Online Estimation and Control in Delta Domain Using Time Moments

Thesis submitted in partial fulfillment of the requirement for the award of degree of

MASTER OF ENGINEERING

In

ELECTRONIC INSTRUMENTATION & CONTROL

Submitted By

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DECLARATION

I hereby certify that the work which is being presented in the thesis entitled, **"Online Estimation & Control in Delta Domain using Time Moments "** in partial fulfillment of the requirements for the award of degree of Master of Engineering in Electronic Instrumentation and Control Engineering submitted in Electrical and Instrumentation Engineering, Department of Thapar University, Patiala, is an authentic record of my own work carried out under the supervision of **Mr. Souvik Ganguli (Assistant Professor)** and refers other researcher's works which are duly listed in the reference section.

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

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Nitin Khurana

ABSTRACT

The present thesis is concerned with controller design of linear discrete-time systems modeled in the delta domain. Time moment method, a tool so far used for model order reduction is applied to compute online time moments from the input-output response of the plant and reference model to estimate the controller parameters adaptively.

Traditionally, discrete-time sampled data systems are represented using shift-operator parameterization. Such parameterizations are not suitable at fast sampling rates. An alternative parameterization using the so-called delta operator maintains a close correspondence to its continuous-time counterpart at fast sampling rate. Using delta operator parameterization it is possible to unify both discrete-time and continuous-time theory.

The coefficients of the discrete-time transfer function are similar to the corresponding ones in continuous-time and it becomes easier to tune controller parameters for improved dynamic performance.

The thesis addresses a novel method to compute the time moments in delta domain. At fast sampling limit, the delta operator model tends to the analog dynamic system model. The δ-operator model can be applied to a wide range of discrete-time systems, from sampled-data systems with coarse sampling intervals to rapidly sample near continuous-time systems. Moreover, it has certain numerical advantages compared to the shift operator parameterization. It therefore provides a unified framework for system studies, where continuous-time results can be achieved from the discrete-time description of the system.

In the first part of the thesis, computation of the first three moments in the delta domain k_0, k_1 and k_2 are carried out. Numerical computation of these time moments has been carried out using MATLAB code.

In the second part of the thesis, time moments obtained in the first part are used in different control schemes like PCMS, PTCS, and PTCFS to match the time moments of the compensated plant with the desired response. At the end, the different control schemes give the same results in continuous domain as well as in delta domain.

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LIST OF ACRONYMS

AMM	Approximate Model Matching
EMM	Exact Model Matching
LTI	Linear Time-Invariant System
MRAC	Model Reference Adaptive Control
PTCFS	Plant Time Moment Controller with Feedback Scheme
PCMS	Plant Command Modifier Scheme
PPCMS	Padé–adapted Plant Command Modifier Scheme
PTCS	Plant Time Moment Controller Scheme
SISO	Single Input Single Output

CHAPTER 1

INTRODUCTION

1.1 Adaptive Control

According to Webster's dictionary, "to adapt" means to change a behavior to conform to new circumstances. Intuitively, an adaptive controller is thus a controller that can modify its behavior in response to changes in the dynamics of the process and the character of the disturbances. Since ordinary feedback was introduced for the same purpose, the question of the difference between feedback control and adaptive control immediately arises. Over the years there have been many attempts to define adaptive control. Truxal [1] proposed to define an adaptive system as a physical system which has been designed with an adaptive viewpoint. Other definitions were proposed by early workers in the field of [2-3]. Since, ordinary feedback also attempts to reduce the effects of disturbances and plant uncertainty. As defined in [4] an adaptive controller is a controller with adjustable parameters and a mechanism for adjusting the parameter. Simply speaking, an adaptive control system consists of two closed loops. One loop is a normal feedback with the process and the controller. The other loop is the parameter adjustment loop.

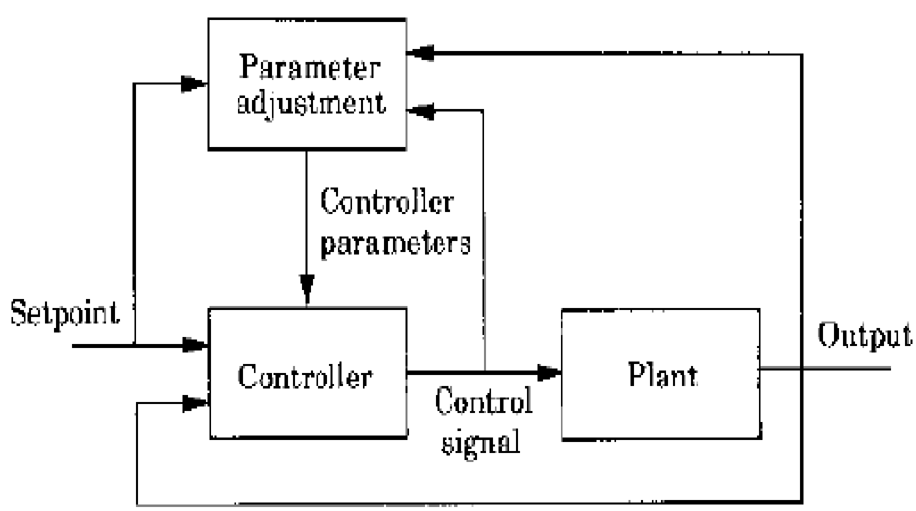


Figure 1.1 Block diagram of an adaptive system.

The parameter adjustment loop is often slower than the normal feedback loop. Adaptive systems can be profitably used to design control systems with improved performance and functionality. If the process operating conditions or the environment changes significantly, the controller may then have to be re-tuned. If the change occurs frequently, then adaptive control techniques should be considered. An adaptive control system is one in which the controller parameters are adjusted automatically to compensate for changing process conditions.

Control of fully known deterministic, LTI dynamical systems has received wide attention for many decades a lot of study and surveys have been performed. Among the different type of adaptive schemes traditionally four such schemes namely self oscillating, gain scheduling, auto tuning, model reference adaptive control (MRAC) are in wide use in system theory.

1.2.1 about Model Reference Adaptive Control

The model reference adaptive control (MRAC) may be regarded as an adaptive scheme in which the desired performance is expressed in terms of a reference model, which gives the desired response to a command signal. Alternatively, if it is specified (as a map) as the response of a reference model to an input from a permissible class, then the control scheme is referred to as model reference adaptive control (MRAC) [5]. The block diagram of the controller is shown below in the figure 1.2.

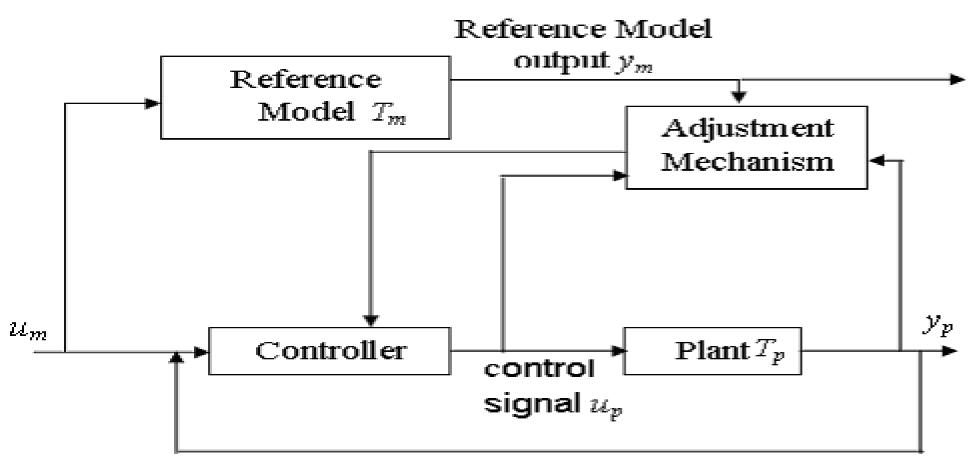


Figure 1.2 Block Diagram of a Model Reference Adaptive Control (MRAC)

The system has an ordinary feedback loop composed of the process and the controller, which is called the inner loop and another feedback loop that changes the controller parameters known as the outer loop. The parameters are changed on the basis of feedback from the error, which is the difference between the output of the system and the output of the reference model.

There are two approaches of MRAC:

- Indirect MRAC
- Direct MRAC

In indirect approach, the plant is taken as represented by an explicit model, the unknown parameters of which are estimated using on-line system identification techniques. This requires knowledge of the order of the plant or at least of an upper bound on the order of the plant. In the absence of knowledge of even a reasonable upper bound on the order, an order estimation technique has to face a difficult situation. These approaches are feasible only in situations where speed of adaptation and plant stability is not critical.

In direct MRAC, no explicit effort is made to identify the plant but the controller is directly adjusted to minimize the error between the plant and the model output. However, the assurance of the stability of such adaptation largely relies on some restrictive assumptions [6] as enameled below.

- The plant should be minimum phase (zeros or poles restricted to the left half of the s-plane).
- The order or the upper bound on the order of the plant should be known apriority.
- The reference model should be stable and minimum phase; and
- The input should be piecewise continuous and bounded.

There are two ways of parameter adjusting mechanisms both of these must satisfy the conditions stated above.

These two ways are:

- Using MIT rule (or Gradient method) or
- Applying Lyapunov's stability theory.

The block diagram of MRAC in figure 1.2 can also be represented as in figure below

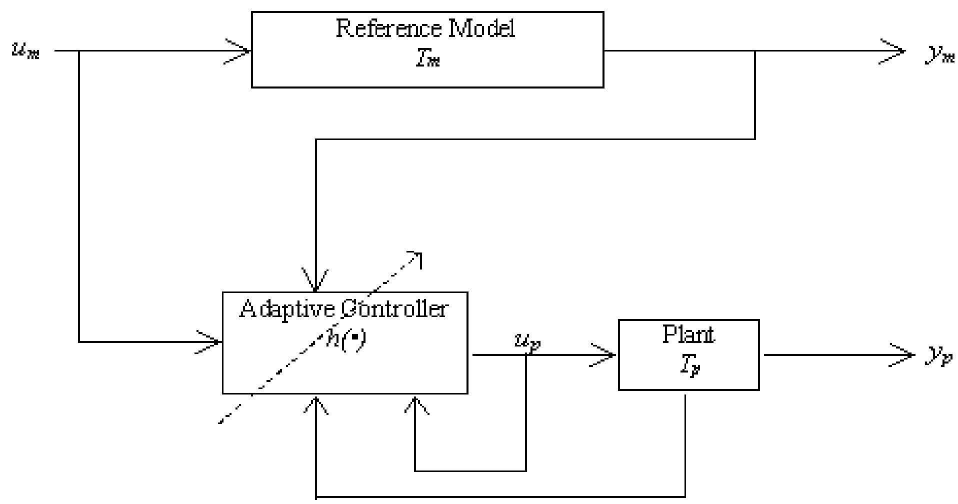


Figure 1.3 Schematic diagram of Model Reference Adaptive Control

1.2.2 MRAC Design Using MIT Rule

The MIT rule is the original approach to model-reference adaptive control. The name is derived from the fact that it was developed at the Instrumentation Laboratory (now the Draper Laboratory) at MIT by Whittaker. To present the MIT rule, let us consider a closed loop system in which the controller has one adjustable parameter θ. The desired closed-loop response is specified by a model whose output is y_m. Let e be the error between the output y_p of the closed-loop system and the output y_m of the model. One possibility is to adjust parameters in such a way that the cost function

$$J(\theta) = \frac{1}{2} e^2 \qquad \text{----- (1)}$$

is minimized. To make J small, it is reasonable to change the parameters in the direction of the negative gradient of J that is,

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} = -\gamma \, e \frac{\partial e}{\partial \theta}, \qquad \text{----- (2)}$$

This is the celebrated MIT rule. The partial derivative $\frac{\partial e}{\partial \theta}$, which is called the sensitivity derivative of the system, which shows how the error is influenced by the adjustable parameter. If it is assumed that the parameter changes are slower than the other variables in the system, then the derivative $\frac{\partial e}{\partial \theta}$ can be evaluated under the assumption that θ is constant. There are many alternatives to the cost function given by the above equation. If it is chosen to be,

$$J(\theta) = |e| \qquad \text{----- (3)}$$

The gradient gives

$$\frac{d\theta}{dt} = -\gamma \frac{\partial e}{\partial \theta} sign(e) \qquad \text{----- (4)}$$

The first MRAC that was implemented was based on the above formula.

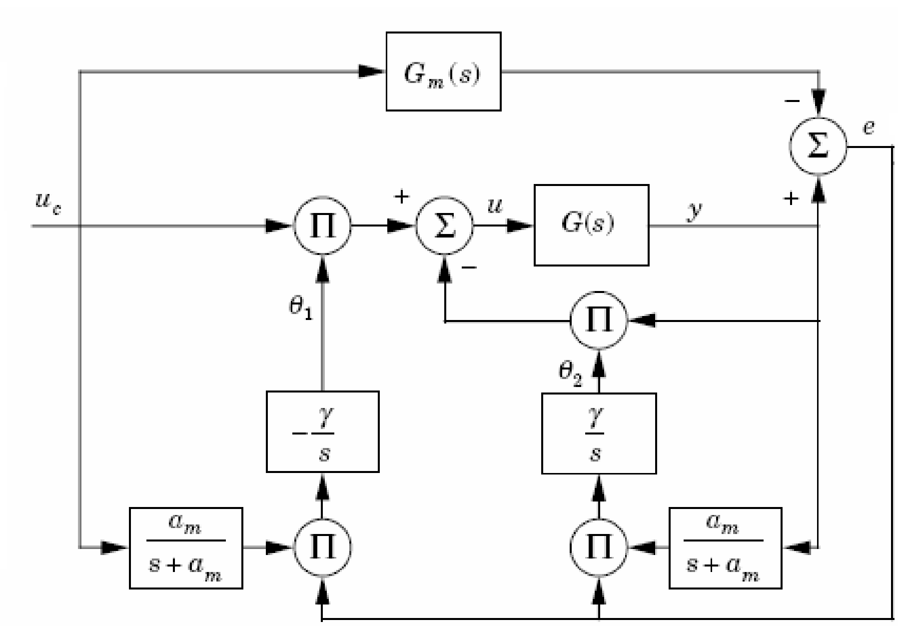


Figure 1.4 Block Diagram of a MRAC Based on MIT rule for a First Order System

While adjusting many parameters the equation (2) can also be used then the symbol θ should be interpreted as a vector and $\frac{\partial e}{\partial \theta}$ as the gradient of the error with respect to the parameters.

- Tracking error: $e = y_p - y_m$

- Cost function: $J(\theta) = \frac{1}{2} e^2$
- Update rule: $\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} = -\gamma \, e \frac{\partial e}{\partial \theta}$ change in θ is proportional to the negative gradient of J .
- Different cost functions can be chosen
- Ex $J(\theta) = | e(\theta) |$

$$\frac{d\theta}{dt} = -\gamma \frac{\partial e}{\partial \theta} sign(e) \qquad \text{----- (5)}$$

where,

$$sign(e) = \begin{cases} 1 & e > 0 \\ 0 & e = 0 \\ -1 & e < 0 \end{cases} \qquad \text{----- (6)}$$

- From cost function and MIT rule, control law can be formed.
- MIT does not guarantee error convergence or stability in some cases.
- γ is usually kept small.
- Tuning γ is crucial for adaptation rate and stability.

1.2.3 MRAC Design Using Lyapunov Stability Theorem

Lyapunov's stability theory can be used to construct algorithms for adjusting parameters in adaptive systems. This theory was proposed by Parks. For this parameter adjustment, first derive a differential equation for the error, $e = y - y_m$. This differential equation contains the adjustable parameters. Then attempt to find a Lyapunov function and an adaptation mechanism such that the error will go to zero. When using the Lyapunov theory for adaptive systems, usually $\frac{dV}{dt}$ is used to find only negative semi definite. Hence it is required to determine the error equation and a Lyapunov function with a bounded second derivative. To meet the parameter convergence, it is necessary to impose further conditions, such as persistently excitation and uniform observability, on the reference signal and the system.

Consider a simple example.

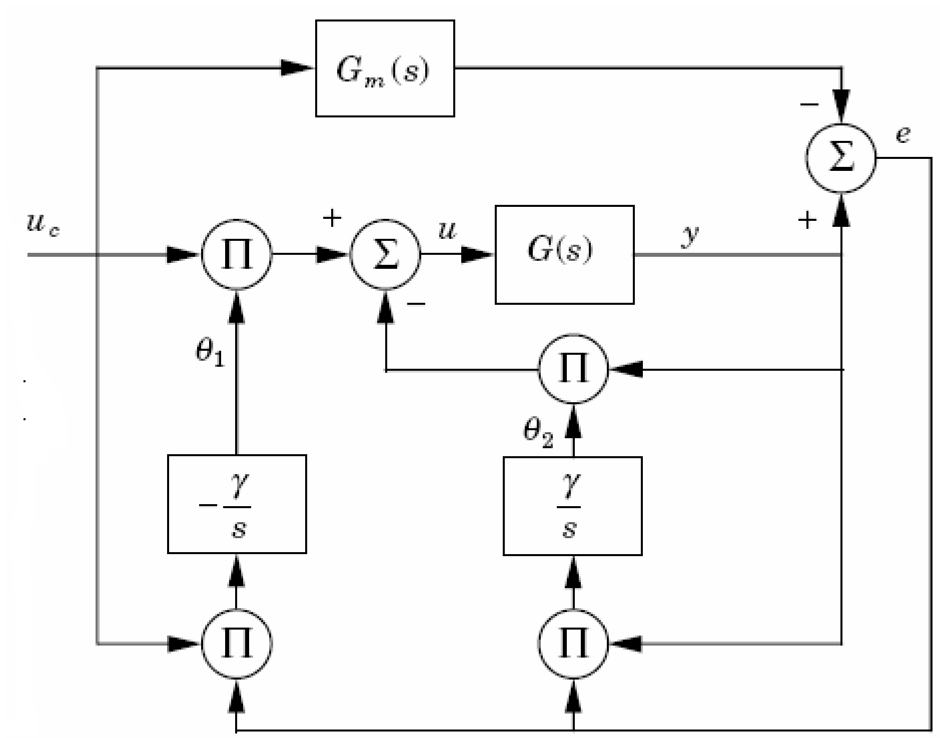


Figure 1.5 Block Diagram of a MRAC Based on Lyapunov rule for a first order system.

Example: First-order MRAC system based on stability theory.
The desired response is given by

$$\frac{dy_m}{dt} = -a_m y_m + b_m u_c \qquad \text{----- (7)}$$

where $a_m > 0$ and the reference signal is bounded. The process is described by,

$$\frac{dy}{dt} = -ay + bu \qquad \text{----- (8)}$$

the controller is

$$u = \theta_1 u_c - \theta_2 y \qquad \text{----- (9)}$$

the error is

$$e = y - y_m$$

To make the error small, it is natural to derive a differential equation for the error. We get,

$$\frac{de}{dt} = -a_m e - (b\theta_2 + a - a_m)y + (b\theta_1 - a_m)u_c \qquad \text{----- (10)}$$

The error goes to zero if the parameters are equal to the values given by the equations. The parameter adjustment mechanism that will drive the parameters θ_1 and θ_2 to their desired values is given as under:

$$V(e,\theta_1,\theta_2) = \frac{1}{2}\left(e^2 + \frac{1}{b\gamma}(b\theta_2 + a - a_m)^2 + \frac{1}{b\gamma}(b\theta_1 - b_m)^2\right) \qquad \text{----- (11)}$$

where $b\gamma > 0$

The function $V(\cdot)$ is zero when 'e' is zero and the controller parameters are equal to the correct values. For the function to qualify as a Lyapunov function the derivative $\frac{dV}{dt}$ must be negative. The derivative is,

$$\frac{dV}{dt} = e\frac{de}{dt} + \frac{1}{\gamma}(b\theta_2 + a - a_m)\frac{d\theta_2}{dt} + \frac{1}{\gamma}(b\theta_1 - b_m)\frac{d\theta_1}{dt}$$

$$= -a_m e^2 + \frac{1}{\gamma}(b\theta_2 + a - a_m)(\frac{d\theta_2}{dt} - \gamma y_e) + \frac{1}{\gamma}(b\theta_1 - b_m)(\frac{d\theta_1}{dt} + \gamma u_c e) \qquad \text{----- (12)}$$

If the parameters are updated as,

$$\frac{d\theta_1}{dt} = -\gamma u_c e \qquad \text{----- (13)}$$

$$\frac{d\theta_2}{dt} = \gamma y_e \qquad \text{----- (14)}$$

Then, putting (13) and (14) in (12) we get

$$\frac{dV}{dt} = -a_m e^2 \qquad \text{----- (15)}$$

The derivative of V with respect to time is thus negative semi definite but not negative definite. This implies that $V(t) \le V(0)$ and thus that e, θ_1 and θ_2 must be bounded. This implies that $y = e + y_m$ is also bounded.

To use Lyapunov's second theorem we determine

$$\frac{d^2V}{dt^2} = -2a_m e \frac{de}{dt} = -2a_m e(-a_m e - (b\theta_2 + a - a_m)y + (b\theta_1 - b_m)u_c) \qquad \text{-----} \ (16)$$

Since u_c, e and y are bounded, it follows that is bounded; hence $\frac{dV}{dt}$ is uniformly continuous. From theorem it now follows that the error 'e' will go to zero. However, the parameters will not necessarily converge to their correct values; it is shown only that they are bounded. To have parameter convergence, it is necessary to impose conditions on the excitation of the system.

The adaptation rule given in (15) is similar to the MIT rule given by equation (2), but the sensitivity derivatives are replaced by other signals.

CHAPTER 2

LITERATURE SURVEY

In this section, a literature survey of the evolution of δ-operator modeling and control with a view to its eventual application to model order reduction and model matching type of controller design problems is presented. The discussion mainly focuses on the issues that are relevant to this research and is by no means an exhaustive exposition of all the available contribution to these theories.

We start with an introduction to the development of δ-operator and control followed by a brief account of the early efforts in addressing the model order reduction problem in continuous time (s-domain) and discrete time (z-domain) systems. Then a brief discussion on controller design methods focusing on model matching based controller design methodologies for continues and discrete time systems.

In the works accumulated over the last four decades, a good volume of literature is available in the area of discrete time system studies using the shift operator, and in the complex domain using the associated z-transformation [7-12]. The last decade has brought a major development in linear system theory from the design and implementation perspective. The development is in the δ-operator formulation that make it possible to understand both continues and discrete time system identification, modeling and control within a unified frame work, while substantially improving the numerical robustness with respect to the traditional shift operator representation of discrete time system.

Central to the development of δ-operator theory is modeling of the system using the discrete differential or 'finite difference' operator. The calculus finite difference has its roots in the works on numerical analysis developed during the early part of 17^{th} century.

By Middleton and Goodwin [13] the application of this operator in control system studies was not widespread till 1985 when Middleton and Goodwin [14] introduced and redefined it as the δ-operator.

In an earlier work by Tschauner [15] defined the δ-operator as the discrete approximation of the differential operator, the application of which converts all the relationships of sample-data systems to their continuous-time counterparts at the fast sampling limit.

As early as in the sixties by Gupta [16] defined ξ-transform as the shifted z-transform with a very important property. As a period of sampling tends to zero, the ξ-transform becomes the Laplace transform with a slight modification. The so called ξ-transform is the complex domain version of δ-operator. The ξ-transform is advantageous because all continues system properties can be obtained from study to the discrete model. Other than this revelation, the actual benefits of its use in control systems analysis and design was fully explored by Middleton and Goodwin [17].

Mukhopadhyay [18] et al. have presented a new class of discrete-time models for continuous time system called γ-operator model which originate from the z-transfer function and have shown the δ-operator formulation as a member to his class [19]. The reliable and superior robust numerical properties of δ-operator have widened the area of application in many areas of systems, information and control. To highlight the important application of δ-operator in systems and control and literature on classical and modern control [20], predictive control, adaptive control, robust and optimal control, signal processing and identification etc. are worth mentioning.

From the literature survey it concluded that important area in system theory like model order reduction of large scale systems and controller design using the model reduction philosophy in the δ-domain are yet to be investigated. To fill up this gap, in this topic we consider the model order reduction and controller design problems of large scale linear discrete time invariant systems in the complex δ-domain. The δ-domain is a complex transformed domain, which is obtained by transforming the time domain δ-operator model by a complex domain variable γ and is called δ-transformation. δ-operator representation of discrete time systems have unique feature which exhibits combined properties of the discrete time systems as well its continues time representation at fast sampling limit and also shows superior numerical conditioning as compared to the corresponding shift operator representation. Both of these properties have been well exploited in analysis, design and simulation studies.

Tschauner [15] defined the delta operator **(**or incremental difference operator) in [14] as the discrete approximation of the differential operator. Over the years, engineers have recognized that application of the delta operator leads to reliable and robust numerical algorithms for computer control [17] and [21], adaptive control [22-23] the intuitive notion [1] of an adaptive controller is an algorithm which is capable of initially tuning itself and of retuning itself in the event that the process characteristics subsequently change. There has been intense interest in these algorithms for more than three decades. The algorithms are typically highly nonlinear and this has meant that the associated theory has taken some time to mature.

A major break-through occurred in the late 1970's when a number of researchers, e.g., [2]-[5], established bounded input bounded state **(**BIBS) stability of adaptive controllers applied to linear time-invariant systems. Later it was argued [6] that this theory was inadequate since it could be shown that the algorithms would fail if blindly applied to systems having small un-modeled dynamics. This latter problem has been the subject of intense study and several results have recently appeared applicable to situations where the nominal plant is time-invariant, It has also been established that persistency of excitation of the system states leads to exponential stability and that this, in turn, gives local robustness to un-modeled dynamics and time variations. However, in many practical circumstances, the set point is not persistently exciting and it is frequently undesirable to inject additional perturbations. Moreover, there remain some unresolved issues concerning how to guarantee persistency of excitation in the presence of potentially destabilizing feedback as it occurs in adaptive control. Persistency of excitation in the presence of time variations alone has recently been resolved in subject to a stability assumption. The extension to limited classes of time-varying plants (without persistency of excitation).

In this parameters are stochastic, but with constant expected value. In time-varying parameters which converge exponentially fast are considered.

The first important investigation and research about 2-D models was made by Marzetta [24] where the so-called reflection coefficients are introduced. A new 2-D lattice filter structure in the so-called delta domain, which avoids possible numerical instabilities that may occur in traditional *q*-domain based algorithms has been developed.

For the last 25 years, lattice filters and related structures have been an important research area both in 1-D and 2-D signal processing. 1-D approaches are already well known and well developed.

The first important investigation and research about 2-D models was made by Marzetta where the so-called reflection coefficients are introduced.

One of the most attentions drawing work has been done by Parker and Kayran [25] where quarter-plane autoregressive lattice parameter modeling of 2-D fields has been developed and four prediction error fields have been generated.

For several decades, extensive work and research have been performed on Levinson and Schur type algorithms which are based in the *q*-domain. However, such algorithms can lead to numerical instabilities under finite precision implementation when fast sampled or ill-conditioned data is used. Delta domain (DD) based algorithms, which alleviate such problems, are especially suitable for on-line implementation and thus more desirable for adaptive signal processing applications.

On-line Levinson and Schur type RLS algorithms have been developed using the so called backward delta operator. Application areas of DD based algorithms are adaptive signal processing, adaptive identification and control**,** modeling fast sampled data [26], unifying continuous and discrete.

Kalman filtering [27], in the statistical problem of estimating the bandwidth parameter of a Gauss-Markov (GM) process from a realization of fixed and finite duration T at selectable sampling interval Δ is addressed. As the observation time T , is fixed and finite, the variance of estimated autocorrelation and continuous-time parameter does not vanish as Δ approaches 0. This necessitates a second-order Taylor expansion in deriving the parameter estimator bias and variance. The second-order Taylor expansion produces better bias and variance results than a first-order one does. The distribution of the estimator is also discussed. According to the gradient change of the variance, a key result is that three sample size regions, which are termed finite, large, and very large, corresponding to substantial, gradual, and very slight decrease in the variance of the parameter estimator, respectively, are quantified, high-speed digital signal processing and control and formulating discrete dynamic robot models incorporated delta operator models in design software. The delta operator creates the rapprochement between analog and discrete dynamic system models and establishes the natural framework to investigate the behavior of discrete dynamic models in the fast sampling limit.

Time Moment Matching method is based on determining a set of time functions of the high-order system and matching them with those of the reduced model. The number of the time-moments matched depends on the desired order of the reduced model [28-29]. The more is the number of the time-moments matched, the more accurate the low-order approximate will be. In the case of continued fraction expansion (CFE) method, the degree of the numerator of the ROM obtained is always either equal to or one less than the degree of the denominator. However, such a restriction doesn't exist in the time-moment matching method. Extension of the moment-matching method to the reduction of multivariable systems is given in [30].

Bosely et al. [31], and Lal and Mitra [28] have discussed the similarity between the CFE (continued fraction expansion) and moment matching methods. A mixed method of model-order reduction is introduced, which is based on the linear state-space representation of the original system. The derivation of its transfer function matrix expression uses the Leverrier algorithm and the application of well known partial-expansion techniques together with the concept of dominant eigen values on the transfer function of the system. The method is general, relatively simple to apply and yields fairly accurate reduced-order models.

Parthasarathy and Singh [32] used markov parameters and time-moments for the minimal realization of symmetric transfer function matrices.

Seinfeld and Lapidus [33] summarized the theory and applications of the moment-matching method. When the process is mathematically too complex to model from the fundamental physical and chemical laws, empirical models can be obtained from experimental dynamic data. Characterizing and estimating the parameters of this model from its input/output experimental data is known as process identification.

Characterizing a process by an empirical model from its input/output experimental data is known as process identification. One may judge from input/output data if the system needs to be identified by a linear model or non-linear model. Mainly, if the output satisfies the superposition principle, that is, the response of the system to the sum of two inputs is the same as the sum of the response to the individual inputs, then a linear model will be adequate. Otherwise we need to identify the system by a non-linear model.

From the Literature Survey it also concluded that with the development of the δ-operator, interest has been rekindled among present day research workers to have a relook at discrete-time systems for control systems analysis/synthesis, design and implementation the concepts of true digital control, which rejects the idea that a digital control should be initially designed in continuous-terms. Rather it suggests that the designer should consider the design from a digital-sample data standpoint, near continuous time operation is required.

Many books have been written on the subject of process identification, see for example, Bendat (1990), Box and Jenkins (1970), Box and Draper (1987), Eykhoff (1974), Graups (1972), Ljung (1980), Mehra and Lainiotis (1976), Ray and Lainiotis (1987), Sage and Melsa (1971), Seinfield and Lapidus (1974), Sinha and Kuszta (1988), Soderstrom and Stoica (1989) and Unbehauen and Rao (1987). The main task is to identify a suitable linear model to represent the data. Ha.ving suggested the model, methods of parameter estimation can then be used and statistical methods are then called for to test the adequacy of the proposed model.

CHAPTER 3

DELTA OPERATOR REPRESENTATION

3.1 Introduction

Traditionally discrete-time sampled data systems are represented using shift-operator parameterization. Such parameterization is not suitable at fast sampling rates. An alternative parameterization using the so-called delta operator maintains a close correspondence to its continuous-time counterpart at fast sampling rate. Using delta operator parameterization it is possible to unify both discrete-time and continuous-time theory.

3.2 Delta operator Parameterization

Here an alternative formulation of discrete-time systems, called δ - operator parameterization is discussed. One of the most important works on such parameterization is due to Middleton and Goodwin [17], where the following points of motivation for this alternative discrete-time operator are given:

- It highlights similarities rather than differences between discrete-time and continuous-time systems. This allows physical continuous-time insights to be useful also in the discrete-time case.
- It allows a unified system theory to be developed in which discrete-time and continuous-time results can be derived simultaneously rather than as two special cases.
- Most continuous-time results can be obtained as a limiting case (when the sampling period tends to zero) of the discrete-time results.
- It is possible to use short sampling periods without incurring numerical difficulties such as a high sensitivity to round-off errors in coefficient representation.
- The coefficients of the discrete-time transfer function are similar to the corresponding ones in continuous-time and it becomes easier to tune controller parameters for improved dynamic performance.
- The frequency and transient responses of the continuous-time system can be accurately estimated from the discrete-time system.
- It offers substantial numerical advantages especially at high sampling rates.

3.3 Delta Operator

The δ -operator [34-36] is defined in the time-domain as

$$\delta = \frac{q-1}{\Delta} \quad \text{----- (17)}$$

where Δ the sampling period and q is the forward shift operator. Operating δ on a differential signal x (t) gives

$$\delta x(t) = \frac{x(t+\Delta)-x(t)}{\Delta} \quad \text{----- (18)}$$

It is straight forward to see that

$$\lim \Delta \to 0 \;\; \delta\text{x(t)} = \frac{d}{dt}x(t) \quad \text{----- (19)}$$

which demonstrates the close relationship between the discrete-time δ -operator and the continuous-time differential operator $\frac{d}{dt}$ at high sampling rates. Note that (17) is a simple linear transformation and thus system modeling using δ -operator parameterization offers exactly the same flexibility as q-operator parameterization, i.e. the class of describable systems is not changed.

Similar relation exists in the complex domain as well. The delta transform operator γ is defined as

$$\gamma = \frac{z-1}{\Delta} \quad \text{----- (20)}$$

where z is complex domain transform operator for discrete-time system, like the Laplace transform operator for continues-time system. Since eqn. (20) is a linear transform relation which shifts and scales the complex z-domain by $(-\frac{1}{\Delta})$ and $(\frac{1}{\Delta})$ to the complex δ -domain in γ -plane, therefore all the linear system properties and parameterizations of complex z-domain can be transformed to the δ -domain with equal flexibility as offered by the z-domain. The complex δ -domain variable is represented here by γ . The notation δ is a time domain operator called here as δ -operator and the notation γ is a complex domain variable called here as δ -domain variable for clarity and distinction between time and frequency domain analysis, design and simulation studies.

3.4 Significance of Time Moments in Delta Domain

Time moment, a tool for model order reduction is applied here to compute the online time moments in delta domain from the input-output response of the plant. A unified framework is required to obtain a reduced model from a high order system which is discrete at coarse sampling frequency and continuous at very high sampling frequency. At fast sampling limit, delta time moments converge to the corresponding time moments of the continuous-time systems leading to a unified treatment for discrete-time and continuous-time reduction methods simultaneously.

3.5 Estimation of Time Moments in Delta Domain

As first step towards adaptive control of the unknown plant, Sivaramakumar [37] has proposed a time-moment estimation scheme. No more than the plant input and output is available for this purpose. Following his steps, we have derived the corresponding time-moments in delta domain [38].

Consider a plant in delta domain as $G(\gamma)$. Let its impulse response be $g(k\Delta)$. Let y be the output of G excited by u. Then

$$Y(\gamma) = G(\gamma)U(\gamma) \qquad \text{----- (21)}$$

Since G is asymptotically stable, it permits a series expansion in terms of its time moments $\{k_i\}$ as

$$G(\gamma) = k_0 + k_1\gamma + k_2\gamma^2 + ... = \sum_{i=0}^{\infty} k_i \gamma^i \qquad \text{----- (22)}$$

The problem is to obtain estimates of $\{k_i\}$ from on-line measurements, up to the current time of u and y.

Consequently, $U(\gamma)$ is defined as follows:

$$U(\gamma) = \Delta \sum_{k=0}^{\infty} U(k\Delta)(1+\Delta\gamma)^{-k} \qquad \text{----- (23)}$$

$$= \Delta \sum_{k=0}^{\infty} U(k\Delta) - \Delta\gamma \sum_{k=0}^{\infty} (k\Delta)U(k\Delta) + \Delta\gamma^2 \sum_{k=0}^{\infty} k(k+1)\Delta^2 U(k\Delta) + ... \qquad \text{----- (24)}$$

$$= \Delta \sum_{k=0}^{\infty} U(k\Delta) - \Delta\gamma \sum_{k=0}^{\infty} (k\Delta)U(k\Delta) + \Delta\gamma^2 \sum_{k=0}^{\infty} k(k+1)\Delta^2 U(k\Delta) + ... \qquad \text{----- (25)}$$

Similarly,

$$Y(\gamma) = \Delta \sum_{k=0}^{\infty} Y(k\Delta)(1+\Delta\gamma)^{-k} \qquad \text{----- (26)}$$

$$= \Delta\left[Y(0)+Y(\Delta)+Y(2\Delta)+...\right] - \Delta^2\gamma\left[Y(\Delta)+2Y(2\Delta)+...\right] + \Delta^3\gamma^2\left[Y(\Delta)+3Y(2\Delta)+...\right]+... \qquad \text{----- (27)}$$

$$= \Delta\sum_{k=0}^{\infty} Y(k\Delta) - \Delta\gamma\sum_{k=0}^{\infty}(k\Delta)Y(k\Delta) + \Delta\gamma^2\sum_{k=0}^{\infty}k(k+1)\Delta^2 Y(k\Delta) + ... \qquad \text{----- (28)}$$

Equating $Y(\gamma) = G(\gamma)U(\gamma)$, we have

$$k_0 = \frac{\sum_{k=0}^{\infty} Y(k\Delta)}{\sum_{k=0}^{\infty} U(k\Delta)}$$

$$k_1 = \frac{-\Delta\left[\sum_{k=0}^{\infty} kY(k\Delta) - k_0\sum_{k=0}^{\infty} kU(k\Delta)\right]}{\sum_{k=0}^{\infty} U(k\Delta)}$$

$$k_2 = \frac{\Delta^2\sum_{k=0}^{\infty} k(k+1)Y(k\Delta) - k_0\Delta^2\sum_{k=0}^{\infty} k(k+1)U(k\Delta) + k_1\Delta\sum_{k=0}^{\infty} kU(k\Delta)}{\sum_{k=0}^{\infty} U(k\Delta)} \qquad \text{----- (29)}$$

CHAPTER 4

CONTROLLER DESIGN IN THE DELTA DOMAIN

4.1 Introduction

In system identification problem the moment matching technique is a well-established technique. This method was mainly used in case of off-line system identification. But in the moment matching technique is used for online system identification. In online system identification the estimation of the time moments requires the knowledge of input and output of the system. Now by using the estimates k_0, k_1, k_2 are first, second & third time moments respectively. Now this time moments obtained are used in control schemes to match the time moments of the compensated plant with the desired response.

4.2 Plant command modifier scheme in delta domain

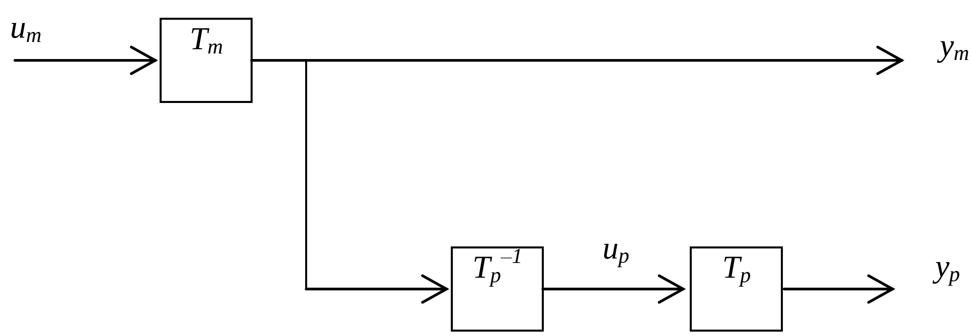


Figure: 4.1: Block diagram of PCMS

The plant command modifier scheme as proposed in [6] has been modified in this section in the delta domain with the goal to study the control scheme and the application of the online estimation scheme.

The basis on which this PCMS is built is shown in the figure 4.1. Suppose T_p^{-1} is available. Then u_p could have been obtained as

$$u_p = (T_p^{-1} T_m) u_m \qquad \text{----- (29)}$$

to get $y_p = y_m$ .Obviously, T_p^{-1} will be unstable in the event of T_p having nonminimum phase zero(s). To overcome this problem, a method involving time moments has been proposed.

Let $\{k_{i,p}\}$ denote the time moments of the unknown plant T_p .Then

$$T_p(\gamma) = k_{i,p}\gamma^i \qquad \text{----- (30)}$$

Regardless of the number of its zero in excess over its poles. T_p^{-1} permits an expansion in terms of its time moments { q_i } as

$$T_p^{-1}(\gamma) = q_{i,p}\gamma^i \qquad \text{----- (31)}$$

Now, $T_pT_p^{-1}$ =1 leads to

$$\sum_{i=0}^{\infty} k_{i,p}\gamma^i \sum_{i=0}^{\infty} q_{i,p}\gamma^i = 1 \qquad \text{----- (32)}$$

Collecting the coefficients of like powers of γ yields

$$k_{0,p}q_0 + (k_{1,p}q_0 + k_{0,p}q_1)\gamma + (k_{2,p}q_0 + k_{1,p}q_1 + k_{0,p}q_2)\gamma^2 + \_\_\_ = 1 \qquad \text{----- (33)}$$

and hence

$$\begin{aligned} &k_{0,p}q_0 = 1, \\ &k_{1,p}q_0 + k_{0,p}q_1 = 0, \\ &k_{2,p}q_0 + k_{1,p}q_1 + k_{0,p}q_2 = 0 \end{aligned} \qquad \text{----- (34)}$$

and so on. This is same as

$$\begin{aligned} &k_{0,p}q_0 = 1, \\ &\sum_{j=0}^{i} k_{i-j,p}q_j = 0 \text{ where i=1, 2, -------} \end{aligned} \qquad \text{----- (35)}$$

Thus exact matching would have been possible. Even this is infeasible. In fact, the resulting $T_p^{-1}T_m$ will be improper in continuous time and anti-casual in discrete time. From the foregoing it is clear that, in general, exact model matching is ruled out even in principle. The goal has to be necessarily settled down to approximate model matching.

For the T_p^{-1} of figure 4.1 has been replaced by a P_c - an approximation to T_p^{-1}. The hard constraint therefore turns out to be that $P_c T_m$. This P_c will have

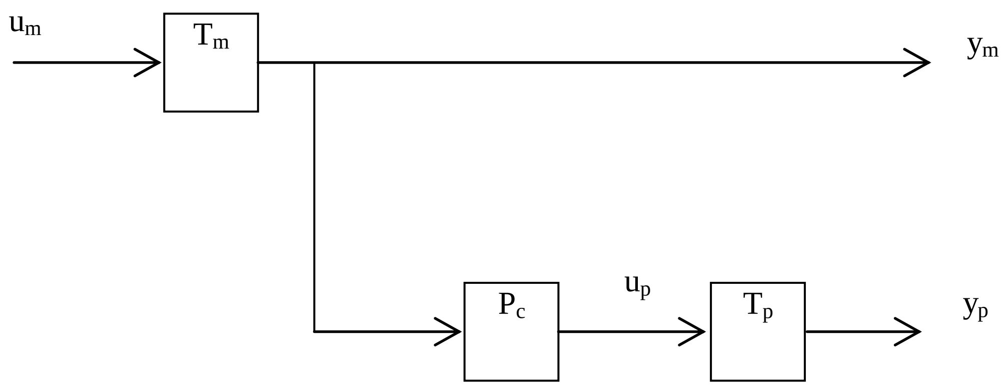


Figure: 4.2 Schematic for PCMS

To be specified in terms of T_m and the measurable quantities u_m, y_m, u_p and y_p. Further, it is necessary that P_c satisfy the following conditions.

- $(P_c T_m)$ is proper even when the relative degree of T_p exceeds the relative degree of T_m. It may be noted that P_c itself be proper is not required.
- P_c is stable even if T_p happens to be nonminimum phase.

It is natural to desire that the overall scheme be easy to implement. Equation (35) suggest a possible means of approximating T_p^{-1} as P_c. The approximation has been effected at two levels:

- First, the infinite series expansion of T_p^{-1} has been truncated.
- Secondly, finitely many coefficients of P_c have been through estimates of time moments of T_p.

4.3 Padé-adapted Plant Command Modifier Scheme in Delta Domain

Padé approximation was originally introduced by Padé and subsequently used by several researchers for order reduction of high order systems.

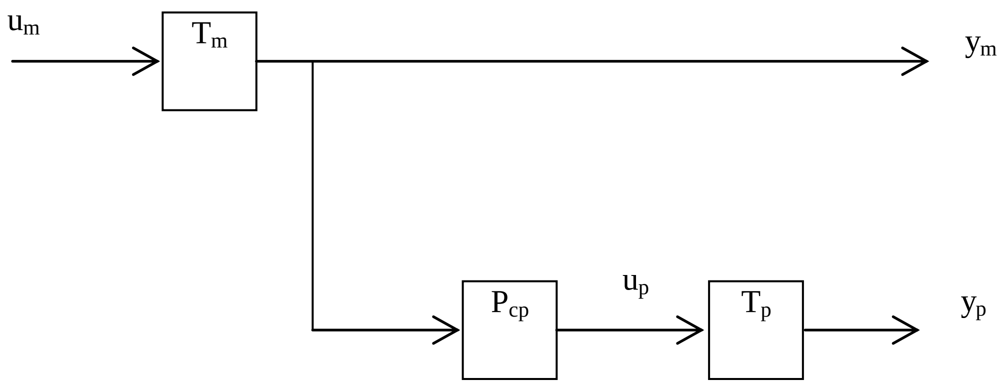


Figure 4.3 Schematic for PPCMS

Here,

$$P_c(\gamma) = f_0 + f_1\gamma + ... + f_n\gamma^n$$

with,

$$f_0 = \frac{1}{\hat{k}_{0,p}},$$

$$f_i = \frac{\sum_{j=0}^{i-1} \hat{k}_{i-j,p} f_j}{\hat{k}_{0,p}} \text{ where i=1, 2, 3......,}\eta$$

Now proceed to analyze the implications of this representation with $\mu \geq \nu$

$$P_c(\gamma) = f_0 + f_1\gamma + ... + f_\eta\gamma^\eta \ = \frac{b_0 + b_1\gamma + ... + b_\nu\gamma^\nu}{a_0 + a_1\gamma + ... + a_\mu\gamma^\mu} \text{ through } \gamma^{\mu+\nu}$$

Without loss of generality, $A(\gamma)$ can be chosen monic, i.e. $a_\mu = 1$ set $\mu = \nu = \eta$ .Cross multiplying and equating the coefficients of like powers of γ leads to

$$Fa = b,$$

$$F = \begin{bmatrix} f_0 & 0 & 0........ & 0 \\ f_1 & f_0 & 0........ & 0 \\ \vdots & \vdots & \vdots & \vdots \\ f_\eta & f_{\eta-1} & \vdots & f_0 \end{bmatrix}$$

where,

$$a = [a_0 \;\; a_1.....a_{\eta-1}]^T$$

and

$$b = [b_0 \;\; b_1.....b_{\eta-1}]^T$$

The identities obtained by equating the coefficients of γ^i , i= η+1, η+2… 2η have been neglected. The consequence is that with the compensator taking the form

$$P_{cp} = \frac{B}{A}$$

4.4 Plant Time Moment Controller Scheme in Delta Domain

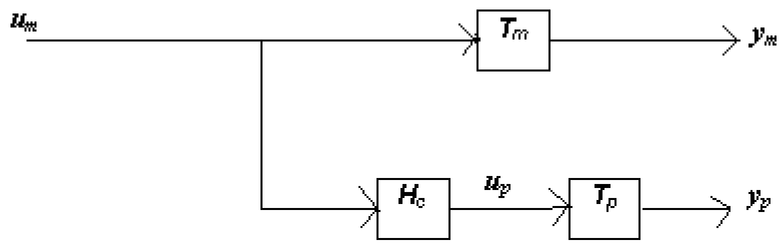


Figure 4.4 Schematic for PTCS

The AMM (Approximate Model matching) problem so far has been approached through an exclusively cascade compensator P_c or P_{cp} realized as an approximate inverse of the unknown plant T_p^{-1}. In the process no information of the reference model other than the settling times of its various modes has been utilized. More information of T_m may be gainfully exploited as in the sequel. When dealing with a known plant and a reference model, an approach to match the responses of the model with that of the plant for any reference model input is to insert a cascade compensator H_c. Presently, not only is T_p unknown, exact matching may be precluded without an unstable compensator in the event of T_p having zero(s) in the R.H.S. of the s-plane. This apart, even it T_p was known, the order of the compensator required may be too large. This happens when the order of T_p far exceeds that of T_m a situation often times encountered in practice. Hence as seen earlier, EMM (Exact Model Matching) condition $T_m = T_p H_c$ is ruled out in general.

So, start with a realistic approach that what can be best achieved in the case of approximate not exact match between y_p and y_m. Our endeavor will be desired H_c such that $T_p H_c$ approximates T_m in the sense of matching the first few time–moments.

Express H_c and T_m about $\gamma = 0$ respectively as

$$H_c(\gamma) = \sum_{i=0}^{\infty} h_i \gamma^i \quad \text{and} \quad T_m(\gamma) = \sum_{i=0}^{\infty} k_{i,m} \gamma^i$$

Along the same lines as in PCMS, set

$$\sum_{i=0}^{\infty} k_{i,m}\gamma^{i} = \sum_{j=0}^{\infty} k_{j,p}\gamma^{j} \sum_{l=0}^{\infty} h_{l}\gamma^{l}$$

Truncating this identity up to $i=\eta$ on the L.H.S., replacing $\{k_{j,p}\}$ by their estimates $\left\{\hat{k}_{j,p}\right\}$ and following the steps similar to those in PCMS finally results in

$$\sum_{j=0}^{i} k_{i-j,p}h_{j} = k_{i,m}; i = 1,2,......\eta$$

$\hat{K}_p$ h= k_m

where,

$$\hat{K}_p = \begin{bmatrix} \hat{k}_{0,p} & 0 & 0 \text{-------} 0 & \\ \hat{k}_{1,p} & \hat{k}_{0,p} & 0 \text{-------} 0 & \\ \vdots & \vdots & & \\ \hat{k}_{\eta,p} & \hat{k}_{\eta-1,p} & & \hat{k}_{0,p} \end{bmatrix}$$

$h = [\, h_0\, h_1 \text{----------} h_{\eta-1} 1]^T$

and

$k_m = [k_{0,m}\, k_{1,m} \text{------} k_{\eta-1,m}\, k_{\eta,m}]^T$

As seen earlier $\hat{k}_{o,p}$ and hence, $k_{o,p}$ are finite and nonzero. The matrix K_p is non singular. Here can solve for a unique h only, which is necessary and sufficient to match the first $(\eta+1)$ time moments. The next step is to realize H_c by following the design procedure.

$$H_c(\gamma) \;=\; h_0 + h_1\gamma + \;-\; + \gamma^{\eta} = \frac{B(\gamma)}{A(\gamma)}$$

$$= \mathrm{b}_0 + \mathrm{b}_1\gamma + \;---+ \mathrm{b}_\nu\gamma^\nu a_0 + a_1\gamma + \;------- \;+\; a_\mu\gamma^\mu$$

Through $\gamma^{\mu+\nu}$

Set a_μ=1. Cross multiplying and equating the coefficients of like powers of γ yields.

$$Ha = b$$

where,

$$H = \begin{bmatrix} h_0 & 0 & 0........ & 0 \\ h_1 & h_0 & 0........ & 0 \\ : & : & : & : \\ h_\eta & h_{\eta-1} & : & h_0 \end{bmatrix}$$

a = $[a_0 \;\; a_1 \text{----------} a_{\eta-1}]^T$ and b = $[b_0 \;\; b_1 \text{------} b_{\eta-1} \;\; b_\eta]^T$... and so on.

Specify a then computed b.

4.5 Plant Time Moment Controller with Feedback Scheme in Delta Domain

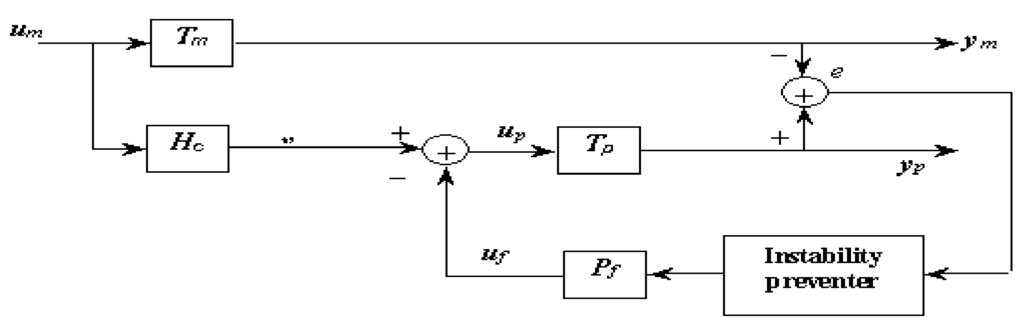


Figure 4.5 Implementation of PTCFS

PTCS developed in the delta domain is exclusively a cascade compensation scheme. While it is true that the time moment estimates are computed from online measurements of y_p, u_m and y_m. The actual disparity $(y_m - y_p)$ is neither computed nor used to modify the plant command u_p. As even an upper bound on the plant order is unknown, issues such as feedback, or even estimation, of state-variables do not arise at all. Output feedback can be used. But with the plant quite unknown, there is the risk of destabilizing the plant. Considering the well known advantages of feedback, we shall now devise alternative approaches which employ output feedback while taking steps to tackle the instability problem.

In this PTCF scheme, v is generated as $v = H_c u_m$. It is similar to the PTCS except that in which we use instability preventer block as a feedback with P_f.

CHAPTER 5

RESULTS AND DISCUSSION

The computation of the first three moments in delta domain k_0, k_1 and k_2 are carried out. The result obtained seems satisfactory. Numerical computation of these time moments can be carried out using MATLAB code.

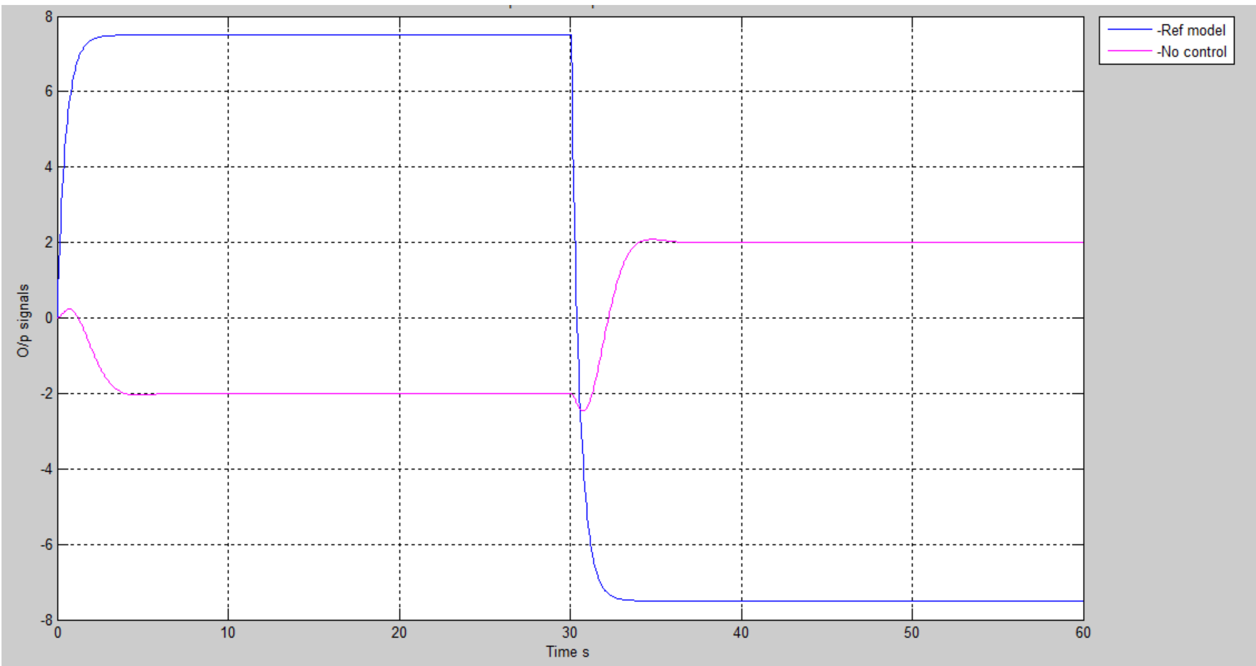
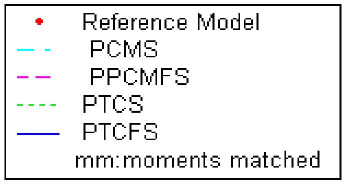


Figure 5.1 Minimum phase SISO plant with no control



Legends for plots with controls

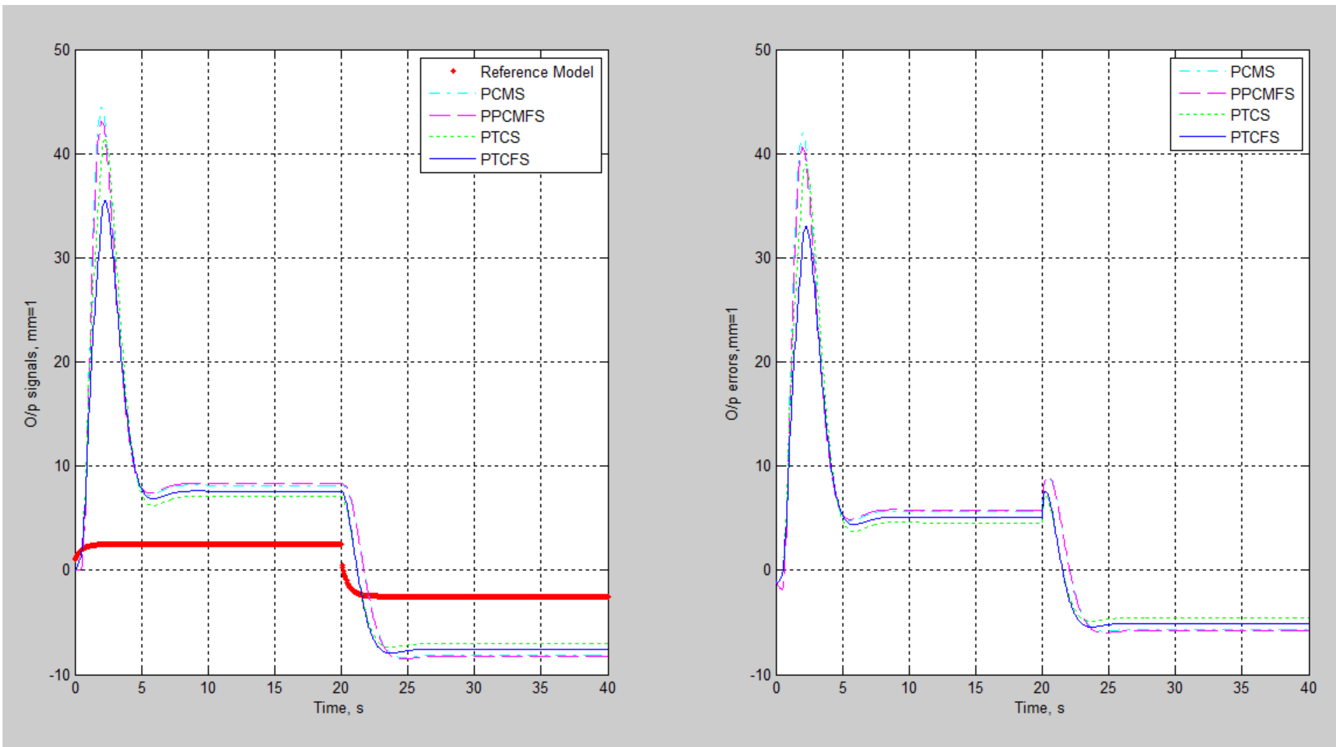


Figure 5.2 Minimum phase SISO plant with different controls; matching one moment.

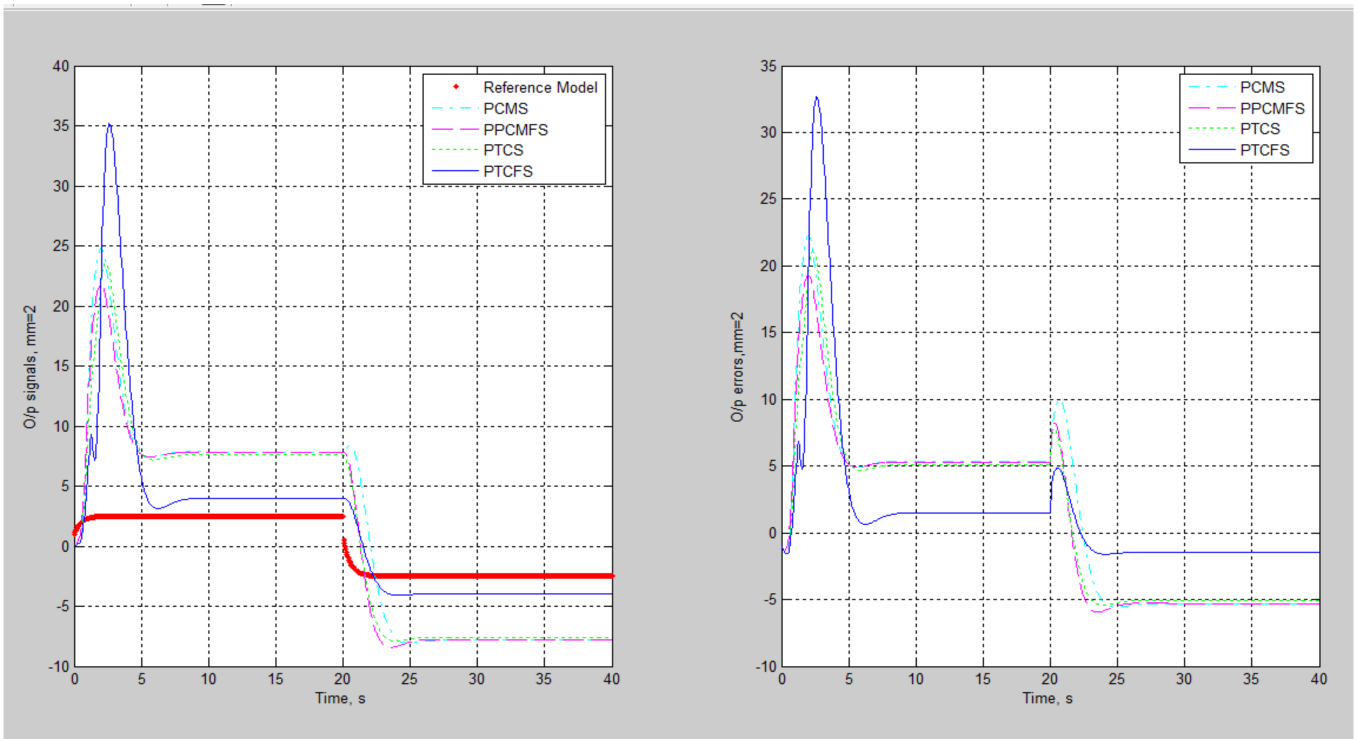


Figure 5.3 Minimum phase SISO plant with different controls; matching two moments.

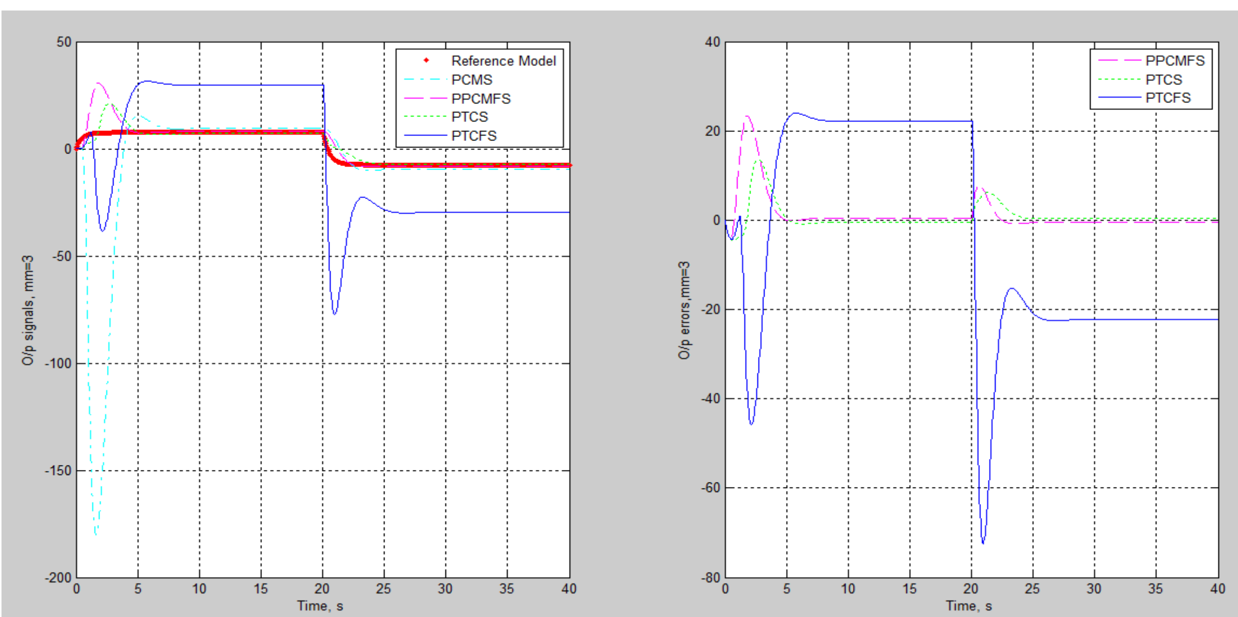


Figure 5.4 Minimum phase SISO plant with different controls; matching three moments.

Analysis

In the figures from 5.2 to 5.4 above the 1, 2 and 3 moments are matched for all the four different control schemes. The scalar corrections are incorporated in all the four control schemes. The model following is fast, the initial overshoot may be prohibited. In the second half cycle the overshoot is highly reduced and consequently faster model following is found.

Figure 5.2 describes the simplest case with just one moment matched with the same control schemes, when the two and three moments are matched the performances are as shown in the figures 5.3 and 5.4 respectively.

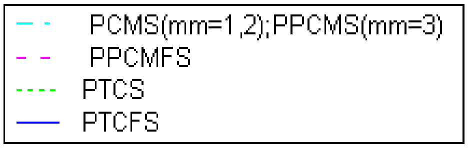
In the figure 5.1 above the plant with no control can be seen. The legends for the plots with controls are given above after figure 5.1.

The initial overshoot in the first half cycle is reduced in the two moment matching case with respect to one moment matching but the model following time is not delayed in the positive half cycle of the input. This may be due to the reason of adding an additional time moment.

Although, the performance in the negative half cycle of the input remains almost the same for all the different moments matched. When the three moments are matched the initial overshoot in the positive half cycle for the different controls get further reduced. But again the model following time is the same and it is not delayed.

When the three moments are matched (mm=3), PCMS is replaced by the PPCMS scheme. Because the PCMS scheme performs poorly. The PTCFS scheme appears to be the best of all the schemes.

Legends for IAE plots



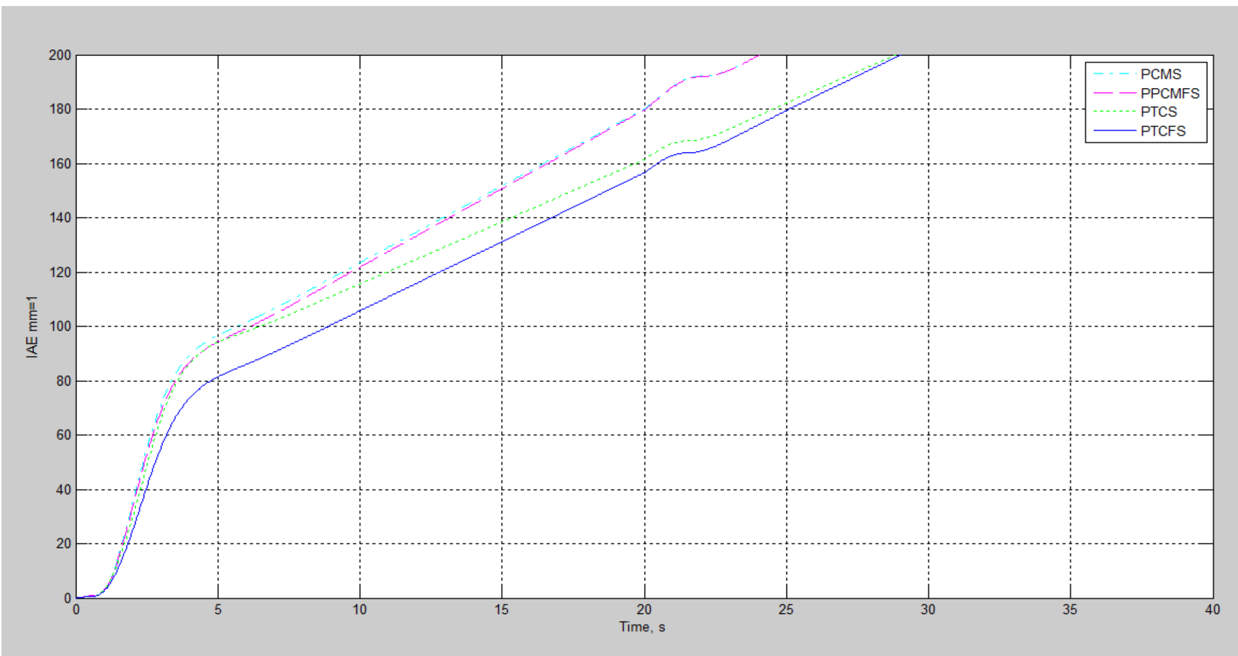


Figure 5.5 Minimum phase SISO plant with different controls; IAE matching one moment

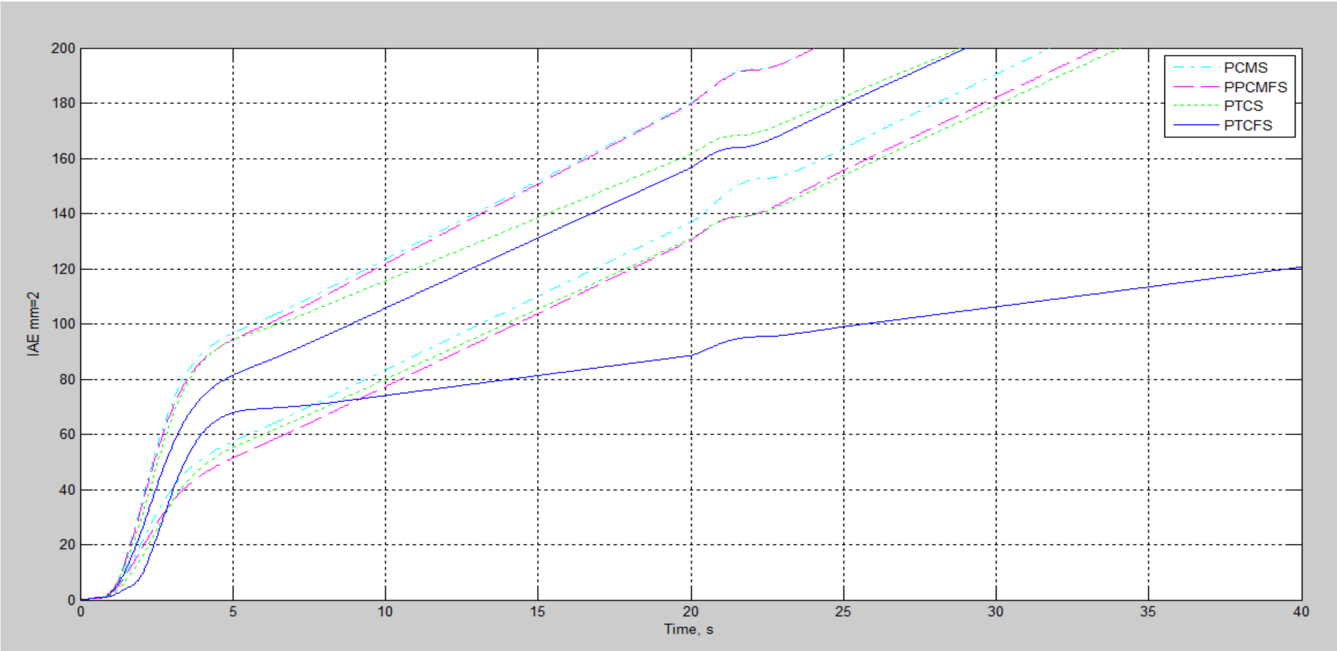


Figure 5.6 Minimum phase SISO plant with different controls; matching two moments.

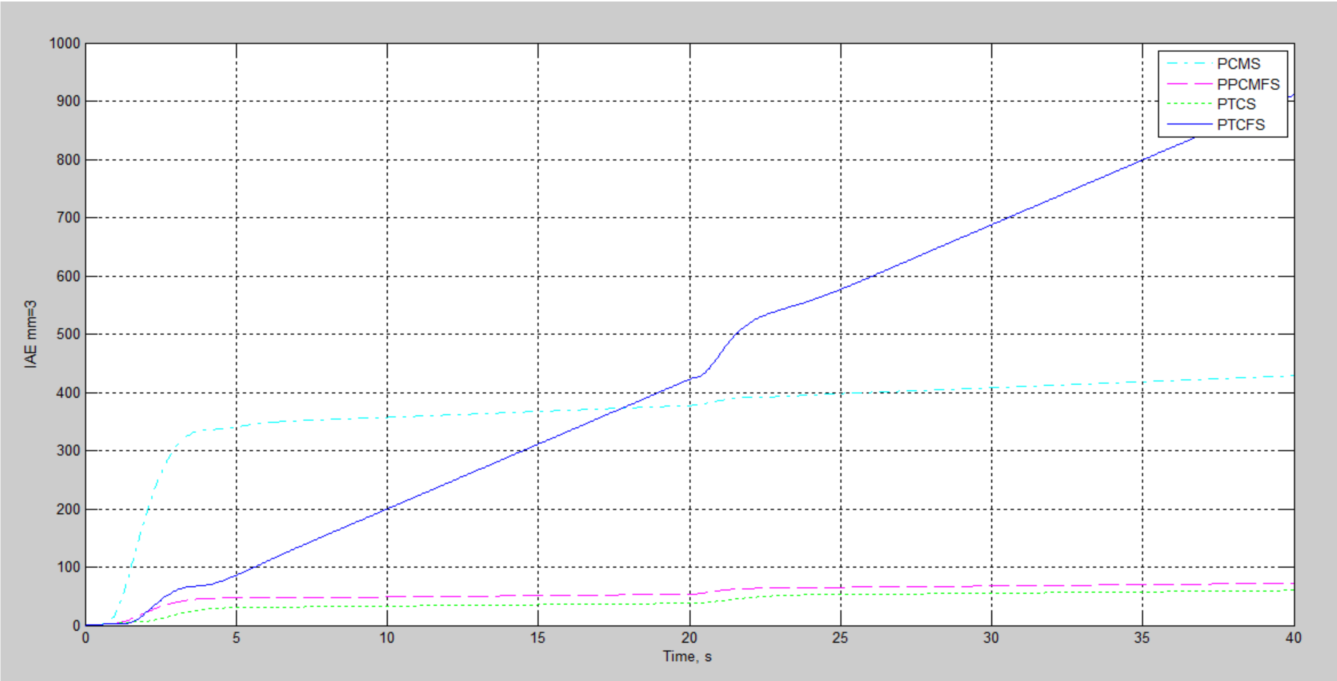


Figure 5.7 Minimum phase SISO plant with different controls; IAE matching three moments.

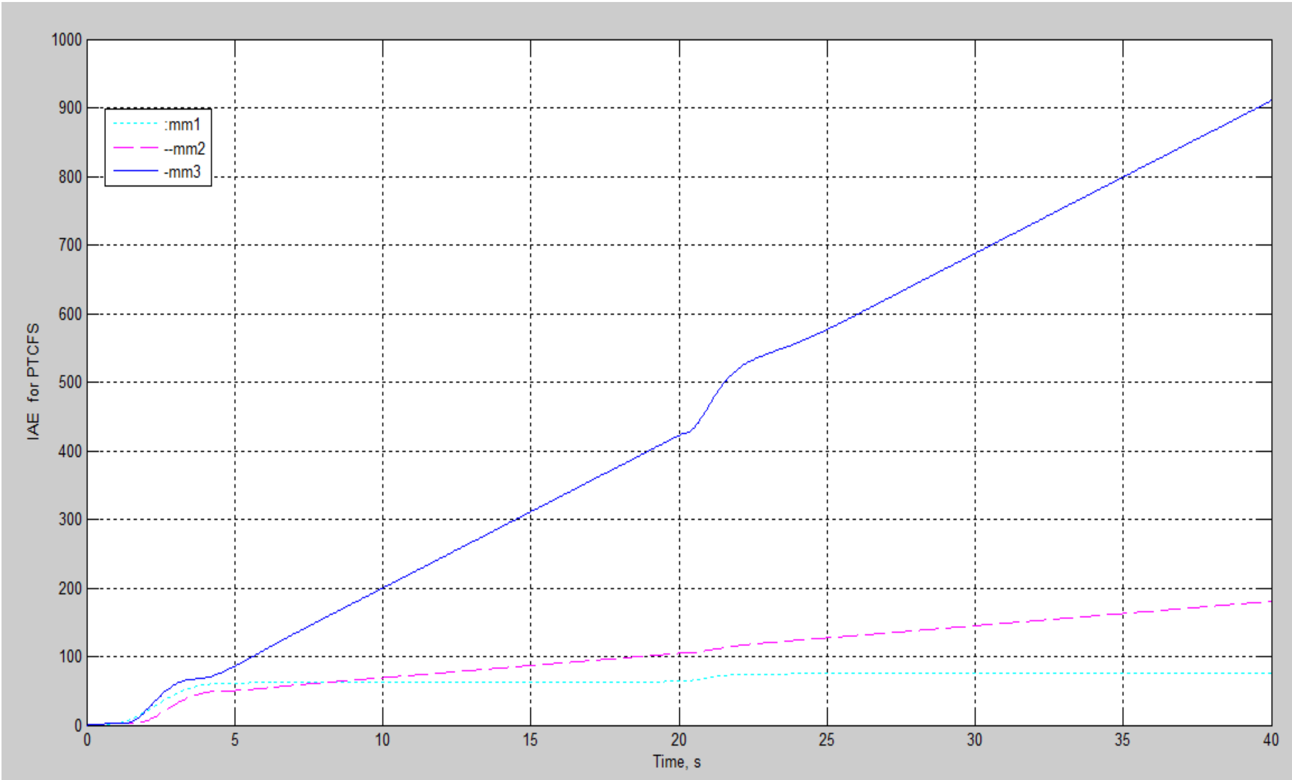


Figure 5.8 Minimum phase SISO plant with PTCFS; IAE comparison with one two and three moments matched.

To compare the performances of the control schemes the performance measuring index used as integral absolute error (IAE). A head to head comparison of the two schemes is made, the number of moments matched considered is equal. From the above comparison we again get the information that the PTCF scheme again performs the best because it has the least IAE.

To make a head to head comparison of control algorithms we have considered the number of moments matched equal. Figures 5.2, 5.3 and 5.7 show the corresponding story. It clearly shows that for each of mm=1, mm=2 and mm=3, PTCFS is consistently performing the best of all. Now a question arises that whether it true that the overall improved model is following is realizable by going for matching moments up to higher orders. The figure 5.8 is providing the evidence.

Now we choose the best control scheme among all the schemes i.e. PTCFS scheme. The information is clear that PTCFS with three moments matching ranks first, superior to two and one moment matching. Thus, for the unknown minimum phase plants with corrections of estimates, there lies two independent rules among the various control schemes.

The first of the two lies in this sequence of schemes– PCMS to PPCMFS to PTCS to PTCFS. This is evident from the careful examination of the figure 5.5, 5.6 and 5.7.The second one is in going in for matching of more and more moments.

The above observations follow from taking a closer look at the terminal values of IAE in figures 5.5, 5.6 and 5.7 and more directly from figure 5.8.

Note: For mm=1 and mm=2, we show PCMS. For mm=3, PPCMS takes the place of PCMS. This is the meaning of the first item of the legend for figures 5.5, 5.6 and 5.7.

The outcome of our study so far as the minimum phase systems are concerned,

- The various control schemes proposed, PTCFS performs to be the best because it has the least IAE which is evident from the results obtained from the figures 5.5, 5.6 and 5.7
- In the PTCFS control scheme, there is a steady improvement with every additional time moment matched which is evident from the figure 5.8

CHAPTER 6

FUTURE SCOPES OF WORK

- Time domain model order reduction of delta operator parameterized discrete-time systems is an open area for further research.

- The model matching controller design procedures developed in this work are not directly applicable to nominally unstable systems. This aspect merits further investigation.

- It has been proved from the results that more moments will be matched more the performances will be improved. But the order of the controller will become higher and higher. Another possible alternative would be to first design a low order controller. If it is not satisfactory, the compensated plant may now be treated as the unknown plant and using the same reference model, the very same schemes can be done on it to achieve the model following. We would then be applying two conceptually similar adaptive control loops, one over the other.

- As a future scope of work we can extend this discussion for multi-variable systems as well.

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Appendix A

Matlab codes for some of the results presented

%% First moment matching mm1 %%

```
NUM=[1 5];
DEN=[1 2];
delta=0.05;                %sampling interval
t1=0:delta:20;
t2=20.05:delta:40;
t=[t1 t2];                 %time range%
u1=1.0*ones(size(t1));
u2=-1.0*ones(size(t2));
um=[u1 u2];                % step input
ym=lsim(NUM,DEN,um,t);
yp1=pcmsmm1(0);
yp2=ppcmfsmm1(0);
yp3=ptcsmm1(0);
yp4=ptcfsmm1(0);
e1=yp1'-ym;
e2=yp2'-ym;
e3=yp3'-ym;
e4=yp4'-ym;
I1=iae(e1,delta);
I2=iae(e2,delta);
I3=iae(e3,delta);
I4=iae(e4,delta);
figure(1)
subplot(1,2,1)
plot(t,ym,'.r')
hold on
```

```
plot(t,yp1,'-.c')
hold on
plot(t,yp2,'--m')
hold on
plot(t,yp3,':g')
hold on
plot(t,yp4,'-b')
hold on
xlabel('Time, s')
ylabel('O/p signals, mm=1')
%% legend('.ReferenceModel','-.PCMS','--PPCMFS',':PTCS','-PTCFS','mm:moments matched',-
10)
grid on
subplot(1,2,2)
plot(t,e1,'-.c')
hold on
plot(t,e2,'--m')
hold on
plot(t,e3,':g')
hold on
plot(t,e4,'-b')
hold on
xlabel('Time, s')
ylabel('O/p errors,mm=1')
% legend('-.pcms','--ppcmfs','.ptcs','-ptcfs')
grid on
figure(2)
plot(t,I1,'-.c')
hold on
plot(t,I2,'--m')
hold on
```

```
plot(t,I3,':g')
hold on
plot(t,I4,'-b')
hold on
xlabel('Time, s')
ylabel('IAE mm=1')
% legend('-.PCMS(mm=1,2);PPCMS(mm=3)','--PPCMFS',':PTCS','-PTCFS',-1)
v=[0 40 0 200];
axis(v)
grid on
```

Appendix B

%% Second moment matching mm2 %%

```
NUM=[1 5];
DEN=[1 2];
delta=0.05;        %sampling interval
t1=0:delta:20;
t2=20.05:delta:40;
t=[t1 t2];      %time range%
u1=1.0*ones(size(t1));
u2=-1.0*ones(size(t2));
um=[u1 u2];       % step input
ym=lsim(NUM,DEN,um,t);
yp1=pcmsmm2(1);
yp2=ppcmfsmm2(1);
yp3=ptcsmm2(1);
yp4=ptcfsmm2(1);
e1=yp1'-ym;
e2=yp2'-ym;
e3=yp3'-ym;
e4=yp4'-ym;
I1=iae(e1,delta);
I2=iae(e2,delta);
I3=iae(e3,delta);
I4=iae(e4,delta);
figure(1)
subplot(1,2,1)
plot(t,ym,'.r')
hold on
plot(t,yp1,'-.c')
hold on
plot(t,yp2,'--m')
```

```
hold on
plot(t,yp3,':g')
hold on
plot(t,yp4,'-b')
hold on
xlabel('Time, s')
ylabel('O/p signals, mm=2')
%    %    legend('.Reference    Model','-.PCMS','--PPCMFS',':PTCS','-PTCFS','mm:moments
matched',-10)
grid on
subplot(1,2,2)
plot(t,e1,'-.c')
hold on
plot(t,e2,'--m')
hold on
plot(t,e3,':g')
hold on
plot(t,e4,'-b')
hold on
xlabel('Time, s')
ylabel('O/p errors,mm=2')
% legend('-.pcms','--ppcmfs','.ptcs','-ptcfs')
grid on
figure(2)
plot(t,I1,'-.c')
hold on
plot(t,I2,'--m')
hold on
plot(t,I3,':g')
hold on
plot(t,I4,'-b')
```

```
hold on
xlabel('Time, s')
ylabel('IAE mm=2')
% legend('-.pcms','--ppcmfs',':ptcs','-ptcfs',-10)
grid on
```

Appendix C

%% Third time moment matching mm3 %%

```
NUM=[15];
DEN=[1 2];
delta=0.05;        %sampling interval
t1=0:delta:20;
t2=20.05:delta:40;
t=[t1 t2];      %time range%
u1=1.0*ones(size(t1));
u2=-1.0*ones(size(t2));
um=[u1 u2];       % step input
ym=lsim(NUM,DEN,um,t);
yp1=ppcmsmm3(2);
yp2=ppcmfsmm3(2);
yp3=ptcsmm3(2);
yp4=ptcfsmm3(2);
e1=yp1'-ym;
e2=yp2'-ym;
e3=yp3'-ym;
e4=yp4'-ym;
I1=iae(e1,delta);
I2=iae(e2,delta);
I3=iae(e3,delta);
I4=iae(e4,delta);
figure(1)
subplot(1,2,1)
plot(t,ym,'.r')
hold on
plot(t,yp1,'-.c')
hold on
plot(t,yp2,'--m')
```

```
hold on
plot(t,yp3,':g')
hold on
plot(t,yp4,'-b')
hold on
xlabel('Time, s')
ylabel('O/p signals, mm=3')
% % legend('.Reference Model','-.PCMS','--PPCMFS',':PTCS','-PTCFS','mm:moments matched',-10)
grid on
subplot(1,2,2)
% plot(t,e1,'-.c')
% hold on
plot(t,e2,'--m')
hold on
plot(t,e3,':g')
hold on
plot(t,e4,'-b')
hold on
xlabel('Time, s')
ylabel('O/p errors,mm=3')
% legend('-.pcms','--ppcmfs','.ptcs','-ptcfs')
grid on
figure(2)
plot(t,I1,'-.c')
hold on
plot(t,I2,'--m')
hold on
plot(t,I3,':g')
hold on
plot(t,I4,'-b')
```

```
hold on
xlabel('Time, s')
ylabel('IAE mm=3')
% legend('-.pcms','--ppcmfs',':ptcs','-ptcfs',-10)
grid on
```