PROBABILISTIC ANALYSIS OF RECEIVED COMPOSITE WIRELESS SIGNALS FOR RANDOM NUMBER OF MULTIPATHS

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DECLARATION

I hereby declare that the work, which is being presented in the thesis, entitled **"PROBABILISTIC ANALYSIS OF RECEIVED COMPOSITE WIRELESS SIGNALS FOR RANDOM NUMBER OF MULTIPATHS"** in partial fulfillment of the requirements for the award of degree of Master of Engineering in Electronics and Communication Engineering at Electronics and Communication Engineering Department of Thapar University, Patiala, is an authentic record of my own work carried out under the guidance of Dr. Amit Kumar Kohli.

I have not submitted the matter presented in the thesis for the award of any other degree of this or any other university.

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ABSTRACT

In wireless communication, the received random signal is composed of the sum of several random sinusoidal signals e.g., multipath fading or interference in communication channels, clutter and target cross section in radars, wave propagation in random media/channels and light scattering. According to the correspondence between a random wireless signal and a random vector, the sum of random vectors can be considered as an abstract mathematical model. It is desired to obtain the probability density function (pdf) of the length of the resulting vector.

This report presents a technique to obtain pdf for the most general cases in which the lengths of vectors are arbitrary dependent random variables. The obtained pdf is in the form of definite integral, which may be inappropriate for analytic manipulations and numerical computations. Therefore, an appropriate infinite integral form called laguerre expansion is derived. The obtained results are applied in computing the scattering cross-section of random scatterers resulting from a small number of constant amplitude scatterers each having a random phase, and also achieve expressions for pdf of scattered signal intensity. The methods for computing the coefficients of aforementioned infinite series are also discussed. An arbitrary random nonzero real number β is incorporated; whose appropriate value can reduce truncation error.

This report is mainly focused on the computation of probability density function of the received composite wireless signals for random number of multipaths in a single antenna element of multi-input-multi-output wireless systems, and also selecting an optimum value of β to minimize the truncation error.

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LIST OF ABBREVIATIONS

$\alpha_n(t)$	Attenuation factor
$\tau_n(t)$	Propagation Factor
δ	Kronecker Delta Function
σ_τ	Delay Spread
BS	Base Station
B_c	Coherence Bandwidth
B_D	Doppler Spread
B_s	Bandwidth of Signal
CF	Characteristic Function
CW	Continuous Wave
EM	Electromagnetic
HF	High Frequency
MIMO	Multiple-input Multiple-output
MRC	Maximal Ratio Combining
MISO	Multiple-input Single-output
P_o	Outage Probability
Pdf	Probability Density Function
PDF	Probability Distribution Function
RF	Radio Frequency
RVs	Random Variables
Rx	Receiver
SAR	Synthetic Aperture Radar
SIMO	Single-input Multiple-output
SISO	Single-input Single-output
SNR	Signal to Noise Ratio
SSs	Subscriber Stations
T_c	Coherence Time
T_s	Symbol Period

Tx	Transmitter
UHF	Ultra high Frequency
VHF	Very High Frequency

CHAPTER 1

INTRODUCTION

In wireless communications, rapid small-scale fading of the received radio signal envelope seriously degrades system error probability performance. This small-scale or short-term or multipath fading is due to the superposition of multiple reflections of a transmitted electromagnetic wave by local scatterers surrounding a wireless receiver. Several multipath models have been suggested to describe the statistical characteristics of this fading process. In all of these works, the probability density functions (pdf) of signal amplitude and phase are the two key functions that have been studied, since they are useful for system design, such as the choice of diversity, equalization, and error control coding techniques, and error probability performance analysis.

Small-scale fading experienced by a wireless receiver causes dramatic fluctuations in received signal strength as a receiver moves over a relatively small local area. One of the most common methods for characterizing a fading channel is the use of a probability density function (pdf), which represents the probability density of the received signal strength. The shape of a pdf determines the performance of a wireless receiver in the presence of noise and interference. Proper characterization of fading pdfs also impacts the design and use of diversity schemes, equalization methods, and error-correction coding used for a communications link .

We generally encounter a random signal that is composed of the sum of several random wireless signals. For example multipath fading in communication channels [1], clutter and target cross section in radars [2], interference in communication system, wave propagation in random media and channels and light scattering [3]. Sometime it is required to calculate the envelope probability density function (PDF) of the sum of these random wireless signals. According to the correspondence between a random wireless signal and a random vector, the sum of random vectors can be considered as an abstract mathematical modal. Now it is desired to obtain the pdf of the length of the resulting vector. Here the pdf is obtained for most general case in which the lengths of vectors are arbitrary dependent random variables. This pdf is in the definite integral form, which may be inappropriate for analytic manipulations and computationals. So an appropriate infinite Laguerre expansion is also derived. Finally the obtained results are applied to a typical example in computing the scattering cross section of random scatterers resulting from a small number of constant amplitude scatterers each having a random phase. Also we drive expressions for pdf of scattered signal intensity. Here a arbitrary random non-zero positive number $\boldsymbol{\beta}$ is introduced whose value can be chosen in such a way so that truncation error can be minimized which is the main problem.

This technique for calculating pdf can be applied in transmission of wireless signals through multiple antenna systems. In multiple antenna system such as multiple-input multiple-output system (MIMO), each antenna element receives a number of composite random signals for multipaths. So the pdf of these multipath wireless signals can be obtained by using the techniques, which are to discussed in this thesis.

1.1 Introduction to Multipath Fading

The most troublesome and frustrating problem in receiving radio signals is variations in signal strength, most commonly known as Fading.
multipath is simply a term used to describe the multiple paths a radio wave may follow between transmitter and receiver. Such propagation paths include the ground wave, ionospheric refraction, reradiation by the ionospheric layers, reflection from the earth's surface or from more than one ionospheric layer.

Fading is used to describe the rapid fluctuations of the amplitudes, phases, or multipath delays of a radio signal over short period of time or travel distance. Multipath fading is occurred due to interference between two or more versions of the transmitted signal which arrive at receiver at slightly different times [4].

Multipath in the radio channel creates small-scale fading effects. The three most important effects are:-

1. rapid changes in signal strength over a small travel distance or time interval
2. random frequency modulation due to varying Doppler shifts on different multipath signals
3. time dispersion(echoes) caused by multipath propagation delays.

1.1.1 Slow and Fast Fading

The terms slow and fast fading refer to the rate at which the magnitude and phase change imposed by the channel on the signal changes. The coherence time is a measure of the minimum time required for the magnitude change of the channel to become decorrelated from its previous value.

Slow fading arises when the coherence time of the channel is large relative to the delay constraint of the channel. In this regime, the amplitude and phase change imposed by the channel can be considered roughly constant over the period of use. Slow fading can be caused by events such as shadowing, where a large obstruction such as a hill or large building obscures the main signal path between the transmitter and the receiver [4]. The amplitude change caused by shadowing is often modeled using a log-normal distribution with a standard deviation according to the log-distance path loss model.

Thus , a signal undergoes slow fading if

$$\mathbf{T_s << T_c} \text{ and}$$

$$\mathbf{B_s >> B_D}$$

Where B_D is Doppler spread, T_c is coherence time.

Fast fading occurs when the coherence time of the channel is small relative to the delay constraint of the channel. In this regime, the amplitude and phase change imposed by the channel varies considerably over the period of use.

Thus , a signal undergoes fast fading if

$$\mathbf{T_s > T_c} \text{ and}$$

$$\mathbf{B_s < B_D}$$

In a fast-fading channel, the transmitter may take advantage of the variations in the channel conditions using time diversity to help increase robustness of the communication to a temporary deep fade. Although a deep fade may temporarily erase some of the information transmitted, use of an error-correcting code coupled with successfully transmitted bits during other time instances (interleaving) can allow for the erased bits to be recovered.

In a slow-fading channel, it is not possible to use time diversity because the transmitter sees only a single realization of the channel within its delay constraint. A deep fade therefore lasts the entire duration of transmission and cannot be mitigated using coding.

1.1.2 Flat and Frequency-selective Fading

In flat fading, the coherence bandwidth of the channel is larger than the bandwidth of the signal. Therefore, all frequency components of the signal will experience the same magnitude of fading. Thus, a signal undergoes flat fading if

$$\mathbf{B_c} \gg \mathbf{B_s} \text{ and}$$

$$\mathbf{T_s} \gg \boldsymbol{\sigma_\tau}$$

Where T_s is the reciprocal of bandwidth (symbol period) and B_s is the Bandwidth of of signal. And σ_τ and B_c are the rms delay spread and coherence bandwidth respectively, of the channel. The characteristics of flat fading channel is shown in below figure.

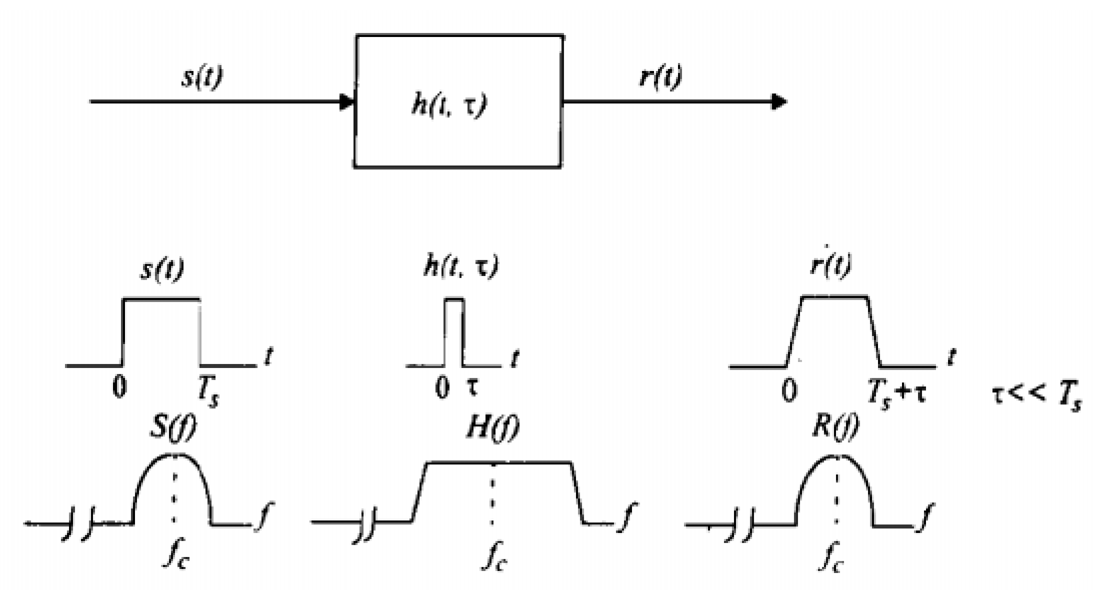

Fig 1.1 flat fading channel characteristics

In frequency-selective fading, the coherence bandwidth of the channel is smaller than the bandwidth of the signal. Different frequency components of the signal therefore experience decorrelated fading. Thus a signal undergoes frequency selective fading if

$$\mathbf{B_c < B_s} \text{ and}$$

$$\mathbf{T_s < \sigma_\tau}$$

The characteristic of frequency selective fading is illustrated in below figure.

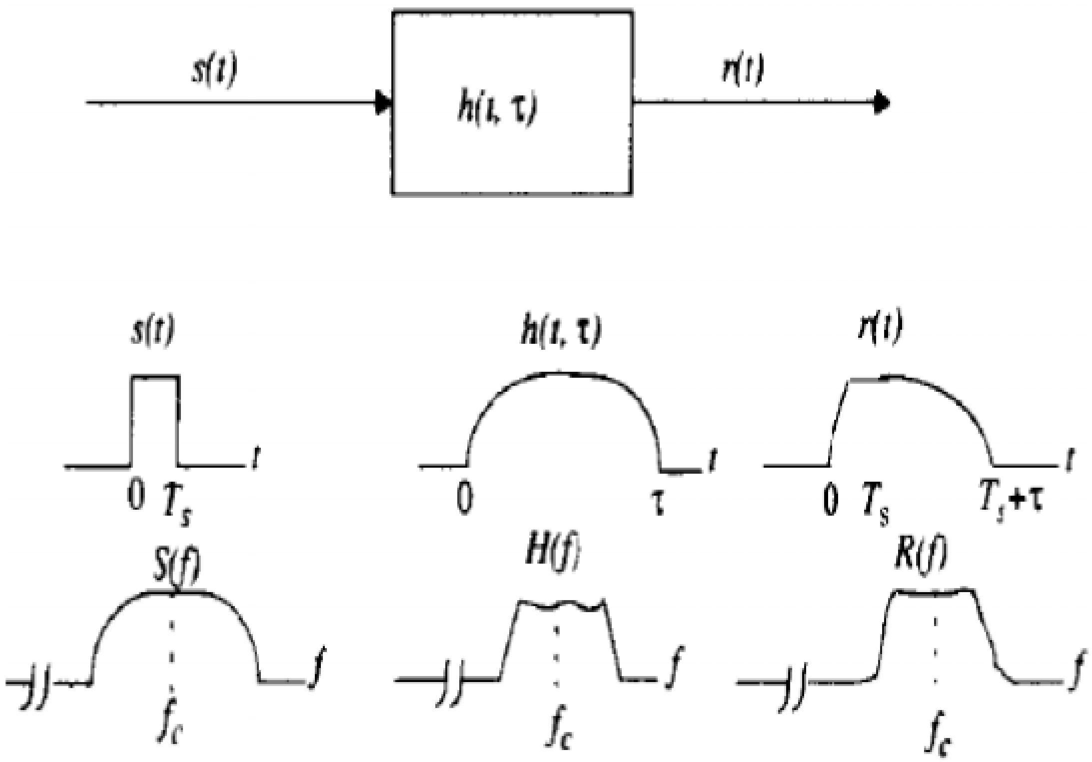

Fig 1.2 frequency selective fading characteristics

Multipath fading usually degrades the performance of communication systems, severely. To describe this random phenomenon, it is necessary to derive expressions for the PDF of the received random signal envelope in multipath fading channels at receiver. Here, two expansions for the envelope PDF's are derived under general channel characteristics: an infinite power series and an infinite Laguerre series [7].

When a single unmodulated carrier (i.e. with constant envelope) is transmitted in a multipath fading channel, it breaks into several multipath components. These multipath components add vectorially (according to their amplitudes and phases), and a rapidly fluctuating envelope is experienced by the receiver. Generally speaking, and for a fixed instant of time, the number of multipath components. The amplitudes and the phases of multipath components are all random variables. Thus, due to the multipath fading phenomenon, the constant envelope of the transmitted carrier changes to a stochastic process; and so, a probabilistic model should be developed to describe this rapidly fluctuating envelope. A useful probabilistic model is the random vector model, since each multipath component can be considered as a random vector, with random length and angle [7].

If a closed form formula for the envelope probability density function (PDF) in a multipath fading channel is available, then it can be used to prelct analytically the performance of communication systems with various modulation schemes, in the presence of multipath fading. It also facilitates the design of accurate channel simulators. Thus, it is very important to obtain a mathematically tractable formula for $f(r)$, the envelope PDF, under general conditions the expression for pdf of length of resulting vector can be expanded in terms of an infinite series containing Laguerre polynomials. Laguerre polynomials are a set of orthogonal polynomials on the positive real axis. In this series, we introduce a arbitrary nonzero real number (β) purposely. The advantage of a variable (β), instead of predetermined constant is that β can be selected in a such a way to minimize the truncation error.

1.2 Introduction to Random Variables and Probability Density Function

1.2.1 Random Variables

Usually it is more convenient to associate numerical values with the outcomes of an experiment than to work directly with a non-numerical description. A random variable is a function that associates a unique numerical value with every outcome of an experiment. The value of the random variable will vary from trial to trial as the experiment is repeated.

There are two types of random variable-discrete and continuous. A random variable has either an associated probability distribution (discrete random variable) or probability density function (continuous random variable).

1.2.2 Expected Value

In probability theory and statistics, the expected value (or expectation value, or mathematical expectation, or mean, or first moment) of a random variable is the integral of the random variable with respect to its probability measure. For discrete random variables this is equivalent to the probability-weighted sum of the possible values, and for continuous random variables with a density function it is the probability density - weighted integral of the possible values. Stating the expected value gives a general impression of the behavior of some random variable without giving full details of its probability distribution (if it is discrete) or its probability density function (if it is continuous). The expected value of a random variable X is symbolised by E(X) or μ.

If X is a continuous random variable with probability density function f(x), then the expected value of X is defined by:

$$\mu = E(x) = \int xf(x)dx$$

If X is a discrete random variable with possible values $x_1, x_2, x_3, ..., x_n$, and $p(x_i)$ denotes $P(X = x_i)$, then the expected value of X is defined by:

$$p(x_i) = P(X = x_i)$$
$$\sum p(x_i) = 1$$

1.2.3 Probability Distribution

A discrete probability distribution function associates a list of probabilities with each possible value of a discrete random variable. The probability distribution function is thus used to model the probabilities of a discrete random variable and is also known as a probability mass function. The probabilities of a continuous random variable are modeled using continuous distribution functions, also known as probability density functions (pdf's).

The probability distribution of a discrete random variable X is a function which gives the probability p(xi) that the random variable equals xi, for each value x_i:

$$p(x_i) = P(X = x_i)$$

It satisfies the following conditions:

1. $0 \le p(x_i) \le 1$
2. $\sum p(x_i) = 1$

1.2.4 Probability Density Function (pdf)

The probability density function of a continuous random variable is a function which cane integrated to obtain the probability that the random variable takes a value in a given interval. Let X be a continuous random variable and f(x) is a Probability density function

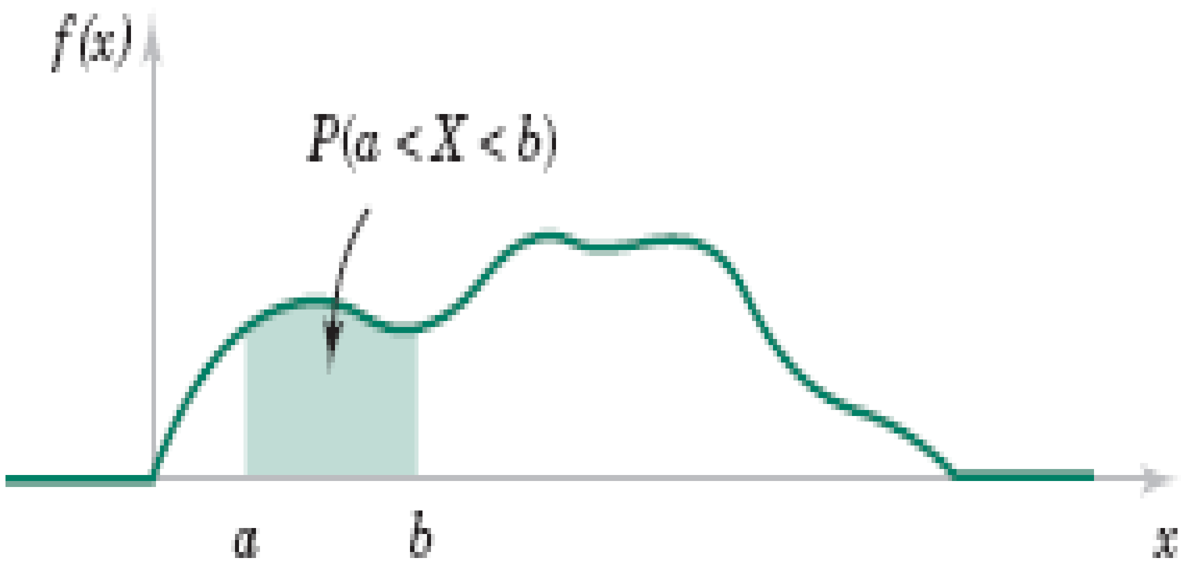

Fig 1.3: Probability density function of random variable X

For Random variable X, a PDF is a function such that

$$f(x) \geq 1$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$P(a \leq X \leq b) = \int_{-\infty}^{\infty} f(x)\,dx$$

1.2.5 Gaussian PDF

If a random variable X is normally distributed with mean m(or μ) and variance σ^2, then its probability density function (pdf) is given as:-

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(X-m)^2/2\sigma^2}$$

If the number of events is very large, then the Gaussian distribution function may be used to describe physical events. The Gaussian distribution is a continuous function which approximates the exact binomial distribution of events. The Gaussian distribution shown is normalized so that the sum over all values of x gives a probability of 1.

The Normal or Gaussian pdf (1.1) is a bell-shaped curve that is symmetric about the mean μ .

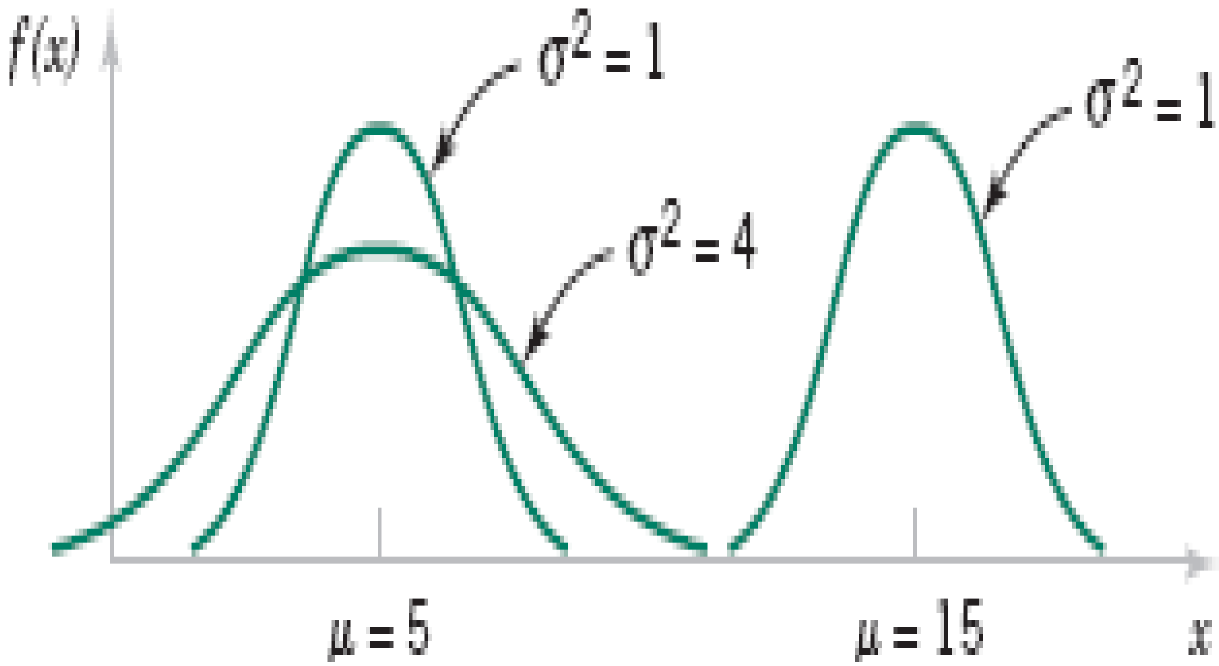

Fig 1.4 Normal probability density functions for selected values of the parameters μ and σ^2

1.2.6 Characteristics Function

The characteristics function of a Random variable X is defined as

$$\psi(\omega) = E[\exp(j\omega x)]$$

$$\psi(\omega) = \int_{-\infty}^{\infty} f(x)\exp(j\omega x)\,dx$$

Where $f(x)$ is PDF of X. This function is maximum at the origin because $f(x) \geq 0$:

$$|\psi(\omega)| \leq \psi(0) = 1$$

1.2.7 Random Vectors

A random vector is a finite-dimensional formal vector of random variables. The random vector can be written either as a column or row of random variables, depending on its context and use. So if $X_1, X_2, X_3 \ldots$ and so on are random variables, then

$$\mathbf{X} = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{pmatrix} = (X_1, X_2, \ldots, X_n)^{\mathrm{T}}$$

is a random (column) vector.

The distribution of a random vector $X = (X_1\ X_2 \ldots\ldots X_n)$ is defined to be the joint distribution of its coordinates $(X_1\ X_2 \ldots\ldots X_n)$

$$F_x(x) = F_{X1,\ldots\ldots Xn}(x_{1,}\ldots\ldots x_n).$$

Any random sinusoidal signal can be considered as a random vector, i. e., a vector with random length and angle. Thus, the sum of random sinusoidal signals changes to the sum of random vectors. If there are N vectors with length A_i's and angle Φ_i's where N, A_i's and Φ_i's are random variables. Now it is desired to obtain the probability density function (pdf) of A, as follow

$$A\exp(j\Phi_i) = \sum_{i=1}^{n} A_i \exp(j\Phi_i) = X + jY$$

The pdf of A also represents the univariate envelope pdf of the sum of random sinusoidal signals.

The resulting pdf of A, obtained under various assumptions and conditions, may be summarized as:-

1. N is a random variable [5]
2. Φ_i's have non-uniform pdf's on [0,Π] [6]
3. N is a deterministic variable and Φ_i's have uniform pdf on [0,Π] ([5], [7])
4. X and Y have a joint Gaussian pdf [35].

5. X and Y have a joint non-Gaussian pdf. [36].

The following methods are used for obtaining the pdf of A:-

1. Infinite expansion in terms of the Fourier-Bessel series
2. Infinite expansion in terms of the Laguerre series
3. Various analytic approximations
4. Recursive.

Problem Formulation

This thesis presents the following work:

1. Calculation of probability density function of received composite wireless signals for a random number of multipath using infinite series expansion.
2. Minimization of truncation error.

Organisation of Thesis

This thesis report is organized in four chapters

Chapter 1 summarizes the basic problem statement of the research work and give the overview of maultipath fading and basic probability theory.

In Chapter 2, we will introduce the basic concept of multiple antenna system (SISO. SIMO, MISO & MIMO) and characterization of fading multipath channels.

In chapter 3, we illustrate the radar scattering and obtain the pdf for different radar scattering elements.

In chapter 4, we derive the general formula for calculating pdf for a random wireless signals received from multipaths.

In chapter 5, the simulation results are obtained for a application of radar scattering. All simulations are done using MATLAB.

Finally, we conclude our work by studying various simulation results.

CHAPTER 2

DIGITAL COMMUNICATION THROUGH MULTIPATHS

2.1 Introduction to Multiple Antenna Systems

In the last few years wireless services have become more and more important. Wireless communication system design was until recently thought to have been limited in practice by time and bandwidth. The discovery that space, obtained by increasing the number of transmit and receive antennas, can also effectively generate degrees of freedom, and hence expand the range of choices made available to the design offers system designers important new opportunities. Likewise the demand for higher network capacity and performance has been increased. Several options like higher bandwidth, optimized modulation or even code-multiplex systems offer practically limited potential to increase the spectral efficiency. Multiple antenna system utilize space-multiplex by using antenna arrays to enhance the efficiency in the used bandwidth.

2.1.1 SISO System

SISO (single input, single output) refers to a wireless communications system in which one antenna is used at the source (transmitter) and one antenna is used at the destination (receiver).

SISO systems are the simplest antenna technology. With single antennas used, single frequencies are vulnerable to space limits and frequency fading. SISO systems are sometimes troubled by multipath effects. Electromagnetic wavefronts are dispersed when they encounter signal-path obstructions like buildings, hills, tunnels, valleys and utility wires. In such cases, the scattered electromagnetic waves take many paths to reach the destination. The late arrival of scattered portions of the signal causes problems such as fading, cut-out (cliff effect), and intermittent reception (picket fencing).

In a digital communications system, it can cause a reduction in data speed and an increase in the number of errors. SISO is relatively simple and cheap to implement and it has been used age long since the birth of radio technology. It is used in radio and TV broadcast and our personal wireless technologies (e.g. Wi-Fi and Bluetooth).

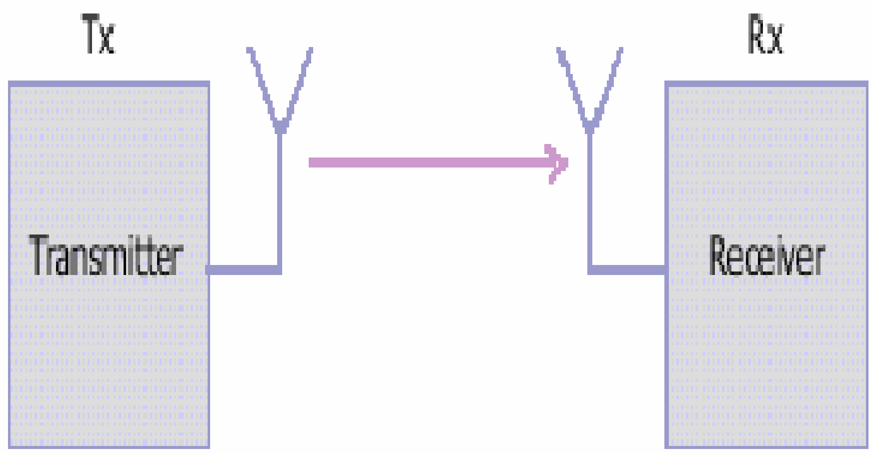

Fig 2.1 SISO antenna system

2.1.2 SIMO System

The term "SIMO" (**S**ingle **I**nput **M**ultiple **O**utput), as applied to wireless technology, refers to the smart antenna technology that uses a single antenna at the transmitter side and multiple antennas at the receiver side for improved performance and security. The antennas are combined to minimize errors and optimize data speed. The source (transmitter) has only one antenna. The receiver can either choose the best antenna to receive a stronger signal or combine signals from all antennas in such a way that maximizes SNR (Signal to Noise Ratio). The first technique is known as switched diversity or selection diversity. The latter is known as maximal ratio combining (MRC).

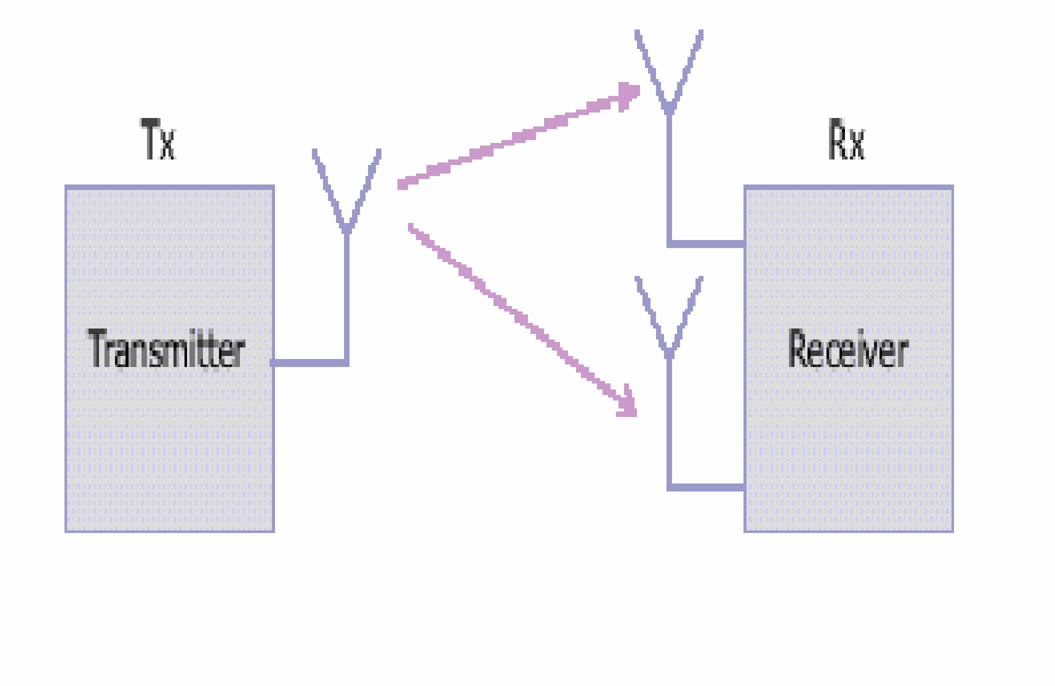

Fig 2.2 SIMO antenna system

The use of two or more antennas at the destination can reduce the trouble caused by multipath wave propagation. An early form of SIMO, known as diversity reception, has been used by military, commercial, and shortwave radio operators at frequencies below 30 MHz since the First World War. The other forms of smart antenna technology include Single Input Single Output (SISO), Multiple Input Multiple Output (MIMO) and Multiple Input Single Output (MISO).

2.1.3 MISO System

A system which uses multiple antennas at the transmitter and a single antenna at the receiver is named Multiple Input Single Output (MISO). The antennas are combined to minimize errors and optimize data speed.

Multiple antennas of either SIMO or MISO are usually placed at a base station (BS). This way, the cost of providing either a receive diversity (in SIMO) or transmit diversity (in MISO) can be shared by all subscriber stations (SSs) served by the BS. We assume the receiver has perfect knowledge of the channel but the transmitter only knows the channel distribution. The knowledge of the channel allows the transmitter to transmit with a signal covariance that maximizes the signal-to-noise ratio (SNR) at the receiver, thus increasing the mutual information.

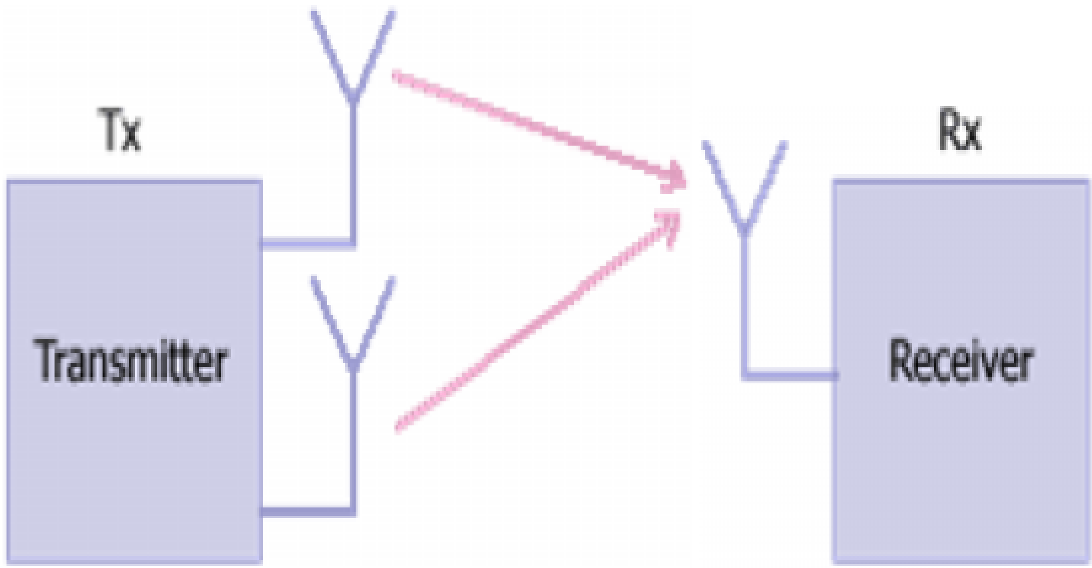

Fig 2.3: MISO antenna system

2.1.4 MIMO System

MIMO (multiple-input multiple output) wireless systems are those that have multiple antenna elements at both transmitter and receiver. Key feature of MIMO systems is the ability to turn multipath propagation, traditionally a pitfall of wireless transmission, into a benefit for the user [12],

MIMO effectively takes advantage of random fading and when available, multipath delay spread, for multiplying transfer rates. Core Idea in MIMO system is space-time signal processing in which time (the natural dimension of digital communication data) is complemented with the spatial dimension inherent in the use of multiple spatially distributed antennas. Band limited wireless channels are narrow pipes that do not accommodate rapid flow of data [14]. Deploying multiple transmit and receive antennas broadens this data pipe. Information theory provides measures of capacity, and the standard approach to increasing data flow is linear processing at the receiver [15], [16].

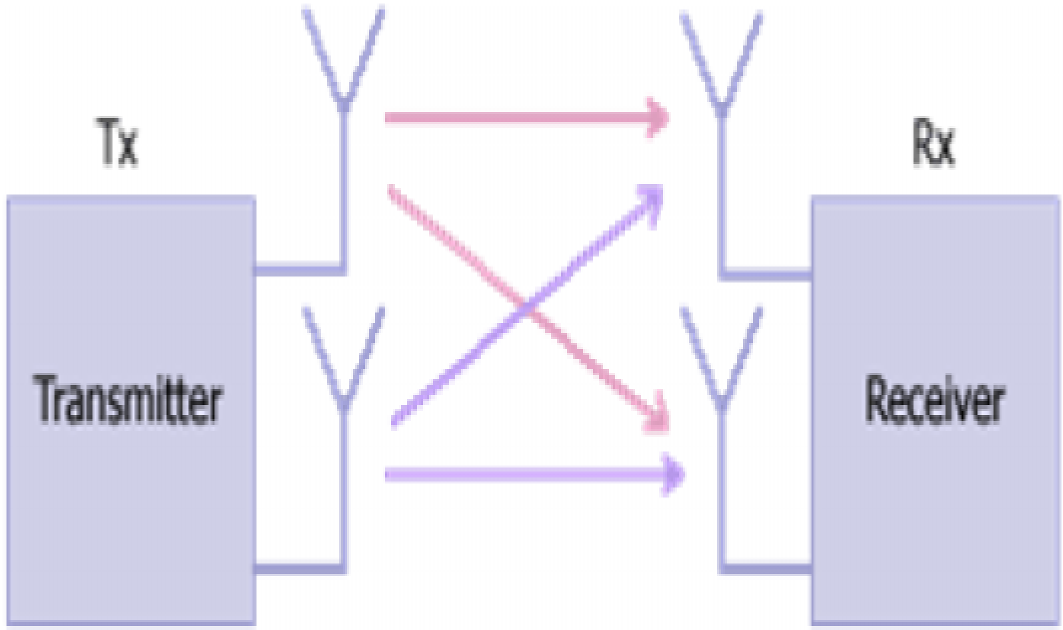

Fig 2.4: MIMO antenna System

Diversity is the key topic in mobile communications. It is principally used to combat fading. The basic idea is that if different copies of the same signal are available then there is a high probability that at least one of them is of a good quality. Of course choosing the best copy and rejecting all others is not the optimal solution. This brings us to the problem of choosing the best way to combine all of them [17].

The popular forms of diversity are:

1. Temporal diversity: Replicas of the information bearing signal are transmitted in different time slots, where the separation between the time slots is greater than the coherence of the channel.
2. Frequency diversity: In this case, replicas of the information bearing signal are transmitted in different frequency bands, where the separation between the frequencies is greater than the coherence bandwidth of the channel.

3. Antenna (spatial) diversity: It has been observed that antennas with a spacing of more than half a wavelength leads to spatially uncorrelated channels. The transmission of the replicas of the information-bearing signal over these uncorrelated spatial channels leads to spatial diversity.

Note that not all kinds of diversity are always feasible. For example a slowly fading channel (with a long coherence time) can not support temporal diversity with practical interleaving depths. Similarly, frequency diversity is not feasible when the coherence bandwidth of the channel is comparable to the bandwidth of the signal employed. However, irrespective of the channel characteristics, antenna diversity can always be exploited as long as there is sufficient spacing between the antennas.

Temporal and frequency types of diversity normally introduce redundancy in time and/or frequency domain, and therefore induce loss in bandwidth efficiency [18]. Typical examples of spatial diversity are multiple transmit and/or receiver, multiple antenna communication rely on space diversity to mitigate fading without necessarily sacrificing precious bandwidth resources; thus, they become attractive solution for broadband wireless application. Compare to single antenna transmission, multiple antenna transmissions increase the channel capacity.

A simple space diversity scheme is to use multiple antennas at the receiver. This scheme does not involve loss of bandwidth. The optimal way to combine the outputs of different antennas is the Maximal Ratio combining. At the same time remote unit are supposed to be small light weighted pocket communicators. Inevitably, the pocket communicators must remain simple. Also, antenna diversity at a mobile handset is more difficult to implement because of electromagnetic interaction of antenna elements on small platforms and the expense of multiple down conversion RF paths. Furthermore, the channels corresponding to different antennas are correlated, with the correlation factor determined by the distance as well as the coupling between the antennas. Typically, the second antenna is inside the mobiles handset, resulting in signal attenuation at the second antenna. This can cause some loss in diversity benefit. All these factors motivate the use of multiple antennas at the base station for transmission. This method is called transmit diversity [19].

Current transmission schemes over MIMO channels typically fall into two categories:

1. data rate maximization
2. diversity maximization

The first kind focuses on improving the average capacity behavior and second kind tries to minimize the outage probability, or equivalently maximize the outage capacity.

In these systems we analyze that each antenna receives a number of multipath random signals. So it is required to calculate probability density function for random wireless signals through multipaths received at receiving antenna.

So in this chapter we discuss the digital signaling over fading multipath channels.

Characterization of Fading Multipath Fading Channels

In this chapter, we consider the signal design and receiver performance for more complex channels i.e. channels having randomly time-variant impulse responses. This characterization serves as a model for signal transmission over many radio channels such as shortwave ionospheric radio transmission in 3-30 MHz frequency band (HF), tropospheric scatter radio communications in 300-3000MHz frequency band (UHF) and 3000-30000 MHz frequency band (VHF). The time-variant responses of these channels are a consequence of the constantly changing physical characteristics of the media. We shall begin our treatment of digital signaling over fading multipaths channels by first developing a statistical characterization of the channel.

If we transmit a very short pulse or impulse over a time-varying channel multipath channel, the received signal is appeared as a train of pulses shown in fig., [40]. Hence, one characteristic of a multipath medium is the time spread introduced in the signal that is transmitted through the channel.

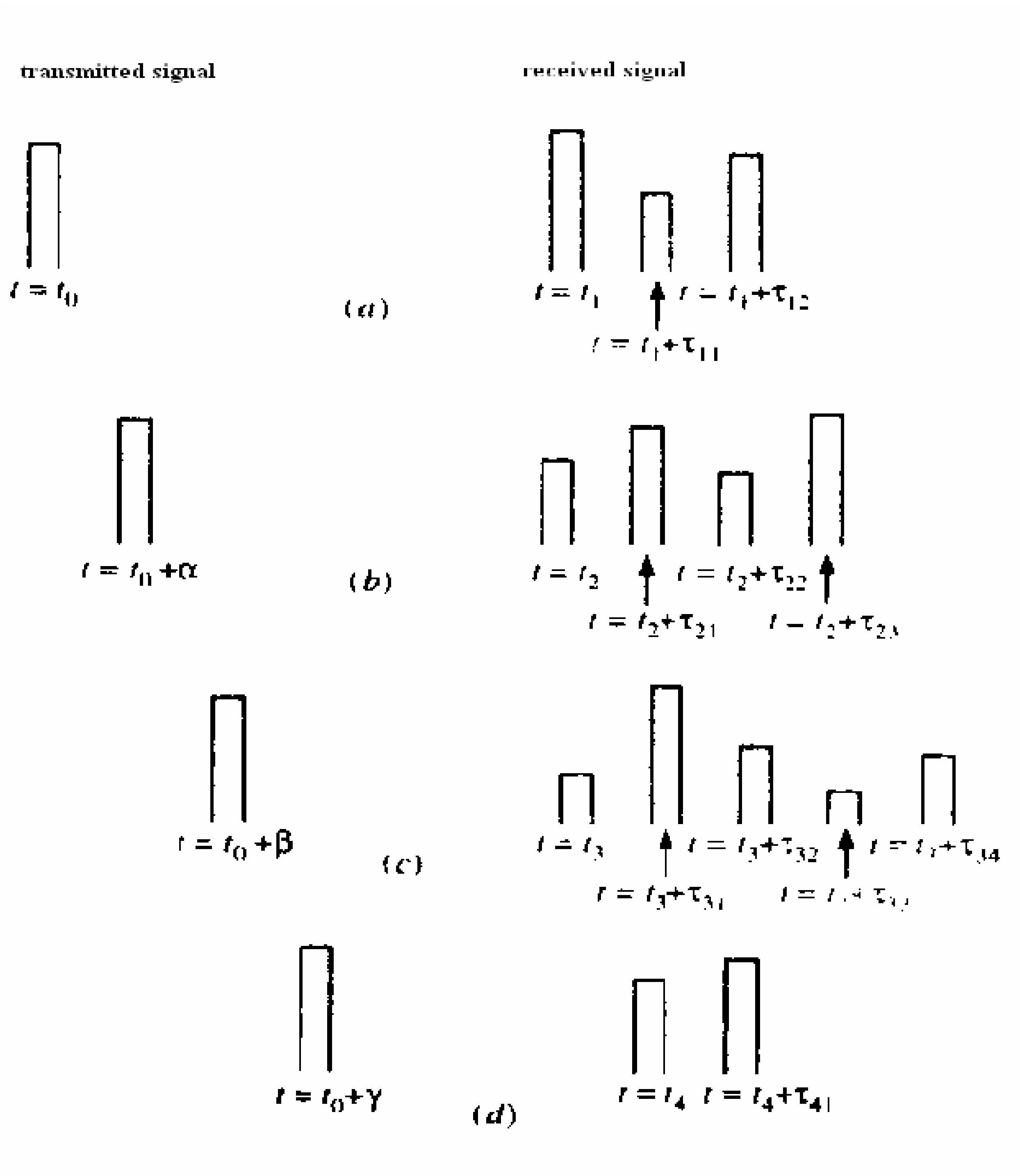

Fig 2.5 Example of response of time-variant multipath channel to a very narrow pulse

Now lets examine the effects of the channel on the transmitted signal that is represented as

$$s(t) = \mathrm{Re}[s_i(t)\exp(j2\pi f_c t)] \tag{2.1}$$

We assume that there a number of multiple paths. Associated with each multipath there is a propagation delay and attenuation factor. Thus the received bandpass signal may be expressed as

$$x(t) = \sum_n \alpha_n(t)s(t-\tau_n(t)) \tag{2.2}$$

Where $\alpha_n(t)$ is the attenuation factor for the signal received on n[th] path and $\tau_n(t)$ is the propagation delay for n[th] path. Here the value of n is not given i.e. it may be an random quantity. So it is desired to calculate the pdf (probability density function) of received vector of length n.

Substitution s(t) from (2.1) into (2.2) the following result can be obtained

$$x(t) = \mathrm{Re}\left\{\left[\sum_n \alpha_n(t)\exp(-j2\pi f_c\tau_n(t)s_i(t-\tau_n(t))\right]\exp(j2\pi f_c t)\right\}$$

(2.3)

The equivalent lowpass received signal is,

$$r_i(t) = \sum_n \alpha_n(t)\exp(-j2\pi f_c\tau_n(t)s_i(t-\tau_n(t)) \tag{2.4}$$

Where $r_i(t)$ is the response of the equivalent lowpass channel to equivalent lowpass signal $s_i(t)$. It follows that equivalent lowpass channel is described by the time-variant impulse response

$$c(\tau;t) = \sum_n \alpha_n(t)\exp(-j2\pi f_c\tau_n(t)\delta(t-\tau_n(t))) \qquad (2.5)$$

For some channels, such as the tropospheric scatter channel, it is more appropriate to view the received signal as consisting of a continuum of multipath components. In such cases, the received signal x(t) is expressed in integral form

$$x(t) = \int_{-\infty}^{\infty} \alpha(\tau;t)s(t-\tau)d\tau \qquad (2.6)$$

Where $\alpha(\tau;t)$ denotes the attenuation of the signal components at delay τ and time instant t. now putting value from (2.1) into (2.6),

$$x(t) = \mathrm{Re}\left\{\left[\int_{-\infty}^{\infty} \alpha(\tau;t)\exp(-j2\pi f_c\tau s_i(t-\tau)d\tau\right]\exp(j2\pi f_c t)\right\} \qquad (2.7)$$

Since the integral in (2.7) represents the convolution of the $s_i(t)$ with an equivalent lowpass time-variant impulse response $c(\tau;t)$, it follows that,

$$c(\tau;t) = \alpha(\tau;t)\exp(-j2\pi f_c\tau) \qquad (2.8)$$

Where $c(\tau;t)$ represents the response of the channel at time t due to an impulse applied at time $(t-\tau)$.

Now let us consider the transmission of an unmodulated carrier at frequency f_c. Then $s_i(t) = 1$ for all t, and, hence, the received signal for the case of discrete multipath reduces to

$$r_i(t) = \sum_n \alpha_n(t)\exp(-j2\pi f_c\tau_n(t)$$

$$= \sum_n \alpha_n(t)\exp(-j\theta_n(t)$$

$$= A(t)\exp(j\Phi(t)) \quad (2.9)$$

Where $\theta_n(t) = 2\pi f_c \tau_n(t)$. Thus, the received signal consists of the sum of a number of time-variant vectors (phasors) having amplitudes $\alpha_n(t)$ and phases $\theta_n(t)$. Note that large dynamic changes in the medium are required for $\alpha_n(t)$ to change sufficiently to cause a significant change in the received signal. On the other hand $\theta_n(t)$ will change by 2π rad with relatively small motions of the medium. This implies that the received signal is modeled as a random process. When there are a large number of paths, the central limit theorem can be applied i.e. $r_i(t)$ can be modeled as complex-valued Gaussian random process. Akin to the problem mentioned in [39], we will derive an expression for the pdf of A in terms of an infinite series containing Laguerre polynomials in subsequent chapters.

CHAPTER 3

RADAR SCATTERS

Complex radar targets are often modeled as a number of individual scattering elements randomly distributed throughout the spatial region containing the target. While it is known that as the number of scatterers grows large the distribution of the scattered signal power or intensity is asymptotically exponential, this is not true for a small number of scatterers. We study the statistics of measured power or intensity, and hence scattering cross section, resulting from a small number of constant amplitude scatterers each having a random phase. We derive closed-form expressions for the probability density function (pdf) of the scattered signal intensity for one, two, and three scatterers having arbitrary amplitudes [23]. For $n > 3$ scatterers, we derive expressions for the pdf when the individual scatterers have identical constant amplitudes and independent random phases.

A common model for complex or extended radar targets is to consider them to consist of a collection of randomly distributed scattering elements. Each scattering element making up the extended target is assumed to be a point target or isotropic scatterer, and each scattering element within the radar resolution cell under consideration contributes a component to the total echo signal from that resolution cell. The statistics of the resulting radar cross-section arising from the interfering scattered components from the target are difficult to derive. While the distribution of the energy scattered from a collection of Rayleigh scatterers or a collection of Rayleigh scatterers plus a constant scatterer can be easily derived, the statistical description of a fixed number of scatterers with constant (nonrandom) amplitudes, randomly distributed in space, is not generally known. While it is known that for a large number of scatterers, the resulting scattering ensemble will exhibit Rayleigh scattering if the scatterers are randomly scattered throughout a region whose dimensions are large compared to the wavelength of the illuminating radiation, this is not the case when the number of scatterers is small.

In this section, we derive expressions for the probability distribution of the power or intensity of the scattered signal from, or equivalently the radar cross section of, a collection of constant scatterers randomly distributed in space within a radar resolution cell. We first derive exact closed-form expressions for the probability density function (pdf) of radar cross section arising from one, two, and three constant-amplitude scatterers (based on a single look) as a function of the scatterers' amplitudes using a recursive algorithm. For four or more equal amplitude scatterers, we derive an orthonormal series representation of the pdf in terms of exponentially weighted Laguerre polynomials. For the multi-look case, we obtain a closed-form expression for the distribution of the sum of the intensities of each of the individual looks for two coherent scatterers based on two looks.

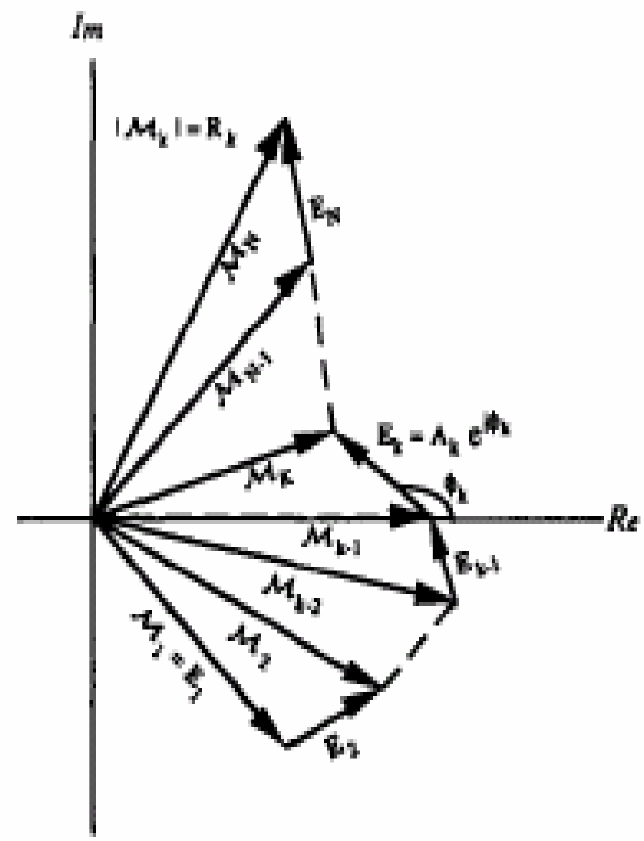

Fig 3.1 Geometry of radar scattering

We expect the pdf expressions and approximations derived in this thesis to be useful in extraction and estimation of surface roughness information from radar measurements of surfaces whose scattering characteristics are dominated by a relatively small number of scatterers per resolution cell. Specifically, the closed-form expressions derived for the probability density function of the intensity and their associated orthogonal series expansions will be the key to formulating parametric estimators to determine the surface characteristics of synthetic aperture radar (SAR) images from partially developed speckle measurements.

3.1 Scattering Cross-Section Model

3.1.1 Single-Look Model

In this model, a radar resolution cell is assumed to contain a collection of n elemental point scatterers randomly distributed throughout the resolution cell with each scatterer position distributed independently of the positions of other scatterers. For example, the scatterers could be randomly distributed on a surface being imaged by an imaging radar and falling within the particular resolution cell of interest. Each backscattered electric field component Ej from the jth scatterer, **j** = **1,** . .., **n**, has a constant amplitude $\mathbf{A_j}$ proportional to the size or reflectance strength of the jth scatterer and a random phase $\mathbf{\Phi_j}$ uniformly distributed over the interval **[O, 2 π]**

$$E_j = A_j e^{i\Phi_j}$$

The random phases of the elementary backscattered electric fields are also assumed to be statistically independent, because the random spatial locations of the individual scatterers are statistically independent. We assume that the number of elementary scatterers n is fixed, although n could in general be considered random, in which case the distributions we derive for fixed n would become conditional intensity distributions conditioned on having a given number of scatterers within a resolution cell. The superposition of the radar returns from each of the n elementary scatterers gives rise to the total backscattered electric field from the resolution cell as

$$M_n = \sum_{j=1}^{n} E_j \quad (3.1)$$

The overall intensity measurement of the radar target is proportional to the square of the magnitude of M, (intensity) given by

$$S_n = \left| \sum_{j=1}^{n} A_j e^{i\phi_j} \right|^2 \quad (3.2)$$

3.1.2 Multi-Look Model

In the multilook model, L-independent diversity measurements are taken over the resolution cell by the radar. These diversity measurements can be made by changing the carrier frequency of the radar or moving the radar with respect to the target between measurements to randomize the phases of the returns from the individual scatterers. The multilook technique involves the noncoherent sum of L statistically independent single realizations of the intensity measurements $S_{nl}(l = 1,2,..L)$ in (2) at each resolution cell

$$T_{nL} = \sum_{l=1}^{L} S_{nl} \quad (3.3)$$

3.2 PDF of Single-Look Intensity Measurement

3.2.1 Recursive Method

Fig. 1 shows the geometry of the scattering problem we are examining which can be described as a random walk of phasor components in the complex plane.

Let the electric field reflected from the jth elementary scatterer be denoted by

$$E_j = A_j e^{i\Phi_j} \tag{3.4}$$

Where A_j is the scatter amplitude assumed to be a fixed constant, and phase ϕ_j is uniformly distributed on the interval {0,2π). The electric field resulting from the coherent sum of k elementary scatterers by

$$M_k = \sum_{j=1}^{k} E_j \tag{3.5}$$

and its magnitude by R_k for k = 1,2,…..n. Then we define the radar cross section as $S_k = R_k^2$.

Assume the intensity measurement S_{k-1} is known. We then can assume that the phase associated with M_{k-1} is zero. Now we can write,

$$\begin{aligned} S_k &= \left| R_{k-1} + A_k e^{j\phi_k} \right|^2 \\ &= R_{k-1}^2 + A_k^2 + 2A_k R_{k-1}\cos(\phi_k) \\ &= S_{k-1} + A_k^2 + 2A_k\sqrt{S_{k-1}}\cos(\phi_k) \end{aligned} \tag{3.6}$$

The conditional cumulative distribution function of S_k conditioned on s_{k-1} can be written as

$$\begin{aligned} F_{S_k/S_{k-1}}(s_k/s_{k-1}) &= P(S_k \le s_k / s_{k-1} = s_{k-1}) \\ &= P(A_k^2 + s_{k-1} + 2A_k\sqrt{s_{k-1}}\cos(\phi_k) \le s_k \end{aligned}$$

$$= 1 - P\left(\left|\phi_k\right| > \cos^{-1}\left(\frac{s_k - A_k^2 - s_{k-1}}{2A_k\sqrt{s_{k-1}}} \right) \right)$$

$$= 1 - \frac{1}{\pi}\arccos\left(\frac{s_k - A_k^2 - s_{k-1}}{2A_k\sqrt{s_{k-1}}} \right) \qquad (3.7)$$

Differentiating (3.7) with respect to S_k, we get the conditional pdf, i.e.

$$p_{S_k/S_{k-1}}(s_k/s_{k-1})$$

$$= \begin{cases} \dfrac{1}{\pi\sqrt{\left(\left(\sqrt{s_k}+A_k\right)^2 - s_{k-1}\right)\left(s_{k-1} - \left(\sqrt{s_k}-A_k\right)^2\right)}} \\ \left(\sqrt{s_k}-A_k\right)^2 \le s_{k-1} \le \left(\sqrt{s_k}+A_k\right)^2 \\ 0, elewhere. \end{cases} \qquad (3.8)$$

Using Bayes' rule, the probability density function of S_k could be written as

$$p_{s_k}(s_k) = \int_0^\infty p_{S_k/S_{k-1}}(s_k \left| s_{k-1}\right.) p_{k-1}(s_{k-1}) ds_{k-1} \qquad (3.9)$$

Hence, the probability density function of the intensity measurement S_n could be recursively determined by successive applications of (8) and (9) for k = 2,3, . . , ***n*.** In the following analysis, we assume the relational order $A_{k-1} \le A_k$ for the scatterers amplitudes (k = 2,3,. . . , n).

(a) Exact PDF for the Case of a Single Scatterer

When measurements are taken over a resolution cell consisting of a single scatterer, it is straightforward to show that $S_1 = A_1^2$,

and hence

$$p_{s_1}(s_1) = \delta(s_1 - A_1)^2 \tag{3.10}$$

(b) Exact PDF for the Case of two Scatterers

For the case when the resolution cell consists of two scatterers (n = **2),** substituting (3.10) and (3.8) into (3.9) (with **k** = **2)** and applying the sifting property of the Dirac delta function yields

$$p_{s_2}(s_2) == \begin{cases} \dfrac{1}{\pi\sqrt{\left(\left(\sqrt{s_2}+A_2\right)^2 - A_1^2\right)\left(A_1^2 - \left(\sqrt{s_2}-A_2\right)^2\right)}} \\ \left(\sqrt{s_2}-A_2\right)^2 \le A_1^2 \le \left(\sqrt{s2}+A_2\right)^2 \\ 0, elewhere. \end{cases} \tag{3.11}$$

Or, the constraint $\left(\sqrt{s_k}-A_k\right)^2 \le s_{k-1} \le \left(\sqrt{s_k}+A_k\right)^2$ can be replaced by $\left(A_1 - A_2\right)^2 \le s_2 \le \left(A_1 + A_2\right)^2$.

Now equation (3.11) becomes

$$p_{s_2}(s_2) == \begin{cases} \dfrac{1}{\pi\sqrt{\left(s_2-(A_1-A_2)^2\right)\left(\left(A_1+A_2\right)^2-s_2\right)}} \\ \left(A_1-A_2\right)^2 \le s_2 \le \left(A_1+A_2\right)^2 \\ 0, elewhere. \end{cases} \tag{3.12}$$

3.2.2 Orthonormal Laguerre Polynomial Representation

In this, the pdf is expanded as a series of Orthonormal Laguerre Polynomials. The pdf $p_{s_n}(s_n)$ can be shown to be asymptotically exponential as *n* grows large

$$p_{s_n}(s_n) \sim \frac{1}{nA_0^2}\exp\left(-\frac{s_n}{nA_o^{\ 2}}\right) I_{[0,\infty)}(s_n) \tag{3.13}$$

Where the indicator function is defined as

$$I_A(s) = 1 \text{ for s} \in \text{A}$$
$$0 \text{ for s} \in \text{A}$$

Now using Gram – Charlier type of expansion and write $p_{s_n}(s_n)$ as a series of exponentially weighted orthonormal Laguerre polynomials given by

$$p_{s_n}(s_n) \sim \frac{1}{nA_0^2}\exp\left(-\frac{s_n}{nA_o^{\ 2}}\right). \cdot\left(1+\sum_{m=1}^{M_n} c_m L_m\left(\frac{s_n}{nA_o^{\ 2}}\right)\right) I_{[0,\infty)}(s_n) \tag{3.14}$$

where $L_m(s)$ are Laguerre polynomials defined by their expansion in powers of s, [22]

$$L_m(s) = \sum_{j=0}^{m} \frac{(-1)^j m!}{(m-j)!\,j!} s^j \tag{3.15}$$

Where M_n is the number of Laguerre polynomials terms in Gram-Charlier expansion. The first few Laguerre polynomials are:

$$L_0(s) = 1$$
$$L_1(s) = 1 - s$$
$$L_2(s) = 1 - 2s + s^2/2.$$

The Laguerre polynomials obey the following orthogonality condition with respect to an exponential weighting function

$$\int_0^\infty \exp(-x) L_m(x) L_k(x)\, dx = \delta_{mk} \tag{3.16}$$

Where δ_{mk} is the Kronecker delta function. In defining the Gram-Charlier expansion, the upper limit M, in the sum in (3.14) should actually be replaced by infinity; however, in practice it is only necessary to use a relatively small number of terms M,. We address the number of terms *M,* required for the case of *n* scatterers.

The coefficients c_m measure the departure of the pdf $p_{s_n}(s_n)$ from a pure exponential law and are to be determined. Let us consider the expression

$$\int_0^\infty \left(p_{sn}(x) - \frac{1}{nA_0^2} \exp\left(-\frac{x}{nA_o{}^2} \right) \right) L_m\left(\frac{x}{nA_o{}^2} \right) dx$$

$$= \sum_{k=1}^{M_n} c_k \int_0^\infty \frac{1}{nA_0^2} \exp\left(-\frac{x}{nA_o^2}\right).L_k\left(\frac{x}{nA_o^2}\right)L_m\left(\frac{x}{nA_o^2}\right)dx$$

$$= \sum_{k=1}^{M_n} c_k \delta_{km}$$

$$= c_m$$

after applying the orthogonality condition of (3.16). Using the property,

$$\int_0^\infty \frac{1}{nA_0^2} \exp\left(-\frac{x}{nA_o^2}\right).L_m\left(\frac{x}{nA_o^2}\right)dx = 0 \tag{3.17}$$

It follows that

$$c_m = E\left(L_m\left(\frac{s_n}{nA_o^2}\right)\right) \tag{3.18}$$

The series coefficients can be approximated using a maximum likelihood estimator equal to the sample mean of a random sample of scattering cross sections $\{s_{nk}\}$

$$c_m = \frac{1}{k}\sum_{k=1}^{K} L_m\left(\frac{s_{nk}}{nA_0^2}\right) \tag{3.19}$$

In this chapter, we discussed the statistics of the radar cross-section, or equivalently the intensity of the scattered field, of a small collection of random scatterers. The goal was to determine either the exact pdf for a given number n of scatterers with amplitudes $a_1,a_2,..a_n$ or an accurate approximation to the pdf that would be useful in numerical computation. We derived exact closed-form expressions for the intensity's pdf as a function of the scatterers' constant amplitudes when the number of scatterers within a resolution cell is one, two, and three, and a single look is taken over the resolution cell.

When the number of scatterers within a resolution cell was greater than three, we used a Gram-Charlier type of expansion and represented the pdf as an orthonormal series of exponentially-weighted Laguerre polynomials; in the case of the Gram-Charlier approximations, it was assumed that all scatterers had equal amplitude. The series coefficients were estimated using a maximum likelihood estimator.

CHAPTER 4

PROBABILISTIC ANALYSIS OF RECEIVED WIRELESS SIGNALS

In wireless communications, rapid small-scale fading of the received radio signal envelope seriously degrades system error probability performance. This small-scale or short-term or multipath fading is due to the superposition of multiple reflections of a transmitted electromagnetic wave by local scatterers surrounding a wireless receiver. Several multipath models have been suggested in to describe the statistical characteristics of this fading process. In all of these works, the probability density functions (pdf) of signal amplitude and phase are the two key functions that have been studied, since they are useful for system design, such as the choice of diversity, equalization, and error control coding techniques, and error probability performance analysis. Thus, One of the most common methods for characterizing a fading channel is the use of a probability density function (pdf), which represents the probability density of the received signal strength. The shape of a pdf determines the performance of a wireless receiver in the presence of noise and interference.

In a multipath fading channel the transmitted signal travels through several different paths to the receiver. In each path, amplitude and phase of the signal vary in a random manner. It is common to consider the number of paths as a large constant and to model random fluctuations of the phase by the uniform probability density function (PDF). However, these assumptions are not realistic in many cases. In this, a general multipath fading channel with random number of paths and nonuniform phase distributions is considered and it is shown that the envelope fluctuates according to a gamma PDF. It is also shown that the parameters of this gamma PDF are directly related to the physical parameters of the channel. Due to the realistic assumptions made in the derivation, the gamma PDF is a promising candidate for accurate modeling of envelope statistics in multipath fading channels.

4.1 Receiver Model

The summation of constant-amplitude waves with independent random phases provides a useful mathematical description of narrow-band local area UHF and microwave propagation. The CW complex baseband voltage, A, as seen by a receiver has the following form:

$$A\exp(j\Phi_i) = \sum_{i=1}^{n} A_i \exp(j\Phi_i) = X + jY \qquad (4.1)$$

The A_i's are the amplitudes of multipath waves and the Φ_i's are their corresponding phases. For local-area propagation, the phase variables, Φ_i are treated as statistically independent random phase variables, uniformly distributed over the interval.

The envelope pdf is crucial because it determines the range of received signal strengths. Since received power is proportional to the square of received envelope, the envelope pdf can determine the ultimate Shannon channel capacity of a fading wireless link.

4.2 Calculation of Envelope pdf

This section reviews the basic method for generating envelope pdfs for received voltage signal. But before deriving the formula for calculating envelope pdf, it is necessary to understand following terms:

4.2.1 Reduced Wave Grouping

In free space, it is possible to decompose the propagating waves into a collection of plane-wave voltages at the terminals of an antenna. In a local area, those voltages are primarily due to homogeneous plane waves*,* with the sum being written in the form of (1) [41]. From this point, we can group parts of the summation in (1) into three different categories.

1. **Specular Component:-** A specular component is a single term,{ $A_i\exp(j\Phi_i)$} , in the summation of (1), representing one arriving multipath wave. The phase of a specular component is random, but the envelope is constant.

2. **Nonspecular Component:-** A nonspecular component is a group of two or more terms in the summation of (1), representing more than one multipathwaves arriving at the receiver.

3. Diffuse Component:- A diffuse component is a nonspecular component with numerous individual waves, each carrying power that is negligible compared to the total average power of the diffuse component. In a local area, it is possible to write the channel as the sum of a finite number of specular waves and a nonspecular component.

Each specular wave represents a multipath wave with relatively large signal strength. The diffuse component represents a group of relatively weak multipath waves, whose individual signal power can be assumed to be insignificant compared to the total diffuse power.

In summary, we are motivated to derive an alternate envelope pdf for an arbitrary *N* that can be computed efficiently. By using an orthogonal expansion, we derive a pdf expression as a series of Laguerre polynomials and in terms of their moments in Section II. We show that this pdf formula can be computed efficiently, and it reduces to several previously known fading envelope pdf formulas.

4.2.3 General Random Vector Problem and its associated PDF

Consider n random vectors with lengths A_i's and angles Φ_i 's, Where n is a deterministic variable. For I =1,….,n, Φ_i's are independent random variables with uniform pdf's on [0,2π], A_i's are arbitrary dependent positive random variables, and A_i's are independent of Φ_i's. Summation of these random vectors results in a random vector with length A and angle Φ, defined as:-

Now we have

$$A\exp(j\Phi_i) = \sum_{i=1}^{n} A_i \exp(j\Phi_i) = X + jY$$

$$X = A\cos\phi = \sum_{i=1}^{n} A_i \cos\phi_i$$

$$Y = A\sin\phi = \sum_{i=1}^{n} A_i \sin\phi_i \tag{4.2}$$

The joint characteristic function of X and Y is defined as

$$\psi_{XY}(u,v) = E_{XY}[\exp(juX + jvY]$$

Where E is the mathematical expectation. It can be written in terms of A_i and Φ_i given as:

$$\psi_{XY}(u,v) = E_{A_1 \ldots\ldots A_n, \phi_1 \ldots\ldots \phi_n}[\exp(juX + jvY] \tag{4.3}$$

$$= E_{A_1 \ldots\ldots A_n}\left[E_{\phi_1 \ldots\ldots \phi_n}[\exp(juX + jvY].[A_1 \ldots\ldots A_n]\right]$$

We know that A_i's are independent of Φ_i's, so condition in (4.3) can be written as

$$\psi_{XY}(u,v) = E_{A_1 \ldots\ldots A_n}\left[E_{\phi_1 \ldots\ldots \phi_n}\left[\exp(juX + jvY\right]\right] \tag{4.4}$$

Putting values of X and Y from (4.2) in (4.4), we get

$$\psi_{XY}(u,v) = E_{A_1 \ldots\ldots A_n}\left[E_{\phi_1 \ldots\ldots \phi_n}\left[\prod_{i=1}^{n} \exp(\text{ju } A_i \cos\phi\text{i} + \text{jv } A_i \sin\phi\text{i})\right]\right] \tag{4.5}$$

Due to independence of Φ_i's , (4.5) can be simplified as,

$$\psi_{XY}(u,v) = E_{A_1 \ldots\ldots A_n}\left[\prod_{i=1}^{n} \exp(\text{ju } A_i \cos\phi\text{i} + \text{jv } A_i \sin\phi\text{i})\right] \tag{4.6}$$

By introducing the new variables ρ and θ and in terms of u and v as

$$u = \rho \cos\theta \quad \text{and}$$
$$v = \rho \sin\theta$$
$$\rho = \sqrt{u^2 + v^2} \tag{4.7}$$

Using trigonometry identity

$$\cos C \cos D + \sin C \sin D = \cos(C - D) \text{ we can write}$$
$$u \cos \Phi_i + v \sin \Phi_i = \rho \cos(\Phi_i - \theta),$$

now equation (4.6) can be written as,

$$\psi_{XY}(u,v) = E_{A_1 \ldots A_n}\left[\prod_{i=1}^{n}\int_0^{2\pi} \exp(jA_i\rho\cos(\Phi_i - \theta)) f_{\Phi_i}(\Phi_i) d\Phi_i\right] \tag{4.8}$$

Where $f_{\Phi_i}(\Phi_i)$ is the pdf of Φ_i. The uniform distribution of each Φ_i on [0, 2π] means $f_{\Phi_i}(\Phi_i) = 1/2\pi$.

Now the integral form of the zeroth- order Bessel function is defined as,

$$j_o(z) = (1/2\pi)\int_0^{2\pi} \exp(jz\cos t)dt$$

Using this formula, equation (4.8) reduces to the following form:

$$\psi_{XY}(u,v) = E_{A_1 \ldots A_n}\left[\prod_{i=1}^{n} J_o(A_i\rho)\right]$$
$$= \Lambda(\rho) = \Lambda(\sqrt{u^2 + v^2}) \tag{4.9}$$

As we know the random n-vector X = (X_1,. .,X_n) is said to be spherically symmetric (SS) if its joint characteristic function (CF) can he expressed as a function of the quadratic form upu', where u = (u_1,….,u_n) and p is an n x n positive definite matrix.
Based on the definition of jointly spherically symmetric random variables [35], the functional form of $\Psi_{XY}(u,v)$ in terms of $\sqrt{u^2+v^2}$ implies that X and Y are jointly spherically symmetric random variables.

Normally, characteristic functions are designed to be Fourier transform pairs. However, we desire to calculate the envelope statistics of (4.1). Therefore, the envelope pdf, , and its characteristic function, , are defined as Hankel transform pairs,

$$\psi_{XY}(u,v) = \int_0^\infty f_A(a) J_0(zt)\,dt$$

$$f_A(a) = t\int_0^\infty \psi_{XY}(z) J(zt) z\,dz$$

Thus, it can be deduced that A and Φ_i are independent; Φ_i has a uniform pdf on, A and has the following pdf:

$$\begin{aligned} f_A(a) &= a\int_0^\infty \rho J_0(a\rho) \wedge (\rho)\,d\rho \\ &= aH_{0a}\{\Lambda(\rho)\} \end{aligned} \tag{4.10}$$

where $H_{0a}\{G(\xi)\}$ is the zeroth-order Hankel transform of G(.) defined as

$$H_{0z}\{G(\xi)\} = \int_0^\infty \xi J_0(z\xi) G(\xi)$$

Sometime it is called fourier-bessel transform. The Hankel transform of a function G(ξ) is valid at every point at which G(ξ) is continuous provided that the function is defined in (0, ∞), is piecewise continuous and of bounded variation in every finite subinterval in (0, ∞).

Generalized Laguerre expansion of $f_A(a)$

We want to derive the generalized Laguerre expansion [38], which gives the fading distributions $f_X(x)$ in terms a given set of orthogonal functions *{Un(x)}*, which is to be specified shortly.

The generalized Laguerre expansion is defined as,

$$f_X(x) = \sum_{n=0}^{\infty} C_n U_n(x); 0 \leq x \leq \infty$$

for any random variable *x* defined on the interval $\{0,\infty\}$. Now, our main objective is to determine the coefficients *Cn*.

The sum of two randomly-phased sine waves and Gaussian noise arises in various fields of communications. In this, a Laguerre series and also a power series are introduced, for the envelope PDF of this random process. Moreover, tight upper bounds are derived for the truncation error of these two infinite series. Comparison of these two upper bounds shows that the Laguerre series is superior to the power series; because for a fixed number of terms, it yields minimum truncation error.

Laguerre polynomials are a set of orthogonal polynomials on the positive real axis. So they make a useful basis for expanding the pdf of positive random variables. Based on the properties of the Hankel transform [36], it can be shown that

$$\Lambda(\rho) = H_{0\rho}\left[\frac{f_A(a)}{a}\right]$$

$$= \int_0^{\infty} J_0(\rho a) f_A(a) da = E_A[J_0(a\rho)] \qquad (4.11)$$

The following generating function for the laguerre polynomials [22] is given as:

$$\exp(\sigma) j_0(2\sqrt{\tau\sigma}) = \sum_{M=0}^{\infty} \frac{L_m(\tau)\sigma^m}{m!} \qquad (4.12)$$

Where $L_m(.)$ is the laguerre polynomial of order m. Assuming $\tau = \beta A^2$ and $\sigma = (\rho^2/4\beta)$, equation (4.12) gives the following parametric expansion for $J_0(A\rho)$, [21]:

$$J_0(A\rho) = \sum_{m=0}^{\infty} \frac{1}{m!(4\beta)^m} L_m(\beta A^2)\rho^{2m} \cdot \exp(-\frac{\rho^2}{4\beta}); \beta \neq 0 \tag{4.13}$$

where is β an arbitrary nonzero real number, introduced on purpose.

Now, by inserting (4.13) into (4.11), we can obtain

$$\Lambda(\rho) = \sum_{m=0}^{\infty} \frac{1}{m!(4\beta)} E_A[L_m(\beta A^2)]\rho^{2m} \exp(-\frac{\rho^2}{4\beta}), \beta \neq 0 \tag{4.14}$$

Now substitute the value of $\Lambda(\rho)$ from (4.14) into (4.10),

$$f_A(a) = a\sum_{m=0}^{\infty} \frac{1}{m!(4\beta)} E_A[L_m(\beta A^2)].$$

$$.\int_0^{\infty} \rho^{2m+1} J_0(a\rho)\exp(-\frac{\rho^2}{4\beta})d\rho \quad , \quad \beta \neq 0 \tag{4.15}$$

Now assume,

$$I = \int_0^{\infty} \rho^{2m+1} J_0(a\rho)\exp(-\frac{\rho^2}{4\beta})d\rho \tag{A}$$

From table of integral [40], it is known that

$$\int_0^{\infty} e^{-x^2} . x^{2n+u+1} J_u(2x\sqrt{z})dx = \frac{n!}{2} e^{-z} z^{\frac{1}{2}u} L_n^u(z) \tag{B}$$

Now substituting $\frac{\rho^2}{4\beta} = x^2 \Rightarrow \rho d\rho = (4\beta)xdx$ in (A),

$$I = \int_0^\infty (\sqrt{4\beta}.x)^{2m} J_0(a\rho)e^{-x^2}.\rho d\rho$$

$$= \int_0^\infty (4\beta)^{m+1}(x)^{2m+1} J_0(a\rho)e^{-x^2} dx$$

$$= (4\beta)^{m+1}\int_0^\infty (x)^{2m+1} J_0(a\rho)e^{-x^2} dx$$

$$= \frac{(4\beta)^{m+1}.m!}{2}\exp(-\beta a^2)L_m(\beta a^2)$$

Hence, $\int_0^\infty \rho^{2m+1} J_0(a\rho)\exp(-\frac{\rho^2}{4\beta})d\rho$

$$= \frac{(4\beta)^{m+1}.m!}{2}\exp(-\beta a^2)L_m(\beta a^2), \quad 0<\beta<\infty \tag{4.16}$$

Now putting (16) into (4.15), the following result can be obtained,

$$f_A(a) = 2\beta a\exp(-\beta a^2)\sum_{m=0}^{\infty} C_m L_m(\beta a^2), \qquad 0<\beta<\infty \tag{4.17}$$

Where by definition ,

$$C_m = E_A[L_m(\beta A^2)]$$

It should be mentioned that the infinite series for $f_A(a)$ in (4.17) is obtained just by employing the fact that $f_A(a)$ and $\Lambda(\rho)$ constitute a Hankel transform pair [see (10)], along with the use of the Laguerre generating function in (4.12). In fact, not only the pdf of A but also the pdf of an arbitrary positive random variable P can be expressed similar to (4.17) . The functional form of $\Lambda(\rho)$ in (4.9) only affects the coefficients in C_m (4.17).

The main advantage of a variable β , instead of a predetermined constant, lies on the fact that β can be selected in such a way to minimize the truncation error of (4.17). Truncation error is arised due to finite number of N.

4.2.4 Closed–form Formula for C_m

Definition of the th-order Laguerre polynomial implies that,

$$L_m(z) = \sum_{m=0}^{\infty} \frac{(-1)^k m!}{(m-k)!(k!)^2} z^k \tag{4.19}$$

So C_m in (4.18) can be written as

$$C_m = \sum_{m=0}^{\infty} \frac{(-\beta)^k m!}{(m-k)!(k!)^2} \mu_n^{(2k)} \tag{4.20}$$

Where $\mu_n^{(2k)}$ is the 2kth moment of A, which resulted from the sum of n random vectors

$$\mu_n^{(2k)} = E_A[A^{2k}] \tag{4.21}$$

Inspection of (4.20) shows that $\mathbf{C_m}$ is a linear combination of $\mu_n^{(2k)}$'s. Hence, for computing $\mathbf{C_m}$, it is useful to obtain a closed-form formula for $\mu_n^{(2k)}$.

For $\beta \to \pm\infty$, the expansion in (4.13) reduces to the Maclaurin series of $J_0(A\rho)$, and simplified to,

$$\Lambda(\rho) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k!)^2 4^k} \mu_n^{(2k)} \rho^{2k} \qquad (4.22)$$

Comparison of (4.22) with the Maclaurin series of $\Lambda(\rho)$ reveals that

$$\mu_n^{(2k)} = -(4)^k \frac{(k!)^2}{(2k)!} \partial^{2k} \Lambda(\rho) / \partial \rho^{2k} \qquad (4.23)$$

When calculation of $E_{A1....An}$ in (4.9) is possible in a closed form, $\mu_n^{(2k)}$'s and consequently C_m, can be computed via (4.23) and (4.20), respectively. It should be mentioned that (23) holds not only for A but also for an arbitrary positive random variable P, where $\Lambda(\rho) = E_p[J_0(P_\rho)]$. However, the corresponding $\Lambda(\rho)$ for A is presented in (4.9).

In (4.23) we must compute multiple derivatives of a function. For the case in which A_i's are independent, there is a recursive relation for $\mu_n^{(2k)}$.

Let us consider N independent C.S.(circular symmetric) pairs $(X_1,Y_1),(X_2,Y_2),\ldots,(X_n,Y_n)$. Define the moments V_i^{2k} and μ_j^{2k} by

$$v_i^{2k} = E[(X_i^2 + Y_i^2)^k], \quad i = 1,2,\ldots,N, \; k=1,2\ldots\ldots$$

$$\mu_j^{2k} = E\left\{\left[\left(\sum_{i=1}^{j} X_i\right)^2 + \left(\sum_{i=1}^{j} Y_i\right)^2\right]^k\right\} \quad J = 1,2\ldots.N, \; K = 1,2\ldots$$

In this way, we can derive a simple expression for the Moments μ_j^{2k} in terms of the moments $\{v_i^{2k,}$ $j = 0,\ldots,n;\ i =1, 2\ldots,N\}$, which are assumed known.

This relation is given as,

$$\mu_j^{2k} = v_1^{2k} \qquad l = 1, \quad k = 0,1\ldots \quad \text{And}$$

$$\mu_j^{2k} = \sum_{i=0}^{k} \left(\frac{k!}{i!(k-i)!} \right)^2 \mu_{l-1}^{2i} v_l^{(2k-2i)}$$

$$l = 2,\ldots n \ , k = 0,1\ldots \qquad (4.24)$$

In which v_l^{2k} is defined as

$$v_l^{2k} = \mathrm{E}_{A_1}[A_l^{2k}],$$

$$l = 1,\ldots\ldots n, \ k = 0,1,2\ldots.. \qquad (4.25)$$

4.3 Results Applied to a Typical Example

The results obtained can be applied to a random vector problem describing the scattering cross section of a small number of random scatterers. a random vector model is employed to investigate the statistical behavior of the scattering cross section, when the number of scatterers is small.

Complex radar targets are often modeled as a number of individual scattering elements randomly distributed throughout the spatial region containing the target [32],[31]. While it is known that as the number of scatterers grows large the distribution of the scattered signal power or intensity is asymptotically exponential, this is not true for a small number of scatterers. We study the statistics of measured power or intensity, and hence scattering cross section, resulting from a small number of constant amplitude scatterers each having a random phase. We derive closed-form expressions for the probability density function (pdf) of the scattered signal intensity for one, two, and three scatterers having arbitrary amplitudes. Here it is considered the sum of n random vectors with length A_i's and angles Φ_i's, where n is a deterministic variable, $A_1 = \ldots\ldots A_n = A_0$, where A_0 is a positive deterministic variable, and Φ_i's are independent random variables with uniform pdfs On $[0,2\pi]$.

Based on the above assumptions, an orthonormal Laguerre polynomial representation is presented for the pdf of A,

$$S_n = \left| \sum_{j=1}^{n} A_j e^{i\phi_j} \right|^2$$

By noting $S_n = A^2$ and using (17) and (18) assuming $\beta = \frac{1}{nA_0^2}$, the pdf of S_n can be written as,

$$f_{s_n}(s_n) = \frac{1}{nA_0^2} \exp\left(-\frac{s_n}{nA^2}\right) \sum_{m=0}^{\infty} c_m L_m \left(\frac{s_n}{nA_0^2} \right) \qquad (4.26)$$

Where c_m is defined as

$$c_m = E_{s_n}\left[L_m \left(\frac{s_n}{nA^2} \right) \right] \qquad (4.27)$$

Note that there is the following relationship between the c_m and the C_m defined as,

$$c_m = C_m \Big|_{\beta = 1/nA_0^2} \qquad (4.28)$$

Using (4.20) c_m can be calculated, assuming $\beta = 1/nA_0^2$ and using the formula for μ_n^{2k} .

A maximum likelihood estimator can be used to approximate the coefficients of c_m.

Thus, the envelope pdf of sum of random sinusoidals obtained in this chapter is of important in various applications. Based on the results obtained by this method, the time-consuming Monte-Carlo simulations for determining the envelope pdf can be completely avoided. Moreover, this approach expresses the envelope pdf just in terms of polynomials, while the Fourier-Bessel series expands the envelope pdf in terms of Bessel functions which is definitely more complicated than polynomials from both numerical and analytical points of view. Thus, based on these results, and either numerically or analytically, performance of various modulation and coding schemes in general multipath fading channels can be assessed, efficient detection procedures may be developed in radars assuming various clutter pdf's and for different target cross section pdf's, error probability in the presence of several interferers can be determined, suitable speckle reduction techniques can be developed against different scattering conditions.

CHAPTER 5

SIMULATION RESULTS

The simulation results obtained in this thesis are verified using MATLAB. Here the required probability density functions (pdf) are obtained for the application of Radar scatters. The exact and approximated coefficients (maximum-likelihood estimator) are used to calculating pdf for various scatters (n). The exact coefficients and maximum-likelihood estimator shown in equations (3.18) and (3.19) respectively.

The intensity measurements $\{s_{nk}\}$, k= 1,2, . . , *K,* were generated using computerized Monte-Carlo simulation. Specifically, a pseudo-random number generator, employing the inverse distribution function method, is used to generate a large number (K = 1000) of statistically independent scattering ensembles made up of *n* unit-amplitude statistically independent random point scatterers with phases uniformly distributed over the interval **$[0,2\pi)$.**

The recursive expressions for the exact pdf in (3.8) and (3.9) and the orthogonal series expansion in (3.14) were numerically implemented for four, five, six, seven, and eight unit-amplitude scatterers. The exact pdfs were numerically computed using the recursive integration method.

For single-look intensity measurement, we note that the exact pdf approaches an exponential distribution as the number of the scatterers is gradually increased from four to seven, as illustrated in Figs. 5.4-5.7. For the case of four scatterers, a relatively high number of series terms *(M* = 17) is needed to approximate the pdf up to a 5% maximum relative error. An increase in the number of terms *M to* **20** or **25** merely reduces the maximum relative error to 4%. It takes as many as 35 terms to obtain a significantly lower maximum relative error of 2% at the cost of increasing the computational complexity of the Laguerre polynomial expansion. Considering the trade-off between computational complexity and relative error, we use only 17 terms in the series expansion and accept an increased maximum relative error of 5%. Increasing the number of scatterers from four to only five scatterers significantly reduces the number of series terms from 17 to seven while maintaining a maximum relative error of 5%. As the number of scatterers is further increased to six and seven scatterers, the pdf's converge faster to an exponential distribution, and only three and two series terms are needed, respectively, for a remarkably low maximum relative error of 2%. Furthermore, the graph of the Laguerre series expansion is almost indistinguishable from the exact pdf for the case of seven scatterers. Only one series term is needed for the case of eight scatterers to maintain a maximum relative error of 2%.

The below figure shows the exact probability density function for two unit amplitude scatterers ($A = A_2 = 1$). The pdf p(s) has singularities at points, $(A_1 - A_2)^2$ and $(A_1 + A_2)^2$ and at other points it is minimum.

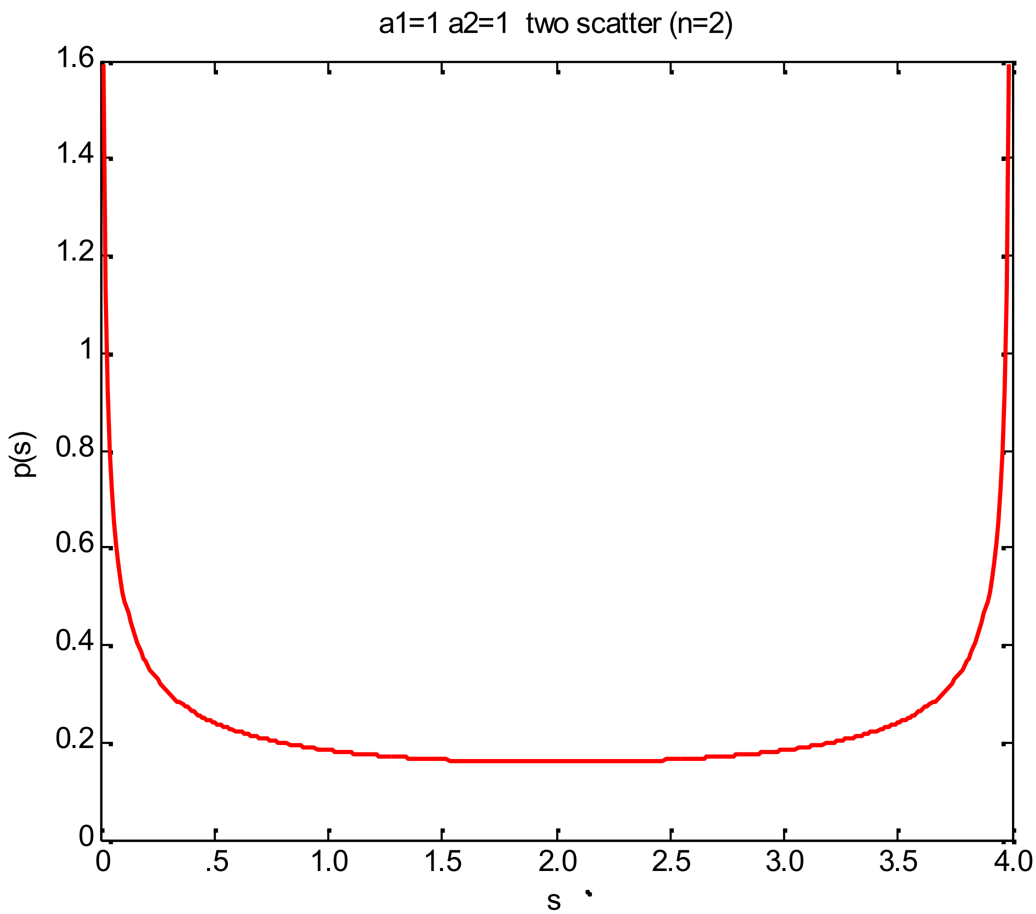

Fig 5.1: Exact Probability Density of Intensity For Two Unit-Amplitude Scatterers

Similarly the figures 5.2 and 5.3 show the pdf for two scatterers having unequal amplitude and similarly the plots for pdf p(s) have singularities at $(A_1 - A_2)^2$ and $(A_1 + A_2)^2$.

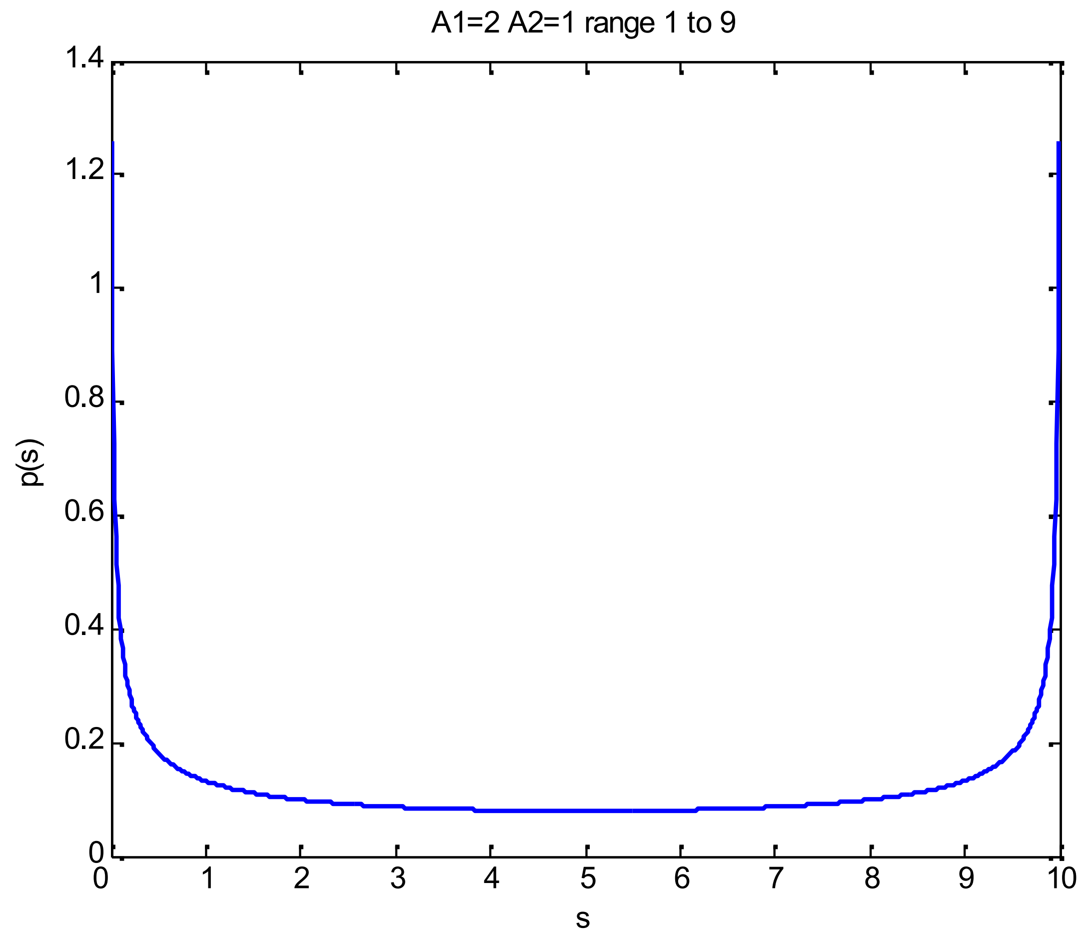

FIG 5.2: Exact Probability Density of Intensity For Two Different-Amplitude Scatterers (A1=2, A2=1)

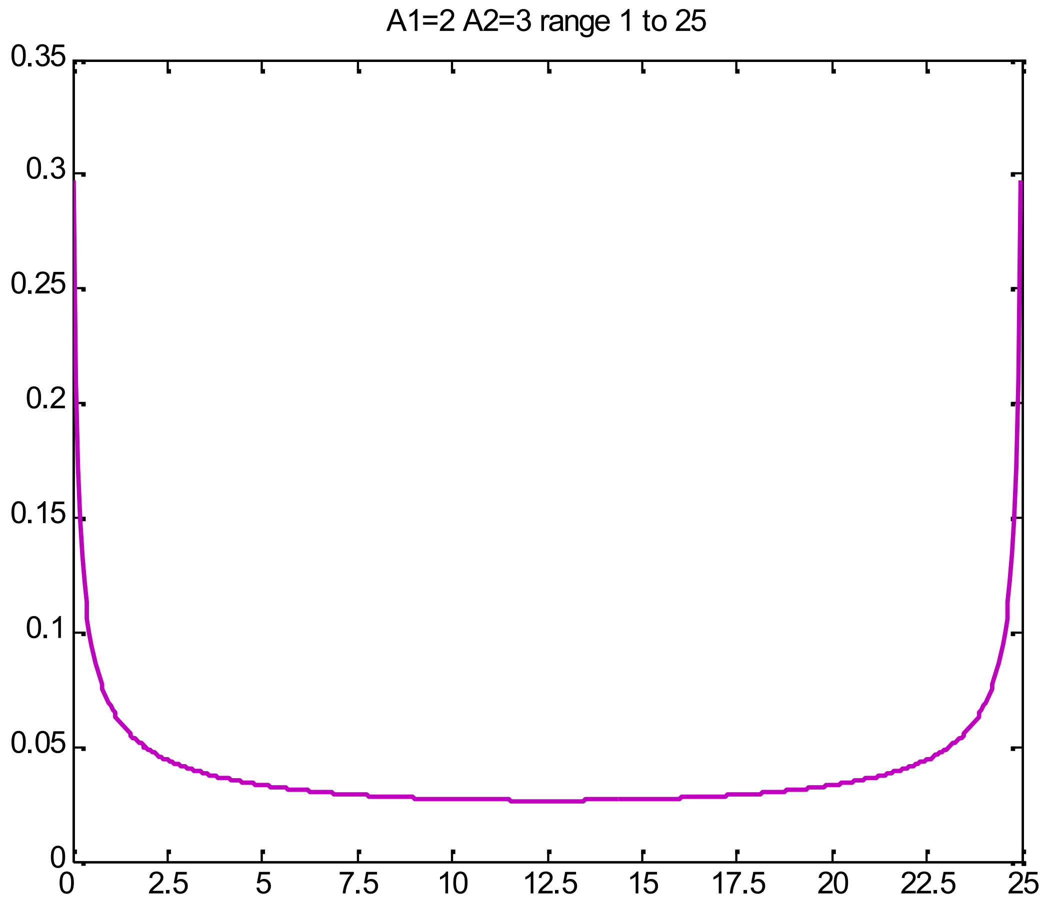

FIG 5.3: Exact Probability Density of Intensity For Two Different-Amplitude Scatterers (A1=2, A2=3)

The below figure shows the exact probability density function for different numbers of unit amplitude scatterers i.e. n=4,5,6,7. For single-look intensity measurement, it is found that the exact pdf approaches an exponential distribution as number of the scatterers is gradually increased from 4 to 7.

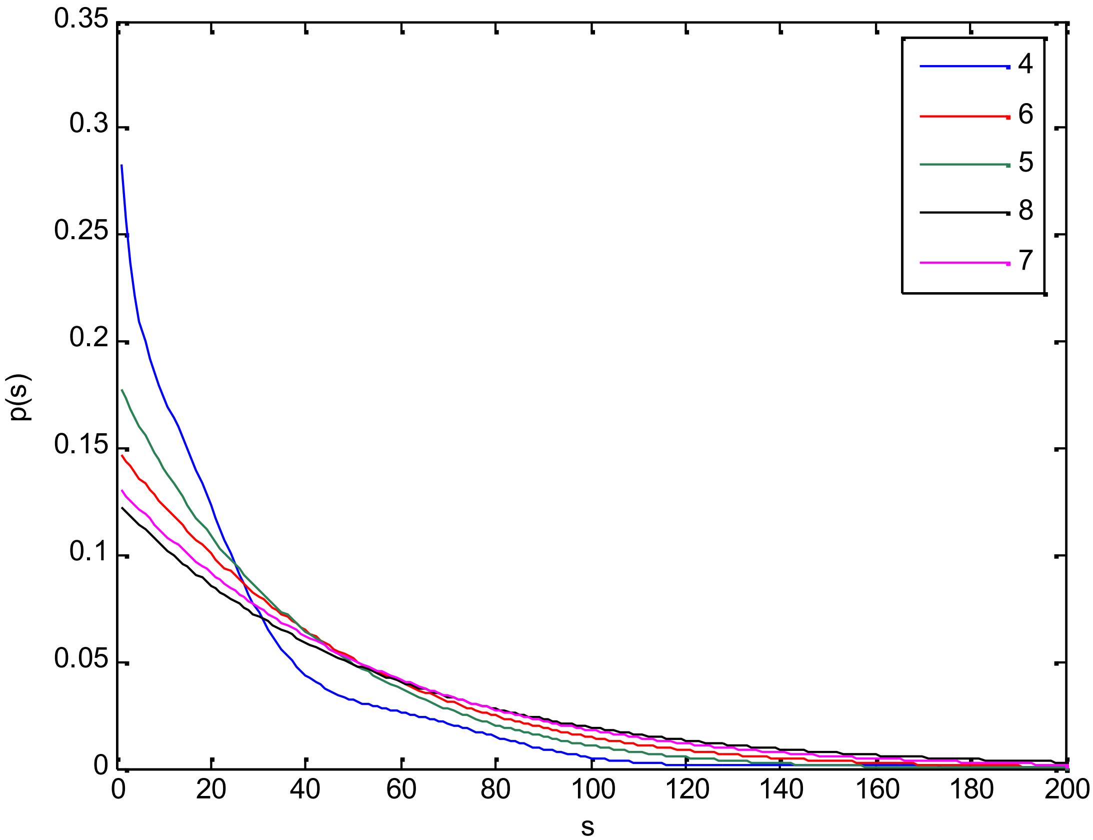

FIG 5.4: : Exact Probability Density of Intensity as Number (N) of Unit Amplitude Scatterers Increase (N=4,5,6,7,8).

The following figure shows the corresponding laguerre series expansions of exact pdf for different number of unit scatterers. The pdf for different value of n is almost similar to exact pdf obtained in fig 5.4.

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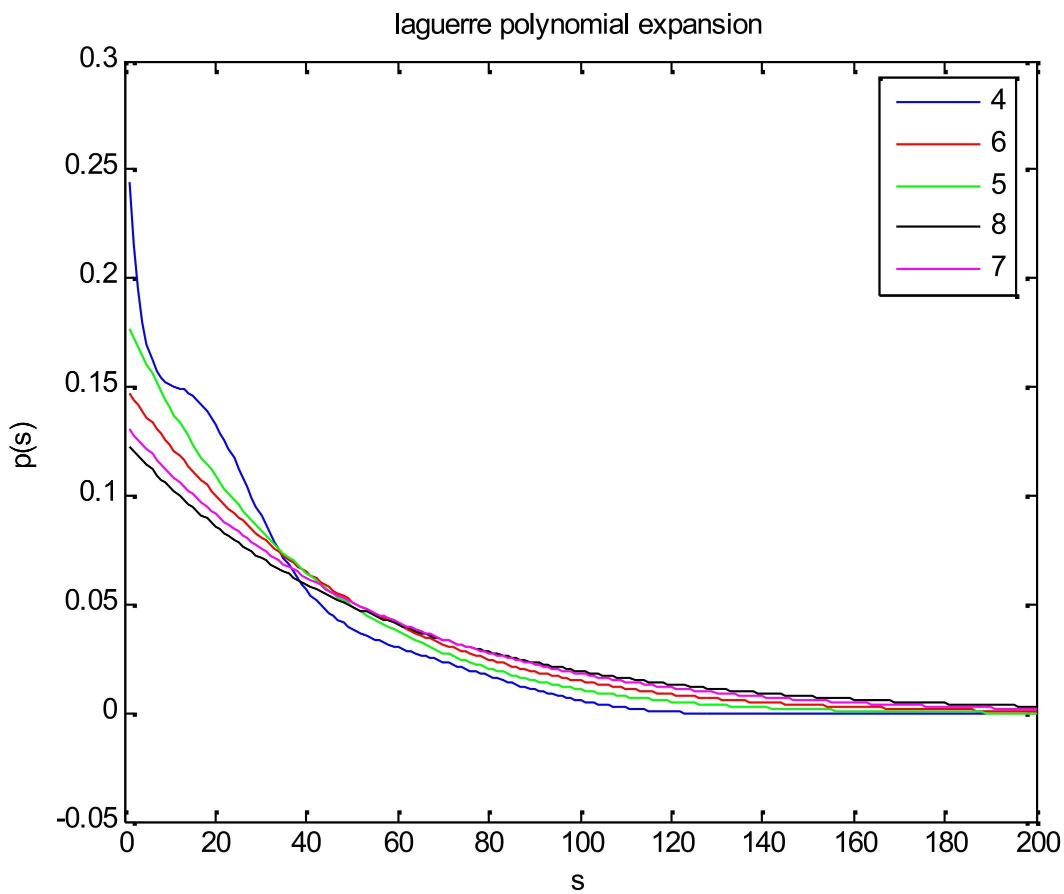

FIG 5.5: Probability Density of Intensity By Laguerre Polynomial Expansion as Number (N) of Unit Amplitude Scatterers Increase (N=4,5,6,7,8).

The following figure shows the graph of exact pdf for four single look unit amplitude scatterers versus its corresponding series expansion. For the case of four scatterers, a relatively high number of series terms(M = 17) is required to approximate the pdf up to a 5% maximum relative error.

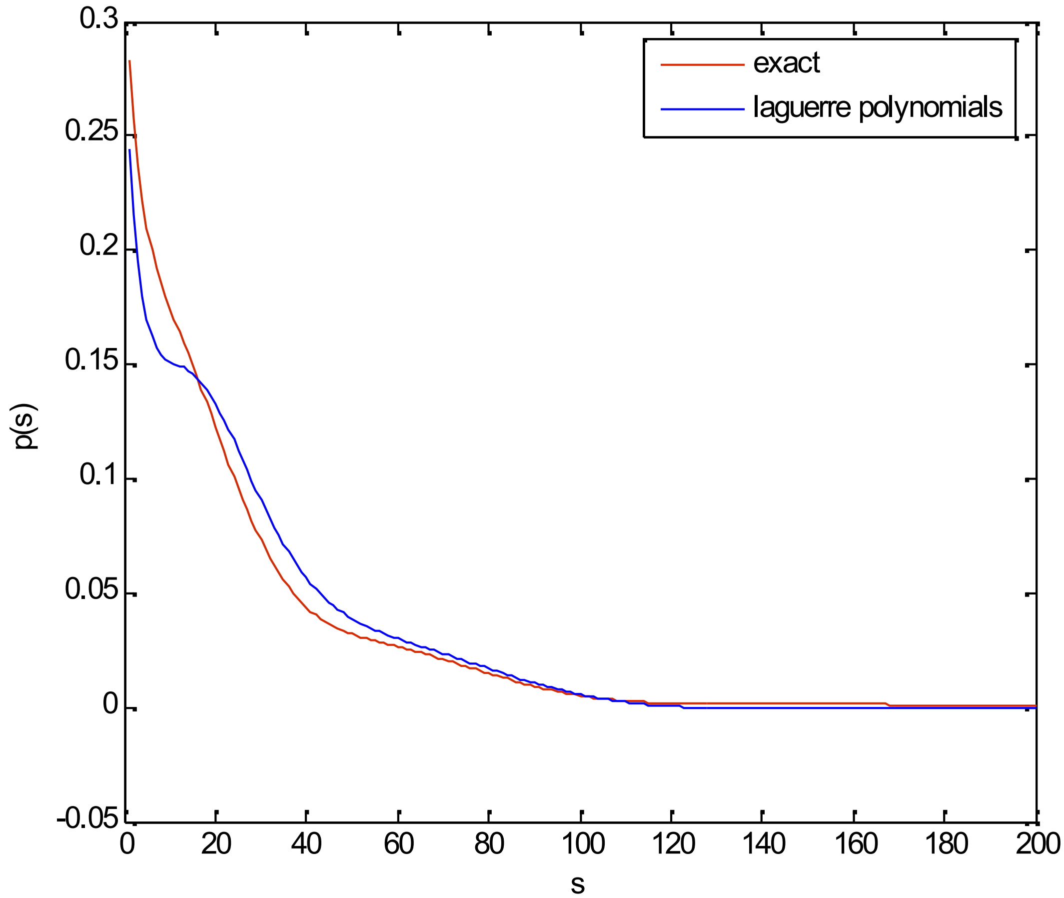

FIG 5.6: Exact Probability Density of Intensity For N=4 Versus The Laguerre Polynomial Expansion (M=17)

The following figure shows the graph of exact pdf for seven single-look unit amplitude scatterers versus its corresponding series expansion. In this case, the graph of the laguerre series expansion is almost indistinguishable from the exact pdf. This figure shows that as the numbers of scatterers increased to seven, the pdf's converge faster to an exponential distribution, and only two series terms are needed for remarkably low maximum relative error of 2%.

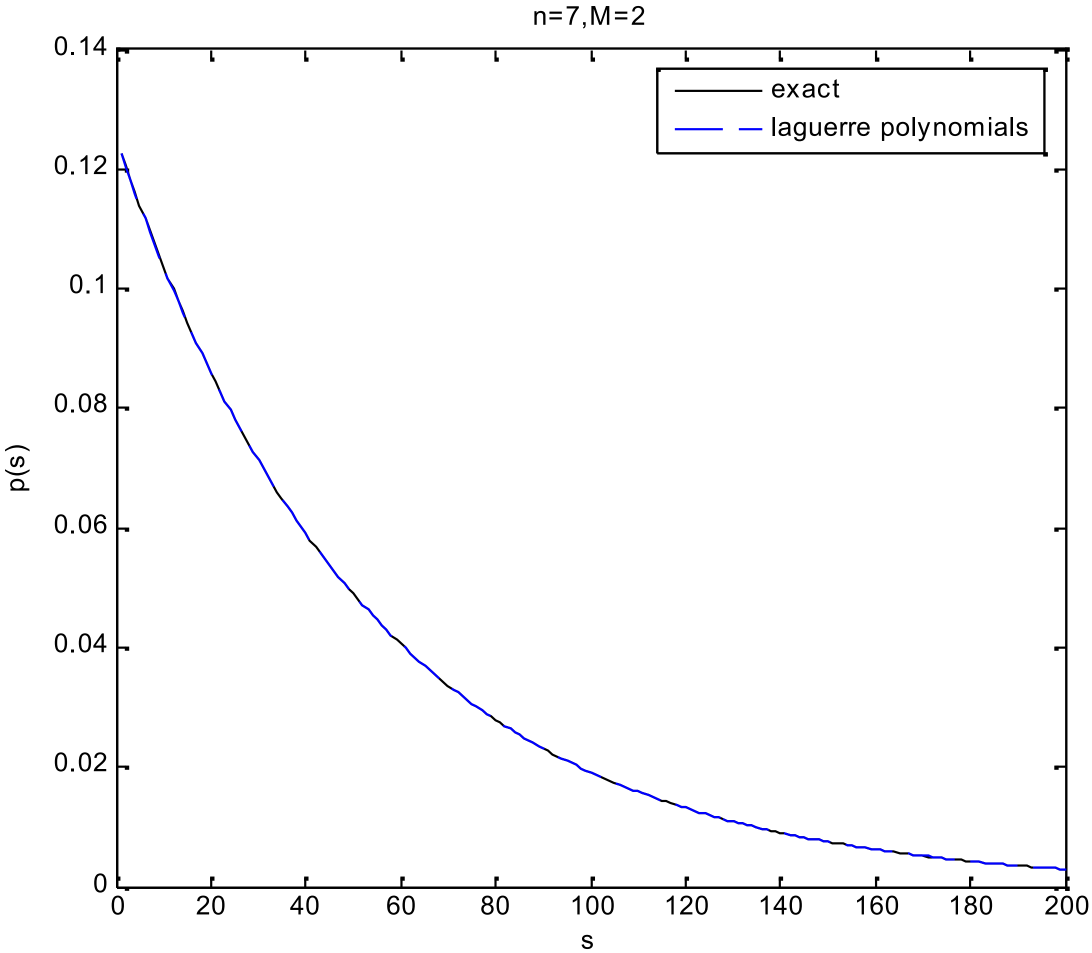

FIG 5.7: Exact Probability Density of Intensity For N=7 Versus The Laguerre Polynomial Expansion (M=2)

CHAPTER 6

CONCLUDING REMARKS

In this thesis, it is mentioned that the envelope probability density function (pdf) of the sum of random wireless signals of the importance in several application mainly in wireless communication. A most general case for the amplitudes of random wireless signals is considered and closed-form expressions are obtained for calculating the pdf. Instead of definite integrals, which are inappropriate, an infinite laguerre series is derived. Thus, it is shown that our approach expresses the envelope pdf just in terms of polynomials. Based on results obtained, it is found that the time-consuming Monte-Carlo simulation for determining envelope pdf is completely avoided. The results are applied to a radar scatter problem and achieve the expressions for pdf of random scattered signal intensity. Based on these results, the performance of multipath fading channel is also discussed. We also discussed the characterization of multipath fading channel in Multiple antenna system such as Multi-input multi-output (MIMO) considering a single antenna element.

In the problem of radar scatterers, we considered the statistics of radar cross section of a small collection of random scatterers. Here we derived exact closed-form expressions for the intensity's pdf **as** a function of the scatterers' constant amplitudes when the number of scatterers within a resolution cell is one, two, and three, and a single look is taken over the resolution cell. The series coefficients are estimated using a maximum likelihood estimator and the number of series terms was tabulated. Here also the value of an arbitrary random nonzero real number β is chosen in such a way so that the truncation error should be minimized.

For single-look intensity measurement, we noted that the exact pdf approaches an exponential distribution as the number of the scatterers is gradually increased from four to seven. For the case of four scatterers, a relatively high number of series terms ($M = 17$) is needed to approximate the pdf up to a 5% maximum relative error. An increase in the number of terms M to **20** or **25** merely reduces the maximum relative error to 4%. It takes as many as 35 terms to obtain a significantly lower maximum relative error of 2% at the cost of increasing the computational complexity of the Laguerre polynomial expansion. Considering the trade-off between computational complexity and relative error, we use only 17 terms in the series expansion and accept an increased maximum relative error of 5%. Increasing the number of scatterers from four to only five scatterers significantly reduces the number of series terms from 17 to seven while maintaining a maximum relative error of 5%. As the number of scatterers is further increased to six and seven scatterers, the pdf‘s converge faster to an exponential distribution, and only three and two series terms are needed, respectively, for a remarkably low maximum relative error of 2%. Furthermore, the graph of the Laguerre series expansion is almost indistinguishable from the exact pdf for the case of seven scatterers. Only one series term is needed for the case of eight scatterers to maintain a maximum relative error of 2%.

REFERENCES

1. W. C. Y. Lee, Mobile Communications Engineering. New York: Mc- Graw-Hill, 1982.
2. M. I. Skolnik, Introduction to Radar Systems, 2nd ed. Tokyo, Japan: McGraw-Hill, 1980.
3. J. K. Jao and M. Elbaum, "First-order statistics of a non-Rayleigh fading signal and its detection," *Proc. IEEE*, vol. 66, pp. 781–789, 1978.
4. Theodore S. Rappport, "wireless communication principles and practice" 2^{nd} ed. Phi.
5. P. Beckmann, Probability in Communication Engineering. New York Harcourt, Brace & World, 1967.
6. B. T. Irons and K. D. Donohue, "Probability of erasure in non-Rayleigh fading channels-A simulation study," *IEEE Trans. Commun*., vol. 43 pp. 1246–1247, 1995.
7. A. Abdi and S. Nader-Esfahani, "A general pdf for the signal envelope in multipath fading channels using Laguerre polynomials," in *Proc. IEEE Veh. Technol. Conf.,* Atlanta, GA, 1996, pp. 1428–1432.
8. A.Abdi and S. Nader-Esfahani, "An optimum Laguerre expansion for the envelope pdf of two sine waves in Gaussian noise," in *Proc. IEEE Southeastcon Conf*., Tampa, FL, 1996, pp. 160–163.
9. M. K. Simon, "On the probability density function of the squared envelope of a sum of random phase vectors," *IEEE Trans Commun*., vol. COM-33, pp. 993–996, 1985.
10. C. Lucas and A. Abdi, "Nonparametric estimation of the pdf of signal envelope in a multipath fading channel with random number of paths (N) using Monte Carlo simulation" (in Persian*), Amirkabir J. Sci. Technol*., vol. 6, no. 24, pp. 316–328, 1994.
11. D. W. Schaefer and P. N. Pusey, "Statistics of non-Gaussian scattered light," *Phys. Rev. Lett*., vol. 29, pp. 843–845, 1972.

12. G. J. Foschini and M. J. Gans, “On limits of wires communications in fading environment when using multiple antennas,” *Wireless Pers. Commun*., vol. 6, Mar. 1998, pp. 311-335.
13. G. J. Foschini, “Layered space-time architecture for wireless communication in fading environment when using multielement antennas,” *Bell Labs Tech. J*., 1996, pp. 41-59
14. E. telatar, “Capacity of multiantenna Gaussian channels,” *AT&T Bell Lab., Tech*. Memo. June 1995.
15. G. Raleigh and J.M. Cioffi, “Spatial temporal coding for wireless communications,” *IEEE Trans. Commun*., Vol. 46, 1998, pp. 357-366.
16. H. bolcskei, D. Gesbert, A.J. Paulraj, “on the capacity of OFDM based spatial multiplexing systems,” *IEEE Trans. Commun*., vol. 50, Feb. 2002, pp. 225-234.
17 J. C. Guey, M. P. Fitz, M. R. Bell, and W. Y. Kuo, “Signal design for transmitter diversity wireless communication systems over Rayleigh fading channel,” In *Proc. IEEE VTC,* 1996, pp.136-140.
18. A. Wittneben, “A new bandwidth efficient transmit antenna modulation diversity scheme for linear digital modulation,” in *Proc. IEEE 1993*, pp. 1630-1634.
19. Girish Ganesan and Petre Stoica, “ Space-time diversity Using Orthogonal and Amicable Orthogonal Designs,” Wireless personal communication 18, pp. 165-178., 2001.
20. B. T. Irons and K. D. Donohue, “Probability of erasure in non-Rayleigh fading channels-A simulation study,” *IEEE Trans. Commun*., vol. 43, pp. 1246–1247, 1995.
21. A. Abdi and S. Nader-Esfahani, “A general pdf for the signal envelope in multipath fading channels using Laguerre polynomials,” *in Proc. IEEE Veh. Technol. Conf*., Atlanta, GA, 1996, pp. 1428–1432.
22. A. Abdi and S. Nader-Esfahani, “An optimum Laguerre expansion for the envelope pdf of two sine waves in Gaussian noise,” in *Proc. IEEE Southeastcon Conf*., Tampa, FL, 1996, pp. 160–163.
23. J. S. Daba and M. R. Bell, “Statistics of the scattering cross-section of a small number of random scatterers,” *IEEE Trans. Antennas Propagat*., vol. 43, pp. 773–783, 1995.

24. R. Esposito and L. R.Wilson, "Statistical properties of two sine waves in Gaussian noise," *IEEE Trans. Inform. Theory*, vol. IT-19, pp. 176–183, 1973.
25. C.W. Helstrom, "Distribution of the sum of two sinewaves and Gaussian noise," *IEEE Trans. Inform. Theory*, vol. 38, pp. 186–191, 1992.
26. E.Jakeman and P. N. Pusey, "A model for non-Rayleigh sea echo," *IEEE Trans. Antennas Propagat*., vol. AP-24, pp. 806–814, 1976.
27. H. W. Lorber and J. D. Burns, "The SQUAD: A versatile and accurate scheme for measuring signal strengths," *IEEE Trans. Inform. Theory*, vol. IT-19, pp. 37–43, 1973.
28. M. Nesenbergs, "Error probability for multipath fading—The "slow and flat" idealization," *IEEE Trans. Commun. Technol*., vol. COM-15, pp. 797–805, 1967.
29. R. Price, "An orthonormal Laguerre expansion yielding Rice's envelope density function for two sine waves in noise," *IEEE Trans. Inform. Theory*, vol. 34, pp. 1375–1382, 1988.
30. R. S. Raghavan, "A model for spatially correlated radar clutter," *IEEE Trans. Aerosp. Electron. Syst*., vol. 27, pp. 268–275, 1991.
31. S. O. Rice, "Probability distributions for noise plus several sine waves- The problem of computation," *IEEE Trans. Commun*., vol. COM-22, pp. 851–853,1974.
32. G. V. Trunk and S. F. George, "Detection of targets in non-Gaussian sea clutter," *IEEE Trans. Aerosp. Electron. Syst*., vol. AES-6, pp. 620–628, 1970.
33. K. A. Norton, L. E. Vogler,W. V. Mansfield, and P. J. Short, "The probability distribution of the amplitude of a constant vector plus a Rayleigh distributed vector," *Proc. IEEE*, vol. 43, pp. 1354–1361, 1955.
34. S. M. Zabin and G. A. Wright, "Nonparametric density estimation and detection in impulsive interference channels-Part I: Estimators," *IEEE Trans. Commun*., vol. 42, pp. 1684–1697, 1994.
35. J. Goldman, "Detection in the presence of spherically symmetric random vectors," *IEEE Trans. Inform. Theory*, vol. IT-22, pp. 52–59, 1976.
36. R. V. Churchill, Operational Mathematics, 3rd ed. Tokyo, Japan: Mc- Graw-Hill, 1972.

37. S. M. Candel, "Dual algorithms for fast calculation of the Fourier–Bessel transform," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-29, pp. 963–972, 1981.
38. I.S. Reed, "On the use of Laguerre polynomials in treating the envelope and phase components of narrow-band Gaussian noise," *IEEE Trans. Inform. Theory*, vol. IT-5, pp. 102–105, 1959
39. Ali Abdi, student member, IEEE,"on the PDF of the sum of random vectors", *IEEE trans. On commun*. Vol. 48, No.1, Jan 2000.
40. I. S. Gradshteyn and I. M. Ryzhic, *Table of Integrals, Series, and Products*, A. Jeffrey, Ed. New York: Academic, 1980.