2}\left(\rho_{rr}u_1+\rho_{r\theta}U_1\right),\\ \frac{\partial S_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial S_{\theta\theta}}{\partial \theta}+\frac{\partial S_{\theta z}}{\partial z}+\frac{2S_{r\theta}}{r}-P\frac{\partial w'_{z}}{\partial r}=\frac{\partial^2}{\partial t^2}\left(\rho_{rr}v_1+\rho_{r\theta}V_1\right),\\ \frac{\partial S_{rz}}{\partial r}+\frac{1}{r}\frac{\partial S_{\theta z}}{\partial \theta}+\frac{\partial S_{zz}}{\partial z}+\frac{S_{rz}}{r}-P\frac{\partial w'_{\theta}}{\partial r}=\frac{\partial^2}{\partial t^2}\left(\rho_{rr}w_1+\rho_{r\theta}W_1\right),\end{array}\right\} \tag{3.3.1}$$

and

$$\left.\begin{array}{l}\frac{\partial S}{\partial r}=\frac{\partial^2}{\partial t^2}\left(\rho_{r\theta}u_1+\rho_{\theta\theta}U_1\right),\\ \frac{\partial S}{\partial \theta}=\frac{\partial^2}{\partial t^2}\left(\rho_{r\theta}v_1+\rho_{\theta\theta}V_1\right),\\ \frac{\partial S}{\partial z}=\frac{\partial^2}{\partial t^2}\left(\rho_{r\theta}w_1+\rho_{\theta\theta}W_1\right),\end{array}\right\} \tag{3.3.2}$$

where (U_1,V_1,W_1) denotes the displacement components of liquid, (u_1,v_1,w_1) denotes the corresponding displacement components of the solid, S represents stress in liquid, S_{rr} , $S_{\theta\theta}$, S_{zz} , S_{rz} , $S_{r\theta}$ and $S_{\theta z}$ are the stress components,

$$\left.\begin{array}{l}w'_r=\frac{1}{2r}\left(\frac{\partial w_1}{\partial \theta}-r\frac{\partial v_1}{\partial z}\right),\\ w'_{\theta}=\frac{1}{2}\left(\frac{\partial u_1}{\partial z}-\frac{\partial w_1}{\partial r}\right),\\ w'_z=\frac{1}{2r}\left(\frac{\partial r v_1}{\partial r}-\frac{\partial v_1}{\partial \theta}\right),\end{array}\right\} \tag{3.3.3}$$

represents the components of rotational vector w'. The stress-strain relations for the porous layer is mentioned below:

$$\left.\begin{array}{l}S_{rr}=(A+P)e_{rr}+(A-2N+P)e_{\theta\theta}+(F+P)e_{zz}+Q_{\varepsilon},\\ S_{\theta\theta}=(A-2N)e_{rr}+Ae_{\theta\theta}+Fe_{zz}+Q_{\varepsilon},\\ S_{zz}=Fe_{rr}+Fe_{\theta\theta}+Ce_{zz}+Q_{\varepsilon},\\ S_{r\theta}=2Ne_{r\theta},S_{\theta z}=2Ge_{\theta z},S_{rz}=2Ge_{zr},\end{array}\right\} \tag{3.3.4}$$

where C, F, A are the elastic constants of this medium, G and N are shear modulii along z and r direction, respectively and Q_{ε} is proportion of coupling between the difference in volume of solid and fluid. The connection between stress vector S and fluid weight P' is

$$-S=fP' \tag{3.3.5}$$

where f is the porosity of layer. The mass density of total is:

$$\rho' = \rho_s + f(\rho_w - \rho_s) \tag{3.3.6}$$

The torsional wave is indicated by

$$\left.\begin{aligned} &u_1 = 0, w_1 = 0,\\ &v_1 = v(r,z,t),\\ &U_1 = 0, W_1 = 0,\\ &V_1 = V_1(r,z,t), \end{aligned}\right\} \tag{3.3.7}$$

which gives strain components as:

$$\left.\begin{aligned} &e_{rr} = 0, e_{\theta\theta} = 0,\\ &e_{zz} = 0, e_{zr} = 0,\\ &2e_{\theta z} = \tfrac{\partial v_1}{\partial z},\\ &2e_{r\theta} = \tfrac{\partial v_1}{\partial r} - \tfrac{v_1}{r}, \end{aligned}\right\} \tag{3.3.8}$$

Now, using eq. (3.3.7) and (3.3.8) in (3.3.4), we get two stress components, i.e.

$$\left.\begin{aligned} &S_{\theta z} = G\tfrac{\partial v_1}{\partial z},\\ &S_{r\theta} = N\left(\tfrac{\partial v_1}{\partial r} - \tfrac{v_1}{r}\right), \end{aligned}\right\} \tag{3.3.9}$$

Substituting above relations in eq.(3.3.1) and eq.(3.3.2), we get

$$\left(N - \frac{P}{2}\right)\left(\frac{\partial^2 v_1}{\partial r^2} + \frac{1}{r}\frac{\partial v_1}{\partial r} - \frac{v_1}{r^2}\right) + G\frac{\partial^2 v_1}{\partial z^2} = \frac{\partial^2}{\partial t^2}\left(\rho_{rr}v_1 + \rho_{r\theta}V_1\right) \tag{3.3.10}$$

and

$$(\rho_{r\theta}v_1 + \rho_{\theta\theta}V_1) = k_1 \tag{3.3.11}$$

Now, eliminating V_1 from equations (3.3.10) and (3.3.11), we get

$$\left(N - \frac{P}{2}\right)\left(\frac{\partial^2 v_1}{\partial r^2} + \frac{1}{r}\frac{\partial v_1}{\partial r} - \frac{v_1}{r^2}\right) + G\frac{\partial^2 v_1}{\partial z^2} = d'\frac{\partial^2 v_1}{\partial t^2} \tag{3.3.12}$$

where $d' = \rho_{rr} - \frac{\rho_{r\theta}^2}{\rho_{\theta\theta}}$. Equation (3.3.12) indicates that shear wave velocity along radial heading is:

$$\sqrt{\frac{N - \frac{P}{2}}{d'}} = \sqrt{\frac{1-\zeta}{d}c_1^2}$$

where $\zeta = \frac{P}{2N}$ and $d = \frac{d'}{\rho'}$. The arrangement of eq.(3.3.12) is of the structure:

$$v_1 = V_1^*(z)J_1(kr)e^{i\omega t} \tag{3.3.13}$$

and hence eq.(3.3.12) takes the form

$$\frac{\mathrm{d}^2 V_1^*}{\mathrm{d}z} + m_1^2 V_1^* = 0 \tag{3.3.14}$$

where $m_1 = k\sqrt{pd\left(\frac{c^2}{c_1^2} - \frac{1-\zeta}{d}\right)}$ and $p = \frac{N}{G}$ and hence the solution of eq.(3.3.14) is

$$V_1^*(z) = A_1 cos(m_1 z) + A_2 sin(m_1 z)$$

Hence, the displacement segment of anisotropic porous layer is obtained as:

$$v_1 = (A_1 cos(m_1 z) + A_2 sin(m_1 z))J_1(kr)e^{i\omega t} \tag{3.3.15}$$

3.4 Displacement in anisotropic homogeneous half-space

If θ and r be circumferential and radial coordinates and wave is moving along radial direction, dynamical equations of motion are given by:

$$\left.\begin{array}{l} \frac{\partial\sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta} + \frac{\partial\sigma_{rz}}{\partial z} + \frac{\sigma_{rr}-\sigma_{\theta\theta}}{r} = \rho\frac{\partial^2 u_2}{\partial t^2}, \\ \frac{\partial\sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta} + \frac{\partial\sigma_{\theta z}}{\partial z} + \frac{2\sigma_{r\theta}}{r} = \rho\frac{\partial^2 v_2}{\partial t^2}, \\ \frac{\partial\sigma_{rz}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{\theta z}}{\partial\theta} + \frac{\partial\sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r} = \rho\frac{\partial^2 w_2}{\partial t^2}, \end{array}\right\} \tag{3.4.1}$$

The stress-strain relation is given by

$$\sigma_{ij} = \lambda\Omega\delta_{ij} + 2\mu e_{ij} \tag{3.4.2}$$

where λ and μ are Lame's constant, $e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$ and $e_{ii} = \Omega$. The displacement components are

$$\left.\begin{array}{l} u_2 = 0, \\ w_2 = 0, \\ v_2 = v_2(r,z,t), \end{array}\right\} \tag{3.4.3}$$

Using equation (3.4.2) and (3.4.3), the equation of motion in absence of body force is obtained as

$$\frac{\partial \sigma_{r\theta}}{\partial r}+\frac{\partial \sigma_{\theta z}}{\partial z}+\frac{2\partial \sigma_{r\theta}}{r}=\rho\frac{\partial^2 v_2}{\partial t^2} \tag{3.4.4}$$

The stresses are associated to displacement in elastic medium by:

$$\sigma_{r\theta}=\mu\left(\frac{\partial v_2}{\partial_r}-\frac{v_2}{r}\right), \sigma_{z\theta}=\mu\frac{\partial v_2}{\partial z} \tag{3.4.5}$$

Using the above relation, equation (3.4.4) reduces to

$$\mu(z)\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right)v+\frac{\partial}{\partial z}\left(\mu(z)\frac{\partial v_2}{\partial z}\right)=\rho(z)\frac{\partial^2 v_2}{\partial t^2} \tag{3.4.6}$$

For wave propagationg along radial direction and changing harmonically in time, we get

$$v_2=V_2(z)J_1(kr)e^{i\omega t} \tag{3.4.7}$$

where k denotes the wave number, $J_1(kr)$ is bessel function of first kind and first order and ω is angular frequency and therefore equation (3.4.7) takes the form

$$\frac{\mathrm{d}^2V_2}{\mathrm{d}z^2}+\frac{\mu'(z)}{\mu(z)}\frac{\mathrm{d}V_2}{\mathrm{d}z}-k^2\left(1-\frac{c^2}{c_s^2}\right)V_2(z)=0 \tag{3.4.8}$$

where c represents the torsional wave velocity and c_s is shear wave velocity.

Solution for lower half-space

The heterogeneity of lower half-space is portrayed as:

$$\mu=\mu_2\,, \rho=\rho_2 \tag{3.4.9}$$

Using (3.4.9), the equation of motion changes to

$$\frac{\mathrm{d}^2V}{\mathrm{d}z^2}-k^2m_2V_2(z)=0 \tag{3.4.10}$$

where $m_2=1-\frac{c^2}{c_2^2}$ and c_2 is shear velocity. The consequence of eq(3.4.10) is:

$$V_2(z)=A_3e^{-kz\sqrt{m}_2} \tag{3.4.11}$$

Therefore, the result for homogeneous anisotropic half-space is

$$v_2=A_3e^{-kz\sqrt{m}_2}J_1(kr)e^{i\omega t} \tag{3.4.12}$$

3.5 Boundary conditions

1. The upper half-space of porous layer is stress free and hence density is zero i.e.

$$\mu_1 \frac{\partial v_1}{\partial z} = 0 \; at \; z = -H$$

2. The stress component, displacement component are continuous at the irregular interface of layer at $z = \varepsilon\, F(r)$ i.e.

$$v_1 = v_2 \; , \; \mu_1 \frac{\partial v_1}{\partial z} = \mu_2 \frac{\partial v_2}{\partial z}$$

3.6 Dispersion relation

By using the boundary conditions, we get the below mentioned phase velocity equations

$$\mu_1 \left(A_1 m_1 sin(m_1 H) + A_2 m_1 cos(m_1 H)\right) = 0 \tag{3.6.1}$$

$$A_1 cos(m_1 \varepsilon F(r)) + A_2 sin(m_1 \varepsilon F(r)) = A_3 e^{-k\varepsilon F(r)\sqrt{m}_2} \tag{3.6.2}$$

$$\mu_1 \left(A_1 m_1 sin(m_1 \varepsilon F(r)) + A_2 m_1 cos(m_1 \varepsilon F(r))\right) = \mu_2 A_3 k\sqrt{m}_2 e^{-k\varepsilon F(r)\sqrt{m}_2} \tag{3.6.3}$$

Now, eliminating the arbitrary constants from above obtained phase velocity equations, we get

$$\begin{vmatrix} \mu_1 m_1 sin(m_1 H) & \mu_1 m_1 cos(m_1 H) & 0 \\ cos(m_1 \varepsilon F(r)) & sin(m_1 \varepsilon F(r)) & -e^{-k\varepsilon F(r)\sqrt{m}_2} \\ -\mu_1 m_1 sin(m_1 \varepsilon F(r)) & \mu_1 m_1 cos(m_1 \varepsilon F(r)) & \mu_2 k\sqrt{m} e^{-k\varepsilon F(r)\sqrt{m}_2} \end{vmatrix} = 0$$

which is reduced to

$$tan(qH) = \frac{cos(m_1 \varepsilon F(r))\mu_2 k\sqrt{m}_2 - sin(m_1 \varepsilon F(r))\mu_1 m_1}{sin(m_1 \varepsilon F(r))\mu_2 k\sqrt{m}_2 + cos(m_1 \varepsilon F(r))\mu_1 m_1} \tag{3.6.4}$$

The generalized dispersion relation of torsional wave in presence of irregularity is obtained as:

$$\tan\left[kH\sqrt{\frac{N}{G}\frac{d'}{\rho'}\left(\frac{c^2}{c_1^2}-\frac{1-\zeta}{d}\right)}\right]$$

$$=\frac{\mu_2 k\sqrt{m}_2 cos\left(k\varepsilon h\left(1-\frac{r^2}{b^2}\right)\sqrt{\frac{N}{G}\frac{d'}{m_1'}\left(\frac{c^2}{c_1^2}-\frac{1-\zeta}{d}\right)}\right)-\mu_1 m_1 sin\left(k\varepsilon h\left(1-\frac{r^2}{b^2}\right)\sqrt{\frac{N}{G}\frac{d'}{m_1'}\left(\frac{c^2}{c_1^2}-\frac{1-\zeta}{d}\right)}\right)}{\mu_2 k\sqrt{m}_2 sin\left(k\varepsilon h\left(1-\frac{r^2}{b^2}\right)\sqrt{\frac{N}{G}\frac{d'}{m_1'}\left(\frac{c^2}{c_1^2}-\frac{1-\zeta}{d}\right)}\right)+\mu_1 m_1 cos\left(k\varepsilon h\left(1-\frac{r^2}{b^2}\right)\sqrt{\frac{N}{G}\frac{d'}{m_1'}\left(\frac{c^2}{c_1^2}-\frac{1-\zeta}{d}\right)}\right)} \tag{3.6.5}$$

3.7 Validation of the problem

Case-I: If there is no irregularity at lower interface i.e. h → 0, then dispersion relation (3.6.5) changes to

$$\tan\left(kH\sqrt{\frac{N}{L}\left(\frac{1}{1-\frac{P}{2L}}\right)d\left(\frac{c^2}{c_1^2}-\frac{1-\frac{P}{2N}}{d}\right)}\right)=\frac{\mu_2}{\mu_1}\frac{\sqrt{1-\frac{c^2}{c_2^2}}}{\frac{N}{L}\left(\frac{1}{1-\frac{P}{2L}}\right)\left(\frac{c^2}{c_1^2}-\frac{1-\frac{P}{2N}}{d}\right)}$$

Case-II: Substituting N → L→ μ , then the above eq. is transformed to

$$\tan\left(kH\sqrt{\frac{d}{1-\frac{P}{2L}}\left(\frac{c^2}{c_1^2}-\frac{1-\frac{P}{2N}}{d}\right)}\right)=\frac{\mu_2}{\mu_1}\frac{\sqrt{1-\frac{c^2}{c_2^2}}}{\frac{N}{L}\left(\frac{1}{1-\frac{P}{2L}}\right)\left(\frac{c^2}{c_1^2}-\frac{1-\frac{P}{2N}}{d}\right)}$$

Case-III: If there will be no initial stress in layer i.e. $\frac{P}{2L}$ = $\frac{P}{2N}$ = 0, then the above obtained equation changes to

$$\tan\left(kH\sqrt{d\left(\frac{c^2}{c_1^2}-\frac{1}{d}\right)}\right)=\frac{\mu_2}{\mu_1}\frac{\sqrt{1-\frac{c^2}{c_2^2}}}{\sqrt{d\left(\frac{c^2}{c_1^2}-\frac{1}{d}\right)}}$$

.

Case-IV: If there is no porosity i.e. f → 0, then d → 1 and hence, we get

$$\tan\left(kH\sqrt{\frac{c^2}{c_1^2}-1}\right) = \frac{\mu_2}{\mu_1}\frac{\sqrt{1-\frac{c^2}{c_2^2}}}{\sqrt{\frac{c^2}{c_1^2}-1}}$$

The above equation regards to standard dispersion relation of torsional wave in anisotropic porous layer and homogeneous half-space.

3.8 Numerical results

In view of the dispersion relation obtained in eq.(3.6.5), numerical calculations are performed to exhibit the impact for various estimations of d, $P/2L$, $P/2N$, h/H and r/b on propagation of torsional waves in permeable layer lying over a homogeneous half-space.Fig(3.2) to Fig(3.6) is plotted for undimensioned phase velocity c/c_1 versus undimensioned wave number kH. The values of $N/L = 0.4$, $N/G = 0.8$, $\mu_2/\mu_1 = 3.0$, $c/c_2 = 0.2$ and $\varepsilon = 0.02$ is kept fixed in all figures.

Fig(3.2) represents the effect of porosity d. The estimation of porosity d for the bends 1,2 and 3 is supposed as 0.54, 0.56 and 0.58 respectively and the other parameters remains constant. It is noticed that as porosity factor expands the phase speed is decreases.

Fig(3.3) gives variation of dimensionless wave number kH against phase velocity of torsional wave for distinct values of $P/2L$. The value of $P/2L$ for bend 1, bend 2 and bend 3 is taken as 0.2, 0.3 and 0.4 individually. It's been noticed that phase velocity diminishes rapidly with increment in anisotropic factor N/G.

Fig(3.4) gives the variation of undimensioned wave number KH against phase velocity of torsional wave for distinct values of $P/2N$. The estimation of $P/2N$ for the curves 1,2 and 3 are choosen as 0.2, 0.3 and 0.4 individually. It is seen that phase velocity diminishes with increase of $P/2N$.

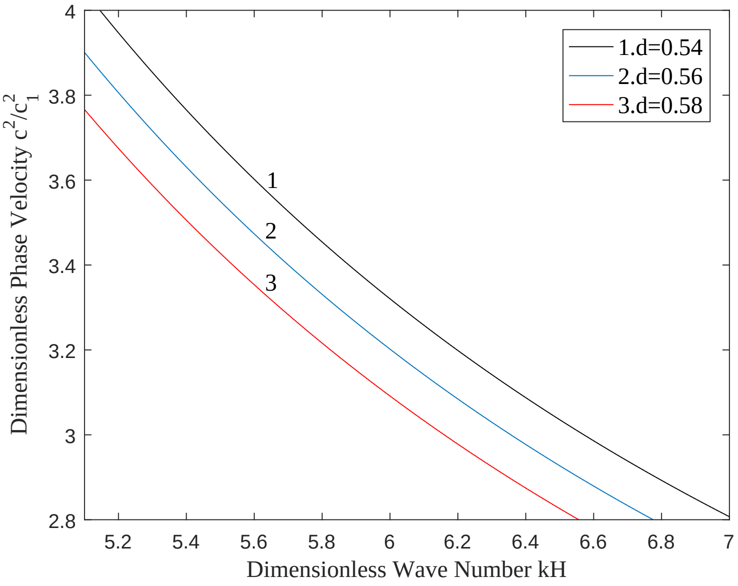

Figure 3.2: Dimensionless phase velocity c^2/c_1^2 versus dimensionless wave number kH for the different values of d = 0.54,0.56,0.58

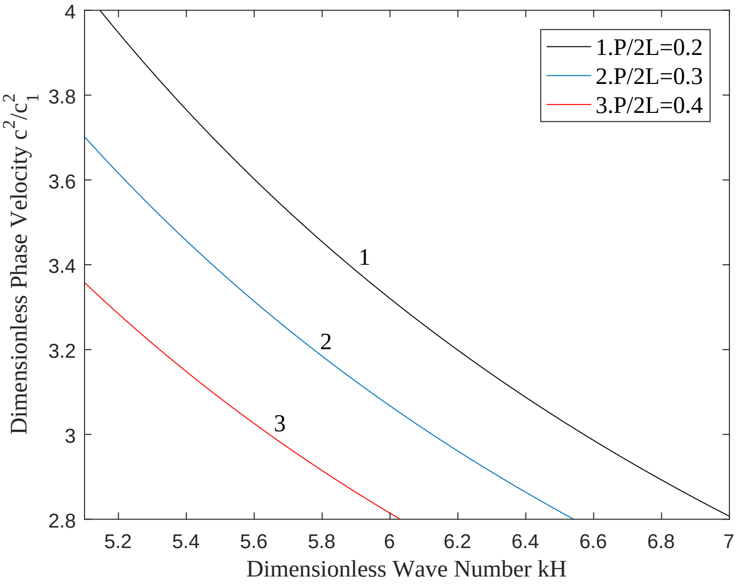

Figure 3.3: Dimensionless phase velocity c^2/c_1^2 versus dimensionless wave number kH for the different values of $P/2L = 0.2,0.3,0.4$

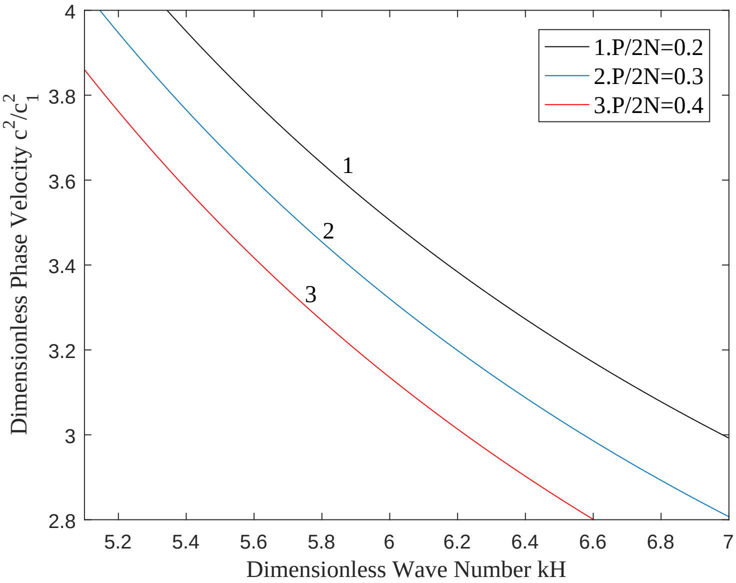

Figure 3.4: Dimensionless phase velocity c^2/c_1^2 versus dimensionless wave number kH for the different values of $P/2N = 0.2,0.3,0.4$

Fig(3.5) shows the impact of the proportion h/H on propagation of torsional wave in permeable layer. The estimation of proportion h/H is taken as 4,5 and 6 for the curves 1,2 and 3 individually. It has been seen that phase velocity of torsional surface wave diminishes as undimensioned proportion h/H increases.

Fig(3.6) represents the effect of r/b. The value of r/b is choosen to be as 0.2, 0.4 and 0.6 individually. It is observed that phase velocity of torsional wave increases as the estimation of r/b increases.

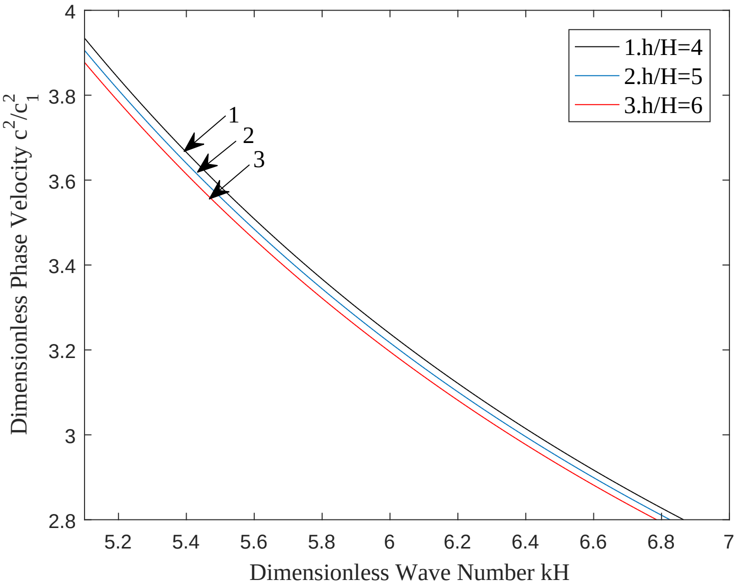

Figure 3.5: Dimensionless phase velocity c^2/c_1^2 versus dimensionless wave number kH for the different values of $h/H = 4,5,6$

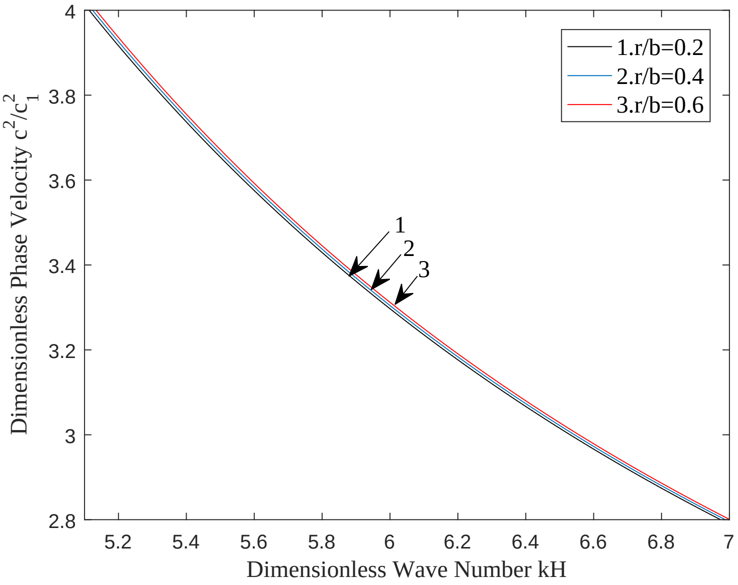

Figure 3.6: Dimensionless phase velocity c^2/c_1^2 versus dimensionless wave number kH for the different values of $r/b = 0.2,0.4,0.6$

3.9 Conclusions

Torsional surface wave propagation in an at first focused porous layer lying over a homogeneous half-space has been considered and dispersion relation is acquired analytically. The dynamic impact of depth of heterogeneity parameters and irregularity on torsional wave's phase velocity has been observed graphically. It is noticed that at interface of layer and half-space within the sight of parabolic irregularity, it essentially influenced torsional wave's phase velocity in an homogeneous half-space. Case-1 describes the dispersion relation of torsional wave in the absence of irregularity at lower interface. Case-2 describes the dispersion relation when rigidity and density is kept same. Case-3 represents the dispersion relation when there will be no initial stress in layer. Case-4 represents the dispersion relation when the porosity is negligible i.e. porosity is taken zero. From the above cases we observe that, phase speed of torsional wave reduces with the increase of porosity factor d and anisotropic factor N/G. Also, with increase of initial sress $P/2L$, $P/2N$ and dimensionless parameter h/H , the phase velocity of torsional wave decreases. Phase velocity of torional wave increases as estimation of r/b increases. The present examination is useful for the seismologist and geologist for the examination of Earth's inside with various outcomes of minerals.

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