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Functionally graded piezoceramic ultrasonic transducers

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Abstract
Ultrasonic piezoelectric transducers that are employed in various applications are desired to produce a broadband frequency spectrum. However, interference from the sympathetic pulses generated by the transducer limits the duration of the waveform to a very short time. This paper discusses grading of the transducer as a means of alleviating the sympathetic pulses. A simple one-dimensional model based on the spectral approach has been presented. The piezoelectric constant e_{33} is graded in various manners and their performances are evaluated. The signal qualities are evaluated through their Euclidean distances from the applied voltage pulse. Linearly graded transducers show the best results.

1. Introduction

Piezoelectric ceramics are widely used for manufacturing ultrasonic transducers due to their low acoustic impedance, high electromechanical coupling constant, versatile forms and low cost. Conventional transducers have a uniform piezoelectric ceramic monolith. It has been observed that the bandwidth of the frequency spectrum produced by these transducers is limited due to the sympathetic waves generated at the free surfaces. The fidelity of the signal is improved using digital signal processing (DSP). In the recent past, some excellent methods have been proposed for producing short time waveform ultrasonic pulses alleviating the need of DSP. Almost all of these methods are based on gradation of the piezoelectric plate transducer. Various processes are being used for fabricating and studying these graded transducers. Mitchell and Redwood [1, 2] successfully studied the sound generated by non-uniform piezoelectric material way back in the late 1960s. Chapeau-Blondeau and Greenleaf [3] explained the behavior of piezoelectric transducer with a non-uniform distribution of piezoelectric coefficient within its bulk and derived an expression for the receiving transfer function of the transducer. Yamada *et al* [4–7] focused their studies on the performance of a linearly and an exponentially graded piezoelectric material using an equivalent network analysis. All the above findings have clearly shown the effectiveness of the piezoelectrical grading. Yamada *et al* [8] have maintained a temperature gradient at the time of manufacturing the piezoelectric plate to produce a graded piezoceramic plate. Ichinose *et al* [9] fabricated FGM transducers with individual layers of piezoelectric ceramics calcined powder or a ceramic green sheet composed of the calcined powder with some organic binder, plasticizer and solvent. Transducers produced by both the methods are capable of generating a high fidelity signal.

In this paper, the performance of a piezoelectric transducer that has a graded piezoelectric constant (e_{33}) has been evaluated numerically. The optimal grading for the highest bandwidth has been explored. The conventional numerical models such as the finite element are not efficient for such problems due to the necessity of very fine meshing and very small time steps. Therefore, a numerical model based on the spectral approach [10] is proposed. In this method, the necessity of fine meshing is avoided by transforming the temporal signal into a frequency spectrum. A superconvergent element is generated by using exponential shape functions. Several gradation curves have been evaluated to determine the optimal one. The relative performances for different gradings have been evaluated by determining their Euclidean distances from the input voltage.

2. Piezoelectric plate modeling

A PZT plate of thickness 0.75 mm is used for modeling the transducer (figure 1). The properties of the PZT plate are

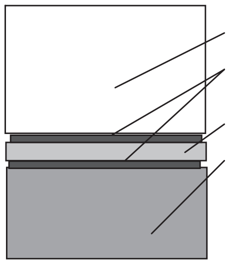

Figure 1. Model of the piezoelectric ultrasonic transducer.

Table 1. Physical properties of PZT.

Elastic modulus, E (GPa)	49
Density, ρ (kg m^{-3})	7500
Wave velocity (m s^{-1})	2556
Piezoelectric constant e_{33} (N V^{-1} m^{-1})	25

presented in table 1. The electrodes are fixed on the two surfaces of the piezoelectric plate. The transducer plates are affixed to the backing layer of the same material. The backing material absorbs the sympathetic signals arising out of the transducer. The backing material is simulated in our model as a semi-infinite plate that does not reflect any signal back into the transducer. The excitation is caused due to a voltage (V) impulse applied through the electrodes in the piezoelectric plate. Using Tiersten's equation, the response of the PZT plate can be expressed as

$$S_i = s_{ij}^{\varepsilon} T_j + e_{mi} V_m \tag{1}$$

where S is the mechanical strain, T is the mechanical stress, e_{mi} is the piezoelectric constant and V is the electric potential, ε is the permittivity and the subscripts i, j and m denote the direction of the stress, strain and electric potential.

However, PZT is very weakly coupled. Thus, neglecting the coupling, the impulsive stress, per unit length, in the x direction on the piezoelectric plate, can be represented as

$$\sigma(x, t) = E e_{33}(x) V(t) \tag{2}$$

where E is the modulus of elasticity of the piezoelectric plate, and $e_{33}(x)$ is the piezoelectric constant in the 33 direction.

It may be noted that for a graded plate the piezoelectric constant e_{33} is a function of the position in the plate. In the present paper, the piezoelectric constant e_{33} is assumed to follow the power law

$$e_{33} = e_0 (x/d)^n \tag{3}$$

where e_0 is the piezoelectric constant in the 33 direction at $x = 0$. d is the thickness of the plate; n is the power law exponent.

For modeling the gradation in the piezoelectric constant e_{33} in our system, discretely layered PZT plates with 10 layers, each 0.075 mm, are used. This gradation is considered fine enough for producing results very close to the continuous grading.

3. Proposed analysis

The characteristics of wave propagation problems are that the frequency content of the exciting force is very high. Therefore, a very fine mesh of finite elements is necessary for adequately modeling the problem. This problem can be alleviated by using frequency based methods such as the spectral method [10] instead of the time based techniques. In this method, the governing partial differential wave equation is reduced to a set of ordinary differential equations. Their solution is easier than that of the original differential equation. However, often approximate solutions are sought. The transformation is effected by fast Fourier transforms (FFT). These solutions to the governing equations are used as shape functions for spectral element formulation. The advantage of this method is that the inertial effects are exactly represented and hence often exact solutions are obtained. In addition, the shape function is, most of the time, superconvergent and very few elements are required to model the system. In the present case the number of elements is equal to the number of inner layers of the graded transducer.

3.1. Reconstruction of waves

The significance of the spectral approach to waves along with the differential equation is that once the signal is characterized at one position in space then it is known at all positions, and therefore, propagating it becomes a fairly simple matter. The solution obtained by the spectral method is in the frequency domain and to get back into the time domain the inverse FFT is obtained. It should be observed that while using FFT for inversion the transformed solution in the frequency domain is calculated up to the Nyquist frequency and the rest is obtained by imposing the condition that it must be the complex conjugate of the initial part. This is done to ensure that the reconstructed history is real.

3.2. Spectral approach

The governing wave motion equation is given by

$$\frac{\partial}{\partial x}\left\{EA\frac{\partial u}{\partial x}\right\} = \rho A \frac{\partial^2 u}{\partial t^2} \tag{4}$$

where E is the elastic modulus, ρ is the mass density and A is the cross-sectional area of the plate. Considering that both the modulus and the area do not vary with position, the displacement solution to the above equation for a plate of length L, according to the spectral approach [10], can be represented by

$$u(x, t) = \sum_n \hat{u}_n(x, \omega_n) \mathrm{e}^{\mathrm{i}\omega_n t} \tag{5}$$

where ω_n represents the frequency and the spectral displacement $\hat{u}_n$ has the following solution:

$$\hat{u}_n = \mathbf{A}\mathrm{e}^{-\mathrm{i}k_n x} + \mathbf{B}\mathrm{e}^{\mathrm{i}k_n (L-x)} \tag{6}$$

where k is the wavenumber and $\mathbf{A}$ and $\mathbf{B}$ are undetermined amplitudes at each frequency, determined by applying boundary conditions. In conventional finite element methods,

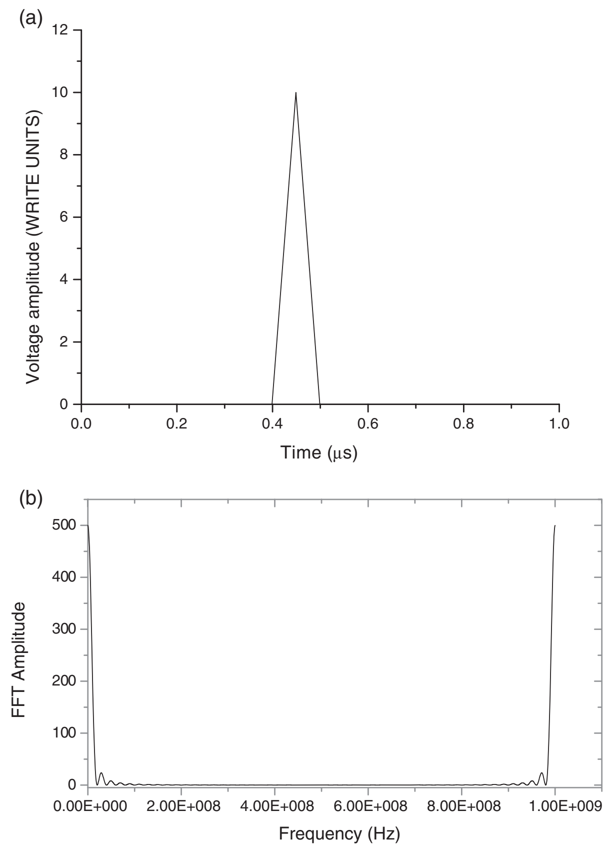

Figure 2. (a) Time waveform of the input voltage excitation. (b) Frequency spectrum of the input voltage excitation.

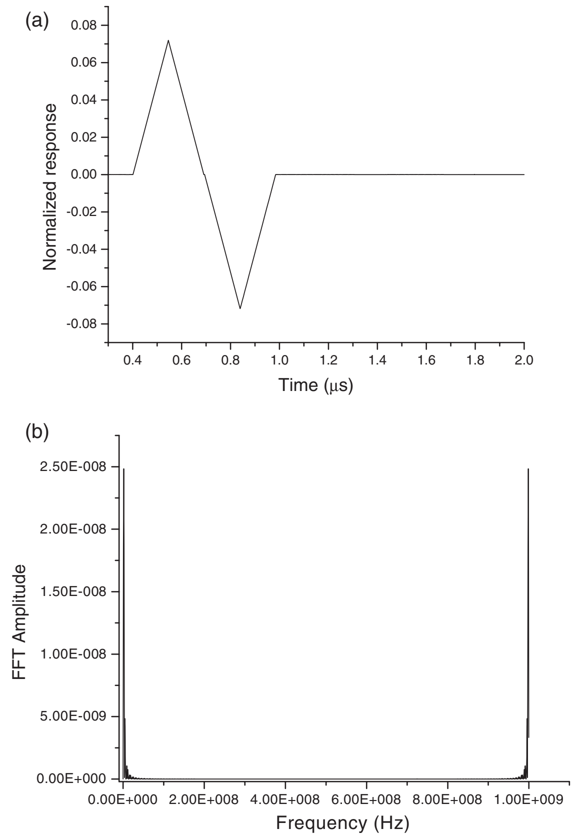

Figure 3. (a) Normalized time waveform for conventional transducers. (b) Frequency spectrum of a conventional transducer.

the expression on the right in equation (3) is called the shape function, but here it is a function of the frequency ω_n. These functions are the exact solutions of the governing partial differential equation of the wave motion. The above function can be written in terms of nodal displacements $u_1 = u(0)$ and $u_2 = u(L)$ of the plate. Hence, we can establish a relation between the unknown parameters **A** and **B** and the nodal degrees of freedom. After inverting the relation this can be arranged into the following form:

$$\{\mathbf{A}\} = [Q]\{\hat{u}\} \tag{7}$$

where $[Q]$ is

$$[Q] = \frac{1}{(1 - \mathrm{e}^{-\mathrm{i}2kL})}\begin{bmatrix} 1 & -\mathrm{e}^{-\mathrm{i}kL} \\ -\mathrm{e}^{-\mathrm{i}kL} & 1 \end{bmatrix}. \tag{8}$$

Using these matrices, nodal forces can be calculated by differentiation of the displacements and can be expressed in matrix form as

$$\begin{Bmatrix} \hat{F}_1 \\ \hat{F}_2 \end{Bmatrix} = \frac{EA}{L}\frac{\mathrm{i}kL}{(1 - \mathrm{e}^{-\mathrm{i}2kL})}\begin{bmatrix} 1 & -\mathrm{e}^{-\mathrm{i}kL} \\ -\mathrm{e}^{-\mathrm{i}kL} & 1 \end{bmatrix}\begin{Bmatrix} \hat{u}_1 \\ \hat{u}_2 \end{Bmatrix}. \tag{9}$$

This relation is of the form $F = [k]\{\hat{u}\}$ where k is the frequency dependent dynamic stiffness for the plate. It is symmetric and real.

The assemblage of these matrices can be done in the same manner as in conventional FEM methods to determine the displacements. Several cases that explain the utility of the proposed model are discussed in the next section.

4. Numerical results

The present system is illustrated in figure 1. The PZT plate is supported by backing strips at one end. At the other end a medium is placed that represents the material through which the stress wave is propagated. The measurement point is within this medium. The aim of the numerical experiment is to determine the effect of grading the PZT plate on the generated stress wave. The time waveform and the frequency spectrum of the applied voltage spike excitation are shown in figures 2(a) and (b) respectively. The duration of the input impulse is 0.1 μs.

Case I: uniform PZT plate

First, the uniform PZT plate is examined. Figures 3(a) and (b) show the time waveform and frequency spectra respectively for a conventional transducer. Two peaks (figure 3(a)) are noticed due to the waves originating from both the faces of the piezoelectric plate. Clearly, the input waveform could not be replicated in this case. We will first investigate the effect of the duration of the applied voltage spike on the generated waveform. For this purpose, we compare the normalized time waveform response for the 0.1 μs (figure 4(a), 0.29 μs (figure 4(b)) and 0.5 μs (figure 4(c)). The normalized displacement and normalized input voltage spike are determined using

$$\hat{\mathbf{u}} = \frac{u}{|u|} \tag{10}$$

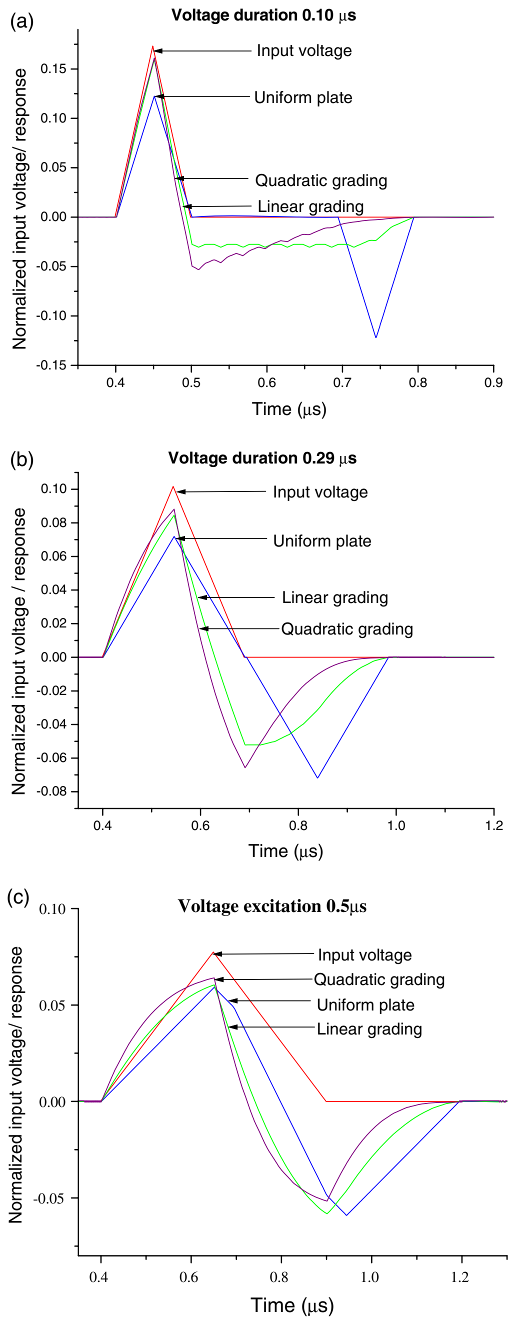

Figure 4. Normalized time waveforms of input excitation and response.

(This figure is in colour only in the electronic version)

$$\hat{\mathbf{V}} = \frac{V}{|V|} \tag{11}$$

where V is the input voltage spike. The magnitudes $|u|$ and $|V|$ are calculated using the square root of the sum of squares (SRSS) method. Normalized input voltages are also shown in the same figures to compare the efficacy of the transducer to replicate the input signal.

It may be noted that the travel time for the wave, from one end of the transducer to the other end, is 0.2934 μs. This is the critical pulse duration. When the duration is far lower than the critical duration we obtain two separate pulses. The pulses start interacting with each other when the duration is more than the critical duration. As a result, the generated waveform is distorted. This also limits the efficacy of the transducer above a particular spike duration or below a particular input frequency. The efficacy goes down rapidly when the spike duration is greater than the travel time. This limits the bandwidth of the waveform produced by the transducer. One strategy for improving the performance of the transducer is to grade its piezoelectric property so that the waveform generated from the far end is of smaller amplitude. This strategy is evaluated in the next section.

Case II: graded PZT plate

The PZT plate is modeled in 10 layers with discretely graded piezoelectric constant e_{33}. The voltage impulses applied across their ends are the same. Linear and quadratic variations of the piezoelectric constant e_{33} are investigated. The goal is to alleviate the pulse originating from the far face. The effect of the duration of the input impulse on the response obtained from graded transducer is also explored (figure 4(a)–(c)). The duration of the input impulse must be chosen judiciously, based on the plate thickness. Figure 4(a) shows the plots of normalized response measured for various durations for input impulse for linear and quadratic gradations. It can be observed from these figures that only one major peak is observed when the PZT plate is graded. The other pulse has diminished considerably. However, the pulse fidelity depends on the degree of grading. From figure 4(b), it can be noticed that the generated signal distorts due to grading. This is due to the reflected waves, from the inner layers. There are two effects of signal distortion due to grading shortening of the pulse duration and distortion of the pulse shape. The shortening of the pulse duration can easily be compensated by applying the input pulse of suitably higher duration. However, the distortion of the pulse shape is not easy to alleviate. The distortion is proportional to the degree of grading. Therefore, the quadratically graded transducer distorts the signal more than the linearly graded one. The degree of distortion is proportional to the pulse duration. To compare the efficiency of the graded media to replicate the input pulse, the Euclidean distance (t) between the input pulse and the generated pulse has been calculated:

$$t = \sqrt{\sum (\hat{\mathbf{V}} - \hat{\mathbf{u}})^2}. \tag{12}$$

The histogram of the Euclidean distances for various gradings is plotted in figure 5. It is clear that for a long range of pulse durations the signal produced by the linearly graded transducer has the least Euclidean distance. Therefore, the linear gradation is most efficient in replicating the input signal. The superiority of the graded transducers over the uniform ones is tangible for pulse durations below the critical duration. The Euclidean distance for the linear gradation is always lower than that for the quadratic gradation. Therefore linearly graded transducers are most desirable.

5. Conclusions

It has been demonstrated that the signal fidelity of a PZT transducer deteriorates with increase in the pulse duration.

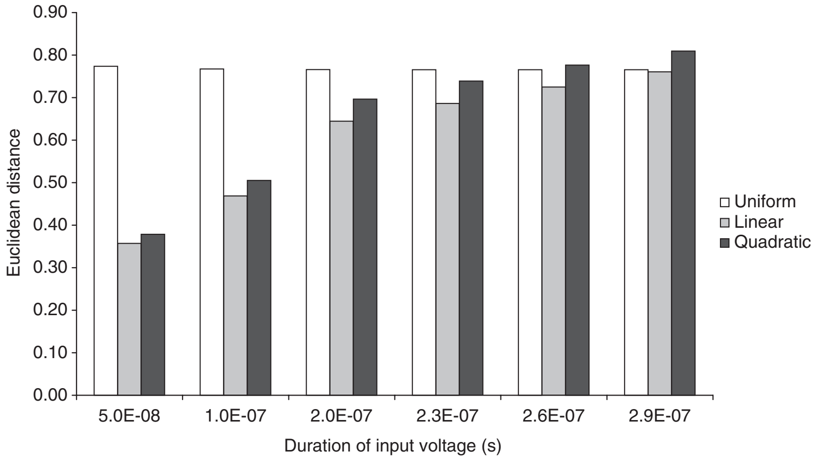

Figure 5. Euclidean distance bars for various duration of voltage excitation.

The fidelity can be improved by grading PZT plate. In this investigation the gradation is done in the piezoelectric constant e_{33}. Linear and quadratic gradations have been examined for various durations of the input pulse. Although the pulse generated at the far end can be alleviated by grading, the resulting signal is distorted. The performance of the transducers has been compared by measuring the Euclidean distances between the input pulse and the generated pulses. The linearly graded transducer has the best overall performance.

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