Controlled Scallop Height Tool Path Generation for 3-Axis Vertical CNC Machining of STL Surfaces

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CERTIFICATE

I hereby declare that the thesis titled "**Controlled Scallop Height Tool Path Generation for 3-Axis Vertical CNC Machining of STL Surfaces**" is an authentic record of my study carried out as requirements for the award of the degree of **Master of Engineering** in **CAD/CAM Engineering** at **Thapar University, Patiala** under the supervision of **Mr. Ravinder Kumar Duvedi**, Assistant Professor, Mechanical Engineering Department, Thapar University, Patiala during July, 2014. The matter embodied in this report has not been submitted in partial or full to any other university or institute for the award of any degree.

Date: 15/07/2014

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It is certified that the above statement made by the student is correct to the best of my/our knowledge and belief.

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Dedicated to
My respected mother
Late Smt. Raj Rani

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Rajiv

Abstract

Sculpture surface machining is widely used in these days to make intricate products. NC machining of sculpture surfaces require effective tool path positioning and planning. Formation of scallops between two neighboring tool paths is the main problem that occurs while NC machining which directly results increase in machining time. Although several methods are using for sculpture surface machining for tool path generation with ball nose ended milling cutters but Iso-parametric, Iso-planar and Iso-scallop methods are commonly used. Among all these methods iso-scallop method is widely used because it requires less machining time and produces uniform surface roughness. It is also important to identify the minimum NC tool path length for iso-scallop machining. For small path segments of tool paths, the machining time would rapidly increase, especially for high speed machining. Ideal segment length and side step are thus necessary for proper iso scallop machining.

In this present work, scallop height measure in real and global sense is obtained by application of iso-scallop strategy. For controlled scallop height, proper orientation of cutter path and planning is required. For tool path positioning and planning, “ball drop” and iso-scallop methods respectively are used here to develop a new methodology.

It is concluded that scallops are difficult to remove but in this present work, by calculating the maximum scallop height with ball ended milling cutters to achieve controlled scallop height tool path on triangulated surfaces is done. Mathematical calculations of tool path positioning and planning are also done with real sense. To achieve controlled scallop height tool path, bisection method is used.

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Nomenclature

C_L	= Cutter location point
C_C	= Cutter contact point
S	= Parametric surface
u, v	= Parameters of parametric surface
$\widehat{N}$	= Normal vector
x, y, z	= Cartesian coordinates
T_1	= Initial tool position
T_2	= Final tool position
P_1, P_2, P_3 and P_4	= Vertices of triangle
R	= Radius of ball end mill
$\hat{t}_l$	= Tool axis direction
$\hat{u}$, $\hat{v}$, $\widehat{w}$	= Direction vectors
I_c	= Intersection point of two spheres
M	= Midpoint of two cutter location on neighboring tool paths
S_s	= Side step value
l	= Half of the distance between two cutter locations
I_{stl}	= Intersection point on STL surface
s, t	= Barycentric coordinates
h	= Scallop height (Distance between I_c and I_{stl})
F_s	= Forward step value
S_{sl}	= Side step left value
S_{si}	= Side step initial value
S_{sr}	= Side step right value
ϵ	= User defined scallop value
$C_{L(ij)}$	= Cutter location of i^{th} tool path taken along x-axis direction for j^{th} feed forward value along y-axis direction
$C_{L(i+1,j)}$	= Cutter location of $(i+1)^{th}$ tool path taken along x-axis direction for j^{th} feed forward value along y-axis direction
h_{ij}	= Scallop height of i^{th} tool path taken along x-axis direction for j^{th} feed forward value along y-axis direction

Chapter 1

Introduction

Manufacturing of sculptured surfaces is the major domain of interest in the integrated CAD/CAM system. For machining of intricate surfaces, it may consist of three parts: (1) a surface description method; first of all surface is described on which machining is done. Sculptured surface machining is a typical method hence it is very important to describe surface (STL, Parametric, Bezier and B-spline to name a few) for efficient machining. (2) a tool path generation method (iso-planar, iso-parametric and iso-scallop to name a few) which are used to generate the tool path for sculptured surface machining in order to produce minimum path shift within the specified tolerances; and (3) a tool control method to position the tool for machining of intricate surfaces using CNC machine tools. Manufacturing of intricate surfaces on CNC machine tools with the specified tolerances need tool path generation and planning of tool movements. It is necessary to differentiate the cutter interference inadvertently for generating the tool path to intricate surfaces. Tool interference generally occurs while machining of sculptured surfaces. There is overcutting (gouging) and undercutting (formation of scallops) while machining of such intricate surfaces. Formation of scallops is the area of interest to be studied which enhances the machining time and also unnecessarily wastage of manpower in bench-working.

1.1 Triangulated surfaces

Most of manufacturers of intricate products rely on triangulated surfaces for tool path planning now days. Triangulated surfaces known as Stereo Lithography (STL) format. STL format consists of vertices and normal of the facets and set of triangles. Industry users like this format because it simplifies input into analysis algorithms, data transfer, algorithms such as tool path planning and positioning, creation of 3D printed models, and provides desired accuracy in parts made using triangulated models. STL format is used for part design output in most of the Computer-aided design packages.

To produce physical 3D models on CAD/CAM systems from a STL file is the original use of the STL file format. Part models can be enhanced in STL files. Hence STL files are popular these days due to above advantages and its present use in today's industry.

1.2 Cutter location and cutter contact point

Cutter location and cutter contact point are very important in tool methodology.

Figure 1.1 shows the Cutter location point (C_L) and cutter contact point (C_C). Cutter location point (C_L) is the control point on tool. Cutter contact point (C_C) is the point on surface of work piece.

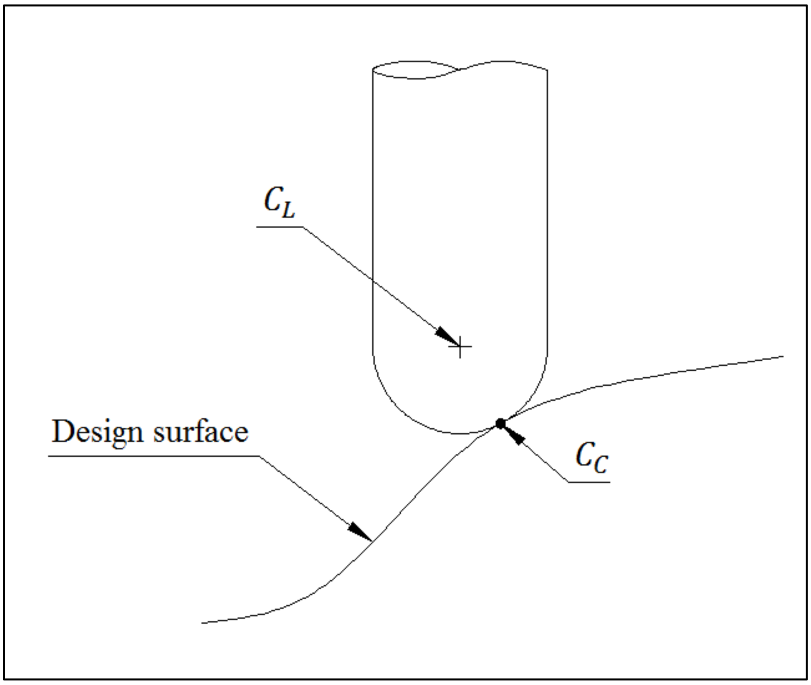

Figure 1.1: Cutter location and cutter contact point

1.3 Proposed work

In this present work, “Ball drop” method is used for identifying cutter location data for two neighboring tool footprint locations. The cutter location data is used to identify the scallop height produced because of side step value between two adjoining cutter locations. This scallop height estimate has been made using a new concept which uses the “Ray triangle intersection concept”. Now if this estimated scallop height is more than given tolerance then side step value has to decrease. This estimate of scallop height is used to find appropriate side step value using “Bisection Method” as discussed in chapter 3.

Chapter 2

Literature Review

In this present work, study on NC tool path positioning and planning, iso-scallop surfaces with triangular surfaces (Stereo lithography format) is reviewed here. Review on surface finish of scallop is taken into consideration.

2.1 Sculptured surface machining with STL format

Sculptured surface is defined as a collection or sum of interconnected and bounded parametric patches together with blending and interpolation formulas. Sculptured surface machining is a typical process. Various products like dies, moulds, aerodynamics and turbine blades are manufactured with intricate surfaces. Efficient machining of these products has become very important. Hence sculptured surfaces gaining importance now days. There are different types of representation of surfaces such as parametric representation of synthetic surfaces which includes Bezier, B-Spline, Offset, Triangular patches and Non-uniform rational B-Spline (NURBS). Among these Triangular patches has been widely accepted in industries for quite some time which is known as Stereo lithography (STL) format. As STL format is supported by many types of software, direct STL models can be created by reverse engineering process [1].

2.2 Tool positioning strategies

It is very important to position the tool before planning the tool path. There are various tool positioning strategies which are discussed below

2.2.1 "Ball drop" tool position method

Manos et al. [2] describe how to locate a tool using "ball drop" method on SCALM (Single controlled axis lathe mill). Ball-nosed end mill is used on triangulated surfaces. First of all find the position of the tool. But the position of the tool axis at various points is fixed. To calculate the position of the tool along the tool axis to avoid the gouging of tool, C_L point is generated. In "Ball drop" tool position method as the name suggests, ball is dropped along the tool axis and first point of contact is determined. The tool used is ball end mill for machining of generated tool path. The generation of tool path also influences the tool position to produce surface which is free from scallops or minimum of scallops. Once the C_L points are generated by dropping a series of ball along the tool axis at constant interval of time, all the points are stored in file which later on use for the generation of tool path. Small facets are formed by the points grouped in triangles. An approximation of the desired surface is represented by the triangles. There is process of knowing the best tool depth in four stages as:

(i). Shadow check

(ii). Triangles check

(iii). Edges check

(iv). Points check

Variations on standard closest point computations are based on the last three checks.

2.2.2 Offset tool position method

To generate offset surfaces is the approximation of CL surfaces requires traditional solid modeling approach to NC machining. Though it is simple process, there are many shortcomings. Firstly, it is not simple problem for generation of offset surfaces. B-Rep model is the most popular representational form of a boundary representation. It is complex and computationally expensive operation to offset trimmed non-uniform rational B-splines (NURBS). Second, complex self-intersection and global-intersection problems can easily create the offset of multiple trimmed surfaces. Third, for generation of C_L points for ball end mills, the uniform offset of surfaces is only useful. Besides the automatically programmed tools (APT) cutters, the offset of C_L surfaces for fillet-end mills is a more difficult problem. Overall, surface offsetting by the traditional solid modeling approach to NC tool-path generation is difficult in calculation and ineffective in computation [1].

2.2.3 Two stage cutter path position method

Chih-Ching Lo [3] describes two-stage cutter-path positioning for ball-end milling of wall-bounded and concave surfaces. In this method, for two different stages of surface machining, two cutters with different sizes are utilized, respectively. At the first stage, as the cutter-interference region(s) is skipped, the larger cutter is utilized for efficient surface machining. At the second stage, for the skipped residual region, the smaller cutter is utilized. To avoid the cutter interference, the sizes of the two cutters are determined and the total length of the machining paths is minimized. The cutter paths are scheduled to control the height of the scallop formed by two adjacent machined paths within an allowable limit in order to achieve surface quality. The path interval should be small enough in order to meet this requirement, and consequently, tool path increases. Theoretically, by using a large cutter the path interval can increase or reduce the desired tool paths. For machining a concave surface or a wall-bounded surface a large cutter may cause cutter interference. To achieve an interference-free machining, one approach is to select a cutter that is small enough. Another approach is to utilize a large cutter for machining and skip the cutter interference region by using modern

CNC machining centers that provide high-speed automatic tool-change capabilities, and then a small cutter is utilized to cut the interference or the remaining region. This approach is known as a two stage cutter-path positioning method.

2.3 Tool path planning methods

Tool path planning is very important aspect while machining the sculptured surfaces. Traditionally iso-planar and iso-parametric methods are commonly used. Lasemi et al. [5] describe traditional tool path generation methods with recent developments in CNC machining of sculptured surfaces. Park and Chung [6] describe the procedure through which 3-axis NC tool paths can be directly generated from measured data. In Fig 2.1 side step (S_s) is defined as the horizontal distance between two adjacent tool paths and forward step (F_s) is defined as the finite increment between the successive tooth feeds. Isoplanar tool path lines projected on required machined surface is also shown.

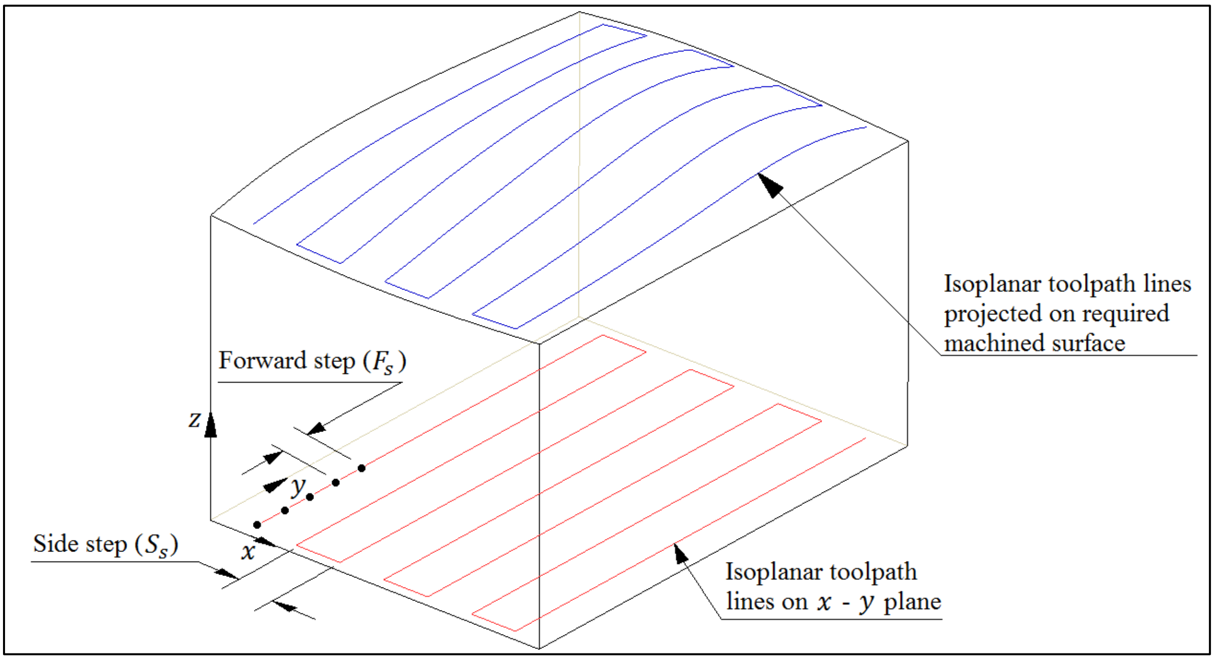

Figure 2.1: Iso-planar tool path lines projected on required machined surface

2.3.1 Iso-planar Method

For CNC machining of sculptured surfaces, iso-planar (Cartesian) technique has been widely used. To generate iso-planar tool path in Cartesian space, surfaces are intersected with parallel planes. Iso-planar method consists of iso-planar lines which are parallel in the entire tool path. The tool path length is decided on the basis of side step or path interval which is the distance between the two consecutive parallel iso-planar lines. This is one of the commonly used traditional methods because of its robustness. Various surfaces like triangular surfaces, pocketing and trimmed surfaces can be machined using iso-planar method. S.Ding et al. [7] describe the procedure that disables the disadvantages of the iso-planar method. To check the tool path length and time required for machining, intersected planes are the major consideration in iso-planar method.

2.3.2 Iso-parametric method

Loney and Ozsoy [8] addressed iso-parametric paths first time. u and v are the two parameters considered in iso-parametric method. The surface $S(u, v)$ on which the cutter contact (C_C) points are produced by having one parameter constant among two, beside the other parameter is known as parametric surface. The cutter contact points generated on parametric surfaces are directly used in the generation of tool path hence this method is well known for machining of sculptured surfaces. In this method undercutting (scallop height) constraint controls the path interval (side step). Tool paths consisting of trimmed surfaces are difficult to generate in iso-parametric method [9].

2.3.3 Iso-scallop method

Conventional path intervals are led by iso-parametric and iso-planar methods to regulate the scallop height [7,10]. Scallop height is the maximum height of uncut material measured normal to the surface. Scallop height is close to design constraint in very few points of surface while in other areas due to high surface quality, there is non-optimal time for machining. Suresh and Yang [11] describe the iso-scallop machining first time. Iso-scallop method is far better than iso-planar and iso-parametric methods which lead to constant scallop height.

Feng and Li [10] describe 3-axis machining of sculptured surface for iso-scallop generation of tool path. By having iso-scallop height, redundant machining is minimized. Hence there is reduction of path length as compared with the other two methods. Path interval is defined as the horizontal distance between two neighboring tool paths, which results in the scallop of the machined surface.

Eungki Lee [12] describes contour offset approach with iso-scallop height to spiral tool path generation. It focuses on tool path generation for high speed machining (HSM). To prepare high quality cutting path in order to accomplish machining to a high speed, which minimizes the tool retractions, controls the scallop height and make the cutting load constant. For the sculptured surface being machined, the tool path generation algorithm starts with the contour offset technique. The scallop height should not be larger than the permissible surface roughness. Side step is obtained by the offset distance. Spiral path is generated between the contour offset curves. The ability to cut continuously and to keep the iso-scallop height in high speed machining (HSM) is the main advantage of this approach.

Chen et al. [13] describe the generation of scallop mechanism with ball nosed end milling process. The feed-interval scallop and the pick-interval scallop are two kinds of scallops generated on the machined surface. In high speed machining (HSM), the feed-interval scallop must be given priority.

Agarwal et al. [14] describe genetic algorithm for optimization of NC iso-scallop sculptured surface machining. For reducing machining time while applying iso-scallop machining through application of a genetic algorithm (GA) by adjusting the position of the master cutter path (MCP) or primary cutter path.

Chen and Song [15] describe 3-axis CNC milling for iso-cusped tool paths. Iso-cusped tool paths over sculptured surfaces reducing manual grinding and can save manufacturing time by eliminating redundant machining and, while other tool path patterns cannot. So this pattern is widely accepted in industries.

Lee et al. [16] describe generation of tool path based on mesh for machining of iso-scallop-height. Mesh surface has become popular because of simple geometric computation and more robust than parametric surface. Hence it is easy to control and eliminate obstacles in the generation of tool path.

2.4 Conclusion

From the above literature review, “ball drop” tool positioning method is appropriate for triangular surfaces. For tool path planning, iso-scallop method is considered. Scallops are formed while machining which are manually removed by grinding and benchwork. To remove scallops through benchwork involves lots of time. Hence to avoid scallops during NC machining, controlled scallop height should maintained. To maintain controlled scallop height, first of all it is essential to calculate maximum scallop height. Patel et al. [18] describe the concept of scallop height calculation upto some extent but the way they drive the concept is not appropriate. So, in this present work maximum scallop height is calculated and then by using bisection method controlled scallop height tool path is maintained. The work is carried with stereo lithography (STL) surface.

Chapter 3

Methodology

Tool path generation is important aspect in NC machining. Iso-planar and iso-parametric methods are commonly used for the generation of tool path. In iso-parametric method, a parametric surface $S(u, v)$ is considered where points are generated known as C_C points along the other parameter in which one parameter is constant. However the path interval is controlled by the scallop height constraint. Uneven distribution of scallops and machining efficiency is affected by unequal path intervals between neighboring paths generated because of constant step in the parametric domain. As far the iso-planar method is concerned, it is used for triangular meshed models, compound and trimmed surfaces. To control the scallop height, iso-planar method and the iso-parametric method lead to conventional path intervals. Based on iso-scallop method, scallop heights are constant on the machined surface. For generation of iso-scallop surfaces, developments have been focused with recent methods. Surface finish quality is achieved by controlling the scallop height formed by two neighboring tool paths within a predefined machining tolerance.

In this present work, maximum scallop height is calculated and then by using bisection method controlled scallop height tool path is maintained. The work is carried with stereo lithography (STL) surface.

3.1 Stereo lithography (STL) data format

Two types of STL files that are commonly used are ASCII (American standard code for information interchange) and Binary STL files. STL files store 3D facets (3D triangles) and their normal. ASCII STL files are easy to read with most text editors. Errors can be easily detected in STL files. Computer systems can easily read ASCII STL files as compare to Binary STL files. Binary STL files are more compact and faster to read. Binary STL files are mostly used because of the smaller file size when STL files are transferred over a network. Most rapid prototyping machines that produce 3D physical models are using STL files which become the standard for Many CAD/CAM systems. Usually a 3D design program (Solidworks, Pro/Engineer, Mesh Lab) is used to create/modify/view the file. STL files are generally use because of robustness. Robustness is considered as a variation capability with the different surfaces and machines. By working with multi-patch surfaces and their continuity conditions, a robust system can be generated which can work for both parametric and tessellated models. With different topologies and parameters, a robust system should allow for selecting among a variety of tool paths. The structure of text STL format as follows

Solid Part_name

facet normal N_x N_y N_Z

outer loop

vertex x_1 y_1 z_1

vertex x_2 y_2 z_2

vertex x_3 y_3 z_3

endloop

endfacet

----

endsolid Part_name

N_x N_y N_Z are normal vectors. (x_1 y_1 z_1), (x_2 y_2 z_2) and (x_3 y_3 z_3) are coordinates of vertices.

Parametric surfaces are widely used to represent a CAD model in computer-aided design (CAD) systems. Triangles and their normal vectors are contained by an STL file. Triangular surfaces are formed by connecting the discretized points on the part surface with vectors in 3-dimensional Cartesian space. In order to improve the accuracy in approximating the desired surface, number of points should increase. Today most CAD/CAM system support STL translation because of its simplicity and use in various engineering fields. Because of advance technology, computer systems are now more powerful according to technical specification of central processing unit. Hence it is easy to handle the transfer and processing of large STL files with such high technology computer system as compare to old system where computation speed and memory allocation is the barrier. Complex models are created because of rapid growth of the reverse engineering application in which very large STL files are stored because of latest three-dimensional scanning technology. Hence it is right to say triangulated surfaces are become popular because of their simplicity in computations and need for generating tool path in present CAD/CAM system.

3.2 Tool position Method

In the “ball drop” method, a ball nose end mill is used where a ball is dropped along tool axis as a sphere. To represent cutter location free from overcutting, the point of contact between the surface and ball which is known as first point is selected. In order to generate tool path a series of ball drops at regularly pre-determined intervals are executed and stored in a file. There are three possible states for the point of contact between cutter and surface: (1) planar contact, (2) edge contact, and (3) vertex contact as shown in Fig. 3.1. Topography of the surface will decide the point of contact. Three checks are generally performed. As mentioned by Manos et al. [2], the last three checks are variations on standard closest-point computations. When the cutter is dropped onto the surface, to find the optimal distance between the tool and the surface of contact, these checks are used for calculating the best tool depth. Best tool depth can be determined by the following three stages:

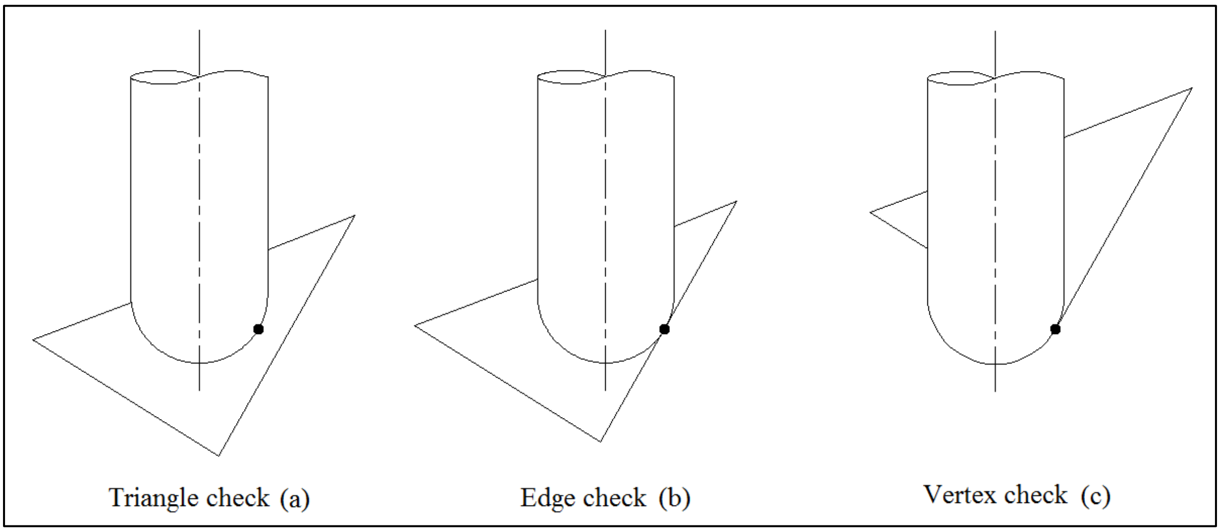

Figure 3.1: Cases for first contact with a surface. a Plane, b Edge, c Vertex

3.2.1 Triangle check (For Plane)

This is the first check to be performed to identify tool axis

$$T(u) = T_1 + u(T_2 - T_1) \quad \textbf{(3.1)}$$

having vertices P_1, P_2, and P_3 of a triangle. Since the tool contacts the triangle tangentially to the plane, the center of the tool lies along the tool axis and, to get the following equations:

$$T_1 + u(T_2 - T_1) = P_1 + (P_2 - P_1)t + (P_3 - P_1)s + \widehat{N}r \quad \textbf{(3.2)}$$

where s and t specify the barycentric coordinates and r is the radius of the tool.

Where

$$\hat{N} = \frac{(P_2 - P_1) \times (P_3 - P_1)}{|(P_2 - P_1) \times (P_3 - P_1)|}$$

In scalar form equation (3.2) can be written as:

$$\left(T_{x_2} - T_{x_1}\right)u - \left(P_{x_2} - P_{x_1}\right)t - \left(P_{x_3} - P_{x_1}\right)s = P_{x_1} - T_{x_1} + N_x r \quad \textbf{(3.3)}$$

$$\left(T_{y_2} - T_{y_1}\right)u - \left(P_{y_2} - P_{y_1}\right)t - \left(P_{y_3} - P_{y_1}\right)s = P_{y_1} - T_{y_1} + N_y r \quad \textbf{(3.4)}$$

$$\left(T_{z_2} - T_{z_1}\right)u - \left(P_{z_2} - P_{z_1}\right)t - \left(P_{z_3} - P_{z_1}\right)s = P_{z_1} - T_{z_1} + N_z r \quad \textbf{(3.5)}$$

To find u, use the formula as:

$$u = \frac{\Delta_u}{\Delta}$$

$$\Delta_u = \begin{vmatrix} P_{x_1} - T_{x_1} + N_x r & P_{x_2} - P_{x_1} & P_{x_3} - P_{x_1} \\ P_{y_1} - T_{y_1} + N_y r & P_{y_2} - P_{y_1} & P_{y_3} - P_{y_1} \\ P_{z_1} - T_{z_1} + N_z r & P_{z_2} - P_{z_1} & P_{z_3} - P_{z_1} \end{vmatrix}$$

By solving this determinant we get:

$$\left(T_{x_1} - P_{x_1} - N_x r\right)\left[P_{z_1}\left(P_{y_2} - P_{y_3}\right) + P_{z_2}\left(P_{y_3} - P_{y_1}\right) + P_{z_3}\left(P_{y_1} - P_{y_2}\right)\right] - (T_{y_1} - P_{y_1} - N_y r)\left[P_{z_1}\left(P_{x_2} - P_{x_3}\right) + P_{z_2}\left(P_{x_3} - P_{x_1}\right) + P_{z_3}\left(P_{x_1} - P_{x_2}\right)\right] + (T_{z_1} - P_{z_1} - N_z r)\left[P_{y_1}\left(P_{x_2} - P_{x_3}\right) + P_{y_2}\left(P_{x_3} - P_{x_1}\right) + P_{y_3}\left(P_{x_1} - P_{x_2}\right)\right] \quad \textbf{(A)}$$

$$\Delta = \begin{vmatrix} T_{x_2} - T_{x_1} & P_{x_2} - P_{x_1} & P_{x_3} - P_{x_1} \\ T_{y_2} - T_{y_1} & P_{y_2} - P_{y_1} & P_{y_3} - P_{y_1} \\ T_{z_2} - T_{z_1} & P_{z_2} - P_{z_1} & P_{z_3} - P_{z_1} \end{vmatrix}$$

By solving this determinant we get:

$$\left(T_{x_1} - T_{x_2}\right)\left[P_{z_1}\left(P_{y_2} - P_{y_3}\right) + P_{z_2}\left(P_{y_3} - P_{y_1}\right) + P_{z_3}\left(P_{y_1} - P_{y_2}\right)\right] - \left(T_{y_1} - T_{y_2}\right)\left[P_{z_1}\left(P_{x_2} - P_{x_3}\right) + P_{z_2}\left(P_{x_3} - P_{x_1}\right) + P_{z_3}\left(P_{x_1} - P_{x_2}\right)\right] + \left(T_{z_1} - T_{z_2}\right)\left[P_{y_1}\left(P_{x_2} - P_{x_3}\right) + P_{y_2}\left(P_{x_3} - P_{x_1}\right) + P_{y_3}\left(P_{x_1} - P_{x_2}\right)\right] \quad \textbf{(B)}$$

Now

$$u = \frac{A}{B}$$

To find t, use the formula as:

$$t = \frac{\Delta_t}{\Delta}$$

$$\Delta_t = -\begin{vmatrix} T_{x_2} - T_{x_1} & P_{x_1} - T_{x_1} + N_x r & P_{x_3} - P_{x_1} \\ T_{y_2} - T_{y_1} & P_{y_1} - T_{y_1} + N_y r & P_{y_3} - P_{y_1} \\ T_{z_2} - T_{z_1} & P_{z_1} - T_{z_1} + N_z r & P_{z_3} - P_{z_1} \end{vmatrix}$$

Expanding along C_2

$(P_{x_1} - T_{x_1} + N_x r)[(T_{y_2} - T_{y_1})(P_{z_3} - P_{z_1}) - (P_{y_3} - P_{y_1})(T_{z_2} - T_{z_1})] - (P_{y_1} - T_{y_1} + N_y r)[(T_{x_2} - T_{x_1})(P_{z_3} - P_{z_1}) - (P_{x_3} - P_{x_1})(T_{z_2} - T_{z_1})] + (P_{z_1} - T_{z_1} + N_z r[\big(T_{x_2} - T_{x_1}\big)\big(P_{y_3} - P_{y_1}\big) - (P_{x_3} - P_{x_1})(T_{y_2} - T_{y_1})]$ **(C)**

Now

$$t = \frac{C}{B}$$

To find s, use the formula as:

$$s = \frac{\Delta_s}{\Delta}$$

$$\Delta_s = -\begin{vmatrix} T_{x_2} - T_{x_1} & P_{x_2} - P_{x_1} & P_{x_1} - T_{x_1} + N_x r \\ T_{y_2} - T_{y_1} & P_{y_2} - P_{y_1} & P_{y_1} - T_{y_1} + N_y r \\ T_{z_2} - T_{z_1} & P_{z_2} - P_{z_1} & P_{z_1} - T_{z_1} + N_z r \end{vmatrix}$$

Expanding along C_3

$-(P_{x_1} - T_{x_1} + N_x r)[(T_{y_2} - T_{y_1})(P_{z_2} - P_{z_1}) - (P_{y_2} - P_{y_1})(T_{z_2} - T_{z_1})] + (P_{y_1} - T_{y_1} + N_y r)[(T_{x_2} - T_{x_1})(P_{z_2} - P_{z_1}) - (P_{x_2} - P_{x_1})(T_{z_2} - T_{z_1}) - (P_{z_1} - T_{z_1} + N_z r)[\big(T_{x_2} - T_{x_1}\big)\big(P_{y_2} - P_{y_1}\big) - (T_{y_2} - T_{y_1})(P_{x_2} - P_{x_1})]$ **(D)**

Now

$$s = \frac{D}{B}$$

3.2.2 Edge Check

This is the second check to be performed in which the equation of line is

$$T(u) = T_1 + u(T_2 - T_1)$$

will touch the edge

$$P(t) = P_1 + t(P_2 - P_1)$$

Where P_1 and P_2 are the vertices of edge acts tangentially and satisfy the equation:

$$T_1 + u(T_2 - T_1) = P_1 + t(P_2 - P_1) + \widehat{N}r \qquad \textbf{(3.6)}$$

Where:

$$\widehat{N} = \frac{(T_2 - T_1) \times (P_2 - P_1)}{|(T_2 - T_1) \times (P_2 - P_1)|}$$

In scalar form equation (3.6) can be written as:

$$\left(T_{x_2} - T_{x_1}\right)u - \left(P_{x_2} - P_{x_1}\right)t - \left(P_{x_1} - T_{x_1} + N_x r\right) = 0$$

$$\left(T_{y_2} - T_{y_1}\right)u - \left(P_{y_2} - P_{y_1}\right)t - \left(P_{y_1} - T_{y_1} + N_y r\right) = 0$$

By solving we get

$$u = \frac{\left[\left(P_{x_2} - P_{x_1}\right)\left(P_{y_1} - T_{y_1} + N_y r\right) - \left(P_{y_2} - P_{y_1}\right)\left(P_{x_1} - T_{x_1} + N_x r\right)\right]}{\left(T_{x_1} - T_{x_2}\right)\left(P_{y_2} - P_{y_1}\right) + \left(P_{x_2} - P_{x_1}\right)\left(T_{y_2} - T_{y_1}\right)}$$

$$t = \frac{\left[\left(T_{y_1} - T_{y_2}\right)\left(P_{x_1} - T_{x_1} + N_x r\right) + \left(T_{x_2} - T_{x_1}\right)\left(P_{y_1} - T_{y_1} + N_y r\right)\right]}{\left(T_{x_1} - T_{x_2}\right)\left(P_{y_2} - P_{y_1}\right) + \left(P_{x_2} - P_{x_1}\right)\left(T_{y_2} - T_{y_1}\right)}$$

3.2.3 Vertex check

This is the third check to be performed, the equation of line is

$$T(u) = T_1 + u(T_2 - T_1)$$

At vertex P_i, the tool contacts and satisfy the following equation:

$$|P_i - [(1 - u)T_1 + uT_2]| = r$$

3.3 Scallop height calculation

Scallop height is measured as the maximum height of uncut material by virtue of two feed direction of a cutter measured normal to the surface.

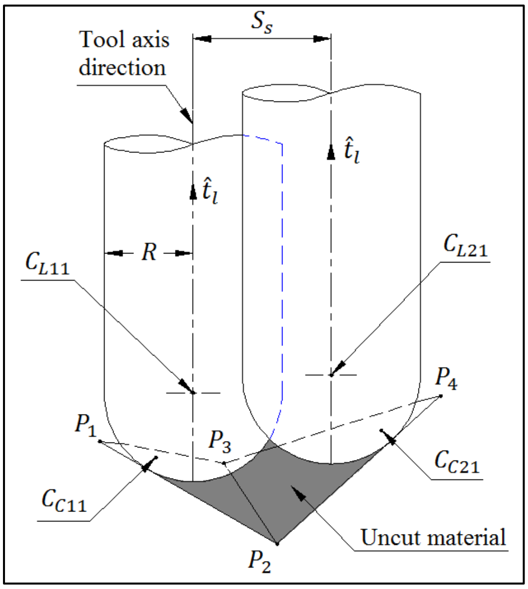

Figure 3.2: Cutter location and cutter contact points on adjacent tool paths

Points of contact in two adjacent tool paths C_{C11} and C_{C21} shown in Fig. 3.2 may not be lying on the vertical plane containing point C_{L11} and C_{L21}. The filled region of Fig. 3.2 shows the uncut material between two adjacent tool paths. Therefore a better measure for surface finish can be determined using a vertical plane through the cutter location positions C_{L11} and C_{L21} of two adjacent tool path directions separated by side step value of S_s.

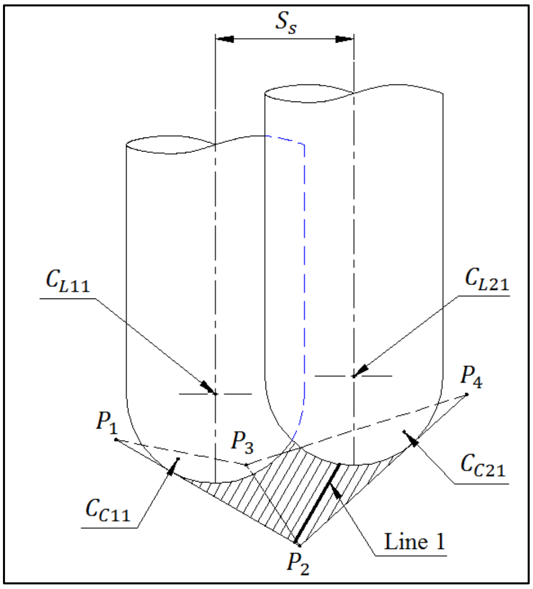

Figure 3.3: Scallop height when measured normal to triangle $P_1P_2P_3$

Figure 3.3 represents uncut region to determine scallop height. Since scallop height is measured normal to the surface, so to determine scallop height from surface normal of first triangle, the indicated longest line in Fig. 3.3 shows the scallop.

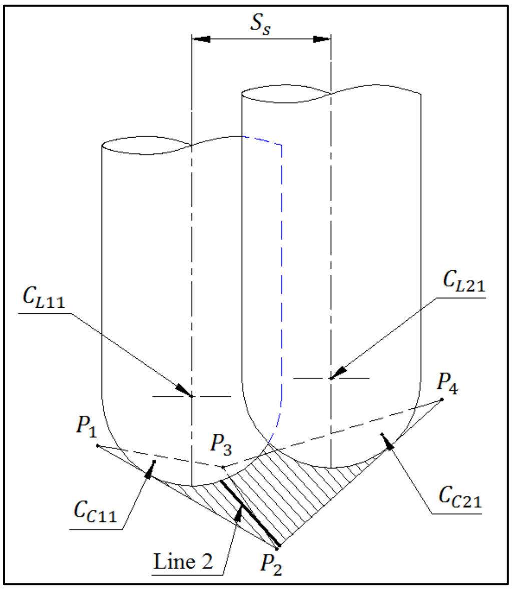

Figure 3.4: Scallop height when measured normal to triangle $P_2P_4P_3$

Neither of Line 1 (Fig. 3.3) and Line 2 (Fig. 3.4) represent the true scallop height for actual surface because the actual curved surface has been approximated with STL triangulation. The Fig. 3.5 shows the cut sectioned surface passing near STL triangles. Good approximation of scallop height between two adjacent tool passes by using the location of C_{L11} and C_{L21} as shown in Fig. 3.5.

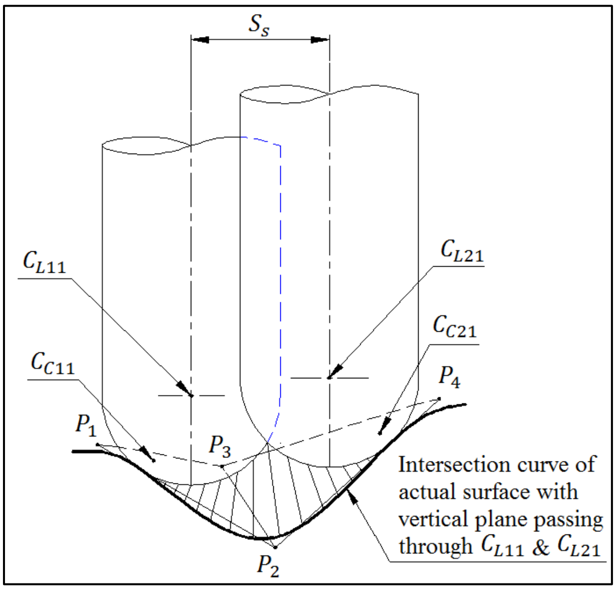

Figure 3.5: Actual surface section by plane passing through C_{L11} and C_{L21}

Figure 3.5 shows the actual surface section by plane passing through C_{L11} and C_{L21}.

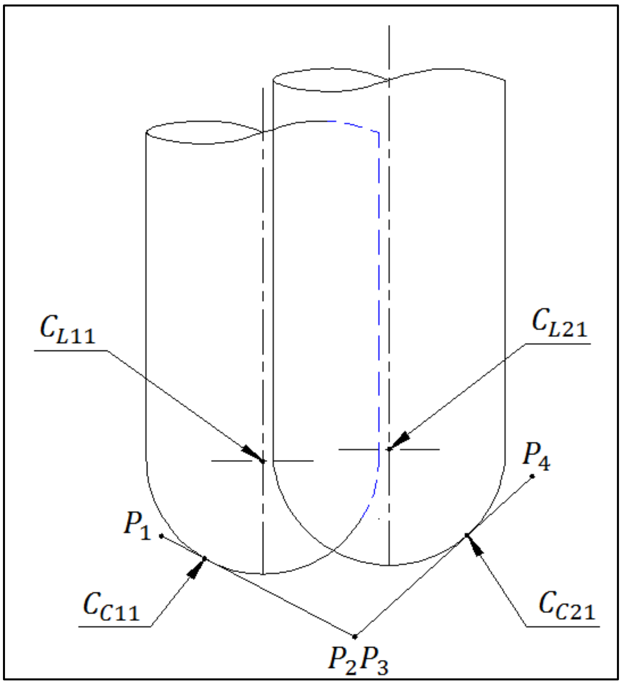

Figure 3.6: Line view of triangles

Figure 3.6 shows line view of triangles observing normal to P_2P_3 common edge (presented as point) on plane. In this side step value is changed.

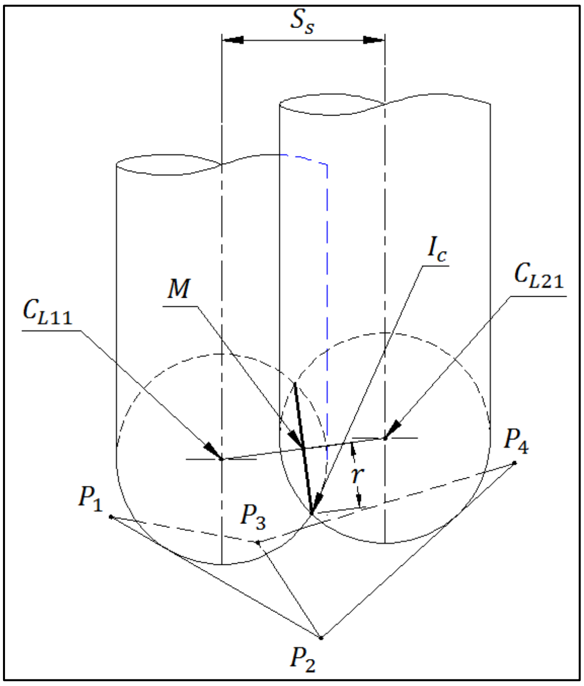

Figure 3.7: Intersection curve of spheres through C_{L11} and C_{L21}

The intersection curve of spheres through C_{L11} and C_{L21} is a circle whose radius can be determined as:

$$r = \sqrt{R^2 - l^2}$$

Where, $l = |C_{L11} - M|$

To determine point I_c, which can be determined as given below:

Vertical plane through C_{L11} and C_{L21} is normal to feed direction. M is mid point of C_{L11} and C_{L21}. Now make a line through C_{L11} and C_{L21}. Further determine the intersection of two projected circles, representing ball end mill projected on plane through C_{L11} and C_{L21} normal to feed forward direction as shown by I_c in Fig. 3.7.

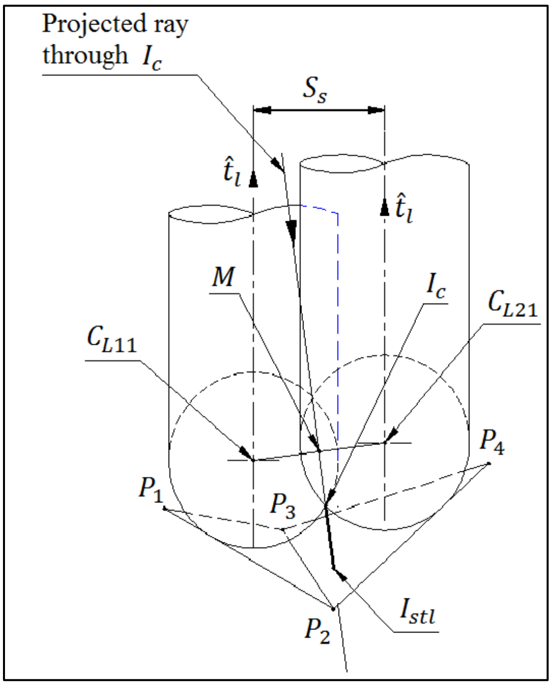

Figure 3.8: Projection of ray through I_c

Figure 3.8 shows projection of ray through I_c to meet I_{stl} on STL surface on one of the triangle $P_2P_4P_3$.

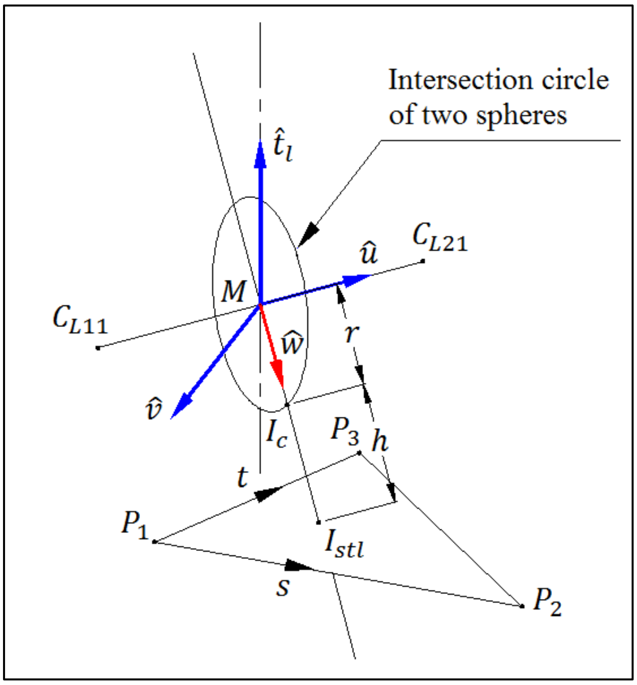

Figure 3.9: Intersection circle of two spheres with their centers S_s distance apart

I_c can be determined as:

$$I_c = M + \hat{w}.r$$

Where

$$\hat{u} = \frac{M - C_{L11}}{|M - C_{L11}|}$$

$$\hat{t}_l = \{0,0,1\}$$

$$\hat{v} = \frac{\hat{u} \times \hat{t}_l}{|\hat{u} \times \hat{t}_l|}$$

$$\hat{w} = \frac{\hat{u} \times \hat{v}}{|\hat{u} \times \hat{v}|}$$

Now drop a ray through point I_c in the direction of $\hat{w}$ on STL surface and determine the intersection point with STL surface (I_{stl}) as shown in Fig. 3.9. Points I_c, M, C_{L11}, C_{L21} are on the same plane.

$|I_c - I_{stl}|$= Magnitude of uncut material in two passes of tool through C_{L11} and C_{L21} which are S_s distance apart.

To determine I_{stl}, drop a ray through point I_c in the direction of $\hat{w}$ on STL surface.

Now

$$I_{stl} = P_1 + s(P_2 - P_1) + t(P_3 - P_1)$$

Where

$$0 \le (s, t, s + t) \le 1$$

Also, $I_c + h.\hat{w} = I_{stl}$

Therefore

$$I_c + h.\hat{w} = P_1 + s(P_2 - P_1) + t(P_3 - P_1)$$

or $h.\hat{w} + s(P_1 - P_2) + t(P_1 - P_3) = P_1 - I_c$

$$h.\hat{w} + s.A + t.B = C$$

where $A = (P_1 - P_2), B = (P_1 - P_3), C = P_1 - I_c$

or

$$\begin{bmatrix} \hat{w}_x & A_x & B_x \\ \hat{w}_y & A_y & B_y \\ \hat{w}_z & A_z & B_z \end{bmatrix} \begin{bmatrix} h \\ s \\ t \end{bmatrix} = \begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix}$$

or

$$\begin{bmatrix} h \\ s \\ t \end{bmatrix} = \begin{bmatrix} \hat{w}_x & A_x & B_x \\ \hat{w}_y & A_y & B_y \\ \hat{w}_z & A_z & B_z \end{bmatrix}^{-1} \begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix}$$

Check if s, t are satisfying the conditions that

$$0 \le (s, t, s + t) \le 1$$

Then I_{stl} lies inside the triangle else, it is not. If above condition is true then h can be taken as the solution.

Comparing the intersection of ray with all triangles, for solution, the highest h is the maximum scallop height.

3.4 Constant scallop height tool path

For constant scallop height tool path minimum path shift is required. There are different path topologies required for tool path planning. Path topology and the methods of tool path planning affect the machining time. Among all the tool path topologies, raster tool path planning is effectively used. For iso-scallop machining of raster tool path planning frequent feed rate shift is not required. Raster tool path planning is shown in Fig. 3.10.

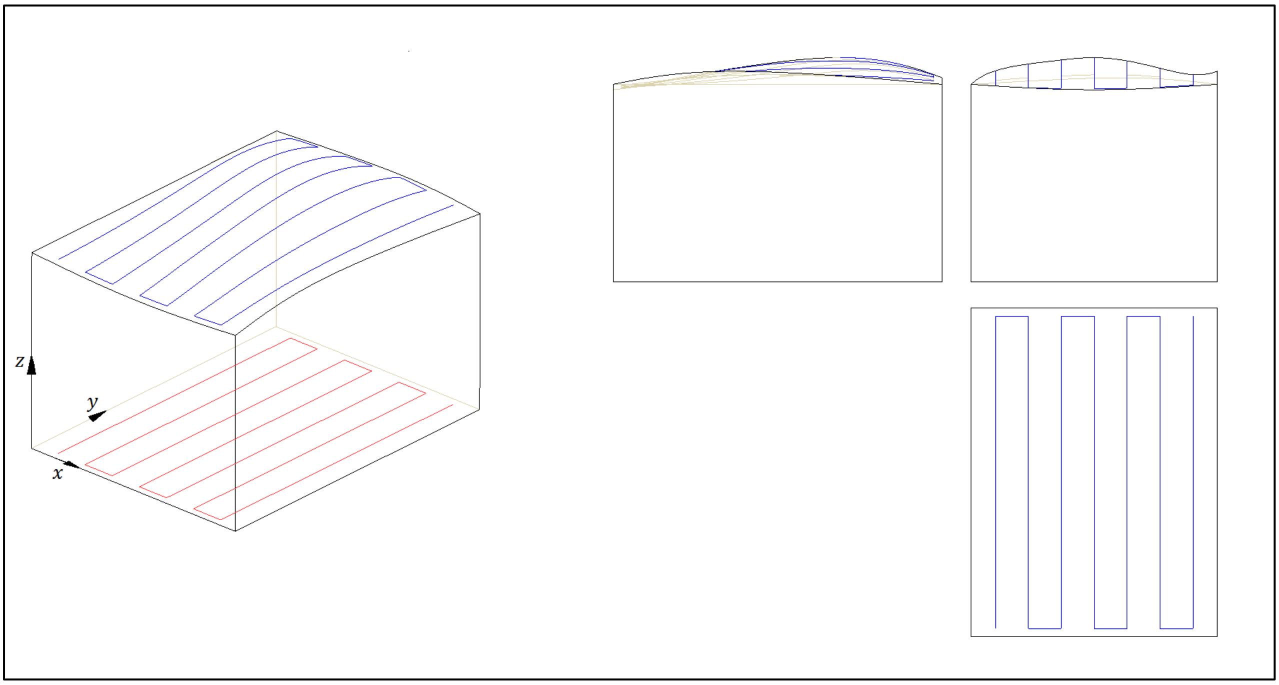

Figure 3.10: Isometric and Orthographic views of Iso-planar tool path lines and machined surface

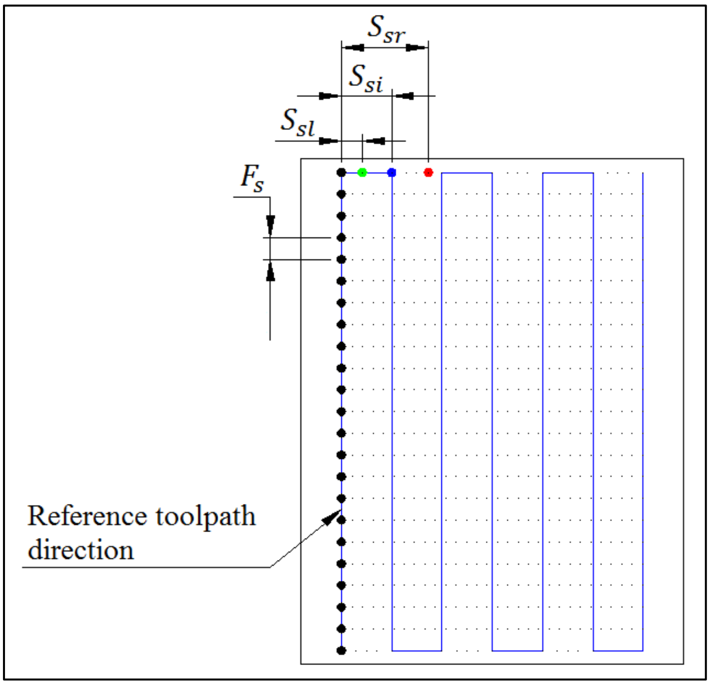

Figure 3.11: Iso-scallop tool path positioning

The iso-scallop tool path positioning shown in Fig. 3.11 in which first line shows reference tool path and in second line calculation of tool path foot print to determine z for which x and y are known.

3.5 Bisection Method:

Forward step (F_s) is shown on reference tool path in Fig. 3.11. S_{sl}, S_{si} and S_{sr} are side step left, side step initial and side step right values respectively. First consider the Bisection (Binary search) Method which is based on the Intermediate Value Theorem (IVT). Suppose a continuous function f, defined on $[S_{sl}, S_{sr}]$ is given with f (S_{sl}) and f (S_{sr}) of opposite sign. By the IVT, there exists a point $S_{si} \in (S_{sl}, S_{sr})$ for which f (S_{si}) = 0. It will be assumed that the root in this interval is unique. Hence

$$S_{si} = (S_{sl} + S_{sr})/2$$

Continue with this process so as to achieve the constant scallop height tool path as shown in Fig. 3.12.

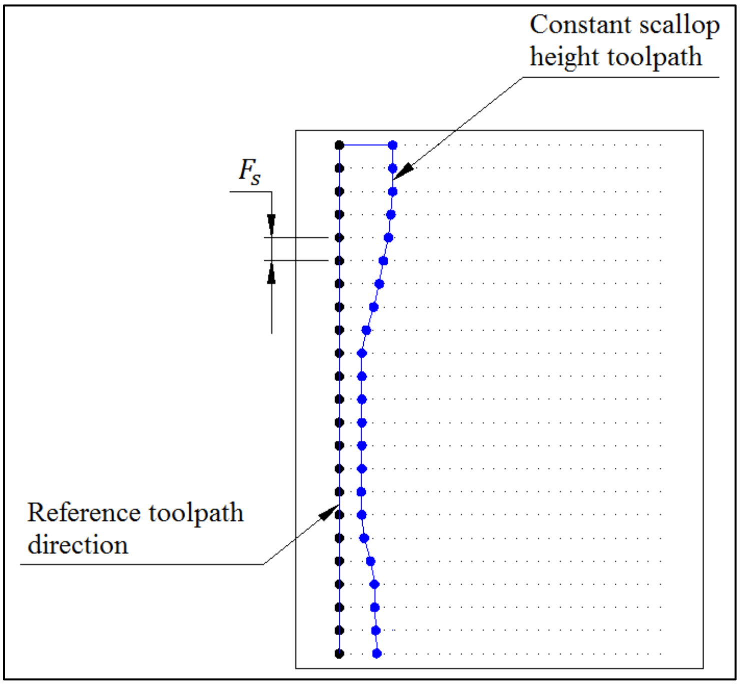

Figure 3.12: Constant scallop height tool path

If ϵ is user defined scallop value and $C_{L(ij)}$ is cutter location of i^{th} tool path taken along x-axis direction for j^{th} feed forward value along y-axis direction. The $C_{L(ij)}$ tool location has been taken as reference to adjust the $C_{L(i+1,j)}$ by shifting the x -axis increment (side step increment) so that the scallop height h_{ij} obtained between $C_{L(i,j)}$ and $C_{L(i+1,j)}$ tool positions can be made equal to user defined tolerance (ϵ). If h_{ij} is equal to user defined tolerance (ϵ), then it is ok otherwise iterate S_{si}.

3.6 Flow charts

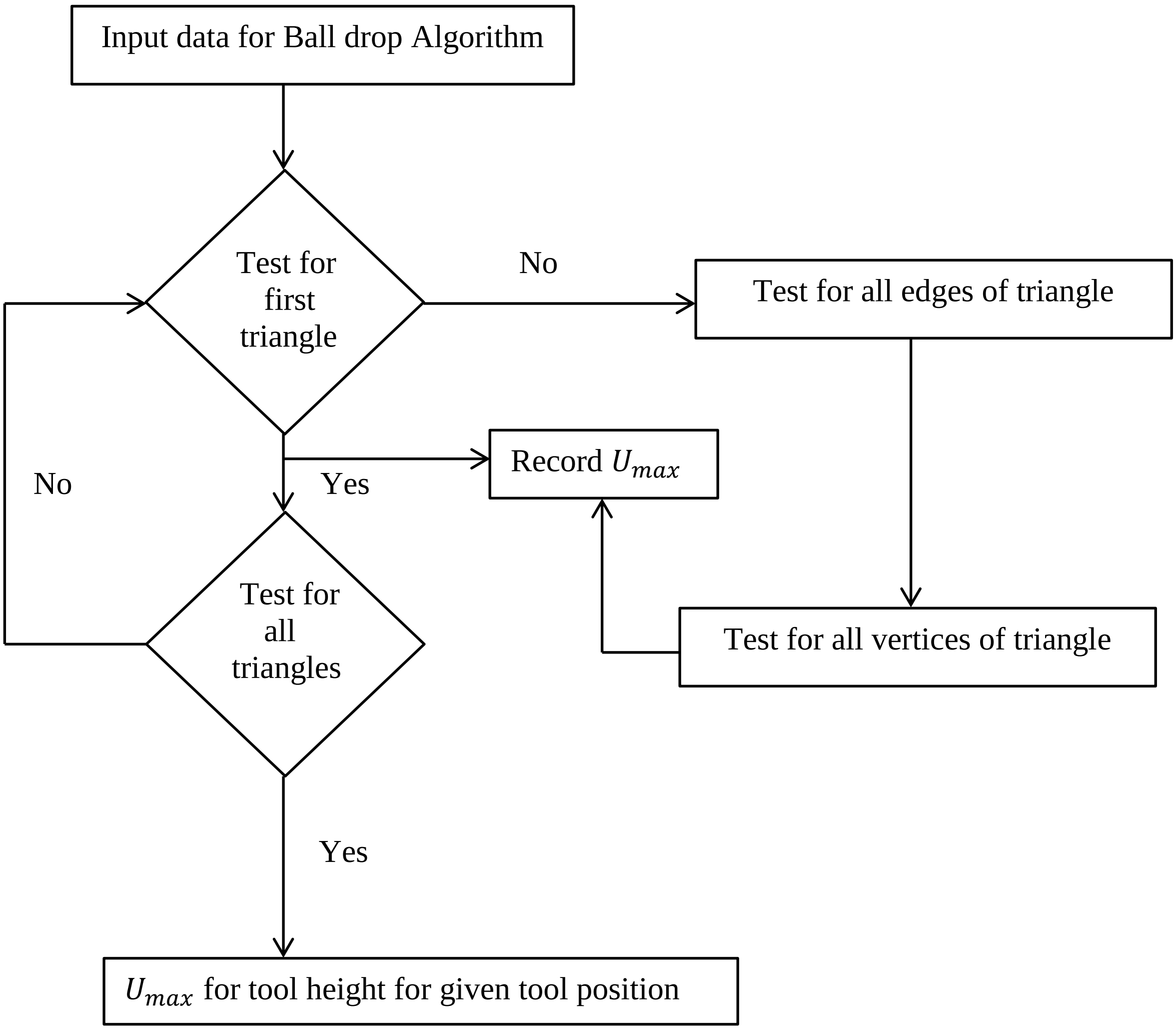

Figure 3.13: Flow chart of ball drop method (Module 1)

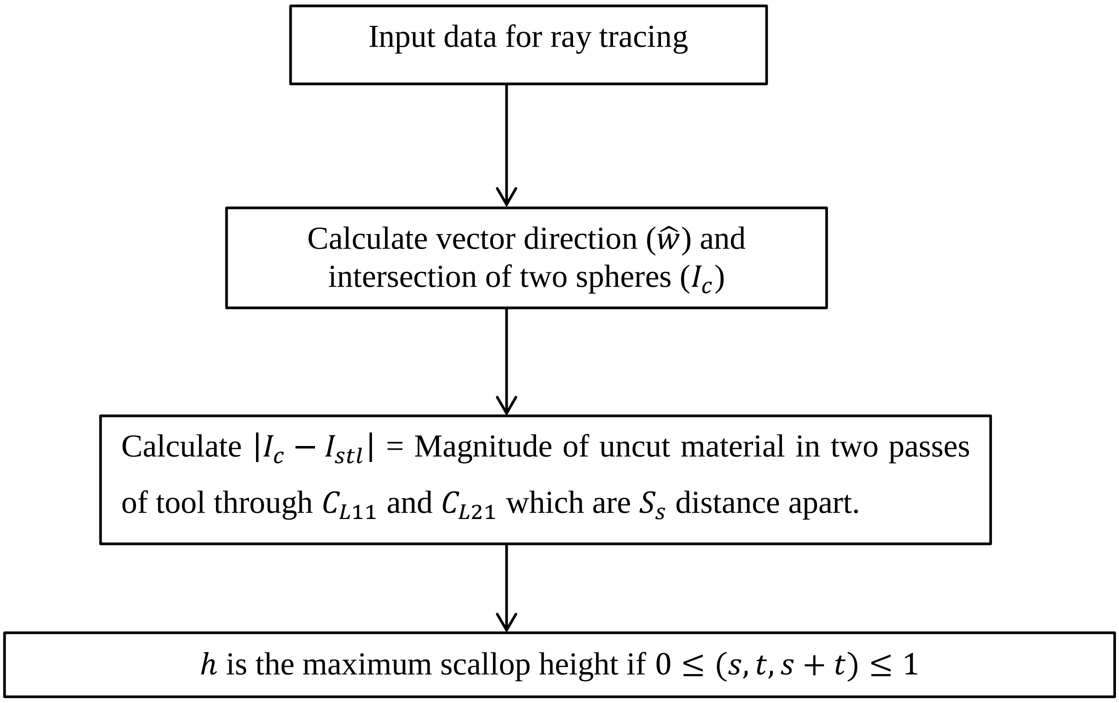

Figure 3.14: Flow chart of ray tracing (Module 2)

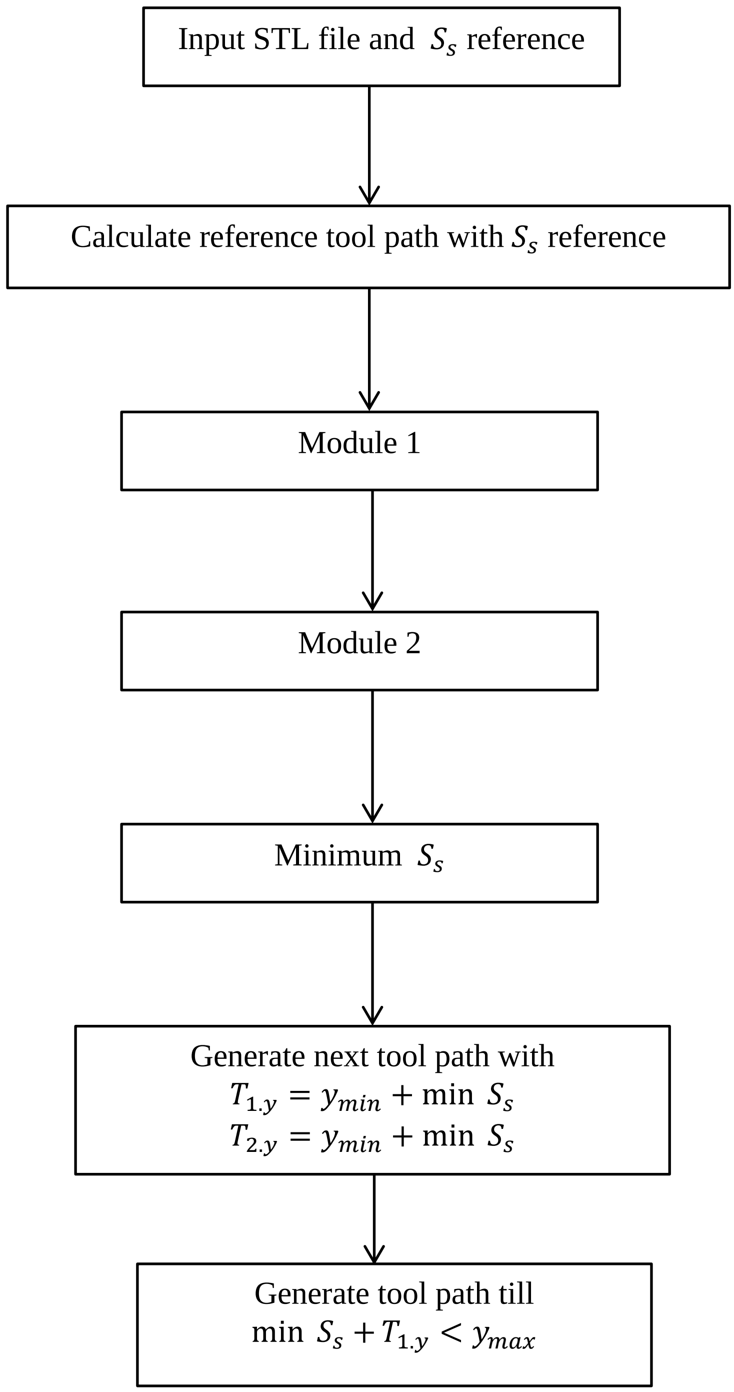

Figure 3.15: Overall flowchart

Chapter 4

Results and Discussions

It is difficult to maintain the constant scallop height. So in this present work scallop height is controlled by taking minimum side step value. Also minimum scallop height required from the machined surface cannot be less than the surface tolerance used to generate the input STL surface. ϵ is the user defined scallop height value. While CNC machining, if ϵ decreases then number of passes increases.

4.1 Bezier test part

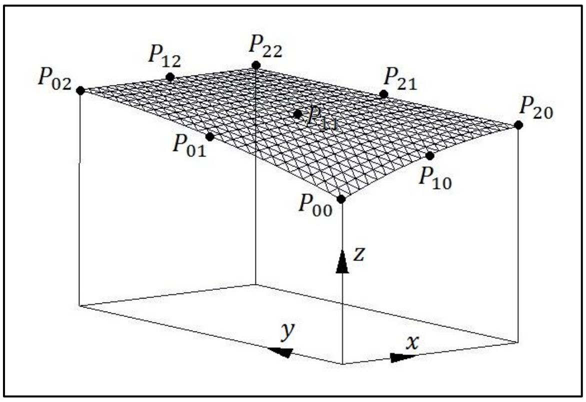

Figure 4.1: Bezier test part $25 \times 50 \times 25$ for $UV = 0.05$

Figure 4.1 shows the test part which is being taken for analyzing the scallop height variation while tool path planning. The size of the part is $25 \times 50 \times 25$. Coordinates of points are shown in table 4.1.

Table 4.1: Coordinates of Bezier test part

Points	P_{00}	P_{01}	P_{02}	P_{10}	P_{11}	P_{12}	P_{20}	P_{21}	P_{22}
x-Coordinates	0.0	0.0	0.0	12.5	12.5	12.5	25.0	25.0	25.0
y-Coordinates	0.0	25.0	50.0	0.0	24.0	50.0	0.0	25.0	50.0
z-Coordinates	19.0	24.0	25.0	24.0	24.5	25.0	25.0	25.0	25.0

4.1.1 $x - y$ plots of Bezier test part

By executing the C++ program, TP file is generated which is then used for $x - y$ plots as shown in the following figures.

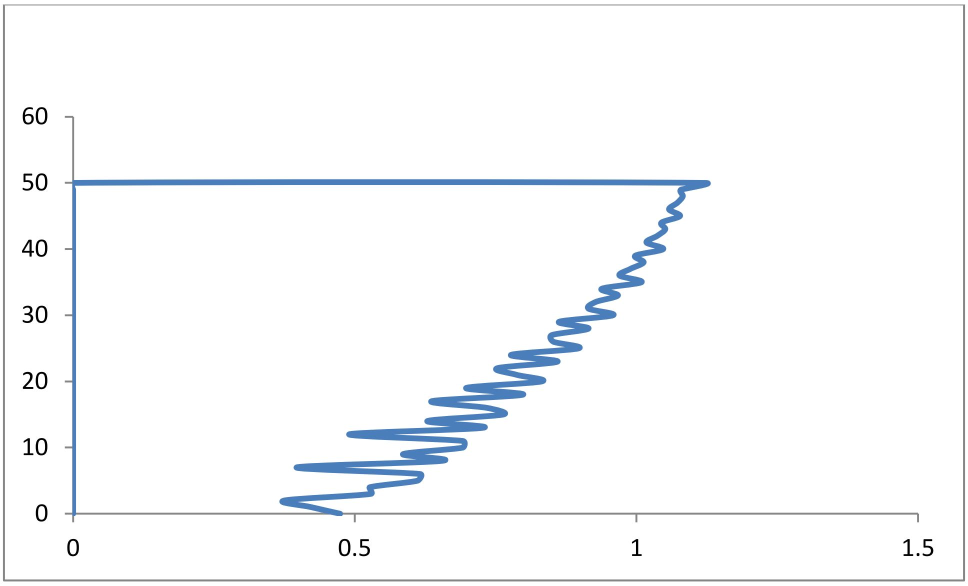

Figure 4.2: Single pass tool path variation for $\epsilon = 0.05$

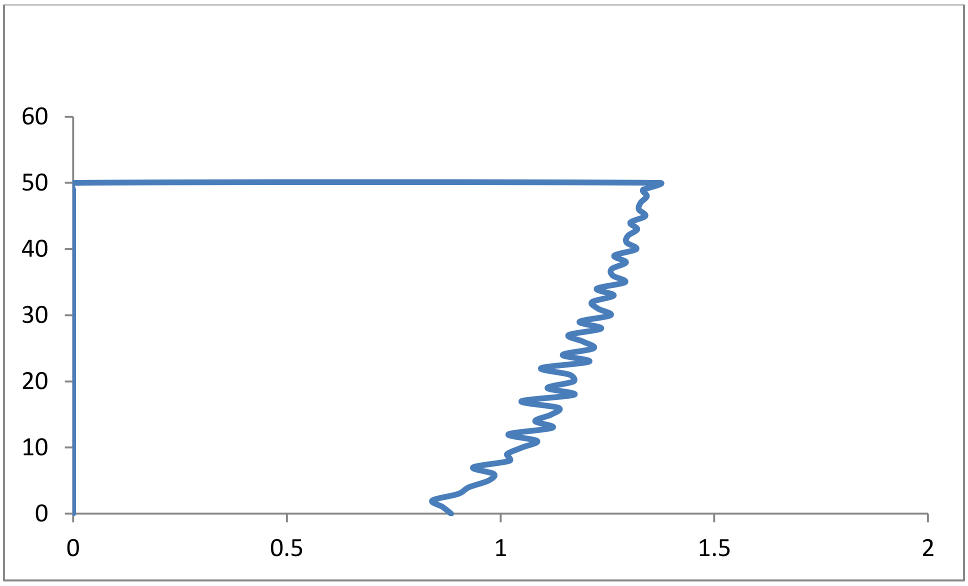

Figure 4.3: Single pass tool path variation for $\epsilon = 0.075$

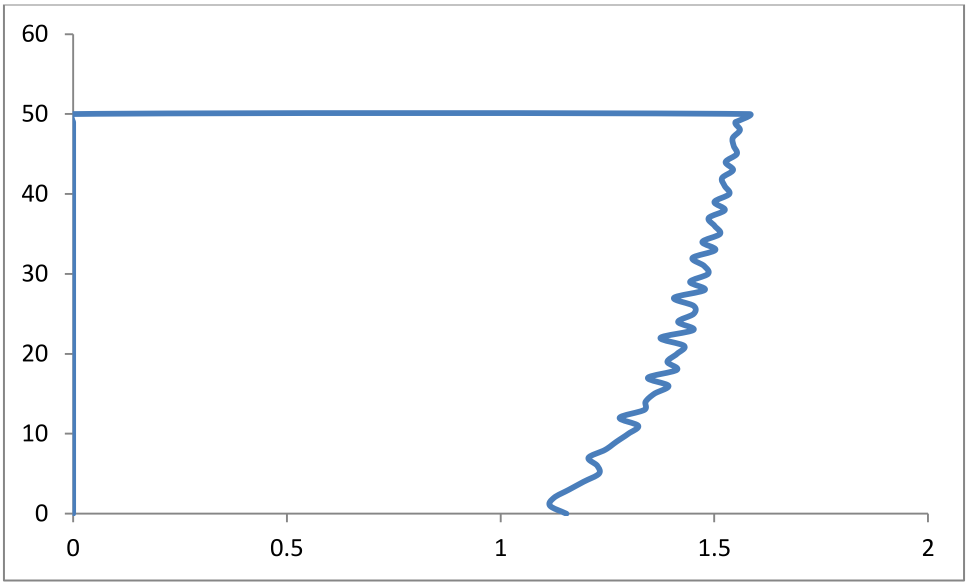

Figure 4.4: Single pass tool path variation for $\epsilon = 0.1$

Figures 4.2, 4.3 and 4.4 show single pass tool path variation for 0.05, 0.075 and 0.1values of ϵ. The haphazard tool path generated moves away in x-axis as the values of ϵ increases which directly interrupt the feed or forward step. Due to very less forward step, it is difficult to maintain constant machining which adversely increase machining time.

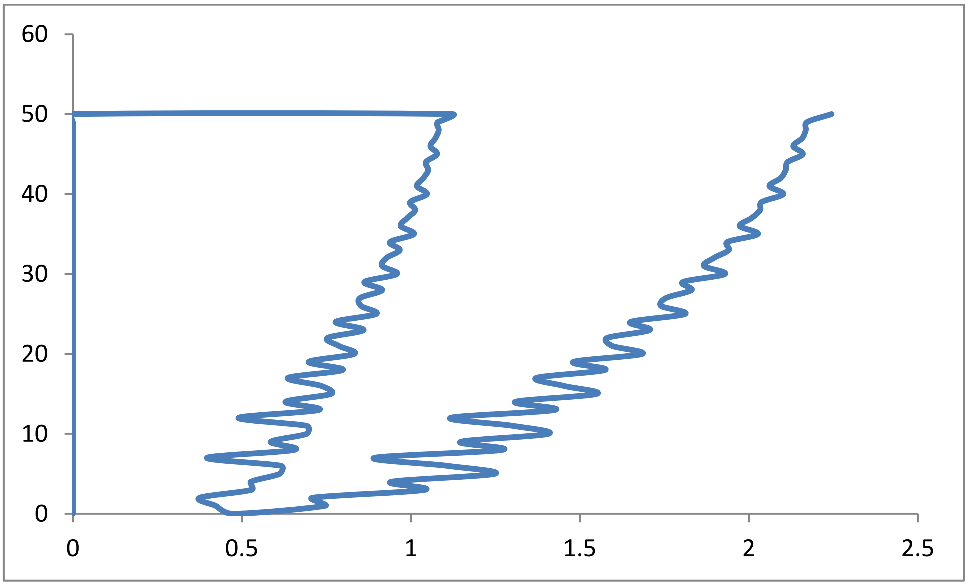

Figure 4.5: Two passes tool path variation for $\epsilon = 0.05$

Figure 4.5 shows two passes tool path variation which generates more haphazard tool path and away from first pass which directly interrupts the machining.

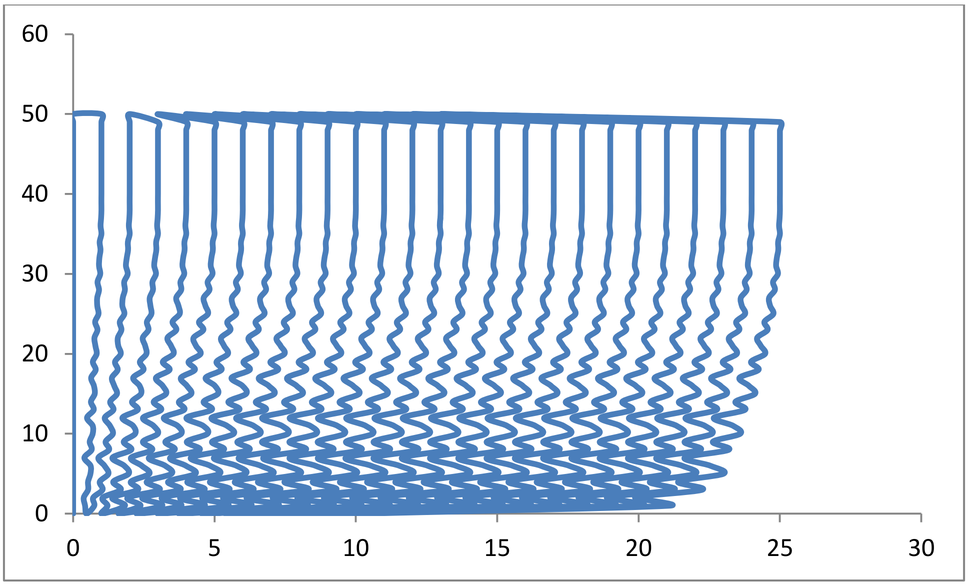

Figure 4.6: Complete tool path variation for $\epsilon = 0.05$

Figure 4.6 shows complete tool path variation while maintaining the scallop height. Haphazard tool path is formed and every next path is away from previous one in x-axis and in y-axis when tool moves from top to bottom, tool path becomes messier.

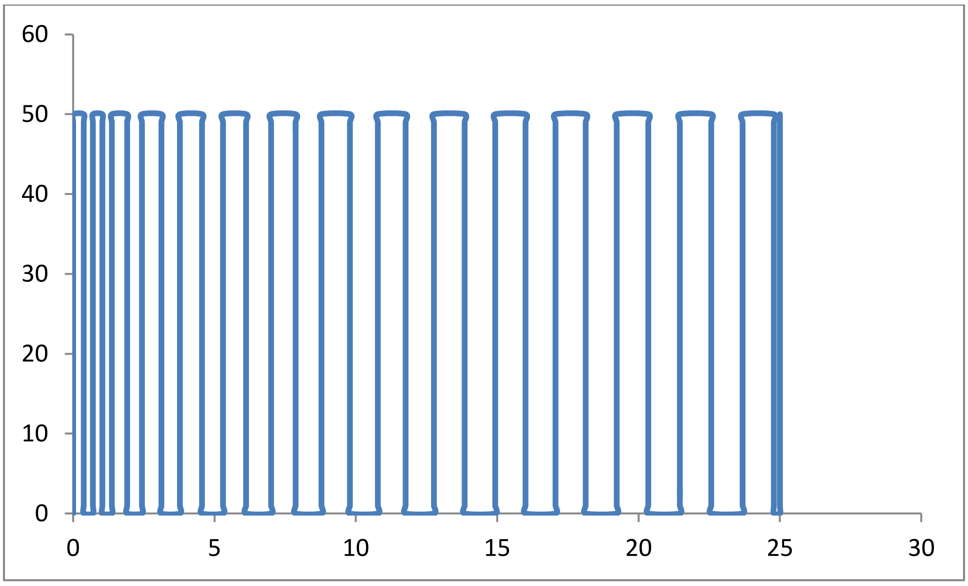

Figure 4.7: Controlled scallop height tool path for $\epsilon = 0.05$

Figure 4.7 shows controlled scallop height tool path. It is controlled by taking minimum side step value (S_s) for every next tool path and regenerate the tool path with this value. It is clear from Fig. 4.7 tool path is narrow at the start and also at the end.

4.1.2 Controlled scallop height plots

Obj files are generated while executing the file in simulator. For controlling the scallop height ray tracing is done. Z-buffer technique is used to capture the data. Tool path is generated in simulator available in SIDC lab, Thapar University Patiala.

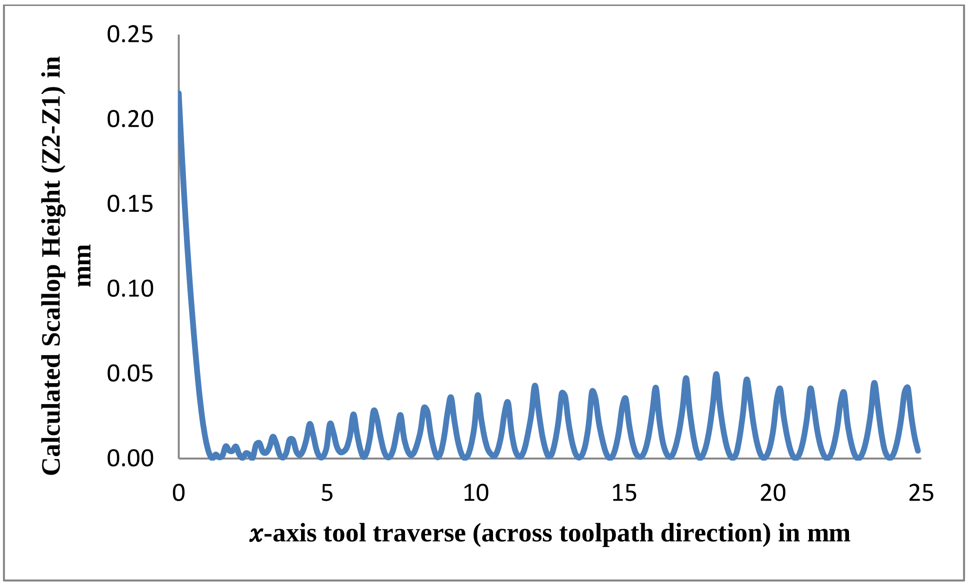

Figure 4.8: Controlled scallop height at y = 1.0158 mm for $\epsilon = 0.05$

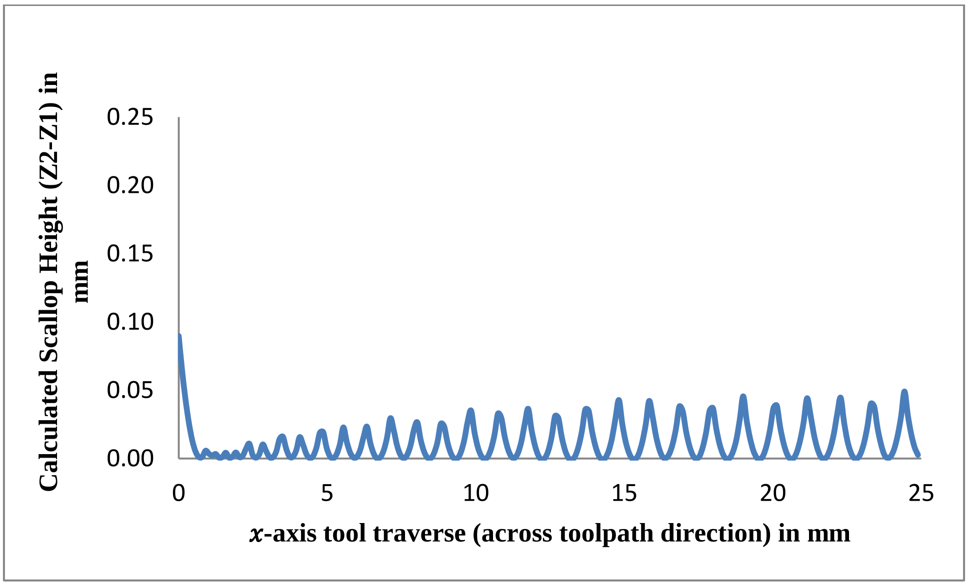

Figure 4.9: Controlled scallop height at y = 12.3025 mm for $\epsilon = 0.05$

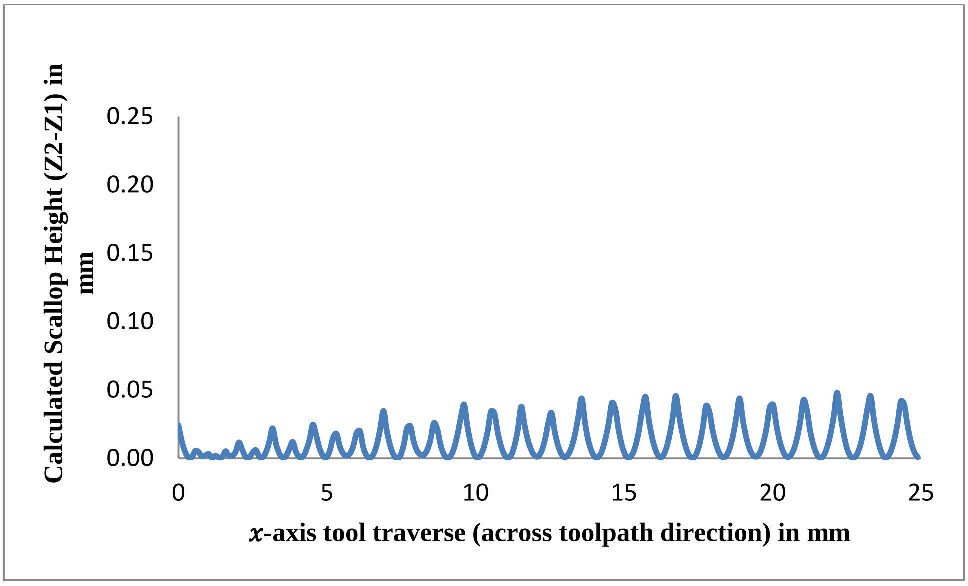

Figure 4.10: Controlled scallop height at y = 23.5891 mm for $\epsilon = 0.05$

Figures 4.8, 4.9 and 4.10 show the controlled scallop height for different values of y. ϵ is equal to 0.05. The x-axis and y- axis coordinates denote tool traverse and calculate scallop height $(z_2 - z_1)$ in mm respectively. It is clear that scallop height is controlled as it varies smoothly.

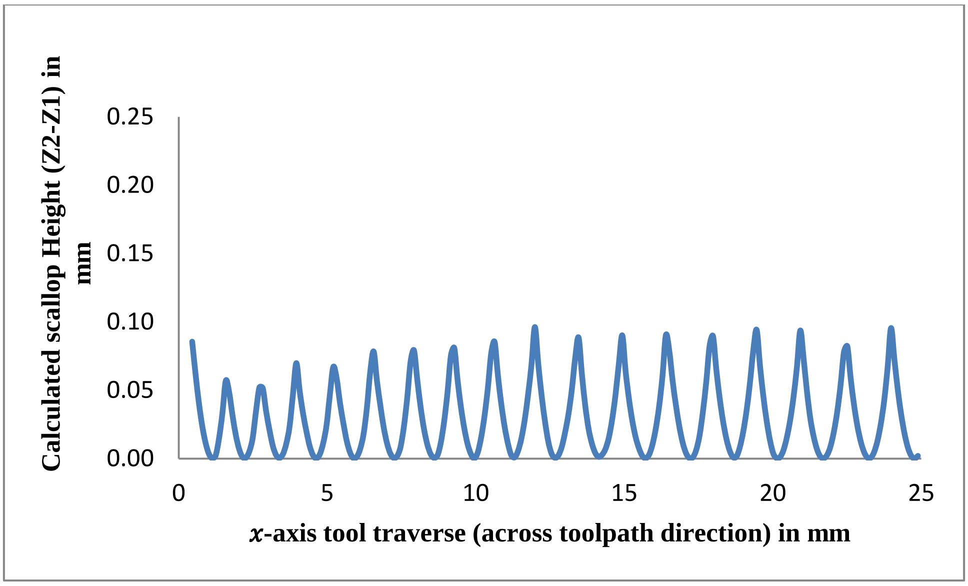

Figure 4.11: Controlled scallop height at y = 1.0158 mm for $\epsilon = 0.1$

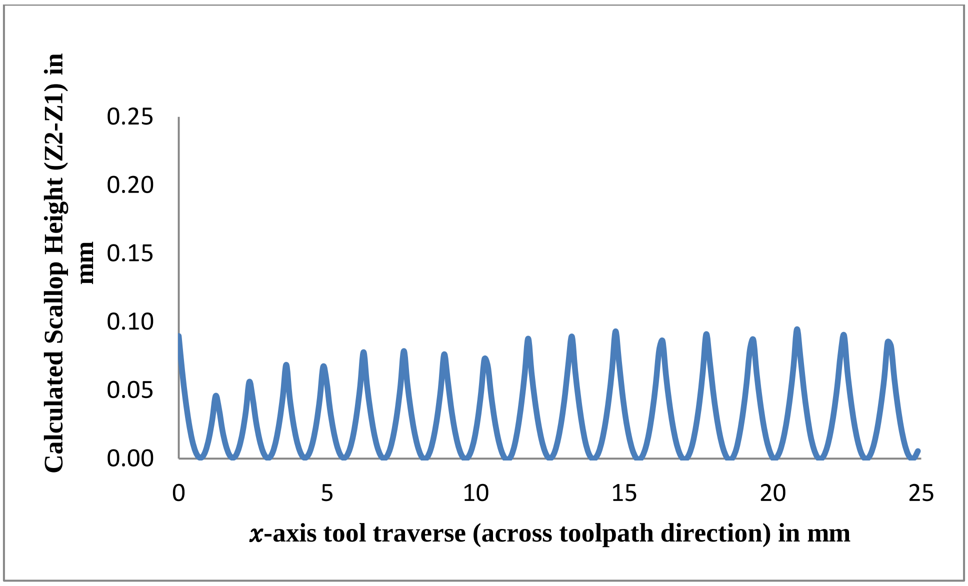

Figure 4.12: Controlled scallop height at y = 12.3025 mm for $\epsilon = 0.1$

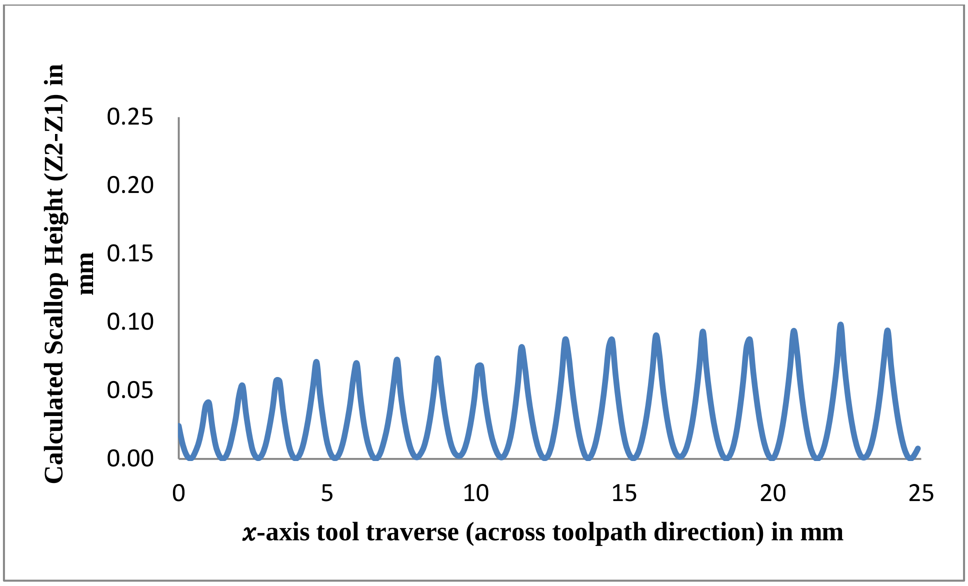

Figure 4.13: Controlled scallop height at y = 23.5891 mm for $\epsilon = 0.1$

Figures 4.11, 4.12 and 4.13 show the controlled scallop height for different values of y. ϵ is equal to 0.1. The x-axis and y- axis coordinates denote tool traverse and calculate scallop height $(z_2 - z_1)$ in mm respectively. It is clear that scallop height is controlled as it varies smoothly.

Table 4.2: Data for test part for different ϵ

ϵ (mm)	Time for program execution (seconds)	Number of tool passes	Number of tool positions
0.1	4.828	19	969
0.09	4.466	21	1071
0.08	4.713	21	1071
0.07	6.162	23	1173
0.06	7.057	27	1377
0.05	8.025	31	1581

Time for program execution, number of tool passes and number of tool positions for different ϵ is shown in table 4.2. Surface formed with a discretization along x and y direction with $\Delta x = 2.5$ and $\Delta y = 2.5$. Total numbers of facets are 443. Tool path with F_s and S_s having value of 1.0mm generated in 0.603 seconds with 1404 tool positions having 28 passes.

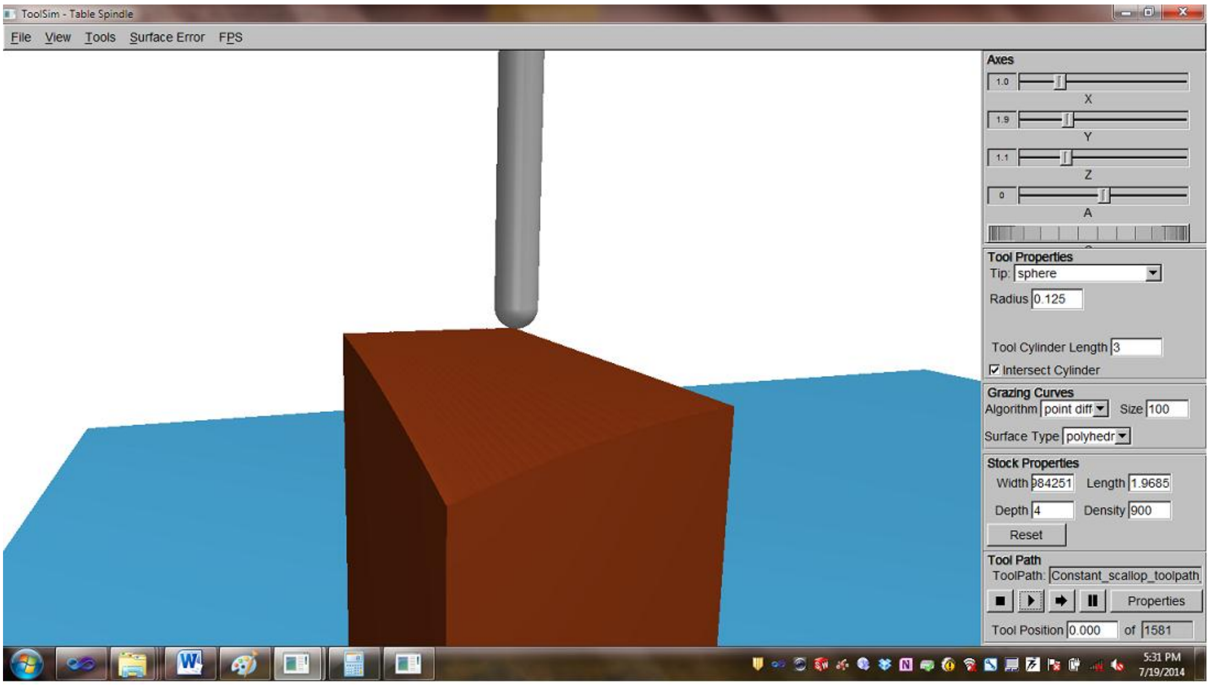

Figure 4.14: Test part in toolsim simulator

Figure 4.14 shows test part in toolsim simulator. The test part discussed in section 4.1 is simulate in toolsim simulator. Simulator is available in SIDC Lab, Mechanical Engineering Department, Thapar University, Patiala.

4.2 Concave and convex Bezier surfaces

In order to compare results with other parts, concave and convex Bezier parts are taken for analysis of scallop height during tool path planning.

Concave Bezier surface

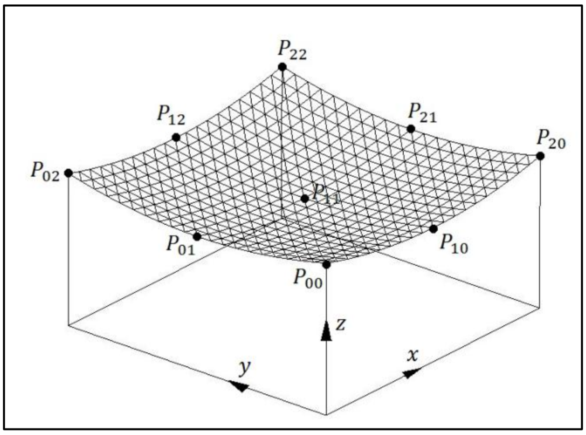

Figure 4.15: Concave Bezier surface $50 \times 50 \times 25$ with $UV = 0.05$

Figure 4.15 and 4.16 shows the concave surface which is being taken for analyzing the scallop height variation while tool path planning. The size of the part is $50 \times 50 \times 25$. Coordinates of points are shown in table 4.3. Different discretization has been taken to compare the results of tool path variations.

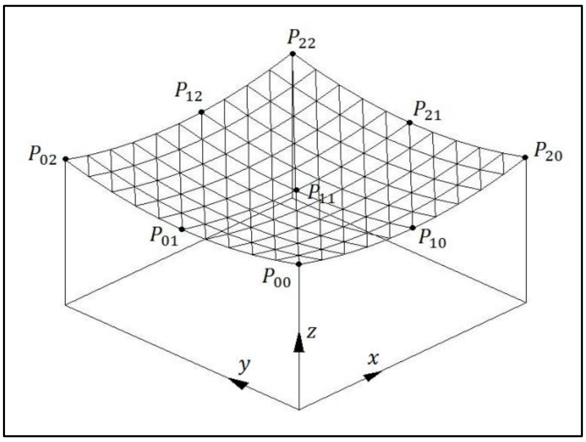

Figure 4.16: Concave Bezier surface $50 \times 50 \times 25$ with $UV = 0.1$

Table 4.3: Coordinates of concave Bezier surface

Points	P_{00}	P_{01}	P_{02}	P_{10}	P_{11}	P_{12}	P_{20}	P_{21}	P_{22}
x-Coordinates	0.0	0.0	0.0	25.0	25.0	25.0	50.0	50.0	50.0
y-Coordinates	0.0	25.0	50.0	0.0	25.0	50.0	0.0	25.0	50.0
z-Coordinates	25.0	19.0	25.0	19.0	15.0	19.0	25.0	19.0	25.0

Concave Bezier surface $\mathbf{50 \times 50 \times 25}$ for $\boldsymbol{UV = 0.05}$

Time for program execution, number of tool passes and number of tool positions for different ϵ for concave part with 0.05 discretization are shown in table 4.4. Surface formed with a discretization along x and y direction with $\Delta x = 2.5$ and $\Delta y = 2.5$. Total number of facets equals to 886. The radius of tool used is 3.175 mm.

Table 4.4: Data for concave surface for different ϵ for $\boldsymbol{UV = 0.05}$

ϵ (mm)	Time for program execution (seconds)	Number of tool passes	Number of tool positions
0.1	9.011	57	2907
0.09	10.536	67	3417
0.08	14.6	93	4743

Concave Bezier surface $\mathbf{50 \times 50 \times 25}$ with $\boldsymbol{UV = 0.1}$

Surface formed with a discretization along x and y direction with $\Delta x = 5$ and $\Delta y = 5$. Total number of facets = 246. Data is shown in table 4.5.

Table 4.5: Data for concave surface for different ϵ for $\boldsymbol{UV = 0.1}$

ϵ (mm)	Time for program execution (seconds)	Number of tool passes	Number of tool positions
0.1	2.969	53	2703
0.09	3.374	61	3111
0.08	4.16	75	3825

In both the cases of concave surface, for $\epsilon < 0.08$, methodology doesn't work because minimum side step value is becoming very small. Program generates lot of tool path data and number of machine passes.

Convex Bezier surface

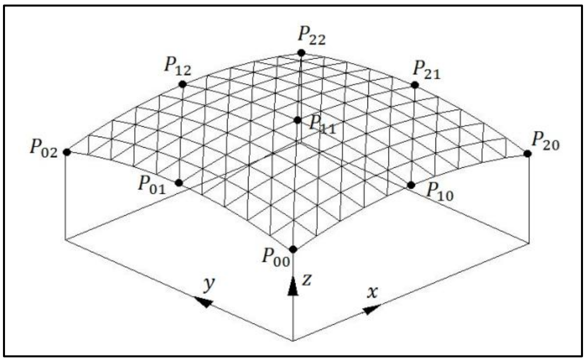

Figure 4.17: Convex Bezier surface $50 \times 50 \times 25$ with $UV = 0.05$

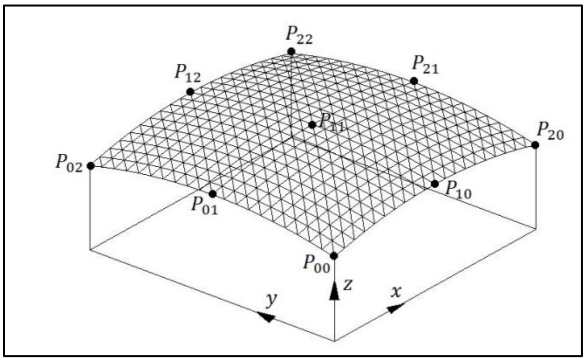

Figure 4.18: Convex Bezier surface $50 \times 50 \times 25$ with $UV = 0.1$

Figure 4.17 and 4.18 shows the convex surface having size of $50 \times 50 \times 25$. Table 4.6 shows the coordinates of points of convex Bezier surface.

Table 4.6: Coordinates of convex Bezier surface

Points	P_{00}	P_{01}	P_{02}	P_{10}	P_{11}	P_{12}	P_{20}	P_{21}	P_{22}
x-Coordinates	0.0	0.0	0.0	25.0	25.0	25.0	50.0	50.0	50.0
y-Coordinates	0.0	25.0	50.0	0.0	25.0	50.0	0.0	25.0	50.0
z-Coordinates	15.0	21.0	15.0	21.0	25.0	21.0	15.0	21.0	15.0

Convex Bezier surface $50 \times 50 \times 25$ for $UV = 0.05$

Time for program execution, number of tool passes and number of tool positions for different ϵ for convex part with 0.05 discretization are shown in table 4.7. Surface formed with a discretization along x and y direction with $\Delta x = 2.5$ and $\Delta y = 2.5$. Total number of facets equals to 886. The radius of tool used is 3.175 mm.

Table 4.7: Data for convex surface for different ϵ for $UV = 0.05$

ϵ (mm)	Time for program execution (seconds)	Number of tool passes	Number of tool positions
0.1	9.468	55	2805
0.09	10.672	65	3315
0.08	16.517	87	4437

Convex Bezier surface $50 \times 50 \times 25$ with $UV = 0.1$

Surface formed with a discretization along x and y direction with $\Delta x = 5$ and $\Delta y = 5$. Total number of facets = 246. Data is shown in table 4.8.

Table 4.8: Data for convex surface for different ϵ for $UV = 0.1$

ϵ (mm)	Time for program execution (seconds)	Number of tool passes	Number of tool positions
0.1	3.437	51	2601
0.09	3.904	59	3009
0.08	4.137	73	3723

In both the cases of convex surface, for $\epsilon < 0.08$, methodology doesn't work because minimum side step value is becoming very small. Program generates lot of tool path data and number of machine passes. However by reducing the radius of tool, ϵ will be less than 0.08 which depends on curvature of surface.

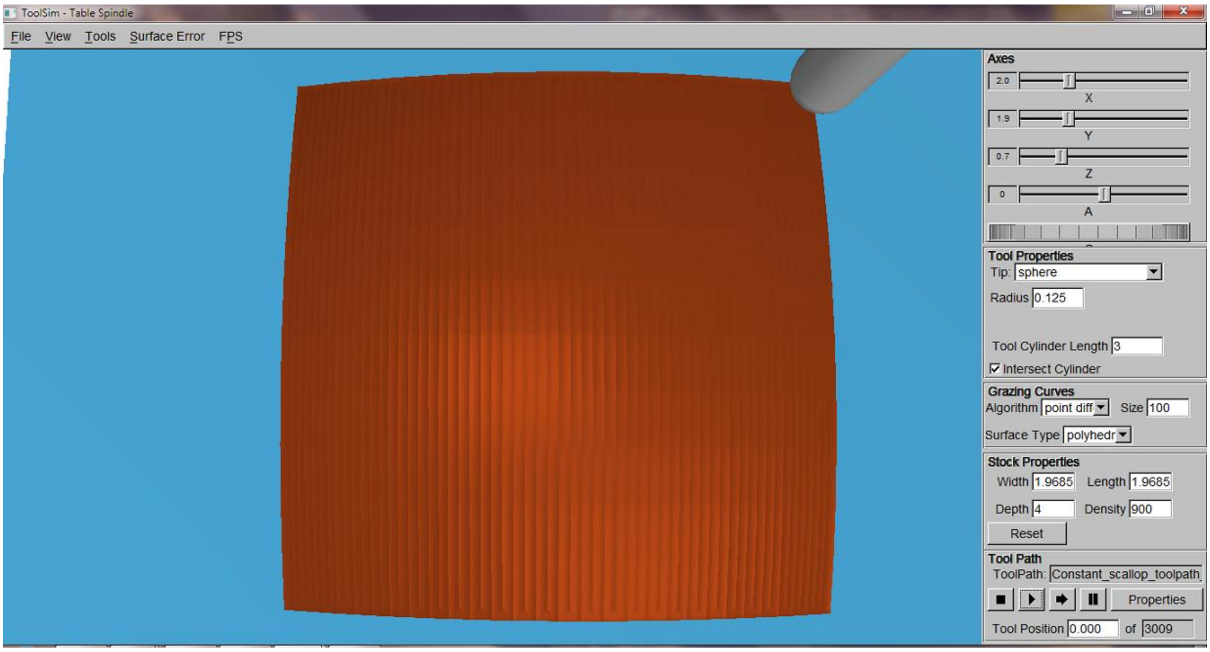

Figure 4.19: Convex Bezier surface in toolsim simulator with $UV = 0.1$

Figure 4.19 shows the convex Bezier surface in toolsim simulator which shows clearly the generation of controlled scallop height tool path. The value of ϵ taken here is 0.09 and discretization of 0.1. The data density of 900 is shown here.

4.3 Conclusion and Scope for further study

While tool path positioning and planning, it is very important to control the scallop height. It is very difficult to keep the scallop height constant. Based on above methodology first calculate the maximum scallop height and then control the scallop height by using "Bisection method" with ball end mill. The tool path haphazardly produced results in very less feed or forward step (F_s) which increases machining time. Hence to control the scallop height new methodology is being developed in this present work where minimum side step value (S_s) is used to generate the next tool paths. To validate this methodology, results of Bezier test part, Bezier concave and convex surfaces are compared in chapter 4.

Scope for further study

Methodology for constant scallop height tool path generation using toroidal or radius end mill for 5-axis machining may be considered for future work. Various other surfaces like Non-uniform rational B-Spline (NURBS) with offset method can be considered for generation of constant scallop height tool path.

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