SPATIO-TEMPORAL SOLITON DYNAMICS IN INHOMOGENEOUS NONLINEAR MEDIA

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I dedicate this thesis to my loving FAMILY who has always been caring and supportive towards me.

CERTIFICATE

I hereby certify that the work which has been presented in this thesis entitled, **"Spatio-temporal Soliton dynamics in inhomogeneous Nonlinear media,"** submitted in partial fulfillment of the requirements for the award of degree of **Master of Science in Physics at Thapar university**, Patiala, is an authentic record of my own work carried out under the supervision of **Dr. Soumendu Jana, Asst. Prof., SPMS** and refers other researcher's work which are duly listed in reference section. The matter embodied in this thesis has not been submitted for the award of any other degree of this or any other university.

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CHAPTER-I

1. INTRODUCTION

The basic requirement of present era is tremendous amount of data transfer and extremely high rate of data processing. Day-by-day with the advancement of technology the demand of data transfer and processing is increasing in a zooming rate. Frequently the current age is mentioned as the age of information. A huge effort has already been made for the development of the field. To cope with the extraordinary data handling demand a continuous and fast evolution in technology is required. Nonlinear optics provides the science behind such cutting age technology.

After the invention of lasers, nonlinear optics [1-5] has emerged as the most sought after subject in all the frontiers of science by both theoreticians and experimentalists. Indeed, nonlinear optics is one of the vital cores of recent scientific advancements and it continues to be so for future research work. Nonlinear optics has stirred many phenomena like fabrication of new nonlinear materials, harmonic generations, optical solitons, parametric amplification, stimulated Raman scattering, self-induced transparency, modulational instability, etc, which find a myriad of applications ranging from high data transmission in optical communication, switching, amplifiers, pulse reshaping, pulse compression, tunable lasers to encoded message transmission. Notable among these exciting phenomena is the concept of optical solitons, pioneered by Hasegawa of Japan. It revolutionized the scope of telecommunication world, mainly optical fiber communication (OFC), like never before and solitons are nowadays perceived to be the carriers of communication signals in near future. In recent years optical soliton fiber communication has attracted much interest in the academic and industrial worlds. In fact, the continuous pursuit of both methodological and technological innovation has led to the realization that conventional linear models of real systems suffer from severe limitations. Today the potential of soliton engineering is recognized worldwide with research groups actively working on this topic in every part of the globe. Perhaps, solitons are the most elegant and complex structures produced by nature in the realm of nonlinearity, which in fact attract theoreticians. On the other hand, for experimentalists, the main appeal is the prospect of applications of solitons in telecommunication, pulse compression, logic gates, optical switching and so on. Anderson, Lisak and Berntson present a variational approach to nonlinear Schrodinger equations in nonlinear optics and Rayleigh–Ritz optimization method is used to find the approximate solutions of different nonlinear evolution equations in optics.

Optical solitons are localized electromagnetic waves that propagate stably in nonlinear media with group-velocity dispersion (GVD) and/or diffraction. Temporal solitons in single mode optical fibers are the prototypical optical solitons; these were predicted theoretically in 1973 [6] and first observed experimentally in 1980 [7]. Extensive research since then has led to the current development of telecommunication systems based on solitons Compared to the work on temporal solitons, progress in the area of multidimensional (spatial or spatiotemporal) optical solitons has been much slower. Like temporal solitons, in cubic ($\chi^{(3)}$) nonlinear media these waves are also governed by nonlinear Schrodinger equations (NLSE). It has long been understood that self-focusing as a result of the (cubic) Kerr nonlinearity could compensate for the spreading of a beam due to diffraction, but the resulting balance is unstable in > 1 dimension the beam tends to diffract, collapse, or disintegrate into multiple filaments. However, processes not included in the NLSE may stabilize self-trapped beams. Spatial solitons were first produced in liquid CS_2, where an interference grating was employed to stabilize the solitons [8] and light filaments were observed [9] in resonant propagation through an atomic vapor, where the nonlinearity is saturable. One-dimensional (1D) spatial solitons of the NLSE were generated in a glass waveguide in 1990 [10] Studies of spatial solitons began to make rapid progress in the1990s, when two new types of soliton-supporting nonlinear-optical interactions were identified. Segev et al [11] predicted that the photorefractive effect in electro-optic materials could be used to create a saturable nonlinear index of refraction that would support soliton formation. Photorefractive solitons were observed soon afterwards [12], and since then a variety of such solitons has been reported in 1D and 2D [13]. At nearly the same time, there was a resurgence of interest in an effective cubic nonlinearity that is produced by the interaction of two or three waves in quadratic ($\chi^{(2)}$) nonlinear media [14]. The renewed interest was based on the recognition that large, effective third-order nonlinearities of controllable sign can be produced. An additional property is that the effective nonlinearity saturates, so self-focusing collapse can be avoided in quadratic media [15]. Thus, quadratic media possess the properties required for multidimensional soliton formation [16], and numerous theoretical treatments of solitons in quadratic media appeared in the early 1990s [17]. Torruellas et al first observed stationary spatial solitons [18], and Di Trapani and co-workers produced temporal solitons in quadratic media [19]. Very recently, Di Trapani et al reported the observation of vortex solitons in quadratic media [20]. Photorefractive and quadratic solitons are profoundly different from solitons in one dimensional Kerr media. As an illustration of the differences, in quadratic media the soliton actually consists of two fields at different frequencies, coupled and mutually trapped by the nonlinear interaction. Because they are solutions of non-integrable systems, solitons in quadratic and photorefractive media can participate in a variety of phenomena not available to temporal solitons in optical fibers. For example, soliton fission, annihilation, and stable orbiting in three dimensions are all possible.

One of the major goals in the field of soliton physics is the production of light fields that are localized in all three dimensions of space as well as time, which we will refer to as 3D spatiotemporal solitons (STS). These result from the simultaneous balance of diffraction and GVD by self-focusing and nonlinear phase-modulation, respectively. The possibility of such pulses in multiple dimensions was considered by Silberberg, who is generally credited with coining the term ‘light bullets’ to describe them. It is well-known that 3D STS are unstable against collapse in cubic nonlinear media, but solutions may be stabilized if the nonlinearity is saturable, or if additional nonlinear processes such as multiphoton ionization exist to arrest the collapse favored by self-focusing. Their scientific importance has motivated a number of theoretical studies of STS in quadratic media. The first experimental studies of optical STS were reported last year.

1.1 A BRIEF HISTORY OF NONLINEAR OPTICS

- **Maiman (1960)**, invention of the laser. Charles H. Townes was in 1964 awarded the Nobel Prize for the invention of the ammonia laser.[39]
- **Franken et al. (1961)**, First observation ever of nonlinear optical effects, *second harmonic generation* (SHG). (Franken et al. detected ultraviolet light (λ=347.1 nm) at twice the frequency of a ruby laser beam (λ=694.2 nm) when this beam traversed a quartz crystal. Second harmonic generation is also the first nonlinear effect ever observed where a coherent input generates a coherent output [40].
- **Terhune et al. (1962)**, First observation of third harmonic generation (THG). Noteworthy is that in their seminal experiment, Terhune et al. detected only about a thousand THG photons per pulse, at λ=231.3 nm, corresponding to a conversion of one photon out of about 10^{15} photons at the fundamental wavelength at λ=693.9 nm [41].
- **E. J. Woodbury and W. K. Ng (1962)**, first demonstration of stimulated Raman scattering. **Armstrong et al. (1962)**, formulation of the general permutation symmetry relations in nonlinear optics [42-43].
- **A. Hasegawa and F. Tappert (1973)**, the first theoretical prediction of soliton generation in optical fibers Transmission of stationary nonlinear optical pulses in dispersive optical fibers: I, Anomalous dispersion; II Normal dispersion [44].
- **H. M. Gibbs et al. (1976)**, first demonstration and explaination of optical bistability [45].

- **L. F. Mollenauer et al. (1980)**, first confirmation of soliton generation in optical fibers. The first reported observation of solitons was though made in 1834 by John Scott Russell, a Scottish scientist and later famous Victorian engineer and shipbuilder, while studying water waves in the Glasgow-Edinburgh channel. Recently, many advances in nonlinear optics has been made, with a lot of efforts with fields of, for example, Bose-Einstein condensation and laser cooling; these fields are ,however, a bit out of focus from the subjects of this course, which can be said to be an introduction to the 1960s and 1970s advances in nonlinear optics. It should also be emphasized that many of the effects observed in nonlinear optics, such as the Raman scattering, were observed much earlier in the microwave range**.**

1.2 APPLICATIONS AND IMPORTANCE OF NLO

There is a flurry of development of nonlinear optical effects in the last nearly five decades. The most important among them are the light propagation and soliton formation in optical fibers, waveguides and bulk media. The temporal, spatial and spatiotemporal soliton propagation attracted considerable attention for years [24-37]. They are self-trapped light beams or pulses that have aroused tremendous interest owing to their immense potential for technological applications. In particular, they have been contemplated for future bits in long-haul communications [22].

Use of solitons for switching is also suggested and experimentally demonstrated [23]. The developments of erbium doped fiber amplifier have added momentum to the ongoing activities. High bit rate data transfer at a distance of thousands of kilometers using optical solitons and lumped erbium-doped fiber amplifiers are under intense investigations. Soliton propagation in monomode optical fibers is a characteristic of Kerr nonlinearity with anomalous dispersion. The main problem of the application of solitons in optical communication systems is the problem of soliton stabilization. Because of fiber loss, the essential condition of balance between nonlinearity and dispersion satisfies only over a small length of the fiber, and as the soliton propagates down the fiber the strength of nonlinearity decreases, thereby offsetting the essential balance. A large number of theoretical and experimental investigations have addressed the issue of soliton stability. Among them, the most promising scheme owes to the recent and rapid development of erbium doped fiber amplifier.

Another important nonlinear optical phenomenon is the spatial soliton propagation. They have been experimentally observed in different materials such as CS_2 waveguides, single mode glass waveguides and AlGaAs waveguides. Recently they have been observed in polydiacetelene paratolune sulfonate (PTS) at a very low power level of 12 watt only. Spatial soliton emission and excitation of nonlinear surface waves are also under intense investigation. These self-trapped waves have very promising practical applications in communication technology. Most of the past research interest on spatial and temporal soliton propagation in optical fibers and waveguides are normally confined to the Kerr medium, which is manifestation of small nonlinear coefficient and nonresonant interactions. However, higher order nonlinearities become important at moderate intensities in materials with large higher order susceptibilities. Experimental results of nonlinear absorption in semiconductor-doped glass (SDG) and other composite materials show that nonlinearity saturates at moderate intensities. SDG fibers and waveguides have drawn attention lately. In order to investigate optical pulse propagation in SDG fibers and waveguides, two different forms of nonlinearities have been considered, particularly, saturating form of nonlinearity and cubic quintic form of nonlinearity. Higher order nonlinear effects would not only modify known properties of spatial and temporal solitons, but may also lead to completely new phenomena under certain circumstances. They promise new possibilities, as higher order nonlinearity may be helpful in achieving stable propagation. Therefore, they require serious attention.

Thus, this thesis presents an investigation on the influence of higher order nonlinearity on the propagation characteristics of spatiotemporal soliton.

1.3 MOTIVATION

Spatio-temporal soliton has been investigated in different media in different situation usually in kerr media, also its propagation in a quintic nonlinear medium has also been studied,3D spatio-temporal bright vortex soliton in a bulk dispersive, cubic quintic optical medium has been studied also 2D and 3D spatio-temporal spinning solitons cubic and defocusing quintic nonlinearity have been also investigated by several workers. Though spatiotemporal solitons have been investigated in different media such as uniform Kerr, quadratically nonlinear and graded index Kerr media. The effect of inhomogeneity on the dynamics of spatio-temporal soliton has been investigated [38]. However, spatio-temporal soliton propagation can be studied in many other systems with other soliton profile. Thus we propose to investigate the spatio-temporal soliton propagation in inhomogeneous nonlinear media with cosh-Gaussian soliton profile.

1.4 OBJECTIVE

- To study the existence and stability of spatio-temporal soliton in inhomogeneous nonlinear media with cosh-Gaussian soliton profile.
- To investigate the effect of the inhomogeneity of the media in spatio-temporal soliton propagation.

Before discussing our model and investigation it would be worth to describe the fundamentals of nonlinear optics, particularly soliton formation. The next CHAPTER-II contains the same.

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CHAPTER-II

2. INTRODUCTION TO NONLINEAR OPTICS

Nonlinear optical (NLO) materials play a major role in nonlinear optics and in particular they have a great impact on information technology and industrial applications. In the last decade, however, this effort has also brought its fruits in applied aspects of nonlinear optics. This can be essentially traced to the improvement of the performances of the NLO materials. The understanding of the nonlinear polarization mechanisms and their relation to the structural characteristics of the materials has been considerably improved. The new development of techniques for the fabrication and growth of artificial materials has dramatically contributed to this evolution. The aim is to develop materials presenting large nonlinearities and satisfying at the same time all the technological requirements for applications such as wide transparency range, fast response, and high damage threshold. But in addition to the processability, adaptability and interfacing with other materials improvements in nonlinear effects in devices, led the way to the study of new NLO effects and the introduction of new concepts. Optical solitons, optical switching and memory by NLO effects, which depend on light intensity, are expected to result in the realization of pivotal optical devices in optical fiber communication (OFC) and optical computing which make the maximum use of light characteristics such as parallel and spatial processing capabilities and high speed. Some materials change light passing through them, depending upon orientation, temperature, light wavelength etc. (red light, lower wavelength) releasing one photon of accumulated higher energy (blue and green light, higher wavelength). NLO materials typically have a distinct crystal structure, which is anisotropic with respect to electromagnetic radiation. The importance of nonlinear optics is to understand the nonlinear behavior in the induced polarization and to analyze and to control its impact on the propagation of light in the matter

2.1 BASICS OF NONLINEAR OPTICS

Optics is an important part of everyday life. Light flows or propagate through empty space, as well as through material objects, and provides us with visual information about our world. The familiar effects of reflection, refraction, diffraction, absorption, and scattering explain a wide variety of visual experiences common to us, from the focusing of light by a simple lens to the colors seen in a rainbow. Remarkably, these can be explained by assigning a small set of optical parameters to materials. Under the ordinary experiences of everyday life, these parameters are constant, independent of the intensity of light that permits observation of the optical phenomena. This is the realm of what is called linear optics. The invention of the laser gave rise to the study of optics at high intensities, leading to new phenomena not seen with ordinary light such as the generation of new colours from monochromatic light in a transparent crystal, or the self-focusing of an optical beam in a homogeneous liquid. At the intensities used to generate these types of effects, the usual optical parameters of materials cannot be considered constant but become functions of the light intensity. The science of optics in this regime is called nonlinear optics.

2.2 ORIGIN OF NONLINEAR OPTICS

Nonlinear optics is the study of phenomena that occur as a consequence of the modification of the optical properties of a material system by the presence of light. Typically, only laser light is sufficiently intense to modify the optical properties of a material system. The beginning of the field of nonlinear optics is often taken to be the discovery of second-harmonic generation by Franken *et al.* (1961), shortly after the demonstration of the first working laser by Maiman in 1960.∗ Nonlinear optical phenomena are “nonlinear” in the sense that they occur when the response of a material system to an applied optical field depends in a nonlinear manner on the strength of the optical field. For example, second-harmonic generation occurs as a result of the part of the atomic response that scales quadratically with the strength of the applied optical field. Consequently, the intensity of the light generated at the second-harmonic frequency tends to increase as the square of the intensity of the applied laser light.

In order to describe more precisely what we mean by an optical nonlinearity, let us consider how the dipole moment per unit volume, or polarization *P(t)*, of a material system depends on the strength *E(t)* of an applied optical . In the case of conventional (i.e. linear) optics, the induced polarization depends linearly on the electric field strength in a manner that can often be described by the relationship:

$$P = \in_0 \chi^{(1)} E(t) \tag{1}$$

where the constant of proportionality $\chi^{(1)}$ is known as the linear susceptibility and $\in_0$ is the permittivity of free space. In nonlinear optics, the optical response can often be described by generalizing Eq. (1) by expressing the polarization P(t) as a power series in the field strength E(t) as:

$$\begin{aligned} P(t) &= \in_0 \left[\chi^{(1)} E(t) + \chi^{(2)} E^{(2)}(t) + \chi^{(3)} E^{(3)}(t) + ...\right] \\ &\equiv P^{(1)}(t) + P^{(2)}(t) + P^{(3)}(t) + ... \end{aligned} \tag{2}$$

The quantities $\chi^{(2)}$ and $\chi^{(3)}$ are known as the second- and third-order nonlinear optical susceptibilities, respectively. For simplicity, we have taken the fields *P(t)* and *E(t)* to be scalar quantities in writing Eqs (1) and (2). In writing Eqs (1) and (2) in the forms shown, we have also assumed that the polarization at time *t* depends only on the instantaneous value of the electric field strength. The assumption that the medium responds instantaneously also implies (through the Kramers–Kronig relations) that the medium must be lossless and dispersion less. We shall see how to generalize these equations for the case of a medium with dispersion and loss. In general, the nonlinear susceptibilities depend on the frequencies of the applied fields, but under our present assumption of instantaneous response, we take them to be constants. We shall refer to $P^{(2)}(t) = \epsilon_0\chi^{(2)}E^{(2)}(t)$ as the second-order nonlinear polarization and to $P^{(3)}(t) = \epsilon_0 \chi^{(3)}E^{(3)}(t)$ as the third-order nonlinear polarization. We shall see later that physical processes that occur as a result of the second-order polarization $P^{(2)}$ tend to be distinct from those that occur as a result of the third-order polarization $P^{(3)}$. In addition, to that second-order nonlinear optical interactions can occur only in noncentrosymmetric crystals—that is, in crystals that do not display inversion symmetry. Since liquids, gases, amorphous solids (such as glass), and even many crystals display inversion symmetry, $\chi^{(2)}$ vanishes identically for such media, and consequently such materials cannot produce second-order nonlinear optical interactions. On the other hand, third-order nonlinear optical interactions (i.e. those described by a $\chi^{(3)}$ susceptibility) can occur for both centrosymmetric and noncentrosymmetric media.

The reason why the polarization plays a key role in the description of nonlinear optical phenomena is that a time-varying polarization can act as the source of new components of the electromagnetic field.

2.3 TYPES OF NONLINEARITIES

2.3.1 Kerr Nonlinearity

The Kerr effect is a nonlinear optical effect occurring when intense light propagates in crystals and glasses but also in other media such as gases. Its physical origin is a nonlinear polarization generated in the medium, which itself modifies the propagation properties of the light. The Kerr effect is the effect of an instantaneously occurring nonlinear response, which can be described as modifying the refractive index. In particular, the refractive index for the high intensity light beam itself is modified according to

$$\Delta n = n_2 I$$

with the nonlinear index n_2 and the optical intensity I. The n_2 value of a medium can be measured e.g. with the z-scan technique. Note that in addition of (a purely electronic nonlinearity), electrostriction can significantly contribute to the value of the nonlinear index [1-2]. The electric field of light causes density variations (acoustic waves) which themselves influence the refractive index via the photo elastic effect. That mechanism, however, occurs on a much longer time scale and is thus relevant only for relatively slow power modulations, but not for ultra short pulses. Fused silica, as used e.g. for silica fibers, has a nonlinear index of $\approx 3 \times 10^{-16}\,\mathrm{cm}^2/\mathrm{W}$. For soft glasses and particularly for semiconductors, it can be much higher, because it depends strongly on the bandgap energy. The nonlinearity is also often negative for photon energies above roughly 70% of the bandgap energy (self-defocusing nonlinearity). The time and frequency-dependent refractive index change leads to self-phase modulation and Kerr lensing, for different overlapping light beams also to cross-phase modulation. Note that the effective refractive index increase caused by some intense beam for *other* beams is twice as large as that according to the equation shown above, assuming that both beams are in the same polarization state. The description of the Kerr effect via an intensity-dependent refractive index is actually based on a certain approximation, valid for light with a small optical bandwidth. For very short and broadband pulses, a deviation from this simple behavior can be observed, which is called self-steepening. It reduces the velocity with which the peak of the pulse propagates (i.e. it reduces the group velocity) and thus leads to an increasing slope of the trailing part of the pulse. This effect is relevant e.g. for super continuum generation. Furthermore, the strength of the Kerr effect is known to saturate at very high optical intensities. At extremely high optical intensities, there may not be a further increase of refractive index in proportion to the intensity, but a saturation and even substantial decrease of refractive index [3]. This can be understood as an effect of multiphoton ionization, leading to induced losses, which are related to additional phase changes via Kramers–Kronig relations [4-5]. A nonlinear polarization with delayed (non-instantaneous) response cannot be simply described as a modification of the refractive index. Its effect is called Raman scattering, and is not considered to be part of the Kerr effect.

2.3.2 Cubic-Quintic Nonlinearity

The propagation and stability of STSs solitons under the combined influence of dispersion, diffraction, diffraction, self-phase modulation in a medium possessing cubic quantic nonlinearity is investigated [6]. The general form of this kind of nonlinearity can be expressed as

$$n(i)= n_p\, i^p + n_{2p} i^{2p}$$

where p is positive constant. The nonlinear coefficients n_p and n_{2p} possess opposite signs i.e. $n_p n_{2p}<0$. When $n_p>0$ and $n_{2p}<0$ this situation will lead to self-focusing and self-defocusing effects respectively. The effective nonlinear coefficient is the resultant of the two when $p=1$ then that nonlinearity will be called as cubic quintic nonlinearity which can be expressed as

$$n(i)= n_o + n^2 i + n_4 I^2$$

where n_4 is the fifth order nonlinear coefficient as it corresponds to fifth order nonlinear susceptibility through the relation $n_4 = 5/16 n_o\, \chi^{(5)}$. The cubic quintic materials are less in number as compared to kerr nonlinear materials. The cubic quintic nonlinearity is also called as parabolic nonlinearity eg– para toluene sulfonate it possess high value of n_4. Other e.g.s are $CdS_X Se_{1-x}$ doped glass, semiconductor doped glass, semiconductor double doped glass.

2.4 DESCRIPTIONS OF NONLINEAR OPTICAL PROCESSES

There are variety of nonlinear optical processes, the brief qualitative descriptions of a number of nonlinear optical processes is as follows:

2.4.1 NONLINEAR OPTICAL PHENOMENA:

- Second harmonic generation (SHG), or frequency doubling, generation of light with a doubled frequency (half the wavelength), two photons are destroyed creating a single photon at two times the frequency.

- Third harmonic generation (THG), generation of light with a tripled frequency (one-third the wavelength), three photons are destroyed creating a single photon at three times the frequency.
- High harmonic generation (HHG), generation of light with frequencies much greater than the original (typically 100 to 1000 times greater)
- Sum frequency generation (SFG), generation of light with a frequency that is the sum of two other frequencies (SHG is a special case of this)
- Difference frequency generation (DFG), generation of light with a frequency that is the difference between two other frequencies
- Optical parametric amplification (OPA), amplification of a signal input in the presence of a higher-frequency pump wave, at the same time generating an *idler* wave (can be considered as DFG)
- Optical parametric oscillation (OPO), generation of a signal and idler wave using a parametric amplifier in a resonator (with no signal input)
- Optical parametric generation (OPG), like parametric oscillation but without a resonator, using a very high gain instead
- Spontaneous parametric down conversion (SPDC), the amplification of the vacuum fluctuations in the low gain regime
- Optical rectification (OR), generation of quasi-static electric fields.
- Nonlinear light-matter interaction with free electrons and plasmas.
- Soliton formation.

2.5 HISTORY OF SOLITON FORMATION

Long ago, in August 1834 [7] John Scott Russell, a naval architect, was working for the Scottish Canal companies to establish the possibility of rapid steamboat transit on canals. As part of this investigation, he was observing a boat being pulled along, rapidly, by a pair of horses. For some reason, the horses must have stopped the boat rather suddenly. What happened next was to change science in the most dynamic way. The stopping of the boat caused a very strong wave to be generated. This wave, in fact, a significant hump of water stretching across the rather narrow canal, rose up at the front of the boat and proceeded to travel, quite rapidly, down the canal. Russell, immediately, realized that the wave was something very special. It was alone in the sense that it sat on the canal with no disturbance to the front or the rear, nor did it die away until he had followed it for quite a long way. The word alone is synonymous with solitary and Russell soon referred to his observation as the Great Solitary Wave.

The word solitary is now routinely used, indeed even the word solitary tends to be replaced by the more generic word soliton. Once the physics behind Russell's wave is understood, however, solitons, of one kind or another, appear to be everything but it is interesting only partially by his contemporaries. What Russell saw was that the solitary waves has speed proportional to their amplitude and that they pass through each other [8] without destruction, or change. Airy [9] was not convinced with the Russell results, Airy claimed that large amplitude waves would self-steepen and break up. In the meantime, in 1847, Stokes [10] worked out that, for deep water, CW (periodic) waves (not localized) with finite amplitude can exist without breaking up i.e. in mathematical language they have permanent form. The reason which was missing from Airy was that the observation of Russell needed a study of the delicate balance between dispersion and nonlinearity. This point was appreciated by both Boussines(1871) [11] and Rayleigh 1876 [12] but even they never arrived at the term that we now know i.e. Nonlinear differential equation, for which

Russell's wave is a solution. It took until 1895 for two Dutchmen, Korteweg and de Vries to produce this differential equation. One of the solutions of the KdV equation is the solitary wave observed by Russell. Normally, a hump of water like Russell's solitary wave is thought of as a packet of waves, all travelling with different speeds. This way of looking at things comes from Fourier, but it is a linear viewpoint with the end result being the destruction of the hump due to dispersion. Large amplitudes mean lots of power, so the waves in the packet are now forced to interact with one another. The forces trying to restore equilibrium are no longer just proportional to the height of the wave and the speed now depends on displacement; this is nonlinear system. In a balanced situation, the power of the wave acts against dispersion. The wave then remains intact and a soliton is born. This idea of nonlinearity was ignored for many years. Indeed, the natural philosophers of that time seemed blissfully unaware that our world is really rather a nonlinear one. Russell, on the other hand, was convinced of the importance of his wave, and its curiosity, right up to his death in 1882. But it took much later research on waves in crystal lattices, by Zabusky and Kruskal in 1965, to make plain the elegant generic nature of Russell's observation to a wider public, and to emphasise that nonlinearity is important to the propagation of such waves; Zabusky and Kruskal realized that the form of their equations was exactly like that of Kortweg and de Vries.

The work done in 1965, demonstrated that solitary waves retain their shape, even after collision with each other. It was fascinating that Russell had quietly observed this 130 years ago. The 1965 authors invented the name soliton for these waves, just to emphasize that although is a solitary wave it retains its identity, even after a collision. The name has been coined keeping the analogy with fundamental particles (like electron, protons etc.) due to its particle type nature.

In the course of past several years, a new level of understanding has been achieved about conditions for the existence, stability, and generation of spatiotemporal optical solitons, which are non-diffracting and non-dispersing wave packets propagating in nonlinear optical media. Experimentally, effectively two-dimensional (2D) spatiotemporal solitons that overcome diffraction in one transverse spatial dimension have been created in quadratic nonlinear media. With regard to the theory, fundamentally new features of light pulses that self- trap in one or two transverse spatial dimensions and do not spread out in time, when propagating in various optical media, were thoroughly investigated in models with various nonlinearities. Stable vorticity-carrying spatiotemporal solitons have been predicted too, in media with competing nonlinearities. In 1973 soliton research reached to a turning point when Akira Hasegawa and Tappert [13].

It theoretically predicted the possibility of optical solitons in optical fiber. Linn Mollenauer, Roger Stolen and Jim Gordon have experimentally detected fiber soliton in 1980 [14]. These two discoveries, in fact, started a new era of optical communication systems that promise to revolutionize the global communication scenario. As of today, soliton based optical communication systems assure to provide ultra-high bit rate data transfer as high as several Tera bit/s.

My concern is only optical solitons, which are probably the most significant productive- application of nonlinear optics. Notable point is that, nonlinearity in any system is usually considered as perturbation that ultimately deteriorates the system performance. But beauty of soliton physics is that herein nonlinearity is intelligently used to reap benefit. Since its first observation in 1980, optical solitons have been studied in various systems and conditions that give rise to a large variety of solitons. Optical solitons are mainly classified into two broad categories, namely temporal soliton and spatial soliton, depending on their confinement in time or space domain. Temporal solitons are pulses that remain confined in time domain, whereas, the spatial solitons are self-trapped localized beams that are confined in transverse direction of propagation. A third kind of soliton, that is localized in both time as well as space domain is named as spatiotemporal solitons. There are many sub groups of solitons that include, to mention but a few; photorefractive soliton, quadratic soliton, vortex soliton, bright soliton, dark soliton, propeller soliton, vector solitons, gap-solitons, zig-zag solitons, snake-solitons etc.

2.5.1 FORMATION OF SOLITON

A general property of electromagnetic wave packets is that they tend to spread out as they evolve. A fundamental cause for this is that distinct frequency components, which are superposed to create the wave packet, propagate with different velocities and or in different directions. An example is the transverse spreading of a laser beam due to diffraction. Similarly, light pulses spread in time as they propagate in a material medium, as, due to the group-velocity dispersion (GVD), each Fourier component of the pulse has a different velocity. These examples pertain to linear propagation of beam or pulses. Nonlinear effects generally accelerate the disintegration of a wave packet. However, under special conditions, nonlinearity may compensate the linear effects. The resulting balanced localized pulse or beam of light, that propagates without decay, is generally known as a soliton. Thus, optical solitons are localized electromagnetic waves that propagate stably in nonlinear media with dispersion, diffraction or both.

2.5.2 TYPES OF SOLITON

2.5.2.1 Spatial Soliton

Spatial solitons originate from a balance of self-focusing and diffraction. In this case, a laser beam experiences the Kerr-induced nonlinear contribution to the index of refraction that follows the spatial profile of the intensity, and thus acts as an effective lens. In one transverse dimension, the diffraction formally resembles the anomalous dispersion in the temporal domain; therefore the intensity-dependent lens can exactly compensate diffraction, and the resulting beam may propagate without spreading or self-compression. Such a beam has the form of a stipe in a planar waveguide, as was first observed by

Aitcheson et al 1990. In view of the above-mentioned mathematical similarity of the one-dimensional diffraction to anomalous chromatic dispersion, the existence of soliton solutions (which may also be regarded as (1+1)D objects, where, this time, the first '1' refers to the transverse spatial coordinate, x, rather than t in the case of temporal solitons) in this case is not surprising. However, (2+1)D spatial solitons (self-formed cylindrical beams) in media with the Kerr (cubic, or χ^3) nonlinearity are unstable, unlike their (1+1)D counterparts, because two-dimensional fluctuations may destroy the balance between the nonlinearity and diffraction in that case.

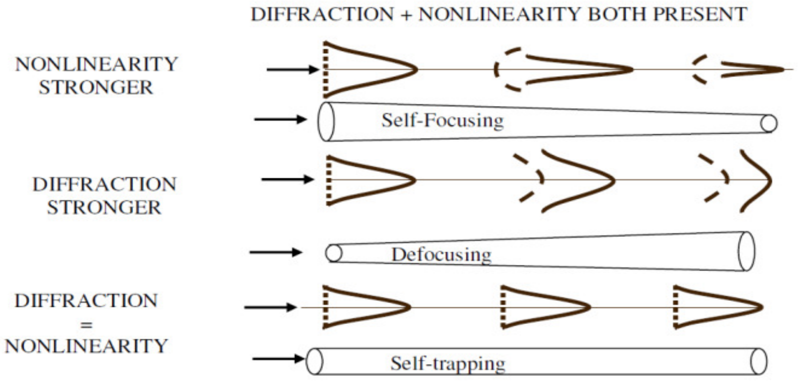
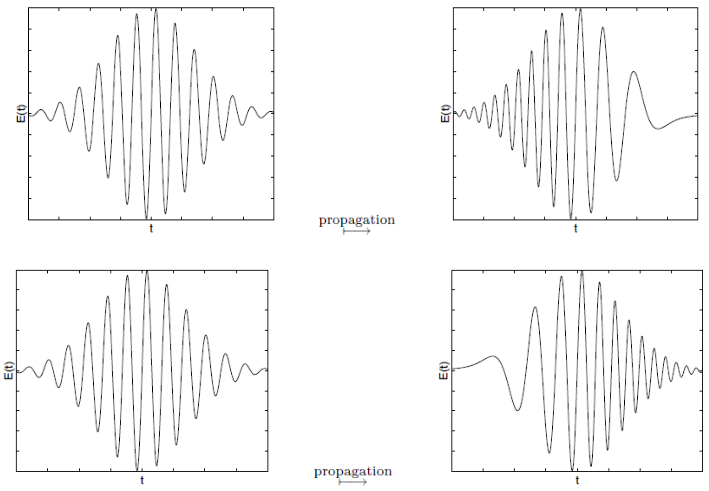


Fig.1 Schematic showing the spatial beam profiles (solid line) and phase fronts (dashed line) for (a) beam self-focusing, (b) normal beam diffraction, and (c) soliton propagation.

2.5.2.2 Temporal Soliton

Temporal soliton in single mode fibers are the prototypical optical solitons; these were predicted in 1973 (Hasegawa and Tappert 1973), and first observed experimentally in 1980. The formation of such temporal solitons can be understood intuitively as follows. In an optical fiber (or any other suitable optical medium), the effective index of refraction n depends on the intensity I, so that $n= n_0 + \Delta n(I)$, where, in first approximation, the small nonlinear correction is proportional to the intensity, $\Delta n(I)=n_2I$. Usual materials feature self-focusing, which implies $n_2>0$, and instantaneous nonlinear response of the medium (the latter means that there is no temporal delay between $\Delta n(I)$ and I such materials are referred to as Kerr media. In a more general case, the medium is self-focusing or self-defocussing in the case of, respectively, $d(\Delta n)/dI>0$ or $d(\Delta n)/dI<0$. Temporal solitons[15-48] are light pulses that neither broaden nor squeeze in both temporal and spectral domain. When a pulse propagates through a nonlinear dispersive medium, both dispersion and nonlinearity induced SPM produce chirps. Under appropriate condition, these chirps could be of opposite nature. In a perfectly counterbalancing condition, the opposite chirps cancel each other and the pulse propagates with constant temporal as well as spectral width. This localized pulse is known as temporal soliton. Sometimes it is called longitudinal soliton also. To have a much more clear insight into soliton formation, consider a propagating optical pulse in self-focusing Kerr nonlinear media, which is anomalously dispersive.



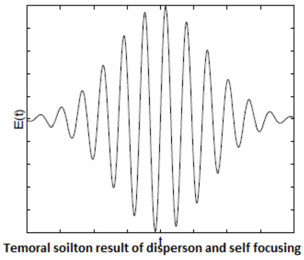


Fig.2 Formation of temporal soliton; balance between GVD and Kerr nonlinearity.

represents the mechanism of formation of temporal soliton. When, dispersion (anomalous) is dominant, the leading edge of the pulse gets blue shifted and the trailing edge gets red shifted. This causes a temporal spreading of the pulse. On the other hand, when nonlinearity is dominant, a chirp, which is opposite to that due to GVD, develops because of SPM. SPM broadens the pulse in spectral domain with inclusion of newly generated frequency, without broadening the pulse in temporal domain. Since the nonlinear medium anomalously dispersive, a perfect counter balance between these two chirps keeps both spectral and temporal widths constant that leads to formation of temporal soliton. Temporal soliton is mandatory (1+1) dimensional. This means a waveguide (e.g., optical fiber) whose longitudinal dimension (along propagation length) rather than transverse dimension is significant for light propagation, can only host temporal solitons.

2.5.2.3 Spatiotemporal Soliton

Spatiotemporal soliton (STS) is optical pulse self-trapped both in space and time under the simultaneous influence of diffraction, dispersion, self-focusing and self-phase modulation. Such pulses maintain their shape simultaneously in space and time. They are also known as light bullets. Light bullets are useful in optical digital logic, beam deflection, beam scan and all optical switching. STS have been investigated by many workers. Though a 3D light bullet has not been experimentally detected yet, its 2D version has been observed in a bulk quadratic medium. In a uniform cubic or Kerr medium these 2D and 3D light bullets are unstable. Three-dimensional spatio-temporal bright vortex soliton in a bulk dispersive cubic quantic optical medium has been predicted recently. In addition, 2D and 3D spatio-temporal spinning solitons with focusing cubic and defocussing quantic nonlinearity have also been investigated by several workers [49]. STSs have been investigated in different media such as uniform Kerr quadratically nonlinear and graded index Kerr media.

2.6 STABILTY

Spatiotemporal soliton in inhomogeneous, dispersive nonlinear media using graded-index Kerr medium has been proved to be stable. In that variational approach was used to solve the multidimensional, inhomogeneous, nonlinear Schrodinger equation. It was verified by means of a full numerical analysis and was showed that such solitons are observable.

My goal is to study spatiotemporal soliton in 3D in inhomogeneous media using cosh-Gaussian soliton profile, so that stable soliton can be formed. But to make soliton stable in 3D is very challenging.

2.7 METHODOLOGY

Optical solitons have been shown to form and propagate inside a nonlinear Kerr medium. They are called temporal or spatial solitons depending on whether their shape remains intact in time or in one space dimension. Mathematically, wave propagation in a Kerr medium is governed by the nonlinear Schrodinger equation (NLSE), which can be solved exactly in (1+1)- dimensions by using the inverse scattering method. Let us consider pulse propagation in a general nonlinear, dispersive medium with special emphasis on a self-defocusing medium with normal dispersion. An example of such a medium occurs for visible or near infrared light propagating inside a semiconductor. In our system we analyze the possibility of formation of stable spatiotemporal solitons in an inhomogeneous kerr medium, for which the refractive index is of form

$$n(r,\omega) = n_0(\omega + n_1(x^2 + y^2) + n_2|E|^2) \tag{1}$$

Where the homogeneous part $n_o(\omega)$ takes into account chromatic dispersion, n_2 is the nonlinear parameter responsible for self-focusing or self-defocusing, and n_1 governs the change in refractive index in the transverse dimensions x and y. Here we present the variational method which is the reproduction of the ref [50].

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CHAPTER-III

3. MATHEMATICAL DEVELOPMENT

In our system we explore the possibility of formation of stable spatiotemporal solitons in an inhomogeneous cubic-quintic medium, for which the refractive index is of form

We will take NLSE as

$$i\frac{dU}{dz}+\frac{1}{2}\left(\frac{d^2U}{dx^2}+\frac{d^2U}{dy^2}\right)+\frac{\delta}{2}\frac{d^2U}{d\tau^2}+\frac{s}{2}(x^2+y^2)U+\nu|U|^2U+\gamma|U|^4U=0 \qquad (1)$$

Where U is varying envelope with respect to z i.e. z is varying w.r.t to space. U is varying in transverse spatial dimensions x and y coordinate and U is varying in time , s is inhomogeneity parameter in transverse spatial dimensions and υ it defines by what factor cubic nonlinearity is changing and γ defines by what amount quantic nonlinearity is changing. When we put $s=0$ then inhomogeneity will vanish, similarly when we will put υ and $\gamma=0$, cubic and quantic nonlinearity terms will vanish. $\delta= \pm 1$ refers respectively anomalous or normal GVD (group velocity dispersion), $s= \pm 1$ refers respectively anti-guiding or guiding medium, $\upsilon= \pm 1$ represents self-focusing or self-defocusing respectively. γ is the relative strength of the quintic nonlinearity.

There exist several ways to solve a NLSE, broadly analytical methods and numerical approach. There are many famous methods to obtain exact solution of NLSE, for example, inverse scattering [1]. A-K-N-S method [2] Bäcklund transformation method [3] and Hirota bi-linear method [4]. One inadequacy of these methods is that, they generally cannot be useful for nonintegrable systems. However there are several approximate analytical methods those are capable to do that job. For example, paraxial ray approximation method [5-7]. Variational approximation method [8] moment method [9] collective variable method [10-12] tanh method [13]. The advantage of these approximate methods is that they are capable to provide substantial and quick insight of the systems. Variational method is one such popular method for solving NLSE. Also full numerical methods are widely used to solve NLSE.

Variation method requires the determination of Lagrangian density. For our NLSE the Lagrangian density can be written as follows:

$$L = \frac{i}{2}\left(U \frac{dU^*}{dz} - U^* \frac{dU}{dz} \right) + \frac{1}{2}\left[\left| \frac{dU}{dx} \right|^2 + \left| \frac{dU}{dy} \right|^2 \right] + \frac{\delta}{2}\left| \frac{dU}{d\tau} \right|^2 - \frac{s}{2}(x^2 + y^2)|U|^2 - \frac{\nu}{2}|U|^4 - \frac{\gamma}{3}|U|^6 \qquad (2)$$

Now we will use the following trial function, which is a coshyperbolic Gaussian, to solve this NLSE

$$U = A(z)\cosh(\Omega t)\exp(-(1+iC)t^2)/2t^2{}_0 \exp\left(-\frac{x^2}{2a^2} - \frac{y^2}{2b^2} \right) \exp(i(\theta t^2 + \alpha x^2 + \beta y^2 + \phi)) \qquad (3)$$

where, A(z) is a parameter depending on z, which can be calculated by normalizing the trial function i.e. $\int |U|^2 dxdyd\tau = E$ represents constant pulse energy. The parameters *α & β, Ω, a & b*, θ, ϕ all are varying with z represent, respectively spatial chirp along x and y coordinate, temporal width, spatial width along x and y coordinate, temporal chirp, and phase associated with the pulse.

The spatial profile of the trial function has been depicted in Fig.1 and 2.

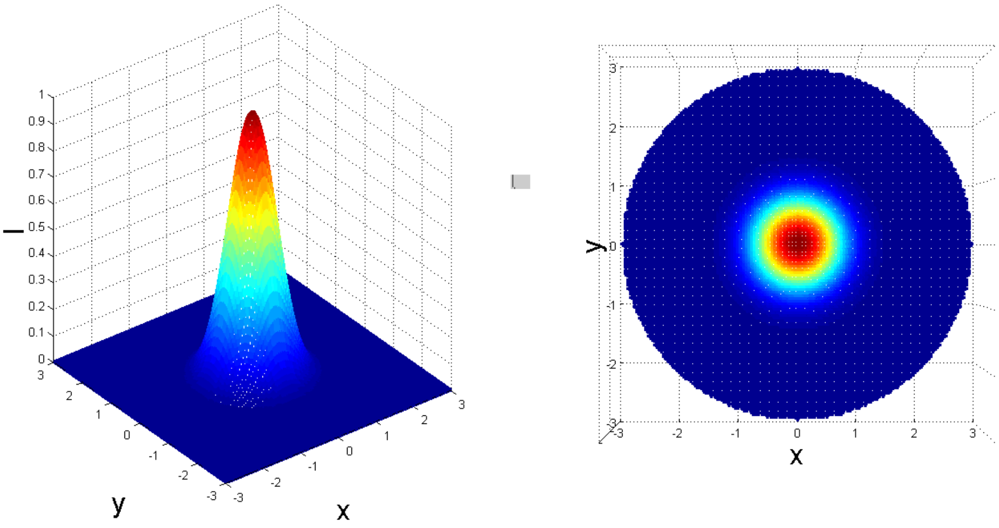

Fig.1 Spatial profile of the cosh-hyperbolic Gaussian function for symmetric beam width *m=1; r=1; a=1;b=1.0;*

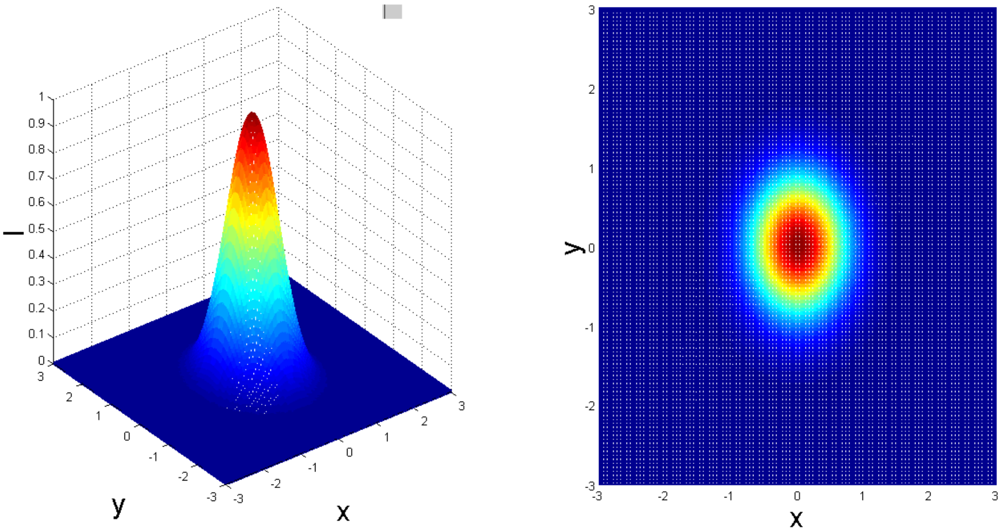

Fig.2 Spatial profile of the coshyperbolic Gaussian function for asymmetric beam width *m=1; r=1; a=1;b=1.0;*

The temporal profile shows the cosh Gaussian characteristics of the pulsed beam, which is shown in Fig.3 With the increase in cosh parameter Ω_0 the central deep will increase.(why?)

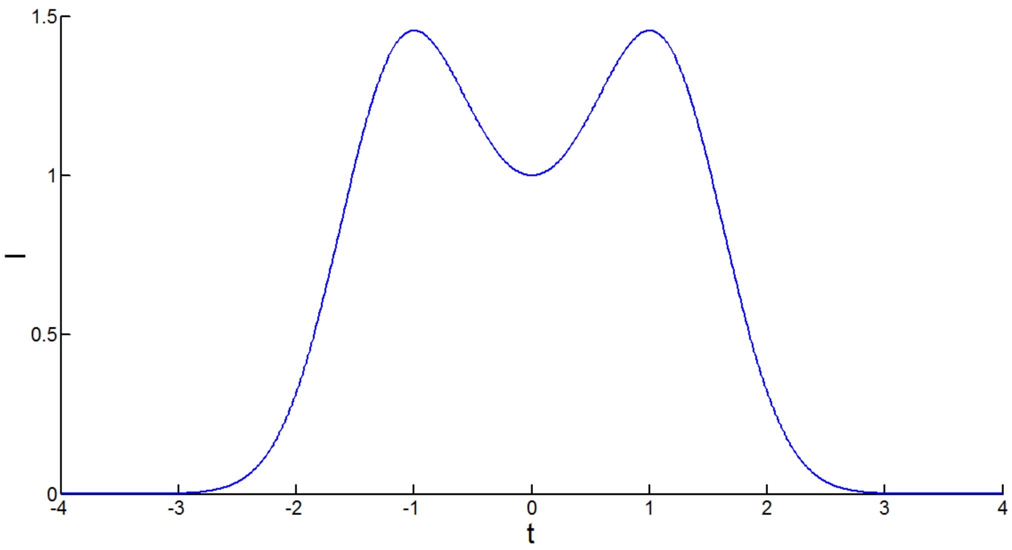

Fig. 3 Temporal profile of the cosh-hyperbolic Gaussian function. $A_0 = 1; C = 0; \Omega_0 = 1.2; T_0 = 1$.

We now adopt the standard procedure of variational method. At the first hand we calculate the reduced Lagrangian $L = \int dxdyd\tau L(U, U)$ by using Eq (2) and (3) as follows:

$$< L >= \frac{d\theta}{dz}\frac{(A^2abt^3{}_0\Pi^{3/2})}{2} + \frac{d\alpha}{dz}\frac{(A^2a^3bt\,{}_0\Pi^{3/2})}{2} + \frac{d\beta}{dz}\frac{(A^2a\,b^3t\,{}_0\Pi^{3/2})}{2} + \frac{d\phi}{dz}(A^2abt\,{}_0\Pi^{3/2}) +$$

$$+(4\alpha^2+\frac{1}{a^4})\frac{(A^2a^3bt{}_0\Pi^{3/2})}{4}+(4\beta^2+\frac{1}{b^4})\frac{(A^2a\,b^3t{}_0\Pi^{3/2})}{4}+A^2\delta\theta^2abt^3{}_o\Pi^{3/2}-\frac{sA^2a^3bt\Pi^{3/2}}{4}-\frac{sA^2a\,b^3t\Pi^{3/2}}{4}$$

$$-\frac{\nu A^4abt_0\Pi^{3/2}}{\sqrt{3}}-\frac{17\sqrt{2}A^6\gamma abt_0\Pi^{3/2}}{24} \tag{4}$$

This equation has been obtained by putting C=0, Ω=0.

We do the following simplification using $A=\sqrt{\frac{E}{abt_o\Pi^{3/2}}}$, where, $E=\int\left|U\right|^2dxdyd\tau$ (which represents constant pulse energy) and the Lagrangian takes the following form:

$$<L>=\frac{d\theta}{dz}\frac{(Et^2{}_0\ )}{2}+\frac{d\alpha}{dz}\left(\frac{a^2E}{2}\right)+\frac{d\beta}{dz}\left(\frac{b^2E}{2}\right)+\frac{d\phi}{dz}E+(4\alpha^2+\frac{1}{a^4})(\frac{a^2E}{4})+(4\beta^2+\frac{1}{b^4})(\frac{b^2E}{4})+\frac{\delta E}{4t_0^2}+$$

$$\delta\theta^2t_0^2E-\frac{sa^2E}{4}-\frac{sb^2E}{4}-\frac{E^2\nu}{\sqrt{3}}\frac{}{abt_0\Pi^{3/2}}-\frac{\sqrt{2}}{24}\frac{17\gamma E^3}{(a^2b^2t^2{}_0\Pi^3)} \tag{5}$$

We then use the Euler-Lagrangian equations to obtain following set of equations. These show the dynamics of spatial and temporal widths of the pulsed beam.

$$\frac{d^2a}{dz^2}=\frac{1}{a^3}+sa-\frac{2\nu E}{\sqrt{3}}\frac{}{a^2bt_0\Pi^{3/2}}-\frac{17\sqrt{2}E^2\gamma}{6a^3b^2t_0^2\Pi^3} \tag{6}$$

$$\frac{d^2b}{dz^2}=\frac{1}{b^3}+sb-\frac{2\nu E}{\sqrt{3}}\frac{}{ab^2t_0\Pi^{3/2}}-\frac{17\sqrt{2}E^2\gamma}{6a^2b^3t_0^2\Pi^3} \tag{7}$$

$$\frac{d^2t_0}{dz^2}=\frac{\delta^2}{t_0^3}-\frac{2\nu\delta E}{\sqrt{3}abt_0^2\Pi^{3/2}}-\frac{17\sqrt{2}E^2\gamma\delta}{6a^2b^2t_0^3\Pi^3} \tag{8}$$

Equations (6), (7), (8) have been used to explore the existence and stability of stationary states.

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CHAPTER-IV

4. RESULTS AND DISCUSSION

Equations (6), (7), (8) are solved to find the variation of the beam widths during propagation of the pulsed beam. Fig:3 depicts the feature. We calculated the condition for stable propagation by considering $\frac{d^2a}{dz^2}=0$, $\frac{d^2b}{dz^2}=0$ and $\frac{d^2t_0}{dz^2}=0$. The corresponding parameters have been used as an initial condition. Also we considered zero initial divergence of the beam.

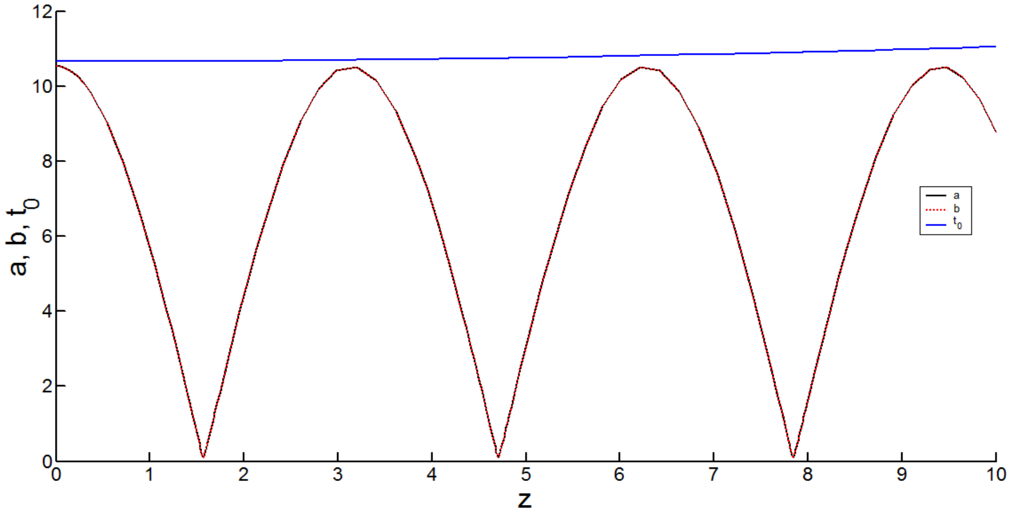

Fig.1 Variation of spatial widths a and b (a=b) and temporal width t_o w.r.t propagation distance for $s=-1; \nu=1; \delta=1; \gamma=-1; E_n=10$

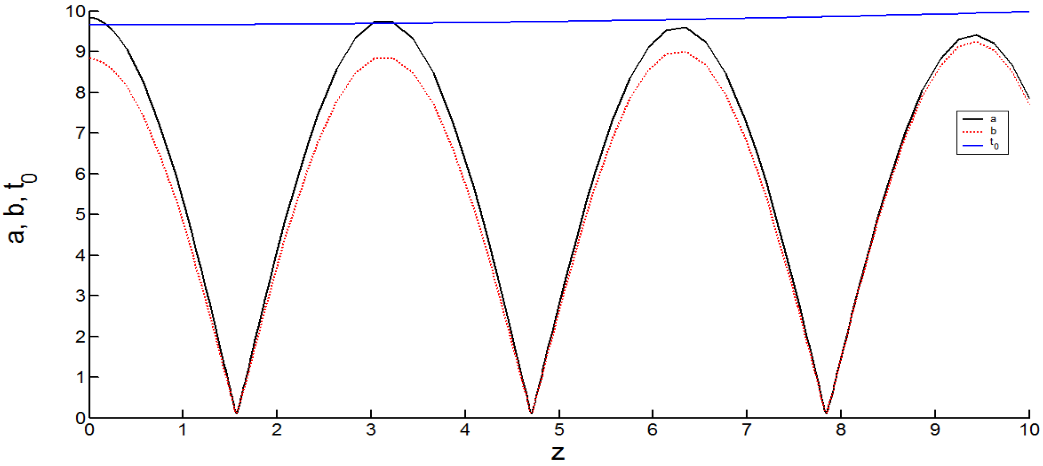

Fig.2 Variation of spatial widths a, b and temporal width t_0 w.r.t propagation distance for $s=-1; \nu=1; \delta=1; \gamma=-1; E_n=10$

The Fig.1 and 2 shows the stable propagation of the pulsed beam for symmetric and asymmetric spatial beam width respectively. In both cases spatial widths oscillates periodically, which resembles with breather soliton. The temporal width is almost stable during this propagation. This is the most important finding of our investigation. This is also the first objective of our work.

In order to address the second objective, i.e., to study the effect of inhomogenity in the nonlinear medium we draw the following plots with the same initial conditions as that of Fig.4 and 5. Fig.6 shows quasi-stable spatio-temporal soliton propagation when *s=0, i.e.,* in absence of inhomogenity. However, for s=1, i.e., self-focusing medium shows no indication of such stable or quasi-stable soliton propagation in the same initial condition. This has been described in Fig.7. Although temporal width can be stabilized, the spatial width increases extremely high. This means the pulsating beam collapse quickly due to self-focusing.

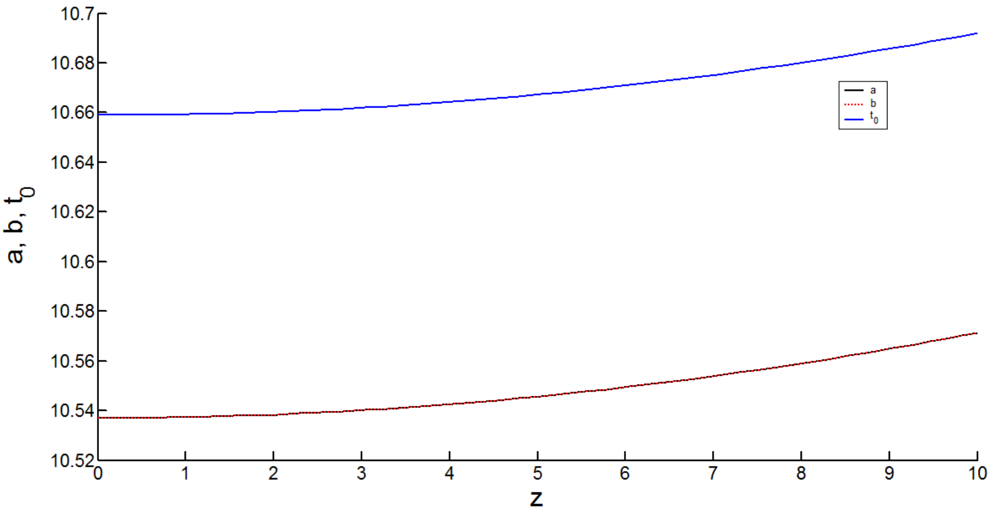

Fig.3 Variation of spatial widths a, b and temporal width t_0 w.r.t propagation distance

$$s=0; \nu=1; \delta=1; \gamma=-1; E_n=10$$

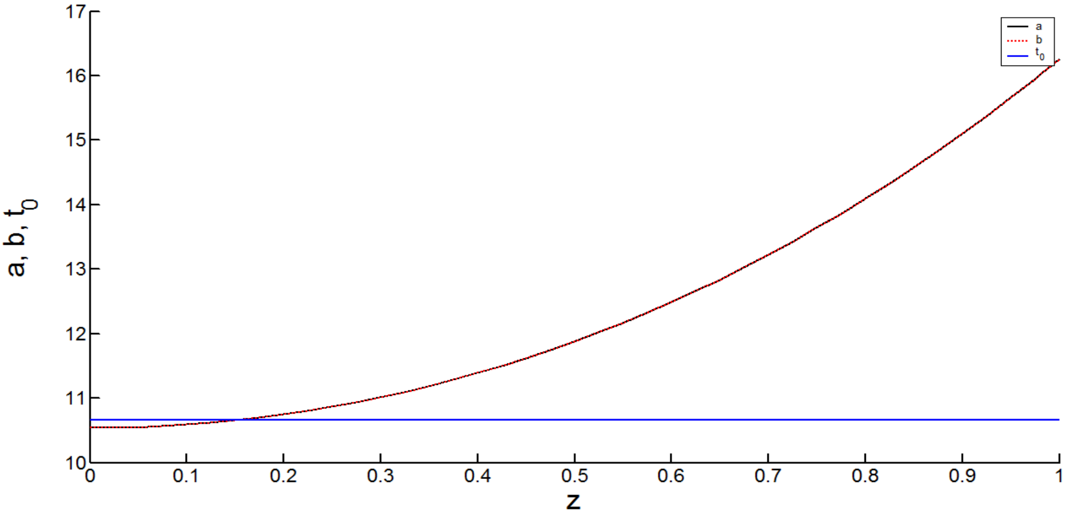

Fig.4 Variation of spatial widths a, b and $s=1; \nu=1; \delta=1; \gamma=-1; E_n=10$ temporal width t_0 w.r.t propagation distance z ,having values as

For further stability analysis we follow the standard method of potential analysis. We construct a scalar potential V of the system using the relationship

$$\frac{d^2a}{dz^2} = -\frac{dV}{dz}, \frac{d^2b}{dz^2} = -\frac{dV}{dz} \; and \; \frac{d^2t_0}{dz^2} = -\frac{dV}{dz}$$

Thus from V all these equations (6), (7), (8) are derivable.

We find

$$\mathrm{V} = \frac{a^2+b^2}{2a^2b^2} - \frac{s(a^2+b^2)}{2} + \frac{\delta^2}{2t_0^2} - \frac{2E\nu(\delta+1)}{\sqrt{3}\Pi^{3/2}abt_0} - \frac{17\sqrt{2}E^2\gamma(\delta+1)}{12\Pi^3a^2b^2t_0} \qquad (10)$$

This scalar potential is now examined to investigate the existence of solitons and their stability. For better handling we switch to spherical coordinates. Employing $a = \rho \sin\theta \cos\theta \quad b = \rho \sin\theta \sin\phi$ and $t_0 = \rho\cos\theta$, V becomes as follows at θ= 45° (i.e., along axis of symmetry)

$$V - \left(\frac{l}{\cos^2\phi \sin^2\phi}\right) = -\frac{s}{4l} + \delta^2 l^2 - l^3\left(\frac{34E^2\gamma}{3\Pi^3\cos^2\phi\sin^2\phi}\right)(\sqrt{2}\delta+1) - l^{3/2}(2+\delta)\frac{4\sqrt{2}\nu E}{\sqrt{3}\Pi^{3/2}\sin\phi\cos\phi}$$

(11)

where $l = \dfrac{1}{\rho^2}$.

The above is a set of two equations, we consider the LHS as $F_1(l)$ and the RHS as $F_2(l)$.

$$F_1(l) = V - \left(\frac{l}{\cos^2\phi\sin^2\phi}\right)$$

$$F_2(l) = -\frac{s}{4l} + \delta^2 l^2 - l^3\left(\frac{34E^2\gamma}{3\Pi^3\cos^2\phi\sin^2\phi}\right)(\sqrt{2}\delta+1) - l^{3/2}(2+\delta)\frac{4\sqrt{2}\nu E}{\sqrt{3}\Pi^{3/2}\sin\phi\cos\phi}$$

$F_1(l)$ is a family of straight line (considering V as a parameter) and the otrher $F_2(l)$ is the nonlinear curve. We plot both of them on a same plane to identify the region of stable soliton. Fig.5

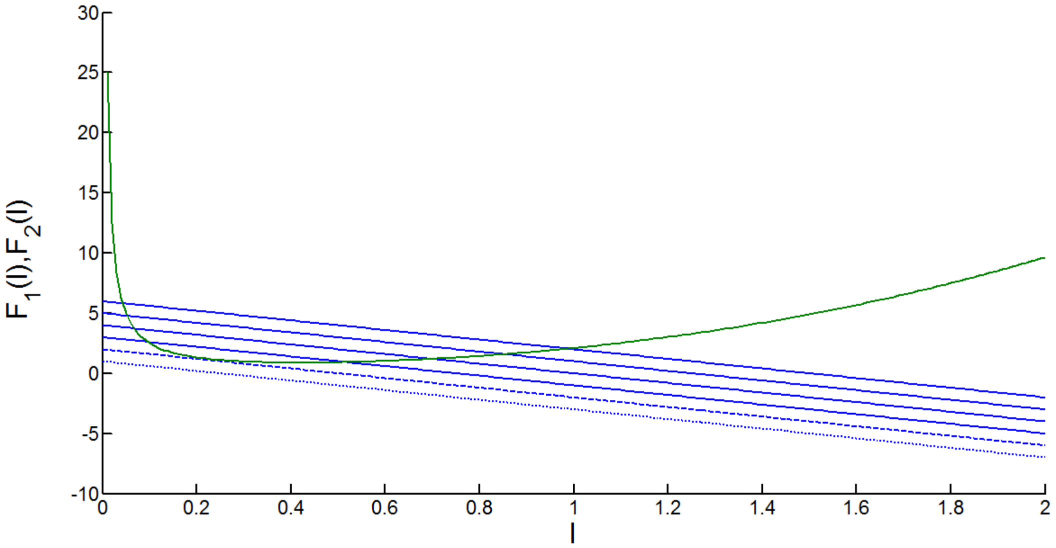

Fig.5 Plot showing the self -trapped region for $s=-1$ and different V (1-6). Other parameters are $\phi=\frac{\Pi}{4};\gamma=1;E=1;\nu=-1;\delta=-1$

$F_1(l)$ (indicated by solid lines) those cut $F_2(l)$ twice for some particular values of *V*. This indicates that the beam width will oscillate periodically (i.e., soliton can be achieved) for those cases. Whereas below some value of V no intersection point , i.e., no soliton will be found. The Hamiltonian *H* equations corresponding to the system described by eq. (6), (7), (8) can be calculated using eq. (10) as follows:

$$H=\frac{(\dot{a}^2+\dot{b}^2+\dot{t}_0{}^2)}{2}+\frac{a^2+b^2}{2a^2b^2}-\frac{s(a^2+b^2)}{2}+\frac{\delta^2}{2t_0^2}-\frac{2E\nu(\delta+1)}{\sqrt{3}\Pi^{3/2}abt_0}-\frac{17\sqrt{2}E^2\gamma(\delta+1)}{12\Pi^3a^2b^2t_0} \qquad (13)$$

We further try to find the effective width of the spatiotemporal soliton as follows: We found that

$$\frac{d^2(a^2+b^2+t_0^2)}{dz}=4H+4s(a^2+b^2)+\frac{4\nu E\delta}{\sqrt{3}\Pi^{3/2}ant_0}-\frac{17\sqrt{2}E^2\gamma}{3\Pi^3a^2b^2t_0^2}$$

This leads to the following relationship:

$$\rho^2=2Hz^2-\frac{4s(a_0^3+b_0^3)z}{3}+\frac{s(a_0^4+b_0^4)}{3}-2\rho_0\dot{\rho}_0+\mathrm{C}$$

where, $\rho^2=a^2+b^2+{t_0}^2$ is the effective beam width parameter &

$$C=\frac{4\nu E\delta}{\sqrt{3}\Pi^{3/2}}\ln\frac{R_1}{R_1(0)}+\frac{17\sqrt{2}E^2\gamma}{18\Pi^3}\left(\frac{1}{R_2^2(0)}-\frac{1}{R_2^2}\right)$$

We find the critical length of self-focusing (z_c)/ collapse length by setting $\dot{\rho}=0$, which reads $z_c=\dfrac{\frac{4s(a_0^3+b_0^3)}{3}\pm\sqrt{\frac{4}{3}s(a_0^3+b_0^3)^2-8HC}}{4H}$;

where, $R_1^2=abt_0$ also $R_2^4=a^2b^2t_0^2$.

Since in presence of quintic nonlinearity the beam collapsing has been arrested, the beam collapse length should be infinity. This leads to the condition that for self-trapped beam propagation $4H=0$. This condition, in turn, yields the threshold energy corresponding to self-trapping of the beam:

$$E_{th}=\frac{6\Pi^{3/2}abt_0\left[\frac{2\nu(1+\delta)^{1/2}}{\sqrt{3}}\pm\frac{1}{\Pi^{3/2}}\left(\sqrt{\frac{4\nu^2}{3}-\frac{17\sqrt{2}\gamma(s-\frac{1}{a^2b^2})(a^2+b\ \ )}{6}}\right)\right]}{17\sqrt{2}\gamma(1+\delta)^{1/2}}$$

Following Fig.6 shows the variation of the threshold energy with spatial beam width for different inhomogeneity. For all kinds of medium E_{th} increases almost monotonically. Fig.6 and 7 shows the variation of the threshold energy with temporal beam width for different inhomogenity. Here E_{th} increases linearly with the increasing temporal width. These plots will be helpful for the experiment to estimate the initial beam energy for soliton formation.

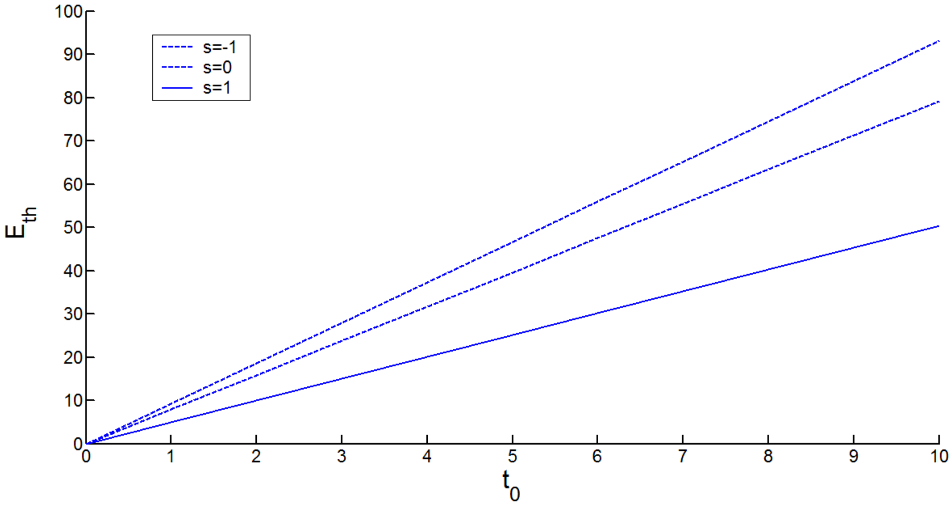

Fig.6 The variation of the threshold energy with spatial beam widths for different inhomogeneity. $E = 1; \gamma = -1; \nu = -1; \delta = -1$

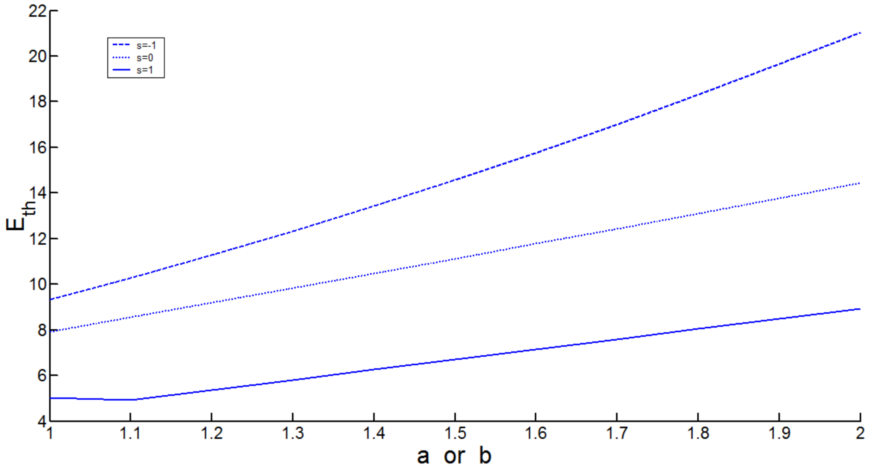

Fig.7 The variation of the threshold energy with temporal beam widths for different inhomogeneity $E = 1; \gamma = 1; \nu = -1; \delta = 1$.

4.1 Conclusion:

We found spatiotemporal soliton in inhomogeneous medium in quintic nonlinearity. We identified the medium, which can create better soliton. We also showed the effect of inhomogeneity. The threshold beam energy for stable spatio-temporal soliton formation has been calculated. This study will be helpful for experimental investigation to get an idea about the zone of stable soliton. This can be utilized to realize spatio-temporal soliton in all optical data processing and computing etc.
