Effect of Coriolis Force on The Equillibrium Structures of Rotationally and Tidally Distorted White Dwarf and Prasad Model of Stars

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Aishni Singla
(Roll No. 301003002)

Under the supervision of
Dr. A. K. Lal

School of Mathematics and ComputerApplications
Thapar University, Patiala –147004
PUNJAB-INDIA

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Certificate

I hereby certify that the work which is being presented in the thesis entitled "Effect of Coriolis Force on the Equilibrium Structures of Rotationally and Tidally Distorted White Dwarf and Prasad Models of Stars" in fulfilment of the requirements for the award of degree of Master of science, School of Mathematics and Computer Applications, Thapar University, Patiala is an authentic record of my own work carried out under the supervision of Dr. A. K. Lal.

The matter presented in the thesis has not been submitted for the award of any other degree of this or any other university.

(Aishni Singla)

This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

(A. K. Lal)
Associate Professor,
School of Mathematics and Computer Applications,
Thapar University, Patiala
Punjab, India

Countersigned by:

Dr. S.S.Bhatia
(Professor & Head)
School of Mathematics & Computer Applications
Thapar University, Patiala.

Dr. S.K Mohapatra
Dean of Academic Affairs
Thapar University
Patiala.

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(Aishni Singla)

Abstract

In the present thesis an attempt has been made to extend the work of Pathania (37). He studied the effect of inclusion of Coriolis force on the equilibrium structures as well as periods of small adiabatic barotropic modes of oscillations of rotating stars and stars in binary system. Earlier, while computing the effects of rotational and tidal distortions on the equilibrium structures and the eigenfrequencies of radial and non-radial modes of oscillations of rotating stars in binary systems, Mohan and Saxena (29,30) and Mohan et al. (33,34,31,32) did not explicitly account for the effect of Coriolis force and they analyse these problems in inertial frame of reference only. Pathania (37) studied the problems in rotating frame of reference and used the methodology to determine the equilibrium structures of rotationally and tidally distorted polytropic models of stars. In the present study we have tried to investigate the effects of inclusion of Coriolis force on the equilibrium structures of rotationally and tidally distorted White dwarf and Prasad model of stars.

Thesis consists of three chapters. Chapter I is introductory in nature. In this chapter we briefly discuss the modified expression for Roche equipotentials incorporates the effect of Coriolis force. The system of differential equation governing the equilibrium structures of rotationally and tidally distorted stars incorporating the effect of Coriolis forces which were earlier developed by Pathania (37), is also discussed. A brief survey of the literature available on the subject and summary of the work presented in the succeeding chapters of the thesis are also given in this chapter.

In chapter II we first discuss the concept of white dwarf model of stars and then methodology discussed in chapter II is used to study the equilibrium structures and certain other parameters of rotationally and tidally distorted white dwarf model of stars for different values of $\frac{1}{\phi_0^2}$ as 0.01, 0.05, 0.1, 0.2, 0.5, 0.8. The results thus obtained are compared with the undistorted model in which the effect of Coriolis force has not been there.

In chapter III we first discuss the concept of Prasad model of stars and then apply the methodology discussed in chapter II to study the equilibrium structures and certain other parameters of rotationally and tidally distorted Prasad model of stars. Certain conclusion based on the present analysis has also been drawn.

CONTENTS

CHAPTER – I

INTRODUCTION

This chapter is introductory in nature. In section 1.1, a brief survey of literature available on the subject is presented. Basic equations determining the equilibrium structures of gaseous sphere are presented in section 1.2. Section 1.3 deals with the averaging technique of Kippenhahn and Thomas which plays an important role in the present study. In section 1.4, Modified expression for Roche equipotentials incorporates the effect of coriolis force earlier discussed by Pathania (38) is presented. The equilibrium structure of rotationally and tidally distorted stars incorporating the effect of coriolis forces is discussed in section 1.5.A brief summary of the work presented in succeeding chapters is finally presented in section 1.6.

1.1 BRIEF SURVEY OF LITERATURE

A star is a huge gaseous sphere in hydrostatic and thermal equilibrium radiating huge amount of energy generated by thermonuclear reactions. Most of the theoretical studies about the equilibrium structures of stars have been carried out in literature by assuming the star to be an undistorted spherical gaseous sphere. (Extensive literature is now available on this subject Chandrasekhar (8),Rosseland (39), Schwarzschild (44), Eddington (12), Mentzel et al. (27), Cox and Guili (11), Kippenhahn and Weigert (18), Clement (10), Kopal (20,21), Abhyankar (3)).

Theoretically a star can be considered as a gaseous sphere both in hydrostatic and thermal equilibrium. Most of the stars are rotating about their own axis. This rotation can be a solid body rotation as well as differential rotation in which different parts of the star are rotating with different angular velocities. Equilibrium structure of rotating stars get distorted by the tidal effects alone if the star is not rotating and by the combined tidal and rotational effects. Kippenhahn and Thomas(17) suggested a practical way of analysing the effects of rotation and tidal distortions on the equilibrium structures of stars by approximating the actual equipotentials surfaces of the star by Roche equipotentials.

Reese et al (38) have studied the acoustic oscillations of rapidly rotating stars taking into account the effects of coriolis forces and centrifugal forces. Roche model of a star subject to non uniform rotation and demonstrated that the effects of small uniform rotation have greater significance than the actual values of rotational velocities themselves. Mohan et al (32) have proposed a method to compute the eigenfrequencies of small adiabatic baratropic modes of oscillations of differentially rotating stars polytropic and white dwarf models of stars.

The equilibrium structure of stars appearing in binary system or a multiple system will be tidally distorted gaseous sphere if it is not rotating and a rotationally and tidally distorted if the star is rotating as well. Attempts have been made in literature to determine the effects of rotation and tidal distortions on equilibrium structure of stars in binary and multiple systems. In a series of Chandrasekhar (5, 6, 7) developed a first order analysis which he applied to study the rotational problems, the tidal problem and the binary star problem. Later, Kippenhahn and Thomas (17) suggested a practical way of analysing the effects of rotation and tidal distortions on the equilibrium structures of stars by approximating the actual equipotentials surfaces of the stars by Roche equipotentials. Lal et al (24) have discussed the equilibrium structures of rotationally and tidally distorted primary component of binary stars taking into account the effect of mass variation inside the star. Vorontsov (50) developed a perturbation theory to calculate the influence of slow differential rotation on the adiabatic nonradial modes of stellar oscillations. He analysed the effects of Coriolis force and ellipticity simultaneously using the perturbation technique for Hamiltonion operator which is developed upto second order in eigenfrequences and to first order in eigenfunctions.

Kopal (20) introduced a system of coordinates, known as Roche coordinates, to study the problem of rotating stars in binary system. Mohan and Saxena (33,34) use the Kippenhahn and Thomas (17) averaging technique in conjuction with Kopal's results on Roche equipotentials to determine the combined effects of rotation and tidal distortions on the equilibrium structures and oscillations of the polytropic model of stars. This approach is presented in Saxena (43). Later this approach was also used by Mohan and Aggarwal (28) to study the effects of rotation and tidal distortions on the structure and periods of small adiabatic oscillations of Prasad and composite model of stars. The technique was formalized by Mohan et al (33,34) and used to study the problems of equilibrium structures and oscillations of rotationally and tidally distorted main sequence stars. Lal (22) studied in detail the equilibrium structures and periods of oscillations of differentially rotating stellar models. Later on Singh and Sharma (47) also studies the oscillations of differentially rotating stars in binary system. Lal et al (23) applied this technique to study the equilibrium structures of differentially rotating and tidally distorted white dwarf model of stars. Equilibrium structures of these type of white dwarf stars, assumed to be primary components of binary systems, are being influenced by the combined effects of differential rotation as well as the gravitational effects of the companion star causing tidal distortions.

Most of the authors have studied pulsation of stars having solid body rotation. The influence of uniform rotation on the global structure of the white dwarf models have been considered by Chandrasekhar (9), Suda (49), Lal et al (24). The most detailed models of uniformly rotating white dwarf are due to Anand (1), Roxburg (47), Mohaghan (35), Anand et al. (2). Some of the authors such as Ostriker and Tassaul (36), Shapiro and Teukolsky (45) have noted the stability analysis of uniformly rotating white dwarf stars. Smart and Monaghan (48) and Blinnikov (4) extensively analysed the models of zero temperature white dwarf in non-uniform rotation, Hachisu et al. (13) studied the fate of merging double white dwarf and presented a numerical method, Lal et al (24) presented a method for computing equilibrium structure of differentially rotating white dwarf models of stars. Adiabatic radial pulsation of zero temperature white dwarfs have been discussed by various authors such as Sauvenier-Goffin (42) using the equation of state of completely degenerate stellar matter. Singh and Das (46) studied the radial oscillations of cold and hot white dwarf for different values of central degeneracy parameters $\frac{1}{\phi_0^2}$ and temperature T. Harper and Rose (14) obtained the solution of Pekeris equation for non radial oscillations for white dwarf, hot white dwarfs and $10\,M_0$ models.

For instance comments have generally been made that the approaches based on Roche equipotential for computing the equilibrium structure of stars in binary systems carry out their studies in fixed frame of reference and do not account for Coriolis forces which is expected to arise in such cases when rotating frame of reference are used. Jain and Sharma (15) have studied the effect of rotation and tidal distortions on the equilibrium structures of gaseous spheres in the presence of the Coriolis forces, such a study has practical relevance in astrophysics where it is expected to help in understanding the problems of stellar stability and stellar variability of rotating stars as well as stars in binary and multiple system. Later, Pathania (37), studied the effect of Coriolis force besides the centrifugal and gravitational forces. He developed expression for the Roche equipotential of rotating star in a binary system in a rotating frame of reference to explicitly include the effect of coriolis force besides the centrifugal and gravitational forces. The methodology developed is next used to determine the effects of coriolis force on the shapes of Roche equipotential surfaces and position of Roche limit for different types of binary stars. Pathania et al (26) also studied the effects of coriolis forces on the equilibrium structures of rotationally and tidally distorted polytropic models of stars.

1.2 BASIC EQUATIONS DETERMINIG THE EQUILIBRIUM STRUCTURE OF GASEOUS SPHERE

The system of basic equations of the equilibrium structure of gaseous sphere in hydrostatic and thermal equilibrium, are well established in literature. These equations pertain to the problem of the equilibrium structure of stellar models. Let P and ρ denote the pressure and the density at a point, respectively, distant r from the center of the sphere.

1.2.1 Mass conservation

Let $M(r)$ be the mass contained within radius r, then the mass contained with the shell from r to $r+dr$ is then

$$M(r+dr)-M(r)=\rho(r)\,dV \tag{1.1a}$$

where $dV=4\pi r^2 dr$ is the volume of the shell. Now the left-hand-side can be rewritten as,

$$\frac{dM(r)}{dr}dr=\rho(r)4\pi r^2 dr \tag{1.1b}$$

Cancelling out the factor dr, we obtain

$$\frac{dM(r)}{dr}=\rho(r)4\pi r^2 \tag{1.1c}$$

1.2.2 Hydrostatic equilibrium

For a star in hydrostatic equilibrium, gravity is balanced by pressure. Now,for a small element in a shell from r to $r+dr$. The area of this element is dA.

A coordinate system,where the radial component increases outwords, establishes.

Then the pressure force acting on the inner side of radius r is positive, while the pressure force acting on the outer side of radius $r+dr$ is negative.

The total pressure force is then

$$P(r)dA-P(r+dr)dA=[P(r)-P(r+dr)]dA=-\frac{dP}{dr}dr\,dA \tag{1.2a}$$

The gravity force, due to spherical symmetry, ponts toward the centre and therefore has a negative sign i.e.

$$-\frac{GM_r dm}{r^2} \tag{1.2b}$$

where dm is the mass of the element. Obviously

$$dm = \rho(r)\,dV = \rho(r)\,dA\,dr \tag{1.2c}$$

Therefore the gravity is given by

$$-\frac{GM_r \rho(r) dA\,Dr}{r^2} \tag{1.2d}$$

So from Newton's second law, we have

$$\rho(r) dA\,dr\frac{d^2r}{dt^2} = -\frac{dP}{dr}dr\,dA - \frac{GM_r \rho(r) dA\,dr}{r^2} \tag{1.2e}$$

We can cancel out the factors dr and dA, and arrive at:

$$\rho(r)\frac{d^2r}{dt^2} = -\frac{dP}{dr} - \frac{GM_r \rho(r)}{r^2} \tag{1.2f}$$

If the star is in hydrostatic equilibrium, then the sum of all forces must vanish, i.e.,

$$-\frac{dP}{dr} - \frac{GM_r \rho(r)}{r^2} = 0 \tag{1.2g}$$

Moving the second term to the right hand side, we have

$$\frac{dP}{dr} = -\frac{GM_r \rho(r)}{r^2} \tag{1.2h}$$

1.2.3 Energy conservation

Stars lose energy via radiation. The radiation loss must be balanced by energy generated by nuclear reactions. The energy conservation equation express this in mathematical terms as:

Let $L(r)$ be the energy flow across the sphere with radius r, in units of W,then the net energy loss in the shell from r to $r + dr$ is

$$L(r+dr)-L(r)=\frac{dL(r)}{dr}dr \tag{1.3a}$$

If ε is the energy generation per kg, then the total energy generated in the shell is

$$dE=\varepsilon\rho(r)4\pi r^2 dr \tag{1.3b}$$

For the gas to be in thermal equilibrium, the radiation loss must be equal to the energy gain from nuclear burning. Therefore we have

$$\frac{dL(r)}{dr}dr=\varepsilon\rho(r)4\pi r^2 dr \tag{1.3c}$$

dr cancels out, so we have

$$\frac{dL(r)}{dr}=\varepsilon\rho(r)4\pi r^2 \tag{1.3d}$$

In problems, where the thermal properties of the model are either not to be investigated or are not important, the equilibrium structure of the gaseous sphere may be determined by solving equations (1.1c and 1.2h) using some suitable equations of state together with boundary conditions

At the centre $r=0,\ M(r)=0$

At the surface $r=0, M(r)=0, P=0, or P_s, \rho=0 or \rho_s$

A number of the theoretical as well as numerical studies regarding the equilibrium structure of gaseous spheres, particularly those which have particular reference to the problems of the equilibrium structures of the stars are available in literature (Chandrasekhar (8), Schwarzschild (44), Eddington (12), Menzal et al. (27), Cox and Giuli (11), Kippenhahn and Weigert (18)).

1.3 AVERAGING TECHNIQUE OF KIPPENHAHN AND THOMAS

In order to study the effects of rotation and tidal distortions on the equilibrium structure of gaseous spheres, Kippenhahn and Thomas (17) developed the concept of topologically equivalent spherical surfaces corresponding to actual equipotential surfaces of a rotationally and tidally distorted model. They define on these equivalent spherical surfaces, quantities such as $\overline{f}, \overline{g}$ etc. Which denote certain averages of the quantities f, g, respectively on the actual equipotential surfaces.

Let ψ denotes the total potential (gravitation, rotation and tidal forces) of a rotationally and tidally distorted model at an arbitrary point $P(x, y, z)$ then

$\psi(x, y, z)$ constant is an equipotential surface. Now let V_ψ be the volume enclosed by the equipotential surface.

ψ = constant and S_ψ be the surface area of this equipotential surface. For any function $f(x, y, z)$ they defines $\overline{f}$ as its mean value over the equipotential surfaces ψ = constant by the relation

$$\overline{f} = \frac{1}{S_\psi} \int_{\psi = constant} f \, d\sigma \tag{1.4}$$

where $d\sigma$ denotes the surface element of the equipotential surface $\psi =$ constant. Clearly $\overline{f}$ is a function of equipotential surface ψ only and can be obtained as equation (1.3) for each equipotential surface ψ = constant. Kippenhahn and Thomas (17) also define a variable r_ψ in analogy with the radius of sphere by the relation

$$V_\psi = \frac{4}{3} \pi r_\psi^3 \tag{1.5}$$

Also by definition

$$S_\psi = \int_{\psi = constant} d\sigma \tag{1.6}$$

So obviously , in general, S_ψ is not equal to $4\pi r_\psi^3$. Kippenhahn and Thomas (17) define a function $g(x, y, z)$ by the relation

$$g = \frac{d\psi}{dn} \tag{1.7}$$

This g correspondsto the force of gravity of a sphere. The distance dn between the two neighbouring surfaces ψ = constant and $\psi + d\psi$ = constant is, in general , not a constant (i.e. not same at all points of the surface). They used (1.6) to compute the mean values, i.e. , $\bar{g}$ and $\overline{g^{-1}}$ with help of relations

$$\begin{Bmatrix} \bar{g} = \dfrac{1}{S_\psi} \displaystyle\int_{\psi=const} \dfrac{d\psi}{dn} d\sigma \\ \overline{g^{-1}} = \dfrac{1}{S_\psi} \displaystyle\int_{\psi=const} \left(\dfrac{d\psi}{dn}\right)^{-1} d\sigma \end{Bmatrix} \tag{1.8}$$

Both $\bar{g}$ and $\overline{g^{-1}}$ are functions of ψ only and represent the value of g and g^{-1} respectively over the topologically equivalent spherical surface. The volume dV_ψ between the surface ψ = constant and $\psi + d\psi$ = constant is given as

$$dV_\psi = \int_{\psi=const} dn\, \mathrm{d}\sigma = \int_{\psi=const} \left(\frac{d\psi}{dn}\right)^{-1} dn = S_\psi \overline{g^{-1}} d\psi \tag{1.9}$$

Kippenhahan and Thomas also defined nondimensional parameters u, v and w as

$$u = \frac{S_\psi}{4\pi r_\psi^2},\ v = \frac{\bar{g}\, r_\psi^2}{GM_\psi},\ w = \frac{\overline{g^{-1}} GM_\psi}{r_\psi^2} \tag{1.10}$$

where M_ψ is the mass enclosed by equipotential surface ψ = constant.

Equations (1.3) to (1.8) are purely mathematical definitions, which was applied by Kippenhahn and Thomas (17) to gravitational fields of gaseous spheres, distorted by tidal and rotational forces. In hydrostatic equilibrium the equipotential surfaces are also equipressure and equidensity surfaces. So, on an equipotential surface, the pressure P_ψ and density ρ_ψ are also constant.

Using these concepts, and from equation (1.4) the mass dM_ψ between the equipotential surface ψ = constant and $\psi + d\psi$ = constant is given as

$$dM_\psi = dV_\psi\, \rho_\psi = 4\pi r_\psi^2 \rho_\psi\, dr_\psi \tag{1.11}$$

Thus, we get

$$\frac{dM_\psi}{dr_\psi} = 4\pi r_\psi^2 \rho_\psi \qquad (1.12)$$

From equation (1.10) and (1.11) we have

$$d\psi = \frac{d\psi}{dV_\psi} dV_\psi = (\frac{dV_\psi}{d\psi})^{-1} \frac{dM_\psi}{\rho_\psi} = \frac{dM_\psi}{S_\psi \overline{\overline{g^{-1}}} \rho_\psi} \qquad (1.13)$$

Using relations (1.10), we get

$$d\psi = \frac{GM_\psi dM_\psi}{4\pi r_\psi^4 \rho_\psi uw} \qquad (1.14)$$

The conditions for hydrostatic equilibrium, $dP_\psi / d\psi = -\rho_\psi$, can now be written with equation (1.10) in the form

$$\frac{dP_\psi}{dM_\psi} = -\frac{GM_\psi}{4\pi r_\psi^4} f_p \qquad (1.15)$$

where $f_P = \frac{1}{uw} = \frac{4\pi r_\psi^4}{GM_\psi} \frac{1}{S_\psi \overline{\overline{g^{-1}}}}$

the factor f_p is a function of ψ only. If ψ is known the euipotential surface can be determined, and then consequently values of $S_\psi, r_\psi, \overline{g}$ and $\overline{g^{-1}}$ for each equipotential surface can be obtained simply from the geometry of the equipotentials. The mass M_ψ which depends on the density distribution ρ_ψ can be determined by integrating the equation (1.12).

Similarly the other structure equations derived by Kippenhahn and Thomas (17), which includes the effects of rotation and tidal distortions on the equilibrium structure of gaseous spheres are as follows.

For chemically homogeneous spheres, the nuclear energy generation rate ε depends only upon density ρ_ψ and the temperature T_ψ and are, therefore, constant on equipotential surface. Thus, if L_ψ is the energy which passes per second through the equipotential surface ψ = constant, then

$$\frac{dL_\psi}{dM_\psi} = \varepsilon \tag{1.16}$$

Using equation (1.12), it can be written as

$$\frac{dL_\psi}{dr_\psi} = 4\pi r_\psi^2 \rho_\psi \varepsilon \tag{1.17}$$

Also $$\frac{dT_\psi}{dM_\psi} = -\frac{3\kappa L_\psi}{64\pi^2 ac T_\psi^3 r_\psi^4} f_T \tag{1.18}$$

Using equation (1.11).the equation can be expressed as

$$\frac{dT_\psi}{dr_\psi} = -\frac{3\kappa \rho_\psi L_\psi}{16\pi ac T_\psi^3 r_\psi^2} f_T \tag{1.19}$$

where $f_T = \frac{1}{u^2 vw}$

equations (1.12), (1.15), (1.16) and (1.18) which are the four basic equations governing the equilibrium structure of a gaseous sphere distorted by rotational and tidal forces may now be collected together and written as

$$\frac{dM_\psi}{dr_\psi} = 4\pi r_\psi^2 \rho_\psi \tag{1.20a}$$

$$\frac{dP_\psi}{dM_\psi} = -\frac{GM_\psi}{4\pi r_\psi^4} f_p \tag{1.20b}$$

$$\frac{dL_\psi}{dM_\psi} = \varepsilon \tag{1.20c}$$

and $$\frac{dT_\psi}{dM_\psi} = -\frac{3\kappa L_\psi}{64\pi^2 ac T_\psi^3 r_\psi^4} f_T \tag{1.20d}$$

where $f_P = \frac{1}{uw}$ and $f_T = \frac{1}{u^2 vw}$

these reduce to the normal equations used for determining the equilibrium structures of spherical models of stars when distorted parameters u, v, w are set one each. The boundary conditions which the above equations has to satisfy are

$$M_{\psi} = 0,\ L_{\psi} = 0 \tag{1.21a}$$

At the centre $r_{\psi} = 0,\ M_{\psi} = M_0,\ L_{\psi} = L_{\psi s},\ P_{\psi} = 0, T_{\psi} = 0$ or $P_{\psi} = P_{\psi s},\ T_{\psi} = T_{\psi s}$

At the free surface $r_{\psi} = R_{\psi}$ (1.21b)

Where M_0 is the total mass of the model and $L_{\psi s}, P_{\psi s}, T_{\psi s}$ are the values of $L_{\psi}, P_{\psi}, T_{\psi}$ respectively, on the outermost equipotential surface.

1.4 MODIFIED EXPRESSION FOR ROCHE EQUIPOTENTIALS INCORPORATES THE EFFECT OF CORIOLIS FORCES

An expression for the Roche equipotential surface, which incorporates the effect of coriolis forces in addition to the centrifugal and gravitational force, is obtained by Pathania (37), which is given as following.

The total potential at a point inside the primary component , which experiences the effect of coriolis force besides the gravitational and centrifugal forces, is given as

$$\psi = \frac{GM_0}{r} + \frac{GM_1}{r_2} + \frac{1}{2}(\vec{\Omega} \times \vec{r_1}).(\vec{\Omega} \times \vec{r_1}) + \vec{V}.(\vec{\Omega} \times \vec{r_1}) \tag{1.22}$$

Here M_0 and M_1 are the masses of the two components of binary system (Fig 2.1), separated by a distance D. The primary component of mass M_0 is supposed to be much massive than its companion star of mass M_1 $(M_0 >> M_1)$ which for all practical purposes is regarded as a point mass. $\vec{V}$ is the velocity of the particle of unit mass at point P(x,y,z) with respect to rotating frame of reference rotating with Ω be the angular velocity of revolution of system about a line parallel to z-axis which passes through the centre of gravity C of the system and is perpendicular to the xy plane. r , r_2 and r_1 represent the distances of a point from centres of gravity of the primary and secondary stars and the centre of gravity of the system respectively. In a binary system, the primary star is rotating about axis OZ with angular velocity Ω_1 as well as revolving about an axis parallel to z-axis oassing through the common centre of mass C with angular velocity Ω. The point P when it is inside the primary will experience the effects of Coriolis force besides the gravitational and centrifugal forces. In equation (1.22) the first two terms corresponds to the gravitational potential which arises due to the primary and secondary component of binary system and third term is due to centrifugal force. These three terms are same as earlier obtained by Kopal (20) in his studies to the problems of Roche Model and its

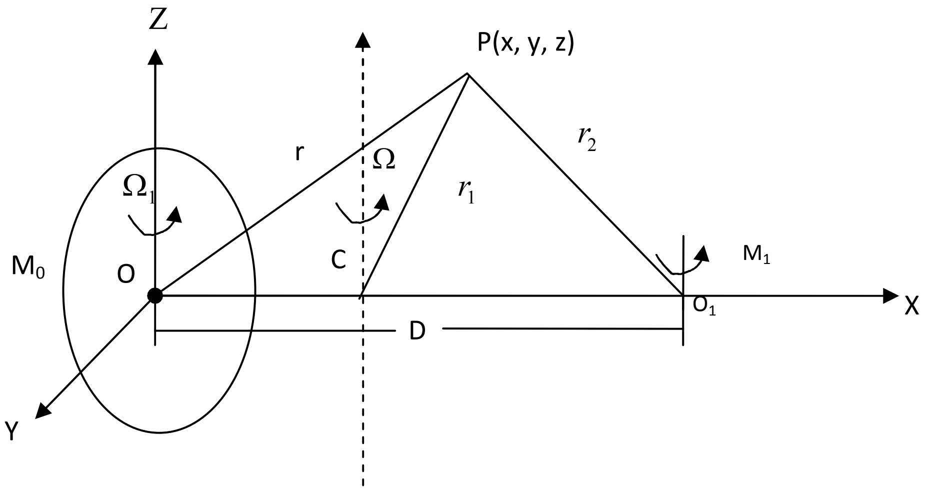

Fig 1.1: Axis of reference for a binary system

applications to close binary system. Now, the fourth term $\vec{V}.(\vec{\Omega}\times\vec{r_1})$ represents the contribution of the coriolis force to the potential at a point, where $\vec{V}$ is the tangential component of velocity of this particle in the rotating frame of reference.

In the earlier studies carried out on Roche equipotentials by Kopal (20) and Mohan and Saxena (29), the contribution of this last term $\vec{V}.(\vec{\Omega}\times\vec{r_1})$, which arises on account of coriolis force, has been neglected.

The modified expression for potential at a point P(x.y,z) inside the star in Cartesian form as

$$\psi = \frac{GM_0}{r} + \frac{GM_1}{r_2} + \frac{\Omega^2}{2}\left[(x-d_1)^2 + y^2\right] + \left(\Omega\Omega_1 - \Omega^2\right)\left(x^2 + y^2 - xd_1\right) \qquad (1.23)$$

where Ω_1 is the angular velocity of a rotating point inside the star.

Following Kopal (20), equation (1.23) may be expressed in nondimensional form as

$$\psi^* = \frac{1}{r^*} + \frac{q}{\sqrt{1-2\lambda r^* + r^{*\,2}}} + \frac{\Omega^{*2}}{2}\left[r^{*2}\left(1-v^2\right) + d_1^{*2} - 2\lambda r^* d_1^*\right]$$
$$+ \left(\Omega_p^{*\,2} - \Omega^{*2}\right)\left[r^{*2}\left(1-v^2\right) - \lambda r^* d_1^*\right] \tag{1.24}$$

If we assume that the angular velocity Ω is identical with Keplerian angular velocity Ω_k where $\Omega_k^{\ 2} = G(M_0 + M_1)/D^3$ then in terms of the nondimensional variables used by us, we get a relation $\Omega^{*2} = 2n = (q+1)$. Using this relation, equation (1.24) can be written in more simplified form as

$$\psi^{**} = \frac{1}{r^*} + q\left[\frac{1}{\sqrt{1-2\lambda r^* + r^{*\,2}}} - \alpha\,\lambda\,r^*\right] + \beta\,n\,r^{*2}\left(1-v^2\right) \tag{1.25}$$

where

$$\alpha = \sqrt{n_1/n}\ ,\ \beta = 2\sqrt{n_1/n} - 1\ ,\ \Omega_p^{\ *2} = 2n_1 \text{ and } \psi^{**} = \frac{D\psi}{GM_0} - \frac{M_1^{\ 2}}{2M_0(M_0 + M_1)}$$

For binary system rotating synchronously, the angular velocity due to rotation and revolution are same, that is, $\vec{\Omega_1} = \vec{\Omega}$ and the terms containing Coriolis force will not appear explicitly.

In such cases equation (1.24) reduces to

$$\psi^{**} = \frac{1}{r^*} + q\left[\frac{1}{\sqrt{1-2\lambda r^* + r^{*\,2}}} - \lambda\,r^*\right] + n\,r^{*2}\left(1-v^2\right) \tag{1.26}$$

which agrees with expression earlier obtained by Kopal (20).

Also there is no coriolis force in pure tidal case and hence therefore expression for ψ^{**} in this case is identical to its earlier obtained by Kopal (20) and can be obtained directly from (1.26) by putting $n = 0$.

In case of pure rotation also, coriolis force is not generated as there is no revolution on centre of the star and hence no rotataing frame of reference. In such a case the expression for Roche equipotential for a a purely rotating star which is not subject to tidal effects of the companion star becomes

$$\psi^* = \frac{1}{r^*} + n\, r^{*2}\left(1 - \nu^2\right) \tag{1.27}$$

which can be obtained from (1.27) just by putting $q = 0$ or from (1.25) by setting $q = 0$, $\beta = 1$. This expression is same as earlier discussed by Kopal (14) for pure rotating stars.

Thus on explicit inclusion of coriolis force, expression for Roche equipotential gets modified from the earlier one obtained by Kopal (20) only in the case of nonsynchronous binaries. In case of synchronous binaries, purely rotating and purely tidally distorted stars, there is no change.

1.5 EQUILLIBRIUM STRUCTURES OF ROTATIONALLY AND TIDALLY DISTORTED STARS INCORPORATING THE EFFECT OF CORIOLIS FORCES

As we discussed above, the modified expression for Roche equipotentials explicitely incorporates the effect of coriolis forces, besides the centrifugal and gravitational forces. Adopting an approach similar to the one used by Kopal (20) and Mohan et al (33), Pathania (37) defined a nondimensional variable r_0 by the relation

$$r_0 = \frac{1}{\psi^{**} - q} \tag{1.28}$$

Following Kopal (20), variables (r, θ, ϕ) on the surface of the modified Roche equipotentials (1.25) can be shown to be connected through the relation

$$\begin{aligned} r = r_0[&1 + \lambda q t r_0^2 + a_0 r_0^3 + (qP_3 + 2\lambda^2 q^2 t^2) r_0^4 + (qP_4 + 5a_0\lambda q t) r_0^5 + (qP_5 + 3a_0^5 + 6\lambda q^2 t P_3) r_0^6 \\ &+ (qP_6 + 7a_0 q P_3 + 7\lambda q^2 t P_4) r_0^7 + (qP_7 + 8a_0 q P_4 + 8\lambda q^2 t P_5 + 4q^2 P_3^2) r_0^8 \\ &+ (qP_8 + 9a_0 q P_5 + 9\lambda q^2 t P_6 + 9q^2 P_3 P_4) r_0^9 + (qP_9 + 10a_0 q P_6 + 10\lambda q^2 t P_7 \\ &+ 5q^2\{P_4^2 + 2P_3 P_5\}) r_0^{10} + \cdots] \end{aligned} \tag{1.29}$$

where $a_0 = qP_2 + \beta n(1 - \nu^2)$, $t = 1 - \alpha$ and $P_j = P_j(\lambda)$ denote Legendre polynomial. As earlier upto second order of smallness in n, n_1 and q, and terms upto r_0^{10} in r_0 are retained in (1.29). Relation (1.29) incorporates the effect of coriolis force and can be used to obtain the shapes of various Roche equipotential surfaces $\psi^{**} =$ constant.

Again following Kopal (20) and Mohan et al (33) , expressions for volume V_ψ, surface area S_ψ and value of radial distance r_ψ of a point on the equipotential surface ψ^{**} = constant inside the primary can be shown to be

$$V_\psi = \frac{4\pi}{3} D^3 r_0^3 [1 + 2(\beta n) r_0^3 + 3q^2 t^2 r_0^4 + \left(\frac{12}{5} q^2 + \frac{32}{5} (\beta n)^2 + \frac{8}{5} (\beta n) q \right) r_0^6 + \frac{15}{7} q^2 r_0^8 + 2q^2 r_0^{10} + ...] \tag{1.30}$$

$$S_\psi = 4\pi D^2 r_0^2 [1 + \frac{4}{3}(\beta n) r_0^3 + \frac{5}{3} q^2 t^2 r_0^4 + \left(\frac{7}{5} q^2 + \frac{56}{15} (\beta n)^2 + \frac{14}{15} (\beta n) q \right) r_0^6 + \frac{9}{7} q^2 r_0^8 + \frac{11}{9} q^2 r_0^{10} + ...] \tag{1.31}$$

and

$$r_\psi = D\ r_0 [1 + \frac{2}{3}(\beta n) r_0^3 + q^2 t^2 r_0^4 + \left(\frac{4}{5} q^2 + \frac{76}{45} (\beta n)^2 + \frac{8}{15} (\beta n) q \right) r_0^6 + \frac{5}{7} q^2 r_0^8 + \frac{2}{3} q^2 r_0^{10} + ...] \tag{1.32}$$

Inverting above equation (1.32) we have

$$r_0 = r_\psi^* [1 - \frac{2}{3}(\beta n) r_\psi^{*3} - q^2 t^2 r_\psi^{*4} - \left(\frac{4}{5} q^2 - \frac{4}{45} (\beta n)^2 + \frac{8}{15} (\beta n) q \right) r_\psi^{*6} - \frac{5}{7} q^2 r_\psi^{*8} - \frac{2}{3} q^2 r_\psi^{*10} + ...] \tag{1.33}$$

where $r_\psi^* = r_\psi / D$, r_ψ^* being the non dimensional form of r_ψ . And here $t = 1 - \alpha$.

In absence of the effect of coriolis forces ($\alpha = \beta = 1$) hence these reduces to expressions earlier obtained by Mohan et al (33).

Using the modified expressions for Roche equipotential and other parameters obtained above, which incorporates the effect of coriolis force besides gravitational and centrifugal forces, the values of distortions parameters u, v, w, f_P *and* f_T now become

$$u = [1 - \frac{1}{3} q^2 t^2 r_\psi^{*4} - \left(\frac{1}{5} q^2 + \frac{4}{45} (\beta n)^2 + \frac{2}{15} (\beta n) q \right) r_\psi^{*6} - \frac{1}{7} q^2 r_\psi^{*8} - \frac{1}{9} q^2 r_\psi^{*10} + ...] \tag{1.34}$$

$$v = [1 - \frac{4}{3}(\beta n) r_\psi^{*3} - \left(\frac{2}{5} q^2 + \frac{8}{45}(\beta n)^2 + \frac{4}{15}(\beta n) q \right) r_\psi^{*6} - \frac{5}{7} q^2 r_\psi^{*8} - q^2 r_\psi^{*10} + ...] \quad (1.35)$$

$$w = [1 + \frac{4}{3}(\beta n) r_\psi^{*3} + 3q^2 t^2 r_\psi^{*4} + \left(\frac{18}{5} q^2 + \frac{152}{45}(\beta n)^2 + \frac{12}{15}(\beta n) q \right) r_\psi^{*6} + \frac{30}{7} q^2 r_\psi^{*8} + 5q^2 r_\psi^{*10} + ...] \quad (1.36)$$

$$f_P = [1 - \frac{4}{3}(\beta n) r_\psi^{*3} - \frac{8}{3} q^2 t^2 r_\psi^{*4} - \left(\frac{17}{5} q^2 + \frac{68}{45}(\beta n)^2 + \frac{34}{15}(\beta n) q \right) r_\psi^{*6} - \frac{29}{7} q^2 r_\psi^{*8} - \frac{44}{9} q^2 r_\psi^{*10} + ...] \quad (1.37)$$

$$f_T = [1 - \frac{7}{3} q^2 t^2 r_\psi^{*3} - \left(\frac{14}{5} q^2 + \frac{56}{45}(\beta n)^2 + \frac{28}{15}(\beta n) q \right) r_\psi^{*6} - \frac{23}{7} q^2 r_\psi^{*8} - \frac{34}{9} q^2 r_\psi^{*10} + ...] \quad (1.38)$$

where $r_\psi^* = r_\psi / D$ is the nondimensional form of r_ψ and terms upto second order of smallness in n, n_1 and q and terms upto r_0^{10} in r_ψ are retained . These reduce to expressions earlier obtained in Mohan et al (33) by setting $\alpha = \beta = 1$.

The system of differential equations governing the equilibrium structure of a rotationally and tidally distorted model which incorporates the effects of coriolis force besides the gravitational and centrifugal forces can be expressed as

$$\frac{dM_\psi}{dr_0} = 4\pi D^3 \rho_\psi r_0^3 f_1 \quad (1.39a)$$

$$\frac{dP_\psi}{dr_0} = -\frac{GM_\psi}{Dr_0^2} \rho_\psi f_2 \quad (1.39b)$$

$$\frac{dL_\psi}{dr_0} = 4\pi \varepsilon D^3 \rho_\psi r_0^2 f_1 \quad (1.39c)$$

$$\frac{dT_\psi}{dr_0} = -\frac{3\kappa L_\psi \rho_\psi}{16\pi D a c T_\psi^3 r_0^2} f_3 \quad (1.39d)$$

with $r_0 = \dfrac{1}{\psi^{**} - q}$

Here f_1, f_2 are certain functions of distortion parameters n, n_1, q and r_0 and incorporates the effect of coriolis force in addition to centrifugal and gravitational forces on the equilibrium structure equations of rotationally and tidally distorted stellar model (In case of no distortion $f_1 = f_2 = 1$). Explicit expression for distortion parameters $f_1 = \frac{dr_\psi}{dr_0}\frac{r_\psi^2}{D^3}$, $f_2 = \frac{f_P}{r_\psi^2}\frac{dr_\psi}{dr_0}D$, $f_3 = \frac{f_1}{r_\psi^2}\frac{dr_\psi}{dr_0}D$ and r_0 when terms upto second order of smallness in n, n_1, q and upto r_0^{10} in r_0 are retained, are given as

$$f_1 = [1 + 4(\beta n)r_0^3 + 7q^2t^2r_0^4 + \left(\frac{36}{5}q^2 + \frac{96}{5}(\beta n)^2 + \frac{24}{5}(\beta n)q\right)r_0^6 + \frac{55}{7}q^2r_0^8 + \frac{26}{3}q^2r_0^{10} + ...] \tag{1.40a}$$

$$f_2 = [1 + \frac{1}{3}q^2t^2r_0^4 + \left(\frac{3}{5}q^2 + \frac{4}{5}(\beta n)^2 + \frac{2}{5}(\beta n)q\right)r_0^6 + \frac{6}{7}q^2r_0^8 + \frac{10}{9}q^2r_0^{10} + ...] \tag{1.40b}$$

$$f_3 = [1 + \frac{4}{3}(\beta n)r_0^3 + \frac{2}{3}q^2t^2r_0^4 + \left(\frac{6}{5}q^2 + \frac{224}{45}(\beta n)^2 + \frac{4}{5}(\beta n)q\right)r_0^6 + \frac{12}{7}q^2r_0^8 + \frac{20}{9}q^2r_0^{10} + ...] \tag{1.40c}$$

and

$$r_0 = r_\psi^*[1 - \frac{2}{3}(\beta n)r_\psi^{*3} - q^2t^2r_\psi^{*4} - \left(\frac{4}{5}q^2 - \frac{4}{45}(\beta n)^2 + \frac{8}{15}(\beta n)q\right)r_\psi^{*6} - \frac{5}{7}q^2r_\psi^{*8} - \frac{2}{3}q^2r_\psi^{*10} + ...] \tag{1.40d}$$

where r_ψ^* is the nondimensional value of the radius of topologically equivalent spherical surface. Effects of coriolis force appear in these expressions through α and β . The boundary conditions now become

At the centre $r_0 = 0$, $M_\psi = 0$

And at the surface $r_0 = r_{0s}$, $M_\psi = M_0$, $P_\psi = 0$ or $P_{\psi s}$

Here M_0 is the total mass of the model and $P_{\psi s}$ is the value of P_{ψ}, on the outermost equipotential surface, $\psi^{**} =$ constant.

At the surface, $r_0 = r_{0s}$

$$(1.41)$$

where $r_{0s} = \dfrac{1}{\psi_s^{**} - q}$

ψ_s^{**} being the nondimensional value of the total potential ψ^{**} on the outermost equipotential surface of the rotationally and tidally distortec stellar model.

1.6 THE PRESENT WORK

In case of binary system besides the gravitational and centrifugal forces, coriolis forces also come into picture. While computing the effects of rotational and tidal distortions on the equilibrium structures and the eigenfrequencies of radial and nonradial modes of oscillations of rotating stars and stars in binary systems, Mohan and Saxena (29,30) and Mohan et al (33,34,31,32) approach does not explicitly accounts for the effect of coriolis forces. Keeping this in view, Pathania (37) analysed the effects of inclusion of coriolis force, besides the gravitational and centrifugal forces, on equilibrium structures of rotating stars and stars in binary system using a rotating frame of reference. This methodology is here used in determining equilibrium structure of rotationally and tidally distorted polytropic model of stars.

The approximation of exact equipotential surface of rotationally and tidally distorted stars by corresponding Roche equipotential by Mohan, Saxena and Aggarwal (29,30) may not be very much justified for the white dwarf stars. However still, it will be of interest to see how the equilibrium structures of rotationally and tidally distorted white dwarf model is affected with the incorporation of Coriolis force. In fact, Lal et al (25) applied Roche technique to study the equilibrium structures of differentially rotating white dwarf models of stars but did not study the equilibrium structures of rotationally and tidally distorted white dwarf models of stars. The study is expected to help in better understanding the limitations of Mohan et al (33,34,31,32) approach to determine the structure of theoretical models of stars.

Chaptrwise summary of the work presented in the subsequent chapters of this thesis is as follows:

In chapter II we studied the equilibrium structures of rotationally and tidally distorted white dwarf model of stars subject to effect of Coriolis force, for various choices of the of rotational and tidal distortion parameters. The results thus obtained, for degeneracy parameter $\frac{1}{\phi_0^2}$ as 0.01,0.05,0.1,0.2,0.5,0.8, is compared with the undistorted model where effect of Coriolis forces is not taken.

In chapter III we studied the effect of incorporation of Coriolis force on the equilibrium structures of rotationally and tidally distorted Prasad model and developed the expressions for the density, mass and pressure. And, hence the conclusions based on our study is discussed here.

CHAPTER – II

EFFECT OF CORIOLIS FORCES ON THE EQUILIBRIUM STRUCTURES OF ROTATIONALLY AND TIDALLY DISTORTED WHITE DWARF MODELS OF STARS

A White dwarf, also called a degenerate dwarf, is a small star composed mostly of electron-degenerate matter. They are very dense; a white dwarf mass is comparable to that of sun and its volume is comparable to that of earth. Its faint luminosity comes from the emission of stored thermal energy.

Observations show that most of the stars in the binary system are known to be rotating about their axes as well as revolving around their common centre of mass. The influence of uniform rotation on the global structure of white dwarf models had been considered by Chandrashekhar (8), James (16), Roxburgh (40) etc. Their results show that solid body rotation does not induce any substantial change in the global structure of degenerate dwarfs.

In the present chapter, the concept of white dwarf model of stars is presented in section 2.1.The methodology developed in section 1.5 is then used in section 2.2 to compute the inner structures of rotationally and tidally distorted white dwarf models of rotating stars and stars in binary system. In section 2.3 mathematical expressions for volumes and surface areas of rotationally and tidally distorted white dwarf models of stars is obtained. Numerical computations have next been performed in section 2.4 to obtain the inner structures and values of other parameters of certain rotationally and tidally distorted white dwarf models of stars for different values of η_u. Final conclusions based on present results are summarized in section 2.5.

2.1 WHITE DWARF MODEL OF STARS

White dwarf models of stars have frequently been used in literature to depict the inner structures of realistic low mass stars in their last stage of evolution. In a white dwarf model, the pressure P and the density ρ at an arbitrary point in the model are given by the relation

$$P = A\,f(x) \quad \text{and} \quad \rho = Bx^3 \tag{2.1}$$

where

$$A = 6.01\times 10^{22},\ B = 9.82\times 10^{5}\,\mu_e,\ f(x) = x(2x^2-3)\left(x^2+1\right)^{1/2} + 3\sinh^{-1} x$$

where $x = \frac{P_0}{mc}$ is a relativistic constant (Here P_0 denotes the momentum, m is the mass of the particle, c is the velocity of light and μ_e is the mean molecular weight per electron).

The equilibrium structure of a white dwarf model can be shown to be governed by the nonlinear differential equation

$$\frac{1}{\eta^2}\frac{d}{d\eta}(\eta^2\frac{d\phi}{d\eta}) = -(\phi^2 - \frac{1}{\phi_0^{\ 2}})^{3/2} \tag{2.2}$$

subject to the boundary conditions

$$\phi = 1\,,\ \frac{d\phi}{d\eta} = 0 \text{ at the center } \eta = 0 \tag{2.3a}$$

and

$$\phi = \frac{1}{\phi_0} \text{ at the surface } \eta = \eta_u \tag{2.3b}$$

The basic difference between the white dwarf model and the polytropic models is that the equation (2.2) of white dwarf model does not admit of a homology constant, and hence each mass has a density distribution characteristics of itself which cannot be inferred from the density distribution in a configuration of a different mass.

2.2 EQUILIBRIUM STRUCTURES OF ROTATIONALLY AND TIDALLY DISTORTED WHITE DWARF MODEL OF STARS

In this section we consider the feasibility of using the approach developed in section 1.5 , to determine the inner structures and equilibrium configurations of certain rotationally and tidally distorted white dwarf model of stars which incorporates the effect of coriolis force besides the gravitational and centrifugal forces.

Suppose a white dwarf model is subject to rotation and tidal distortion then its structure becomes a rotationally and tidally distorted white dwarf model. Following the approach of section 1.5, we may approximate the equipotential surfaces of such a model by modified Roche equipotential surfaces which incorporate the effect of coriolis force besides the centrifugal and gravitational forces.

Let P_ψ and ρ_ψ denote the pressure and the density respectively on the equipotential surface ψ^{**} = constant of distorted white dwarf model. If we assume that the distorted model is also a completely degenerate white dwarf model, then P_ψ and ρ_ψ of such a configuration will be connected through the relations of the type

$$P_\psi = Af(x) \quad \text{and} \quad \rho_\psi = Bx^3 \tag{2.4}$$

The equations (1.39a) and (1.39b) which govern the hydrostatic equilibrium structure of a rotationally and tidally distorted gaseous sphere can be combined together to yield

$$\frac{1}{r_\psi^2}\frac{d}{dr_\psi}\left[\frac{r_\psi^2}{\rho_\psi} u\, w \frac{dP_\psi}{dr\psi}\right] = -4\pi G\rho_\psi \tag{2.5}$$

Using relations (2.4) and substituting $(x^2+1) = \phi_0^2\,\phi_\psi^2$, it reduces to

$$\frac{\ell^2}{r_\psi^2}\frac{d}{dr_\psi}\left[r_\psi^2\, u\, w \frac{d\phi_\psi}{dr_\psi}\right] = -(\phi_\psi^2 - \frac{1}{\phi_0^2})^{3/2} \tag{2.6}$$

where

$$\ell^2 = \frac{2A}{\pi G B^2 \phi_0^2}$$

In case the white dwarf model is rotationally and tidally distorted, the values of r_ψ , u, and w needed in equation (2.6) are provided by (1.34) , (1.35) , (1.36) respectively. It may be noted that the approximation of equipotential surfaces by Roche equipotentials does not basically alter the structures of the white dwarf model because in the absence of any distortion ($u = w = 1$), equation (2.6) reduces to the usual structure of white dwarf given in equation (2.4).

To obtain the equilibrium structure of a rotationally and tidally distorted model, (2.6) has to be integrated numerically subject to the boundary conditions

$$\phi_\psi = 1,\ \frac{d\phi_\psi}{dr_\psi} = 0 \text{ at the centre } r_\psi = 0, \tag{2.7a}$$

and

$$\phi_\psi = \frac{1}{\phi_0} \text{ at the surface } r_\psi = R_\psi \tag{2.7b}$$

The values of r_ψ on the outermost equipotential surface of the distorted white dwarf model is given by

$$r_\psi = \ell \eta_u \tag{2.8}$$

where η_u is the value of η when ϕ equals $\frac{1}{\phi_0}$ for the undistorted model.

If we set $\frac{D}{\ell} = \frac{\eta_u}{K}$ (where D is the distance between the centres of the primary and secondary components of the binary system, K is an arbitrary constant) in equation (2.6), the differential equation governing the equilibrium structure of a rotationally and tidally distorted white dwarf model incorporating the effects of Coriolis force can be written explicitly in nondimensional form as

$$\frac{d}{dr_0}[A(r_0,n,q)\frac{d\phi_\psi}{dr_0}] = -\frac{\eta_u^2}{K^2}\left(\phi_\psi^2 - \frac{1}{\phi_0^2}\right)^{3/2} r_0^2 B(r_0,n,q) \tag{2.9}$$

where

$$A(r_0,n,q) = \frac{r_0^2}{f_2} = r_0^2\,[1 - \frac{1}{3}q^2t^2r_0^4 - \left(\frac{3}{5}q^2 + \frac{4}{5}(\beta n)^2 + \frac{2}{5}(\beta n)q\right)r_0^6 - \frac{6}{7}q^2r_0^8 - \frac{10}{9}q^2r_0^{10} + ...]$$

$$B(r_0,n,q) = f_1 = [1 + 4(\beta n)r_0^3 + 7q^2t^2r_0^4 + \left(\frac{36}{5}q^2 + \frac{96}{5}(\beta n)^2 + \frac{24}{5}(\beta n)q\right)r_0^6 + \frac{55}{7}q^2r_0^8$$

$$+ \frac{26}{3}q^2r_0^{10} + ...]$$

with $r_0 = \frac{1}{\psi^{**} - q}$, $t = 1-\alpha$, $\alpha = \sqrt{n_1/n}$ and $\beta = 2\alpha - 1$.

In above expressions terms upto second order of smallness in n, n_1, q and upto r_0^{10} in r_0 are retained.

In the absence of coriolis force (on setting $\alpha = \beta = 1$) the expressions $A(r_0,n,q)$ and $B(r_0,n,q)$ reduce to their corresponding expressions when the effects of coriolis force was not explicitly considered.

Equation (2.9) has to be solved subject to the boundary conditions (2.3) which now become:

At the centre: $r_0 = 0,\ \phi_\psi = 1, \dfrac{d\phi_\psi}{dr_0} = 0$, and (2.10a)

At the surface: $r_0 = r_{0s},\ \phi_\psi = \dfrac{1}{\phi_0}$ (2.10b)

r_{0s} being the value of r_0 at the outer surface (r_{0s} and r_0 , they both are dimensionless quantities).

Equation (2.9) subject to the boundary conditions (2.10) determines the equilibrium structure of rotationally and tidally distorted white dwarf model, which accounts for the effect of coriolis force in addition to the centrifugal and gravitational forces on the equipotential surfaces.

In order to determine the numerical solution of the second order nonlinear differential equation (2.9) subject to the boundary conditions (2.10) . we integrate equation (2.9) for certain choice of the values $1/\phi_0^2,\ K, n, n_1, q, \eta_u$ with the boundary conditions (2.10). the integration continues till ϕ_ψ equals to $1/\phi_0$. The value of r_0 for which ϕ_ψ becomes $1/\phi_0$, determines the outermost free surface r_{0s} of the model. Once the solutions of equation (2.9) is obtained , we know the values of ϕ_ψ for various values of nondimensional independent variable r_0 varying from zero to r_{0s}.

2.3 COMPUTATION OF VOLUME, SURFACE AREA AND OTHER PHYSICAL PARAMATERS OF DISTORTED WHITE DWARF MODELS OF STARS

In this section, we present the explicit expressions for computing the volume, surface area and other physical parameters of rotationally and tidally distorted white dwarf model. Following the approach given in section 1.5. The pressure P_ψ and the density ρ_ψ on various equipotentials of the distorted model may now be obtained through the relations (2.4) in the same manner as done for undistorted white dwarf model. Also, the radius r_ψ of the topologically equivalent spherical surface ψ^{**} = constant can be determined from equation (1.40) and (2.10) and is given by

$$r_\psi = \left(\frac{\ell \eta_u}{K}\right) r_0 [1 + \frac{2}{3}(\beta n) r_0^3 + q^2 t^2 r_0^4 + \left(\frac{4}{5}q^2 + \frac{76}{45}(\beta n)^2 + \frac{8}{15}(\beta n) q\right) r_0^6 + \frac{5}{7} q^2 r_0^8 + \frac{2}{3} q^2 r_0^{10} + ...] \tag{2.11}$$

The total volume enclosed by a rotationally and tidally distorted white dwarf model which accounts for coriolis force as well is given by

$$V_\psi = \frac{4\pi}{3}\left(\frac{\ell \eta_u}{K}\right)^3 r_{0s}^3 [1 + 2(\beta n) r_{0s}^3 + 3 q^2 t^2 r_{0s}^4 + \left(\frac{12}{5} q^2 + \frac{32}{5}(\beta n)^2 + \frac{8}{5}(\beta n) q\right) r_{0s}^6 + \frac{15}{7} q^2 r_{0s}^8 + 2 q^2 r_{0s}^{10} + ...] \tag{2.12}$$

And the is surface area of this rotationally and tidally distorted white dwarf model is given as

$$S_\psi = 4\pi \left(\frac{\ell \eta_u}{K}\right)^2 r_{0s}^2 [1 + \frac{4}{3}(\beta n) r_{0s}^3 + \frac{5}{3} q^2 t^2 r_{0s}^4 + \left(\frac{7}{5} q^2 + \frac{56}{15}(\beta n)^2 + \frac{14}{15}(\beta n) q\right) r_{0s}^6 + \frac{9}{7} q^2 r_{0s}^8 + \frac{11}{9} q^2 r_{0s}^{10} + ...] \tag{2.13}$$

In the absence of coriolis force ($\alpha = \beta = 1$), the expressions for V_ψ, S_ψ and r reported in equation (2.12-2.13), respectively reduce to their corresponding expressions obtained by Kopal (20) and Mohan and Saxena (29) who did not explicitly account the effects of coriolis force.

The relations (2.12-2.13) determine the volume and surface area of rotationally and tidally distorted white dwarf model when terms upto second order of smallness in n , n_1 and q and terms upto r_{0s}^{10} in r_{0s} are retained . in case we need the volume and the surface area of some inner equipotential surface of the distorted model then we need to replace r_{0s} by the appropriate value of r_0 for that surface in the above relations (2.12-2.13).

2.4 NUMERICAL COMPUTATIONS

To obtain the inner structure, the volume and surface area of rotationally and tidally distorted white dwarf model, equation (2.9) has to be integrated numerically subject to the boundary conditions (2.10) for the specified values of the parameters $1/\phi_0^2$, K, n, n_1, q, η_u which denote respectively the parameter, the ratio of the undistorted radius of the primary to the distance between the centres of the primary and secondary, the nondimensional measure of angular velocity of rotation, parameter having coriolis force effect, the ratio of the mass of the companion to the mass of the primary and the radius of the undistorted white dwarf model. In case of white dwarf model which is the primary component of a binary system ,the value of K has to be chosen in such a way that the outermost surface of the primary component lies well within the Roche lobe otherwise the two components of the binary will coalesce (cf. Kopal (20), page 11).

For obtaining the numerical solutions, equation (2.9) has been integrated numerically using fourth-order Runge-kutta method for the specified values of the input parameters. A series solution should preferably be developed near the centre. Such a series solution for present case is given by

$$\phi_\psi = 1 - \frac{Y^3}{6}\frac{\eta_\mu^2}{K^2}r_0^2 + \frac{Y^3}{40}\frac{\eta_\mu^4}{K^4}r_0^4 - \frac{1}{90}\left(12(\beta n) + 2q^2t^2\right)\frac{\eta_\mu^2}{K^2}Y^3 r_0^5$$
$$-\left(\frac{5}{27}q^2t^2 + \frac{1}{360}\frac{\eta_\mu^4}{K^4} + \frac{1}{1008}Y^3\frac{\eta_\mu^4}{K^4}\right)Y^3\frac{\eta_\mu^2}{K^2}r_0^6 + \left(\frac{1}{16} + \frac{1}{140}(\beta n) + \frac{1}{840}q^2t^2\right)Y^3\frac{\eta_\mu^4}{K^4}r_0^7$$

(2.14)

where $Y = \left(1 - \frac{1}{\phi_0^2}\right)^{1/2}$.

Taking the starting values from this series solution at r_0 = 0.005, numerical integration of equation (2.9) was then carried forward using Runge-Kutta method of fourth order. Using a step length of 0.005, numerical integration was continued till ϕ_ψ first become zero. Relations (2.12-2.13) were then used to determine the volume and surface area of the distorted white dwarf model of stars.

Numerical results obtained for different values of the input parameters are tabulated in Tables 2.1 to 2.3 .the value of parameter has been taken as one for the rotationally distorted model and 0.5 for rotationally and tidally distorted models. (the chosen value of K provides the outer-most surface of the model well within the Roche lobe for each considered case). In Table 2.1, we present the values of ϕ_ψ , P_ψ , ρ_ψ for various values of η_μ. The values of the volume and surface areas obtained for certain distorted models are next presented in Tables 2.2. In Table 2.3 we have analysed the effect of different choices for values of n , n_1 and $q$on the volumes and surface areas of rotationally and tidally distorted white dwarf models of stars.

2.5 ANALYSIS OF THE RESULTS

Our results in Tables 2.1 show that in the case of uniformly rotating, nonsynchronous binaries and synchronous binaries, the values of θ_ψ, P_ψ and ρ_ψ increases with the increase in the value of degeneracy parameter $\frac{1}{\phi_0^2}$ as 0.01, 0.05, 0.1, 0.2, 0.5, 0.8.

The volumes and surface areas of white dwarf model of stars are calculated in Table 2.2, showing the values of volume and surface area decreases with the increase in the value of degeneracy parameter $\frac{1}{\phi_0^2}$ as 0.01, 0.05, 0.1, 0.2, 0.5 and value of ϕ_ψ gradually increases as $\frac{1}{\phi_0^2}$ approaches 0.8.

Results in Table 2.3 show that, in the case of non-synchronous binaries when the values of parameter n of revolution and parameter q of tidal effects are kept fix than with an increase in the value of parameter n_1, the angular velocity of primary, there is an increase in the values of volumes and surface areas of white dwarf of models of stars, fixing the value of degeneracy parameter $\frac{1}{\phi_0^2}$. However, the volumes and surface areas are lesser than the values in the case of undistorted model when the value of n_1 is less than $n/4$. However when parameters n_1 and q are kept fix, than for values of n <0.5 there is increase in the values of volumes and surface area. When the values of n becomes greater than $4n_1$ these values become even less than their corresponding values for the undistorted models. Again when the rotational parameters n and n_1 are kept fixed and values of q varies, there is an increase in values of the volumes and surface areas of white dwarf models of stars, but not appreaciably.

Thus our present study has shown that whereas earlier results of Kopal (20) and Mohan and Saxena (29) already account for the effect of coriolis force in case of uniformly rotating and synchronous binaries, even in the case of non-synchronous binaries explicit inclusion of coriolis force in the equation of structure of white dwarf model does not produce any appreciable effect on the equilibrium structure of white dwarf models of such stars.

Table 2.1 Effect of inclusion of coriolis force on the equilibrium structures of rotationally and tidally distorted white dwarf models of stars.

$x=\frac{r_0}{r_{0s}}$	Uniformly Rotating			Non synchronus binary			Synchronus binary		
	n= 0.05, q= 0.0, n_1 = 0.05			n=0.05, q=0.2, n_1 =0.05			n=0.55, q=0.1, n_1 =0.55		
	ϕ_ψ	ρ_ψ	$P_\psi\times10^{28}$	ϕ_ψ	ρ_ψ	$P_\psi\times10^{28}$	ϕ_ψ	ρ_ψ	$P_\psi\times10^{28}$
$1/\phi_0^{\,2}$=0.01 , η_u=5.3571									
0.0	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)
0.2	0.8422 (0.8390)	0.5936 (0.5870)	0.5797 (0.5711)	0.8393 (0.8390)	0.5896 (0.5870)	0.5718 (0.5711)	0.8374 (0.8390)	0.5834 (0.5870)	0.5665 (0.5711)
0.4	0.5578 (0.5523)	0.1678 (0.1627)	0.1057 (0.1015)	0.5530 (0.5523)	0.1638 (0.1626)	0.1020 (0.1015)	0.5378 (0.5523)	0.1498 (0.1627)	0.0907 (0.1015)
0.6	0.3357 (0.3310)	0.0330 (0.0319)	0.0116 (0.0109)	0.3317 (0.3310)	0.0321 (0.0319)	0.0011 (0.0109)	0.3063 (0.3310)	0.0246 (0.0319)	0.0076 (0.0109)
0.8	0.1917 (0.1897)	0.0044 (0.0042)	0.0006 (0.0006)	0.1900 (0.1897)	0.0042 (0.0042)	0.0006 (0.0006)	0.1620 (0.1897)	0.0021 (0.0042)	0.0002 (0.0006)
1.0	0.0999 (0.0999)	0.0000 (0.0000)	0.0000 (0.0000)	0.0999 (0.0999)	0.0000 (0.0000)	0.0000 (0.0000)	0.0999 (0.0999)	0.0000 (0.0000)	0.0000 (0.0000)
$1/\phi_0^{\,2}$=0.05 , η_u=4.4601									
0.0	1.0000 (1.0000)	1.0000 (1.0000)	0.4009 (0.4005)	1.0000 (1.0000)	1.0000 (1.0000)	0.4009 (0.4005)	1.0000 (1.0000)	1.0000 (1.0000)	0.4009 (0.4005)
0.2	0.8923 (0.8900)	0.6969 (0.6904)	0.2525 (0.2496)	0.8902 (0.8900)	0.6911 (0.6904)	0.2499 (0.2496)	0.8892 (0.8900)	0.6886 (0.6904)	0.2487 (0.2496)
0.4	0.6702 (0.6651)	0.2723 (0.2655)	0.0691 (0.0667)	0.6658 (0.6651)	0.2665 (0.2655)	0.0671 (0.0667)	0.6534 (0.6651)	0.2501 (0.2655)	0.0614) (0.0667)
0.6	0.4666 (0.4621)	0.0741 (0.0714)	0.0103 (0.0109)	0.4628 (0.4621)	0.0718 (0.0714)	0.0110 (0.0109)	0.4380 (0.4621)	0.0577 (0.0714)	0.0076 (0.0109)
0.8	0.3206 (0.3185)	0.0131 (0.0126)	0.0008 (0.0008)	0.3189 (0.3185)	0.0127 (0.0126)	0.0008 (0.0008)	0.2886 (0.3185)	0.0065 (0.0126)	0.0002 (0.0008)
1.0	0.2235 (0.2236)	0.0000 (0.0000)	0.0000 (0.0000)	0.2236 (0.2236)	0.0000 (0.0000)	0.0000 (0.0000)	0.2236 (0.2236)	0.0000 (0.0000)	0.0000 (0.0000)
$1/\phi_0^{\,2}$=0.1 , η_u=4.069									
0.0	1.0000 (1.0000)	1.0000 (1.0000)	0.8643 (0.8643)	1.0000 (1.0000)	1.0000 (1.0000)	0.8645 (0.8643)	1.0000 (1.0000)	1.0000 (1.0000)	0.8649 (0.8643)
0.2	0.9169 (0.9132)	0.7441 (0.7386)	0.5887 (0.5832)	0.9151 (0.9132)	0.7417 (0.7386)	0.5860 (0.5832)	0.9134 (0.9132)	0.7371 (0.7386)	0.581 (0.5832)
0.4	0.7317 (0.7268)	0.3365 (0.3284)	0.1924 (0.1856)	0.7300 (0.7268)	0.3330 (0.3284)	0.1900 (0.1856)	0.7171 (0.7268)	0.3122 (0.3284)	0.1729 (0.1856)
0.6	0.5495 (0.5445)	0.1061 (0.1020)	0.0360 (0.0340)	0.5483 (0.5445)	0.1052 (0.1020)	0.0356 (0.0340)	0.5223 (0.5445)	0.08417 (0.1020)	0.0255 (0.0340)
0.8	0.4109 (0.4083)	0.0212 (0.0202)	0.0031 (0.0036)	0.4117 (0.4083)	0.2146 (0.0202)	0.0038 (0.0036)	0.3792 (0.4083)	0.1070 (0.0202)	0.0010 (0.0036)
1.0	0.3162 (0.3162)	0.0000 (0.0000)	0.0000 (0.0000)	0.3162 (0.3162)	0.0000 (0.0000)	0.0000 (0.0000)	0.3162 (0.3162)	0.0000 (0.0000)	0.0000 (0.0000)

$1/\phi_0{}^2$=0.2 , η_u=3.7271									
0.0	1.0000 (1.000)	1.0000 (1.0000)	0.1570 (0.1569)	1.0000 (1.000)	1.0000 (1.0000)	0.1576 (0.1569)	1.0000 (1.000)	1.0000 (1.0000)	0.1570 (0.1569)
0.2	0.9399 (0.9383)	0.7989 (0.7846)	0.1143 (0.1132)	0.9385 (0.9383)	0.7851 (0.7846)	0.1133 (0.1132)	0.9382 (0.9383)	0.7839 (0.7846)	0.1131 (0.1132)
0.4	0.8005 (0.7964)	0.4091 (0.4000)	0.0439 (0.0424)	0.7970 (0.7964)	0.4013 (0.4000)	0.0424 (0.0424)	0.7891 (0.7964)	0.3840 (0.4000)	0.0399 (0.0424)
0.6	0.6516 (0.6473)	0.1488 (0.1432)	0.0096 (0.0091)	0.6479 (0.6473)	0.1440 (0.1432)	0.0092 (0.0091)	0.6287 (0.6473)	0.1205 (0.1432)	0.0069 (0.0091)
0.8	0.5316 (0.5292)	0.0332 (0.0317)	0.0009 (0.0008)	0.5296 (0.5292)	0.0319 (0.0317)	0.0009 (0.0008)	0.5032 (0.5292)	0.0171 (0.0317)	0.0003 (0.0008)
1.0	0.4472 (0.4472)	0.0000 (0.0000)	0.0000 (0.0000)	0.4472 (0.4472)	0.0000 (0.0000)	0.0000 (0.0000)	0.4472 (0.4472)	0.0000 (0.0000)	0.0000 (0.0000)

$1/\phi_0{}^2$=0.5 , η_u=3.533									
0.0	1.0000 (1.0000)	1.0000 (1.0000)	0.7293 (0.7270)	1.0000 (1.0000)	1.0000 (1.0000)	0.7275 (0.7270)	1.0000 (1.0000)	1.0000 (1.0000)	0.7273 (0.7270)
0.2	0.9728 (0.9720)	0.8439 (0.8395)	0.5694 (0.5648)	0.9722 (0.9720)	0.8401 (0.8395)	0.5654 (0.5648)	0.9720 (0.9720)	0.8393 (0.8395)	0.5647 (0.5648)
0.4	0.9055 (0.9033)	0.5121 (0.5025)	0.2620 (0.2544)	0.9036 (0.9033)	0.5038 (0.5025)	0.2554 (0.2544)	0.8997 (0.9033)	0.4810 (0.5025)	0.2423 (0.2544)
0.6	0.8265 (0.8239)	0.2217 (0.2140)	0.0696 (0.0658)	0.8243 (0.8239)	0.2151 (0.2140)	0.0663 (0.0658)	0.8136 (0.8239)	0.1844 (0.2140)	0.0518 (0.0658)
0.8	0.7576 (0.7560)	0.0569 (0.0542)	0.0077 (0.0071)	0.7563 (0.7560)	0.0547 (0.0542)	0.0072 (0.0071)	0.7405 (0.7560)	0.0301 (0.0542)	0.0027 (0.0071)
1.0	0.7070 (0.7070)	0.0000 (0.0000)	0.0000 (0.0000)	0.7070 (0.7070)	0.0000 (0.0000)	0.0000 (0.0000)	0.7071 (0.7070)	0.0000 (0.0000)	0.0000 (0.0000)
$1/\phi_0{}^2$=0.8 , η_u=4.0446									
0.0	1.0000 (1.0000)	1.0000 (1.0000)	0.2731 (0.2730)	1.0000 (1.0000)	1.0000 (1.0000)	0.2733 (0.2730)	1.0000 (1.0000)	1.0000 (1.0000)	0.2713 (0.2730)
0.2	0.9909 (0.9906)	0.8674 (0.8640)	0.2200 (0.2186)	0.9907 (0.9906)	0.8644 (0.8640)	0.2188 (0.2186)	0.9906 (0.9906)	0.8633 (0.8640)	0.2183 (0.2186)
0.4	0.9676 (0.9669)	0.5633 (0.5550)	0.1090 (0.1064)	0.9670 (0.9669)	0.5561 (0.5550)	0.1067 (0.1064)	0.9655 (0.9669)	0.5985 (0.5550)	0.1013 (0.1064)
0.6	0.9392 (0.9384)	0.2634 (0.2561)	0.0314 (0.0299)	0.9385 (0.9384)	0.2570 (0.2561)	0.0302 (0.0299)	0.9344 (0.9384)	0.2210 (0.2561)	0.0235 (0.0299)
0.8	0.9135 (0.9130)	0.0716 (0.0692)	0.0036 (0.0034)	0.9131 (0.9130)	0.0695 (0.0692)	0.0034 (0.0034)	0.9069 (0.9130)	0.0377 (0.0692)	0.0011 (0.0034)
1.0	0.8944 (0.8944)	0.0000 (0.0000)	0.0000 (0.0000)	0.8944 (0.8944)	0.0000 (0.0000)	0.0000 (0.0000)	0.8944 (0.8944)	0.0000 (0.0000)	0.0000 (0.0000)

Here $\rho_\psi = \frac{\rho_0}{\rho} = [(\phi_\psi^2 - \frac{1}{\phi_0^2})/(1-\frac{1}{\phi_0^2})]^{3/2}$, and the values in parenthesis are the values for undistorted white dwarf model.

Table 2.2: Volumes and surface areas of rotationally and tidally distorted white dwarf

Type of model	n	n_1	q	Volume ($V_\psi \times 10^2$)	Surface Area ($S_\varphi \times 10^2$)
$1/\phi_0^{\ 2}$=0.01 , η_u = 5.3571					
Undistorted	0.0	0.0	0.0	6.4398	3.6063
Uniformly Rotating	0.02	0.02	0.0	6.6198	3.6731
	0.05	0.05	0.0	6.9162	3.7823
Non synchronus Binary	0.05	0.01	0.2	6.4544	3.6103
	0.05	0.05	0.2	6.5060	3.6304
	0.05	0.1	0.2	6.5390	3.6421
Synchronus Binary	0.525	0.525	0.05	7.0881	3.8453
	0.550	0.550	0.1	7.1258	3.8589
$1/\phi_0^{\ 2}$=0.05 , η_u = 4.4601					
Undistorted	0.0	0.0	0.0	3.7164	2.4997
Uniformly Rotating	0.02	0.02	0.0	3.8130	2.5428
	0.05	0.05	0.0	3.9805	2.6170
Non synchronus Binary	0.05	0.01	0.2	3.7247	2.5025
	0.05	0.05	0.2	3.7511	2.5149
	0.05	0.1	0.2	3.7803	2.5275
Synchronous Binary	0.525	0.525	0.05	4.0604	2.6522
	0.550	0.550	0.1	4.0802	2.6609
$1/\phi_0^{\ 2}$=0.1 , η_u = 4.069					
Undistorted	0.0	0.0	0.0	2.8219	2.0805
Uniformly Rotating	0.02	0.02	0.0	2.8926	2.1151
	0.05	0.05	0.0	3.0077	2.1710
Non synchronus Binary	0.05	0.01	0.2	2.8274	2.0824
	0.05	0.05	0.2	2.8474	2.0927
	0.05	0.1	0.2	2.8678	2.1024
Synchronus Binary	0.525	0.525	0.05	3.0723	2.2023
	0.550	0.550	0.1	3.0862	2.2089
$1/\phi_0^{\ 2}$=0.2 , η_u = 3.7271					
Undistorted	0.0	0.0	0.0	2.1687	1.7456
Uniformly Rotating	0.02	0.02	0.0	2.2209	1.7735
	0.05	0.05	0.0	2.3059	1.8186
Non synchronus Binary	0.05	0.01	0.2	2.1729	1.7471
	0.05	0.05	0.2	2.1876	1.7554
	0.05	0.1	0.2	2.2026	1.7632
Synchronous Binary	0.525	0.525	0.05	2.3528	1.8434
	0.550	0.550	0.1	2.3633	1.8489

$1/\phi_0^{\ 2}$=0.5 , $\eta_u = 3.533$					
Undistorted	0.0	0.0	0.0	1.8472	1.5685
Uniformly Rotating	0.02	0.02	0.0	1.8896	1.5924
	0.05	0.05	0.0	1.9585	1.6310
Non synchronus Binary	0.05	0.01	0.2	1.8502	1.5696
	0.05	0.05	0.2	1.8622	1.5767
	0.05	0.1	0.2	1.8749	1.5837
Synchronus Binary	0.525	0.525	0.05	1.9963	1.6521
	0.550	0.550	0.1	2.0051	1.6570
$1/\phi_0^{\ 2}$=0.8 , $\eta_u = 4.0466$					
Undistorted	0.0	0.0	0.0	2.7715	2.0557
Uniformly Rotating	0.02	0.02	0.0	2.8337	2.0863
	0.05	0.05	0.0	2.9349	2.1359
Non synchronus Binary	0.05	0.01	0.2	2.7760	2.0571
	0.05	0.05	0.2	2.7931	2.0660
	0.05	0.1	0.2	2.8122	2.0751
Synchronous Binary	0.525	0.525	0.05	2.9898	2.1624
	0.550	0.550	0.1	3.0031	2.1691

Table 2.3 Effect of the choice of values of n, n_1 and q on the volumes and surface areas of rotationally and tidally distorted white dwarf models of stars

Model No.		Volume ($V_\psi \times 10^2$)	Surface Area ($S_\psi \times 10^2$)	Volume ($V_\psi \times 10^2$)	Surface Area ($S_\psi \times 10^2$)
		$1/\phi_0^2$=0.01 , $\eta_u = 5.3571$		$1/\phi_0^2$=0.05 , $\eta_u = 4.4601$	
	n_1	$n = 0.10, q = 0.20$			
1	0.00	6.3427	3.5663	3.6603	2.4720
2	0.03	6.4641	3.6142	3.7303	2.5052
3	0.05	6.4996	3.6278	3.7508	2.5146
4	0.07	6.5267	3.6380	3.7665	2.5217
5	0.10	6.5636	3.6517	3.7878	2.5312
6	0.12	6.5851	3.6596	3.8002	2.5367
7	0.15	6.6183	3.6718	3.8193	2.5451
8	0.20	6.6668	3.6894	3.8473	2.5573
	n	$n_1 = 0.10, q = 0.20$			
1	0.01	6.6622	3.6755	3.8447	2.5477
2	0.03	6.5626	3.6494	3.7872	2.5296
3	0.05	6.5589	3.6495	3.7850	2.5296
4	0.07	6.5607	3.6505	3.861	2.5304
5	0.10	6.5636	3.6517	3.7878	2.512
6	0.20	6.5461	3.6450	3.7777	2.5265
7	0.40	6.4584	3.6119	3.7271	2.5036
8	0.50	6.4049	3.5918	3.6962	2.4895
	q	$n = 0.10, n_1 = 0.20$			
1	0.00	7.9847	4.1760	4.6079	2.8946
2	0.01	6.6040	3.6673	3.8111	2.5420
3	0.05	6.6037	3.6671	3.8109	2.5418
4	0.10	6.6075	3.6683	3.8131	2.5427
5	0.15	6.6110	3.6694	3.8151	2.5434
6	0.20	6.6183	3.6718	3.8193	2.5451
7	0.30	6.6320	3.6760	3.8273	2.5480
8	0.40	6.6527	3.6825	3.8392	2.5525

		$1/\phi_0^2$=0.1 , $\eta_u = 4.069$		$1/\phi_0^2$=0.2 , $\eta_u = 3.7271$	
	n_1	$n = 0.10, q = 0.20$			
1	0.00	2.7984	2.0688	2.1506	1.7341
2	0.03	2.8325	2.0851	2.1768	1.7494
3	0.05	2.8446	2.0912	2.1861	1.7545
4	0.07	2.8626	2.1001	2.2000	1.7620
5	0.10	2.8699	2.1037	2.2055	1.7650
6	0.12	2.8784	2.1078	2.2121	1.7684
7	0.15	2.8910	2.1139	2.2218	1.7736
8	0.20	2.9102	2.1230	2.2365	1.7812
	n	$n_1 = 0.10, q = 0.20$			
1	0.01	2.9091	2.1155	1.6557	1.4529
2	0.03	2.8695	2.1024	1.6332	1.4439

3	0.05	2.8698	2.1024	1.6323	1.4439
4	0.07	2.8696	2.1034	1.6332	1.4446
5	0.10	2.8699	2.1037	1.6334	1.4448
6	0.20	2.8632	2.1002	1.6296	1.4426
7	0.40	2.8292	2.0834	1.6103	1.4308
8	0.50	2.8083	2.0730	1.5984	1.4237
	q	$n = 0.10, n_1 = 0.20$			
1	0.00	2.4048	2.3653	1.9379	1.6245
2	0.01	2.8993	2.1184	1.6502	1.4549
3	0.05	2.9001	2.1187	1.6506	1.4551
4	0.10	2.9008	2.1190	1.6510	1.4553
5	0.15	2.9033	2.1200	1.6524	1.4560
6	0.20	2.9056	2.1210	1.6537	1.4567
7	0.30	2.9126	2.1239	1.6577	1.4586
8	0.40	2.9217	2.1277	1.6629	1.4612

		$1/\phi_0^2$=0.5 , $\eta_u = 3.533$		$1/\phi_0^2$=0.8 , $\eta_u = 4.0446$	
	n_1	$n = 0.10, q = 0.20$			
1	0.00	1.8328	1.5588	2.7500	2.0429
2	0.03	1.8530	1.5713	2.7802	2.0593
3	0.05	1.8609	1.5759	2.7921	2.0654
4	0.07	1.8669	1.5794	2.8011	2.0699
5	0.10	1.8757	1.5843	2.8142	2.0764
6	0.12	1.8812	1.5874	2.8245	2.0804
7	0.15	1.8882	1.5913	2.8331	2.0855
8	0.20	1.9001	1.5978	2.8509	2.0941
	n	$n_1 = 0.10, q = 0.20$			
1	0.01	1.8894	1.5922	2.8491	2.0867
2	0.03	1.8754	1.5833	2.8138	2.0751
3	0.05	1.8749	1.5837	2.8131	2.0755
4	0.07	1.8754	1.5841	2.8139	2.0761
5	0.10	1.8757	1.5843	2.8142	2.0764
6	0.20	1.8719	1.5821	2.8085	2.0734
7	0.40	1.8514	1.5703	2.7778	2.0580
8	0.50	1.8382	1.5628	2.7581	2.0482
	q	$n = 0.10, n_1 = 0.20$			
1	0.00	2.1902	1.7624	3.2861	2.3098
2	0.01	1.8848	1.5897	2.8279	2.0834
3	0.05	1.8853	1.5899	2.8286	2.0838
4	0.10	1.8858	1.5901	2.8294	2.0840
5	0.15	1.8868	1.5906	2.8308	2.0846
6	0.20	1.8882	1.5913	2.8331	2.0855
7	0.30	1.8921	1.5931	2.8388	2.0879
8	0.40	1.8973	1.5955	2.8467	2.0911

CHAPTER – III

EFFECT OF CORIOLIS FORCE ON THE EQUILIBRIUM STRUCTURE OF ROTATIONALLY AND TIDALLY DISTORTED PRASAD MODELS OF STARS

In the present chapter we use the methodology developed in chapter I, section 1.5, to determine the equilibrium structures of rotationally and tidally distorted Prasad model. This model is often used in astrophysics to represent the inner structures of certain type of stars.

The concept of Prasad models of stars is presented in section 3.1. the methodology developed in section 1.5 is then used in section 3.2 to compute the inner structures of rotationally and tidally distorted Prasad models of stars. Mass equation is also developed in this section 3.2. Scope for further study on this type of model of stars is discussed in section 3.3.

3.1 PRASAD MODEL OF STARS

In Prasad model the density distribution follows the law $\rho = \rho_c(1 - x^2), \rho_c$ being the value of density ρ at the centre and x the non-dimensional measure of the distance from the centre. Prasad model have often been used in literature to analyse the problems of stellar structures and stellar pulsations.

3.2 EQUILIBRIUM STRUCTURES OF ROTATIONALLY AND TIDALLY DISTORTED PRASAD MODEL

In this section we consider the feasibility of using the approach developed in section 1.5 , to determine the inner structures and equilibrium configurations of certain rotationally and tidally distorted Prasad model which incorporates the effect of coriolis force besides the gravitational and centrifugal forces.

Suppose, we assume that the primary component of binary system behaves as a Prasad model and is subject to rotation and tidal distortion then its structure becomes a rotationally and tidally distorted Prasad model. Following the approach of section 1.5, we may approximate the equipotential surfaces of such a model by modified Roche equipotential surfaces which incorporate the effect of coriolis force besides the centrifugal and gravitational forces.

Let r_ψ denotes the radius of the topologically equivalent spherical model corresponds to an equipotential surface ψ^{**} = constant, discussed in section 1.4, of this rotationally and tidally distorted Prasad model. Also, let R_ψ be the value of r_ψ on the outermost surface corresponding to equipotential surface ψ^{**} = constant of this rotationally and tidally distorted model.

As discussed in section 1.5, r_ψ and R_ψ are written as

$$r_\psi = D\ r_0\,[1+\frac{2}{3}(\beta n)r_0^3 + q^2t^2r_0^4 + \left(\frac{4}{5}q^2 + \frac{76}{45}(\beta n)^2 + \frac{8}{15}(\beta n)q\right)r_0^6 + \frac{5}{7}q^2r_0^8 + \frac{2}{3}q^2r_0^{10} + ...] \tag{3.1}$$

$$R_\psi = D\ r_{0s}[1+\frac{2}{3}(\beta n)r_{0s}^3 + q^2t^2r_{0s}^4 + \left(\frac{4}{5}q^2 + \frac{76}{45}(\beta n)^2 + \frac{8}{15}(\beta n)q\right)r_{0s}^6 + \frac{5}{7}q^2r_{0s}^8 + \frac{2}{3}q^2r_{0s}^{10} + ...] \tag{3.2}$$

where $r_0 = \dfrac{1}{\psi^{**} - q}$, $t = 1-\alpha$, $\alpha = \sqrt{n_1/n}$ and $\beta = 2\alpha - 1$.

Further let ρ_ψ denote the value of density on an equipotentials surfaces. Hence, the density distribution law of rotationally and tidally distorted Prasad model is given by

$$\rho_\psi = \rho_c\left(1 - \frac{r_\psi^2}{R_\psi^2}\right) \tag{3.3}$$

On substituting the value of r_ψ from equation (3.1) into equation (3.3) we get

$$\rho_\psi = \rho_c(1 - \frac{D^2r_0^2}{R_\psi^2}(1+\frac{4}{3}(\beta n)r_0^3 + 2q^2t^2r_0^4 + \left(\frac{8}{5}q^2 + \frac{172}{45}(\beta n)^2 + \frac{16}{15}(\beta n)q\right)r_0^6 + \frac{10}{7}q^2r_0^8 + \frac{4}{3}q^2r_0^{10} + ...)) \tag{3.4}$$

On substituting the value of ρ_ψ from (3.4) in (1.39a) of chapter I and integrate w.r.t r_0 and using the fact that $M_\psi = 0$ at centre $r_0 = 0$ we get

$$M_\psi = \frac{4\pi D^3 r_0^3 \rho_c}{3}(1 - \frac{3D^2}{5R_\psi^2}r_0^2 + 2(\beta n)r_0^3 + 3q^2t^2r_0^4 - \frac{2(\beta n)D^2}{R_\psi^2}r_0^5 + (\frac{12}{5}q^2 + \frac{32}{5}(\beta n)^2 + \frac{8}{5}(\beta n)q - \frac{3q^2t^2D^2}{R_\psi^2})r_0^6 - ((\frac{12}{5}q^2 + \frac{116}{15}(\beta n)^2 + \frac{8}{5}(\beta n)q)\frac{D^2}{R_\psi^2} - \frac{15}{7}q^2)r_0^8 - (\frac{180}{91}\frac{q^2D^2}{R_\psi^2} - 2q^2)r_0^{10} + ...) \tag{3.5}$$

where M_ψ is the mass enclosed by equipotential surface ψ = constant.

Similarly on substituting the value of ρ_ψ from equation (3.4) and M_ψ from (3.5) in (1.39b) of chapter I and integrate w.r.t. r_0 we get

$$P_\psi = \frac{2\pi G D^2 \rho_c^2}{3}(K - r_0^2 + \frac{4}{5}\frac{D^2}{R_\psi^2}r_0^4 - \frac{4}{5}(\beta n)r_0^5 - (\frac{10}{9}q^2t^2 + \frac{1}{5}\frac{D^4}{R_\psi^4})r_0^6 + \frac{32}{21}\frac{(\beta n)D^2}{R_\psi^2}r_0^7$$
$$-(\frac{69}{100}q^2 + \frac{41}{25}(\beta n)^2 + \frac{1}{2}(\beta n)q - \frac{32}{15}\frac{q^2t^2D^2}{R_\psi^2})r_0^8 - \frac{28}{45}\frac{(\beta n)D^4}{R_\psi^4}r_0^9 + ((\frac{3}{5}q^2 + \frac{4}{3}(\beta n)^2 + \frac{46}{125}(\beta n)q$$
$$-\frac{22}{25}\frac{q^2t^2D^2}{R_\psi^2})\frac{D^2}{R_\psi^2} + ((\frac{33}{25}q^2 + \frac{652}{225}(\beta n)^2 + \frac{46}{75}(\beta n)q)\frac{D^2}{R_\psi^2} - \frac{3}{5}q^2))r_0^{10} + ...)$$

(3.6)

Where P_ψ is the pressure enclosed by equipotential surface ψ = constant and K is constant of integration whose value may be calculated using boundary conditions say P_ψ = 0 at $r_0 = r_{0s}$. This yield

$$K = r_{0s}^2 - \frac{4}{5}\frac{D^2}{R_\psi^2}r_{0s}^4 + \frac{4}{5}(\beta n)r_{0s}^5 + (\frac{10}{9}q^2t^2 + \frac{1}{5}\frac{D^4}{R_\psi^4})r_{0s}^6 - \frac{32}{21}\frac{(\beta n)D^2}{R_\psi^2}r_{0s}^7$$
$$+(\frac{69}{100}q^2 + \frac{41}{25}(\beta n)^2 + \frac{1}{2}(\beta n)q - \frac{32}{15}\frac{q^2t^2D^2}{R_\psi^2})r_{0s}^8 + \frac{28}{45}\frac{(\beta n)D^4}{R_\psi^4}r_{0s}^9 - ((\frac{3}{5}q^2 + \frac{4}{3}(\beta n)^2 + \frac{46}{125}(\beta n)q$$
$$-\frac{22}{25}\frac{q^2t^2D^2}{R_\psi^2})\frac{D^2}{R_\psi^2} + ((\frac{33}{25}q^2 + \frac{652}{225}(\beta n)^2 + \frac{46}{75}(\beta n)q)\frac{D^2}{R_\psi^2} - \frac{3}{5}q^2))r_{0s}^{10} + ...)$$

(3.7)

Similarly the volume V_ψ and surface area S_ψ of rotationally and tidally distorted Prasad model is given as

$$V_\psi = \frac{4\pi}{3}D^3r_0^3[1 + 2(\beta n)r_0^3 + 3q^2t^2r_0^4 + \left(\frac{12}{5}q^2 + \frac{32}{5}(\beta n)^2 + \frac{8}{5}(\beta n)q\right)r_0^6 + \frac{15}{7}q^2r_0^8 + 2q^2r_0^{10} + ...]$$

(3.8)

$$S_\psi = 4\pi D^2r_0^2[1 + \frac{4}{3}(\beta n)r_0^3 + \frac{5}{3}q^2t^2r_0^4 + \left(\frac{7}{5}q^2 + \frac{56}{15}(\beta n)^2 + \frac{14}{15}(\beta n)q\right)r_0^6 + \frac{9}{7}q^2r_0^8 + \frac{11}{9}q^2r_0^{10} + ...]$$

(3.9)

3.3 CONCLUSIONS

The expressions (3.4), (3.5), (3.6), (3.8) and (3.9) determine the density, mass pressure, volume and surface area of rotationally and tidally distorted Prasad models subject to the effect of Coriolis force. Then physical parameters can be computed in the units of ρ_c, $\frac{4}{3}\pi D^3\rho_c \times 10^{-3}$, $2\pi G D^2 \rho_c^2 \times 10^{-2}$, $\frac{4}{3}\pi D^3$ and $4\pi D^2$ respectively. On substituting ($\alpha = \beta = 1$) in the expressions (3.4), (3.5), (3.6), (3.8) and (3.9), one can determine the equilibrium structure of rotationally and tidally distorted Prasad models of starsin absence of Coriolis force.

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